Negative Swap Spreads and Limited Arbitrage

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Abstract

Since October 2008 fixed rates for interest rate swaps with a thirty year maturity have been mostly below treasury rates with the same maturity. Under standard assumptions this implies the existence of arbitrage opportunities. This paper presents a model for pricing interest rate swaps where frictions for holding bonds limit arbitrage. I show analytically that negative swap spreads should not be surprising. In the calibrated model, swap spreads can reasonably match empirical counterparts without the need for large demand imbalances in the swap market. Empirical evidence is consistent with the relation between term spreads and swap spreads in the model. Keywords: Swap spread, limited arbitrage, fixed income arbitrage (JEL: G12, G13).

1   Introduction

Interest rate swaps are the most popular derivative contracts. According to the Bank for International Settlements, for the first half of 2015, the notional amount of such contracts outstanding was 320 trn USD. In a typical interest rate swap in USD, a counterparty periodically pays a fixed amount in exchange for receiving a payment indexed to LIBOR. Since October 2008, the fixed rate on swaps with a thirty year maturity has typically been below treasuries with the same maturity, so that the spread for swaps relative to treasuries has been negative. What in 2008 may have looked like a temporary disruption related to the most virulent period of the financial crisis has persisted to date, see Figure 1.

Negative swap spreads are challenging for typical asset pricing models as they seem to imply a risk-free arbitrage opportunity. By investing in a treasury bond and paying

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Figure 1: Swap Spreads. *Difference between fixed swap rate and treasury yield of same maturity. Units are in percent.*

the lower fixed swap rate, an investor can generate a positive cash flow. With typical repo financing for the bond, the investor would also receive a positive cash flow from the difference between LIBOR and the repo rate. If the position is held to maturity, this represents a risk-free arbitrage. In reality, a shorter horizon exposes the investor to the risk of an even more negative swap spread. Capital requirements and funding liquidity also make such an investment risky. While there seem to be good reasons for why arbitrage would be limited in this case, there are no equilibrium asset pricing models consistent with negative swap spreads.

This paper develops a model for pricing interest rates swaps that features limited arbitrage. In the model, dealers invest in fixed income securities. A dealer can buy and sell risk-less debt with different maturities, as well as interest rate swaps. Debt prices are exogenous, the model prices swaps endogenously. Without frictions, the price of a swap equals its no-arbitrage value, and the swap spread has to be positive. When frictions limit the size of the dealer's fixed income investments, swaps cannot be fully arbitraged, and swaps are priced with state prices that are not fully consistent with bond prices.

My main finding is that with limited arbitrage, negative swaps spreads are not surprising anymore, even without explicit demand effects. With frictions, dealers have smaller bond

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positions and are less exposed to long-term interest rate risk. They require less compensation to hold the exposure to the fixed swap rate and, therefore, the swap rate is lower. In the model, in the limit as frictions become more extreme, the unconditional expectations of the swap rate and LIBOR are equalized. With long-term treasury rates typically larger than LIBOR, the swap spread would then naturally be negative. Equivalently, because the TED spread is typically smaller than the term spread, the swap spread would be negative. Quantitatively, with moderate frictions, the model can produce thirty-year swap spreads in the range observed since October 2008. A key implication of the model is that, conditional on short term rates, term spreads are negatively related to swap spreads. Empirical evidence consistent with this regularity is presented.

Practitioners have advanced a number of potential explanations for why swap spreads have turned negative, the so-called swap spread *inversion*. Consistently among the main reasons is the notion that stepped-up banking regulation in the wake of the global financial crisis has made it more costly for banks to hold government bonds. For instance, Bowman and Wilkie (2016) at Euromoney magazine write on this topic: "... there is little doubt about the impact of regulation – primarily the leverage ratio and supplementary leverage ratio – on bank balance-sheet capacity and market liquidity. ... The leverage ratio has made the provision of the repo needed to buy treasuries prohibitively expensive for banks." As it has become more costly for banks to hold treasuries, apparent arbitrage opportunities can persist. In my model, it is costly for dealers to hold treasuries and this reduces the size of their bond positions. This leads to the possibility that swaps are no longer priced in line with treasuries.[1] A key insight provided by my model is that with arbitrage limited in this way, swap spreads should naturally be negative, even in the absence of explicit demand effects.

A large literature has developed models with limited arbitrage where frictions faced by specialized investors can affect prices. For instance, Shleifer and Vishny (1997) consider mispricing due to the limited capital of arbitrageurs, Dow and Gorton (1994) study the impact of holding costs when traders have limited horizons. Other examples include Garleanu, Pedersen and Poteshman (2009) on pricing options when risk-averse investors cannot hedge perfectly, Gabaix, Krishnamurthy and Vigneron (2007) on the market for mortgage-backed securities, and Vayanos and Vila (2009) who price long-term bonds with demand effects. Liu and Longstaff (2004) analyze portfolio choice for arbitrageurs with collateral constraints, and Tuckman and Vila (1992) with holding costs. For a survey of this literature, see Gromb and Vayanos (2010). As in most of these papers, in my model specialized investors determine

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[1]Duffie (2016) documents increased financial intermediation costs for US fixed income markets due to the tightening of leverage ratio requirements for banks. Du, Tepper and Verdelhan (2016) show that deviations from covered interest parity have persisted since 2008 and relate these to increased banking regulation.

the price of some security with other prices given exogenously. So far, this literature has not considered interest rate swaps.

Empirical studies have documented the drivers of swap spreads with factor models, in particular Liu, Longstaff and Mandell (2006) and Feldhuetter and Lando (2008). More recently, Hanson (2014) documents the relation between MBS duration and swap spreads. Gupta and Subrahmanyam (2000) study swap prices relative to the prices of interest rate futures, and Eom, Subrahmanyam and Uno (2002) the links between USD and JPY interest rate swaps. Collin-Dufresne and Solnik (2001) focus on the impact of the LIBOR panel selection for swap pricing. Johannes and Sundaresan (2007) theoretically and empirically find an increase in swap rates due to collateralization. More recent studies that focus on the period with negative swap spreads include Smith (2015) who analyzes the principal components in swap spreads, and Klinger and Sundaresan (2016) who document a relation between pension funds duration hedging and negative swap spreads.

My paper contributes to the literature by developing a model that determines swap spreads with limited arbitrage. It is shown analytically and quantitatively that the model can plausibly explain negative swap spreads. The model is also shown to be consistent with additional empirical evidence on the relation between swap spreads and term spreads. In my model long-term debt and swaps are modelled with geometric amortization, a feature used for tractability in models for corporate debt or sovereign debt, following Leland (1998). The model remains challenging numerically because it includes a dynamic portfolio problem with potentially large short and long positions in multiple securities with incomplete markets that needs to be combined with the pricing of swap contracts with a long maturity. Only a global solution seems to be able to offer the required numerical precision.

In the rest of the paper the model is first presented, followed by analytical characterizations of the arbitrage-free case and the case with frictions. Section 4 contains the model's quantitative implications and additional empirical evidence on swap spreads. Section 5 concludes.

2 Model

A dealer with an infinite horizon invests in bonds and swaps. Bond prices are exogenous, the swap price is endogenous. The model is driven by the exogenous prices for the bonds and inflation. Long-term bonds and swaps have geometric amortization with a given maturity parameter.

2.1 Available assets

The dealer chooses among three securities: short-term risk-free debt (which we can think of as treasury or repo), long-term default-free debt (treasury bonds), and a fixed-for-floating interest rate swaps. The risk averse dealer takes prices as given and maximizes the lifetime utility of profits. Prices for swaps are determined in equilibrium to clear the swap market. The demand for swaps is assumed to come from endusers such as corporations and insurance companies. Swap contracts are free of default risk, as they nowadays are mostly collateralized. The fixed swap rate can differ from the long-term bond with the same maturity because the floating leg pays LIBOR which typically exceeds the short-term treasury rate. A process for the LIBOR rate is assumed. Because holding bonds is costly, the dealer cannot perfectly arbitrage between securities, and this creates an additional wedge between fixed swap rates and rates for long-term bonds.

Short-term riskless debt pays one unit of the numeraire (the dollar) next period and has a current price of

$$q_{ST}(z) = \exp(-y_{ST}(z)),$$

with y_{ST} the log of the short rate. The exogenous z follows a finite-state Markov process.

LIBOR debt pays one unit of the numeraire next period. The price of LIBOR debt is

$$q_{LIB}(z) = \exp(-y_{LIB}(z))$$

with the log yield

$$y_{LIB}(z) = y_{ST}(z) + \theta(z).$$

We can think of θ as the so-called TED ("Treasury Euro-Dollar") spread, where LIBOR is referred to as the Euro-Dollar rate. Historically, 3-month TED spreads have never been negative; the model will satisfy this property. The TED spread can be thought of as compensating for some disadvantage of bank debt relative to the risk-free debt. This could be reduced liquidity or higher default risk. Explicitly modelling the sources of this spread would be conceptually straightforward, but would burden computations, without an obvious benefit for the current analysis.

Long-term default-free debt pays

$$c_{LT} + \lambda$$

per period, where c_{LT} is the coupon and λ the amortization rate, implying an average maturity of $1/\lambda$. In the next period the owner of the bond gets

$$c_{LT} + \lambda + (1 - \lambda) q'_{LT}(z'),$$

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where q'_{LT} is the market price of the long-term bond next period. The price of this bond is related to its yield to maturity, $\exp(y_{LT}) - 1$, which after solving for the infinite sum can be written as

$$q_{LT}(z) = \frac{c_{LT} + \lambda}{\exp(y_{LT}(z)) - 1 + \lambda}.$$

Clearly, with the bond at par, $q_{LT}(z) = 1$, we have $c_{LT} = \exp(y_{LT}(z)) - 1$. We model the exogenous yield process as

$$y_{LT}(z) = y_{ST}(z) + \tau(z),$$

with $\tau(z)$ the stochastic term spread. Note that this relation between q_{LT} and y_{LT} is without loss of generality; λ and c are constants.

Swaps pay a constant coupon in exchange for LIBOR. To enter a swap contract, a price m is paid. This price captures mark-to-market gains and losses for the swap. In particular, next period, the fixed rate receiver of the swap gets

$$c_{Sw} - (\frac{1}{q_{LIB}(z)} - 1) + (1 - \lambda) m',$$

with c_{Sw} the fixed coupon rate. The maturity parameter for the swap, λ, is the same as for the long-term debt. This could easily be changed, but given our focus on the spreads of swaps and bonds with the same maturity, does not seem useful. My way of modelling an interest rate swap with geometric amortization is inspired by Leland's (1998) model for long-term debt. As in this case, the advantage of this representation is that the swap does not age, and the model does not require swaps with multiple maturities.

The coupon rate of the swap can be set so that for a given state of the economy the swap has a market value of zero, $m = 0$. The coupon rate c_{Sw} for which the current price of the swap equals zero is called the swap rate, y_{Sw}. The swap spread is defined as

$$y_{Sw} - (\exp(y_{LT}) - 1).$$

Empirically, swaps have zero initial value, and new swap contracts are continuously offered with fixed coupon rates so that the contract value is zero. In the general model, where the net demand facing the dealer, $d(z)$, is not zero, we can think that the model has only one swap, whose coupon does not change but that is traded at its mark-to-market value. The dealer then only trades the swap with this fixed coupon, and not new at-market swaps. Having a new at-market swap every period would create an infinite dimensional state variable, and make the model intractable. For the special case with a net demand of swaps facing the dealer that is zero for all periods, $d(z) = 0$, the existence of swaps does not

affect the equilibrium. Therefore, new swaps, with normalized coupons can be continuously introduced and priced, and a time-series of swap rates can be generated in the model. Given this obvious advantage, I focus the numerical analysis on this special case.

Long or short positions for bonds are costly to hold for the dealer. Specifically, the cost for holding short-term debt is given by

$$h\left(\alpha'_{ST}\right) = \frac{\kappa_{ST}}{2}\left(\alpha'_{ST}\right)^2. \tag{1}$$

The cost is incurred in the current period, with $\kappa_{ST} > 0$ the cost parameter and α'_{ST} the amount of the short-term bond bought this period and held into next period. Similarly, the cost for long-term debt is

$$j\left(\alpha'_{LT}\right) = \frac{\kappa_{LT}}{2}\left(\alpha'_{LT}\right)^2. \tag{2}$$

These costs capture financing and regulatory costs and create the frictions that limit perfect arbitrage. Other functional forms are possible, but do not seem essential.

It would be straightforward to introduce a cost function for holding swaps. To keep the analysis focused, this is not done here explicitly. The costs and the consequences for equilibrium swap prices would depend to a large extent on the exogenous demand the dealer is facing. For my quantitative analysis the net demand is zero. With a quadratic cost function, the marginal cost would then be zero and there would be no impact on swap prices. The more general case is considered in the appendix.

2.2 Maximization, equilibrium, and solution

The dealer maximizes lifetime utility of profits by selecting short and long bonds, swaps and payouts. Specifically, the dealer solves

$$V\left(\omega, z\right) = \max_{c, \alpha'_{ST}, \alpha'_{LT}, s'} u\left(c\right) + \beta\left(\omega, z\right) E\left(V\left(\omega', z'\right)\right)$$

subject to

$$c = \omega - \alpha'_{ST} q_{ST}\left(z\right) - \alpha'_{LT} q_{LT}\left(z\right) - s'm - h\left(\alpha'_{ST}\right) - j\left(\alpha'_{LT}\right)$$

and

$$\omega' = \frac{\alpha'_{ST}}{e^{\mu(z')}} + \frac{\alpha'_{LT}}{e^{\mu(z')}}\left[c_{LT} + \lambda + \left(1 - \lambda\right) q_{LT}\left(z'\right)\right] + \frac{s'}{e^{\mu(z')}}\left[c_{Sw} - \left(\frac{1}{q_{LIB}\left(z\right)} - 1\right) + \left(1 - \lambda\right) m'\right] + \pi\left(z'\right).$$

$\pi\left(z\right)$ are other profits, $\mu\left(z'\right)$ is the log inflation rate, and s the amount of the swap. Payouts or consumption c are valued with momentary utility $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$, and the discount factor is

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of the Uzawa-Epstein type, $\beta\left(\omega, z\right) = \left(1 + \bar{c}\right)^{-\nu}$, with $\nu > 0$, and $\bar{c}$ equilibrium consumption which the dealer does not internalize. With this specification the dealer discounts the future more when wealth and consumption/payouts are high. Wealth accumulation is favored when wealth is low and limited when wealth is high. This helps make the model more tractable numerically, but does not directly produce pricing frictions. This specification is popular for inducing stationarity in small open models with incomplete markets, following Mendoza (1991) and Schmitt-Grohe and Uribe (2003).

In equilibrium

$$s' = -d\left(z\right)$$

where $d\left(z\right)$ is the net market demand for the fixed receiver swap.

The state vector includes the level of dealer equity/wealth ω, and the exogenous state that determines bond prices, inflation and possibly swap demand $\left(y_{ST}\left(z\right), \tau\left(z\right), \theta\left(z\right), \mu\left(z\right), d\left(z\right), \pi\left(z\right)\right)$.

First-order conditions for bonds and swaps are given by

$$h_1\left(\alpha'_{ST}\right) = \beta E \frac{u_1\left(c'\right)}{u_1\left(c\right) e^{\mu\left(z'\right)}} - q_{ST}\left(z\right), \tag{3}$$

$$j_1\left(\alpha'_{LT}\right) = \beta E \left(\frac{u_1\left(c'\right)}{u_1\left(c\right) e^{\mu\left(z'\right)}} \left[c_{LT} + \lambda + \left(1 - \lambda\right) q_{LT}\left(z'\right)\right]\right) - q_{LT}\left(z\right), \tag{4}$$

$$m = \beta E \left(\frac{u_1\left(c'\right)}{u_1\left(c\right) e^{\mu\left(z'\right)}} \left[c_{Sw} - \left(\frac{1}{q_{LIB}\left(z\right)} - 1\right) + \left(1 - \lambda\right) m'\right]\right). \tag{5}$$

As is clear from the first-order conditions for short-term and long-term debt, holding costs introduce a wedge in the dealer's Euler equations. As a consequence, the price of the swap – given in equation 5 – is typically not equal to its no-arbitrage value.

3   Analytical characterization

Several properties of the swap spread can be derived analytically. Analytical expressions also help understand some of the quantitative findings. I focus on three cases. First, some properties of the no-arbitrage case are reviewed. Second, it is shown how frictions for holding short-term and long-term debt affect swap prices in general, and for the specific frictions considered in my quantitative model. For the third case, it is shown how with very strong frictions a negative swap spread should be expected.[2]

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[2]In this section, yields are compounded per period, while in the rest of the paper they are continuously compounded. This is for convenience. The notation does not explicitly aknowledge this difference.

3.1 No-arbitrage case

In this subsection, the swap spread is characterized explicitly when arbitrage is ruled out, and it is shown why a limited arbitrage approach is needed to produce a negative swap spread. Specifically, ruling out arbitrage, this section establishes that if the TED spread (three-month LIBOR minus three-month treasury) is constant, the swap spread is equal to that constant value, and otherwise, if TED is nonnegative, the swap spread also needs to be nonnegative.

Rewriting the dealer's first-order condition for the swap for a more general state-price process, explicit sequential time-indexing, and with an analytically more convenient additive notation for the TED spread, the price of the swap is given as

$$
m_t = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1 \right) - \theta_t + (1 - \lambda) m_{t+1} \right] \right).
$$

Ruling out arbitrage implies that Λ_t prices not only swaps, but also short-term and long-term debt. Under this assumption, and after some algebra (detailed in the appendix), the value of the swap can be written as

$$
m_t = \left(c^{Sw} + \lambda \right) \Omega_t \left(\{1\} \right) - 1 - \Omega_t \left(\{\theta_t\} \right), \tag{6}
$$

with

$$
\Omega_t \left(\{x_t\} \right) = \sum_{j=0}^{\infty} (1 - \lambda)^j E_t \frac{\Lambda_{t+1+j}}{\Lambda_t} x_{t+j}.
$$

Ω_t is the present value of a sequence of geometrically declining, potentially random, payoffs x, that are paid out with a one period lag. Intuitively, the term $c^{Sw} \Omega_t \left(\{1\} \right)$ captures the annuity value of receiving the fixed coupon, while $1 - \lambda \Omega_t \left(\{1\} \right)$ represents the value of a floating rate note paying the risk-free short rate adjusted for the amortization payments. The last term in (6) represents the present value of the sequence of TED spreads.

Defining the (at-market) swap rate y_t^{Sw} as

$$
0 = \left(y_t^{Sw} + \lambda \right) \Omega_t (1) - 1 - \Omega_t \left(\{\theta_t\} \right),
$$

implies

$$
y_t^{Sw} = \frac{1 + \Omega_t \left(\{\theta_t\} \right)}{\Omega_t (1)} - \lambda.
$$

Consider a long term-bond with the same amortization rate as the swap and whose price

can be written as

$$q_t^{LT} = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{LT} + \lambda + (1 - \lambda) q_{t+1}^{LT} \right] \right) = \left(c^{LT} + \lambda \right) \Omega_t (1) .$$

Combined with the implicit definition of the yield from above, $q_t^{LT} = \frac{c^{LT} + \lambda}{y_t^{LT} + \lambda}$,

$$y_t^{LT} = \frac{1}{\Omega_t (1)} - \lambda.$$

The swap spread then equals

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t (\{\theta_t\})}{\Omega_t (1)}. \tag{7}$$

As is clear from equation (7), if $\theta_t = \theta$,

$$y_t^{Sw} - y_t^{LT} = \theta.$$

If $\theta_t \geq 0$,

$$y_t^{Sw} - y_t^{LT} \geq 0.$$

To summarize these results, ruling out arbitrage, the swap spread equals the present value of a TED annuity scaled by the present value of a constant annuity at 1. If TED is non-negative, the swap spread is non-negative. If TED is constant, the swap spread equals the constant TED spread. Clearly, without violation of arbitrage, the swap spread cannot be negative. In my model, arbitrage is limited by the holding costs for bonds. Instead of the geometrically amortizing structures, the same argument can be made with standard swaps and bullet bonds. In this subsection, there is no advantage of using the geometrically amortizing bond; it is essential however for numerical tractability.

3.2   Bond holding frictions

I now assume that there are frictions for the one-period debt and for the long-term debt with the same maturity as the swap. Note that this is more general than the quantitative model presented in Section 2 because no assumptions are made about whether the dealer trades bonds for other maturities and because the friction is more general.

As above, assume that the value of the swap satisfies

$$m_t = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1 \right) - \theta_t + (1 - \lambda) m_{t+1} \right] \right) .$$

Contrary to the no-arbitrage case, the dealer's marginal valuations, Λ, are now no longer necessarily consistent with the prices of risk-free debt.

Assume that frictions for short-term debt affect the relation between the market price and the dealers marginal valuations such that

$$q_t^{ST} = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - \mathfrak{h}_t,$$

where $\mathfrak{h}_t$ is a wedge coming from frictions of holding/selling short-term debt. For instance, this could be the derivative of a convex holding cost function, as in my quantitative model from the previous section, and $\mathfrak{h}_t$ could be positive or negative. Alternatively, $\mathfrak{h}_t$ could be capturing the Lagrange multiplier of a borrowing constraint, in which case it would be negative.

To achieve analytical tractability, I will now work with a first-order approximation of $1/q_t^{ST}$ around $\mathfrak{h} = 0$ and $1/E\left(\frac{\Lambda'}{\Lambda}\right) = 1$. The price of the swap can then be written as

$$m_t = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{Sw} - \frac{1}{E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right)} + h_t + 1 - \theta_t + (1 - \lambda) m_{t+1} \right] \right) + o \left(\mathfrak{h}_t, E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \right),$$

and, going forward, for notational ease, the approximation error will be omitted from the equations. After some algebra, the swap rate satisfies

$$y_t^{Sw} = \frac{1 + \Omega_t \left(\{ \theta_t + \mathfrak{h}_t \} \right)}{\Omega_t (1)} - \lambda.$$

Considering a pricing equation for the debt with the same maturity as the swap that is similarly distorted

$$q_t^{LT} = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{LT} + \lambda + (1 - \lambda) q_{t+1}^{LT} \right] \right) - \mathfrak{j}_t,$$

where $\mathfrak{j}_t$ is the implied frictional marginal cost of holding one unit, which again can be positive or negative.

After some algebra, the long-term yield can be written as

$$y_t^{LT} = \frac{1}{\Omega_t (1) - \frac{1}{(c^{LT} + \lambda)} \Omega_t \left(\left\{ \frac{\Lambda_t}{\Lambda_{t+1}} \mathfrak{j}_t \right\} \right)} - \lambda.$$

The frictional cost $\mathfrak{j}_t$ is here multiplied by $\frac{\Lambda_t}{\Lambda_{t+1}}$ to account for the fact that, unlike for the other uses of $\Omega(.)$, $\mathfrak{j}_t$ is not lagged.

The swap spread now becomes

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)} + \frac{\Omega_t\left(\{\mathfrak{h}_t\} - \frac{1}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\mathfrak{j}_t\right\}\right)}{\Omega_t\left(1\right)}, \tag{8}$$

highlighting its dependence on current and future frictional marginal costs of short-term and long-term debt. Clearly, these frictional costs have the ability to produce negative swap spreads.

If there are no frictions for these two types of debts, ST and LT, then the swap spread is priced by the dealer's valuations Λ implicit in the present value operator $\Omega_t\left(\{.\}\right)$. Without complete markets, the dealer's valuations do not necessarily equal those of other market participants. Through this channel, demand effects can also affect the price of the swap. However, such market incompleteness or segmentation cannot produce a negative swap spread as long as θ_t has a zero probability of becoming negative. To produce negative swap spreads, the dealer needs to be subject to frictions either for the short rate or the long rate corresponding to the maturity of the swap. The intuitive reason why this has to be the case is that the payout of a swap can be replicated syntethically with a combination of these two types of debt, one-period debt and debt with the same maturity as the swap.

For additional insights, I am now specializing the frictional cost terms to replicate my quantitative model. In this case,

$$\mathfrak{h}_t = \kappa_{ST}\alpha_{t+1}^{ST}, \text{ and } \mathfrak{j}_t = \kappa_{LT}\alpha_{t+1}^{LT}.$$

Marginal costs are linearly increasing in the size of the positions, and

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)} + \frac{\Omega_t\left(\kappa_{ST}\left\{\alpha_{t+1}^{ST}\right\} - \frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\alpha_{t+1}^{LT}\right\}\right)}{\Omega_t\left(1\right)}. \tag{9}$$

This equation shows that if the dealer has a long position in long-term bonds, an increase in the frictional cost – every else equal – lowers the swap spread. As an example, consider the impact of an increase in the term spread, everything else equal. In response, the dealer will increase the position on long-term debt and typically reduce the position in short-term debt. Together, this will lead to a lower swap spread. Other terms in the equation can change too, but they are not likely to overturn this relation. In particular, as q_t^{LT} declines, it also contributes to a lower swap spread. Movements in the dealer's valuations cannot change the sign of the effect, they only reweigh the different periods' effects. Possibly, the movement in $\frac{\Lambda_t}{\Lambda_{t+1}}$ multiplying the position can go in the opposite direction. However, a higher term

spread implies higher future expected short rates, which on average lead to increases in the $\frac{\Lambda_t}{\Lambda_{t+1}}$ terms. The quantitative model confirms this negative relation between term spreads and swap spreads.

Equation 9 also shows that if the dealer's positions are smaller in absolute value, everything else equal, the swap spread will be less distorted by holding costs. This will be the case when the dealer's equity is low. The calibrated model can be used to quantify these relations.

3.3 Swap pricing with very strong frictions

To illustrate the main mechanism that allows the model to produce negative swap spreads, I am now presenting an analytic characterization of the quantitative model for the case where bond holdings costs are very large.

In the limit, as holding costs increase, the dealer will not hold any bonds. That is, as κ_{LT} and κ_{ST} get larger, with $d = 0$ and constant endowment of other profits, $\pi(z) = \pi$, consumption/payouts tend to equal endowment and be constant. In this limiting case, the price of the swap is given by

$$m_t = \beta E_t \left(\frac{1}{\mu_{t,1}} \left[c^{Sw} - y_t^{LIB} + (1 - \lambda) m_{t+1} \right] \right).$$

The at-market swap rate for which $m_t = 0$ satisfies

$$y_t^{Sw} = \frac{\sum_{j=1}^{\infty} \beta^j (1 - \lambda)^{j-1} E_t \left(\frac{1}{\mu_{t,j}} y_{t+j-1}^{LIB} \right)}{\sum_{j=1}^{\infty} \beta^j (1 - \lambda)^{j-1} E_t \frac{1}{\mu_{t,j}}}. \tag{10}$$

In the quantitative model, inflation uncertainty does not play a big role. To get a sharper characterization, consider the case with no inflation uncertainty, $\mu_{t,j} = \mu^j$. Taking unconditional expectations,

$$E\left(y_t^{Sw}\right) = \sum_{j=1}^{\infty} w^j E\left(y_{t+j-1}^{LIB}\right),$$

and

$$E\left(y_t^{Sw}\right) = E\left(y_t^{LIB}\right) = E\left(y_t^{ST}\right) + E\left(\theta_t\right)$$

because the weights, w^j, implicitly defined by Equation 10, are constant with $\mu_{t,j} = \mu^j$ and add up to one. Therefore, in this case, the swap rate equals the unconditional expected value of LIBOR, or equivalently, the unconditional expected short rate plus the TED spread.

For comparison, the unconditional mean of the long-term treasury yield can be written

as

$$E\left(y_t^{LT}\right) = E\left(y_t^{ST}\right) + E\left(\tau_t\right)$$

where $E\left(\tau_t\right)$ is the unconditional mean term spread. Combining the two, the unconditional expectation of the swap spread equals

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\left(\theta_t\right) - E\left(\tau_t\right). \tag{11}$$

Historically, based on time-series averages, $E\left(\theta_t\right) = 0.6\%$ in annualized terms, and $E\left(\tau_t\right) = 1.7\%$, so that

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\left(\theta_t\right) - E\left(\tau_t\right) = -1.1\%.$$

Therefore, in the limiting case for which very strong frictions prevent arbitrage, the expected swap spread should be roughly -1%. Intuitively, the high holding costs drive down the dealer's bond positions and reduce his exposure to long-term interest rate risk. The swap is perceived to be less risky, and the required fixed rate declines. Of course, this is an extreme and unrealistic benchmark case. Nevertheless, it shows that in a world with limited arbitrage possibilities one should not be surprised by low or negative swap spreads, even without the need for strong demand effects.

4   Quantitative analysis

In this section the model is calibrated and solved numerically. Quantitative model implications for swap prices are presented. I show that as bond holding costs are increased, the swap spread declines away from its arbitrage-free benchmark. The calibrated model has no difficulty generating negative swap spreads even without explicit demand pressure. The section concludes with additional empirical evidence in support of the model.

4.1   Parameterization

Processes for bond prices and inflation are specified so that the model matches key empirical facts. A period in the model is a quarter. The joint process for the short rate, the term spread, inflation and the TED spread,

$$\left[y_{ST}\left(z\right), \tau\left(z\right), \mu\left(z\right), \theta\left(z\right)\right],$$

is based on an estimated first-order vector autoregression. The data is for US treasuries with 3-month and 30-year maturities and CPI inflation covering 1960-Q1 to 2015-Q3. TED

Symbol	Parameter	Value
$\bar{y}_{ST}$	Short rate level	0.01156
$\bar{\tau}$	Term spread level	0.00429
$\bar{\mu}$	Inflation level	0.00938
$\bar{\theta}$	TED spread level	0.00158
γ	Risk aversion	2
ν	Discount elasticity	1
$1/\lambda$	Maturity of long-term debt and swap	120

Table 1: Model Parameters.

spreads are available from 1986-Q1 onwards. TED spreads do not significantly enter the other three variables' equations. Innovations in the TED spread have very low correlations with the short rate and the term spread; I set these correlations to zero. The elements in the transition matrix that are not statistically significant are set to zero and equations are re-estimated with the zero restrictions.

As shown below, all series are quite persistent, and the only significant off-diagonal interaction terms go from lagged inflation to the short rate and from lagged inflation to the TED spread. The transition matrix is

$$
\begin{bmatrix}
.91 & 0 & .07 & 0 \\
0 & .87 & 0 & 0 \\
0 & 0 & .76 & 0 \\
0 & 0 & .06 & .72
\end{bmatrix}
$$

and the covariance matrix for the innovations

$$
10^{-6} \times
\begin{bmatrix}
5.1 & -3.4 & 4.3 & 0 \\
 & 3.2 & -2.6 & 0 \\
 & & 24.8 & -1.1 \\
 & & & 0.5
\end{bmatrix}.
$$

The VAR is approximated by a finite-state Markov chain following Gospodinov and Lkhagvasuren (2014) with a total of $3^4 = 81$ possible realizations. Two sets of adjustments are made to the Markov chain obtained in this way. First, it is made sure that there are no arbitrage opportunities between the short-term and long-term bonds. This requires a slight reduction in the term spread for the highest realization of the long-term yield. Second, realizations for the TED spread are limited by a lower bound of 0.0003, which corresponds to the lowest historical end-of-quarter value. This is to make sure that negative swap spreads

cannot come from negative TED spreads, which a linear VAR does not rule out. This requires increasing negative TED realizations to 0.0003 and adjusting small positive realizations so as to keep the unconditional expectation of the TED spread at the targeted (per quarter) level of $\bar{\theta} = 0.00158$.

The average maturity of the swap and long-term debt, $1/\lambda$, corresponds to 120 quarterly periods, that is 30 years. Risk aversion is set to 2. The elasticity parameter of the discount rate equals $\nu = 1$. With this value, for the benchmark case, the dealer has long/short positions about 90% of the time; for lower values, more wealth is accumulated and positive positions for both bonds are more common. For the benchmark case, the cost parameter for short term debt is set to $\kappa_{ST} = 0$; I consider several values for the cost parameter for long-term debt κ_{LT}. The model is solved globally with an algorithm that shares features with Judd (1992) and Stepanchuk and Tsyrennikov (2015).

4.2 Model properties

Swap spreads with a thirty-year maturity from the model are compared to their empirical counterparts in Table 2. Unconditional expectations and standard deviations are presented for a set of values for κ_{LT}, the holding cost parameter for the long-term debt, with short-term debt costs at $\kappa_{ST} = 0$. As κ_{LT} increases, arbitrage becomes more costly and the unconditional mean of the swap spread goes from positive 44 basis points with a low cost of $\kappa_{LT} = .0001$ to a negative -52 basis points for the highest cost presented of $\kappa = .005$. Clearly, the model has no difficulty producing realistic negative values for swap spreads. The pattern shown in Table 2 is consistent with the analytical characterization for the high friction case: as arbitrage becomes more costly, swap rates decline and spreads become negative. To match the post-2008 empirical levels, the cost parameter should be slightly larger than for the intermediate case with $\kappa = 0.001$.

Inspecting the mechnism derived in Equation (9) suggests a back-of-the envelop approximation for the swap spread of

$$E\left(y_t^{Sw} - y_t^{LT}\right) \approx E\left(\theta_t\right) + \kappa_{ST} E\left(\alpha_{t+1}^{ST}\right) - \kappa_{LT} E\left(\alpha_{t+1}^{LT}\right).^3 \tag{12}$$

For the benchmark case with $\kappa_{LT} = 0.001$ and $\kappa_{ST} = 0$, the unconditional mean of the long-term bond position is $E\left(\alpha_{t+1}^{LT}\right) = 1.77$, a bit more than twice the dealer's average wealth level. In this case, with $E\left(\theta_t\right) = 63.2$ (in annualized basis points), Equation (12) implies

---

[3]In particular, consider a first-order approximation with respect to θ_t, α_{t+1}^{ST} and $\frac{\Lambda_t}{\Lambda_{t+1}}\alpha_{t+1}^{LT}$ around their unconditional expectations. Take unconditonal expectations, and with $E\left(1/q_t^{LT}\right)$ close to 1 and $E\frac{\Lambda_t}{\Lambda_{t+1}}\alpha_{t+1}^{LT}$ close to $E\alpha_{t+1}^{LT}$, we have (12).

	E(30Y SS)	Std(30Y SS)	E(TED)
Data			
7/1997 − 9/2008	57	27	58
10/2008 − 10/2015	−18	12	35
Model			
$\kappa_{LT} = 0.0001$	44	12	63
$\boldsymbol{\kappa_{LT}= 0.001}$	**−7**	**45**	**63**
$\kappa_{LT} = 0.005$	−52	82	63
Constant TED	−14	57	63
Constant Inflation	−17	45	63
$\kappa_{ST} = 0.001$	−2	64	63
Higher risk aversion, $\gamma = 4$	−16	44	63
Lower discount elast., $\nu = 0.8$	−14	50	63

Table 2: Swap Spreads in the Data and the Model. Units are annualized basis points. The cost parameter for long-term debt is $\kappa_{LT} = 0.001$ for the last five cases.

an annualized swap spread of $63.2 − .001 \times 1.77 \times 40000 = −7.6$. which is almost exactly the model-implied value reported in Table 2. More generally, simulations show (12) to be a decent approximation.

Given the monotonicity of the expected swap spread in the cost parameter κ_{LT} shown in Table 1, we can directly focus on the size of the marginal cost $\kappa_{LT}E\left(\alpha_{t+1}^{LT}\right)$ needed to explain a given expected swap spread. For instance, to match the post 10/2008 values for the average TED spread and swap spread of 35 and 18 basis points, respectively, requires a marginal cost of

$$\begin{aligned} \kappa_{LT}E\left(\alpha_{t+1}^{LT}\right) &= E\left(\theta_t\right) - E\left(y_t^{Sw} - y_t^{LT}\right) \\ &= 35 + 18 = 53 \text{ bps}. \end{aligned}$$

As one of the holding costs for long-term bonds, large US banks are subject to the Supplementary Leverage Ratio (Bowman and Wilkie (2016), Duffie (2016)) that requires them to hold 5% of tier 1 capital against total assets. Assuming a required return on equity of 10%, the implied cost would be $5\% \times 10\% = 50$ basis points. Therefore, this requirement, if binding, would have a cost roughly in the order of magnitude needed for the model to match the expected swap spread. Even if this capital requirement is not binding, banks would likely want to consider a precautionary buffer. The are other potential costs that have increased since the financial crises including risk-weighted capital requirements and FDIC insurance premiums, in addition to the restrictions due to the Volcker rule. Overall, it appears that the extent of the friction in the model is empirically plausible. Note also that because equation

(12) includes no model parameters beyond κ_{LT}, this conclusion is robust to changing other model parameters such as risk aversion and the time-discount factor.

For a value of the holding cost of $\kappa_{LT} = .001$ which allows the model to approximately match the level of the swap spread post 2008, the model implied standard deviation of 45 basis points is higher than its empirical counterpart. Part of the discrepancy between the model and the data can be attributed to the relatively short post-2008 sample, as swap spreads in the model (and in the data) are quite persistent.

Table 2 includes five additional cases that illustrate the sensitivity to changes in parameter values. Risk in inflation and risk in the TED spread have relatively moderate effects on the swap spread. When short-term debt is subject to the same holding cost as the long-term debt, $\kappa_{ST} = 0.001$, the swap spread increases moderately. With the additional cost on short-term debt, the dealer's wealth position and long-term bond holdings are somewhat lower, contributing to this higher swap spread as suggested by Equation (12). The position in short-term debt is close to zero on average in this case.

4.3 Additional empirical evidence

A key model mechanism is the relation between the term spread and the swap rate. As shown in Subsection 3.3, in the limiting case where the dealer holds no long-term bonds, the swap spread declines one for one with the term spread, because the risk premium in long-term bonds does not get incorporated into the price of the swap. In the time series dimension, in the model, this relation is not very strong unconditionally. However, conditional on a given level of the short rate, the term spread is robustly negatively related to the swap spread. Figure 2 shows how term spreads and swap spreads are related conditional on given levels of the short rate. In particular, each set of lines of a given color corresponds to a fixed level of the short rate. The dealer's equity, ω, is also a state variable, but its impact on the swap rate is relatively less important. The figure shows swap spreads for the mean level of equity for a given combination of the term spread and the short rate. Inflation and the TED spread do not significantly affect this relationship. Indeed, for a given color, multiple lines represent different inflation rates and TED spreads; these lines are very close to each other. The rest of this section presents empirical evidence that is consistent with this model property.

Table 3 displays the coefficients from regressing swap spreads with different maturities on a number of factors one would expect to be relevant. As suggested by my model, this includes the term spread between thirty-year and three-month treasuries, three-month treasuries, and the TED spread. In line with Feldhuetter and Lando (2008) and Hanson (2014), the duration of mortgage backed securities (MBSD) is included as it appears to capture hedging demands
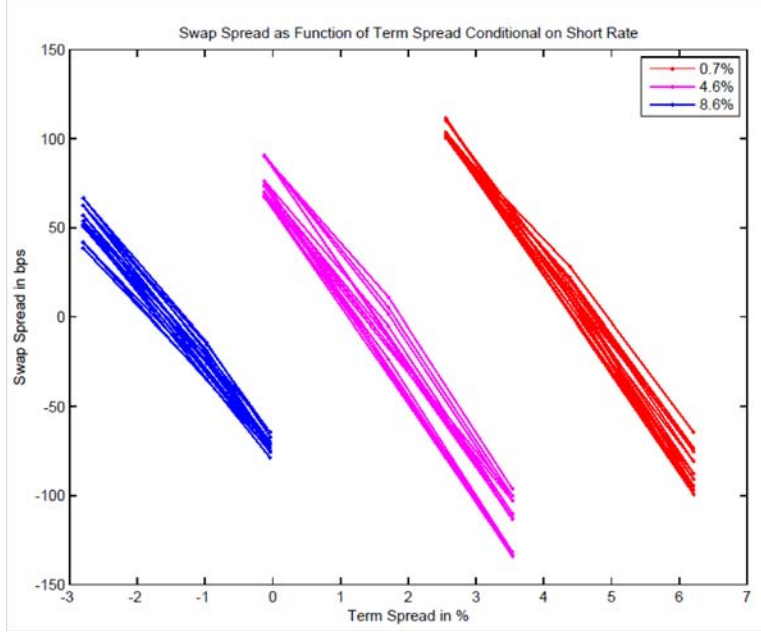
Figure 2: Swap Spread as a Function of Term Spread Conditional on Short Rate. *The holding cost parameters equal* $\kappa_{LT} = 0.001$ *and* $\kappa_{ST} = 0$.

for investors in MBS.

Table 3 covers the period 7/1997 to 10/2015. Regressions are based on monthly growth rates for all the variables. Consistent with the model, the term spread, TERM, is negatively related to the thirty-year swap spread and significant at the 1% level. The relationship becomes slightly weaker for shorter maturities.

As expected, the TED spread factors in positively. It is also not surprising that this relationship is weaker for longer maturities, as the swap spread is driven by the sequence of future TED spreads over the period to maturity of the swap. The current TED spread should be relatively less important for longer maturities. Consistent with the prior research cited above, MBSD is positively related to the swap spread.

Table 4 presents the same regression for the post-crisis sample. For the thirty-year maturity, and the other longer maturities, the negative link to the term spread remains. Somewhat unexpectedly TED now is significantly negatively related to the longer maturity swap spreads.

Regressor \ Swap Maturity	2	5	10	20	30
TERM	−0.038**	−0.042**	−0.093***	−0.085***	−0.128***
TED	0.242***	0.123***	0.057***	0.030	0.039
MBSD	0.028***	0.044***	0.065***	0.048***	0.073***
3MTB	0.016	0.026	0.030	−0.010	0.024
adj. R2	0.48	0.18	0.21	0.06	0.18

Table 3: Swap Spread Regressions 7/1997 - 10/2015. All variables are in monthly growth rates. TERM stands for the difference between the 30 year Treasury rate minus the 3 month rate, TED is the TED spread, and 3MTB the 3 month treasury rate, MBSD is the duration of mortgage backed securities. Significance levels: *** 1 %, ** 5 %, * 10 %.

Regressor \ Swap Maturity	2	5	10	20	30
TERM	−0.022	−0.020	−0.126***	−0.137***	−0.183***
TED	0.381***	0.225***	−0.050	−0.137***	−0.214***
MBSD	0.005	0.004	0.052***	0.052***	0.068***
3MTB	−0.007	0.250**	0.275***	0.365**	0.418**
adj. R2	0.44	0.38	0.40	0.27	0.28

Table 4: Swap Spread Regressions 10/2008 - 10/2015. All variables are in monthly growth rates. TERM stands for the difference between the 30 year Treasury rate minus the 3 month rate, TED is the TED spread and 3MTB the 3 month treasury rate, MBSD is the duration of mortgage backed securities. Significance levels: *** 1 %, ** 5 %, * 10 %.

5   Conclusion

Negative swap spreads are inconsistent with an arbitrage-free environment. In reality, arbitrage is not costless. I have presented a model where specialized dealers trade swaps and bonds of different maturities. Costs for holding bonds can put a price wedge between bonds and swaps. I show a limiting case with very high bond holding costs, expected swap spreads should be negative. In this case, no term premium is required to price swaps, and this results in a significantly lower fixed swap rate. As a function of the level of bond holding costs, the model can move between this benchmark and the arbitrage-free case. The quantitative analysis of the model shows that under plausible holding costs, expected swap spreads are consistent with the values observed since 2008. Demand effects would operate in the model but are not explicitly required for these results.

My model can capture relatively rich interest rate dynamics. Conditional on the short rate, the model predicts a negative link between the term spread and the swap spread. The paper has presented some empirical evidence consistent with this property.

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Appendix A: Swap value

Iterating on the different parts in the square brackets of

$$m_t = E_t \left(\frac{\Lambda_{t+1}}{\Lambda_t} \left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1 \right) - \theta_t + (1 - \lambda) m_{t+1} \right] \right),$$

gives for the first term

$$c^{Sw} E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} + (1 - \lambda) \frac{\Lambda_{t+2}}{\Lambda_t} + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} .. \right\}$$

$$= c^{Sw} \sum_{j=1}^{\infty} (1 - \lambda)^{j-1} E_t \frac{\Lambda_{t+j}}{\Lambda_t}$$

$$\equiv c^{Sw} \Omega_t (1),$$

where Ω_t is the present value of a geometrically declining annuity. For the second part with the $- \left(\frac{1}{q_t^{ST}} - 1 \right)$ terms, assuming that Λ_t prices short-term debt,

$$E_t \left\{ \begin{array}{c} -1 + \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) - (1 - \lambda) \frac{\Lambda_{t+1}}{\Lambda_t} \\ + (1 - \lambda) \frac{\Lambda_{t+2}}{\Lambda_t} - (1 - \lambda)^2 \frac{\Lambda_{t+2}}{\Lambda_t} \\ + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} ... = \end{array} \right\}$$

$$E_t \left\{ \begin{array}{c} -1 - (1 - \lambda) \frac{\Lambda_{t+1}}{\Lambda_t} - (1 - \lambda)^2 \frac{\Lambda_{t+2}}{\Lambda_t} ... \\ + \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) + (1 - \lambda) \frac{\Lambda_{t+2}}{\Lambda_t} + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} ... = \end{array} \right\}$$

$$E_t \left\{ \begin{array}{c} -1 - (1 - \lambda) \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} + (1 - \lambda)^1 \frac{\Lambda_{t+2}}{\Lambda_t} + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} ... \right\} \\ + \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} + (1 - \lambda)^1 \frac{\Lambda_{t+2}}{\Lambda_t} + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} ... \right\} = \end{array} \right\}$$

$$= -1 + \lambda \Omega_t (1).$$

And the third part

$$E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \theta_t + (1 - \lambda) \frac{\Lambda_{t+2}}{\Lambda_t} \theta_{t+1} + (1 - \lambda)^2 \frac{\Lambda_{t+3}}{\Lambda_t} \theta_{t+2} .. \right\} =$$

$$\sum_{j=1}^{\infty} (1 - \lambda)^{j-1} E_t \frac{\Lambda_{t+j}}{\Lambda_t} \theta_{t+j-1} \equiv \Omega_t (\{\theta_t\}).$$

Combining terms yields

$$
\begin{aligned}
m_t &= c^{Sw}\Omega_t(1) - 1 + \lambda\Omega_t(1) - \Omega_t(\{\theta_t\}) \\
&= \left(c^{Sw} + \lambda\right)\Omega_t(1) - 1 - \Omega_t(\{\theta_t\}).
\end{aligned}
$$

Appendix B: Swap holding costs

With an explicit holding costs for swaps, the pricing equation can be written as

$$
m_t = E\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - y_t^{LIB} + (1-\lambda)m_{t+1}\right]\right) - i_t
$$

where i_t is the marginal cost from a cost function with a timing in line with the bond cost functions used. This implies that

$$
m_t = c^{Sw}\Omega_t(1) - 1 + \lambda\Omega_t(1) - \Omega_t\left(\left\{\theta_t + h_t + \frac{\Lambda_{t+1}}{\Lambda_t}i_t\right\}\right),
$$

and the swap spread

$$
y_t^{Sw} - y_t^{LT} = \frac{\Omega_t(\{\theta_t\})}{\Omega_t(1)} + \frac{\Omega_t(\{h_t\}) - \frac{1}{q_t^{LT}}\Omega_t^{Cum}(\{j_t\})}{\Omega_t(1)} + \frac{\Omega_t\left(\left\{\frac{\Lambda_{t+1}}{\Lambda_t}i_t\right\}\right)}{\Omega_t(1)}.
$$

The last term differentiates this from equation (8) in the main text. With a quadratic cost, $\frac{\kappa_{Sw}}{2}(s')^2$, the marginal cost is $i_t = \kappa_{Sw}s_{t+1}$, and the additional price wedge depends on the process for current and future swap holdings.