

International Monetary Theory: Mundell-Fleming Redux.*

- Preliminary and Incomplete. Do not Circulate -

Markus K. Brunnermeier and Yuliy Sannikov[†]

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Abstract

We build a two-country model, in which currency values are endogenously determined. Risk plays a key role - including idiosyncratic risk that creates precautionary savings demand for money, and sector productivity risk that leads to fluctuations of prices in tradable goods and determines currency risk profiles. Agents prefer to hold their country's currency, as its value is more aligned with the price of the local consumption basket, and hold foreign currency only to hedge export risk. The value of the local currency can be very sensitive to monetary policy in the large country. However, even with an open capital account, there is a corridor within which the small country can conduct its monetary policy - the width of this corridor depends on the large country's policy.

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[†]Brunnermeier: Department of Economics, Princeton University, markus@princeton.edu, Sannikov: Stanford GSB, sannikov@gmail.com

1 Introduction

What governs exchange rates, the relative value of money between two countries? Is the monetary policy of a country open to international capital flows independent or is it primarily a serf of international spillovers and spillbacks? When should a country give up its independent monetary policy and simply target its exchange rate or even form a currency board? How much foreign currency reserves should a country hold in order to establish an optimal currency risk profile for its citizens?

To answer these and related questions we develop a dynamic two-country model in which the value of money in each country is endogenously determined. The real value of money, prices and the exchange rate are risky and driven by productivity and monetary policy shocks in both countries. Different types of currencies can co-exist and their risk profiles are endogenously determined.

Our model follows a long tradition in international macro that tries to address these important questions. The seminal Mundell-Fleming model studies a small open economy version of Hick's IS-LM workhorse model. It emphasizes the trilemma that one can only pick two of three desired features: independent monetary policy, free capital flows and fixed exchange rate. The Mundell-Fleming model is essentially static and abstracts from any risk considerations. Money has value due to exogenously specific liquidity preferences. Prices are sticky. Obstfeld and Rogoff (1995) develop a two-country New Keynesian model. Importantly, they add a time dimension. Prices are rigid à la Calvo and monopolistic competitions allows firms to earn a mark-up. Money is valuable since citizens derive utility from real money balances - a reduced-form modeling device to capture the transaction role of money. The risk analysis is limited to impulse response function of e.g. the exchange rate after a one time unanticipated shock.

In this paper both the risk and the time dimension play a prominent role. Specifically, we consider a two-country model with full risk dynamics in which the value of money arises - similar to Bewley models - purely from financial frictions. More specifically, we consider a two-country model with flexible prices based on Brunnermeier and Sannikov (2016a). In both countries, one large and one small open economy country, citizens choose a portfolio consisting of physical capital, which is subject to uninsurable idiosyncratic shocks, and money. Money pays no dividends but serves as a store of value. Since money is free of idiosyncratic risk, it takes on an insurance role. Monetary policy affects this insurance role and agents' portfolio choice by making money holding more or less attractive. Money of

the large country, which we assume is at its steady state and faces no aggregate shock, is our “global money”. The focus of our analysis is the small open economy (SOE) and its “local money.” As risk profiles of both currencies may differ, citizens in the SOE may want to hold both forms of currency. Citizens in the small country want to consume mostly local (non-tradable) goods. Local money is a better store of value and hedge against idiosyncratic risk since it better reflects risk of the local basket.

However, a small portion of goods is produced for export - those goods are sold to the large country, and the revenues are used to buy a basket of global goods. The productivity of local exporters is subject to shocks. As a result, the consumption basket of global goods that can be obtained in exchange for exports is fluctuating. In other words, there is risk due to the world prices of export goods, due to shocks to productivity, due to global demand. We call this “export risk” - local agents face this risk as well as individual idiosyncratic risks. Local citizens can self-insure against export risk by holding global money. After a negative export shock, the prices of imported global goods relative to local goods / incomes rise (that is, the real exchange rate deteriorates). If local agents have global money savings, they can spend it to smooth their consumption of imported global goods. The optimum holdings of global money depend on the level of export risk.

Local citizens in the small country are also exposed to idiosyncratic “rainy day” risk, just like the agents in the large country. In the absence of local currency, the agents in the small country could be holding global money to self-insure against idiosyncratic risk. However, if idiosyncratic shocks are sufficiently large, optimal money holdings would result in over-hedging of export risk. That is, agents in the small country consume a basket of mostly local goods, and relative to it, global money is risky. These conditions leave room for local money - if there is a currency held by other local citizens, then each individual in the small country would hold global money only to the extent that he desires to hedge export risk. This would provide some insurance against idiosyncratic risk - additionally each individual would hold local money to protect against idiosyncratic risk further. Local currency can have value in this environment even if expected return from holding local currency is less than that from holding global currency, because for local agents global currency is risky, and hence commands a risk premium.

In such an environment, how do shocks affect portfolio weights? What about exchange rates? The portfolio weight on local currency depends on holdings of global money. When those holdings are below steady state, portfolio weight on local currency is greater. This is because local and global money are imperfect substitutes. A positive export shock leads to

the depreciation of the global money because locals can produce export goods more effectively. Local money appreciates in value relative to global money / global goods. Moreover, because of a lower supply of global money for self insurance, the demand for local money rises, and so local money appreciates in value even relative to local goods. In contrast, a negative export shock leads to appreciation of the global currency relative to local.

How does large country's monetary policy spill over to small country? For example, a (permanent) loosening in monetary policy by the large country lowers the return on global money holdings. Small country's citizens reduce their global money holdings and the whole risk-dynamics shifts.

Our analysis paints a more nuanced picture of the classic Mundell-Fleming trilemma. Since both currencies have different risk profiles, they are only imperfect substitutes. With an open capital account and flexible exchange rate, monetary policy has still some maneuvering space, though smaller. Local inflation decreases local money holdings, tends to increase portfolio weight on capital but also increases portfolio weight on global money. Local government can affect the portfolio weight on local money between zero (e.g. extreme inflation) and some bound - that bound depends on monetary policy / inflation rate of the large country. This bound can be lifted if the central bank closes the capital account and does not allow its citizens to hold global money, but robs its citizens of the ability to hedge against "export risk." To soften this the small country's central bank can back its currency by holding global money as reserves.

This paper is organized as follows. Section 2 provides an introduction to the way we model money. It describes the large country in isolation, and derives the risks generated within the large country that affect the small country. Section 3 provides a model of the small country, and characterizes equilibrium there in the absence of policy. Section 4 provides numerical example, and gives a sense of equilibrium dynamics without policy. It also provides intuition, via back-of-the-envelope calculations, about conditions that allow two types of money to co-exist. Section 5 discusses the range of monetary policies that the small country can implement with an open capital account, and with control over foreign currency holdings in the small country. While some policies have real effects, we also uncover two irrelevance results regarding policy classes that have only nominal and no real effects. Section 6 (HIGHLY INCOMPLETE AT THIS POINT) addresses the question of optimal policy in our setting.

2 The Large Country: Money, Self-insurance and Risk

In this section we present a basic model of money. The model derives its foundations from Samuelson (1958) and Bewley (1980), but it is based more closely on Brunnermeier and Sannikov (2016a) and (2016b). Money plays the role of a store of value when agents face uninsurable idiosyncratic risk. The value of money depends on the overall risk exposure. The presence of money leads to distortions that affect welfare. In particular, money creates an opportunity to *save* but not *invest*. The optimal money supply depends on the level of idiosyncratic risk: when the risk is sufficiently large then inflation can improve welfare by encouraging investment in real projects.

The main takeaways of this section are as follows. First, it is the basic link between idiosyncratic risk and the value of money. Second, the value of money can be affected by policy - i.e. inflation created through money printing or deflation - if money has fiscal backing. Policy affects money supply, and hence insurance of idiosyncratic risk as well as distortions. We study the planner's problem of providing insurance against self-reported idiosyncratic risk shocks. We show that among policies that provide uniform insurance across agents (that is, policies which do not condition on reported histories of shocks of individual agents), the optimal policy can be implemented through monetary policy with constant inflation. This result highlights the relationship between monetary policy and insurance. This insight helps us understand monetary policy in the small country, where agents face not only idiosyncratic risk, but also risk generated within the large country. Third, lastly, we derive the risks generated within the large country, which affect the small country. Economic shocks within the large country affect relative prices of global goods, and therefore spill over to the prices of the small country's exports and imports. In addition, shocks within the large country affect the risk profile of large country's currency.

In formulating our two-country model, we assume that the small country is negligible in size compared to the large country, so we model the large country as a stand-alone entity as if it is the only country in the world.

Denote by K_t^* the total supply of capital in the large country. Agents can use capital to produce either the local good at a constant rate a^* per unit of capital or either one of two global goods at rates $b_{1,t}^*$ and $b_{2,t}^*$, respectively. Agents can freely choose which good to produce with their capital. We will specify the aggregate stochastic processes for $b_{1,t}^*$ and $b_{2,t}^*$, as well as the idiosyncratic shocks that individuals face, later. We assume that idiosyncratic shocks, which cancel out in the aggregate, are the same regardless of which good the agents

choose to produce.

Agents have logarithmic and Cobb-Douglas utility functions from consuming the three goods, of the form

$$(1 - \alpha) \log c_l + \alpha \log(c_1^\beta c_2^{1-\beta}),$$

where $\alpha, \beta \in (0, 1)$ are parameters, and c_l, c_1 and c_2 are consumption of the local and the two global goods, respectively.

For tractability (to maintain stationarity), we assume that the local good is used to build new capital. In the aggregate capital evolves according to

$$\frac{dK_t^*}{K_t^*} = (\Phi(\iota_t^*) - \delta) dt,$$

where ι_t^* is the investment rate, per unit of capital. The common discount rate is ρ .

Agents can allocate their wealth between capital, an asset that carries idiosyncratic risk, and money. The baseline assumption is that money is an infinitely divisible asset, which does not pay dividends and is available in fixed supply (a good, but not perfect, example of such an asset is gold).

If the agent's portfolio weight on money is θ_t^* then we can compute the rates of investment and growth in the large country as follows. Agents must be indifferent between producing the local good or either one of the global goods. Hence, expressed in terms of the local good, the value of all goods produced must be $a^* K_t^*$, and the value of all consumption, $(a^* - \iota_t^*) K_t^*$. Since agents with logarithmic utility consume at the rate ρ times their net worth, total wealth must be $(a^* - \iota_t^*) K_t^* / \rho$ and the price of capital per unit, $q_t^* = (1 - \theta_t^*)(a^* - \iota_t^*) / \rho$. Thus, the optimal investment rate must satisfy

$$\Phi'(\iota_t^*) \underbrace{\frac{(1 - \theta_t^*)(a^* - \iota_t^*)}{\rho}}_{q_t^*} = 1. \quad (2.1)$$

We conclude that the rate of investment ι_t^* together with the growth in the large country are functions of the agents' portfolio share of money θ_t^* at any moment of time. This observation highlights the negative relationship between money holdings and investment.

Furthermore, since consumption is ρ times net worth, the dividend yield on the entire wealth portfolio is ρ . Under the baseline assumption that money pays no dividends, the dividend yield on capital must be $\rho / (1 - \theta_t^*)$.

Now, let us discuss idiosyncratic risk. In Brunnermeier and Sannikov (2016a) and Di

Tella (2015), shocks hit capital held by individual agents, as if capital suddenly becomes more or less productive. This assumption is also used in He and Krishnamurthy (2013), where the dividend produced by capital is a geometric Brownian motion (hence, shocks to the productivity are permanent and tied to capital). In Brunnermeier and Sannikov (2016b), shocks are to cash flows, and this is also a common assumption in corporate finance literature - see DeMarzo et. al. (2012), for example. One can imagine various other ways in which shocks depend on the fraction of output that is consumed or invested. A general formulation, which captures all of the above possibilities, is to assume that agents face idiosyncratic risk $\tilde{\sigma}^*(q^*)$, where q^* is the price of capital in terms of the local good. With capital shocks, $\tilde{\sigma}^*(q^*)$ is a constant, and with cash flow shocks, $\tilde{\sigma}^*(q^*)$ is inversely proportional to q^* since cash flow risk is absorbed by the entire value of capital. Importantly, we follow the literature in assuming, somewhat unrealistically, that idiosyncratic shocks scale with the size of the agents' capital portfolios, i.e. there is no diversification from holding a large portfolio. The interpretation is that each agent operates a particular business, and the shocks hit the entire business.

Let us characterize the stationary equilibrium, in which money portfolio share $\theta^* > 0$ is a constant. (Since money is a bubble, there is also always an equilibrium in which money is worthless, and many nonstationary equilibria in between). To get optimal portfolio weights, we use the condition for log utility that the difference between expected returns of any two assets has to be explained by the covariance between the difference in risks and the risk of the agent's net worths. This condition holds regardless of the numeraire. Then individual net worth is subject to idiosyncratic risk of $(1 - \theta^*)\tilde{\sigma}^*(q^*)$ and, if measured in terms of the local good as the numeraire, no aggregate risk. Money and capital have the same capital gains rates of $\Phi(\iota_t^*) - \delta$. Taking into account the difference in dividend yields and risk, the pricing condition for capital relative to money is

$$\frac{\rho}{1 - \theta^*} = (1 - \theta^*)\tilde{\sigma}^*(q^*)^2, \quad (2.2)$$

$$\text{where } q^* = \frac{(1 - \theta_t^*)(a^* - \iota_t^*)}{\rho}. \quad (2.3)$$

In the special case that $\tilde{\sigma}^*(q^*)$ is a constant function, $\theta^* = 1 - \sqrt{\rho}/\tilde{\sigma}^*$, and there exists an equilibrium in which money is held and hence has value as long as $\tilde{\sigma}^* > \sqrt{\rho}$. In the special case that there are no investment adjustment costs, i.e. $\Phi(\iota^*) = \iota^*$, we have that $q^* = 1$ and therefore $\theta^* = 1 - \sqrt{\rho}/\sigma^*(1)$. Portfolio share of money is increasing in the level

of idiosyncratic risk.

Let us briefly discuss the relationship between the local good, global goods and the aggregate good. Notice that a units of the local good can be used to buy $b_{1,t}^*$ units of global good 1 and $b_{2,t}^*$ units of global good 2. Hence, a^* units of the local good can be converted to $(a^*)^{1-\alpha}(b_{1,t}^*)^{\alpha\beta}(b_{2,t}^*)^{\alpha(1-\beta)}$ units of the aggregate good. As a result, per unit of capital, the production of aggregate consumption good is

$$(a^* - l_t^*)(b_t^*)^\alpha, \quad \text{where} \quad b_t^* = \frac{(b_{1,t}^*)^\beta (b_{2,t}^*)^{1-\beta}}{a^*}.$$

Thus, we see can think of the large country effectively as a local-good economy, but with a multiplier on consumption that comes from the productivity of global goods.

2.1 Monetary Policy as Insurance

Direct insurance policies. In our setting, there is welfare loss from idiosyncratic risk. Let us look at policies that aim to improve welfare by providing insurance to the agents directly in a moneyless economy. The extent of insurance that the planner can provide depends on observability, and the range of deviations available to individual agents. If idiosyncratic shocks are observable, and if the planner can control the agents' investment rates, then it is possible to insure idiosyncratic risk perfectly and obtain first best. If agents can reduce investment to increase consumption and portray lower capital accumulation rate to be the result of idiosyncratic shocks, then first best cannot be attained - insurance leads to investment distortions. If agents can hide capital from the planner, the range of deviations is even larger, and the set of implementable outcomes is smaller.

We derive the optimal policy from a class that treats all agents equally (i.e. such that insurance level is independent of individual agents' histories) and implement the optimal policy by a specific monetary policy. This result highlights the role of monetary policy in redistributing risk, but at the cost of creating distortions.

Consider the following environment, in which the social planner can give insurance to individuals. Assume that idiosyncratic shocks to individuals' capital or consumption are not observable, but the planner observes the level of capital managed by each individual. As a result, the planner cannot distinguish between capital created through investment or as a result of a shock. The planner can implement transfers among individuals based on observables. We would like to study the optimal social contract in this setting, and to

compare attainable outcomes with those that result in equilibrium with money, under various monetary policies.

Denote by ψ_t^* the fraction of risk that individual agents retain (the same across all agents) under the insurance policy. Denote by \hat{q}_t^* the shadow price of capital after insurance. Then, a unit of investment creates $\Phi'(\iota_t^*)$ units of capital before insurance and $\psi_t^* \Phi'(\iota_t^*)$ after, so the first-order condition for the optimal investment level is

$$1 = q_t^* \Phi'(\iota_t^*), \quad \text{where} \quad q_t^* = \psi_t^* \hat{q}_t^* \quad (2.4)$$

is the shadow price of capital before insurance. Hence, the market-clearing condition for consumption goods is

$$a^* - \iota(q_t^*) = \rho \hat{q}_t^*. \quad (2.5)$$

Equations (2.4) and (2.5) determine the prices of capital q_t^* and \hat{q}_t^* as well as the rate of investment ι_t^* as functions of the insurance provided by the planner.

Welfare depends on consumption per unit of capital, expected growth of capital held by individuals and idiosyncratic risk exposure, $\psi_t \tilde{\sigma}^*(q_t^*)$ per unit of capital.¹ From Brunnermeier and Sannikov (2016a), welfare of an agent initially endowed with one unit of capital can be expressed as

$$E \left[\int_0^\infty e^{-\rho t} \left(\log(a^* - \iota(q_t^*)) + \frac{\Phi(\iota(q_t^*)) - \delta}{\rho} - \frac{(\psi_t^*)^2 \tilde{\sigma}^*(q_t^*)^2}{2\rho} \right) dt \right] + \alpha E \left[\int_0^\infty e^{-\rho t} \log(b_t^*) dt \right].$$

We see that optimal ψ_t^* is independent of time or the level of b_t^* , and it must maximize

$$\log(a^* - \iota^*) + \frac{\Phi(\iota^*) - \delta}{\rho} - \frac{(\psi^*)^2 \tilde{\sigma}^*(q^*)^2}{2\rho}, \quad (2.6)$$

with q^* and ι^* determined by (2.4) and (2.5). That is, if the planner is restricted to providing insurance to all agents that is independent of individual histories, the planner would choose to not condition on time or aggregate shocks, and the resulting optimal policy is characterized

¹Idiosyncratic risk depends on the price of capital before insurance, q_t^* , as that price determines investment rate.

by constant ψ^* .² Given this allocation, capital (and net worth) of any agent follows

$$\frac{dk_t}{k_t} = \frac{dn_t}{n_t} = (\Phi(\iota^*) - \delta) dt + \psi^* \tilde{\sigma}^*(q^*) d\tilde{Z}_t. \quad (2.7)$$

The planner insures a portion $1 - \psi^*$ of the shocks, and since these shocks cancel out in the aggregate, this policy respects the resource constraint.

Now, we ask the question of whether this policy remains incentive compatible even if agents could hide capital, i.e. individual agents could divert some of the capital pretending to get an adverse idiosyncratic shock, and manage the diverted capital secretly (absorbing all idiosyncratic risk). It turns out that we can answer this question fairly easily. By diverting capital, the agent can claim a loss, so the shadow price of diverted capital is the price q_t^* before insurance. We ask the questions, then, (1) what is the risk and return of hidden capital, and how does it compare to the risk and return of legitimate capital and (2) under what conditions do the agents prefer to put zero portfolio weight on the diverted capital?

Legitimate capital has return

$$dr_t^{K,*} = \underbrace{\frac{a^* - \iota^*}{\hat{q}^*}}_{\rho} dt + (\Phi(\iota^*) - \delta) dt + \psi^* \tilde{\sigma}^*(q^*) d\tilde{Z}_t.$$

The optimal investment rate for diverted capital is also ι^* , since the price is q^* , hence the return on diverted capital is

$$d\hat{r}_t^{K,*} = \underbrace{\frac{a^* - \iota^*}{q^*}}_{\rho/\psi^*} dt + (\Phi(\iota^*) - \delta) dt + \tilde{\sigma}^*(q^*) d\tilde{Z}_t.$$

Net worth risk under the assumption that there is no capital diversion is $\psi^* \tilde{\sigma}^*(q^*)$. Therefore, the condition for the optimal portfolio weight on illegitimate capital to be 0 is

$$E[d\hat{r}_t^{K,*} - dr_t^{K,*}] \leq \text{Cov}(d\hat{r}_t^{K,*} - dr_t^{K,*}, \psi^* \tilde{\sigma}^*(q^*) d\tilde{Z}_t) \quad \text{or} \quad \rho \leq (\psi^*)^2 \tilde{\sigma}^*(q^*)^2. \quad (2.8)$$

²It is an open question whether the planner could improve the social outcome by conditioning on individuals histories of shocks. If so, then the planner can not only choose the level of insurance of each individual, but also make transfers that are functions of wealth, subject to the aggregate resource constraint. We conjecture that yes, a policy of this more general form can be welfare improving, because the cost of insuring agents who have suffered adverse shocks in terms of their contribution to overall economic growth is lower. However, this is an open question.

Whether this condition holds under the policy that maximizes (2.6) depends on model parameters. For example, with $\Phi(\iota^*) = \iota^*$, i.e. in the absence of investment adjustment costs, we have $q^* = 1$ and we can find through a bit of algebra that (2.8) holds if and only if

$$\tilde{\sigma}^*(1) > 2\sqrt{\rho}.$$

Optimal Monetary Policy. We show next that the optimal insurance policy can be implemented in a decentralized way in an equilibrium with money and capital. Suppose that individuals can hold capital and money. Money supply is controlled by the policy maker, and monetary policy can be either inflationary or deflationary. Under inflationary policy, the planner prints money, at a rate proportional to the total money supply, and distributes it to individuals proportionately to their wealth. Under deflationary policy, the planner imposes a proportionate wealth tax, and uses the proceeds to repurchase money and remove it permanently from circulation.

Under policy, the market-clearing condition for consumption (2.3) as well as the optimal investment equation (2.1) still hold, but the pricing equation (2.2) may no longer hold. Rather, the policymaker can raise θ^* relative to its equilibrium level through a deflationary policy, or lower it through an inflationary policy.

Since equilibrium equations (2.3) and (2.1) are equivalent to (2.4) and (2.5), with $1 - \theta^* = \psi^*$, and since the equilibrium law of motion of individual net worth is identical to (2.7), we conclude that monetary policy can implement the outcome of any social insurance scheme with constant ψ^* .

Recall that the optimal policy is robust to the agent's ability to hide capital if and only if condition (2.8) holds. Observe also that if condition (2.8) holds then the corresponding monetary policy is inflationary: it makes capital more attractive to hold relative to the constant-money-supply equilibrium. If (2.8) fails, then the optimal policy is deflationary. Hence, optimal monetary policy is easier to enforce if it is inflationary, i.e. it is robust to a greater set of deviations by individual agents.

To sum up, we can think of monetary policy as providing uniform insurance. Any insurance carries moral hazard: in this case it distorts the agents' incentives to invest. Optimal policy is based on the trade-off between insurance and incentives, and may be inflationary or deflationary. Inflationary policy is more robust to agents' deviations. Finally, even though productivity parameters $b_{1,t}^*$ and $b_{2,t}^*$ are stochastic, optimal policy is stationary.

2.2 Return and Return on Global Money

We now address the risks generated within the large country that propagate to the small country. We present the full model of the small country in the next section. The small country can produce only the first global good, and must import the second global good from the large country. Agents in the small country have incentives to self-insure against productivity shocks and shocks to the prices of goods by holding large country's currency. Therefore, here we (1) derive how the price of global good 1 exported by the small country fluctuates relative to the price of aggregate global good and (2) characterize the return on large country's money, expressed in terms of the global aggregate good as the numeraire.

First, $b_{1,t}^*$ units of global good 1 buy $(b_{1,t}^*)^\beta (b_{2,t}^*)^{1-\beta}$ units of the aggregate global good. Hence, the price of the aggregate global good in terms of good 1 is

$$(b_{1,t}^*/b_{2,t}^*)^{1-\beta}. \quad (2.9)$$

Second, let us derive the return on large country's currency. In terms of the local good, money in the large country has risk-free return $r^* dt$, which may be less than or greater than the natural rate

$$(\Phi(\iota^*) - \delta) dt$$

depending on whether monetary policy is deflationary or inflationary.

Now, the price of the local good in terms of the global good is

$$b_t^* = \frac{(b_{1,t}^*)^\beta (b_{2,t}^*)^{1-\beta}}{a^*}.$$

Hence, the return on money in terms of the global good is

$$r^* dt + \frac{db_t^*}{b_t^*}. \quad (2.10)$$

We wait until the next section to impose specific assumptions on the processes $b_{1,t}^*$ and $b_{2,t}^*$ that the price of the aggregate global good in terms of good 1, as well as the return on the *global currency* (i.e. large country's money).

3 The Small Country and the Two Currencies

Now we turn to the small country: our main object of interest. The model of the small country is similar to that of the large country, except that the small country does not have the technology to produce global good 2, and so it must trade. Agents in the small country can use capital to produce either the non-tradable local good at rate a per unit of capital, or global good 1 at rate $b_{1,t}$. In trade with the large country, the small country takes the prices of global goods as given. The local good is used for investment.

The size of the basket of aggregate global good that can be traded for global good 1 produced by a single unit of capital in the small country, given the price ratio (2.9), is given by

$$b_t = b_{1,t}(b_{2,t}^*/b_{1,t}^*)^{1-\beta}.$$

In what follows, we can imagine that one unit of local capital produces b_t units of the aggregate global good directly - although in the back of our minds we know that these units are obtained from trade, by producing global good 1 first. The stochastic process that b_t follows, from the point of view of local agents, is of primary importance. Local agents face this risk, and they can insure themselves against this risk partially by holding global money.

Assume that the stochastic processes for $b_{1,t}$, $b_{1,t}^*$ and $b_{2,t}^*$ are such that b_t follows a geometric Brownian motion, i.e.

$$db_t/b_t = \mu^b dt + \sigma^b dZ_t,$$

where Z is a standard Brownian motion. This will always be the case if $b_{1,t}$, $b_{1,t}^*$ and $b_{2,t}^*$ follow possibly correlated geometric Brownian motions.

Also, assume that process b_t^* also follows a geometric Brownian motion, so that the return on global money (2.10) can be written in the form

$$dr_t^G = \mu^G dt + \sigma^G dZ_t + \sigma^{G,*} dZ_t^*, \quad (3.1)$$

where Brownian motions Z and Z^* are taken to be independent. Return (3.1) is expressed in the units of the aggregate global good.

Agents in the small country have utility

$$(1 - \alpha) \log c_l + \alpha \log c_g,$$

when they consume the local good at rate c_l and the *aggregate* global good at rate c_g . The common discount rate is ρ .

Capital held by individual agents is subject to idiosyncratic shocks of the size $\sigma(q_t)$, where q_t is the price of capital in terms of the local good. There is local money, which is an infinitely divisible asset in fixed supply, which pays no dividends. Agents can hold local money as well as global money (i.e. money of the large country) to self insure themselves against idiosyncratic shocks, as well as shocks to b_t . This completes our model description, absent monetary policy within the small country.

The determinants of value of the two currencies. Before even writing this model, we asked ourselves several questions. Is it at all possible for different types of money have value in equilibrium, even though money is a bubble? Would it not be arbitrary which type of money “survives” in equilibrium, and would it not be knife-edge for two types of money to co-exist? If it is indeed possible for several currencies to co-exist, and if the conditions for this to happen are not knife-edge, then what may be the fundamental determinants of currency value, to such an extent that it is possible to determine exchange rates and analyze currency risk?

In the equilibrium we derive, we are able to determine the values of two countries’ currencies, their risk and the exchange rate process. What ties the currency to a country? We imagine a perturbation of the model in which each country’s currency gives a small benefit to local citizens, as the local currency is used for local transactions. This small perturbation is like a dividend on local currency - and hence, as Sims (1994) observes (in his setting, the small dividend is backed explicitly by taxes), this perturbation selects the equilibrium in which money has value in a one-country model. With two countries, it is possible for two currencies to coexist in one model.

Why, and under what conditions, do individuals in the small country want to hold both local and global currencies? The rough intuition is that the local currency is tied to the local economy, and thus it is less risky relative to the local consumption basket than the global currency. Thus, to self-insure against idiosyncratic shocks, local agents want to hold local money, but to self-insure against shocks to b_t , they want to hold global money. Of course, relative holdings of local and global currencies in the local agents’ portfolios depend on expected growth in each country, as well as monetary policy. There are conditions in which local agents prefer to hold exclusively one currency. However, there is also a set of parameter values of positive measure where two currencies co-exist.

With this introduction, we proceed to solve for the equilibrium in the small country’s

economy, taking as given the risks generated within the large country.

Production and consumption of global goods, and global money savings. Denote by $\hat{\alpha}_t$ the fraction of capital dedicated to the production of global good 1, which can be traded for aggregate global good or global money. Denote by $\xi_t b_t K_t$ the local consumption of the aggregate global good. Let G_t be the value of local holdings of global money, expressed in the units of the aggregate global good. Then

$$\frac{dG_t}{G_t} = dr_t^G + \frac{(\hat{\alpha}_t - \xi_t)b_t K_t}{G_t} dt.$$

In this paper we restrict $G_t \geq 0$, i.e. local agents cannot borrow global currency.³

State Variable. Unlike in the large country, the equilibrium in the small country is nonstationary as it depends on savings of the global currency. The relevant state variable is the ratio of global currency savings to potential productivity of the local economy, i.e.

$$\nu_t = \frac{G_t}{b_t K_t}.$$

From the laws of motion of G_t , b_t and K_t , it follows

$$\begin{aligned} \frac{d\nu_t}{\nu_t} &= dr_t^G + \frac{\hat{\alpha}_t - \xi_t}{\nu_t} dt - \frac{db_t}{b_t} - (\Phi(\nu_t) - \delta) dt + \sigma^b(\sigma^b - \sigma^G) dt = & (3.2) \\ & \frac{\hat{\alpha}_t - \xi_t}{\nu_t} dt + \underbrace{(\mu^G - \mu^b + \sigma^b(\sigma^b - \sigma^G) + \delta)}_M dt - \Phi(\nu_t) dt + (\sigma^G - \sigma^b) dZ_t + \sigma^{G,*} dZ_t^*. \end{aligned}$$

As we continue deriving the equilibrium conditions, the reader can confirm that six parameters of this model, μ^G , μ^b , δ , σ^b , σ^G and $\sigma^{G,*}$ enter the equilibrium equations through only two combinations, M and

$$\sigma^\nu = \sqrt{(\sigma^G - \sigma^b)^2 + (\sigma^{G,*})^2},$$

which is the volatility of ν .

Returns and Asset Pricing. It is useful to express returns in terms of the portfolio weights θ_t on local money and ζ_t on global money. We postulate the following law of motions

³It is interesting to study what happens if borrowing is allowed, but in this case default must occur with positive probability, and so the solution would depend on model specification regarding how default is treated.

for θ_t and ζ_t ,

$$\frac{d\theta_t}{\theta_t} = \mu_t^\theta dt + \sigma_t^\theta dZ_t + \sigma_t^{\theta,*} dZ_t^*$$

and

$$\frac{d\zeta_t}{\zeta_t} = \mu_t^\zeta dt + \sigma_t^\zeta dZ_t + \sigma_t^{\zeta,*} dZ_t^*.$$

Also, denote by n_t the individual wealth of a citizen of and N_t the aggregate wealth in the small country.

The asset-pricing conditions with log utility are based on the principle that the difference in expected returns between any two assets is explained by the covariance between the difference in risk and net worth risk. Importantly, we can use any numeraire that is convenient to express the returns. It is convenient to use the entire country wealth N_t as the numeraire. Then, relative to N_t , then the portfolio return of an individual agent has dividend yield equal to the consumption rate ρ and idiosyncratic risk, i.e. the return on individual wealth is

$$dr_t^n = \rho dt + \tilde{\sigma}^n d\tilde{Z}_t, \quad (3.3)$$

where $\tilde{\sigma}^n = (1 - \zeta - \theta)\tilde{\sigma}(q_t)$ is idiosyncratic risk exposure. The returns on global and local money are, respectively

$$dr_t^{MG} = \frac{\xi_t - \hat{\alpha}_t}{\nu_t} dt + \frac{d\zeta_t}{\zeta_t} \quad \text{and} \quad dr_t^{ML} = \frac{d\theta_t}{\theta_t}.$$

Hence, pricing global money relative to the full net worth portfolio, we obtain

$$E[dr_t^n - dr_t^{MG}] = Cov(dr_t^n - dr_t^{MG}, dr_t^n) \Rightarrow \rho - \frac{\xi_t - \hat{\alpha}_t}{\nu_t} - \mu^\zeta = (\tilde{\sigma}^n)^2. \quad (3.4)$$

Likewise, pricing local money relative to the net worth portfolio, we obtain

$$E[dr_t^n - dr_t^{ML}] = Cov(dr_t^n - dr_t^{ML}, dr_t^n) \Rightarrow \rho - \mu^\theta = (\tilde{\sigma}^n)^2. \quad (3.5)$$

By expressing returns in terms of portfolio weights, we were able to obtain rather simple asset-pricing conditions (3.4) and (3.5). We can solve these equations numerically via the time step of the iterative method described in Brunnermeier and Sannikov (2016c). These are differential equations for portfolio weights ζ and θ . We also need additional static conditions to determine ξ , $\hat{\alpha}$ and the price of capital q on the entire state space.

Market Clearing, Capital Allocation and Optimal Investment. Aggregate wealth,

expressed in the units of the tradable good basket, is G_t/ζ_t . We also know that global good consumption is fraction α of the value of total consumption. Hence, total consumption is $\xi_t b_t K_t/\alpha$, and the market-clearing condition for consumption is

$$\rho \frac{G_t}{\zeta_t} = \frac{\xi_t b_t K_t}{\alpha} \quad \text{or} \quad \xi_t = \frac{\alpha \rho \nu_t}{\zeta_t}. \quad (3.6)$$

Let P_t^l and P_t^g be the prices of local and global goods, in terms of the combined good according to the Cobb-Douglas function with weights $1 - \alpha$ and α . Then

$$\frac{\xi_t b_t K_t}{\alpha} P_t^g = \frac{((1 - \hat{\alpha}_t)a - \iota_t) K_t}{1 - \alpha} P_t^l$$

is total consumption expenditure. Moreover, optimal production allocation in the small country implies

$$\frac{b_t P_t^g}{a P_t^l} = 1$$

if both goods are produced in the local economy, i.e. $\hat{\alpha}_t \in (0, 1)$, since the agents must be indifferent. However, if capital share devoted to global good production $\hat{\alpha}_t$ is 0 or 1, then this ratio can be less than 1 or greater than 1, respectively. Hence,

$$\frac{b_t P_t^g}{a P_t^l} = \frac{(1 - \hat{\alpha}_t)a - \iota_t}{1 - \alpha} \frac{\zeta_t}{a \rho \nu_t} \begin{cases} = 1 & \text{if } \hat{\alpha}_t \in (0, 1), \\ \leq 1 & \text{if } \hat{\alpha}_t = 0 \\ \geq 1 & \text{if } \hat{\alpha}_t = 1 \end{cases} \quad (3.7)$$

Investment depends on the price of capital q_t in terms of the local good. Wealth measured in the local good is $(G_t/\zeta_t)P_t^g/P_t^l$. Hence, the value of capital in terms of the local good is

$$q_t K_t = (1 - \zeta_t - \theta_t) \frac{G_t}{\zeta_t} \frac{P_t^g}{P_t^l} \Rightarrow q_t = a \nu_t \frac{1 - \zeta_t - \theta_t}{\zeta_t} \underbrace{\frac{(1 - \hat{\alpha}_t)a - \iota(q_t)}{1 - \alpha} \frac{\zeta_t}{a \rho \nu_t}}_{1 \text{ if } \hat{\alpha}_t \in (0,1)} \quad (3.8)$$

We can determine consumption rate ξ_t , capital allocation $\hat{\alpha}_t$, and the price of capital q_t that guides investment from portfolio weights ζ_t and θ_t as follows. First, guess that $\hat{\alpha}_t \in (0, 1)$, set $q_t = a \nu_t (1 - \zeta_t - \theta_t)/\zeta_t$ and test whether $\hat{\alpha}_t$ implied by (3.7) is indeed between 0 and 1. If not, then we set $\hat{\alpha}_t$ to the nearest endpoint of the interval, and solve for q_t instead from (3.8). After that, we compute ξ_t from (3.6).

When money has value. We finish this section by asking questions about money

holding in equilibrium. We can identify analytically (without solving for full equilibrium dynamics) conditions when only local money is held in equilibrium, and conditions when global money is held (and local money may be held as well). The following proposition characterizes the relevant cases.

Proposition 1. *Let*

$$A_1 = \mu^G - \mu^b + \sigma^b(\sigma^b - \sigma^G) + \delta - \Phi(\iota) = M - \Phi(\iota) \quad \text{and} \quad A_2 = \tilde{\sigma}^2(q) - \rho,$$

where ι is the rate of investment and q is the price of capital at the point where $\nu_t = 0$. If $A_1 \leq 0$ and $A_2 > 0$, then local money has value in equilibrium and $\nu = 0$ is the absorbing state, i.e. local agents choose to not hold any global money. Otherwise, if $A_1 + A_2 > 0$ then individuals will accumulate global money, and local money may or may not have value in equilibrium.

Proof. If locals had no access to global money, then the equilibrium with local money, whenever it has value, is characterized by the equations of Section 2.1, i.e.

$$\Phi'(\iota)q = 1, \quad \rho = (1 - \theta)^2 \tilde{\sigma}(q)^2 \quad \text{and} \quad \rho q = (1 - \theta)(a - \iota). \quad (3.9)$$

For this to be the case, we need $A_2 < 0$. If individuals also had access to global money, then local money would still have value and no global money would be held at the steady state if the optimal portfolio weight on global money is 0. To see when this is the case, notice that the return on global money in terms of the global good is given by (3.1) and the return on local money under the assumption that $\nu_t = 0$ is the steady state is $(\Phi(\iota) - \delta) dt$ in the local good and

$$(\Phi(\iota) - \delta) dt + \mu^b dt + \sigma^b dZ_t$$

in the global good, since the price of the local good in terms of the global good is proportional to b_t . The optimal portfolio weight on global money is 0 if

$$\Phi(\iota) - \delta + \mu^b - \mu^G \geq (\sigma^b - \sigma^G)\sigma^b, \quad (3.10)$$

or $A_1 \leq 0$. Hence, $A_2 < 0$ and $A_1 \leq 0$ is the condition for local money to have value and global money to not be held at the steady state.

If we identify now the condition where neither money is held, then we can back out the region where global money is held and local money may be held. If neither money is used

in equilibrium, then the return on agents' net worth is

$$\underbrace{\rho dt + (\Phi(\iota) - \delta) dt + \tilde{\sigma}(q) d\tilde{Z}_t + \mu^b dt + \sigma^b dZ_t}_{\text{return in terms of local good}}$$

in terms of global good basket. The optimal portfolio weight on global money is indeed 0 if

$$\rho + \Phi(\iota) - \delta + \mu^b - \mu^G \geq (\sigma^b - \sigma^G)\sigma^b + \tilde{\sigma}^2(q).$$

This condition is equivalent to $0 \geq A_1 + A_2$. (Outside of the region where $A_2 < 0$ and $A_1 \geq 0$), this is where neither money has value, and if $A_1 + A_2 > 0$ then global money must have value and local money may or may not have value. \square

We must acknowledge that the conditions of Proposition 1 depend on endogenous variables, particularly the rate of local investment and local growth. However, these are variables that can be found from static equations, without solving for the full equilibrium dynamics, under the assumption that $\nu_t = 0$ is an absorbing state.

We do not provide explicit conditions for the boundary where global money has value, and local money may or may not have value. That boundary depends on global money accumulation, and the extent to which global money fulfills money demand. We suspect that the precise condition depends on the full equilibrium dynamics, although we may be able to provide a sufficient condition for local money to not have value, and we can provide an approximate back-of-the-envelope condition to guarantee that both types of money have value (see Section 4).

4 Examples and Discussion.

Consider parameters of the local economy $\rho = 5\%$, $\tilde{\sigma} = 0.3$, $\alpha = 0.2$, $\mu^b = 1\%$, $\sigma^b = 0.15$, $\mu^G = 2.2\%$, $a = 0.13$, $\delta = 0.02$, $\sigma^G = \sigma^{G,*} = 0$ and $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$ with $\kappa = 2$. Thus, $M = 0.0545$ and $\sigma^\nu = 0.15$.

Then Figure 1 illustrates the equilibrium in this example. Top left panel shows the drift of ν , which is positive for low holdings of global money, but eventually becomes negative. Notice the kink in the left panels. When ν is low then a portion of capital is dedicated to producing global good 1 for export, while the remaining capital is used to produce the non-tradable local good. When ν is high enough then all capital is used to produce the

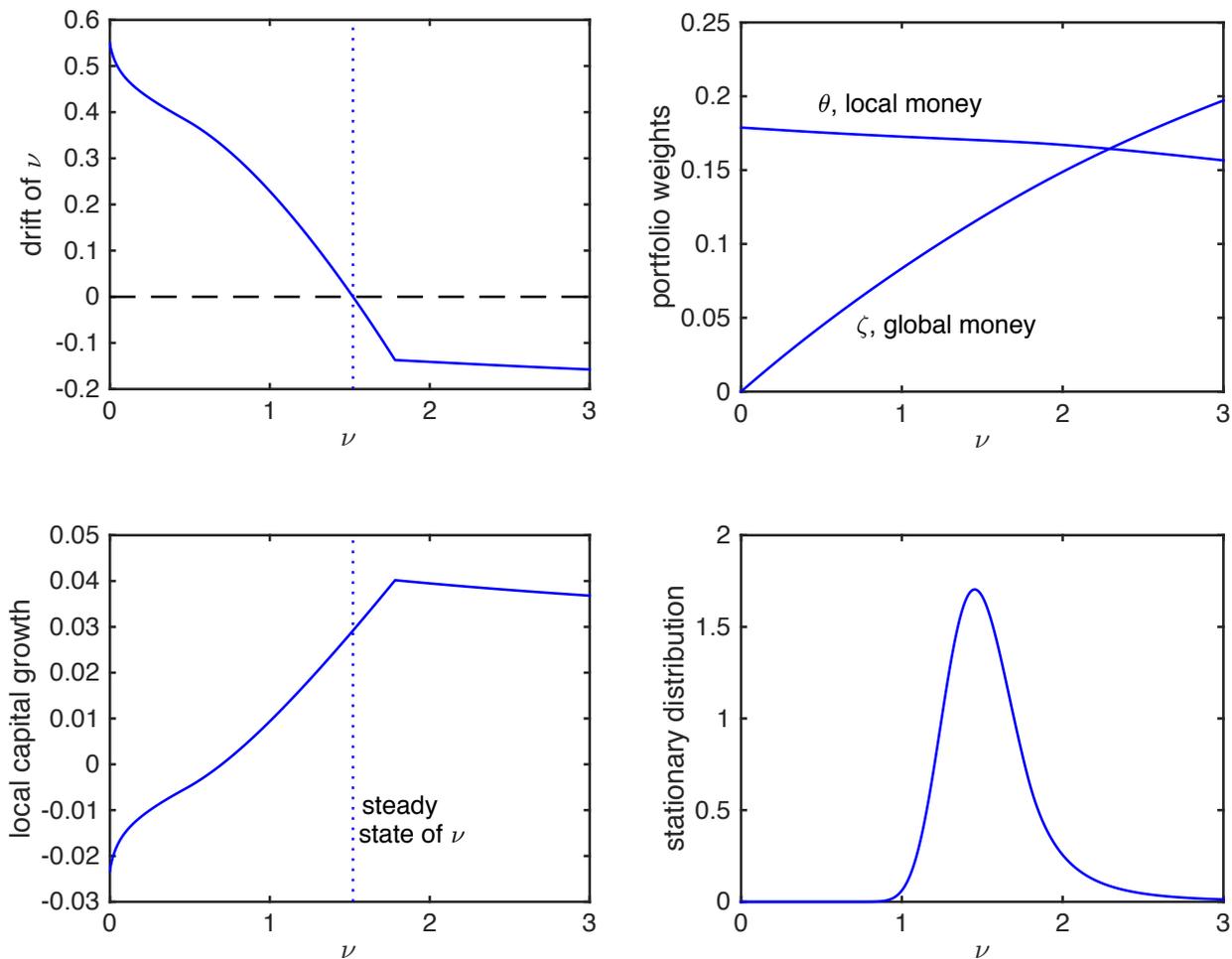


Figure 1: Equilibrium.

local good, because agents prefer to use their savings to buy the global good. The kink corresponds to the boundary between these two regimes.

In this example, the “steady state,” i.e. the level of ν where the drift is 0, lies deep in the region where both goods are produced locally. In fact, most of the stationary distribution, illustrated in bottom right panel, lies in the region where both goods are produced locally. Variable ν moves around the steady state due to shocks to b_t , i.e. a combination of shocks to local productivity and shocks to relative global goods prices.

Bottom left panel shows the growth rate of local capital, which is related to but different from the GDP growth rate. Expected GDP growth in this model is a bit higher because of productivity growth with respect to global good production. As ν rises until the kink, more

capital is devoted to the production of the local good, which is used for investment, hence investment rises.

Top right panel shows the portfolio weights on global and local money. As ν rises, naturally the portfolio weight on global money rises. As total money supply (global and local) increases, the value of local money and hence its portfolio weight, falls. Portfolio weight of global money is a concave function - as the amount of global money held increases, its value per unit falls.

Global and local money are imperfect substitutes, and money demand depends on the riskiness of the two types of money as well as money returns. In this example $A_2 > 0$ and $A_1 > 0$ so parameters fall in the region where global money has value and local money may have value, according to Proposition 1. Both types of money are held in equilibrium, even though local growth is higher than the expected return on global money. Agents use global money as a hedge against the shocks to b_t .

In equilibrium agents can use both types of money regardless of how the two countries' growth rates or returns on the two currencies compare. Currency demand depends on currency returns, and also their risk profiles. Global money provides insurance, possibly imperfect, for shocks to export revenues while the risk of local money is more aligned with the local consumption basket. Proposition 1 suggests that the demand for global money is guided by

$$M = \mu^G - \mu^b + \sigma^b(\sigma^b - \sigma^G) + \delta.$$

It is instructive to identify the parameter changes that lead to a decrease in M , and thus lower demand for global currency within the small country. The demand for global money falls if the expected return μ^G on global money falls, if growth μ^b of export revenues rises, or if local growth rises, e.g. because depreciation falls. These are the return effects. Furthermore, we can look at changes in the risk parameters that keep $(\sigma^\nu)^2 = (\sigma^b - \sigma^G)^2 + (\sigma^{G,*})^2$ fixed. Demand for global money falls if global money becomes riskier relative to goods the small country wants to export, i.e. $\sigma^{G,*}$ rises while the riskiness of exports relative to money $\sigma^b - \sigma^G$ falls, or of the risk of exports σ^b falls while $\sigma^b - \sigma^G$ stays fixed.

To illustrate how a decrease in M affects equilibrium, we consider a loosening of the large country's monetary policy, reflected in the lower return on global money of $\mu^G = 2.1\%$. Figure 2 compares equilibrium under the old set of parameters and the new set.

The left panel tells us that lower μ^G leads to a lower drift of ν_t , and thus lower "steady state." In the right panel we see that local growth is higher at all levels of ν_t when μ^G is lower, because greater consumption out of global money savings at any level leads to a higher

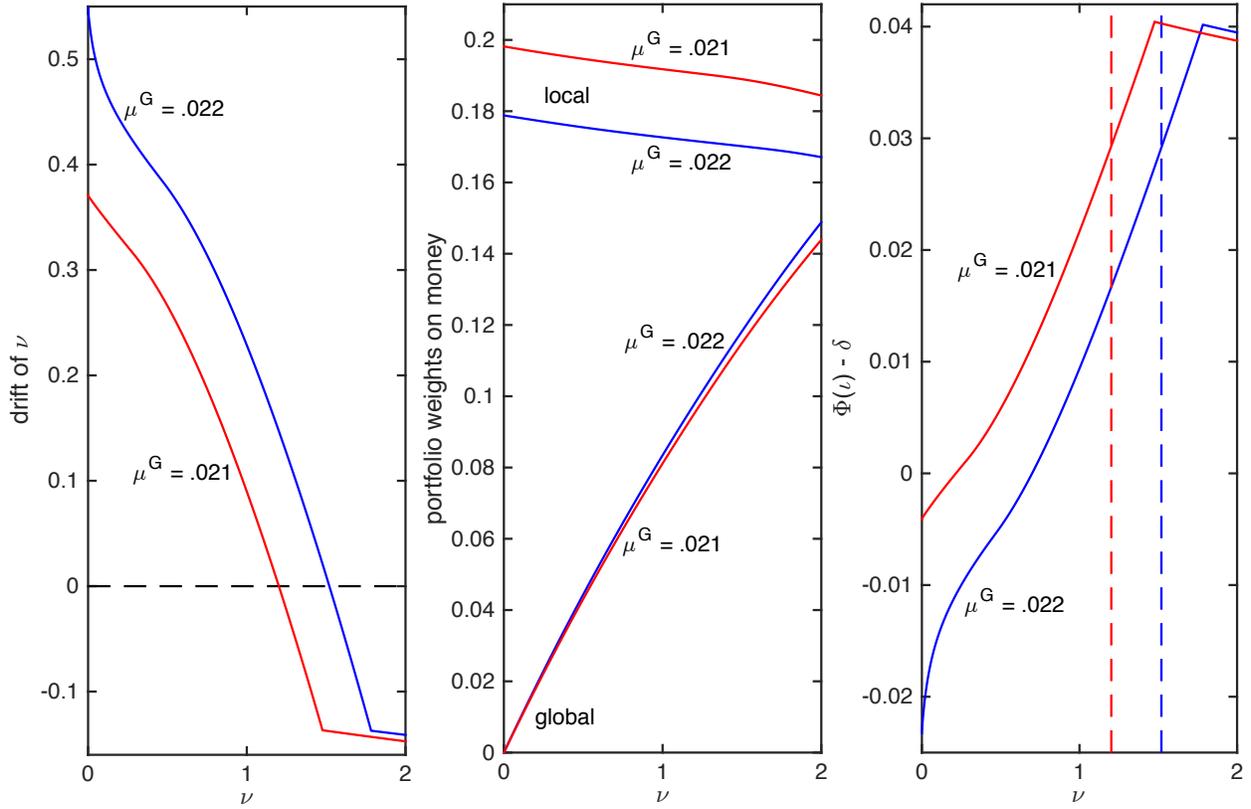


Figure 2: Lowering of μ^G .

production of local good and greater investment. However, at the steady state, growth is virtually the same for both parameter values, $\Phi(\nu) - \delta \approx 2.92\%$. The vertical dashed lines in the right panel locate the steady state of ν_t in the two scenarios.

In the middle panel we see that, with lower μ^G , global money becomes slightly less valuable to the locals. That is not surprising. However, what may appear surprising at first is the significant increase in the value of local money. The reason for the significant increase is that local money value takes into account the demand for local money at the steady state of the system, and global money holdings at the steady state drops significantly when μ^G drops.

We finish this section with back-of-the-envelope calculations for conditions, under which both global and local money have value in equilibrium in the small country. These conditions give us crude, but valuable intuition. We compare these conditions with the actual range of parameters under which both types of money have value.

Range in which global and local money have value: a back-of-the-envelope calculation. For the sake of this approximation, we imagine that the price of the local good in terms of the global good is proportional to b_t , and we ignore endogenous risk (such as changes in q_t due to movements of ν_t). We use the local good as numeraire for the expressions below. The fundamental risks of capital, local money and global money are, respectively,

$$\tilde{\sigma}(q) d\tilde{Z}, \quad 0 \quad \text{and} \quad (\sigma^G - \sigma^b) dZ_t + \sigma^{G,*} dZ_t^*.$$

The expected return on global money can be approximated as

$$\mu^G - \mu^b + \sigma^b(\sigma^b - \sigma^G).$$

The expected returns on capital and local money can be approximated as

$$\frac{\rho}{1 - \theta - \zeta} + \Phi(\iota) - \delta \quad \text{and} \quad \Phi(\iota) - \delta,$$

where we attach the total dividend yield in the economy (at rate ρ times net worth) to capital.

Net worth has idiosyncratic risk of $(1 - \theta - \zeta)\tilde{\sigma}(q)$ and aggregate risk

$$\zeta((\sigma^G - \sigma^b) dZ_t + \sigma^{G,*} dZ_t^*).$$

Pricing local money relative to global,

$$\underbrace{\mu^G - \mu^b + \sigma^b(\sigma^b - \sigma^G) + \delta - \Phi(\iota)}_{A_1 = M - \Phi(\iota)} = \zeta(\sigma^\nu)^2 \quad \Rightarrow \quad \zeta = \frac{M - \Phi(\iota)}{(\sigma^\nu)^2}.$$

Pricing capital relative to local money,

$$\frac{\rho}{1 - \theta - \zeta} = (1 - \theta - \zeta) \tilde{\sigma}^2(q) \quad \Rightarrow \quad 1 - \theta - \zeta = \frac{\sqrt{\rho}}{\tilde{\sigma}(q)}. \quad (4.1)$$

Hence, our conditions for both types of money to have value in equilibrium are

$$M > \Phi(\iota) \quad \text{and} \quad \frac{\sqrt{\rho}}{\tilde{\sigma}(q)} + \frac{M - \Phi(\iota)}{(\sigma^\nu)^2} < 1, \quad (4.2)$$

where our approximation for ι is given by the market-clearing condition

$$\frac{\rho q}{1 - \theta - \zeta} = a - \iota(q).$$

These conditions suggest that quantity A_1 of Proposition 1 is related to the demand for global money, while quantity A_2 is related to total money demand. Thus, this rough calculation complements Proposition 1. To sum up, both local and global money have value in equilibrium if $\tilde{\sigma}(q) > \sqrt{\rho}$, and if global money is not so attractive (as measured by $M - \Phi(\iota)$) that it crowds out local currency.

Let us see to what extent this approximation is valid. For our example, (4.1) predicts that $1 - \theta - \zeta = 0.745$. In actuality, at the steady state, $1 - \theta - \zeta = 0.7104$. The approximation seems quite precise. However, the approximation is less good at predicting demand for local money, as it gives $\zeta = 0.0815$ while in practice at the steady state $\zeta = 0.1195$. What about the range of parameter values μ^G for which global and local money have value? The low boundary of $\mu^G = 0.0201$ is pinned down precisely by (4.2) - see Proposition 1. For the upper boundary value of μ^G where global money crowds out local money, the approximation gives us 0.0258. In numerical simulations, this happens later, at $\mu^G = 0.0255$.

5 Policy Range and the Mundell-Fleming Trilemma

In this section, we study the room for monetary policy that the small country has within this framework. Does the small country have any room to maneuver with its own currency given an open capital account? What about with a closed capital account? What difference does it make if the small country holds global-currency reserves? To what extent can the small country affect the risk profile of its own currency? Is it possible, and does it make sense, to peg the small country's currency to the dollar?

In order to address these questions, we start by modifying some of our equilibrium equations to take policy into account. First, consider what happens if the central bank prints local money at rate π_t per unit of local money outstanding, and allocates it to population proportionately to capital holdings. This has no effect on the law of motion of individual wealth (3.3) but the return on local money has to be modified to

$$dr_t^{ML} = \frac{d\theta_t}{\theta_t} - \pi_t dt.$$

Hence, the pricing equation for local money (3.5) has to be modified to

$$\rho + \pi_t - \mu^\theta = (\tilde{\sigma}^N)^2. \quad (5.1)$$

Local Inflation: An Example. We can see the effect that this policy has on an example. For our baseline set of parameters, consider the policy that sets $\pi_t = 0.3\%$. Then Figure 3 illustrates the effects that policy has on equilibrium.

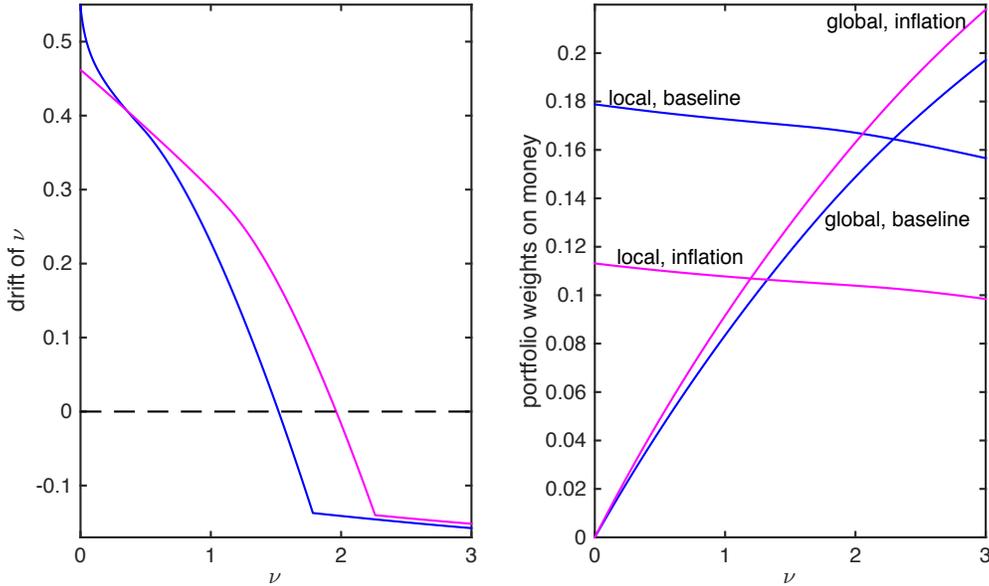


Figure 3: Inflation policy.

The left panel shows that with inflationary policy with respect to the local currency, locals accumulate more global currency: the steady state shifts to the right. The right panel illustrates the effects of the policy on money portfolio weights. With local inflation, the value of local currency falls. This effect is quite pronounced. The value of global currency rises slightly relative to local goods. Local agents accumulate more global currency to fill in the money demand that stems from idiosyncratic risk. However, global currency has an imperfect risk profile for the locals' savings: at the steady state individual agents end up over-hedged with respect to export risk. The total portfolio share on local and global money at the steady state is $\theta + \zeta = 0.2815$, whereas in our baseline scenario it was $\theta + \zeta = 0.2896$. Inflation leads to higher growth at the steady state at the cost of less perfect insurance of idiosyncratic risk: we have $\Phi(\iota) - \delta = 3.14\%$, whereas in the baseline scenario we had

$$\Phi(\iota) - \delta = 2.92\%.$$

Open Capital Account and the Range of Local Monetary Policy. In the presence of global currency that individual agents can hold freely, the power of local monetary policy is limited because global currency is an imperfect substitute for local currency. However, there is still room to maneuver even with completely open capital account. Up until about $\pi = 0.52\%$, local money has declining but positive value. For higher values of π , local monetary policy has no bite as local money becomes worthless. Through inflation, local monetary authority can raise local growth at the steady state up to $\Phi(\iota) - \delta = 3.52\%$.

When the large country carries out loose monetary policy, so that expected return on global currency μ^G is lower, the small country has more room to maneuver. In the extreme μ^G is so low that locals choose not to hold global currency - the same outcome is attained by closing the capital account and preventing local agents from global currency holdings. If so, then at the steady state $\nu = 0$, we have $\theta = 0.2546$ when $\pi = 0$, i.e. the local government refrains from inflating. Total money holding are less than our baseline scenario with an open capital account, when extra demand for global currency comes from the locals' hedging demand with respect to export risk. Lower money holdings overall lead to higher growth of $\Phi(\iota) - \delta = 3.35\%$. Local monetary authority can boost growth further through inflation. In fact, local money retains value all the way to $\pi = 4\%$, the level at which

$$\sqrt{\rho + \pi} = \tilde{\sigma}.$$

Beyond that point, policy has no bite, the price of capital reaches the level of

$$q = \frac{\kappa a + 1}{\kappa \rho + 1}$$

given our functional form $\Phi(\iota) = \log(\kappa \iota + 1)/\kappa$, and steady-state growth in our example reaches the maximum level of $\Phi(q)/\kappa - \delta = 4.79\%$. To sum up, higher inflation in the global currency or closed capital account give the small country much more room to maneuver in local monetary policy, with respect to inflation choice.

In the opposite direction, the local country can strengthen its currency through fiscal backing, but taxing capital and distributing the proceeds as interest on local currency. This policy corresponds to a negative value of π_t in equation (5.1).

Other Possibilities for Taxes and Subsidies. One may also ask the question of how equation (5.1) is affected if the monetary authority distributes proceeds of seignorage

in other ways, e.g. proportionately to (1) wealth invested in capital and local money or (2) all wealth or (3) per capita. Or, local money can be backed by taxes in any of the three ways above. If the distribution is proportionate to wealth in capital and local money, then the pricing equation for local money has to be modified to

$$\rho + \frac{1 - \zeta_t - \theta_t}{1 - \hat{\zeta}_t} \pi_t - \mu^\theta = (\tilde{\sigma}^N)^2,$$

since only the portion of money printed to be distributed to capital matters. Thus, in this case the range of real outcomes that can be implemented by setting π_t remains the same - there are only nominal differences. If the printing of local currency is distributed proportionately to wealth, then equation (5.1) is modified by adding coefficient $1 - \theta_t$ in front of π_t . The pricing equation for global money also has to be modified by the dividend yield that global money held by locals gets from seignorage. In general, through taxes and subsidies, the local policy-maker can modify the pricing equations (3.5) and (3.4) in an arbitrary way to affect the amount of global and local currency held by local agents. Then the policy maker can control the risk exposures of local agents to idiosyncratic and export risk. Recognizing the link between these risk exposures and investment distortions, we can think about optimal policy. We address this question in Section 6.

Finally, if the policy-maker distributes the proceeds of seignorage per capita, then one may guess that it may be possible to fully insure idiosyncratic risk. After all, individuals whose wealth has been virtually wiped out by idiosyncratic shocks can get transfers. Not so fast.. It turns out that policy of local money-printing and distribution per capital has no effect whatsoever if it is anticipated by agents, with sufficiently frictionless markets. Agents will anticipate these transfers in their investment decisions, and act as if they already had received the transfers. For this equivalence result to hold, it is essential that the agents should be able to borrow against future transfers. This observation is in the spirit of Modigliani-Miller and Ricardian equivalence - that certain financing policies have no real effects because they get undone by investors.

Dollarization/Currency Peg. Can the small country affect the risk profile of the local currency, and is it desirable? At the extreme, the small country can do away with local currency and let its citizens use the global currency instead. This outcome is obtained if the small country sets π_t above the critical level, which makes the local currency worthless.

Figure 4 shows the outcome that obtains in the absence of local currency for our baseline parameters. Local agents accumulate much more of the global currency and the growth rate

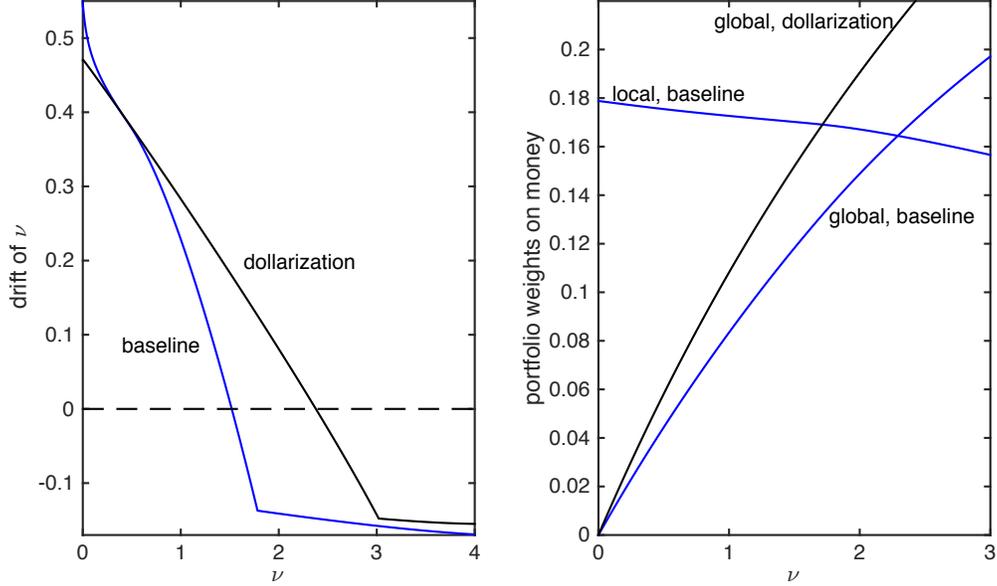


Figure 4: Equilibrium without Local Currency.

is $\Phi(\iota) - \delta = 3.52\%$. This outcome does not seem attractive from the point of view of welfare since the global currency is a risky way to save to self-insure idiosyncratic shocks.

Can the local monetary authority peg the local currency to the global currency? Yes, but such a peg would require strong fiscal backing if the monetary authority wants to create a positive supply of local currency without holding the global currency as reserves. There is always a positive probability that the local economy shrinks below any positive initial outstanding amount of local currency pegged to the global currency. Given this possibility, to maintain the peg, the government of the local country must tax and use the proceeds to remove some of the local currency out of circulation. This has to be difficult, distortionary, and it also robs the local citizens of the benefit of using their local currency to self insure.

Foreign-Currency Reserves. Can the local policy-maker affect the risk profile of local currency by holding foreign-currency reserves? Can this policy give value to the local currency even in the range where local currency would be worthless on its own?

Consider reserve holdings in the absence of fiscal policy, i.e. the monetary authority neither taxes wealth to back local currency, nor raise seignorage and give the proceeds to its citizens. Then we have an irrelevance result: local citizens treat the currency held by the central bank as reserves as if they had the money in their portfolios. The equilibrium is characterized by the same equations (3.5) and (3.4), except with modified definitions of

ζ_t and θ_t . Specifically, ζ_t denotes the portfolio weight of all global currency held by local agents and the local central bank, while θ_t denotes the portfolio weight of all local currency in circulation, net of the the portfolio weight of global currency held as reserves at the local central bank.

The central bank chooses the portfolio weight of country's wealth ζ'_t to be held as foreign currency reserves backing the local currency. The central bank can adjust ζ'_t through open-market operations, i.e. by issuing local currency to buy some of the global currency held by individuals, and, in reverse, by selling some of its reserves to take local currency out of circulations. All of these actions have no effect on ζ_t and θ_t defined above. In nominal terms, the portfolio weight of individual agents on local currency, which is backed by foreign reserves, is $\theta_t + \zeta'_t$. The portfolio weight on global currency is ζ'_t . Foreign-reserve policy affects the nominal value and risk profile of the local currency, but it has no real effects. The irrelevance result holds even if $\zeta'_t > \zeta_t$ if local agents can borrow foreign currency from abroad against their local currency holdings. The irrelevance result implies that without fiscal backing, in the parameter range where local currency is worthless on its own, the central bank cannot create a local currency that has greater value than that of foreign reserves it holds.

Reserve policy can have real effects with restrictions on financial markets. If local agents cannot borrow foreign currency from abroad, then by choosing ζ'_t above its equilibrium level, the central bank can increase foreign-currency holdings within the country. If, in addition, local agents are not allowed to hold foreign currency, then the central bank can hold it for them by choosing ζ'_t above or below its equilibrium level. Hence, restrictions on capital flow give a lot more flexibility to the central bank. By choosing reserves ζ'_t together with restrictions on capital flows, as well as the rate of money-printing π_t , the central bank can full control portfolio weights ζ_t and θ_t .

6 Optimal Policy.

In this section we consider what the policy maker of the local country can do to improve welfare. We use the intuition we built in Section 2.1 of monetary policy as affecting the agents' insurance against idiosyncratic risk. In particular, we start by considering a problem, in which the planner can directly provide insurance against idiosyncratic risk and control the holdings of global money savings in the local economy. Later on, we discuss the implementation of the direct policy through appropriate monetary policy, as well as limitations. The

planner can control insurance against idiosyncratic shocks that local agents get by the inflation policy with respect to local money. The planner can control local holdings of the global currency by holding foreign reserves that back local currency, through capital controls, or through various combinations of taxes and subsidies.

Specifically, suppose that the planner can directly control the fraction of idiosyncratic risk ψ_t that the agents hold, as well as the consumption rate of the global good ξ_t . Then savings of the global good within the small country follow

$$\frac{d\nu_t}{\nu_t} = \frac{\hat{\alpha}_t - \xi_t}{\nu_t} dt + M dt - \Phi(\iota_t) dt + (\sigma^G - \sigma^b) dZ_t + \sigma^{G,*} dZ_t^*. \quad (6.1)$$

The following proposition shows that the policy problem of the planner as an optimal stochastic control problem.

Proposition 2. *The problem of the policy maker is an optimal stochastic control problem with controls ψ_t and ξ_t that drive the motion of ν_t given by (6.1) and objective*

$$E \left[\int_0^\infty e^{-\rho t} \left(\log \frac{\xi_t b_t}{\alpha} + (1 - \alpha) \log \frac{P^g}{P^l} + \frac{\Phi(\iota_t) - \delta}{\rho} - \frac{\psi^2 \sigma(\psi \hat{q})^2}{2\rho} \right) dt \right]. \quad (6.2)$$

Controls map to the ratio P^g/P^l , the price of capital and investment rate (tied by the relationship $1 = \psi_t \hat{q}_t \Phi'(\iota_t)$) as follows. If equation

$$(1 - \hat{\alpha})a = \frac{1 - \alpha}{\alpha} a \xi_t + \iota(\psi \hat{q}), \quad \text{where} \quad \hat{q} = \frac{a \xi}{\rho \alpha} - a \nu,$$

leads to $\hat{\alpha} > 0$ then \hat{q} and ι are determined by these equations and $P^g/P^l = a/b_t$. Otherwise $\hat{\alpha} = 0$, investment and \hat{q} are determined by

$$\rho \hat{q}_t = \frac{a - \iota_t(\psi_t \hat{q}_t)}{1 - \alpha} \frac{\xi_t - \rho \alpha \nu_t}{\xi_t} \quad \text{and} \quad \frac{P_t^g}{P_t^l} = \frac{a - \iota_t}{1 - \alpha} \frac{\alpha}{\xi_t b_t}.$$

Proof. These equations come from the market-clearing condition for consumption, as well as the condition that agents must be indifferent between producing local and global goods if they produce both. Details are to be completed. \square

We should emphasize again that optimal policy depends on the degree of control we assume policy maker has over individual agents. Proposition 2 characterizes the optimal

policy assuming that the planner has full control over global money savings and spending by the small country, e.g. if the planner chooses the level of global currency reserves and local agents cannot hold global currency. Then local currency is backed in part by reserves of the global currency - the planner can affect the insurance that local agents can get by holding local money by setting inflation (i.e. printing local currency and distributing it to individuals proportionately to their capital holdings, as in Section 2.1.

We can then ask the questions of sustainability. What would happen if individuals could hold global currency on their own? This would limit the set of tools available to the policy maker, especially with respect to creating inflation in the small country to boost investment. On the opposite end, the flexibility to generate deflation may be limited by incentive-compatibility considerations, if local agents can hide capital from taxation (or if taxation leads to distortions). There may also be a discrepancy between global currency reserves that the planner wants to hold and the amount of global currency that individual agents would accumulate on their own.

(TO BE COMPLETED)

7 Conclusion

(to be completed)

8 Bibliography

(HIGHLY INCOMPLETE)

Bewley, T. (1980) “The Optimum Quantity of Money”, in *Models of Monetary Economics*, eds. J. Kareken and N. Wallace. Minneapolis, Minnesota: Federal Reserve Bank.

Brunnermeier, M. K. and Y. Sannikov (2016a) “The I Theory of Money”, working paper, Princeton University.

Brunnermeier, M. K. and Y. Sannikov (2016b) “On the Optimal Inflation Rate,” *American Economic Review Papers and Proceedings*, vol 106(5), pages 484-489

Brunnermeier, M. K. and Y. Sannikov (2016c) “Macro, Money and Finance: A Continuous-Time Approach”, in *Handbook of Macroeconomics*, Vol. 2B, John B. Taylor and Harold Uhlig, eds., North Holland.

DeMarzo, P., M. Fishman, Z. He and N. Wang, (2012) “Dynamic Agency and q Theory of Investment,” *Journal of Finance* 67, pp. 2295-2340.

Di Tella, S. (2016) “Uncertainty Shocks and Balance Sheet Recessions.” Forthcoming at the *Journal of Political Economy*.

Fleming, J. Marcus (1962) “Domestic financial policies under fixed and floating exchange rates.” IMF Staff Papers, 9, 369-379. Reprinted in Cooper, Richard N., ed. (1969) International Finance. New York: Penguin Books.

He, Z., and A. Krishnamurthy (2013) “Intermediary Asset Prices”, *American Economic Review*, 103(2), 732-770.

Mundell, Robert A. (1963) “Capital mobility and stabilization policy under fixed and flexible exchange rates.” *Canadian Journal of Economic and Political Science*. 29 (4), 475-485. Reprinted in Mundell, Robert A. (1968) International Economics. New York: Macmillan.

Obstfeld, M. and K. Rogoff (1995) “Exchange Rate Dynamics Redux,” *Journal of Political Economy*, 102, pp 624-660.

Samuelson, P. A. (1958) “An exact consumption loan model of interest, with or without the social contrivance of money.” *Journal of Political Economy*, 66, 467-82.

Sims, C. (1994) “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory*, 4, 381-399.

A Computing Equilibria: Numerical Details

B Proofs