The Macroeconomic Effects of Trade Policy*

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Abstract

We study the short-run macroeconomic effects of trade policies that are equivalent in a frictionless economy, namely a uniform increase in import tariffs and export subsidies (IX), a value-added tax increase accompanied by a payroll tax reduction (VP), and a border-adjustment of corporate profit taxes (BAT). Using a dynamic New Keynesian open-economy framework, we show that IX and BAT policies are equivalent, and the unilateral implementation of either policy boosts output and inflation even under flexible exchange rates. Although these policies may have no allocative effects under specific assumptions – as the exchange rate appreciates enough to fully offset the effects on trade prices – we argue that the conditions required for such neutrality are very unlikely to hold in practice (even approximately). Finally, we show that VP policies have substantially different effects than IX or BAT policies under a wide range of assumptions – including about monetary policy and price-setting – and are likely to be contractionary rather than expansionary for output.

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1 Introduction

There is a longstanding debate about how trade policies can stimulate the macroeconomy. In the context of evaluating the merits of remaining on the gold standard during the early phases of the Great Depression, Keynes (1931) argued that the U.K. could derive a similar degree of stimulus from raising import tariffs and reducing export tariffs as through devaluing the pound against gold. However, Mundell (1961) questioned whether this mercantilist prescription would stimulate demand in economies with floating exchange rates, arguing that for the latter economies "equilibrium in the balance of payments is automatically maintained by variations in the price of foreign exchange".

In this paper, we examine the short-run macroeconomic effects of alternative trade policies in a New Keynesian open-economy framework that builds on contributions by Galì and Monacelli (2005) and Farhi, Gopinath, and Itskhoki (2014). We begin by analyzing how Keynes' proposal of a uniform increase in import tariffs and export subsidies (IX henceforth) would play out under different monetary policy regimes, and then consider alternative tax policies that may also affect traded goods prices even without directly taxing imports or subsidizing exports.

The first key finding of our analysis is that IX policies tend to boost domestic output and inflation *even under flexible exchange rates.* While IX policies clearly stimulate demand under fixed exchange rates – as hypothesized by Keynes and corroborated by Farhi et al. (2014) – our finding that these policies are also stimulative under flexible exchange rates contrasts sharply with the conventional view, in which the exchange rate appreciates enough to fully offset any allocative effects of import and export tariffs on the domestic economy.[1]

We highlight that the conditions under which IX policies are "neutral," i.e., have no allocative effects, appear extremely restrictive and hence unlikely to hold in practice. Specifically, neutrality requires that the IX policies are unanticipated, understood as permanent, and do not trigger retaliatory actions by foreign countries; that valuation effects associated with the nominal exchange rate appreciation exactly offset changes in fiscal revenues originating from the policy (i.e., there is no trade in domestic currency denominated assets);

[1] See, for instance, the orginal contribution by Lerner (1936), Mundell (1961) and, more recently, Constinot and Werning (2017).

and, finally, that the exchange rate passthrough to import prices is full and immediate (often referred in the literature as producer currency pricing).

We explore the implications of relaxing these conditions for neutrality, and show that the long-run effects of the trade policy actions on the exchange rate play a central role in determining its allocative effects. Using a Markov-switching framework, our paper considers two mechanisms that cause the exchange rate to revert to its initial level in the long-run: first, an eventual abandonment of the policy; and second, retaliation by foreign countries. In both cases, the policy has sizeable stimulative effects. Intuitively, the tariff policies resemble a familiar "IS curve" shock under these conditions: without any change in the domestic real interest rate, the real exchange rate must also remain unchanged, so that the higher tariffs and subsidies show through fully to trade prices, and provide a strong boost to net exports. In contrast, if trade policy actions are expected to last forever and there is no retaliation by other countries – as typically assumed in the literature – the expected long-run exchange rate appreciation puts immediate upward pressure on the exchange rate even without any interest rate rise. This long-run appreciation markedly damps the shift in aggregate demand that would occur at any given interest rate, offsetting it completely in the special case in which the neutrality conditions hold so that output is unaffected.

Our paper also considers how the quantitative effects of IX policies depend on key structural features of the economy, including monetary and exchange rate policies. While the stimulus to output is comparatively larger under fixed exchange rates, these policies provide a sizable boost to output in the near-term even under a standard Taylor rule provided that policymakers "look through" any transient spike in consumer prices. Our Markov-switching framework is helpful in illustrating how beliefs about the persistence of trade policy actions, or about the likelihood of near-term retaliation, influence the size and persistence of the output response. We also show how empirically relevant frictions such as habit persistence in consumption and local currency pricing tend to amplify the response of output to IX policies.

We then turn our attention to the analysis of two tax policies that are often considered equivalent to IX. In particular, we first study the effects of an increase in value-added taxes accompanied by a reduction in employer payroll contributions (VP), a policy that has been proposed as a possible way to reproduce the effects of IX and a nominal exchange rate devaluation through an internal fiscal adjustment (see, for example, Farhi et al. (2014)). We also analyze the effects of a border-adjustment of corporate profit taxation (BAT). Border-adjustment of taxes is an issue widely studied in the context of value-added taxation and flexible prices.[2] More recently, several authors, including Auerbach and Holtz-Eakin (2016), Auerbach et al. (2017), have argued that a border adjustment of corporate taxation would be equivalent to VP and thus fully compliant with WTO rules.

We find that while the import tariffs and export subsidies stimulate GDP, boost inflation, and induce domestic interest rates to rise, a combination of a higher VAT and a rise in the payroll subsidy to employers (VP policy) tends to have contractionary effect on aggregate demand and inflation, at least under a Taylor-style interest rate rule. The contractionary effects of VP are particularly large when the monetary policy reaction function is fairly unresponsive, as occurs if the central bank puts a substantial weight on exchange rate stability.

We discuss two key assumptions responsibile for the contractionary effects of VP. First, we assume that pre-tax prices are sticky, so that VAT increases are immediately passed through to consumer prices. Second, we assume once again that agents perceive some chance the VP policy will be reversed. The upshot of this assumption is that consumers would face a higher real interest rate if policy rates were unchanged and pre-tax goods prices were also unchanged (since households would expect the prices of goods to be lower at some point in the future). Thus, policy rates would have to decline to keep aggregate demand (and hence output) at its pre-shock level, and the exchange rate to depreciate. Since a standard Taylor rule does not provide enough accommodation to stabilize the economy, output contracts and inflation falls, and the contraction is much more severe under an exchange rate peg.

These results may seem surprising in light of Farhi et al. (2014) which show that, under fixed exchange rates, VP provides equivalent stimulus to output and inflation as IX or an exchange rate devaluation. The key reason for the dramatic difference in results is that we assume that consumer prices adjust quickly to the VAT – so prices are sticky in pre-tax terms – whereas Farhi et al. (2014) assume that consumer prices are sticky inclusive of the VAT. We view our contribution as highlighting the sensitivity of their equivalence results to this

[2] See, for example, Meade (1977), Grossman (1980), and Feldstein and Krugman (1990), Costinot and Werning (2017).

pricing feature.

Finally, we also show that in our framework a border adjustment of corporate taxation is equivalent to IX policies, and, as a consequence, differs substantially from VP policies. Intuitively, the BAT eliminates the deductibility of imports from profits, thus acting like a tariff, and exempts exports, thus acting like an export subsidy. Consequently, the BAT in general provides stimulus exactly like IX policies and has no allocative effects only under the fairly extreme assumptions.

The paper is organized as follows. Section 2 describes the model. Section 3 discusses conditions for equivalence of the IX, VP, and BAT policies as well as the macroeconomic effects of such policies. Section 4 concludes.

2 Model

The benchmark economy features a home (H) country and a foreign (F) country. Agents in the economy include households, retailers, producers of intermediate goods, and the government. The next sections describe the optimization problems solved by each agent. Foreign variables are denoted with an asterisk.

2.1 Households

Households in the home country derive utility from a final good consumption (C_t) and disutility from labor (N_t). Households trade noncontingent nominal bond B_{Ht} and B_{Ft} denominated in the home and foreign currency respectively. The households maximizes expected lifetime utility

$$\mathbb{E}_0 \Sigma_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1}$$

subject to the budget constraint

$$P_t C_t + B_{Ht} + \varepsilon_t B_{Ft} = R_{t-1} B_{Ht-1} + \varepsilon_t R_{t-1}^{*} B_{Ft-1} + W_t N_t + \widetilde{\Pi}_t + T_t \tag{2}$$

where P_t is the consumer price index, R_{t-1} is the domestic nominal interest rate, R_{t-1}^{*} is the foreign nominal interest rate, ε_t is the nominal exchange rate (defined as the price of one unit of foreign currency in terms of units of home currency), W_t is the wage rate, $\widetilde{\Pi}_t$ is the aggregate profit of the home firms assumed to be owned by the home consumers, T_t is a lump-sum transfer from the government. We assume for simplicity that the period utility function is

$$U(C, N) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{\eta+1} N_t^{1+\eta} \tag{3}$$

Optimality requires the standard conditions:

$$N_t^{\eta} C_t^{\sigma} = \frac{W_t}{P_t} \tag{4}$$

$$1 = \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{P_t}{P_{t+1}} R_t \right] \tag{5}$$

$$1 = \beta \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \frac{\varepsilon_{t+1}}{\varepsilon_t} R_t^* \right] \tag{6}$$

where $\Lambda_{t,t+1} = \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}$ is the real stochastic discount factor of the home household. The corresponding optimality conditions for foreign household holdings of bonds are

$$1 + \chi \left(B_{Ht}^* - \bar{B} \right) = \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^* \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} R_t \right] \tag{7}$$

$$1 = \beta \mathbb{E}_t \left[\Lambda_{t,t+1}^* \frac{P_t^*}{P_{t+1}^*} R_t^* \right] \tag{8}$$

where $\chi \in \{0, \infty\}$ determines the costs for the foreign household of holding home currency denominated bonds in excess of a given long-run value $\bar{B}$. Thus, when $\chi = 0$ foreign households can costlessly adjust their holdings of B_{Ht}^*, whereas when $\chi = \infty$ holdings of B_{Ht}^* are fixed at their long-run value at all times (i.e. $B_{Ht}^* = \bar{B}$). These conditions, together with (6), imply the risk-sharing condition

$$\mathbb{E}_t \left\{ \left[\Lambda_{t,t+1} \frac{Q_{t+1}}{Q_t} - \Lambda_{t+1}^* \right] \frac{P_t^*}{P_{t+1}^*} \right\} = 0 \tag{9}$$

where Q_t is the real exchange rate expressed as the price of the foreign consumption bundle in home currency relative to the price of the domestic consumption bundle, that is

$$Q_t = \varepsilon_t \frac{P_t^*}{P_t} \tag{10}$$

2.2 Retailers

Competitive home retailers combine home and foreign intermediate goods to produce the final consumption good according to the constant-elasticity-of-substitution (CES) aggregator

$$C_t = \left[\omega_H^{\frac{1}{\theta}} y_{Ht}^{\frac{\theta-1}{\theta}} + (1-\omega_H)^{\frac{1}{\theta}} y_{Ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{11}$$

where $\theta \geq 0$ determines the elasticity of substitution between home and foreign intermediates and $\omega_H \in [0.5, 1]$ governs home bias. The home good (y_{Ht}) and the foreign good (y_{Ft}) consist of CES aggregators over home and foreign varieties

$$y_{Ht} = \left[\int_0^1 y_{Ht}(i)^{\frac{\gamma-1}{\gamma}} di\right]^{\frac{\gamma}{\gamma-1}} \tag{12}$$

$$y_{Ft} = \left[\int_0^1 y_{Ft}(i)^{\frac{\gamma-1}{\gamma}} di\right]^{\frac{\gamma}{\gamma-1}} \tag{13}$$

Profit for the home retailers are

$$\Pi_t^R = (1-\tau_t^\pi)(P_t C_t - P_{Ht} y_{Ht} - P_{Ft} y_{Ft}) \tag{14}$$

where τ_t^π is the tax rate on profits. Prices of imported goods, P_{Ft}, are inclusive of tariffs (τ_t^m).

Given the CES structure of these aggregators, the home and foreign good demand functions are characterized by

$$y_{Ht} = \omega \left[\frac{P_{Ht}}{P_t}\right]^{-\theta} C_t \tag{15}$$

$$y_{Ft} = (1-\omega) \left[\frac{P_{Ft}}{P_t}\right]^{-\theta} C_t \tag{16}$$

$$y_{Ht}(i) = \left[\frac{P_{Ht}(i)}{P_{Ht}}\right]^{-\gamma} y_{Ht} \tag{17}$$

$$y_{Ft}(i) = \left[\frac{P_{Ft}(i)}{P_{Ht}}\right]^{-\gamma} y_{Ht} \tag{18}$$

The home-country price indexes consistent with the CES aggregators are

$$P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega)\, P_{Ft}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{19}$$

$$P_{Ht} = \left[\int_0^1 P_{Ht}\,(i)^{1-\gamma}\, di\right]^{\frac{1}{1-\gamma}} \tag{20}$$

$$P_{Ft} = \left[\int_0^1 P_{Ft}\,(i)^{1-\gamma}\, di\right]^{\frac{1}{1-\gamma}} \tag{21}$$

2.3 Producers

Each country features a continuum $i \in [0,1]$ of monopolistically-competitive firms producing different varieties of intermediate goods. Producers use the technology

$$Y_{Ht}\,(i) = A_t Z_t(i) N_t^{\alpha}\,(i) \tag{22}$$

with $0 < \alpha \leq 1$. A_t is the aggregate country-wide level of technology and $Z_t(i)$ is the idiosyncratic level of technology. Producers use labor $N(i)$ as the only input of production. Total production is sold both domestically and abroad

$$y_{Ht}(i) + y_{Ht}^{*}(i) = Y_{Ht}\,(i) \tag{23}$$

at price $P_{Ht}\,(i)$ and $P_{Ht}^{*}\,(i)$, respectively.[3] After tax profits of firm i are

$$\Pi_t^i = (1-\tau_t^{\pi})\left[P_{Ht}(i) y_{Ht}(i) + \frac{(1+\varsigma_t^x)}{(1+\tau_t^{m*})}\varepsilon_t P_{Ht}^{*}(i) y_{Ht}^{*}(i) - W_t N_t(i)\right] \tag{24}$$

where ς_t^x is the export subsidy and τ_t^{m*} are import tariffs levied in the foreign economy.

Firm i sets prices as in Calvo (1983): In any given period, it can adjust its price with probability $(1-\zeta_P)$ and maintains the same price as in the previous period with probability

[3] As indicated before, all prices are inclusive of tariffs levied in the two countries.

ζ_P. Therefore, firm i's domestic price evolves according to:

$$P_{Ht}(i) = \begin{cases} \overline{P}_{Ht}(i) & \text{w/prob } (1-\zeta_P) \\ \\ P_{Ht-1}(i) & \text{w/prob } \zeta_P \end{cases} \tag{25}$$

In our benchmark specification, we assume that firms set export prices in their domestic currency (PCP).[4] Hence, the price in foreign currency of exported goods $P_{Ht}^{*}(i)$ adjusts in order to equalize net unit revenues across domestic and foreign markets:

$$P_{Ht}^{*}(i) = \frac{(1+\tau_t^{m*})}{(1+\varsigma_t^{x})} \frac{P_{Ht}(i)}{\varepsilon_t} \tag{26}$$

Firm i chooses a reset price, $\overline{P}_{Ht}(i)$, to maximize the expected present discounted value of profits conditional on no price change

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \left[\Lambda_{s,t} \Pi_s^i \right] \tag{27}$$

subject to its production technology (22), the evolution of prices in (25) and (26), retailers' demand in the home market (17), and an analogous demand schedule in the foreign market.

The reset price $\overline{P}_{Ht}(i)$ satisfies the following optimality condition

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{s-t} \Lambda_{s,t} Y_{Ht}(i) P_{Hs} (1-\tau_s^{\pi}) \left[\overline{P}_{Ht}(i) - \frac{\gamma}{\gamma-1} \frac{W_s}{\alpha A_s Z_s(i) N_s(i)^{\alpha-1}} \right] = 0 \tag{28}$$

Expression (28) indicates that the adjusted price $\overline{P}_{Ht}(i)$ is a constant markup over the weigthed-average expected future marginal costs during the period for which the price will be in effect.

Similarly, foreign firm j sets price $\overline{P}_{Ft}^{*}(j)$ in the foreign market according to

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{*s-t} \Lambda_{s,t}^{*} Y_{Ft}^{*}(j) P_{Fs}^{*} (1-\tau_s^{\pi*}) \left[\overline{P}_{Ft}^{*}(j) - \frac{\gamma}{\gamma-1} \frac{W_s^{*}}{\alpha A_s^{*} Z_s^{*}(i)^{*} N_s^{*}(j)^{\alpha-1}} \right] = 0 \tag{29}$$

and lets the price for the home market $P_{Ft}(j)$ adjust in order to equalize net unit revenues

[4] We later explore the implications of alternative pricing assumptions, such as local currency pricing (LCP).

across markets

$$P_{Ft}(j) = \frac{(1+\tau_t^m)}{(1+\varsigma_t^{x*})} P_{Ft}^*(j)\varepsilon_t \tag{30}$$

Combining the evolution of firm $i's$ price in (25) with the equation for the domestic price index (20) and using the law of large numbers, we derive a forward-looking Phillips curve

$$\pi_{Ht} = \left[\zeta_P + (1-\zeta_P)\left(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{31}$$

where domestic inflation (π_{Ht}) depends on future marginal costs through the optimal reset price $\bar{P}_{H,t}$.

2.4 Government Policy

Fiscal policy in the home and foreign country is characterized by a vector of taxes and subsidies

$$s_t = (\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \varsigma_t^{x*}) \tag{32}$$

We assume that $s_t \in S$ is a finite state Markov chain process and Ω is the associated transition probability matrix, with element $\Omega_{i,j}$ indicating the probability to move from state j to state i. For simplicity, we do not consider changes in corporate profit taxes in our experiments (i.e. $\tau_t^\pi = \tau_t^{\pi*} = \overline{\tau}$ for $t \geq 0$).

This specification for fiscal policy in the two countries is particularly appealing as it allows to consider a wide range of policy configurations and dynamics. For instance, a large literature has devoted much attention to the stability of the Lerner Symmetry Theorem (Lerner, 1936) that establishes the neutrality of permanent increases import tariffs and export subsidies.[5] In our framework, these policy changes can be modelled as a unanticipated transition to an absorbing state where only tariffs and subsidies in the home country vary. Our formulation is also useful to study dynamics associated with turnover in governments and policy reversals which could make policy changes to be perceived by agents as transient. Finally, and more

[5] See McKinnon (1966) and, more recently, Costinot and Werning (2017) and Linde' and Pescatori (2017). The Lerner Symmetry Theorem is also a relevant result for the analysis of neutrality of border tax adjustments, as in Meade (1974), Grossman (1980), and Auerbach et al. (2017).

importantly for the focus of our analysis, our formulation is a laboratory to analyze the effects of potential retaliatory actions by foreign governments.[6]

To complete the description of fiscal policy we assume that the government balances its budget in every period:

$$\tau_t^m \varepsilon_t P_{F,t}^* - \varsigma_t^x \varepsilon_t P_{H,t}^* + \frac{\tau_t^\pi}{1+\tau_t^\pi}\widetilde{\Pi}_t^\pi + T_t^I = T_t \tag{33}$$

Monetary policy in the home country follows the interest rate rule

$$R_t = \frac{1}{\beta}\left[\pi_{Ht}\frac{(1-\tau_t^v)}{\left(1-\tau_{t-1}^v\right)}\right]^{\varphi_\pi} (\tilde{y}_t)^{\varphi_y} \left(\varepsilon_t - \bar{\varepsilon}_t\right)^{\varphi_\varepsilon} \tag{34}$$

where φ_π is the weight on domestic price inflation (π_{Ht}) and φ_y is the weight on the output gap $(\tilde{y}_t)$. The parameter $\varphi_\varepsilon \in \{0, M\}$ governs the sensitivity of the interest rate rule to changes in the nominal exchange rate.[7] When $\varphi_\varepsilon = 0$, the home interest rate responds exclusively to fluctuations in domestic inflation and output gaps. When $\varphi_\varepsilon = M$, the home interest rate rule also responds to deviations of the nominal exchange rate from a target exchange rate.

3 Equilibrium

Define an initial condition for home holdings of bonds and individual producer prices in the domestic market

$$x_0 = \left(B_{H-1}R_{-1}, B_{F-1}R_{-1}^*, P_{H-1}(i), P_{F-1}^*(i)\right)$$

Definition. *Given an initial state x_0, a stochastic process for fiscal policy $\{S, \Omega\}$ and international transfers $\left\{T_t^I\right\}$,*

an equilibrium consists of an allocation $\{C_t, C_t^*, N_t,$

[6] Ossa (2014, 2016) present estimates about the effects of cooperative and noncooperative commercial policies in multi-country and multi-industry general equilibrium models of international trade. Although Ossa is able to characterize the optimal trade policies, his analysis abstracts from dynamic considerations which are the focus of this paper.

[7] See Benigno *et al.* (2007) for a discussion of interest rate rules that maintain a fixed exchange rate.

$N^*_t, B_t, B^*_t, Y_{Ht}, Y_{Ft}, Y^*_{Ft}, Y^*_{Ht}\}_{t\geq 0}$,*individual producer prices* $\{P_{Ht}(i), P_{Ft}(i), P^*_{Ft}(i), P^*_{Ht}(i),$ $\bar{P}_{Ht}(i), \bar{P}^*_{Ft}(i)\}_{t\geq 0}$, *and aggregate prices* $\{P_t, P^*_t, W_t, W^*_t, R_t, R^*_t, P_{Ht}, P_{Ft}, P^*_{Ft}, P^*_{Ht}, \varepsilon_t, Q_t\}_{t\geq 0}$ *such that*

1. *Given prices, the allocation solves the maximization problems of households and retailers (i.e. it satisfies optimality conditions (4) − (6) and (15) − (16) and the analogous conditions in the foreign country);*

2. *Individual producer prices maximize producers profits (i.e. they satisfy conditions (25), (26)* and *(28) and the analogous conditions in the foreign country);*

3. *Prices clear all markets. Specifically:*

   Price indexes satisfy (19) − (21) and the analogous conditions in the foreign country;

   Nominal rates are determined according to (66) and an analogous rule in the foreign country;

   Labor markets clear:

$$N_t = \int N_t(i)\,di \tag{35}$$

   where $N_t(i)$ is firms employment demand as determined by its prices, its production function and retailers demand schedules;

   The bond markets clear

$$\varepsilon_t B_{Ft} - B^*_{Ht} = \varepsilon_t B_{Ft-1} R^*_{t-1} - B^*_{Ht-1} R_{t-1} + NX_t \tag{36}$$

   where net exports (NX_t) are defined as

$$NX_t = \frac{P^*_{Ht} y^*_{Ht}}{1 + \tau^{m*}_t} \varepsilon_t - \frac{P_{Ft} y_{Ft}}{(1 + \tau^m_t)} \tag{37}$$

4 Macroeconomic Effects of Trade Policy

In this section we study the macroeconomic effects of an increase in import tariffs and export subsidies (IX) in the home country and investigate how these effects depend on the exchange rate regime. Our main result is that as long as the effects of IX on the real exchange rate are eventually reversed, the unilateral implementation of IX has potentially large expansionary effects on the home economy and recessionary spillovers in the foreign economy. The appreciation of the currency, in other words, does not fully offset the stimulus provided by these trade policies.

We discuss the intuition for this result in a flexible-price version of our benchmark economy by comparing the effects of a permanent implementation of IX to those of a transitory implementation of the same policy. We then present two mechanisms that imply exchange rate reversal and, consequently, an only partial offset of the macroeconomic stimulus associated with IX. First, we consider the possibility that political turnover in the home country leads to an eventual abandonment of the policy. Second, we analyze the scenario where the implementation of IX in the home country triggers retaliatory actions by the foreign economy. We argue that both mechanisms produce dynamics similar to a transitory change in policy instruments and analyze the critical role of monetary policy, including the exchange rate regime, for the short-run transmission of IX policies.

We conclude by specifying the exact conditions under which, in a flexible exchange rate regime, the IX policy has no effects on the real allocation as its stimulative effects on net exports and output are fully offset by the appreciation of the exchange rate, an argument originally put forward in Mundell (1961). Quantitatively, reversal of the exchange rate effects appears to be the most relevant assumption to deliver sizable allocative effects of IX.

4.1 Parameter Values

The parameters values used in our baseline experiments are listed in Table 1. Given our focus on the transmission mechanism of trade policies, we consider standard values commonly used in the literature.[8]

[8] See, for instance, Galì (2008).

Table 1

	Parameter	Value
Discount factor	β	0.99
Risk aversion	σ	1.0
Frisch elasticity of labor supply	η^{-1}	1.0
Labor share	α	0.36
Price stickiness	ζ_P	0.95
Trade elasticity	θ	1.2
Inflation weight in the rule	φ_π	1.25
Output gap weight in the rule	φ_y	0.125

4.2 Permanent vs Transitory IX in a Flex-Price Economy

Under floating exchange rates, the allocative effects of IX policies depend critically on their long-run effects on the real exchange rate. Specifically, IX policies may have no effect on net exports and output if the long-run expected appreciation of the real exchange rate is as large as the shift in policy instruments, as recognized in the seminal analysis of Mundell (1961). Conversely, when IX policies exert little or no effect on the real exchange rate in the long run, they tend to be expansionary in the short run.

To illustrate this point, we begin by comparing the case of a (unanticipated) permanent implementation of IX with an alternative scenario in which IX is expected to eventually be reversed. For simplicity, here we restrict our attention to the benchmark economy under flexible prices so that details about monetary policy do not have any bearings on the transmission of these policies.[9]

The top panel of Figure 1 provides a graphical representation of the effects of a permanent IX of size δ (i.e. $\tau^m = \varsigma^x = \delta$) on net exports and the real exchange rate.[10] The "NX" locus (solid blue line) shows a negative relation between real net exports and the real exchange rate and can be interpreted as the demand for home savings. This schedule is derived from

[9] We also assume that $\chi = \infty$ and $B^*_{Ht} = B_{Ht} = 0$, so that all aggregate savings by the home country are invested in foreign currency denominated bonds B_{Ht}.

[10] The figure is derived by solving the model nonlinearly under perfect foresight.

the definition of net exports and the PCP conditions expressed in real terms

$$\frac{P^*_{Ht}}{P^*_t} = \frac{P_{Ht}}{P_t}\frac{1+\tau^{m*}_t}{1+\varsigma^x_t}\frac{1}{Q_t} \tag{38}$$

$$\frac{P_{Ft}}{P_t} = \frac{P^*_{Ft}}{P^*_t}\frac{1+\tau^m_t}{1+\varsigma^{x*}_t}Q_t \tag{39}$$

The other key determinant of the real exchange rate and net exports is the supply of savings in the home country (the solid red line). This schedule is derived from the uncovered interest parity condition linking the home real interest rate to the expected appreciation of the domestic currency $\left(\frac{Q_{t+1}}{Q_t}\right)$ and to foreign real interest rates[11]

$$\mathbb{E}_t\left\{\left[\Lambda_{t,t+1}\frac{Q_{t+1}}{Q_t} - \Lambda^*_{t+1}\right]\right\} = 0 \tag{40}$$

The supply of savings schedule slopes upward because – holding constant the foreign real interest rate and real exchange rate level Q_{t+1} expected in the future – a higher level of the exchange rate today implies a larger expected depreciation of the home currency (or smaller appreciation), that is, a higher real interest rate and a lower level of desired consumption.

The implementation of a permanent IX policy shifts the "NX" schedule outward (dashed blue line). A visual inspection of (38) and (39) reveals that, for any given level of the real exchange rate (Q_t), higher tariffs make imports more expensive while higher subsidies make exports cheaper in the foreign country market. Consequently, home-country households substitute away from the more expensive imports and exports rise, resulting in higher net exports (point A).

The shift in the saving supply schedule (dashed red line) depends on how Q_{t+1} is affected, which can be interpreted – in this simple heuristic framework – as a proxy for the effect on the long-run value of the real exchange rate. In the case of a permanent policy change, long-run balanced trade requires that the real exchange rate will have to appreciate by exactly δ so as to offset the changes in import tariffs and export subsidies, as implied by (38) and (39). For any given level of the current real exchange rate (Q_t), however, the rise in the long-run real exchange rate would translate into a lower real interest rate, and hence lower desired savings exactly when net exports are expanding. Accordingly, the real exchange rate must

[11] For ease of notation, we do not consider the covariance terms involved in the UIP condition.

immediately appreciate to its long-run value to bring the demand and supply of savings into balance. As Mundell recognized (1961), the exchange rate rises enough to fully offset the stimulus to real net exports from the IX policies.

The bottom panel of Figure 1 provides a graphical representation of the effects of a transitory IX. While the initial shift of the "NX" schedule is comparable to the previous case, the transient nature of this experiment has fundamentally different effects on the saving supply schedule. Specifically, agents now expect the "NX" schedule, and hence the real exchange rate, to return to its pre-shock level in the long run. Thus, the inward shifts of the home saving supply schedule is smaller than in the previous case and the new short-run equilibrium – at point E_1 – is essentially determined by the outward shift in the NX schedule. As a consequence, the short-run appreciation of the real exchange only dampens, but does not completely offset, the expansion in net exports.

Taken together, this simple flexible price model is suggestive of two more general points. First, under conditions in which the real exchange rate is expected to appreciate permanently, the impact effect of IX policies on the real exchange rate is likely to be of commensurate magnitude: if not, the real interest rate would fall, reducing desired home savings at the same time that the IX policies were driving up home net exports. This strong “gravitational force” in favor of large immediate appreciation of the real exchange rate – and little response of net exports – is likely to hold across a wide range of models given the nature of the forces driving it, even if the model is not consistent with the full exchange rate offset implied by the simple framework. Second, the effects of IX policies can well be markedly different under conditions in which the real exchange rate eventually reverts to it pre-shock level. In this case, net exports are likely to rise, with the effects on output and domestic inflation depending on monetary policy and other features of the modeling framework, as we will next explore.

4.3 Policy Reversal and Retaliation

We now consider two different international trade policy configurations that imply that the exchange rate effects of a unilateral implementation of IX vanish in the long run. We first consider an unanticipated implementation of the IX policy in the home country that may be subsequently reversed with some probability $1 - \rho$. This assumption captures the possibility that a new government in the future may unwind the trade policy adopted by the incumbent government. In this case, the international trade policy regime belongs to one of two different states, $s_t \in S^T = \left\{s^{NT}, s^{IX}\right\}$. In the first state $\left(s^{NT}\right)$ no country levies any taxes. In the second state $\left(s^{IX}\right)$, the home country unilaterally raises import tariffs and export subsidies by the same amount δ. Hence, the only non-zero elements of the state s^{IX} are $\tau_t^m = \varsigma_t^x = \delta$.

We assume that the transition probability matrix governing the evolution of s_t is as follows

$$\Omega^T = \begin{bmatrix} 1 & 0 \\ 1-\rho & \rho \end{bmatrix} \tag{41}$$

The matrix Ω^T implies that implementation of the IX policy is completely unexpected and, conditional on being implemented, it is reversed with probability $(1-\rho)$.[12]

Second, we consider the possibility that the implementation of IX in the home country triggers retaliation by the foreign country. We model this environment by assuming that the trade policy regime belongs to one of three different states, $s_t \in S^R = \left\{s^{NT}, s^{IX}, s^{TW}\right\}$. The first two states are as described above. In the third state $\left(s^{TW}\right)$, the foreign country retaliates with a symmetric policy, that is, $\tau_t^m = \varsigma_t^x = \tau_t^{m*} = \varsigma_t^{x*} = \delta$. In this case the transition probability matrix is:

$$\Omega^R = \begin{bmatrix} 1 & 0 & 0 \\ (1-\pi)(1-\rho) & \rho & \pi(1-\rho) \\ 1-\varphi & 0 & \varphi \end{bmatrix} \tag{42}$$

The matrix Ω^R implies that the implementation of IX in the home country is either reversed by the home country autonomously with probability $(1-\pi)(1-\rho)$, or it triggers a retaliation by the foreign country with probability $\pi(1-\rho)$. Once the foreign country retaliates, the economy transitions back to a no-tax regime with probability $1-\varphi$, while with probability φ it remains in the trade war regime. Notice that we are assuming that the foreign country does not autonomously abandon its retaliatory policies so that a trade war can only be reversed by a coordinated reversal of policies in both countries. Equivalently, a unilateral IX implementation is unanticipated both from the no-policy state s^{NT} and from the trade-war state s^{TW}.

[12] The special case of $\rho = 1$ represents the typical experiment considered in the literature of a unilateral permanent tax change.

Under our assumption of symmetric retaliatory response, the distance between the equilibrium allocation under retaliation and the equilibrium allocation with policy reversal can be summarized by the economic effects of two offsetting international transfers, as formalized below.

Lemma 1 *A unilateral implementation of IX with policy reversal, i.e.* s_t *governed by* $\left\{S^T, \Omega^T\right\}$, *implements the same equilibrium allocation as a unilateral implementation that triggers retaliation, i.e.* s_t *governed by* $\left\{S^R, \Omega^R\right\}$, *coupled with international transfers that satisfy:*

$$T_{t_1}^I = -\frac{\delta}{1+\delta}\left[B_{f,t_1-1}\frac{R_{t_1-1}^*}{\pi_{t_1}^*}\varepsilon_{t_1} - B_{ht_1-1}\frac{R_{t_1-1}}{\pi_{t_1}}\right]$$

$$T_{t_2}^I = \delta\left[B_{f,t_2-1}\frac{R_{t_2-1}^*}{\pi_{t_2}^*}\varepsilon_{t_2} - B_{ht_2-1}\frac{R_{t_2-1}}{\pi_{t_2}}\right]$$

where t_1 *is the first time the economy transits to the retaliation state* s^{TW} *and* $t_2 > t_1$ *is the first time it leaves the retaliation state* s^{TW}.

Proof. See Appendix.

The intuition of this lemma can be easily understood by considering the special case of a permanent transition to a trade war regime starting from balanced trade. In this case, $T_{t_1}^I = 0$ and $T_{t_2}^I$ never occurs. The effects of the trade war can then be analyzed by considering the laws of one price in the home and foreign country

$$\frac{P_{Ht}^*}{P_t^*} = \frac{P_{Ht}}{P_t}\frac{1+\tau_t^{m*}}{1+\varsigma_t^x}\frac{1}{Q_t} \tag{43}$$

$$\frac{P_{Ft}}{P_t} = \frac{P_{Ft}^*}{P_t^*}\frac{1+\tau_t^m}{1+\varsigma_t^{x*}}Q_t \tag{44}$$

Under symmetric retaliation,

$$\frac{1+\tau_t^{m*}}{1+\varsigma_t^x} = \frac{1+\tau_t^m}{1+\varsigma_t^{x*}} = 1 \tag{45}$$

so that the net export schedules are unaffected and the equilibrium allocations coincide exactly.

When the home country has a positive net foreign asset position after a transition to s^{NT} at time t, however, an implementation of IX will generate fiscal revenues. Given its positive net foreign asset position, the home country will expect to run trade deficits in the future so that tariff revenues will exceed export subsidies. Symmetrically, the foreign economy will suffer losses from its implementation of IX. Consequently, a transfer of resources that corrects this international wealth redistribution is needed to implement the same allocation under policy reversal and retaliation.[13]

The important takeaway from Lemma 1 is that for reasonable levels of net foreign assets, the difference between the economic effects of retaliation and policy reversal is tiny. This is because the size of the two offsetting one-time transfers, $T^I_{t_1}$ and $T^I_{t_2}$, is given by a percentage $\frac{\delta}{1+\delta}$ of net foreign assets which, under reasonable calibrations, account for only a very small portion of countries total wealth. Assuming a net foreign asset position of around 40 percent and annual GDP of about 4 percent of wealth, the size of each transfer turns out to be in the order of magnitude of $\delta * .4 * .04$, resulting in second order effects on the allocation.

In what follows we will focus on the case in which the real exchange rate reversal is a consequence of retaliatory behavior by the foreign economy. That is we consider a special case of $\{S^R, \Omega^R\}$ in which $\pi = 1$ and $\varphi = \rho$.

$$\Omega^R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho & (1-\rho) \\ 1-\rho & 0 & \rho \end{bmatrix} \tag{46}$$

As explained above, any choice of π and φ would yield almost identical predictions for the effects of IX. Our focus on this specific policy configuration is only guided by the fact that we regard future retaliatory behavior to be a very plausible mechanism to deliver exchange rate reversal.

4.3.1 IX policy with retaliation

We now turn to the effects of an IX policy implementation that triggers retaliation abroad, as captured by $\{S^R, \Omega^R\}$. Figure 3 shows the response of the real exchange rate, net exports, and domestic output in the home country to a uniform increase in import tariffs and export subsidies of 10 percentage points under a flexible exchange rate regime. The solid line shows the response under an expected retaliation lag of five years ($\rho = .95$) while the dotted line

[13] We develop this argument further below in section 4.3.2.

portrays the response under a permanent unilateral IX policy $(\rho = 1)$. For simplicity, and without loss of generality, the no-tax state is assumed to be absorbing $(\pi = 0)$ in both cases so that the initial transition is an unexpected shock.

The figure illustrates the key role played by the long run behavior of the exchange rate that we described above. When the unilateral implementation of IX is permanent, agents expect the real exchange to remain appreciated forever. Hence, the upward shift of the saving supply perfectly offsets the increase in the net export schedule and the nominal exchange rate needs to jump immediately to its long-run value. The only effect of IX is a permanent appreciation of the real exchange rate.

When we consider the possibility that the foreign country can retaliate, however, the IX policy affects the allocation in the short run. Agents anticipate that the foreign government will eventually implement its own IX policy, pushing both countries to the initial steady state of no taxes. Thus, the saving effect in the home country will not completely offset the trade effect and net exports will increase. The appreciation of the exchange rate will only attenuate the mercantilist element of the IX policy but will not prevent an increase in net exports and a short-run expansion in domestic output.]

Our previous discussion did not rely on any assumption about monetary policy or price stickiness. Here we compare our benchmark economy with sticky prices and flexible exchange rates with two extreme cases, specifically a flexible price economy and an economy with sticky prices under a fixed exchange rate regime, in order to shed some light on the direction of these effects. As shown in the top row of Figure 4, any transitory IX policy will stimulate net exports irrespective of whether prices are sticky or flexible and whether the exchange rate is pegged or free to float. Nonetheless, different specifications of these details will affect the quantitative response of aggregate demand, prices, and interest rates.

Starting with the benchmark economy with sticky prices (blue lines), higher tariffs raise the price of domestic imports and induce consumers to switch away from imported goods and towards home-produced goods. Consumer price inflation jumps because of the pass-through of tariffs into prices, but firms in the home country are unable to raise their prices in response to demand switching of home consumers. The nominal interest rate rises and restrains the relative increase in the demand of domestic goods, leaving consumption of the home good essentially unaffected. The moderate rise in domestic prices together with the increase in the real exchange rate translate into a decrease in export prices and a boom in exports and output. The IX policy has stimulative effect in home country by diverting global demand towards home varieties and, as a consequence, negative spillovers to the foreign country. Foreign output decreases and lower export prices in the home country push foreign inflation down, calling for lower interest rates.

In the flexible price economy (red lines), the expenditure switching effect of IX away from foreign goods and towards home-produced goods is met with an immediate increase in prices of home goods. As a result, policy rates increase more than in the benchmark economy depressing domestic demand for the home good, which actually falls in this case. The home output increase is thus smaller than in the benchmark economy as it is only supported by higher exports.

Finally, when the home exchange rate is pegged (yellow lines), nominal interest rates actually decrease since the negative spillovers of IX to the foreign economy typically require lower policy rates. In this case, firms' inability to raise prices is coupled with a monetary policy that imports the expansionary stance of the foreign economy in order to keep nominal rates fixed. As a result, the appreciation of the real exchange rate is much smaller, as both prices and nominal exchange rates are not allowed to vary flexibly, and the home economy experiences a larger boom in both domestic consumption of the home good and domestic exports.

In sum, unilateral IX policies tend to appreciate the exchange rate, boost net exports, and increase domestic production. These effects do not depend on assumptions about pricing or the exchange rate regime. IX policies have negative spillovers to foreign output and inflation.

4.3.2 Neutrality of IX: a very fragile result

In the sections above we discussed how the prospect of retaliatory behavior by foreign economies can result in substantial stimulative effects of IX at home. For pedagogical purposes we conducted our analysis under a set of assumptions that ensured that, absent the prospect of retaliation, IX would have no allocative effects. While we regard the assumption of eventual retaliation as a highly plausible scenario in the case of a unilateral implementation of IX, there are other important departures from our baseline assumptions that will also break the neutrality result.

The purpose of this section is twofold. First, we spell out the assumptions that are needed in order to obtain neutrality in the absence of retaliatory prospects or policy reversal. Second, we study the effect of relaxing the other conditions that are needed to obtain neutrality. Overall, we conclude that while the exact neutrality result requires very specific assumptions, the prediction of small effects of a permanent implementation of IX that does not trigger retaliation is rather robust.

Proposition 1. *Let* $x_0 = \left(B^*_{H-1}R_{-1}, B_{F-1}R^*_{-1}, P_{H-1}(i), P^*_{F-1}(i), s^0\right)$ *be the initial condition. In an economy with flexible exchange rates, a unilateral implementation of IX of size* δ *has no allocative effect if*

1. *It is unanticipated, permanent, and there is no probability of retaliation;*

2. *Foreign holdings of home currency are always zero:* $B^*_{H-1} = 0 = \bar{B}$ and $\chi = \infty$;

3. *Export prices are set in producer currency (PCP) or prices are flexible*

Appendix B contains a formal proof of proposition 1. Here we describe the basic intuition behind this result in the special case in which at time 0 a unilateral IX policy is implemented and only the home country actively uses trade policy instruments (i.e. $\tau^{m*}_t = \varsigma^{x*}_t = 0$).

Neutrality to a policy change requires that relative prices must remain unchanged. Under PCP, export prices satisfy the law of one price (for convenience, in real terms)

$$\frac{P^*_{Ht}}{P^*_t} = \frac{P_{Ht}}{P_t}\frac{1}{\left(1+\varsigma^x_t\right)Q_t} \tag{47}$$

$$\frac{P_{Ft}}{P_t} = \frac{P^*_{Ft}}{P^*_t}\left(1+\tau^m_t\right)Q_t \tag{48}$$

Equations (47) and (48) imply that relative prices remain unchanged only if a δ increase in import tariffs and export subsidies causes an exchange rate appreciation of the same exact size. Let $Q_t(\delta)$ denote the real exchange rate under an IX policy of size δ, neutrality then requires

$$\frac{1}{Q_t(\delta)} = \frac{1}{Q_t(0)}(1+\delta) \tag{49}$$

At this point we have to check that this appreciation of the exchange rate does not imply violations in other equilibrium conditions. Changes in the real exchange rate affect directly the two optimality conditions for holdings of foreign currency denominated bonds, (6) and (7) , and the equilibrium in the balance of payments.

Equations (6) and (7) are unaffected if, and only if, condition 1 is satisfied: since optimal holdings of foreign currency denominated bonds depend on future exchange rate appreciation, a permanent appreciation of the real exchange rate does not affect demand schedules. That is, under condition 1, equation (49) implies $\frac{Q_{t+1}(\delta)}{Q_t(\delta)} = \frac{Q_{t+1}(0)(1+\delta)}{Q_t(0)(1+\delta)}$.

As for the balance of payment, we can rewrite equation (36) in (foreign good) real terms and, using condition 2, obtain

$$\frac{B_{Ft}}{P_t^*} = \frac{B_{Ft-1}}{P_{t-1}^*}\frac{R_{t-1}^*}{\pi_t^*} + \frac{P_{Ht}^*}{P_t^*}y_{Ht}^* - \frac{P_{Ft}}{P_t}\frac{y_{Ft}}{(1+\tau_t^m)\,Q_t\,(\tau_t^m)} \tag{50}$$

Equation (50) shows that aggregate home savings in foreign bonds are also unaffected under (49) . Therefore, , the original allocation is still an equilibrium after the implementation of IX.

It might seem surprising that the home household is keeping consumption, C_t, unchanged while the value of its savings in the foreign market, $Q_t\frac{B_{Ft}}{P_t^*}$, declines with the real exchange rate appreciation. The reason for this result is that, under condition 1, IX induces two perfectly offsetting changes in two different components of households wealth. On the one hand, the real exchange rate appreciation decreases the value of home holdings of foreign bonds, thus generating losses of size

$$[Q_t\,(\delta) - Q_t\,(0)]\,\frac{B_{Ft-1}}{P_{t-1}^*}\frac{R_{t-1}^*}{\pi_t^*} = -\frac{\delta}{1+\delta}Q_t\,(0)\,\frac{B_{Ft-1}}{P_{t-1}^*}\frac{R_{t-1}^*}{\pi_t^*} \tag{51}$$

On the other hand the IX policy generates fiscal revenues whenever the home country has a trade deficit since in this case revenues from tariffs exceed subsidies to exporters. The wealth increase associated to these fiscal revenues is given by

$$E_t\sum_{i\geq 0}\left(\prod_{j=1}^{i}\frac{1}{R_{t+j}^A}\right)\frac{\tau_{t+i}^m}{1+\tau_{t+i}^m}\left(p_{ft+i}y_{ft+i} - Q_{t+i}\,(0)\,p_{ht+i}^*y_{ht+i}^*\right) = \frac{\delta}{1+\delta}NFA_t \tag{52}$$

which in the case of a permanent unilateral IX policy, $\tau_{t+i}^m = \delta$, is proportional to the present discounted value of future trade deficits. As equation (52) indicates, in equilibrium the present discounted value of future trade deficits is exactly equal to the initial net foreign asset position.

Under condition 2, the net foreign asset position is just given by foreign bond holdings $Q_t \frac{B_{Ft-1}}{P^*_{t-1}} \frac{R^*_{t-1}}{\pi^*_t}$, which implies that the increase in home wealth through higher fiscal revenues exactly offsets the decline in wealth induced by losses on foreign holdings

$$-\left[Q_t\left(\delta\right) - Q_t\left(0\right)\right] \frac{B_{Ft-1}}{P^*_{t-1}} \frac{R^*_{t-1}}{\pi^*_t} = \frac{\delta}{1+\delta} NFA_t$$

leaving consumption unchanged.

The argument above also shows why condition 2 of no foreign holdings of home currency denominated bonds is necessary for neutrality. When $B^*_{Ht-1} > 0$ net foreign assets are given by

$$NFA_t = Q_t \frac{B_{Ft-1}}{P^*_{t-1}} \frac{R^*_{t-1}}{\pi^*_t} - \frac{B^*_{Ht-1}}{P_{t-1}} \frac{R_{t-1}}{\pi_t}$$

which implies that the sensitivity of home households wealth to a real exchange rate appreciation through its effect on net foreign assets is bigger than the level of net foreign assets, i.e. the home country has a leveraged exposure to foreign exchange rate variations. In this case

$$-\left[Q_t\left(\delta\right) - Q_t\left(0\right)\right] \frac{B_{Ft-1}}{P^*_{t-1}} \frac{R^*_{t-1}}{\pi^*_t} > \frac{\delta}{1+\delta} NFA_t$$

Therefore, given an unchanged path for future trade deficits, an exchange rate appreciation of the same size of the policy, i.e. as in (49), would induce a decrease in home households wealth as the increase in fiscal revenues is not enough to offset capital losses on holdings of foreign bonds. This negative wealth effects on home households induces them to decrease their saving supply so that in equilibrium the exchange rate appreciates by less and the the trade balance increases.

These effects are portrayed in Figure 4 that shows the response of the economy to a permanent unilateral implementation of IX when condition 2 is not satisfied. In particular we assume that in the initial state trade is balanced but countries hold offsetting position in domestic and foreign currency denominated bonds, i.e. $B_{F_{-1}} = B^*_{H-1} > 0$. In the simulation we set B^*_{H-1} to two times the value of annual GDP. For comparison we also plot the response of our baseline economy. As explained above, positive foreign holdings of home currency denominated bonds results in a shifting in of home saving supply schedule. This dampens the long run effects on the exchange rate and results in a permanent increase in the trade balance at home that is enough to pay interests on its negative net foreign asset position.

Finally, to understand the effects of relaxing condition 3 we also study the effects of a permanent IX policy under the assumption of LCP pricing. Notice that, even though the allocation is unchanged under conditions 1-3, foreign producers are immediately reducing the home currency price they charge on home imports, $\frac{P_{ft}}{1+\tau_t^m}$ However, when firms cannot flexibly adjust the foreign currency price of their exports in response to variations in exchange rates and taxes, foreign exporters will only gradually reduce their prices so consumer prices of home imports will initially spike causing a large increase in import prices and an overshooting of the real exchange rate as the home bundle becomes temporarily more expensive. This movement in relative prices implies that in the short run the home economy experiences a boom in output mostly driven by an increase in domestic demand of the home produced good, as home retailers switch away from more expensive foreign inputs. Exports also increase on impact since the exchange rate rises by more than export subsidies, domestic exporters would also like to increase their prices but their inability to immediately do so results in a temporary increase in exports.

5 Equivalence and Neutrality of IX, BAT, and VP Policies

In Section 4, we analyzed the stimulative effects of IX policies with an emphasis on policy reversal and retaliation as sources of partial exchange rate offset. Here we turn our attention to the relation between IX policies and other government policies often considered as having the same macroeconomic effects of IX, specifically a fiscal devaluation implemented through VAT-cum-payroll subsidy (VP) as well as a border adjustment of corporate taxation (BAT).

The equivalence between IX and VP policies for countries in a fixed exchange rate regime appears to be a consolidated result among both practitioners and academics. The main intuition for this argument is that increases in VAT rates affect traded prices exactly as increases in import tariffs and export subsidies after taking into account that a commensurate payroll subsidy compensates domestic firms for the adverse effects of higher VAT rates. Consequently, VP should generate large expansionary effects as IX when a country pegs its exchange rate. In light of this argument, numerous governments have attempted to provide macroeconomic stimulus by implementing these forms of fiscal devaluations, including more recently in Germany (2006), in France (2012), and at least partially in Portugal in the context of the 2011-2014 EU-IMF Economic Stabilization Program. From a theoretical perspective, Farhi et al. (2014) provide exact conditions under which VP is not only equivalent to IX, but it also implements the same allocation of an exchange rate devaluation when a country adheres to a fixed exchange rate regime or a currency union. A number of quantitative and empirical papers have also provided estimates for the pro-competitive effects of fiscal devaluations, including Lipińska and Von Thadden (2012), Franco (2013), Gomes et al. (2016) to name a few.

The equivalence between IX and BAT has been established since Lerner (1936) and, more recently, Meade (1977), Grossman (1980), and Feldstein and Krugman (1990). Interestingly, much of this literature focused on the *neutrality* of BAT in the context of static trade models with flexible prices. Not surprisingly, then, the recent discussion on reforms of the U.S. corporate tax system has seen an intense debate on the domestic and international effects of a shift towards border-adjusted taxation, including its compatibility with WTO rules.[14] An important thesis of the proponents of the BAT is that, despite its equivalence with an import tariff and an export subsidy, the value of the dollar would immediately jump by exactly the necessary magnitude to offset any effect of trade.

In this section, we use our framework to study the relation among these three policies and provide conditions for equivalence and neutrality. We show that, generically, IX and BAT are equivalent whereas VP is not. The latter result hinges on the insight that the incidence of value-added taxes depends critically on the formulation of the nature of nominal rigidities, a result reminiscent of Poterba et al (1986). Conditions for neutrality of the three policies appear very restrictive.

5.1 BAT and VP in the Benchmark Economy

We first present the description of the benchmark model expanded to include VP and BAT tax instruments. For simplicity, we focus on the key equations affected by these policies.

[14] See, for example, Auerbach et al. (2017).

Retailers. Profit for the home retailers are

$$\Pi_t^R = (1-\tau_t^v)(1-\tau_t^\pi)\left\{P_t C_t - P_{Ht} y_{Ht} - \frac{P_{Ft}}{(1-\tau_t^\pi BAT_t)} y_{Ft}\right\} \tag{53}$$

where τ_t^v is the value-added tax rate, τ_t^π is the tax rate on profits, and $BAT_t \in \{0,1\}$ indicates whether profit taxes are adjusted at the border or not. The border adjustment implies that the cost of imported goods (y_{Ft}) cannot be deducted from profits. Prices are inclusive of value-added taxes and, in the case of imported goods, are also inclusive of home tariffs (τ_t^m).

Given the CES structure of the aggregators (11), (12), and (13) , the foreign good demand is

$$y_{Ft} = (1-\omega)\left[\frac{P_{Ft}}{(1-\tau_t^\pi BAT_t) P_t}\right]^{-\theta} C_t \tag{54}$$

and the home-country consumer price index consistent with the CES aggregators is

$$P_t = \left[\omega P_{Ht}^{1-\theta} + (1-\omega)\left(\frac{P_{Ft}}{1-\tau_t^\pi BAT_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{55}$$

Producers. After tax profits of firm i are

$$\Pi_t^i = (1-\tau_t^\pi)\left[(1-\tau_t^v) P_{Ht}(i) y_{Ht}(i) + \frac{(1+\varsigma_t^x)}{(1+\tau_t^{m*})(1-\tau_t^\pi BAT_t)} P_{Ht}^*(i) y_{Ht}^*(i) - (1-\varsigma_t^v) W_t N_t(i)\right] \tag{56}$$

where ς_t^v is a payroll subsidy, ς_t^x is the export subsidy, and τ_t^{m*} are import tariffs levied in the foreign economy. Expression (56) indicates that export sales are excluded from the definition of profits when the corporate profit tax is adjusted at the border. Similarly, export sales are not subject to the VAT.

Firm i sets prices as in Calvo (1986) so that, in any given period, it can adjust its price with probability $(1-\zeta_P)$ and maintains the same price as in the previous period with probability ζ_P. We assume that, absent any price adjustment by the firm, changes in VAT rates are fully passed through to the retailers and, as a consequence, consumers. Therefore, firm $i's$ domestic price inclusive of VAT evolves according to:

$$P_{Ht}\left(i\right) = \begin{cases} \overline{P}_{Ht}(i) & \text{w/prob } (1-\zeta_P) \\ \\ P_{Ht-1}\left(i\right) \frac{(1-\tau^v_{t-1})}{(1-\tau^v_t)} & \text{w/prob } \zeta_P \end{cases} \tag{57}$$

where ς^v_t is a payroll subsidy, ς^x_t is the export subsidy, and τ^{m*}_t are import tariffs levied in the foreign economy. In our benchmark specification, we assume that firms set export prices in their domestic currency (PCP).[15] Hence, the price in foreign currency of exported goods $P^*_{Ht}(i)$ adjusts in order to equalize net unit revenues across markets:

$$P^*_{Ht}\left(i\right) = \frac{\left(1+\tau^{m*}_t\right)\left(1-\tau^{\pi}_t BAT_t\right)\left(1-\tau^v_t\right)}{\left(1+\varsigma^x_t\right)} \frac{P_{Ht}(i)}{\varepsilon_t} \tag{58}$$

where ς^v_t is a payroll subsidy, ς^x_t is the export subsidy, and τ^{m*}_t are import tariffs levied in the foreign economy.[16]

Firm i chooses a reset price, $\overline{P}_{Ht}(i)$, to maximize the expected present discounted value of profits conditional on no price change

$$\mathbb{E}_t \Sigma^{\infty}_{s=t} \zeta^{s-t}_P \left[\Lambda_{s,t} \Pi^i_s\right] \tag{59}$$

subject to its production technology (22), the evolution of prices in (57) and (58), retailers' demand in the home market (17), and an analogous demand schedule in the foreign market.

The reset price $\overline{P}_{Ht}(i)$ satisfies the following optimality condition

$$\mathbb{E}_t \Sigma^{\infty}_{s=t} \zeta^{s-t}_P \Lambda_{s,t} \Omega_{s,t} \left[\overline{P}_{Ht}(i)\left(1-\tau^v_t\right) - \left(1-\varsigma^v_s\right) \frac{\gamma}{\gamma-1} \frac{W_s}{\alpha A_s Z_s(i) N_s(i)^{\alpha-1}}\right] = 0 \tag{60}$$

where $\Omega_{s,t} = Y_{Ht}\left(i\right) P_{Hs}\left(1-\tau^{\pi}_s\right)$. Expression (60) indicates that the adjusted price $\overline{P}_{Ht}(i)$ is a constant markup over the weigthed-average expected future marginal costs during the period for which the price will be in effect.

[15] Our results are robust to alternative pricing assumptions, such as local currency pricing (LCP). We explore the implications of LCP in Section XX..

[16] We do not study the case in which foreign economies raise VAT taxes.

Similarly, foreign firm j sets price $\overline{P}^*_{Ft}(j)$ in the foreign market according to

$$\mathbb{E}_t \Sigma_{s=t}^{\infty} \zeta_P^{*s-t} \Lambda^*_{s,t} Y^*_{Ft}(j) P^*_{Fs} \left[\overline{P}^*_{Ft}(j) - \frac{\gamma}{\gamma - 1} \frac{W^*_s}{\alpha A^*_s Z^*_s(i)^* N^*_s(j)^{\alpha-1}} \right] = 0 \tag{61}$$

and lets the price for the home market $P_{Ft}(j)$ in order to equalize net unit revenues across markets:

$$P_{Ft}(j) = \frac{(1+\tau^m_t)}{(1+\varsigma^{x*}_t)(1-\tau^v_t)} P^*_{Ft}(j)\varepsilon_t \tag{62}$$

Using the evolution of firm $i's$ price in (57) in the P_H price index equation (20) and using the law of large numbers we derive a forward-looking Phillips curve

$$\pi_{Ht} = \left[\zeta_P \left(\frac{1-\tau^v_{t-1}}{1-\tau^v_t} \right)^{1-\gamma} + (1-\zeta_P) \left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{63}$$

where domestic inflation, π_{Ht}, depends on future marginal costs through the optimal reset price $\bar{P}_{H,t}$. This expression also reveals that, in the presence of nominal price rigidities ($\zeta_P > 0$), a VAT rate increase translates directly into higher domestic price inflation because of our assumption of full pass through of taxes.

Government Policy. Fiscal policy in the home and in the foreign country is characterized by a vector of fiscal instruments

$$s_t = (\tau^m_t, \varsigma^x_t, \tau^v_t, \varsigma^v_t, \tau^\pi_t, BAT_t, \tau^{m*}_t, \varsigma^{x*}_t) \tag{64}$$

We assume that $s_t \in S$ is a finite state Markov chain and Ω is the associated transition probability matrix, with element $\Omega_{i,j}$ indicating the probability to move from state i to state j. In our experiments below, we consider different configurations of S and Ω to capture alternative scenarios for the evolution of fiscal policy at home and possible retaliatory behavior abroad.

To complete the description of fiscal policy we assume that the government balances its budget in every period:

$$\tau^m_t \varepsilon_t P^*_{F,t} - \varsigma^x_t \varepsilon_t P^*_{H,t} + \frac{\tau^p_t}{1+\tau^p_t} \widetilde{\Pi}^\pi_t + \frac{\tau^v_t}{1+\tau^v_t} P^*_{F,t} - \frac{\varsigma^v_t}{1+\varsigma^v_t} W_t N_t = T_t \tag{65}$$

Monetary policy in the home country follows the interest rate rule

$$R_t = \frac{1}{\beta}\left[\pi_{Ht}\frac{(1-\tau_t^v)}{(1-\tau_{t-1}^v)}\right]^{\varphi_\pi}(\tilde{y}_t)^{\varphi_y}(\varepsilon_t - \bar{\varepsilon}_t)^{\varphi_\varepsilon} \tag{66}$$

where φ_π is the weight on domestic price inflation (π_{Ht}) and φ_y is the weight on the output gap $(\tilde{y}_t)$. The parameter $\varphi_\varepsilon \in \{0, M\}$ governs the sensitivity of the interest rate rule to changes in the nominal exchange rate.[17] When $\varphi_\varepsilon = 0$, the home interest rate responds exclusively to fluctuations in domestic inflation and output gaps. When $\varphi_\varepsilon = M$, the home interest rate rule proactively responds to deviations of the nominal exchange rate from a target exchange rate.

Balance of payment. The only other equation that is affected is the condition determining equilibrium in the balance of payment, that becomes

$$\varepsilon_t B_{Ft} - B_{Ht}^* = \varepsilon_t B_{Ft-1} R_{t-1}^* - B_{Ht-1}^* R_{t-1} + \frac{P_{Ht}^* y_{Ht}^*}{1+\tau_t^{m*}}\varepsilon_t - \frac{(1-\tau_t^v)\, P_{Ft} y_{Ft}}{(1+\tau_t^m)} \tag{67}$$

6 Equivalence Results

We begin our comparison of the effects of IX, BAT, and VP by focusing on the special case of permanent policy changes under producer currency pricing and flexible exchange rates also discussed in the previous section. Under these assumptions, the three policies are equivalent and neutral as the real exchange rate appreciate just enough to completely offset any stimulative effect of these policies on net exports and output.

The appreciation of the real exchange rate, however, originates from different sources, namely an immediate jump in the nominal exchange rate under IX or BAT, and an adjustment in the domestic price level under VP. A direct implication of this observation is that when we consider the case of fixed nominal exchange rates, the policies are not equivalent anymore. In this case IX and BAT act like a fiscal devaluation and stimulate output, while VP remains neutral.

We next turn our attention to the more general case of transitory policy changes. We show that, for IX and BAT, the same qualitative effects of a permanent change with PCP that materialize under a fixed exchange rate regime extend to both fixed and variable exchange

[17] See Benigno *et al.* (2007) for a discussion of interest rate rules that maintain a fixed exchange rate.

rate regimes and arbitrary pricing conventions (e.g. LCP). IX and BAT are always equivalent, stimulate net exports and domestic output, and reduce foreign output. These policies still appreciate the real exchange rate but this effect only partially offsets the boost to output and net exports. VP is not equivalent to IX or BAT, however. With a significant level of nominal price rigidity or under fixed exchange rates, VP is contractionary even in the home country.

6.1 A special case : Equivalence and Neutrality

We start by considering the case in which policies are expected to be permanent and retaliation by foreign economies is ruled out.

Further, we assume that monetary policy targets domestic price inflation and the output gap, while exchange rates are perfectly flexible:

$$R_t = \frac{1}{\beta}\left[\pi_{Ht}\frac{(1-\tau_t^v)}{\left(1-\tau_{t-1}^v\right)}\right]^{\varphi_\pi}(\tilde{y}_t)^{\varphi_y} \tag{68}$$

The interest rate rule in *(68)* implies that monetary policy policy sees through any transitory increase in consumer price inflation due to VAT changes or import tariffs.

In this case we can state the following proposition.

Proposition 2. *Under the assumptions of Proposition 1 and if monetary policy is described by (68), a permanent unexpected IX policy*

$$IX = \{\tau_s^m, \varsigma_s^x\}_{s\geq t} \quad s.t. \quad \tau_s^m = \varsigma_s^x = \delta \tag{69}$$

a permanent unexpected BAT policy

$$BAT = \{\tau_s^\pi\}_{s\geq t} \quad s.t. \quad \tau_s^\pi = \frac{\delta}{1+\delta} \tag{70}$$

and a permanent unexpected VP policy

$$VP = \{\tau_s^v, \varsigma_s^v\}_{s\geq t} \quad s.t. \quad \tau_s^v = \varsigma_s^v = \frac{\delta}{1+\delta} \tag{71}$$

have no effect on the real allocation and induce a real exchange rate appreciation of size $\frac{\delta}{1+\delta}$.

Appendix 1a contains the complete proof of Proposition 1. Here we explain the intuition behind equivalence of IX and VP[18]. As discussed above, under the assumptions of Proposition 1 IX will be neutral on the equilibrium allocation. To understand why VP will also be neutral notice that the two laws of one price

$$\frac{P_{Ht}^{*}}{P_t^{*}}=\frac{(1-\tau^{v})}{(1+\varsigma^{x})}\frac{P_{Ht}}{P_t}\frac{P_t}{\varepsilon_t P_t^{*}}=\frac{(1-\tau^{v})}{(1+\varsigma^{x})}\frac{P_{Ht}}{P_t}\frac{1}{Q_t} \tag{72}$$

$$\frac{P_{Ft}}{P_t}=\frac{(1+\tau^{m})}{(1-\tau^{v})}\frac{P_{Ft}^{*}}{P_t^{*}}Q_t \tag{73}$$

imply that IX and VP induce the same expansion of domestic exports and contraction of foreign exports. This is because the VAT increase acts like an import tax and the VAT deductibility of exports acts as an export subsidy.

The same exact argument developed in Section **??** to explain how the balance of payment equilibrium determines the exchange rate response to IX can be applied directly also to the case of the VP policy, with an important difference. In the case of VP, in fact, the tax changes affect two additional equilibrium conditions. First, the optimality condition of the home firm i requires that a VAT increase is accompanied by a payroll subsidy in order to prevent any distortion in the supply of the home varieties

$$\mathbb{E}_t\Sigma_{s=t}^{\infty}\zeta_P^{s-t}\Lambda_{s,t}Y_{Ht}(i)\,P_{Hs}(1-\tau^{\pi})\left[\overline{P}_{Ht}(i)(1-\tau^{v})-(1-\varsigma^{v})\frac{\gamma}{\gamma-1}\frac{W_s}{\alpha A N_s(i)^{\alpha-1}}\right]=0 \tag{74}$$

Intuitively, the VAT increase reduces the firm's marginal revenue, $\overline{P}_{Ht}(i)(1-\tau^{v})$, for any given price $\overline{P}_{Ht}(i)$ paid by the consumer. Payroll subsidies (ς^{v}) ensure that this reduction in marginal revenues is offset by an equal reduction in marginal costs .

Second, under our assumption that VAT taxes are fully passed through to the consumer, the Phillips curve for domestic price inflation

$$\pi_{Ht}=\left[\zeta_P\left(\frac{(1-\tau_{t-1}^{v})}{(1-\tau_t^{v})}\right)^{1-\gamma}+(1-\zeta_P)\left(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{75}$$

indicates that a VAT increase boosts domestic price inflation π_{Ht}. Given our assumption that monetary policy policy sees through any increase in consumer price inflation due to

[18] As shown in Section 3.2, IX and BAT are always equivalent in our environment. Hence, we decided to simplify notation and focus on the relation between IX and VP here.

VAT changes, neutrality of VP follows by letting all prices under VP increase by $\frac{1}{1-\tau_t^v} = \frac{\delta}{1+\delta}$

$$P_{H,t}^{VP} = \frac{P_{H,t}^{IX}}{1-\tau_t^v}; \quad P_{F,t}^{VP} = \frac{P_{F,t}^{IX}}{1-\tau_t^v}; \quad P_t^{VP} = \frac{P_t^{IX}}{1-\tau_t^v}$$

In other words, under VP the real exchange rate appreciation is achieved through an adjustment in the price level

$$Q_t^{VP} = \frac{1+\delta}{\delta} Q_t = \varepsilon_t \frac{P_t^*}{P_t^{VP}}$$

The different response of inflation and the nominal exchange rate under IX and VP in a flexible exchange rate regime is key to understand the response of the economy when the nominal exchange rate is fixed. In a fixed exchange rate regime, IX (and BAT) will in general stimulate net exports and output as the inability of the nominal exchange rate to appreciate does not allow the real exchange rate to fully offset the net export stimulus of the policy. Indeed, IX (and BAT) in this case will implement the same allocation of a currency devaluation as conjectured by Keynes (1931) and formalized in the fiscal devaluation literature, such as Farhi et al. (2014). VP, in contrast, remains neutral irrespective of the monetary policy regime as the nominal exchange rate is constant even under a flexible exchange rate regime. The proposition below formally states this result. The proof is in the Appendix.

Proposition 2. *If the exchange rate regime is fixed ($\varphi_\varepsilon = M$) and prices are set in producer currency (PCP), a permanent unexpected IX policy*

$$IX = \{\tau_s^m, \varsigma_s^x\}_{s \geq t} \quad s.t. \quad \tau_s^m = \varsigma_s^x = \delta \tag{76}$$

and a permanent unexpected BAT policy

$$BAT = \{\tau_s^\pi\}_{s \geq t} \quad s.t. \quad \tau_s^\pi = \frac{\delta}{1+\delta} \tag{77}$$

have the same allocative effects of a once and for all unexpected currency devaluation of size δ. A *permanent unexpected VP policy of the same size*

$$VP = \{\tau_s^v, \varsigma_s^v\}_{s \geq t} \quad s.t. \quad \tau_s^v = \varsigma_s^v = \frac{\delta}{1+\delta} \tag{78}$$

has no effect on the real allocation.

6.2 The general case

In this section we study to the case in which the fiscal adjustments are perceived to be transitory, perhaps because of threat of retaliation. We show that the equivalence between IX and BAT generalizes to transitory changes and arbitrary price setting conventions (i.e. PCP, LCP). In contrast, with nominal rigidities, relaxing any of the assumptions in Proposition 1 will result in different allocative effects of VP and the other two policies. The proposition below states this result formally:

Proposition 3. *Under full pass-through of taxes, the policies*

$$IX = \{\tau_s^m, \varsigma_s^x\}_{s\geq t} \quad s.t. \quad \tau_s^m = \varsigma_s^x = \delta_s \tag{79}$$

and

$$BAT = \{\tau_s^\pi\}_{s\geq t} \quad s.t. \quad \tau_s^\pi = \frac{\delta_s}{1+\delta_s} \tag{80}$$

implement the same allocation. Generically, the policy

$$VP = \{\tau_s^v, \varsigma_s^v\}_{s\geq t} \quad s.t. \quad \tau_s^v = \varsigma_s^v = \frac{\delta_s}{1+\delta_s} \tag{81}$$

does not implement the same allocation as IX or BAT. The three policies are equivalent only if prices are flexible or when the change is permanent and firms set prices in producer currency (See Proposition 1).

Appendix A.1 presents a formal proof of Proposition 3. The intuition for the equivalence of IX and BAT can be summarized by the observation that the non-deductibility of imports acts like an import tariff whereas the exemption of export sales acts like an export subsidy. Nonetheless, this observation is not sufficient as the IX and BAT policies appear to distort, respectively, the supply and demand of foreign good in the home country. The assumption of full pass through of import taxes ensures that the supply shift under IX is exactly symmetric to the demand shift under BAT, regardless of the specific pricing convention.[19] Therefore, the allocation under BAT will be identical to the allocation under IX with the only difference that import prices will be lower under BAT:

[19] If import tariffs are not fully passed through, as for example in FGI, BAT and IX would not be equivalent under LCP.

$$\frac{P_{Fs}^{BAT}}{(1-\tau_s^{\pi})} = P_{Fs}^{BAT}(1+\tau_t^m) = P_{Fs}^{IX}$$

The intuition for the lack of equivalence between VP and IX (or BAT) in the general case follows the same argument as in Proposition 2 above. Under VP, and given our assumption of full pass through of VAT, the slow response of domestic producers in adjusting (pre-tax) prices leads to an increase in consumer prices of the domestic good at home $(P_{H,t})$. This increase depresses domestic demand of the home variety and limit the competitiveness boost coming from the VAT deductibility of exports.

Finally, Figure 9 shows the same exercise with a VAT cum payroll increase of $\frac{.1}{1+.1}$. Again, we can focus on the domestic market for the home good to understand the differences between IX and VP. Under VP, when domestic firms are unable to freely adjust prices, the VAT is fully passed through to consumers causing a big jump in CPI inflation. This results in a sizeable drop in the home consumption of the domestic good, Y_H, and a smaller boost to net exports associated with higher export prices and smaller relative increase in import prices. When prices are rigid enough, this can cause output to actually contract in response to VP.

7 Conclusion

TBA

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8 Appendix A. Proof of Lemma 1

Lemma 1 *Under balanced trade, a transition to state s^{TW} at time t_1 has the same effects on the equilibrium allocation as a transition to a state s^{NT}. If trade is not balanced two international transfers are needed for the equivalence: a transfer at time t_1 when the economy transits to s^{TW}*

$$T_t = -\frac{\delta}{1+\delta}\left[B_{f,t-1}\frac{R^*_{t-1}}{\pi^*_t}\varepsilon_t - B_{ht-1}\frac{R_{t-1}}{\pi_t}\right]$$

and another symmetric transfer at time t_2 when the economy leaves state s^{TW}

$$T_{t^{NT}} = \delta\left[B_{f,t^{NT}-1}\frac{R^*_{t^{NT}-1}}{\pi^*_{t^{NT}}}\varepsilon_{t^{NT}} - B_{ht^{NT}-1}\frac{R_{t^{NT}-1}}{\pi_{t^{NT}}}\right]$$

Proof. Let $\left\{\Psi\left(s^t\right)\right\}_{s^t\in(S^T)^t,t\geq 0}$ denote an equilibrium allocation with no international transfers and no retaliation, i.e. $T\left(s^t\right)=0\ \forall s^t \in \left(S^T\right)^t$ and $\forall t\geq 0$, $S^T=\left\{s^{NT},s^{IX}\right\}$.

Consider the process with retaliation $\left\{S^R,\Omega^R\right\}$ and define a sequence of function μ_t : $\left(S^R\right)^t \to \left(S^T\right)^t$ as follows: $\forall s^t=(s_1,...,s_t,...)\in\left(S^R\right)^t$, $\mu_t\left(s^t\right)=\tilde{s}^t=(\tilde{s}_1,...,\tilde{s}_t,....)\in\left(S^T\right)^t$ where $\forall i\geq 1$

$$\tilde{s}_i = \begin{cases} s_i & if\ s_i \neq s^{TW} \\ s^{NT} & if\ s_i = s^{TW} \end{cases}$$

that is function μ_t maps all histories in which a trade war occurs into a history in which instead of a trade war we have no taxes.

Consider now an allocation $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t\in(S^R)^t,t\geq 0}$ such that, for each element $\tilde{\varkappa}$ of allocation $\tilde{\Psi}$, other than bond holdings, we have

$$\tilde{\varkappa}\left(s^t\right)=\varkappa\left(\mu_t\left(s^t\right)\right) \qquad \forall s^t\in\left(S^R\right)^t,\quad \forall t\geq 0 \tag{82}$$

where $\varkappa$ is the corresponding element of the equilibrium allocation Ψ without trade wars. That is, all quantities and prices, apart from bond holdings, do not depend on whether retaliation has ever occurred. In fact (82) and the definition of μ readily imply that

$$\tilde{\varkappa}\left(s^t\right)=\tilde{\varkappa}\left(\mu_t\left(s^t\right)\right)$$

since $\mu_t\left(\mu_t\left(s^t\right)\right)=\mu_t\left(s^t\right)$, i.e. if an history has no retaliation μ does not modify it.

Bond holdings satisfy $\forall s^t = (s_1, ..., s_t)$

$$\tilde{B}_f\left(s^t\right)\tilde{\varepsilon}\left(s^t\right) - \tilde{B}_h\left(s^t\right) = \begin{cases} \left[B_f\left(\mu_t\left(s^t\right)\right)\varepsilon\left(\mu\left(s^t\right)\right) - B_h\left(\mu_t\left(s^t\right)\right)\right] & if\ s_t \neq s^{TW} \\ \frac{1}{1+\delta}\left[B_f\left(\mu_t\left(s^t\right)\right)\varepsilon\left(\mu_t\left(s^t\right)\right) - B_h\left(\mu_t\left(s^t\right)\right)\right] & if\ s_t = s^{TW} \end{cases} \tag{83}$$

We want to show that $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t\in\left(S^R\right)^t, t\geq 0}$ is an equilibrium when international transfers satisfy

$$\tilde{T}\left(s^t\right) = \begin{cases} 0 & if\ s_{t-1} \neq s^{TW} \text{ and } s_t \neq s^{TW} \\ -\frac{\delta}{1+\delta}\left[\tilde{B}_{f,t-1}\frac{\tilde{R}^*_{t-1}}{\tilde{\pi}^*_t}\tilde{\varepsilon}_t - \tilde{B}_{ht-1}\frac{\tilde{R}_{t-1}}{\tilde{\pi}_t}\right] & if\ s_{t-1} \neq s^{TW} \text{ and } s_t = s^{TW} \\ \delta\left[\tilde{B}_{f,t-1}\frac{\tilde{R}^*_{t-1}}{\tilde{\pi}^*_t}\tilde{\varepsilon}_t - \tilde{B}_{ht-1}\frac{\tilde{R}_{t-1}}{\tilde{\pi}_t}\right] & if\ s_{t-1} = s^{TW} \text{ and } s_t \neq s^{TW} \end{cases} \tag{84}$$

where we use tilde to denote elements of allocation $\tilde{\Psi}_t$.

It is straightforward to check that if Ψ_t is an equilibrium equation then $\tilde{\Psi}_t$ satisfies all static equations. This follows by construction for any s^t such that $s_t \neq s^{TW}$. When $s_t = s^{TW}$ the only static conditions that need to be checked are the laws of one price. Considering the law of one price for domestic goods at an history s^t such that $s^t = \left(s^{t-1}, s^{TW}\right)$ we see that

$$\frac{\tilde{P}^*_H\left(s^t\right)}{\tilde{P}^*\left(s^t\right)} = \frac{P^*_H\left(\mu\left(s^t\right)\right)}{P^*\left(\mu\left(s^t\right)\right)} = \frac{P_H\left(\mu\left(s^t\right)\right)}{P\left(\mu\left(s^t\right)\right)}\frac{1}{Q\left(\mu\left(s^t\right)\right)} \tag{85}$$

$$= \frac{\tilde{P}_H\left(s^t\right)}{\tilde{P}\left(s^t\right)}\frac{1+\delta}{1+\delta}\frac{1}{\tilde{Q}\left(s^t\right)} \tag{86}$$

where the first and third equality follow from (82) and the second from the fact that Ψ is an equilibrium. An analogous arguemt shows that all static conditions are satisfied.

Consider now the balance of payment equilibrium which we rewrite as follows

$$\tilde{A}_t = \tilde{A}_{t-1}\tilde{r}^a_t + N\tilde{X}_t + \tilde{T}_t$$

where

$$\tilde{A}_{t-1} = \tilde{B}_{f,t-1}\tilde{\varepsilon}_{t-1} - \tilde{B}_{ht-1}$$

$$r^a_t = \frac{\left[\tilde{B}_{f,t-1}\frac{\tilde{R}^*_{t-1}}{\tilde{\pi}^*_t}\tilde{\varepsilon}_t - \tilde{B}_{ht-1}\frac{\tilde{R}_{t-1}}{\tilde{\pi}_t}\right]}{\tilde{A}_{t-1}}$$

Take any history $\tilde{s}^{\infty} = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(S^R\right)^{\infty}$ such that $s_i = s^{TW}$ $\exists i$. Let t_1 and t_2 satisfy $s_{t_1} = s^{TW}$, $s_{t_1 - 1} \neq s^{TW}$,$s_{t_2} \neq s^{TW}$, $s_{t_2-1} = s^{TW}$. At t_1 we have

$$\begin{aligned} \tilde{A}_{t_1} &= \tilde{A}_{t_1-1}\tilde{r}^a_{t_1} + N\tilde{X}_{t_1} + \tilde{T}_{t_1} \\ &= A_{t_1-1}r^a_{t_1} + \frac{NX_{t_1}}{1+\delta} - \frac{\delta}{1+\delta}A_{t_1-1}r^a_{t_1} \\ &= \frac{A_{t_1-1}r^a_{t_1} + NX_{t_1}}{1+\delta} \\ &= \frac{A_{t_1}}{1+\delta} \end{aligned} \tag{87}$$

where, with abuse of notation we let $A_{t_1-1} = A\left(\mu\left(s^{t_1-1}\right)\right)$ and analogously for other variables. Notice that (83) implies $\tilde{A}\left(s^{t_1-1}\right) = A\left(\mu\left(s^{t_1-1}\right)\right)$ given the definition of t_1. Also (82) implies $N\tilde{X}_{t_1} = \frac{NX_{t_1}}{1+\delta}$ so that the second equality follows.

As long as the trade war is in place we have: $\forall s$ $t_1 < s < t_2$

$$\begin{aligned} \tilde{A}_s &= \tilde{A}_{s-1}\tilde{r}^a_s + N\tilde{X}_s \\ &= \frac{A_s}{1+\delta} \end{aligned} \tag{88}$$

And when it ends, at t_2, we have

$$\begin{aligned} \tilde{A}_{t_2} &= \tilde{A}_{t_2-1}\tilde{r}^a_{t_2} + N\tilde{X}_{t_2} + \tilde{T}_{t_2} \\ &= \frac{A_{t_2-1}r^a_{t_2}}{1+\delta} + NX_{t_2} + \frac{\delta}{1+\delta}A_{t_2-1}r^a_{t_2} \\ &= A_{t_2-1}r^a_{t_2} + NX_{t_2} \\ &= A_{t_2} \end{aligned} \tag{89}$$

where we are using again (83) and (82) as in (87).

9 Appendix B. Proof of Proposition 1

We start by giving a broad definition of a unilateral IX policy.

Definition 1. *Let the evolution of trade policy at home and abroad* $s_t = (\tau^m_t, \varsigma^x_t, \tau^{m*}_t, \varsigma^{x*}_t)$ *be determined by the stochastic process* $\{S, \Omega\}$. *A unilateral implementation of IX of size* δ *which happens with state dependent probability* p^{IX} *starting from* $\{S, \Omega\}$, *is described by a new stochastic process* $\left\{\tilde{S}, \tilde{\Omega}\right\}$ *such that* $\tilde{S} = S \cup S^{IX} \cup S^{TW}$

$$S^{IX} = \sigma^{IX}_{\delta}\left([S]\right) \tag{90}$$

$$\sigma_\delta^{IX}\left(\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \tau_t^{x*}\right) = \left(\left(1+\tau_t^m\right)\delta - 1, \left(1+\varsigma_t^x\right)\delta - 1, \tau_t^{m*}, \varsigma_t^{x*}\right)$$

$$S^W = \sigma_\delta^W\left([S]\right) \tag{91}$$

$$\sigma_\delta^{IX}\left(\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \tau_t^{x*}\right) = \left(\left(1+\tau_t^m\right)\delta - 1, \left(1+\varsigma_t^x\right)\delta - 1, \left(1+\tau_t^{m*}\right)\delta - 1, \left(1+\varsigma_t^{x*}\right)\delta - 1\right)$$

and

$$\tilde{\Omega} = \left[\begin{array}{cc} diag\left(1-\pi^{IX}\right)\Omega & diag\left(\pi^{IX}\right)\Omega \\ \Omega^R & \Omega^{IX} \end{array}\right] \tag{92}$$

$$\tilde{\Omega} = \left[\begin{array}{ccc} \left(1-\pi^{IX}\right)\Omega & \pi^{IX}\Omega & 0 \\ \pi\left(1-\rho\right)\Omega & \rho\Omega & \left(1-\pi\right)\left(1-\rho\right)\Omega \\ \left(1-\varphi\right)\Omega & 0 & \varphi\Omega \end{array}\right] \tag{93}$$

where the ordering of states in the matrix $\tilde{\Omega}$ is the obvious one.

When IX is implemented, import tariffs and export subsidies proportionally increase at home by δ, that is the economy transits from a given state $s_t = \left(\tau_t^m, \varsigma_t^x, \tau_t^{m*}, \tau_t^{x*}\right)$ in the original set S to the corresponding element $\sigma^{IX}\left(s_t\right) = \left(\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}\right)$ in S^{IX} that satisfies $\frac{1+\tilde{\tau}_t^m}{1+\tau_t^m} = \frac{1+\tilde{\varsigma}_t^x}{1+\varsigma_t^x} = \delta$ as given by (90). Similarly, S^{TW}, captures retaliation: $\sigma^W\left(s_t\right) = \left(\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tilde{\tau}_t^{m*}, \tilde{\tau}_t^{x*}\right) \in S^{TW}$ satisfies $\frac{1+\tilde{\tau}_t^m}{1+\tau_t^m} = \frac{1+\tilde{\varsigma}_t^x}{1+\varsigma_t^x} = \frac{1+\tilde{\tau}_t^{m*}}{1+\tau_t^m} = \frac{1+\tilde{\varsigma}_t^{x*}}{1+\varsigma_t^{x*}} = \delta$. The definition above encompasses the possibility that the policy change is anticipated, with π^{IX} indicating the probability of implementing IX.

Proposition 1. *Let $x_0 = \left(B_{H-1}^* R_{-1}, B_{F-1} R_{-1}^*, P_{H-1}\left(i\right), P_{F-1}^*\left(i\right), s^0\right)$ be the initial condition. In an economy with flexible exchange rates, a unilateral implementation of IX of size δ has no allocative effect if*

1. *It is unanticipated, permanent, and there is no probability of retaliation;*

2. *Foreign holdings of home currency are always zero: $B_{H-1}^* = 0 = \bar{B}$* and $\chi = \infty$;

3. *Export prices are set in producer currency (PCP) or prices are flexible*

Proof.

Condition 1 implies that $\pi^{IX} = 0$ and $\rho = 1$. In this case we can focus on a reduced state space given by $\tilde{S} = S \cup S^{IX}$ and

$$\tilde{\Omega} = \left[\begin{array}{cc} \Omega & 0 \\ 0 & \Omega \end{array}\right] \tag{94}$$

Let $\left\{\Psi\left(s^t\right)\right\}_{s^t\in\left(S^T\right)^t, t\geq 0}$ denote an equilibrium allocation before the IX implementation, i.e. when s_t is governed $\{S,\Omega\}$.

Consider the process with unilateral IX $\left\{\tilde{S},\tilde{\Omega}\right\}$ and define a sequence of functions $\mu_t : \left(\tilde{S}\right)^t \to (S)^t$ as follows: $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t, ...) \in \left(\tilde{S}\right)^t$, $\mu_t\left(\tilde{s}^t\right) = s^t = (s_1, ..., s_t, ...) \in (S)^t$ where $\forall i \geq 1$

$$s_i = \begin{cases} \tilde{s}_i & if\ \tilde{s}_i \in S \\ \left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_i) & if\ \tilde{s}_i \in S^{IX} \end{cases}$$

that is function μ_t maps all histories in which IX is implemented into a history in which IX is not implemented.

Consider now an allocation $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t\in\left(\tilde{S}\right)^t, t\geq 0}$ with an unanticipated permanent IX such that, for each element $\tilde{\varkappa}$ of allocation $\tilde{\Psi}$, other than the nominal exchange rate, we have

$$\tilde{\varkappa}\left(\tilde{s}^t\right) = \varkappa\left(\mu_t\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(\tilde{S}\right)^t, \ \ \forall t \geq 0 \tag{95}$$

where $\varkappa$ is the corresponding element of the equilibrium allocation Ψ without IX. The nominal exhange rate satisfie $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t)$

$$\tilde{\varepsilon}\left(\tilde{s}^t\right) = \begin{cases} \varepsilon\left(\mu_t\left(\tilde{s}^t\right)\right) & if\ s_t \in S \\ \frac{\varepsilon\left(\mu_t\left(\tilde{s}^t\right)\right)}{1+\delta} & if\ s_t \in S^{IX} \end{cases} \tag{96}$$

We want to show that $\left\{\tilde{\Psi}\left(s^t\right)\right\}_{s^t\in\left(S^R\right)^t, t\geq 0}$ is an equilibrium.

It is straightforward to check that if Ψ_t is an equilibrium equation then $\tilde{\Psi}_t$ satisfies all static equations. This follows by construction for any s^t such that $s_t \in S$. When $s_t \in S^{IX}$ the only static conditions that need to be checked are the laws of one price. Considering the law of one price for domestic goods at an history $\tilde{s}^t$ such that $\tilde{s}_t = (\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S^{IX}$ and letting $\left(\sigma_\delta^{IX}\right)^{-1}(\tilde{s}_t) = (\tau_t^m, \tau_t^x, \tau_t^{m*}, \tau_t^{x*}) \in S$ we see that

$$\frac{\tilde{P}_H^*\left(\tilde{s}^t\right)}{\tilde{P}^*\left(\tilde{s}^t\right)} = \frac{P_H^*\left(\mu\left(\tilde{s}^t\right)\right)}{P^*\left(\mu\left(\tilde{s}^t\right)\right)} = \frac{P_H\left(\mu\left(\tilde{s}^t\right)\right)}{P\left(\mu\left(\tilde{s}^t\right)\right)}\frac{1+\tau_t^{m*}}{1+\sigma_t^x}\frac{1}{Q\left(\mu\left(\tilde{s}^t\right)\right)} \tag{97}$$

$$= \frac{\tilde{P}_H\left(\tilde{s}^t\right)}{\tilde{P}\left(\tilde{s}^t\right)}\frac{1+\tau_t^{m*}}{\left(1+\sigma_t^x\right)}\frac{1}{\tilde{Q}\left(\tilde{s}^t\right)(1+\delta)} \tag{98}$$

where the first and third equalities follow from (95) and (96), which together imply $Q\left(\mu\left(\tilde{s}_t\right)\right) = \tilde{Q}\left(\tilde{s}_t\right)(1+\delta)$. And the second equality follow from the fact that Ψ is an equilibrium. An analogous arguemt for the law of one price abroad shows that all static conditions are satisfied.

Consider now the balance of payment equilibrium which we rewrite in real (foreign good) terms as

$$\frac{\tilde{B}_{Ft}}{\tilde{P}_t^*} = \frac{\tilde{B}_{Ft-1}}{\tilde{P}_{t-1}^*}\frac{\tilde{R}_{t-1}^*}{\tilde{\pi}_t^*} + \frac{\tilde{P}_{Ht}^*}{\tilde{P}_t^*}\frac{\tilde{y}_{Ht}^*}{1+\tau_t^{m*}} - \frac{\tilde{P}_{Ft}}{\tilde{P}_t}\frac{\tilde{y}_{Ft}}{\left(1+\tilde{\tau}_t^m\right)\tilde{Q}_t}$$

clearly this equation is satisfied when $s_t \in S$. When $\tilde{s}_t = \left(\tilde{\tau}_t^m, \tilde{\varsigma}_t^x, \tau_t^{m*}, \tau_t^{x*}\right) \in S^{IX}$ we have

$$\begin{aligned}
\frac{\tilde{B}_{Ft}}{\tilde{P}_t^*} &= \frac{B_{Ft}}{P_t^*} = \frac{B_{Ft-1}}{P_{t-1}^*}\frac{R_{t-1}^*}{\pi_t^*} + \frac{P_{Ht}^*}{P_t^*}\frac{y_{Ht}^*}{1+\tau_t^{m*}} - \frac{P_{Ft}}{P_t}\frac{y_{Ft}}{\left(1+\tau_t^m\right)Q_t} \\
&= \frac{\tilde{B}_{Ft-1}}{\tilde{P}_{t-1}^*}\frac{\tilde{R}_{t-1}^*}{\tilde{\pi}_t^*} + \frac{\tilde{P}_{Ht}^*}{\tilde{P}_t^*}\frac{\tilde{y}_{Ht}^*}{1+\tau_t^{m*}} - \frac{\tilde{P}_{Ft}}{\tilde{P}_t}\frac{\tilde{y}_{Ft}}{\left(1+\tau_t^m\right)\left(1+\delta\right)\frac{Q_t}{1+\delta}} \\
&= \frac{\tilde{B}_{Ft-1}}{\tilde{P}_{t-1}^*}\frac{\tilde{R}_{t-1}^*}{\tilde{\pi}_t^*} + \frac{\tilde{P}_{Ht}^*}{\tilde{P}_t^*}\frac{\tilde{y}_{Ht}^*}{1+\tau_t^{m*}} - \frac{\tilde{P}_{Ft}}{\tilde{P}_t}\frac{\tilde{y}_{Ft}}{\left(1+\tilde{\tau}_t^m\right)\tilde{Q}_t}
\end{aligned}$$

where we abuse notation by denoting $\tilde{B}_{Ft} = \tilde{B}_F\left(\tilde{s}^t\right)$ and $B_{Ft} = B\left(\mu_t\left(\tilde{s}^t\right)\right)$ and analogously for all other variables. The first and third equality follow from (95) the second from the fact that Ψ is an equilibrium and the last one from the fact that (95) and (96) imply $Q\left(\mu\left(\tilde{s}_t\right)\right) = \tilde{Q}\left(\tilde{s}_t\right)\left(1+\delta\right)$.

Inspecting all of the other dynamic equations we observe that since the allocation is unchanged, no taxes enter any of those equations and the exchange rate only enters as a ratio, all equations will be satisfied by $\tilde{\Psi}_t$ since they are satisfied by Ψ_t.

We start by giving a definition of a permanent unexpected implemnation of IX, BAT and VP.

Definition 2. *Let the evolution of trade policy at home and abroad*

$$s_t = \left(\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, BAT_t, \tau_t^{m*}, \varsigma_t^{x*}\right)$$

be determined by the stochastic process $\{S, \Omega\}$ *that satisfies* $BAT = 0\ \forall s \in S$. *A unilateral implementation of IX of size* δ *is described by a new stochastic process* $\left\{\tilde{S}, \tilde{\Omega}\right\}$ *such that* $\tilde{S} = S \cup S^{IX}$

$$S^{IX} = \sigma_\delta^{IX}\left([S]\right) \tag{99}$$

$$\sigma_\delta^{IX}\left(\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, BAT_t, \tau_t^{m*}, \varsigma_t^{x*}\right) = \left(\left(1+\tau_t^m\right)\delta - 1, \left(1+\varsigma_t^x\right)\delta - 1, \tau_t^v, \varsigma_t^v, \tau_t^\pi, BAT_t, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}\right)$$

$$\tilde{\Omega} = \begin{bmatrix} \left(1-\pi^{IX}\right)\Omega & \pi^{IX}\Omega \\ \left(1-\rho\right)\Omega & \rho\Omega \end{bmatrix} \tag{100}$$

where the ordering of states in the matrix $\tilde{\Omega}$ is the obvious one.

An *Implementation of BAT and an implementation VP of size* $\frac{\delta}{1+\delta}$ *are described by stochastic processes* $\left\{\tilde{S}^{BAT},\tilde{\Omega}\right\}$ *and* $\left\{\tilde{S}^{VP},\tilde{\Omega}\right\}$ *respectively such that* $\tilde{S}^{BAT}=S\cup S^{BAT}$ *and* $\tilde{S}^{VP}=S\cup S^{VP}$ where

$$S^{BAT}=\sigma_{\delta}^{BAT}\left([S]\right) \tag{101}$$

$$\sigma_{\delta}^{IX}\left(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,0,\tau_t^{m*},\varsigma_t^{x*}\right)=\left(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,1,\tau_t^{m*},\varsigma_t^{x*}\tau_t^{m*},\varsigma_t^{x*}\right)$$

$$S^{VP}=\sigma_{\delta}^{VP}\left([S]\right) \tag{102}$$

$$\sigma_{\delta}^{IX}\left(\tau_t^m,\varsigma_t^x,\tau_t^v,\varsigma_t^v,\tau_t^\pi,0,\tau_t^{m*},\varsigma_t^{x*}\right)=\left(\tau_t^m,\varsigma_t^x,\left(1+\tau_t^v\right)\frac{\delta}{1+\delta}-1,\left(1+\varsigma_t^v\right)\frac{\delta}{1+\delta}-1,\tau_t^\pi,1,\tau_t^{m*},\varsigma_t^{x*}\tau_t^{m*},\varsigma_t^{x*}\right)$$

Proposition 2. *If the exchange rate regime is flexible ($\varphi_\varepsilon=0$), monetary policy is described by (68) and prices are set in producer currency (PCP), the following are equivalent:*

1. *A permanent unexpected IX policy of size δ*
2. *A permanent unexpected BAT policy when corporate taxes are $\bar{\tau}=\frac{\delta}{1+\delta}$*
3. *A permanent unexpected VP policy of size $\frac{\delta}{1+\delta}$*

These three policies have no effect on the real allocation and induce a real exchange rate appreciation of size $\frac{\delta}{1+\delta}$.

Proof. In the case of a permanent unexpected IX policy of size δ the transition matrix becomes

$$\tilde{\Omega}=\begin{bmatrix}\Omega & 0\\ 0 & \Omega\end{bmatrix} \tag{103}$$

Let $\left\{\tilde{\Psi}\left(\tilde{s}^t\right)\right\}_{\tilde{s}^t\in\left(\tilde{S}\right)^t,t\geq 0}$ denote an equilibrium allocation with a permanent IX implementation of size δ, i.e. when s_t is governed $\left\{\tilde{S},\tilde{\Omega}\right\}$.

Consider the processes with BAT and VP$\left\{\tilde{S}^{BAT},\tilde{\Omega}\right\}$ and $\left\{\tilde{S}^{VP},\tilde{\Omega}\right\}$ and define sequence of functions $\mu_t^{BAT}:\left(\tilde{S}^{BAT}\right)^t\rightarrow\left(\tilde{S}\right)^t$ and $\mu_t^{VP}:\left(\tilde{S}^{VP}\right)^t\rightarrow\left(\tilde{S}\right)^t$ as follows: $\forall s^t=(s_1,...,s_t)\in\left(\tilde{S}^{BAT}\right)^t$, $\mu_t^{BAT}\left(s^t\right)=\tilde{s}^t=\left(\tilde{s}_1,...,\tilde{s}_t\right)\in\left(\tilde{S}\right)^t$ where $\forall i\geq 1$

$$\tilde{s}_i=\begin{cases} s_i & if\ s_i\in S\\ \sigma_{\delta}^{IX}\left(\left(\sigma_{\delta}^{BAT}\right)^{-1}\left(\tilde{s}_i\right)\right) & if\ s_i\in S^{BAT}\end{cases}$$

and $\forall s^t = (s_1, ..., s_t) \in \left(\tilde{S}^{VP}\right)^t$, $\mu_t^{BAT}\left(s^t\right) = \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(\tilde{S}\right)^t$ where $\forall i \geq 1$

$$\tilde{s}_i = \begin{cases} s_i & if\ \ s_i \in S \\ \sigma_\delta^{IX}\left(\left(\sigma_\delta^{VP}\right)^{-1}(\tilde{s}_i)\right) & if\ \ s_i \in S^{VP} \end{cases}$$

that is functions μ_t^{BAT} and μ_t^{VP} maps all histories in which BAT and VP are implemented in histories in which IX is implemented instead.

Consider now an allocation $\left\{\tilde{\Psi}^{BAT}\left(s^t\right)\right\}_{s^t \in \left(\tilde{S}^{BAT}\right)^t, t \geq 0}$ with an unanticipated permanent BAT implementation such that, for each element $\tilde{\varkappa}^{BAT}$ of allocation $\tilde{\Psi}^{BAT}$, other than import prices $\tilde{P}_{Ft}^{BAT}$, we have

$$\tilde{\varkappa}^{BAT}\left(\tilde{s}^t\right) = \tilde{\varkappa}\left(\mu_t^{BAT}\left(\tilde{s}^t\right)\right) \qquad \forall \tilde{s}^t \in \left(\tilde{S}^{BAT}\right)^t, \ \ \forall t \geq 0 \tag{104}$$

where $\tilde{\varkappa}$ is the corresponding element of the equilibrium allocation $\tilde{\Psi}$ with IX. Import prices satisfy $\forall \tilde{s}^t = (\tilde{s}_1, ..., \tilde{s}_t) \in \left(S^{BAT}\right)^t$

$$\tilde{P}_F^{BAT}\left(\tilde{s}^t\right) = \begin{cases} \tilde{P}_F\left(\mu_t^{BAT}\left(\tilde{s}^t\right)\right) & if\ \ \tilde{s}_t \in S \\ \left(1 - \tau_t^\pi\right)\tilde{P}_F\left(\mu_t^{BAT}\left(\tilde{s}^t\right)\right) & if\ \ \tilde{s}_t \in S^{BAT} \end{cases}$$

We want to show that $\left\{\tilde{\Psi}^{BAT}\left(s^t\right)\right\}_{s^t \in \left(S^R\right)^t, t \geq 0}$ is an equilibrium.

The static condition that are affected by BAT and IX are the two laws of one price, retailers optimal demand of imports and the price index, equations (54) and (55). $\forall \tilde{s}^t = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in \left(S^{BAT}\right)^t$

$$\begin{aligned} \tilde{y}_{Ft}^{BAT} &= \tilde{y}_{Ft} = (1 - \omega)\left[\frac{\tilde{P}_{Ft}}{\tilde{P}_t}\right]^{-\theta}\tilde{C}_t \\ &= (1 - \omega)\left[\frac{\tilde{P}_{Ft}^{BAT}}{\tilde{P}_t^{BAT}}\frac{1}{1 - \tau_t^\pi}\right]^{-\theta}\tilde{C}_t^{BAT} \end{aligned} \tag{105}$$

$$\tilde{P}_t^{BAT} = \tilde{P}_t = \left[\omega\tilde{P}_{Ht}^{1-\theta} + (1 - \omega)\left(\tilde{P}_{Ft}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{106}$$

$$= \left[\omega\left(\tilde{P}_{Ht}^{BAT}\right)^{1-\theta} + (1 - \omega)\left(\frac{\tilde{P}_{Ft}^{BAT}}{1 - \tau_t^\pi BAT_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{107}$$

where we abuse notation to let $\tilde{y}_{Ft}^{BAT} = \tilde{y}_F^{BAT}\left(\tilde{s}^t\right)$ and $\tilde{y}_{Ft} = \tilde{y}_F\left(\mu_t^{BAT}\left(\tilde{s}^t\right)\right)$ and analogously for all other variables.

Turning to the laws of one price at home and aborad, equations (58) and (62), we have $\forall \tilde{s}^t$ such tath $\tilde{s}_t = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in S^{BAT}$ and $\mu_t^{BAT}\left(\tilde{s}^t\right) = s^t$ such that $s_t = ((1+\tau_t^m)(1+\delta) - 1, (1+\varsigma_t^x)(1+\delta) - 1, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*})$

$$\begin{aligned} \tilde{P}_{Ht}^{*BAT} &= \tilde{P}_{Ht}^* = \frac{(1+\tau_t^{m*})(1-\tau_t^v)}{(1+\varsigma_t^x)(1+\delta)} \frac{\tilde{P}_{Ht}(i)}{\tilde{\varepsilon}_t} \\ &= \frac{1}{(1-\tau_t^\pi)(1+\delta)} \frac{(1+\tau_t^{m*})(1-\tau_t^v)(1-\tau_t^\pi)}{(1+\varsigma_t^x)} \frac{\tilde{P}_{Ht}^{BAT}(i)}{\tilde{\varepsilon}_t^{BAT}} \end{aligned} \tag{108}$$

$$\tilde{P}_{Ft}^{BAT} = \tilde{P}_{Ft}(1-\tau_t^\pi) = \frac{(1+\tau_t^m)(1+\delta)(1-\tau_t^\pi)}{(1+\varsigma_t^{x*})(1-\tau_t^v)} \tilde{P}_{Ft}^* \varepsilon_t \tag{109}$$

when

$$(1-\tau_t^\pi)(1+\delta) = 1 \tag{110}$$

equations (108) and (109) imply that the two laws of one price (58) and (62) are satisfied. Under our assumption that $\tau_t^\pi = \frac{\delta}{1+\delta}$ equation (110) is satisfied.

Consider now the balance of payment equilibrium which we rewrite in real terms in the no IX and no BAT case as

$$\frac{B_{Ft}}{P_t^*} Q_t - \frac{B_{Ht}}{P_t} = \frac{B_{Ft-1}}{P_{t-1}^*} \frac{R_{t-1}^*}{\pi_t^*} Q_t - \frac{B_{Ht-1}}{\tilde{P}_{t-1}} \frac{R_{t-1}}{\pi_t} + \frac{P_{Ht}^*}{P_t^*} \frac{y_{Ht}^*}{1+\tau_t^{m*}} Q_t - \frac{P_{Ft}}{P_t} \frac{y_{Ft}}{(1+\tau_t^m)}$$

and take $\tilde{s}^t$ such that $\tilde{s}_t = (\tau_t^m, \varsigma_t^x, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 1, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}) \in S^{BAT}$ and $\mu_t^{BAT}\left(\tilde{s}^t\right) = s^t$ such that $s_t = ((1+\tau_t^m)(1+\delta) - 1, (1+\varsigma_t^x)(1+\delta) - 1, \tau_t^v, \varsigma_t^v, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*})$

$$\begin{aligned} \frac{\tilde{B}_{Ft}^{BAT}}{\tilde{P}_t^{BAT*}} \tilde{Q}_t^{BAT} - \frac{\tilde{B}_{Ht}^{*BAT}}{\tilde{P}_t^{BAT}} &= \frac{\tilde{B}_{Ft}}{\tilde{P}_t^*} \tilde{Q}_t - \frac{\tilde{B}_{Ht}}{\tilde{P}_t} &(111) \\ &= \frac{\tilde{B}_{Ft-1}}{\tilde{P}_{t-1}^*} \frac{\tilde{R}_{t-1}^*}{\tilde{\pi}_t^*} \tilde{Q}_t - \frac{\tilde{B}_{Ht-1}}{\tilde{P}_{t-1}} \frac{\tilde{R}_{t-1}}{\tilde{\pi}_t} + \frac{\tilde{P}_{Ht}^*}{\tilde{P}_t^*} \frac{\tilde{y}_{Ht}^*}{1+\tau_t^{m*}} \tilde{Q}_t - \frac{\tilde{P}_{Ft}}{\tilde{P}_t} \frac{\tilde{y}_{Ft}}{(1+\tau_t^m)(1+\delta)} &(112) \\ &= \frac{\tilde{B}_{Ft-1}^{BAT}}{\tilde{P}_{t-1}^{BAT*}} \frac{\tilde{R}_{t-1}^{BAT*}}{\tilde{\pi}_t^{BAT*}} \tilde{Q}_t^{BAT} - \frac{\tilde{B}_{Ht-1}^{BAT}}{\tilde{P}_{t-1}^{BAT}} \frac{\tilde{R}_{t-1}^{BAT}}{\tilde{\pi}_t^{BAT}} + \frac{\tilde{P}_{Ht}^{BAT*}}{\tilde{P}_t^{BAT*}} \frac{\tilde{y}_{Ht}^{BAT*}}{1+\tau_t^{m*}} \tilde{Q}_t^{BAT} - \frac{\tilde{P}_{Ft}^{BAT}}{\tilde{P}_t} \frac{\tilde{y}_{Ft}}{(1+\tau_t^m)} &(113) \end{aligned}$$

where the last equality uses that as long as $\tau_t^\pi = \frac{\delta}{1+\delta}$ we have $(1+\delta)(1-\tau_t^\pi) = 1$.

No other equilibrium equation is affected by tariffs, export subsidies, BAT or import prices so that $\left\{\tilde{\Psi}^{BAT}\left(s^t\right)\right\}_{s^t \in (S^R)^t, t \geq 0}$ is an equilibrium.

Let's now turn to equivalence with VP.

Consider an allocation $\left\{\tilde{\Psi}^{VP}\left(s^t\right)\right\}_{s^t\in\left(\tilde{S}^{VP}\right)^t,t\geq 0}$ with an unanticipated permanent VP implementation such that, for each element $\tilde{\varkappa}^{VP}$ of allocation $\tilde{\Psi}^{VP}$, other than domestic prices $\left(\tilde{P}^{VP}_{Ht},\tilde{P}^{VP}_{Ft},\tilde{P}^{VP}_{t}\right)$ and the exchange rate $\tilde{\varepsilon}^{VP}_t$, we have

$$\tilde{\varkappa}^{VP}\left(\tilde{s}^t\right)=\tilde{\varkappa}\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right) \qquad \forall\tilde{s}^t\in\left(\tilde{S}^{VP}\right)^t,\quad \forall t\geq 0 \tag{114}$$

where $\tilde{\varkappa}$ is the corresponding element of the equilibrium allocation $\tilde{\Psi}$ with IX. Prices satisfy $\forall\tilde{s}^t=(\tilde{s}_1,....,\tilde{s}_t)\in\left(S^{VP}\right)^t$ and for each domestic firm i

$$\frac{\tilde{P}^{VP}_H(i)\left(\tilde{s}^t\right)}{\tilde{P}_H(i)\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right)}=\frac{\tilde{P}^{VP}_H\left(\tilde{s}^t\right)}{\tilde{P}_H\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right)}=\begin{cases}1 & if\ \tilde{s}_t\in S\\ (1+\delta) & if\ \tilde{s}_t\in S^{VP}\end{cases}$$

$$\frac{\tilde{P}^{VP}_F(i)\left(\tilde{s}^t\right)}{\tilde{P}_F(i)\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right)}=\frac{\tilde{P}^{VP}_F\left(\tilde{s}^t\right)}{\tilde{P}_F\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right)}=\begin{cases}1 & if\ \tilde{s}_t\in S\\ (1+\delta) & if\ \tilde{s}_t\in S^{VP}\end{cases}$$

$$\frac{\tilde{P}^{VP}\left(\tilde{s}^t\right)}{\tilde{P}\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right)}=\begin{cases}1 & if\ \tilde{s}_t\in S\\ (1+\delta) & if\ \tilde{s}_t\in S^{VP}\end{cases}$$

and the exchange rate

$$\tilde{\varepsilon}^{VP}\left(\tilde{s}^t\right)=\begin{cases}\tilde{\varepsilon}\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right) & if\ \tilde{s}_t\in S\\ (1+\delta)\,\tilde{\varepsilon}\left(\mu^{VP}_t\left(\tilde{s}^t\right)\right) & if\ \tilde{s}_t\in S^{VP}\end{cases}$$

We want to show that $\left\{\tilde{\Psi}^{VP}\left(s^t\right)\right\}_{s^t\in\left(S^R\right)^t,t\geq 0}$ is an equilibrium.

The two laws of one price are again straightforward: $\forall\tilde{s}^t$ such that

$$\tilde{s}_t=\left(\tau^m_t,\varsigma^x_t,1-\frac{(1-\tau^v_t)}{(1+\delta)},1-\frac{(1-\tau^v_t)}{(1+\delta)},\tau^\pi_t,0,\tau^{m*}_t,\varsigma^{x*}_t\tau^{m*}_t,\varsigma^{x*}_t\right)\in S^{VP}$$

and $\mu^{VP}_t\left(\tilde{s}^t\right)=s^t$ such that $s_t=\left((1+\tau^m_t)(1+\delta)-1,(1+\varsigma^x_t)(1+\delta)-1,\tau^v_t,\varsigma^v_t,\tau^\pi_t,0,\tau^{m*}_t,\varsigma^{x*}_t\tau^{m*}_t,\varsigma^{x*}_t\right)$ we have

$$\begin{aligned}\tilde{P}^{*VP}_{Ht} &= \tilde{P}^*_{Ht}=\frac{(1+\tau^{m*}_t)(1-\tau^v_t)}{(1+\varsigma^x_t)(1+\delta)}\frac{\tilde{P}_{Ht}(i)}{\tilde{\varepsilon}_t}\\ &= \frac{(1+\tau^{m*}_t)(1-\tau^v_t)}{(1+\varsigma^x_t)(1+\delta)}\frac{\tilde{P}^{VP}_{Ht}(i)}{\tilde{\varepsilon}^{VP}_t}\end{aligned} \tag{115}$$

$$\begin{aligned}\tilde{P}^{VP}_{Ft} &= \tilde{P}_{Ft}(1+\delta)=(1+\delta)\frac{(1+\tau^m_t)(1+\delta)}{(1+\varsigma^{x*}_t)(1-\tau^v_t)}\tilde{P}^*_{Ft}\tilde{\varepsilon}_t\\ &= \frac{(1+\tau^m_t)(1+\delta)}{(1+\varsigma^{x*}_t)(1-\tau^v_t)}\tilde{P}^*_{Ft}\tilde{\varepsilon}^{VP}_t\end{aligned}$$

The balance of payment equilibrium condition is also satisfied since

$$\begin{aligned}
\frac{\tilde{B}_{Ft}^{VP}}{\tilde{P}_t^{VP*}}\tilde{Q}_t^{VP} - \frac{\tilde{B}_{Ht}^{*VP}}{\tilde{P}_t^{VP}} &= \frac{\tilde{B}_{Ft}}{\tilde{P}_t^*}\tilde{Q}_t - \frac{\tilde{B}_{Ht}^*}{\tilde{P}_t} && (116)\\
&= \frac{\tilde{B}_{Ft-1}}{\tilde{P}_{t-1}^*}\frac{\tilde{R}_{t-1}^*}{\tilde{\pi}_t^*}\tilde{Q}_t - \frac{\tilde{B}_{Ht-1}}{\tilde{P}_{t-1}}\frac{\tilde{R}_{t-1}}{\tilde{\pi}_t} + \frac{\tilde{P}_{Ht}^*}{\tilde{P}_t^*}\frac{\tilde{y}_{Ht}^*}{1+\tau_t^{m*}}\tilde{Q}_t - \frac{\tilde{P}_{Ft}}{\tilde{P}_t}\frac{(1-\tau_t^v)\,\tilde{y}_{Ft}}{(1+\tau_t^m)\,(1+\delta)} && (117)\\
&= \frac{\tilde{B}_{Ft-1}^{VP}}{\tilde{P}_{t-1}^{VP*}}\frac{\tilde{R}_{t-1}^{VP*}}{\tilde{\pi}_t^{VP*}}\tilde{Q}_t^{VP} - \frac{\tilde{B}_{Ht-1}}{\tilde{P}_{t-1}}\frac{\tilde{R}_{t-1}}{\tilde{\pi}_t} + \frac{\tilde{P}_{Ht}^{VP*}}{\tilde{P}_t^{VP*}}\frac{\tilde{y}_{Ht}^{VP*}}{1+\tau_t^{m*}}\tilde{Q}_t^{VP} - \frac{\tilde{P}_{Ft}^{VP}}{\tilde{P}_t^{VP}}\frac{(1-\tau_t^v)\,\tilde{y}_{Ft}^{VP}}{(1+\tau_t^m)\,(1+\delta)}
\end{aligned}$$

Now consider the optimality condition for the price of the domestic good at home at any $\tilde{s}^t$ such that

$$\tilde{s}_t = \left(\tau_t^m, \varsigma_t^x, 1-\frac{(1-\tau_t^v)}{(1+\delta)}, 1-\frac{(1-\tau_t^v)}{(1+\delta)}, \tau_t^\pi, 0, \tau_t^{m*}, \varsigma_t^{x*}\tau_t^{m*}, \varsigma_t^{x*}\right) \in S^{VP}$$

we have that

$$\tilde{\mathbb{E}}_t^{VP}\Sigma_{s=t}^{\infty}\zeta_P^{s-t}\tilde{\Lambda}_{s,t}^{VP}\tilde{Y}_{Ht}^{VP}(i)\,\tilde{P}_{Hs}^{VP}(1-\tau^\pi)\left[\overline{\tilde{P}}_{Ht}^{VP}(i)\frac{(1-\tau_s^v)}{1+\delta} - \frac{(1-\varsigma_s^v)}{1+\delta}\frac{\gamma}{\gamma-1}\frac{\tilde{W}_s^{VP}}{\alpha A\tilde{N}_s^{VP}(i)^{\alpha-1}}\right] \quad (118)$$

$$\tilde{\mathbb{E}}_t\Sigma_{s=t}^{\infty}\zeta_P^{s-t}\tilde{\Lambda}_{s,t}\tilde{Y}_{Ht}(i)\,\tilde{P}_{Hs}(1+\delta)(1-\tau^\pi)\left[\overline{\tilde{P}}_{Ht}(i)(1-\tau_s^v) - (1-\varsigma_s^v)\frac{\gamma}{\gamma-1}\frac{\tilde{W}_s}{\alpha A\tilde{N}_s(i)^{\alpha-1}}\right] \quad (119)$$

Finally, consider domestic good inflation

$$\pi_{Ht} = \left[\zeta_P\left(\frac{(1-\tau_{t-1}^v)}{(1-\tau_t^v)}\right)^{1-\gamma} + (1-\zeta_P)\left(\frac{\bar{P}_{H,t}}{P_{H,t-1}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \quad (120)$$

When the policy is implemented, *i.e.* $\tilde{s}^t$ s.t. $s_t \in S^{VP}$ and $s_{t-1} \in S$, π_{Ht} also jumps by $(1+\delta)$:

$$\begin{aligned}
\tilde{\pi}_{Ht}^{VP} &= \left[\zeta_P\left(\frac{(1-\tau_{t-1}^v)}{(1-\tau_t^v)}(1+\delta)\right)^{1-\gamma} + (1-\zeta_P)\left(\frac{\overline{\tilde{P}}_{Ht}^{VP}}{\tilde{P}_{H,t-1}^{VP}}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\\
&= \left[\zeta_P\left(\frac{(1-\tau_{t-1}^v)}{(1-\tau_t^v)}(1+\delta)\right)^{1-\gamma} + (1-\zeta_P)\left(\frac{\overline{\tilde{P}}_{Ht}}{\tilde{P}_{H,t-1}^{VP}}(1+\delta)\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\\
&= \tilde{\pi}_{Ht}(1+\delta)
\end{aligned}$$

however our assumption that monetary policy sees trhough transient changes in inflation due to taxes implies that the nominal rates are unaffected:

$$\begin{aligned}\tilde{R}_t^{VP} &= \frac{1}{\beta}\left[\tilde{\pi}_{Ht}^{VP}\frac{(1-\tau_t^v)}{\left(1-\tau_{t-1}^v\right)}\frac{1}{1+\delta}\right]^{\varphi_\pi}\left(\tilde{y}_t^{VP}\right)^{\varphi_y} \\ &= \frac{1}{\beta}\left[\tilde{\pi}_{Ht}\frac{(1-\tau_t^v)}{\left(1-\tau_{t-1}^v\right)}\right]^{\varphi_\pi}\left(\tilde{y}_t^{VP}\right)^{\varphi_y} \\ &= \tilde{R}_t\end{aligned}$$

.

Figure 1: Exchange Rate and Trade Balance

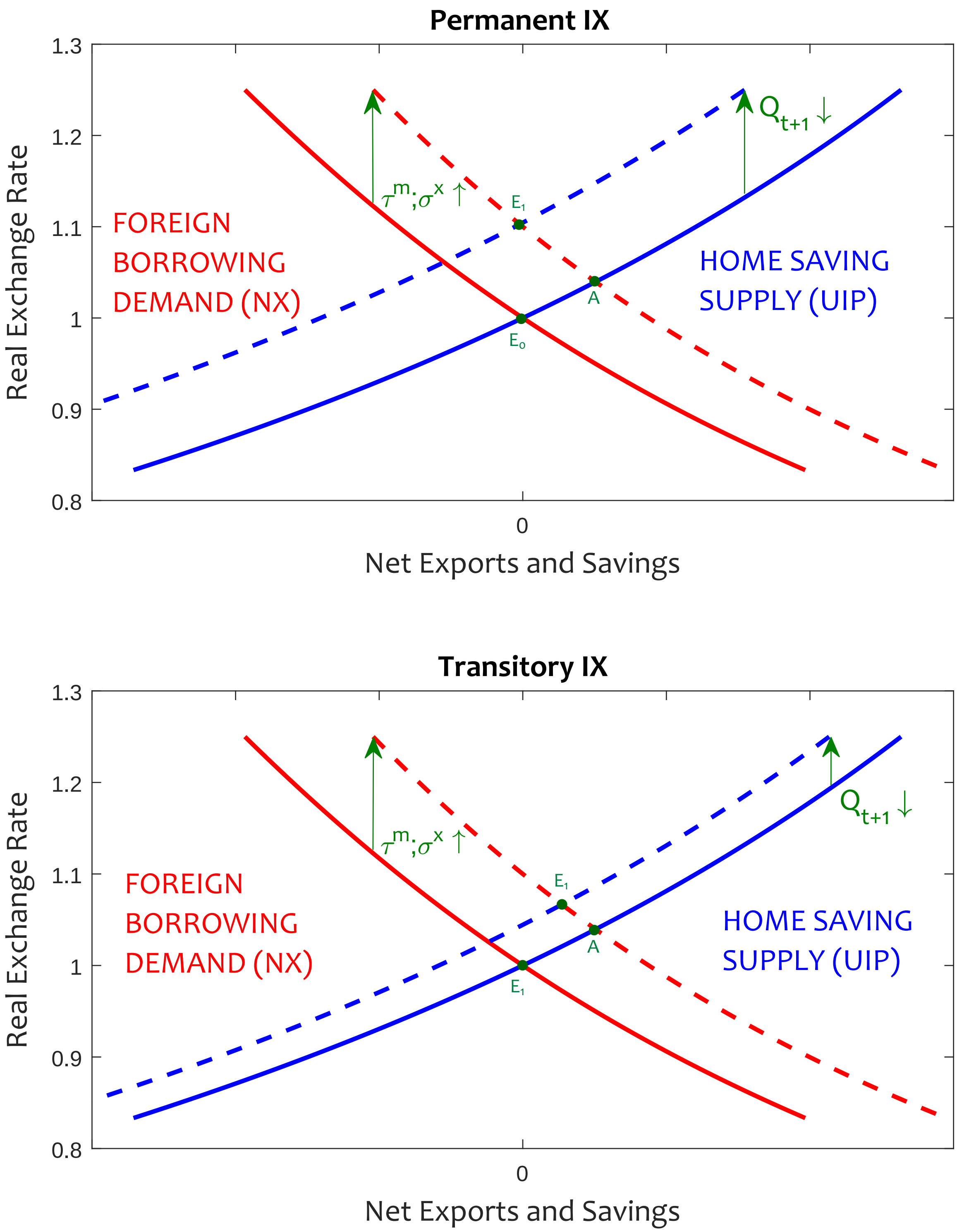

Figure 2: Permanent IX vs. IX with Retaliation

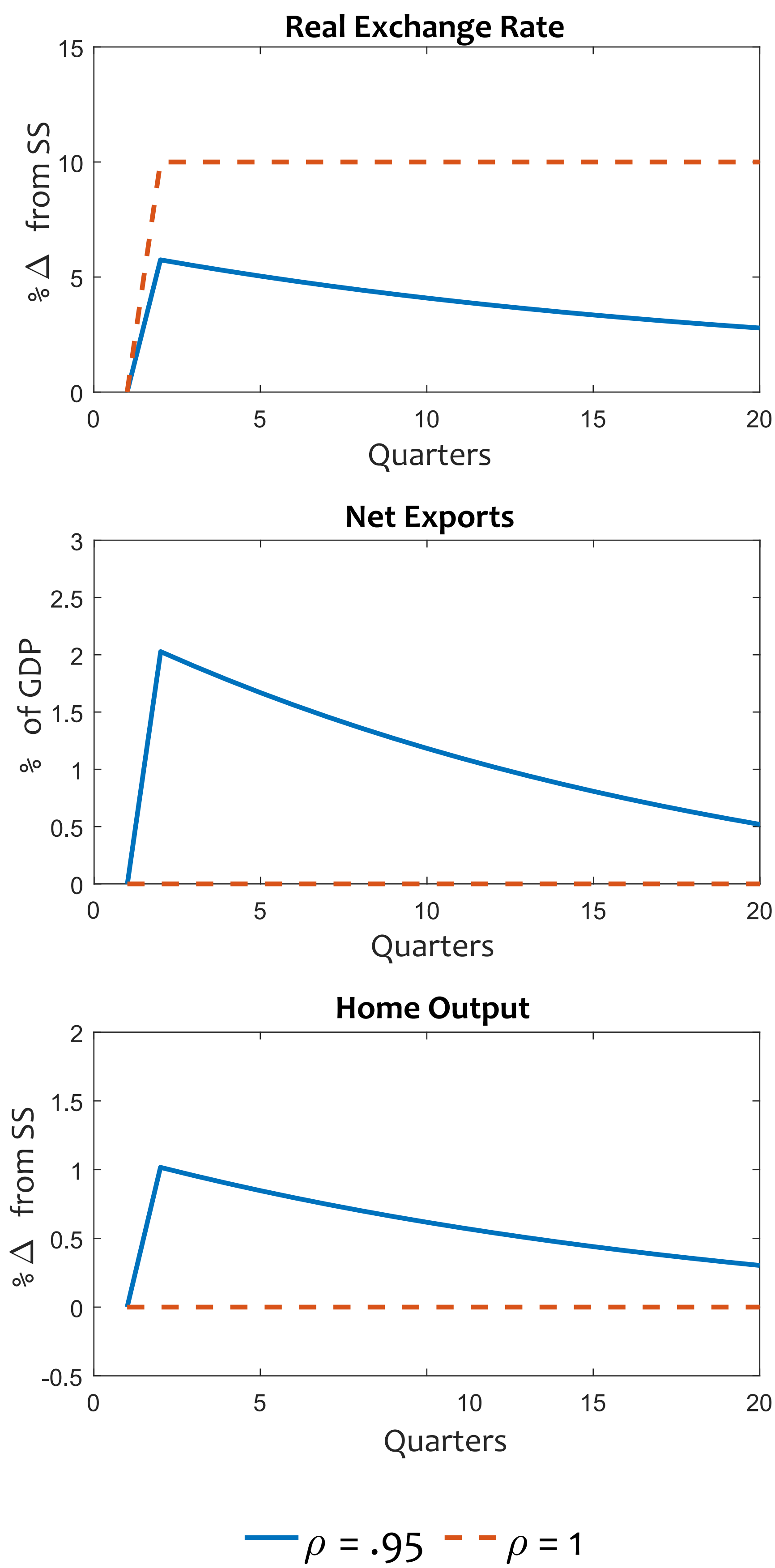

Figure 3: Macroeconomic Effects of IX with Retaliation (ρ = 0.95)

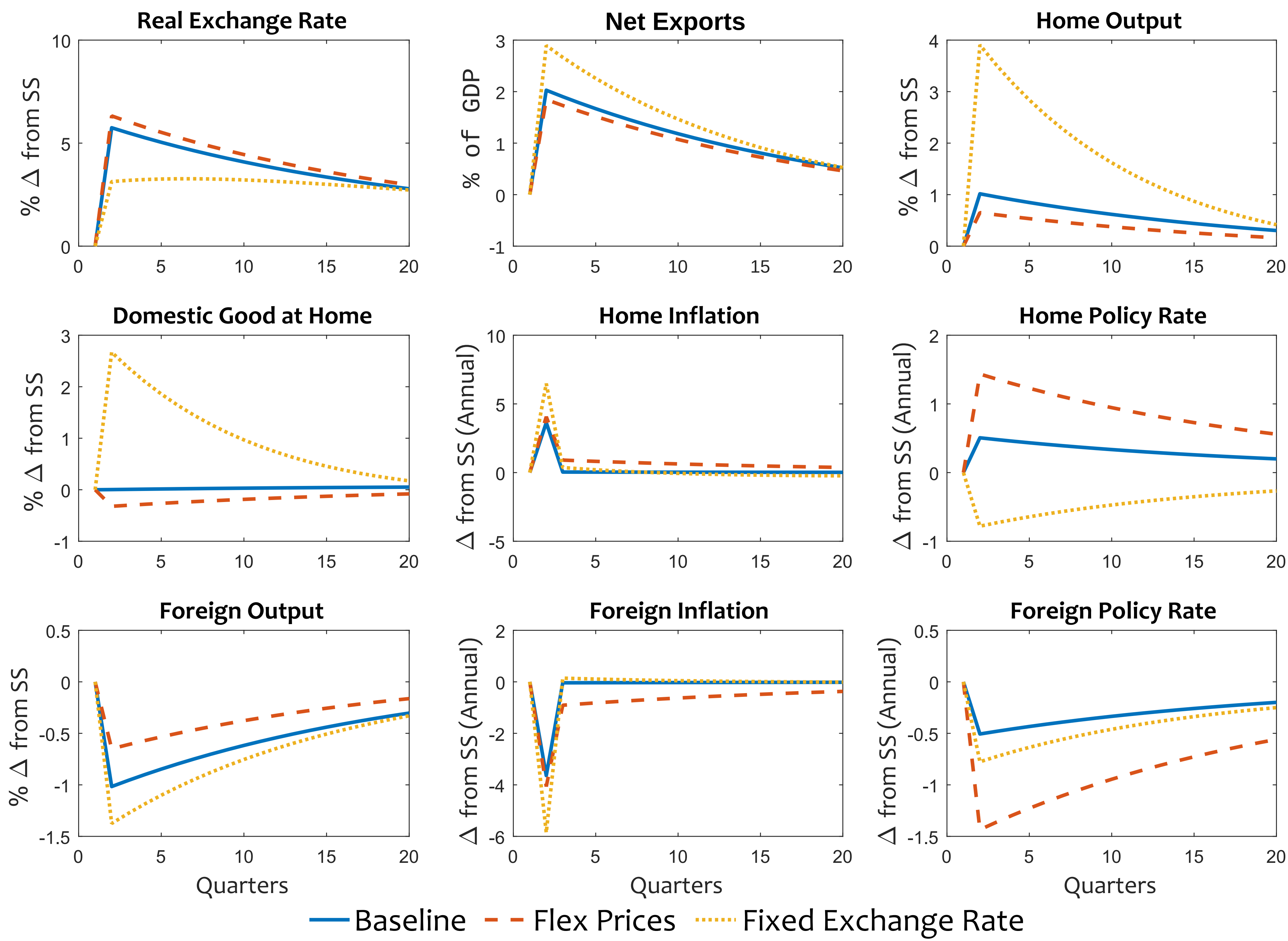

Figure 4: Permanent Unilateral IX With and Without Foreign Holdings of Home Bonds

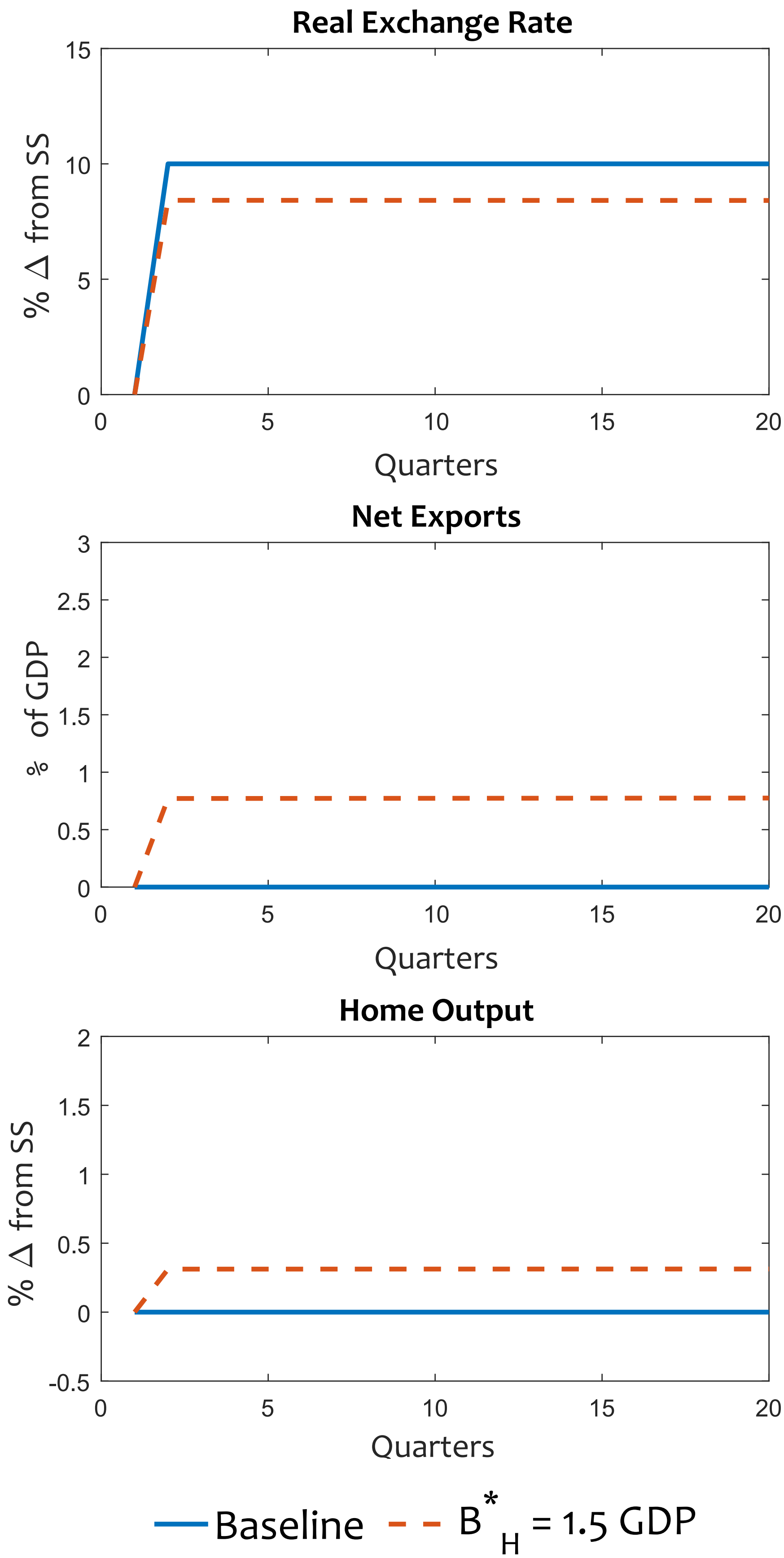

Figure 5: Permanent Unilateral IX, PCP vs. LCP

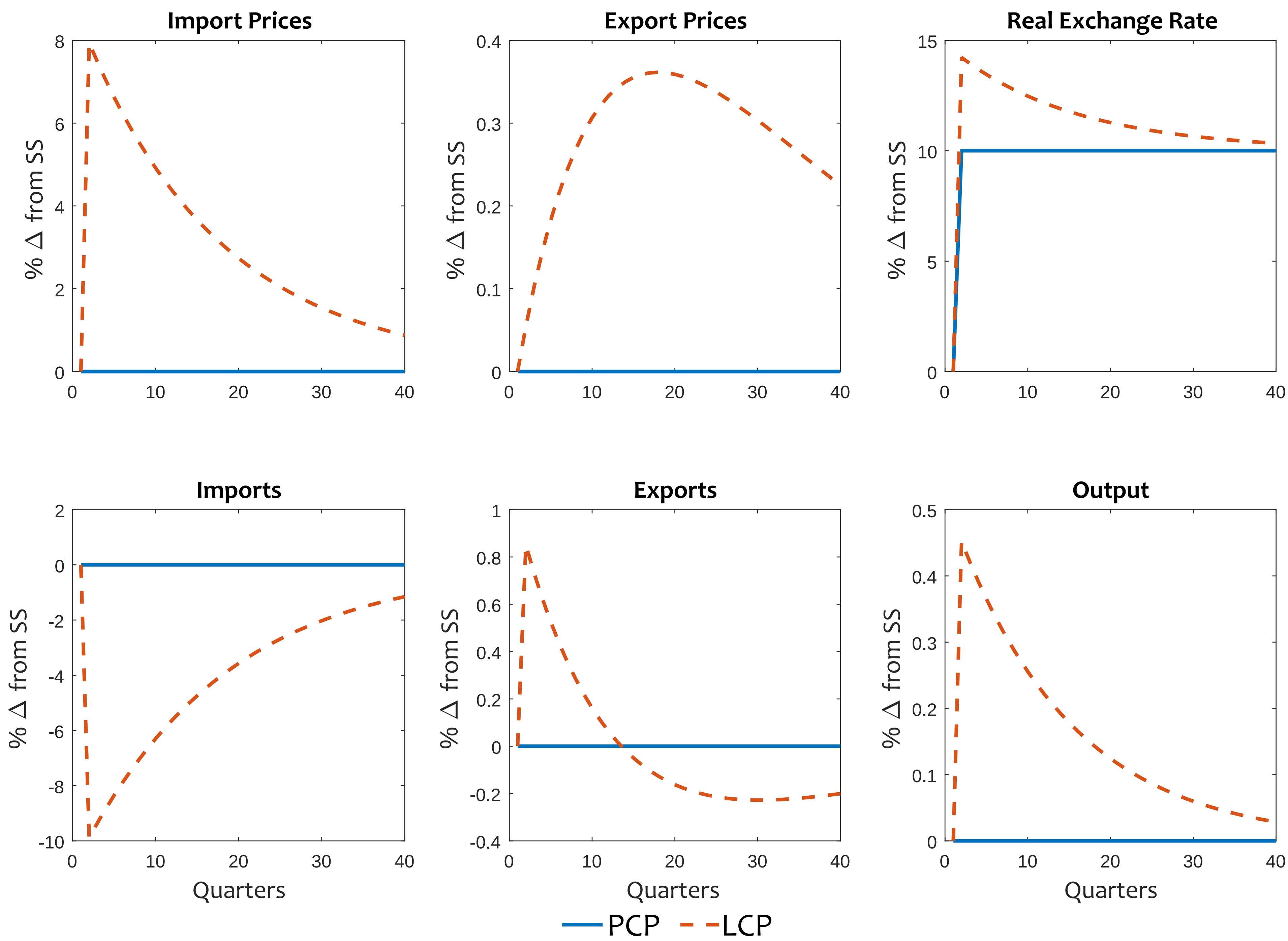

Figure 6: VP vs IX (PCP)