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Investing Public Pensions in the Stock Market: Implications for Risk Sharing and Asset Prices

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I. Introduction

Policymakers worldwide are contemplating investing public pension assets in the stock market -- or allowing their citizens to make this choice -- and many countries have recently begun to do so. This is motivated by concerns that existing public systems will be unable to provide benefits to a rapidly aging population without sharp increases in taxes on future workers, and by the perception that the higher average return on stocks could help alleviate these pressures. Economists have also suggested that including stock market investments in public pension plans could improve risk sharing within and between generations. The purpose of this paper is to assess the implications of such policies for asset prices and risk sharing qualitatively and quantitatively, using a calibrated overlapping-generations model.

Although there may be legitimate reasons to include stock market investments in public pension systems, there are a number of problems with the simple line of reasoning that emphasizes the higher average rate of return. Two main points are well established in the literature but bear repeating. First, shifting pension fund investments from government securities to stocks may provide little or no incentive for additional savings (Abel, 1999, 2000). Hence aggregate economic growth, and the resources available to pay future pension benefits, may be largely unchanged; the consequences are primarily distributional. Furthermore, raw comparisons between average stock market returns and the returns to pensioners under the current system are misleading on several counts. They confuse investment returns with flows determined by program rules, and do not adjust for the risk characteristics of different investments (Geanakoplos, Mitchell and Zeldes (1998)).

Investing public pension system assets in the stock market may, however, shift the allocation of risk and return within and between generations. Whether this will improve or worsen risk sharing will depend on the allocation of risk prior to reform, and the details of how the pension policy is implemented. Because of the heterogeneity in wealth, income and preferences, and the complexity of current and proposed policies, a calibrated general equilibrium model may provide some insights on these issues. To this end, the implications for risk sharing and asset prices of various pension reforms are explored using of an overlapping-generations (OLG) model with many heterogeneous agents. The model is calibrated to match the stylized aggregate and cross-sectional features of the U.S. economy. The focus is on the implications of policy changes similar to those currently under consideration in the U.S.[1]

Our analysis is most closely related to that of Diamond and Geanakoplos (1999), who emphasize the importance of the potential distributional effects of including stock market investments in public pension systems. In a related OLG framework, they show that adding stock market risk to pension benefits can be welfare improving under particular assumptions about preferences, endowments, and market structure. We extend their analysis by quantifying the effects on prices and welfare, emphasizing the interaction of government investment policy with tax and debt policy, and by considering a wider set of explanations for why low-income households have low savings rates and low stock market participation rates in the first place. Structurally the model has many similarities to Donaldson, Constantinides and Mehra (2000b), who value a put option on equity-contingent benefit payments. A number of studies (see Bonn (1999), Storesletten et. al. (1999), and references therein) also have looked at the risk-sharing implications of various pension-system reform proposals also using an OLG framework. Our analysis differs in its emphasis on stock market investments, and on how those investments and tax policy interact with borrowing constraints and non-contractible income to affect the cross-sectional distribution of risk.

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[1] For simplicity, we do not distinguish between stock market investments via a government trust fund or via private accounts. While the structure may have important implications for the efficiency and transparency of the system, either have equivalent implications in this stylized model.

In brief, we find that the introduction of a new "pay as you go" social security system has a large impact on desired saving rates and hence asset prices.[2] Whether or not the government invests in stocks, however, has little influence on rates of return once the system is established. This is true even with very limited stock market participation prior to the governments investing in stocks. Whether the risk of government investments is borne by pension beneficiaries or by taxpayers also has little effect on equilibrium prices. These policy differences can, however, have significant effects on the welfare of different wealth cohorts.

The paper is organized as follows. Section I lays out the model. Section 2 describes the calibration and policy experiments. Section 3 presents and interprets the results. Section 4 concludes.

II. The Model

This section presents an overlapping-generations model with heterogeneous agents. The model will be used to analyze some of the general equilibrium effects of shifting government pension system investments into the stock market, and to illustrate the variety of distributional outcomes that may occur depending on the details of policy implementation.[3] The model also is used to illustrate the cross-sectional welfare implications of different policies. For instance, by calibrating the model to the cross-sectional distribution of income in the U.S. economy, and by assuming that low-income individuals do not have direct access to the stock market, we evaluate the effects of making public pension payments contingent on market returns and thereby exposing low-income individuals to stock market risks and returns. Technically, the model generalizes Heaton and Lucas (2000) by adding a government sector and public pension system to an

[2] This point was emphasized by Huggett and Ventura (1995).

overlapping-generations model in which heterogeneous agents are subject to, borrowing constraints, and stock market participation is limited.

II.A. Structure

In each time period, *t*, a generation of *J* types of young agents is born. They live for four periods, earning income and paying taxes in the first two periods, earning income, paying taxes, and investing in the third, and consuming retirement income from social security and investments in the fourth. Agents are distinguished by their preference parameters, lifetime income profile (which in turn affects taxes and pension benefits), and by whether they have access to the stock market in the third period of life (referred to as "middle age"). We abstract from bequests and inheritances.

A recursive utility specification is used, which distinguishes the elasticity of intertemporal substitution from the coefficient of risk aversion (Epstein and Zin (1989)). It accommodates heterogeneity across agents in risk aversion and time preference. Lifetime utility is given by:

$$\ln[v^i(j,t)] = (1-\beta_j)\ln(C^i(j,t)) + \frac{\beta_j}{1-\alpha_j}\ln\left(E\left[v^{i+1}(j,t+1)^{1-\alpha_j} \mid F(t)\right]\right)$$

for i=1, 2, 3, and $v^4(j,t) = \ln\left(C^4(j,t)\right)$

(1)

where $\beta_j > 0$ and $\alpha_j > 0$. $F(t)$ is the information available in period *t*, common to all agents. $C^i(j,t)$ denotes the consumption of agent of type *j* at age *i*, (*j*=1,2, ... , *J;* *i*=1,2,3,4) at time t. The

[3] For tractability production is assumed to be exogenous, so the model cannot be used to evaluate how aggregate investment would be affected by policy changes. If, as we find, equilibrium asset returns are similar across policy experiments, it is likely that in a model with endogenous production, aggregate investment would be largely unaffected also. See Abel (1999) for an example of a production economy where this is the case.

elasticity of intertemporal substitution is fixed at one, and the coefficient of relative risk aversion is $\alpha_{j.}$

Each agent of type j and age i is endowed with random nonmarketed income $Y^i(j,t)$, at time t. "Nonmarketed income" refers to all income that by assumption cannot be easily contracted upon. This includes income from sources such as wages, privately held businesses, service flows from owner occupied housing, private defined benefit pensions, and net transfers from the government (primarily social security and health benefits net of taxes). It excludes income from publicly traded stocks and bonds, which is contractible.

In the first two periods of life saving is precluded by assumption,[4] and consumption equals income net of taxes. In the third period of life a savings decision is made. As discussed below, this structure preserves the computational simplicity of a two-period OLG model, while accommodating more realistic life cycle patterns of consumption and savings. The middle-aged can invest in financial assets to provide additional income in retirement. There are two types of securities available: a risky stock and a one-period risk-free bond. At time t the stock represents a claim to stochastic future dividends $D(t+n)$: n = 1,2, ... The total supply of stock is normalized to one. The supply of risk-free government bonds depends on the government's policy. Private bonds are in zero net supply.

Nonmarketed income risk is assumed to have both an aggregate component and an idiosyncratic component. Aggregate nonmarketed income at time t is denoted by $Y^a(t)$ where:

$$Y^a(t) = \sum_{i=1}^{4} \sum_{j=1}^{J} Y^i(j,t). \tag{2}$$

In equilibrium, this aggregate endowment, plus dividends, equals time t aggregate consumption.

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[4] This is assumed for tractability, and is consistent with evidence in Gourinchas and Parker (2002) and other studies of life cycle savings. For plausible parameterizations, it can be verified that most young

Middle-aged agents maximize expected utility through their savings decision, subject to a constraint that depends on the agent's access to financial markets. Let $P^s(t)$ be the price of the stock at time t and $P^b(t)$ be the price of a bond that pays one unit of consumption at time $t+1$ for sure. If an agent of type j has access to both financial markets, then the agent's flow wealth constraints are:

$$C^3(j,t) = Y^3(j,t) - S(j,t)P^s(t) - B(j,t)P^b(t) \tag{3a}$$

and

$$C^4(j,t+1) = Y^4(j,t+1) + S(j,t)\left[P^s(t+1) + D(t+1)\right] + B(j,t) \tag{3b}$$

where $S(j,t)$ is the stock purchased by type j agents in middle age, and $B(j,t)$ is the face value of the bonds purchased by type j agents in middle age, both at time t. For the subset of agents assumed to have access only to the bond market, constraint (3a) is replaced by:

$$C^3(j,t) = Y^3(j,t) - B(j,t)P^b(t) \tag{4a}$$

and (3b) is replaced by:

$$C^4(j,t+1) = Y^4(j,t+1) + B(j,t). \tag{4b}$$

Young agents do not save or borrow by assumption, implying that for i = 1 and 2,

$$C^i(j,t) = Y^i(j,t). \tag{4c}$$

Total government spending, including social security payments, $SS(t)$, other spending, $G(t)$, and any stock purchases, is financed by tax receipts or debt issuance. This implies a flow budget constraint for the government:

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agents optimally would save little (except for precautionary reasons) due to the upward sloping age-income profile.

$$P^b(t)B(t) = B(t-1) + G(t) + SS(t) + P^s(t)S(g,t) - S(g,t-1)\left[P^s(t) + D(t)\right] - \sum_{i=1}^{4}\sum_{j=1}^{J}\tau^i(j,t) \quad (5)$$

The function *SS(t)* represents total public pension system payments at time *t*, which may or may not be contingent on financial market realizations. *B(t)* is government debt held by the public, and S*(g,t)* are stocks held in the trust fund. This equation reflects actual transactions that occur between the government and the rest of the economy.

The time path of debt held by the public depends on the policy rules linking taxes to other state variables. To obtain a stationary, rational-expectations equilibrium, the government's policy with respect to public debt, taxes, other spending, and pension payments must also be stationary. In the calibrations we consider a variety of stationary government policies, the details of which are explained below.

The notional debt held in the social security trust fund is denoted by $\delta(t)$. It is notional rather than real because, as in the case of the U.S. social security system, it represents a claim held by the government on itself. Trust fund assets evolve according to:

$$P^b(t)\delta(t) + P^s(t)S(g,t) = \delta(t-1) + T^{SS}(t) + (D(t) + P^s(t))S(g,t-1) - SS(t) \quad (6)$$

where $T^{SS}(t)$ is the portion of total tax revenues earmarked for the trust fund. Equation (6) should be thought of as an accounting identity that defines $\delta(t)$, which has no real effect on the economy. Nevertheless trust fund debt is often a focus of policy interest and can be tracked using (6).

An equilibrium of the model is given by processes for stock and bond prices $P^s(t)$: $t = 0,1, \ldots$ and $P^b(t)$: $t = 0,1, \ldots$ such that markets clear:

$$\sum_{j=1}^{J} S^*(j,t) = 1 - S(g,t) \tag{7a}$$

and

$$\sum_{j=1}^{J} B^*(j,t) = B(t). \tag{7b}$$

and that for middle-aged agents, $\{S^*(j,t),\ B^*(j,t)\}$ maximizes (1) for each t and j, subject to (3a) and (3b) for those agents can trade in both markets or subject to (4a) and (4b) for agents that can only trade in the bond market. Further, the government budget constraint (5) must be satisfied. In some of the calibration exercises described below, some or all of middle-aged agents also are subject to a borrowing constraint, $B(j,t) \geq 0$, and a short sales constraint $S(j,t) \geq 0$.

It is convenient to adopt notation that reflects the stationarity assumption. We assume that the growth rate of nonmarketed income, $Y^a(t)$, and dividend income, $D(t)$, is such that the growth rate of aggregate income is a stationary process. Consistent with this, we can write $Y^i(j,t) = \varepsilon^i(j,t)Y^a(t)$ and $D(t) = d(t)Y^a(t)$, where $\varepsilon^i(j,t)$ denotes the share of individual j's income in aggregate income at age i, and $d(t)$ gives the dividend relative to aggregate nonmarketed income. A stationary government debt policy implies that the face value of debt held by the public at time t can be represented by $\omega(t)Y^a(t)$, and therefore $B(j,t) = b(j,t)\omega(t)Y^a(t)$ where $b(j,t)\omega(t)$ represents the quantity of "rescaled" bonds purchased by agents of type j in middle age. The share of stocks owned by the government, $S(g,t)$, is also restricted to be stationary. This implies that one can look for an equilibrium in which stock prices also scale with aggregate income $P^s(t) = p^s(t)Y^a(t)$.

III. Calibration

In this section we describe the inputs into the calibrated model, the policy experiments considered, and how welfare comparisons are made. Section III.A contains a description of the parameterization of the non-government component of income processes and the preference specification. These quantities are chosen to reflect the cross-sectional distribution of income and asset holdings, and to be consistent with the gross features of historical data on stock returns, the risk-free rate, and the driving processes for non-government income and dividends. Section III.B summarizes previous findings on the relation between income, wealth, and stock holdings, and discusses alternative assumptions about stock market participation and preferences that potentially allow the model to capture those stylized facts. Section III.C describes features of the current social security system that are incorporated into the model, and the policy alternatives that are considered. Section III.D describes how the welfare comparisons are implemented.

The model cannot be solved in closed form, but is solved numerically using standard techniques. The fact that agents only solve a two-period optimization problem makes it feasible to assume considerable heterogeneity in the cross-section.[5] However, instabilities can arise when trying to solve for government policies that accommodate shocks with debt rather than immediate tax changes, which limits the scope of policies considered in this draft.

As is generally the case when a consumption-based model is used to predict asset returns, the equity premium puzzle presents a potential problem. If the model predicts a smaller equity premium than the historical data indicates might be expected, the predicted benefit of increased access to the stock market could be understated. Similarly to Constantinides, Donaldson and Mehra (2002a and b), imposing borrowing constraints on the young and taking into account the sharply rising age-income profile concentrates risky asset holdings in older households. This has the effect of raising the predicted premium to a level considerably higher than in most consumption-based models, although still lower than observed in historical data. Assuming higher volatility in dividends and income than observed in the data, as described below, further increases the predicted premium and the potential gains from improved risk-sharing.

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[5] The Matlab programs are available upon request.

III.A Income and Preferences

We begin by defining the aggregate state. Let $\gamma(t) \equiv ln(Y^a(t)/Y^a(t-1))$ be the growth rate of aggregate nonmarketed income at time *t*. The aggregate state of the economy is defined as $z(t) \equiv [\gamma(t), d(t); \omega(t), S(g,t)]'$, which is assumed to follow a Markov chain. The aggregate income processes $\gamma(t)$ and $d(t)$ are held constant, whereas the evolution of policy as summarized by $\omega(t)$ and $S(g,t)$ varies across experiments.

The model is calibrated so that one period corresponds to 15 years. This allows the first two periods of an agent's life to loosely represent the working years between ages 18 and 48, the third period between ages 48 and 63 to represent the later working years in which significant retirement savings accumulate, and the final 15-year period to represent time spent in retirement. We assume a 15-year average (log) growth rate in aggregate income of 25.5 percent with a standard deviation of 20.33 percent. Letting γ take on the values 0.0517 and 0.4583 with equal probability discretizes this distribution. This approximation is based on the observed annual (log) growth in real aggregate consumption over the period 1889 to 1995, which averaged 1.7 percent with a standard deviation of 3.5 percent. Assuming that annual income growth is independently and identically distributed (abstracting from the slight negative auto-correlation in that series), and that the standard deviation is 1.5 times the observed standard deviation, yields the assumed distribution.[6]

The dividend share is scaled to represent the capital share in total income, which historically averages approximately 30 percent. Following Heaton and Lucas (2000) and consistent with historical data, we assume that only half of this capital income is tradable in the stock market. The non-publicly traded portion of capital income, generated for instance by

[6] As discussed earlier, increasing the volatility of aggregate consumption to increase the equity premium is also used by Constantinides et. al. (2002a), Heaton and Lucas (2000), and Abel (1999). This assumption may affect the quantitative importance of improved risk sharing, but we do not anticipate that it will change the qualitative implications of the model.

private business holdings, is accounted for in nonmarketed income. Accounting for capital income this way emphasizes that along with labor income, it is a significant source of uninsurable risk for some households. Since dividends in the model are scaled relative to nonmarketed income, *d(t)* averages 18 percent. In most of the calculations *d(t)* is assumed to equal either 11 percent or 25 percent with equal probability, reflecting the higher variability of dividends than of output.[7]

The relative nonmarketed income of an agent of type j at age i at time t is given by:

$$\varepsilon^i(j,t) = \varepsilon_x{}^i(j,t) + \varepsilon_p{}^i(j,t) \tag{8}$$

where

$$\sum_{i=1}^{4}\sum_{j=1}^{J}\varepsilon^i(j,t) = 1 \tag{9}$$

Nonmarketed income is further broken down into a private component, $\varepsilon_x{}^i(j,t)$ representing income from sources such as labor and privately held businesses, and a policy component, $\varepsilon_p{}^i(j,t)$ reflecting public pension system payments and any other assumed taxes or monetary transfers. The distribution of $\varepsilon_x{}^i(j,t)$, is important for several reasons. First, it captures the cross-sectional distribution of income across the population and over the life cycle. In some specifications, it also affects the severity of distortions in the portfolio choices of low-income households due to the interaction of borrowing or participation constraints and tax policy. Although it could be incorporated, in this analysis we abstract from the potential effects of idiosyncratic risk in the last period of life on asset prices, since in our earlier work this effect was found to have little quantitative impact (Heaton and Lucas, 2000).

[7] This variability is greater than the observed value for dividends or earnings, but is also employed to

The cross-section of private income for middle-aged agents is chosen to span the distribution of social security income replacement rates in retirement, as reported by the Social Security Administration. The income distribution for middle-aged agents and corresponding replacement rates are shown in Table 1. The life-cycle pattern of income is assumed to take the same shape for all agents. Income in the first two periods is scaled relative to income when middle-aged, based on average age/earnings profile estimated in Parker and Gourinchas (1999). The period 1 income share is 77% of period 3 income share, and the period 2 income share is 92% of the period 3 income share, for each type. Private income in the fourth period is set to zero, so that all non-investment income derives from social security.

To summarize the interaction of the policy component and private component of income processes, the old only have income from public pension benefits and private savings, so ${\varepsilon_x}^4(k,t)$ =0 and ${\varepsilon_p}^4(k,t)$ is based on the replacement rates in Table 1. As discussed below, to reflect the partial indexation of benefits to wage growth in the social security system, ${\varepsilon_p}^4(k,t)$ multiplies a weighted average of $Y^a(t\text{-}1)$ and $Y^a(t)$, with 75 percent of the weight on income at *t-1*. In the third period the distribution of $\varepsilon_x(j,t)$ is as reported in Table 1, and in the first and second period it equals .77 and .92 of this value for each type. For the non-elderly the effect of policy on income is only through taxes: $\varepsilon_p(j,t) = -\tau(t)\varepsilon_x(j,t)$ where $\tau(t)$ is the equilibrium tax rate.

In the base case, preferences are parameterized with βj = 0.99[15] and α_j = 5 for all *j*. The possibility of differential risk aversion on the part of low-income households is also considered, as is heterogeneity in time preference. These parameters, shown in Tables 2 and 3, are chosen to reflect the possibility that poorer individuals are more impatient and/or more risk averse than their

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generate a larger equity premium.

wealthier brethren. This results in potentially lower savings rates and stock market participation rates even in the absence of market imperfections or policy-induced incentives for less saving.

III.B. Relation between Income, Wealth and Stock Holdings

In practice stock holdings are concentrated in wealthy households. For instance, tabulations from the Survey of Consumer Finances (SCF) show that the top 10 percent of U.S. households in terms of wealth held approximately 80 percent of outstanding stocks in 1995. Stock holdings are also concentrated among high-income households, but the concentration is not as high as when conditioning on wealth; the top 10 percent of U.S. households in terms of income held approximately 40 percent of stock in 1995 (Heaton and Lucas, 2000). These high concentration rates have persisted, despite the popularization of stock market investing. Retirement savings are also concentrated in high-income households. Diamond and Geanakoplos (1999) report that for the bottom quintile of the income distribution only 6 percent of retiree income is from private pensions and other assets, while for the top quintile of retirees 46 percent is from pensions and assets.

In this model the cross-sectional distribution of savings and stock holdings is endogenous. There are several ways to induce heterogeneity in the ratio of stocks to income that captures the gross features of the empirically observed relation between these quantities. Those include assuming heterogeneity in preferences, distortionary government policies, participation constraints (often attributed to the high fixed costs of participation), or heterogeneity in the anticipated growth and variability of income. Depending on what is assumed, the welfare implications of a given policy can vary significantly.

Since there is not a well-established explanation for the observed pattern of heterogeneity in stock holdings, we consider several parameterizations that generate a positive correlation between income and stock holdings, and compare their implications for equilibrium prices and welfare under the various policy alternatives. The possibility that low-income households lack access to equity markets or find it prohibitively expensive to participate is reflected by considering specifications in which below-median income households are precluded from stock market participation. We refer to these as "limited participation" scenarios. Another possibility that is straightforward to explore in this model is heterogeneity in preference parameters. For the preferences assumed, when all agents share common utility parameters and in the absence of borrowing constraints or a government sector, it is a well-known that portfolio shares are independent of wealth. The observed positive correlation between income and stock holdings, however, suggests that there may be a correlation between risk aversion and income. The positive correlation between income and saving also suggests the possibility that the subjective discount factor is correlated with income. To examine these possibilities, results under the alternative utility parameterizations in Tables 2 (heterogeneous risk aversion) and Table 3 (heterogeneous time preference) are presented for each experiment, with the ranges in the tables ranging over fairly conventionally assumed values.

III.C. Alternative Policy Regimes

As a benchmark, the model is solved with no government sector for the various parameterizations for preferences and assumptions about the extent of stock market participation. We then consider five main policy regimes (not all completed in this draft). The first is a stylized representation of the current U.S. system. In the second the government holds some stocks in the trust fund, but social security remains a defined benefit system and any surpluses or deficits are offset with immediate tax changes on the working population. The third case is similar to the second, but surpluses or deficits are absorbed by government debt except in extreme deficit or surplus conditions, which trigger tax rate changes. (This case is not completed for the current draft.) The fourth case features stocks in the trust fund with benefits partially contingent on stock market realizations. The fifth case is similar to the fourth, except that there is a floor on benefits, financed by contemporaneous tax increases (also not completed).

These policy regimes are chosen to span the range of possibilities for this type of reform. Certain assumptions are maintained across all the policy regimes considered. Because the focus is the public pension system, other taxes and government expenditures are always set to zero (or equivalently, are assumed to completely offset each other each period for each agent).

Government Debt. Debt held by the public is fixed initially at $B(0)$ and evolves according to equation (5). We set $B(0)$ to approximately 1% of aggregate annual labor income in the base case without any social security, and also in the stylized model of the current social security system. Policy regime changes involving government purchases of stock initially increase the public debt outstanding by an amount equal to the value of stocks purchased.

Trust Fund. As discussed above, trust fund debt can be tracked in the model. Despite the fact that independent of the size of the trust fund, stocks must be funded by issuing debt to the public or raising taxes, in policy proposals the trust fund is often considered the amount available for government investment in stocks.   For this reason, we use the trust fund to scale the size of the initial government purchase of stocks. Debt in the trust fund is set initially at $\delta(0)$, and evolves according to equation (6). The initial level of trust fund debt is set using SSA's projected ratio of trust fund assets to expenditures in 2005, which is 350 percent in their intermediate case. Based on that ratio, and the fact that in the model on average retired agents receive social security income equal to 13.3 percent of aggregate private income, $\delta(0)$ is set to 46.5 percent of aggregate exogenous income. Relative to the stock market, this amounts to the government holding about 13 percent of shares outstanding, a sufficiently large share to expect measurable effects could arise.

Benefits. In the policy scenarios with defined benefits, the benefit levels are based on the current U.S. social security system. According to the Social Security Administration (SSA), people retiring at age 65 in 2000 received a first-year benefit that ranged from a 64 percent replacement of average income for someone who earned 45 percent of national average earnings, to a 40 percent replacement rate for someone who earned 160 percent of national average income. For an average earner, the replacement rate was 47 percent.[8] Accordingly, benefits are set as a progressive function of the realization of the exogenous income shock as described in Table 1. We consider replacement rates ranging from 80 percent for the lowest income group to 20 percent for the wealthiest group.[9] The fraction of the population assumed to fall into each group is also based on income distribution statistics from the SSA.

In applying the replacement rates in the model, an adjustment is made for the fact that there is generally some real growth in benefits for retirees, but benefits do not keep up with the growth of the economy.[10] We assume that the replacement rates apply to a weighted average of aggregate nonmarketed income when middle-aged and old, with a weight of .75 on income when middle-aged and .25 on income when old. The dependence of benefits on lagged income causes the proportion of income transferred to the elderly to vary over time, even with a constant population. This transfers aggregate risk from old to young through the tax system, as emphasized below. Adding explicit demographic shocks would magnify this risk.

Tax and debt policy. As discussed earlier, the extent to which shocks to promised benefit payments are offset by immediate taxes is a policy choice. In the initial set of experiments, all such shocks are financed with a uniform and immediate change in tax rates on the non-elderly population.

[8] The rules underlying those effective rates are that 90 percent of average monthly income is replaced up to $561, 32% is replaced between $561 and $3,381, and 15% is replaced over $3,381.

[9] Proposals that involve imposing taxes on Social Security income for the high-income elderly could be modeled by reducing the replacement rate for the target groups, but no such experiment is considered in this version of this paper.

[10] Under current law, benefits are indexed to inflation and real growth until retirement, and then indexed only to inflation.

An alternative is for the government to use debt policy to partially redistribute shocks across generations. There is a limit, however, on the extent to which debt can be used to smooth. Absorbing all shocks with debt to maintain a completely constant tax rate is generally not feasible because the accumulated effects of high benefit payments or low stock returns can lead to exploding debt. Computationally, finding a stationary feedback rule under which debt buffers equity return shocks is challenging.

To represent policies where debt is used to smooth tax rates, we assume a quadratic policy rule where tax rates increase nonlinearly with debt levels. The parameters are chosen to generate fairly constant tax rates in most states of the world, but higher or lower tax rates when debt levels move toward extreme values. This can be interpreted as a deliberate policy of avoiding changes in tax rates except in situations of very high deficits or surpluses. For policy scenarios with constant (or near constant) tax rates, "payroll" taxes are assessed as a near constant fraction of exogenous income in the first three periods of life. [11] The tax rate is chosen so that the ratio of debt held by the public to output is approximately stationary. That rate varies across policy experiments depending on whether stocks are included in the trust fund, and on other parameters affecting equilibrium returns.

Participation constraints. To assess the importance of stock market participation constraints, for each policy regime we first allow all types to participate in the stock market. We then consider the alternative that the 33% of middle-aged households with incomes below the median are excluded from the stock market. The non-constrained groups can borrow a fraction of their income, providing private debt securities which along with the government debt are purchased by constrained agents.

[11] This is a realistic depiction social security taxes, except that annual collections are capped.

We now describe the parameters distinguishing the alternative policy regimes. We do not look at transition dynamics --calculations are as if each policy regime is in the steady state. We hope to use the model to address transition issues in a future draft.

1. The current social security system. Current law specifies non-contingent constant benefit rules and constant tax rates. An equilibrium policy cannot have this feature, since it implies potentially unlimited imbalances between the present value of taxes and benefits. Instead we assume that benefit rates are fixed, but that surpluses or deficits are offset by a contemporaneous change in the payroll tax rate on the non-elderly population.

2. Stocks in the trust fund and defined benefits. At time 0 approximately 50% of the assumed initial debt in the trust fund is sold to the public and used to buy stocks held by the government. Benefits remain progressive and independent of stock returns. Contemporaneous tax rate changes on the working population are again the means by which surpluses or deficits in the social security system are offset. It is assumed that trust fund balances are maintained at a constant proportion of aggregate exogenous income, with the government's share of stock fixed at 13 percent of the total (consistent with the government continuing to hold the initial number of shares purchased). Any debt held by the public also grows proportionally with the economy. Deviations from these target ratios are avoided by adjusting the taxes on the working population. For instance, if tax revenues plus trust fund returns fall short of promised benefits, the taxes on the working population are raised to generate sufficient revenues to meet the shortfall. Similarly, tax rates are lowered when revenues exceed expenditures.

3. Stocks in the trust fund, defined benefits, and tax smoothing with debt. The third set of policy experiments is similar to the second, except that discrepancies between benefits and revenues from taxes and trust fund income are partially financed by issuing or buying back debt, rather than by adjusting taxes. The idea is to use government debt to spread stock market risk over more generations. Comparing the second and third scenario should be revealing about the extent to which the government can spread stock market risk more broadly than can the market if households are non-dynastic. (In this version of the paper we do not report this case.)

4. Stocks in the trust fund and contingent benefits. 50 percent of the initial debt in the trust fund is again sold to the public and used to buy stocks held in the trust fund. Benefits, however, are partially contingent on the realization of trust fund returns. To maintain most of the progressivity of the fixed benefit structure under current policy, each beneficiary receives a payoff with a fixed component proportional to the assumed payoff in the fixed benefit case. In addition there is a variable component that depends on the realization of the stock market over the previous 15 years. On a risk-adjusted basis, total benefits are similar to those under the status quo social security system. The fraction of the promised benefit replaced by a risky return is assumed to be equal across types, and the progressive benefit structure is maintained. To minimize the number of state variables, it is assumed that the entire trust fund investment in stocks is liquidated each period. The government delivers excess returns to beneficiaries, and reinvests in the stock market so as to maintain the target ratio of stocks in the trust fund. This policy results in a considerable portion of pension benefits contingent on stock market returns, and likely overstates the transfer of risk relative to viable proposals, which call for a relatively small reallocation. Hence it can be interpreted as putting an upper bound on price and welfare effects. As in the second set of experiments, immediate tax rate changes are used to buffer shocks to promised benefits arising from macroeconomic variability affecting the non-stock market contingent component.

*4. Stocks in the trust fund and contingent benefits with a floor. T*he fifth policy experiment will be structured similarly to the fourth, but with a floor on benefits financed by higher contemporaneous taxes when stock market realizations are low.

IIID. Welfare Calculations

To evaluate whether effect on welfare of a type j agent under an alternative policy regime, we calculate the constant percentage change in income each period of life that would make the present discounted value of lifetime utility equal to that under the parameterization for the current social security system. Algebraically, this implies solving for ζ^{j} such that lifetime utility under the status quote system, with income process $\varepsilon^{i}(j,t) = \varepsilon_{x}{}^{i}(j,t) + \varepsilon_{p}{}^{i}(j,t) + \varsigma^{i}$ is equal to lifetime utility under an alternative regime with income $\varepsilon^{i}(j,t) = \varepsilon_{x}{}^{i}(j,t) + \varepsilon_{p}{}^{i}(j,t)$. The asset return process reflects the equilibrium in each case, so the welfare effects include the fact that returns change across steady states.

IV. Results

IV.A. No Social Security Benchmark

Equilibrium prices and quantities in the benchmark case with no social security and stock market participation by all middle-aged agents are reported in Table 4A. Panel 1 assumes homogeneous preference parameters, Panel 2 assumes heterogeneity in risk aversion, Panel 3 assumes heterogeneity in time preference, and Panel 4 assumes heterogeneity in both parameters. With homogeneous preferences, the equity premium is 1.9 percent, the risk-free rate is 4.6 percent, and the standard deviation of stock returns is 7.5 percent. Asset returns for the alternative preference specifications are similar.

Although the equity premium is smaller than it has been historically, so is the variance of stock returns. Further increasing the assumed variability of dividends or income, or increasing the coefficient of relative risk aversion, would increase both. The parameterization chosen is consistent with the view that in the future, the equity premium may be lower. It also is consistent with the view that the true variable of interest is the return on capital more broadly defined (i.e., a mix of stock and corporate bond returns).

The risk-free rate is close to constant because the non-policy component of the aggregate state is i.i.d.. The small amount of variability is a result of tax policy that varies with the quantity of government debt, which in turn varies slightly with the aggregate state. The saving rate of the middle-aged is high (46 percent of income) because all consumption when old must be financed out of these savings (there is no social security and the young do not save).

The last three columns of Table 4A show the effect of heterogeneous preferences that are correlated with income. Savings rates are largely unaffected by variation in the risk aversion coefficient, but as would be expected, are sensitive to the assumed rate of time preference. Savings by all income groups are held primarily in stocks, due to relatively small amount of government bonds assumed to be available (about 11% of annual GDP). Notice that the portfolio share held in stock is quite sensitive to variation in the coefficient of relative risk aversion, ranging from 29% to 140%. This suggests that risk aversion differences could explain a significant amount of cross-sectional variation.

Tables 4B illustrates the effect of limiting participation in the stock market to middle-aged agents with at or above median income, Table 4C further restricts participation to the 37% of the population above the median group. Even with the highly restricted participation in Table 4C, the equity premium only increases by .5 percentage points relative to when all middle-aged hold stocks. This is consistent with the findings of Heaton and Lucas (2000) that non-participation only has a significant effect on asset returns when it is considerably more severe than what is observed empirically. Note also that in this model, even in the base case there is considerable non-participation because the young do not save anything. Finally, although non-participation has a minor effect on returns, it may have a considerable effect on the welfare implications of adopting policies that allow more complete risk sharing, a possibility that is explored below.

IV.B. Current Policy

Table 5 reports equilibrium prices and quantities for the case of the stylized representation of the current U.S. system (policy regime 1). The addition of social security has a significant effect on asset returns, the level and cross-section of savings rates, and on the cross-section of portfolio weights.

With homogeneous preferences, the predicted risk-free rate increases by 1.5 percent to 6.1 percent, and the equity premium falls by .1% to 1.8% percent. The variability of the risk-free rate increases because of the variability in marginal rates of substitution induced by changes in tax rates to meet social security surpluses or deficits that arise due to partially fixed benefits relative to current output. The intuition for these price effects is that the expectation of receiving social security reduces the demand to save, driving up the required rate of return on all financial assets. Social security has two partially offsetting effects on the allocation of risk. It provides the equivalent of a risk-free bond, increasing the tolerance for stock market risk by guaranteeing a minimum income in retirement. At the same time, due to the progressive benefit structure, saving and hence stock holdings are more concentrated in the portfolios of the wealthy. The increase in the concentration of risk tends to increase the required return on equities. For these parameters, the net effect of a higher risk-free rate and higher return to stocks is to slightly decrease the equity premium.

By providing a floor on income in retirement, social security depresses personal savings rates. The weighted average saving rate falls from 46% in the benchmark case to 34.7% with the addition of social security. Further, the progressive benefit structure creates sharp differentials in the saving rate across income groups. With homogeneous preferences the saving rate ranges from 22 percent for the lowest income group to 36 percent for the highest income group. These results are consistent with those of Hubbard, Skinner and Zeldes (1995), who attribute much of the differential in savings rates by income status to social insurance programs, and with Huggett and Ventura (1995), who emphasize the disincentive effects of social security on savings.

While the progressive benefit structure helps explain differential saving rates, it has the counterfactual implication that with homogeneous preferences, low income agents hold a much larger percentage of their savings in stocks. This is because their relatively high expected level of social security payments substitute for bonds. Assuming preference heterogeneity reverses this effect, implying a stock share of 41% for the lowest income group and 118% for the highest income group. The assumed heterogeneity in discount rates also widens the range of savings rates from 18 percent to 39 percent.

The last two columns report the results when stock market participation is limited to those earning the median income or above. Similarly to the base case, this tends to increase the equity premium slightly and changes portfolio composition, but has little effect on private savings rates.

IV.C. Stock Investments with Defined Benefits and Tax-Financed Shocks

Table 6 reports equilibrium prices and quantities for the case in which stocks on average comprise 50 percent of the trust fund, there are defined benefits, and surpluses or deficits are offset with temporary rate tax changes. The effect on asset returns is modest. In comparison to the current social security system, the risk-free rate increases by 0.3 percent and the equity premium falls by 0.1 percent when preferences are homogeneous, and the changes are similar for alternative participation and preference assumptions.

The price effects can be interpreted by considering the effect on the incidence of risk. The relatively wealthy bear less stock market risk in retirement, since portfolio risk decreases and public pension risk remains the same. The risk borne by the middle- and low-income elderly is largely as before since the dominant effect is that their pension benefits are unchanged. Because tax revenues are assumed to absorb the additional stock market risk in the trust fund, the working population faces greater exposure to market risk and return. Interestingly, because young workers do not trade in the asset markets, the increased tax risk does not directly affect asset prices. Tax rate changes do affect the investment behavior of the middle-aged, both because of a wealth effect and because taxes affect the relative size of expected retirement benefits and current income.

The reduction in the stock market risk borne by the middle-aged probably explains the drop in the risk-free rate -- less risk results in a lower precautionary demand for savings, and a lower required premium on stock. The variability of the bond return increases since the more variable taxes on the middle-aged increase the variability in the marginal rate of substitution. As for the previous experiments, assuming either limited participation or heterogeneous preferences (or a combination of the two) results in more realistic portfolio structures, but has relatively small effects on prices or savings behavior. The welfare implications of this case will be discussed in the next draft.

IV.D. Stock Investments with Defined Benefits and Partially Debt-Financed Shocks

The next draft also will report equilibrium prices and quantities for the case in which stocks comprise 50 percent of the trust fund, there are defined benefits, and repurchasing or issuing debt finances surpluses or deficits (policy regime 3).

IV.D. Stock Investments with Fully Contingent Benefits

This forth policy regime most closely resembles current proposals for social security reform. Table 7 reports equilibrium prices and quantities for the case in which stocks comprise 50 percent of the trust fund, and benefits reflect the stock market risk in the trust fund, with any gains or losses in the government's stock portfolio distributed through social security benefits. Table 8 compares the welfare of different income groups with that under the status quo social security system.

To the extent that the policy change is offset in private portfolio choices, retirees have similarly risky income as under current policy. This is because when participation is unrestricted, the middle-aged can offset the additional risk in promised benefits by decreasing private stock holdings -- their personal portfolios on average are safer since they hold government debt in place of the stock sold to the government, but their public pension benefits will be riskier by an offsetting amount.

There is a significant shift in risk through the tax system, however, since the young now provide less consumption insurance to the old via the tax system. As in Table 6, this shift in risk away from the wealthy middle-aged may account for the higher interest rates than under the current system, due to a reduced precautionary demand. Overall, the rate of return differences are minor whether under the status quo, with stocks in the trust fund and defined benefits, and with stocks in the trust fund and risk-sensitive benefits.

The improved intergenerational risk-sharing is reflected by the welfare calculations of the fraction of labor income workers under the current system would give up to move to the fourth policy regime with benefits contingent on the stock market. Table 8 reports gains for all income cohorts of between 11.5% and 12.5% of income under homogeneous preferences. Gains range from 5.7% for the low income group to 19.8% for the high income group with heterogeneous preferences. The dispersion in gains increases with limited participation, with constrained agents relatively better off. This likely is explained by the benefit of increased stock market exposure in retirement. Interestingly, this gain is small relative to the benefit of improved risk-sharing across generations through the tax system.

V. Discussion and Conclusions

The calibration results highlight the fact that the allocation of risk and return that results from investing pension fund assets in the stock market is determined by three main factors: (1) the effect on individual portfolio composition, (2) the effect on the distribution of benefits, and (3) the effect on the distribution of tax liabilities across generations.

Effect on individual portfolio risk. The effect of the government buying stocks financed with the sale of debt is that middle-aged individuals will on average hold more debt and fewer stocks in their portfolios. Stock holdings currently are concentrated in the portfolios of relatively wealthy and older households, while most middle- and lower-income young families have a small fraction of their wealth in the stock market. If the government were to purchase a sizeable quantity of stocks, it would have to be disproportionately from high wealth and older households.

Effect on benefit risk. Investing pension fund assets in stocks may lead to riskier retirement benefits, but there is no necessary connection. For instance, most private firms with defined benefit pensions finance their plans with a mixture of risky stocks and bonds, with the risk affecting firm shareholders but not plan beneficiaries. Many discussions of social security reforms that include stock market investments assume that benefits will be sensitive to realized asset returns. If, however, a government trust fund were invested in stocks, and if realized returns were low, fixed benefits could still be delivered by adjusting government debt or taxes. The example considered here suggested that such a policy could improve risk-sharing by reducing the aggregate risk absorbed by the young, but not by increasing the consumption risk of the old.

Conversely, although social security is currently structured as a defined benefit system, the rules could be modified so that benefits were contingent on asset market returns, without any physical investment in stocks. In general, it is the rules of the system and the political process, not the underlying investments, which fundamentally determine benefit risk.

Effect on tax risk. If the government assumes stock market risk but does not pass it on to pensioners, some of the risk ultimately will be transferred to taxpayers. Since retirees generally are taxed at a lower-than-average rate, workers would bear increased risk to the extent they do not offset it through portfolio rebalancing. The precise timing and incidence of tax risk, however, will depend on the specific policy. If, for instance, if shortfalls are financed initially by issuing more government debt to the public, then that debt may not be repaid with tax revenue for many years. The immediate consequence is a drop in the consumption of the middle-aged, who must be induced to purchase the debt in return for increasing their consumption in old age.

The results of our simulations suggest that predicted asset returns are fairly insensitive to whether stocks are held in the social security trust fund, independent of the details of policy implementation. The implications for risk-sharing are quite sensitive to these details, and the preliminary welfare calculations suggest the effects can be large.

The finding that asset prices are largely unaffected by shifting trust fund investments into stock market can be explained as follows. Currently the U.S. social security system interacts with capital markets only in so far that less government debt is issued to the public when payroll taxes exceed pension benefits, and more public debt is issued when taxes fall short of expenses. If the trust fund were to substitute purchases of risky private securities for purchases of public debt, individual investors would shift private savings from stocks to government securities, in effect doing an asset swap with the pension system.

After the asset swap it would appear that individuals hold safer, lower return portfolios than previously, and that the trust fund portfolio has greater risk and return. This, however, is an incomplete analysis, as can be seen by considering an economy with a representative agent. If stock returns are too low to meet promised pension benefits, the representative agent will ultimately bear this risk, since either his pension benefits must be cut or his taxes increased to meet the shortfall. Similarly, if returns are greater than anticipated, the pension system will have a surplus that will either be used to reduce taxes or pay additional pension benefits, again allocating the risk back to the representative agent. Aggregate risk, which includes pension, tax and portfolio risk, is unchanged. If there is no change in aggregate risk, the marginal rates of substitution governing asset prices are unaffected.

Of course, thinking about pension systems in terms of a representative agent is unsatisfactory in many ways, and the point of the model in this paper is to examine the implications of heterogeneity. As explored in the above simulations, the additional risk and returns assumed by the government to some extent can be reallocated across generations and income groups. These simulations suggest, however, that these distributional considerations are less important for asset prices than the fact that aggregate risk is unaffected. In part, the small effects can be attributed to the relatively small amount of stocks held by or on behalf of poorer households in either case.

The simulations suggest that these types of policies can have a significant effect on the distribution of risk across age and income groups. Whether such outcomes will actually occur, however, depends on whether such policies are politically feasible. It has been observed (Brooks and James, 1999) that the risks associated with various pension structures are political as well as economic. For instance, a public system that stipulates a defined benefit may fail to deliver one in the face of demographic pressures; political opposition to tax increases may lead to legislated benefit cuts. A public pension system in which stock market investments put retirees at risk also have political risk. In the event of a stock market downturn, the government may shift the risk back to future taxpayers rather than allow retiree benefits to be cut below a minimally acceptable level. The model introduced here could be modified to reflect such scenarios, but this is left for future research.

Table 1: Progressive Fixed Benefit Schedule

Type	Income when middle-aged (relative to median)	Benefit replacement rate	Percent of population
1	0.10	.80	5.8
2	0.65	.65	27.3
3	1.00	.50	29.6
4	1.35	.35	17.2
5	1.90	.20	20.1

Table 2: Subjective Discount Rates and Relative Incomes When Preferences are Heterogeneous

Income relative to the median	Discount rate (annual)
0.10	.98
0.65	.985
1.00	.99
1.35	.995
1.90	1.0

Table 3: Risk Aversion Parameter and Relative Incomes When Preferences are Heterogeneous

Income relative to the median	Coefficient of relative risk aversion
0.10	13
0.65	9
1.00	6
1.35	4
1.90	3

Table 4A: Baseline Model Results (No Social Security)

	Homogeneous preferences	heterogeneous risk aversion	Heterogeneous time preference	heterogeneous risk aversion and time preference
Asset Returns*				
$E(R_S)$	6.5%	6.5%	6.3%	6.4%
$\sigma(R_S)$	7.5%	7.5%	7.3%	7.5%
$E(R_B)$	4.6%	4.8%	4.3%	4.8%
$\sigma(R_B)$	0.2%	0.2%	0.2%	0.3%
$E(R_s-R_b)$	1.9%	1.7%	2.0%	1.6%
Savings /income by relative income				
0.10	46%	46%	42%	42%
0.65	46%	46%	44%	44%
1.00	46%	46%	46%	46%
1.35	46%	46%	48%	48%
1.90	46%	46%	50%	50%
Stock/saving by relative income				
0.10	95%	29%	95%	28%
0.65	95%	42%	95%	42%
1.00	95%	65%	95%	64%
1.35	95%	102%	95%	100%
1.90	95%	140%	95%	138%

*Reported returns are annualized. The reported means are ln(1 + 15-year return)/15, and the standard deviations are the standard deviation of ln(1 + 15-year return)/ $\sqrt{15}$.

Table 4B: Baseline Model Results (No Social Security, Limited Participation Including Median)

	Homogeneous preferences	heterogeneous risk aversion	Heterogeneous time preference	heterogeneous risk aversion and time preference
Asset Returns*				
$E(R_S)$	6.5%	6.5%	6.4%	6.4%
$\sigma(R_S)$	7.5%	7.6%	7.6%	7.5%
$E(R_B)$	4.4%	4.7%	4.3%	4.7%
$\sigma(R_B)$	0.2%	0.8%	0.7%	0.2%
$E(R_s-R_b)$	2.1%	1.8%	2.1%	1.7%
Savings /income by relative income				
0.10	46%	46%	42%	42%
0.65	46%	46%	44%	44%
1.00	46%	46%	46%	46%
1.35	46%	46%	48%	48%
1.90	46%	46%	50%	50%
Stock/saving by relative income				
0.10	0%	0%	0%	0%
0.65	0	0%	0%	0%
1.00	114%	70%	112%	68%
1.35	114%	110%	112%	107%
1.90	114%	153%	112%	148%

*Reported returns are annualized. The reported means are ln(1 + 15-year return)/15, and the standard deviations are the standard deviation of ln(1 + 15-year return)/ $\sqrt{15}$.

Table 4C: Baseline Model Results (No Social Security, Limited Participation Excluding Median)

	Homogeneous preferences	heterogeneous risk aversion	Heterogeneous time preference	Heterogeneous risk aversion and time preference
Asset Returns*				
$E(R_S)$	6.5%	6.5%	6.4%	6.4%
$\sigma(R_S)$	7.5%	7.5%	7.5%	7.5%
$E(R_B)$	4.1%	4.5%	4.0%	4.7%
$\sigma(R_B)$	0.2%	0.2%	0.2%	0.2%
$E(R_s-R_b)$	2.4%	2.00%	2.4%	1.7%
Savings /income by relative income				
0.10	46%	46%	42%	42%
0.65	46%	46%	44%	44%
1.00	46%	46%	46%	46%
1.35	46%	46%	48%	48%
1.90	46%	46%	50%	50%
Stock/saving by relative income				
0.10	0%	0%	0%	0%
0.65	0%	0%	0%	0%
1.00	0%	0%	0%	0%
1.35	169%	136%	163%	130%
1.90	169%	190%	163%	183%

*Reported returns are annualized. The reported means are ln(1 + 15-year return)/15, and the standard deviations are the standard deviation of ln(1 + 15-year return)/ $\sqrt{15}$.

Table 5: Stylized Current U.S. System (Policy Regime 1)

	Complete Participation		Limited Participation	
	homogeneous preferences	heterogeneous preferences	Homogeneous preferences	Heterogeneous preferences
Asset Returns*				
$E(R_S)$	7.9%	7.8%	8.0%	7.8%
$\sigma(R_S)$	8.4%	8.4%	8.4%	8.3%
$E(R_B)$	6.1%	6.2%	5.9%	6.1%
$\sigma(R_B)$	1.7%	1.7%	1.7%	1.7%
$E(R_s-R_b)$	1.8%	1.6%	2.1%	1.7%
Savings/income by relative income				
0.10	22%	18%	21%	18%
0.65	26%	24%	25%	23%
1.00	29%	29%	29%	29%
1.35	33%	34%	32%	34%
1.90	36%	39%	36%	39%
Stock/saving by relative income				
0.10	147%	41%	0%	0%
0.65	124%	53%	0%	0%
1.00	105%	72%	129%	77%
1.35	91%	98%	110%	105%
1.90	79%	118%	95%	126%

Table 6: Stocks in Trust Fund, Defined Benefits, and Variable Taxes (Policy Regime 2)

	Complete Participation		Incomplete Participation	
	homogeneous preferences	heterogeneous preferences	Homogeneous preferences	heterogeneous preferences
Asset Returns				
$E(R_S)$	8.1%	7.9%	8.2%	8.0%
$\sigma(R_S)$	8.4%	8.4%	8.5%	8.4%
$E(R_B)$	6.4%	6.6%	6.3%	6.5%
$\sigma(R_B)$	2.6%	2.7%	2.6%	2.7%
$E(R_s\text{-}R_b)$	1.7%	1.3%	1.9%	1.5%
Savings/income by relative income				
0.10	22%	18%	21%	17%
0.65	25%	23%	24%	22%
1.00	28%	28%	28%	28%
1.35	31%	33%	31%	33%
1.90	34%	38%	34%	38%
Stock/saving by relative income				
0.10	126%	34%	0%	0%
0.65	106%	45%	0%	0%
1.00	91%	63%	110%	67%
1.35	78%	86%	95%	91%
1.90	69%	103%	82%	109%

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Table 7: Stocks in Trust Fund, Variable Benefits, and Variable Taxes (Policy Regime 4)

	Complete Participation		Incomplete Participation	
	homogeneous preferences	**heterogeneous preferences**	**Homogeneous preferences**	**heterogeneous preferences**
Asset Returns				
$E(R_S)$	8.1%	8.0%	7.9%	7.9%
$\sigma(R_S)$	8.4%	8.5%	8.2%	8.3%
$E(R_B)$	6.3%	6.4%	6.3%	6.1%
$\sigma(R_B)$	2.0%	2.7%	1.8%	1.8%
$E(R_s-R_b)$	1.8%	1.6%	2.0%	1.7%
Savings/income by relative income				
0.10	22%	18%	20%	17%
0.65	25%	24%	24%	22%
1.00	28%	29%	28%	28%
1.35	33%	34%	31%	33%
1.90	35%	38%	35%	38%
Stock/saving by relative income				
0.10	109%	16%	0%	0%
0.65	96%	38%	0%	0%
1.00	87%	63%	116%	74%
1.35	79%	94%	102%	107%
1.90	73%	115%	90%	132%

Table 8: Consumption Adjustment

	Complete Participation		Incomplete Participation	
	homogeneous preferences	**heterogeneous preferences**	**Homogeneous preferences**	**heterogeneous preferences**
Group				
1	12.5%	5.7%	14.4%	
2	12.3%	8.7%	11.3%	
3	12.0%	12.2%	9.3%	
4	11.8%	16.0%	8.9%	
5	11.5%	19.8%	8.6%	

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