Subjective Mortality Risk and Bequest Motivation

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July, 2004
(Preliminary)

Abstract: This paper investigates whether subjective expectation about future mortality affect bequest motives in the presence of a borrowing constraint. We estimate a dynamic life-cycle model based on subjective survival rates and wealth from the panel dataset of Asset and Health Dynamics among Oldest Old. We find that the bequest motives are small on average, which indicates that most bequests are in fact involuntary or accidental. Moreover, parameter estimates using subjective mortality risks can perform better in predicting out-of-sample wealth levels than estimates using life table mortality risks, suggesting individuals' decisions in consumption and saving are consistent to what they believe in their own mortality risks.

KEYWORDS: subjective mortality risk bequest motives, median regression

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I. Introduction

It is known in the literature that a significant portion of household wealth is passed from one generation to another by bequest. According Kotlikoff and Summers (1981), 80% household-held capital was inherited. Gale and Scholz (1994) estimate that total bequests were $105 billion in the U.S. in 1986. Hurd and Smith (2002) find that the elderly anticipate leaving roughly 40% of their wealth in bequests. Kotlikoff (1988) claims that inherited wealth plays an important and perhaps dominant role in U.S. wealth accumulation. Thus it is conceivable that bequests may hold a key answer to the social security problem that baby boomers may face: they may eventually receive significant estates from their parents such that their dependence on social security may be reduced.

However, predicting whether a large portion of wealth will be passed from one generation to the next generation requires knowledge of the motives for bequests.[2] As pointed out in the literature (Kotlikoff 1988; Hurd 1989), a large amount of bequeathed wealth does not necessarily imply a substantial motive for bequests. Without a well-functioning annuity market, people will have to save against mortality risk, and the resulting bequests are involuntary. If most bequests are in fact involuntary or accidental, the value of the bequeathed wealth may decrease in the future as the annuity market further develops.[3] In addition, it is also possible that people may change their perceptions of stock market risks after the recent crash of the market. In that case, more people may move into annuities, and the total amount of bequeathed wealth will decrease.[4]

There is no consensus in the literature on the significance of bequest motives. Some people (Bernheim 1987; Kotlikoff and Summers, 1988) argue that the bequest motive is important while others (Hurd 1989) claim that it is almost zero, and most bequests are accidental or involuntary.

It is well known that subjective expectations about future events are important factors to understand individual economic behaviors, such as saving, consumption and investment.

[2] Various incentives for bequest are offered in the literature. Some argue that bequests serve as incentives to younger generations to provide appropriate care for older generations (Cox 1987; Bernheim, Shleifer and Summers, 1985). Others argue that bequests are mainly motivated by altruism.
[3] Poterba (1997) documents that variable annuity premium payments increased by a factor of five during the period 1988-1993.
[4] The S&P 500 index peaked on August 2001 at 1517.7. Since then, it has dropped to 879.8 at the end of 2002.

However, few available information or data on individual subjective expectations limits the application of economic models to explain individual actions. Our main goal in this paper is to investigate the empirical relevance of subjective survival rates for bequest motives. . More specifically, we want to exam if and to what extent subjectively expected mortality risks are correlated with bequest motives in the presence of a borrowing constraint for single elders. We estimate a life cycle model with uncertain lifetime as, developed by Yaari (1965) and Hurd (1989). Instead of applying the commonly used life tables to approximate individual survival expectations, we adopt the estimated individual subjective survival curves from Gan, Hurd and McFadden (2003, henceforth GHM).

Empirical estimates that are based on life-table survival curves are likely to be biased. For example, consider a typical utility function:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

where c_t is the consumption at time t, and γ is the risk aversion parameter. The first order condition in a common formulation (without a bequest motive) is:

$$\Delta \ln c_t \approx (r + \ln \beta + \Delta \ln s_t)/\gamma + f(X_t),$$

where X_t represents some socio-demographic and/or economic variables, r is the interest rate, and β is the time discount factor. s_t is the subjective survival probability at time t so that $-\Delta \ln s_t$ is the mortality hazard rate. If s_t is not measured but it is correlated with X_t, we have a classic problem of endogeneity. If s_t is measured with error, the parameter estimate of γ will be biased.

A large panel dataset, the Asset and Health Dynamics among Oldest Old (AHEAD) collects data on people who were born between 1890 and 1923 and their spouses (regardless of age) including information on individuals' expectations of a wide range of future events. Respondents in the survey are asked about their expectations of chances to live to a certain age. Earlier work, such as Hurd and McGarry (1995, 2002) and GHM have looked at the subjective probabilities regarding survival rates. These papers have found that, on average, individual subjective survival probabilities are consistent with life tables and that they vary appropriately with known risk factors. Therefore, there is important information content in these responses on subjective survival probabilities.

However, the subjective survival probabilities suffer serious focal response problems: some individuals tend to give responses of 0.0 and 1.0. These focal responses cannot be directly used in analyzing life-cycle models where survival probabilities are required. To eliminate focal biases, GHM suggest a Bayesian update method. For each individual in the AHEAD data set, GHM estimate an “optimism” index. Compared to the life table survival probability, an individual may overestimate or underestimate his/her survival probability. The estimated “optimism” indices show significant individual heterogeneity, and can be applied to derive individuals’ subjective survival probabilities without focal biases.

The rest of the paper is organized as follows. In Section 2, we introduce a life-cycle model with bequests. Our emphasis is on how to estimate such a model. Section 3 presents the estimation results. In particular, Section 3.1 introduces the data that will be used in the paper. Three key variables are used in the empirical variables: wealth, income and subjective survival probabilities. In Section 3.2, we present parameter estimates based on various estimation methods. Section 3.3 calculates the bequest incentives based on estimates from Section 3.2. In Section 3.4, we conduct out-of-sample predictions and simulate the consumption and wealth trajectories under various sets of parameter estimates. Finally, we summarize the results of this paper and discuss several issues for future research in Section 4.

II. The Model

Our starting point is the standard life-cycle model with bequest as in Yaari (1965) and Hurd (1989). Let the utility function of a retired individual be:

$$\sum_{t=0}^{N} \beta^t U(c_t) s_t + \sum_{t=0}^{N} \beta^t B(w_{t+1}) m_{t+1} \tag{1}$$

where s_t is the subjective probability that the individual will be alive at time *t*. m_{t+1} is the subjective mortality rate at time $t + 1$: $m_{t+1} = s_t - s_{t+1}$. The subjective maximal number of periods an individual can survive is *N*. The time discount factor is denoted as β. Consumption at time *t* is denoted as c_t, and the wealth at the beginning of time *t* is denoted as w_t. The first term in (1) is the present value of utility from consumption conditional on survival; and the second term in (1) is the present value of the utility from leaving a bequest of w_{t+1} conditional dying at $t + 1$. The utility from bequest, $B(w_{t+1})$, is increasing in w_{t+1}.

This model only applies to singles. The corresponding model for couples is much more complicated because it has to account for bequeathing by a couple to the next generation, and also for providing to a surviving spouse.[5]

As in Hurd (1989), we further assume a borrowing constraint such that bequeathable wealth cannot become negative. The constraint imposed on borrowing indicates that future Social Security benefits cannot be used as collateral for a consumption loan. This constraint arises from the fact that all heads of households in the sample are older than 70 years old in 1993 when the survey started, and in the U.S., Social Security benefits cannot be used as collateral. Such a constraint imposes important boundary condition in our analysis. Equation (2) lists the budget constraint at time *t*:

$$w_t = (1+r)w_{t-1} + A_{t-1} - c_{t-1} \geq 0, \qquad (2)$$

where A_{t-1} is annuity income at time *t*-1.

It is typical in this literature to assume a constant risk aversion utility function $U(c_t) = c_t^{1-\gamma} / (1-\gamma)$. The income from annuities such as Social Security is assumed to be constant. The marginal utility of a bequest, denoted as α, is dependent on how many children the person has:

$$B_w \equiv \alpha \equiv \frac{\partial B}{\partial w} = 1_{\text{children}}\left(\alpha_0 + \alpha_1 * \text{No. of children}\right), \qquad (3)$$

where 1_{children} is an indicator function. The assumption that the bequest motive exists only if the person has any children is important to identify the model. Otherwise, the identification may only come from the functional form assumptions.

The maximal age that a person may live, denoted as *N*, is obtained when the person's subjective survival rate $s_t < 1e\text{-}4$. Different agents have different maximum ages *N* since their subjective survival rates are different. Given the interest rate *r*, income *A*, and the parameter values of β, γ, and α, the paths of wealth are always contingent on the initial wealth w_0. However, the paths of consumption may not dependent on the initial wealth w_0. The solution to the optimization problem depends on whether the borrowing constraint is binding or not. The analysis of the solution of the discrete model is similar to that of the continuous model in Hurd (1989). Here we only state how to estimate the model.

[5] Estimating the couple's bequest motive is our next research objective.

Estimating the model requires at least two waves of wealth data for each individual. We use wealth data in wave 2 and wave 3 to estimate the model. The wave 4 wealth data is used for out-of-sample prediction.[6] The wealth level in wave 2 serves as the initial wealth w_0. We use backward induction to find the trajectories of the wealth and consumption. For a given set of parameter values β, γ, and α, we can obtain the trajectories of wealth $\{w_t^b, t = 1, \cdots, N+1\}$, where the superscript b indicates the value is calculated from backward induction. We then compare w_3^b at the trajectory with the observed wave 3 wealth w_3^*. We use the subscript 3 because in our data set the interval between the two waves of wealth is 3 years. The parameter set that minimizes the difference between w_3^b and w_3 are our estimates.

There are three types of consumptions paths corresponding to low, medium, and high wealth. We discuss these three different cases in the discrete model:

(1) In the first case, the bequest is strictly positive even if the individual survives to the greatest age possible: i.e., $w_{N+1} > 0$. Then the consumption trajectory satisfies:

$$c_t^{-\gamma} s_t = \alpha \sum_{i=t}^{N} \beta^{i-t} (1+r)^{i-t} m_{i+1} \tag{3a}$$

The trajectory in (3a) and actually initial wealth, w_0, satisfy

$$w_{t+1} = (1+r)^{t+1} w_0 + \sum_{i=0}^{t} (1+r)^{t-i} (A_i - c_i) > 0 \,. \tag{3b}$$

Equation (3a) shows that consumption trajectory is dependent of subjective survival rate but is independent of initial wealth w_0 if the wealth level at $N+1$ is strictly positive. This occurs because the marginal utility from consumption (left-hand-side) at time t has to equal to the present value of the marginal utility from bequest, which is assumed to be independent of wealth level. The wealth trajectory, w_t^b, can be calculated from the equation (3b), which shows that wealth trajectories vary according to the initial wealth w_0. Figure 1-1 shows the typical consumption and wealth trajectories. As illustrated, while wealth monotonically increases and consumption monotonically decreases, they may exhibit other patterns. The only requirement for this case is the wealth is strictly positive at any time in this person's life span.

[6] There is strong evidence that wave 1 wealth data in AHEAD underestimate the stock ownership and hence the value of the stock wealth.

The minimal level of initial wealth that corresponds to the consumption path (3a) is w_0^*, given by:

$$w_0^* = \sum_{i=0}^{N} (1+r)^{-i-1}(c_i - A) > 0 .$$

Any initial wealth larger than $w_0 > w_0^*$ will produce a consumption path $\{c^*\}$ as in (3a), and will lead to $w_{N+1} > 0$. Note that both N and w_0^* vary as individual subjective survival rate varies.

(2) In the second case, although the bequest is zero at the time of death, ($w_{N+1}= 0$), the borrowing constraint is not binding; that is, the wealth level is strictly positive for any $t<N+1$. The consumption path satisfies:

$$c_t^{-\gamma} s_t = \beta(1+r)c_{t+1}^{-\gamma} s_{t+1} + \alpha m_{t+1}, \text{ for } t = 0,1,\ldots N-1 \quad (4a)$$

$$w_{N+1} = (1+r)^{N+1} w_0 + \sum_{i=0}^{N} (1+r)^{N-i}(A_i - c_i) = 0 \quad (4b)$$

$$w_t > 0, \text{ for } t = 1,2,\ldots N. \quad (4c)$$

Equation (4b) states that the consumption trajectory should lead to zero wealth level at time N +1: the person will leave no bequest should he or she live to the greatest age possible. Figure 1-2 illustrates one case where wealth reaches zero exactly at the maximum possible age. Consumption in Figure 1-2 first increases and then decreases as mortality risk becomes large. However, it is possible that consumption monotonically decreases if the time discount factor is small.

There will be a range of initial wealth and associated consumption paths that satisfy (4a), (4b) and (4c). The intuition for this result will be discussed when we provide estimation algorithm (Step 2 in the algorithm). Let w_0^* be the largest of these values so that any value of w_0 larger than w_0^* leads to $w_{N+1} > 0$ and the consumption path will be independent of w_0. Let $\hat{w}_0$ be the smallest of those values so that any smaller value of initial wealth causes the wealth to reach 0 before N +1. Let $\{\hat{c}\}$ and $\{\hat{w}\}$ be the individual's consumption and wealth trajectories associated with $\hat{w}_0$, and $\{c^*\}$ and $\{w^*\}$ be the individual's consumption and wealth trajectories associated with w_0^*. Therefore, in the case of medium wealth, the consumption trajectory must lie between $\{\hat{c}\}$ and $\{c^*\}$, and the wealth trajectory must lie between $\{\hat{w}\}$ and $\{w^*\}$.

(3) Lastly, we consider the case that the borrowing constraint is binding. Let T be the time when bequeathable wealth is exhausted. The consumption path is found from the solutions to four equations, (5a)-(5d):

$$c_t = A \text{, for } t = T, \cdots, N \text{,} \tag{5a}$$

$$c_t^{-\gamma} s_t = \beta(1+r) c_{t+1}^{-\gamma} s_{t+1} + \alpha\, m_{t+1} \text{, for } t = 0, 1, \ldots T-2 \text{,} \tag{5b}$$

$$w_T = (1+r)^T w_0 + \sum_{i=0}^{T-1} (1+r)^{T-1-i} (A_i - c_i) = 0 \,. \tag{5c}$$

$$w_t > 0 \text{, for } t = 1, 2, \ldots T-1. \tag{5d}$$

In this case consumption and wealth will eventually decline. Figures 2-3 and 2-4 illustrate two possible consumption and wealth trajectories in this case.

Each individual in our sample has a different subjective survival curve. Therefore, every individual's critical value of wealth is different. We search to find out his/her critical wealth value, and then calculate his/her consumption and wealth trajectories. Our objective is to find a set parameter values that minimize the difference between the predicted second wave wealth, w_3^b, and the observed second wave wealth, w_3. We consider two different objective functions: mean square loss function and the absolute value loss function.

$$\min_{\alpha,\beta,\gamma} \sum_i \left(w_{3i} - w_{3i}^b\right)^2 \tag{6a}$$

$$\min_{\alpha,\beta,\gamma} \left\{ \sum_i \left| w_{3i} - w_{3i}^b \right| \right\} \tag{6b}$$

The mean square loss function in (6a) is the one used in Hurd (1989). The absolute value loss function in (6b) corresponds to median regression. The advantage for median regression over the mean regression is that median regression is robust to outliers.

We apply the Quasi-Newton method to mean square loss objective function (6a) and Nelder-Mead Simplex method to absolute value loss objective function (6b). For any given set of parameters, β, γ, and α, we need to find the predicted wave 3 wealth for each individual. Following is the detailed algorithm to find w_3^b.

Step 1: Check the high wealth case, in which a strictly positive bequest is left at the maximum age of life, i.e., $w_{N+1} > 0$.

(1) From equation (3a), we calculate the consumption trajectory $\{c_t^b, t = 0, \cdots N\}$.

(2) Substitute the trajectory of consumption $\{c_t^b, t = 0, \cdots N\}$ into Equation (3b) to get the wealth trajectory $\{w_t^b, t = 1, \cdots, N+1\}$.

(3) If for all $t \in \{1, 2, \ldots, N\}$, $w_t^b \geq 0$ and $w_{N+1}^{\ b} > 0$, then report w_3^b and go to next observation; else go to *Step 2*.

Step 2: Check the medium wealth case, in which the wealth at the end of maximum age of life is zero, i.e., $w_{N+1} = 0$, and at all other time periods $t \leq N$, $w_t > 0$. We use backward induction to get the consumption and wealth trajectories.

(1) From (4a), c_t ($t = 0, \ldots, N$ -1) is a function of c_N by recursive iteration: $c_t = c_t(c_N)$. Substitute the trajectory of consumption $\{c_t(c_N), t = 0, \ldots, N -1\}$ into Equation (4b) such that wealth level in (4b) now is only a function of c_N . In particular, we have:

$$w_{N+1}(c_N, w_0) = 0 \tag{7}$$

Given observed w_0, we can solve (7) to get c_N, denoted as c_N^b . Given c_N^b , we can apply (4a) to iteratively find out $\{c_t^b, t = 0, \cdots, N-1\}$. However, if we do not know w_0, we will have many values of c_N and w_0 such that (7) are satisfied. Among them, the higher bound w_0^* is the maximum of w_0 such that (7) is satisfied and $c_t > 0$ for all $t < N+1$; the lower bound $\hat{w}_0$ is the smallest w_0 such that (7) is satisfied and $c_t > 0$ for all $t < N+1$.

(2) If for all $t \in \{0, 1, \ldots, N\}$, $c_t^b > 0$, then calculate the wealth trajectory $\{w_t^b, t = 1, \cdots, N\}$ from Equation (2); else go to *Step 3*.

(3) If for all $t \in \{1, 2, \ldots, N\}$, $w_t^b > 0$, then report w_3^b and go to next observation; else go to *Step 3*.

Step 3: Check the low wealth case, in which the wealth reaches zero at a time period $T \leq N$. We search all over the possible T from the backward. The method is similar to *Step 2*.

(1) Let $T = N$. From (5b), c_t ($t = 0, \ldots, T$ -2) is a function of $c_{T\text{-}1}$ by recursive iteration: $c_t = c_t(c_{T\text{-}1})$. Substitute the trajectory of consumption $\{c_t(c_{T\text{-}1}), t = 0, \ldots, T -2\}$ into Equation (5c) such that (5c) now is only a function of $c_{T\text{-}1}$. Solve the equation: $w_T = 0$ to get $c_{T\text{-}1,}$ denoted as c_{T-1}^b . We can get the consumption trajectory $\{c_t^b, t = 0, \cdots, N\}$ by applying (5b) with given c_{T-1}^b .

(2) If for all $t \in \{0, 1, \ldots, T-1\}$, $c_t^b > 0$, then calculate the wealth trajectory $\{w_t^b, t = 1, \cdots, T-1\}$ from Equation (2); else let $T = T$-1, and repeat (1) - (2).

(3) If for all $t \in \{1, 2, \ldots, T-1\}$, $w_t^b > 0$, then break from the cycle, report w_3^b and go to next observation; else let $T = T$-1, and repeat (1) - (3).

Finally, we briefly discuss how to estimate the covariance matrix. Let the parameter set be denoted as *δ=(γ, β, α)'*, and let the covariance matrix be Ω. It is straightforward to obtain the covariance matrix for estimates based (6a). The covariance matrix from median regression in (6b) is given by:

$$\Omega = \frac{1}{4 f_u^2(0)} E\left[\left(\frac{\partial w_3^b}{\partial \delta}\right)\left(\frac{\partial w_3^b}{\partial \delta}\right)'\right],$$

where $f_u(0)$ is the density of the error term u evaluated at 0. The error term u is defined as $u = w_3 - w_3^b$. Empirically, we first conduct a non-parametric kernel regression, and then evaluate the obtained density function at 0 to get $f_u(0)$. The expectation part can be calculated by sample average. Since no explicit solutions exist for the derivative $\partial w_3^b / \partial \delta$, numerical derivatives are used in the calculation.

III. Data and Estimation Results

3.1. Data

Our data set consists of the second, third and fourth waves of the AHEAD sample.[7] To select our sample, we use the following sample selection criteria: (1) Because the model in this paper applies only to singles, our sample only includes people who are alive and who are singles in both wave 2 and wave 3. (2) Total wealth or non-housing wealth is non-negative in wave 2 and wave 3. (3) Responses to the survival probability question in wave 2 are valid. When total wealth is used as one of the selection criterion, the number of valid observations is 1,903. When we consider non-housing wealth, the number of observations decreases to 1,752. Among these valid observations in wave 1 and wave 2, only 1,460 of them are still valid in wave 3.

[7] There is some evidence that the first wave of AHEAD under-reported asset holdings.

Three key variables are used in this paper: household wealth, income, and individual subjective survival curves. We now discuss these three variables in detail.

(1) The Wealth and Income Data

The AHEAD survey is a panel survey of older Americans. The wave 1 survey of AHEAD was conducted in 1993. The initial sample of AHEAD includes a sample of people who were 70 years old or more in 1993 (and their spouses regardless of ages). The wave 2 survey was conducted in 1995, and waves 3 and wave 4 were conducted in 1998 and 2000, respectively. The AHEAD data set provides more than 10 categories of wealth data. It is well-known in the literature that often a large portion of people do not provide valid responses on wealth questions (Juster and Smith, 1997; Chand and Gan, 2003). AHEAD uses a sequence of questions to bracket a wealth item. Although this technique is very successful in reducing non-response rates, it requires serious effort to impute the wealth values. Chand and Gan (2002) discuss various imputation methods. The imputed wealth data used in this paper are obtained from Adams *et al* (2003) who impute three waves of wealth and two waves of income. In Table 1, we list summary statistics of the total wealth and the wealth net of housing wealth. For each wave of wealth, we list the mean, median, variance, minimum and maximum values. From Table 1, mean wealth decreases slightly between wave 2 and wave 3 but decreases significantly between wave 3 and wave 4. Between wave 2 and wave 3, the average total wealth decreases 4.5% while the non-housing wealth decreases by 2.5%. Between wave 3 and wave 4, the mean total wealth declines 18% and the non-housing wealth declines 30%. The pattern for the median wealth is different from the mean wealth. Between wave 2 and wave 3, the median wealth decrease 14% and 15% for total wealth and non-housing wealth. However, between wave 3 and wave 4, there is a slightly increase for the median total wealth with rate 5.8%. The decreasing rate for non-housing wealth between wave 3 and 4 is 6.2%.

As Table 1 indicates, the median wealth is less than half of the mean wealth, reflecting the positive skewness that exists in the asset distribution. More specifically, the median is respectively 35%, 32% and 48% of the mean for the wave 2, 3 and 4 total wealth, and respectively 20%, 14%, and 19% of the mean for the wave 2, 3, and 4 non-housing wealth.

In Table 2, we list age, the number of children and income. The average age of respondents in the second wave is 79 years of old. Although heads of households in our sample have to be at least 72 years in wave 2, their spouses who may be younger are also included in the sample. The number of people in our sample who are younger than 72 years old is 46 (2.63% of the sample). Among all the people in our sample, 80.2% have children. The average number of children in our sample is 2.55. One household has 16 children. Second wave income is used as a measure of people's annuity income. The mean income level is $18,107 with a large standard deviation of $22,873.

(2) Individual subjective survival probability

In this paper, for each individual, we construct two survival curves: the life-table survival curve and the subjective survival curve. The life-table survival curve is directly obtained from the life table. The subjective survival curve is obtained from GHM. Here we briefly describe the subjective survival curve. One innovation in two recent surveys (Health and Retirement Study and AHEAD) is that they include questions about respondent's subjective probabilities about events in the future. In particular, each respondent is asked about his/her perceived probability of surviving to a target age that is between 10 and 15 years in the future. Although Hurd and McGarry (1995, 2002) show that on average these subjective probabilities are generally consistent with life tables, at the individual level, they suffer a serious problem. In all age groups, a substantial fraction of respondents give responses of 0.0 and 1.0. These responses cannot represent the respondents' true probabilities. GHM develop a model to recover each individual's "true" subjective probability.

Given the same age and sex, different people may have very different subjective survival probabilities. Some of the difference may relate to the health and wealth situations of individuals, some may simply be reflect personality. For each individual in their data set (AHEAD), GHM estimate an "optimism" index. Compared to the life table survival probability, an individual may overestimate or underestimate his/her survival probability. The estimated "optimism" index in GHM shows that significant individual heterogeneity exist in the AHEAD population. In a simple life cycle model, GHM show that ignoring individual heterogeneities may result in bias estimates. In this paper, we apply both the subjective survival probability developed in GHM and the life table survival probability.

Four different "optimism" indices were estimated in GHM, representing four different specifications. In this paper, we use the "unconstrained hazard-scaling" index.[8] In particular, let the current age of individual i be a. His subjective survival probability to age $a+t$ is given by:

[8] We select this index because it has the best predictive power of actual survival experience among all four indices.

$$s_{ia}(t) = \exp\left(-\int_0^t \lambda_{ia}(a+r)dr\right),$$

where $\lambda_{ia}(a+t)$ is the hazard function at age $a+t$. Further, let the individual's life table hazard be $\lambda_{i0}(a+t)$. The "unconstrained hazard-scaling" in GHM assumes that: $\lambda_{ia}(a+t)=\psi_i\lambda_{i0}(a+t)$ where ψ_i is the individual's optimism index. If $\psi_i>1$, this individual is said to be "pessimistic"; if $\psi_i<1$, then this person is "optimistic". Table 2 has the summary statistics of this index estimated fro responses in wave 2.

In Table 2, the mean and median of ψ_i are .659 and .663, respectively. People in this sample are on average more optimistic about their survival probabilities than the life table implies. A more optimistic person may save more than a life-table person would do. If we use an observed sequence of wealth to estimate our model, the estimates based on subjective survival curves should indicate a lower time discount factor and/or lower bequest motive than the estimates based on life tables.

3.2. Estimation Results

In Table 3, we estimate our model by using non-housing wealth and by assuming a fixed interest rate r = 0.04. We will test the robustness of our estimates later by using different interest rates. In Panel (A) of Table 3, we apply median regression to estimate the model using both subjective and life-table survival curves. Although the marginal utility of bequests is estimated to be almost zero in both cases, other parameter estimates vary significantly. Using life-table survival curve yields a higher time discount rate than using subjective survival curves. This is expected because people subjectively overestimate their survival probabilities. They behave accordingly by saving more to prepare for a longer lifespan, rather than valuing future consumption more than current-period consumption as implied by the estimates based life-table survival curves.

Panel (B) in Table 3 lists the estimates when the mean regression method is used. The marginal utilities of bequest in this panel are much larger than those estimated in Panel (A), which imply strong bequest motives. Another observation in Panel (B) is that the time discount factor is estimated to be significantly larger than 1, indicating that people value future consumption more than current consumption. Between the two sets of the estimates from mean regressions, the time discount factor is higher when the life table survival curve is used.

It is important to note that in a life-cycle model of time-varying survival probabilities, a time discount factor that is larger than 1 does not imply necessarily non-stationary growth in either consumption or wealth. Kocherlakota (1990) shows that it is possible that people still prefers current consumption to future one even with $\beta>1$, as long as the output or income grows at a rate that is sufficiently high. Kocherlakota's discussion is based on an infinitely lived representative agent. In our model, individual agent has constant income levels. From equation (1), even with $\beta>1$, the rate of consumption growth will turn negative at the time when the hazard rate $-\Delta lns_t$ is large enough.

The reason to have such an unusual time discount factor is that non-housing wealth during the sample period declined by only 2.5%. Apparently because of no significant difference in bequest motives between those who have children and those who do not have children, the marginal utility of bequest is almost always zero. Given the constant interest rate at .04, matching such a small decrease in wealth requires the individual to have tremendous incentives to save. This large saving incentive has to come from a large time discount factor. One major drawback, we suspect, is the interest rate we use: the return to capital investment may not be at 4% during our sample period. However, how to formally incorporate varying interest rate requires a model of portfolio choice. The dataset does not have enough information to estimate such a model.

In summary, mean regression yields very different parameter estimates from median regression. Specifically, mean regression suggests very large desired bequests while the median regression implies almost zero bequest motives. In addition, life table mortality risk yields a slightly larger bequest motives than subjective mortality risk. .

In Table 6, we list results from median regressions with varying interest rates. The risk-averse parameters and the time discount factor are very close to the reference value when interest rate changes from .02 to .06. Within this range of interest rates the marginal utility of bequests is very small.

In the following section, we will try to understand the economic significance of the bequest motives by some simulation exercises.

3.3 Bequest Simulations

Among the three parameters we estimate, it is relatively easy to understand the economic significance of the risk-averse parameter *γ* and the time discount factor *β*. To understand the effect of *γ* and *β* on bequests, consider a familiar consumption growth equation in the absence of the bequest motive of equation: $\Delta \ln c_t \approx (r + \ln \beta + \Delta \ln s_t)/\gamma$. Given *γ >0*, a larger *β* lowers bequests because it raises the value of the consumption growth rate. The consumption growth equation indicates that effect of *γ* on bequest depends on whether consumption is increasing. At a period of increasing consumption, a lower *γ* lowers the bequest because of it raises the consumption growth rate. However, when consumption is decreasing, a larger *γ* increases the bequest motive since it lowers the value of consumption growth (raises the magnitude of the consumption growth). It is important to note that a change in bequests because of a change in either *γ* or *β* is a change in accidental bequest. A non-accidental bequest is measured by the marginal utility of bequest, *α*. The larger the value of *α*, larger the bequest motive.

Two methods measure the economic significance of marginal utility of bequest, α:

$$\sum (1+r)^{-t}\left[\hat{w}_t(\hat{\alpha}) - \hat{w}_t(0)\right]m_t \tag{8a}$$

$$\sum \left[\hat{w}_t(\hat{\alpha}) - \hat{w}_t(0)\right]a_t \tag{8b}$$

where $\hat{\alpha} \equiv 1_{\text{children}}\left(\hat{\alpha}_0 + \hat{\alpha}_1 \cdot \text{No of children}\right)$. In (8a) and (8b), $\hat{w}_t(\hat{\alpha})$ is the optimal wealth trajectory given initial wealth and the estimated values of parameters. The term $\hat{w}_t(0)$ is defined in the similar way except that the marginal utility of bequests is zero. Equation (8a) and (8b) represent two different ways to understand the effect of bequests. In (8a), we calculate the present value of bequests. In (8b), we calculate the population difference in wealth holdings with and without a bequest motive. In another words, (8b) represents the effect of a bequest motive on the population wealth holdings. In Table 4, we calculate the effect of a bequest motive for a particular individual: a male at age 79 whose initial wealth is $35,000 and whose income is $12,000. The individual has two children. The optimistic index of this individual is 0.6594.

The results in Table 4 are presented in three different panels, grouped by their estimation methods. In the first three rows, (R1)-(R3), we let the marginal utility of bequests vary. In particular, row (R1) corresponds to a bequest motive estimated from (A1) in Table 3 where subjective mortality risk is used. We let time discount factor vary in rows (R4)-(R6), and let the risk averse parameter vary in rows (R7)-(R9). The marginal utility of bequest parameter has significant impact on the level of desired bequest and on the difference in wealth holdings. In rows (R1)-(R3) where the risk averse parameter (γ) and the time discount factor (β) are estimated using the median regression, the desired bequest rises from almost zero to $32,316 and the difference in wealth holding increases from $1 to $514,790 when the marginal utility of bequests increases from 2.47E-06 to 0.1. The effect of varying the marginal utility of bequests on desired bequests and on the difference in wealth holdings is very large.

In rows (R4)-(R6), we allow the time discount factor vary while keeping risk averse parameter constant. The marginal utility of bequest is constant at 0.001. In this case, desired bequests increases from $2.58 to $1,408 when the time discount factor increases from 0.7 to 1.3. Finally, in rows (R7)-(R9), we consider the effect of risk averse parameter γ. A larger γ implies a more risk averse agent. When γ increases from 0.5 to 2.0, the desired bequest increases from $5.80 to $518.5.

In summary, a higher marginal bequest motive, larger time discount factor, and larger risk averse parameter all increase the level of desired bequests significantly. A modest increase in either of the three variables may lead to a very large increase in desired bequests and in differences in wealth holdings.

3.4 Consumption/Wealth Trajectory and Out-of-Sample Predictions:

A typical way to evaluate parameter estimates from different methods is to conduct out-of-sample predictions. We used wealth data in wave 2 and wave 3 to obtain parameter estimates. We will now use the estimated parameters to predict the wealth values in wave 4, and compare the predicted wealth to observed wealth in wave 4. Table 5 has the comparison results. Each column in Table 5 reports various sums of errors based on a given set of parameter estimates. The column number, A1, A2, B1, or B2, corresponds to the estimates listed in Panel A and Panel B in Table 3. These estimates differ in their estimation method and their survival probabilities. The out-of-sample calculation is based on the same survival probability as the parameter estimates are. For example, if the set of parameters is obtained based on subjective survival probability, the out-of-sample calculation is also based on the subjective survival probability.

Parameter estimates in Column (A1) and (A2) are from median regressions while Column (B1) and (B2) are from mean regressions. Not surprisingly, (A1) and (A2) have smaller absolute errors and smaller mean square errors than (B1) and (B2), regardless of error types.

Furthermore, (A1) and (A2) have a lower sum of absolute errors for low wealth people and a larger sum of absolute errors for high wealth people than (B1) and (B2). This is expected because mean square regressions tend to fit high-wealth observations better because the large wealth values are magnified by the square operation.

Results in Table 5 can also be used to evaluate the advantage of using subjective survival probability instead of life-table survival probability. When the median regressions are used, parameter estimates based on subjective survival probability (A1) produce lower sums of mean square errors and lower sum of absolute errors in out-of-sample prediction of wealth than estimates based on life-table survival curves. When the mean regression method is used, parameter estimates based on subjective survival curves do not have a significant advantage in predicting fourth wave wealth comparing to ones based on life-table survival curves. Based on these results, we conclude that median regression is better than mean regression, and subjective survival probability better describes individual saving and bequest decisions than the life-table survival probability.

Finally, to better understand how people's consumption and wealth vary, we apply estimates from Table 3 to simulate a hypothetical person's consumption and wealth trajectories in Figure 2. The hypothetical person we consider is: single male at age 79 with an optimistic index of .6594. He has two children. His initial wealth and income are assumed at the median values in Table 2. In addition, the parameter set for Figure 3 is obtained from the median regression in Table 3. Figure 3 illustrates that For the person with median wealth his wealth decreases and reaches zero at age 85. His consumption level is highest when he starts at age 79, and decreases until he reaches age 85. From age 85, the person's wealth reaches zero and his consumption equals to his annuity income at $12,000. If the person dies before age 85, he leaves some bequest. However, such bequest is accidental since his bequest motive is essentially zero. In all these cases, since the person values future utility lower than current utility, his consumption level peaks at the first year and then decreases until it reaches his annuity income level.

IV. Conclusions

This paper investigates if and to what extent bequest motives exists for a sample of single elderly people. Our main goal in this paper is to estimate a classical life-cycle model with bequests, as in Yaari (1965) and Hurd (1989) for elderly with individual-specific subjective survival curves. In almost any life-cycle models, individual mortality risk is an important factor that affects people's decisions. Previous literature assumes the individual mortality risk is the same as the life-table mortality risk, ignoring the apparent individual heterogeneity in their mortality risk. This assumption may cause biases in parameter estimates. This paper applies the individual subjective survival probability model developed in an earlier paper (GHM). Their subjective survival probabilities have significant variations across individuals, and can better predict actual survival experience than life tables. We find that the estimation results from mean regressions differ significantly from median regression results. Most importantly, mean regression yields very large desired bequests while the median regression implies almost zero bequest motives. In addition, we find that life table mortality risk yields a little bit larger bequest motives than subjective mortality risk.

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Table 1: Summary Statistics of Wealth
(Being alive and single in the 2nd and 3rd waves; wealth is not negative; not missing subjective survival question; in 1995 dollars)

	wave 2		wave 3		wave 4	
	total wealth	Non-housing Wealth	Total Wealth	non-housing wealth	total wealth	non-housing wealth
mean	221,728	173,042	211,760	168,634	174,428	118,112
median	78,500	35,000	67,190	23,364	70,746	22,500
std dev	1,416,500	1,446,572	1,299,766	1,253,508	404,712	317,598
minimum	0	0	0	0	-52,632	-157,895
maximum	43,325,000	43,225,000	36,794,393	31,186,916	8,368,421	5,679,825
No. of obs	1903	1752	1903	1752	1460	1460

Table 2: Summary Statistics

	Mean	std dev	Median	min	max
Age of respondents in 1995	79	5.21	78	63	92
Income in wave 2					
Sample of 1903 observations	17,764	22,146	12,000	468	466,000
Sample of 1752 observations	18,107	22,873	12,000	468	466,000
Percentage who have children	80.2%				
Number of children	2.5514	2.3028	2	0	16
Survival probabilities					
optimum index (ψ)	0.6594	0.1176	0.6631	0.4385	1.0906
subjective 3-year survival prob	0.8911	0.0509	0.9026	0.6225	0.9893
life-table 3-year survival prob	0.8347	0.0844	0.8592	0.4175	0.9790
no. of observations in the sample	1752				

Table 3: Estimation Results:
(Marginal Utility of Bequest = 1_{child}*($\alpha_0 + \alpha_1$ * No. of kids),
interest rate = .04, non-housing wealth)

	estimation method	subjective or life table	risk averse parameter (γ)	time discount rate (β)	marginal utility of bequest (α_0)	marginal utility of bequest (α_1)
A1	median	subjective	0.9855	0.9420	3.8067e-7	1.0431e-6
			(0.0519)	(0.0028)	(8.957e-5)	(4.6931e-5)
A2	median	life table	0.7403	1.0045	7.6864e-4	2.1185e-5
			(0.1275)	(0.0044)	(8.601e-4)	(1.7597e-4)
B1	mean	subjective	0.7870	1.0546	1.0008	1.0022
			(1.544)	(0.8767)	(0.1525)	(0.925)
B2	mean	life table	0.7634	1.0763	0.9986	0.8941
			(1.295)	(0.6890)	(0.2316)	(0.7546)

Table 4: Economic Significance of Marginal Utility of Bequest
(For a hypothetical person: male, age 77, 2 kids, optimist index = 0.6594,
initial wealth = \$35,000, income = \$12,000)

rows	Risk averse parameter (γ)	time discount rate (β)	Marginal utility of bequest ($\alpha_0 + 2*\alpha_1$)	Desired bequest	Difference in wealth holdings
R1	0.9855	0.942	2.4669e-6	\$0.05	\$1.17
R2	0.9855	0.942	.001	\$21.12	\$477.22
R3	0.9855	0.942	.1	\$32,316	\$514,790
R4	0.9855	0.70	.001	\$2.59	\$57.26
R5	0.9855	1.00	.001	\$80.48	\$1,434
R6	0.9855	1.20	.001	\$1,408	\$18,238
R7	0.	0.9420	.001	\$5.80	\$116.7
R8	1.5	0.9420	.001	\$129.5	\$2,413
R9	2	0.9420	.001	\$518.5	\$9,463

Table 5: Results from Out-of-Sample Predictions

models	med reg (subjective) (A1)	med reg (life table) (A2)	mean reg (subjective) (B1)	mean reg (life table) (B2)
mean square error	6.5230e8	1.1248e9	2.6798e9	2.7650e9
absolute error	1.5489e5	1.6440e5	2.6789e5	2.6744e5

[1] select the sample in the first quartile of the 3rd wave.

Table 6: Robust Test with Median Regression Results
(varying interest rates, subjective survival rate, non-housing wealth)

interest rate used (r)	risk averse parameter (γ)	time discount rate (β)	marginal utility of bequest (α_0)	marginal utility of bequest (α_1)
0.02	0.8933	1.0151	1.7789e-5	1.8797e-6
	(0.1960)	(0.0061)	(3.3e-3)	(7.9283e-4)
0.03	0.8053	1.0049	7.2723e-6	3.57e-6
	(0.1797)	(0.0050)	(2.8102e-3)	(8.4822e-4)
0.04	0.9855	0.9420	3.8067e-7	1.0431e-6
	(0.0519)	(0.0028)	(8.957e-5)	(4.6931e-5)
0.05	0.9783	0.94	9.7635e-46	1.3841e-50
	(0.2420)	(0.0163)	(2.6350e-020)	(4.8609e-020)
0.06	0.9007	0.9293	9.1176e-48	1.468e-44
	(0.0289)	(0.0029)	(3.1365e-21)	(6.1125e-21)

Figure 1-1 Illustration of the Positive Bequest Case

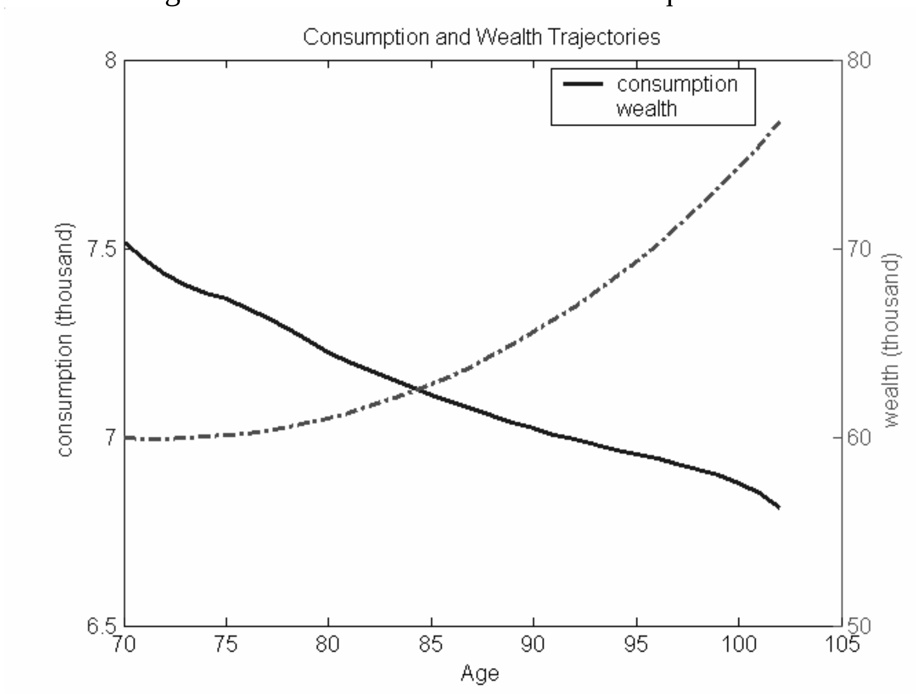


Figure 1-2 Illustration of the Zero Bequest Case
(No Borrowing Constraint Binding)

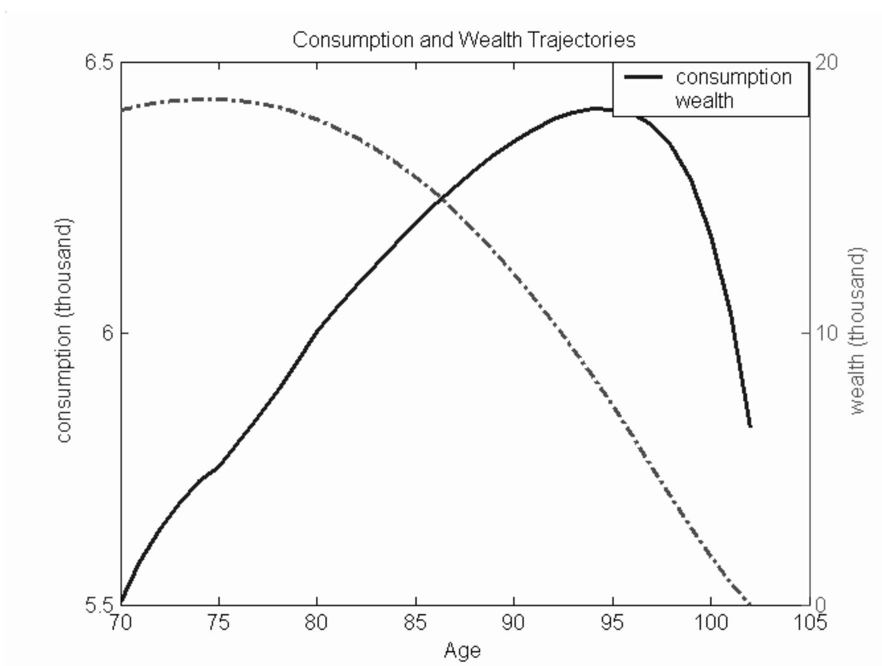
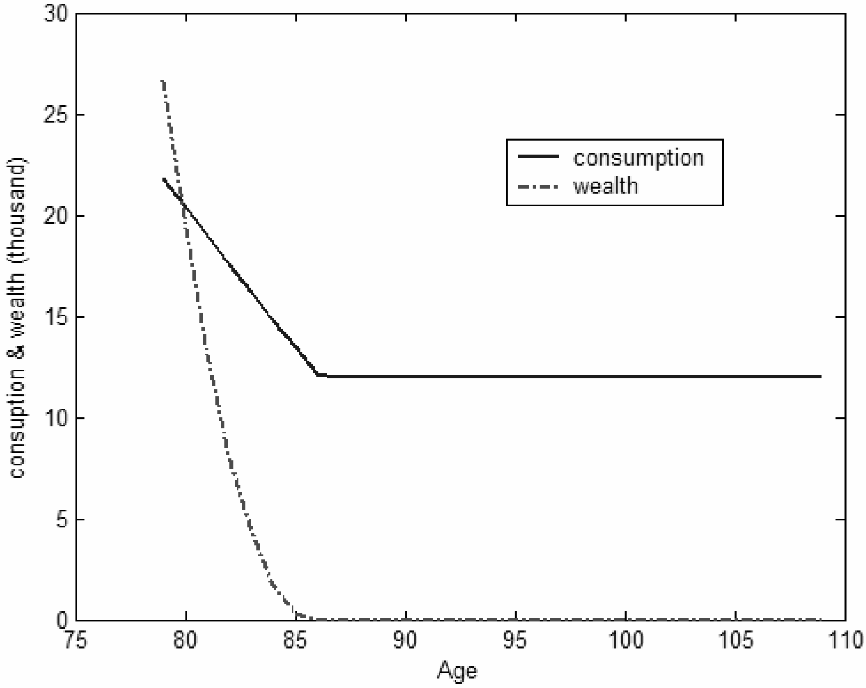


Figure 2: Consumption and Wealth Trajectories at Median Wealth Level [a]



[a] a hypothetical person: male, age 79, 2 kids, optimal index .6594, initial wealth $35,000, income $12,000; risk averse γ = 0.9855, time discount β = 0.9420, bequest motive: α_0 =3.8067e-7, α_1 =1.0431e-6; desired bequest is $0.05, and difference in wealth holdings is $1.17.