Buried by aggregation: excavating the dynamics of investment, employment and uncertainty

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Abstract

Micro evidence on investment and employment has uncovered stark evidence of lumpy intermittent behaviour. This is consistent with non-convex adjustment cost models in which uncertainty plays a central role in driving firm behaviour. This lumpy micro-behaviour, however, is buried by cross-sectional and time series aggregation at the firm level, obscuring any underlying structural model. To excavate beneath this aggregation we construct a model with capital, labour, and time varying uncertainty, which we explicitly aggregate to yearly firm values and structurally estimate against Compustat using indirect inference. This model then provides a laboratory to examine the impact of policy changes and shocks on investment and employment. Three experiments are considered: (i) temporarily raising uncertainty as in OPEC I and II, and 9/11/2001; (ii) permanently increasing labour adjustment costs from US to EU levels; and (iii) cutting capital taxes by 1%.

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1. Introduction

There is now a large body of evidence demonstrating that investment and employment at the plant level is lumpy and intermitted rather than smooth and continuous. This is starkly documented by, for example, Doms and Dunne (1993) for investment and Davis and Haltiwanger (1992) for employment[1]. This type of lumpy behaviour is consistent with models of non-convex adjustment costs, where adjustment rules take on a threshold form, with infrequent rates of adjustment and rich time series dynamics[2].

In these models uncertainty plays a major role in determining the gap between thresholds and the responses of investment and employment, and so is central to the impact of these non-convexities on firm behaviour. The aim of this paper is to understand the impact of uncertainty on the adjustment dynamics of investment and employment. We do this by constructing a rich model of firm behaviour which we use to try and estimate the underlying demand and adjustment cost parameters. This approach is innovative in that we explicitly model and address three central issues which play an important role in shaping observed behaviour under uncertainty:

Time Varying Uncertainty

The literature on investment and uncertainty has, for analytical tractability, focused on cross-sectional changes in uncertainty rather than time series variation. While this is essential for delivering analytical results it misses out on important time series variation. For example, uncertainty varies over time at the macro level due to shocks like OPEC I and II, and 9/11, at the industry level due to shocks like regulatory changes and new technologies, and at firm level due to shocks from demand and factor supplies. In this paper we model an underlying demand process with time varying uncertainty.

Labour and Capital Adjustment Costs

The empirical literature on investment and labour demand has, again for tractability, focused on estimating either investment or labour demand by making

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[1] Other work on investment includes Coooper and Haltiwanger (1993), Caballero, Engle and Haltiwanger (1995), Cooper, Haltiwanger and Power (1999) and Cooper and Haltiwanger (2003), and on employment Hammermesh (1989), Caballero and Engel (1993), Caballero, Engel and Haltiwanger (1997) and Cooper, Haltiwanger and Willis (2004)..

[2] See, Bertola (1988), Pindyck (1988), Dixit (1989), Abel and Eberly (1996), and Eberly and Van Mieghem (1997) on models of (partial) irreversibility, Caballero and Leahy (1996) on fixed cost models of investment and and Abel and Eberly (1994) who combine these together in a Tobin's Q framework.

the assumption that the other factor of production are perfectly flexible[3]. But both capital and labour are typically costly to adjust, and potentially becoming increasingly so as workers become more skilled and ICT goods become an increasing share of physical capital. Since capital and labour are linked in production, modelling their join dynamics will be important. In this paper we jointly solve for the optimal investment and employment behaviour with adjustment costs for each factor.

Aggregation across time and units

While micro-level data appears to be lumpy and intermittent firm data is smooth and continuous, suggesting that cross-sectional aggregation obscures the underlying non-convexities. Furthermore, even at the truly micro-level there is the additional problem of temporal aggregation, because the frequency of data collection is generally much lower than that of decision making. Firms often make investment and employment decisions on a weekly or monthly basis but data is typically collected quarterly or annually[4]. We explicitly build aggregation across units and across time into our model and estimation routine.

The paper starts in section 2 by building a model of firm behaviour, section 3 discusses our data and estimation routine. Section 4 discusses these results, while section 5 provides some policy simulations while section 6 concludes.

2. The Model

2.1. Overview

We model a firm as a collection of a very large, but fixed, number of production units. Each unit faces an iso-elastic demand curve for its product which is produced with Cobb-Douglas technology. Both demand and technology are affected by multiplicative shocks described by a geometric random walk with time varying drift and uncertainty. These shocks have a unit specific idiosyncratic component and a common firm component. We work in discrete time.

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[3] One of the exceptions was Shapiro (1986), who estimated a joint investment and employment demand equation with quadratic adjustment costs for each factor.

[4] For evidence of cross–sectional and time series aggregation see tables (A.1) and (A.2) in the Appendix. From this it appears that even within lumpy micro-level data there is evidence of cross-sectional aggregation with zeros occuring much more frequently in smaller single production units, reflecting the fact that even establishment level data may conceal several different plants each with multiple production lines.

Firms can adjust their capital stock, labour force and average hours. Capital and labour adjustment entails adjustment costs, while hours can be freely raised or lowered, but at the penalty of a higher hourly wage rate. Since our aim is to model infrequent and lumpy investment and employment at the micro-unit level these adjustments costs include a fixed cost and partial irreversibility component, as well as a more traditional convex cost component. As is standard in non-convex models the resulting microeconomic policy is one of inaction interspersed with periods of (potentially large) investment and employment bursts. Aggregation across units within the firm and across time up to yearly values obscures this lumpiness but nevertheless plays an important role.

2.2. The Production unit

Each production unit has a revenue function $R(Y, K, L, H)$

$$R(Y, K, L, H) = Y^{1-\alpha-\beta} K^{\beta} (L \times H)^{\beta} \qquad (2.1)$$

which nests a Cobb-Douglas production function in capital (K), labour (L) and hours (H) and an iso-elastic demand curve. Demand and technology conditions are combined into an index (Y) - henceforth called demand conditions[5]. Wages are determined by undertime and overtime hours around the standard weekly norm of 40 hours, following the form and parameters used in Cooper, Haltiwanger and Willis (2004)

$$w(H) = w_0 + w_1 \times (H - 40) + w_2 \times (H - 40)^2 \qquad (2.2)$$

Working hours can be costlessly adjusted (at the cost of higher hourly wages rates), so overtime acts as a short-run pressure valve in response to demand changes.

The demand conditions parameter Y evolves as a geometric random walk with a time-varying auto-regressive drift and variance

$$
\begin{array}{llll}
Y_t & = & Y_{t-1} \times (1 + \mu_t + \sigma_t V_t) & V_t \sim N(0, 1) & (2.3) \\
\mu_t & = & \rho_\mu(\mu^* - \mu_{t-1}) + \sigma_\mu X_t & X_t \sim N(0, 1) & (2.4) \\
\sigma_t & = & \rho_\sigma(\sigma^* - \sigma_{t-1}) + \sigma_\sigma Z_t & Z_t \sim N(0, 1) & (2.5)
\end{array}
$$

This generates permanent demand shocks with short run cycles of faster and slower growth, and short-run cycles of lower and higher uncertainty. Rapid growth and

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[5]Note that this unitary homogeneity in (Y,K,L) can be achieved from any iso-elastic demand curve and Cobb-Douglas production function by an arbitrary redefinition of the demand and technology shocks.

high uncertainty this period implies rapid expected growth and higher uncertainty next period, but in the long run all rates converge in expectation to the average level. This is consistent with the typical assumptions over the short and long run profile of firm growth in the finance valuation literature (i.e. Brealey and Myers, 2003, Copeland et al. 2003). It also implies a log-normal cross-sectional distribution of firm sizes, and also Gribrat's law that size and growth rates are independent, which while statistically rejected on large panels of data is a reasonable economic first approximation[6].

The third piece of technology determining the firms activities are the investment and employment adjustment costs. There is a long literature investigating and explaining investment and employment adjustment costs[7] which typically focuses on three terms, all of which we include in our specification:

1) Partial irreversibilities

The resale price of capital is usually less than the purchase price due to a combination of transactions costs, the market for lemons phenomena and the physical costs of resale. Labour partial irreversibility derives from hiring, training and firing costs. So we model

$$\text{Purchase price capital} = \quad B > S \quad = \text{Resale price capital} \qquad (2.6)$$
$$\text{PDV hiring 1 worker} = \quad H > F \quad = \text{PDV saved by firing 1 worker} \quad (2.7)$$

2) Fixed disruption costs

When new workers are added into the production process and new capital installed some downtime may result, involving a fixed cost loss of output. For example the factory may need to close for a few days while a refit is occurring, or hiring new workers will require fixed costs of advertising, interviewing and training. We model this in terms of units of lost output

$$\text{Investment disruption costs} \quad = \quad FCI \times R(Y, K, L, H) \times |I| > 0 \quad (2.8)$$
$$\text{Hiring disruption costs} \quad = \quad FCE \times R(Y, K, L, H) \times |E| > 0 \quad (2.9)$$

3) Quadratic adjustment costs

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[6] See Scherer (1980) for evidence on the widespread phenomenon of log-normal distributions of firm sizes. Evans (1987) and Dunne et al. (1989) test Gibrat's law on a large sample of firms and statistically reject it, finding a typical coefficient of around -0.03 on the regression of growth in logged sales on lagged logged sales. This implies an auto-regressive logged sales coefficient of 0.97, which is economically close to our coefficient of 1 in the geometric random walk.

[7] See, for example, Nickell (1977) and Hammermesh (1996).

The costs of investment and employment may also be related to the rate of adjustment due to higher costs for more rapid changes

$$\text{Quadratic investment cost} \quad = \quad \lambda_I K (\frac{I}{K})^2 \qquad (2.10)$$

$$\text{Quadratic hiring costs} \quad = \quad \lambda_E L (\frac{E}{L})^2 \qquad (2.11)$$

The combination of all adjustment costs is defined by the firm's adjustment cost function, $C(Y, L, I, E)$.

2.2.1. The Value Function

Investment and employment enters production immediately. At the end of each period capital and labour depreciates by δ_K and δ_L respectively so that, for example, $K_{t+1} = (K_t + I_t)(1 - \delta_K)$. The firms optimization problem can then be simplified by noting that the revenue function, adjustment cost function $C(Y, L, I, E)$, depreciation schedules and expectations operators are all jointly homogenous of degree one in (K, L, Y). This allows us to normalize by one state variable. Thus, after dividing (K, L, Y) by K the firms maximization can be stated as

$$
\begin{aligned}
Q(y_t, l_t, \mu_t, \sigma_t) \quad = \quad \max_{i,e,H} R(\frac{y_t}{1+i}, \frac{l_t(1+e)}{1+i}, H) - C(y_t, l_t, i, e) - \frac{l_t(1+e)}{1+i} w(H) \quad (2.12) \\
+ \frac{(1-\delta_K)(1+i)}{1+r} E[Q(y_{t+1}, l_{t+1}, \mu_{t+1}, \sigma_{t+1})]
\end{aligned}
$$

where lower case variables are $l = \frac{L}{K}$, $y = \frac{Y}{K}$, $i = \frac{I}{K}$ and $e = \frac{E}{L}$, and $Q(y, l, \mu, \sigma)$ is Tobin's Q. Normalizing by K is an essential step, which enables us to remove one major state variable from our problem, rendering our numerical solution routine computationally feasible.

2.3. Optimal investment and employment

The model is too complex to solve analytically and requires numerical simulation. However analytical results can be used to show the solution has a unique valued continuous solution and an (almost everywhere) unique policy function[8]. This

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[8]Application of Stokey and Lucas (1989) for quadratic and partially irreversible investment costs, Caballero and Leahy (1996) for the extension to fixed costs, and Alvarez and Stokey (1998) for unbounded homogeneous functions.

means any numerical results we derive will be convergent to the unique solution. Results for an illustrative set of parameters are displayed in figure (**??**) overleaf to illustrate the resultant policy functions. The plot contains 4 lines plotted in $(\frac{Y}{K}, \frac{Y}{L})$ space for values of the fire and hire thresholds (left and right lines) and the sell and buy capital thresholds (top and bottom lines). The inner region is the region of inaction where ($E = 0$ and $I = 0$) while outside that region investment and hiring will be taking place according to the optimal values of E and I. The gap between the investment/disinvestment thresholds is higher than between the hire/fire thresholds due to the higher irreversibilities of capital.

Figure (2.2) displays the same lines for two different values of current uncertainty - $\sigma = 0.1$ in the "inner box" of lines and $\sigma = 0.3$ for the "outer box" of lines. It can be seen that the comparative static intuition that higher time varying uncertainty also increases real options is confirmed here, suggesting that large changes in σ can have quantitatively important impacts on investment and labour demand behaviour even with time varying uncertainty.

Interestingly, re-computing these thresholds with permanent (time invariant) uncertainty results in a stronger impact on the investment and employment thresholds. So the standard comparative static results on changes in uncertainty will tend to over predict the expected impact of time changing uncertainty. The reason is that firms evaluate the uncertainty of their discounted value of marginal returns over the lifetime of the project, so high current uncertainty only matters to the extent that it drives up long run uncertainty. When uncertainty is mean reverting high current values have a lower impact on expected long run values than if uncertainty were constant.

2.4. Firm level investment

2.4.1. Cross-Sectional Aggregation

Aggregation up to the firm level is a difficult problem and several simplifying assumptions need to be taken. We assume each firm owns a number of production units. These units can be thought of as different production plants, different geographic markets, or different functions and divisions within the same firm. Production units are subject to both the common firm level demand process outlined above and a plant level idiosyncratic demand process with mean 0 and variance σ_U^2.

Firms own a sufficiently large number of these production units that any single unit level shock has no significant impact on firm behaviour - that is full
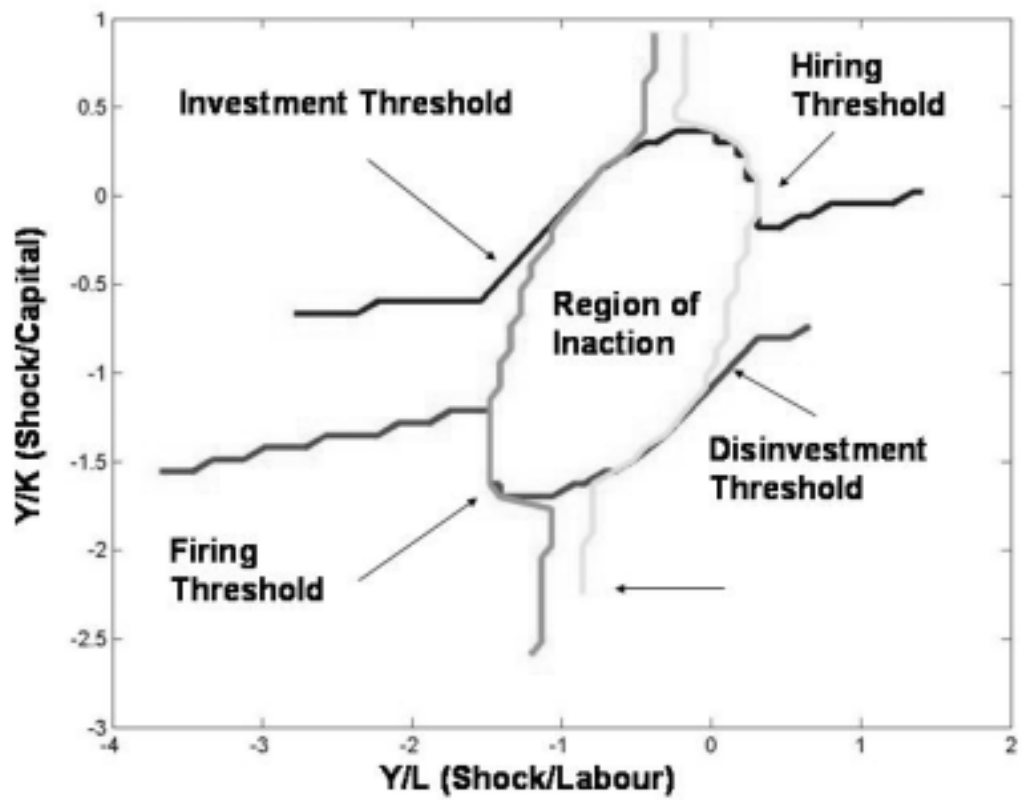
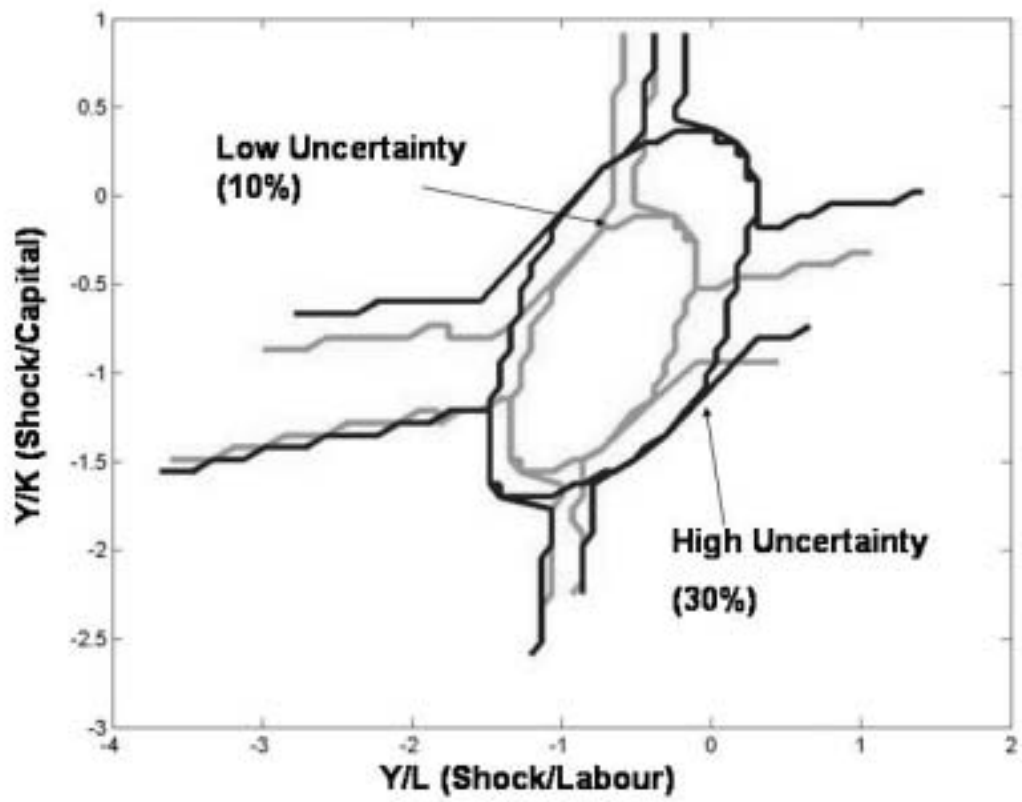Figure 2.1: Investment, Disinvestment, Hiring and Firing Thresholds

Figure 2.2: Thresholds with low and high uncertainty

aggregation occurs. Units independently optimized to determine investment and employment, following the optimal behaviour outline above, with firm values being stochastic aggregates[9]. Thus all linkages across units within the same firm are modelled by common shocks to the demand process in levels, growth or uncertainty. Thus, to the extent that units are linked over and above these common shocks we are assuming they independently optimize due to bounded rationality and/or localized incentive mechanisms (i.e. managers being assessed only on their own units P&L).

Our intuition is that a relaxation of this assumption to allow more complex interaction between units within firms - for example through internal capital markets or sales to common output markets - would reduce the smoothing of aggregation. This will be similar in fashion to a lower variance in the unit level shock σ_U^2. Thus, while our estimated value for σ_U^2 may be downwardly biased, since this is estimated as a free parameter it will help to capture the first-order impact of linkages across units. Thus, the behaviour of the model should hopefully approximate a more complex model with some interactions between units.

2.4.2. Time series aggregation

We model demand and technology shocks, investment decisions and employment decisions on a monthly basis. Management meetings in a typical business-unit generally happen at quarterly, monthly or even weekly frequencies, with this depending on market conditions, technology and management practices. The main point for us is that they almost invariably happen at a higher frequency than that of data collection, and that we need to explicitly model this. Since one of our main variables of interest - uncertainty - is measured in Compustat using monthly returns variance we selected a monthly frequency for modelling purposes, although in section X we experiment with estimation on quarterly data to investigate the role of time-series aggregation.

To generate a modelled counterpart to Compustat "flow" figures found in the accounting P&L, such as sales and capex, are added up across the year while "stock" figures found in the balance sheet, like the capital stock and labour force, are taken as year end values. Thus we aim to match up our modelled data as closely as possible to the Compustat empirical data.

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[9] This follows the approach of Bertola and Caballero (1994), Caballero and Engel (1999) and Abel and Eberly (1999) who driven by the need for analytical tractability also assume a large number of indepedently maximising units or lines of capital to model macro, industry and firm level investment respectively.

3. Empirical Evidence

3.1. Data

The data is a panel of firm from the US Compustat dataset from 1983-2001 inclusive which was extracted and initially constructed using the same procedure as Bond and Cummins (2004). The data was cleaned to remove mergers and acquisitions by dropping observations with large jumps in the sales, employment and capital stock figures. The sample was also trimmed to focus on large continuing firms by keeping only firms with at least 15 years of data and over 500 employees. This was done to reduce the impact of entry and exit and focus on larger more aggregated firms[10]. The summary statistics are detailed in the table (3.1) below.

Table 3.1: Descriptive Statistics for the Compustat Data Set.

	mean	median	stan. dev.	min.	max.
$(I_{i,t}/K_{i,t-1})$ (investment rate)	0.191	0.144	0.189	-0.595	1.997
$\Delta Y_{i,t}/Y_{i,t-1}$ (real sales growth rate)	0.073	0.048	0.215	-0.658	1.971
$dL_{i,t}/L_{i,t-1}$ (employment rate)	0.052	0.020	0.223	-0.659	2.000
$Y_{i,t}$ (real sales, 1990 \$M)	3855	652	12360	23.887	168,920
$L_{i,t}$ (employment, 1000's)	20.1	4.788	52.02	0.500	876.8
$\sigma_{i,t}$ (variance monthly returns,%)	33.13	32.32	11.66	6.581	80.779
observations per firm	17.3	18	1.04	15	18

3.1.1. The measurement of uncertainty

Measuring uncertainty is a tricky issue. What we need is a proxy variable which is well correlated with true uncertainty - σ_t^2 - which can be generated in simulated data and also in the Compustat data, and is measurable at an annual frequency. Fortunately, there is one variable that meets these exacting requirements, which is the variance of monthly share returns within each (accounting) year. Monthly share returns are calculated in the simulation as net cash flow plus capital gains per

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[10]While this focus on larger continuing firms reduces the need to model entry and exit decisions it does undoubtedly introduce a selection bias here. This selection halved the sample size from 1073 to 521 firms, although it only reduced the total number of employees and sales covered by 22.4% and 21.6% respectively.

month per $ of value, and its yearly standard deviation has a correlation with true uncertainty of 0.788. Monthly share returns are calculated in Compustat (data item RIXM) as net cash-flow (dividends, buy-backs and rights issues) plus capital gains per $ of value[11]. Thus our uncertainty proxy is both well correlated with true uncertainty and also measured on the same basis and at the same frequency in Compustat and in the simulated data.

3.2. Estimation Approach

This model has no analytical closed form so can not be written down in a form which would enable standard regression estimation. Instead estimation of the underlying parameters is achieved by a methodology called indirect inference (see Gourierioux, Monfort and Renault 1996, and Smith, 1993). The basis of this methodology is a regression (hereafter termed the reduced form regression) which is run on both actual and simulated data. The simulated data set is created by solving the dynamic programming problem given a vector of parameters. The resulting policy functions are then used to create a panel data set comparable to Compustat. The structural parameters are chosen so that the coefficients of the reduced form regression from the simulated data are "close" to the estimates from the actual data.

The choice of a reduced form regression is thus a crucial piece of the analysis. For our purposes, the reduced form regression needs to satisfy two criteria. First, the parameters of the regression should be "informative" about our underlying structural parameters. That is, as the structural parameters are varied the regression coefficients should be responsive[12]. Second, the reduced form regression should summarize relevant aspects of the investment and employment decisions. As emphasized above, the basic insights of the recent literature on non-convexities is that they imply rich cross-sectional non-linearities and time-series dynamics in the relationship between investment, employment and fundamentals, and uncertainty plays a central role in this. This is used to focus on two sets of reduced

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[11]This share returns variance measure has been previously used by Leahy and Whited (1996) and Bloom, Bond and VanReenen (2003). Share returns volatility are also significantly correlated at between 0.3 to 0.4 (depending on the calculation) with the dispersion of IBES analysts forecasts for each firm-year, which is the other typical firm-level uncertainty proxy used in the literature (see Bond et al, 2004).

[12]The formal condition is that there exists a one-to-one mapping between the structural parameters and the moments calculated from the data (the conceptual parallel to the rank condition in standard regression).

form regression equations: the first estimates the dynamics and non-linearities of investment and employment responses, and the second estimates the impact of uncertainty levels and changes on investment and employment.

Focusing first on the on the dynamics and non-linearities we estimate

$$i_{i,t} = \alpha_1 i_{i,t-1} + \alpha_2 s_{i,t-1} + \alpha_3 s_{i,t-1}^+ + \alpha_i \qquad (3.1)$$

$$e_{i,t} = \beta_1 e_{i,t-1} + \beta_2 s_{i,t-1} + \beta_3 s_{i,t-1}^+ + \beta_i \qquad (3.2)$$

where $i_{i,t}$, $e_{i,t}$ and $s_{i,t}$ are the investment, employment and sales growth rate of firm i in period t. The term $s_{i,t-1}^+$ is the sales growth rate multiplied by a indicator which is 1 for positive sales growth rate and zero otherwise. This is included to pick up any non-linearities in response to positive and negative sales changes, reflecting the importance of asymmetric responses in Compustat[13]. Lagged values are used to prevent the feedback from current investment and employment decisions into changes in current sales. Finally the terms α_i and β_i are included to control for unobserved heterogeneity[14]. The ability to condition for this in an indirect inference framework motivates our choice of this as an estimating strategy over matching unconditional moments.

Focusing secondly on the impact of uncertainty levels and changes we estimate

$$i_{i,t} = \alpha_1 sd_{i,t-1} + \alpha_2 \Delta sd_{i,t-1} \qquad (3.3)$$

$$e_{i,t} = \alpha_1 sd_{i,t-1} + \alpha_2 \Delta sd_{i,t-1} \qquad (3.4)$$

where $sd_{i,t}$ and $\Delta sd_{i,t}$ are the lagged level and change in logged returns uncertainty. This specification is chosen because of the central role that uncertainty plays in determining micro unit level thresholds, with α_1 and α_2 the elasticity of investment/employment to uncertainty. Combining all four reduced form regression equation together yields ten parameters in total.

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[13]An alternative specification which we experimented with was instead to include a sales growth squared term following, for example, Cooper and Haltiwanger (2002). We found in our Compustat data, however, that the coefficient on this squared term was extremely sensitive to the data cleaning routine used to remove mergers and acquisitions (and very large measurement errors) while the co-efficient on $s_{i,t-1}^+$ was not.

[14]Clearly there is unobserved heterogeneity in Compustat. To match our model with data requires us either to build the unobserved differences across plants into our analysis or to purge Compustat of these. We have chosen the latter approach, although in principle an exercise explicitly trying to build in heterogeneity through parameter variation across firms could be undertaken.

3.3. Estimation Procedure

The econometric problem consists of estimating the parameters that characterize (a) firms production function and mark-up, (b) the distribution of adjustment costs, (c) the wage curve, (d) the firm demand process, and (e) the unit level demand process. For tractability we need to limit the number of parameters being estimated and so take the starting values from the literature or Compustat wherever possible and use iteration to check their impact on the results.

These starting values (and their sources) are: (i) hiring cost of 1 month and firing cost of 2 months wages (Nickell, 1986 and XXXX); (ii) capital resale loss of 25% (Ramey and Shapiro, 2001); (iii) mean demand drift rate 5% (Compustat, average growth of real sales); (iv) mean-reversion in demand drift of 0.65 (Compustat, calculated from 5 year correlation of sales growth[15]); and (v) mean, variance and mean-reversion of demand uncertainty 50%, 30% and 0.30 (Compustat, estimated from a Garch regression on monthly returns index data).

This leaves six parameters for estimation $\Theta = (FCI, FCE, \lambda_I, \lambda_E, \sigma_\mu^2, \sigma_U^2)$, the fixed and quadratic adjustment cost parameters for investment and employment, the variance of the demand drift and the unit level uncertainty. With ten reduced form regression parameters and six estimation parameters the estimation is over-identified, and these parameters can be estimated as follows. For an arbitrary value of Θ the dynamic program is solved and policy functions are generated. using these policy functions the decision rule is simulated and given arbitrary initial conditions to create a simulated Compustat. The reduced form regressions (3.1), (3.2), (3.3) and (3.4) are then estimated on the simulated Compustat. Let $\Psi^S(\Theta)$ denote this vector of stacked regression coefficients ignoring the fixed effects, and further let Ψ^C represent the same stacked vector of coefficients from Compustat.

The estimated value $\widehat{\Theta}$ then minimizes the weighted distance between the actual and simulated regression coefficients. Formally, we solve

$$\Gamma(\Theta) = \min_{\Theta}[\Psi^C - \Psi^S(\Theta)]'W[\Psi^C - \Psi^S(\Theta)] \tag{3.5}$$

where W is a weighting matrix. We use the optimal weighting matrix given by the inverse variance-covariance of the regression coefficients from our true Compustat regressions (3.1), (3.2), (3.3) and (3.4), calculated through bootstrapping[16].

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[15]A 5 year gap was taken to allow the impact of adjustment costs on the dynamics of sales to dissipate.

[16]The bootstrap estimates these regressions 1000 times on Compustat data (drawn with replacement) to generate the VCV between these 1000 coefficient vector estimates.

Of course, the $\Psi^S(\Theta)$ function is not analytically tractable. Thus, the minimization is performed using numerical techniques. Given the potential for discontinuities in the model and the discretization of the state space, we used a simulated annealing algorithm to perform the optimization.

3.4. Results

The model is simulated on a grid of $(\frac{Y}{K}, \frac{L}{K}, \mu, \sigma)$ of dimension (40,40,3,3) with an hours grid of (36, 40, 44). The number of units per firm is set at 100 as a computationally feasible approximation to full aggregation[17]. This currently takes about 2 hours to converge, which is too slow for simulated annealing so **RESULTS ARE CURRENTLY CALIBRATED RATHER THAN ESTIMATED** at present while we speed up the program and wait for a more powerful PC (presently 2003 vintage). Calibration results are based on experimentation with around 40 different parameter choices, thus are not fully optimized with no standard-errors or over-identification tests.

For interpretation alongside the results for the calibrated parameter set we also display results from two illustrative parameters sets:

Investment Costs Only

This model assumes labour is fully flexible and only capital incurs adjustment costs. This is included to understand the impact of only modelling the adjustment costs for one factor of demand (capital in this example) on the behaviour of both factors. That is, to what extent will making the common assumption in the literature that labour is fully flexible and only capital incurs adjustment costs generate sufficient smoothing and non-linearities to fit both the investment and employment series? In this draft version of the paper we present results for same parameters set as our calibrated model except without any employment costs. While making direct comparisons easier, this analysis of the investment costs only case is not a "fair" test of the model since the parameters are not optimally chosen. So the next steps when the speeded up simulation is running will be to estimate the optimal parameters for an investment only adjustment cost function and compare this directly to our results for investment and employment adjustments costs.

Quadratic Costs, no aggregation

This model assumes that only quadratic costs for investment and employment occur (so no fixed-costs or partial irreversibilities) and that the firm only operates

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[17]It appears that adding any more units beyond 100 has no significant impact on the reduce form regression parameters, and so mimics full aggregation. We are currently looking into this.

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one unit. This is included to understand to what extent would a typical convex adjustment costs model of firm fit our investment and employment data. By assuming quadratic adjustment costs at the firm level we can generate smooth firm level data without resorting to aggregation, so this tests to need for aggregation to generate smoothing. In addition with quadratic-costs only uncertainty has no real-options impact so a second test is on the impact of uncertainty in the model. That is if we remove any real options effects of uncertainty what is its role in this model in explaining investment and employment.

The parameter vectors for the calibrated model and the two comparison models are outlined in table (3.2) below. This includes both the free calibrated parameters and the values for prior fixed parameters if these are changed across models.

Table 3.2: Key parameters in the simulated model

Parameters	Free (Calibrated)						Prior		
	σ_μ^2	σ_U^2	λ_I	λ_E	FCI	FCE	$S - B$	$H + F$	units
Calibrated	5%	30%	0.75	0	0	2/52	0.25	3/12	100
Investment costs only	5%	30%	0.75	0	0	0	0.25	0	100
Quadratic costs only	5%	30%	0.75	0.5	0	0	0	0	1

3.4.1. Investment and Employment dynamics

Table (3.3) reports the estimated reduced form regression coefficients for Compustat and our three versions of the model. Turning first to Compustat there is clear evidence of strong dynamics in investment, which even after removing fixed effects, shows a large coefficient on lagged investment as well as lagged sales. Employment appears to show less dynamics with a low negative coefficient on the lag, although this is still significant. One issue for estimating dynamic employment equations is measurement error[18]. Since the employment rate is a differenced transformation of employment levels, measurement error in the levels will generate a negative (and potentially large) bias in the autoregressive employment rate coefficient, which provides one potential explanation for the seemingly low degree of auto-correlation which we explore below.

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[18] Of course by introducing fixed-effects was are also biasing down the coefficient on the lagged dependent variable, but this is of order $\frac{1}{T}$ (Nickell, 1983) and we have around 18 years per firm on average.

Looking next at asymmetries the large and significant coefficient on the positive sales growth term $s_{i,t-1}^+$ for both investment and employment provides evidence of a strong non-linearities.

Table 3.3: Aggregation Obscuring Zero Investment Episodes.

Equation	Investment rate, $i_{i,t-1}$			Employment rate, $e_{i,t}$		
Coefficient	$i_{i,t-1}$	$s_{i,t-1}$	$s_{i,t-1}^+$	$e_{i,t-1}$	$s_{i,t-1}$	$s_{i,t-1}^+$
Compustat	0.202*	0.046*	0.118*	-0.032*	-0.076*	0.207*
Calibrated	0.212*	-0.021*	0.139*	0.034*	-0.011	0.154*
Investment Costs Only	0.100*	-0.016	0.249*	0.150*	-0.185*	0.213*
Quadratic Cost Only	0.576*	-0.045*	-0.115*	0.610*	-0.121*	-0.159*

Note: * indicates coefficient is significant at the 5% level

For the calibrated model comparing to the investment results to Compustat the match is reasonably good, with a similarly strong dynamics (indicated by the coefficient on lagged investment) and a high level of asymmetries (indicated by the coefficient on $s_{i,t-1}^+$). The fit of the employment equation is far less satisfactory displaying a low but positive autoregressive coefficient and less asymmetries than implied by the Compustat data. One potential rationale is insufficiently high values for the fixed costs, which tends to generate negative auto-correlation. Although we could raise the value of this parameter the current value of 2 weeks of global revenue seems high so this is unappealing, and so instead in section (3.4.3) we investigate an alternative explanation based on measurement error.

In the third row we report the results for a model with investment costs only. Interestingly both the investment and the employment coefficients change, with investment becoming less autoregressive and more asymmetric because employment adjustment costs play an important role in driving investment responses. The reason is the auto-regressive investment coefficient is driven partly by lagged investment proxying for the distribution of Y/L (demand conditions/labour) across units between their hiring and firing thresholds. Since this distribution is auto-correlated, indirectly drives investment and is not proxied by sales it will be picked up partly by lagged investment. Without these employment adjustment costs the hiring and firing thresholds are the same, so there is no distributional dynamics, reducing the size of the lagged investment coefficient and so increasing the size

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of the direct sales coefficient. Turning to the employment equation, despite the absence of any adjustment costs, this still displays substantial dynamics with a large autoregressive coefficient and a strongly asymmetric response. This suggests that investment costs can lead through to significant employment dynamics.

Finally in the fourth row the quadratic adjustment cost model with only one unit per firm displays extremely strong autocorrelated dynamics for both investment and employment. It also appears to display a negative response to lagged shocks with negative asymmetries, although this is driven by the high-coefficient on lagged investment and employment. In the absence of any lagged investment and employment terms the coefficients on $s_{i,t-1}$ and $s_{i,t-1}^+$ rise to 0.425 (0.015) and 0.05 (0.02) for investment and 0.410 (0.016) and 0.021 (0.021) for employment, which is positive with small or insignificant asymmetry. Hence, while the quadratic model can generate smooth firm level investment behaviour without the need for cross-sectional aggregation is generates (at least for this parameter choice) investment and employment dynamics which do not match those observed in Compustat

3.4.2. Uncertainty levels and changes

Table (3.4) reports the estimates from the reduced form uncertainty levels and changes regression. Looking at row one for the Compustat regressions we can see a positive correlation with the level of uncertainty and a negative correlation with the change in uncertainty, and this is closely replicated in the second row for the calibrated model. The reason for the negative coefficient on the change in uncertainty is that higher uncertainty moves the thresholds apart and reduces the responsiveness of investment and employment. Since the median value of both of investment and employment is positive the short-run net effect of an increase (decrease) in uncertainty is positive (negative). This is supported by a sample split around median investment and employment in Compustat and the simulated data, where in all four cases the response to changes in uncertainty for the bottom 50% was much lower than for the top 50% and generally insignificant or positive[19]. The positive coefficient on the uncertainty levels term appears to derive from a correlation between large positive shocks and high values of current uncertainty.

In the third row the results for the model with investment costs shows a sim-

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[19]In Compustat the coefficient (standard error) on lagged change in uncertainty for investment and employment below the median is 0.012 (0.0018) and 0.017 (0.004) respectively. In the callibrated data the same coefficients are -0.001 (0.001) and -0.001 (0.001).

ilar level of positive levels and negative coefficients for investment, but with a lower levels and insignificant change coefficients for employment. For investment this arises from the real-options effects on investment discussed above, but for employment since there are no adjustment costs this only arises from a weaker indirect effect from investment. Finally in the fourth row the signs are reversed for the quadratic model, which is due to the absence of any real options effects and concavity on the value function leading to a net negative levels effect.

Table 3.4: Investment and Employment Response to Uncertainty

Equation	Investment rate, $i_{i,t}$		Employment rate, $e_{i,t}$	
Coefficient	$sd_{i,t-1}$	$\Delta sd_{i,t-1}$	$sd_{i,t-1}$	$\Delta sd_{i,t-1}$
Compustat	0.039*	-0.019*	0.027*	-0.017*
Calibrated	0.036*	-0.012*	0.033*	-0.010*
Investment Only	0.033*	-0.010*	0.016*	-0.002
Quadratic Only	-0.018*	0.009*	-0.018*	0.009*

Note: * indicates coefficient is significant at the 5% level

3.4.3. Measurement error in employment levels

An issue for understanding the dynamics of employment is the role of measurement error. As a non-financial measure it is typically provided in company accounts to provide a guide to firm size and so is often heavily rounded (31% of employment figures end in 00 versus only 1% of sales figures). Since employment growth rates are generated from changes in employment levels errors in levels will generate negative correlations in first differences[20].

To understand the potential impact of this we added normally distributed iid measurement error with a mean absolute deviation of 2.5% (benchmarked for the median firm of 5,000 that rounds its employees numbers to the nearest against 50) to our simulated data. From table (3.5) it is clear that even this moderate level of measurement error generates negative auto-correlation, with the lagged coefficients in all three models falling substantially. There is also a much closer overall correspondence between Compustat and the calibrated model, suggesting

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[20]For capital this is less of an issue since iid errors in the measured capex and the initial capital stock will not lead to any strong bias in the estimated dynamics.

a model with reasonable employment adjustment costs and small iid measurement error in employment levels could be consistent with observed data.

Table 3.5: Impact of measurement error on estimated employment dynamics

Equation	Employment		
Coefficient	$i_{i,t-1}$	$s_{i,t-1}$	$s^+_{i,t-1}$
Compustat	-0.032*	-0.076*	0.207*
Calibrated + measurement error	-0.037*	0.075*	0.193*
Investment Only + measurement error	0.025*	-0.055*	0.244*
Quadratic Only + measurement error	0.354*	0.084*	-0.056*

Note: * indicates coefficient is significant at the 5% level

3.5. Robustness Checks

3.5.1. Estimation on quarterly Compustat data

Compustat also provides quarterly data on sales and investment (but not employment), and the monthly returns index which can be used to generate a quarterly returns variance. Thus, to investigate the time aggregation issue further we simulate quarterly sales, investment and returns variance data and compare the investment regressions coefficients (3.1) and (3.3) to those from estimation on similar quarterly Compustat data.

Table 3.6: Quarterly estimation on actual and simulated investment

Equation	Investment rate, $i_{i,t}$			Investment rate, $i_{i,t}$	
Coefficient	$i_{i,t-1}$	$s_{i,t-1}$	$s^+_{i,t-1}$	$sd_{i,t-1}$	$\Delta sd_{i,t-1}$
Compustat					
Calibrated + measurement error					
Investment Only + measurement error					
Quadratic Only + measurement error					

3.5.2. Estimation by industry sector

4. Policy Simulations

This model can be used to study the micro distributional and macro aggregated investment and employment impact of shocks and policy changes. The three experiments we run here are: firstly the effect of a large once-off temporary rise in uncertainty; secondly a permanent rise in the level of employment adjustment costs, and finally a rise in taxation modelled by introducing taxes into the value function (2.12). To evaluate this we take a steady-state distribution of firms, expose them to the shock/policy change, and compare the resultant behaviour to the simulated counterfactual with no shocks/policy changes.

4.1. Rise in uncertainty following a large macro-shock

4.2. Bringing US Labour Adjustment Costs to European Levels

4.3. Reducing corporation tax by 1%

5. Conclusions

A. Appendix

Table A.1: Aggregation Obscuring Zero Investment Episodes.

% zero investment	Structures	Equipment	Vehicles	Total
Firms	5.9	0.1	n.a.	0.1
Establishments (All)	46.8	3.2	21.2	1.8
Establishments (Single Plants)	53.0	4.3	23.6	2.4
Establishments (Small, Single Plants)	57.6	5.6	24.4	3.2

Note: Source UK ARD and Datastream

Table A.2: Aggregation Reduces Time Series Volatility.

% standard deviation growth rates	Quarterly	Yearly
Sales	43.2	25.0
Investment	45.6	24.3

Note: Source Compustat

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