

Default Risk, the Real Exchange Rate and Income Fluctuations in Emerging Economies

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Abstract

Volatile and countercyclical country interest rates and dollar-denominated debt are common features of emerging economies. This paper develops a two-sector small open economy model to study endogenous default risk and its interaction with the real exchange rate and income. Default probabilities and country risk premia depend on incentives for repayment. Default occurs in equilibrium because asset markets are incomplete. The model predicts that default incentives and default premia are higher in recessions, as observed in the data. The reason is that in a recession, a risk averse borrower finds it more costly to repay non-contingent debt and is more likely to default. In a quantitative exercise, the model matches several features of the Argentinian economy and can account for the dynamics observed during the recent default episode. Prior to default, the economy experiences higher interest rate premia, capital outflows, real exchange rate depreciation, and collapses in consumption. An important feature of the model is that economies with relatively small tradable sectors have higher incentives to default on dollar denominated debt and thus have larger default probabilities.

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1 Introduction

Emerging markets tend to have volatile business cycles and experience economic crises more frequently than developed economies. Recent evidence suggests that this may be related to cyclical changes in the access to international credit. In particular, emerging market economies face volatile and highly counter-cyclical interest rates, usually attributed to counter-cyclical default risk.¹ In addition, interest rates and default risk are also systematically correlated with real exchange rate devaluations: devaluations increase the probability of default and default increases the probability of devaluations. The exposure of the economy to such devaluations is magnified by high levels of liability dollarization - that is, "dollar" denominated debt, leveraged to a large extent on domestic income and assets.² Figure (1) illustrates these correlations by plotting aggregate consumption, real exchange rates, interest rates and the current account for Argentina. In December 2001, Argentina defaulted on its international debt and fell into a deep economic crisis. During the crisis, consumption and real exchange rates collapsed, interest rates increased, and the current account experienced sharp reversals.³ This evidence indicates that a priority for theoretical work in emerging markets macroeconomics must be understanding markets for international credit and in particular the joint analysis of default risk, interest rates and real exchange rates.

This paper develops a stochastic general equilibrium model with endogenous default risk. The model allows the study of the relation between interest rates, real exchange rates and output, shedding light on potential mechanisms generating the co-movements described above. The terms of international loans are endogenous to domestic fundamentals and depend on incentives to default. The paper extends the approach developed by Eaton and Gersovitz (1981) in their seminal study on international lending, by analyzing how endogenous default probabilities and fluctuations in output are related. In addition, the model considers a default penalty that is limited to temporary financial autarky, and introduces a nontradable sector to analyze dynamics of interest rates and real exchange rates. The paper characterizes the equilibrium country interest rate schedule and its relation with output, and provides conditions under which default can be an equilibrium outcome. In a quantitative exercise the model is applied to analyze the default experience of Argentina. The results show that the model can

¹Neumeyer and Perri (2001) and Uribe and Yuen (2003) document the counter-cyclicality of country interest rates for Argentina, Brazil, Ecuador, Mexico, Peru, Philippines, and South Africa.

²Reinhart (2002), Calvo and Reinhart (2000), Hausmann et. al (2001) document the high levels of liability dollarization in emerging markets foreign debt and its effect on crises and devaluations.

³Mendoza (2002) documents similar dynamics for all the emerging market crises in the 1990's.

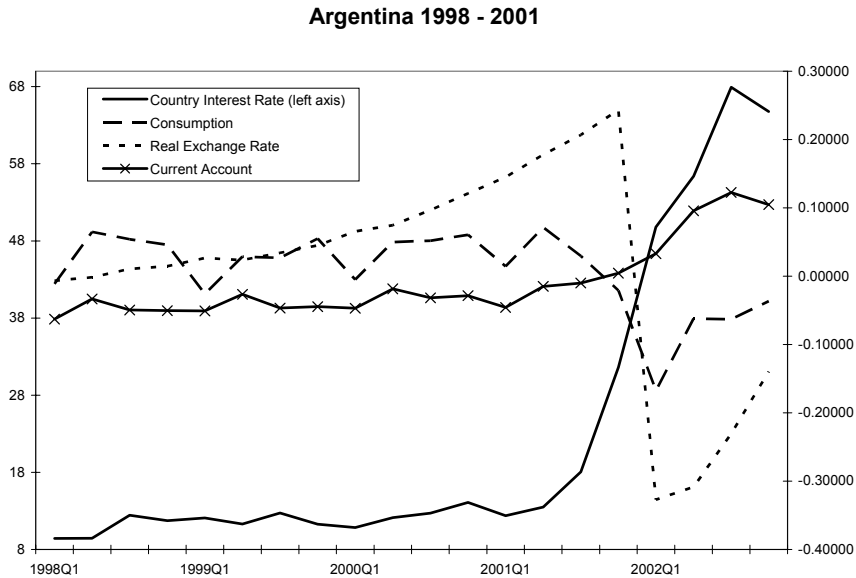


Figure 1: Consumption and real exchange rate data are log, and current account data are reported as percentage of output. These series are filtered with a Hodrick-Prescott filter.

account for the dynamics of consumption, interest rates, current account and the real exchange rate.

The model in this paper accounts for the empirical regularities in emerging markets as an equilibrium outcome of the interaction between risk-neutral competitive creditors and a risk averse borrower that has the option to default. The borrower is a small open economy with two sectors, tradables and nontradables. The government in this economy can buy or sell tradable denominated bonds with foreign creditors. The model starts from the premise that default probabilities are endogenous to the economy's incentives to default and they affect the equilibrium interest rate the economy faces. Default is an equilibrium outcome of the model because the asset structure is incomplete since it includes only one period discount bonds that pay a non-contingent face value. Asset incompleteness is necessary in this framework to study time-varying default premia due to equilibrium default. With non-contingent assets, risk neutral competitive lenders are willing to provide debt contracts that in some states will result in default by charging a higher premium on these contracts. In addition to more closely reflecting the actual terms of international contracts where foreign debt is largely dollar denominated and contracted at non-contingent interest rates, this market structure has the potential to deliver

counter-cyclical default risk, since repayment of non-contingent, nonnegotiable loans in low output, low consumption times is more costly than repayment in boom times. The existence of nontradable goods is important because even though foreign debt cannot be used to smooth consumption of nontradable goods, nontradable output fluctuations affect repayment incentives and equilibrium interest rates through changes in the real exchange rate.

In the first part of the paper, a simpler version of the model with *i.i.d.* shocks is considered in order to characterize analytically the equilibrium properties of credit markets and in particular the conditions under which default is an equilibrium outcome. It is shown that default is more likely the lower the output of tradables. This result contrasts with standard participation constraint models that have a complete set of assets, which have the feature that default incentives are higher in good times. The key intuition for why asset market incompleteness can reverse this outcome is that after a long series of low endowment shocks, an economy with incomplete markets can experience capital outflows in bad times. Risk averse agents with large debt holdings that can experience capital outflows have more incentives to default in times of low shocks. In essence, in times of low output, the asset available to the economy does not help agents smooth consumption, thus it is not very valued and default can be preferable than repayment. A similar intuition holds for the relationship between the level of nontradable output and default, although here it depends on how nontradable shocks affect the marginal utility of tradable consumption. We find that default is more tempting for low nontradable shocks when a low nontradable shock increases the marginal utility of tradable consumption. When this condition holds, capital outflows more costly for lower nontradable shocks and thus default more likely.

In the quantitative part of the paper, the model is calibrated to Argentina to study its recent default episode. The model can match the observed business cycle correlations in Argentina. Specifically, it can account for the negative correlations between output and consumption with interest rates. It also matches the positive correlation between real exchange rate devaluations and interest rates, and the positive correlation between the current account and interest rates. We find that endogenous time varying default probabilities can account for about 15 percent of the volatility in interest rates observed in Argentina. The model matches the data in that before a default occurs, the economy faces high interest rates and features sharp reductions in capital inflows, sharp reversals from large current account deficits into much smaller deficits or even surpluses, and collapses in consumption and real exchange rates. The relative sizes of the tradable and nontradable sectors are also very important. We find that relatively large nontradable sectors reduce the economy's commitment for debt repayment and further limit the

financial integration of economies. A model with only a tradable sector would face much lower equilibrium interest rates and default would be a much less likely outcome.

The paper is related to several studies from different strands of the macroeconomics and finance literatures. Some papers have looked at the relation between interest rates and business cycles. Neumeyer and Perri (2001) model the effect that exogenous interest rate fluctuations have on business cycles and find that interest rate shocks can account for 50% of the volatility of output. Uribe and Yue (2003) construct an empirical VAR to uncover the relationship between country interest rates and output, and then estimate with a theoretical model this relationship. They find that country spreads explain 12% of movements in output, and that output explains 12% in the movements of country interest rates. These papers, however, do not model endogenous country spreads responding to default probabilities.

The debt contractual arrangement in this paper is related to the optimal contract arrangements in the presence of commitment problems such as the analysis by Kehoe and Levine (1993) and Kocherlakota (1996). These studies, however, assume that a complete set of contingent assets are available and search for allocations that are efficient subject to a lack of enforceability. Alvarez and Jermann (2000) show that in this framework each state-contingent asset is associated with a state-contingent borrowing limit. This limit is such that in each state the borrowing country prefers to repay loans rather than default. While it is useful to be able to characterize allocations under the efficiency benchmark, this market structure may not be useful as a framework for understanding actual emerging markets. First, default never arises in equilibrium, so default risk premia are never observed. Second, default incentives in this class of models are typically higher in periods of high output, which is when efficiency dictates loan repayment. These features put these models at odds with the empirical evidence regarding default risk in emerging markets where bond yields are counter-cyclical and where debt prices reflect largely the risk of default. This paper delivers the correct empirical prediction because it assumes an incomplete set of assets where default occurs with a positive probability. In this regard the paper is specially related to the analysis on unsecured consumer credit with the risk of default by Chatterjee, Corbae, Nakajima and Rios Rull (2002) where they model equilibrium default in an incomplete markets setting.

Other authors have developed models to study liability dollarization and have concentrated on how the level and volatility of output and relative price of nontradables affect the ability to pay dollar debt. Calvo (1998) shows how collapses in the price of nontradables due to constraints on tradable denominated debt can lead to crises by limiting the ability for debt repayment. Mendoza (2002) examines a stochastic environment where households face a liquidity

requirement borrowing constraint (in which debt cannot be larger than an exogenous fraction of current income) to study emerging markets crises. He shows how crises can be the outcome of the equilibrium dynamics of an economy with imperfect credit markets. In these models though, the borrowing constraints do not guarantee that debtors have incentives to repay debts. In addition, they do not model the endogeneity of interest rates and the relation between interest rates and liability dollarization.

The focus in this paper is on understanding how the level and volatility of tradable and nontradable output affect incentives to default and thus equilibrium country interest rates in an environment of liability dollarization. Results match the empirical facts in that default incentives are higher when the economy has large debt positions, is in recession, and has relatively small tradable sectors.

The paper is organized as follows: section 2 presents the theoretical model, section 3 characterizes the equilibrium, section 4 assesses the quantitative implications of the model in explaining the data, and section 5 concludes.

2 The Model Economy

Consider small open economy with two sectors, a tradable sector and a nontradable sector. A set of one period discount bonds is available to the government of the small open economy at contingent prices. Debt contracts are not enforceable as the government can choose to default in its debt contracts if it finds it optimal to do so. If the economy defaults in its debts, it is assumed to be temporarily excluded from international financial markets and to lose a portion of its endowment during the autarky periods. The economy trades discount bonds with risk neutral, competitive foreign creditors.

Households are identical and have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^T, c_t^N) \tag{1}$$

where $0 < \beta < 1$ is the discount factor, c^T and c^N are consumption of the tradable and nontradable goods, and $u(\cdot)$ is concave, strictly increasing, and twice continuously differentiable. Households receive stochastic endowment streams of tradable y^T and nontradable output y^N . The exogenous state vector of sector-endowment shocks is defined as $y \equiv (y^T, y^N) \in Y$. Shocks are assumed to have a compact support and to be a Markov process drawn from a probability

space $(y, B(y))$ with a transition function $f(y'|y)$. In addition households receive a transfer of tradable goods from the government in a lump sum fashion. Households trade in the spot market tradable and nontradable goods with p^N being the relative price of nontradable goods.

The government is benevolent in that its objective is to maximize the utility of households. The government has access to international financial markets, where it can buy one period discount bonds B' denominated in terms of tradables at price q . The government also decides whether to repay or default on its debt. The bond price function $q(B', y)$ depends on the size of the bond B' and on the aggregate shock y because default probabilities depend on both. A purchase of a discount bond with a positive value for B' means that the government has entered into a contract where it saves $q(B', y) B'$ units of period t tradable goods to receive $B' \geq 0$ units of tradable goods the next period. A purchase of a discount bond with negative face value for B' means that households have entered into a contract where they receive $q(B', y) B'$ units of period t tradable goods, and promise to deliver, conditional on not declaring default, B' units of tradable goods the next period. The government rebates back to the households all the proceedings from its international credit operations in a lump sum fashion.

When the government chooses to repay its debts, the tradable sector market clearing condition is the following:

$$c^T = y^T + B - q(B', y)B'$$

The market clearing condition for the nontradable sector, requires that nontradable consumption equals nontradable output at all times.

$$c^N = y^N$$

Given that the government is benevolent, it effectively uses international borrowing to smooth consumption. But the idiosyncratic income uncertainty induced by y^T and y^N cannot be insured away with the set of bonds available to the economy, which pay a time and state invariant amount of tradable goods only. That is, asset markets in this model are incomplete not only because of the endogenous default risk, but also because of the set of assets available.

If the government defaults, it is assumed that current debts are erased and that it is not able to save or borrow. The government will remain in financial autarky for a stochastic number of periods. There is an exogenous constant probability θ that the government can reenter the market. This is a simple way to model that governments that default on their international debt lose access to international financial markets only temporarily. In addition, in the period when

the government defaults, endowments fall a proportion $(1 - \lambda)$, where $\lambda \leq 1$. The assumption that default may reduce output can be rationalized by the common view that after default there is a disruption in the countries' ability to engage in international trade, and this reduces the value of output (Cole and Kehoe 2000, Conklin 1998, Obstfeld and Rogoff 1989). By having a choice of θ and λ , the model simply gives greater flexibility in terms of what the default punishment is.

When the government chooses to default market clearing conditions require that consumption equals output :

$$c^T = \lambda y^T$$

$$c^N = \lambda y^N$$

Foreign creditors have access to an international credit market in which they can borrow or lend as much as needed at a constant international interest rate $r > 0$. Creditors have perfect information regarding the economy's endowment processes and can observe each period the endowment levels.⁴ Creditors are assumed to be risk neutral and behave competitively. Creditors engage in Bertrand competition, offering contracts to the government that gives them expected zero profits.

They choose loans B' to maximize expected profits ϕ , taking as given prices:

$$\phi = -qB' + \frac{(1 - \delta)}{1 + r}B' \tag{2}$$

where δ is the probability of default.

For positive levels of foreign asset holdings, $B' \geq 0$, the probability of default is zero and thus the price of a discounted bond will be equal to the opportunity cost for investors. For negative asset holdings, $B' < 0$, the equilibrium price accounts for the risk of default that creditors are facing, that is, the price of a discount bond will be equal to risk adjusted opportunity cost.⁵ This requires that bond prices satisfy:

$$q = \frac{(1 - \delta)}{1 + r} \tag{3}$$

⁴We assume for simplicity that each period the government enters into a debt contract with only one creditor. This can be generalized by assuming that creditors can observe previously written contracts and contracts that are written first have to be honored first.

⁵Risk adjustment in this framework is not due to compensation for risk aversion as lenders are risk neutral. It reflects the risk neutral compensation for a lower expected payoff.

The probability of default δ is endogenous to the model and depends on the government incentives to repay debt. Since $0 \leq \delta \leq 1$, the zero profit requirement implies that bond prices q lie in the closed interval $[0, (1+r)^{-1}]$. The country gross interest rate can be interpreted as the inverse of the discount bond price, $1+r^c = \frac{1}{q}$.

The timing of decisions within each period is as follows. The government starts with initial assets B , observes the endowment shock y , and decides whether to repay its debt obligations or default. If the government decides to repay, then taking as given the schedule bond price $q(B', y)$ the government chooses B' subject to the resource constraints. Then creditors taking as given q choose B' . Finally consumption of tradable and nontradable goods c^T, c^N takes place.

3 Recursive Equilibrium

We define a recursive equilibrium in which the government does not have commitment and in which the government, foreign creditors and households act sequentially. Given aggregate states $s = (B, y)$, the policy functions for the government B' , the price function for bonds q and the policy functions for the consumers c^T, c^N together with the relative price p^N determine the equilibrium.

Households' problem is static. They observe their endowment shocks and taking as given the aggregate states, government transfers and the nontradable price they choose optimal consumption. Their first order condition equates the marginal rate of substitution between nontradable and tradable consumption to the relative price.

$$\frac{u_{c^N}(c^T, c^N)}{u_{c^T}(c^T, c^N)} = p^N \quad (4)$$

Given that purchasing power parity (PPP) is assumed to hold for the tradable sector, the real exchange rate for this small economy is the domestic consumption price index p^c which is a function of the nontradable price, p^N , and the tradable price, which is normalized to 1.

Foreign creditors in the model are risk neutral and behave competitively. They lend the amount of bonds wanted by the government as long as the price satisfies

$$q(B', y) = \frac{(1 - \delta(B', y))}{1 + r}$$

Foreign creditors in the model are pretty passive, they lend bonds as long as the gross return on the bonds equals $1 + r$.

The government observes the aggregate endowment shocks y , and given initial foreign assets B , choose whether to repay or to default. If the government chooses to repay its debt obligations and remain in the contract, then it chooses the new level of foreign assets B' . The government understands that the price of new borrowing $q(B', y)$ depends on the states y and on its choice of B' . The government also understands that its choices will affect the households choices c^T , c^N and p^N and internalizes the domestic market clearing conditions. The government objective is to maximize the lifetime utility of households.

Define $v^o(B, y)$ as the value function for the government who has the option to default, and that start the current period with assets B and endowment shocks y . The government decides on whether to default or repay its debts, to maximize the welfare of households. Note that only when the government has debt (i.e. negative assets) could the default option be optimal.

Given the option to default, $v^o(B, y)$ satisfies:

$$v^o(B, y) = \max \{v^c(B, y), v^d(y)\} \quad (5)$$

where $v^c(B, y)$ is the value associated with not defaulting and staying in the contract and $v^d(y)$ is the value associated with default.

When the government defaults, the economy will be in temporary financial autarky; θ is the probability that it will regain access to international credit markets. The value of default is given by the following:

$$v^d(y) = u(\lambda y^T, \lambda y^N) + \beta \int_{y'} [\theta v^o(0, y') + (1 - \theta)v^d(y')] f(y'|y) dy' \quad (6)$$

If the government defaults endowments fall, and consumption equal output.

When the government chooses to remain in the credit relation, the value conditional on not defaulting is the following

$$v^c(B, y) = \max_{(B')} \left\{ u(y^T - q(B', y)B' + B, y^N) + \beta \int_{y'} v^o(B', y') f(y'|y) dy' \right\} \quad (7)$$

The government decides on optimal policies B' to maximize utility. The decision to remain in the credit contract and not default is a period by period decision. That is, the expected value from next period forward incorporates the fact that the government could choose to default in the future. The government also faces a lower bound on debt, $B' \geq -Z$, which prevents the government from running ponzi schemes but is otherwise not binding in equilibrium.

The government default policy can be characterized by default sets and repayment sets. Let

$A(B)$ be the set of y 's for which repayment is optimal when assets are B such that⁶:

$$A(B) = \{y \in Y : v^c(B, y) > v^d(y)\}$$

and $D(B) = \tilde{A}(B)$ be the set of y 's for which default is optimal for a level of assets B , such that:

$$D(B) = \{y \in Y : v^c(B, y) \leq v^d(y)\} \quad (8)$$

The value function can then be represented more precisely by the following dynamic problem where the government decides on optimal borrowing taking into account that its choice on assets may imply defaulting in some states.

$$v^c(B, y) = \max_{(B')} \left\{ u(y^T - q(B', y)B' + B, y^N) + \beta \left[\int_{A(B')} v^c(B', y') f(y'|y) dy + \int_{D(B')} v^d(y') f(y'|y) dy \right] \right\}$$

The above centralized government borrowing problem can be decentralized in multiple ways, with the simplest being lump sum taxation as presented here. In a separate appendix, it is shown that the above problem can also be decentralized by letting households borrow directly from foreign creditors. The government in this case makes the economy wide default decision, and levies a savings tax that gives households the right incentives for holding the optimal level of debt. For simplicity in the exposition we have assumed here that the government has access to lump sum taxes.

Having developed the problem for each of the agents in the economy, the equilibrium is defined.

Definition 1. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $c^T(s)$ and $c^N(s)$, nontradable price $p^N(s)$ (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$ and default sets $D(B)$, and (iii) the price for bonds $q(s, B')$, such that:*

1. *Taking as given the government policies, policy functions for households $c^T(s)$, $c^N(s)$, and the relative price $p^N(s)$ satisfy the households optimization problem and domestic market clearing conditions hold.*

⁶We assume that if the government is indifferent between repayment and default, default is chosen.

2. Taking as given the bond price function $q(B', y)$, the government's policy functions $B'(s)$, repayment sets $A(B)$ and default sets $D(B)$, satisfy the government optimization problem.
3. Bonds prices $q(B', y)$ are such that they reflect the government default probabilities and they are consistent with creditor's expected zero profits such that the loan market clears.

The equilibrium bond price function $q(B', y)$ has to be consistent with government's optimization and with expected zero profits for foreign creditors. That is, q correctly assesses the probability of default of the government.⁷

Default probabilities $\delta(B', y)$ and default sets $D(B')$ are then related in the following way:

$$\delta(B', y) = \int_{D(B')} f(y'|y) dy' \quad (9)$$

When default sets are empty, $D(B') = \emptyset$, equilibrium default probabilities $\delta(B', y)$ are equal to 0. That is, the economy with assets B' never chooses to default for all realization of the endowment shocks. When $D(B') = Y$, default probabilities $\delta(B', y)$ are equal to 1. More generally, default sets are shrinking in assets, as the following proposition shows:

Proposition 1. (Default sets are shrinking in assets) *For all $B^1 < B^2$, if default is optimal for B^2 , in some states y , then default will be optimal for B^1 for the same states y , that is $D(B^2) \subseteq D(B^1)$.*

Proof. See Appendix.

This result is similar to Eaton and Gersovitz (1981) and Chatterjee, et al. (2002). Intuitively, the result follows from the property that the value of staying in the contract is increasing in B , and that the value of default is independent of B . As assets decrease, the value of the contract monotonically decreases, while the value of default is constant. Thus, if default is preferred for some level of assets B , for a given state y , the value of the contract is less than the value of default. As assets decrease, the value of the contract will be even lower than before, and so default will continued to be preferred.

Since stochastic shocks are assumed to have a bounded support and the value of the contract is monotonically decreasing as assets fall, there exists a level of B that is low enough, such that for all endowment shocks default is optimal and default sets are equal to the entire endowment

⁷Chatterjee et al (2001) prove the existence of an equilibrium price schedule in a similar environment for their work on consumer default risk. We conjecture that the existence proof for this model follows that of Theorem 4 in their paper.

set. On the other hand, given that default can only be preferable when assets have a negative value (i.e. when the government is holding debts), there exists a level of assets $B \leq 0$, such that default sets are empty. These two properties of default sets can be summarized as follows.

Definition 2. Denote \underline{B} the upper bound of assets for which the default set constitutes the entire set and let \overline{B} be the lower bound of assets for which default sets are empty, where $\underline{B} \leq \overline{B} \leq 0$ due to Proposition 1.

$$\underline{B} = \sup \{B : D(B) = Y\}$$

$$\overline{B} = \inf \{B : D(B) = \emptyset\}$$

For asset holdings greater than \overline{B} , default is never the optimal for all y and equilibrium bond prices are equal to $(1+r)^{-1}$ because default probabilities are zero.⁸ For asset holdings $B \leq \underline{B}$ default is always optimal and equilibrium prices for these bonds are zero because default probabilities are one. Given that default sets are shrinking in the level of assets, condition (9) implies that equilibrium default probabilities are decreasing in B' , and the equilibrium price function $q(B', y)$ is an increasing function of B' . Lower levels of assets will be associated with larger default probabilities, and thus discount prices for those bonds will be lower to compensate risk neutral investors for a lower expected payoff. That is, larger loans are generally more expensive. Equilibrium bond prices are also contingent on the endowment shocks, because the probability distribution from which shocks are drawn the next period depends on today's shocks. Since the risk of default varies with the level of debt and depends on the stochastic structure of the endowment shocks, competitive risk neutral pricing requires that equilibrium bond prices depend on both B' and y .

3.1 Case of *i.i.d.* Tradable Endowment Shocks

This section characterizes the equilibrium bond price function and the default decision for the case of a constant nontradable endowment and *i.i.d.* tradable endowment shocks. When endowment shocks are *i.i.d.*, equilibrium bond prices are independent of the endowment realization and only a function of the level of assets $q(B')$, because today's shock gives no information on the likelihood of tomorrow's shock. We will assume, without loss of generality, that $\lambda = 1$, no

⁸Zhang (1997) introduced this debt limit as the no default debt limit in his work on participation constraints under incomplete markets.

output loss in autarky, and $\theta = 0$, financial autarky is permanent after default. The results can be generalized for other parameters of λ and θ .

Proposition 2. *Default incentives are stronger the lower the tradable endowment. For all $y_1^T < y_2^T$, if $y_2^T \in D(B)$, then $y_1^T \in D(B)$.*

Proof. See Appendix.

This result comes from the property that utility is increasing and concave in tradable consumption and by noting that default can be optimal only if under no default the economy experiences net capital outflows ($B - q(B')B' < 0$). In fact, when for some state default is optimal, there are no contracts available to the government such that the economy can experience capital inflows given that level of debt for all states. Given that utility is increasing and concave in consumption, and that the economy is not able to borrow more when it has the low endowment, repayment is more costly in this low endowment state and thus default more likely.

Intuitively, the asset available to the economy is not a very useful insurance instrument for a highly indebted economy, because in times of a low endowment it cannot raise enough resources to smooth consumption. Thus, the asset the economy is giving up is not very valuable and default may be preferable in times of low endowments.

Endowment shocks have generally two opposing effects on default incentives. When output is high, the value of default is relatively high, increasing default incentives. But at the same time, when output is high, the value of repayment is relatively higher too, decreasing default incentives. With an incomplete set of assets, and *i.i.d.* shocks, the latter effect dominates and thus default is more likely the lower the tradable endowment.

This result contrasts with the participation constraint models that have a complete set of contingent assets. These models have the feature that default incentives are higher in times of good endowment shocks. In fact, for small open economy models with participation constraints and a complete set of contingent assets, default incentives are always higher in the good states because only autarky is affected by the current endowment, as the value of staying in the contract is constant in the long run and independent of the economy's specific endowment.

Proposition 3. *If default sets are non-empty, then they are closed intervals where only the upper bound depends on the level of assets:*

$$D(B) = [\underline{y}, y^*(B)] \quad \text{for } B \leq \bar{B} \quad (10)$$

where $y^*(B)$ is a continuous, non-increasing function of B , such that:

$$y^*(B) = \left\{ \begin{array}{ll} y^*(B) : v^d(y^*(B)) = v^c(B, y^*(B)) & \text{for } \underline{B} \leq B \leq \overline{B} \\ \bar{y} & \text{for } B < \underline{B} \end{array} \right\} \quad (11)$$

Proof. See Appendix.

Proposition 3 proves that for $B \in [\underline{B}, \overline{B}]$, there will be a unique y^* where the contract value and continuation value cross. Default sets can then be characterized solely by a closed interval where only the upper bound is a function of the level of assets. For a given a level of assets $B \in [\underline{B}, \overline{B}]$, default will be optimal for endowment levels less than or equal to $y^*(B)$, and repayment will be optimal for endowment levels greater than $y^*(B)$. The function $y^*(B)$ is the default boundary that divides the y, B space into the repayment and default regions.⁹

The first order condition of the government with respect to asset holdings can be presented more sharply in this case by the following condition:

$$\frac{\partial[q(B')B']}{\partial B'} u_{cT} = \beta \int_{A(B')} v_{B'}^c(B', y') f(y') dy' \quad (12)$$

Equation (12) equates the marginal utility of consumption today to the expected marginal value of wealth tomorrow for the states where repayment is optimal. The marginal cost from borrowing a loan of size B' in the current period is the expected marginal disutility of consumption from repaying that loan the following period. As opposed to standard intertemporal conditions for models without default, here that cost is only experienced if in the following period the government choose to repay. That is, the cost of repaying is realized for the set of y 's, $A(B') = (y^*(B'), \bar{y}]$, for which repayment is optimal when the economy has assets equal to B' .

Given that default sets are such that only the upper bound depends on the level of debt, the equilibrium price function $q(B)$ can then be characterized by the following condition:

$$q(B) = \frac{1}{1+r} [1 - F(y^*(B))] \quad (13)$$

where F is the cumulative probability distribution of the stochastic endowments.

Equilibrium bond prices will fall in three ranges. For asset levels above \overline{B} prices are equal to the inverse of the risk free rate. For asset levels less than \underline{B} , prices are zero. For intermediate

⁹In continuous time optimal stopping problems, it has been shown that under special cases not only the function is continuous at the boundary, but present a 'smooth pasting' condition. Which would imply that the derivatives with respect to y of the continuation and default values are equal at the boundary.

asset levels, $B \in [\underline{B}, \overline{B}]$ prices will be increasing in the level of assets because $y^*(B)$ is decreasing in this range. Note that the bond price function will be crucially dependent upon the probability distribution of the endowment shocks.

If initial bond values are B then the probability of default at every point of the state space is given by $F(y^*(B))$, which is greater than zero in the space $B \in [-\infty, \overline{B}]$. The next issue to be addressed is whether the economy would ever choose a B' such that $D(B') \neq \emptyset$. That is if in the ergodic distribution of assets a point exists where default has a positive probability. That is, if initial bond holdings are larger than \overline{B} , whether the model can have default as an equilibrium outcome. To clarify this issue, imagine the economy happens to start at $B \leq \underline{B}$, then in that period the economy would default with probability one. But given that discount prices are zero in this range, if the economy's initial bond holdings are greater than \overline{B} , the economy would never choose as optimal asset holdings levels of $B' \leq \underline{B}$, because it would get nothing this period, and would incur a liability the following period. The range of B' for which default can potentially be an equilibrium outcome is limited to $(\underline{B}, \overline{B}]$, because here is where default sets are non-empty and equilibrium prices are different from zero.

Intuitively, the necessary condition for default to be an equilibrium outcome of the model is that the equilibrium price function does not decrease "too fast" as assets decrease. For default to potentially be an equilibrium outcome, the economy must be able to find it optimal to borrow bonds less than or equal to \overline{B} such that the economy is exposed to the risk of default. And for this to ever be an optimal decision, the economy should be able to increase total resources borrowed $q(B)B$, that is, have a higher level of consumption, by choosing a lower level of assets at lower price. This means that the equilibrium total resources borrowed $q(B)B$ needs to be increasing for some $B \in (\underline{B}, \overline{B})$. Figure (2) helps visualize this issue.

The slope of $q(B)B$ for $B > \overline{B}$ equals $(1+r)^{-1}$ because bond prices are constant and default probabilities are zero. The slope of $q(B)B$ for $B < \underline{B}$ is equal to zero, because bond prices are zero. For the intermediate range $(\underline{B}, \overline{B})$ the slope at equilibrium prices is:

$$\frac{\partial[q(B)B]}{\partial B} = \frac{1}{(1+r)} \left\{ [1 - F(y^*(B))] - f(y^*(B)) \cdot B \frac{\partial y^*}{\partial B} \right\} \quad (14)$$

Note that the sign of this derivative is ambiguous because bond positions $B \in (\underline{B}, \overline{B})$ are negative, and $y^*(B)$ is decreasing in B .¹⁰ In fact if there exists some $B^* \in (\underline{B}, \overline{B})$ for which $\frac{\partial[q(B^*)B^*]}{\partial B^*} = 0$ this level of assets corresponds to the endogenous borrowing limit in the model.

¹⁰In general $q(B)B$ may not be differentiable at the points \underline{B} and \overline{B} .

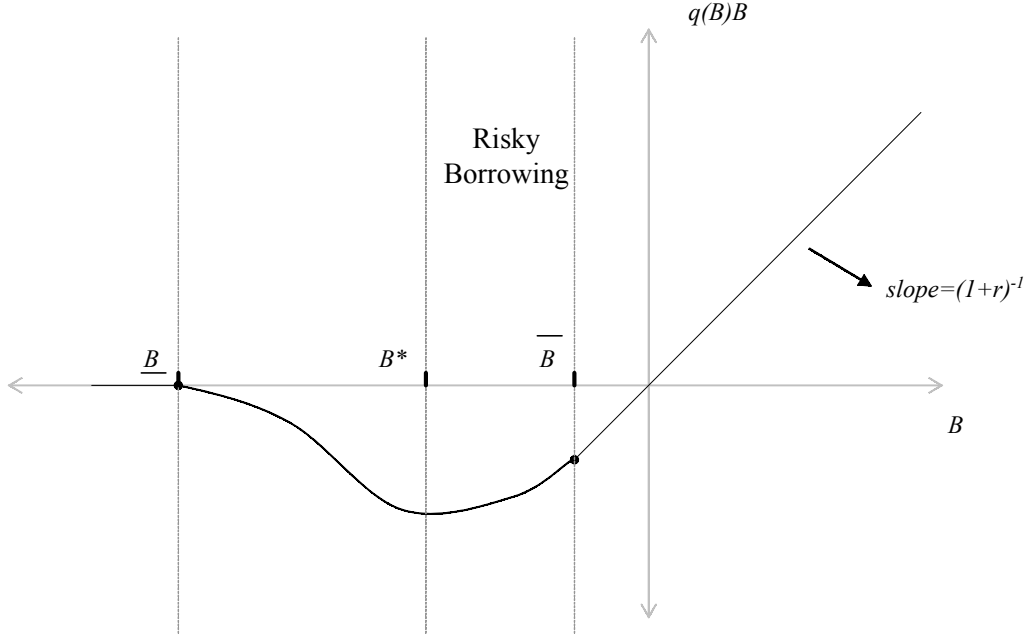


Figure 2: Total Resources Borrowed

The government would never find optimal to choose a level of assets below B^* because it can always find some other contract such that consumption the current period increases by the same amount while incurring a smaller liability for next period. The region that is relevant for risky borrowing and thus for default to be an equilibrium outcome of the model is then $B \in (B^*, \bar{B})$.

The necessary condition for the above derivative to be positive within a range depends on the hazard function (i.e. $\frac{f(y)}{[1 - F(y)]}$) of the probability distribution in the neighborhood of \underline{y} relative to how fast the upper bound on the default sets increase with debt. The following proposition summarizes these findings:

Proposition 4. *Default can be an equilibrium outcome of the model for all probability distributions over $[\underline{y}, \bar{y}]$ satisfying the property:*

$$\lim_{B \rightarrow \bar{B}^-} \frac{1}{h(y^*(B))} > \lim_{B \rightarrow \bar{B}^-} B \frac{\partial y^*(B)}{\partial B}$$

where $h(y)$ is the hazard function of the distribution.

Proof. When the above condition holds, $\frac{\partial [q(B)B]}{\partial B} > 0$ in the neighborhood of \bar{B} from the left. Given that $y^*(B)$ is continuous by Proposition 3, total resources borrowed increase for

lower levels of assets in the region where assets carry a default premium. \square

The hazard function $h(y^*(B))$ represents the instantaneous probability of default at B for $B \leq \bar{B}$. The above condition requires that the instantaneous probability of default as B approaches \bar{B} from the left is sufficiently small such that as B decreases (debt increases), the total resources borrowed increase. Due to Proposition 3, $y^*(\bar{B}) = \underline{y}$, and thus the condition is a restriction on the probability of the bad endowment shock. The government might then be willing to take on a risky loan, because in periods of low endowments it can increase consumption.

A sufficient condition for default to be an equilibrium outcome of the model is the following:

Corollary 4.1. *Default can be an equilibrium outcome of the model for all probability distributions over $[\underline{y}, \bar{y}]$ satisfying the property:*

$$\lim_{y \rightarrow \underline{y}} h(y) = 0$$

where $h(y)$ is the hazard function of the distribution.

Proof. See Appendix.

For all probability distributions satisfying the above property, the model will present a region in the state space where engaging in risky borrowing can increase consumption or $\frac{\partial[q(B)B]}{\partial B} > 0$, making default a positive probability event.

3.2 Case of *i.i.d* Nontradable Endowments

Now the role of volatile nontradable endowment is explored and its effect on default incentives. Here it is assumed that nontradable endowments follow an *i.i.d.* stochastic process, and without loss of generality that $\lambda = 1$, and $\theta = 0$.

Proposition 5. *Default incentives are stronger for low nontradable endowments if the cross derivative of the utility function is negative. For all $y_1^N \leq y_2^N$, if $y_2^N \in D(B)$, then $y_1^N \in D(B)$ if $u_{c^T c^N} < 0$.*

Proof. See Appendix.

Given that assets are tradable denominated only, nontradable fluctuations affect default decisions by their effect on the utility of tradable consumption. When $u_{c^T c^N} < 0$, a low nontradable shock will tend to increase the marginal utility of tradable consumption, giving the

government incentives for borrowing. In the region of default the economy experiences capital outflows and the fact that households cannot borrow as much as desired because of high interest rates and debt limits is relatively more costly for households who experience a low nontradable endowment if the above condition holds. Thus, default is more likely in the low nontradable states because repayment of tradable denominated loans is more costly.

For CES preferences it is well known that sign of this cross derivative depend on the relative magnitudes of the elasticity of intratemporal substitution between tradables and nontradables versus the intertemporal elasticity of substitution (Obstfeld and Rogoff 1996). When the elasticity of intratemporal substitution between tradables and nontradables is greater the intertemporal elasticity of substitution the cross derivative is negative, and otherwise its positive. Intuitively, decisions on optimal asset holdings depend on nontradable output fluctuations due to the effect on the relative price of nontradables. The intertemporal elasticity through time and intratemporal elasticity between tradable and nontradable consumption pull the time path of nontradable prices in opposite directions. When the intratemporal elasticity is greater than the intertemporal elasticity a low nontradable shock today tends to produce a decreasing time path in the nontradable price, which gives incentives for borrowing in the current period. So that under CES preferences the relation between default and nontradable output depend on the relative elasticities.

4 Simulations

In this section the model is solved numerically to asses its quantitative implications regarding business cycle properties of interest rates, real exchange rates and consumption together with default episodes. The main issue of interest for the quantitative analysis is to address the question of whether a model with endogenous default probabilities and interest rates can account for the empirical regularities observed in emerging markets regarding the high volatility of interest rates and real exchange rates, the negative correlation of interest rates with output and consumption, and the negative correlation between interest rates and real exchange rates. In addition, sensitivity analysis is performed on the degree of persistence of shocks, preference parameters, and default penalties. The benchmark model is calibrated to match certain features of the Argentinian economy.¹¹

¹¹The model is solved by a value function iteration algorithm that allows for the bond price vector to be endogenous. Specifically, endogenous and exogenous states are discretized, and the model is solved by iterating

The following utility function is used in the numerical simulations:

$$u(c_t^T, c_t^N) = \frac{c(c_t^T, c_t^N)^{1-\sigma}}{1-\sigma},$$

where $c(c_t^T, c_t^N)$ is the constant elasticity of substitution aggregator

$$\left[\omega (c_t^T)^{-\eta} + (1 - \omega) (c_t^N)^{-\eta} \right]^{-\frac{1}{\eta}}.$$

4.1 Calibration and Data in Argentina

The parameters of the benchmark model are calibrated to mimic some of the empirical regularities in the Argentinian economy or taken from other emerging markets studies. For the preference parameters, the risk aversion coefficient is set to 5, which is a standard value for real business cycles studies in emerging markets. The elasticity of substitution between tradable and nontradable consumption $1/(1 + \mu)$ is taken from Gonzales and Neumeyer (2003), where they estimate the elasticity for Argentina to be equal to 0.48.

To calibrate the relative sizes of the tradable and nontradable sectors in Argentina, we follow the standard methodology of assessing the degree of tradability of goods by computing the share of total trade (exports plus imports) of each sector as a percentage of total sectoral gross output. We find that the agricultural, manufacturing and energy sectors have a high degree of tradability, with an average share in this period of 0.38, 0.78 and 0.34 respectively. Nontradable output includes construction and all service sectors, which have a degree of tradability of less than 5%. An interesting fact to note is that in Argentina the size of the tradable sector is small, constituting only 26% of output. We normalize mean $y^T = 1$, and then set mean y^N equal to 2.78.

The weight on tradable consumption in the CES aggregator, ω , is set to normalize the relative price of nontradables to be equal to one in steady state when the economy is in autarky.

The time preference parameter, β , and the probability of reentering financial markets after default, θ , are calibrated to match default rates in emerging markets. Reinhart (2001) estimates an unconditional annual probability of one default episode for a sample of 56 emerging countries from 1970-1999 to be 0.0665. If crises were equally probable across countries, this would imply a per country default probability of roughly 0.0012.¹²

on the value function for an initial guess of the bond price vector. The bond price vector is updated with a Gauss-Seidel algorithm, using the creditors equilibrium zero profit condition. The procedure is repeated until the convergence criterion is met for the bond price vector.

¹²Beim and Calomiris (2001) report default episodes for a list of countries. They report two episodes of

For the benchmark calibration, the fraction of output lost in times of default, λ , is set equal to 0.02, which is the percent in output contraction estimated by Puhan and Sturzenegger (2002) following the default episodes in the 1980's in Latin America. In the sensitivity analysis we explore the effects of other default penalties.

Shocks to tradable and nontradable output are assumed to be independent and have a common first order autocorrelation coefficient. The standard deviation from each shock is set to match the sectoral output volatility observed in Argentina. The autocorrelation coefficient for both sectors is set to 0.4, which is the autocorrelation coefficient in the aggregate output series for Argentina. Each shock process is discretized into a 5 state Markov chain by using a quadrature based procedure (Hussey and Tauchen 1991). Table 1 summarizes the parameter values.

Table 1.

Parameter Values		
Risk Aversion	$\sigma = 5$	
Elasticity of Substitution	$1/(1 + \eta) = 0.48$	Gonzales and Neumeyer (2003)
Weight on CES	$\omega = 0.1061$	Normalization $p^N = 1$ in autarky steady state
Tradable Share	$y^N/y^T = 2.78$	$y^N/y^T = 2.78$ Argentina
Std dev y^N, y^T	$\sigma^N = 0.04$ $\sigma^T = 0.056$	Argentina sectoral volatility
Autocorrelation y^N, y^T	$\rho^N = \rho^T = 0.4$	Argentina output autocorrelation
Risk Free Interest Rate	$r = 0.01$	US quarterly interest rate
Output loss in default	$1 - \lambda = 0.02$	Sturzenegger (2003)
Probability of Re-entry	$\theta = 0.7$	Def prob = 0.0012 Reinhart (2001)
Discount Factor	$\beta = 0.88$	

Table 2 presents business cycle statistics for Argentinian data. The data are quarterly real series taken from the Ministry of Finance (MECON) for 1993 to 2002.¹³ Real exchange rates are constructed from dollar nominal exchange rates using the ratio of the consumer price index for Argentina and the US. The country interest rate series are taken from the dataset constructed

sovereign defaults in Argentina's foreign debt for 1900-2000: one in 1956 where Argentina defaulted on their suppliers credits in the post-Peron budget crises, and another in 1982 where it defaulted on its foreign bank loans in the midst of another budget crises. In 2001 Argentina defaulted a third time in their Brady Bonds after the recent crisis. This would give a higher default probability for Argentina of 0.03 annually.

¹³The series are constructed such that all variables are consistent with the model's statistics. Specifically, aggregate output and consumption are denominated in terms of tradables and sectoral output is denominated in real terms deflated by the appropriate price deflator.

by Neumeyer and Perri and MECON. Output and consumption data are log, and the current account data are reported as a percentage of output. All data are filtered with the Hodrick-Prescott filter.

Aggregate output, tradable output and nontradable output are all negatively correlated with interest rates. This negative relation is stronger for aggregate output. Tradable output is slightly more volatile than nontradable output, and both are correlated with aggregate output. Consumption is as volatile as nontradable output, and negatively correlated with interest rates. Real exchange rates are the most volatile series, and highly negatively correlated with interest rates. Periods of low real exchange rates are accompanied by higher country interest rates.

Table 2.
Business Cycle Statistics for Argentina

	Std Dev	Corr Y	Corr R
Interest Rates	0.0781	-0.8626	
Real Exchange	0.1104	0.8324	-0.8332
Consumption	0.0498	0.7320	-0.6631
Current Account	0.0237	-0.7393	0.8267
Tradable Output	0.0653	0.3802	-0.2491
Nontradable Output	0.0489	0.6644	-0.5585
Aggregate Output	0.0935		-0.8626

4.2 Simulation Results

Table 3 presents business cycle statistics for the benchmark calibration of the model. Business cycle statistics are mean values from 100 simulations of 40 observations each. The simulated data are log and filtered equally as the Argentinian data. In addition, the denomination of the statistics in the model is consistent with the data.

Overall the model can match several features of the Argentinian economy. Aggregate output is defined in terms of tradables, and is more volatile than sectoral output. Aggregate consumption defined in terms of tradables is as volatile as output, and more volatile than sectoral output. The volatility of interest rates is lower than the one observed for Argentina. Note though, that interest rate volatility in this model is endogenous and it is only due to volatile default premium. Risk neutral pricing of the endogenous risk of default can explain 15% of the volatility of country interest rates. The only mechanism in the model for volatile interest rates are varying default probabilities due to volatile endowments. The model does not

address other sources of interest rate volatility such as the volatility in the international interest rate and the feedback and magnifying effects that volatile interest rates can have on output.

The real exchange rate is the most volatile variable in the model, being more volatile than aggregate output and twice as volatile as nontradable output matching the real exchange rate volatility in the data. The volatility of the real exchange rate comes from the exogenous volatility of nontradable output and the endogenous volatility of tradable consumption through equation (??). Endogenous time varying interest rates and debt limits make tradable consumption much more volatile in this model, which is the main driving mechanism of the high volatility of the real exchange rate.

The model can also match the correlations observed in the data. The correlations between consumption and sectoral and aggregate output are positive and in line with the data. On the downside, the model does not match the negative correlation between the current account and output observed in the data. Debt is used in the model to smooth output variations. Households generally want to run down their assets in periods of low output, and engage in precautionary savings in periods of relatively high output. Thus, as in any insuring type model of debt without investment, current account and output should be positively correlated. However, this model produces a very low positive correlation, once the risk of default is modeled. This is because bonds in this model are sometimes not very good insurance assets as interest rates are high exactly in periods of low output.

Table 3.

Business Cycle Statistics for the Benchmark Model

	Std Dev	Corr Y	Corr R
Interest Rates	0.0107	-0.3562	
Real Exchange Rate	0.1160	0.9502	-0.3507
Consumption Tradable	0.0598	0.9476	-0.3412
Consumption	0.0904	0.9741	-0.3567
Current Account	0.0044	0.0832	0.0789
Nontradable Output	0.0451	0.0673	0.1268
Tradable Output	0.0654	0.9130	-0.3231
Aggregate Output	0.1084		-0.3562
Default Rate	0.0012		
Debt Limit (% tradable output)	0.1645		

Correlations with interest rates are overall consistent with the data, although the magnitudes are lower. Interest rates are negatively correlated with aggregate and tradable output, consumption and real exchange rates in the business cycle, and are slightly positively correlated with the current account. The model does not match the negative correlation between nontradable output and interest rates and produces a slight positive correlation. However, when nontradable output is measured in terms of tradables, the correlation becomes highly negative, because of the negative correlation between nontradable prices and interest rates.

The endogenous borrowing limit for this economy is equal to 16.5% percent of tradable output or 5% percent of total output. That is, the price of loans above this limit would be zero because default would be a certain outcome. Note that the calibration of the model is able to match the default rates observed in emerging markets. Default is a rare outcome in the model, because forward-looking rational agents insure against it. Nevertheless, default is a positive probability event that affects equilibrium allocations and prices. Note that in the model there is a direct matching from default probabilities to interest rates due to risk neutral lenders. The mean annual spread (defined as the difference between the country interest rate and the risk free interest rate, $r^c - r$) in the limiting distribution of assets conditional on not defaulting is 1.6 % in the model, which accounts for 23% of the spread observed in Argentina of 7%.

The fact that default probabilities do not account for all the yield spread in bonds is a well known puzzle in the finance literature on corporate defaultable bonds. Huang and Huang (2003) find that in investment grade bonds, default probabilities and credit risk account only for 19% of the yield observed in defaultable bonds. In fact, for shorter maturity bonds, they find that default risk accounts for almost nothing of the yield.¹⁴

Figure (3) plots the equilibrium price schedule faced by the economy for the highest and lowest tradable endowments, holding constant the nontradable output, and for the highest and lowest nontradable sector endowments, holding the tradable endowment constant. Bond prices are an increasing function of assets. That is, larger levels of debt are associated with higher interest rates. For values of debt of up to 10% of mean tradable output, the economy faces low interest rates. Effectively, it is charged the risk free interest rate for these loans. This is because agents in the economy have no incentives to default when holding relatively smaller amounts of debt regardless of the state. At this level, default incentives start to increase in the economy giving rise to higher interest rates. At debt levels of about 16% of tradable output, households refuse to pay any liabilities for all endowment shocks and thus the probability of default for this

¹⁴The majority of the finance studies on defaultable bonds use reduce form models that take as exogenous the process for the default probabilities.

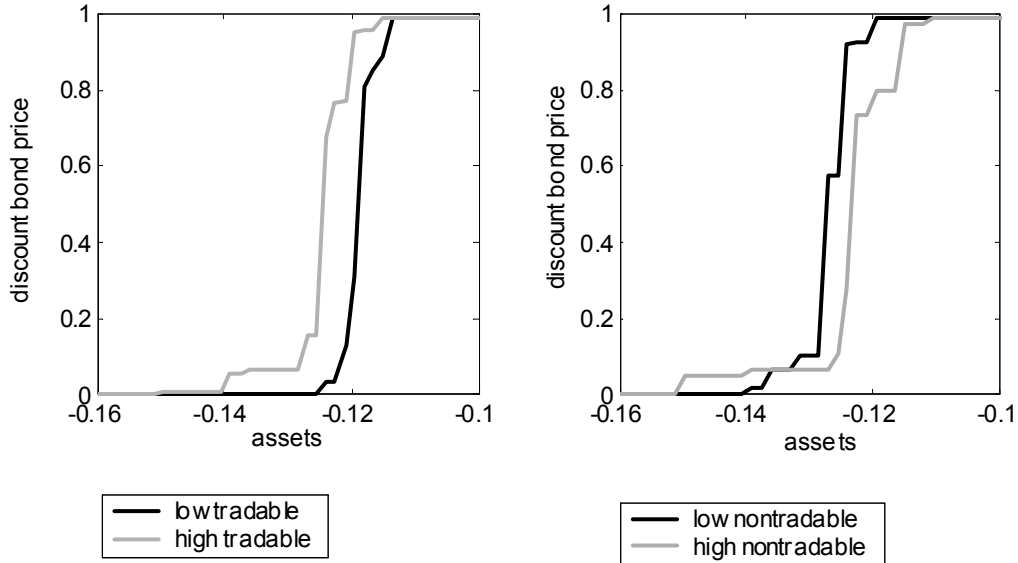


Figure 3: Bond Prices

debt level or higher is 1. This is why discount bond prices for these levels of debt or higher are zero.

A feature of the model is that it produces a narrow range for asset positions that carry positive but finite risk premia (i.e. the range $B \in (\underline{B}, \overline{B}]$). This is because the range of assets that have positive risk premia is determined by the B 's that make the default value and the contract value equal across states and this difference does not vary significantly. Below we explore how alternative punishment specifications can alter the range for risky lending.

Note that the theoretical results in Proposition 3 for *i.i.d.* shocks hold in the calibrated example that features persistent shocks. The economy is more likely to default in low tradable endowment states, and thus face a higher interest rate premium in these states. For all levels of debt, the economy faces lower discount bond prices (higher interest rates) when it is hit by a negative tradable endowment shock as shown in figure (3). However, the relation between interest rates and nontradable endowments in the calibrated example with persistent shocks is no longer monotonic. That is, for a range of bonds, discount prices are lower for high nontradable endowments, and for another range of bonds, prices are lower for low nontradable endowments. This suggests that the relation between nontradable endowments and interest rates is quite

sensitive to the level of persistence of the shocks. For *i.i.d.* shocks, the relation between default incentives and nontradable endowments is monotonic and for CES preferences dependent on the relative elasticities, but once shocks deviate from the *i.i.d.* case, this is no longer the case. This is the reason why the benchmark model produces a slightly positive correlation between nontradable endowments and interest rates. However, the relation between nontradable output measured in terms of tradables and interest rates is negative and monotonic. As shown below, the model has the feature that high interest rates are associated with low real exchange rates and this effect dominates when we look at the relationship between nontradable output measured in terms of tradables and interest rates.

Figure (4) plots equilibrium interest rates as a function of aggregate output denominated in tradables for bond positions ranging from -0.10 to -0.14. Aggregate output in this economy is endogenous because of the endogenous relative price of nontradables. When the economy is in a state where default premium is positive, the economy faces higher interest rates in periods of low aggregate output. Given that default is more likely if the economy faces a recession, the probability that it faces a recession tomorrow is greater with persistent shocks when the economy faces a negative shocks today, and thus a higher premium is charged to this larger default probability.

The negative relation between interest rates and output is also observed in the business cycle of the simulated economy. Correlations of interest with output are all negative, although the magnitudes are smaller than in the data.

Figure (5) shows the equilibrium price of nontradables as a function of assets for the highest and lowest tradable and nontradable shocks. As the economy runs down its assets, the real exchange rate depreciates because equilibrium tradable consumption is lower through equation (??). In the region where the economy faces high interest rates, the real exchange rate collapses rapidly up to the point where default is the optimal decision. That is, real exchange devaluations are greatest when the economy faces high interest rates. Dynamics are similar when looking at tradable and nontradable shocks. However, the schedule of the equilibrium nontradable price for the low tradable shocks is below that for high tradable shocks, and the schedule for low nontradable shocks is above that for high nontradable shocks. Correlations from the simulated data also present this negative correlation between interest rates and real exchange rates.

An interesting feature of the model is that capital outflows (*i.e.* $y^T - c^T$) can be larger in a recession because default probabilities are high. When interest rates are low, debt is a good

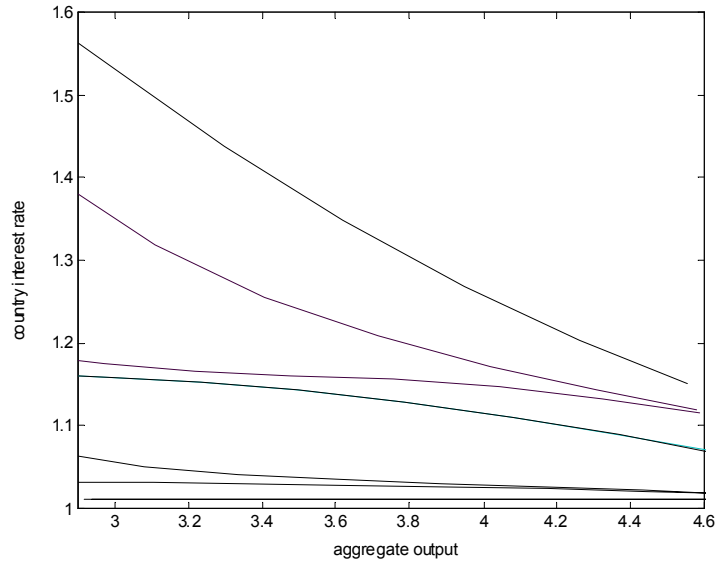


Figure 4: Interest Rates and Aggregate Output

insurance instrument: capital outflows are large in good times because the economy saves, and are low in bad times because the economy borrows. However, when the economy is highly indebted and interest rates are high due to high default probabilities, debt becomes a less good insurance instrument. In fact, given that interest rates are countercyclical, the economy can face larger capital outflows in a recession. This result is similar to Atkeson's (1991) result, where he shows that in an insurance model of debt that features moral hazard and unenforceability of debt contracts, the optimal debt contract will feature capital outflows in a recession. Here, what drives the result is the incompleteness of assets and the fact that default is an equilibrium outcome.

Figure (6) plots equilibrium capital outflows as a function of aggregate output for bond positions with positive default premia. This feature of the model matches the empirical regularity that emerging markets in crises are not able to use the international markets for smoothing and experience net capital outflows.

To summarize, these are the main findings of the benchmark model:

1. Output disturbances can explain a significant portion of the volatility of country interest rates by affecting default premium.

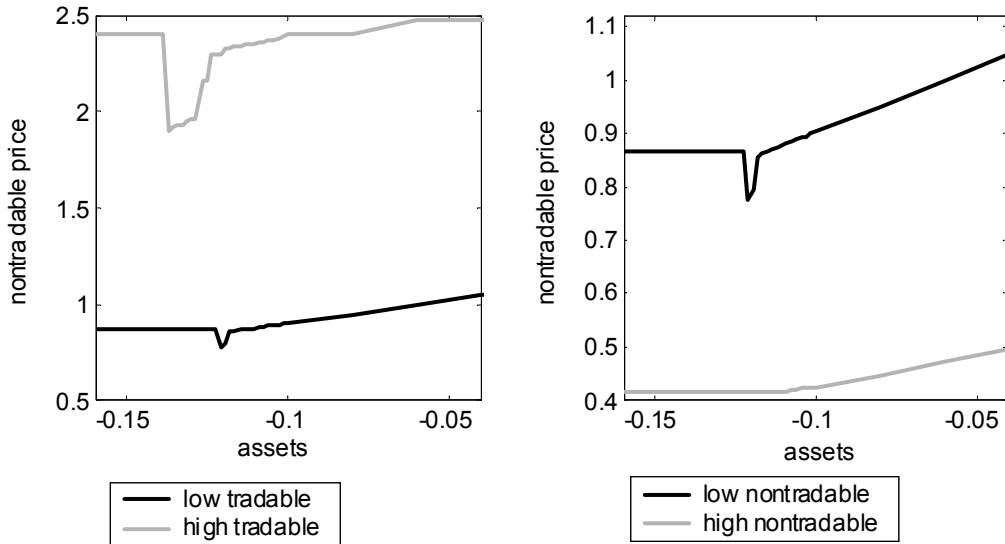


Figure 5: Nontradable Prices

2. Country interest rates are higher when the economy is in a recession.
3. Higher interest rates and real exchange rate devaluations move together.
4. When default premium is positive, the economy experiences larger capital outflows in a recession.

Figure (7) plots the time series dynamics of the economy around a default.¹⁵ In period 32 the economy chooses to default on its debts. Prior to defaulting the economy experiences increases in interest rates and collapses in real exchange rates and consumption. The model can generate dynamics similar to the ones observed in Argentina's default episode.

4.3 Experiments

We now explore alternative specifications and parameters to understand the important drivers of the mean, volatility and counter-cyclical of interest rates and default probabilities in the

¹⁵This time series plot is generated from the limiting distribution of assets. We followed the equilibrium optimal decisions through time, and presented a case where the economy defaulted on its debt.

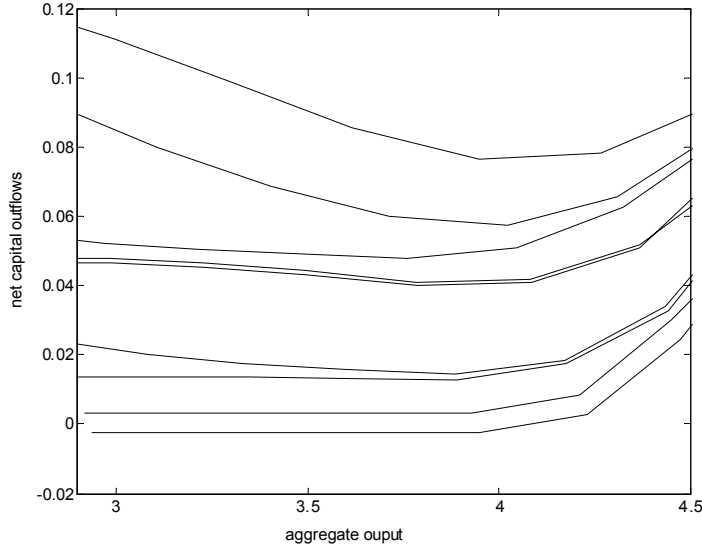


Figure 6: Capital Outflows and Aggregate Output

model.

4.3.1 Sensitivity Analysis

The theoretical results in this paper are for *i.i.d.* shocks; here it is explored numerically how sensitive the results are to introducing different degrees of autocorrelation in the endowment processes. Table 4 presents statistics regarding credit market conditions for various levels of persistence. Default rates, mean spreads and interest rate volatility are higher for more persistent shocks because the range of $B \in [\underline{B}, \overline{B}]$ that carry positive default premia is larger as contract and default value differ more across states. But with persistent shocks, default incentives seem to be higher in good times, as shown by the positive correlation between output and interest rates for highly persistent shocks. That is, because although repayment is more costly in the bad state, tending to make the output-interest rate correlation negative, the default value is relatively higher in good times if shocks are persistent. This gives the economy greater default incentives in boom times, which tends to make the correlation positive.

As table 4 shows, the coefficient of risk aversion has important effects on credit market conditions. Higher degree of risk aversion produces monotonically looser debt limits. This is because consumption smoothing is more valued for more risk averse households and thus more borrowing can be supported without having incentives to default. The relationship between

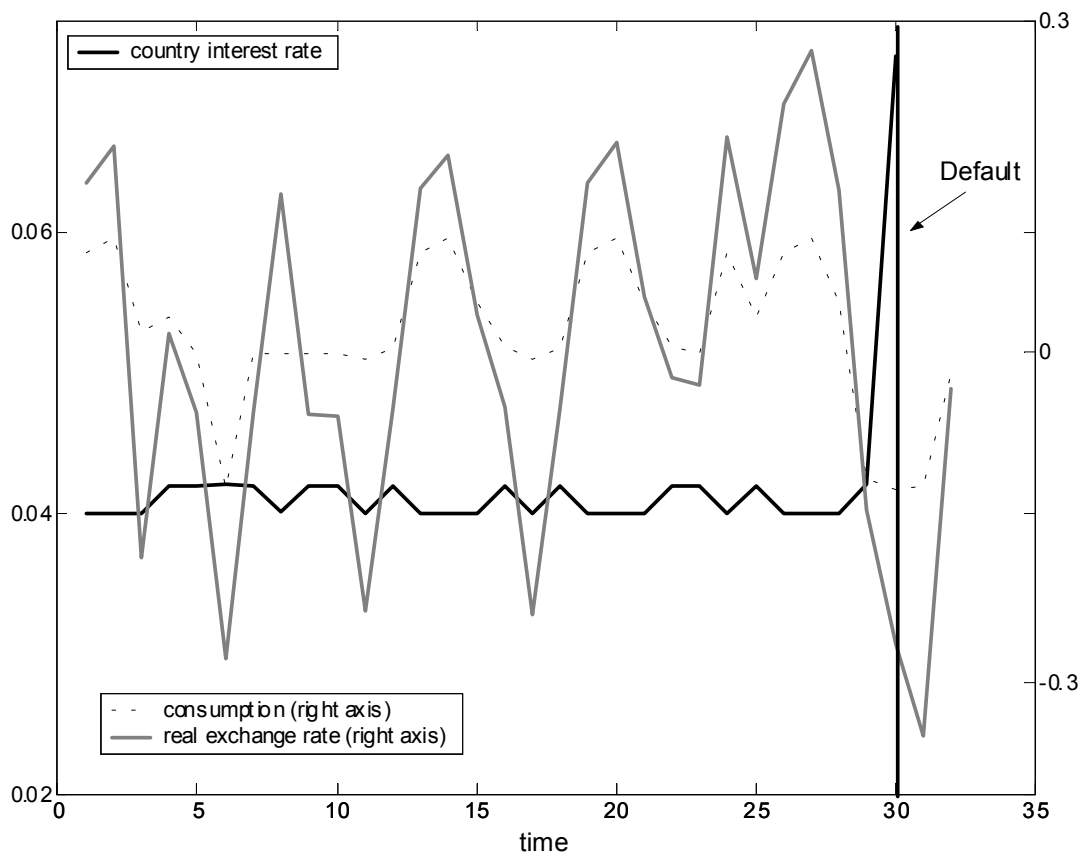


Figure 7: Dynamics Prior to Default

risk aversion and default probabilities, spreads and interest rate volatility is not monotonic. We find that the model can generate the highest mean spread for a risk aversion coefficient of about 10. The intuition for this result is that risk aversion plays a double role in the model. Higher risk aversion increases the concavity of the utility function and thus makes repayment of non-contingent debt more costly in recessions, tending to increase default probabilities and spreads. But high risk aversion also increases the amount of precautionary savings the economy engages in, which would tend to decrease default probabilities and interest rates because the economy engages more actively in insuring against this outcome. Correlations of interest rates and output decrease with higher degree of risk aversion, and correlations of interest rates with real exchange rates become positive for higher degrees of risk aversion. Note that risk aversion is also the inverse of the intertemporal rate of substitution. The correlation of interest rates and exchange rates becomes positive because as the intertemporal elasticity decreases (i.e. high coefficient of risk aversion), the probabilities of default are higher for lower nontradable shocks. Low nontradable shocks tend to appreciate the real exchange rate by increasing the nontradables' price.

Table 4 also presents sensitivity analysis for various default penalties. Borrowing limits become looser for economies that face higher default penalties, either coming from larger output losses or longer time in financial autarky. We find that default probabilities, mean spreads, and interest rate volatility are higher the larger the probability of the economy regaining access to international credit markets after default. Interest rate correlations with output and the real exchange rates do not seem to be particularly sensitive to changing the length of time the economy is penalized. Regarding the percentage of output loss in default, we find that with the benchmark calibration an economy that is penalized with only financial autarky for a small number of periods may not be able to support any borrowing in equilibrium because probabilities of default increase too fast once the economy starts holding any debt. Nevertheless, we find that for other calibrations, an economy being penalized with only temporary financial autarky presents positive default probabilities in equilibrium.

Table 4
Sensitivity Analysis

	Mean($r^c - r$)	Default Rate	Std Dev (r^c)	Debt limit	Corr(y, r^c)	Corr(p^c, r^c)
Shocks autocorrelation						
$\rho = 0$	0.04	0.0001	0.0001	0.18	-0.3950	-0.1513
$\rho = 0.4$	1.6	0.0012	0.0107	0.1645	-0.3562	-0.3507
$\rho = 0.8$	2.8	0.0014	0.0138	0.358	0.1487	0.3207
Risk Aversion						
$\sigma = 2$	0.32	0.0002	0.0015	0.136	-0.6217	-0.6683
$\sigma = 5$	1.6	0.0012	0.0107	0.1645	-0.3562	-0.3507
$\sigma = 10$	3.2	0.0014	0.0118	0.272	-0.1244	0.0638
$\sigma = 15$	2.8	0.0010	0.0069	0.347	-0.0302	0.1163
Default Penalties						
$\theta = 0.5$	0.8	0.0009	0.0089	0.245	-0.5743	-0.4643
$\theta = 0.7$	1.6	0.0012	0.0107	0.1645	-0.3562	-0.3507
$\theta = 0.9$	2.0	0.0013	0.0112	0.1580	-0.5075	-0.4656
$\lambda = 1$	0	0.00	0	0	0	0
$\lambda = 0.98$	1.6	0.0012	0.0107	0.1645	-0.3562	-0.3507
$\lambda = 0.95$	0.9	0.0009	0.0074	0.3682	-0.3701	-0.2895

4.3.2 One Sector Model

In this experiment the benchmark model is compared with a one sector endowment model where all output is composed of tradable goods. The mean and volatility of output in the one sector model is such that it equals the two sector benchmark aggregate output. We find that this economy would have no default in the long run distribution with the benchmark calibration. However, an interesting result from this experiment is that this economy would face a looser debt limit of 17.2% of aggregate output, whereas in the benchmark calibration the debt limit is 5% of aggregate output. That is, the economy is able to access greater credit when output is composed of tradable goods only. The intuition of this result is that, from an incentives perspective, an economy with larger tradable sectors will benefit more from greater access to international borrowing to smooth fluctuations. This tends to suggest that economies with relatively larger tradable sectors would have greater access to international credit markets.

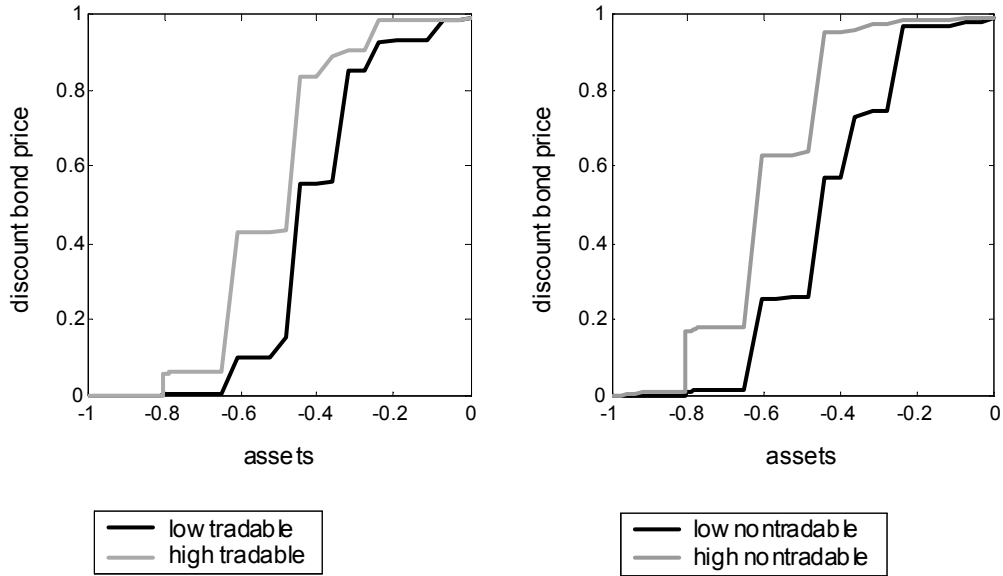


Figure 8: Bond Prices – Constant Default Penalties

4.3.3 Constant Default Penalties

The main limitation of the model in explaining interest rate spread and volatility is that for all specifications it produces a small region of assets that have positive but finite risk premium, because default probabilities increase rapidly once the economy reaches the debt limit \bar{B} . This is because the level of debt that makes the continuation value equal the default value does not vary too much across states. One way to address this issue is by assuming that the default penalty is constant across endowment states. In this last experiment we explore this avenue. Specifically, it is assumed that if the economy decides to default, the economy’s endowment falls one standard deviation from the mean level.

Figure (8) shows the bond price functions for the tradable and nontradable shocks that are one standard deviation above and below the mean level. As the figure shows, these types of penalties can increase dramatically the range for loans that in equilibrium have a positive but finite risk premium (i.e. $B \in (\underline{B}, \bar{B}]$).

Table 5 shows that the type of penalties for defaulting can have significant effects on default probabilities and equilibrium interest rates. With the benchmark calibration, the default probability increases to 3.2% , interest volatility increases to 11.5 , debt limits are much looser

and equal about 100% of tradable output, and the correlation of output and interest rates is negative. Overall, this specification for penalties gives the model greater flexibility in being able to capture the high spreads observed in emerging markets foreign debt.

Table 5

Constant Default Penalties

(Endowments fall to one standard deviation below the mean levels)

Mean($r^c - r$)	Default Rate	Std Dev (r^c)	Debt limit	Corr(y, r^c)	Corr(p^c, r^c)
10.84	0.0267	0.0987	0.9875	-0.3494	-0.1252

One way to rationalize the assumption that endowments fall to a low level as opposed to a proportional fall is to consider that default may trigger trade sanctions. Most trade inflows in emerging markets are composed of intermediate inputs. If inputs are not very substitutable in production, then a limit on these products will cause output to fall to a low level, regardless of what the current productivity shock is. That is, the drop in output will not be proportional after default but to an equally low level. This experiments suggest that more evidence is needed regarding the type of punishments economies encounter after defaulting on their international debt.

5 Conclusion

This paper models endogenous default risk in a stochastic dynamic framework of a small open economy that has two sectors and that features liability dollarization. The paper presents a model where interest rates respond to output fluctuations through endogenous time-varying default probabilities. The main contributions of the paper are three. First, it studies analytically the relationship between default and output in an environment of incomplete markets and provides conditions for default to be an equilibrium outcome. Second, it explores quantitatively the predictions of the model in explaining the volatility and counter-cyclicality of interest rates observed in emerging markets. In a calibration exercise, output fluctuations can account for 15% of the volatility of interest rates and 20% of the mean interest rate spread in Argentina, by affecting endogenous default risk premia. In addition, the model can match the negative correlations of interest rates with output and consumption observed in the data. Third, it explores the effects that fluctuations and relative sizes of nontradable sectors have on incentives

to default on tradable denominated debt. The model matches the data in that real exchange rate devaluations are correlated with high interest rates. An empirical implication of the model is that economies with relatively small tradable sectors have higher incentives to default on tradable denominated debt and thus face higher interest rates.

Even though the model is able to explore default incentives and can match interest rate counter-cyclicalities, it cannot match the magnitude of the interest rate spread observed in the data of emerging markets. This is because the model assumes that creditors are risk neutral and it lacks a feedback mechanism through which volatile and high interest rates affect output. Interesting extensions of the model would be addressing these issues by exploring alternative specifications for creditors, modeling default penalties as a bargaining outcome, and adding a feedback from interest rates to production. These issues remain to be explored.

References

- [1] Aiyagari, S. R. (1993), "Explaining Financial Markets Facts: The Importance of Incomplete Markets and Transactions Costs," Federal Reserve Bank of Minneapolis Quarterly Review 17, 17-31.
- [2] Alvarez, F., and U. J. Jermann (2000) "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, v. 68(4), 775-798.
- [3] Atkeson, A. (1991), "International Lending with Moral Hazard and Risk of Repudiation," *Econometrica*, v. 59 (4), 1069-89.
- [4] Betts, C., and T. J. Kehoe (2001), "Tradability of Goods and Real Exchange Rate Fluctuations," Working Paper, University of Minnesota
- [5] Beim, D., and C. Calomiris (2001). *Emerging Financial Markets*. New York: McGraw-Hill, Irvin.
- [6] Bulow, J., and K. Rogoff (1989), "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, v. 97, 155-178.
- [7] Caballero, R., and A. Krishnamurthy (2000), "Dollarization of Liabilities: Underinsurance and Domestic Financial Underdevelopment," NBER WP #7740.
- [8] Calvo, G. (1998), "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops," *Journal of Applied Economics*, v.1, 35-54.
- [9] ———, A. Izquierdo, and E. Talvi (2002), "Sudden Stops, the Real Exchange Rate and Fiscal Sustainability: Argentina's Lessons," Working Paper, Research Department, Inter-American Development Bank.
- [10] ———, and C. Reinhart (2002), "Fear of Floating," *Quarterly Journal of Economics*, Vol. CXVII, No. 2, 379-408.
- [11] Chatterjee, S., D. Corbae, M. Nakajima, and J. Rios Rull (2002), "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," Working Paper, University of Pennsylvania.
- [12] Chuhan, P., and F. Sturzenegger (2003), "Default Episodes in the 1990's: What have we learned?," Working Paper, Universidad Torcuato Di Tella.

- [13] Cole, H., and T. Kehoe (2000), "Self-Fulfilling Debt Crises," *Review of Economic Studies*, 67, 91-116.
- [14] Conklin J. (1998), "The Theory of Sovereign Debt and Spain under Philip II," *Journal of Political Economy*, , vol. 106 (3), 483-513
- [15] Eaton, J., and M. Gersovitz (1981), "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, v. XLVII, 289-309.
- [16] Gibson, R., and S. Sundaresan (2001), "A Model of Sovereign Borrowing and Sovereign Yield Spreads," Working Paper, Columbia University.
- [17] Gonzales-Rosada, M., and P. A. Neumeyer (2003), "The Elasticity of Substitution in Demand for Non-tradable Goods in Latin America. Case Study: Argentina," Working Paper, Universidad Torcuato Di Tella.
- [18] Hamann, F. (2002), "Sovereign Risk and Real Business Cycles in a Small Open Economy," Working Paper, North Carolina State University.
- [19] Hausmann, R., U. Panizza, and E. Stein (2001), "Why countries float the way they float?" *Journal of Development Economics*, v.66 (2), 387-414.
- [20] Huang, J., and M. Huang (2003), "How Much of the Corporate-Treasury Yield-Spread is due to Credit Risk?" Working Paper, Stanford University.
- [21] Huggett, M. (1993), "The risk free rate in heterogenous-agents incomplete-insurance economies," *Journal of Economic Dynamics and Control* v.17, 953-969.
- [22] Hussey, R., and G. Tauchen (1991), "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica*, v 59 (2), 371-396.
- [23] Kehoe, T. J., and D. K. Levine (1993), "Debt-Constrained Asset Markets." *Review of Economic Studies* 60 868-88.
- [24] Kletzer, K. M., and B. D. Wright (2000), "Sovereign Debt as Intertemporal Barter," *American Economic Review*. 90(3), 621-639.
- [25] Mendoza, E. G. (2002), "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop." In *Preventing Currency Crises in Emerging Markets*, Frankel, Jeffrey and Sebastian Edwards eds. Chicago: University of Chicago Press.

- [26] — (2001), “The Benefits of Dollarization when Stabilization Policy Lacks Credibility and Financial Markets are Imperfect,” *Journal of Money, Credit and Banking* v33.
- [27] Neumeyer, P., and F. Perri (2001), “Business Cycles in Emerging Economies: The Role of Interest Rates,” Working Paper, Department of Economics, New York University.
- [28] Obstfeld M., and Rogoff K.(1989), "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review* 79, 43-50.
- [29] — (1996), *Foundations of International Macroeconomics*, Cambridge, Massachusetts, MIT Press.
- [30] Oviedo, M. (2003) "Macroeconomic Risk and Banking Crises in Emerging Market Countries: Business Cycles with Financial Crashes ," Working Paper, Iowa State University.
- [31] Reinhart, C. (2002), "Default, Currency Crises and Sovereign Credit Ratings," NBER Working Paper #8738.
- [32] Uribe, M., and Yue V. (2003), "Country Spreads and Emerging Markets: Who Drives Whom?" Working Paper, University of Pennsylvania.
- [33] Zame, W. (1993), "Efficiency and the Role of Default When Security Markets are Incomplete," *American Economic Review*, 83, No. 5, 1142-1164.
- [34] Zhang, H. (1997) “Endogenous Borrowing Constraints with Incomplete Markets,” *The Journal of Finance*, 52, 2187-2209.

6 Appendix A

Proposition 1. *Default is more likely the higher the level of debt: For $B^1 < B^2$, if default is optimal for B^2 , in some states y , then default will be optimal for B^1 for the same states y , that is $D(B^2) \subset D(B^1)$.*

This result is similar than Eaton and Gersovitz (1981) and Corbae et al (2001).

For all $\{y^T, y^N\} \in D(B^2)$, $u(y^T, y^N) + \beta Ev^d(y') > u(y^T + B^2 - qB', y^N) + \beta Ev^o(B', y')$. Since $y^T + B^2 - qB' > y^T + B^1 - qB'$ for all B' , thus $u(y^T + B^2 - qB', y^N) + \beta Ev^o(B', y') > u(y^T + B^1 - qB', y^N) + \beta Ev^o(B', y')$. That is the value of the contract under no default is increasing in foreign asset holdings. Hence $u(y^T, y^N) + \beta Ev^d(y') > u(y^T + B^1 - qB', y^N) + \beta Ev^o(B', y')$, which implies that $\{y^T, y^N\} \in D(B^1)$ ■

The Case of i.i.d. Endowment Shocks

Proposition 2. *Default incentives are stronger the lower the tradable endowment. For all $y_1^T < y_2^T$, if $y_2^T \in D(B)$, then $y_1^T \in D(B)$.*

In order to prove proposition 2, we will first prove the Lemma 2.1

Lemma 2.1 . *If for some B , the default set is non empty $D(B) \neq \emptyset$, then there are no contracts available for the economy $\{q(B'), B'\}$ such that the economy can experience capital inflows, $B - q(B')B' > 0$*

This is a proof by contradiction.

Suppose there are contracts $\{q(B'), B'\}$ available to the economy such that $B - q(B')B' > 0$. But that the economy choose under the contract utility some \widehat{B} to maximize utility such that $B - q(\widehat{B})\widehat{B} < 0$ and then finds default to be the optimal option because $u(y^T, y^N) + \beta Ev^d(y') > u(y^T + B - q(\widehat{B})\widehat{B}, y^N) + \beta Ev^o(\widehat{B}, y')$.

Now note that under all contracts $\{q(B'), B'\}$ such that $B - q(B')B' > 0$ staying in the contract is always preferable to default because $Ev^o(B', y') \geq Ev^d(y')$, and $u(y^T + B - q(B')B', y^N) > u(y^T, y^N)$. This implies that \widehat{B} cannot be the maximizing level of assets and then find default to be optimal, because it is a contradiction.

Thus, if $D(B) \neq \emptyset$, then \exists some $y \in Y$, such that $v^d(y) \geq v^c(B, y)$, or equivalently, $u(y) + \beta Ev^d(y') \geq u(y + B - q(B')B') + \beta Ev^o(B', y')$.

Given that B' is chosen to maximize the value of the contract, then if default is preferable, it must be the case that not only $B - q(B')B' < 0$, but that \nexists a contract available $\{q(B'), B'\}$ such that $B - q(B')B' > 0$ □

Now we prove proposition 2.

If $y^2 \in D(B)$ then by definition $u(y_2^T, y^N) + \beta Ev^d(y') \geq u(y_2^T + B - q(B')B', y^N) + \beta Ev^o(B', y')$

If

$$\begin{aligned} & u(y_2^T + B - q(B^2)B^2, y^N) + \beta Ev^o(B^2, y') - \\ & \{u(y_1^T + B - q(B^1)B^1, y^N) + \beta Ev^o(B^1, y')\} > \\ & u(y_2^T, y^N) + \beta Ev^d(y') - \{u(y_1^T, y^N) + \beta Ev^d(y')\} \end{aligned} \quad (15)$$

then $y^2 \in D(B)$, implies $y^1 \in D(B)$,

Now it is necessary to show that expression (15) holds.

Given that shocks are iid, the right hand side of equation (15) simplifies to

$$[u(y_2^T, y^N)] - [u(y_1^T, y^N)]$$

Because of utility maximization:

$$u(y_2^T + B - q(B^2)B^2, y^N) + \beta Ev^o(B^2, y') \geq u(y_2^T + B - q(B^1)B^1, y^N) + \beta Ev^o(B^1, y')$$

Thus if

$$\begin{aligned} & u(y_2^T + B - q(B^1)B^1, y^N) + \beta Ev^o(B^1, y') - \\ & \{u(y_1^T + B - q(B^1)B^1, y^N) + \beta Ev^o(B^1, y')\} > \\ & \{[u(y_2^T, y^N)] - u(y_1^T, y^N)\} \end{aligned} \quad (16)$$

holds then through transitivity expression (15) holds.

Simplifying (16):

$$u(y_2^T + B - q(B^1)B^1, y^N) - u(y_1^T + B - q(B^1)B^1, y^N) > [u(y_2^T, y^N)] - [u(y_1^T, y^N)]$$

Now, note that due to Lemma 2.1, if $y^2 \in D(B)$ then $B - q(B')B' < 0$ for all available $\{q(B'), B'\}$ thus $B - q(B^1)B^1 < 0$.

Hence, given that utility is increasing and strictly concave in both arguments, then (16) holds, which implies that $y^1 \in D(B)$. \square

Proposition 3. *If default sets are non-empty, then they are closed intervals where only the upper bound depends on the level of debt:*

$$D(B) = [\underline{y}, y^*(B)] \quad \text{for } B \leq \bar{B}$$

where $y^*(B)$ is a continuous, non-increasing function of B , such that:

$$y^*(B) = \left\{ \begin{array}{ll} y^*(B) : v^d(y^*(B)) = v^c(B, y^*(B)) & \text{for } \underline{B} \leq B \leq \bar{B} \\ \bar{y} & \text{for } B < \underline{B} \end{array} \right\}$$

Proof:

For $B < \underline{B}$, $D(B) = Y$, so that $y^*(B) = \bar{y}$

For $\underline{B} \leq B \leq \bar{B}$ let $\Psi(B, y) \equiv v^d(y) - v^c(B, y)$

$$\Psi(B, y) = u(y) + \beta E v^d(y') - u(y^T + B - q(B')B'(B, y), y^N) - \beta E v^o(B'(y, B), y')$$

$$\begin{aligned} \frac{\partial \Psi}{\partial y^T} &= u_{c^T}(y^T, y^N) - u_{c^T}(y^T + B - q(B')B'(B, y), y^N) + \\ &\quad \frac{\partial B'(B, y)}{\partial y^T} \left[\frac{\partial [q(B')B']}{\partial B'} u_{c^T}(y^T + B - q(B')B'(B, y), y^N) - \beta \frac{\partial [E v^o(B'(y, B), y')]}{\partial B'} \right] \end{aligned}$$

The term in brackets is exactly the first order condition of the government's problem and thus it is equal to zero. Thus,

$$\frac{\partial \Psi}{\partial y^T} = u_{c^T}(y^T, y^N) - u_{c^T}(y^T + B - q(B')B'(B, y), y^N)$$

The sign of the above derivative depends on whether $B - q(B')B'$ is greater or less than zero. In general, this can be positive or negative because of the insurance type use of debt. But for all $B \in [\underline{B}, \bar{B}]$, default sets are non-empty, and so $B - q(B')B' < 0$ due to Lemma 2.1

Thus for $B \in [\underline{B}, \bar{B}]$, $\frac{\partial \Psi(B, y)}{\partial y^T} < 0$.

Given that default sets are non-empty and strictly less than the entire endowment space for $B \in (\underline{B}, \overline{B}]$, then for some y default is preferable, and for some y repayment is preferable within this range. But given that $\Psi(B, y)$ is monotonically decreasing in y for all $B \in [\underline{B}, \overline{B}]$, then there exists a unique y^* such that for value $y \leq y^*(B)$ default is preferable, and for $y > y^*(B)$ repayment is preferable, where $v^c(B, y^*(B)) = v^d(y^*(B))$. And thus default sets can be characterized by closed intervals where only the upper bound depends on the level of debt.

Now using the implicit function theorem :

$$\frac{\partial y^T}{\partial B} = \frac{v_B^c(B, y)}{v_{y^T}^d(y) - v_{y^T}^c(B, y)} = \frac{v_B^c(B, y)}{u_{c^T}(y^T, y^N) - u_{c^T}(y^T + B - q(B')B'(B, y), y^N)} < 0.$$

which says that for $B \in [\underline{B}, \overline{B}]$, $y^*(B)$ is decreasing in B . \square

Corollary 4.1. *Default can be an equilibrium outcome of the model for all probability distributions over $[\underline{y}, \overline{y}]$ satisfying the property:*

$$\lim_{y \rightarrow \underline{y}} h(y) = 0$$

where $h(y)$ is the hazard function of the distribution.

Proof.

The condition that the slope of $q(B) \cdot B$ be positive stated in terms of the hazard function of the distribution:

$$\frac{\partial [q(B)B]}{\partial B} = [1 - F(y^*(B))] - f(y^*(B)) B \frac{\partial y^*}{\partial B} > 0$$

or

$$\frac{1}{h(y^*(B))} > B \frac{\partial y^*}{\partial B}$$

Note that

$$\lim_{B \rightarrow \overline{B}^-} y^*(B) = \underline{y}$$

due to Proposition 3. Thus for distributions satisfying the above condition:

$$\lim_{B \rightarrow \overline{B}^-} \frac{1}{h(y^*(B))} = \infty$$

The only thing we need to prove now is that the $\lim_{B \rightarrow \bar{B}^-} \frac{\partial y^*(B)}{\partial B}$ is finite

$$\lim_{B \rightarrow \bar{B}^-} \frac{\partial y^*(B)}{\partial B} = \lim_{B \rightarrow \bar{B}^-} \frac{v_B^c(B, y)}{u'(y^T, y^N)c_{c^T} - u'(y^T + B - q(B')B', y^N)c_{c^T}}$$

The numerator of the above expression is finite and positive for finite B . Note that the only y which we need to consider the limit, is \underline{y} . And specifically that

$$\lim_{B \rightarrow \bar{B}^-} B - q(B'(B))B'(B, \underline{y}) < 0.$$

which holds by continuity due to Lemma 2.1. \square

Proposition 5. *Default incentives are stronger for low nontradable endowments if the cross derivative of the utility function is negative. For all $y_1^N \leq y_2^N$, if $y_2^N \in D(B)$, then $y_1^N \in D(B)$ if $u_{c^T c^N} < 0$.*

Using the exact same strategy than the one used in proposition for proposition 2 the condition needed to prove the proposition simplifies to depending only on period utility. That is if :

$$[u(y^T + B - q(B^1)B^1, y_2^N)] - [u(y^T + B - q(B^1)B^1, y_1^N)] > [u(y^T, y_2^N)] - [u(y^T, y_1^N)] \quad (17)$$

then for all $y_1^N < y_2^N$, if $y_2^N \in D(B)$, then $y_1^N \in D(B)$

Rearranging:

$$[u(y^T, y_1^N)] - [u(y^T + B - q(B^1)B^1, y_1^N)] > [u(y^T, y_2^N)] - [u(y^T + B - q(B^1)B^1, y_2^N)] \quad (18)$$

where $B - q(B')B' < 0$ for $B \in [\underline{B}, \bar{B}]$.

Let:

$$\Psi(y, B) = [u(y^T, y^N)] - [u(y^T + B - q(B^1)B^1, y^N)]$$

$$\frac{\partial \Psi(y, B)}{\partial y^N} = \frac{\partial u(y^T, y^N)}{\partial y^N} - \frac{\partial u(y^T + B - q(B^1)B^1, y^N)}{\partial y^N}$$

If $\frac{\partial^2 u(c^T, c^N)}{\partial c^T \partial c^N} < 0$ then $\frac{\partial \Psi(y, B)}{\partial y^N} < 0$, which then makes equation (18) hold. \square