

# Financial Markets and Wages\*

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## Abstract

We study the optimal long-term wage contract between firms and workers in the presence of financial constraints. Firms that are financially constrained promise an increasing wage profile, that is, they pay lower wages today in exchange of higher future wages once they become unconstrained and operate at a larger scale. In equilibrium, constrained firms are on average smaller and pay lower wages. In this way the model generates a positive relation between firm size and wages. The model also captures other empirical regularities such as the lower wages paid by fast growing firms.

*JEL classification:* G31, J31, E24

*Key words:* Investment financing, long-term contracts, wages

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## 1 Introduction

The fact that large firms pay higher wages is a well-known stylized fact. Brown and Medoff (1989) and Oi and Idson (1999) provide a review of the empirical studies. In this paper we ask whether financial factors—in addition to other considerations proposed in the theoretical literature—can contribute to explaining the dependence of wages on the size of the employer.

Our interest in understanding the importance of financial factors for the firm size-wage relation is motivated by a set of regularities about the link between the financial characteristics of firms and their size. In general, the view that emerges from the financial literature is that smaller and younger firms face tighter financial constraints, either in the form of lower ability to raise funds or in the form of higher cost of funds. In spite of these regularities, the role played by financial market imperfections in generating the firm size-wage relation has not been studied in the theoretical literature.

We develop a model in which firms sign optimal long-term (implicit) contracts with workers as in Harris and Holmstrom (1982) and Holmstrom (1983). Due to limited enforceability, external investors are willing to finance the firm only in exchange of collateralized capital. If the funds supplied by external investors are limited—that is, the firm is financially constrained—the optimal wage contracts offered by the firm to the workers will be characterized by an increasing wage profile. By paying lower wages today, the firm is able to generate higher cash-flows in the current period which relax the tightness of the financial constraints. Because firms with tighter constraints operate at a sub-optimal scale—which then they gradually expand until they become unconstrained—small firms pay on average lower wages than large firms. Therefore, the model generates a positive relation between the size of the firm and the average wages it pays to workers (the firm size-wage relation). At the same time, because constrained firms grow in size, the model also captures the empirical regularity that fast growing firms pay lower wages.

There are two features in the model that explain why firms are able to *implicitly* borrow from workers beyond what they can borrow from external investors. First, if a worker quits, the firm loses some sunk investment. This could derive from recruiting costs or training expenses that enhance the job specific human capital of the worker. The firm's loss of valuable investment endows the worker with a punishment tool which is not available to external investors. Second, a worker provides effort in the working place only if he or she believes that the effort will be rewarded by the firm. But when the firm

renege its wage promises, worker's confidence is lost, and the worker prefers to quit, since he or she expects the firm to renege the wage promises also in future periods. These punishment mechanisms, in conjunction with the risk that a worker may quit in the event of repudiation, guarantee that the firm will never renege the long-term wage contract.

The plan of the paper is as follows. In the next section we review the main empirical and theoretical contributions to the study of the firm size-wage relation. Section 3 describes the basic theoretical framework and characterizes the firm's policy and dynamics. Section 4 extends the model to allow for firm's entrance and exit and workers' turnover and derives the labor market equilibrium. The properties of the model are then studied numerically in Section 5. Section 7 describes how the optimal long-term contract can be sustained as a sub-game perfect equilibrium of the strategic interaction between the firm and each individual worker. Section 8 discusses the robustness of some simplifying assumptions. Section 9 concludes.

## 2 Empirical regularities and existing theories

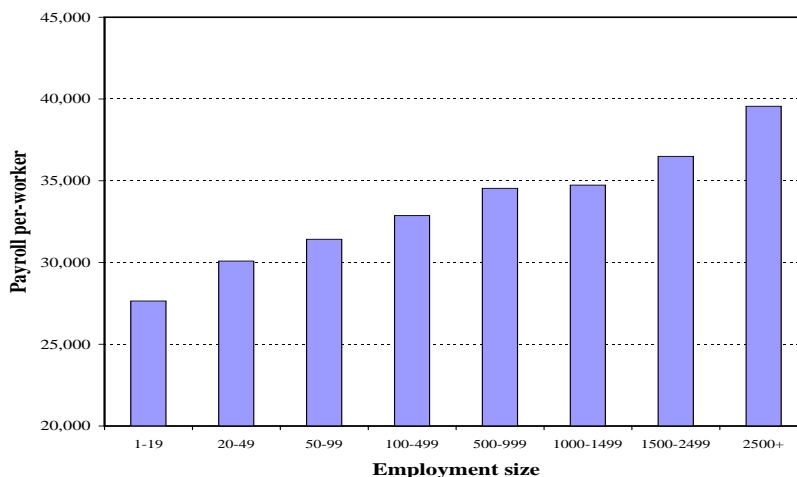
Before describing our theoretical framework, we briefly review the main empirical regularities and theoretical contributions to the study of the firm size-wage relation. The review of the theoretical literature shows that the effect of firm size on wages is still an unresolved puzzle while some of the empirical findings suggest that financial factors could play an important role.

### 2.1 Empirical regularities

Figure 1 plots the payroll per-worker for different size classes of firms, which is increasing in the size of firms. This is the typical pattern in almost all industries and is robust to the introduction of several controls for worker's and firm's characteristics. See Brown and Medoff (1989) and Oi and Idson (1999).

There are many factors that could generate the positive relation between firm size and wages. For instance, the fact that larger firms employ more skilled workers. However, using matched employer-employee data, recent studies have reached the conclusion that the effect of firm size on wages is mostly explained by variation in firms' characteristics rather than workers' characteristics. In particular Abowd and Kramarz (2000) report that both in France and in the US, variation in firms' characteristics explains about 70

Figure 1: Firm size and wages in 2001.



Source : U.S. Small Business Administration, Office of Advocacy

per cent of the firm size-wage differential. In addition to this result, there are other important findings in the empirical literature that are relevant for our paper. We summarize them below.

1. *Fast growing firms pay lower wages.* Bronars and Famulari (2001) report that employment growth has a negative effect on wages in a regression that controls for several workers' and firms' characteristics. See also Hanka (1998).
2. *Firms that are in financial distress have lower employment and pay lower wages.* Nickell and Wadhvani (1991) document the negative relation between debt and employment. Other studies provide some evidence that indicators of financial pressure are associated with lower wages. See Nickell and Nicolitsas (1999), Hanka (1998), Blanchflower, Oswald, and Garrett (1990).
3. *In some studies firm size is no longer significant after controlling for the financial conditions of the firm.* Hanka (1998) finds that size (as measured by total assets) ceases of being significant after controlling for productivity (ROA and assets per employee) and financial distress variables (debt over assets ratio).
4. *The link between firm age and wages is not clear-cut.* Doms, Dunne, and Troske (1997) find that the effect of firm age on wages is positive if we

do not control for worker's characteristics but it becomes negative (albeit not significant) if we control for worker's experience. The same pattern is documented by Troske (1999) and Brown and Medoff (2001).

5. *Indirect indicators point out that small firms tend to be more financially constrained.* Small firms pay fewer dividends and have higher value of Tobin's  $q$ . They rely more on bank financing and their growth is sensitive to cash flows. See for example Fazzari, Hubbard, and Petersen (1988), Gilchrist and Himmelberg (1996), Ross, Westerfield, and Jordan (1993) and Smith (1977). See also Kumar, Rajan, and Zingales (1999) for cross-countries evidence on how financial factors affect the size of firms.

These empirical results are important to evaluate our theoretical contribution to the explanation of the firm size-wage relation. Before presenting the theoretical model, however, we summarize the existing theoretical contributions and how they relate to the empirical findings.

## 2.2 Existing theories

There are several explanations in the theoretical literature for the firm size-wage relation but none of them are entirely satisfactory. This view is clearly stated in Troske (1999) who concludes: "*After testing several possible explanations we are still left with the question: why do large firms pay higher wages?*". Following is a brief description of the main theoretical contributions and their limits.

1. *Sorting of high quality workers in large firms.* If this was the basic mechanism, then the firm size-wage relation should become insignificant if we control for workers' quality. However, after controlling for several workers' characteristics, the effect of firm's characteristics remains large, see for example Brown and Medoff (1989) and Abowd and Kramarz (2000). The model studied in Kremer and Maskin (1996) emphasizes the complementarity that arises from matching high quality workers in the same firm. In this way, the effect of sorting on wages could possibly translate into a firm's fixed effect that any single worker's characteristic fails to capture. Yet, the inclusion of measures of average workers' quality into a standard wage regression does not reduce the size of the firm size-wage effect (see Bayard and Troske (1999)). Thus, sorting of high quality workers in large firms can explain only part of the size effect.

2. *Efficiency wages.* In an efficiency wage model a la Shapiro and Stiglitz (1984), large firms may pay higher wages because detecting shirking is more difficult. Some empirical evidence is not fully consistent with this explanation. For example, there are no differences in the magnitude of the firm size-wage effect between production and non-production workers (see Brown and Medoff (1989)) or supervisory and non supervisory workers (see Troske (1999)). Moreover, the magnitude of the effect does not change after conditioning on the number of workers receiving incentive pay (again, see Brown and Medoff (1989)).

3. *Wage bargaining.* In bargaining models, wages increase with the net surplus generated by the job and with the bargaining power of workers. This theory can explain why wages are positively related to the size of the firm only if either the bargaining power of workers or the value of the job increases with the firm's size. However, the inclusion of variables that proxy for the bargaining power of workers, such as union-density or union-coverage, or the inclusion of variables that proxy for the value of the job such as firm's profit, firm's capital or severance payments, do not eliminate the significance of the firm size-wage effect (see Brown and Medoff (1989)).

4. *Burdett and Mortensen's model.* In Burdett and Mortensen (1998) firms face a trade-off between paying high wages to attract and retain a large number of workers or paying low wages but with fewer workers hired and retained. In equilibrium there are firms that pay low wages and remain small and firms that pay high wages and become large. This model, however, does not seem to capture the fact that fast growing firms tend to pay lower wages. In fact, firms that grow faster are the ones that pay higher wages. It should be point out, however, that this is only a conjecture since the firm dynamics generated by this model has not been fully explored. Similar considerations apply to the model studied in Burdett and Coles (2003).

The goal of our paper is to provide an additional explanation for the firm size-wage relation in which financial markets frictions play a central role. The importance of financial factors for the firm size-wage relation has not been investigated in the literature, although Oi and Idson (1999) and Brown and Medoff (1989) hint a potential link. They conjecture that financial market imperfections can lead to a greater cost of capital for small firms, which induce them to choose lower capital intensity. In a model in which workers have some bargaining power over the surplus of the firm, this would imply lower wages paid by smaller (constrained) firms. This mechanism, however,

does not resolve the puzzle because the firm size effect remains significant even if we control for the capital intensity and the productivity of the firm. The financial mechanism proposed in our paper does not rely on the capital intensity of the firm and explain why firms of different size have different access to financial markets.

### 3 The basic model

We start describing a simple model in which firms face a deterministic problem and they live forever. This model allows us to emphasize some of the key features of the general model studied in Section 4.

Consider a risk-neutral infinitely lived entrepreneur with initial wealth  $a_0$  and with lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t$$

where  $\beta$  is the intertemporal discount factor and  $c_t$  is consumption.

The entrepreneur has the managerial skills to run an investment project that generates revenues  $y = A \cdot N$ . The variable  $N$  denotes the number of hired workers and  $A$  is a constant. The project is subject to the capacity constraint  $N \leq \bar{N}$ . In the general model studied in Section 4, the capacity constraint  $\bar{N}$  is allowed to differ across entrepreneurs or firms.

The employment of each worker requires two types of fixed investment: fungible investment  $\kappa_f$  and worker-specific investment  $\kappa_w$ . The first type of investment,  $\kappa_f$ , has an external value and can be resold at no cost. The second type,  $\kappa_w$ , represents the cost incurred by the firm for recruiting and training a new worker for the specific job, and it is lost if the worker quits or is fired. We will denote by  $\kappa = \kappa_f + \kappa_w$  the sum of the two components. The total capital accumulated at the end of time  $t$  by a firm created at time zero is  $\kappa \sum_{\tau=0}^t n_{\tau}$ , where  $n_{\tau}$  is the number of workers hired at time  $\tau$  (who start producing at time  $\tau + 1$ ). The output produced by the firm at  $t + 1$  is  $A \sum_{\tau=0}^t n_{\tau}$ .

Workers are infinitely lived with lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \ell_t], \quad U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

where  $\beta$  is the discount factor,  $\sigma$  is the coefficient of relative risk aversion,  $c_t$  is consumption, and  $\ell_t \in \{0, \bar{\ell}\}$  denotes the utility of leisure which is forgone

when the worker provides working effort. The assumption that there is some forgone utility is relevant only for the analysis of renegotiation studied in Section 7. In equilibrium, workers always provide effort. Therefore, in the analysis that precedes Section 7 we simply impose  $\ell_t = 0$ .

Workers do not have any assets and can not borrow by pledging their future labor income. For the moment we also assume that workers cannot save, and therefore, consumption is simply equal to their wages. In Section 8 we will discuss the conditions under which workers would not save even if they were allowed to.

Funds are provided by *investors* who are risk-neutral and discount future payments at rate  $r$ . The supply of funds of each investor is infinitesimal, but the aggregate number of investors is large enough to guarantee that the aggregate supply of funds is perfectly elastic at rate  $r$ . This implies that financial markets are perfectly competitive and the equilibrium interest rate is  $r$ . We assume that  $1/(1+r) \geq \beta$ . This guarantees that internal financing does not dominate external financing.<sup>1</sup>

The capital investment  $\kappa$  necessary to employ a worker is what creates the financial need. Using the renegotiation idea of Hart and Moore (1994) and Kiyotaki and Moore (1997), we assume that the entrepreneur can borrow only the amount that can be collateralized. In case of liquidation, investors can seize only the fungible capital. Therefore,  $\kappa_f$  is the only capital that can be used as a collateral. Since the collateral must also guarantee the interests on the loan, the firm can borrow at most  $\bar{\kappa}_f = \kappa_f/(1+r)$ , per each worker. The borrowing limit, then, can be written as  $b_t \leq \bar{\kappa}_f \sum_{\tau=0}^t n_\tau$ , where  $b_t$  denotes the debt level contracted at time  $t$ .

When a new worker is hired, the firm signs a long-term contract that specifies the whole sequence of wages. By assuming that the labor market is *competitive*, the initial promised utility provided by the contract to the worker is equal to the utility that the worker would earn by re-entering the labor market (reservation value). This value, denoted by  $q_{res}$ , is exogenous in this simple version of the model. For the moment we assume that the firm cannot renegotiate the wage contract in future periods. In Section 7 we will characterize the conditions under which the firm never reneges on its promises and the contract can be supported as a sub-game perfect equilibrium of the

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<sup>1</sup>The difference between the market discount factor,  $1/(1+r)$ , and  $\beta$  is intended to capture the fact that the securities placed in hands of (a large number of) investors tend to have greater liquidity and integrate into better diversified portfolios than those privately held by the entrepreneur.



repeated game played by the firm with each individual worker.

### 3.1 The firm's problem

If the initial wealth of the entrepreneur  $a_0$  is small, the firm starts with an initial employment that is smaller than  $\bar{N}$ . As the firm retains its earnings and accumulate internal wealth, the firm hires more workers and eventually it reaches the optimal scale  $\bar{N}$ .

Let  $\{w_{t,t+j}\}_{j=1}^{\infty}$  be the sequence of wages that the firm promises to the workers hired at time  $t$ . Here  $w_{t,t+j}$  denotes the wage paid at time  $t+j$  to workers hired at time  $t$ . Then the total wage payments at time  $t+1$  are  $\sum_{\tau=0}^t n_{\tau} w_{\tau,t+1}$ . Let  $a_t$  denote the *net worth* at the end of period  $t$ —that is, after production and after the payment of wages and interests. The sum of the firm's net worth,  $a_t$ , and debt financing,  $b_t$ , equals the sum of firm's capital,  $\kappa \sum_{\tau=0}^t n_{\tau}$ , and dividend payments,  $d_t$ . Thus,  $d_t = a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau}$ .

Given the initial assets  $a_0$ , the firm maximizes the discounted value of the entrepreneur's consumption, which always equals dividends since the entrepreneur is at least as impatient as the market,  $\beta \leq 1/(1+r)$ . Thus, at time zero, the firm chooses the whole sequence of debt, employment and wages, that is  $\{b_t, n_t, \{w_{t,t+j}\}_{j=1}^{\infty}\}_{t=0}^{\infty}$ , to solve the problem:

$$V(a_0) = \max \sum_{t=0}^{\infty} \beta^t \left( a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \right) \quad (1)$$

subject to

$$a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \geq 0, \quad (2)$$

$$b_t \leq \bar{\kappa}_f \sum_{\tau=0}^t n_{\tau}, \quad (3)$$

$$\sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) \geq q_{res}, \quad (4)$$

$$a_{t+1} = (\kappa + A) \sum_{\tau=0}^t n_{\tau} - \sum_{\tau=0}^t n_{\tau} w_{\tau,t+1} - (1+r)b_t, \quad (5)$$

which all have to hold for any  $t \geq 0$ . Constraint (2) imposes the non-negativity of dividends. This results from the limited liability of the en-

trepreneur together with the non-negativity of consumption. Constraint (3) imposes the borrowing limit and (4) is the worker's participation constraint. This imposes that the sequence of wages offered to each cohort of new recruits cannot be smaller than their reservation value  $q_{res}$ .<sup>2</sup> Finally, constraint (5) defines the law of motion for the end-of-period net worth.

Let  $\gamma_t$  and  $\lambda_t n_t$  be the lagrange multipliers associated with the constraints (2) and (4), respectively. Then Appendix A shows that the first order conditions imply that

$$\lambda_\tau U_c(w_{\tau,t}) = 1 + \gamma_t, \quad (6)$$

where  $U_c$  denotes the marginal utility of consumption. The variable  $\lambda_\tau$  is the marginal cost to the firm of providing one unit of utility to a worker hired at time  $\tau$ . Thus the term  $\lambda_\tau U_c(w_{\tau,t})$  represents the marginal cost of reducing wages. The term  $1 + \gamma_t$  is the value of one additional unit of internal funds. Therefore, equation (6) says that the optimal wage policy of the firm is such that the marginal cost of reducing wages is equal to the marginal value of internal funds. In other words the firm borrows from a worker until the cost of borrowing from him is equal to the marginal value of internal funds.

The multiplier  $\gamma_t$  captures the tightness of financial constraints and depends on the firm's net worth  $a_t$ . If  $a_t$  is small, the financial needs of the firm are high which imply that the value of an extra unit of internal funds is large. As the firm retains earnings, its assets increase over time and the variable  $\gamma_t$  converges to zero. Then, equation (6) implies that:

**Property 1** *The wage received by each worker grows over time until the firm becomes unconstrained, that is,  $\gamma_t = 0$ .*

Equation (6) also implies that the ratio of marginal utilities between workers of different cohorts remain constant over time. Indeed, if we evaluate (6) for two different cohorts indexed by  $\tau_1$  and  $\tau_2$ , and we divide side by side we obtain

$$\frac{U_c(w_{\tau_1,t})}{U_c(w_{\tau_2,t})} = \frac{\lambda_{\tau_2}}{\lambda_{\tau_1}}.$$

Since the right-hand-side does not depend on  $t$ , this condition implies that:

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<sup>2</sup>Since the worker could always quit, the participation constraint should be imposed not only when the worker is hired, but also in all future periods. However, as shown below, wages never decrease. Therefore, if the participation constraint is satisfied when the worker is hired, it will also be satisfied at any future date.

**Property 2** *The ratios of marginal utilities between workers of different cohorts remain constant over time.*

In the next section we take advantage of this property to rewrite the problem recursively with a limited number of state variables. Rewriting the problem recursively is convenient for solving the model, especially when in the next section we will study a more general version of the model.

### 3.2 Recursive formulation of the firm's problem

Let  $q_{\tau,t} = E \sum_{j=1}^{\infty} \beta^j U(w_{\tau,t+j})$  be the expected lifetime utility promised at the end of time  $t$  to a worker hired at time  $\tau$ , with  $\tau \leq t$ . Notice that  $q_{\tau,t}$  follows the recursive form

$$q_{\tau,t} = \beta [U(w_{\tau,t+1}) + q_{\tau,t+1}] \quad (7)$$

with  $q_{\tau,\tau} = q_{res}$ .

With the utility function  $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ , Property 2 implies that the ratios of wages paid to workers of different cohorts remain constant over time. This property also implies that the ratios of lifetime utilities promised to different cohorts of workers remain constant over time. Thus, if we consider the last and the first cohort of workers hired by the firm, we have that, at any given point in time, their relative lifetime utilities and wages are linked by

$$\frac{q_{t,t}}{q_{0,t}} = \left( \frac{w_{t,t+1}}{w_{0,t+1}} \right)^{1-\sigma} = \frac{q_{res}}{q_{0,t}},$$

where the last equality uses the fact that  $q_{t,t} = q_{res}$ . Inverting the second equality provides an expression for the wage ratio between the cohort hired at time  $t$  and the cohort hired at time zero, which reads as

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left( \frac{q_{res}}{q_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(q_{0,t}).$$

From now on we omit the zero subscript to identify the first cohort of workers. Therefore,  $w_t$  and  $q_t$  denote the time- $t$  wage and promised utility of the first cohort of workers. The total wage payments paid by the firm at time  $t$  can be written as  $H_t w_t$ , where

$$H_t = \sum_{\tau=0}^{t-1} \psi(q_{\tau}) n_{\tau},$$

which evolves recursively as

$$H_{t+1} = H_t + \psi(q_t)n_t. \quad (8)$$

Once we know  $H_t$  and the utility promised to the first cohort of workers,  $q_t$ , the determination of the whole wage structure paid by the firm at time  $t + 1$  only requires the determination of the wage for the first cohort of workers, that is  $w_{t+1}$ . This allows us to write the firm's problem recursively with a limited number of state variables. Specifically, the firm's problem can be written as:

$$V(a, q, N, H) = \max_{b, w', q', N' \leq \bar{N}} \left\{ a + b - \kappa N' + \beta V(a', q', N', H') \right\} \quad (9)$$

subject to

$$a + b - \kappa N' \geq 0, \quad (10)$$

$$b \leq \bar{\kappa}_f N', \quad (11)$$

$$q = \beta [U(w') + q'], \quad (12)$$

$$a' = \kappa N' + AN' - H'w' - (1 + r)b, \quad (13)$$

$$H' = H + \psi(q)(N' - N), \quad (14)$$

where  $N$  denotes the current employment of the firm and the prime denotes the next period value. Thus  $N' - N$  is the change in employment, that is, the number of workers hired in the current period (who start producing in the next period). Equation (9) is the Bellman equation. Constraints (10) and (11) impose the non negative constraint on dividends and the borrowing limit, respectively. Equation (12) is the promise-keeping constraint for the first cohort of workers hired. Finally, equations (13) and (14) characterize the law of motion of the state variables  $a$  and  $H$ , respectively.

Let  $\gamma$  and  $\lambda H'$  denote the lagrange multipliers associated with constraints (10) and (12), respectively. Then Appendix B shows that the first order conditions of the above problem imply that

$$\lambda U_{w'} = 1 + \gamma', \quad (15)$$

$$\lambda = \lambda'. \quad (16)$$

The first condition is analogous to (6) while the second simply says that the lagrange multiplier for the worker's participation constraint is constant over time.

These two conditions characterize the wage dynamics of the firm. As observed in the previous section, the lagrange multiplier  $\gamma$  decreases over time until it becomes zero. From equation (15) we can see that the wage paid to the first cohort of workers increases over time until  $\gamma' = 0$ . Because the wages paid to all other cohorts of workers are proportional to the wage paid to the first cohort, we also have that the average wages increase over time until  $\gamma' = 0$ . Wages differ across workers of different cohorts. In fact, because all workers start with  $q = q_{res}$ , after which the promised utility grows over time, older workers receive higher wages than younger workers. One of the predictions of the model is that the wage profile of constrained (young) firms is steeper than the wage profile of mature (old) firms.

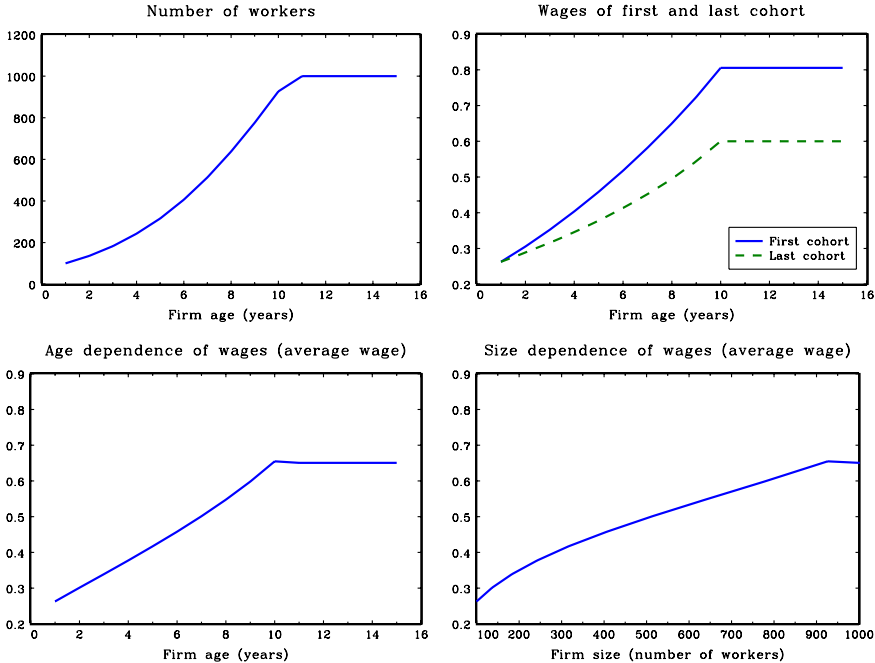
Once the firm becomes unconstrained, that is,  $\gamma = 0$ , the firm would like to increase employment beyond  $\bar{N}$ , but the capacity constraint binds.

Figure 2 shows some of the properties of the model with a numerical example based on the following parameter values:  $r = 0.03$ ,  $\beta = 0.934$ ,  $\sigma = 1$ ,  $q_{res} = U(0.6)/(1 - \beta)$ ,  $\bar{N} = 1,000$ ,  $A = 1$ ,  $\kappa = 2.8$ ,  $\kappa_f/\kappa = 0.3$  and  $a_0$  is such that the initial size of the firm is 10 percent the maximum scale. This is obtained by setting  $a_0 = 196$ . The numerical example considered here is provided only for illustrative purposes. A formal calibration exercise will be conducted in Section 5 after the specification of the general model.

The first panel of Figure 2 plots the employment dynamics. The firm starts with an initial employment of 100 workers and then gradually grows over time until it reaches the optimal size  $\bar{N} = 1,000$ . The transition takes place in 11 periods. The second panel plots the wage profile of the first cohort of workers (those hired at time 0) and the initial wage paid to newly hired workers. The wage profile of the first cohort of workers (continuous line) is increasing until the firm reaches the unconstrained status. The dashed line shows the wage earned in the first period of employment by workers of different cohorts. As the firm gets closer to the optimal scale, it offers higher initial wages, and therefore, the wage profile of newer workers is less steep overall.

The third panel plots the average wage paid by the firm as a function of its age and the fourth panel the average wage as a function of its size (measured by the number of employees). The average wage increases with the size and age of the firm. This is a direct consequence of the fact that, when the firm is young and constrained, it operates at a suboptimal scale and offers an increasing profile of wages.

Figure 2: Employment dynamics and wage patterns over age and size.



In this simple model the profile of wages is fully captured by the age of the firm. In other words, once we control for age, the size of the firm is irrelevant because there is a one-to-one mapping between size and age. However, in a cross section of firms, size will have an independent effect. This is because firms may have different capacities  $\bar{N}$  and they can start with different initial assets  $a_0$ . In order to capture the firm size effect on wages in a cross-section of firms, we need to specify the whole industry structure, including entrance and exit. Let's turn then to the specification of the general model.

#### 4 General model and labor market equilibrium

We now extend the model along several dimensions: we allow for (a) firm heterogeneity in technology  $\bar{N}$  and initial wealth  $a_0$ ; (b) firm exit and entry; (c) turnover of workers at the firm level. The first extension allows us to generate a size distribution of firms close to the data. The second guarantees that at each point in time there is a fraction of firms that are financially

constrained. The third is introduced for robustness.

We assume that there is a probability  $1 - p$  that an investment project becomes obsolete and the firm exits. Exiting firms are replaced by new entrant firms managed by new entrepreneurs. New entrepreneurs draw the project capacity  $\bar{N}$  from the distribution  $\Gamma(\bar{N})$ . The mass of workers is  $L$  while the mass of firms (entrepreneurs) is normalized to 1.

The initial wealth of new entrepreneurs could be correlated with the project capacity. For instance, entrepreneurs with more promising projects may be able to raise more funds initially by pooling a larger number of founders. Alternatively, we can think that the probability of drawing large capacity projects increases with the ability of the entrepreneur, which in turn may be related to his initial wealth. To formalize this idea in a simple manner, we assume that there is a unique relation between the project capacity  $\bar{N}$  and the initial wealth of the entrepreneur, taking the simple form  $a_0 = \alpha \cdot \bar{N}^\rho$ . The parameters  $\alpha$  and  $\rho$  determine the degree of financial tightness for new firms, as a function of projects capacity. Given the linearity of the production function and the borrowing limit, the financial tightness of a new firm is captured by the ratio:

$$FTI \equiv \frac{(\kappa - \bar{\kappa}_f) \cdot \bar{N}}{a_0} = \frac{(\kappa - \bar{\kappa}_f) \cdot \bar{N}^{1-\rho}}{\alpha}.$$

where  $FTI$  stands for Financial Tightness Index. The numerator is the total capital that must be financed internally when the firm operates at the optimal scale  $\bar{N}$ . The denominator is the value of initial net worth. When this ratio is greater than 1 the firm is financially constrained. Lower values of  $\alpha$  increases the financial tightness for all new firms while the parameter  $\rho$  differentiates the tightness across different types of firms. When  $\rho = 1$ , the tightness is independent of the firm's capacity. When  $\rho < 1$ , firms with larger capacity face tighter constraints.

The last assumption is that workers may die with some probability  $1 - \eta$ . This feature implies that firms loose some workers at any point in time and there will be workers' turnover. To keep the model tractable we assume that  $1 - \eta$  is also the fraction of workers that the firm loses in every period, as if the firm employs a continuum of workers. Of course, this is a simplification but it is convenient to keep the firm problem tractable. With this assumption, in fact, the only source of uncertainty for the firm is the event in which the technology becomes obsolete and the firm exits—that represents an absorbing state for the dynamics of the firm.

## 4.1 Recursive problem

Given the initial assets  $a_0$  and the project capacity  $\bar{N}$ , the problem solved by an active entrepreneur is similar to the problem studied in the previous section although now we have to specify what happens to the wage contracts when the project becomes obsolete.

When the investment project becomes obsolete, all workers lose their jobs and any claim toward the current employer. By re-entering the labor market, they will get the reservation utility  $q_{res}$ . In the implementation analysis conducted in Section 7 we show that this is the only equilibrium outcome of the strategic interaction between the worker and the firm when the project becomes obsolete.<sup>3</sup> The promise-keeping constraint can then be written as:

$$q_{\tau,t} = \beta \left[ U(w_{\tau,t+1}) + \eta \cdot p \cdot q_{\tau,t+1} + \eta \cdot (1-p) \cdot q_{res} \right]$$

Here the assumption is that the survival of the worker and the viability of the project is observed after paying the current wage (but before the new investment). Consequently, the current wage is not renegotiated.<sup>4</sup>

For the analysis that follows it will be convenient to rescale the promised utility  $q_{\tau,t}$  by the constant term  $\eta\beta(1-p)q_{res}/(1-\eta p\beta)$  so as to define

$$z_{\tau,t} = q_{\tau,t} - \frac{\eta\beta(1-p)q_{res}}{1-\eta p\beta}$$

Using this rescaled variable, the promise-keeping constraint becomes:

$$z_{\tau,t} = \beta \left[ U(w_{\tau,t+1}) + p z_{\tau,t+1} \right]. \quad (17)$$

Since the ratios of marginal utilities between different cohorts of workers is constant over time (i.e. Property 2 remains valid), the advantage of using

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<sup>3</sup>In principle the entrepreneur could promise extra future payments to the worker if the firm is liquidated. However, the promises of these payments are not credible. Indeed, when the technology becomes obsolete, any sunk investment is inevitably lost and there is no cost for the firm from renegotiating the contract. Consequently, the worker's continuation utility becomes  $q_{res}$ .

<sup>4</sup>Notice that a worker who is employed by a firm that becomes unviable at time  $t$  receives his wage from the current employer and the training from the new one. In equilibrium the worker never experiences any spell of unemployment.



$z$ , rather than  $q$ , is that the wage ratio between a new worker and the first cohort of workers satisfies

$$\frac{w_{t,t+1}}{w_{0,t+1}} = \left( \frac{z_{t,t}}{z_{0,t}} \right)^{\frac{1}{1-\sigma}} = \psi(z_t)$$

which identifies the constant relative wage earned by the workers hired at time  $t$ . Notice that we maintained the convention to omit the zero subscript to identify the first cohort of workers.

The law of motion for the state variable  $H$  defined in the previous section becomes

$$H' = \eta H + \psi(z)(N' - \eta N) \quad (18)$$

where  $N' - \eta N$  is the number of workers hired in the current period.

Since only a fraction  $\eta$  of workers remain in the firm from one period to the next, the law of motion for the next period value of the firm's asset is:

$$a' = \kappa N' + AN' - H'w' - (1+r)b - \kappa_w(1-\eta)N', \quad (19)$$

where the last term accounts for the fact that a fraction  $(1-\eta)$  of workers exit the firm with consequent loss of worker's specific human capital.

The recursive representation is similar to that of section 3.2, once we use  $z$  as a state variable in place of  $q$  and we use the law of motion (17) for  $z_{0,t} \equiv z_t$ , and (18) and (19) to characterize the evolution of  $H$  and  $a$ , respectively. The full description of the firm's problem and the derivation of the first order conditions are in Appendix C. We are now able to define a steady state labor market equilibrium.

**Definition 1** *A steady state labor market equilibrium is defined by: (i) A distribution (measure) of firms  $M(a, z, N, H, \bar{N})$ ; (ii) A reservation utility  $q_{res}$ ; (iii) A transition function for the distribution of firms. Such that: (a) The transition function is consistent with the firm policies, the probability distribution of initial capacities,  $\Gamma(\bar{N})$  and the initial distribution of wealth  $a_0 = \alpha \bar{N}^p$ ; (b) The demand of labor  $\int N \cdot dM(a, z, N, H, \bar{N})$  equals the fixed supply of workers  $L$ ; (c) The next period distribution generated by the transition function is equal to the current distribution.*

Notice that, although the reservation value  $q_{res}$  is endogenously derived to clear the labor market, the interest rate  $r$  is exogenous in the model (and equal to the subjective discount rate of investors).

## 5 Numerical analysis and simulated regression

We first parameterize the model at the yearly frequency and then report the results from running wage regressions similar to those considered in the empirical literature.

**Parametrization** The interest rate on secured debt is set to  $r = 0.03$  and the intertemporal discount factor to  $\beta = 0.934$ . This implies a discount rate for the entrepreneurs equal to  $1/\beta - 1 \approx 0.07$ , which is close to the post-war stock market return in the U.S. economy. The risk-aversion parameter is set to  $\sigma = 1$  (log-utility). We will conduct a sensitivity analysis with respect to this parameter. The per-worker investment  $\kappa$  is chosen to have a capital-output ratio of 2.8. With the normalization  $A = 1$ , this requires  $\kappa = 2.8$ . The non-sunk fraction of capital  $\kappa_f/\kappa$  determines the leverage of the firm. We set  $\kappa_f/\kappa = 0.3$  which is consistent with the average leverage of Compustat companies. The probability of firms' death is set to  $1-p = 0.0286$ . This is the aggregate employment losses due to the death of firms observed in the 2001 data for the U.S. economy (see the footnote to Table 1 for the data source). The survival probability of workers is set to  $\eta = 0.9778$ . This corresponds to a working life duration of about 45 years, which is consistent with the calibration of explicit life-cycle models such as Auerbach and Kotlikoff (1987) and Rios-Rull (1996).<sup>5</sup>

We assume that the employment capacity  $\bar{N}$  can take eight values. These values and the corresponding probabilities  $\Gamma(\bar{N})$  are determined jointly with the parameters  $\alpha$  and  $\rho$  in the function  $a_0 = \alpha \cdot \bar{N}^\rho$  characterizing the initial assets of new firms. We choose these parameters to replicate, as close as possible, the size distributions of incumbent firms and newly created firms observed in the U.S. economy in 2001 and to yield a capital income share of 40 percent. The U.S. distribution of new and incumbents firms, released by the Small Business Administration, is reported in Table 1.

We use a simulated method of moments to pin down these parameters. More specifically, we minimize the square errors between the moments of the distribution we try to match (Table 1 plus the capital income share) and the moments generated by the model.<sup>6</sup> Table 2 reports the estimated dis-

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<sup>5</sup>The typical assumption is that agents start working at the age of 20 years old and retire at age 65.

<sup>6</sup>The size distribution reported in Table 1 gives us 20 independent moments. With the addition of the capital income share we have 21 moments to match but only 17 parameters:

Table 1: Size distribution of firms in the U.S. economy, 2001.

<i>Firm size (Employees)</i>	<i>Firms</i>	<i>Employees</i>	<i>Employees Firms</i>
<b>New firms</b>			
1-19	95.37%	53.28%	3.3
20-499	4.58%	37.66%	48.0
500+	0.05%	9.06%	1,022.7
<i>Total</i>	100.00%	100.00%	5.8
<b>All firms</b>			
1-19	87.46%	17.90%	4.7
20-49	7.94%	10.27%	30.0
50-99	2.53%	7.43%	68.4
100-499	1.72%	14.26%	192.4
500-999	0.17%	5.13%	689.0
1,000-1,499	0.06%	3.02%	1,217.4
1,500-2,499	0.05%	3.84%	1,915.8
2,500+	0.07%	38.13%	12,074.1
<i>Total</i>	100.00%	100.00%	23.2

*Source:* Small Business Administration, Office of Advocacy, data from U.S. Census Bureau, Statistics of U.S. Businesses. <http://www.sba.gov/advo/stats/data.html>.

tribution of new projects and their initial financial tightness. The estimated parameters imply that firms with larger projects face higher initial tightness. This is a consequence of the fact that the distribution of new firms shown in Table 1 is much more concentrated toward small firms than the distribution of incumbent firms. The values of the other two parameters are  $\alpha = 1.860$  and  $\rho = 0.716$ .

**Simulated regression** From the steady state distribution, we extract a random sample of firms and estimate the following regression:

$$\ln(\text{Wage}_{i,j}) = \bar{\alpha} + \alpha_T \cdot \text{WorkerTenure}_{i,j} + \alpha_{T^2} \cdot \text{WorkerTenure}_{i,j}^2 + \alpha_A \cdot \text{FirmAge}_j + \alpha_S \cdot \ln(\text{FirmSize}_j) + \alpha_G \cdot \text{FirmGrowth}_j$$

where the index  $i$  identifies the worker and  $j$  the firm where the worker is employed. This specification is similar to the one used in the empirical literature although we include a smaller set of control variables consistent

eight values of  $\bar{N}$ , seven probabilities  $\Gamma(\bar{N})$ , plus  $\alpha$  and  $\rho$ . Once we have the values of these parameters we also have the labor supply. The implied value is  $L = 27.2$ .

Table 2: Distribution of new projects and financial tightness.

$\bar{N}$	$\Gamma(\bar{N})$	$FTI$
5.9	0.81887	1.74
31.7	0.11367	2.81
53.0	0.03634	3.26
189.8	0.02671	4.68
602.0	0.00237	6.50
1,148.5	0.00074	7.81
1,866.4	0.00058	8.96
17,875.6	0.00071	17.04

with the structure of our model. The estimation results are reported in Table 3 with  $t$ -statistics in parenthesis.

The first column reports the coefficient estimates when all variables are included in the regression. All the estimates are statistically significant. Of special interest are the coefficients of firm's size and growth. The estimates for these two parameters are consistent with the findings of the empirical literature. In particular, while the size of the firm has a positive effect on wages, firm's grow has a negative one. We discuss in details each of the coefficient estimates below.

*The firm size effect:* Firms that are large in the current period are those that experienced tight financial constraints in the past. Therefore, they were operating at suboptimal (smaller) scales. In order to accelerate their grow, these firms paid low wages in the past in exchange of higher future wages. Now that they are unconstrained and larger, they pay higher wages in fulfillment of their previous promises. This generates a positive correlation between firm's size and wages. In quantitative terms the effect of the firm's size is important and comparable to those found in the empirical literature. Brown and Medoff (1989) survey the empirical literature on the employer size effect and report a coefficient of log-firm-size that ranges from 0.01 to 0.03 (see their Table 1). Similar results are reported by Bronars and Famulari (2001). The findings of Bronars and Famulari are particularly relevant for us since, as in our simulated regression, they study the effects on wages of firm size after controlling for firm growth. If we compare firms that are in the size class 1-19 (whose average size is 4.7) with firms that employ more than 2,500 employees (whose average size is 12,074), then our estimates imply that the

Table 3: Wage equation estimation from model-generated data.

<i>Description</i>	(1)	(2)	(3)	(4)
Constant	-0.5583 (-174.7)	-0.5771 (-165.5)	-0.5159 (-181.5)	-0.5360 (-217.0)
Worker tenure	0.0068 (30.2)	0.0031 (13.5)	- -	- -
Worker tenure <sup>2</sup> /1,000	-0.0343 (-10.9)	-0.0330 (-9.6)	- -	- -
Firm age	-0.0031 (-45.9)	- -	-0.0006 (-13.9)	- -
Firm log-size	0.0105 (31.8)	0.0073 (20.7)	0.0084 (23.5)	0.0077 (21.4)
Firm growth	-0.6720 (-43.7)	-0.5382 (-32.4)	-0.7788 (-49.7)	-0.6869 (-47.9)
<i>R-square</i>	0.372	0.239	0.231	0.216
<i>Observations</i>	10,005	10,005	10,005	10,005

*Notes: t-statistics in parenthesis.*

average wage paid by the second group of firms is about 8 percent higher than the wage paid by the first group of firms.

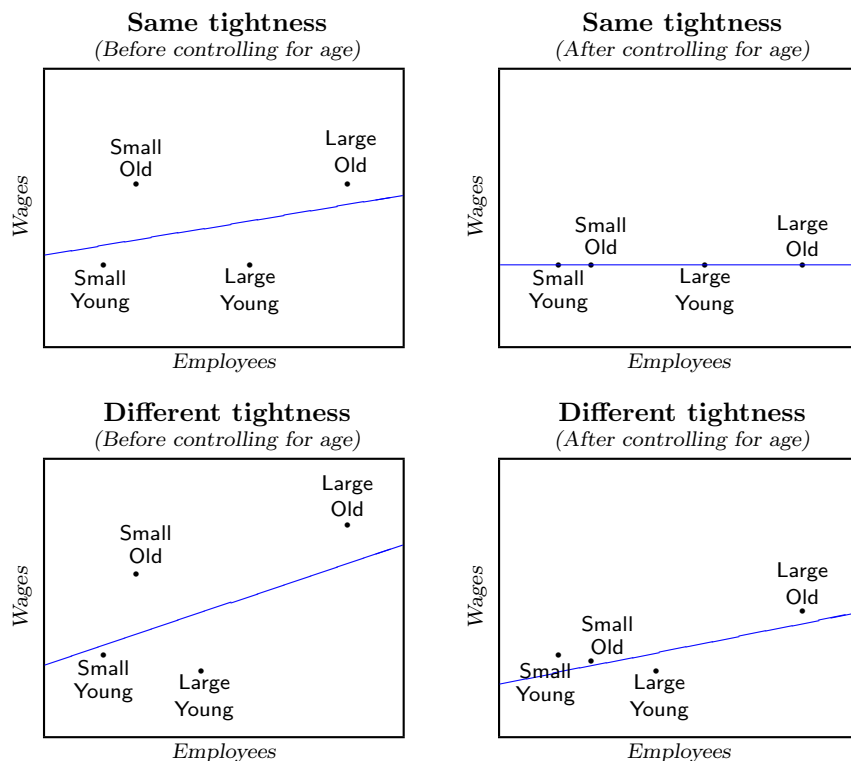
It is important to emphasize that the key factor to generate a firm size-wage effect is not simply the presence of financial constraints but the fact that these constraints are tighter for high capacity firms. Our estimated value of  $\rho$  is 0.716, which implies that the financial tightness of new firms with the largest  $\bar{N}$  is almost 10 times the tightness of firms with the smallest  $\bar{N}$  (see Table 2). If instead all new firms faced the same financial tightness, that is,  $\rho$  was equal to one, then the differences in wages would be fully captured by the age of the firm. Indeed if we constrain  $\rho$  to be one and we control for firm age, then the coefficient estimate for the firm size becomes insignificant.<sup>7</sup>

To further illustrate the intuition behind this result, consider the following example. Suppose that there are only two types of firms: low capacity and high capacity firms and refer to the first type of firms as “Small” and to the second type as “Large”. Suppose that firms live for two periods. When

<sup>7</sup>On the other hand, the sign and significance of this coefficient is not affected by  $\alpha$ . The parameter  $\alpha$  affects the growth effect but not the size effect.

young they are financially constrained. When old they are unconstrained and operate at the optimal scale. This implies that young firms pay lower wages and operate at a smaller scale. Figure 3 plots the wages and size for these two types of firms, when young and when old. The top panels are for the case in which all firms face the same financial tightness when young, that is,  $\rho = 1$ . The bottom panels are for the case in which high capacity firms face tighter constraints when young, that is,  $\rho < 1$ .

Figure 3: Financial tightness and firm size-wage relation.



When  $\rho$  is equal to one (top panels), the differential in wages between young and old firms is the same for small and large firms. Therefore, a dummy variable that differentiates young firms from old firms would be sufficient to account for the wage differential. In other words, after conditioning by age, there is no relation between firm size and wages. This is shown in the right-hand-side panel of Figure 3. When instead  $\rho$  is less than one, the wage profile is steeper in large firms than in small firms (see the bottom panels of Figure

3). In this case, an age dummy is unable to fully capture the wage differential and there still remains a positive correlation between firm size and wages, even after controlling for the age of the firm.

*The firm growth effect:* The second important result is the negative effect of firm growth on wages. The intuition for this result arises naturally from the discussion above: firms that grow are firms with binding financing constraints. Because of these constraints, growing firms pay lower wages today in exchange of higher future wages when they are unconstrained and operate at their optimal scale. Quantitatively, the coefficient is not very different from those estimated in the empirical literature. Bronars and Famulari (2001) report a coefficient of firm growth that ranges from -0.4 to -0.35 (see their Table 4).

*Tenure and firm age:* The other two variables included in the regression is the worker's tenure and the age of the firm. The positive effect of the worker's tenure derives from the fact that the wages paid by constrained firms increase over time, and therefore, with the tenure of workers. The return to tenure is smaller than the one estimated by Topel (1991), but comparable to the effect estimated by Altonji and Shakotko (1987) where, however, the coefficient is not statistically significant. The estimated coefficient for firm's age is negative. However, the magnitude of this coefficient depends on the variables we include in the regression. For instance, if we exclude worker's tenure, the coefficient of firm's age decreases significantly and it would turn out to be positive if we also excluded firm size from the regressors. In brief the unconditional correlation between wage and firm age is positive while it becomes negative after controlling for some workers and firms characteristics. The fact that the relation between firm age and wages depends on the variables included in the regression is consistent with the results of the empirical literature for which the effect of age is not clear cut (see Section 2.1).

**Sensitivity analysis:** Table 4 reports the estimates for alternative values of the coefficient of risk aversion  $\sigma$ . When  $\sigma = 0.5$  (low concavity), the firm-size wage effect increases more than 20 percent. In this case, the wages of firms with more than 2,500+ employees are about 10 percent higher than the wages paid by firms in the first size class 1-19. This derives from the fact that the cost of offering an increasing wage profile is smaller when the intertemporal elasticity of substitution for workers is high. Consequently, firms offer a steeper wage profile and the firm size-wage effect and the firm

growth-wage effect are stronger. The opposite is true when  $\sigma = 2.0$ . In the limit case in which  $\sigma = \infty$ , all firms would pay a constant wage and the model would not generate any wage differential.<sup>8</sup>

Table 4: Sensitivity analysis

<i>Description</i>	(1) $\sigma = 0.5$	(2) $\sigma = 1.0$	(3) $\sigma = 2.0$
Constant	-0.5548 (-120.6)	-0.5583 (-174.7)	-0.5488 (-301.8)
Worker tenure	0.0071 (21.8)	0.0068 (30.2)	0.0043 (33.5)
Worker tenure <sup>2</sup> /1,000	-0.0257 (-5.7)	-0.0343 (-10.9)	-0.0239 (-13.2)
Firm age	-0.0039 (-40.0)	-0.0031 (-45.9)	-0.0018 (-46.6)
Firm log-size	0.0130 (27.3)	0.0105 (31.8)	0.0069 (36.1)
Firm growth	-1.2324 (-61.3)	-0.6720 (-43.7)	-0.2821 (-28.7)
<i>R-square</i>	0.412	0.372	0.325
<i>Observations</i>	10,045	10,005	10,133

*Notes:* *t*-statistics in parenthesis.

**Firm size and wage-tenure profile** The theoretical analysis has shown that average wages increase with workers' tenure. An important question is whether the wage-tenure profile differs across firms of different sizes. Using data from the Benefits Supplement to the Current Population Survey (CPS), Hu (2003) shows that the wage-tenure profile for white collar workers is steeper in large firms. Our theoretical model is fully consistent with this finding. Following Hu (2003), we estimate a regression equation that relates

<sup>8</sup>There is a limit to how small  $\sigma$  can be. If this parameter is very small, then the wage profile becomes so steep that large-unconstrained firms pay much higher wages than the ones offered to new workers. This implies that the firm may have an incentive to renegotiate the wage contract: the gains from replacing the worker (and paying lower wages) could exceed the loss in sunk capital. With  $\sigma = 0.5$  the non-renegotiation condition is no longer satisfied, as we will show in Section 7.

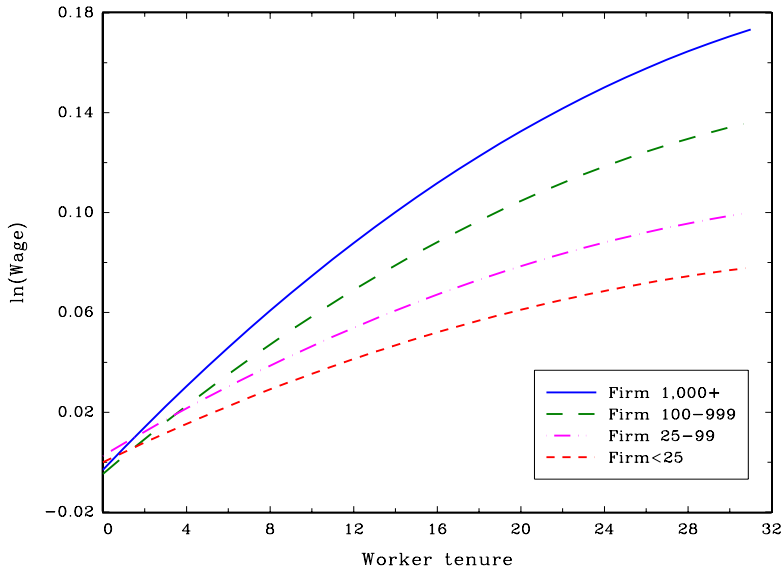


the log-wage to a quadratic polynomial of worker’s tenure, for four size classes of firms: less than 25 employees, 25-99 employees, 100-999 employees, 1,000 and more employees. Formally, the regression equation is:

$$\begin{aligned} \ln(\text{Wage}) &= \sum_{j=1}^4 \gamma_j \cdot \text{SizeDummy}_j + \sum_{j=1}^4 \delta_j \cdot (\text{SizeDummy}_j \cdot \text{WorkerTenure}) \\ &+ \sum_{j=1}^4 \lambda_j \cdot (\text{SizeDummy}_j \cdot \text{WorkerTenure}^2) \end{aligned}$$

where the index  $j$  identifies the four size classes of firms described above.<sup>9</sup>

Figure 4: Wage-Tenure profile for different size classes of firms.



The estimated wage-tenure profiles for the four size classes of firms are plotted in Figure 4. The wage-tenure profile of workers in large firms is steeper than in small firms, which is consistent with the data. This result may appear counterintuitive at first: because firms that pay increasing wages are those with binding financial constraints—and therefore, they operate at

<sup>9</sup>Hu also includes other workers’ characteristics which we do not include because in our model all workers are alike.

a sub-optimal scale—we may have inferred that small firms have steeper wage profiles. This would be the case if all firms have the same capacity  $\bar{N}$ . But in the model the size of firms depends not only on the financial status, but also on their technological capacity. Given our estimate of  $\rho$ , large capacity firms are on average more constrained than small capacity firms. This implies that large capacity firms offer a steeper wage profile. Because they are on average larger than small capacity firms, the model generates a positive relation between the slope of the wage-tenure profile and the size of firms.

## 6 Job-to-job flows and employer vs occupational tenure

In this section we extend the model to overcome two apparent shortcomings of our analysis. First, the turnover of workers generated by the model tends to be too small, since it is just the result of workers and firms death. In reality, a substantial fraction of workers switch their occupation from one employer to the other without any unemployment spell. See, for example, Akerlof, Rose, and Yellen (1988) and Fallick and Fleischman (2001). Another important feature of our model is that *employer tenure* is an important determinant of wages. However, a recent paper by Kambourov and Manovski (2002) argues that the tenure of a worker with an employer has little importance in the determination of wages. What matters is the *occupational tenure*—*i.e.*, the experience in a particular occupation even if with multiple employers. To address these issues, we extend the model to allow for occupation-to-occupation flows.

We make the following assumptions. First, in each period a firm is able to contact a measure  $m$  of workers, who already hold a job, and who do not require any training to become productive in the firm. We interpret these workers as holding jobs in the same occupation as that offered by the new employer. Because the worker does not change occupation, he or she can transfer the (occupation) specific human capital to the new employer. This allows the firm to save on the investment cost  $\kappa_w$ .

Second, the firm is able to attract the worker simply by offering the utility earned with the current employer. As in Burdett and Mortensen (1998) and Burdett and Coles (2003), the worker is unable to let the current and new employers compete over his services, and the poaching firm has all the bargaining power. The microfoundations for these assumptions, based on the existence of some renegotiation costs, are developed in Hashimoto (1981)

and Anderlini and Felli (2001).<sup>10</sup> Notice that the new employer is willing to offer a promised utility higher than  $q_{res}$  because it saves on the training cost  $k_w$ .

To keep the model tractable, we also make two further simplifying assumptions. First, the matching technology is balanced as in Burdett and Vishwanath (1988), in the sense that the number of workers contacted by the firm is proportional to its size, that is,  $m = \chi N$ . This implies that each employed worker has a probability  $\chi$  of being contacted by another employer offering a job in the same occupation. The idea behind this matching technology is that larger firms are more visible and find easier to contact workers. Notice that the mass of workers contacted by a firm and the workers contacted in the firm is not stochastic. This implies that the mass of workers poached by the firm is equal to the mass of workers leaving the firm. Here we are proceeding as if we can apply some law of large numbers. Second, we assume that firms contact only workers employed in firms of the same capacity ( $\bar{N}$ ) and cohort (tenure). The idea is that workers employed in firms of the same type and cohort are more likely to have transferable skills. This implies that the promised utilities of workers who quit the firm are exactly equal to the utilities of new hired workers. As a result, the state variables of the firm do not change.

The previous assumptions are simple abstractions that allow us to keep the model tractable. Thanks to these assumptions the firm's problem does not change. However, we can now distinguish between employer tenure and occupational tenure. We then have the result that employer tenure is no longer relevant for the determination of wages once we control for occupational tenure.

A possible calibration target for the job-to-job flow is  $\chi = 0.15$ . Together with  $\eta = 0.9778$  and  $p = 0.9714$  chosen previously, this implies that about 80

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<sup>10</sup>The idea goes as follows. The poaching firm makes a *take or leave it* offer to the contacted worker. This offer is private information and it can not be verified by the current employer in the absence of some worker's effort. The current employer would match the external offer if the worker demands to renegotiate the contract. However, the worker has to incur a utility cost  $e > 0$  in order to make the wage offer verifiable. This cost is sunk when renegotiation starts. Thus if the contract is renegotiated the current employer would simply offer to the worker a promised utility just above the utility that he would obtain by quitting. This generates an hold-up problem and the worker never tries to renegotiate the contract. Anticipating this, the poaching firm offers an expected utility slightly higher than the utility that the worker would earn by remaining with the current employer and the worker quits.

percent of workers have more than one year of tenure with the same employer. This is the number reported by Farber (1999, Table 3) for the U.S. economy. The estimation of the wage equation gives the following coefficients:

$$\begin{aligned} \ln(\text{Wage}) = & -0.8614 + 0.0476 \cdot \text{WorkerTenure} - 0.0016 \cdot \text{WorkerTenure}^2 \\ & (-110.8) \quad (27.1) \quad (-10.7) \\ & -0.0024 \cdot \text{FirmAge} + 0.0091 \cdot \ln(\text{FirmSize}) - 0.496 \cdot \text{FirmGrowth} \\ & (-29.4) \quad (27.4) \quad (-28.7) \end{aligned}$$

These numbers are not very different from the case in which there is not job-to-job flows as reported in Table 3.<sup>11</sup>

## 7 Contracts implementation

In the analysis of the long-term contract we have assumed that the firm never reneges the promised wages. This could be problematic because wages and promised utilities increase until the firm becomes unconstrained. More specifically, a new hired worker starts with  $q_t = q_{res}$  but then he or she receives  $q_{t+j} \geq q_{res}$ , for all  $j > 0$ . Because new workers can be hired with initial utility  $q_{res}$ , the firm may have an incentive to renege promises that exceed  $q_{res}$ . The goal of this section is to discuss the conditions that prevent the firm from renegotiating the long-term contract. We then discuss why collateralized debt is the only form of external financing for the firm.

Before continuing, it will be convenient to summarize the timing of the model. First workers decide whether to provide effort—which has a cost  $\bar{\ell}$  in forgone utility—and whether to quit the firm. Then production takes place and the firm observes whether the worker has provided effort. At this point the firm could renege its wage promises. Afterwards, the firm decides whether to renegotiate the debt. Renegotiation entitles the investors to seize the firm’s assets. After the payment of the wages and the repayment of the debt, the survival of the firm and the workers are observed.

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<sup>11</sup>Even though the firm’s problem does not change with the addition of job-to-job flows, the tenure of workers with the same employer is shorter on average. Consequently, the coefficient estimates are not the same.

## 7.1 Worker-firm relationship

If both the worker and the entrepreneur cooperate (the worker by exerting effort and the entrepreneur by paying the promised wage), output is produced and the worker earns the promised income. The only Nash Equilibrium of each period sub-game is the one in which the firm reneges its promises and pays zero wages and the worker, anticipating that, withdraws effort and quits. In the repeated game, however, cooperation can be sustained through trigger strategies, provided that replacing the worker is sufficiently costly for the firm. Specifically, suppose that the worker and the firm follow these strategies (which for simplicity are specified independently of the investors' past history):

- **Worker:** The worker provides effort as long as the firm pays the contracted wages. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), the worker withdraws effort and quits.
- **Firm:** The firm pays the contracted wages as long as the worker provides effort. If one of the two parties has reneged sometimes in the past (either the worker has shirked or the firm has paid a wage different from the one contracted), it sets the wage to zero.

The equilibrium associated with these strategies is sub-game perfect. To see this, let's consider first the worker. Providing low effort would trigger a wage cut which forces the worker to quit the firm and be left with the reservation value  $q_{res}$  starting from the next period. But the utility from doing so,  $U(0) + \bar{\ell} + \eta q_{res}$ , is not bigger than the utility obtained from providing effort, that is,  $U(w_t) + \eta p q_t + \eta(1-p)q_{res}$ . Thus, along the equilibrium path, the worker never shirks and quits. If the firm has sometimes paid a different wage from the one contracted, quitting is optimal since the firm would pay a zero wage both today and in the future.

Consider now the firm. When the firm expects the worker to quit tomorrow, setting the wage to zero today is always the firm's best response. Thus, given each worker's strategy, paying zero wages is optimal when the worker has sometimes shirked. Along the equilibrium path, the firm never finds optimal to deviate from the promised long-term contract because, if the firm reneges its wage promises, the worker quits and the firm loses the

sunk investment  $\kappa_w$ . Therefore, the assumptions that part of the investment is worker-specific, is key to prevent the firm from renegotiating the contract.

The fact that the replacement of an existing worker is costly for the firm, creates an indirect form of “collateral” for workers. This allows the firm to borrow from the workers beyond what it can formally borrow from external investors. Of course, there is a limit to this. If the worker’s utility becomes very large, the loss of sunk investment may be smaller than the gains from reducing the wage obligations (by reneging the long-term contract and hiring a new worker). This may happen if  $\bar{\kappa}_f/\kappa$  is close to 1 and the initial assets of the firm,  $a_0$ , are small. In this paper we have implicitly assumed that  $\bar{\kappa}_f/\kappa$  is sufficiently small and  $a_0$  sufficiently large so that this never arises in equilibrium. This condition, in particular, is satisfied in the numerical analysis of Section 5.

To show that the non-renegotiation condition is satisfied in the numerical exercises, Table 5 reports the maximum gains that can be obtain by replacing an existing worker (and paying lower wages afterwards). The maximum possible gains are for firms with the largest capacity  $\bar{N}$  that are unconstrained.<sup>12</sup> These firms are paying the highest wages to the first cohort of workers, that is, the workers hired when firms were first created. Denote the wage paid to this cohort by  $w_{max}$ . A firm could replace these workers with new workers receiving a constant wage  $w_{res}$ . This is the wage that gives the reservation utility  $q_{res}$ .<sup>13</sup> By doing so, the firm would save  $w_{max} - w_{res}$  in wage payments in each period. The expected discounted value of these payments are:

$$RG(\mathbf{P}) \equiv \frac{\beta(w_{max} - w_{res})}{1 - \beta(1 - \chi)\eta p}$$

where  $RG$  stands for Renegotiation Gains and  $\mathbf{P}$  are the model’s parameters. Notice that the term  $\beta(1 - \chi)\eta p$  becomes the discount factor of the gains for the firm because the worker remains in the firm with probability  $(1 - \chi)\eta p$ : the worker does not quit the firm with probability  $1 - \chi$ , he survives with probability  $\eta$ , and the firm remains in operation with probability  $p$ . The renegotiation gains are compared with the loss of workers specific capital  $\kappa_w$ .

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<sup>12</sup>One can check that the maximal promised utility such that the firm does not find optimal to renegotiate the contract is decreasing in the age of the firm. This together with the fact that the promised utility of workers increases with tenure (till the firm becomes financially unconstrained), proves that temptation to renegotiate is the highest when the firm is financially unconstrained, which justifies our approach.

<sup>13</sup>That is  $w_{res}$  satisfies  $q_{res} = \beta U(w_{res})/(1 - \eta\beta)$ .

As shown in Table 5, for all curvatures of the utility function used in the quantitative section of the paper, the renegotiation gains are smaller than the loss in sunk capital  $\kappa_w = 1.935$ . In computing these numbers we have used  $\chi = 0.15$ .

Table 5: Renegotiation gains for different curvatures of the utility function.

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 2.0$
$RG(\mathbf{P})$	2.605	1.820	1.045

The table also shows that the renegotiation gains increase as we reduce the curvature of the utility function and  $\sigma$  becomes smaller. This is because the profile of wages becomes steeper. This explains why for very small values of  $\sigma$  the non-renegotiation condition is no longer satisfied.

## 7.2 Investors-firm relationship

Suppose that in the case in which the entrepreneur renegotiates the debt contract (or defaults), investors have the ability to liquidate the assets of the firm but cannot exclude the entrepreneur from participating in financial markets. In other words, the entrepreneur can get new financing from other investors. Furthermore, let's assume that if the firm is able to refinance investment, the firm is also able to retain the hired workers. This implies that the investment in recruitment and training is not lost.

Under the above conditions, collateralized debt is the only form of external financing for the firm. To see this, suppose on the contrary that the firm is able to borrow above the value of the collateral. If so, after receiving the loan, the entrepreneur would always renegotiate down the part of the debt in excess of the collateral and then obtain a new (identical) financial contract from other investors. Anticipating this, only secured loans will be offered.

## 8 Workers' savings

In studying the optimization problem of the firm, we have made the extreme assumption that workers cannot save. We now discuss the plausibility of this assumption.

The possibility that workers could loose their existing jobs with the consequent fall in continuation utility, creates an incentive for accumulating assets. However, if the workers' return from savings is sufficiently small relative to the intertemporal discount factor  $\beta$ , and the likelihood of a negative shock  $1 - p$  is small, then the worker will not save.

Suppose that workers can accumulate assets with return  $r$  but they cannot borrow against future wages, that is, their wealth cannot be negative. The optimal saving decision of an individual worker is characterized by the following first order condition:

$$U_c(c_t) \geq \eta\beta(1+r) \left[ p \cdot U_c(c_{t+1}) + (1-p) \cdot E(U_c(\tilde{c}_{t+1})) \right]$$

where  $c_t$  and  $c_{t+1}$  are consumptions when the worker is employed and  $\tilde{c}_{t+1}$  is consumption when the worker re-enters the labor market after loosing the job. Notice that  $\tilde{c}_{t+1}$  is not known at time  $t$  because it depends on the financial status of the new employer, which explains the expectation operator  $E(\cdot)$ . If the condition holds with strict inequality, the worker chooses to hold zero assets, that is, the workers' borrowing constraint is binding. In the numerical exercises conducted in Section 5, the above condition always holds with strict inequality, implying that workers never find optimal to accumulate assets.

To show this, let's consider the case of a worker that is currently receiving the highest possible wage, which we denote by  $w_{max}$ . This is the oldest worker of an unconstrained large-capacity firm. Because the firm is unconstrained, the wage paid to the worker is constant as long as he or she remains employed. Therefore,  $c_t = c_{t+1} = w_{max}$ . If the worker looses the job, the new wage depends on the financial status of the new employer, which is unknown ex-ante. The probability distribution is given by the economy-wide distribution of starting wages offered by hiring firms. Using the specific functional form for the utility function, the worker's first order condition can be rewritten as:

$$S(\mathbf{P}) \equiv 1 - \eta\beta(1+r) \left[ p + (1-p) \cdot E \left( \frac{\tilde{c}_{t+1}}{w_{max}} \right)^{-\sigma} \right] \geq 0$$

where  $\mathbf{P}$  denotes the parameters of the model. The condition  $S(\mathbf{P}) > 0$  guarantees that all workers do not save, independently of their current status.

Table 6 reports the value of  $S(\mathbf{P})$  for alternative values of  $\sigma$ . As shown in the table, the no-saving condition is always satisfied. Therefore, given the parameters used in the paper, the assumption that workers cannot save is simply a reduced form of the environment in which they are allowed to save



but they deliberately decide not to do so. The conditions that guarantee this result is the assumption that the return on savings  $r$  is small relative to the intertemporal discounting  $\beta$ , and the assumption that workers cannot hold negative assets.

Table 6: No-saving condition for different curvatures of the utility function.

	$\sigma = 0.5$	$\sigma = 1.0$	$\sigma = 2.0$
$S(\mathbf{P})$	0.041	0.030	0.021

## 9 Conclusion

In this paper we have studied how financial constraints affect the compensation structure of workers. Firms that are financially constrained find optimal to offer an upward profile of wages in order to alleviate their financial restrictions. Because large firms are more likely to have experienced a history of financial tightness with low wage payments, they have to pay high wages after becoming unconstrained. This mechanism can generate a positive correlation between firm size and wages (the firm size-wage relation). Our theory is also consistent with other empirical observations. In particular, the fact that fast growing firms—which in our model are those financially constrained—pay lower wages.

In offering an upward profile of wages, firms are implicitly borrowing from workers. This rises the question of why firms are able to borrow from workers beyond what they can borrow from external investors. In our model this is possible because workers can use punishment mechanisms that are not available to external investors. An external investor can punish the debtor only by confiscating the firm’s physical assets, which represents the only collateral that the firm can use to raise funds in financial markets. But the firm can expand its debt capacity by using another form of implicit “collateral” in the hands of workers. If a worker quits, the firm loses the job-specific investment. This gives the worker a credible punishment tool in the event of repudiation that is not available to investors. The cost of replacing the worker—due to the sunk nature of the investment—guarantees

that the long-term wage contract between the worker and the firm is never renegeed and allows the firm to use the wage policy to finance its growth.

In practice firms may defer the compensation of workers not only by paying wages that are increasing with tenure, but also by giving workers assets, such as stock options, whose value increases as the firm grows and becomes financially unconstrained. Indeed, stock options are often awarded to workers, such as middle-run managers, secretaries and clerks, whose effort is likely to have negligible effects on the value of the firm. In these cases stock options are unlikely to provide better incentive to workers and may be regarded as a form of deferred compensation of the kind emphasized in this paper. In accordance with this interpretation, Blasi, Kruse, and Bernstein (2003) estimate that the stock options yielded particularly high profits to workers hired before their companies went public—*i.e.*, companies that were likely to be financially constrained when they awarded the options.

## A Characterization of the firm's problem

Let  $\gamma_t$ ,  $\mu_t$ ,  $\lambda_t n_t$  and  $\theta_t$  denote the lagrange multipliers associated with the constraints (2), (3), (4) and (5) respectively. Then the Lagrangian can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{aligned} & \left( a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \right) + \\ & \gamma_t \left[ a_t + b_t - \kappa \sum_{\tau=0}^t n_{\tau} \right] + \\ & \mu_t \left[ \bar{\kappa}_f \sum_{\tau=0}^t n_{\tau} - b_t \right] + \\ & \lambda_t n_t \left[ \sum_{j=1}^{\infty} \beta^j U(w_{t,t+j}) - q_{res} \right] + \\ & \theta_t \left[ \sum_{\tau=0}^t (\kappa + A - w_{\tau,t+1}) n_{\tau} - (1+r)b_t - a_{t+1} \right] \end{aligned} \right\}.$$

The first order conditions with respect to  $w_{\tau,t}$  and  $a_t$ , for  $t \geq 1$ , are

$$\beta \lambda_{\tau} U_c(w_{\tau,t}) = \theta_{t-1}, \quad \forall \tau \leq t \quad (20)$$

and

$$\theta_{t-1} = \beta(1 + \gamma_t), \quad (21)$$

respectively. Using (21) to substitute for  $\theta_{t-1}$  in (20) yields (6) in the text.

## B First order conditions for the recursive formulation of the basic model

The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= a + b - \kappa N' + \beta V(a', q', N', H') \\ &+ \gamma [a + b - \kappa N'] \\ &+ \mu [\bar{\kappa}_f N' - b_t] \\ &+ \lambda H' [\beta(U(w') + q') - q] \end{aligned}$$

where  $\gamma$ ,  $\mu$  and  $\lambda H'$  are lagrange multipliers. The problem is also subject to the laws of motion for the next period value of  $a$  and  $H$ , that is, constraints (13) and (14), respectively.

The first order conditions are:

$$b : \quad 1 + \gamma - \mu = \beta(1 + r)V_{a'} \quad (22)$$

$$w' : \quad V_{a'} = \lambda U_{c'} \quad (23)$$

$$q' : \quad V_{q'} + \lambda H' = 0 \quad (24)$$

$$N' : \quad \beta \left[ (\kappa + A - \psi(q)w')V_{a'} + V_{N'} + \psi(q)V_{H'} \right] \geq (1 + \gamma)\kappa - \mu\bar{\kappa}_f \quad (25)$$

where the last condition is satisfied with equality if  $N' < \bar{N}$ . The envelope conditions are:

$$V_a = 1 + \gamma \quad (26)$$

$$V_q = -\beta\psi_q(N' - N)[w'V_{a'} - V_{H'}] - \lambda H' \quad (27)$$

$$V_N = \beta\psi(q)[w'V_{a'} - V_{H'}] \quad (28)$$

$$V_H = -\beta[w'V_{a'} - V_{H'}] \quad (29)$$

Equation (15) in the text comes from using (26) to substitute for  $V_a$  in (23). We now show that the above conditions also imply that  $\lambda = \lambda'$ .

By substituting (26) in (29) we get:

$$-V_H = \beta[(1 + \gamma')w' - V_{H'}] \quad (30)$$

From (23) we have that  $(1 + \gamma')w' = \lambda(w')^{1-\sigma} = \lambda(1 - \sigma)U(w')$ , which substituted in (30) yields

$$-V_H = \beta[(1 - \sigma)\lambda U(w') - V_{H'}]. \quad (31)$$

Now consider the promise-keeping constraint  $q = \beta[U(w') + q']$ . Multiplying the left and right-hand side by  $(1 - \sigma)\lambda$  we get:

$$(1 - \sigma)\lambda q = \beta[(1 - \sigma)\lambda U(w') + (1 - \sigma)\lambda q'] \quad (32)$$

Equations (31) and (32) imply:

$$-V_H = (1 - \sigma)\lambda q \quad (33)$$

$$-V_{H'} = (1 - \sigma)\lambda q' \quad (34)$$

Updating the first term we also have that:

$$-V_{H'} = (1 - \sigma)\lambda' q' \quad (35)$$

Condition (34) and (35) then imply that  $\lambda = \lambda'$ .

### C Recursive formulation of the general model

The problem solved by a firm with capacity  $\bar{N}$  can be written recursively as follows: More specifically:

$$V(a, z, N, H) = \max_{\substack{b, w', z', \\ N' \leq \bar{N}}} \left\{ d + \beta \left[ p \cdot V(a', z', N', H') + (1 - p) \cdot L' \right] \right\} \quad (36)$$

subject to

$$d = a + b - \kappa N' \geq 0 \quad (37)$$

$$b \leq \bar{\kappa}_f N' \quad (38)$$

$$z = \beta [U(w') + \eta p z'] \quad (39)$$

$$a' = \kappa N' + AN' - H' w' - (1 + r)b - \kappa_w (1 - \eta) N' \quad (40)$$

$$H' = \eta H + \psi(z)(N' - \eta N) \quad (41)$$

$$L' = \kappa_f N' + AN' - H' w' - (1 + r)b \quad (42)$$

where  $L$  is the liquidation value of the firm, which consists of the sum of its physical capital and its current profits minus the value of debt.

Let  $\gamma$ ,  $\mu$  and  $\lambda H'$  be the lagrange multipliers associated with the constraints (37), (38), and (39), respectively. Following the same steps as in Appendix B we obtain the first order conditions:

$$b : \quad 1 + \gamma - \mu = \beta(1 + r)(1 + p\gamma') \quad (43)$$

$$w' : \quad 1 + p\gamma' = \lambda U_{c'} \quad (44)$$

$$z' : \quad V_{z'} + \eta \lambda H' = 0 \quad (45)$$

$$\begin{aligned}
N' : \quad & \beta \left[ (1 + p\gamma') (\kappa + A - \psi(z)w' - (1 - \eta)\kappa_w) \right. \\
& \left. + p(V_{N'} + \psi(z)V_{H'}) - \eta(1 - p)\kappa_w \right] \geq (1 + \gamma)\kappa - \mu\bar{\kappa}_f
\end{aligned} \tag{46}$$

where the last equation is satisfied with equality if  $N' < \bar{N}$ . Notice that (43), (44) and (46) make use of the envelope condition  $V_a = 1 + \gamma$ . The remaining envelope conditions are:

$$V_z = \beta\psi_z(N' - \eta N) \left[ pV_{H'} - (1 + p\gamma')w' \right] - \lambda H' \tag{47}$$

$$V_N = -\eta\beta\psi(z) \left[ pV_{H'} - (1 + p\gamma')w' \right] \tag{48}$$

$$V_H = \eta\beta \left[ pV_{H'} - (1 + p\gamma')w' \right] \tag{49}$$

## D Computation of the equilibrium

**Solving for the firm's problem:** For given  $\bar{N}$  and  $q_{res}$ , the firm problem is solved backward starting from the state in which the firm is unconstrained. Let's assume that the firm takes  $T$  periods to become unconstrained. Therefore, we know that  $N_{T+1} = \bar{N}$  and  $\gamma_T = \gamma_{T+1} = 0$ .

We start by guessing the value of  $w_{T+1}$  and  $H_{T+1}$ . Using the first order condition  $1 = \lambda U_c(w_{T+1})$ , we determine the lagrange multiplier  $\lambda$ . Using the promise-keeping constraint  $z_T = \beta[U(w_{T+1}) + \eta p z_{T+1}]$ , and imposing  $z_T = z_{T+1}$ , we determine the (transformed) promised utility at time  $T+1$ . Using condition (47) with the terminal condition  $V_{H,T} = V_{H,T+1}$ , we determine the partial derivative of the value function with respect to  $H$ . Finally, we determine  $b_T$  using the borrowing limit  $b_T = \bar{\kappa}_f N_{T+1}$  and  $\mu_T$  using the first order condition  $\mu_T = 1 + \gamma_T - \beta(1 + r)(1 + p\gamma_{T+1})$ . At this point we have all the terminal conditions to solve the problem backward at each point  $t = T, T-1, \dots, 0$ . The solution at each point  $t$  is determined as follows:

1. Using the budget constraint with  $d_t = 0$ , we determine the firm's assets:

$$a_t = \kappa N_{t+1} - b_t$$

2. The wage  $w_t$  is determined using the first order condition:

$$1 + p\gamma_t = \lambda U_c(w_t)$$

3. We now determine the variables  $N_t$ ,  $H_t$  and  $b_{t-1}$  using the laws of motion for  $a_t$ ,  $H_{t+1}$ , and the borrowing limit:

$$\begin{aligned} a_t &= (\kappa + A)N_t - H_t w_t - (1 + r)b_{t-1} - \kappa_w(1 - \eta)N_t \\ H_{t+1} &= \eta H_t + \psi(z_t)(N_{t+1} - \eta N_t) \\ b_{t-1} &= \bar{\kappa}_f N_t \end{aligned}$$

4. The values of  $V_{H,t}$  and  $z_{t-1}$  are determined using condition (47) and the promise-keeping constraint, that is:

$$\begin{aligned} V_{H,t} &= -\eta\beta(1 + p\gamma_{t+1})w_{t+1} + \eta\beta pV_{H,t+1} \\ z_{t-1} &= \beta[U(w_t) + \eta p z_t] \end{aligned}$$

5. The values of  $\mu_{t-1}$  and  $\gamma_{t-1}$  are then determined using the first order conditions for debt and employment, that is:

$$\begin{aligned} 1 + \gamma_{t-1} - \mu_{t-1} &= \beta(1 + r)(1 + p\gamma_t) \\ \beta \left[ (1 + p\gamma_t) \left( \kappa + A - \psi(z_{t-1})w_t - (1 - \eta)\kappa_w \right) + p \left( \psi(z_{t-1}) - \psi(z_t) \right) V_{H,t} \right. \\ &\quad \left. - \eta(1 - p)\kappa_w \right] = (1 + \gamma_{t-1})\kappa - \mu_{t-1}\bar{\kappa}_f \end{aligned}$$

After solving for all  $t = T, T - 1, \dots, 0$ , we check two conditions: Whether  $z_0 = z_{res}$  and  $H_1 = N_1$ . The second condition implies that  $N_0 = H_0 = 0$ . If these two conditions are not satisfied, we change the guesses for  $w_{T+1}$  and  $H_{T+1}$  until convergence.

In the solution of the model we also solve for the initial assets  $a_0$ . If  $a_0$  is bigger than the initial assets, we increase  $T$ . This takes advantage of the fact that smaller are the initial assets of the entrepreneur and longer is the transition to the unconstrained status.

**Labor market equilibrium:** To compute the labor market equilibrium we start by guessing the equilibrium value of  $z_{res}$ . Given this value we solve for the firm's problem for all values of  $\bar{N}$ . The procedure to solve for the firm's problem has been described above. After finding the invariant distribution of firms, we find the aggregate demand of labor and we check the clearing condition in the labor market. We update  $z_{res}$  until the labor market clears.

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