The Gravity of Unit Values

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Very Preliminary

Abstract

We introduce quality differentiation into a standard quantitative, general equilibrium model of international trade. The framework allows bilateral trade to vary both at the margin of quantity and of unit value. We estimate the parameters of the model using bilateral data on trade flows and on unit values in trade. The model captures a number of regularities in the data.

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1 Introduction

Quantitative work in international work has advanced on several fronts in the last 15 years. One line of research has developed global general equilibrium models to understand the determinants of bilateral trade flows and their implications for welfare.[1] Another literature has delved into trade data to ask how total bilateral exports decompose into various margins, such as that between number of products and sales per product, and how sales per product decompose into quantity and unit value. These studies have revealed a number of robust and intriguing regularities.[2]

While both lines of research have been extremely fruitful, they remain somewhat at odds with each other. Capturing a complex world with a general equilibrium system has required assumptions inconsistent with richer countries paying more for the same product and richer countries charging more for the same product, two of the most robust regularities to emerge from this second line of inquiry.

This paper seeks to reconcile the two by developing a simple general equilibrium framework that consistent these regularities. The model delivers the same aggregate relationships governing bilateral trade that emerge in a standard general equilibrium framework, in particular that of Eaton and Kortum (2002). Hence it is consistent with previous work that used this framework to study the determinants of trade, quantify the gains from trade, and perform counterfactuals. But it can also explain how these aggregate relationships can decompose into quantities and unit values in a way that covary systematically with income level.

In line with previous work, we associate differences in unit values of a product with differences in the quality of a variety of product.[3] We allow for two dimensions of quality, which we call vertical and horizontal. Horizontal quality perfectly substitutes for quantity, and is equally valued by all users of the variety, whether a household using the variety for final consumption or a firm using the variety as an intermediate input. Vertical quality complements quality. As a consumer chooses to spend more on a variety, the

[1]Relatively early examples are Anderson and van Wincoop (2002), who pursue an Armington approach, Eaton and Kortum (2002) whose approach is Ricardian, and quantitative papers building on Melitz (2003), such as Chaney (2006) and Eaton, Kortum, and Kramarz (2011).

[2]Early contributions here are by Hummels and Klenow (2005), which we build on very directly, and Hallak (2006).

[3]Aside from Hummels and Klenow (2005) and Hallak (2006), other authors making this connection are Schott (2004), Khandelwal (2010), Hallak and Schott (2011), Baldwin and Harrigan (2011), Hummels and Skiba (2004), Choi, Hummels, and Xiang (2009), and Bekkers, Francois, and Manchin (2012). Of course different unit values may reflect something other than quality differences. Lashkaripour (2016) develops a general equilibrium multicountry version of the Krugman (1979) model in which different classes of goods have different elasticities of substitution, so their producers charge different Dixit-Stiglitz markups. This framework can also explain some of the empirical regularities we address here.

increased spending is divided between more effective quantity and higher vertical quality. Aggregation across varieties is CES, as is standard in the literature. Hence at the level of total spending our model delivers the same observations as a standard model without quality differentiation. Different producers are endowed with an ability to make products of different horizontal quality, but increasing vertical quality requires using more inputs.

We estimate the parameters of the model using data on trade flows and unit values from COMTRADE . We then simulate the model to show how it can deliver decompositions of trade into the extensive and intensive margins, and unit values and quantities similar to those measured by Hummels and Klenow (2005).

Our theoretical framework builds on the theoretical literature on quality differentiation in international trade. Early on Flam and Helpman (1987) developed a two-country, two-good general equilibrium framework that explained why a rich country might both produce and demand a good of higher quality. More recently Fajgelbaum, Grossman, and Helpman (2011) provided a much richer framework that allowed for many goods and countries.

Applying these approaches to the problem at hand poses two challenges, however. These models employ a discrete framework-choice in which the buyer is contemplating buying only a single unit of the good. They thus do not allow for increased spending on a variety to reflect a combination of more quantity and higher quality. Second, with only a single dimension of quality, if rich countries both prefer higher quality goods and are better at making them, rich countries should have larger market shares in other rich countries than in poor ones, and vice versa. This pattern is not one that we observe in the data. Our framework can deal with each issue.

Closest to our paper, to our knowledge, is Feenstra and Romalis (2014). Like them we are also interested in explaining unit values in terms of a quantitative general equilibrium framework. Theirs builds on the Melitz (2003) model. Their framework is a vehicle less in keeping with standard general equilibrium modeling in that they introduce a specific trade cost as well as the iceberg costs commonly used in the literature. It consequently does not imply a standard homothetic gravity specification for aggregate trade.

We proceed as follows. Section 2 presents our data and revisits the empirical regularities pursued before. Section 3 presents our model and Section 4 our estimation of it. In section 5 we use our model and estimates to evaluate our model's ability to capture the margins of trade.

2 Overview of the Data

We use data provided by the CEPII website, originating from the United Nations Comtrade database and cleaned by Gaulier and Zignago (2010). CEPII provides physical quantities and values of bilateral trade flows by 6-digit HS product categories annually from 1995 to 2007. For 184 countries we have matching data for GDP and population from the World Bank Development Indicators. Appendix A.1 provides a list.[4]

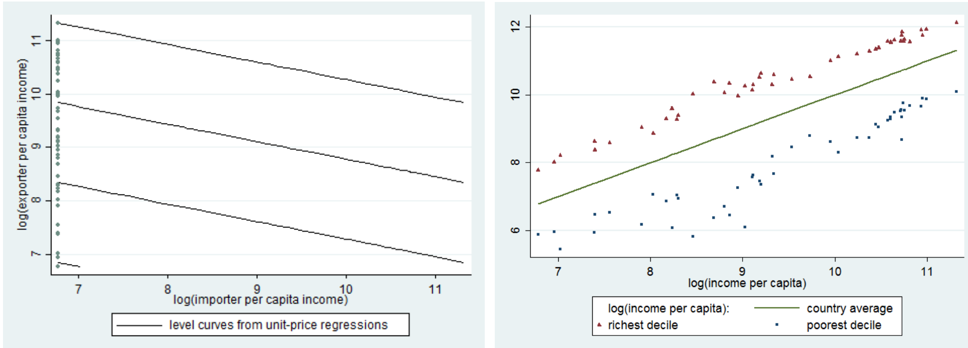
Table 1 reports results of some basic regressions of unit values (in logarithms) against, in various combinations, fixed effects, exporter and importer GDP per capita and exporter and importer total GDP (all in logarithms), and, in all cases, bilateral distance (again in logarithms). Robust across the various relevant specifications is that unit values increase in both importer and exporter per capita income.[5] The exporter effect is larger. For all products the point estimates of the elasticity with respect to exporter per capita income range between 0.183 and 0.187, while point estimates of the elasticity with respect to importer per capita GDP range between 0.052 and 0.065. To give a sense of these magnitudes, U.S. per capita income is 45 times that of India. These elasticities imply that, when selling to the same importer, exports from the United cost twice as much per unit as exports from India, while, when buying from the same exporter, the United States pays 20% more per unit than India.

Columns (6) and (7) split products into manufacturing and non-manufacturing. The elasticity for exporter per capita income is larger for manufacturing products, with the opposite the case for importer per capita income. We don't find the differences sufficient to warrant separating the two groups of products in what follows.

Our core analysis develops a framework that can incorporate these roles for importer and exporter per capita income into a standard quantitative general equilibrium framework. Also significant in the regressions in Table 1 is the effect of distance on unit price, with an elasticity ranging from 0.05 to 0.07. This relationship is the well-known

[4]In this section we describe some basic features of the data using the full sample of 184 countries. In the estimation of our model, as reported in sections 4 through **??**, we use a subsample of the 50 largest countries in the 2007 cross section, both to speed up computation and to reduce the number of zeros per exporter-importer-product triad, allowing for a better comparison of unit values. Countries in the estimation subsample import and export most 6-digit t HS product categories. The estimation subsample combines Hong Kong and China, and Malaysia and Singapore into single country units. We exclude Kuwait because of its specialization in oil. Silva, Tenreyro (2006), Helpman, Melitz, Rubinstein (2008), Baldwin and Harrigan (2011) and Eaton, Kortum and Sotelo (2013) study zeros in trade flows. See also section 5.

[5]Many researchers ahead of us have reported similar relationships. For a positive elasticity of unit value with respect to importer per capita income see, e..g., Hallak (2006), Feenstra and Romalis (2014). For the positive elasticity of unit value with respect to exporter per capita income among U.S. imports see Schott (2014) and Khandelwal (2010). For an analysis closer to ours see Hanson (????).



(a) Level curves of unit prices

(b) Income distributions

Figure 1: Evidence for two dimensions of quality

Alchian-Allen "Washington apples effect" documented by Hummels and Skiba (2004). An extension of our core framework is able to capture this relationship as well. Because this extension adds some complexity we delay it to a later section.[6]

Figure 1(a) shows iso-unit-value curves implied by the estimates in column (3) of table 1. The x-axis is exporter per capita income and y-axis importer per capita income. (Dots along the y-axis indicate the 50 countries in the subsample used to estimate the model, indicating that the axes span these countries.) Each point on a curve corresponds to a combination of importer and exporter per capita income that delivers the same predicted unit value. The curves indicate an overlap in the unit values of high, middle, and low-income exporters depending on where they export. For example, in 2007, income per capita was \$45,600 in the United States, \$9,700 in Mexico and \$1,900 in Indonesia. Regression coefficients imply that the United States sells goods to the poorest countries at the same unit values that Mexico sells to the richest countries. And Mexico sells goods to the poorest countries at the same unit values that Indonesia sells to the richest.

Our analysis focuses on differences in income per capita between countries while ignoring differences within countries, based on evidence that differences in income per capita within countries are much smaller than differences between countries. For each country, figure 1(b) plots the income per capita of the poorest and richest decile of the population as a function of the country's average income per capita.

Appendix A.2 presents additional moments. It augments the results in Table 1 by separating products, years and countries. The coefficients on importer and exporter per capita income are typically positive and statistically significant, and have magnitudes

[6]While estimates in Table 1 of the elasticity of unit value with respect to **total** exporter GDP is not significantly different from zero our regressions imply an elasticity with respect to importer total GDP significantly above zero and around 0.015. We leave the exploration of this slight but interesting empirical relationship to future research.

Table 1: Pooled price regrssion

	pooled by imp-prod-year (1)	pooled by exp-prod-year (2)	pooled by product-year (3)	GDP controls (4)	geography (5)	manufacturing only (6)	non-manuf. only (7)	weighted (8)
exporter GDP per capita	0.183		0.185	0.187	0.185	0.199	0.123	0.132
	(0.017)		(0.016)	(0.021)	(0.021)	(0.022)	(0.010)	(0.020)
importer GDP per capita		0.062	0.065	0.053	0.052	0.050	0.073	0.042
		(0.023)	(0.008)	(0.007)	(0.008)	(0.008)	(0.008)	(0.012)
exporter GDP				-0.003	-0.002	-0.001	0.000	
				(0.014)	(0.013)	(0.015)	(0.007)	
importer GDP				0.014	0.015	0.017	0.000	
				(0.005)	(0.005)	(0.005)	(0.004)	
distance	0.070	0.062	0.062	0.060	0.051	0.057	0.073	0.042
	(0.013)	(0.012)	(0.012)	(0.012)	(0.014)	(0.014)	(0.008)	(0.011)
no. observations	3.3E+07	3.3E+07	3.3E+07	3.3E+07	3.3E+07	2.7E+07	4.3E+06	3.3E+07
R-squared	0.379	0.162	0.162	0.163	0.163	0.162	0.199	0.113
product-year-importer fixed effects	yes	no	no	no	no	no	no	no
product-year-exporter fixed effects	no	yes	no	no	no	no	no	no
product-year fixed effects	no	no	yes	yes	yes	yes	yes	yes
controls for common language, border	no	no	no	no	yes	no	no	no
manufacturing only	no	no	no	no	no	yes	no	no
non-manufacturing only	no	no	no	no	no	no	yes	no
regression weighted by value	no	no	no	no	no	no	no	yes
standard deviations								
ln(GDP per capita)		0.214						
ln(GDP)		0.103						
ln(distance)		0.101						
ln(unit price), demeaned by product		1.14						

similar to table 1.

A straightforward explanation for the positive elasticities of unit value with respect to both exporter and importer per capita incomes is that higher unit value reflects higher quality, and that rich countries specialize in the production of higher quality products and demand higher quality products. A problem with this explanation, if quality has only one dimension, is that rich countries would then have systematically higher market shares in rich countries than in poor ones. This pattern is not one we see in the data. The theoretical framework we develop in the nest section accounts for both the positive elasticities of unit value with respect to both exporter and importer per capita income and the ability of rich countries to compete on par with poor ones in all markets.

3 The Model

Our model begins with basic Ricardian ingredients. The world has N countries, indexed by $i, n = 1, ..., N$, each endowed with a measure L_i of workers who are also the households in the economy. A worker can perform different jobs within a country but can't change countries. A worker in country i earns a wage w_i determined in equilibrium. There is a composite output available in amount Y_n in country n that can be used either by households for final consumption or by firms as an intermediate.

Output consists of a continuum Ω of varieties indexed by $\omega \in [0, 1]$. A unit of variety ω has two dimensions of quality: A vertical dimension $q(\omega) \in [0, \infty)$ complements quantity $y(\omega) \in [0, \infty)$ while a horizontal dimension $Q(\omega) \in [0, \infty)$ perfectly substitutes for quantity. Examples of vertical quality differentiation might be Robert Parker's rating of a wine or the precision of a machine tool. Examples of horizontal quality might be the heating value of a ton of coal, the durability of a light bulb, or the caffeine content of a cup of coffee.

3.1 Aggregation

We now turn to how individual varieties aggregate into the composite output. To simplify notation we temporarily ignore the international dimension of the problem and suppress country subscripts.

Varieties combine to form aggregate output according to the function

$$Y = \left[\int_{\omega \in \Omega} \tilde{u}(\omega)^{\beta} d\omega \right]^{1/\beta} \tag{1}$$

where the variety-specific benefit is:

$$\tilde{u}(\omega) = \left[(Q(\omega)y(\omega))^{\rho} + q(\omega)^{\rho}\right]^{1/\rho}.$$

Here $\beta \leq 1$ governs the elasticity of substitution between varieties while $\rho \leq 1$ governs the elasticity of substitution between quantity and the vertical dimension of quality.[7]

The cost of $y(w)$ units of vertical quality $q(\omega)$ of variety ω is

$$x(\omega) = y(\omega)q(\omega)^{\gamma}c(\omega). \tag{2}$$

Here $\gamma > 0$ is a parameter reflecting the cost of producing higher vertical quality and $c(\omega) > 0$ is the cost of creating one unit of variety ω of quality $q(\omega) = 1$, which is determined in equilibrium.

An agent with a budget X seeks to maximize (1) subject to (2). We split the problem into two parts. We first ask, for a particular variety ω with given horizontal quality $Q(\omega)$, how to choose $q(w)$ and $y(w)$ maximize the benefit $\tilde{u}(\omega)$ given spending $x(\omega)$ on this variety. We then ask how the consumer should allocate his budget X across spending on each variety $x(\omega)$.

3.1.1 Quality versus quantity

Since we first focus on a given variety, we temporarily drop the ω argument. If the consumer has chosen to spend x on this variety, the problem is:

$$\max_{y,q} \left[(Qy)^{\rho} + q^{\rho}\right]^{1/\rho} \tag{3}$$

subject to:

$$yq^{\gamma}c \leq x \tag{4}$$

To satisfy the second-order conditions for a minimum we need to impose the condition that $\rho < 0$.[8] Taking the ratio of the two first-order conditions gives:

$$q = \gamma^{1/\rho}Qy, \tag{5}$$

[7] To guarantee an interior solution we impose the additional restriction that $\rho < 0$ below.

[8] Graphically, the budget constraint:

$$x \geq yq^{\gamma}c$$

has a surface that is Cobb-Douglas in q and y. For a tangency to represent a minimum requires that the indifference curve:

$$\bar{u} = \left[(Qy)^{\rho} + q^{\rho}\right]^{1/\rho}$$

have an elasticity of substitution strictly below 1.

which, upon substitution into the problem above reduces it to:

$$\max_{y}(1+\gamma)^{1/\rho}Qy \tag{6}$$

subject to:

$$y^{1+\gamma}Q^{\gamma}c \leq x.$$

Defining the term:

$$A = \gamma^{\gamma/[\rho(1+\gamma)]}$$

the implied quantity is:

$$y = A^{-1}\left(\frac{x}{c}\right)^{1/(1+\gamma)} Q^{-\gamma/(1+\gamma)}$$

with corresponding quality:

$$q = A^{1/\gamma}\left(\frac{Qx}{c}\right)^{1/(1+\gamma)}.$$

The price per unit is then:

$$p = Ac\left(\frac{Qx}{c}\right)^{\gamma/(1+\gamma)}$$

and the benefit is:

$$u = A^{-1}(1+\gamma)^{1/\rho}\left(\frac{Qx}{c}\right)^{1/(1+\gamma)}$$

Instead of working with the unit cost c of vertical quality $q = 1$ we introduce:

$$v = \frac{Q}{c},$$

representing horizontal quality relative to unit cost of variety ω. We can then write these expressions more compactly as functions of x and v:

$$
\begin{aligned}
y &= A^{-1}Q^{-1}(xv)^{1/(1+\gamma)} && (7)\\
q &= A^{1/\gamma}(xv)^{1/(1+\gamma)} && (8)\\
p &= Ax^{\gamma/(1+\gamma)}v^{-1/(1+\gamma)}Q && (9)\\
u &= A^{-1}(1+\gamma)^{1/\rho}(xv)^{1/(1+\gamma)} && (10)
\end{aligned}
$$

Note that the parameter γ governs how an increase in spending on $\gamma/(1+\gamma)$.the variety gets divided into quantity and quality, with quantity having an elasticity $1/(1+\gamma)$ and a higher price to buy higher vertical quality having an elasticity $\gamma/(1+\gamma)$.

3.1.2 How much of a variety?

Having solved for the benefit $u[x(\omega), v(\omega)]$ of spending an amount $x(\omega)$ on variety ω we turn to the problem of how much to spend on each variety. Specifically we solve the problem:

$$\max_{x(\omega)} \left[\int_{\omega \in \Omega} u \left[x(\omega), v(\omega) \right]^{\beta} \right]^{1/\beta}$$

where:

$$u \left[x(\omega), v(\omega) \right] = A^{-1} (1 + \gamma)^{1/\rho} \left[x(\omega) v(\omega) \right]^{1/(1+\gamma)}$$

subject to:.

$$\int_{\omega \in \Omega} x(\omega) d\omega \leq X$$

The solution gives us:

$$x(\omega) = \frac{v(\omega)^{\beta/(1+\gamma-\beta)}}{V} X \tag{11}$$

where:

$$V = \int_{\omega' \in \Omega} v(\omega')^{\beta/(1+\gamma-\beta)} d\omega'. \tag{12}$$

From (11) and its substitution into expressions (7) through (10), we can write:

$$\begin{aligned}
x(\omega) &= v(\omega)^{\beta/(1+\gamma-\beta)} \frac{X}{V} && (13)\\
y(\omega) &= A^{-1} v(\omega)^{1/(1+\gamma-\beta)} \left(\frac{X}{V} \right)^{1/(1+\gamma)} Q(\omega)^{-1} && (14)\\
q(\omega) &= A^{1/\gamma} v(\omega)^{1/(1+\gamma-\beta)} \left(\frac{X}{V} \right)^{1/(1+\gamma)} && (15)\\
p(\omega) &= A v(\omega)^{-(1-\beta)/(1+\gamma-\beta)} \left(\frac{X}{V} \right)^{\gamma/(1+\gamma)} Q(\omega) && (16)\\
u(\omega) &= A^{-1} (1 + \gamma)^{1/\rho} v(\omega)^{1/(1+\gamma-\beta)} \left(\frac{X}{V} \right)^{1/(1+\gamma)} && (17)
\end{aligned}$$

where we continue to take horizontal quality $Q(\omega)$ as given.

We can then solve for Y as a function of X and V:

$$\begin{aligned}
Y &= \left[\int_{\omega \in \Omega} u(\omega)^{\beta} d\omega \right]^{1/\beta}\\
&= A^{-1} (1 + \gamma)^{1/\rho} \left(\frac{X}{V} \right)^{1/(1+\gamma)} \left[\int_{\omega \in \Omega} v(\omega)^{\beta/(1+\gamma-\beta)} d\omega \right]^{1/\beta}\\
&= A^{-1} (1 + \gamma)^{1/\rho} X^{1/(1+\gamma)} V^{(1+\gamma-\beta)/[\beta(1+\gamma)]} && (18)
\end{aligned}$$

3.1.3 Determinants of horizontal quality and the distributions of efficiency

We now turn to the determination of horizontal quality $Q(\omega)$ and the distribution of the variety-specific cost parameter $c(\omega)$. We now make our multicountry setting explicit by denoting an origin country by i and a destination country by n.

We assume that vertical quality $Q(\omega)$ depends only on origin, so that if country n buys variety ω from country i, $Q_n(\omega) = Q_i$. As in Eaton and Kortum (2002), if country n buys variety ω from country i then

$$c(\omega) = \frac{d_{ni}c_i}{z_i(\omega)}$$

where c_i is the unit cost of a bundle of inputs in country i, d_{ni} is the iceberg cost of shipping a unit from country i to country n, and $z_i(\omega)$ is country i's efficiency producing variety ω. The measure of varieties with efficiency $z_i(\omega) \leq z$ is

$$F_i(z) = \exp(-T_i z^{-\theta}).$$

We define

$$c_{ni} = \frac{d_{ni}c_i}{Q_i},$$

the cost of inputs in source i adjusted source i's horizontal quality and iceberg transport costs to n, and:

$$v_{ni}(\omega) = \frac{z_i(\omega)}{c_{ni}}.$$

An agent in country n sources variety ω from country i if

$$i = \arg \max_{i'=1,\dots,N} \{v_{ni'}(\omega)\}.$$

Using the distribution of z, the share of goods that country n sources from country i is

$$\pi_{ni} = \frac{T_i c_{ni}^{-\theta}}{\Phi_n} \tag{19}$$

where

$$\Phi_n = \sum_{i'} T_{i'} c_{ni'}^{-\theta}. \tag{20}$$

Note that, despite the nonhomothetic intricacies introduced by the quality dimensions of our model, it delivers the same trade-share equation (19) as the homothetic Eaton-Kortum (2002) model.

The distribution of $v_{ni}(\omega)$ conditional on i being the lowest cost supplier to country n is

$$
\begin{aligned}
G_n(v) &= \Pr\left(V_{ni} \leq v | \, i = \arg\max_{k \leq N}\{v_{ni}\}\right) \\
&= \pi_{ni}^{-1}\int_0^{vc_{ni}Q_i^{-1}} \theta T_i z^{-\theta-1}\exp\left(-\Phi_n c_{ni}^{\theta} z^{-\theta}\right) dz \\
&= \frac{T_i Q_i^{\theta} c_{ni}^{-\theta}}{\Phi_n \pi_{ni}}\left[\exp\left(-\Phi_n c_{ni}^{\theta} z^{-\theta}\right)\right|_{z=0}^{vc_{ni}Q_i^{-1}} \\
&= \exp(-\Phi_n v^{-\theta}).
\end{aligned}
\tag{21}
$$

As in Eaton and Kortum (2002), the distribution G_n is independent of source i so that the unconditional distribution $v_n(\omega)$ in equation (12) has the same cumulative distribution $G_n(v)$. We can use (21) to solve:

$$
V_n = \int v_n(\omega)^{\beta/(1+\gamma-\beta)} d\omega = \int v^{\beta/(1+\gamma-\beta)} dG_n(v) = \Gamma_0 \Phi_n^{\beta/[\theta(1+\gamma-\beta)]} \tag{22}
$$

where:

$$
\Gamma_0 = \Gamma\left(1 - \frac{\beta}{\theta(1+\gamma-\beta)}\right)
$$

and Γ is the gamma function.

Substituting this expression into (18) yields:

$$
Y_n = \Gamma_0^{(1+\gamma-\beta)/[\beta(1+\gamma)]} A^{-1}(1+\gamma)^{1/\rho} X_n^{1/(1+\gamma)} \Phi_n^{1/[\theta(1+\gamma)]} \tag{23}
$$

It will be useful below to invert this expression to solve for the expenditure X_n required to achieve an aggregate Y_n:

$$
X_n = \Gamma_0^{-(1+\gamma-\beta)/\beta} A^{1+\gamma}(1+\gamma)^{-(1+\gamma)/\rho} Y_n^{1+\gamma} \Phi_n^{-1/\theta} \tag{24}
$$

We introduce the term:

$$
\epsilon(\omega) = v_n(\omega)\Phi_n^{-1/\theta} \tag{25}
$$

which has the distribution:

$$
\begin{aligned}
\Pr[E &\leq \epsilon] = \Pr[v_n(\omega) \leq \epsilon \Phi_n^{1/\theta}] \\
&= \exp(-\epsilon^{-\theta})
\end{aligned}
\tag{26}
$$

which depends only on the parameter θ so is independent of exporter i and importer n.

Inserting (25) and (12) into expressions (13) through (17) gives us:

$$\begin{aligned}
x(\omega) &= \Gamma_0^{-1}\epsilon(\omega)^{\beta/(1+\gamma-\beta)}X_n && (27)\\
y(\omega) &= A^{-1}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}X_n^{1/(1+\gamma)}Q_i(\omega)^{-1} && (28)\\
q(\omega) &= A^{1/\gamma}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}X_n^{1/(1+\gamma)} && (29)\\
p(\omega) &= A\Gamma_0^{-\gamma/(1+\gamma)}\epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)}\Phi_n^{-1/[\theta(1+\gamma)]}X_n^{\gamma/(1+\gamma)}Q_i(\omega) && (30)\\
u(\omega) &= A^{-1}(1+\gamma)^{1/\rho}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}X_n^{1/(1+\gamma)} && (31)
\end{aligned}$$

Note that each magnitude depends on a constant, the realization of $\epsilon(\omega)$, the buyer's total spending X_n, the buyer's price index term Φ_n, and, for quantity and price, the seller's horizontal quality $Q_i(\omega)$.

3.2 Consumption

Since our economy is perfectly competitive, total income in country n is simply w_nL_n. Since we assume balanced trade and income equality, spending per worker is simply $X_n = w_n$. Household utility aggregates consumption of individual varieties according to (1).

We can then use expressions (27) through (31) to get expressions, for variety ω, of total household consumption spending, total quantity demanded, vertical quality, price, and per household benefit:

$$\begin{aligned}
x_n^C(\omega) &= \Gamma_0^{-1}\epsilon(\omega)^{\beta/(1+\gamma-\beta)}w_nL_n && (32)\\
y_n^C(\omega) &= A^{-1}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}w_n^{1/(1+\gamma)}L_nQ_i(\omega)^{-1} && (33)\\
q_n^C(\omega) &= A^{1/\gamma}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}w_n^{1/(1+\gamma)} && (34)\\
p_n^C(\omega) &= A\Gamma_0^{-\gamma/(1+\gamma)}\epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)}\Phi_n^{-1/[\theta(1+\gamma)]}w_n^{\gamma/(1+\gamma)}Q_i(\omega) && (35)\\
u_n^C(\omega) &= A^{-1}(1+\gamma)^{1/\rho}\Gamma_0^{-1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\Phi_n^{1/[\theta(1+\gamma)]}w_n^{1/(1+\gamma)} && (36)
\end{aligned}$$

3.3 Intermediates

Inputs are a combination of labor and intermediates, with intermediates combining varieties according to (1). Total input per worker is $m^{1-\alpha}$ where m is the use of intermediates per worker. We can think of inputs as provided by a set of competitive input providers who combine labor and intermediates to minimize the cost of an input bundle.

An input provider in country n who hires l workers and buys m bundles of materials for each worker has total cost $l(w_n + X(m))$ where w is the wage rate $X(m)$ is the cost of a bundle of materials defined in equation (24).

We can thus solve for the cost of a unit of inputs c_i in country i from the cost minimization problem of an input provider in country n. The provider chooses labor l and intermediates per worker m to minimize the cost of producing one unit of inputs:

$$\min_{l,m} l(w + X(m))$$

subject to providing 1 unit of inputs or:

$$lm^{1-\alpha} = 1. \tag{37}$$

With perfect competition, the solution to this problem is the cost c_n used for the production of final goods above.

Substituting $X(m)$ from equation (24) and (37) into the objective function, the problem becomes:

$$\min_{l} w_n l + \Gamma_1 \Phi^{-1/\theta} l^{-(\alpha+\gamma)/(1-\alpha)}$$

where:

$$\Gamma_1 = \Gamma_0^{-(1+\gamma-\beta)/\beta} A^{1+\gamma} (1+\gamma)^{-(1+\gamma)/\rho}.$$

The first order condition for a minimum is:

$$w_n - \Gamma_1 \Phi^{-1/\theta} \left(\frac{\alpha+\gamma}{1-\alpha}\right) l^{-(1+\gamma)/(1-\alpha)} = 0.$$

Solving for l:

$$l = \left(\frac{\alpha+\gamma}{1-\alpha} \frac{\Gamma_1 \Phi_n^{-1/\theta}}{w_n}\right)^{(1-\alpha)/(1+\gamma)}.$$

From the constraint $lm^{1-\alpha} = 1$:

$$m = \left(\frac{\alpha+\gamma}{1-\alpha} \frac{\Gamma_1 \Phi_n^{-1/\theta}}{w_n}\right)^{-1/(1+\gamma)}.$$

From (24), spending on intermediates per worker is:

$$\begin{aligned} X(m) &= \Gamma_1 \Phi_n^{-1/\theta} m^{1+\gamma} \\ &= \frac{1-\alpha}{\alpha+\gamma} w_n. \end{aligned} \tag{38}$$

The cost of providing a unit of inputs is

$$c_n = l(w_n + X(m)) = \Gamma_1^{1-\tilde{\alpha}} \tilde{\alpha}^{-\tilde{\alpha}} (1-\tilde{\alpha})^{-(1-\tilde{\alpha})} w_n^{\tilde{\alpha}} \Phi_n^{-(1-\tilde{\alpha})/\theta}$$

where

$$\tilde{\alpha} = \frac{\alpha + \gamma}{1 + \gamma}$$

is the share of labor in production, since:

$$\begin{aligned} \frac{wl}{wl + lX(m)} &= \frac{\alpha + \gamma}{1 + \gamma} = \tilde{\alpha} \\ \frac{lX(m)}{wl + lX(m)} &= \frac{1 - \alpha}{1 + \gamma} = 1 - \tilde{\alpha} \end{aligned}$$

We can insert $X(m)$ from (38) into expressions (27) through (31) to derive spending, quantity, quality, price and benefit of variety ω when used as an intermediate:

$$x^M(\omega) = \Gamma_0^{-1} \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right) \epsilon(\omega)^{\beta/(1+\gamma-\beta)} w_n L_n \tag{39}$$

$$y^M(\omega) = A^{-1} \Gamma_0^{-1/(1+\gamma)} \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)} \epsilon(\omega)^{1/(1+\gamma-\beta)} \Phi_n^{1/[\theta(1+\gamma)]} w_n^{1/(1+\gamma)} L_n Q_i(\omega)^{-1} \tag{40}$$

$$q^M(\omega) = A^{1/\gamma} \Gamma_0^{-1/(1+\gamma)} \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)} \epsilon(\omega)^{1/(1+\gamma-\beta)} \Phi_n^{1/[\theta(1+\gamma)]} w_n^{1/(1+\gamma)} \tag{41}$$

$$p^M(\omega) = A \Gamma_0^{-\gamma/(1+\gamma)} \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\gamma/(1+\gamma)} \epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)} \Phi_n^{-1/[\theta(1+\gamma)]} w_n^{\gamma/(1+\gamma)} Q_i(\omega) \tag{42}$$

$$u^M(\omega) = A^{-1} (1+\gamma)^{1/\rho} \Gamma_0^{-1/(1+\gamma)} \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)} \epsilon(\omega)^{1/(1+\gamma-\beta)} \Phi_n^{1/[\theta(1+\gamma)]} w_n^{1/(1+\gamma)} \tag{43}$$

3.4 Unit Values in Bilateral Trade

We now turn to what our model predicts about the unit value of a variety exported from source i to destination n, .which we can express as:

$$p_{ni}(\omega) = \frac{x_n^C(\omega) + x^M(\omega)}{y_n^C(\omega) + y^M(\omega)}$$

From (33) and (40):

$$y^M(\omega) = \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)} y_n^C(\omega)$$

From (32) and (39):

$$x^M(\omega) = \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right) x_n^C(\omega)$$

so that we can write:

$$\begin{aligned}
p_{ni}(\omega) &= \frac{(1+[(1-\tilde{\alpha})/\tilde{\alpha}])}{\left(1+[(1-\tilde{\alpha})/\tilde{\alpha}]^{1/(1+\gamma)}\right)} \cdot \frac{x_n^C(\omega)}{y_n^C(\omega)} \\
&= \left(\frac{\tilde{\alpha}^{-\gamma/(1+\gamma)}}{\tilde{\alpha}^{1/(1+\gamma)} + (1-\tilde{\alpha})^{1/(1+\gamma)}}\right) p_{ni}^C(\omega) \\
&= \left(\frac{\tilde{\alpha}^{-\gamma/(1+\gamma)}}{\tilde{\alpha}^{1/(1+\gamma)} + (1-\tilde{\alpha})^{1/(1+\gamma)}}\right) A\Gamma_0^{-\gamma/(1+\gamma)} \epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)} \Phi_n^{-1/[\theta(1+\gamma)]} w_n^{\gamma/(1+\gamma)} Q_i
\end{aligned} \tag{44}$$

4 Estimation

Section 4.1 uses trade flows to estimate multilateral resistance terms Φ. Section 4.2 uses prices to estimate parameters γ and θ.

4.1 Trade flows and Φ

We estimate the Φ_n's exploiting equation (19) using data on trade flows, GDP, and distance. We parameterize iceberg costs as

$$d_{ni} = dist_{ni}^{\delta^g} \tag{45}$$

for $i \neq n$, where $dist_{ni}$ is the distance between i and n. Here δ^g is a parameter that relates distance to trade costs.

Given data on country n's total final production GDP_n (corresponding to $w_n L_n$,in our model) its total absorption X_n (taking into account its intermediate demand and its deficit) is:

$$X_n = \frac{Y_n}{\tilde{\alpha}} + D_n$$

where D_n is country n's deficit. For all countries we set the value-added share $\tilde{\alpha} = 0.5$.

Trade shares are then:

$$\pi_{ni} = \frac{X_{ni}}{X_n}$$

for $i \neq n$, and:

$$\pi_{nn} = \frac{X_n - \sum_{i' \neq n} X_{ni'}}{X_n}.$$

For all $n \neq i$, we regress:

$$\log\left(\frac{\pi_{ni}}{\pi_{nn}}\right) = A_n + B_i + \delta^g \log dist_{ni} + \varepsilon^X_{ni} \tag{46}$$

where A_n is an importer fixed effect, B_i is an exporter fixed effect, and ε^X_{ni} is the residual.[9] Equivalent to Waugh (2010), and in contrast to Eaton and Kortum (2002), we attribute country-level differences in openness to differences in internal trade costs (d_{nn}). Under this interpretation, equation (19) implies that fixed effects correspond to:

$$\begin{aligned} A_n &= -\log\left(T_n Q_n^\theta c_n^{-\theta}\right) - \log(d_{nn}^{-\theta}) \\ B_n &= \log\left(T_n Q_n^\theta c_n^{-\theta}\right) \end{aligned}$$

A consistent estimate of Φ_n is then

$$\hat{\Phi}_n = \exp(-\tilde{A}_n) + \sum_{i \neq n} \exp(\tilde{B}_i + \tilde{\delta}^g \log dist_{ni}), \tag{47}$$

where $\tilde{x}$ denotes the estimate of x.

4.2 Unit Prices

We think of varieties ω in our model as very finely defined product. If varieties in our model corresponded to 6-digit HS categories in the data, our model would incorrectly predict that, for any product an importer would by from only one source. We reconcile this discrepancy between theory and data by thinking of 6-digit product categories in the CEPII data as corresponding to a finite set of varieties ω in our model, with varieties within a product are measured in the same units. Taking logs of the price equation (44), delivers the linear equation for each:

$$\log p_{nik} = \delta_k + \delta_1 \log \Phi_n + \delta_2 \log w_n + \log(Q_i) + \varepsilon^P_{nik} \tag{48}$$

for each product category k, where the product fixed effect δ_p accounts for the units in which it is measured and ε^P_{ni} is a residual. If we attribute the residual to variation across realizations of $\epsilon(\omega)$ in equation (42), it corresponds to:

$$\varepsilon^P_{nik} = -\frac{1-\beta}{1+\gamma-\beta} \sum_{\omega \in \Omega^k} s_{nik}(\omega) \ln \epsilon_{ni}(\omega)$$

[9] The discussion in Anderson and vanWincoop (2004) on assumptions on the residual term hold here.

Table 2: Results from price regression (48)

dependent var. → independent var. ↓	$\log p_{ni}(\omega)$ (1)	$\log p_{ni}(\omega)$ (2)	importer fixed effect from (2) (3)
importer Φ_n	-0.036		-0.037
	(0.018)		(0.037)
importer per capita income	0.059		0.066
	(0.029)		(0.015)
exporter fixed effect	yes	yes	no
importer fixed effect	no	yes	no
product fixed effect	yes	yes	no
R-squared	0.71	0.71	0.31
number of observations	2,020,317	2,020,317	50
implied parameters			
γ	0.06		0.07
θ, not adjusted	25.9		25.4
θ, adjusted for attenuation	20.2		19.7

Standard errors are clustered by importer and exporter. See Cameron, Gelbach, Miller (2011) for multiway clustering.

where Ω^k is the set of varieties constituting product k, $s_{nik}(\omega)$ is the share of variety ω in i's exports to n of product k, and $\epsilon_{ni}(\omega)$ is the realization of $\epsilon(\omega)$.Defining $\varepsilon = \log \epsilon$, ϵ is distributed Gumbel distribution with cumulative function

$$H(\varepsilon) = \mathrm{e}^{-\mathrm{e}^{-\beta_1(\epsilon-\beta_2)}}$$

where:

$$\beta_1 = \frac{\theta(\gamma + 1 - \beta)}{1 - \beta} > 0$$

and β_2 is an added parameter to ensure mean zero. In estimating this equation we measure w_i as the income per capita of country i, Φ_n are our estimates from equation (47), and we capture $\log Q_i$ with exporter fixed effects. From our theoretical framework, the coefficients δ_1 and δ_2 correspond to:

$$\delta_1 = -\frac{1}{\theta(1+\gamma)},$$
$$\delta_2 = \frac{\gamma}{1+\gamma}.$$

Table 2 shows the results from the OLS regression (48). Column (1) reports a regression of unit values on the estimated multilateral resistance terms $\hat{\Phi}_n$ above, importer per capita spending, exporter fixed effects and product dummies. We calculate per capita spending of country n as its per capita income plus its trade deficit divided by population. The estimates satisfy the model's restriction that $\delta_2 \in (0,1)$ and $\delta_1 < 0$. Columns (2) and (3) estimates the regression (48) in two stages: Column (2) regresses prices on importer dummies, exporter dummies and product dummies, and column (3) regresses the with importer fixed effects rather than $\hat{\Phi}_n$ and per capita GDP. Parameter estimates change very little.[10]

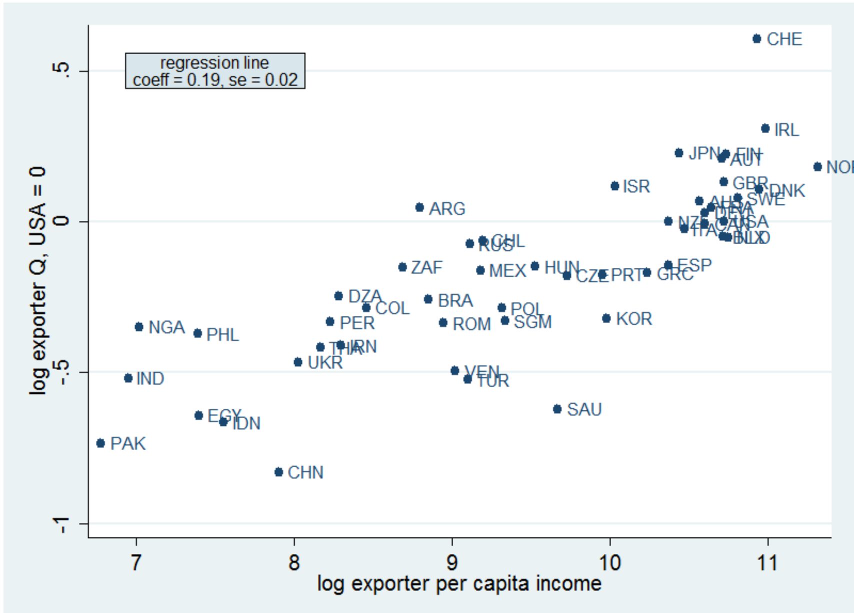
The last three rows of the table show the model parameters implied by coefficients δ_1 and δ_2. Using specification (1), estimated

$$\gamma = \frac{\delta_2}{1-\delta_2} = 0.06$$

The coefficient δ_1 is biased downward due to attenuation: We do not observe Φ_n and use instead its estimated value $\hat{\Phi}_n$. Appendix B estimates this attenuation bias. Adjusting

[10]Results barely change if we replace importer per capita spending with importer per capita GDP. Results change significantly if we force trade within countries to be frictionless, $d_{nn} = 1$, as in Eaton and Kortum (2002). Recalculating $\hat{\Phi}_n$ assuming $d_{nn} = 1$, we get an estimate of δ_2 that is small and positive, contradicting the model. The rationale for this result is that if $d_{nn} = 1$, the estimated $\hat{\Phi}_n$ is correlated with importer GDP per capita, which has a positive coefficient in the price regressions in table 1 above.

Figure 2: Fixed exporter effects from table 2, column (1)



for attenuation yields an implied estimate of the elasticity of trade with respect to trade cost, $\theta = 20$, which is significantly higher than the literature. Three points are in order. First, the coefficient δ_2 is not precisely estimated. If we add just one standard deviation to the parameter, we obtain an estimate of $\theta = 11$ which is in the upper range of estimates in the literature. Furthermore, previous estimates of this elasticity based on prices are biased downward if goods are quality-differentiated. These estimates typically infer a low elasticity of trade with respect to trade costs whenever products with high unit value sell in large volumes, but this inference does not hold if such varieties have higher quality. Third, we can re-write the price equation as

$$\log p_{nik} = \delta_k - \frac{1}{\theta} \log \Phi_n + \delta_2 \log \left(w_n \Phi_n^{1/\theta} \right) + \log(Q_i) + \varepsilon_{ni}^P$$

Unit prices increase with real income per capita, $w_n \Phi_n^{1/\theta}$, at a rate δ_2 and *in proportion* to the price index after controlling for nonhomotheticities. Although we find this theoretical result intuitive, table 2 indicates that unit prices are only weakly correlated with the price index implied by the gravity equation of trade flows.

Figure 2 plots the exporter fixed effects from table 2, specification (1), against income per capita. The coefficient on the regression line is, not surprisingly, positive, statistically significant and similar to the coefficient on exporter per capita income on table 1, section 2 above.

5 Product Heterogeneity

Section 2 above documented the increasing relation between unit values and both importer and exporter per capita income in aggregate data. Section 5.1 describes the heterogeneity in this relation across 6-digit HS product categories in the data.

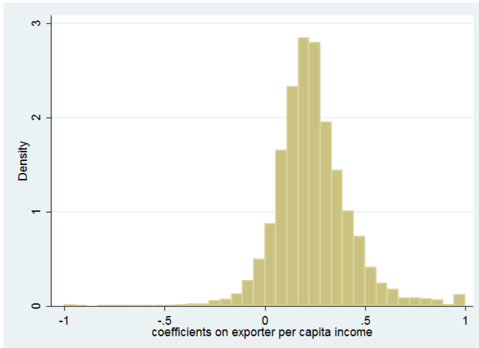
Rather than explaining heterogeneity across products with product-specific parameters, we instead see how far we can go attributing product heterogeneity to random variation in products' underlying varieties. To do so we simulate the model in terms of underlying randomness in varieties and then group varieties into products to replicate the distribution of the number of different importer-exporter pairs per product in the data. Although heterogeneity is not directly targeted in the estimation, the model accounts for a small but non-negligible amount of product heterogeneity. The procedure implies that the simulated model generates not only predictions for trade volumes and prices but also predictions on the extensive margin of trade. Section 5.3 shows that the model does quite well in replicating the data on the quantity and unit value margins of exports in the cross-section and proceeds to analyze how countries adjust their margins of production, intensive, extensive, and quality, when subjected to counterfactual foreign shocks.

5.1 Product heterogeneity in the data

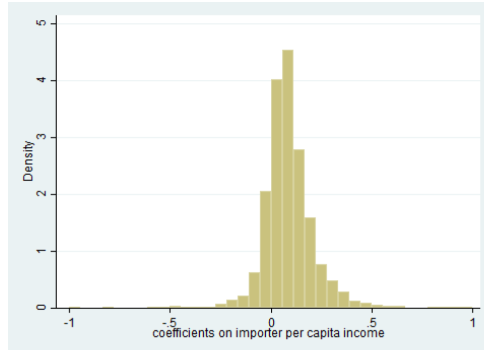
Separately, for each product with more than 20 different importer-exporter pairs, we regress unit prices on importer per capita income and exporter per capita income. This section describes the results from these 4750 regressions.[11] Figure 3 plots the histogram of the estimated coefficients: 82% of the coefficients on importer per capita income are positive and 94% of the coefficients on exporter per capita income. The median coefficients, 0.07 for importer per capita income and 0.22 for exporter per capita income, are consistent with table 1 above. But coefficients vary significantly across products. The standard error across products is 0.13 for coefficients on importer per capita income and 0.21 for exporter per capita income.

There are several approaches to capture this heterogeneity in the model. The first is to assume that model parameters differ across products. This approach is infeasible because only 1% of products have all 50 exporters, which are required to estimate gravity equations and recover multilateral resistance terms Φ. An alternative approach is to assume that parameters vary by broad sectoral classifications, but not by individual products within

[11] Results are almost identical whether we control for distance or not. We use only the subsample of 50 countries in 2007 used in the estimation. With the cut off of 20 observations, we drop 4% of products that account for less than 0.1% of product-importer-exporter observations.



(a) Exporter



(b) Importer

Figure 3: Histogram of coefficients on importer and exporter per capita income

Table 3: Between-sector effect on unit prices

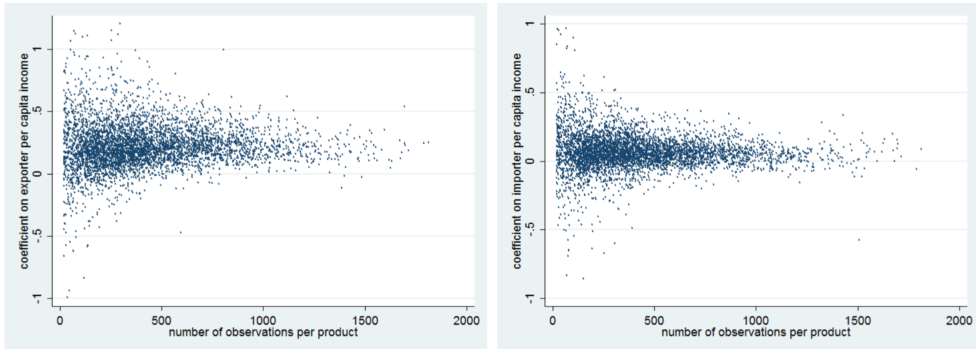
	sectoral classification					
	HS1	HS2	HS3	HS4	HS5	HS6
number of sectors	10	96	173	1229	3418	4750
coef. on importer per capita income	0.02	0.14	0.19	0.44	0.49	1.00
coef. on exporter per capita income	0.09	0.24	0.26	0.40	0.19	1.00

For each HS6 code, we regress unit prices on distance, importer per capita and exporter per capita income. The table reports the share of the variance across coefficients that is between sectors, where the sector is defined as broadly as 1 digit in the first column, 2 digits in the second column and so on.

sectors. To evaluate this approach, we decompose the variance of coefficients in figure 3 and report on table 3 the share of this variance that occurs between sectors. The between share is less than 10% when sectors are classified by broad one-digit sectors, and less than 50% when sectors are defined by narrow 5-digit sectors. These results suggest that systematic variation across broad sectors does not explain the across-product heterogeneity.

As discussed above, we instead take the approach that products in the data are random, finite collections of varieties ω in the model. Unlike the first two approaches, if the number of products is sufficiently large (or a continuum), the model still delivers the aggregate gravity equation of trade flows and unit prices estimated in section 4.[12] Heterogeneity across products arises in the modified model because unit prices vary with variety-specific terms $\epsilon(\omega)$. Figure 4 supports this view. The figure plots the regression

[12]The gravity equation of trade flows depends only on a common parameter θ across products. So, in principle, we can allow parameter γ to be product-specific, but without a closed-form solution for λ, we cannot separately run the price regression by product and so we cannot identify parameters γ and θ.



(a) Exporter per capita income (b) Importer per capita income

Figure 4: Coefficients on importer and exporter per capita income as functions of number of observations in regression

coefficients from figure 3 against the number of observations in each regression. The variation in coefficients at least in part seems to arise because of the small number of importer-exporter observations per product: The variation is visibly larger for regressions with few observations, the standard error of coefficients for regressions with more than 1000 observations is about half of the overall standard error.[13]

5.2 Estimating product heterogeneity

5.2.1 Estimating β

We first estimate β, and in section 5.2.2, we conduct numerical simulations. Solving for $\epsilon(\omega)$ in the price equation (42) and substituting it into the spending equation (39) gives us a relationship between spending on a variety $x_{ni}(\omega)$ and price $p_{ni}(\omega)$ which we can write in a log-linear form as:

$$\log x_{ni}(\omega) = \delta_n + \delta_i + \delta_\omega + \delta_3 \log p_{ni}(\omega) + \varepsilon^X_{ni}(\omega)$$

We can aggregating across varieties within a product to get a product level expression:

$$\log x_{nik} = \delta_n + \delta_i + \delta_k + \delta_3 \log p_{nik} + \varepsilon^X_{nik}$$

where δ_n, δ_i and δ_k are, respectively, importer, exporter and product fixed effects, and $\delta_3 = -\beta/(1-\beta)$ is a parameter to be estimated that corresponds to the elasticity of spending with respect to prices. The term ε^X_{nik} picks up measurement error. To account

[13]The figure do not show coefficients less than -1 or greater than 1, less than 0.2% of coefficients.

Table 4: Estimate of elasticity of spending with respect to prices

independent var → dependent var ↓	unit price$_{nik}$ OLS	dependent var ↓	value$_{nik}$ OLS	value$_{nik}$ IV
instrument	0.1765	unit price$_{nik}$	-0.088	-0.443
	(0.0007)		(0.001)	(0.007)
importer fixed effect	yes		yes	yes
exporter fixed effect	yes		yes	yes
product fixed effect	yes		yes	yes
R-squared	0.70		0.25	0.10
number of observations	1,991,640		2,020,317	1,991,640

The table shows the results from estimating the elasticity of spending with respect to prices. Observations are specific to importer n, exporter i and product k.The instrument is the average price of exporter i of the same product to all other importers. All variables are in logs. The first column shows the regression of the instrument on price, the instrumented variable. The second column shows the biased OLS result and the third column is the IV.

for potential demand shifts in country n for product k, we instrument the price p_{nik} with the average price of exporter i in product k to destinations different from n. In the model, the price from i will be particularly low to all destinations whenever it has a large draw of z.

Table 4 shows results. The first column shows that the instrument has power, it is highly correlated with prices even after controlling for all fixed effects. In the last two columns, the estimated coefficient δ_3 is -0.09 with OLS and -0.44 with IV. The small OLS regression suggests large measurement error in prices. The coefficient in the IV implies an elasticity of demand with respect to prices of -1.44, which is lower but not far from other estimates. The implied β is 0.31.

5.2.2 Numerical simulations

Simulation procedure. We partition varieties ω into products k. Each product contains a finite collection of varieties ω. If there are a continuum of products, , all macro-level predictions of the model hold, but finite and random realizations of $z_i(\omega)$ within each product generate heterogeneity across products. This interpretation is analogous to Eaton, Kortum, Sotelo (2013), and we follow their strategy of combining macro-level estimates with numerical simulations.

The procedure to generate data from the model has seven steps. Steps 1-5 group varieties into products, requiring only parameters from the gravity equation of trade flows in section 4.1. Steps 6 and 7 require estimates of β and θ from unit-price data.

1. Randomly draw a vector
$$v_i(\omega) = \left(\frac{Q_i z_i(\omega)}{c_i}\right)^{\theta}$$
from a Fréchet distribution with cumulative function $H_i(v) = \exp(-e^{B_i} v^{-1})$ where B_i is the estimated exporter fixed effect of the gravity regression of trade flows (46).

2. For each importer and exporter, calculate
$$\tilde{v}_{ni}(\omega) = \left(\frac{Q_i z_i(\omega)}{d_{ni} c_i \Phi_n^{\frac{1}{\theta}}}\right)^{\theta} = \frac{v_i(\omega)\widehat{d_{ni}^{-\theta}}}{\hat{\Phi}_n}$$
where
$$\log \widehat{d_{ni}^{-\theta}} = \begin{cases} \delta_0^g + \delta_1^g \log D_{ni} & \text{if } n \neq i \\ -(B_n + A_n) & \text{otherwise} \end{cases}$$
δ_0^g and δ_1^g are the geography terms in the gravity regression (46). The division of v_i by Φ_n at this stage is irrelevant, but it is useful in simulating prices below.

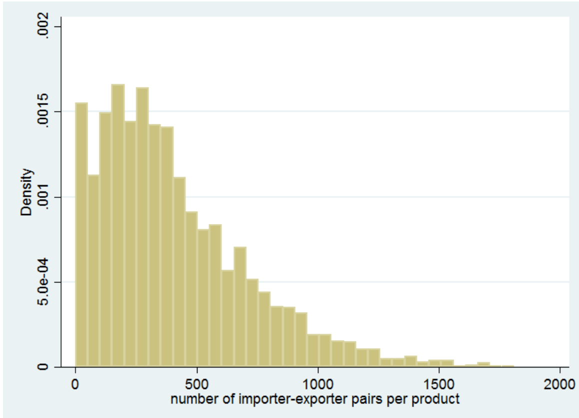
3. For each importer and variety, select the preferred source $i_n^*(\omega) = \arg\max_i\{\tilde{v}_{ni}(\omega)\}$.

4. Repeat steps 1-3 for 250,000 simulated varieties ω, and drop all observations for which importer equals exporter.

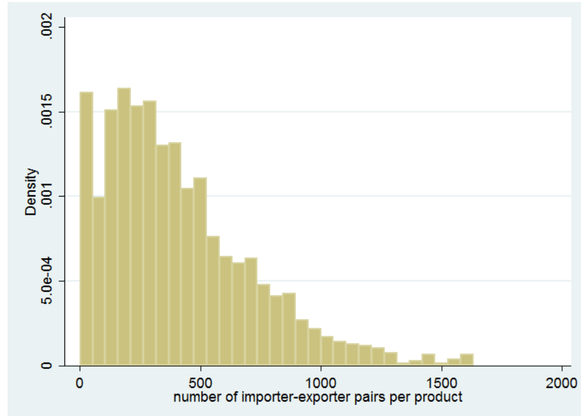
5. Randomly group varieties ω into products to match the distribution of the number of distinct importer-exporter pairs per product in the data. The procedure for grouping varieties is nonparametric and presented in appendix C.

6. For each importer-exporter-variety triad, calculate unit prices and trade flows from equations (16) and (13) where
$$\varepsilon = \tilde{v}_{ni}(\omega)^{\frac{\beta-1}{\gamma+1-\beta}}$$
In this step, we take the estimated β from section 5.2.1 and consider three values for $\theta = 20, 10, 5$, because our estimated $\widehat{\theta} = 20$ is higher than the literature and it has large standard errors.

7. Calculate unit prices per importer-exporter-product triad to be the average of unit prices across varieties, weighted by quantities whenever more than one variety per product have the same importer-exporter pair.



(a) Data



(b) Model (in sample)

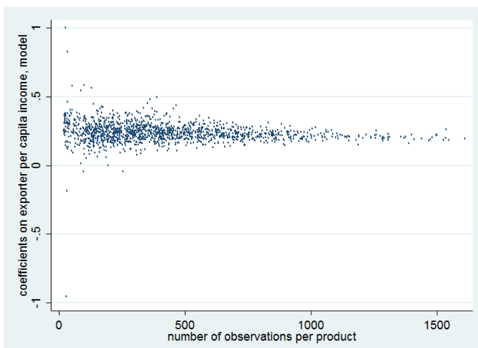
Figure 5: Histogram of number of observations per product

Simulation results. Figure 5 shows the distribution of observations per product in the data and in the model. Since the non-parametric procedure in step 5 is quite flexible, the distributions in the data and the model are similar. Using the model-generated data, we repeat the aggregate unit price regression (48), and by product, we regress unit prices on importer and exporter per capita income. Table 5 summarizes product heterogeneity in the model. Column (1) shows the standard errors of residuals in the aggregate price regression. Columns (2) and (3) present the standard error across coefficients of importer and exporter per capita income in regressions run separately by product. Again suggesting a lot of noise in unit prices in data, the model accounts for a small share of the variation in the residual in the aggregate price regression. But in the per-product regression, the standard errors across coefficients on exporter per capita income in the model are about one-third of the standard error in the data, and they are about one-tenth for coefficients on importer per capita. We consider these fractions to be large as noise and measurement error were never targeted. In both the data and the model, the number of exporters per product is typically smaller than the number of importers. In the model this difference implies that the standard error in coefficients on exporter per capita income is larger than in the coefficients on importer per capita income.

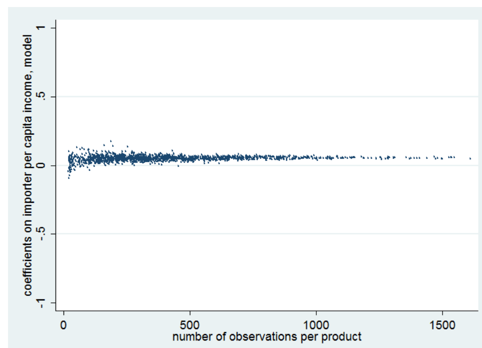
Analogous to figure 4 in the data, figure 6 plots the coefficients of unit price regressions run separately by product as a function of the number of observations in the regression. Not surprisingly, variations coefficients across products are much smaller in the model than the data, and they occur mostly in regressions with a small number of observations. What is more surprising is that the variation in the coefficient on exporter per capita income does not completely go away even when there are more than 1000 observations

Table 5: Heterogeneity in data and model (out of sample)

	standard error of residuals in aggregate price regression	standard error in coefficients of price regressions run by product	
		coeff. on exporter per capita income	coeff. on importer per capita income
	(1)	(2)	(3)
data	**1.90**	**0.21**	**0.13**
$\theta = 20$	0.05	0.06	0.011
$\theta = 10$	0.10	0.06	0.014
$\theta = 5$	0.21	0.08	0.020



(a) Exporter per capita income



(b) Importer per capita income

Figure 6: Coefficients on importer and exporter per capita income as functions of number of observations in regression, simulated data

per product.

5.3 Counterfactuals and margins of adjustment

The gravity model of trade flows implies that countries' exports grow roughly in proportion to their total income. Hummels and Klenow (2005) decompose this growth into an extensive margin (number of products) and an intensive margin (value per product). They further decompose the intensive margin into unit values and quantities. Our model above speaks to these margins of adjustment. It is developed to match unit values in trade data. As with Eaton and Kortum (2002) and Melitz (2003), variation in trade volumes occurs both through the extensive margin of varieties, since selection implies that the distribution of efficiency in what is actually bought by an importer does not depend on the exporter. But the mapping from variety ω to products is not one-to-one in the numerical simulations above. So, the expansion of varieties within a product in the simulated model is observationally equivalent to an increase in the intensive margin. Section 5.3.1 describes the fit of the model in the cross-section, and section **??** presents counterfactuals.

5.3.1 Margins of adjustment in the cross-section

We follow Hummels and Klenow (2005) in our definitions of intensive (IM) and extensive margins (EM). We take the world as the reference region. Denote with k an individual product, K the complete set of products, and K_{ni} the set of products exported from country i to country n. The value of world trade in product k is x_k, and the value exported from country i to country n in product k is x_{nik}. We define margins as

$$EM_{ni} = \frac{\sum_{k\in K_{ni}} x_k}{\sum_{k\in K} x_k}$$
$$IM_{ni} = \frac{\sum_{k\in K_{ni}} x_{nik}}{\sum_{k\in K_{ni}} x_k}$$

Trade flow from country i to country n normalized by total world trade is the product of the two margins:

$$\frac{X_{ni}}{X_{world}} = EM_{ni} \times IM_{ni}.$$

In the data and simulated model, we calculate the extensive and intensive margins for each importer-exporter pair. Table 6 shows how these margins change with exporter characteristics. Panel A refers to the data and panel B to the simulated model. In the first column, we regress separately X_{ni}, EM_{ni}, and IM_{ni} on importer fixed effects and exporter total income (all in logs). By construction the coefficients on the regressions where the dependent variables are EM_{ni}, and IM_{ni} sum to the coefficients on regression where the dependent variable is X_{ni}. To illustrate the decomposition of the intensive margin into quantity and price, the last row regresses unit prices on importer and product fixed effects and on exporter income. In the data and in the model, total trade increases with exporter total income with an elasticity of 1.08. In the data, about 55% (=0.6/1.08) of this response comes through the intensive margin, while in the model, the intensive margin accounts for about 34% of the response. In both the data and the model, unit prices account for only about 4% of the response.

The second and third columns substitutes the independent variable exporter income in all regressions with exporter income per capita and exporter population. Again by construction, the coefficient on total income in the first column lies between the coefficients on income per capita and population. As others have found, the elasticity of exports with

Table 6: Decomposition of exports into margins

Panel A: Data			
	independent variable		
dependent var ↓	exporter GDP	exporter GDP/L	exporter L
X_{ni}	1.08	1.21	0.99
EM_{ni}	0.48	0.63	0.38
IM_{ni}	0.60	0.58	0.61
p_{nik}	0.04	0.16	-0.04

Panel B: Model			
	independent variable		
dependent var ↓	exporter GDP	exporter GDP/L	exporter L
X_{ni}	1.08	1.20	1.00
EM_{ni}	0.71	0.83	0.63
IM_{ni}	0.37	0.38	0.37
p_{nik}	0.04	0.16	-0.04

The table shows the decomposition of countries' exports into extensive, intensive margins and unit prices. All variables are in logs. Regressions where the dependent variables are X_{ni}, EM_{ni} or IM_{ni} have importer fixed effects, and unit-price regressions have importer and product fixed effects.

respect to income per capita is larger than the elasticity with respect to population. The model in panel B captures this restriction because of geography, high-income countries are geographically closer to other large countries, and because the model allows for country-specific internal costs d_{ii}.[14] In the data and in the model, the elasticity of total trade flows with respect to exporter per capita income and population differ almost exclusively due to the extensive margin. Richer countries export a greater variety of products, but do not have larger sales per product. In the model, this result arises because the variety-specific term ϵ in the trade flow equation (27) does not depend on exporter once we control for importer fixed effects.

The finding that the elasticity of the intensive margin with respect to income per capita and with respect to population are the same masks the distinction between sales growth through quantity and unit prices. The last column shows that sales per product increases with population only through quantity, while about one third of the elasticity of the intensive margin with respect to income per capita is attributed to unit price increases. The numbers on table 6A are very similar to Hummels and Klenow (2005) who study only exports to the United States.

[14] One can show that the model with domestic trade costs delivers the same specifications for trade flows as Waugh (2011), who allows importing trade costs to depend on country fixed effects. Our estimates imply that domestic trade costs d_{ii} are generally higher for larger and for poorer countries.

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All labels of sections, equations, figures and tables not preceded by a letter refer to the main text.

A Appendix: Data

Appendix A.1 presents the list of countries in the data. Appendix 2 presents moments that supplement the findings in section 2.

A.1 List of countries

Table A.1 lists all countries in the data described in section 2. Table A.2 presents summary statistics for the subsample of 50 countries used in the estimation, sections 4 through **??**.
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Table A.1: List of countries, full sample

Albania	Djibouti	Kyrgyz Republic	Rwanda
Algeria	Dominica	Lao PDR	Sao Tome and Principe*
Angola	Dominican Republic	Latvia	Saudi Arabia
Antigua and Barbuda*	Ecuador	Lebanon	Senegal
Argentina	Egypt, Arab Rep.	Lesotho	Seychelles
Armenia	El Salvador	Liberia	Sierra Leone
Aruba*	Equatorial Guinea	Libya	Singapore
Australia	Eritrea	Lithuania	Slovak Republic
Austria	Estonia	Macao	Slovenia
Azerbaijan	Ethiopia	Macedonia	Solomon Islands
Bahamas, The	Fiji	Madagascar	South Africa
Bahrain*	Finland	Malawi	Spain
Bangladesh	France	Malaysia	Sri Lanka
Barbados	French Polynesia*	Mali	St. Kitts and Nevis
Belarus	Gabon	Malta	St. Lucia
Belgium-Luxembourg	Gambia, The	Marshall Islands	St. Vincent and the Grenadines
Belize	Georgia	Mauritania	Sudan
Benin	Germany	Mauritius	Suriname
Bermuda	Ghana	Mexico	Swaziland
Bhutan	Greece	Moldova	Sweden
Bolivia	Greenland*	Mongolia	Switzerland
Bosnia and Herzegovina	Grenada	Morocco	Syrian Arab Republic
Botswana	Guatemala	Mozambique	Taiwan*
Brazil	Guinea	Namibia	Tajikistan
Bulgaria	Guinea-Bissau	Nepal	Tanzania
Burkina Faso	Guyana	Netherlands	Thailand
Burundi	Haiti	New Caledonia*	Togo
Cambodia	Honduras	New Zealand	Tonga
Cameroon	Hong Kong, China	Nicaragua	Trinidad and Tobago
Canada	Hungary	Niger	Tunisia
Cape Verde	Iceland	Nigeria	Turkey
Cayman Islands*	India	Norway	Turkmenistan
Central African Republic	Indonesia	Oman*	Uganda
Chad	Iran, Islamic Rep.	Pakistan	Ukraine
Chile	Iraq*	Palau	United Arab Emirates*
China	Ireland	Panama	United Kingdom
Colombia	Israel	Papua New Guinea	United States
Comoros	Italy	Paraguay	Uruguay
Congo, Dem. Rep.	Jamaica	Peru	Uzbekistan
Congo, Rep.	Japan	Philippines	Vanuatu
Costa Rica	Jordan	Poland	Venezuela
Cote d'Ivoire	Kazakhstan	Portugal	Vietnam
Croatia	Kenya	Puerto Rico*	Yemen, Rep.
Cyprus	Kiribati	Qatar	Yugoslavia*
Czech Republic	Korea, Rep.	Romania	Zambia
Denmark	Kuwait	Russian Federation	Zimbabwe*

* Countries with some years missing.

Table A.2: Summary of subsample in estimation

	country	GDP (US$ MI)	GDP per capita	population (000)	rank of GDP/cap in full sample
1	United States	13,751,400	45,642	301,290	10
2	Japan	4,384,250	34,313	127,771	20
3	China, Hong-Kong	3,589,339	2,708	1,325,237	93
4	Germany	3,317,370	40,324	82,268	16
5	United Kingdom	2,772,030	45,442	61,001	11
6	France	2,589,840	41,970	61,707	14
7	Italy	2,101,640	35,396	59,375	19
8	Spain	1,436,890	32,017	44,879	22
9	Brazil	1,333,270	7,013	190,120	57
10	Canada	1,329,880	40,329	32,976	15
11	Russian Federation	1,290,080	9,079	142,100	50
12	India	1,176,890	1,046	1,124,787	119
13	Korea, Rep.	1,049,240	21,653	48,456	27
14	Mexico	1,022,820	9,715	105,281	48
15	Australia	820,974	39,066	21,015	17
16	Netherlands	765,818	46,750	16,381	8
17	Turkey	655,881	8,984	73,004	51
18	Belgium-Luxembourg	502,213	45,221	11,106	12
19	Sweden	454,310	49,662	9,148	7
20	Indonesia	431,933	1,914	225,630	104
21	Poland	424,790	11,143	38,121	44
22	Switzerland	424,367	56,207	7,550	6
23	Norway	388,412	82,480	4,709	2
24	Saudi Arabia	383,587	15,879	24,157	33
25	Austria	373,192	44,880	8,315	13
26	Malaysia, Singapore	353,669	11,358	31,138	43
27	Greece	313,354	27,995	11,193	23
28	Denmark	311,579	57,051	5,461	5
29	Iran, Islamic Rep.	286,058	4,028	71,021	76
30	South Africa	283,743	5,930	47,851	61
31	Argentina	262,451	6,644	39,503	59
32	Ireland	259,018	59,324	4,366	4
33	Finland	244,661	46,261	5,289	9
34	Thailand	236,615	3,533	66,979	83
35	Venezuela, RB	228,071	8,299	27,483	52
36	Portugal	222,758	20,998	10,608	28
37	Colombia	207,786	4,724	43,987	71
38	Czech Republic	173,958	16,833	10,334	32
39	Romania	165,976	7,703	21,547	55
40	Nigeria	165,921	1,121	147,983	116
41	Israel	163,957	22,835	7,180	26
42	Chile	163,878	9,875	16,595	47
43	Philippines	144,043	1,624	88,718	107
44	Pakistan	142,893	879	162,481	128
45	Ukraine	142,719	3,069	46,509	89
46	Hungary	138,757	13,799	10,056	37
47	New Zealand	135,667	32,086	4,228	21
48	Algeria	134,304	3,967	33,853	79
49	Egypt	130,476	1,630	80,061	106
50	Peru	107,291	3,763	28,508	82

A.2 Appendix: Complementary moments

This appendix supplements the findings in section 2 with additional data moments.

Unit price regressions by product-year. Section 2 uses pooled regressions, on table 1, to show that within finely-defined product categories, unit prices are increasing in importer and exporter per capita income. Here we run the corresponding regressions separately by product, year and country.

For each importer, product and year with 15 or more exporters, we regress unit prices on exporter per capita income and distance (all in logs). Table A.3A summarizes the results: The first column shows the number of regressions run per year. The second column shows the percentage of product-importer pairs with positive coefficients on exporter per capita income, the third column is the percentage of coefficients that are positive and statistically significant at a 10% level. The last column shows the median coefficient across regressions. The coefficient on exporter per capita income is positive in about 86% of regressions and it is positive and statistically significant at a 10% level in 57%. Analogous regressions of unit prices on importer per capita income by exporter-product-year tuple appear on panel B. The median coefficients on panel A are similar to the coefficients on exporter per capita income on table 1, while the median coefficients on panel B are similar to the coefficients on importer per capita income.

Table A.3: Regressions by product, country and year

Panel A: Regressions of unit prices on exporter per capita income by importer-product-year				
year	# of regressions	% of positive coefficients	% of positive, signif. at 10% coeff.	median coefficient
2007	62,204	86	53	0.18
2006	59,622	85	52	0.16
2005	56,495	86	54	0.17
2004	48,434	87	57	0.20
2003	43,290	87	57	0.20
2002	38,828	86	56	0.19
2001	36,597	86	55	0.19
2000	37,941	85	54	0.18
1999	35,799	87	58	0.20
1998	36,218	87	58	0.19
1997	35,378	86	56	0.19
1996	33,555	87	59	0.20
1995	30,123	88	61	0.21
Panel B: Regressions of unit prices on importer per capita income by exporter-product-year				
year	# of regressions	% of positive coefficients	% of positive, signif. at 10% coeff.	median coefficient
2007	65,986	66	32	0.05
2006	64,980	67	32	0.05
2005	63,171	65	31	0.04
2004	56,966	68	35	0.06
2003	53,194	68	35	0.06
2002	49,350	67	33	0.06
2001	48,000	66	33	0.05
2000	49,155	67	34	0.06
1999	47,785	66	33	0.05
1998	48,086	66	34	0.05
1997	47,535	67	35	0.06
1996	46,226	67	35	0.06
1995	43,918	67	34	0.05

All variables are in logs. We use only regressions with more than 15 observations, which implies on average dropping 11% of trade flows per year.

Markups. Suppose that the increasing relation between unit prices and income per capita in the data arose because exporters were simply charging a lower price in poor countries, where consumers are more price-elastic. Then either the price premium paid by poor countries to rich countries' exports would be smaller, or the volume of goods from rich exporters relative to poor exporters would decrease a lot with importer per capita income. Table A.4 shows that neither of these hypotheses hold in the estimation subsample. The first column regresses unit prices on fixed effects for importer, exporter and products, and on an interaction between importer per capita income and exporter per capita income. If poor countries paid a lower price premium for goods exported by rich countries, the interaction term would have a positive coefficient. Instead, the coefficient is statistically insignificant and very close to zero, -0.002, suggesting that the price premium paid for goods produced in rich exporters relative to poor exporters is the same across rich and poor importers. With the same premium and a higher price elasticity, poor importers should demand relatively less from rich exporters. The second column regresses total trade flows for each importer-exporter pair on a fixed effect for importer and exporter, distance and on the same interaction between importer and exporter per capita income. Again, the coefficient on the interaction term is not positive, but small, negative and statistically insignificant.

Note that the markup story above, in addition to not being supported by data, is incomplete. It needs an additional dimension of product differentiation to explain why goods from rich exporters are systematically more expensive. This additional dimension could be our horizontal quality Q or a difference in the elasticity of substitution faced by rich countries' firms, as in Fieler and Harrison (2016).

B Appendix: Attenuation bias

The objective of this appendix is to estimate the attenuation bias of parameter δ_1 in the price regression (48):

$$\ln p_{ni}(\omega) = \delta_\omega + \delta_1 \ln \Phi_n + \delta_2 \ln w_n + \ln(Q_i) + \epsilon.$$

The coefficient δ_1 is biased downward because Φ_n is measured with error. The probability limit of the estimate of δ_1 is

$$\text{plim}\hat{\delta}_1 = \delta_1 \frac{\sigma_\Phi^2}{\sigma_\Phi^2 + \sigma_e^2}$$

Table A.4: Regressions on interaction between importer and exporter per capita income

dependent variable $\rightarrow$ **independent variable** $\downarrow$	(1) p_{nik}	(2) X_{ni}
importer per capita income*exporter per capita income	-0.002	-0.006
	(0.01)	(0.04)
distance	0.14	-1.14
	(0.02)	(0.10)
exporter fixed effect i	yes	yes
importer fixed effect n	yes	yes
product category fixed effect k	yes	-
R-squared	0.71	0.78
number of observations	2,020,317	2,447

All variables are in logs.

where σ_Φ^2 is the variance of Φ and σ_e^2 is the variance in the estimate of Φ. To adjust for attenuation, given our estimate of Φ, we calculate $\sigma_\Phi^2 = \sum_n (\Phi_n - \overline{\Phi})^2$ where $\overline{\Phi}$ is the average Φ across countries. The purpose of this appendix is to describe the estimate of σ_e^2.

We start with the gravity equation

$$\frac{\pi_{ni}}{\pi_{nn}} = S_i + D_n + c\Upsilon_{ni} + \varepsilon \tag{B.1}$$

where S_i is a fixed exporter effect, D_n is a fixed importer effect, and c is a vector of parameters and Υ_{ni} is a vector of geography characteristics. In our baseline specification, has only a constant and the $\log(D_{ni})$ where D_{ni} is the distance between the exporter i's and the importer n's most populous city. In this baseline, following the order of coefficients in Stata, c_1 is the coefficient on distance, and c_2 is the constant. The procedure is similar when we extend the model to allow for other geography variables, e.g., $\log(D_{ni})^2$, common border, common language, etc. We order countries by total GDP and normalize the USA's fixed effects $S_1 = D_1 = 0$.

We calculate trade shares π without assuming balanced trade. We calibrate the value added of production $\beta = 0.5$, approximate gross production as $Y_n = \frac{\mathrm{GDP}_n}{\beta}$ and absorption as $X_n = Y_n - \text{exports} + \text{imports}$. We then take $\pi_{ni} = X_{ni}/X_n$ where X_{ni} is observed trade flows when $n \neq i$, and $\pi_{nn} = \frac{Y_n - \text{exports}}{X_n}$.

The estimate for Φ allowing for internal costs $d_{ii} > 0$ is

$$\hat{\Phi}_n = \exp(-\hat{D}_n) + \sum_{i \neq n} \exp(\hat{c}\Upsilon_{ni} + \hat{S}_i) \tag{B.2}$$

To estimate the variance of Φ_n we use the formula $V(f(x)) = \nabla f(x)\Sigma_x \nabla f(x)^T$ where Σ is the variance-covariance matrix of a vector x, $\nabla f(x)$ is the gradient of function f evaluated at x, and $\nabla f(x)^T$ is its transpose. Take $x = (S_1, S_2, ..., S_N, D_1, ..., D_N, c_1, c_2)$ to be the vector of estimates from the gravity equation (B.1) augmented with $S_1 = D_1 = 0$ and Σ_x be the variance-covariance matrix from the same regression again augmented to account for $S_1 = D_1 = 0$:

$$\Sigma_x = \begin{bmatrix} 0 & 0 & ... & & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & V(S_2) & ... & C(S_2,S_N) & 0 & C(S_2,D_2) & ... & C(S_2,D_N) & C(S_2,c_1) & C(S_2,c_2) \\ ... & & & & ... & & & & & ... \\ 0 & C(S_N,S_2) & ... & V(S_N) & 0 & C(S_N,D_2) & ... & C(S_N,D_N) & C(S_N,c_1) & C(S_N,c_2) \\ 0 & 0 & ... & & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & C(D_2,S_2) & ... & C(D_2,S_N) & 0 & V(D_2) & ... & C(D_2,D_N) & C(D_2,c_1) & C(D_2,c_2) \\ ... & & & & ... & & & & & ... \\ 0 & C(D_N,S_2) & ... & C(D_N,S_N) & 0 & C(D_N,D_2) & ... & V(D_N) & C(D_N,c_1) & C(D_N,c_2) \\ 0 & C(c_1,S_2) & ... & C(c_1,S_N) & 0 & C(c_1,D_2) & ... & C(c_1,D_N) & V(c_1) & C(c_1,c_2) \\ 0 & C(c_1,S_2) & ... & C(c_2,S_N) & 0 & C(c_2,D_2) & ... & C(c_2,D_N) & C(c_2,c_1) & V(c_2) \end{bmatrix}$$

The dimensions of x is $2N + 2$ and Σ_x is $2N + 2 \times 2N + 2$. For each country n, function

$f : \mathbb{R}^{2N+2} \to \mathbb{R}_{++}$ is given by equation (B.2) and its gradient

$$\nabla f(x) = \begin{bmatrix} \exp(\hat{c}\Upsilon_{ni} + \hat{S}_1) & \\ \exp(\hat{c}\Upsilon_{ni} + \hat{S}_2) & \\ ... & \\ 0 & \rightarrow \text{ n}^{th} \text{ position} \\ ... & \\ \exp(\hat{c}\Upsilon_{ni} + \hat{S}_N) & \\ 0 & \\ ... & \\ -\exp(-\hat{D}_n) & \rightarrow \text{ n}^{th} \text{ position} \\ 0 & \\ ... & \\ 0 & \\ \sum_{i \neq n} \log(D_{ni}) \exp(\hat{c}\Upsilon_{ni} + \hat{S}_i) & \\ \sum_{i \neq n} \exp(\hat{c}\Upsilon_{ni} + \hat{S}_i) & \end{bmatrix}$$

C Appendix: Grouping varieties into products in simulations

This appendix explains the procedure for grouping varieties into products in the simulated model of section 5.2.2. Steps 1-4 of the simulation delivers a simulated data with 250,000 varieties, each one with a set of distinct importer-exporter pairs. The objective is to partition these varieties into products in such a way that the number distinct importer-exporter observations per product is similar in the model-generated data and in the data.

Let $s \in \mathbb{N}$ be the number of varieties in a product. We first partition the 250,000 varieties into groups of equal size s, and obtain the density of number of observations per product. Letting $s = 1, 5, 10, 15, 24, 40, 70, 120, 180, 280, 400, 600, 800, 1000, 1400, 1800$ gives us 16 densities, f_s. Let n_s be the number of products of size s in the final partition of products. The expected density in this partition is a weighted average of the original densities f_s:

$$f = \frac{1}{|n_s|} \sum_s n_s f_s$$

We use a simplex algorithm to optimally choose n_s to minimize the distance between the simulated density f and the densities of observations in the data, in figure **??**(a). We experimented a little with the set of numbers s and chose a set that delivered a smooth

Table C.1: Between-sector effect on unit prices

Product size s	share of products (%)
1	0.01
2	0.03
5	0.11
10	0.10
15	0.33
24	1.27
40	3.21
70	8.33
120	10.75
180	11.45
280	16.49
400	11.46
580	13.04
800	5.72
1000	6.22
1400	4.37
1800	7.11
total	**100.0**

density that matched the data well. Table C.1 shows the resulting optimal partition. The first column is the product size, measured in number of varieties, and the second column is the share of products of size s in the final simulated data.