

COMMODITY SHOCKS, FIRM-LEVEL RESPONSES AND LABOR MARKET DYNAMICS

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ABSTRACT

This paper studies the role of labor markets in the transmission of commodity price super cycles. We study the case of Brazil between 1999-2013, a period in which large commodity price fluctuations provided a quasi-natural experiment. We use matched employer-employee data to build regional commodity prices and regional employment and earnings across sectors. Exploratory evidence documents that regional skill premium decreases during a commodity boom. Interestingly, it does not change during the bust. Moreover, regional-level reduced-form models suggest the presence of extensive cross-sectorial spillovers. During a boom, employment in the nontradable and commodity sectors expands while tradable (non-commodity) employment declines. To study the economic mechanisms behind these findings, we introduce a three-sector model of a small open economy with firm heterogeneity, entry and exit decisions, skilled and unskilled labor, and labor market rigidities. Consistent with the evidence, as commodity prices increase the model produces expansions in the commodity and nontradable sectors, a decline in the skill wage premium, and a contraction in the non-commodity exporting sector. During the bust, on the other hand, downward wage rigidity generates dynamic misallocation between sectors, triggering a persistent recession characterized by unemployment and a sluggish recovery of the tradable sector. Empirical evidence using firm-level data confirms the model's main implications.

Keywords: Commodity shocks, labor markets, firm dynamics.

JEL classification: E32, F16, F41

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1. INTRODUCTION

Commodity price cycles are a fundamental source of volatility to emerging markets. Commodities represent more than a quarter of the exports of the median emerging economy and a significant share of government budgets.¹ Price fluctuations influence aggregate economic activity, unemployment rates, and the ability of governments to balance their budgets and to finance welfare programs. Often times, the end of commodity price cycles is associated with deep crises and slow recoveries (Reinhart et al. [2016]). All these features of commodity price cycles suggest that understanding the transmission mechanisms from these prices to the rest of the economy is crucial.

This paper studies the role of labor markets in the transmission of commodity price super cycles throughout the economy, and the role of labor market frictions in generating recessions during the cycle bust. On economic grounds, at least three factors are critical when studying the underlying mechanisms of these processes. First, commodity production in emerging markets is intensive in low-skill labor and output prices are exogenous to the economy. As a consequence, commodity price booms raise the demand for low-skill workers, reduce the skill premium, and generate a negative cost shock to other low-skill-intensive sectors of the economy. Second, wealth effects stimulate the domestic demand. Thus, firms that rely relatively more on domestic sales are likely to benefit from a commodity price boom. Finally, labor markets in emerging markets are frictional, especially at the lower tail of the skill distribution, where minimum wages and firing costs are likely to be binding. These rigidities can potentially keep wages to low-skill workers artificially high long after the commodity super cycle has ended. In this sense, the boom spreads the seeds of a future recession characterized by persistent unemployment and labor misallocation across sectors. To the best of our knowledge, there is no economic model capturing these three channels.

In this context, our contribution to the literature is two-fold. First, we build a three-sector dynamic small open economy model with firm heterogeneity, entry and exit, and skilled and unskilled labor. Firms operating in the commodity good sector compete for a scarce resource and face a fully elastic international demand. Firms in the nontradable sector produce a homogeneous good whose price clears the domestic market. Finally, firms in the tradable sector sell heterogeneous goods and face an exporting decision. In the model, firms in different sectors interact by hiring skilled and

¹Fernández et al. [2017] document commodity export shares across developed and developing countries.

unskilled workers in a common labor market. The only friction in the model is that labor income is downwardly rigid.

We use rich Brazilian matched employer-employee data to calibrate the model.² Brazil is an excellent laboratory to study the interplay of labor markets and commodity prices. First, its economy is a major commodity exporter but its market share in any particular commodity makes any of its regions a price-taker. Second, it is characterized by significant geographic variation in commodity-specialization across local labor markets that we exploit when testing the model.

The calibration exercise highlights the transmission mechanisms through which commodity price cycles affect the economy. During a boom period, earnings in the economy increase for both, skilled and unskilled labor. Because the commodity sector is less intensive in unskilled labor, the skill premium decreases. Additionally, given that the tradable sector is relatively less skill intensive than the nontradable sector, in the model the production cost increases relatively more for the tradable sector. The nontradable sector, on the other hand, benefits from a larger domestic absorption, while large exporters in the tradable sector that depend heavily on the external market fail to fully benefit from these wealth effects. Therefore, the tradable sector contracts and the nontradable sector expands. However, during bust periods, downward wage rigidity plays a critical role. The reallocation of labor across sectors quickly turns into misallocation in presence of downward wage rigidity, generating an aggregate recession with persistent unemployment even when, after the cycle, the commodity prices are never below the pre-boom level. In fact, the effects of downward wage rigidity on welfare are economically important, as the economy with frictions would forgo five times more consumption to avoid the cycle.

Our second contribution to the literature is empirical. Our paper is the first to provide micro-level evidence supporting the microeconomic foundations of the effects of commodity price super cycles over the economy. Specifically, using again Brazilian matched employer-employee, we assess the impact of commodity price fluctuations on the reallocation of labor across sectors, skill premium as well as their influence on firm entry, exit and export responses. We exploit regional variation in exposure to commodity prices. Regional exposure to Brazil's commodity price cycles varies

²As described in the data section, our main source of information is a linked employer-employee census of the universe of formal-sector firms and workers during the period 1999-2013, which we augment with information on firms' exporter status, commodity prices, regional production and total (formal and informal) local employment. In our data, we observe hundreds of millions of job-spells and millions of firms, as well as very detailed information on firms' geographic location and industry, worker characteristics and labor market outcomes.

substantially due to differential degrees of specialization in commodities and differences in the composition of each region’s commodity basket. Given this, we construct region-specific commodity price indices and study the behavior of local labor markets. Our empirical strategy takes into account unobserved heterogeneity both at the region and firm levels, as well as time trends. As predicted by the model, we document region-level employment shifting away from the tradable sector towards commodities as commodity prices rise. We also observe the predicted decline of the skill wage premium, which is due to the positive demand shock experienced by the unskilled-labor-intensive commodity sector as commodity prices rise.

We extend our empirical analysis and examine the response of individual firms to the commodity price shock. Again exploiting regional variation in exposure to the commodity boom, we find a large positive impact on firm-level employment in the commodity sector, especially driven by larger firms. This is accompanied by a decline in tradable employment, especially for exporting firms. Finally, we use our firm-level analysis to study the asymmetric impact of prices on employment during boom and bust periods, finding further evidence in favor of our model.

The rest of the paper is structured as follows. Section 1.1 summarizes the related literature. Section 2 describes the data used in our empirical analysis. Section 3 presents exploratory evidence motivating our conceptual framework. Section 4 introduces our model, while section 5 illustrates the calibration and quantitative results. Section 6 analyzes the response of firm-level employment and entry and exit to commodity price shocks. Section 7 concludes.

1.1. LITERATURE REVIEW

Despite the attention of policymakers, the academic literature on the impact of commodity price fluctuations is divided. On the one hand, the calibrated theoretical macro models that follow [Mendoza \[1995\]](#) suggest that fluctuations in the terms of trade can explain 40% of movements in output. On the other hand, the empirical study by [Schmitt-Grohé and Uribe \[2015\]](#), using data for 38 poor and emerging economies, shows that fluctuations to terms of trade only account for 10% of output movements. Recent papers by [Rodriguez et al. \[2015\]](#) and [Shousha \[2016\]](#) show that the discrepancy is likely to be due to the level of aggregation. In fact, the terms of trade are a basket that obscures the fluctuations of its components. These studies build country specific commodity baskets and uncover a strong relationship between the fluctuations of the prices of commodities

produced by the country and its major macroeconomic variables. For instance [Fernández et al. \[2017\]](#) show that for a sample of 138 countries on average 33% of output fluctuations are due to variations in their country specific commodity baskets, while using a unique aggregate price decreases this number to 10%. Although the pass-through from commodity fluctuations to the aggregate economy is well documented in the literature there is little theoretical and empirical work on the actual mechanism that underlies this contagion. For [Rodríguez et al. \[2015\]](#) commodities are basically a national endowment and a positive commodity shock acts as an endowment shock. [Alberola and Benigno \[2017\]](#) also highlight the wealth effect as the main propagation mechanism of a commodity boom. The permanent effects of the commodity boom are amplified when endogenous growth is included in the tradable sector. [Arezki et al. \[2017\]](#) use a novel database on giant oil discoveries to quantify the importance of news shocks in the economy. In their model, a discovery of oil transpires into the aggregate economy as the news of a positive endowment shock. [Shousha \[2016\]](#) assumes an exogenous link between commodity prices and the interest rate at which the country can borrow. Therefore, commodity booms are transmitted in the economy by lowering the interest rate for every other industry. With respect to this literature, we build a multi sector heterogeneous firm framework with skilled and unskilled labor where changes in the skill premium and difference reliance in the domestic market can provide a micro foundation for the transmission of commodity fluctuations. Moreover, labor market frictions can generate deep recessions at the end of the commodity super cycle, providing a modelling framework to rationalize the empirical regularities of [Reinhart et al. \[2016\]](#).³

Our paper also contributes to the microeconomic trade literature on the impact of external shocks to labor markets. These studies have focused mostly on the impact of trade liberalization or import competition from China on employment and wages. For instance, [Menezes-Filho and Muendler \[2011\]](#) document that the Brazilian trade liberalization displaces workers to the services sector. [Dix-Carneiro and Kovak \[2015b\]](#) also study the effects of trade liberalization in Brazil and document modest reductions in local skill premium due to trade liberalization. [Adão \[2015\]](#) studies the sorting of workers in and out of industries as a response to the fluctuations in commodity prices faced by Brazil during the period 1991-2010. He documents that commodity prices are

³Typically the quantitative international finance literature abstracts from firm heterogeneity when studying the effects of exchange rates, terms of trade, or international financial crises. Some exceptions are [Alessandria et al. \[2013\]](#), [Ates and Saffie \[2016\]](#), and [Gopinath et al. \[2015\]](#).

related to local decreases in the skill premium in Brazil. Our paper is also complementary to a growing literature using variation across local labor markets to identify the impact of trade policy or integration on labor markets. [Autor et al. \[2013\]](#) study the impact of Chinese import competition on the U.S. [Kovak \[2013\]](#) studies the impact on regional wage of Brazil’s trade liberalization. [Costa et al. \[2016\]](#) analyze the impact on local labor markets of the rise in both import competition and demand for Brazilian exports (mainly commodities) from China. Using the 2000 and 2010 cross-sections of Brazil’s demographic census, they find a relative decline in wages in regions facing larger increases in import competition, and a relative wage increase in regions facing a positive demand shock. With respect to this empirical literature, the novelty of this paper is to identify firm level cross sector transmission of commodity prices. A related paper by [Allcott and Keniston \[2014\]](#) studies the effect of oil and gas price shocks in manufacturing firms in the US. They document strong local demand effects that can undo the *Dutch Disease* phenomenon. With respect to their paper, we not only focus on demand spillovers but also on labor cost spillovers driven by changes in skill premium. Because our sample is not restricted to manufacturing, we can exploit cross sectoral differences in skill intensity to identify labor market based spillovers.

From a modeling perspective, the quantitative trade models of heterogeneous firms typically abstract from commodity production by assuming some form of product differentiation. Examples of commodity goods in quantitative trade models are [Costinot et al. \[2016\]](#) and [Sotelo \[2017\]](#). Unlike the quantitative trade literature, our economy is consistent with the international finance tradition. In fact, the model is solved in general equilibrium with well defined current account and trade balance dynamics.

2. DATA DESCRIPTION

Our main source of data is a detailed linked employer-employee census of Brazil’s labor force over the period 1999-2013. This dataset - called RAIS - is an administrative census collected for social security purposes by Brazil’s Labor Ministry.⁴ It encompasses the universe of formal-sector employees. The data is longitudinal, allowing us to track both workers and firms over time.

⁴This data has been used by Alvarez et al. (2016), Dix-Carneiro and Kovak (2015) and Helpman et al. (2016) among others.

We observe very detailed worker characteristics including educational attainment, age, gender, and occupation. We observe firms and plants' industry of employment at the five-digit level of disaggregation according to Brazil's National Classification of Economic Activities (CNAE). This feature is essential as it allows us to observe very narrow commodity categories. Further, we observe workers' and plants' geographic location: the data reports the municipality in which workers and plants are found. This allows us to exploit regional variation in exposure to commodity price changes in our empirical analysis.

For each job spell in each year we observe the exact starting and ending day of a workers' employment in a firm. We can compute the duration of employment and we observe workers average monthly earnings.

Skilled and Unskilled Workers. To match the model, we divide workers into skilled and unskilled categories based on their educational attainment. In the raw data we observe nine groups of attainment ranging from no formal education to tertiary education. We define unskilled workers as those with at most complete secondary education, and skilled workers as those with at least some tertiary education. Nationally, unskilled workers represent about 80 percent of total employment.

Regional Units. Our goal is to define regional units as an approximation to the concept of local labor markets. In the raw data we observe the municipality of each workers' employer. These municipalities are geographically too small to be considered a local labor market. Brazil is divided into about 5500 municipalities, 558 micro-regions, 137 meso-regions, 27 states, and 5 macro-regions. We choose micro-regions as the regional unit that best reflects a local labor market. Throughout the paper we refer to these 558 micro-regions simply as regions. These regional units are defined by Brazil's Statistical Institute (IBGE) and are similar to U.S. commuting zones. They are large enough that interregional migration is fairly small.

Sectors. We define the following sectors in the economy. These are based on Brazil's industrial classification scheme CNAE. The main three sectors of interest to our paper are the commodities sector, the tradable sector, and the nontradable sector. Commodities includes agriculture and mining. Tradables includes all manufacturing industries. Nontradables includes retail and wholesale trade, hotels and restaurants, construction, and transportation.

A fourth sector consists of the government and quasi-public industries, such as education and healthcare. Finally, we group "other services", such as finance and real estate, in a fifth sector,

which we view as fairly different from the industries in the nontradable sector.

Regional Panels of Employment, Wages, and Firm Entry Rates. Using the data described above we create three different panels at the region or region-sector levels which will be used in our empirical analysis. Each of these panels is constructed at a quarterly frequency, allowing us to exploit price dynamics at business-cycle frequency.

The first panel is the **quarterly region panel** which is constructed directly from worker-level records. We define total regional employment as the sum of workers in each region in the last month of each quarter. We define the mean regional wage as the average monthly wage of workers employed in a region, again in the last month of each quarter. Further, we compute total employment and mean wages separately for skilled and unskilled workers. The second panel is the **quarterly region-sector panel** which is also constructed directly from worker-level records in the same way as the previous one, but aggregating worker-level outcomes by sector-region cells. As we discussed earlier, these sectors are commodity, tradable, and nontradable. Finally, we assemble a **quarterly region-sector panel of firm entry and exit rates**. In each region and sector, we compute firm entry and exit rates (from the economy and from exporting).

In each region-sector cell, entry in period t is defined as the number of firms in each region active in t and not in $t - 1$, normalized by the average of the total number of firms in periods $t - 1$ and t , as shown in the following equation:

$$\text{Entry Rate}_t = \frac{\text{Entering Firms}_t}{0.5 * (N_{t-1} + N_t)} \quad (1)$$

Similarly, exit in period t is defined as the number of firms in each region active in $t - 1$ and not in t , normalized by the average of the total number of firms in periods $t - 1$ and t :

$$\text{Exit Rate}_t = \frac{\text{Exiting Firms}_t}{0.5 * (N_{t-1} + N_t)} \quad (2)$$

Firm-Level Panel. Based on our worker-level records we create a firm-level panel. The panel consists of several million firms operating in the three sectors of interest. We compute firms' quarterly employment (total and by skill level) as a firm's employment in the last month of each quarter.

Exporting Firms. We complement our firm-level panel with additional information on exporting

firms for the period 2001 -2013. This data on exporters is obtained from Brazil's SECEX. We assign each firm in our panel a dummy variable for exporting status in each year. (This data is not available at a quarterly frequency).

Table 12 in the appendix reports the number of firms, mean firm employment and mean firm wages in each sector - commodity, tradable, and nontradable - in each of Brazil's five macro-regions. At this aggregate level the table illustrates the regional variation in economic activity we will exploit in our analysis. Appendix table 13 reports mean firm employment and wages in each skill-group by sector. In each of these sectors there is a large gap between the wages with workers with different degrees of educational attainment. The within-firm skill distribution varies significantly across sectors, with a larger share of low-education workers in commodities (especially in agriculture). Table 14 in the appendix reports summary statistics of exporting and non-exporting firms by region and sector in 2000 and 2010. Shares of exporting firms are very low because RAIS is not restricted to firms above a certain size threshold. The table shows that - as usual - exporters are much larger and pay higher wages than non-exporters. This is especially so in 2010, compared to 2001. The difference in employment between exporters and non-exporters is larger in the tradable sector than in commodities.

Commodity Prices. We construct a regional commodity price index based on 14 commodity goods that capture a very large share of commodity employment in Brazil. They span agriculture, mining, and fuel industries and are chosen based on the following criteria: i) we must be able to separately identify workers in these sectors individually in both our linked employer-employee census (RAIS) and in the Brazilian Census of Population and ii) we must be able to match these categories to commodity price data. The list of 14 commodities includes cereals, cotton, sugarcane, soybeans, citric, coffee, cacao, bovine, ovine, poultry, coal, oil and gas, a basket of metallic minerals, and precious metals. The largest commodities in terms of employment are cereals, bovine, coffee and sugarcane and soybeans. There is considerable variation in the geographic distribution of employment in these commodities across Brazil. Further, there is also geographic variation in the share of employment in commodities in each region as is illustrated in figure 14.

We obtain commodity price data for the period 1999-2013 from the World Bank's Global Economic Monitor - Commodities. This dataset has the advantage of tracking commodity prices for a wide number of commodities over a long period of time and with systematic criteria to define the

prices of all of them. The data is initially at a monthly frequency. We construct quarterly price indices based on the last month of each quarter.

We define regional commodity prices as the weighted average of individual commodity prices with the weights given by each region’s employment shares by commodity. Specifically, the commodity price index in region r in period t is

$$p_{rt} = \frac{\sum p_{ct} \cdot e_{cr}}{e_r} \quad (3)$$

where p_{ct} is the price of commodity c in period t , e_{cr} is employment of commodity c in region r in 2000 and e_r is region r ’s total employment in 2000. We obtain the employment shares from Brazil’s population census in year 2000 - the earliest possible year that is close to the start of our sample period. By using the Census of Population for these weights instead of RAIS, we take informal employment into account. There is some employment in commodity-related activities that we cannot directly link to a commodity price. In these cases, we assign this employment to either agriculture or mining, and distribute it equally across the subset of agriculture or mining commodities in our set of 14 commodities for the purpose of creating the regional price index. Changes over time in this regional price index can vary across regions due to differences in the composition of commodities across regions or due to differences in the overall share of regional employment in commodities.

Additional Data Sources:

UNIONIZATION RATES. We use Brazil’s annual household survey (PNAD or Pesquisa Anual por Amostra de Domicilios) to compute state-level measures of unionization, which is one of our measures of wage rigidity used in the empirical analysis below.

INFORMALITY. Also using this household survey we create annual measures of informality at the state level, which we use (in work in progress) to explore the impact of commodity price fluctuations on informal employment.

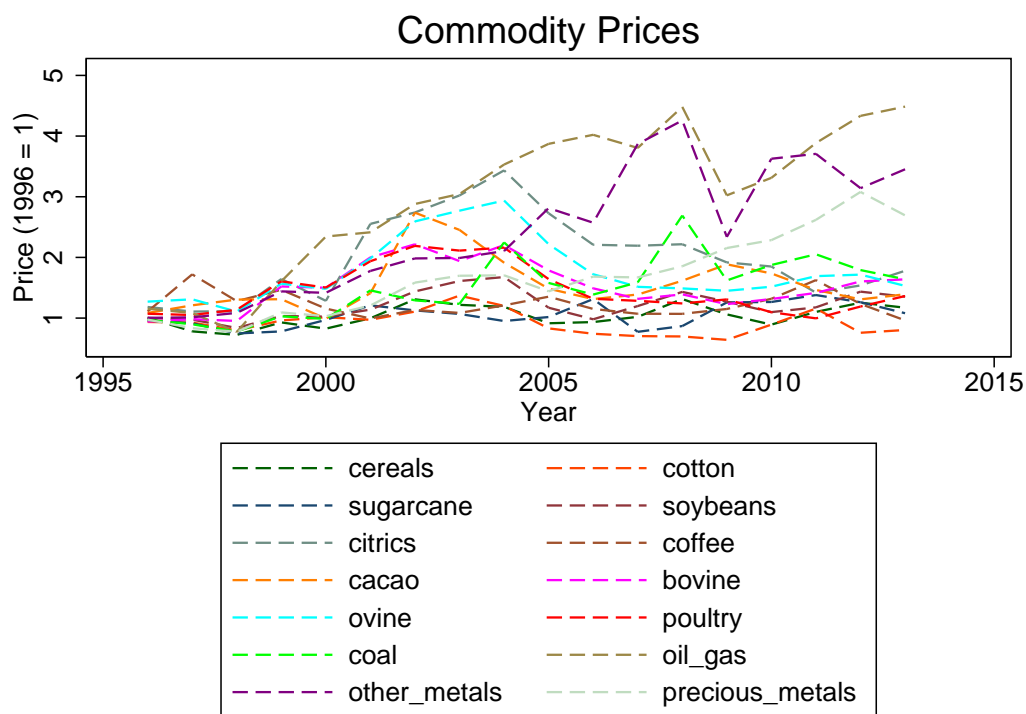
REGIONAL CHARACTERISTICS. Using Brazil’s Agricultural Census (Censo Agropecuario) for 1996 we obtain regional measures of land use. We observe the share of each microregion’s land area used actively in agriculture, as well as potentially fertile land not used in agriculture at the time of the Census. In our empirical results, we explore the heterogeneous response of regions to commodity

prices based on their available agricultural area.

3. EXPLORATORY ANALYSIS

The Brazilian GDP growth rate accelerated from 2002 until 2008. Between 1995 and 2002, the average growth rate was 2.3% and increased to 3.4% between 2002 and 2006 and to 4.5% in the 2006-2010 period. The commodity price boom was a key factor behind this phenomenon. From 2002 until 2008 Brazil's export prices increased 138% and the terms of trade increased 17%. Figure 1 presents the price dynamics for 14 different commodities during the period 1996 and 2013.

Figure 1: Commodities in Brazil: Price Trends

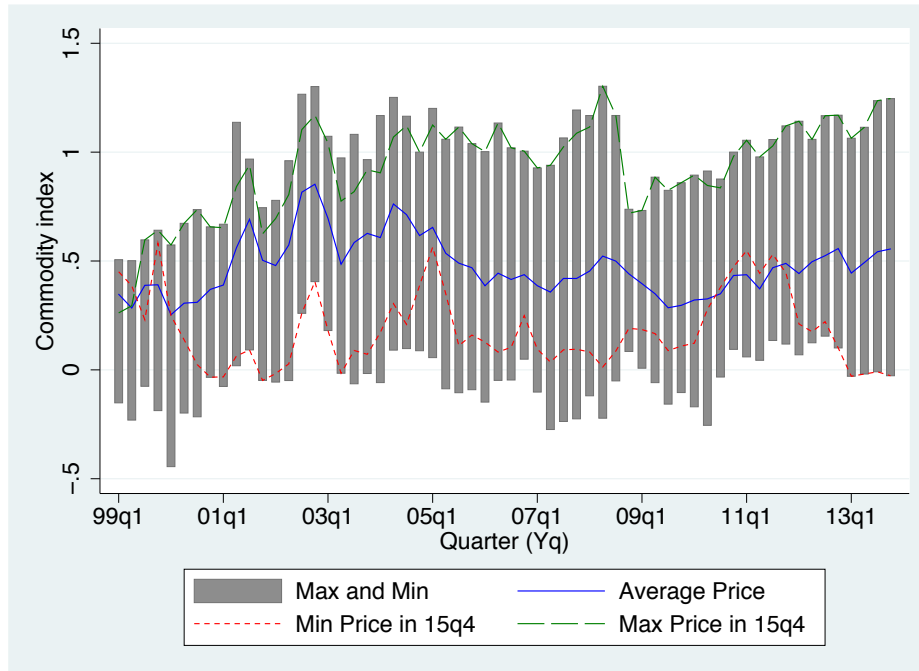


Source: Annual data (year average) for 1999-2013 from the World Banks Global Economic Monitor - Commodities.

Similar patterns emerge when examining a local commodity price index. Figure 2 presents its trend as well as the within-period regional dispersion, confirming the boom experienced by the Brazilian economy during this period. However, this increase was not uniform throughout the

country as regional dispersion increased over time. Given the large degree of regional variation in commodity production, a feature of the Brazilian economy even before the commodity boom (see figure 14 in the appendix), any empirical analysis examining the association between commodity prices and firm or regional responses must take this heterogeneity into account.

Figure 2: Regional Commodity Index: Time and geographic variation

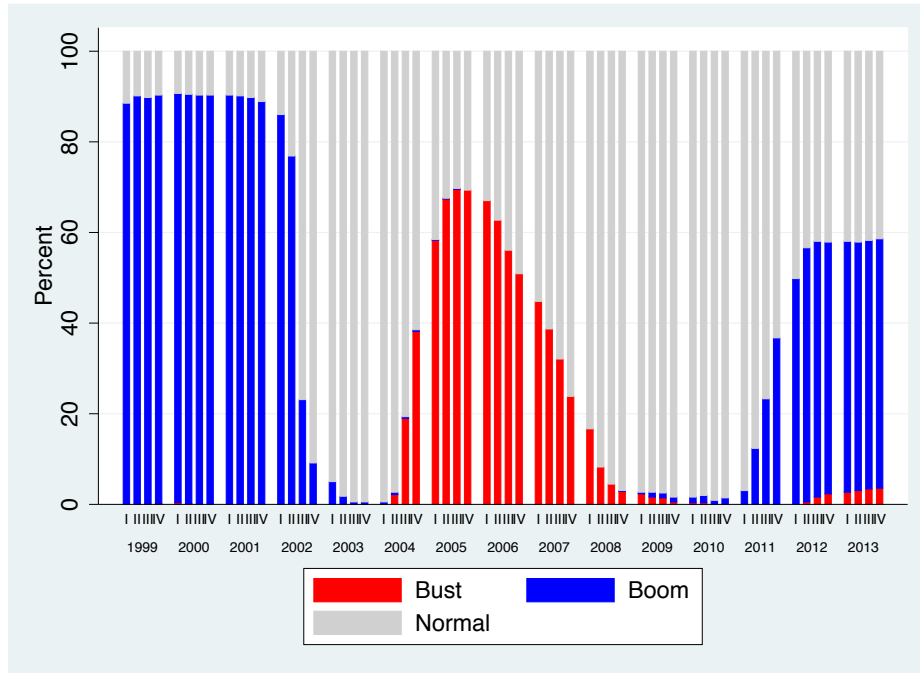


Source: Authors's calculations using data from the World Banks Global Economic Monitor - Commodities.

Figure 3 presents a different perspective of these regional price dynamics. For each quarter in our sample, it displays the proportion of regions experiencing significant changes in their commodity prices.⁵ A region experiences a “boom” when the HP trend of the log regional price increases by 1% between two quarters. Similarly, a “bust” occurs when the price trend decreases by 1%.

⁵For each region, we use detailed information on commodity production and price data to compute regional variation in commodity prices.

Figure 3: Regional Boom and Bust Cycles



Starting in 1999 until 2001, more than 80% of the regions of Brazil experienced “booms”. This period was followed by four years with a large number of regions experiencing “busts” (2004-2008), which in turn were followed again by a period of general price increases with more than 50% of regions experiencing booms. Note that the intensity of these episodes is also heterogeneous between booming regions. Figure 15 in the Appendix displays the cases of Rio de Janeiro and Sao Paulo, two of the largest regions of Brazil.

This paper estimates the impact of these price changes on firm-level responses. However, we motivate our theoretical framework and empirical strategies first analyzing regional- and sectoral-level responses. Three facts emerge from this analysis.

Fact 1: Commodity booms are associated with decreases in the skill premium. Local labor markets adjusted to the boom in commodity prices. Figure 4 displays the trends in the regional skill premium jointly with our commodity price index.

During the first six years of the new millennium, the skill premium in Brazil declined by more than 20%, and although the trend persisted after 2006, it took place at a lower rate. Figure 5 shows the regional distribution of the share of skilled employment for the commodity, tradable and

Figure 4: Fact 1: Regional Skill Premium, Trends



Source: Authors's calculations using data from RAIS .

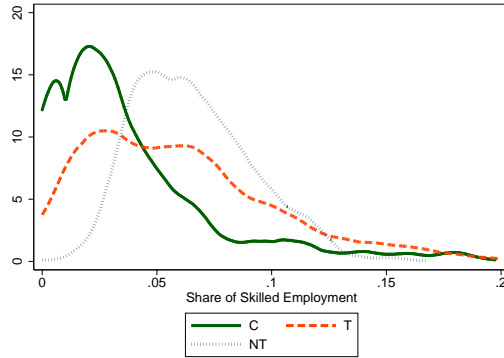
nontradable sectors in Brazil.

Commodity production is considerably more unskilled intensive than the other two sectors, as figure 5 shows. This suggests a potential economic mechanism between the commodity price boom and the decline in the skill premium during this period. Given the intensity with which unskilled labor is used in the commodity sector, an increase in the demand for this input resulting from the boom could have resulted in a lower skill premium. To explore this possibility, we use regional panel of skilled and unskilled wages for the period 1999-2013 to estimate the following regression model:

$$\begin{aligned}
 SP_{r,t} = & \alpha_0 + \alpha_1 PT_{r,t} + \alpha_2 Boom_{r,t} + \alpha_3 Bust_{r,t} + \alpha_4 PT_{r,t} Boom_{r,t} \\
 & + \alpha_5 PT_{r,t} Bust_{r,t} + \alpha_6 PT_{r,t} Int_r + \delta_t + \delta_r + \epsilon_{r,t} \quad (4)
 \end{aligned}$$

where r represents region and t time, and SP is the (log) skill premium. PT represents the (log) commodity index HP trend, and $Boom_{rt}$ ($Bust_{rt}$) is a dummy variable equals to one if region r in period t experienced a boom (bust), and zero otherwise. Int_r is a region-specific dummy capturing

Figure 5: Regional Distribution of the Share of Skilled Employment in Commodities, Tradables and Nontradables



Source: Authors’s calculations using data from RAIS .

the regional commodity intensity. It equals one if commodities were abundant in the region in 1999, and zero otherwise.⁶ δ_r and δ_t represent region and time fixed effects, respectively. $\epsilon_{r,t}$ is the error term. Table 1 presents the results.

The results confirm the suggestive evidence discussed in the context of Figure 4, namely that sizable commodity price increases are associated with skill premium reductions. As expected, the estimated negative effects are stronger among commodity intensive regions.

It is also worth noticing the differences between the estimated impacts of boom and busts. Columns (2) and (3) in Table 1 contain no statistically significant impact of sizable price reductions (busts) on skill premium. This suggests that agents (firms) might adjust differently depending upon the sign of the price change. As discussed below (and confirmed using firm-level data in section 6), labor market rigidities surface as potential channels driving the asymmetry.

Fact 2: Downward wage rigidities. We construct individual-level annual nominal and real earnings growth for full time employees working in the same firm in years $t - 1$ and t , and report the proportion of contracts for which labor income decreases, stays constant (change $< 0.5\%$), or increases. Table 2 contains the results generated for Brazil and compare them to those reported for the US by Kurmann et al. [2016] using the same methodology.

The comparison of columns (1) and (3) with (2) support the idea that downward earnings rigidi-

⁶For each region we compute the share of the overall land devoted (or potentially devoted) to agriculture in 1999. The “Regional Commodity Intensity”= 1 if the region had a share above the median

Table 1: Skill Premium and Commodity Price Index

Variables	(1)	(2)	(3)
(ln) Price _{rt}	0.086 (0.05)	0.111** (0.05)	0.147*** (0.05)
Boom _{rt}		0.081*** (0.02)	0.078*** (0.02)
Bust _{rt}		0.007 (0.01)	0.007 (0.01)
Boom _{rt} x (ln) Price _{rt}		-0.201*** (0.04)	-0.194*** (0.04)
Bust _{rt} x (ln) Price _{rt}		0.008 (0.03)	0.009 (0.03)
Intensity _r x (ln) Price _{rt}	-0.140** (0.06)		-0.111* (0.06)
Constant	1.122*** (0.01)	1.063*** (0.02)	1.066*** (0.02)
Observations	33,455	33,455	33,455
R-squared	0.687	0.689	0.690
Fixed Effects:			
Year	Y	Y	Y
Quarter	Y	Y	Y
Region	Y	Y	Y

Notes: Total number of regions is 558. Regressions are estimated using quarterly information for the period 1999-2013. (ln) Price is the result of the Hodrick-Prescott filter (trend) applied over the original (ln) price series. Boom (Bust) is a dummy variable such that Boom(Bust)= 1 if the actual price increases (decreases) at least 1% relative to its trend between two periods (quarters), = 0 otherwise. Regional Commodity-Intensity captures pre-commodity boom characteristics within a region. More precisely, the variable equals 1 if the share of employment in the commodity sector (within a region) was above the median in 2000, 0 otherwise. This employment share is constructed from Census data. Standard errors (in parentheses) are clustered at the regional level. *** p<0.01, ** p<0.05, * p<0.1.

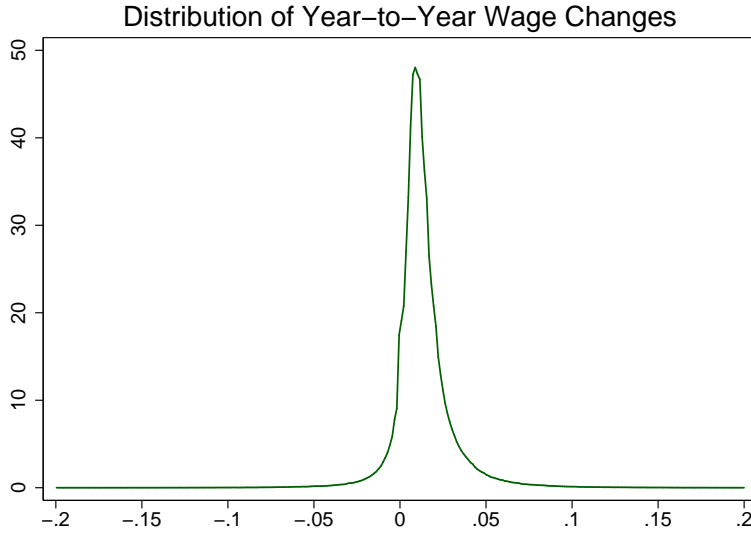
ties are more important in Brazil than the US. To further explore this point, we use our matched employer-employee dataset to construct the histogram for nominal annual earnings differences at the individual level. Figure 15 depicts the resulting distribution.

Table 2: Downward earnings rigidities: Brazil vs. USA

	Brazil - Nominal (1)	USA - Nominal (2)	Brazil - Real (3)
Decrease	5.4	30	14.0
Constant	16.6	5	36.7
Increase	78.0	63	49.2

Source: Columns (1) and (3) are based on authors's calculations using data from RAIS. Column (2) from [Kurmann et al. \[2016\]](#).

Figure 6: Distribution of year-to-year earnings differences



Source: Authors's calculations using data from RAIS.

The estimated average earnings difference is 0.013 and its median 0.011. More importantly, less than 5% of the population reports variations below -0.0058, while more than 75% of the population reports changes of at least 0.0058 (representing 0.29 standard deviations). All in all, downward rigid earnings emerge as a potential explanation for the asymmetric response in skill premium.

Fact 3: Employment is affected by commodity price dynamics. The association between commodity price dynamics and the skill premium could impact the sectoral composition of employment within a region. In fact, [5](#) also shows that the nontradable sector is the less intensive sector in the usage of unskilled labor. Therefore, we would expect that labor costs would be relatively higher

for firms in the tradable sector. Moreover, the local wealth effect generated by the commodity boom should increase domestic demand. This effect should benefit the nontradable sector relatively more than the tradable sector. Therefore, during the commodity boom, we expect positive spillovers to the nontradable sector and negative spillovers to the tradable sector. Therefore, local demand and labor market dynamics could provide a micro-foundation for the so called *Dutch Disease* that is independent of exchange rates as the analysis is within Brazil.

In the spirit of the empirical model for the analysis of the skill premium and commodity prices (equation 4), we postulate the following reduced-form regression for the analysis of sector s 's employment share as a function of commodity prices:

$$\begin{aligned}
 Emp_{s,r,t} = & \beta_{0,s} + \alpha_{1,s}PT_{r,t} + \beta_{2,s}Boom_{r,t} + \beta_{3,s}Bust_{r,t} + \beta_{4,s}PT_{r,t}Boom_{r,t} \\
 & + \beta_{5,s}PT_{r,t}Bust_{r,t} + \beta_{6,s}PT_{r,t}Int_r + \delta_{t,s} + \delta_{r,s} + \epsilon_{r,t,s} \quad (5)
 \end{aligned}$$

where $Emp_{s,r,t}$ is the share of employment of sector s in region r at time t . The right-hand-side variables are defined as above (equation 4). Table 3 presents the results. Columns (1) and (4) confirm that changes in commodity prices increase the share of employment in the commodity sector, particularly for more commodity-intensive regions (as measured before the boom). The magnitudes of the estimated effects range between 0.044 and 0.049. Columns (3) and (6) also suggest the presence of positive spillovers to the nontradable sector as the estimated coefficients are positive and statistically significant among more commodity-intensive regions (0.277 and 0.280, respectively). On the other hand, the estimated coefficients reported in columns (2) and (5) suggest the presence of negative spillovers affecting the share of employment in the tradable sector. It is also worth noting that booms and busts do not seem to correlate with total employment shares. Given the estimated effects of these events on the skill premium (1), firms might adjust at a different margin. Our firm-level results in section 6 provide further identification of this novel transmission mechanism.

To further explore the contagion of commodity price dynamics across sectors within a local labor markets, Table 4 presents the empirical association between employment shares in tradable and nontradable sectors by skill levels as a function of commodity prices.

The findings are consistent with those reported in Table 3, confirming that spillovers to tradable

Table 3: Sector Employment (share) and Commodity Price Index

Variables	Commodity (1)	Tradable (2)	Nontradable (3)	Commodity (4)	Tradable (5)	Nontradable (6)
(ln) Price _{rt}	0.009 (0.01)	-0.006 (0.02)	-0.006 (0.02)	0.010 (0.01)	-0.009 (0.02)	-0.008 (0.02)
Boom _{rt}				0.005 (0.00)	-0.006 (0.01)	-0.003 (0.00)
Bust _{rt}				-0.001 (0.00)	0.001 (0.00)	0.001 (0.00)
Boom _{rt} x (ln) Price _{rt}				-0.002 (0.01)	0.013 (0.01)	0.005 (0.01)
Bust _{rt} x (ln) Price				-0.002 (0.01)	0.001 (0.01)	0.003 (0.00)
Intensity _r x (ln) Price _{rt}	0.027** (0.01)	-0.025* (0.01)	0.018* (0.01)	0.028** (0.01)	-0.026* (0.01)	0.018* (0.01)
Constant _s	0.049*** (0.00)	0.186*** (0.00)	0.277*** (0.00)	0.044*** (0.00)	0.190*** (0.01)	0.280*** (0.00)
Observations	33,419	33,419	33,419	33,419	33,419	33,419
R-squared	0.937	0.966	0.917	0.938	0.966	0.917
Fixed Effects:						
Year	Y	Y	Y	Y	Y	Y
Quarter	Y	Y	Y	Y	Y	Y
Region	Y	Y	Y	Y	Y	Y

Notes: Total number of regions is 558. Regressions are estimated using quarterly information for the period 1999-2013. $Emp_{s,r,t}$ is constructed using five sectors (commodity, tradable, nontradable, other services (mostly financial services and real estate) and government). (ln) Price is the result of the Hodrick-Prescott filter (trend) applied over the original price. Boom (Bust) is a dummy variable such that $Boom(Bust)=1$ if the actual price increases (decreases) at least 1% relative to its trend between two periods (quarters, = 0 otherwise. "Intensity"_r is a dummy variable such that equals one if commodities were abundant in the region in 1999, and zero otherwise. More precisely, for each region we first compute the share of the overall land devoted (or potentially devoted) to agriculture in 1999. The regional commodity intensity, 'Intensity'_r, equals one if the region had a share above the median, and zero otherwise. Standard errors (in parentheses) are clustered at the regional level. *** p<0.01, ** p<0.05, * p<0.1.

Table 4: Contagion: Employment (share) in Tradable and Nontradable Sectors as a function of Commodity Price Index, by Skill Levels

Variables	Unskilled		Skilled	
	Tradable (1)	Nontradable (2)	Tradable (3)	Nontradable (4)
(ln) Price _{rt}	-0.004 (0.03)	0.031 (0.02)	-0.013 (0.02)	-0.001 (0.02)
Boom _{rt}	-0.011 (0.01)	0.001 (0.01)	-0.004 (0.00)	-0.001 (0.00)
Bust _{rt}	-0.010** (0.00)	0.010* (0.01)	0.002 (0.00)	0.002 (0.00)
Boom _{rt} x (ln) Price	0.022* (0.01)	-0.003 (0.01)	0.009 (0.01)	0.001 (0.01)
Bust _{rt} x (ln) Price _{rt}	0.022** (0.01)	-0.023*** (0.01)	0.001 (0.00)	0.004 (0.01)
Intensity _r x (ln) Price _{rt}	-0.057** (0.03)	0.026* (0.01)	-0.024** (0.01)	0.033*** (0.01)
Constant _s	0.224*** (0.01)	0.321*** (0.01)	0.135*** (0.01)	0.201*** (0.00)
Observations	33,408	33,408	33,411	33,411
R-squared	0.947	0.913	0.968	0.914
Fixed Effects:				
Year	Y	Y	Y	Y
Quarter	Y	Y	Y	Y
Region	Y	Y	Y	Y

Notes: Skilled workers are defined as employees with more than 12 years of formal education. Unskilled workers are defined as employees with 12 or less years of formal education. (ln) Price is the result of the Hodrick-Prescott filter (trend) applied over the original price. Boom (Bust) is a dummy variable such that Boom(Bust)= 1 if the actual price increases (decreases) at least 1% relative to its trend between two periods (quarters, = 0 otherwise. ‘Intensity’_r’ is a dummy variable such that equals one if commodities were abundant in the region in 1999, and zero otherwise. More precisely, for each region we first compute the share of the overall land devoted (or potentially devoted) to agriculture in 1999. The regional commodity intensity, ‘Intensity’_r’, equals one if the region had a share above the median, and zero otherwise. Standard errors (in parentheses) are clustered at the regional level. *** p<0.01, ** p<0.05, * p<0.1.

and nontradable sectors are robust across skill levels in more commodity-intensive regions. In addition, for unskilled employment the results imply negative effects of busts on the tradable sector (column (1)) and positive effects on nontradables (column (2)). This is consistent with the hypothesis that firms respond to commodity price dynamics (and skill premium adjustments) by modifying their composition of labor inputs. It also suggests that unskilled workers might be those more affected by downward wage rigidity. These channels would amplify the spillovers across sectors.

In summary, our exploratory analysis suggests that (i) commodity booms decrease skill premium, (ii) downward wage rigidity affects the allocation of resources across regions, and (iii) commodity prices have positive (negative) spillovers to the nontradable (tradable) sector.⁷ Next we introduce a theoretical framework that provides quantitative support to these facts. Our conceptual framework also shed lights on the empirical associations between commodity price dynamics and firm entry and exit decisions. These implications are confirmed using firm-level data in section 6.

4. MODEL

This section introduces a deterministic small open economy model. Its representative consumer demands four types of goods: an imported good, a bundle of differentiated tradable goods, a nontradable good, and a tradable commodity good. The imported good is produced abroad. Its price is exogenous and it is used as the numéraire of the economy. The commodity good is produced by firms with heterogeneous permanent productivity using labor and a scarce natural resource with a decreasing returns to scale technology. Its price is exogenous, while the price of the scarce resource is endogenous. The nontradable good is produced by firms with heterogeneous productivity using labor with a decreasing returns to scale technology as well. Its price is endogenous. Tradable goods are produced by firms with heterogeneous productivity using labor with an increasing returns to scale technology. An endogenous productivity threshold determines which firms operate in

⁷One question that surfaces regarding employment and wage responses to commodity price cycles is the role played informality and regional mobility. As figures 16 and 17 in the appendix show, both regional mobility and the share of informal workers do not fluctuate at business cycle frequency. We see a slight upward trend in mobility and a stable and later downward path of informality during this period, which do not seem to respond to the commodity price boom.

the domestic market and another endogenous productivity threshold determines which firms have access to the external market. The international and the domestic demand for each variety are elastic, and prices are endogenously determined.⁸ The household supplies skilled and unskilled labor inelastically for production and borrows in international markets. Figure 7 shows a diagram of the model economy.

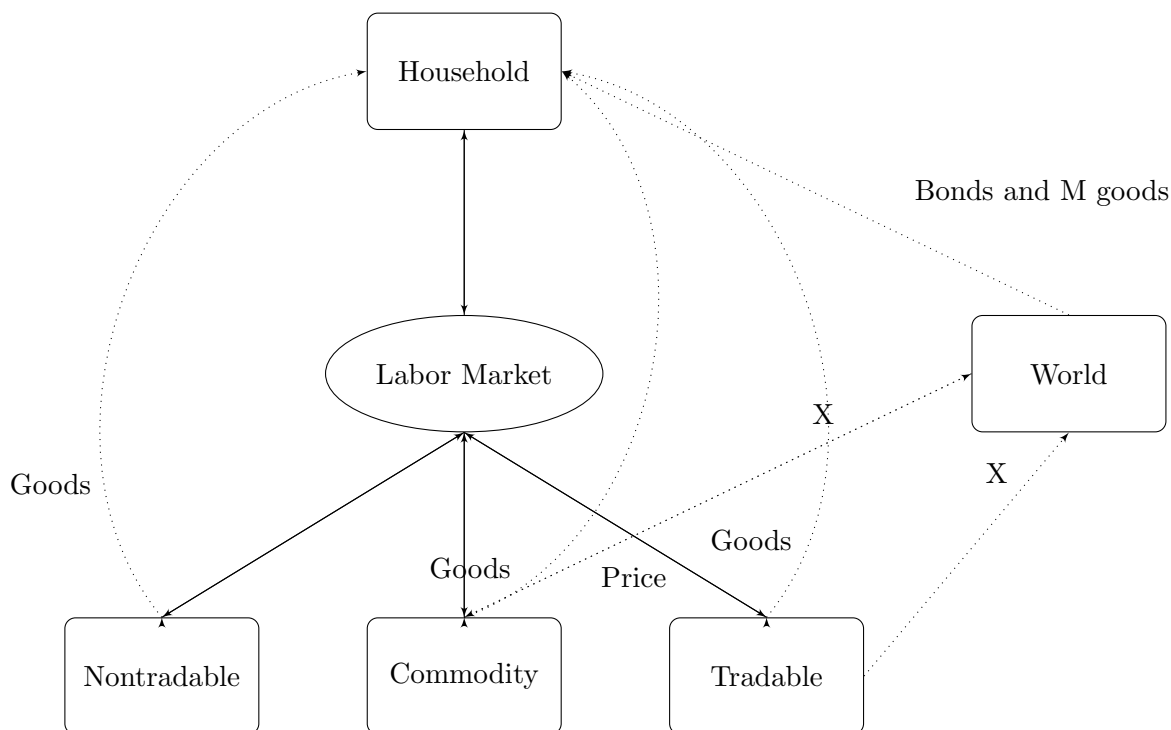


Figure 7: Model Economy

The model nests three heterogeneous-firms models and allows sectors to interact through the labor market. For instance, an increase in the price of the commodity good puts upward pressure on wages. This increases local domestic demand for all goods through an income effect, but it also increases the production cost of every good in the economy. The magnitudes and relative importance of these effects are heterogeneous across sectors. Since the commodity sector is relatively

⁸From a modeling perspective, the international aspect of the model is similar to [Mendoza \[1995\]](#). The commodity and nontradable sectors follow [Hopenhayn \[1992\]](#)'s model of firm dynamics, and the tradable sector follows [Melitz \[2003\]](#). Note that the commodity sector clearly differs from the other two sectors. First, unlike the nontradable good, commodity producers export some of their production at an exogenous price. Second, unlike the tradable sector exporters, commodity producers face an inelastic demand for their exports. Therefore, changes in the international commodity price have a strong passthrough to firm level decisions in the commodity sector as the supply has to do all the adjustment. These strong changes in the intensive (hiring and firing) and extensive (entry and exit) margins of commodity employment can potentially have strong general equilibrium spillovers to the other sectors. Thus, we provide a labor market-based micro-foundation to the *Dutch Disease* phenomenon that is independent of changes in the exchange rate.

more intensive in unskilled workers, the unskilled wage increases more than the skilled wage during the boom, reducing the skill premium. Therefore, sectors that are more skill intensive will see their production cost increase relatively less when compared to less skill intensive sectors. Moreover, the demand channel is especially important for firms selling locally. Therefore, in principle, firms in the nontradable sector should benefit more than firms in the tradable sector from the increase in domestic demand.

The model's dynamics are shaped by the link between commodity prices and the labor market, which in turns implies that labor market frictions have the potential to transform efficient sectoral reallocation into inefficient labor misallocation. To study this channel, we include downward wage rigidity as in [Schmitt-Grohé and Uribe \[2016\]](#) for unskilled labor. We now describe each of the model's components.

4.1. REPRESENTATIVE HOUSEHOLD

The representative household supplies skilled (L^s) and unskilled labor (L^u) inelastically, consumes imported goods (M), nontradable goods (N), the commodity good (C^d), and tradable goods (T^d). In particular, its lifetime utility function is given by:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma^r} \left[\left(M_t^\theta + \alpha^N N_t^\theta + \alpha^C (C_t^d)^\theta + \alpha^T (T_t^d)^\theta \right)^{\frac{1}{\theta}} \right]^{1-\gamma^r}, \quad (6)$$

where $\theta \in (0, 1)$ controls the elasticity of substitution between goods and $\gamma^r > 1$ the inter-temporal elasticity of substitution.

Denote by P_t^* the exogenous commodity good price, P_t^N the endogenous nontradable good price, P_t^T the endogenous tradable bundle price. The exogenous imported good price is normalized to one and used as numéraire. Then, the representative household's budget constraint becomes:

$$M_t + P_t^* C_t^d + P_t^N N_t + P_t^T T_t^d + B_{t+1} \leq w_t^s L^s + w_t^u L^u + P_{R,t} \bar{R} + (1+r^*)B_t + \Pi_t, \quad (7)$$

where bonds are in units of imported goods and their interest rate (r^*) is exogenous. Π_t collects the total profits from commodity, nontradable, and tradable sectors. The tradable bundle consumed

by the household is given by:

$$T_t^d = \left[\int_{\zeta \in \mathfrak{Z}} q_{d,t}(\zeta)^\rho d\zeta \right]^{\frac{1}{\rho}}, \quad (8)$$

where ζ is an index for individual varieties (or operating firms), and \mathfrak{Z} is an index set for individual varieties sold in the domestic market. The bundle price is given by the following aggregation of variety prices:

$$P_t^T = \left[\int_{\zeta \in \mathfrak{Z}} p_t(\zeta)^{1-\sigma} d\zeta \right]^{\frac{1}{1-\sigma}}.$$

Finally, it is useful to define the household's discount factor $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$ where:

$$\lambda_t = \left[M_t^\theta + \alpha^N N_t^\theta + \alpha^C (C_t^d)^\theta + \alpha^T (T_t^d)^\theta \right]^{\frac{1-\theta-\gamma^r}{\theta}} \cdot M_t^{\theta-1} \quad (9)$$

4.2. SECTORS

Commodity. The commodity good is produced by heterogeneous firms according to the following technology:

$$C_{i,t} = C(z_i; R_{i,t}, l_{i,t}^{s,C}, l_{i,t}^{u,C}) = z_i \times R_{i,t}^\xi \times \left(l_i^C(l_{i,t}^{s,C}, l_{i,t}^{u,C}) \right)^\eta,$$

where z_i is a permanent idiosyncratic productivity, $l_{i,t}^{s,C}$ is the skilled labor input, $l_{i,t}^{u,C}$ the unskilled labor input, and $R_{i,t}$ is the resource input used by firm i . Effective labor is defined as

$$l_i^C(l_{i,t}^{s,C}, l_{i,t}^{u,C}) \equiv \left[(\phi^C)^{\frac{1}{\gamma}} (l_{i,t}^{u,C})^{\frac{\gamma-1}{\gamma}} + (1 - \phi^C)^{\frac{1}{\gamma}} (l_{i,t}^{s,C})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}},$$

where $\phi^C \in (0, 1)$ is the share of unskilled labor in the commodity sector and $\gamma > 1$ is the elasticity of substitution between skilled and unskilled labor. Note that the technology has decreasing returns to scale and that firms are heterogeneous in their total factor productivity z_i . The resource input supply is fixed at \bar{R} . There is a fixed operating cost c^c measured in units of output and a fixed entry cost c_e^c measured in units of effective labor. A potential entrant draws its permanent productivity

z_i from a discrete distribution. Let $\mathbf{f}(\mathbf{z})$ and \mathbf{z} be the probability and a payoff vector of dimension n_z , respectively, characterizing this discrete distribution

Profits of firm i in period t are given by:

$$\pi_{i,t}^C(z; \hat{w}_t^C, P_{R,t}, P_t^*) = P_t^* \left[z_i R_{i,t}^\xi (l_{i,t}^c)^\eta - c^c \right] - l_{i,t}^C \hat{w}_t^C - P_{R,t} R_{i,t}, \quad (10)$$

with the price for effective labor given by:

$$\hat{w}_t^C = \left[\frac{\phi^C}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^C}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}}.$$

Firms are subject to an exogenous exit probability δ^C each period. They can also endogenously exit when their values turn negative. The value of a firm is given by:

$$W_t^C(z; \hat{w}_t^C, P_{R,t}, P_t^*) = \max \{0, \pi_t^C + (1 - \delta^C) \Lambda_{t,t+1} W_{t+1}^C(z; \hat{w}_{t+1}^C, P_{R,t+1}, P_{t+1}^*)\}. \quad (11)$$

The operational productivity cut-off \underline{z}_t is implicitly defined by:

$$W_t^C(\underline{z}_t; \hat{w}_t^C, P_{R,t}, P_t^*) = 0, \quad (12)$$

If $\mathcal{F}(z) \equiv \sum_{j=1}^z f(z)$, the time-varying productivity distribution of entrants can be written as:

$$\tilde{\mu}_t^C(z; \underline{z}_t) = \begin{cases} \frac{f(z)}{1 - \mathcal{F}(\underline{z}_t)} & , \text{ if } z \geq \underline{z}_t \\ 0 & , \text{ otherwise.} \end{cases} \quad (13)$$

The free entry condition can be written as:

$$\sum_{j=\underline{i}_t}^{n_z} W_t^C(z_j) f(z_j) = \hat{w}_t^C c_e^c \quad (14)$$

where \underline{i}_t is the position of the cut-off \underline{z}_t . Denote by \mathcal{M}_t^C and $\tilde{\mathcal{M}}_t^C$ the mass of producers and the

mass of potential entrants, respectively. The time-varying distribution of incumbents is given by:

$$\mu_{t+1}^C(z) = \begin{cases} \frac{(1-\delta^C)\mathcal{M}_t^C \mu_t^C(z) + \tilde{\mathcal{M}}_{t+1}^C f(z)}{\mathcal{M}_{t+1}^C} & , \text{ if } z \geq \underline{z}_{t+1} \\ 0 & , \text{ otherwise.} \end{cases} \quad (15)$$

Then, the law of motion that characterize the evolution of \mathcal{M}_{t+1}^C is given by:

$$\mathcal{M}_{t+1}^C = (1 - \delta^C) \mathcal{M}_t^C \sum_{j=i_{t+1}}^n \mu_t^C(z_j) + \tilde{\mathcal{M}}_{t+1}^C \sum_{j=i_{t+1}}^n f(z_j) \quad (16)$$

Note that the natural resource is in fixed supply, and the market clearing condition is:

$$\bar{R} = \mathcal{M}_t^C \sum_{j=i_t}^n R_t(z_j) \mu_t^C(z_j) \quad (17)$$

Therefore, the mass of potential entrants and the natural resource price endogenously adjust so that (14) and (17) hold every period.

Nontradable. The nontradable good is homogeneous and produced by heterogeneous firms with the following technology:

$$N(s_i, l_{i,t}^{u,N}, l_{i,t}^{s,N}) = s_i \times \left(l^N(l_{i,t}^{u,N}, l_{i,t}^{s,N}) \right)^\alpha,$$

As in the previous sector, a potential entrants draws its permanent productivity s_i from a discrete distribution characterized by $\mathbf{h}(\mathbf{s})$ (probability) and \mathbf{s} (payoff vector of length n_s). Effective labor, on the other hand, is defined as:

$$l^N(l_{i,t}^{s,N}, l_{i,t}^{u,N}) \equiv \left[(\phi^N)^{\frac{1}{\gamma}} (l_{i,t}^{u,N})^{\frac{\gamma-1}{\gamma}} + (1 - \phi^N)^{\frac{1}{\gamma}} (l_{i,t}^{s,N})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}},$$

with price

$$\hat{w}_t^N = \left[\frac{\phi^N}{(w^u)^{\gamma-1}} + \frac{1 - \phi^N}{(w^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}}.$$

If we let c_f denote the fixed operating costs (measured in units of outputs), profits can be written

as:

$$\pi^N(l_{i,t}^{u,N}, l_{i,t}^{s,N}; s_i, w_t^u, w_t^s, P_t^N) = P_t^N \left[s_i \left(l^N(l_{i,t}^{u,N}, l_{i,t}^{s,N}) \right)^\alpha - c_f \right] - w_t^u l_{i,t}^{u,N} - w_t^s l_{i,t}^{s,N}, \quad (18)$$

There is an exogenous exit rate δ^N . The period- t value of an incumbent firm is given by:

$$W_t^N(s, \hat{w}_t^N, P_t^N) = \max \{0, \pi_t^N + (1 - \delta^N) \Lambda_{t,t+1} W_{t+1}^N(s, \hat{w}_{t+1}^N, P_{t+1}^N)\}. \quad (19)$$

The operational productivity threshold in the nontradable sector \underline{s}_t is implicitly defined by:

$$W_t^N(\underline{s}_t; \hat{w}_t^N, P_t^N) = 0. \quad (20)$$

The time-varying productivity distribution of entrants is given by:

$$\tilde{\mu}_t^N(s) = \begin{cases} \frac{h(s)}{1 - \mathcal{H}(\underline{s}_t)} & , \text{ if } s \geq \underline{s}_t \\ 0 & , \text{ otherwise} \end{cases} \quad (21)$$

where $\mathcal{H}(s) \equiv \sum_{j=1}^h h(s)$, and the free entry condition is:

$$\sum_{j=\hat{i}_t}^{n_s} W_t^N(s_j; \hat{w}_t^N, P_t^N) h(s_j) = \hat{w}_t^N c_e \quad (22)$$

where c_e is an entry cost (in units of effective labor) and \hat{i}_t is the grid point position associated with the cut-off \underline{s}_t . We denote by \mathcal{M}_t^N and $\tilde{\mathcal{M}}_t^N$ the mass of producers and the mass of potential entrants. The time varying distribution of incumbents is given by:

$$\mu_{t+1}^N(s) = \begin{cases} \frac{(1-\delta^N) \mathcal{M}_t^N \mu_t^N(s) + \tilde{\mathcal{M}}_{t+1}^N h(s)}{\mathcal{M}_{t+1}^N} & , \text{ if } s \geq \underline{s}_{t+1} \\ 0 & , \text{ otherwise.} \end{cases} \quad (23)$$

Then, the law of motion that characterize the evolution of \mathcal{M}_{t+1}^N is:

$$\mathcal{M}_{t+1}^N = (1 - \delta^N) \mathcal{M}_t^N \sum_{j=\hat{i}_{t+1}}^n \mu_t^N(s_j) + \tilde{\mathcal{M}}_{t+1}^N \sum_{j=\hat{i}_{t+1}}^n h(s_j) \quad (24)$$

The time-varying mass of potential entrants and the endogenous price of the nontradable good dynamically make the free entry condition and the market for the nontradable good clear along the equilibrium transition path.

Tradable. Firms in the tradable sector are heterogeneous in their productivity φ . They produce heterogeneous varieties indexed by ζ using the following technology:

$$q_t(\varphi) = q(\varphi; l_t^{s,T}, l_t^{u,T}) = \varphi \times \left(l^T(l_t^{s,T}, l_t^{u,T}) - f \right),$$

where φ is drawn from a discrete distribution. We denote by $\{g(\varphi), \varphi\}$ the pair of probability ($g(\varphi)$) and payoff (φ) vectors of length n_φ that characterize the discrete distribution. f is an operational fixed cost (in units of effective labor) given by:

$$l_t^T = l^T(l_t^{s,T}, l_t^{u,T}) \equiv \left[(\phi^T)^{\frac{1}{\gamma}} (l_t^{u,T})^{\frac{\gamma-1}{\gamma}} + (1 - \phi^T)^{\frac{1}{\gamma}} (l_t^{s,T})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}},$$

with price

$$\hat{w}_t^T = \left[\frac{\phi^T}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^T}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}},$$

Domestic production is subject to $f = f_d$, while production for foreign markets has a higher fixed cost $f = f_x > f_d$. Firms that only produce domestically face the following demand:

$$q_{d,t}(\zeta) = T_t^d \left[\frac{p_t(\zeta)}{P_t^T} \right]^{-\sigma}, \quad (25)$$

while exporting firms face a larger demand given by:

$$q_{ex,t}(\zeta) = q_{d,t}(\zeta) + q_{x,t}(\zeta) = \left[T_t^d (P_t^T)^\sigma + \gamma_0 \right] p_t(\zeta)^{-\sigma},$$

where γ_0 denotes the size of the foreign market. Thus, tradable variety ζ sold in the foreign market is equal to $q_{x,t}(\zeta) = \gamma_0 p_t(\zeta)^{-\sigma}$. For simplicity we assume the same price elasticity for domestic and

foreign demand. Therefore, the marginal revenue for both exporters and non exporters is given by:

$$MR_t(\zeta) = \rho p_t(\zeta) .$$

Regardless of their productivity, firms face a residual demand curve with constant elasticity σ and, as a consequence, choose the same markup $\sigma/(\sigma - 1) = 1/\rho$ over marginal cost \hat{w}_t^T/φ . This yields the following pricing rule:

$$p_t(\varphi) = \frac{\hat{w}_t^T}{\rho\varphi} . \quad (26)$$

The revenue of a firm is given by:

$$r_t(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \begin{cases} r_{d,t}(\varphi) = P_t^T T_t^d \left[\frac{P_t^T \rho\varphi}{\hat{w}_t^T} \right]^{\sigma-1} & , \text{ for non-exporters} \\ r_{d,t}(\varphi) + \gamma_0 \left(\frac{\rho\varphi}{\hat{w}_t^T} \right)^{\sigma-1} & , \text{ for exporters} \end{cases}$$

Firms profits can be written as

$$\pi_t(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \begin{cases} \pi_{d,t}(\varphi) = \frac{r_{d,t}}{\sigma} - \hat{w}_t^T f_d & , \text{ for non-exporters} \\ \pi_{ex,t}(\varphi) = \pi_{d,t}(\varphi) + \frac{r_{x,t}}{\sigma} - \hat{w}_t^T f_x & , \text{ for exporters} \end{cases} \quad (27)$$

Given an exogenous exit rate of δ^T , the value of a firm $j = \{d, ex\}$ is:

$$W_{j,t}^T(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \max \left\{ 0, \pi_{j,t}(\varphi; \hat{w}_t^T, P_t^T, T_t^d) + (1 - \delta^T) \Lambda_{t,t+1} W_{t+1}^T(\varphi; \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d) \right\} , \quad (28)$$

where the continuation value is:

$$W_{t+1}^T(\varphi; \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d) = \begin{cases} W_{d,t+1}^T & , \text{ if } \varphi_{t+1}^* \leq \varphi < \varphi_{x,t+1}^* \\ W_{ex,t+1}^T & , \text{ if } \varphi \geq \varphi_{x,t+1}^* \\ 0 & , \text{ otherwise} \end{cases}$$

The operational (φ_t^*) and exporting ($\varphi_{x,t}^*$) cut-offs are implicitly defined by the conditions:

$$W_{d,t}^T(\varphi_t^*; \hat{w}_t^T, P_t^T, T_t^d) = 0 \quad (29)$$

$$W_{ex,t}^T(\varphi_{x,t}^*; \hat{w}_t^T, P_t^T, T_t^d) = 0. \quad (30)$$

The time-varying productivity distribution of entrants is given by:

$$\tilde{\mu}_t^T(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - \mathcal{G}(\varphi_t^*)} & , \text{ if } \varphi \geq \varphi_t^* \\ 0 & , \text{ otherwise,} \end{cases} \quad (31)$$

where $\mathcal{G}(\varphi) \equiv \sum_{j=1}^g g(\varphi)$. The time-varying probabilities of entering ($p_{in,t}$) and being an exporter ($p_{x,t}$) are:

$$p_{in,t}(\varphi^*) = 1 - \mathcal{G}(\varphi_t^*)$$

$$p_{x,t}(\varphi^*) = \frac{1 - \mathcal{G}(\varphi_{x,t}^*)}{1 - \mathcal{G}(\varphi_t^*)}.$$

The free entry condition is given by:

$$\sum_{j=\hat{l}_t}^{n_\varphi} W_t^T(\varphi_j) g(\varphi_j) = \hat{w}_t^T f_e. \quad (32)$$

where f_e is an entry cost associated with a new productive draw (in units of effective labor) and \hat{l}_t is the grid point position associated with the cut-off φ_t^* . Denote by \mathcal{M}_t^T and $\tilde{\mathcal{M}}_t^T$ the mass of producers and the mass of entrants, respectively. The time-varying distribution of producers is

given by:

$$\mu_{t+1}^T(\varphi) = \begin{cases} \frac{(1-\delta^T)\mathcal{M}_t^T \mu_t^T(\varphi) + \tilde{\mathcal{M}}_{t+1}^T g(\varphi)}{\mathcal{M}_{t+1}^T} & , \text{ if } \varphi \geq \varphi_{t+1}^* \\ 0 & , \text{ otherwise.} \end{cases} \quad (33)$$

Then, the law of motion that characterize the evolution of \mathcal{M}_{t+1}^T becomes:

$$\mathcal{M}_{t+1}^T = (1 - \delta^T) \mathcal{M}_t^T \sum_{j=i_{t+1}}^n \mu_t^T(\varphi_j) + \tilde{\mathcal{M}}_{t+1}^T \sum_{j=i_{t+1}}^n g(\varphi_j) \quad (34)$$

4.3. LABOR MARKET FRICTIONS

Labor-market rigidities are a particularly important in the context of Brazil, one of the most rigid labor markets in Latin America characterized by a wide variety of firing costs and minimum wages. We impose the following restriction in the evolution of wages:

$$w_{t+1}^u \geq \chi w_t^u$$

The following slackness condition emphasizes that unemployment is demand determined.

$$(w_{t+1}^u - \chi w_t^u) (l^u - \bar{l}^u) = 0$$

Note that, either the restriction is not binding and the economy is at full employment, or the restriction binds and there is involuntary unemployment. This is a tractable way to capture the effect of labor frictions at the bottom of the wage and skill distribution.

5. QUANTITATIVE ANALYSIS

5.1. CALIBRATION

Table 5 shows a set of 16 externally calibrated parameters that are consistent with the literature and the regional level data described in Section 2. The discount factor (β) is set to match an annual interest rate of 4%. The parameters governing risk aversion (γ^r), substitution between final goods

(θ), substitution between tradable varieties (σ), and substitution between skilled and unskilled labor (γ) are taken from the trade and international finance literature. The parameters governing the production technologies (ξ , η , and α) are consistent with the heterogeneous firm literature and the importance of land in agriculture. The downward wage rigidity parameter (χ) is taken from [Schmitt-Grohé and Uribe \[2016\]](#). The intensity in the use of skilled and unskilled labor in each sector (ϕ^C , ϕ^N , and ϕ^T) and the abundance of unskilled workers (κ) are consistent with the regional data of Brazil. It is interesting to note that commodity is the more unskilled intensive sector, and that nontradables is the least unskilled intensive sector. Because exit rates do not differ much across sectors, we set the steady state annual exit rate to 12% in every sector.

Table 5: Literature and Data

parameter	description	value	source
β	time discount factor	0.9615	4% annual interest rate
γ^r	risk aversion	1.5	literature
θ	substitution consumption bundle	0.80	literature
σ	tradable sector substitution	4	literature
γ	substitution skilled/unskilled labor	1.41	literature
ξ	resource share, C sector	0.38	literature
η	labor share, C sector	0.45	literature
α	labor share, N sector	0.65	literature
χ	Downward wage rigidity	0.9958	literature
κ	share unskilled labor, aggregate	0.85	Regional Micro Data
ϕ^C	share unskilled labor, C sector	0.95	Regional Micro Data
ϕ^N	share unskilled labor, N sector	0.81	Regional Micro Data
ϕ^T	share unskilled labor, T sector	0.83	Regional Micro Data
δ^C	death rate, C sector	0.12	Regional Micro Data
δ^N	death rate, N sector	0.12	Regional Micro Data
δ^T	death rate, T sector	0.12	Regional Micro Data

Table 6 shows the set of 8 parameters that are internally calibrated to match macro ratios of the Brazilian economy. We set $P^C = P^T = P^M$ in the initial steady state. The total labor supply is adjusted to have a GDP of 100 in the initial steady state. Because this is a small open economy that faces an exogenous interest rate, an initial condition for bond holdings is needed to solve the model. Without loss of generality we assume that the economy starts with zero net foreign assets.⁹ The stock of natural resources is used to match the commodity output share. The size of the foreign market is used to target the export to GDP ratio of the Brazilian economy. The preference

⁹See [Schmitt-Grohé and Uribe \[2003\]](#) for a complete discussion of the fundamental indeterminacy between bonds and consumption in small open economy models.

parameters for commodity and nontradable goods are used to target their respective consumption shares.

Table 6: Calibrated Parameters: Macro Level

parameter	description	value	target	target value
P^C	commodity price	1.0	Normalization	$P^C = 1$
α_T	preference tradable	1.2	Normalization	$P^T = P^C$
\bar{L}	labor supply	39.8	Normalization	$Y = 100$
\bar{B}	steady state bond	0	Trade-balance/GDP	$TB/Y = 0$
\bar{R}	resource supply	7.2	Commodity output share	$C/Y = 0.14$
γ_0	size foreign market	10.2	Exports/GDP	$X/Y = 0.13$
α_C	preference commodity	0.9	Commodity consumption share	$C^d/Y = 0.07$
α_N	preference nontradable	2.4	Nontradable consumption share	$N/Y = 0.52$

To conclude the calibration, we parametrize the fixed costs and the productivity distribution. For simplicity, we assume that every sector has the same underlying productivity distribution. Therefore, the differences in the size distribution of firms are loaded into differences in fixed costs. Although every parameter affects every moment, Table 7 points to some strong relationships between moments and parameters.

Table 7: Calibrated Parameters: Micro Level

parameter	description	value	target	Data	Model
c^c	Fixed cost, C sector	1.0	Top 25% share U employment, C sector	0.82	0.83
c^f	Fixed cost, N sector	1.2	Top 25% share U employment, N sector	0.84	0.60
f_d	Fixed cost, T sector domestic	1.1	Top 25% share U employment, T sector	0.79	0.67
f_x	Fixed cost, T sector foreign	1.8	Fraction of exporters, T sector	0.21	0.21
c_e^c	Fixed entry cost, C sector	1.8	Bottom 25% share U employment, C sector	0.82	0.83
c_e^f	Fixed entry cost, N sector	5.5	Bottom 25% share U employment, N sector	0.84	0.60
f_e	Fixed entry cost, T sector	7.3	Bottom 25% share U employment, T sector	0.79	0.67
μ	Prod. distribution, all sectors	0.5	Share of firms, C sector	0.02	0.07
σ	Prod. distribution, all sectors	0.5	Share of firms, N sector	0.71	0.70

5.2. SOLUTION METHOD

Commodity cycles can be conceptualized as low frequency fluctuations. Therefore, when the DSGE literature centers its analysis at business cycle frequencies, most of the action is lost by construction. In this paper, we take a different perspective. In particular, for a given sequence of commodity prices $\{P_t^*\}_0^\infty$ such that $P_t^* = \bar{P}$ for every $t > T$, we can solve a perfect foresight

transition. Similarly to Alberola and Benigno (2017), in this economy, transitory shock have permanent effects. In fact, during good times the economy accumulates wealth (B). Then, in normal times, the economy can use the associated interests (rB) to finance a negative trade balance and consume more. Due to agents' perfect foresight, the economy immediately jumps to a new level of marginal utility of consumption (λ) and stays there forever. Therefore, in order to solve for the perfect foresight transition, the new level of λ is needed. The solution algorithm can be described in three steps.

Step 1 In the outer loop guess the marginal utility of consumption after the economy transits to the new steady state in T periods.

Step 2 For each level of marginal utility:

2.1 Guess a pair of sequences of skilled and unskilled wages.

2.2 Calculate the implied final steady state at time T .

2.3 Backward loop: Recover operation thresholds of each sector.

2.4 Forward loop: Start at the initial steady state and solve for the evolution of the firm distribution in each sector.

2.5 Use the labor market clearing condition for skilled workers and the slackness condition for unskilled workers to update wages until the sequence of wages converges.

Step 3 Update marginal utility of consumption until bond holdings are stable in the long run.

The algorithm allows for corner solutions during the transition where entry in one or more sectors can be zero.¹⁰ The next subsection studies a 10% increase in the price of the commodity for 4 years, where starting at year 5 the price returns to its original level.

5.3. COMMODITY CYCLES: CONTAGION

Two main channels drive the heterogeneous contagion of commodity super cycles in the model. First, a *cost channel* driven by the changes in skill premium and the differential skill intensity of

¹⁰Details of the algorithm are described in Appendix A3.

the tradable and the nontradable sector. Second, a *demand channel* driven by the wealth effect of the commodity boom and the heterogeneous dependence of domestic demand.

For our quantitative analysis, we assume that the economy is in steady state in period 0, and in period 1 a 10% increase in the commodity price that last until period 5 is revealed to the agents in the economy. The period of high commodity price is denoted by a gray area in each graph. We simulate two economies, one with flexible wages (solid line) and one with downward wage rigidity on the unskilled workers (dashed line).

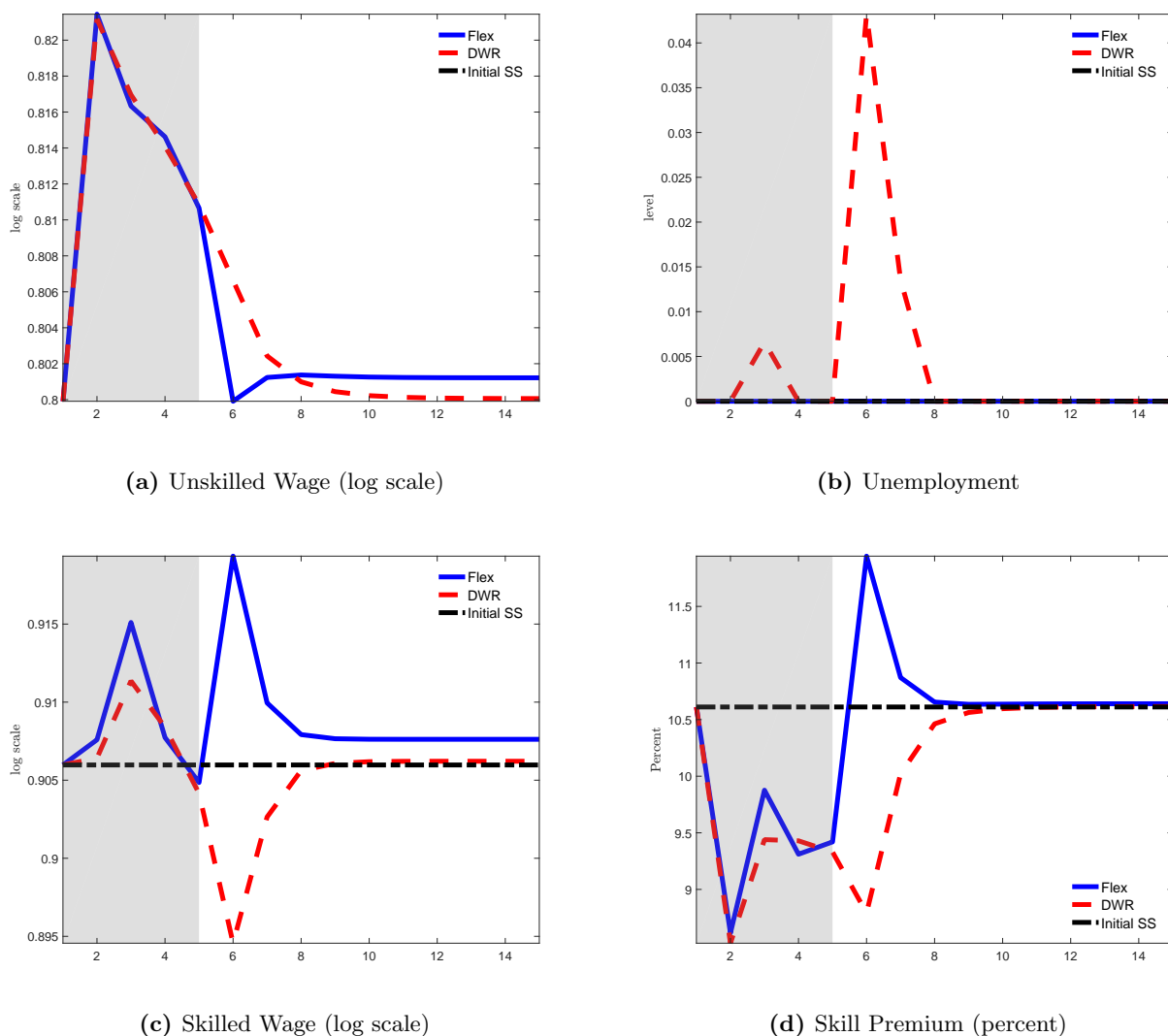


Figure 8: Skill Premium and Commodity Booms

Figure 8 show the evolution of wages and unemployment during the commodity cycle. Let's analyze first the economy without downward wage rigidity. Note that unskilled and skilled wages

increase during the boom. Nevertheless, because commodity production is more intensive in unskilled labor, the increase in the unskilled wage is larger.¹¹ Therefore, the skill premium decreases when commodity prices during the boom and, consequently, the model generates a correlation between skill premium and commodity prices.

On the other hand, because the commodity sector expands during the boom, when the bust starts and the commodity sector contracts, unskilled workers are relatively abundant and their wage drop while the skilled wage overshoots. Thus, with flexible wages, the bust is associated with an increase of the skill premium. The dashed line in Figure 8 shows the effects of downward wage rigidity in this economy. In particular, panel 8b shows that unemployment spikes in two periods, achieving a pick of nearly 5% at the end of the boom. Interestingly, given that the two types of labor are complements, now the skilled wage decreases during the bust in panel 8b. Therefore, the skill premium does not increase during the bust and slowly converges from below to its new level. The differential behavior of unemployment and skill premium are an important determinant in the transmission and the aggregate cost of commodity super cycles in this economy.¹² Figure 9 shows the differential spillover of the commodity boom to total employment in the nontradable and tradable sectors.

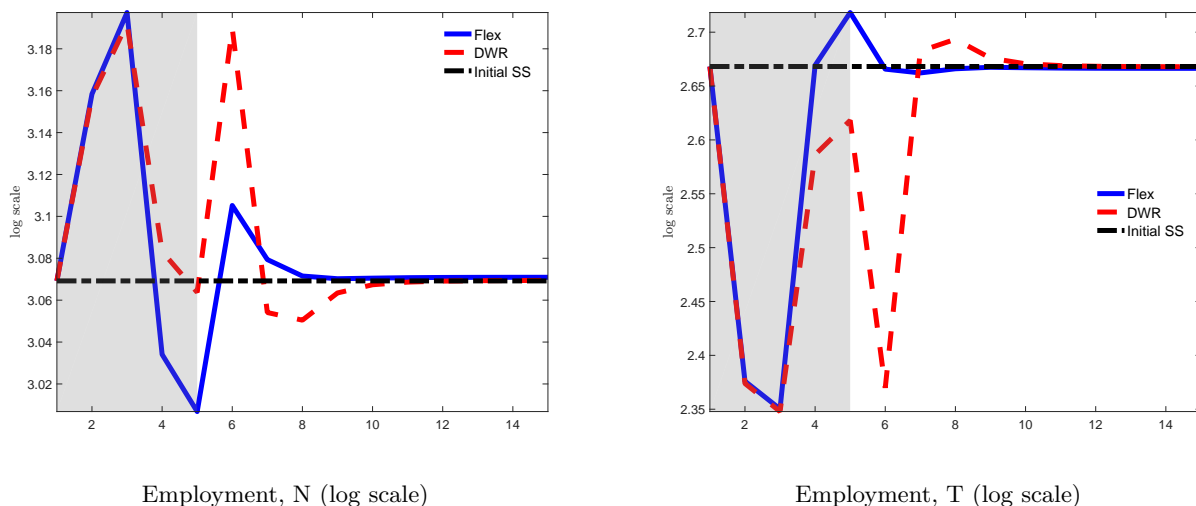


Figure 9: Total Sectoral Employment

¹¹The permanent effects of this transitory shock are clear, none of the wages go back to their initial steady state.

¹²The qualitative analysis should not change if downward wage rigidity is present in both sectors. In fact, given the unskilled intensity of the commodity sector, the solid line in Figures 8c and 8a show that downward wage rigidity would be more active for the unskilled wage.

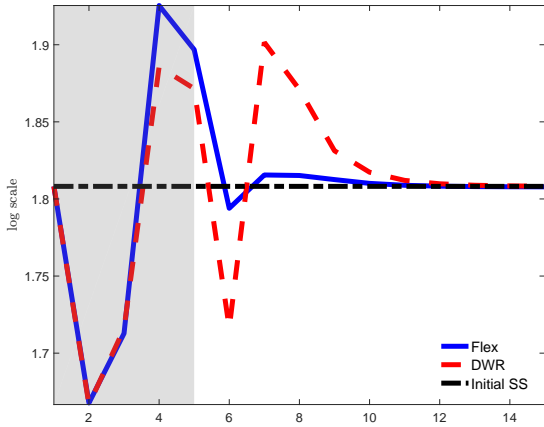
In the absence of downward wage rigidity, nontradable employment expands during the boom while tradable employment decreases. Because of perfect foresight, incumbents and potential entrants anticipate the end of the boom and start adjusting in period 3. The gradual increase of the skill premium in panel 8d after period 3, benefits the tradable sector relatively more than the nontradable sector, triggering a contraction of the nontradable sector and an expansion on the tradable sector at the end of the boom. The dashed lines in Figure 9 shows how downward wage rigidity can affect sectoral reallocation. In fact, the persistent decrease and the slow recovery of the skill premium in Figure 8d under downward wage rigidity, triggers an expansion of the nontradable sector and a further contraction of the tradable sector at the end of the boom. Figure 10 decomposes aggregate employment into average firm employment and total number of firms.

Figures 10a and 10b show the evolution of the average firm employment during the episode. Note that incumbents in the tradable sector decrease their employment during the boom while incumbents on the nontradable sector increase it. Figures 10c and 10d show that the net entry rate in each sector follows a similar pattern. The model then predicts that the adjustment in employment takes place at both, the intensive and the extensive margin.

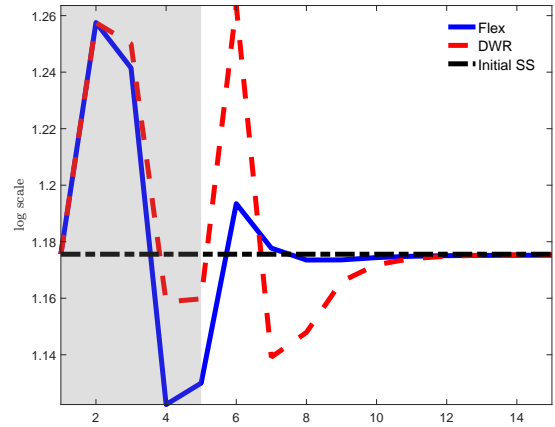
Figures 10e and 10f show the evolution of the mass of active firms in each sector. Interestingly, the allocation disruption triggered by downward wage rigidity can be extremely persistent affecting the mass of firms even 5 years after the boom. In this economy, the differential skill intensity is not the only channel that drives the heterogeneous cross sectoral spillovers of a commodity boom. In fact, because of the wealth effects triggered by the boom, domestic demand also increases. This second source of contagion is also potentially heterogeneous across sectors. In fact, while the nontradable sector depends exclusively in the domestic market, the most productive and larger firms in the tradable market depend much less on the domestic market and have their profits concentrated on their exporting activities. Figure 11 shows the local and foreign profits of the most productive firm in the tradable sector to explain the heterogeneity of the demand channel.

Finally, we focus on the case of the most productive firm on the tradable sector. This firm is always present in both, the domestic and the foreign market. Note that the skill premium (labor cost channel) affects the production cost of every unit of the firm similarly, regardless of the market at which the units are sold. Moreover, because the elasticity of the foreign and domestic demand is the same, there is also no difference in the price of the units. Thus, any difference

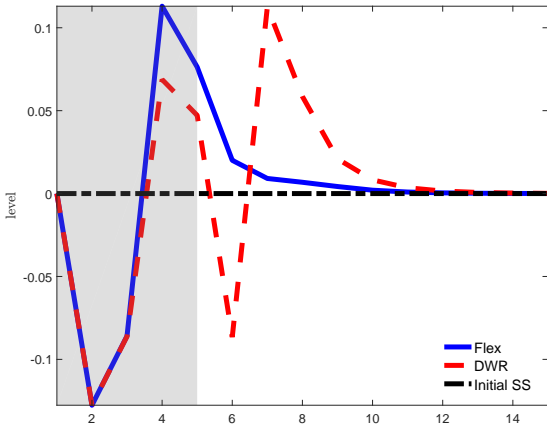
between domestic and foreign profits is due to a change in the relative demand. On the one hand, Figure 11a shows that domestic profits increase during the boom, reflecting the wealth effect of the commodity episode in domestic demand. On the other hand, Figure 11b shows that foreign profits decrease due to the fact that the increase in production cost is not compensated by an increase in foreign demand. Therefore, it is not surprising to see in Figure 11c that the fraction of exporters decreases during the boom. Therefore, the model predicts that larger firms and exporters in the tradable sector should be more affected by the commodity boom than small firms that operate locally.



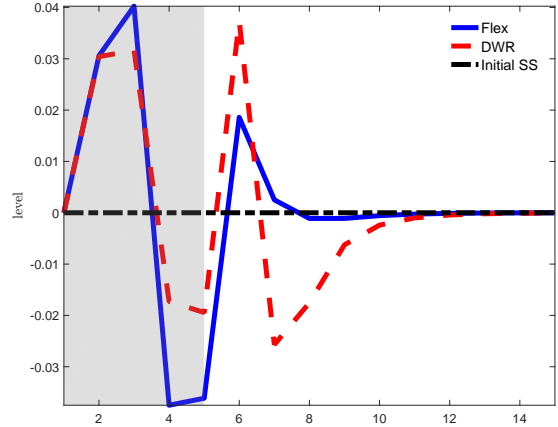
(a) Firm Size, T (log scale)



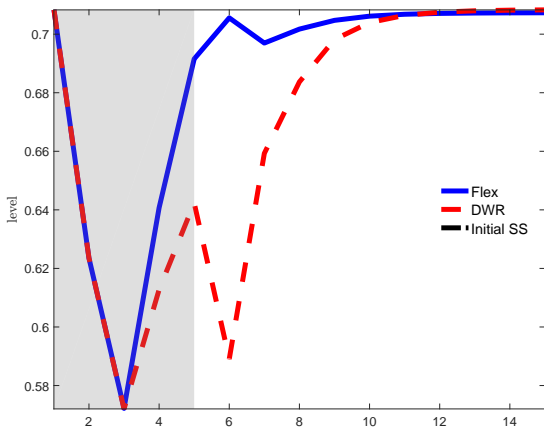
(b) Firm Size, N (log scale)



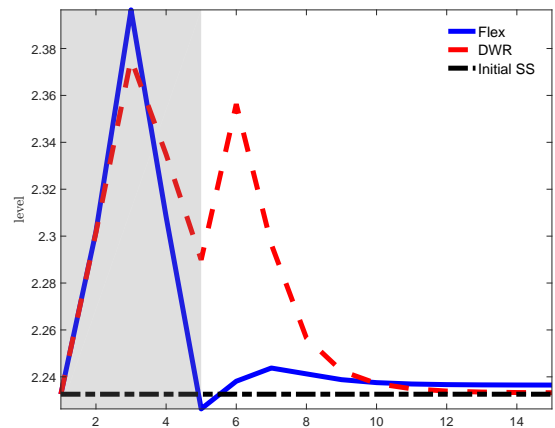
(c) Net Entry, T



(d) Net Entry, N

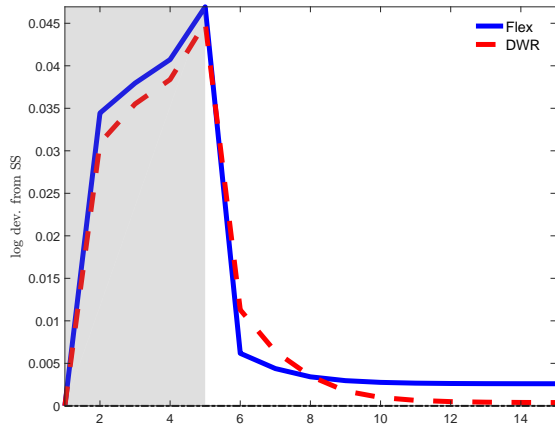


(e) Mass of Firms, T

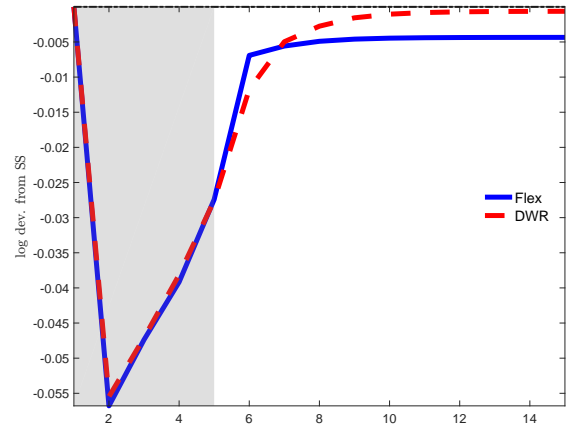


(f) Mass of Firms, N

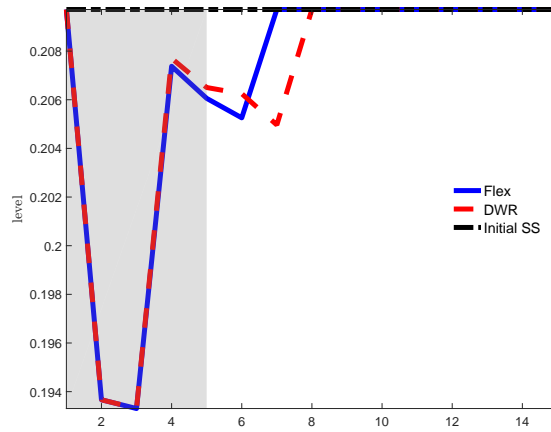
Figure 10: Sectoral Employment: Intensive and Extensive Contagion



Domestic Profits, T (log difference to $t = 0$)



Foreign Profits, T (log difference to $t = 0$)



Fraction of Exporters in T

Figure 11: Domestic and Foreign Profits of the Most Productive Firm in T

5.4 COMMODITY CYCLES: AGGREGATE EFFECTS

Having characterized the main driving forces of the model, we proceed to evaluate the macroeconomic effects of a commodity boom and how costly can downward wage rigidity be during these episodes. Figure 12 shows sectoral output and GDP at current prices.¹³

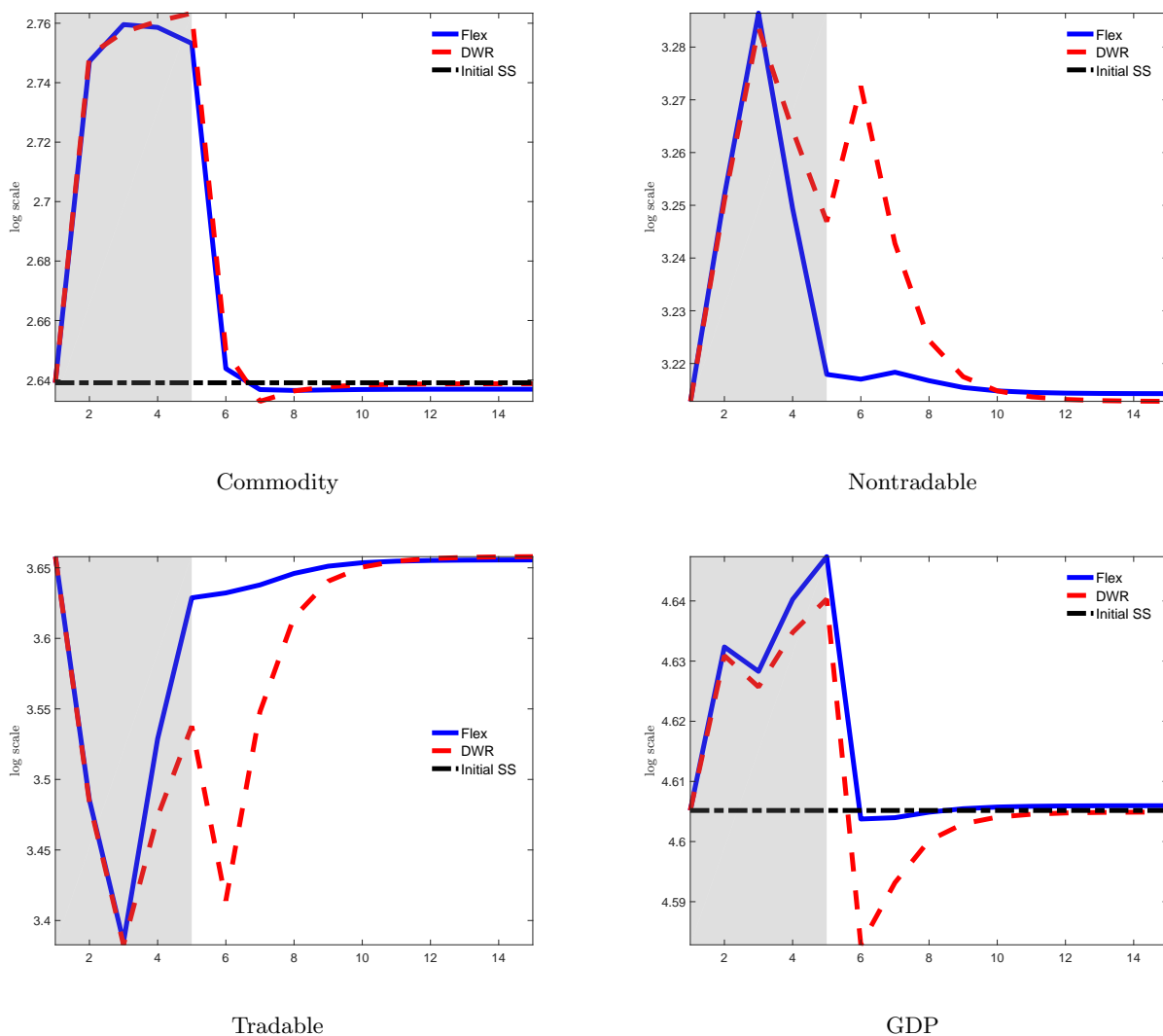


Figure 12: Sectoral Output and GDP at Current Prices

Figure 12a shows the boom and bust of the commodity sector. In line with the highlighted channels above-mentioned, Figure 12b shows an expansion of the nontradable sector during the boom while Figure 12c shows a contraction of the tradable sector during the same period. This

¹³Appendix A2 replicates this analysis at constant prices.

natural reallocation cannot be quickly undone in the presence of downward wage rigidity. Therefore, the dashed lines shows a persistent boom in the nontradable sector and a persistent bust in the tradable sector, well after the commodity cycle has ended. Moreover, aggregate GDP in Figure 12 shows that downward wage rigidity can explain the painful end of commodity super-cycles, characterized by unemployment and deep recessions, documented by Reinhart et al. [2016].

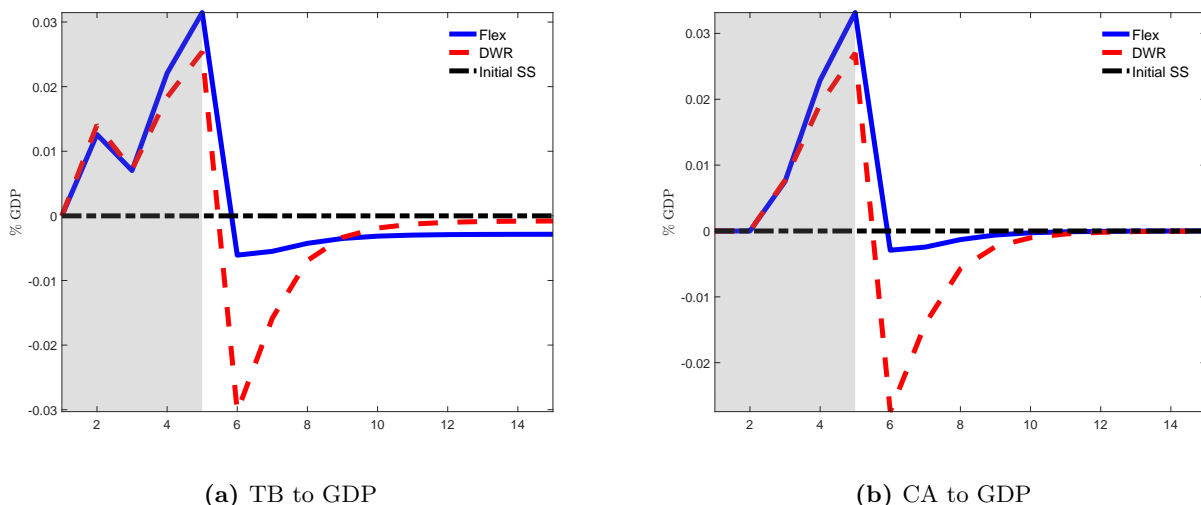


Figure 13: Trade Balance and Current Account

Figure 13 shows a trade balance surplus during the boom that increases the accumulation of wealth. This is consistent with the empirical literature. After the boom, the country uses the accumulated wealth to finance a sequence of trade deficits. Interestingly, downward wage rigidity is associated with a severe current account crisis at the end of the boom. Finally, Table 8 shows the long-run and welfare effects of the episode.

Table 8: Welfare Cost of a Commodity Cycle

Variable	Flexible	DWR
Net Foreign Assets/GDP	7.1%	1.9%
Trade Balance/GDP	-0.3%	-0.1%
Consumption Equivalent Welfare	-0.07%	-0.36%

The permanent effect of the transitory commodity boom can be seen by the long run level of the net foreign assets to GDP ratio and the trade balance to GDP ratio. The flexible economy converges to a new steady state with almost 4 times more foreign assets and that can sustain a three times

larger trade deficit. To abstract from valuation effects, we also compute the consumption equivalent welfare cost of each equilibrium, i.e., the percentage of consumption in every item that the household would forgo in order to avoid the commodity cycle. Although the quantitative experiment focused on a small increase in the price of commodities, the model suggests that commodity cycles are five times more costly in an economy with rigid labor markets. Note that in both cases the number is negative, this means that the episode is costly for the economy as the household is willing to forgo a fraction of its steady state consumption to avoid the commodity cycle. This negative effect is rooted on the fact that the Brazilian economy consumes an important share of their commodity production. In this sense, the model rationalizes the protests in Brazil in the early 2000 to complain about the price of the agricultural goods.

6. FIRM-LEVEL EVIDENCE.

Within-Firm Employment Responses

Section 3 presented aggregate evidence of the economic impacts of the commodity price cycles on aggregate employment. We now turn our analysis to firm-level responses. Importantly, at this level we can control for additional unobserved characteristics, namely firm-fixed effects, which could have biased our previous findings.

We begin by documenting within-firm employment responses to regional commodity price shocks. In particular, we estimate the following regression model:

$$E_{ft} = \beta \cdot PT_{rt} + \gamma_f + \delta_t + \epsilon_{ft}$$

where E_{ft} is the (log) firm employment of firm f in period (quarter) t , PT_{rt} is the (log) commodity price index (its trend) for region r in the period, δ_t represents quarter fixed effects, while γ_f is an unobserved firm-specific component. We cluster standard errors at the firm level.¹⁴

The results are reported in table 9 for each sector and different samples. Columns (1) to (3) correspond to the impact on firms in the commodity sector. Here, the estimated elasticity for the impact of the regional commodity price on within-firm employment is 0.137 (column (1)). The

¹⁴Following the literature, we exclude firms with less than 15 employees. Our results are qualitatively similar when we relax this restriction but less stable.

magnitude of the adjustment increases with firm size: 0.206 for firms with 50+ employees (column (2)) and a 0.238 among firms with 100+ employees (column (3)). Overall, we find a large and significant contemporaneous adjustment in firms' employment in response to commodity prices.

There is also contagion to firms operating in the tradable sector. As we discuss in section 5, these firms face negative cost shocks as regional wages increase in response to commodity price increases. This might lead to a decline in employment. Columns (4) to (7) in table 9 document this effect. A one percent increase in the regional commodity price leads to a 0.111 percent decrease in within-firm employment in the full sample of firms (column (4)). The adjustment is similar for larger firms (columns (5)-(6)).

Firm-level data also allows us to investigate whether negative impact of commodity prices on employment in firms in the tradable sector is stronger among exporters. In order to test this model prediction, we restrict the sample to the period 2001-2013 (for which our firm-level exporter information is available) and estimate:

$$E_{ft} = \beta_0 \times PT_{rt} + \beta_1 \times exporter_{ft} + \beta_2 \times PT_{rt} \times exporter_{ft} + \gamma_f + \delta_t + \epsilon_{ft} \quad (35)$$

where $exporter_{ft}$ is a dummy variable indicating whether the firm f in period t was exporting. Column (7) in table 9 displays the estimated elasticity. A one percent increase in the regional commodity price index leads to 0.037 percent decline in within-firm employment for non-exporters and 0.232 percent decline among exporting firms. This confirms that exporting firms benefit less from the increase in demand brought by a commodity price boom.¹⁵

Finally, columns (8) to (10) show the results in the nontradable sector. In this case, our estimated elasticities for firms operating in the nontradable sector combine these two effects. On the one hand, nontradable firms face the same negative cost-shock faced by their tradable counterparts as commodity prices rise. However, this cost shock is milder as the nontradable sector is more skilled intensive than commodity and tradable sectors. On the other hand, the commodity price boom generates an increase in local demand that benefits local nontraded good producers. Our findings imply that a one percent increase in the regional commodity price index leads to a 0.027

¹⁵As featured by the model, firms selling tradable goods domestically benefit from commodity price booms through increased demand. Even if these goods are sold in different regions of Brazil, regional correlation in commodity price cycles - as seen in the data- lead to this result.

percent increase in employment in the full sample (column (8)). As expected, the adjustments are more than twice as large for firms with 50+employees (0.066) and firms with 100+ employees (0.068) (columns (9) and (10)).

Next, we examine the asymmetric responses of employment to commodity prices during booms and busts. Our model predicts that the asymmetry should depend upon the degree of wage rigidity in the local labor market. We augment equation (35) to include the interaction of the price index with dummy variables capturing periods of boom and bust in each region:

$$E_{ft} = \beta_1 \cdot PT_{rt} + \beta_2 \cdot PT_{rt} \cdot Boom_{rt} + \beta_3 \cdot PT_{rt} \cdot Bust_{rt} + \alpha_1 \cdot Boom_{rt} + \alpha_2 \cdot Bust_{rt} + \gamma_f + \delta_t + \epsilon_{ft} \quad (36)$$

The omitted category corresponds to "normal times". The construction of these variables ($Boom_{rt}$ and $Bust_{rt}$) is described in section 3. As before, we include firm and period fixed effects and cluster standard errors at the firm level. Furthermore, we estimate the regression equation by different regional levels of wage rigidity: above and below-median degrees of wage rigidity.¹⁶

The results for firms in the commodity sector are shown in columns (1) (low-rigidity regions) and (2) (high-rigidity regions) in table 10. The estimated impact of a one percent increase in commodity prices on firms' employment in low-rigidity regions is an increase of 0.054 percent during "normal times", 0.269 percent during the boom, and 0.225 percent during the bust. These figures are in general larger among firms operating in high-rigidity regions: 0.116 in "normal times", 0.343 during boom, and 0.207 during bust.

The differences between the responses to boom and bust (by level of labor market rigidity) are larger for the tradable sector. Here employment falls during the boom and increases during the bust, as predicted by the model. The impact in low-rigidity regions in response to a one percent increase is -0.175 ("normal times"), -0.188 (boom), and -0.121 (bust). The impact in high-rigidity regions is -0.097 ("normal times"), -0.073 (boom), and -0.056 (bust). As discussed in section 5, increases in employment during the bust are softer in relatively more rigid regions.

Finally, in the nontradable sector, we see substantial heterogeneity across regions. While the

¹⁶While we constructed several regional measures of wage rigidity, we use here the share of unionized workers in each state as our wage-rigidity proxy. This measure is based on data for 2001 obtained from Brazil's annual household survey PNAD.

overall impact of commodity prices for nontradable firms employment is positive (as seen earlier in table 9), underlying this result we find a positive impact in high-rigidity regions and a negative impact on low-rigidity regions. In regions with more rigid labor markets, the response to a one percent increase in commodity prices is a 0.129 rise in employment during the boom and much smaller 0.034 percent elasticity during the bust.

Table 9: Commodity Price Shocks and Firm-Level Employment

	COMMODITY			TRADABLE			NONTRADABLE			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Price	0.137*** (0.038)	0.206*** (0.045)	0.238*** (0.049)	-0.111*** (0.019)	-0.0968*** (0.031)	-0.129*** (0.042)	-0.037* (0.020)	0.027** (0.013)	0.066*** (0.025)	0.068*** (0.033)
Exporter							0.274*** (0.010)			
Price x Exporter							-0.195*** (0.019)			
Observations (thousands)	569	280	176	3595	1088	514	3237	6720	1433	629
Fixed Effects:										
Firm	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Quarter	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

NOTES: This table reports the results of the estimation of equation (35). The first three columns correspond to the commodity sector. The next four columns correspond to the tradables sector. The last three columns correspond to the nontradable sector. Columns 1, 4, 7, and 8 include all firms with employment equal or larger than 15 employees. Columns 2, 5, and 9 include all firms with employment equal or larger than 50 employees. Columns 3, 6, and 10 include all firms with employment equal or larger than 100 employees. Standard errors are clustered by firm. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

[htb]

Table 10: Commodity Price Shocks, Firm-Level Employment, and Wage Rigidity

	COMMODITIES		TRADABLES		NONTRADABLES	
	(Low W-R)	(High W-R)	(Low W-R)	(High W-R)	(Low W-R)	(High W-R)
Price	0.054 *** (0.049)	0.116** (0.058)	-0.175 *** (0.028)	-0.097 *** (0.022)	-0.074 *** (0.018)	0.063 *** (0.017)
Price x Boom	0.215*** (0.039)	0.227*** (0.038)	-0.013 (0.015)	0.024** (0.011)	-0.012 (0.008)	0.066 *** (0.009)
Price x Bust	0.171*** (0.037)	-0.018 (0.038)	0.054 *** (0.015)	0.041 *** (0.010)	0.015* (0.008)	-0.029 *** (0.009)
Observations (thousands)	275	293	996	2598	2563	4155
Fixed Effects:						
Firm	Y	Y	Y	Y	Y	Y
Quarter	Y	Y	Y	Y	Y	Y
Boom	Y	Y	Y	Y	Y	Y
Bust	Y	Y	Y	Y	Y	Y

NOTES: This table reports the results of the estimation of equation (37). The first two columns correspond to the commodity sector. The next two columns correspond to the tradables sector. The last two columns correspond to the nontradables sector. Columns 1, 3 and 5 correspond to a sample with regions with low wage rigidity (below median share of unionized workers). Columns 2, 4 and 6 correspond to a sample with regions with high wage rigidity (above median share of unionized workers). Standard errors are clustered by region. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

Commodity Price Shocks and Firm Entry and Exit. We also analyze the impact of regional commodity price dynamics on firm entry and exit. As predicted by the model, we find that a relevant share of aggregate regional employment reallocation occurs at the extensive margin. To study entry and exit at the firm level we define a dummy variable ($Present_{ft}$) indicating whether firm f is active in period t or not. We study entry at an annual frequency. We estimate impact of the regional commodity price index on this dummy variable using the following model:

$$Present_{ft} = \beta \cdot price_{rt} + \gamma_f + \delta_t + \epsilon_{ft} \quad (37)$$

We estimate this equation in first differences, which removes unobserved time-invariant firm and time heterogeneity. Our empirical approach follows [Bustos \[2011\]](#) who studies firm entry into exporting in response to tariff cuts in the context of trade liberalization.

$$\Delta Present_{ft} = \beta \cdot \Delta price_{rt} + \epsilon_{ft} \quad (38)$$

We add to equation (39) controls for industry and regional trends in entry and exit rates by means of industry-year and state-year fixed effects. We estimate this equation separately for each sector using OLS. Column (1) in table 11 shows the results for firms in the commodity sector. The results imply that a one-percent increase in the regional commodity price index leads to a 0.056 percent increase in the probability a firm is active. The result in column (2) for the tradable sector is of similar magnitude but opposite sign. Commodity price booms reduce net entry into the tradable sector. Finally, we do not find a statistically significant impact of prices on net entry in the nontradable sector, as see in column (3). We conclude that while contagion of commodity price cycles to employment in non-commodity sectors occurs through both the extensive and intensive margins in the case of tradable, it occurs only through the intensive margin in the case of the nontradable sector.

Table 11: Commodity Price Shocks and Firm-Level Entry and Exit

	COMMODITIES (1)	TRADABLES (2)	NONTRADABLES (3)
Price	0.056** (0.009)	-0.051*** (0.014)	-0.021 (0.013)
Observations (thousands)	299	2841	7348
Fixed Effects:			
Year-Industry	Y	Y	Y
State-Industry	Y	Y	Y

NOTES: This table reports the results of the estimation of equation (39). Columns 1, 2, and 3 correspond to firms in the commodity, tradable, and nontradable sector. Standard errors are clustered by region. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

7. CONCLUSIONS

Commodity price cycles are a fundamental source of volatility to emerging markets. This paper studies the role of regional labor markets in the transmission of commodity price super cycles, including the causal association between dynamic labor distortions and recessions during cycle busts. On theoretical grounds, we introduce a new three-sector model of a small open economy with firm heterogeneity, entry and exit decisions, and skilled and unskilled labor. We show that during the boom, the commodity and nontradable sectors expand, the skill wage premium narrows, and the non-commodity export sector contracts. During the bust, on the other hand, downward wage rigidity generates dynamic misallocation between sectors, triggering a persistent recession characterized by unemployment and a sluggish recovery of non-commodity exporters. This opens a door for precautionary policy during the boom.

We present empirical evidence to assess these mechanisms. In particular, we study the case of Brazil between 1999-2013, a period in which large commodity price fluctuations provided a clean quasi-natural experiment. We examine linked employer-employee data and exploit variation in prices across commodities and variation across regions in commodity specialization to measure the direct impact of commodity prices on labor market outcomes of both commodity producing

firms and firms in other sectors of the economy. Our empirical evidence supports our theoretical predictions.

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APPENDIX

A.1 DESCRIPTIVE FIGURES AND STATISTICS

Figure 14: Regional Variation in Production:
Share of Employment in Commodities by Region in 2000

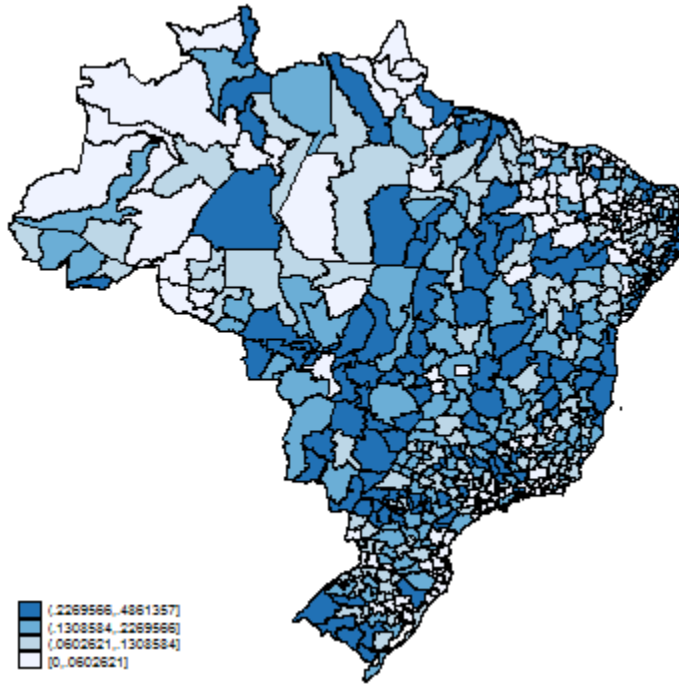
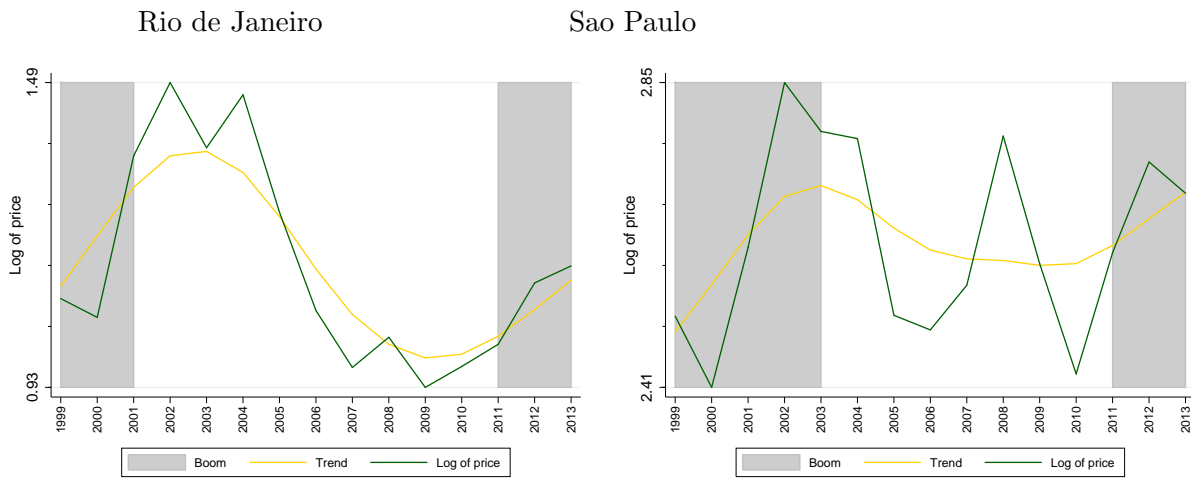


Figure 15: Identifying Regional Commodity Price Booms



Note: We first construct labor shares by sector and region in 1999. We use the shares to construct a weighted price index by region. We apply the HP filter to (log) series, and construct their first differences. When the resulting series change signs from negative to positives and the magnitude is larger than 0.01 (-0.01), we identify a boom (bust).

Table 12: Summary Statistics - Firm Panel

MACRO-REGION	SECTOR	NUMBER OF FIRMS		MEAN EMPLOYMENT		MEAN WAGE	
		2000	2010	2000	2010	2000	2010
North	Commodities	1252	2119	24.7	35.5	329.1	235.0
	Tradables	5691	8358	25.9	32.0	325.2	227.3
	Nontradables	30321	63760	9.2	11.1	284.8	202.3
Northeast	Commodities	4379	6528	42.9	50.0	256.0	205.4
	Tradables	23903	37463	23.5	26.7	256.3	187.8
	Nontradables	134098	261105	8.1	8.8	260.5	185.1
Central-West	Commodities	11082	18972	73.5	57.6	366.8	255.4
	Tradables	114445	142496	24.6	29.7	448.4	283.0
	Nontradables	523076	803805	8.2	9.9	386.9	237.0
Southeast	Commodities	5660	7340	40.5	33.9	339.7	247.4
	Tradables	62757	89978	20.4	22.8	366.8	255.9
	Nontradables	208315	353748	6.0	6.8	353.0	228.8
South	Commodities	2678	4387	55.6	65.3	365.2	268.4
	Tradables	12350	19976	15.8	20.3	297.5	223.6
	Nontradables	69057	136461	7.3	8.6	307.0	214.2

NOTES: This table reports summary statistics on the number of firms and mean firm employment and wages by macro-region and sector in 2000 and 2010. Wages are reported in real Reais of 2000.

Table 13: Summary Statistics

SECTOR	SKILL-GROUP	MEAN EMPLOYMENT		MEAN WAGE	
		2000	2010	2000	2010
Commodities	Unskilled	54.3	48.4	349.2	399.6
Commodities	Skilled	2.0	3.2	1231.6	544.0
Tradables	Unskilled	20.9	23.7	454.0	617.8
Tradables	Skilled	1.9	3.0	2316.6	465.3
Nontradables	Unskilled	7.2	8.2	453.6	564.4
Nontradables	Skilled	0.5	0.8	1519.8	353.8

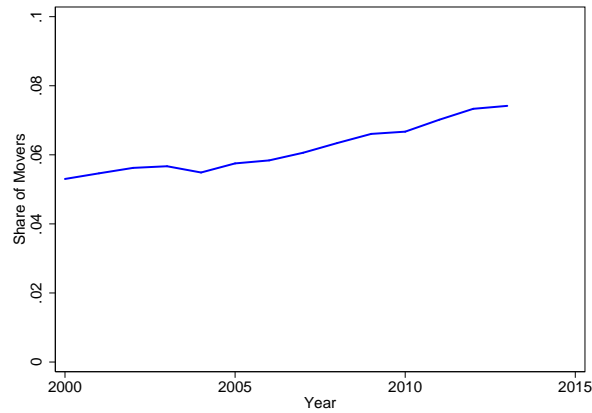
NOTES: This table reports summary statistics on mean firm employment and wages by sectors and skill-groups in 2000 and 2010. Unskilled workers are those with at most complete secondary education. Skilled workers are those with at least some tertiary education. Wages are reported in real Reais of 2000.

Table 14: Summary Statistics on Exporting Firms and Non-Exporting Firms

SECTOR	REGION	SHARE OF EXPORTERS		MEAN EMPLOYMENT				MEAN WAGE			
		2001	2010	EXPORTERS 2001	NON-EXPORTERS 2001	EXPORTERS 2010	NON-EXPORTERS 2010	EXPORTERS 2001	NON-EXPORTERS 2001	EXPORTERS 2010	NON-EXPORTERS 2010
North	Commodities	0.032	0.016	65.0	196.6	23.4	33.0	635.0	484.1	326.8	231.0
	Tradables	0.068	0.041	171.9	316.7	15.4	19.8	521.8	364.7	309.6	221.4
Northeast	Commodities	0.040	0.024	157.6	297.1	41.9	43.9	345.5	277.1	251.0	203.6
	Tradables	0.019	0.013	491.5	765.1	15.3	17.0	557.7	352.8	248.2	185.6
Central-West	Commodities	0.017	0.011	281.5	874.0	69.1	48.5	621.7	594.0	357.5	251.6
	Tradables	0.052	0.049	227.5	290.6	13.4	16.4	929.8	567.3	406.6	268.6
Southeast	Commodities	0.017	0.010	204.6	413.4	35.3	30.1	494.2	343.1	330.8	246.4
	Tradables	0.055	0.039	205.9	286.1	10.2	12.1	559.7	418.4	341.6	249.3
South	Commodities	0.010	0.009	221.1	393.0	56.8	62.4	722.6	534.7	360.0	266.1
	Tradables	0.016	0.011	232.4	643.9	12.2	13.4	434.7	335.5	296.7	222.4

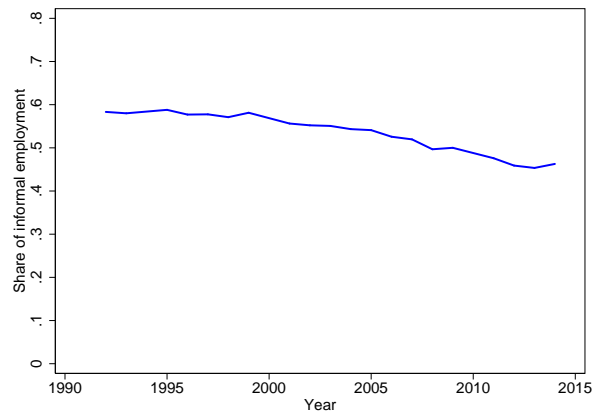
NOTES: This table reports the share of exporting firms by sectors and macro-regions in 2000 and 2010. It also reports mean employment and mean wages in exporting and non-exporting firms. Wages are reported in real Reais of 2000

Figure 16: Evolution of Regional Mobility In Brazil



Note: This figure plots the evolution of regional mobility in Brazil during the period 2000-2013. Using the RAIS worker-level records, we compute for each year the share of workers in each region that were employed in a different region the previous period. As the figure shows, regional mobility is not very responsive to business cycle fluctuations.

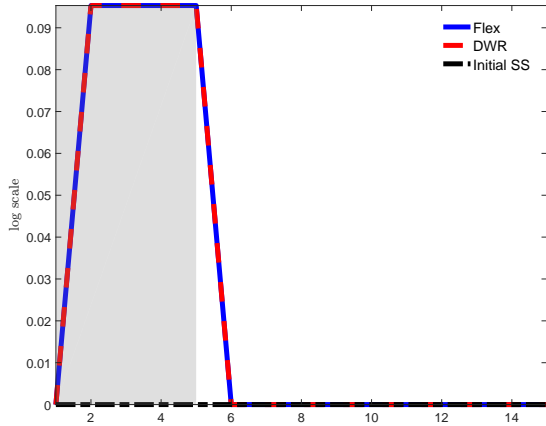
Figure 17: Evolution of Informal Employment In Brazil



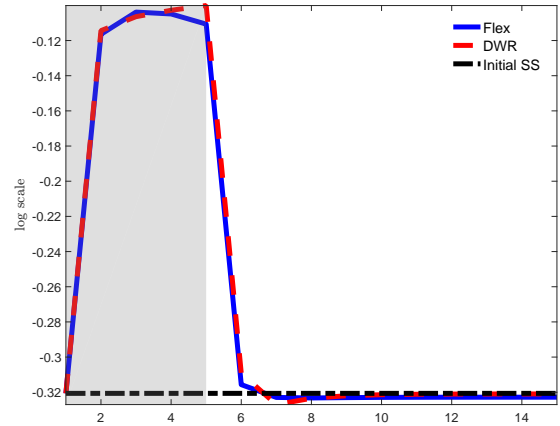
Note: This figure plots the evolution of the share of informal employment in Brazil during the past two decades. The source for this figure is the annual household survey PNAD. As the figure shows, there is a downward trend in informality in Brazil during this period, but informality is not very responsive to business cycle fluctuations.

A.2 REAL OUTCOMES

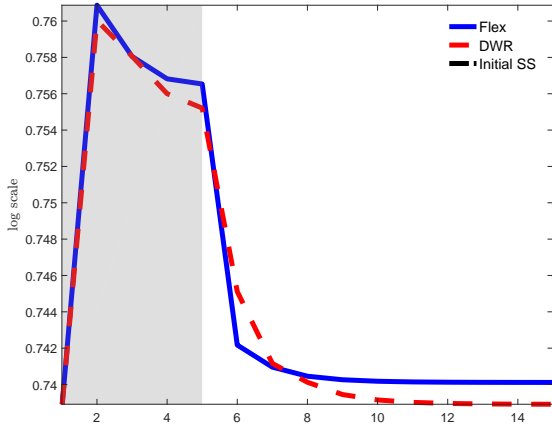
Figure 18 show the evolution of the prices in the economy. Interestingly, the price of the natural resource increases more than the price of the commodity. The nontradable good increases its price by only 2% while the tradable good increases its price by almost 9%.



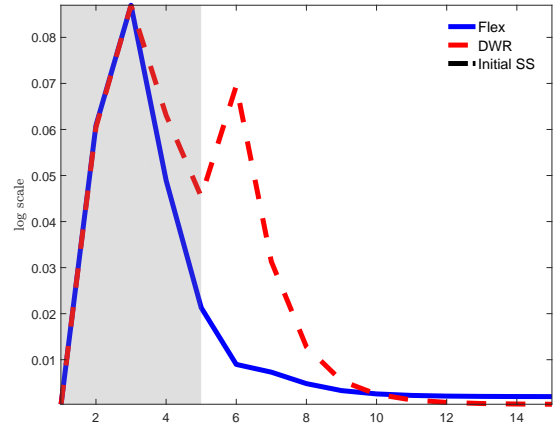
(a) Price, C (log scale)



(b) Price, R (log scale)



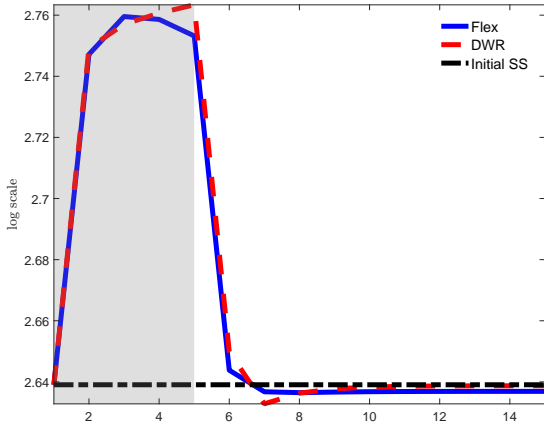
(c) Price, N (log scale)



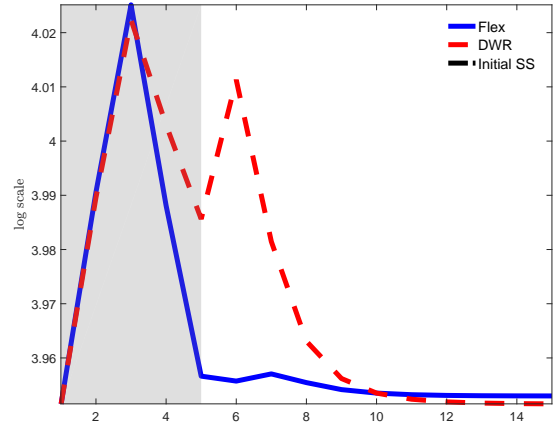
(d) Price, T (log scale)

Figure 18: Prices

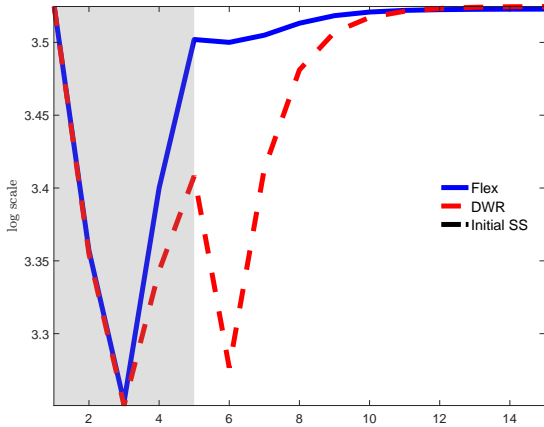
Figure 19 uses steady state prices to evaluate the real effects of the commodity boom. Note that the real expansion of the nontradable and commodity sector are counteracted by a strong real decrease on the tradable sector. Therefore, the total effect on real GDP is persistently negative.



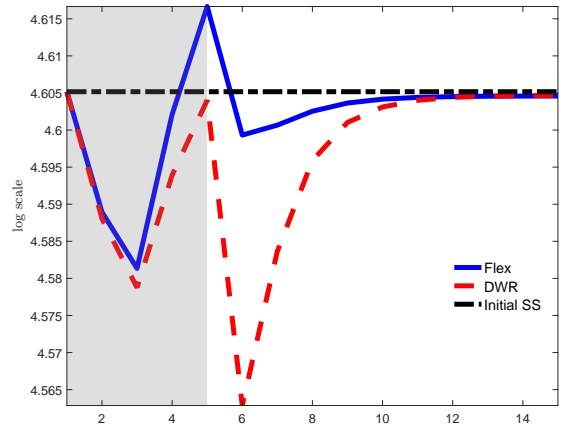
(a) Commodity (log scale)



(b) Nontradable (log scale)



(c) Tradable (log scale)



(d) GDP (log scale)

Figure 19: Sectoral Output and GDP at Steady State Prices

A.3 SYSTEM OF EQUATIONS AND ALGORITHMS

A.3.1 SYSTEM OF DYNAMIC EQUATIONS

$$Exogenous(1) = \{P_t^*\}$$

$$Endogenous(41) = \{N_t, M_t, T_t^d, X_t^T, C_t, C_t^d, P_t^N, P_t^T, P_{R,t}, B_{t+1}, \lambda_t, \Lambda_{t,t+1}, w_t^u, w_t^s, \dots\} = 14$$

$$= \{\hat{w}_t^C, L_t^{s,C}, L_t^{u,C}, W_t^C, \mu_t^C, \mathcal{M}_t^C, \tilde{\mathcal{M}}_t^C, \underline{z}_t, \dots\} = 8$$

$$= \{\hat{w}_t^N, L_t^{s,N}, L_t^{u,N}, W_t^N, \mu_t^N, \mathcal{M}_t^N, \tilde{\mathcal{M}}_t^N, \underline{s}_t, \dots\} = 8$$

$$= \{\hat{w}_t^T, L_t^{s,T}, L_t^{u,T}, W_t^T, W_{d,t}^T, W_{ex,t}^T, \mu_t^T, \mathcal{M}_t^T, \tilde{\mathcal{M}}_t^T, \varphi_t^*, \varphi_{x,t}^*, \dots\} = 11$$

A.3.1.1 HOUSEHOLDS

Optimal Decisions

$$N_t = \left(\frac{\alpha^N}{\alpha^M} \frac{1}{P_t^N} \right)^{\frac{1}{1-\theta}} M_t \tag{39}$$

$$T_t^d = \left(\frac{\alpha^T}{\alpha^M} \frac{1}{P_t^T} \right)^{\frac{1}{1-\theta}} M_t \tag{40}$$

$$C_t^d = \left(\frac{\alpha^C}{\alpha^M} \frac{1}{P_t^*} \right)^{\frac{1}{1-\theta}} M_t \tag{41}$$

$$\lambda_{t+1} = \lambda_t \tag{42}$$

$$\lambda_t = \left[\alpha^M M_t^\theta + \alpha^N N_t^\theta + \alpha^T (T_t^d)^\theta + \alpha^C (C_t^d)^\theta \right]^{\frac{1-\theta-\gamma^T}{\theta}} \alpha^M M_t^{\theta-1} \tag{43}$$

$$\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{44}$$

Aggregate Variables

$$T_t^d = \left[\int_{\zeta \in \mathfrak{Z}} q_{d,t}(\zeta)^\rho d\zeta \right]^{\frac{1}{\rho}}$$

$$P_t^T = \left[\int_{\zeta \in \mathfrak{Z}} p_t(\zeta)^{1-\sigma} d\zeta \right]^{\frac{1}{1-\sigma}}$$

$$L^s = L_t^{s,C} + L_t^{s,N} + L_t^{s,T} \quad (45)$$

$$0 = (L_t^{u,C} + L_t^{u,N} + L_t^{u,T} - L^u)(w_t^u - \gamma^u w_{t-1}^u), \quad (46)$$

$$L^u \geq L_t^{u,C} + L_t^{u,N} + L_t^{u,T}, \quad w_t^u \geq \gamma^u w_{t-1}^u$$

Balance of Payments

$$M_t + B_{t+1} - (1 + r^*)B_t = P_t^* (C_t - C_t^d) + X_t^T \quad (47)$$

A.3.1.2 COMMODITY SECTOR

Optimal Decisions

$$\hat{w}_t^C = \left[\frac{\phi^C}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^C}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}} \quad (48)$$

$$l_{i,t}^C = (\xi^\xi \eta^{1-\xi})^{\frac{1}{1-\xi-\eta}} \left[(\hat{w}_t^C)^{-(1-\xi)} P_{R,t}^{-\xi} P_t^* \right]^{\frac{1}{1-\xi-\eta}} z_i^{\frac{1}{1-\xi-\eta}}$$

$$C_{i,t} = (\xi^\xi \eta^\eta)^{\frac{1}{1-\xi-\eta}} \left[(\hat{w}_t^C)^{-\eta} P_{R,t}^{-\xi} (P_t^*)^{\xi+\eta} \right]^{\frac{1}{1-\xi-\eta}} z_i^{\frac{1}{1-\xi-\eta}}$$

$$R_{i,t} = (\xi^{1-\eta} \eta^\eta)^{\frac{1}{1-\xi-\eta}} \left[(\hat{w}_t^C)^{-\eta} P_{R,t}^{-(1-\eta)} P_t^* \right]^{\frac{1}{1-\xi-\eta}} z_i^{\frac{1}{1-\xi-\eta}}$$

$$l_{i,t}^{s,C} = (1 - \phi^C) \left(\frac{\hat{w}_t^C}{w_t^s} \right)^\gamma l_{i,t}^C$$

$$l_{i,t}^{u,C} = \phi^C \left(\frac{\hat{w}_t^C}{w_t^u} \right)^\gamma l_{i,t}^C$$

$$\pi_{i,t} = (1 - \xi - \eta) (\xi^\xi \eta^\eta)^{\frac{1}{1-\xi-\eta}} \left[(\hat{w}_t^C)^{-\eta} P_{R,t}^{-\xi} P_t^* \right]^{\frac{1}{1-\xi-\eta}} z_i^{\frac{1}{1-\xi-\eta}} - P_t^* c^c$$

Value Function

$$W_t^C(z; \hat{w}_t^C, P_{R,t}, P_t^*) = \max \left\{ 0, \pi_t(z; \hat{w}_t^C, P_{R,t}, P_t^*) + (1 - \delta^C) \Lambda_{t,t+1} \max \left\{ 0, W_{t+1}^C(z; \hat{w}_{t+1}^C, P_{R,t+1}, P_{t+1}^*) \right\} \right\}. \quad (49)$$

Cutoff

$$0 = W_t^C(z_t; \hat{w}_t^C, P_{R,t}, P_t^*) \quad (50)$$

Free entry condition

$$\hat{w}_t^C c_e^c = \sum_{j=i_t}^n W_t^C(z_j; \hat{w}_t^C, P_{R,t}, P_t^*) f(z_j). \quad (51)$$

Distribution of producers law of motion

$$\mathcal{M}_{t+1}^C \mu_{t+1}^C(z) = (1 - \delta^C) \mathcal{M}_t^C \mu_t^C(z) + \tilde{\mathcal{M}}_{t+1}^C f(z) \quad (52)$$

Mass of producers law of motion

$$\mathcal{M}_{t+1}^C = (1 - \delta^C) \mathcal{M}_t^C \sum_{j=i_{t+1}}^n \mu_t^C(z_j) + \tilde{\mathcal{M}}_{t+1}^C \sum_{j=i_{t+1}}^n f(z_j) \quad (53)$$

Aggregate variables

- Total natural resource constraint (Fixed Supply)

$$\bar{R} = \mathcal{M}_t^C \sum_{j=i_t}^n R_t(z_j) \mu_t^C(z_j) \quad (54)$$

- Total skilled and unskilled labor for production

$$L_{p,t}^{s,C} = \mathcal{M}_t^C \sum_{j=i_t}^n l_t^{s,C}(z_j) \mu_t^C(z_j)$$

$$L_{p,t}^{u,C} = \mathcal{M}_t^C \sum_{j=i_t}^n l_t^{u,C}(z_j) \mu_t^C(z_j)$$

- Total skilled and unskilled labor for fixed entry cost

$$L_{e,t}^{s,C} = \tilde{\mathcal{M}}_t^C (1 - \phi^C) \left(\frac{\hat{w}_t^C}{w_t^s} \right)^\gamma c_e^c$$

$$L_{e,t}^{u,C} = \tilde{\mathcal{M}}_t^C \phi^C \left(\frac{\hat{w}_t^C}{w_t^u} \right)^\gamma c_e^c$$

- Total skilled and unskilled labor demand

$$L_t^{s,C} = L_{p,t}^{s,C} + L_{e,t}^{s,C} \quad (55)$$

$$L_t^{u,C} = L_{p,t}^{u,C} + L_{e,t}^{u,C} \quad (56)$$

- Total Commodity Output (net of operational cost)

$$C_t = \mathcal{M}_t^C \sum_{j=\hat{i}_t}^n \left[C_t(z_j) - c_c \right] \mu_t^C(z_j) \quad (57)$$

A.3.1.3 NONTRADABLE SECTOR

Optimal decisions

$$\hat{w}_t^N = \left[\frac{\phi^N}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^N}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}} \quad (58)$$

$$l_{i,t}^N = \left(\alpha s_i \frac{P_t^N}{\hat{w}_t^N} \right)^{\frac{1}{1-\alpha}}$$

$$l_{i,t}^{s,N} = (1 - \phi^N) \left(\frac{\hat{w}_t^N}{w_t^s} \right)^\gamma l_{i,t}^N$$

$$l_{i,t}^{u,N} = \phi^N \left(\frac{\hat{w}_t^N}{w_t^u} \right)^\gamma l_{i,t}^N$$

$$N_{i,t} = \left(\alpha s_i^{\frac{1}{\alpha}} \frac{P_t^N}{\hat{w}_t^N} \right)^{\frac{\alpha}{1-\alpha}}$$

$$\pi_{i,t} = \alpha^{\frac{\alpha}{1-\alpha}} (1 - \alpha) (\hat{w}_t^N)^{-\frac{\alpha}{1-\alpha}} (P_t^N)^{\frac{1}{1-\alpha}} s_i^{\frac{1}{1-\alpha}} - P_t^N c_f$$

Value Function

$$W_t^N(s, \hat{w}_t^N, P_t^N) = \max \{0, \pi_t(s, \hat{w}_t^N, P_t^N) + (1 - \delta^N) \Lambda_{t,t+1} \max \{0, W_{t+1}^N(s, \hat{w}_{t+1}^N, P_{t+1}^N)\}\}. \quad (59)$$

Cutoff

$$0 = W_t^N(\underline{s}_t, \hat{w}_t^N, P_t^N) \quad (60)$$

Free entry condition

$$\hat{w}_t^N c_e = \sum_{j=\underline{i}_t}^n W_t^N(s_j; \hat{w}_t^N, P_t^N) h(s_j). \quad (61)$$

Distribution of producers law of motion

$$\mathcal{M}_{t+1}^N \mu_{t+1}^N(s) = (1 - \delta^N) \mathcal{M}_t^N \mu_t^N(s) + \tilde{\mathcal{M}}_{t+1}^N h(s) \quad (62)$$

Mass of producers law of motion

$$\mathcal{M}_{t+1}^N = (1 - \delta^N) \mathcal{M}_t^N \sum_{j=\underline{i}_{t+1}}^n \mu_t^N(s_j) + \tilde{\mathcal{M}}_{t+1}^N \sum_{j=\underline{i}_{t+1}}^n h(s_j) \quad (63)$$

Aggregate variables

- Total skilled and unskilled labor for production

$$L_{p,t}^{s,N} = \mathcal{M}_t^N \sum_{j=\underline{i}_t}^n l_t^{s,N}(s_j) \mu_t^N(s_j)$$

$$L_{p,t}^{u,N} = \mathcal{M}_t^N \sum_{j=\underline{i}_t}^n l_t^{u,N}(s_j) \mu_t^N(s_j)$$

- Total skilled and unskilled labor for fixed entry cost

$$L_{e,t}^{s,N} = \tilde{\mathcal{M}}_t^N (1 - \phi^N) \left(\frac{\hat{w}_t^N}{w_t^s} \right)^\gamma c_e$$

$$L_{e,t}^{u,N} = \tilde{\mathcal{M}}_t^N \phi^N \left(\frac{\hat{w}_t^N}{w_t^u} \right)^\gamma c_e$$

- Total skilled and unskilled labor demand

$$L_t^{s,N} = L_{p,t}^{s,N} + L_{e,t}^{s,N} \quad (64)$$

$$L_t^{u,N} = L_{p,t}^{u,N} + L_{e,t}^{u,N} \quad (65)$$

- Total output (net of operational cost)

$$N_t = \mathcal{M}_t^N \sum_{j=i_t}^n \left[N_t(s_j) - c_f \right] \mu_t^N(s_j) \quad (66)$$

A.3.1.4 TRADABLE SECTOR

Optimal decisions

$$\hat{w}_t^T = \left[\frac{\phi^T}{(w_t^u)^{\gamma-1}} + \frac{1-\phi^T}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}} \quad (67)$$

$$p_t(\varphi) = \frac{\hat{w}_t^T}{\rho\varphi}$$

$$q_{d,t}(\varphi) = T_t^d \left(\frac{P_t^T \rho\varphi}{\hat{w}_t^T} \right)^\sigma$$

$$q_{x,t}(\varphi) = \gamma_0 \left(\frac{\rho\varphi}{\hat{w}_t^T} \right)^\sigma$$

$$l_{d,t}^{s,T}(\varphi) = (1 - \phi^T) \left(\frac{\hat{w}_t^T}{w_t^s} \right)^\gamma \left[f_d + T_t^d (P_t^T)^\sigma \left(\frac{\rho}{\hat{w}_t^T} \right)^\sigma \varphi^{\sigma-1} \right]$$

$$l_{d,t}^{u,T}(\varphi) = \phi^T \left(\frac{\hat{w}_t^T}{w_t^u} \right)^\gamma \left[f_d + T_t^d (P_t^T)^\sigma \left(\frac{\rho}{\hat{w}_t^T} \right)^\sigma \varphi^{\sigma-1} \right]$$

$$l_{x,t}^{s,T}(\varphi) = (1 - \phi^T) \left(\frac{\hat{w}_t^T}{w_t^s} \right)^\gamma \left[f_x + \gamma_0 \left(\frac{\rho}{\hat{w}_t^T} \right)^\sigma \varphi^{\sigma-1} \right]$$

$$l_{x,t}^{u,T}(\varphi) = \phi^T \left(\frac{\hat{w}_t^T}{w_t^u} \right)^\gamma \left[f_x + \gamma_0 \left(\frac{\rho}{\hat{w}_t^T} \right)^\sigma \varphi^{\sigma-1} \right]$$

$$r_{d,t}(\varphi) = P_t^T T_t^d \left[\frac{P_t^T \rho\varphi}{\hat{w}_t^T} \right]^{\sigma-1}$$

$$\pi_{d,t}(\varphi) = \frac{r_{d,t}(\varphi)}{\sigma} - \hat{w}_t^T f_d$$

$$r_{x,t}(\varphi) = \gamma_0 \left(\frac{\rho\varphi}{\hat{w}_t^T} \right)^{\sigma-1}$$

$$\pi_{x,t}(\varphi) = \frac{r_{x,t}(\varphi)}{\sigma} - \hat{w}_t^T f_x$$

q_d and q_x : optimal output sold in domestic, and foreign market, resp..

$l_j^{s,T}$ and $l_j^{u,T}$: optimal skilled and unskilled labor demand to produce ($j = d, x$) tradable goods.

r_d and r_x : revenue from domestic and foreign market, resp..

π_d and π_x : profits from domestic and foreign market, resp..

Revenue of a firm

$$r_t(\varphi) = \begin{cases} r_{d,t}(\varphi) & , \text{ if nonexporter} \\ r_{d,t}(\varphi) + r_{x,t}(\varphi) & , \text{ if exporter,} \end{cases}$$

Profits of a firm

$$\pi_t(\varphi) = \begin{cases} \pi_{d,t}(\varphi) & , \text{ if nonexporter} \\ \pi_{ex,t} = \pi_{d,t}(\varphi) + \pi_{x,t}(\varphi) & , \text{ if exporter} \end{cases}$$

Value functions

$$W_t^T(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \max \{W_{d,t}^T, W_{ex,t}^T\} \quad (68)$$

$$W_{d,t}^T(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \max \left\{ 0, \pi_{d,t}(\varphi, \hat{w}_t^T, P_t^T, T_t^d) + (1 - \delta) \Lambda_{t,t+1} W_{t+1}^T(\varphi; \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d) \right\} \quad (69)$$

$$W_{ex,t}^T(\varphi; \hat{w}_t^T, P_t^T, T_t^d) = \max \left\{ 0, \pi_{ex,t}(\varphi, \hat{w}_t^T, P_t^T, T_t^d) + (1 - \delta) \Lambda_{t,t+1} W_{t+1}^T(\varphi; \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d) \right\} \quad (70)$$

$$W_{t+1}^T(\varphi; \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d) = \begin{cases} W_{d,t+1}^T & , \text{ if } \varphi_{t+1}^* \leq \varphi < \varphi_{x,t+1}^* \\ W_{ex,t+1}^T & , \text{ if } \varphi \geq \varphi_{x,t+1}^* \\ 0 & , \text{ otherwise} \end{cases}$$

Cut-offs

$$0 = W_{d,t}^T \left(\varphi_t^*; \hat{w}_t^T, P_t^T, T_t^d \right) \quad (71)$$

$$0 = \pi_{x,t} \left(\varphi_{x,t}^*; \hat{w}_t^T \right) \quad (72)$$

Free entry condition

$$\begin{aligned} \hat{w}_t^T f_e &= \sum_{j=\bar{i}_t}^n W_t^T(\varphi_j) g(\varphi_j) \\ &= \sum_{j=\bar{i}_t}^{\bar{i}_t-1} W_{d,t}^T(\varphi_j) g(\varphi_j) + \sum_{j=\bar{i}_t}^n W_{ex,t}^T(\varphi_j) g(\varphi_j) \end{aligned} \quad (73)$$

Distribution of producers law of motion

$$\mathcal{M}_{t+1}^T \mu_{t+1}^T(\varphi) = (1 - \delta^T) \mathcal{M}_t^T \mu_t^T(\varphi) + \tilde{\mathcal{M}}_{t+1}^T g(\varphi) \quad (74)$$

Mass of producers law of motion

$$\mathcal{M}_{t+1}^T = (1 - \delta^T) \mathcal{M}_t^T \sum_{j=\bar{i}_{t+1}}^n \mu_t^T(\varphi_j) + \tilde{\mathcal{M}}_{t+1}^T \sum_{j=\bar{i}_{t+1}}^n g(\varphi_j) \quad (75)$$

Aggregate variables

- Fraction of exporters

$$p_{x,t} = p_x \left(\hat{w}_t^T, P_t^T, T_t^d, \hat{w}_{t+1}^T, P_{t+1}^T, T_{t+1}^d \right) = \frac{1 - G(\varphi_{x,t}^*)}{1 - G(\varphi_t^*)}$$

- Total skilled and unskilled labor demand to produce tradable goods sold in domestic market

$$\begin{aligned} L_{d,p,t}^{s,T} &= \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n l_{d,t}^{s,T}(\varphi_j) \mu_t^T(\varphi_j) \\ L_{d,p,t}^{u,T} &= \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n l_{d,t}^{u,T}(\varphi_j) \mu_t^T(\varphi_j) \end{aligned}$$

- Total skilled and unskilled labor demand to produce tradable goods exported

$$L_{x,p,t}^{s,T} = \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n l_{x,t}^{s,T}(\varphi_j) \mu_t^T(\varphi_j)$$

$$L_{x,p,t}^{u,T} = \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n l_{x,t}^{u,T}(\varphi_j) \mu_t^T(\varphi_j)$$

- Total skilled and unskilled labor for fixed entry cost

$$L_{e,t}^{s,T} = \tilde{\mathcal{M}}_t^T (1 - \phi^T) \left(\frac{\hat{w}_t^T}{w_t^s} \right)^\gamma f_e$$

$$L_{e,t}^{u,T} = \tilde{\mathcal{M}}_t^T \phi^T \left(\frac{\hat{w}_t^T}{w_t^u} \right)^\gamma f_e$$

- Total skilled and unskilled labor demands

$$L_t^{s,T} = L_{d,p,t}^{s,T} + L_{x,p,t}^{s,T} + L_{e,t}^{s,T} \quad (76)$$

$$L_t^{u,T} = L_{d,p,t}^{u,T} + L_{x,p,t}^{u,T} + L_{e,t}^{u,T} \quad (77)$$

- Total tradable good outputs sold in domestic market

$$T_t^d = \left[\mathcal{M}_t^T \sum_{j=\bar{i}_t}^n [q_{d,t}(\varphi_j)]^\rho \mu_t^T(\varphi_j) \right]^{\frac{1}{\rho}} \quad (78)$$

- Total tradable good outputs sold in foreign market

$$T_t^x = \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n q_{x,t}(\varphi_j) \mu_t^T(\varphi_j)$$

- Value of tradable good outputs sold in foreign market

$$X_t^T = \mathcal{M}_t^T \sum_{j=\bar{i}_t}^n p_t(\varphi_j) q_{x,t}(\varphi_j) \mu_t^T(\varphi_j) \quad (79)$$

A.3.1.5 TRANSITION ALGORITHM

Assumptions

- Economy is in stationary equilibrium at $t = 1$, say \mathcal{X}_1 , associated with known P_1^* .
- Economy is in stationary equilibrium at $t = \infty$, say \mathcal{X}_∞ , associated with known P_∞^* .
- The full sequence $\{P_t^*\}_{t=1}^\infty$ is exogenously given.
- Transition is complete after T periods.

Algorithm

1. Fix T .
2. Compute the stationary equilibrium \mathcal{X}_1 , .
3. Compute the stationary equilibrium \mathcal{X}_∞ , and set $\mathcal{X}_T = \mathcal{X}_\infty$.
4. **Outside Loop.** Guess initial marginal utility λ_1 . Given λ_1 we can compute the full sequence $\{\lambda_t, \Lambda_{t,t+1}\}_{t=1}^T$ from (42) and (44).
5. **Inside Loop.** Guess sequence $\{w_t^u, w_t^s\}_{t=1}^T$.
6. **Backward Iteration Loop.** Use time- T value functions and the guessed sequences to find several decision rules. For each $t = T, \dots, 1$.
 - Get w_t^C, w_t^N, w_t^T , from (48), (58), (67).
 - Solve for $P_{R,t}$ using FEC in C sector (51). To do so, we evaluate $W_t^C(z; \hat{w}_t^C, P_{R,t}, P_t^*)$ using (49).
 - Solve for $P_{N,t}$ using FEC in N sector (61). To do so, we evaluate $W_t^N(s; \hat{w}_t^N, P_t^N)$ using (59).
 - Solve for $P_{T,t}$ using FEC in T sector (73). To do so, we evaluate M_t and T_t^d from (43) and (40), and then compute the T sector value functions (68), (69), and (70).
 - Get M_t, N_t, T_t^d, C_t^d from (43), (39), (40), and (41).
 - Get all associated cutoffs $\underline{z}_t, \underline{x}_t, \varphi_t^*, \varphi_{x,t}^*$, from (50), (60), (71), (72).

7. **Forward Iteration Loop.** Use the initial distributions, the guesses and the decision rules computed in the backward loop, to get the time-varying distributions. To do so, we recover the masses of entrants as follows. For each $t = 1, \dots, T$.

- Get the mass of entrants in C sector $\tilde{\mathcal{M}}_t^C$. Note that $P_{R,t}$, \hat{w}_t^C , and \underline{z}_t are known at this point. Plugging the law of motion (52) in the market clearing (54):

$$\begin{aligned}\bar{R} &= \sum_{j=\underline{z}_t}^n R_t(z_j; P_{R,t}, \hat{w}_t^C) \mathcal{M}_t^C \mu_t^C(z_j) \\ &= \sum_{j=\underline{z}_t}^n R_t(z_j; P_{R,t}, \hat{w}_t^C) \left[(1 - \delta^C) \mathcal{M}_{t-1}^C \mu_{t-1}^C(z_j) + \tilde{\mathcal{M}}_t^C f(z_j) \right]\end{aligned}$$

- Get the mass of entrants in N sector $\tilde{\mathcal{M}}_t^N$. Note that N_t , P_t^N , \hat{w}_t^N , and \underline{s}_t are known at this point. Plugging the law of motion (62) in the market clearing (66):

$$\begin{aligned}N_t &= \sum_{j=\underline{s}_t}^n \left[N_t(s_j; P_t^N, \hat{w}_t^N) - c_f \right] \mathcal{M}_t^N \mu_t^N(s_j) \\ &= \sum_{j=\underline{s}_t}^n \left[N_t(s_j; P_t^N, \hat{w}_t^N) - c_f \right] \left[(1 - \delta^N) \mathcal{M}_{t-1}^N \mu_{t-1}^N(s_j) + \tilde{\mathcal{M}}_t^N h(s_j) \right]\end{aligned}$$

- Get the mass of entrants in T sector $\tilde{\mathcal{M}}_t^T$. Note that T_t^d , P_t^T , \hat{w}_t^T , and φ_t^* are known at this point. Plugging the law of motion (74) in the market clearing (78):

$$\begin{aligned}\left[T_t^d \right]^\rho &= \sum_{j=\underline{\varphi}_t}^n \left[q_{d,t}(\varphi_j; T_t^d, P_t^T, \hat{w}_t^T) \right]^\rho \mathcal{M}_t^T \mu_t^T(\varphi_j) \\ &= \sum_{j=\underline{\varphi}_t}^n \left[q_{d,t}(\varphi_j; T_t^d, P_t^T, \hat{w}_t^T) \right]^\rho \left[(1 - \delta^T) \mathcal{M}_{t-1}^T \mu_{t-1}^T(\varphi_j) + \tilde{\mathcal{M}}_t^T g(\varphi_j) \right]\end{aligned}$$

- **Interior Solution:** Given $\tilde{\mathcal{M}}_t^C > 0$, $\tilde{\mathcal{M}}_t^N > 0$, and $\tilde{\mathcal{M}}_t^T > 0$, get \mathcal{M}_t^C , \mathcal{M}_t^N , and \mathcal{M}_t^T , from (53), (63), and (75) (lagged one period).
- Given $\tilde{\mathcal{M}}_t^C$, $\tilde{\mathcal{M}}_t^N$, $\tilde{\mathcal{M}}_t^T$, \mathcal{M}_t^C , \mathcal{M}_t^N , and \mathcal{M}_t^T , get $\mu_t^C(z)$, $\mu_t^N(s)$, and $\mu_t^T(\varphi)$, from (52), (62), and (74) (lagged one period).
- Given \mathcal{M}_t^C and $\mu_t^C(z)$ we can compute C_t from (57).
- Given \mathcal{M}_t^T and $\mu_t^T(\varphi)$ we can compute X_t^T from (79).

- **Check for Corner Solution:** If for any period t the mass of entrants in sector $j = C, N, T$ is negative ($\tilde{M}_t^j < 0$), we impose $\tilde{M}_t^j = 0$, and resolve for the equilibrium price consistent with the sector's market clearing with zero entrants.

8. Update Wage Guesses

- Get labor aggregates from (55), (56), (64), (65), (76), (77).
- Get **implied** w_t^s and w_t^u from both labor market clearings (45) and (46).

$$F^s(w_t^s, w_t^u) = L_t^{s,C}(w_t^s, \hat{w}_t^C) + L_t^{s,N}(w_t^s, \hat{w}_t^N) + L_t^{s,T}(w_t^s, \hat{w}_t^T) - L^s = 0$$

$$F^u(w_t^s, w_t^u) = L_t^{u,C}(w_t^u, \hat{w}_t^C) + L_t^{u,N}(w_t^u, \hat{w}_t^N) + L_t^{u,T}(w_t^u, \hat{w}_t^T) - L^u = 0$$

where

$$\hat{w}_t^C(w_t^s, w_t^u) = \left[\frac{\phi^C}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^C}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}}$$

$$\hat{w}_t^N(w_t^s, w_t^u) = \left[\frac{\phi^N}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^N}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}}$$

$$\hat{w}_t^T(w_t^s, w_t^u) = \left[\frac{\phi^T}{(w_t^u)^{\gamma-1}} + \frac{1 - \phi^T}{(w_t^s)^{\gamma-1}} \right]^{-\frac{1}{\gamma-1}}$$

- Impose DWR on the sequence for w_t^u , if necessary.
- Then, make a new guess for $\{w_t^u, w_t^s\}_{t=1}^T$ using a weighted average of the initial guess and the updated (**implied**) values.
- Go back to step 6 and repeat until convergence.

9. Update Guess for λ_1 .

- Given the sequences of M_t , C_t and X_t^T at converged wages, compute the **implied** sequence for B_{t+1} from (47).
- The solution satisfies the dynamic equilibrium conditions and the transversality condition. If the economy accumulates too much net foreign assets/deb), update the guess for λ_1 accordingly.
- Go back to step 5 and repeat until convergence.