Oil, Equities, and the Zero Lower Bound*

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Abstract

Since 2008, oil and equity returns have moved together much more than they did previously. In addition, we show that both oil and equity returns have become more responsive to macroeconomic news. Before 2008, there is little evidence that oil returns were responsive to macroeconomic news. We argue that these results are consistent with a new-Keynesian model that includes oil and incorporates the zero lower bound on nominal interest rates. Our empirical findings support the model's implication that different rules apply at the zero lower bound.

JEL Classifications: F31, F41, E30, E01, C81

1 Introduction

We document that the rules regarding oil price changes and equity returns changed dramatically in late 2008. Oil and equity price movements became highly correlated, whereas earlier they were typically uncorrelated. Also in contrast to historical experience, oil and equity returns became responsive to macroeconomic news surprises, such as unanticipated changes in nonfarm payrolls. We provide both empirical evidence and theoretical justification that this change in the rules results from nominal interest rates being constrained by the zero lower bound (ZLB). Although a large theoretical literature has argued that the ZLB alters the economic environment, empirical support for this proposition has been lacking, especially for measures of economic activity. As such, our paper's major contribution is to provide

strong evidence of the ZLB altering the economic environment, which has implications for the effectiveness of both fiscal and monetary policy and for the desirability of a fast exit from ZLB episodes.

As can be seen in Figure 1a, the correlation between oil price changes and equity returns increased sharply in 2008. Between 1983 and 2008, the correlation fluctuated around zero, only turning sharply negative in response to events such as the 1990/1991 Gulf War. However, the correlation rose drastically in late 2008, reaching as high as 0.65 in 2010 and then averaging around 0.50 through late 2013. Thereafter, the correlation has moved lower.[1] We provide evidence that this correlation is broad based, with equity returns for a disparate group of sectors all showing an increased correlation with oil prices.

Given that this observed increase in correlation is coincident with the onset of the ZLB period in the U.S. economy, one might wonder whether the ZLB causes this increased correlation. We provide both theoretical and empirical evidence in favor of this causal relationship. We present a formal analysis with a new-Keynesian model that is augmented to include oil.[2] Using our new-Keynesian model, we show that oil price changes and equity returns become more correlated at the ZLB. The mechanism for this increased correlation arises from the monetary authority being constrained at the ZLB. When the ZLB binds, the monetary authority does not respond to changes in inflation at the ZLB. By contrast, away from the ZLB, changes in inflation lead to more than a one-for-one change in the nominal rate. Consequently, movements in inflation have different effects on the real interest rate at the ZLB and away from it. Therefore, shocks have different effects on output, consumption, oil prices and equity prices depending on whether or not the ZLB is binding.

Building on our model's theoretical implications, we provide further empirical evidence of the role of the ZLB by studying how oil and equity returns respond to identified shocks. In particular, we report how much oil and equity returns change on the day of a surprise in U.S. macroeconomic announcements. We identify our shocks as the difference between actual economic announcements (such as nonfarm payrolls) and the average forecast from a week earlier. We show that, in contrast to historical experience, oil and equity returns became and remained responsive to macroeconomic news surprises, such as unanticipated changes in nonfarm payrolls, for several years. These results build and extend on the existing literature. For example, consistent with our finding that the response changes with the onset

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[1]The increased correlation between oil and equities is also discussed by Lombardi and Ravazzolo (2016) and Serletis and Xu (2016). Lombardi and Ravazzolo (2016) are concerned with the implications of time-varying correlation for portfolio allocation. Serletis and Xu (2016) include a time dummy variable starting in late 2008, which they associate with the ZLB. Our work complements these existing studies by providing further theoretical and empirical insight regarding this changing correlation.

[2]Our model is similar in structure to Bodenstein et al. (2013), although they do not consider equity prices.

of the ZLB, Kilian and Vega (2011) report that oil prices do not have statistically significant responses to macroeconomic news surprises over the period from 1983 to 2008. Likewise, using data from 1957 to 2000, Boyd et al. (2005) claim that equities responded positively to bad news in expansions and negatively to bad news in recessions, our results differ in that the increased responsiveness of equity returns post-2008 has outlasted the recession and instead seems to be related to the low level of interest rates.

Although we provide both a consistent theoretical model and supportive empirical evidence that the ZLB caused this changing relationship between oil and equity prices, alternative explanations are conceivable. For example, the increased financialization of commodities or else greater uncertainty related to the financial crisis are possible explanations for this increased correlation.[3] As such, we provide statistical evidence testing the relative merits of explanations based on measures of the ZLB (either a Taylor-rule implied interest rate or the shadow rate of Wu and Xia (2016)) and explanations based on other variables including open interest in oil futures contracts, the VIX, and the uncertainty indexes of Jurado et al. (2015) and Baker et al. (2015). Overall, we find that the variation in sensitivity to macroeconomic news surprises is better explained by measures of the ZLB than by these alternative measures of uncertainty and oil market financialization.

Finally, we provide a structural VAR analysis of the same correlation, albeit at a monthly frequency. In our structural VAR work, the correlation change was not because aggregate demand or supply shocks became more important. Instead, consistent with our ZLB-driven hypothesis, shocks that have an immediate impact on oil prices but not on aggregate demand or oil supply went from causing a negative correlation to a positive correlation.

To summarize, we have presented multiple pieces of empirical evidence that are consistent with the ZLB changing the correlation between oil price changes and equity returns. These results should help focus the debate on what economic models are most appropriate for studying recent conditions.

1.1   Relationship to literature

Our empirical evidence is supportive of a large literature of models, where economic outcomes are different under the ZLB. Our short list of representative papers includes the following. Christiano et al. (2011) and Erceg and Lindé (2014) show that in their models fiscal multipliers were much larger under the ZLB. Likewise, Eggertsson et al. (2014) present a theoretical model, where structural reform, which is normally expansionary, is contractionary when

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[3]The literature on financialization of commodities is large and unsettled. For example, Tang and Xiong (2012) argue that financialization plays an important role in price movements. In contrast, Irwin and Sanders (2012), Fattouh et al. (2013), and Hamilton and Wu (2015) find a more limited role for financialization.

monetary policy is constrained. In the model of Gourinchas (2016), the role of capital flows changes under the ZLB.

Relative to the theoretical literature on the ZLB, the empirical literature testing for ZLB effects is less extensive. Dupor and Li (2015) present some empirical evidence including whether professional forecasters revised their inflation expectations commensurate with their output forecast revisions in response to government stimulus measures. Plante et al. (2016) study the relationship between uncertainty and GDP growth at the ZLB and find that they have been more negatively correlated during the ZLB period and predicted by the new Keynesian model. Wieland (2015) explores whether reductions in oil supply are contractionary at the ZLB and fails to find strong evidence. Garin et al. (2016) study how the economy responds to TFP shocks. They too find a nonsupportive result in that positive productivity shocks actually have a larger positive output effect at the ZLB. However, both papers study how quarterly GDP responds to shocks. As such, the work is limited by the small number of observations under which the ZLB is binding. Our paper complements these previous studies by using higher frequency data based on daily price changes. In addition to providing more observations, high frequency data offers the additional benefits that the timing assumptions are more plausible and that the shocks are more likely to be unanticipated than shocks that are identified at the monthly or quarterly frequency (Ramey, 2016).

One study that does use high-frequency data to study the ZLB is Swanson and Williams (2014), which shows that longer-term interest rates become less responsive to macroeconomic news surprises after 2008, which they attribute to the ZLB. Relative to that paper, a contribution of our work is showing that the ZLB affects not only interest rates by making them less responsive to surprises, but also other asset prices, including oil and equities, by making them more responsive. One important methodological contribution of our paper relative to Swanson and Williams is that, beyond reporting results for time-varying responsiveness as was done by Swanson and Williams, we estimate and test directly the hypothesis that the responsiveness varies with monetary policy conditions, as measured by an interest rate implied by a modified Taylor Rule. Furthermore, we also test alternative hypotheses that attribute the change in responsiveness either to the financialization of oil markets or increased uncertainty in the crisis era, and show that the evidence in favor of the ZLB is stronger.

2 The increased correlation between oil and equities

The correlation between daily oil price changes and equity returns increased markedly in late 2008 (see Figure 1a, Panel (a)). Our measure for the price of oil (Panel (b)) is the closing

value, in dollars per barrel, of the front-month futures contract for West Texas Intermediate (WTI) crude oil for delivery in Cushing, Oklahoma obtained from NYMEX.[4] For equities (also in Panel (b)), we use the Fama-French value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks. Table 1 presents summary statistics for these measures over our sample period, which covers April 6, 1983 through June 30, 2016.

To calculate returns, we drop days with missing values for any of our primary variables of interest: WTI futures and physical spot prices, metals prices, interest rates, and the level of the equity price index implied by the Fama-French equity returns (which include dividends). Then, we calculate "daily" returns as the 100 times the log-difference of these consecutive closing prices, thereby ensuring that the daily returns are calculated over the same period for each variable. Figure 1a depicts the correlation of these daily returns for oil and equities using a rolling sample of one-year.[5]

Using a regression to summarize comovement rather than to indicate causality, we next investigate the time variation in the equity beta of oil returns. Because the rolling one-year window can result in sharp jumps as observations are included or excluded in the window, we measure this time variation using a kernel regression:

$$Oil_t = \alpha(t) + \beta(t)Equity_t + \varepsilon_t, \tag{1}$$

which is estimated by solving

$$\left\{\hat{\alpha}(k), \hat{\beta}(k)\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{t-k}{h}\right)\left(Oil_t - \alpha - \beta Equity_t\right)^2.$$

This model estimates at each time k the $\beta(k)$ which minimizes the weighted regression of oil returns on equity returns, using all available observations. Because the weights on the observations decline as we move further away from $t = k$ in either direction, the intuition is that each estimated $\hat{\beta}(k)$ places more weight on the observed $(Oil_t, Equity_t)$ when t is close to k. As with the rolling correlation, in Figure 2a, we can see a sustained increase in the coefficient estimates of $\hat{\beta}(t)$ in late 2008.

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[4]The series reports the official daily closing prices at 2:30pm in the New York Mercantile Exchange. In contrast, Kilian and Vega (2011) use the daily spot price for WTI crude oil for delivery (freight on board) in Cushing, Oklahoma, as reported by the U.S. Energy Information Administration (EIA). Analyses using the EIA series, or the nearby futures price for Brent crude oil on the ICE exchange obtained from Bloomberg, generate similar results. Of these, we prefer the WTI nearby futures price as its more precise timing allows us to better relate it to the macroeconomic announcements. In supplementary analysis, we also use the WTI far futures price, which we define as the price of the furthest available December contract.

[5]In the appendix, we show that this sustained increase is also visible when using window sizes ranging from one month to three years, and to using returns calculated at the daily, weekly, monthly, and quarterly frequencies.

Next, we use a Chow test for the simple regression of oil returns on equity returns to determine whether there is a break in the oil-equity relationship, and find a statistically significant break date of September 22, 2008. Table 2 reports the estimated equity beta for three sample periods: the full sample, pre-break, and post-break. As shown in Table 2, the coefficient is slightly negative for the pre-break sample, but is large, positive, and significant for the post-break sample. The coefficient of 0.76 in the post-break sample implies that during this period, a daily return of 1 percent on the equity index is associated with oil price change of about 0.76 percent. We find similar results when using our alternative measures of oil prices, including the physical spot price for WTI and the nearby futures price for Brent crude oil. Consistent with the lower variation in far futures prices as compared to nearby futures (reported in Table 1), we find that the results when using the WTI far futures series are qualitatively similar but quantitatively smaller.

To demonstrate that the break in the relationship extends beyond the oil market, we also use the metals spot index constructed by the Commodities Research Bureau. Applying the Chow test to the regression of metals on equity returns also implies a statistically significant break date of September 30, 2008. As with oil, Table 2 shows that the slope coefficient on equity returns is essentially zero for the pre-break sample, but is much larger and statistically significant for the post-break sample. Using the standard Andrews supremum-Wald critical value based upon 15% trimming of the sample as in Stock and Watson (2003), all of these break dates were found to be statistically significant at the 1% level ($F_{crit} = 7.8$). Figure 2a shows the time variation in the metals-equity coefficient estimate, using the same kernel regression approach as in Equation 1.

Finally, to ensure that the increased correlation between oil and equity prices is not being driven by fluctuations in the energy component of the S&P 500, we separately regress oil on each of the twelve Fama-French industry portfolios, determined by SIC codes, as well as on returns for the S&P 500 Ex-Energy index obtained from Bloomberg (Ticker: SPXXEGP). The results of the related Chow tests are presented in Panel B of Table 2. In the pre-break sample, returns in all of the non-energy related sectors are negatively associated with oil prices. Only the energy sector shows a positive, statistically significant relationship before the break in 2008. In contrast, post-break, all of the sectors display a positive and statistically significant relationship similar to that of the energy sector. These results confirm that our finding of an increased correlation between equity prices and oil prices is not being driven exclusively by equity prices for energy producers. Instead, the increased correlation between oil and equity returns is broad-based.

Rather than estimating time-varying coefficients as in Equation 1, we now estimate a model in which the coefficients vary with a measure of the ZLB. Our measure of the ZLB

is the *notional rate*, which we define as the prediction for the federal funds rate using the modified Taylor rule as in Bernanke (April 28, 2015). This notional rate is intended to capture the target federal funds rate implied by the current state of the economy, without censoring due to the ZLB. The rule is specified as

$$\tilde{R} = \pi + y + 0.5(\pi - 2) + 2,$$

where π is inflation, y is the output gap, and $\tilde{R}$ is the notional rate. To measure inflation, p, the modified rule uses the deflator for core personal consumption expenditures (PCE), which excludes food and energy prices. For the output gap, y, it uses estimates prepared by Federal Reserve staff for FOMC meetings through 2009, and then estimates produced and published by the Congressional Budget Office through 2015. We use only real-time data where available. Figure 3a depicts this notional rate along with the actual federal funds rate.

We now estimate the equity beta for oil as a function of the notional rate using the model

$$Oil_t = \alpha(\tilde{R}_t) + \beta(\tilde{R}_t)Equity_t + \varepsilon_t. \tag{2}$$

The estimates of α and β are constructed to solve

$$\left\{\hat{\alpha}(\tilde{R}_k), \hat{\beta}(\tilde{R}_k)\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{\tilde{R}_t - \tilde{R}_k}{h}\right)(Oil_t - \alpha - \beta Equity_t)^2.$$

Figure 3b plots our estimate of $\beta(\tilde{R}_k)$, and provides further evidence that oil and equities have stronger comovement (i.e. $\beta(\tilde{R}_k)$ is larger) when interest rates are low, and in particular, when the notional rate is negative. That is, when the Taylor rule would imply nominal interest rates that are lower than the ZLB, we find that the correlation between oil price changes and equity returns is high.

3 A DSGE model with oil

To study the theoretical effect of the ZLB on the relationship between oil prices and equity returns, we use a small-scale, closed-economy, new-Keynesian model augmented with oil similar to the model in Bodenstein et al. (2013). As in the previous section, we find that oil and equity prices exhibit dramatically different behavior in normal times versus when the nominal interest rate is at its lower bound.

3.1 Households

The model economy is populated by a large number of identical households. The households' utility function values streams of final-goods consumption, C_t, oil consumption, C_t^O, labor services, L_t, and real bond holdings, B_t/P_t, according to a utility function given by

$$E_t \sum_{m=0}^{\infty} \beta^m \left[u \left(C_{t+m}, C_{t+m}^O, L_{t+m}, \mu_{C^O,t+m} \right) + \eta_{t+m} \Xi_{t+m} v \left(\frac{B_{t+m}}{P_{t+m}} \right) \right].$$

Here, η_t is an exogenous and stochastic process that shifts household preferences for risk-free real bond holdings, $\mu_{C^O,t+1}$ is an exogenous and stochastic process that shifts the household preference for oil consumption, and the discount factor, β, satisfies $0 < \beta < 1$. The household faces a per-period budget constraint given by

$$\frac{B_t}{P_t} + \frac{P_t^O}{P_t} C_t^O + C_t + \frac{P_t^K}{P_t} I_{t+1} \leq \frac{W_t}{P_t} L_t + (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + \frac{R_t^K}{P_t} K_t + T_t,$$

where I_{t+1} is investment, K_t are capital holdings, P_t is the price of final goods, P_t^O is the price of oil, P_t^K is the price of capital, W_t is the wage rate, R_t^K is the rental rate of capital, R_t is the nominal interest rate, and T_t are real lump-sum profits, taxes, and transfers.

We include end-of-period risk-free real bond holdings in the utility function as in Fisher (2015) in order to capture the spread between risky and risk-free assets. We couple the bonds in the utility function with the preference shifter, η_t, to allow the spread between real bond holdings and the returns to other assets to change over time. We assume that the function v is increasing, concave, and has some positive and negative support. Moreover, we define $\Xi_t = u_1 \left(\bar{C}_t, \bar{C}_t^O, \bar{L}_t, \mu_{C^O,t} \right)$, where $\bar{x}_t$ means an aggregate quantity taken as given by the household, and normalize v so that the spread in the household's intertemporal Euler equation as identically η_t,

$$1 = \eta_t + \beta E_t \left[\frac{u_1 \left(C_{t+1}, C_{t+1}^O, L_{t+1}, \frac{B_{t+1}}{P_{t+1}} \right)}{u_1 \left(C_t, C_t^O, L_t, \frac{B_t}{P_t} \right)} \frac{1 + R_t}{\pi_{t+1}} \right].$$

Here, $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation of the price of final goods. Changes in η_t represent changes in the spread between the risk-free one-period bond and other assets, and are meant to capture changing preferences for risk-free bond holdings, because of, for example, a flight to safety. Thus, η_t plays an analogous role to the spread shock in Smets and Wouters (2007). We normalize real bonds to be in zero net supply.

We specify the function u so that

$$u\left(C_t, C_t^O, L_t, \mu_{C^O,t}\right) = \log\left[\left((1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{\frac{1}{1+\rho_C}} + (\omega_C)^{\frac{\rho_C}{1+\rho_C}} \left(\frac{C_t^O}{\mu_{C^O,t}}\right)^{\frac{1}{1+\rho_C}}\right)^{1+\rho_C}\right] - \frac{\chi}{1+\phi}L_t^{1+\phi}.$$

These preferences are similar to those used in Bodenstein et al. (2013) in that households value consumption of oil in their utility flow and in that consumption of oil is an imperfect substitute for retail-goods consumption. Here, $\mu_{C^O,t}$ is a preference shifter that affects the utility flow of oil consumption. We interpret shocks to $\mu_{C^O,t}$ as oil demand shocks. For matching our model to the data, it will be useful to define the ex-post return on capital as

$$\frac{P_t^K + R_t^K}{P_{t-t}^K}.$$

Note that we are not taking expectations, because we are measuring realized returns.

3.2   Firms

Perfectly competitive firms produce final output, Y_t, using intermediate inputs, $Y_t(j)$ using a production technology given by

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\varepsilon > 1$. Profit maximization implies demand curves for intermediate goods given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

There is a unit measure of monopolists who produce intermediate goods. They choose their price, $P_j(t)$, to maximize

$$E_t \sum_{m=0}^{\infty} M_{t,t+m} \left[\frac{(1+\tau)P_{t+m}(j) - MC_{t+m}}{P_{t+m}}\left(\frac{P_{t+m}(j)}{P_{t+m}}\right)^{-\epsilon} Y_{t+m} - \frac{\Phi}{2}\left(\frac{P_{t+m}(j)}{P_{t+m-1}(j)} - 1\right)^2\right],$$

where $M_{t,t+m}$ measures the household's time t valuation of real profit flows in period $t+m$, MC_t nominal marginal cost, and τ is a subsidy to offset steady state distortions due to monopoly power. The term $\frac{\phi}{2}\left(P_t(j)/P_{t-1}(j) - 1\right)^2$ represents the cost of price adjustment, as in Rotemberg (1982). Monopolists combine oil inputs, $V^O_t(j)$, with capital and labor to

produce intermediate goods according to

$$Y_t(j) = \left((1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(K_t(j)^\alpha L_t(j)^{1-\alpha} \right)^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O(j)}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{1+\rho_Y}.$$

Monopolists are identical, and in equilibrium set $P_t(j) = P_t$.

In each period oil supply, O_t, is exogenously determined. Market clearing in the oil market requires that the quantity of oil demanded equals the exogenous oil supply:

$$O_t = V_t^O + C_t^O.$$

We assume that the capital stock is fixed and there is no investment. As such, the aggregate resource constraint is given by

$$Y_t = C_t + G_t + \frac{\Phi}{2} (\pi_t - 1)^2.$$

3.3   Government policy

The fiscal authority purchases government consumption, G_t, in each period, and lump sum taxes are set to satisfy the government budget constraint, period-by-period, with $B_t = 0$. The monetary authority sets the nominal interest rate according to a Taylor rule, which is constrained by the ZLB,

$$R_t = \max \{0, R + \theta_\pi (\pi_t - 1)\},$$

where R is the steady state net nominal interest rate. Note that in our specification of the Taylor rule, the monetary authority responds only to final-goods prices, not to oil prices.

3.4   A linearized version of the model for intuition

To gain intuition about how the ZLB affects the dynamics of our model, we consider a log-linearized version, as in Christiano et al. (2011). For ease of exposition, we assume that $\rho_Y = \rho_C = \infty$. The log-linear equilibrium conditions of our model can be written as

$$-\left(\frac{Y}{C} \hat{Y}_t - \frac{G}{C} \hat{G}_t \right) = \hat{\eta}_t - E_t \left(\frac{Y}{C} \hat{Y}_{t+1} - \frac{G}{C} \hat{G}_{t+1} \right) + \hat{R}_t - E_t \hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \kappa \left(\Gamma \left[\hat{Y}_t + \omega_Y \hat{\mu}_{V^O,t} - \omega_Y \hat{O}_t \right] + \frac{G}{C} \hat{Y}_t - \frac{G}{C} \hat{G}_t \right) + \beta E_t \hat{\pi}_{t+1}$$

where $\hat{x}_t$ denotes a log-deviation of x_t from its steady state value, x.[6], $\kappa = \frac{\varepsilon}{\Phi} Y mc$, and $\Gamma = \frac{\varphi+1}{(1-\omega_Y)(1-\alpha)+\omega_Y \frac{CO}{O}(\varphi+1)}$.

We consider an equilibrium as in Eggertsson and Woodford (2003) in that we assume, beginning at time zero, $\hat{\eta}_t = \hat{\eta} > 0$. With probability p, $\hat{\eta}_t$ remains at this level in the next period. With complementary probability $\hat{\eta}_t = 0$ in the next period. In so long as $\hat{\eta}_t = \hat{\eta}$, it will remain at that level in the next period with probability p, with complementary probability will return to its steady state value from then on. Similar to Christiano et al. (2011), while $\hat{\eta}_t = \hat{\eta}$, we assume that $\hat{G}_t = \hat{G}$, $\hat{O}_t = \hat{O}$, and $\hat{\mu}_{VO,t} = \hat{\mu}_{VO}$. When $\hat{\eta}_t = 0$, we assume that $\hat{G}_t = \hat{O}_t = \hat{\mu}_{VO,t} = 0$. Since our model has no state variables (capital is fixed), once $\hat{\eta}_t = 0$ the equilibrium prices and quantities return to their steady state values. Note that when $\hat{\eta}_t > 0$, households gain extra utility from holding bonds, which is to say that they have an increased desire to save through bond holdings and the natural rate of interest is low. For $\hat{\eta}_t$ large enough, the ZLB will bind.

After some algebra, one can show that

$$\hat{\pi} = \frac{\Gamma\kappa(1-p)\left(\omega_Y p\hat{\mu}_{VO} - \omega_Y\hat{O}\right) + \kappa(\Gamma-1)(1-p)\frac{G}{Y}\hat{G} - \hat{\eta} - \hat{R}}{\Delta}.$$

We assume that $\hat{\eta}$ is large enough so that the ZLB binds, meaning $\hat{R}$ is the value of $\hat{R}_t$ at the ZLB. Similar to Carlstrom et al. (2014) and Christiano and Eichenbaum (2012), we define

$$\Delta \equiv (1-p)(1-\beta p) - p\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)\frac{C}{Y}.$$

and require that parameters be chosen so that $\Delta > 0$ in order to guarantee a unique bounded equilibrium at the ZLB. In equilibrium, output is given by

$$\hat{Y} = \frac{\Gamma\kappa\left(p\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)\frac{C}{Y}\right)\left(\omega_Y p\hat{\mu}_{VO} - \omega_Y\hat{O}\right) - (1-\beta p)\left(\hat{\eta} + \hat{R}\right)}{\Delta\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)}$$
$$+ \frac{\kappa\left((1-\beta p)(\Gamma-1)(1-p)\frac{G}{Y} + \Delta\frac{G}{C}\right)\hat{G}}{\Delta\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)}.$$

Note that inflation and output at the ZLB are increasing functions of $\hat{\mu}_{VO}$ and $\hat{G}$ but decreasing functions of $\hat{O}$. Moreover, similar to Christiano et al. (2011), the effect of a change in $\hat{\mu}_{VO}$, $\hat{G}$, or $\hat{O}$ can be quite large because Δ can be very close to zero depending on the value of p and the other parameters of the model. In this sense, output and inflation can be

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[6]In our linearization, we assume that η_t has a zero steady state, and thus $\hat{\eta}_t$ is the deviation of η_t from zero.

sensitive to shocks at the ZLB.

In equilibrium, the price of capital and the price of oil are given by

$$\hat{p}^K = \frac{p\,(1-\beta)}{1-\beta p}\left(\frac{(1-p\beta)}{\kappa}\hat{\pi} + \hat{Y}\right) + \frac{1-p}{1-\beta p}\hat{C}$$

and

$$\hat{p}^O = -\hat{O} + \frac{V^O}{O}\Gamma\left[\hat{Y} + \omega_Y\hat{\mu}_{V^O,t} - \omega_Y\hat{O}_t\right] + \hat{C}.$$

Notice that, because $\hat{Y}$ and hence $\hat{C}$ are increasing in $\hat{\mu}_{V^O}$ and decreasing in $\hat{O}$, $\hat{p}^K$ and $\hat{p}^O$ are also increasing in $\hat{\mu}_{V^O}$ and decreasing in $\hat{O}$. Additionally, so long as the government purchases multiplier is larger than one, $\hat{p}^K$ and $\hat{p}^O$ are both increasing in $\hat{G}$. That is, the price of oil and the price of capital move in the same direction at the ZLB if we change the values of the exogenous variables.

For comparison, we consider an analogous equilibrium in which $\hat{\eta}$ is not large enough to make the ZLB bind. In this case,

$$\hat{\pi} = \frac{\Gamma\kappa\,(1-p)\left(\omega_Y p\hat{\mu}_{V^O} - \omega_Y\hat{O}\right) + \kappa\,(\Gamma-1)\,(1-p)\frac{G}{Y}\hat{G} - \hat{\eta}}{\Delta + \theta_\pi}$$

where θ_π is the parameter on inflation in the Taylor rule and

$$\hat{Y} = \frac{\Gamma\kappa\,((1-p)\,(1-\beta p) - [\Delta + \theta_\pi])\left(\omega_Y p\hat{\mu}_{V^O} - \omega_Y\hat{O}\right) - (1-\beta p)\,\hat{\eta}}{(\Delta + \theta_\pi)\,\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)}$$
$$+ \frac{\kappa\left((1-\beta p)\,(\Gamma-1)\,(1-p)\frac{G}{Y} + [\Delta + \theta_\pi]\frac{G}{C}\right)\hat{G}}{(\Delta + \theta_\pi)\,\kappa\left(\Gamma - 1 + \frac{Y}{C}\right)}.$$

In contrast to an equilibrium at the ZLB, so long as $\theta_\pi > 1$, $\hat{Y}$ is now a decreasing function of $\hat{\mu}_{V^O}$ and an increasing function of $\hat{O}$, meaning that, all else being equal, $\hat{C}$ is also a decreasing function of $\hat{\mu}_{V^O}$ and an increasing function of $\hat{O}$. In addition, the effect of a change in $\hat{G}$ on $\hat{Y}$ is muted relative to the equilibrium at the ZLB because in response to an increase in government spending, inflation rises leading to a more than one-for-one increase in the nominal interest rate. The resulting rise in the real interest rate decreases consumption. The equilibrium effect of the more than one-for-one movements of the nominal interest rate in response to changes in inflation is reflected in the algebra as the Δ always appearing as added to θ_π.

To understand how the correlation between the price of oil and the price of capital can

12

change when the ZLB is not binding as compared to when it is binding, consider an increase in oil supply, $\hat{O}$. In contrast to an equilibrium at the ZLB, output and consumption increase, though the larger oil supply can still drive down real oil prices. The increase in output and consumption pushes up demand for capital rental services, and the price of capital can rise. As a result, the price of capital and the price of oil can move in opposite directions relative to one another in response to a change in $\hat{O}$ away from the ZLB, though at the ZLB the model predicts they will move in the same direction. Thus, oil prices and equity prices can move in qualitatively different ways in response to a change in exogenous variables at the ZLB and away from the ZLB.

3.5 A nonlinear and stochastic version of the model

To simulate a stochastic version of our model so as to generate data to compare with the regressions results reported earlier in the paper, we parameterize and solve the discrete time version of the model. We assume that the household time discount factor is $\beta = 0.985$ and that in steady state the preference parameter $\eta = 0.01$. The parameter η implies a steady-state 4 percent annual premium on risky assets as compared to the risk-free government bond. The parameter β implies a steady-state risk-free real interest rate of 2 percent. We set $\epsilon = 7$, which is well within the range considered in Altig et al. (2011). For price adjustment costs, we set $\Phi = 200$, which implies that a model with Calvo price adjustment friction in which firms update their price with probability of about 0.15 would have an identical linearization as our model. The parameter in the monetary policy rule is set so that the Taylor principle is satisfied, $\theta_\pi = 2$. We set $\rho_Y = \rho_C = -2$, which implies an elasticity of substitution of $\frac{1}{2}$ for the different inputs to production and in the consumption bundle and is in line with estimates reported in Bodenstein et al. (2013). We set $\omega_Y = 0.03$, $\omega_C = 0.02$, and we parameterize χ so that steady state labor supply is 1. The choice of $\omega_Y = 0.03$, $\omega_C = 0.02$ is similar to Bodenstein et al. (2013), and implies that oil is about 4.2 percent of output and also implies that firms use more oil as an input to production that households consume.

For each of the exogenous processes η_t, $\mu_{C,t}$, O_t, and G_t, we assume that the stochastic process governing its evolution is given by

$$\log(x_t/x) = \rho_x \log(x_{t-1}/x) + \sigma_x \epsilon_t^x$$

where x represents the steady state value and ϵ_t^x is a standard normal random variable that is independent of all other processes and over time. For our baseline calibration, we set $\rho_O = \rho_{\mu_{CO}} = 0.5$, $\rho_\eta = 0.9$ and $\rho_G = 0.8$. Additionally, we set $\sigma_O = 0.01$, $\sigma_G = 0.01$, $\sigma_{\mu_{CO}} = 0.03$ and $\sigma_\eta = 0.06$. We assume that the capital stock is fixed ($K_t = \overline{K}$) and

that the steady state capital-labor ratio is 15. We allow households to trade shares of the capital stock, but assume that there is no investment decision by households or firms. For the production function, $\alpha = 0.3$. Finally, we set $\phi = 1$.

We solve the model using a policy iteration methodology similar to the solution strategy introduced by Bizer and Judd (1989) and Coleman (1991). To accommodate the ZLB, we follow Gust et al. (2016) and parameterize two versions of the equilibrium policy functions: one version that assumes $R_t = 0$ and one that assumes $R_t = R + \theta_\pi (\pi_t - 1)$. Expectations are calculated so that if $R + \theta_\pi (\pi_{t+1} - 1) < 1$, then the former functions are operative. Otherwise, the latter functions determine time $t+1$ prices and quantities. When we simulate our model, we use the endogenous interest rate to determine which function is used to calculate the values of the equilibrium variables in our model. In this way, we allow the policy functions for every variable to have a kink at the ZLB.

The policy functions are projected onto Smolyak polynomials as in Judd et al. (2014). We allow up to fourth-order terms in the projection. By solving the model nonlinearly, we avoid the pitfalls associated with linear approximations at the ZLB, which have been documented in Christiano and Eichenbaum (2012) and Braun et al. (2015). By employing a projection onto polynomials, we are implicitly assuming that the nonlinearities in the model (other than the ZLB) are smooth functions of the states. Gust et al. (2016) report that this assumption holds relatively well, even in their large-scale model.

3.6   Shocks at the ZLB

We simulate our model so as to generate data that we can use to analyze the correlation between oil prices and equity returns when interest rates are high and low. When we simulate our model, shocks to η_t predominantly drive the interest rate to the ZLB. To analyze how the correlation between oil prices and equity returns change as the notional interest rate changes, we calculate local correlations. In particular, we define the notional interest rate as

$$\tilde{R}_t = R + \phi_\pi (\pi_t - 1).$$

Notice that the notional rate corresponds to the nominal interest rate prescribed by a Taylor rule, ignoring the ZLB. We compute the mean of a variable, x_t, local to a particular value of $\tilde{R}$ according to

$$\mu_x\left(\tilde{R}\right) = \frac{\sum_i \phi\left(\frac{\tilde{R} - \tilde{R}_i}{h}\right) x_i}{\sum_i \phi\left(\frac{\tilde{R} - \tilde{R}_i}{h}\right)}.$$

where $\phi(\cdot)$ is the standard normal density function and h is a bandwidth parameter. We compute local variances according to

$$\sigma_x^2\left(\tilde{R}\right) = \frac{\sum_i \phi\left(\frac{\tilde{R}-\tilde{R}_i}{h}\right)\left(x_i - \mu_x\left(\tilde{R}\right)\right)^2}{\sum_i \left(\frac{\tilde{R}-\tilde{R}_i}{h}\right)},$$

and covariances according to

$$\sigma_{x,y}\left(\tilde{R}\right) = \frac{\sum_i \phi\left(\frac{\tilde{R}-\tilde{R}_i}{h}\right)\left(x_i - \mu_x\left(\tilde{R}\right)\right)\left(y_i - \mu_y\left(\tilde{R}\right)\right)}{\sum_i \phi\left(\frac{\tilde{R}-\tilde{R}_i}{h}\right)}.$$

Finally, we compute the local correlation as

$$\frac{\sigma_{x,y}\left(\tilde{R}\right)}{\sigma_x\left(\tilde{R}\right)\sigma_y\left(\tilde{R}\right)}.$$

Figure 4 shows the model-implied value of the correlation between oil and equity prices for different values of the notional interest rate. This correlation is the model analogue to the estimation of Equation 2, the results of which were reported in Figure 3b. In our model, when the notional interest rate is low, the correlation between changes in oil prices and equity returns is positive. When the notional interest rate is high, the correlation is negative. We also solve the model ignoring the ZLB. That is, we allow nominal interest rates to be lower than zero. Figure 4 also shows the correlation between oil price changes and equity returns when we ignore the ZLB, and we find that they are negative for all values of the notional interest rate.

To understand why the correlation between oil price changes and equity returns changes at the ZLB, we perform our kernel regression from the previous section using changes in equity returns as the dependent variable and oil supply shocks, ϵ_t^O, as explanatory variables, and the level of the notional rate as the conditioning variable. Specifically, we estimate

$$\left\{\hat{\alpha}(\tilde{R}), \hat{\beta}(\tilde{R})\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{\tilde{R}-\tilde{R}_t}{h}\right)\left(100 \times \log\left[\frac{P_t^K + R_t^K}{P_{t-1}^K}\right] - \alpha - \beta\epsilon_t^O\right)^2.$$

Figure 5a shows the coefficient $\hat{\beta}(\tilde{R})$ from our kernel regression in the DSGE model that includes the ZLB and the model that ignores the ZLB. Notice that the sign of the coefficient between equity returns and oil supply shocks changes for low notional interest rates. When

we ignore the ZLB, we find no such change of sign. The effects of oil supply shocks on oil prices is the same whether or not the ZLB binds because a factor of production becomes more or less scarce. Thus, the effects of oil supply shocks on equity returns changes at the ZLB, but the effect on oil prices is unchanged, which contributes to the change in correlation between oil prices and equity returns.

The intuition from the linearized version of our model explains why the response of equity returns to oil supply shocks changes at the ZLB. In particular, an increase in oil supply increases the marginal product of labor and capital, however it also causes marginal cost to fall, which causes inflation to fall. Away from the ZLB, this decline in inflation causes real interest rates to fall, which increases consumption demand, and pushes up the price of capital. At the ZLB, the decline in inflation causes real interest rates to rise, which reduces consumption demand, and pushes down the price of capital.

Figures 5b and 5c show the coefficients from our kernel regression in the DSGE model that includes the ZLB and the model that ignores the ZLB when we use government spending shocks and oil demand shocks as explanatory variables. Similar to the case of oil supply shocks, the sign of the coefficient changes for low notional interest rates. When we ignore the ZLB, we find no such change of sign. Again, these shocks have the same effect on oil prices whether or not the ZLB binds because a positive shock always increases demand. Thus, the effects of these shocks on equity returns changes at the ZLB, but the effect on oil prices is unchanged, which again contributes to the change in correlation between oil prices and equity returns.

Consistent with our empirical findings, our DSGE model shows that if monetary policy is constrained by the ZLB, then the correlation between oil and equity prices changes. Moreover, at the ZLB, the sign of the response of equity prices to structural shocks also changes. We next explore if this model prediction holds in the data.

4 Estimating the response to macroeconomic news

4.1 Macroeconomic news surprises

To test the new-Keynesian model developed in the previous section, we need to identify shocks. One challenge in the previous literature is that using quarterly data limits the number of observations. Another challenge is that there is an ongoing debate about the plausibility of identifying assumptions. We avoid some of these issues by looking at macro news at the daily frequency.

We measure macroeconomic news using the same approach that has been well-established

in the empirical literature such as Beechey and Wright (2009) and Kilian and Vega (2011). It is important to note that news about macroeconomic announcements is not what macroeconomists would call a "news shock". A Beaudry/Portier-style news shock, as in Barsky et al. (2014) is information about the *future* state of the world. In contrast, our macroeconomic news announcements provide information about the *current* state of the world. We use survey results from Action Economics as the expected U.S. macroeconomic fundamentals. Macroeconomics news is defined as the difference between the announced realization of the macroeconomic aggregates and the survey expectations. We focus on the variables that Swanson and Williams (2014) use in their analysis of interest rate movements during the zero lower bound period: capacity utilization, consumer confidence, core CPI, GDP (advance), initial claims, ISM manufacturing, leading indicators, new home sales, nonfarm payrolls, core PPI, retail sales excluding autos, and the unemployment rate. Following Swanson and Williams (2014), our regression sample begins in January 1990, when all but two of the surprises are available. Our sample ends in June 2016, three and a half years later than that of Swanson and Williams (2014).

Since the units of measurement differ across the news indicators, we follow the common practice in this literature and normalize the surprise component of each news announcement by its full sample standard deviation. This allows the responses to be comparable across all announcements. Therefore, for each indicator j at time t the surprise component s_{jt} is

$$s_{jt} = \frac{(A_{jt} - E_{jt})}{\sigma_j}$$

where A_{jt} denotes the released value of indicator j and E_{jt} refers to the market's expectation of indicator j prior to the announcement. To calculate σ_j, which is the standard deviation of the surprise component $(A_{jt} - E_{jt})$, we use the entire sample period available for each surprise. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that all positive surprises represent stronger-than-expected growth. Summary statistics for the surprise component of each announcement, $(A_{jt} - E_{jt})$, can be found in Table 3.

As discussed in Beechey and Wright (2009), the response of asset prices to news events occurs very rapidly, often completely adjusting within 15 minutes of the announcement. However, as was also noted in Beechey and Wright, although intradaily regressions provide more efficient estimates of the reactions to news announcements, the daily estimators also were consistent. It would seem reasonable to expect a similar result for oil prices. In addition, by using daily data, our results are most comparable to those reported in Kilian and Vega

(2011).[7] Using high-frequency data, Rosa (2014) reports statistically significant results for the responses of oil prices to macroeconomic news over the 1999-2011 sample. However, he does not consider the role of time-variation, which we emphasize here, and which may explain the difference between the results reported in Rosa and those in Kilian and Vega.

4.2   Sensitivity during the ZLB period

We now test whether the sensitivity to macroeconomic news surprises changes during the ZLB period, as would be predicted by our model. Oil and equity returns are calculated as in Section 2, as 100 times the log-difference in daily prices. For interest rates, our dependent variable is the daily change in basis points for the market yield on U.S. Treasury securities at a constant maturity of 2 years. We also include market yields at 1-year and 10-year constant maturity for comparison.

Our estimation procedure is similar to those found in earlier papers, such as Kilian and Vega (2011). For $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$, we estimate the effect of news surprises using the models

$$Y_t = \alpha + \boldsymbol{\beta}\boldsymbol{s_t} + \varepsilon_t \tag{3}$$

$$Y_t = \alpha + \beta S_t + \varepsilon_t. \tag{4}$$

In the first model, $\boldsymbol{s_t} = \{s_{1t}, ...s_{12t}\}$ and $\boldsymbol{\beta} = \{\beta_1, ..\beta_{12}\}$. Each s_{jt} refers to the standardized macroeconomic news surprise for announcement j on day t, and each β_j measures the response of oil or equity returns to a one standard deviation surprise for the corresponding announcement s_j. In the second model, we pool the news surprises to generate $S_t = \sum_{j=1}^{12} s_{jt}$ and then we estimate β, a summary measure of the effect of a generic one standard deviation surprise. By estimating the daily returns around the time of the announcement, we attempt to isolate the immediate reaction of asset prices to the news announcement as much as possible. As discussed earlier, this strategy has already been applied successfully to numerous financial assets in the literature, including in Andersen et al. (2003) and Kilian and Vega (2011).

Both regression models are estimated using data for only those days on which at least one news announcement was made.[8]

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[7]Studies using higher frequency prices include Halova (2012), which looks at how oil and natural gas respond to news about oil and natural gas inventories.

[8]The regression sample includes all days with at least one announcement and with available data for our dependent variables of interest. For each day in our regression sample, we set $s_{jt} = 0$ for those variables without an announcement on that day. In order to prevent these 0's from biasing the coefficients, the s_{jt} are demeaned using the mean of the s_{jt} in the regression sample. We also considered results with all

To get a baseline estimate for responsiveness to surprises, we first report the estimates for the two models over the pre-ZLB era, which covers 1990q1 through 2009q1. In the pre-ZLB columns of Table 4, the generally small coefficient estimates and lack of statistical significance indicate that both oil and equity prices are not responsive to macroeconomic news. In contrast, the larger coefficients and t-statistics for interest rates indicate their responsiveness to surprises over this period.

In contrast, we find strong responsiveness of oil and equities to surprises during the ZLB period, and diminished responsiveness of interest rates, as reported in the ZLB era columns of Table 4. These results are estimated by restricting the sample to the period when the ZLB is strongly binding, defined as the period when the notional rate implied by the Taylor rule is at or below negative 1. The pooled surprises model estimates underscore these findings: the coefficient estimates in the first row of the table show positive significant effects of surprises during the ZLB era for oil and equities, and a much smaller effect of surprises for interest rates during the ZLB era.

Figure 6 summarizes these results, showing the β's from the pooled and nonpooled regression models for oil, equities, and interest rates.

4.3 Time variation

Andersen et al. (2007) report time variation in the responses of equity prices to macroeconomic news before 2008. Likewise, Swanson and Williams (2014) report that the response of interest rates to macroeconomic news varied before 2008. As such, in this section we consider whether we also find similar time variation in the responses of equity price returns and oil prices to the macroeconomic surprises. A simple way to measure the time variation would be to estimate the model in Equation 4 over rolling samples of one year. However, to smooth out our β estimates, we again apply the time kernel as in Equation 1, and use the model

$$Y_t = \alpha(t) + \boldsymbol{\beta(t)s_t} + \varepsilon_t, \tag{5}$$

which is estimated by solving

$$\left\{ \hat{\alpha}(k), \hat{\beta}(k) \right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{t-k}{h}\right) \left(Y_t - \alpha - \boldsymbol{\beta s_t}\right)^2.$$

We also estimate the analogous time kernel regression for the pooled surprise model ($Y_t = \alpha(t) + \beta(t)S_t + \varepsilon_t$), and find that the two models give similar results. For ease of visualization, we plot the $\beta(t)$ for the pooled surprise model in Figure 7. The $\beta(t)$ for oil and

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non-announcement days included. Making the change did not alter our results.

equities are near zero in the early part of the sample, but move sharply positive around the start of the ZLB era, which is shaded in gray. Unsurprisingly, we find a similar pattern for our alternative measures of the oil price, including the far futures and physical spot prices for WTI crude oil and the nearby futures price for Brent crude oil, as shown in Figure 8a.

The timing of the sharp increase in responsiveness to surprises is also consistent with the finding in Swanson and Williams (2014) that the interest rate becomes less sensitive to macroeconomic news around the ZLB era. We replicate the Swanson and Williams result using the model in equation 5. The bottom panel of Figure 7 plots the estimates of our $\beta(t)$ for the 2-year Treasury yields, and Figure 8b adds the analogous results for 1- and 10-year Treasury yields. As found by Swanson and Williams, we can see that interest rates became less responsive to macroeconomic news in the ZLB era. The results also support the finding that the shorter maturity yields were less responsive to macroeconomic news than the longer maturity yields, based on expectations for how long the ZLB period would last. Additionally, we find that both the 1- and 2-year yields become more responsive to news in 2015, likely as market participants expect the end of the ZLB era.

4.4   Kernel regression using the notional rate

The timing of the variation in the response to macroeconomic news announcements points to the ZLB as one likely driving factor. In this section, we test this hypothesis more directly. Rather than estimating coefficients that vary with time, we now use the same kernel regression setup from Equation 5 to estimate coefficients that vary with other underlying, or controlling variables, Z_t:

$$Y_t = \alpha(Z_t) + \boldsymbol{\beta}(\boldsymbol{Z_t})\boldsymbol{s_t} + \varepsilon_t \tag{6}$$

which is estimated by solving

$$\left\{\hat{\alpha}(Z_k), \hat{\beta}(Z_k)\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{Z_t - Z_k}{h}\right)(Y_t - \alpha - \boldsymbol{\beta}\boldsymbol{s_t})^2.$$

In particular, we can estimate how the responsiveness to surprises changes based our estimate of the notional rate from Section 3. When using the notional rate as the kernel variable, $Z_t = \tilde{R}_t$, the coefficients $\hat{\boldsymbol{\beta}}(\tilde{R}_k)$ are estimated by placing more weight on the observed responses to surprises on days when $\tilde{R}_t$ is close to $\tilde{R}_k$. Figure 9a plots the result of the pooled surprises estimation. The results provide direct evidence of the higher sensitivity of oil and equities to macroeconomic news announcements during periods with lower notional rates, and the higher sensitivity of interest rates to macroeconomic news announcements during periods with higher notional rates.

Next, to get a sense of the economic and statistical significance of the improvement in fit, we compare the sum of squared residuals for the unrestricted model in Equation 6 to that of the restricted model, in which the coefficients α and $\boldsymbol{\beta}$ do not vary. The restricted model is estimated using a standard regression of our dependent variables on the surprises, and is equivalent to a kernel regression in which the controlling variable Z_t is equal to a constant for all t. The sum of squared residuals for the restricted (SSR) and unrestricted (SSR(Z)) models are constructed as

$$
\begin{aligned}
SSR(Z) &= \sum_t \left(Y_t - \hat{\alpha}(Z_t) - \hat{\boldsymbol{\beta}}(Z_t)\boldsymbol{s_t} \right)^2 \\
SSR &= \sum_t \left(Y_t - \hat{\alpha} - \hat{\boldsymbol{\beta}}\boldsymbol{s_t} \right)^2 .
\end{aligned}
$$

Our first comparison in Figure 10 illustrates the economic significance of our results by reporting the percent improvement in model fit, calculated as $\frac{SSR - SSR(Z)}{SSR}$. The first bar shows that, when averaging across our three dependent variables of interest, the model fit is improved by just over 3 percent by including the notional rate.

Next, to test for statistical significance of these results, we construct an F-statistic as follows:

$$
F(Z) = \frac{SSR - SSR(Z)}{SSR(Z)}.
$$

To determine the associated p-value, we compare this F-statistic to a distribution of F^{sim} generated using a wild bootstrap procedure. To generate the simulated distribution, we run 1000 simulations. For each simulation i, we use the restricted model estimates for $\hat{\alpha}$, $\hat{\boldsymbol{\beta}}$, and $\hat{\varepsilon}_t$ to generate: $Y_{it}^{sim} = \hat{\alpha} - \hat{\boldsymbol{\beta}}\boldsymbol{s_t} + \nu_{it} * \hat{\varepsilon}_t$. Note that the Y_{it}^{sim} preserves any existing serial correlation in the explanatory variables by leaving the $\boldsymbol{s_t}$ variable fixed and preserves heteroscedasticity by scaling the residuals $\hat{\varepsilon}_t$ by $\nu_{it} = 1 - 2B_{it}$, where B_{it} is a Bernoulli random variable, $B_{it} \sim B(1, 0.5)$.[9] Using these Y_{it}^{sim}, we estimate both the restricted and unrestricted models, and generate the resulting distribution of F-statistics. Finally, we use this distribution to determine how frequently one would observe in this simulated distribution the empirical F-statistic computed using the actual data.

Using this F-statistic and simulated distribution, we test the null hypothesis that the restricted model, in which the coefficients α and $\boldsymbol{\beta}$ do not vary, is equivalent to the unrestricted model, in which the coefficients are allowed to vary with the notional rate, $\tilde{R}_k$. We find statistically significant improvement in model fit for all three of our dependent vari-

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[9]This choice of scaling variable ν_{it} is based on Davidson and Flachaire (2008). If we were to use a scaling variable with a standard normal distribution instead, the fourth moment of the simulated residuals would be artificially exaggerated.

ables. The p-values for oil, equities, and interest rates are 0.095, 0.005, and less than 0.001, respectively.

4.5   The Shadow Rate

The previous section provides strong evidence that oil and equities are more sensitive to macroeconomic news surprises when the notional rate is negative. We now turn to testing alternative hypotheses for these findings. In this section, we test whether the Wu and Xia (2016) shadow rate is a better measure of how constrained monetary policy is by the ZLB. In the next section, we test whether the observed relationships are associated with other market conditions that were roughly coincident with the ZLB period, including elevated market uncertainty and increasing financialization of the oil market.

The Wu-Xia shadow rate is a market-implied driver of the short-term rate that is allowed to be negative during the ZLB period. The shadow rate is estimated using a dynamic term structure model and thus incorporates information from observed longer-term rates during the ZLB era with the historical relationship between short and longer-term rates. As seen in Figure 11a, in contrast to our Taylor rule implied notional rate, the Wu-Xia measure is positive in 2009 and most negative in 2014. The factors affecting this rate include the monetary policy rate, the expected time at the ZLB, and various premia. In particular, unconventional monetary policy (UMP) can lower the shadow rate whereas it would not have the same direct effect on the Taylor-rule implied notional rate.

One potential benefit of using the shadow rate in our kernel regressions–as opposed to our notional rate implied by the Taylor rule–is that it might better capture the effect of unconventional monetary policy (UMP) through observed longer-term rates. In theory, the use of the shadow rate in the kernel regression should help us examine the extent to which UMP is a substitute for interest rate policy. If UMP is a fully effective substitute, it would be equivalent to the model in Section 3 in which the ZLB is not binding, and we should not see heightened sensitivity to shocks in the ZLB period.[10] However, if UMP is only somewhat effective, we would see some additional sensitivity to shocks.

We re-estimate the kernel regression of our three dependent variables on macroeconomic news surprises in Equation 6 using the shadow rate as the controlling variable, and plot the results in Figure 9b. Although we find higher sensitivity of interest rates to macroeconomic news surprises during periods with higher shadow rates, the results for oil and equities are nonmonotonic. Oil and equities sensitivity to surprises increases as the shadow rate falls below zero, but sensitivity then decreases again as the shadow rate gets more negative. The

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[10]Wu and Xia (2016) provide a discussion of how UMP undoes the constraints implied by the ZLB.

rise in sensitivity as the shadow rate first goes negative indicates that the market appears to interpret UMP as being either ineffective or too slow moving to counter macroeconomic news announcements. In contrast, the reduced sensitivity to shocks as the shadow rate gets more negative suggests that UMP is believed to be more effective or flexible during this period.

We also test the model with the shadow rate against the alternative in which the sensitivity to news surprises does not vary. As with the notional rate, we find for the shadow rate that we have about 3 percent improvement in model fit, on average, as seen in the second group of bars in Figure 10. We also have statistically significant improvement in model fit for equity and interest rate responsiveness to surprises, though not for oil. The p-values for oil, equities, and interest rates are 0.172, 0.013, and less than 0.001, respectively.

Finally, we estimate a model that allows the coefficients on the surprises to vary with both the notional rate and the shadow rate.[11] Using this model, we test the null hypotheses that a model including the two rates is equivalent to a model including just the notional rate or just the shadow rate. We find that in general, once the model coefficients are allowed to vary with one of the two rates, the inclusion of the second rate does not result in a statistically significant improvement in model fit. As such, the two rates appear to be substitutes for each other.

In general, the new-Keynesian model suggests that variation in sensitivity to shocks should vary according to whether the monetary authority can fully respond to shocks. Results showing higher sensitivity to shocks when the Taylor rule goes below zero imply that the negative notional rate is a measure of a constrained monetary authority. Under this theory, the Taylor rule estimates tell us how close the notional rate is to "normal policy," and imply how long interest rates are likely to be at the ZLB, and thus non-responsive.

4.6   Uncertainty and financialization

Turning back to alternative theories for the evidence presented thus far, according to the common folk wisdom that all correlations go to one in a crisis, increased uncertainty could be an alternative driver of the elevated oil-equity correlation during the ZLB period as well as the increase responsiveness to macroeconomic news surprises. To test this theory, we use three different measures of uncertainty. First, we use the 90-day moving average of the daily series for economic policy uncertainty from Baker et al. (2015). Second, we use the 90-day horizon measure of financial uncertainty from Jurado et al. (2015). Finally, we use the 90-day

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[11]When using two controlling variables, the coefficients in the model $Y_t = \alpha(Z_{1t}, Z_{2t}) + \boldsymbol{\beta}(\boldsymbol{Z_{1t}}, \boldsymbol{Z_{2t}})\boldsymbol{s_t} + \varepsilon_t$ are estimated by solving $\left\{\hat{\alpha}(Z_{1k}, Z_{2k}), \hat{\beta}(Z_{1k}, Z_{2k})\right\} = \arg\min_{\alpha,\beta} \sum_t \phi\left(\frac{Z_{1t}-Z_{1k}}{h_1}\right) \phi\left(\frac{Z_{2t}-Z_{2k}}{h_2}\right) (Y_t - \alpha - \boldsymbol{\beta}\boldsymbol{s_t})^2.$

moving averages of the VIX, which is a measure of options-implied stock market volatility. According to this measure, market uncertainty began rising in 2007, spiked sharply in 2008 at the height of the financial crisis, and remained elevated for a few years after that. All three of these measures are depicted in Figure 11b.

A second alternative hypothesis is that with increased financialization of oil markets, the greater overlap between oil market and other financial market participants resulted in greater sensitivity of the financial markets for oil to general market conditions. According to this theory, the oil market would react much more strongly to events that earlier would have moved only equity markets. We capture this trend by measuring the 90-day rolling average of the open interest across all futures contracts for WTI oil on NYMEX, as depicted in Figure 11c.

To test these alternative hypotheses, we reestimate the kernel regression of our three dependent variables on macroeconomic news surprises in Equation 6 using each of our alternative controlling variables in turn. We test each of these models against the alternative in which the sensitivity to news surprises does not vary. We also estimate the models using pairs of controlling variables. We test the null hypothesis that a model including the notional rate along with one of the alternative controlling variables is equivalent to a model including just the notional rate or just the alternative controlling variable. Figure 10 depicts the improvement in model fit from including each of these kernel variables against the restricted model alternative. In general, we find smaller improvements in model fit when using these alternative variables.

Table 5 summarizes the hypotheses being tested and the p-values that result from the wild bootstrap procedure for each test. In Panel B1 of Table 5, we generally find that the inclusion of the alternative variables tends to not improve the fit of the model in a statistically significant way against the restricted alternative in which the coefficients are non-varying. The exception is for the inclusion of open interest, which improves model fit for all three dependent variables. In Panels B2 and B3, we find that when allowing the coefficients to vary with the notional rate or the shadow rate, the addition of a second controlling variable sometimes provides a statistically significant improvement in model fit, especially for the interest rate. Lastly, we find in Panels A1 and A2 that even after including the alternative kernel variables, the addition of the notional or shadow rate to the kernel generally results in a statistically significant improvement in model fit. In conclusion, we find that the variation in sensitivity to macroeconomic news surprises for oil, equities, and interest rates is better explained by our measures of constrained monetary policy than by the alternative measures of uncertainty and oil market financialization.

5   Additional Evidence: A Structural VAR

We now explore the shocks behind the increased correlation between oil and equity returns. Although we have shown that oil and equity prices have become more responsive to macroeconomic data announcements, one might wonder if other sources of shocks such as oil supply shocks or aggregate demand shocks are resulting in the change in correlation. In particular, as discussed in Kilian and Park (2009) the source of the shock can determine whether oil price increases are associated with increases or decreases in equity prices. To explore this hypothesis, we use a structural VAR to decompose the observed changes in correlation. The VAR is similar in to Kilian and Park (2009), featuring the four monthly variables: change in global oil production Δq, Kilian's detrended level of global real economic activity A, changes in the real oil price ΔP, and changes in real U.S. equity returns Δs. Given our interest in the correlation between changes in oil prices and equity returns, we estimate the VAR using the first difference of real oil prices, whereas Kilian and Park (2009) estimate their VAR using the level of real oil prices. Using the VAR in levels did not change our results. We identify shocks to the structural VAR using a standard recursiveness assumption, with the following identifying assumptions:

Identifying Assumptions in the Structural VAR

Shock	Immediate Response	Delayed Response
Oil Supply Shock	Δs, P, A, Δq	
Aggregate Demand Shock	Δs, P, A	Δq
Oil Residual Shock	Δs, P	A, Δq
Equity Residual Shock	Δs	P, A, Δq

We remain agnostic about the final two shocks, labeling them as generic residual shocks to the relevant variables. The estimated linear vector autoregression implies the following moving average representation for the h-step ahead forecast errors,

$$y_{t+h} - y_{t+h|t} = \sum_{i=0}^{h-1} \Theta_i w_{t+h-i},$$

where Θ_i is a 4 by 4 matrix of moving average coefficients implied by the estimated VAR and structural factorization of the variance covariance matrix of the reduced-form residuals.

Rather than discussing impulse responses to the structural shock, we instead focus on our paper's central question of what causes changes in the correlation between oil price changes and equity returns. To do so we decompose the correlation into its contribution from the various structural shocks. Although a covariance decomposition is seldom featured in the

literature, to calculate a covariance decomposition is just a straightforward application of textbook calculations for a variance decomposition (see Hamilton (1994)).

This calculation require two kinds of matrices. The first is $MSPE(h)$ the value of the h-step ahead forecast variance covariance matrix conditional on all shocks.

$$
\begin{aligned}
MSPE(h) &= E\left(\left(y_{t+h} - y_{t+h|t}\right)\left(y_{t+h} - y_{t+h|t}\right)'\right) \\
&= \sum_{i=0}^{h-1} \Theta_i I \Theta_i'
\end{aligned}
$$

where I is the identify matrix.

The second is $MSPE_j(h)$,the h-step ahead forecast variance covariance matrix conditional on only the j-shock

$$
MSPE_j(h) = \sum_{i=0}^{h-1} \Theta_i E_j \Theta_i',
$$

where all elements of E_j are equal to zero except the j-th, j-th element, which equals one.

The correlation between oil price changes and equity returns can be defined using terms from these two matrices. In particular, define $\sigma_p(h)$ as the square root of the 3,3 element of $MSPE(h)$, $\sigma_s(h)$ as the square root of the 4,4 element of $MSPE(h)$, and $\sigma_{ps}(h)$: the 3,4 element of $MSPE(h)$. Furthermore, define $\sigma_{ps,j}(h)$ the covariance conditional just on shock j as the 3,4 element of $MSPE_j(h)$. Having defined there terms, we can then write the correlation between oil price changes and equity returns $\rho_{ps}(h)$ as the following equation:

$$
\begin{aligned}
\rho_{ps}(h) &= \frac{\sigma_{ps}(h)}{\sigma_p(h)\,\sigma_s(h)} \\
&= \sum_{j=1} \frac{\sigma_{ps,j}(h)}{\sigma_p(h)\,\sigma_s(h)}.
\end{aligned}
$$

For any shock, the larger is $\sigma_{ps,j}(h)$, the larger is the contribution of the j-th shock to the overall correlation.

Table 6 reports our results. Using the break date based on the observed change in the oil-equity correlation reported earlier, we estimate the VAR using two samples. Consistent with our results for daily data, the overall correlation $\rho_{ps}(h)$ for monthly data increases from negative 0.10 in the early sample to positive 0.25 in the late sample. As reported in Table 6, the correlation change was not because aggregate demand or supply shocks became more important. Instead, consistent with our ZLB-driven hypothesis, shocks that have an

immediate impact on oil prices but not oil demand and supply went from causing a negative correlation to a positive correlation. This 36 point change in the correlation is almost entirely due to the oil residual shock, which has been negative 11 points before 2008 and has been positive 19 points thereafter. The negative effect of these shocks in the pre-ZLB sample is consistent with the empirical results reported in Kilian and Park (2009), whereas the positive effect in the second sample is consistent with how the ZLB alters the response in our DSGE model discussed in Section 3.

6   Conclusion

Starting in late 2008, the correlation between oil price changes and equity returns, which previously had been either small or negative, increased dramatically and remained elevated thereafter. Our main argument is that this correlation change is evidence that the ZLB alters the dynamic behavior of the economy. Our argument is supported by the following observations. First and most obviously, the ZLB becoming binding and the increase in correlation occur at the same time. Second, this jump in correlation is consistent with a standard new-Keynesian model where the ZLB is binding. Third, consistent with theory, we show that oil prices and equity returns became more responsive to economic news announcements when the ZLB binds. Fourth, we consider alternative hypotheses that could alter the responsiveness of oil prices and equity returns related to financialization and uncertainty. Our empirical evidence is that these alternative explanations do not improve the fit relative to conditioning on the ZLB. Finally, we use a structural VAR on monthly data and again find evidence supportive of our ZLB-driven hypothesis.

As such, our results complement and extend the findings of Swanson and Williams for interest rates by showing that activity measures such as oil prices and equity prices are also affected by the ZLB . Our findings are more supportive than some previous empirical work that the ZLB alters the economic environment. We would argue that our results should be preferred given our use of daily data with clearly identified shocks. Our shocks also have the desirable property that they have larger effects than results reported in other studies. For example, a one standard deviation surprise in nonfarm payrolls is associated with an almost 1 percent increase in oil prices, and a 0.2 percent increase in equity prices. These responses are much larger than those reported Wieland (2015) for oil supply shocks. For example, our estimated same-day oil-price response to a one-standard deviation surprise in nonfarm payrolls is similar in magnitude to the Wieland's estimated six month response to a one-standard deviation oil-supply shock.

There is now a large theoretical literature that shows that the ZLB theoretically changes

the dynamic behavior of the economy. However, empirical macroeconomic evidence that the ZLB actually causes these changes has been scarce. The scarcity of evidence for the United States likely reflects that interest rates were well above zero until 2008, and consequently the number of monthly or quarterly observations of macroeconomic variables available to study is small. Our study links the predictions of the new-Keynesian model to our results from daily data, and as such provides further empirical evidence in favor of our model's predictions. As a result, our paper should help motivate further exploration of the effects of the ZLB and related policy implications.

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A   Tables and Figures

Figure 1: Oil and Equities Prices

Panel (a): Rolling correlation between daily oil and equity returns. The date axis marks the end of the one-year rolling window over which the correlation is calculated. Panel (b): Oil price of the front-month futures contract for WTI crude oil in dollars per barrel and level of the equity price index, indexed to April 5, 1983 = 100.

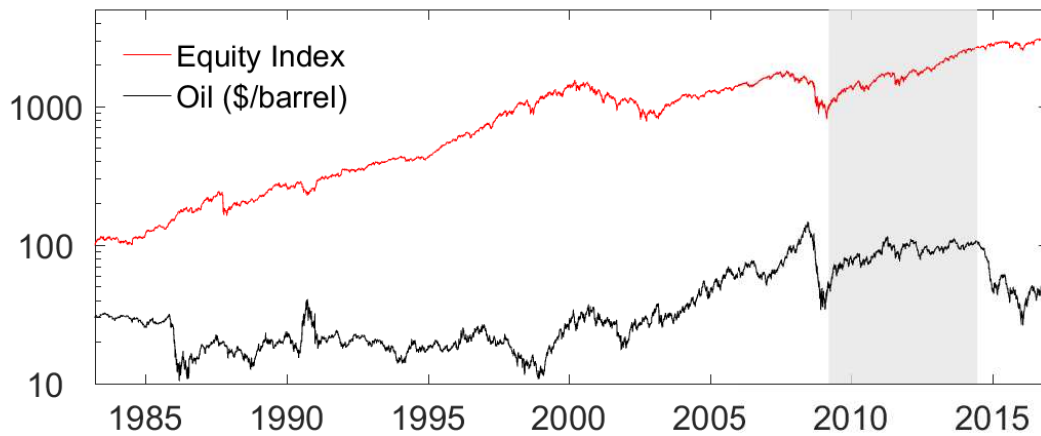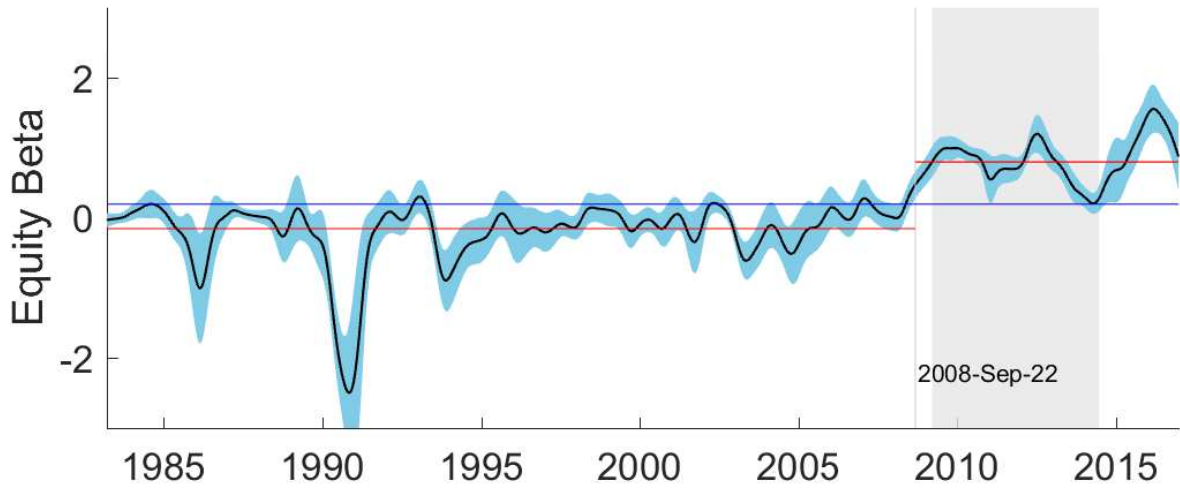(a) Oil-Equity Rolling Correlation
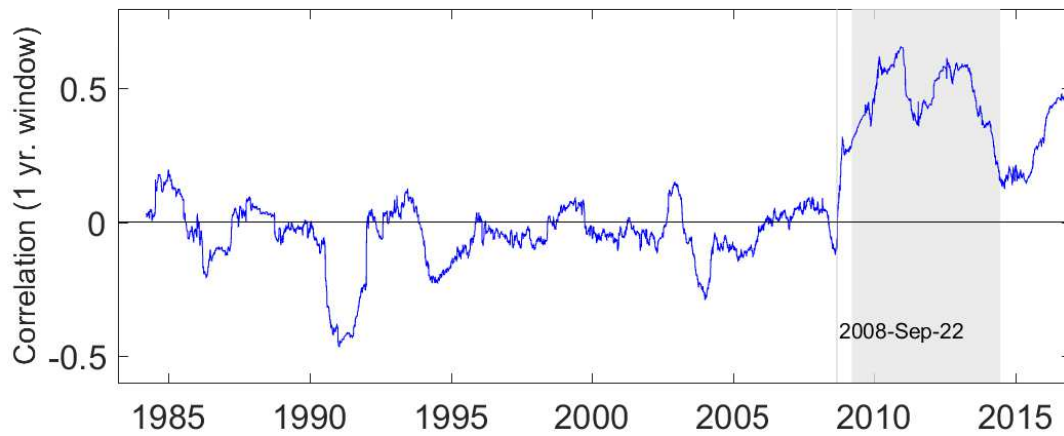
(b) Oil and Equity Prices

Figure 2: Time Varying Equity Betas

Estimate of $\beta(t)$ for oil and metals returns using the kernel regression in Equation 1, $Y_t = \alpha(t) + \beta(t)Equity_t + \varepsilon_t$. The shaded region represents a 90 percent confidence interval for the estimated $\beta(t)$ based on the wild bootstrap as described in Section 4.4.
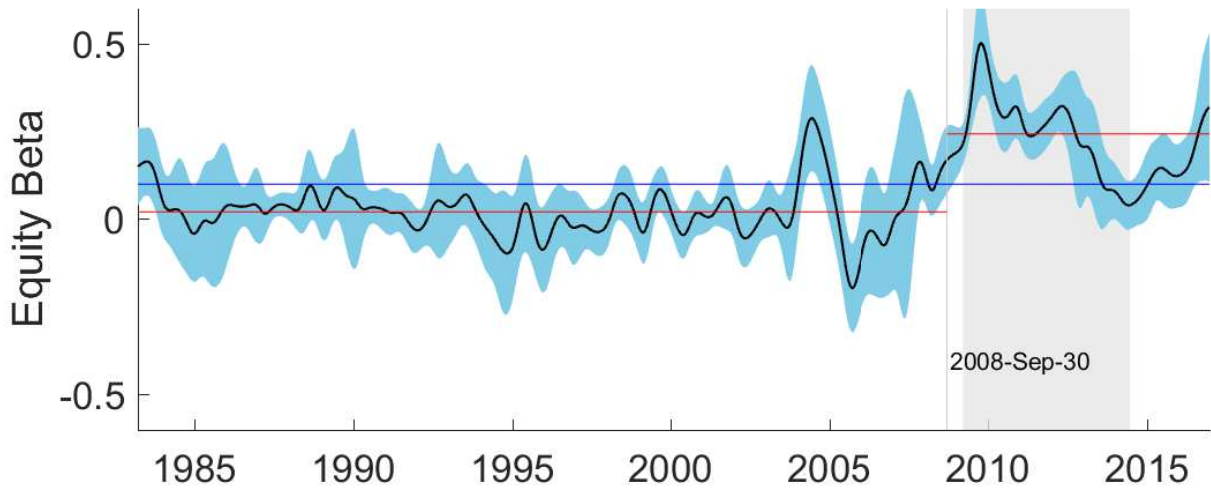
(a) Oil

(b) Metals

Figure 3: Notional Interest Rate

Panel (a): The federal funds rate and the notional interest rate implied by the modified Taylor rule in Equation 2 ((Bernanke, April 28, 2015)). The notional interest rate is intended to capture the target federal funds rate as implied by the current state of the economy, without censoring due to the zero lower bound. Panel (b): Estimate of $\beta(\tilde{R}_k)$ using the kernel regression in Equation 2, $Oil_t = \alpha(\tilde{R}_t) + \beta(\tilde{R}_t)Equity_t + \varepsilon_t$. The shaded region represents a 90 percent confidence interval for the estimated $\beta(\tilde{R}_t)$ based on the wild bootstrap as described in Section 4.4.
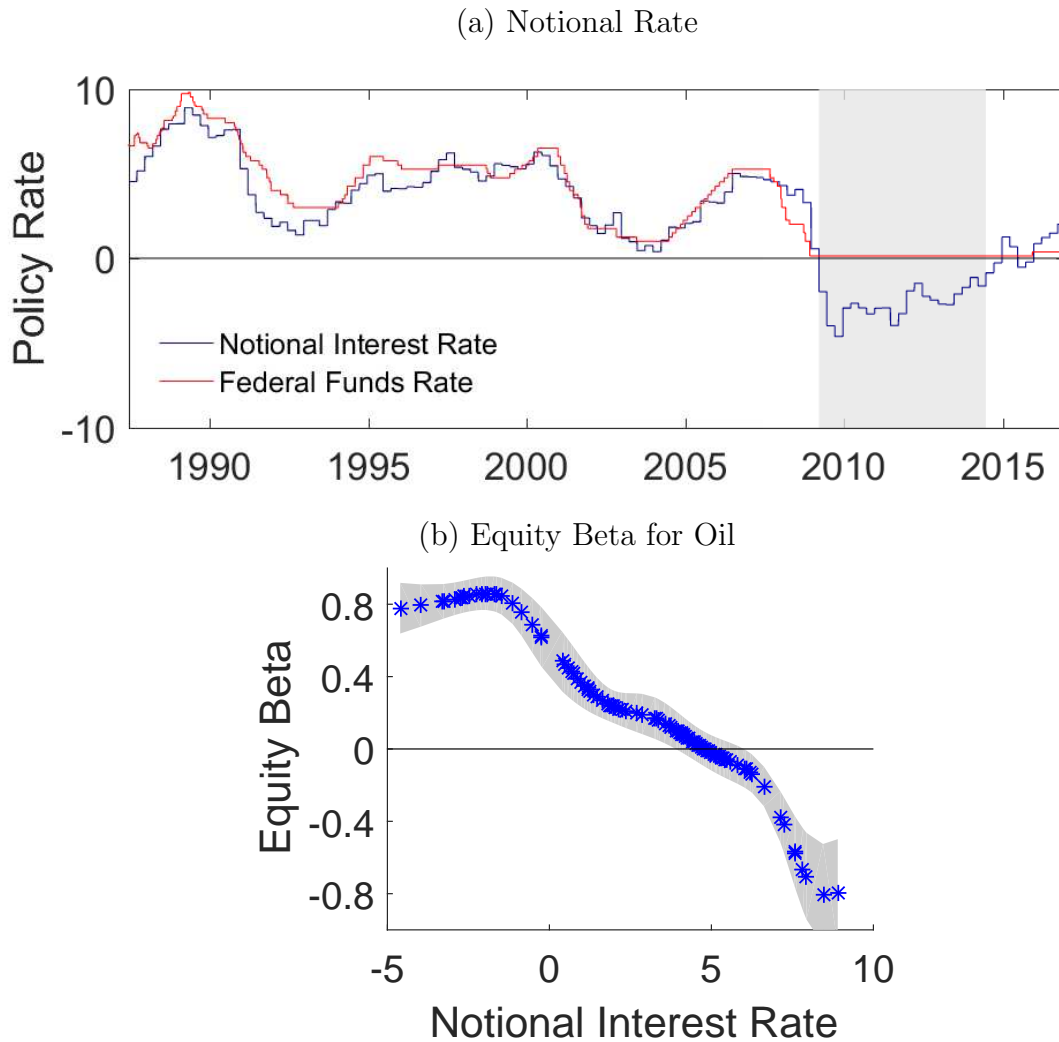
(a) Notional Rate

(b) Equity Beta for Oil

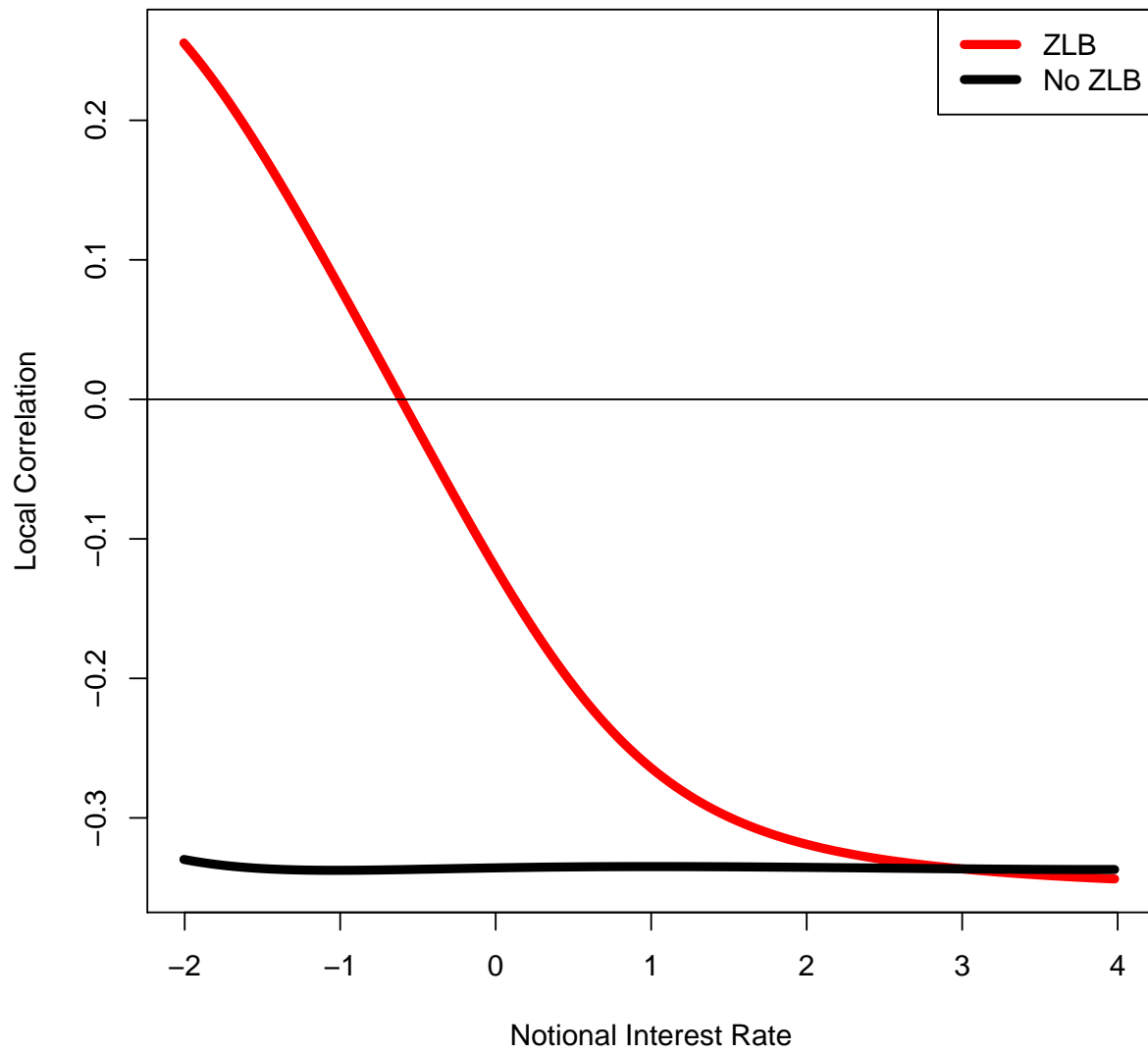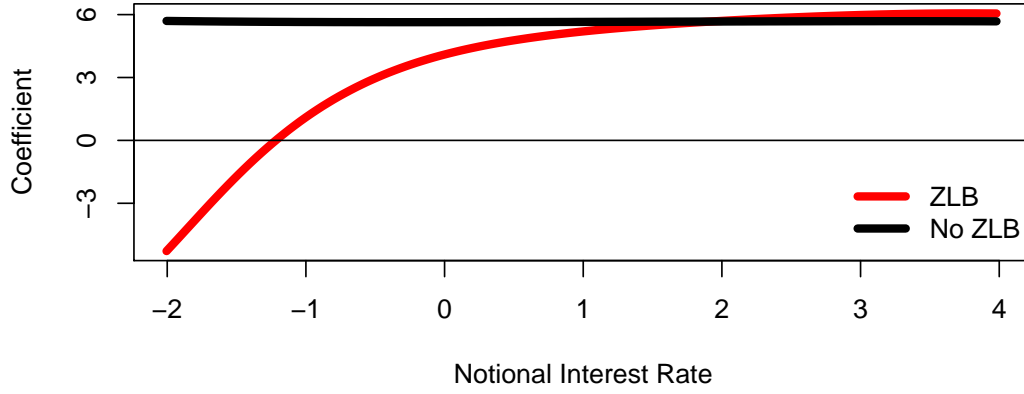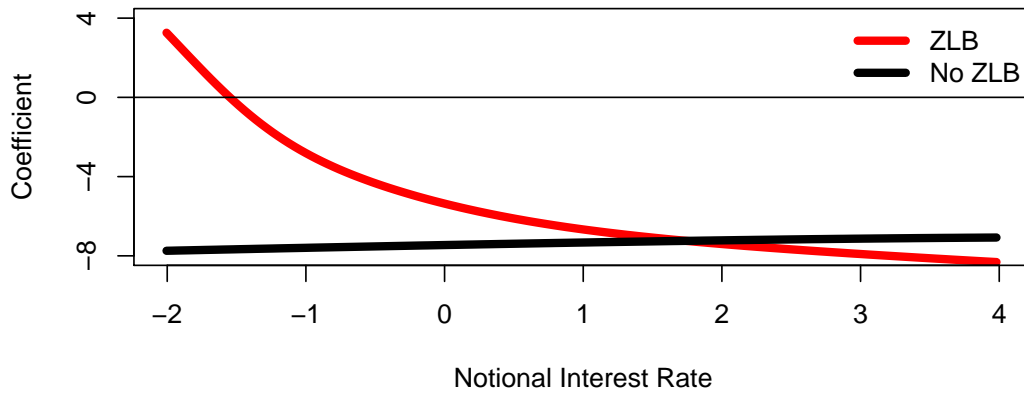Figure 4: Local Correlation Oil Price Change and Equity Return

Figure 5: Equity Return Kernel Regression

(a) Coefficient on Oil Supply Shock

(b) Coefficient on Government Spending Shock

(c) Coefficient on Oil Demand Shock
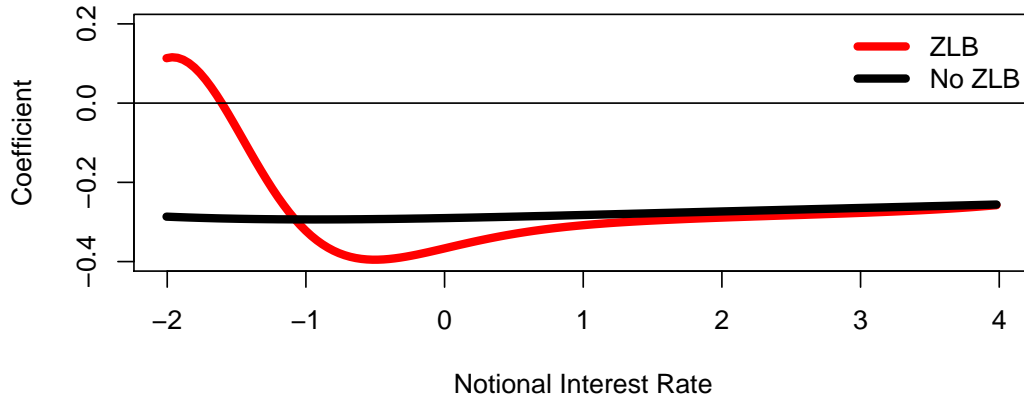
Figure 6: Responsiveness to Surprises in the ZLB era

The bars represent the estimated coefficients from the pre-ZLB (1990q1-2009q1) and ZLB eras (2009q2-2014q2) for the regression $Y_t = \alpha + \boldsymbol{\beta s_t} + \varepsilon_t$, where $\boldsymbol{s_t}$ is the vector of standardized and demeaned news on day t. The announcement codes are defined in Table 4. The final pair of bars in each panel represents the β from the regression using pooled surprises, $Y_t = \alpha + \beta S_t + \varepsilon_t$, where $S_t = \sum_{j=1}^{12} s_{jt}$. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that positive surprises represent a stronger-than-expected economy. See Section 4.2 for more detail, and Table 4 for the associated regression statistics.

(a) Oil

(b) Equity

(c) Interest Rate

Figure 7: Time Varying Responsiveness to Surprises

For each dependent variable $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$, we plot the β estimated from the model $Y_t = \alpha(t) + \beta(t)S_t + \varepsilon_t$. The blue shaded region represents a 90 percent confidence interval for the estimated $\beta(t)$ based on the wild bootstrap as described in Section 4.4, and the gray shaded region represents the ZLB period, defined as the period during which the Taylor rule notional rate is less than negative 1.

(a) Oil

(b) Equity

(c) Interest Rate

41
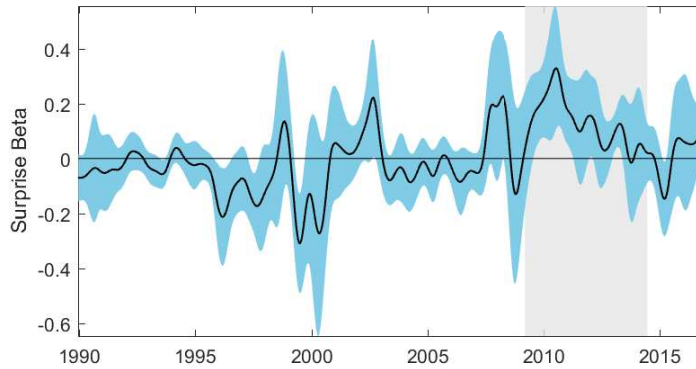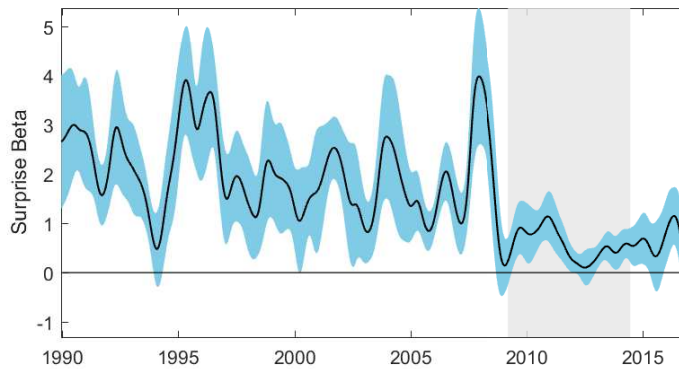
Figure 8: Responsiveness to Surprises

For each dependent variable, Y_t, we plot the $\beta(t)$ estimated using the kernel regression $Y_t = \alpha(t) + \beta(t)S_t + \varepsilon_t$. The gray shaded region represents the ZLB period, defined as the period during which the Taylor rule notional rate is less than negative 1.
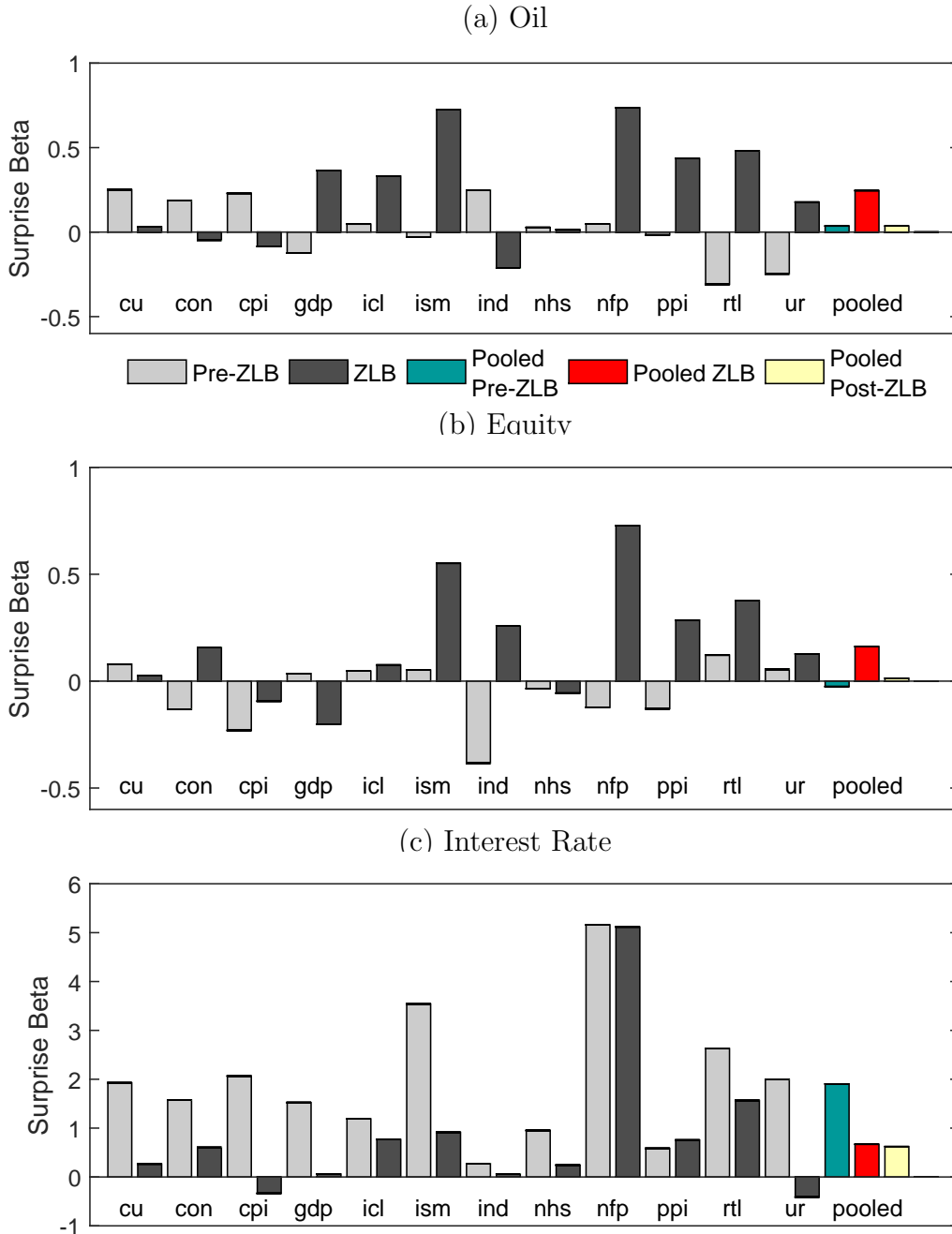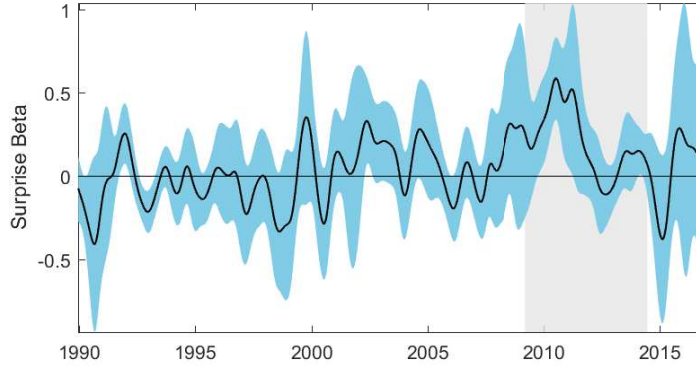
(a) Oil Prices
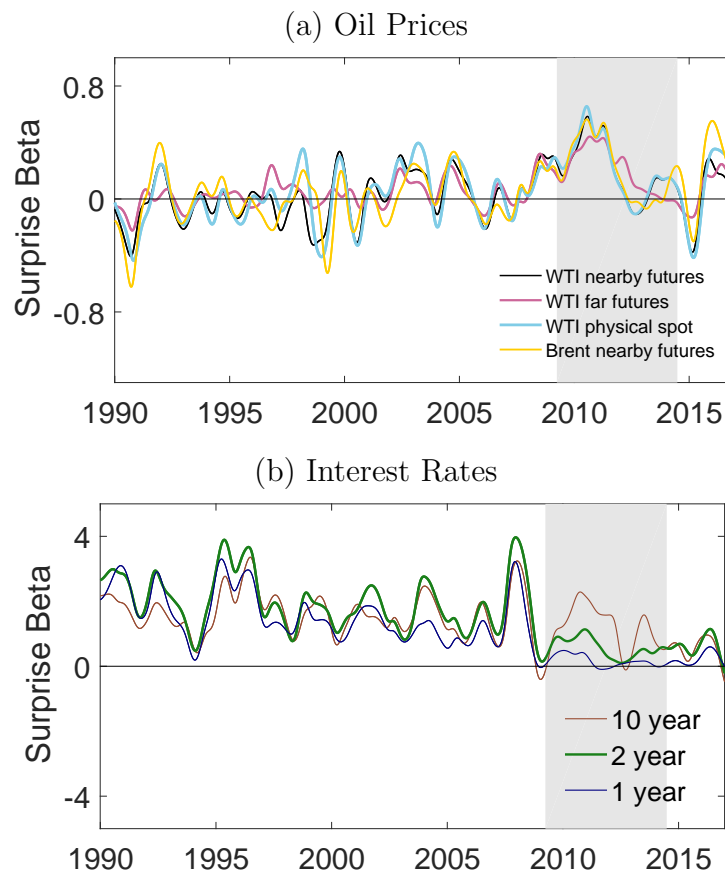
(b) Interest Rates

Figure 9: Varying Responsiveness to Surprises

For each of our dependent variables, $Y_t \in \{Oil_t, Equity_t, InterestRate_t\}$, we estimate the regression $Y_t = \alpha(Z_t) + \beta(Z_t)S_t + \varepsilon_t$, using the notional interest rate (Panel (a)) and then the shadow rate (Panel (b)) as the controlling variable in the kernel regression. The blue shaded region represents a 90 percent confidence interval for the estimated $\beta(Z_t)$ based on the wild bootstrap.

(a) Notional Interest Rate

(b) Shadow Rate
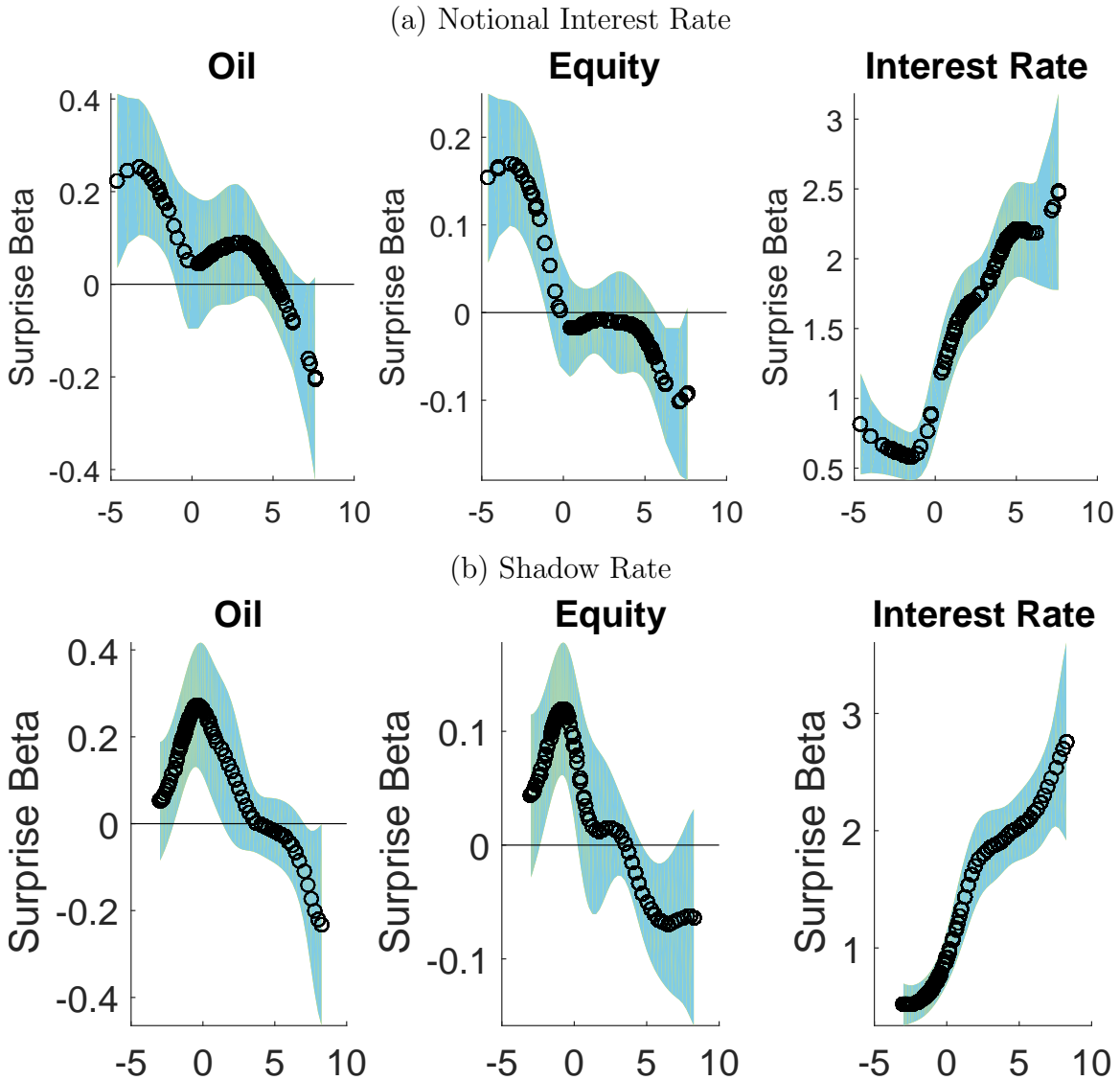
Figure 10: Improvement in Model Fit

These bar charts represent the percent improvement in model fit for the unrestricted model, relative to that of the restricted model, in which the coefficients do not vary. Panel (a) represents the results for the unrestricted model for nonpooled surprises in Equation 6, $Y_t = \alpha(Z_t) + \boldsymbol{\beta}(\boldsymbol{Z_t})\boldsymbol{s_t} + \varepsilon_t$, while Panel (b) represents the results for the unrestricted model with pooled surprises, $Y_t = \alpha + \beta S_t + \varepsilon_t$, where $S_t = \sum_{j=1}^{12} s_{jt}$. These improvements in model fit are calculated for each pairwise combination of dependent variable and controlling variable (as listed in Table 3), and the average improvement for the three dependent variables is also shown for each controlling variable.

(a) Nonpooled Surprises
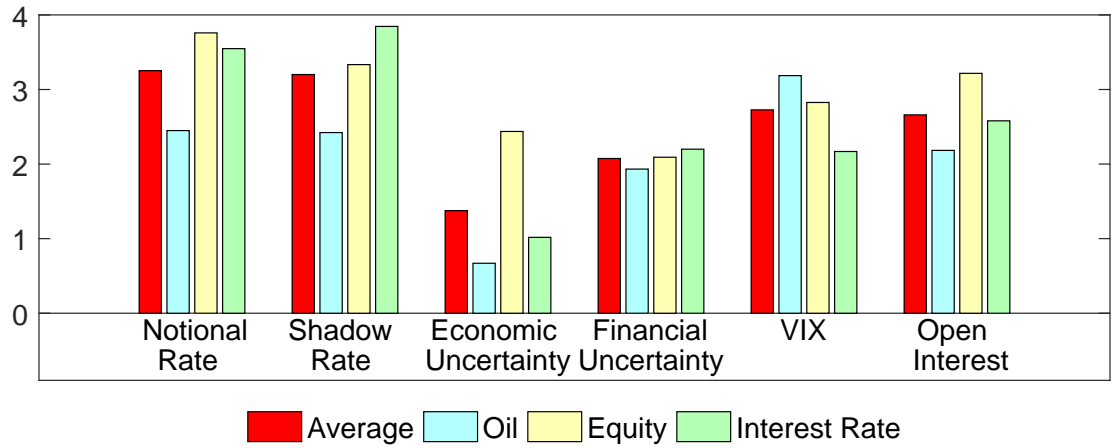
(b) Pooled Surprises

44
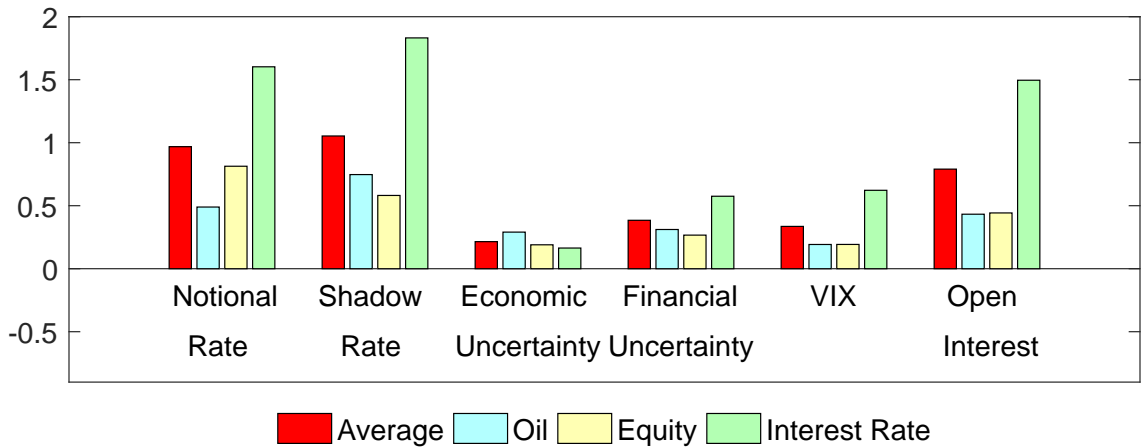
Figure 11: Alternative Theories

Panel (a): Policy rate measures include the Wu and Xia (2016) shadow rate, the notional interest rate implied by the modified Taylor rule in Equation 2 ((Bernanke, April 28, 2015)), and the federal funds rate. Panel (b): Uncertainty measures include the 90-day moving average of the daily series for economic policy uncertainty from Baker et al. (2015), the 90-day horizon measure of financial uncertainty from Jurado et al. (2015), and the 90-day moving average of the VIX. The economic policy uncertainty measure is multiplied by 100 in this panel for ease of comparison to the other two series. Panel (c): The financialization measure is the 90-day moving average of open interest across all maturities of WTI futures contracts, expressed in millions of contracts.

(a) Policy Rates
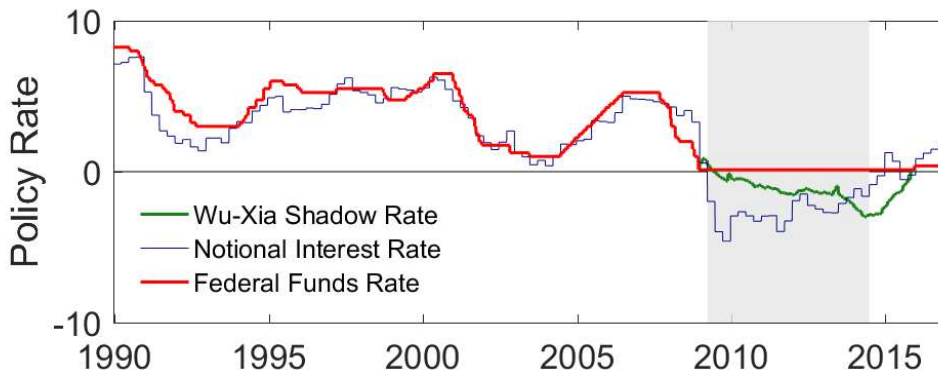
(b) Uncertainty
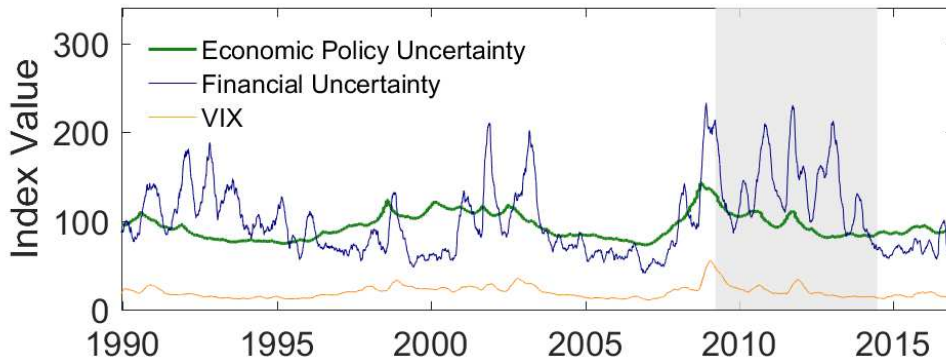
(c) Financialization

Table 1: Summary Statistics

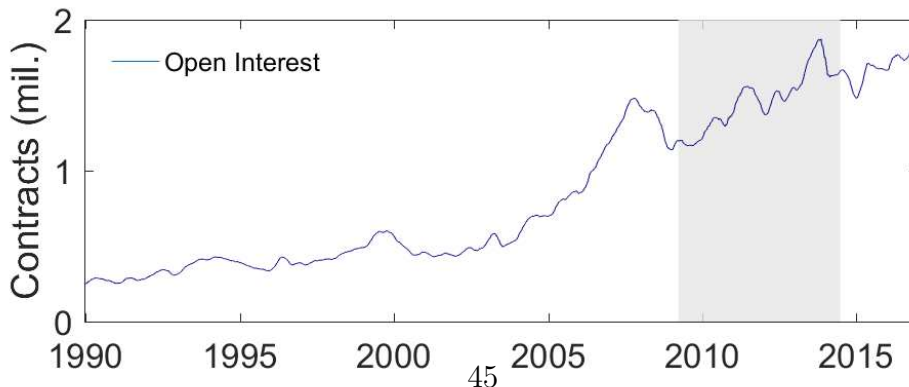| Variable	Obs.	Start Date	Mean	Std. Dev.	Min.	Max.
Panel A: Primary Variables of Interest						
Oil returns (WTI nearby futures)	8339	1983-Apr-06	0.01	2.42	-40.05	22.80
Equity returns	8339	1983-Apr-06	0.04	1.11	-19.13	9.89
Δ Interest rate (2 year)	8339	1983-Apr-06	-0.10	6.22	-84.00	38.00
Panel B: Alternative Measures						
WTI physical spot returns	7665	1986-Jan-03	0.01	2.58	-40.64	21.70
Brent nearby futures returns	8143	1983-May-17	0.00	2.34	-40.71	27.82
WTI far futures returns	8339	1983-Apr-06	0.01	1.38	-10.35	10.80
Metals returns	8339	1983-Apr-06	0.01	0.86	-10.29	9.40
Δ Interest rate (1 year)	8339	1983-Apr-06	-0.10	5.52	-83.00	52.00
Δ Interest rate (10 year)	8339	1983-Apr-06	-0.10	6.45	-75.00	39.00
Panel C: Equity Sector Returns						
Consumer nondurables	8339	1983-Apr-06	0.05	0.96	-18.67	8.83
Consumer durables	8339	1983-Apr-06	0.03	1.48	-20.27	9.12
Manufacturing	8339	1983-Apr-06	0.05	1.23	-22.61	9.55
Energy	8339	1983-Apr-06	0.04	1.47	-21.60	17.24
Chemicals	8339	1983-Apr-06	0.05	1.11	-21.32	9.40
Business equipment	8339	1983-Apr-06	0.04	1.56	-22.43	14.96
Telecommunications	8339	1983-Apr-06	0.04	1.24	-18.26	13.18
Utilities	8339	1983-Apr-06	0.04	0.97	-13.77	12.67
Shops	8339	1983-Apr-06	0.04	1.16	-18.32	10.43
Healthcare	8339	1983-Apr-06	0.05	1.17	-19.71	10.29
Finance	8339	1983-Apr-06	0.04	1.44	-16.06	15.61
Other	8339	1983-Apr-06	0.03	1.20	-18.13	9.47
S&P 500 excl. energy	4701	1998-Jan-02	0.02	1.25	-9.11	10.10

Notes: Oil, equity, and metals returns are calculated as log-differenced prices times 100. The price of oil is the closing value, in dollars per barrel, of the front-month futures contract for West Texas Intermediate (WTI) crude oil for delivery in Cushing, Oklahoma obtained from NYMEX. Equity returns are the Fama-French value-weighted daily return on all NYSE, AMEX, and NASDAQ stocks. Interest rate differences are expressed in basis points. The WTI physical spot price is the daily spot price for WTI crude oil for delivery (freight on board) in Cushing, Oklahoma, as reported by the U.S. Energy Information Administration. The Brent price is the nearby futures price on the ICE exchange, obtained from Bloomberg. The WTI far futures price is defined as the price of the furthest available December contract for WTI crude oil, and is obtained from NYMEX. The metals price is the metals spot index constructed by the Commodities Research Bureau. The 12 industry-specific equity indexes are obtained from the Fama-French data library. The S&P 500 Ex-Energy index is obtained from Bloomberg (Ticker: SPXXEGP).

Table 2: Chow Test Results

		Full Sample		Pre-Break		Post-Break	
	Obs.	Break Date	β	t-stat	β	t-stat	β	t-stat

(Note: column layout — Obs., Break Date, then Full Sample (β, t-stat), Pre-Break (β, t-stat), Post-Break (β, t-stat))

	Obs.	Break Date	β	t-stat	β	t-stat	β	t-stat
Panel A: Regressions on the Full Index of Equity Returns								
WTI nearby futures returns	8339	2008-Sep-22	0.19	(8.04)	-0.16	(-5.33)	0.80	(20.76)
WTI far futures returns	8339	2008-Sep-22	0.10	(7.49)	-0.06	(-3.21)	0.38	(20.23)
WTI physical spot returns	7665	2008-Sep-22	0.21	(8.08)	-0.15	(-4.72)	0.81	(20.93)
Brent nearby futures returns	8143	2008-Sep-22	0.25	(10.73)	-0.11	(-3.68)	0.85	(25.93)
Metals returns	8339	2008-Sep-30	0.10	(11.81)	0.02	(2.14)	0.24	(14.69)
Panel B: Regressions of Oil on Various Equity Sectors								
Consumer nondurables	8339	2008-Sep-22	0.03	(0.94)	-0.29	(-9.31)	0.87	(16.50)
Consumer durables	8339	2008-Sep-22	0.11	(6.31)	-0.17	(-7.50)	0.52	(18.87)
Manufacturing	8339	2008-Sep-22	0.22	(10.16)	-0.14	(-4.98)	0.71	(22.28)
Energy	8339	2008-Sep-30	0.61	(36.51)	0.47	(22.99)	0.97	(35.79)
Chemicals	8339	2008-Sep-22	0.12	(5.21)	-0.21	(-7.49)	0.85	(20.63)
Business equipment	8339	2008-Sep-22	0.08	(4.52)	-0.07	(-3.90)	0.67	(17.93)
Telecommunications	8339	2008-Sep-22	0.10	(4.49)	-0.17	(-6.64)	0.70	(18.22)
Utilities	8339	2008-Nov-21	0.28	(10.21)	-0.03	(-0.97)	0.76	(17.52)
Shops	8339	2008-Sep-22	-0.03	(-1.38)	-0.27	(-10.54)	0.68	(14.72)
Health care	8339	2008-Sep-22	0.01	(0.36)	-0.20	(-7.97)	0.64	(13.88)
Finance	8339	2008-Sep-22	0.10	(5.38)	-0.21	(-8.60)	0.43	(16.28)
Other	8339	2008-Sep-22	0.15	(6.81)	-0.18	(-6.50)	0.69	(19.62)
S&P 500 excl. energy	4701	2008-Sep-22	0.26	(9.16)	-0.18	(-4.58)	0.75	(18.54)

Notes: Panel A reports equity betas from the regression $Y_t = \alpha + \beta Equity_t + \varepsilon_t$. Panel B reports equity sector betas from the regression $Oil_t = \alpha + \beta EquitySector_t + \varepsilon_t$. Full sample observations, β, and t-statistics are reported. The break date, pre-break, and post-break results are estimated after applying the standard Chow test to determine the break date which minimizes the sum of squared errors for regressions run on the pre- and post-break samples. All of these break dates were found to be statistically significant at the 1% level ($F_{crit} = 7.8$) when using the standard Andrews supremum-Wald critical value based upon 15% trimming of the sample as in Stock and Watson (2003).

Table 3: Macroeconomic News Surprises and Controlling Variables

Variable	Obs.	Start Date	Mean	Std. Dev.	Min.	Max.
Panel A: Macroeconomic News Surprises						
Capacity utilization (cu)	345	1988-Apr-18	-0.01	0.35	-1.57	1.40
Consumer confidence (con)	304	1991-Jul-30	0.19	5.13	-14.00	13.30
Core CPI (cpi)	329	1989-Aug-18	0.00	0.11	-0.34	0.40
GDP advance (gdp)	119	1987-Apr-23	0.09	0.75	-1.68	1.80
Initial claims (clm)	1258	1991-Jul-18	0.00	18.19	-85.00	94.00
ISM manufacturing (ism)	321	1990-Feb-01	0.00	1.99	-6.30	7.40
Leading indicators (ind)	443	1980-Feb-29	0.01	0.31	-1.80	2.00
New home sales (nhs)	342	1988-Mar-29	5.02	56.90	-166.00	249.00
Nonfarm payrolls (nfp)	383	1985-Feb-01	-8.20	101.42	-328.00	408.50
Core PPI (ppi)	317	1989-Aug-11	-0.02	0.25	-1.20	1.07
Retail sales ex. autos (rtl)	442	1980-Feb-13	-0.03	0.67	-2.40	5.13
Unemployment rate (ur)	441	1980-Feb-07	0.04	0.16	-0.60	0.60
Panel B: Controlling Variables						
Notional interest rate	7698	1987-Jul-01	2.64	3.20	-4.60	8.90
Wu-Xia shadow rate	7993	1985-Dec-19	3.31	3.21	-2.99	9.81
Economic policy uncertainty	9654	1980-Jan-01	0.94	0.13	0.73	1.42
Financial uncertainty	11598	1985-Mar-31	101.01	38.68	40.95	232.72
VIX	7044	1990-Jan-02	19.73	6.93	10.84	55.03
Open interest	8078	1986-Jan-15	0.77	0.54	0.07	1.92

Notes: For this table only, news surprises are defined as the difference between the announced realization of the macroeconomic aggregates and the survey expectations. Prior to use in regression analysis, each surprise is divided by the full sample standard deviation reported above. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements throughout the paper, so that positive surprises represent a stronger-than-expected economy.

See Equation 2 for details on construction of the notional interest rate. For the remaining controlling variables, we use the shadow rate constructed in Wu and Xia (2016), the 90-day moving average of the daily series for economic policy uncertainty from Baker et al. (2015), the 90-day horizon measure of financial uncertainty from Jurado et al. (2015), the 90-day moving averages of the VIX obtained from Bloomberg, and the 90-day moving average of open interest measured in millions of contracts obtained from the CFTC.

Table 4: Oil, Equity, and Interest Rates Surprise Beta Estimates

	Oil				Equity				Interest rate			
	pre-ZLB era		ZLB era		pre-ZLB era		ZLB era		pre-ZLB era		ZLB era	
	β	t-stat	β	t-stat	β	t-stat	β	t-stat	β	t-stat	β	t-stat
Panel A: Pooled Surprises												
Pooled Surprises	0.04	(0.89)	0.25	(3.55)	-0.03	(-1.24)	0.16	(3.76)	1.91	(16.46)	0.67	(5.56)
R-squared	0.00		0.02		0.00		0.02		0.10		0.05	
Panel B: Individual Surprises												
Capacity utilization (cu)	0.25	(1.64)	0.03	(0.14)	0.08	(1.03)	0.03	(0.18)	1.93	(4.49)	0.26	(0.72)
Consumer confidence (con)	0.19	(1.14)	-0.05	(-0.21)	-0.13	(-1.59)	0.16	(1.14)	1.57	(3.43)	0.60	(1.62)
Core CPI (cpi)	0.23	(1.53)	-0.08	(-0.29)	-0.23	(-3.06)	-0.09	(-0.52)	2.07	(4.94)	-0.33	(-0.69)
GDP advance (gdp)	-0.12	(-0.49)	0.36	(0.73)	0.04	(0.28)	-0.20	(-0.65)	1.52	(2.13)	0.06	(0.07)
Initial claims (clm)	0.05	(0.62)	0.33	(2.80)	0.05	(1.24)	0.08	(1.03)	1.19	(5.64)	0.77	(3.90)
ISM manufacturing (ism)	-0.03	(-0.19)	0.72	(3.03)	0.05	(0.67)	0.55	(3.74)	3.54	(8.31)	0.91	(2.31)
Leading indicators (ind)	0.25	(0.96)	-0.21	(-0.79)	-0.38	(-2.94)	0.26	(1.56)	0.27	(0.38)	0.06	(0.14)
New home sales (nhs)	0.03	(0.20)	0.01	(0.04)	-0.03	(-0.50)	-0.06	(-0.25)	0.95	(2.45)	0.24	(0.41)
Nonfarm payrolls (nfp)	0.05	(0.31)	0.74	(2.05)	-0.12	(-1.61)	0.73	(3.29)	5.16	(12.14)	5.12	(8.60)
Core PPI (ppi)	-0.02	(-0.11)	0.44	(1.50)	-0.13	(-1.74)	0.29	(1.60)	0.58	(1.42)	0.76	(1.58)
Retail sales ex. autos (rtl)	-0.31	(-1.80)	0.48	(1.30)	0.12	(1.40)	0.38	(1.65)	2.63	(5.47)	1.57	(2.56)
Unemployment rate (ur)	-0.25	(-1.38)	0.18	(0.71)	0.05	(0.61)	0.13	(0.82)	1.99	(3.99)	-0.41	(-0.99)
R-squared	0.01		0.04		0.01		0.06		0.15		0.15	

Notes: Panel A reports the β from the regression using pooled surprises, $Y_t = \alpha + \beta S_t + \varepsilon_t$, where $S_t = \sum_{j=1}^{12} s_{jt}$. Panel B reports the individual elements β_j of the vector $\boldsymbol{\beta}$, from the regression $Y_t = \alpha + \boldsymbol{\beta} \boldsymbol{s_t} + \varepsilon_t$, where $\boldsymbol{s_t}$ is the vector of standardized and demeaned news on day t. The parameters β and β_j measure the responses of each dependent variable to a one standard deviation news surprise. 2330 observations are included in the pre-ZLB era regressions (1990q1-2009q1), and 638 observations are included in the ZLB era regressions (2009q2-2014q2). The ZLB era is defined as the period during which the Taylor rule notional rate is less than negative 1. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements, so that positive surprises represent a stronger-than-expected economy. See Section 4.2 for more detail.

49

Table 5: Tests of Varying Coefficients

Explaining variation in the responsiveness to surprises:
$$Y_t = \alpha(.) + \boldsymbol{\beta}(.)\boldsymbol{S_t} + \varepsilon_t, \quad (.) = \{\alpha(.), \boldsymbol{\beta}(.)\}$$

Panel A: Do the ZLB measures improve model fit?

| | Controlling Variables (Z_t) | | Dependent Variable (Y_t) | | |
	Restricted	Unrestricted	Oil	Equity	Int. Rate
A1.	none	$\tilde{R}_k$	0.10 (yes)	0.01 (yes)	0.00 (yes)
	FNU_k	$\tilde{R}_k, FNU_k$	0.02 (yes)	0.01 (yes)	0.00 (yes)
	EPU_k	$\tilde{R}_k, EPU_k$	0.10 (no)	0.06 (yes)	0.00 (yes)
	VIX_k	$\tilde{R}_k, VIX_k$	0.21 (no)	0.03 (yes)	0.00 (yes)
	OI_k	$\tilde{R}_k, OI_k$	0.10 (yes)	0.00 (yes)	0.00 (yes)
	SR_k	$\tilde{R}_k, SR_k$	0.08 (yes)	0.17 (no)	0.19 (no)
A2.	none	SR_k	0.17 (no)	0.01 (yes)	0.00 (yes)
	FNU_k	SR_k, FNU_k	0.08 (yes)	0.00 (yes)	0.00 (yes)
	EPU_k	SR_k, EPU_k	0.31 (no)	0.07 (yes)	0.00 (yes)
	VIX_k	SR_k, VIX_k	0.01 (yes)	0.00 (yes)	0.00 (yes)
	OI_k	SR_k, OI_k	0.21 (no)	0.02 (yes)	0.00 (yes)
	$\tilde{R}_k$	$SR_k, \tilde{R}_k$	0.14 (no)	0.29 (no)	0.07 (yes)

Panel B: Do the alternative controlling variables improve model fit?

| | Controlling Variables (Z_t) | | Dependent Variable (Y_t) | | |
	Restricted	Unrestricted	Oil	Equity	Int. Rate
B1.	none	FNU_k	0.75 (no)	0.18 (no)	0.12 (no)
	none	EPU_k	0.29 (no)	0.38 (no)	0.00 (yes)
	none	VIX_k	0.35 (no)	0.78 (no)	0.10 (no)
	none	OI_k	0.06 (yes)	0.01 (yes)	0.00 (yes)
B2.	$\tilde{R}_k$	$\tilde{R}_k, FNU_k$	0.04 (yes)	0.04 (yes)	0.00 (yes)
	$\tilde{R}_k$	$\tilde{R}_k, EPU_k$	0.16 (no)	0.54 (no)	0.00 (yes)
	$\tilde{R}_k$	$\tilde{R}_k, VIX_k$	0.30 (no)	0.60 (no)	0.00 (yes)
	$\tilde{R}_k$	$\tilde{R}_k, OI_k$	0.12 (no)	0.18 (no)	0.13 (no)
B3.	SR_k	SR_k, FNU_k	0.29 (no)	0.01 (yes)	0.00 (yes)
	SR_k	SR_k, EPU_k	0.28 (no)	0.52 (no)	0.00 (yes)
	SR_k	SR_k, VIX_k	0.06 (yes)	0.31 (no)	0.00 (yes)
	SR_k	SR_k, OI_k	0.12 (no)	0.19 (no)	0.16 (no)

Table 6: Structual VAR Decomposition of the Monthly Oil Price and Equity Resturns Correlation.

		Contribution of			
Sample	$\rho_{ps}(h)$	Oil Supply	Agg. Demand	Oil Resid.	Equity Resid.
1974-2008	-0.10	0.00	0.02	-0.11	-0.01
2009-2016	0.25	0.03	0.01	0.19	0.03

B Appendix Tables and Figures

Figure A.1: Oil and Equity Correlation - Robustness

The rolling window correlations between oil and equity returns are presented here. The four panels illustrate the rolling windows of various lengths (1 month up to 3 years) for returns calculated over daily, weekly, monthly, and quarterly frequencies.

(a) Daily Returns   (b) Weekly Returns

(c) Monthly Returns   (d) Quarterly Returns
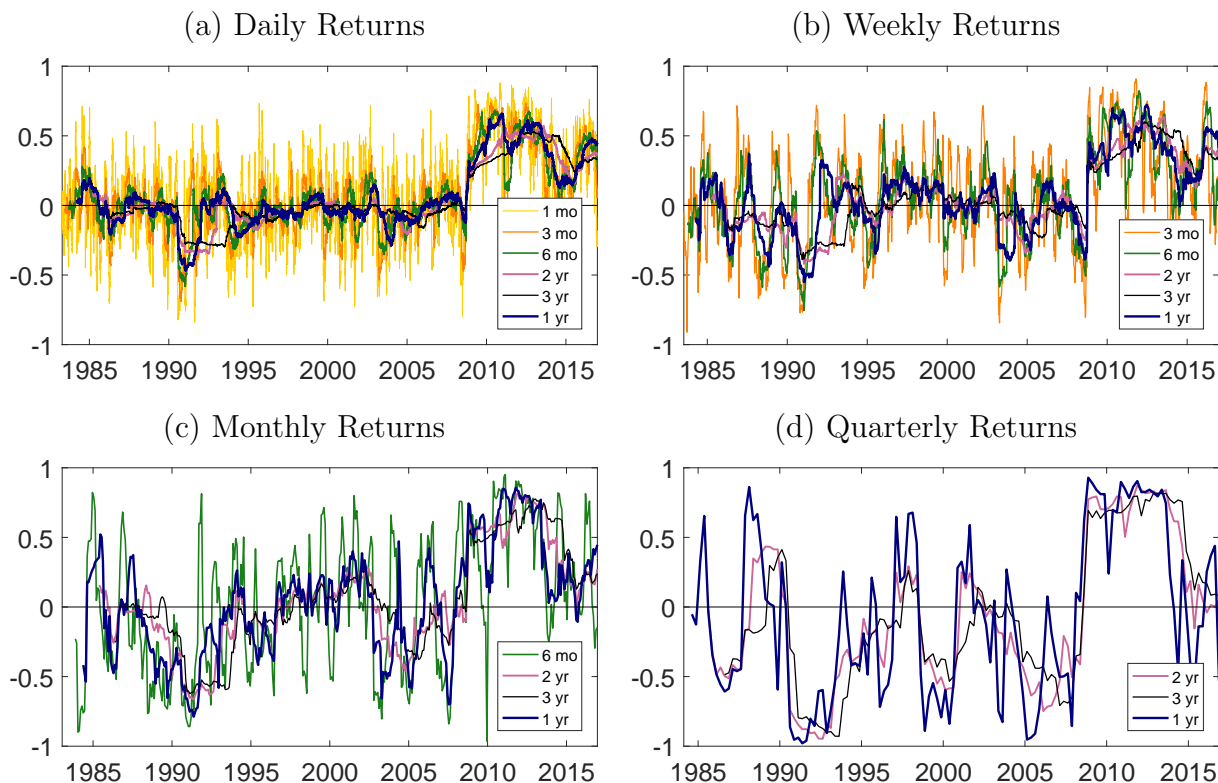
C DSGE Model

In this appendix, we derive the linearized equilibrium conditions from our model.

C.1 Household Problem

Utility is given

$$\mathcal{U}_t = u\left(C_t, C_t^O L_t, \frac{B_t}{P_t}\right) + \beta E_t \mathcal{U}_{t+1}$$

where C_t. The per-period budget constraint is

$$\frac{B_t}{P_t} + \frac{P_t^O}{P_t}C_t^O + C_t + \frac{P_t^K}{P_t}K_{t+1} \le \frac{W_t}{P_t}L_t + R_{t-1}\frac{B_{t-1}}{P_t} + \frac{R_t^K}{P_t}K_t + T_t + \frac{P_t^K}{P_t}K_t$$

where and T_t are taxes and transfers and profits(lump-sum). Note that K_t will be fixed and R_t^K is the nominal rental rate of capital. The first-order conditions are

$$\frac{W_t}{P_t} = -\frac{u_3\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}{u_1\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}$$

$$\frac{P_t^O}{P_t} = \frac{u_2\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}{u_1\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}$$

$$1 = \frac{u_4\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}{u_1\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)} + \beta E_t\left[\frac{u_1\left(C_{t+1}, C_{t+1}^O, L_{t+1}, \frac{B_{t+1}}{P_{t+1}}\right)}{u_1\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}\frac{R_t}{\pi_{t+1}}\right]$$

$$\frac{P_t^K}{P_t} = \beta E_t\left(\frac{R_{t+1}^K}{P_{t+1}} + \frac{P_{t+1}^K}{P_{t+1}}\right)\frac{u_1\left(C_{t+1}, C_{t+1}^O, L_{t+1}, \frac{B_{t+1}}{P_{t+1}}\right)}{u_1\left(C_t, C_t^O, L_t, \frac{B_t}{P_t}\right)}$$

C.2 Retailers

Retailers are perfectly competitive as produce so that

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}}dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

Their demand for any particular good is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}Y_t$$

C.3 Retail goods producers: Rotemberg

There is a measure one of retail good producers. They face the demand curves from the realers. They all face the same problem, so they set $P_t(j) = P_t$. They maximize

$$E_t \sum_{k=0}^{\infty} M_{t,t+k} \left[\frac{(1+\tau)P_{t+k}(j) - MC_{t+k}}{P_{t+k}} \left(\frac{P_{t+k}(j)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} - \frac{\Phi}{2} \left(\frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1 \right)^2 \right]$$

where $M_{t,t+k} \equiv \beta^k \frac{u_{1,t+k}}{u_{1,t}}$ measures the household's valuation of money flows in period $t+k$ relative to period t. The first-order condition for optimality is

$$(\pi_t - 1)\pi_t = \frac{\varepsilon}{\Phi} \left(\frac{MC_t}{P_t} - 1 \right) Y_t + E_t M_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1}$$

C.4 Marginal Cost

The firm solves the following problem

$$\min_{K_t, L_t} W_t L_t + R_t^K K_t + P_t^O O_t$$

subject to the constraint that

$$\left((1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{1+\rho_Y} \geq Y$$

where

$$V_t = K_t^\alpha L_t^{1-\alpha}$$

The first-order conditions are

$$\frac{R_t^K}{P_t} = MC_t \left((1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{\rho_Y} (1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{-\frac{\rho_Y}{1+\rho_Y}} \alpha \left(\frac{L_t}{K_t} \right)^{1-\alpha}$$

$$\frac{W_t}{P_t} = MC_t \left((1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{\rho_Y} (1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{-\frac{\rho_Y}{1+\rho_Y}} (1 - \alpha) \left(\frac{L_t}{K_t} \right)^{-\alpha}$$

$$\frac{P_t^O}{P_t} = MC_t \left((1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{\rho_Y} (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{-\frac{\rho_Y}{1+\rho_Y}} \frac{1}{\mu_{V^O,t}}$$

C.5 Resource Constraint

In the goods market, technology implies

$$C_t + G_t + \frac{\Phi}{2}(\pi_t - 1)^2 = \left((1-\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{1+\rho_Y} = Y_t$$

In the oil market, consumption has to equal supply

$$O_t = V_t^O + C_t^O$$

C.6 Functional Forms

We assume the household preferences are given by

$$u\left(C_t, C_t^O, L_t, \frac{B_t}{P_t} \right) = \log \left(\left((1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{\frac{1}{1+\rho_C}} + \omega_C^{\frac{\rho_C}{1+\rho_C}} \left(\frac{C_t^O}{\mu_{C^O,t}} \right)^{\frac{1}{1+\rho_C}} \right)^{1+\rho_C} \right) - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \eta_t \Xi_t$$

C.7 Linearized equilibrium conditions

We need to determine 14 variables: Y_t, L_t, R_t, r_t^K, K_t, π_t, C_t, C_t^O, V_t, V_t^O, p_t^O, w_t, mc_t, p_t^K.

$$X_t \equiv \left((1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{\frac{1}{1+\rho_C}} + \omega_C^{\frac{\rho_C}{1+\rho_C}} \left(\frac{C_t^O}{\mu_{C^O,t}} \right)^{\frac{1}{1+\rho_C}} \right)^{1+\rho_C},$$

so we need 15 equations. Our 15 equations are

1. The definition of X_t

$$X_t = \left((1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{\frac{1}{1+\rho_C}} + \omega_C^{\frac{\rho_C}{1+\rho_C}} \left(\frac{C_t^O}{\mu_{C^O,t}} \right)^{\frac{1}{1+\rho_C}} \right)^{1+\rho_C}$$

which is linearized to be

$$\hat{X}_t = X^{-\frac{1}{1+\rho_C}} \left[(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C^{\frac{1}{1+\rho_C}} \hat{C}_t + \omega_C^{\frac{\rho_C}{1+\rho_C}} \left(C^O \right)^{\frac{1}{1+\rho_C}} \hat{C}_t^O - \omega_C^{\frac{\rho_C}{1+\rho_C}} \left(C^O \right)^{\frac{1}{1+\rho_C}} \hat{\mu}_{C^O,t} \right]$$

2. The intra-temporal Euler equation

$$w_t = \left(\frac{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{-\frac{\rho_C}{1+\rho_C}}}{X_t^{\frac{1}{1+\rho_C}}} \right)^{-1} \chi L_t^{\varphi}$$

which is linearized to be

$$\hat{w}_t = \varphi \hat{L}_t + \frac{\rho_C}{1+\rho_C} \hat{C}_t - \frac{1}{1+\rho_C} \hat{X}_t$$

3. Household demand for oil consumption

$$p_t^O = \frac{\omega_C^{\frac{\rho_C}{1+\rho_C}} \left(C_t^O\right)^{-\frac{\rho_C}{1+\rho_C}} \mu_{C^O,t}^{-\frac{1}{1+\rho_C}}}{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{-\frac{\rho_C}{1+\rho_C}}}$$

which is linearized to be

$$\hat{p}_t^O = -\frac{\rho_C}{1+\rho_C} \hat{C}_t^O - \frac{1}{1+\rho_C} \hat{\mu}_{C^O,t} + \frac{\rho_C}{1+\rho_C} \hat{C}_t$$

4. Inter-temporal Euler equation

$$1 = \eta_t + \beta E_t \left[\frac{\frac{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_{t+1}^{-\frac{\rho_C}{1+\rho_C}}}{X_{t+1}^{\frac{1}{1+\rho_C}}}}{\frac{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{-\frac{\rho_C}{1+\rho_C}}}{X_t^{\frac{1}{1+\rho_C}}}} \frac{R_t}{\pi_{t+1}} \right]$$

which is linearized to be

$$0 = \hat{\eta}_t + \left[-\frac{\rho_C}{1+\rho_C} E_t\hat{C}_{t+1} - \frac{1}{1+\rho_C} E_t\hat{X}_{t+1} + \frac{\rho_C}{1+\rho_C}\hat{C}_t + \frac{1}{1+\rho_C}\hat{X}_t + \hat{R}_t - E_t\hat{\pi}_{t+1} \right]$$

5. Demand for capital

$$p_t^K = \beta E_t \left(r_{t+1}^K + p_{t+1}^K \right) \frac{\frac{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_{t+1}^{-\frac{\rho_C}{1+\rho_C}}}{X_{t+1}^{\frac{1}{1+\rho_C}}}}{\frac{(1-\omega_C)^{\frac{\rho_C}{1+\rho_C}} C_t^{-\frac{\rho_C}{1+\rho_C}}}{X_t^{\frac{1}{1+\rho_C}}}}$$

which is linearized to be

$$\hat{p}_t^K = (1-\beta) E_t \hat{r}_{t+1}^K + \beta E_t \hat{p}_{t+1}^K - \frac{\rho_C}{1+\rho_C} E_t \hat{C}_{t+1} - \frac{1}{1+\rho_C} E_t \hat{X}_{t+1} + \frac{\rho_C}{1+\rho_C} \hat{C}_t + \frac{1}{1+\rho_C} \hat{X}_t$$

6. Technology for V_t

$$V_t = K_t^\alpha L_t^{1-\alpha}$$

which is linearized to be

$$\hat{V}_t = \alpha \hat{K}_t + (1-\alpha) \hat{L}_t$$

7. Technology for Y_t

$$Y_t = \left((1-\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{\frac{1}{1+\rho_Y}} + (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{\frac{1}{1+\rho_Y}} \right)^{1+\rho_Y}$$

which is linearized to be

$$\hat{Y}_t = Y^{-\frac{1}{1+\rho_Y}} \left[(1-\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V^{\frac{1}{1+\rho_Y}} \hat{V}_t + \omega_Y^{\frac{\rho_Y}{1+\rho_Y}} \left(V^O \right)^{\frac{1}{1+\rho_Y}} \hat{V}_t^O - \omega_Y^{\frac{\rho_Y}{1+\rho_Y}} \left(V^O \right)^{\frac{1}{1+\rho_Y}} \hat{\mu}_{V^O,t} \right]$$

8. The resource constraint

$$Y_t = C_t + G_t + \frac{\Phi}{2} (\pi_t - 1)^2$$

which is linearized to be

$$Y \hat{Y}_t = C \hat{C}_t + G \hat{G}_t$$

9. Oil market clearing

$$O_t = V_t^O + C_t^O$$

which is linearized to be

$$O O_t = V^O \hat{V}_t^O + C^O \hat{C}_t^O$$

10. The pricing equation from the firm

$$(\pi_t - 1) \pi_t = \frac{\varepsilon}{\Phi} (mc_t - 1) Y_t + E_t M_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1}$$

which is linearized to be

$$\hat{\pi}_t = \frac{\varepsilon}{\Phi} Y mc \hat{mc}_t + \beta E_t \hat{\pi}_{t+1}$$

11. Capital is in fixed supply
$$\hat{K}_t = 0.$$

12. Cost minimization for capital
$$r_t^K = mc_t Y_t^{\frac{\rho_Y}{1+\rho_Y}} (1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{-\frac{\rho_Y}{1+\rho_Y}} \alpha \left(\frac{L_t}{K_t} \right)^{1-\alpha}$$

which is linearized to be
$$\hat{r}_t^K = \hat{mc}_t + \frac{\rho_Y}{1+\rho_Y} \hat{Y}_t - \frac{\rho_Y}{1+\rho_Y} \hat{V}_t + (1-\alpha) \left(\hat{L}_t - \hat{K}_t \right)$$

13. Cost minimization for labor
$$w_t = mc_t Y_t^{\frac{\rho_Y}{1+\rho_Y}} (1 - \omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} V_t^{-\frac{\rho_Y}{1+\rho_Y}} (1-\alpha) \left(\frac{L_t}{K_t} \right)^{-\alpha}$$

which is linearized to be
$$\hat{w}_t = \hat{mc}_t + \frac{\rho_Y}{1+\rho_Y} \hat{Y}_t - \frac{\rho_Y}{1+\rho_Y} \hat{V}_t - \alpha \left(\hat{L}_t - \hat{K}_t \right)$$

14. Cost minimization for oil
$$p_t^O = mc_t Y_t^{\frac{\rho_Y}{1+\rho_Y}} (\omega_Y)^{\frac{\rho_Y}{1+\rho_Y}} \left(\frac{V_t^O}{\mu_{V^O,t}} \right)^{-\frac{\rho_Y}{1+\rho_Y}} \frac{1}{\mu_{V^O,t}}$$

which is linearized to be
$$\hat{p}_t^O = \hat{mc}_t + \frac{\rho_Y}{1+\rho_Y} \hat{Y}_t - \frac{\rho_Y}{1+\rho_Y} \hat{V}_t^O - \frac{1}{1+\rho_Y} \hat{\mu}_{V^O,t}$$

15. The monetary policy rule.