College Pricing and Income Inequality

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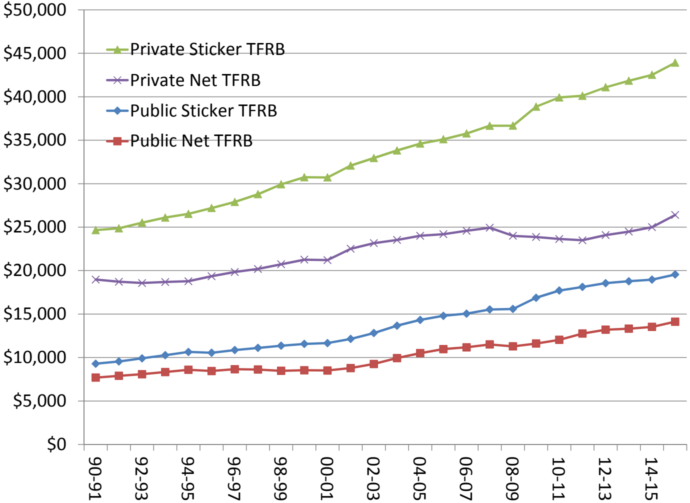
Introduction

- Price of US college tuition has risen fast in recent decades
- At the same time, income inequality has been rising
- Why is tuition rising so fast?
- Are smart low income students being priced out?
- To explore these questions, need a model of the college market
- Key Challenge: College is a club good:
  - Quality (desirability) of a given college depends on attributes (e.g. academic ability) of students who attend
  - Consumers are therefore an input in production

Tuition, Fees, Room & Board (College Board $2015)



Colleges as Clubs

- Club good feature complicates model analysis:
  - two colleges with different student bodies supply different products in different markets
- Lots of college variety $\Rightarrow$ lots of markets in general equilibrium
- $\Rightarrow$ existing literature assumes small number of different college types
  - Epple & Romano (1998), Caucutt (2002), Epple, Romano & Sieg (2006, 2017), Fu (2016), Gordon & Hedlund (2016)
- Potential concerns:
  - Counterfactual $\Rightarrow$ applied analysis difficult
  - Equilibrium existence problems (Scotchmer, 1997)
  - Price-taking assumption questionable – game theoretic oligopolistic price setting more natural

Model: Standard Elements

- Households differ by income and student ability
- Colleges differ by quality
- Quality depends on resources & avg. student ability

Model: Novel Element

Continuous distribution of college quality, with free entry (Ellickson, Grodal, Scotchmer, Zame, 1999)

- Entire distribution of college characteristics and prices can be compared to data
- College distribution can change smoothly and flexibly in response to changing drivers of college demand
- No existence problems
- Price taking natural
- No role for lotteries as in Cole and Prescott (1997) or Caucutt (1999)

Outline

- Model description
- A closed-form example
- Calibration and model-data comparison
- Applications: How do the following affect college pricing and college attendance
  1. Income inequality
  2. Subsidies to public universities
  3. Subsidies to all colleges

Model: Households

- Continuum of measure 2 of households, each containing a parent and a college-age child
- Heterogeneous wrt: (i) income y, (ii) student ability a
- Two ability levels, indexed $i \in \{l.h\}$, $a_l < a_h$, measure 1 of each level
- Continuous distribution for income, CDF $F^i(y)$
- Utility from non-durable consumption c and quality q of the college the child attends

$$u(c, q) = \log c + \varphi \log(\kappa + q)$$

Household Problem

- Make education choice $j \in \{0, 1, 2\}$:
  1. $j = 0$: No college
  2. $j = 1$: Public college, grant toward tuition g_1
  3. $j = 2$: Private college, grant toward tuition $g_2 < g_1$

- Take as given tuition functions $t_j^i(q; y)$

- Given idiosyncratic state (y, i), solve

$$\max_{\{j,c,q \in Q^j\}} u(c, q)$$
$$s.t.$$
$$c + t_j^i(q; y) = y + g_j$$

- Solution: $s^i(y)$, $c^i(y)$, $q^i(y)$

Model: Colleges

- CRS technology for producing education of a given quality
- Quality (per student) reflects:

  (i) average ability of student body

  (ii) consumption good input (per student) e (faculty etc)

$$q = (\eta a_h + (1-\eta) a_l)^{\theta} \, e^{1-\theta}$$

where η is share of student body that is high ability

- Fixed consumption cost R&B ϕ per student admitted

Public versus Private Schools

- Assume all colleges profit maximize
  - minimize cost of supplying given value of education
- Observe income y and child's ability type i, take as given tuition schedules
- Colleges choose private or public status
- Public colleges must keep average tuition below a cap T
- No equilibrium tuition discrimination by income
  - If other colleges charge high income students more, a single profit-maximizing college would skim high income students
  - If other colleges are profit maximizing, a single college charging low income students less would incur negative profits

College Problem

1. Choose quality level
2. Choose public or private model to deliver q
3. Choose input mix and size

Input mix sub-problem for private college supplying mass 1 spots at $q > 0$

$$\max_{\eta,e} \left\{ t_2^h(q)\eta + t_2^l(q)\left(1-\eta\right) - e - \phi \right\}$$
$$s.t.$$
$$q = \left(\eta a_h + \left(1-\eta\right) a_l\right)^{\theta} e^{1-\theta}$$

- Public college problem similar s.t. additional constraint

$$t_1^h(q)\eta + t_1^l(q)\left(1-\eta\right) \leq T$$

Profit Maximization Given $t_j^i(q)$

1. Fix quality q
2. Compute optimal input mix for unconstrained public college

$$\frac{e_1(q)}{\eta_1(q)a_h + (1-\eta_1(q))\,a_l} = \frac{(1-\theta)\left(t_1^l(q) - t_1^h(q)\right)}{\theta \Delta a}$$

3. Check whether avg. tuition exceeds T.
   - If not, only public colleges at quality q
   - Else, compare profit from unconstrained private college to constrained public college, where $\eta_1(q)$ s.t.

$$t_1^h(q)\eta_1(q) + t_1^l(q)\,(1-\eta_1(q)) = T$$

4. Optimal size at each q:

$$\begin{cases} 0 & \text{if} \quad \pi_j(q) < 0 \\ [0,\infty] & \text{if} \quad \pi_j(q) = 0 \\ \infty & \text{if} \quad \pi_j(q) > 0 \end{cases}$$

Equilibrium

$\chi_j(Q)$: measure of students in j type colleges with $q \in Q \subset \mathcal{Q}_j$
Equilibrium is $\chi_j(q)$, $t^i_j(q)$, $\eta_j(q)$, $e_j(q)$, $s^i(y)$, $c^i(y)$, $q^i(y)$ s.t.

1. Given $t^i_j(q)$, $s^i(y)$, $q^i(y)$ & $c^i(y)$ solve household's problem
2. Given $t^i_j(q)$, $\eta_j(q)$ & $e_j(q)$ solve college problem for $j = 1, 2$
3. Zero profits: $\forall Q$, $\pi_j(q) \leq 0 \; \forall q \in Q$ and

$$\int_Q \pi_j(q) d\chi_j(q) = 0$$

4. Market clearing:

$$\sum_{i=h,l} \int c^i(y) dF^i(y) + \sum_{j=1,2} \int \left(e_j(q) + \phi - g_j\right) d\chi_j(q) = \sum_{i=h,l} \int y dF^i(y)$$

$$\int 1_{\{s^h(y)=j, q^h(y)\in Q\}} dF^h(y) = \int_Q \eta_j(q) d\chi_j(q) \qquad \forall Q, j = 1, 2$$

$$\int 1_{\{s^l(y)=j, q^l(y)\in Q\}} dF^l(y) = \int_Q (1 - \eta_j(q)) d\chi_j(q) \qquad \forall Q, j = 1, 2$$

Properties of Tuition Functions

- At each quality level, $t^h(q) < t^l(q)$
  - Otherwise colleges would strictly prefer high ability students
- Tuition is increasing in quality: $q_1 > q_2 \Rightarrow t^i(q_1) > t^i(q_2)$
  - Otherwise no students would choose lower quality college
- Public schools dominate at low quality levels, private at high:
  - At low q, if cap T non-binding, public schools can charge $g_1 - g_2$ more tuition
  - At high q, cap binds tightly $\Rightarrow$ private schools more profitable
- Sorting by income
  - Holding fixed ability, higher income households more willing to pay for higher quality colleges

Parametric Example

- Pure club good model: $\theta = 1 \Rightarrow q = \eta a_h + (1-\eta) a_l$
  - Households sell and buy ability in college market

- Set $\varphi = 1 \Rightarrow u(c, q) = \log c + \log(\kappa + q)$

- No R&B: $\phi = 0$

- No grants, and no public schools
- Uniform income distribution:

$$y \sim U\left[\mu_y - \frac{\Delta_y}{2}, \mu_y + \frac{\Delta_y}{2}\right]$$

$$F^h(y) = F^l(y)$$

- Let $\mu_a = \frac{a_h + a_l}{2}$, $\Delta_a = a_h - a_l$

Questions

1. What are $\chi(q)$, $t^h(q)$, $t^l(q)$?
2. How do these objects depend on Δ_y?
3. How does market for college differ from market for fish?

Digression: Modeling College Like Fish

- Households endowed with a_l or a_h units of ability
- Sell and buy ability at centralized market at per unit price p
- Household problem:

$$\max_{c,q} \{\log(c) + \log(\kappa + q)\}$$
$$s.t.$$
$$c + pq = y + pa_i$$

- Market clearing:

$$p = \frac{\mu_y}{\mu_a + \kappa}$$

- "Tuition" (net price) function:

$$t_F^i(q) = pq - pa_i = (q - a_i)\,\frac{\mu_y}{\mu_a + \kappa}$$

1. Net price functions are linear in q, and
2. Price function does not depend on income inequality Δ_y

The Club Good Model

- College distribution: $\forall Q \subset (a_l, a_h)$

$$\chi(Q) = \frac{2}{\Delta_a}\left(\frac{2}{4+\pi}\right)\int_Q \left[(1-\eta(q))^2+\eta(q)^2\right]^{-2} dq$$

$$\chi(a_h) = \chi(a_l) = \frac{2}{4+\pi} = 0.28$$

- Tuition functions:

$$t^i(q) = \mu_y\left(\frac{q-a_i}{\kappa+q}\right)\left[1-\left(\frac{2}{4+\pi}\right)\frac{\Delta_y}{\mu_y}\arctan\left(1-2\eta(q)\right)\right]$$

- Competitive equilibrium is **Pareto efficient**

1. Distribution of quality independent of $(\mu_y, \Delta_y, \kappa)$
2. Price functions **non-linear** in q
3. Price functions **depend on** Δ_y

Sketch of Solution Method

1. Given any college distribution $\chi(q)$, derive income of households attending college q: $y^i(q;\chi(.))$
2. Given $y^i(q;\chi(.))$, household's FOC gives an ODE that pins down the college tuition function: $t^i(q;\chi(.))$

$$\frac{dt^i(q;\chi(.))}{dq}\frac{1}{y^i(q;\chi(.))-t^i(q;\chi(.))}=\frac{1}{\kappa+q}$$

3. Given $t^i(q;\chi(.))$, derive a college profit function:

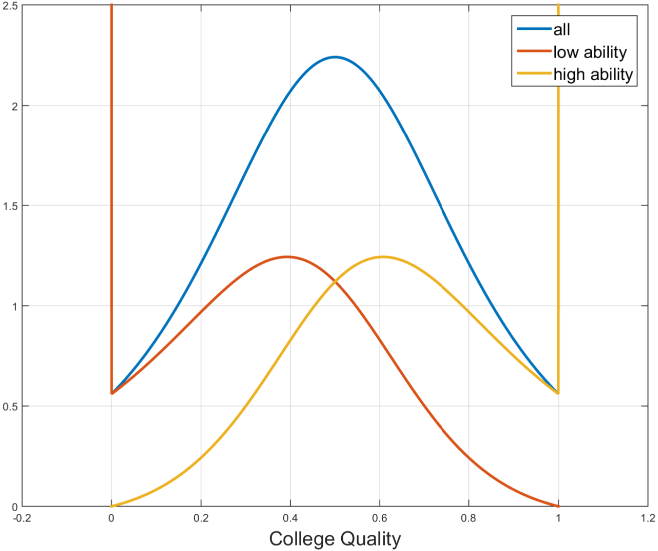
$$\pi(q;\chi(.))=\eta(q)t^h(q;\chi(.))+(1-\eta(q))t^l(q;\chi(.))$$

4. Solve for $\chi(q)$ from the functional equation

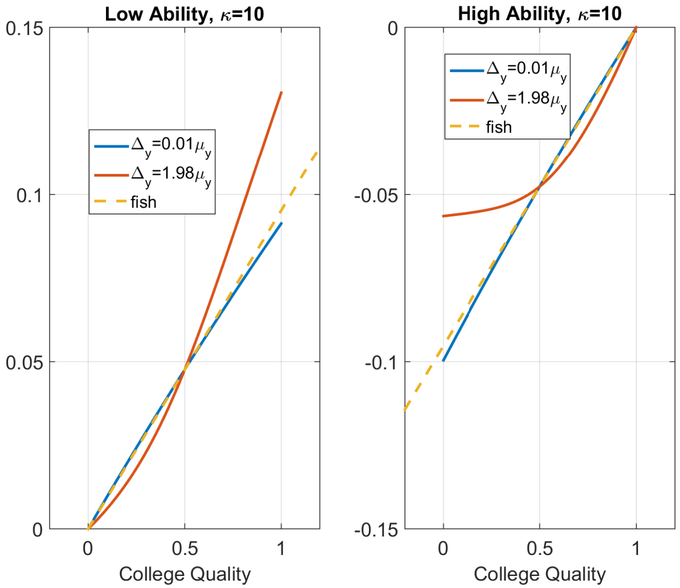
$$\pi(q;\chi(.))=0$$

   - This is a Volterra integral equation of the second kind with degenerate kernels, which has an analytical solution

College Distribution

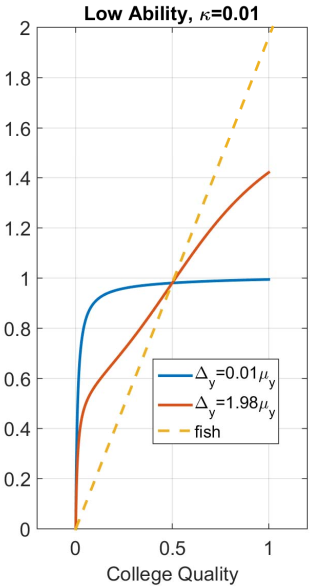


Tuition

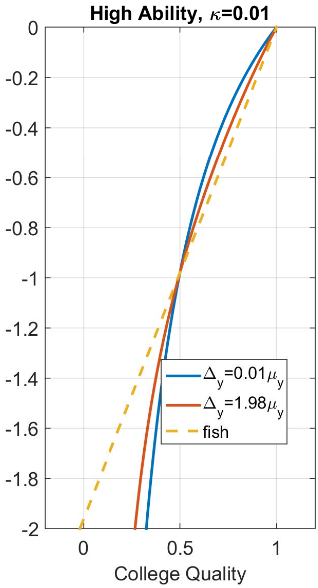


Low Ability, κ=10
High Ability, κ=10
Δ_y=0.01μ_y
Δ_y=1.98μ_y
fish
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0.5
1
College Quality

Tuition



Low Ability, κ=0.01
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
College Quality
Δy=0.01μy
Δy=1.98μy
fish



High Ability, κ=0.01
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
0.5
1
College Quality
Δy=0.01μy
Δy=1.98μy
fish

More Properties of Club Good Equilibrium

1. At any quality level $q \in (0, 1)$ colleges have 2 types of customer:
   - high ability with relatively low income receiving subsidy
   - low ability with higher income paying positive tuition

2. Increasing Δ_y:
   - raises (lowers) $t^l(q)$ for $q \geq (\leq)\, \mu_a$
   - lowers (raises) $t^h(q)$ for $q \geq (\leq)\, \mu_a$
   - raises tuition differential for high q, lowers diff. for low q

Quantitative Example: Calibration

- Income distribution: Pareto Log-Normal:

$$\ln y \sim EMG(\mu^i, \sigma^2, \alpha)$$

- $\sigma^2 = 0.4117$ (SCF, 2007)
- $\alpha = 1.8$ (Piketty-Saez, 2014)
- μ^i s.t. $E[y] = 1$ and

$$\frac{E[y|_{i=h}]}{E[y|_{i=l}]} = \frac{\$67,000}{\$45,000}$$

- (avg. family income conditional on child's AFQT score being above / below median, 1997 NLSY).

Preferences and College Technology

Preferences (φ, κ), Technology: (θ, ϕ)

1. Enrollment: $37.0\% \Rightarrow \kappa = 0.034$
2. Tuition + R&B $\$17,823$ to Agg. Cons. $\Rightarrow \varphi = 0.0235$
3. Room and Board $\$10,881 \Rightarrow \phi = 0.019$
4. Peers vs. goods equally important in quality $\Rightarrow \theta = 0.5$

(targets for 2015-17; all 4 yr colleges)

Preferences and College Technology

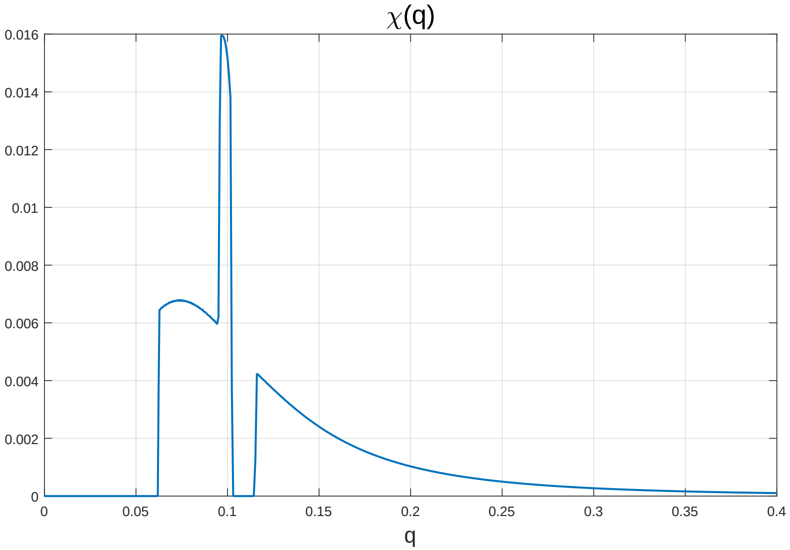
5. Federal and state grant aid: $\$3,204$ for public colleges, $\$2,893$ for private colleges $\Rightarrow g_1 = 0.0057$, $g_2 = 0.0051$
6. Tuition cap T set to replicate public share of 4 year enrollment, $0.695 \Rightarrow T = 0.0250$
7. Ability gap $a_h - a_l$ drives within-school tuition dispersion

- College Board reports avg. price paid net of all subsidies (federal, state and institutional grant aid)
- Assume (i) everyone gets "federal and state grant aid " (ii) all institutional aid goes to high ability

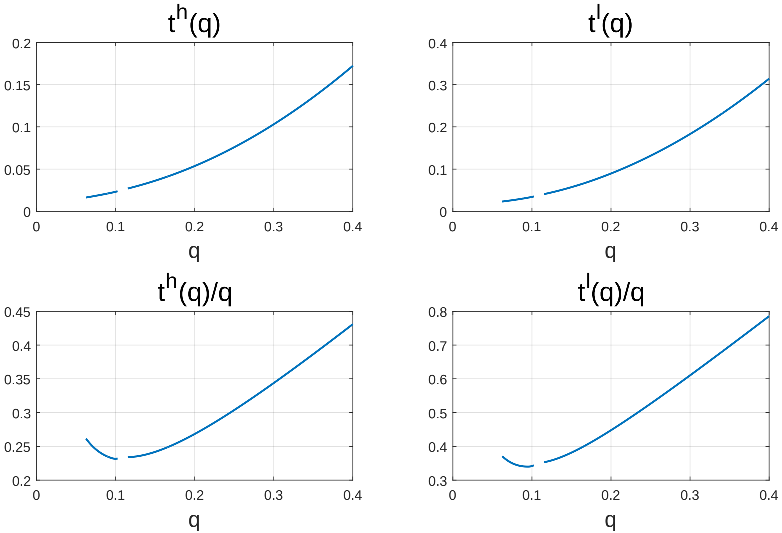
$$\frac{\text{ave. net low ability tuition}}{\text{ave. net tuition}} = \frac{\$24,676}{\$17,823}$$

- $\Rightarrow a_l = 0.275$ $(a_h = 1)$

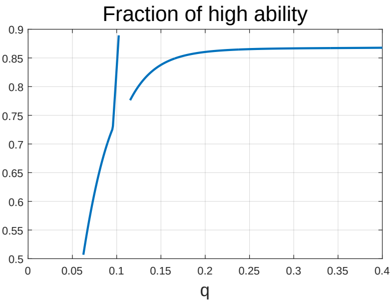
College Quality Distribution

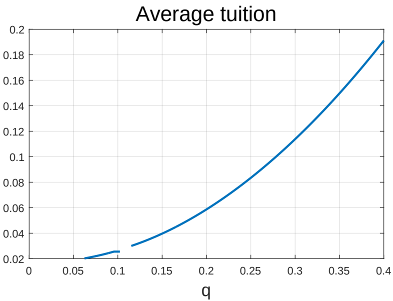


Tuition Schedules

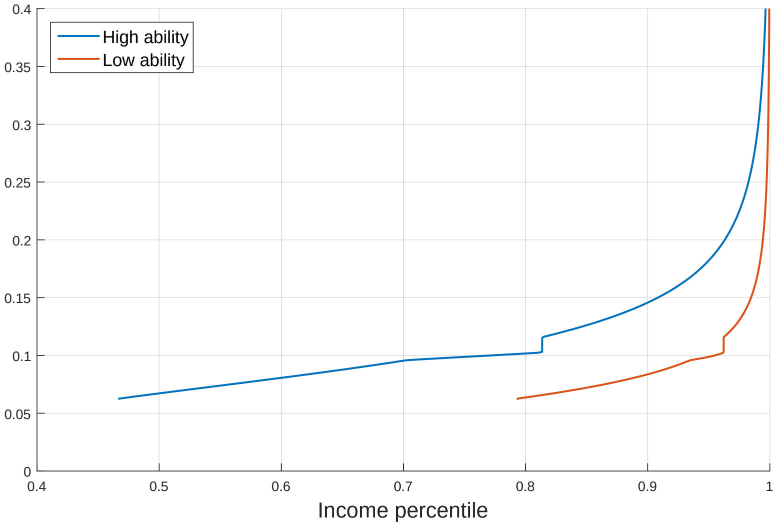


Avg. Ability and Tuition by Quality





Quality by Income Percentile



First Moments: Model and College Scorecard Data

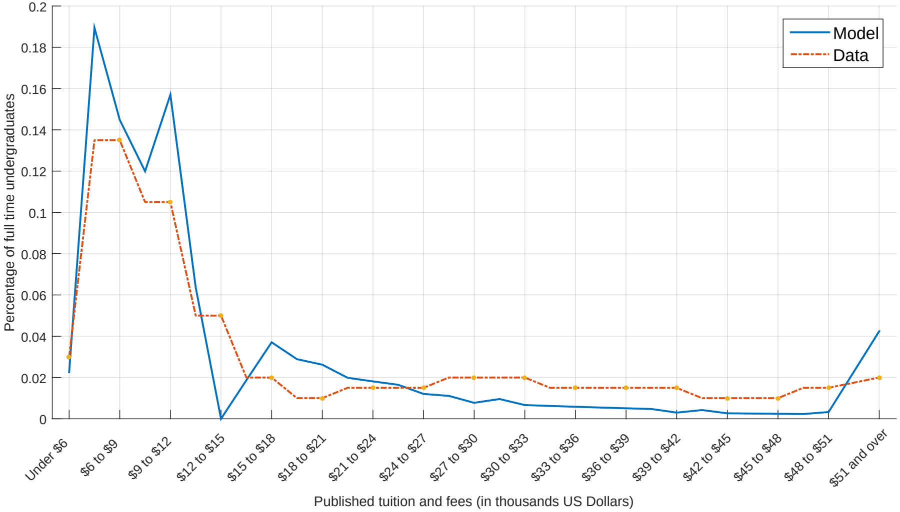
	Model		Data	
	Public	Private	Public	Private
Enrollment	0.258	0.112	0.258	0.112
Sticker TFRB $	19,168	47,018	20,788	41,905
Net TFRB $	12,797	29,373	14,651	25,071
Avg. family income $	54,044	111,763	63,231	77,155
Avg. ability / SAT	0.76	0.88	1,085	1,135

	No Coll.	Public	Private
High ability kids	0.467	0.347	0.186
Low ability kids	0.793	0.169	0.038

College Level Statistics: Model and Data

	Model	Data
var.(log avg. net TFRB)	0.164	0.158
var.(log sticker TFRB)	0.229	0.160
var.(log avg. fam income)	0.331	0.106
var.(log avg. SAT)	0.011	0.012
corr.(log net TFRB, log income)	0.987	0.704
corr.(log net TFRB, log SAT)	0.687	0.383
corr.(log income, log SAT)	0.790	0.591

Tuition Distribution: Model and Data



Experiments

1. Move income distribution back in time from 2014 to 1984:
   - $\alpha^{1984} = 2.7$ instead of $\alpha^{2014} = 1.8$
   - Adjust μ to hold average income fixed

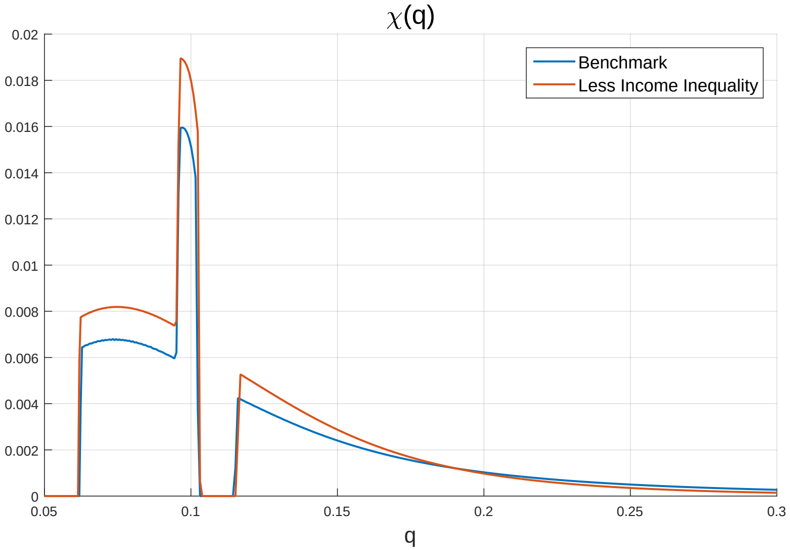
2. Eliminate additional $311 grant for public colleges

3. Eliminate all federal and state grants for colleges ($3,204 for public and $2,893 for private)

Reducing Income Inequality

	Less Inequality			Baseline		
	All	Public	Private	All	Public	Private
Enrollment (%)	44.2	32.1	12.1	37.0	25.8	11.2
Net TFRB $	13,474	11,388	19,055	17,815	12,797	29,373
High abil. part. (%)	63.9	42.9	21.0	53.3	34.7	18.6
Low abil. part. (%)	24.7	21.5	3.2	20.7	16.9	3.8

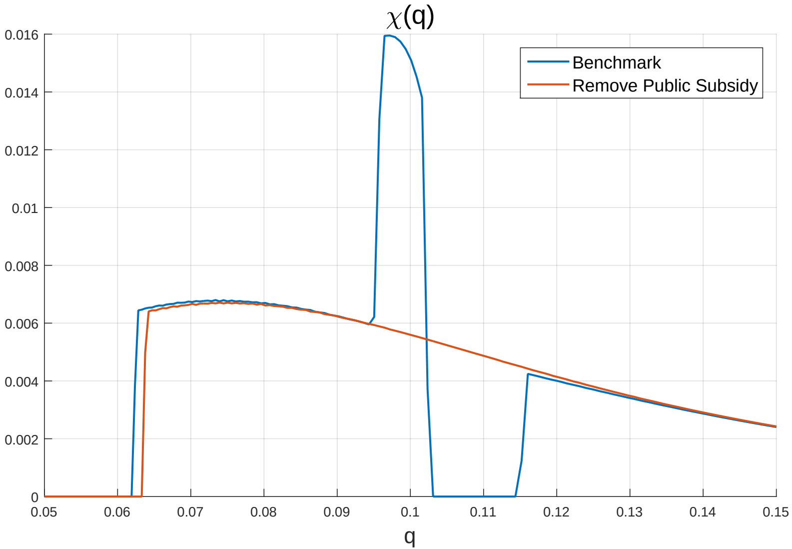
Reducing Income Inequality



Eliminating Public Subsidies

	No Public		Baseline	
	All	All	Public	Private
Enrollment (%)	35.9	37.0	25.8	11.2
Net TFRB $	18,435	17,815	12,797	29,373
High abil. part. (%)	51.8	53.3	34.7	18.6
Low abil. part. (%)	20.0	20.7	16.9	3.8

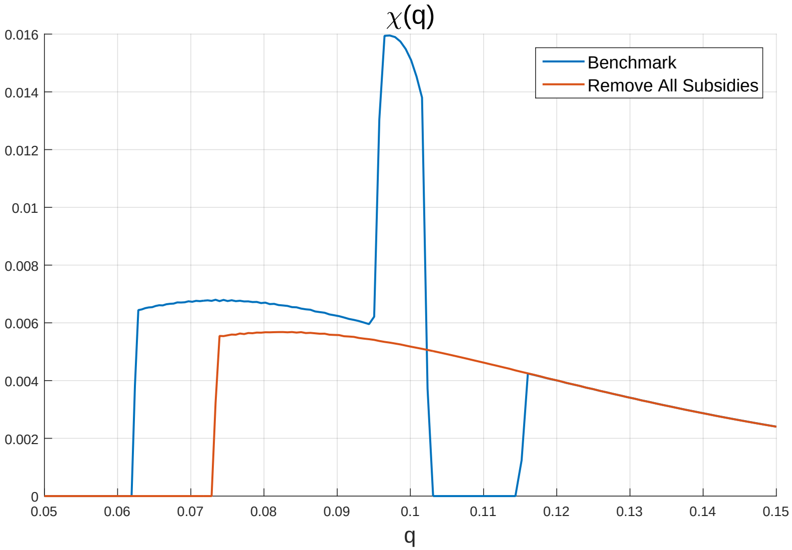
Eliminating Public Subsidies



Eliminating All Subsidies

	No Subsidies	Baseline		
	All	All	Public	Private
Enrollment (%)	27.5	37.0	25.8	11.2
Net TFRB $	23,340	17,815	12,797	29,373
High abil. part. (%)	40.4	53.3	34.7	18.6
Low abil. part. (%)	14.5	20.7	16.9	3.8

Eliminating All Subsidies



Conclusions

- Widening income inequality driving enrollment down, tuition up

  1. rich demand higher quality colleges $\Rightarrow$ average college quality goes up
  2. marginal high ability become poorer and are priced out $\Rightarrow$ high ability students become scarcer and more expensive $\Rightarrow$ increased cost of producing quality

- Small subsidies to public colleges support large public sector, effective in supporting high ability enrollment
- Eliminating all subsidies would drastically shrink college enrollment, push up tuition