The Life-cycle Growth of Plants in Colombia: Fundamentals vs. Distortions.*

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Abstract

We take advantage of rich microdata on Colombian manufacturing establishments to decompose growth over an establishment's life cycle into that attributable to fundamental sources of idiosyncratic growth–physical productivity, demand shocks (firm appeal), and input prices–and distortions that weaken the link between those fundamentals and actual growth. We accomplish this using data on quantities and prices for individual products for each manufacturing establishment. Pooling all ages, measured fundamentals explain around 70% of the variability of output relative to birth level, with the remaining 30% explained by distortions and other unobserved factors. Demand shocks and $TFPQ$ are equally important in the explained part, while input prices play a more minor role. Distortions explain more than 50% of the variance in growth up to age seven, but their contribution falls to less than 25% by around age 20. For the fraction explained by fundamentals, early life growth variation is explained by $TFPQ$ with demand and input prices playing a minor role. But demand is the crucial factor in variation in

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long-run growth, with a contribution that surpases that of $TFPQ$ and unobserved factors by around age 15. In the 2000s compared to the 1980s, two decades separated by a wave of deep structural reforms, the contribution of $TFPQ$ to the variance in life cycle growth grows by around 7 p.p , compensated by a lesser role for input prices and, interestingly, distortions.

Keywords: post-entry growth; TFPQ; demand; distortions.

JEL codes: O47; O14; O39

1 Introduction

The growing availability of detailed firm and establishment level data has allowed researchers to investigate the empirical micro foundations behind sluggish aggregate growth in many low- and middle-income economies. A recent strand of the literature has focused on how businesses grow over their life cycle, uncovering patterns that suggest that less developed economies are characterized by post-entry business growth slower than that observed in developed economies (Hsieh and Klenow, 2014; Buera and Fattal, 2014). This observation has generated interest in understanding the role played by distortions that detach a business' performance from its fundamentals –productivity–, in explaining differential post-entry growth across countries.[1]

Detecting the role of distortions vs. fundamentals is often based on imposing assumptions about technology and demand to draw inferences from dispersion in revenue productivity measures. For example, Hsieh and Klenow (2009) assume Cobb-Douglas technology with constant returns to scale, homogeneous input prices, and a CES demand structure to decompose revenue based measures into fundamentals vs. distortions. Under this assumptions, all dispersion in average products of inputs is attributed to distortions. Beyond imposing structure that may not fit the data in all contexts, this type of approach is limited relative to having firm-level data on prices and quantities (as in Foster, Haltiwanger and Syverson, 2008 and 2016 and Eslava et. al. (2013)). Price and quantity data permit not only direct measurement of fundamentals but also decomposing the fundamentals into a rich set of components including that of technical efficiency (what the literature and we often refer to as physical productivity or $TFPQ$) as well as quality or other demand related factors, and input prices.[2] With direct measurement of these fundamentals, distortions can in turn be measured from the differences between optimal frictionless inputs and outputs (determined using the measured fundamentals) and the actual measured inputs and outputs.

We take advantage of this direct measurement approach for the Colombian manufacturing industry, for which there is rich establishment-level data on prices and quantities for both outputs and inputs over a long period

[1] This approach has grown out of a recent literature highlighting the potentially important role of idiosyncratic distortions in generating misallocation. See, e.g., Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and Bartelsman et. al. (2013).

[2] See, e.g., De Loecker, Goldberg, Khandelwal and Pavnic (2015); Foster, Haltiwanger and Syverson (2008,2016); Hottman, Redding and Weinstein (2016).

of time. Using these data, we conduct a decomposition of growth over an establishment's life cycle into that attributable to fundamental sources of growth (physical productivity, demand shocks, and input prices) and idiosyncratic distortions that weaken the link between those fundamentals and actual growth.[3] Since at least 1982, the Colombian Annual Manufacturing Survey has been recording information on values and quantities for all individual products produced by an establishment and all individual material inputs used by the establishment, besides information on input use and monetary value of production. Establishment level price indices can be constructed using this information. This allows us to measure establishment-level output based on deflating establishment-level revenue with a quality-adjusted establishment-level price deflator rather than the typical approach in the literature of using industry-level deflators. The Manufacturing Survey has census-type coverage of non-micro manufacturing establishments, and allows following each of them longitudinally.

The Colombian Manufacturing Survey also offers rich possibilities in terms of following plants over their life cycle. Age of the establishment from the time of the start of its operations is reported in the survey, and establishments can be followed longitudinally, some of them for over 30 years. The age indicator is not affected by restructuring or changes in ownership. One key advantage of this data infrastructure is that, because we can follow each plant over its own life cycle, we deal with selection bias that affects life cycle growth estimates from cross sectional data. Estimates based on cross-sectional information include in the set of early age observations a series of likely small and unproductive plants that do not make it to the older ages. Own-plant life cycle estimates are not subject to the same problem.

Our approach to decomposing growth driven by fundamentals into the contributions of physical productivity and demand builds on the ideas proposed by Foster et al. (2008), recently applied to the life-cycle growth con-

[3]By physical productivity, we refer to a production function residual, where production is measured as plant-level revenue deflated with a quality adjusted plant-level deflator. The term physical productivity to denote technical efficiency or TFPQ at the plant-level has been recently popularized by Foster, Haltiwanger and Syverson (2008). In their case, the measure is literally physical productivity since they take advantage of physical quantity data for selected products in the US. In our case, we use this same term but it reflects the more traditional concept of measuring output by deflating revenues with a price deflator. However, unlike the vast majority of studies we use a plant-level price deflator based upon the direct collection of plant-level prices, and adjusted to allow for varying quality-appeal across products within a plant.

text by the same set of authors (2016). But we widen the sectoral scope to all (non-micro) manufacturing plants and sectors, which requires an explicit treatment of the multi-product character of most establishments to appropriately separate quantities and prices at the establishment level. We rely on the nested firm-product demand setup proposed by Hottman et al (2016), which allows us to construct quality-adjusted plant price indices to then appropriately measure quantities at the plant level, constructed as values deflated by individual quality-adjusted prices. Beyond our different focus on life-cycle growth over the medium and long run within a business, we also follow a different methodological approach than Hottman et al (2016) to decompose technology vs. demand shocks. While they rely solely on data on prices and quantities, and a structure that allows disentangling cost shocks vs. markups using only these data, we add information on the plant's production process and input prices to obtain measures of fundamentals from the side of technology, separating cost shocks from input prices and physical productivity.

Post entry growth is highly skewed in our data, as it has been shown to be in other contexts (e.g. Haltiwanger, Jarmin and Miranda, 2013). Thus, much of our focus is on the decomposition of the variance in growth across plants at different stages of the life cycle. But our methods also permit quantifying the contribution of fundamentals vs. distortions to first moments such as the mean and median growth as well as high growth (90th percentile) vs. low growth (10th percentile) plants.

In trying to understand the reasons behind slow post-entry productivity growth, much of the focus has been on dimensions external to the business, such as institutions that discourage, or fail to encourage, healthy market selection and investment in productivity growth (e.g.Hsieh and Klenow, 2014). On the side of businesses, meanwhile, the focus has been on efforts conducive to improvements in technical efficiency. For instance, research on managerial practices that impact productivity has focused on production processes and employee management (e.g. Bloom and Van Reenen, 2007; Bloom et al. 2016). Our approach highlights the multidimensional character of growth drivers that are internal to the business. As theory increaslingly devotes attention to endogenous investments in productivity, shedding light on the role of these different dimensions of productivity becomes crucial. The endogenous evolution of technical efficiency vs. demand related factors likely differs, as do frictions to investments in these different dimensions. The optimal design of policies aimed at dealing with such frictions also likely depends on the nature and relative importance of demand side vs. technical efficiency factors in accounting for which businesses succeed.

Our approach should be viewed as a life cycle accounting exercise providing guidance about the relative importance of different fundamentals vs. distortions for a business' choice of scale given fundamentals. As we note above, the patterns of fundamentals such as $TFPQ$ and demand shocks that we detect likely reflect in part endogenous investments in process and product innovation as well as related investments in organizational capital and customer base (e.g. Atkeson and Burstein, 2010; Acemoglu et al. 2014; Hsieh and Klenow, 2014; Foster, Haltiwanger and Syverson, 2016). There may be distortions that impact such endogenous investments, in turn reflected in the evolution of the measured fundamentals. The overall potential role of distortions in explaining to life cycle growth is a composite of their effect on endogenous investments in fundamentals, and their effect on scale for given evolution of fundamentals. Our focus is on the latter. We regard this approach as an important step in the direction of quantifying the overall role of specific fundamentals vs. distortions. As our findings show, the different components have quite different contributions to the variability of life cycle growth across businesses.

For an average manufacturing plant in our Colombian data, compared to its level at birth output has grown by a factor of 2.4 by age 5, almost four-fold by age 10, and ten-fold by age 25. Employment grows at a slower pace, with factors of growth relative to birth of around 1.5, 2, and 3 for ages 5, 10, and 25. Based on comparable data for the US and growth over cohorts, this pace of employment growth in Colombia is approximately half as fast as that in the US.

There is wide dispersion in the patterns of growth across firms, with average growth driven by a small fraction of rapidly growing businesses. We find that, in the long run, such dispersion is mostly explained by dispersion in fundamentals, rather than distortions and other unobserved factors, with $TFPQ$ and demand shocks both playing a crucial role. Pooling all ages, measured fundamentals explain around 70% of the variability of output relative to birth level, with the remaining 30% explained by distortions and other unobserved factors. Of the fraction explained by measured factors, input price growth explains 7 p.p, with demand shocks and $TFPQ$ being equally important in explaining the rest.

Interestingly, distortions are particularly important to explain the variance of growth from birth to early ages, accounting for more than 50% of the variance of growth up to age seven. But, they lose importance for longer horizons, with their contribution to the variance of growth falling to less than 25% for a horizon of 20 years from birth. For the fraction explained by fundamentals, variance in early life growth is explained mostly by $TFPQ$, while demand is the crucial factor in accounting for the variation in long-run growth from birth, with a contribution that surpases that of $TFPQ$ and unobserved factors for ages 15 and beyond.

In terms of other moments, we find that most of the increase in mean growth (over 80%) over a 20- or 30- year life cycle is due to the growth in demand. We also find that based on fundamentals alone, plants should be exhibiting even higher mean growth but other factors such as distortions imply a slower actual growth in production. We also find that there is a large difference between mean and median growth over the life cycle. Underlying this gap is considerable skewness in the growth rate distribution. The 90th percentile grows very rapidly compared to the median while the 10th percentile is closer to the median. At the 10th percentile both $TFPQ$ and demand contribute about equally to growth to age 20-30 (which is negative), while demand is more important for the median and much more for the 90th percentile. Interestingly, it is at the 90th percentile where we find the largest gap between the growth predicted by fundamentals and actual growth. This finding is consistent with distortions playing an especially important role for high growth plants over the life cycle.

We also observe changes in the contribution of technology vs. demand and other factors over time in Colombia. The contribution of TFPQ to the variance in life cycle growth grows by around 7 p.p in the 2000s compared to the 1980s, while the contribution of input prices and distortions falls. Moreover, life cycle growth appears faster in the 2000s compared to the 1980s. Many underlying factors probably changed between those two decades, but a crucial dimension is the implementation of wide market-oriented reforms in the 1990s.

The paper proceeds as follows. Section 2 presents our conceptual framework, defining each of the plant fundamentals that we characterize, and our approach to decompose growth into contributions of those fundamental sources as well as distortions. We then explain the data used in our empirical work, in section 3. Growth over establishments' life cycle in terms of output, employment and other outcomes, which is the object we aim at decomposing, is characterized in section 4). Section 5 explains the approach we use to measure fundamentals. Results for our growth decomposition are presented in section 6. Section 7 presents extensions and robustness analysis. Section 8 concludes.

2 Decomposing firm growth into fundamentals vs distortions

We start with a very simple model of firm optimal behavior given firm fundamentals, to derive the relationship that should be observed between size growth and growth in fundamentals as a firm ages. We also permit firm size to be impacted by distortions. For consistency with the literature on business dynamics, we refer to a business as a "firm", even though the unit of observation for our empirical work is an establishment or plant. The main fundamentals we consider are the productivity of the firm's productive process (often termed TFPQ in the literature) and a demand shock. The conceptual framework below makes clear what we mean by each of these, and the sense in which they are "fundamentals". Beyond measuring TFPQ and demand shocks, we observe unit prices for inputs, in particular material inputs and labor.

In the model, the firm chooses its size optimally given TFPQ, demand shocks, input prices and distortions. As a result, growth over its life cycle is driven by growth in each of them. This is the basis of our analysis. In the spirit of a growth accounting exercise and of much of the literature on firm dynamics, we take growth of fundamentals as exogenous.[4]

We don't explicitly model adjustment frictions but take the shortcut in recent literature on misallocation to permit wedges or distortions between frictionless static first order conditions and actual behavior (e.g. Hsieh and Klenow, 2009). Such distortions and wedges might capture factors such as

[4] For instance, the seminal models of Hopenhayn (1992) and Melitz (2003), and much of the work that has since followed in Macroeconomics and Trade. Endogenous productivity-quality growth has made its way to these models more recently (e.g. Atkenson and Burstein, 2010; Acemoglu et al. 2014; Hsieh and Klenow, 2014; Fieler, Eslava, and Xu, 2016). The firm's efforts to strengthen demand may include investments in building its client base (Foster et al., 2016), and adding new products and/or improving the quality of its pre-existing product lines. Those to strengthen $TFPQ$ may include better management of the production process (e.g. Bloom and Van Reenen, 2007) or acquiring better machines. The results of our decomposition shed light on the relative role and characteristics of each of these accumulation processes, useful for providing guidance about future research that explores the determinants of these fundamentals.

adjustment frictions, technlogical frictions, and distortions arising from regulation.[5] This shortcut enables us to use a simple static model of optimal input determination to frame our analysis of growth between birth and a future period t, where t is far away into the future. We permit the wedges or distortions to vary by firm age which could be viewed as a proxy for permitting adjustment frictions to vary by firm age.

2.1 Technology

Consider a firm indexed by f, that produces output Q_{ft} using a composite input X_{ft} to maximize its profits, with technology

$$Q_{ft} = A_{ft} X_{ft}^{\gamma} = a_{ft} A_t X_{ft}^{\gamma} \tag{1}$$

A_{ft} is the firm's physical total factor productivity $TFPQ$, and γ the returns to scale parameter. In turn, $A_{ft} = a_{ft} A_t$ where A_t is an aggregate technology shock, and a_{ft} is an idiosyncratic component. Equation (1) makes clear that a_{ft} captures the (idiosyncratic) physical efficiency of the productive process: how much physical product the firm expects to obtain from a unit of a basket of inputs, beyond that obtained by the average firm.

Some firms are multiproduct, and for them output Q_{ft} is a composite of individual products (see section 2.2). But though process efficiency is likely to vary across products in the firm, some aspects of it, such as production management and average worker ability for basic tasks, are common to different product lines within the firm. We focus on these firm-level components of efficiency, captured by a_{ft}, a crucial focus when trying to understand growth at the firm level.

2.2 Demand

As in Hottman et al. (2016), in the context of multiproduct firms we define firm output Q_{ft} as a CES composite of individual products $Q_{ft} =$

[5] This shortcut has limitations as the idiosyncratic distortions that we permit don't provide the discipline that formally modeling dynamic frictions imply. See, e.g., Asker, Collard-Wexler and DeLoecker (2014), Decker et. al. (2017) and Haltiwanger, Kulick and Syverson (2017). But it has the advantage in subsuming in a simple measure different types of frictions and distortions.

$\left(\sum_{\Omega_t^f} d_{fjt} q_{fjt}^{\frac{\sigma_J - 1}{\sigma_J}}\right)^{\frac{\sigma_J}{\sigma_J - 1}}$, where q_{fjt} is period t purchases of good j produced by firm f, the weights d_{fjt} reflect consumers' relative preference for different goods within the basket offered by firm f, and Ω_t^f is the basket of goods produced by f in year t. In particular, consumers derive utility from a nested CES utility function, with a CES nest for firms and another for products within firms. Consumer's utility in period t is given by:

$$U\left(Q_{1t}, ..., Q_{Nt}\right) = \left(\sum_{f=1}^{N_{Ft}} d_{ft} Q_{ft}^{\frac{\sigma_F - 1}{\sigma_F}}\right)^{\frac{\sigma_F}{\sigma_F - 1}} \tag{2}$$

$$where\ Q_{ft} = \left(\sum_{\Omega_t^f} d_{fjt} q_{fjt}^{\frac{\sigma_J - 1}{\sigma_J}}\right)^{\frac{\sigma_J}{\sigma_J - 1}} \tag{3}$$

$$s.t. \quad \sum_{f=1}^{N_{Ft}} \sum_{\Omega_t^f} p_{fjt} q_{fjt} = E_t \tag{4}$$

where p_{fjt} is the price of q_{fjt}, and N_{Ft} is the number of firms in period t. We refer to d_{fjt} and d_{ft} as, respectively product (within firm) and firm appeal or demand shocks, defined as in equations 2 and 3: the weight, in consumer preferences, of product fj in firm f's basket of products, and of firm f in the set of firms. Product appeal d_{fjt} captures the valuation of attributes specific to good fj relative to other goods produced by the firm, while firm appeal d_{ft} captures attributes that are common to all goods provided by firm f,, such as the firm's costumer service and average quality of firm f's products. Both firm and product appeal may vary over time. Parameters σ_F and σ_J capture, respectively, elasticities of substitution across firms and across goods produced by the same firm, where σ_J is assumed constant across firms within a sector.

Consumer optimization implies that the period t demand for product fj and the firm revenue are, respectively, given by

$$q_{fjt} = d_{ft}^{\sigma_F} d_{fjt}^{\sigma_J} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_J} \frac{E_t}{P_t} \tag{5}$$

$$and$$

$$R_{ft} = d_{ft}^{\sigma_F} P_{ft}^{1-\sigma_F} \frac{E_t}{P_t^{1-\sigma_F}} \tag{6}$$

where

$$P_{ft} = \left(\sum_{\Omega_t^f} d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma_J}\right)^{\frac{1}{(1-\sigma_J)}} \tag{7}$$

is the firm's exact price index, and $P_t = \left(\sum_{f=1}^{N_F} d_f^{\sigma_F} P_f^{1-\sigma_F}\right)^{\frac{1}{1-\sigma_F}}$ is an aggregate price index. Given the properties of CES demand, $Q_{ft} = \frac{R_{ft}}{P_{ft}}$ (which can be shown using these optimal demands and equation (2)).[6]

Dividing (6) through by P_{ft}, and solving for P_{ft} we obtain demand equation (8), which we use in our empirical application:

$$P_{ft} = D_{ft} Q_{ft}^{-\varepsilon} = D_t d_{ft} Q_{ft}^{-\varepsilon} \tag{8}$$

where $\varepsilon = \frac{1}{\sigma_F}$ is an inverse elasticity of demand, and D_{ft} is a demand shifter, with aggregate and idiosyncratic components $D_t = P_t \left(\frac{E_t}{P_t}\right)^{\varepsilon}$ and d_{ft}. A crucial insight on the measurement of firm appeal emerges from equation (8): d_{ft} is the price charged holding quantities constant, beyond aggregate effects. We refer to d_{ft} generically as the firm's (idiosyncratic) demand shock.

[6] Unlike Hottman et. al. (2016) we do not formally model the decision to add and subtract products. In their setting using UPC code level data, modeling product turnover this is a critical issue. For our purposes, we do make adjustments for product turnover in the manner suggested by Redding and Weinstein (2016). The latter paper does not formally model the decisions to add and substract products but rationalizes the entry and exit of products through assumptions on the patterns of product specific demand shocks. That is, they assume products enter when the product specific demand shock switches from zero to positive and exits when the reverse occurs. We can rationalize product entry and exit in the same manner. We consider multi-product firms mostly for the purpose of obtaining a firm-level price deflator that takes into account multi-product activity.

Inverting this equation and multiplying through by P_{ft} to obtain $R_{ft} = P_{ft}^{1-\frac{1}{\varepsilon}} D_{ft}^{\frac{1}{\varepsilon}}$ (where revenue $R_{ft} = P_{ft}Q_{ft}$), one obtains the analogous interpertation of measured firm appeal (d_{ft}) used by Hottman et al (2016): d_{ft} captures sales holding prices constant. This is akin to quality as defined by Khandelwal (2010), Hallak and Schott (2011), Fieler, Eslava and Xu (2016), and others. Foster et al (2016), in turn, interpret firm appeal as capturing the strength of the business' client base.

2.3 Determination of Firm Size

The firm chooses its scale X_{ft} to maximize profits

$$\underset{X_{it}}{Max} \; (1-\tau_{ft}) P_{ft}Q_{ft} - C_{ft}X_{ft} = (1-\tau_{ft}) D_{ft}A_{ft}^{1-\varepsilon}X_{ft}^{\gamma(1-\varepsilon)} - C_{ft}X_{ft}$$

taking as given A_{ft}, D_{ft}, and unit costs of the composite input, C_{ft}. There may be idiosyncratic distortions τ_{ft}, that affect a firm's choice of size given all of these fundamentals.[7] These distortions capture, for instance, adjustment costs, product-specific tariffs, and size-dependent regulations or taxes. For adjustment costs, we take a reduced-form approach that recognizes costs that break the link between actual adjustment and the "desired adjustment" in an environment where the absence of such costs turns the dynamic problem into a series of static ones, as in our approach.[8] Profit maximization yields optimal input demand of

$$X_{ft} = \left(\frac{\gamma(1-\varepsilon)(1-\tau_{ft}) D_{ft}A_{ft}^{1-\varepsilon}}{C_{ft}} \right)^{\frac{1}{1-\gamma(1-\varepsilon)}} \tag{9}$$

Bear in mind that unit cost shocks also contain aggregate and idiosyncratic components: $C_{ft} = c_{ft}C_t$. In addition, measured $TFPQ$, demand shocks and cost shocks may deviate from a_{ft}, d_{ft} and c_{ft} due to measurement error, shocks realized by the firm after choosing X_{ft}, and other sources

[7] As in Restuccia and Rogerson, 2009 and Hsieh and Klenow, 2009. Further below, we also considered factor-specific distortions that, for given choice of X_{it}, affect the relative choice of a given input with respect to others.

[8] See, for instance, Caballero, Engel and Haltiwanger (1995, 1997), Eslava, Haltiwanger, Kugler, and Kugler (2010).

of noise. We denote noise in each of these three dimensions ($TFPQ$, demand and cost) by α_{ft}, δ_{ft} and ζ_{ft} respectively, and include these noise terms in the derivations below to later help in the interpretation of empirical results.

2.4 Life cycle growth

It follows from equation (9) that input growth over the life cycle of the firm, $\frac{X_{ft}}{X_{f0}}$ where 0 is the year of start of operations for plant f, can be attributed to growth in the different fundamentals:

$$\frac{X_{ft}}{X_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{c_{ft}}{c_{f0}}\right)^{-\kappa_1} \kappa_t \kappa_{ft} \tag{10}$$

where $\frac{d_{ft}}{d_{f0}}$, $\frac{a_{ft}}{a_{f0}}$ and $\frac{c_{ft}}{c_{f0}}$ are, respectively, life cycle growth in idiosyncratic demand shocks, $TFPQ$ and input price shocks. Here, $\kappa_t = \left(\frac{D_t}{D_0}\right)^{\kappa_1} \left(\frac{A_t}{A_0}\right)^{\kappa_2} \left(\frac{C_t}{C_0}\right)^{-\kappa_1}$ captures growth between birth and age t in the aggregate components of fundamentals, and κ_{ft} captures distortions as well as residual variation from noise in fundamentals not observed by the firm at the time of choosing its scale in each period, $\kappa_{ft} = \frac{\delta_{ft}^{\kappa_1} \alpha_{ft}^{\kappa_2} \zeta_{ft}^{-\kappa_1} \left(1-\tau_{ft}\right)^{\kappa_1}}{\delta_{f0}^{\kappa_1} \alpha_{f0}^{\kappa_2} \zeta_{0t}^{-\kappa_1} (1-\tau_{0t})^{\kappa_1}}$. Notice that idiosyncratic distortions τ_{ft} decouple the choice of scale from fundamentals. The distortions that a firm faces may vary as it ages (that is, distortions may be considered age-specific), and thus also decouple life-cycle growth in output from the growth of fundamentals. This aspect is captured in our decomposition in the residual term κ_{ft}. Parameters κ_1, and κ_2, with $\kappa_1 = \frac{1}{1-\gamma(1-\varepsilon)}$, $\kappa_2 = (1-\varepsilon)\,\kappa_1$, are constant across firms that face the same demand elasticity and same factor elasticities in production.

Equation (10) decomposes growth in firm size into the contribution of firm level fundamentals, aggregate effects, and firm level unexpected shocks. An analogous decomposition applies in terms of output, directly derived from $Q_{ft} = A_{ft} X_{ft}^{\gamma}$.Further assuming that $X_{ft} = K_{ft}^{\frac{\beta}{\gamma}} L_{ft}^{\frac{\alpha}{\gamma}} M_{ft}^{\frac{\phi}{\gamma}}$, moreover, we can decompose $\frac{c_{ft}}{c_{f0}}$ into the growth of specific dimensions of input prices, among which two are observed in the data: the price of material inputs, pm_{ft}, and average wage per worker, w_{ft}.[9] There may also be distortions to the use of one input relative to other. Taking these aspects into account, the decomposition of life cycle output growth can be written (see Appendix B):

[9] w_{ft} is measured as payroll divided by the number of workers.

$$\frac{Q_{ft}}{Q_{f0}} = \left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1} \left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1} \chi_t \chi_{ft} \tag{11}$$

Equation (11) is our central object of interest. It decomposes life cycle growth in output into the contribution of different idiosyncratic fundamentals, as well as unobservables, including aggregate effects, distortions, and measurement error. In particular, we focus on four measured souces of fundamental idiosyncratic growth: growth in demand shocks $\frac{d_{ft}}{d_{f0}}$, $TFPQ$ $\frac{a_{ft}}{a_{f0}}$, material input prices $\frac{pm_{ft}}{pm_{f0}}$, and wages $\frac{w_{ft}}{w_{f0}}$. Moreover, $\chi_t = \kappa_t^{\gamma}\left(\frac{A_t}{A_0}\right)$ captures aggregate growth, and $\chi_{ft} = \left(\frac{\left(1-\tau_{ft}\right)^{\kappa_1}\left(1+\tau_{ft}^M\right)^{-\phi\kappa_1}\left(1+\tau_{ft}^L\right)^{-\beta\kappa_1}\delta_{ft}^{\kappa_1}\alpha_{ft}^{1+\kappa_2}\zeta_{ft}^{-\kappa_1}r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{\left(1-\tau_{f0}\right)^{\kappa_1}\left(1+\tau_{f0}^M\right)^{-\phi\kappa_1}\left(1+\tau_{f0}^L\right)^{-\beta\kappa_1}\delta_{f0}^{\kappa_1}\alpha_{f0}^{1+\kappa_2}\zeta_{ft}^{-\kappa_1}r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}}\right)^{\gamma}$ captures residual variation from a number of sources, such as noise in fundamentals not observed by the firm at the time of choosing its scale in each period; growth in unobserved user cost of capital; and changes over the life cycle in distortions faced by the firm, both common across inputs and specific to the use of particular inputs. Here, τ_{ft}^M and τ_{ft}^L are idiosyncratic distortions to the use of materials and labor relative to capital, such as factor-specific adjustment costs, and subsidies/taxes to the use of one input. Beyond growth of fundamentals, equation 11 makes clear that growth over the life cycle also responds to changes over time in the distortions faced by the firm. Age-dependent distortions are a clear example of such changes.[10]

Notice that dispersion in the growth of fundamentals relates to dispersion in the average product of inputs, as well as in $TFPR$, two concepts highlighted in Hsieh and Klenow's work. $TFPR$ has been defined by Foster et al (2008) as $TFPR_{ft} = P_{ft}A_{ft}$. As is apparent, in the absence of idiosyncratic distortons, dispersion in $TFPR$ is driven by that in $TFPQ$, as well as by dispersion in output prices for given $TFPQ$. In particular, assuming $\tau_{ft} = 0$, $TFPR_{ft} = \frac{C_{ft}}{\gamma(1-\varepsilon)}\left(\frac{\gamma(1-\varepsilon)D_{ft}A_{ft}^{1+\varepsilon\gamma}}{C_{ft}}\right)^{\frac{1-\gamma}{1-\gamma(1-\varepsilon)}}$. It is clear from the last expression that the particular case of constant returns to scales, $\gamma = 1$, is one where dispersion in $TFPR$ arises only if there is dispersion in input costs.

[10] Some young firms may, for instance, have more dificulty in accessing financing, or face greater adjustment costs than their older counterparts. Also, many startups enjoy benefits that older firms do not face. This is the case, as an example, of small young firms in Colombia who at times have been exempted from specific labor taxes.

For any $\gamma \neq 1$, meanwhile, $TFPR$ dispersion is also driven by both $TFPQ$ and firm appeal dispersion. This is the case even in absence of distortions to both the process of accumulation of $TFPQ$ and demand, and to the optimal allocation of resources. As noted by Haltiwanger (2016), by allowing for $\gamma \neq 1$, the above derivation explicitly adds idiosyncratic $TFPQ$, demand and cost shocks to potential sources of $TFPR$ dispersion already identified in Hsieh and Klenow's original framework.[11]

3 Data

3.1 Annual Manufacturing Survey

We use data from the Colombian Annual Manufacturing Survey (AMS) from 1982 to 2012. The survey, collected by the Colombian official statistical bureau DANE, covers all manufacturing establishments belonging to firms that own at least one plant with 10 or more employees, or those with production value exceeding a level close to US\$100,000. The unit of observation in the survey is the establishment. A manufacturing establishment (or plant) is a specific physical location where production occurs. Given the nature of the data, in the actual empirical application we refer to the plant rather than the firm. It is worth noting that over 90% of plants in the survey correspond to single-plant firms.

Each establishment is assigned a unique ID that allows us to follow it over time. Since a plant's ID does not depend on an ID for the firm that owns the plant, it is not modified with changes in ownership, and such changes are not mistakenly identified as births and deaths. Plant IDs in the survey were modified in 1992 and 1993. We use the official correspondence that maps one into the other to follow establishments over that period.[12]

[11] Furthermore, dispersion in the average product of inputs is not influenced by demand shock dispersion even under our general assumptions: $\frac{R_{ft}}{X_{ft}} = \frac{C_{it}}{\gamma(1-\varepsilon)}$. Notice that $TFPR$ and average product are equivalent only if $\gamma = 1$. Another case with $TFPR$ dispersion in the absence of distortions is one where demand is not CES. See Haltiwanger, Kulick and Syverson (2017). While we do not permit departures from CES demand, our approach does not suffer from some of the limitations discussed in this latter study since we don't back out $TFPQ$ from measures of $TFPR$ alone.

[12] Though there is supposedly a one-to-one mapping between the two correspondences, there seems to be some degree of mismatch, as suggested by higher measured exit rates in 1991 and 1992 compared to other years, as well as higher measured entry in 1993. DANE

Surveyed establishments are asked to report their level of production and sales, as well as their use of employment and other inputs, their purchases of fixed assets, and the value of their payroll. We construct a measure of plant-level wage per worker by dividing payoll into number of employees. Sector IDs are also reported, at the 3-digit level of the ISIC revision 2 classification.[13] Since 2004, respondents are also asked about their investments in innovation, with bi-annual frequency.

A unique feature of the AMS, crucial for our ability to decompose fundamental sources of growth, is that inputs and products are reported at a detailed level. Plants report separately each material input used and product produced, at a level of disaggregation corresponding to seven digits of the ISIC classification (close to six-digits in the Harmonized System). For each of these individual inputs and products, plants report separately quantities and values used or produced, so that plant-specific unit prices can be computed for both individual inputs and individual outputs. We thus directly observe idiosyncratic input costs for individual materials. Furthermore, by taking advantage of product-plant-specific prices, we can produce plant-level price indices for both inputs and outputs, and as a result generate measures of productivity based on physical output, estimate demand shocks, and consider the role of input prices in plant growth. Details on how we go about these estimations are provided in section 5.

Importantly for this study, the plant's initial year of operation is also recorded–again, unaffected by changes in ownership–. We use that information to calculate an establishment's age in each year of our sample. Though we can only follow establishments from the time of entry into the survey, we can determine their correct age, and follow a subsample from birth. We denominate that subsample, composed of the establishments we observe from birth, as the *restricted life cycle sample*. Based on the restricted life cycle sample, we generate measurement adjustment factors that we then use to

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does report having undertaken efforts to improve actual coverage (compliance) in 1992, which may explain higher entry in 1993, but not higher exit in 1991 and 1992. Even for actual continuers that are incorrectly classified as entries or exits, however, our age variable is correct (see further below). That is, we may fail to properly identify entry into the survey and exit for a fraction of plants over 1992-1993, but this does not lead to mistaken age assignments in our calculations.

[13]The ISIC classification in the survey changed from revision 2 to revision 3 over our period of observation. The three-digit level of disaggregation of revision 2 is the level at which a reliable correspondence between the two classifications exists.

estimate life-cycle growth even for plants that we do not observe from birth (more on this in section 3.3).

With respect to studies that rely on data from economic censuses, one clear limitation of our approach is that we only observe a fraction of establishments from birth (about 30% of establishments in the sample), and that fraction is selected: it corresponds to establishments born at or beyond a given size. Moreover, we only observe establishments that satisfy exclusion criteria based on size, though those criteria cover all SMEs and large establishments, leaving out only microestablishments. And, while being formal is not a criterion for inclusion in the Manufacturing Survey, it is likely that many, of not most, informal establishments are micro, so our results underrepresent informal plants. The crucial upside from these data, however, is that we observe prices and quantities and can follow each establishment longitudinally, and do it at higher frequencies (annual, rather that inter-census). We also observe a census of SMEs and large manufacturing establishments, which are likely to account for a large fraction of any sustained growth actually observed.

We attempt to deal with selection biases using a variety of approaches, from adjusting for expected accumulated life cycle growth at time of entry into the survey, to contrasting our findings for plants observed from birth to analogous figures for all of the other plants in the manufacturing survey. One reassuring piece of information is that a healthy fraction of the firms that own plants recorded in our data (close to 20%) cannot be found in the official business registry, which is suggestive of the ability of the Survey's framework to cover informal establishments that satisfy size inclusion criteria.[14]

Table 1 presents basic descriptive statistics for our sample. It is composed of over 170,000 observations, with numbers of plants per year fluctuating around 7,500. The average plant has just over 50 employees.

[14]See Eslava and Haltiwanger (2017). Figure based on 2002-2009 data. The official business registry, kept by the Federation of Chambers of Commerce, includes all firms that are or ever were registered as merchants (a firm is defined by a tax ID; a firm may own several plants, though as noted most plants in our data belong to single-plant firms). Such registration is mandatory, and necessary to become a government provider and for access to different government services. Business formality in Colombia is frequently measured on the basis of appearing in the business registry.

Table 1: Descriptive Statistics

	Mean	Std. Dev.
Log output	10.573	1.717
Log revenue	11.765	1.556
Employment	52.935	110.184
Log capital	10.134	1.836
Log material expenditure	10.865	1.839
Log material	9.937	1.912
Log input prices (SV)	-0.296	0.809
Log output prices (SV)	-0.062	0.863
Log TPFQ	2.545	1.046
Log Demand shock	7.102	1.653
N (baseline sample)	172,734	
N (life cycle sample)	43,747	

3.2 Plant-level prices

Our ability to separate $TFPQ$ from demand shocks–both defined as in section 2–depends on being able to appropriately capture plant level prices. The exact plant level price index that can be used as a quality-adjusted (or appeal-adjusted) deflator for plant output, $P_{ft} = \left(\sum_{\Omega_t^f} d_{fjt}^{\sigma_J} p_{fjt}^{1-\sigma_J}\right)^{\frac{1}{(1-\sigma_J)}}$, depends on unobservable σ_J and $\{d_{fjt}\}$. We follow here insights from a long and active literature on economically motivated price indices to construct appropriate price indices from observable information.[15] We describe in this section our approach to measure P_{ft}. The underlying derivations and a summary catalogue of definitions are included in Appendix A. In implementing this approach empirically, we assume for the remainder of the analysis that $\sigma_F = \sigma_J$ for plants and products in the same 2-digit sector.[16] For ease of exposition, we refer to this as σ. The nested CES structure is critical for our analysis since it permits us to track product entry and exit at the plant level. Moreover, it permits us to track plant appeal d_{ft} as well as the relative

[15] See Redding and Weinstein (2016), and references therein to Sato (1976), Vartia (1976), and Feenstra (2004), whose insights are key in our derivation.

[16] This assumption is not critical for what follows and we only impose this restriction for the IV based method we use for estimating the demand equation. We will relax this restriction in future drafts.

product appeal across products within the same plant, d_{fjt}. Tracking both is critical for quality adjusted price indices at the plant level.

Denoting by $\Omega^f_{t,t-1}$ the set of goods produced by plant f in both period t and $t-1$, $P^*_{ft} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}\right)^{\frac{1}{(1-\sigma)}}$, and $P^*_{ft-1,\Omega^f_{t,t-1}} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}\right)^{\frac{1}{1-\sigma}}$, we take advantage of Feenstra's (2004) insight, that

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}}\right)^{\frac{1}{\sigma-1}} \frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}} \tag{12}$$

where $s_{fjt} = \frac{p_{fjt} q_{fjt}}{\sum_{\Omega^f_t} p_{fjt} q_{fjt}}$. Building recursively from a base year B this implies:

$$\begin{aligned} P_{ft} &= P_{fB} * \prod_{l=B+1}^{t} \left(\frac{P^*_{fl}}{P^*_{fl-1,\Omega^f_{l,l-1}}}\right) * \left(\prod_{l=B+1}^{t} \frac{\sum_{\Omega^f_{l,l-1}} s_{fjl}}{\sum_{\Omega^f_{l,l-1}} s_{fjl-1}}\right)^{\frac{1}{\sigma-1}} \\ &= \widetilde{P^*_{ft}} * \left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma-1}} \end{aligned} \tag{13}$$

where P_{fB} is the plant-specific price index at the plant's base year B, and we have defined a *consecutive-common-basket price index:* $\widetilde{P^*_{ft}} \equiv P_{fB} * \prod_{l=B+1}^{t} \left(\frac{P^*_{fl}}{P^*_{fl-1,\Omega^f_{l,l-1}}}\right)$ and an adjustment factor (the *Feenstra-adjustment*) $\Lambda^Q_{ft} = \prod_{l=B+1}^{t} \left(\frac{\sum_{\Omega^f_{l,l-1}} s_{fjl}}{\sum_{\Omega^f_{l,l-1}} s_{fjl-1}}\right)^{\frac{1}{\sigma-1}}$.

Defining $s^*_{fjt} = \frac{p_{fjt} q_{fjt}}{\sum_{\Omega^f_{t,t-1}} p_{fjt} q_{fjt}}$, $s^*_{fjt-1,\Omega^f_{t,t-1}} = \frac{p_{fjt-1} q_{fjt-1}}{\sum_{\Omega^f_{t,t-1}} p_{fjt-1} q_{fjt-1}}$, and $\omega_{ft,t-1} = \frac{\frac{(s^*_{fjt} - s^*_{fjt-1,t})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,t}}}{\sum_{\Omega^f_{t,t-1}} \left(\frac{(s^*_{fjt} - s^*_{fjt-1})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,t}}\right)}$ (the "Sato-Vartia" weights, for Sato (1976) and Vartia (1976)), we furthermore rely on the assumption that $\prod_{\Omega^f_{t,t-1}} d^{\omega_{fjt,t-1}}_{fjt} =$

$\prod_{\Omega^f_{t,t-1}} d_{fjt-1}^{\omega_{fjt,t-1}}$ to obtain (see Appendix A)[17]

$$\frac{P^*_{ft}}{P^*_{ft-1,t}} = \prod_{\Omega^f_{t,t-1}} \left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}} \tag{14}$$

After obtaining plant-level price changes for the $\Omega^f_{t,t-1}$ basket of goods in each pair of consecutive years using 14, the consecutive-common-basket price index in log-levels is constructed recursively as $\ln \widetilde{P^*_{ft}} = \widetilde{P^*_{ft-1}} + \ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) = \ln\left(P_{fB} * \prod_{l=B+1}^{t}\left(\frac{P^*_{fl}}{P^*_{fl-1,\Omega^f_{l,l-1}}}\right)\right)$.

The initial level P_{fB}, where B is the base year for plant f, is constructed

[17]The assumption that $\prod_{\Omega^f_{t,t-1}} d_{fjt}^{\omega_{fjt,t-1}} = \prod_{\Omega^f_{t,t-1}} d_{fjt-1}^{\omega_{fjt,t-1}}$, is weaker than the common assumption that individual product appeal at is constant over time, $\frac{d_{fjt}}{d_{fjt-1}} = 1$. We also note that Hottman et. al. (2016) make a related but different normalization in their nested CES structure. They assume $\prod_{\Omega^f_{t,t-1}} d_{fjt}^{1/N_{fjt-1,t}} = \prod_{\Omega^f_{t,t-1}} d_{fjt-1}^{1/N_{fjt,t-1}}$ where $N_{fjt-1,t}$ is the number of common products produced in both $t-1$ and t. The key difference is that Hottman et. al. (2016) normalization uses equal weights based on the number of products firm f produces in both $t-1$ and t. Our normalization may be problematic as Redding and Weinstein (2016) note that the Sato-Vartia weights may be correlated with the relative demand shocks. Our assumption simplifies the relative price change for goods the firm produces in both $t-1$ and t and permits us to construct our plant-level price index for multi-product firms based on observables. We think our assumption is a reasonable approximation given we are implementing this methodology for plant-level price indices while Redding and Weinstein (2016) is implementing their unified price index across all goods in the same product group. Moreover, as noted we make this normalization regarding the relative demand shocks within firms while permitting general d_{ft} to change over time. Thus, even if we assumed that for products that the firm is producing in both $t-1$ and t that the relative demand does not change, it would still be the case that product appeal varies across firms and in turn specific products produced by firms through the variation in d_{ft}. Still as we discuss further in Appendix A, this normalization potentially implies we are not capturing all quality adjustments within firms. Consistent with the discussion in Redding and Weinstein (2016), this implies there may be a potential upward bias in the Sato-Vartia price index that we use, which may induce a mistakenly low growth in quantities and therefore in $TFPQ$. This bias is likely to underestimate the contribution of $TFPQ$ to output growth relative to that of demand shocks.In future drafts of the paper we plan to explore alternatives that will permit us to relax this assumption.

as: $P_{fB} = \prod_{\Omega^f_B} \left(\frac{p_{fjB}}{\overline{p_{jB}}}\right)^{s_{fjB}}$, where $\overline{p_{jB}}$ is the average price of product j in year B across plants, and year B is the first year in which plant f is present in the survey. Notice that this approach takes advantage of cross sectional variability across plants for any given product or input j. In the plant's base year B, $\left(\frac{P_{fjB}}{\overline{P_{jB}}}\right) = 1$ for the average producer of product j. For other plants, it will capture dispersion in price levels around that average.

From 13, to move from our calculated $\widetilde{P^*_{ft}}$ to the exact price index P_{ft}, we need to adjust for the factor $\left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma-1}} = \left(\prod_{l=B+1}^{t} \frac{\sum_{\Omega^f_{l,l-1}} s_{fjl}}{\sum_{\Omega^f_{l,l-1}} s_{fjl-1}}\right)^{\frac{1}{\sigma-1}}$. While this factor can be built from observables, the elasticity of substitution σ is yet to be estimated, and its estimation requires information on P_{ft} (see section 5). We thus work initially with $\widetilde{P^*_{ft}}$ and carry the adjustment factor $\left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma-1}}$ into the derivations of section 5, where its contribution to price variability is flexibly estimated. In particular, we use output prices at the level of the plant to obtain a measure of the plant's output, by deflating the plant's revenue

$$Q^*_{ft} = \frac{R_{ft}}{\widetilde{P^*_{ft}}} = Q_{ft} \left(\Lambda^Q_{ft}\right)^{\frac{1}{\sigma-1}} \tag{15}$$

In logs this implies that $\ln Q^*_{ft} = \ln Q_{ft} + \frac{1}{\sigma-1} \ln \Lambda^Q_{ft}$. That is, by tracking the dynamics of $\ln Q^*_{ft}$ but controlling for $\ln \Lambda^Q_{ft}$we can quantify the contribution of factors that (such as age, fundamentals and distortions) to $\ln Q_{ft}$.

We similarly obtain a measure of materials by deflating material expenditure by plant-level price indices for materials, pm_{ft}. The index $pm_{ft}$is constructed on the basis of information on individual prices and quantities of material inputs, using an analogous approach to that used to construct output prices. The underlying assumption is that M_{ft}, the index of materials quantities used, is a CES aggregate of individual inputs. pm_{ft} is also one of the fundamentals we consider in our decomposition. As is the case with output prices, before estimating the elasticity of substitution we can only build a consecutively-common-basket price index $\widetilde{pm^*_{ft}}$ for plant f, and carry an adjustment factor $\Lambda^M_{ft} = \prod_{l=B+1}^{t} \frac{\sum_{\Omega^{Mf}_{l,l-1}} s_{fml}}{\sum_{\Omega^{Mf}_{l,l-1}} s_{fml-1}}$ to adjust prices for the turnover of materials of different appeals, which is controlled for in estimating the contributions of different factors to plant growth.

In an alternative approach against which we compare our baseline quality-adjusted prices, we examine the robustness of our results to using "statistical" price indices based on either constant baskets of goods, or on divisia approaches. These are discussed in section 7.

3.3 Life cycle growth

We focus on life cycle growth defined as $\frac{Q_{ft}}{Q_{f0}}$, where 0 is the year of start of operations for plant f and t is any post-entry year in which we observe the plant. Suppose at year t plant f is of age a. To avoid confusion between calendar time and plant age when using cross-plant information, in this section we switch to notation $\frac{Q_{fa}}{Q_{f0}}$ rather than $\frac{Q_{ft}}{Q_{f0}}$, keeping in mind that t is the year in which plant f is aged a.

We do not observe all plants from the actual time of their birth, though we do know what that actual year is. To address the problem of missing information when calculating $\frac{Q_{fa}}{Q_{f0}}$ for a plant that was born before entering the survey, we proceed in the following manner:

Suppose B is the age of plant f when we first observe it in the survey. For variable Z ($Z = Q, L, TFPQ, etc$), we estimate size at age a relative to birth as:

$$\frac{Z_{fa}}{Z_{fB}} = \left(\frac{Z_{f,a}}{Z_{f,B}}\right)\overline{\left(\frac{Z_B}{Z_0}\right)}_{life\_cycle} \tag{16}$$

where the last term is an adjustment factor based on what we observe for the sample of plants that we do observe from birth, which we have denominated the *restricted life cycle sample*. That is, $\overline{\left(\frac{Z_B}{Z_0}\right)}_{life\_cycle}$ is relative Z at age B compared to birth averaged over all plants that we observe from birth, controlling for year and sector (three-digit level) effects. We conduct robustness analysis restricting the sample to that of plants observed from birth (the *restricted life cycle sample*), for which we observe actual $\frac{Z_{fa}}{Z_{f0}}$. We restrict all of our analyses to plants born after 1969.

If we just defined $\frac{Z_{fa}}{Z_{f0}} = \left(\frac{Z_{f,a}}{Z_{f,B}}\right)$ as our estimate of post entry growth for age a for plant f we would bias our estimate of actual growth up to a. Since at age $a = B$ the ratio $\left(\frac{Z_{f,a}}{Z_{f,B}}\right)$ is one, the presence of plants that we observe for the first time at age B biases our estimate of average post entry growth for age B towards one (which is, most frequently, downwards), with the size of this bias growing with the number of plants that appear for the first time in our sample at age B. Going to the alternative extreme of simply using $\left(\frac{Z_a}{Z_0}\right)_{life\_cycle}$ as an estimate of the true post-entry growth at age a would also be problematic. On the one hand, it reduces our numbers of observations to about a third of all of the plants that we observe, affecting the precision of our estimates. On the other hand, because the *restricted life cycle sample* is a selected sample of the plants that are born sufficiently large to surpasss the inclusion threshold already at birth, these estimates are biased towards the post-entry growth of these selected plants, which may be faster or slower. By using equation (16) to estimate post-entry growth for plants that enter the sample after birth we expand our sample away from these selected plants.

4 Growth over the life cycle

We start by characterizing outcome growth over the life cycle of a manufacturing establishment (the left hand side of our growth decomposition). Our main outcome is output $Q_{ft} = \frac{R_{ft}}{P_{ft}}$. Because recent literature has focused on life cycle growth in terms of employment, we also describe employment growth for our sample for comparison with that literature.

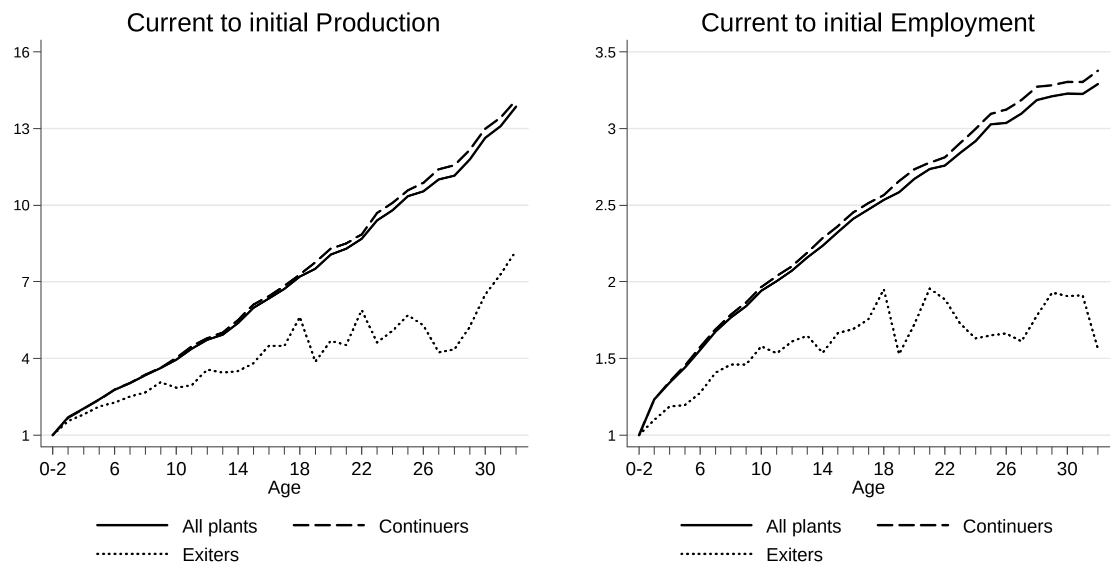
4.1 Average life cycle growth

To characterize output growth for the average establishment in our sample, we estimate a full set of ϕ_{age} coefficients in equation:

$$\frac{Q_{ft}}{Q_{f0}} = \alpha_t + \alpha_s + \sum_{age=3}^{age=30+} \phi_{age} d_{age,f,t} + \varepsilon_{ft} \tag{17}$$

where $\frac{Q_{ft}}{Q_{f0}}$ is the ratio between plant $f's$ output level in year t and the level at plant's birth; $d_{age,f,t}$ is a dummy variable that takes the value of 1 if plant f is of age age in year t; and ε_{ft} is an estimation error. We control for (three-digit) sector effects and aggregate time effects. We define age as the difference between the current year, t, and the year when the plant began its operations, and define the plant's output level at birth Q_{f0} as the average output it reported in ages 0 to 2. By averaging over the plant's first few

Figure 1: Life Cycle Growth



Includes year and sector fixed effects

years in operation we deal with measurement error coming, for instance, from partial-year reporting (e.g. if the plant was in operation for only part of its initial year).

Figure 1, left panel, presents the coefficients associated with different ages in the estimated equation (17). As in the rest of figures, we use a logarithmic scale. The average establishment in our sample grows by a factor of 2.4 in terms of production by age 5, almost four times from birth by age 10, and more than ten times by age 25.[18] For comparison with existing literature on life-cycle growht, the right panel presents analogous results for employment: $\frac{L_{ft}}{L_{0t}}$. By age 5 the average establishment has reached about 1.4 times its initial employment, by age 10 it has almost doubled, and 25 years after birth employment has grown three-fold.

By construction we focus on survivor growth: growth from birth to age a *of plants that have survived to age* a. Because we are able to follow life cycle growth directly at the plant level–by contrast to cross sectional comparisons

[18] More precisely, $\frac{Q_{fa}}{Q_{f0}} = 2.4$ when $a = 5$, $\frac{Q_{fa}}{Q_{f0}} = 3.95$ when $a = 10$, and $\frac{Q_{fa}}{Q_{f0}} = 10.35$ when $a = 25$.

of cohorts–the usual concern that selection drives average growth because size at the initial age is biased down by exits-to-be does not apply. In our case, size at the initial age is that of the same plant which has survived to an older age. It is the case, however, that plants that eventually exit may grow slower than others before they exit and, in that sense, even true life-cycle average growth is affected by selection: if the exiting plant had instead continued to the following age, average growth would be lower. Figure 1 already illustrates that this is indeed the case, since the life-cycle growth of plants that exit does depart significantly, downwards, from that of continuers. But, this growth of plants that exit only affects marginally the overall average. That is, the average patterns described in the previous paragraph are driven by continuous plants (plants of age t that continue on to age $t + 1$). Still, in section 7.3 we also explore how fundamentals relate to selection vs. continuer growth.

To provide perspective about where these average patterns fit in the international spectrum, Figure 2 compares the cross sectional patterns of employment growth with US cross sectional patterns, calculating the two in an analogous manner, and including only manufacturing plants of 10 or more employees. US data is from the publicly available information in the Bureau of the Census' Business Dynamics Database, which shows average size for given age categories. The period is limited to 2002-2012, which is the time span for which we can assign age tags in the US data. The cross sectional version of life cycle growth, used for this graph, is calculated by dividing the average employment level of plants of a given age by the average size of plants at birth.[19] Results indicate that the growth speed of the average US establishment basically doubles that in Colombia for comparable manufacturing plants. For instance, in the US employment in the 16-20 age category more than doubles that of the 0-5 category, while for Colombia the analogous figure is 1.5 times. This is consistent with results in Hsieh and Klenow (2014) indicating that less developed economies are characterized by less dynamic post-entry growth.[20]

Hsieh and Klenow (2009) and Buera and Fattal (2014) attribute such cross-country differences to poor institutions in developing economies, that fail to encourage investments in productivity, as well as healthy market selection. Identifying the actual role that specific institutions play is an interesting area of future research. Within-country changes in institutions, either across businesses or over time (or both) offer a fruitful ground for such exploration, to the extent that they keep constant other factors potentially influencing business dynamics, from the macroeconomic environment to business culture. We undertake that exploration for Colombia, taking advantage of changes in import tariffs, in a separate paper.

The average growth dynamics described above, however, hide considerable heterogeneity. Figure 3 shows different moments of the distribution of

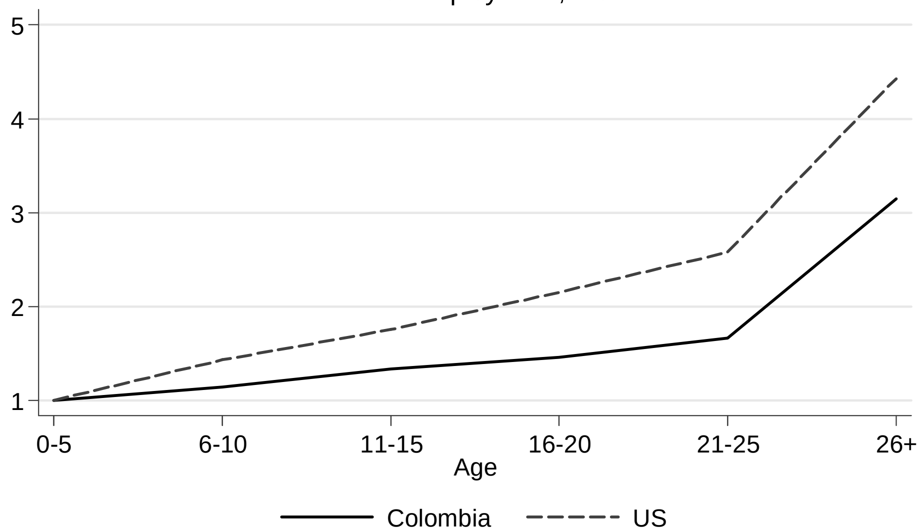
[19] The estimated growth dynamics are considerably dampened in the cross sectional approach compared to the longitudinal one (compare scales in Figures 1 and 2), which hints at the importance of being able to follow individual units longitudinally. Cross-sectional comparisons of cohorts, by contrast to our focus on $\frac{L_{ft}}{L_{f0}}$, implicitly give more weight to plants born larger, which results in the dampened cross-sectional dynamics for Colombia observed in Figure 2b compared to figure 2 (despite the exit of smaller businesses over time):

$$\frac{\overline{L_{age}}}{\overline{L_0}} = \frac{\sum_{i=1}^{N_{age}} L_{i,age}}{\sum_{i=1}^{N_0} L_{i,0}} = \frac{\sum_{i=1}^{N_{age}} \frac{L_{i,age}}{L_{i,0}} * L_{i,0}}{\sum_{i=1}^{N_0} L_{i,0}} = \sum_{i=1}^{N_{age}} \frac{L_{i,age}}{L_{i,0}} \frac{L_{i,0}}{\sum_{i=1}^{N_0} L_{i,0}}$$

In addition, Figure 2 is apt to be much more impacted by selection than Figure 1.

[20] Though similar to Hsieh and Klenow's, our numbers for the US are not identical to theirs, even if we focused on the same year, because of several differences in the calculation. We use data from the Business Dynamics Statistics, which directly records the age of an establishment. It also records employment for establishments of all sizes. Meanwhile, Hsieh and Klenow impute age based on previous appearance in Census, and rely on the Census' approach of imputing employment for small businesses.

Figure 2: Employment over the life cycle of manufacturing plants
Colombian vs. the US, 2002-2012
Current to initial employment, cross-section

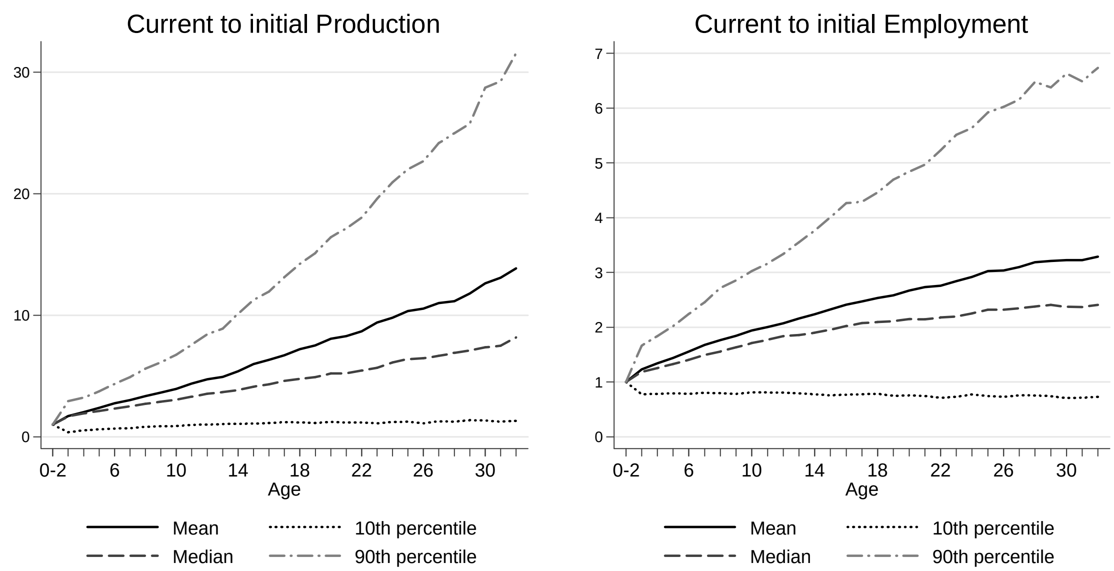


life-cycle growth. For each age the figure depicts the respective moment of the distribution of $\frac{Q_{it}}{Q_{i0}}$ and $\frac{L_{it}}{L_{i0}}$. Median growth falls under mean growth, highlighting the fact that it is a minority of fast-growing plants that drive mean growth; that is, the distribution of plant growth is highly skewed. By age 5, while the average plant has multiplied its output at birth by a 2.4 factor, the plant in the 90th percentile has multiplied it by 3.75, the median plant by 2.1, and the plant in the 10th percentile has shrank to 60% of its original size. At age 10 the 90th percentile of life cycle similarly more more than doubles the median (6.78 rather than 3.1). Employment growth is also characterized by similarly wide dispersion.

Figure 4 further characterizes life cycle growth for other plant characteristics: revenue deflated by an industry level deflator, the capital stock, purchases of material inputs, the share of non production workers and product scope.[21] The capital stock and material inputs grow much faster than output and, especially, than employment (notice the different scales). This

[21] The capital stock is obtained using perpetual inventory methods, initializing at book value of the year the plant enters the survey.

Figure 3: Distribution of Life Cycle Growth



Includes year and sector fixed effects

partly explains why output grows faster than employment: use of other factors is outgrowing that of labor inputs. At age 25 the real capital stock has multiplied by a factor of about 45 with respect to its level at birth for the average plant, and the use of material inputs is almost 20 times that of birth time. As noted, the corresponding figures for output and employment are close to 10 and 3.

Plants also seem to become more sophisticated as they age: both the share of non-production workers and the number of products grow over the life cycle. Ten years after starting operating, the average plant increases the number of 8-digit product categories in which it produces by about 30%. Keep in mind, however, that the level of disaggregation of products in the Manufacturing Survey is insufficient to capture product introduction as captured, for instance, in bar code data (e.g. Hottman et al. 2016). Skill composition also increases, at a slightly larger pace (1.6 times the birth level by age 10).

As with output and employment growth, there is wide dispersion and marked skewness in the patterns described by Figure 4. Mean growth overtakes median growth for all of the plant characteristics presented. At age 25, the 90th percentile of growth doubles the mean for all of the outcomes explored. Some plants also become less sophisticated as they age, as seen in a 10th percentile of life cycle growth below one in both the share of non production workers and product scope, even 25 years after birth.

5 Estimation strategy

5.1 Decomposing firm level growth

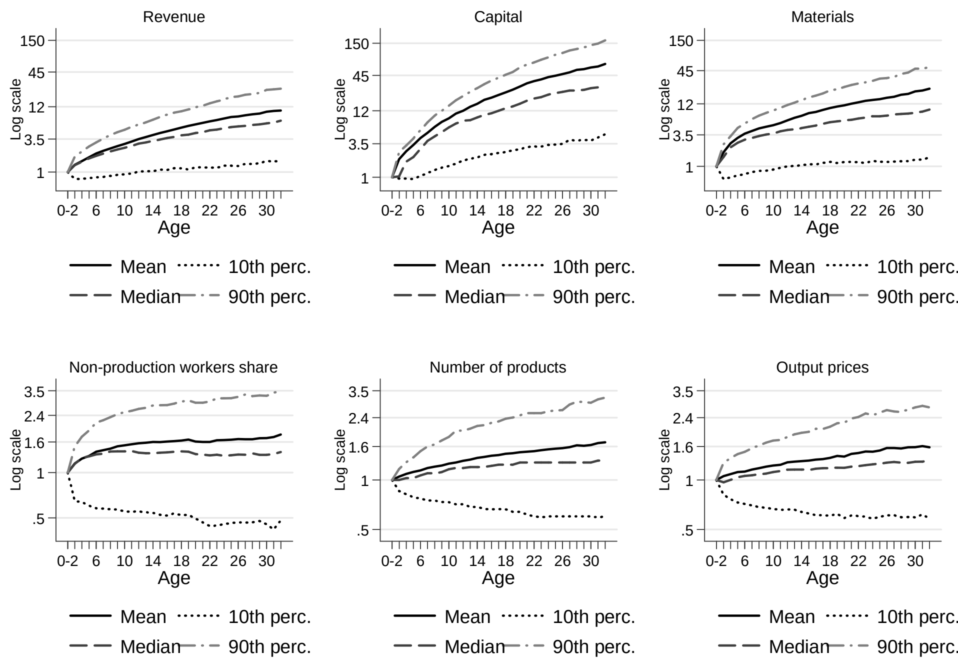
This section explains our approach to estimating the fundamental dimensions into which we then decompose output and input growth. A key feature of our analysis is the availability prices and quantities sold at the product-firm level, constructed as explained in section 3.2. Taking advantage of this key feature, we proceed sequentially in the following way:

5.2 Physical productivity

With physical quantities of output (materials) constructed by deflating nominal output (material expenditures) by the plant-level output price index, and

Figure 4: Distribution of growth: different plant characteristics

Current to inital



direct reports of the number of employees and the stock of physical capital, we estimate plant total $TFPQ$ as the (log) residual of production function (1) $Q_{ft} = A_{ft}X^{\gamma}_{ft}$,where $X_{ft} = K^{\frac{\alpha}{\gamma}}_{ft}L^{\frac{\beta}{\gamma}}_{ft}M^{\frac{\phi}{\gamma}}_{ft}$. A usual concern in the literature is that researchers observe values, rather than quantities, of output and inputs, so that the residual from estimating the (revenue) production function cannot be interpreted as A_{ft} (or $TFPQ$). The fact that we use theory-based plant-level prices to deflate output and materials deals with this concern. We estimate the log production function:

$$\ln Q_{ft} = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} + \ln A_{ft} \tag{18}$$

$Q_{ft} = \left(\frac{R_{ft}}{P_{ft}}\right)$, but we have an estimate of $\widetilde{P}^{*}_{ft}$ rather than P_{ft}. From equation (13) $\ln P_{ft} = \ln \widetilde{P^{*}_{ft}} + \frac{1}{\sigma-1}\ln \Lambda^{Q}_{ft}$, where B is the year in which plant f is initially observed in the survey. We rely on these facts and estimate the coefficients of 18 by estimating:

$$\ln\left(\frac{R_{ft}}{P^{*}_{ft}}\right) = \alpha \ln K_{ft} + \beta \ln L_{ft} + \phi \ln M_{ft} + \rho \ln \Lambda^{Q}_{ft} + \varphi \ln \Lambda^{M}_{ft} + \ln A_{ft} \tag{19}$$

where Λ^{M}_{ft} is the adjustment factor for the prices of materials analogous to Λ^{Q}_{ft}, and $M_{ft} = \frac{\text{materials expenditure}}{\widetilde{pm}^{*}_{ft}}$. See section 3.2 for details.

A remaining concern in estimating the production function is simultaneity bias: the fact that X is chosen as a function of the residual A_{ft}. We estimate the production function for each two-digit sector, using proxy methods. In particular, we follow the approach proposed by Ackerberg, Caves and Frazer (2015, ACF henceforth). Our control function includes lagged materials, current employment, current capital, plant input prices, Λ^{Q}_{ft-1} and Λ^{M}_{ft-1}.[22] Our inclusion of plant input prices follows De Loecker et al (2015), though in our case we take advantage of both output and input plant level prices as deflators.[23]

[22] Declaring labor, besides materials, as a free input, yields somewhat unplausible results for some sectors. Under that assumption, returns to scale are frequently (i.e. for several sectors and some periods of estimation) estimated to be increasing, and the coefficient for labor shoots up. Such implausible results support our prior that treating labor as a free input is not appropriate in the context in which we carry our estimation.

[23] De Loecker et al (2015), deflate output but not inputs using plant-level deflators. This

Table 2: Estimated factor and demand elasticties

Sector	β (L)	α (K)	ϕ (M)	Returns to scale	σ
Overall	0.372	0.173	0.508	1.053	1.403
31	0.296	0.139	0.614	1.049	2.066
32	0.265	0.120	0.609	0.994	1.650
33	0.360	0.154	0.471	0.985	1.321
34	0.629	0.273	0.198	1.101	1.318
35	0.437	0.211	0.464	1.112	1.332
36	0.449	0.187	0.438	1.074	1.613
37	0.437	0.223	0.461	1.121	1.348
38	0.368	0.109	0.585	1.061	1.166
39	0.330	0.180	0.503	1.013	1.582

This table reports estimates of the factor elasticities in the production function, and the demand elasticity in the demand function. The production function is estimated following ACF methods, with lagged materials, current employment, current capital, and plant input prices in the control function. The demand function is estimated using IV methods and TFPQ as an instrument for Q. Sectors are classified at the two digit level of the ISIC classification, revision 2.

We obtain $\widehat{\ln A_{ft}}$ as a residual from this estimation. We then use $\widehat{A_{ft}} = \exp\left(\widehat{\ln A_{ft}}\right)$ as our estimate of total $TFPQ$. Table 2 presents the results of our estimation of the production function, carried at the two-digit level of ISIC revision 2.

5.3 Demand shocks

Our (log) demand shock, $\ln D_{ft}$, is the residual from the (log) demand function (8)

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induces biases that they address by including plant level prices in their control function. Though we do make use of plant-level materials prices, a bias may persist from the lack of access to plant-level capital deflators (not for labor, for which we use physical units). Moreover, the inclusion of additional information in the control function helps deal with the concern that proxy methods based on materials as single invertible proxy may fail to properly identify the coefficient for materials in a gross output production function (Ghandi et al, 2013). We therefore include our plant-level prices in the control function.

$$\ln P_{ft} = \alpha - \varepsilon \ln Q_{ft} + \ln D_{ft} \tag{20}$$

$$\ln \widetilde{P}^*_{ft} + \frac{1}{\sigma - 1} \ln \Lambda^Q_{ft} = \alpha - \frac{1}{\sigma}\left[\ln Q^*_{ft} - \frac{1}{\sigma - 1} \ln \Lambda^Q_{ft}\right] + \ln D_{ft} \tag{21}$$

$$\ln \widetilde{P}^*_{ft} = \alpha - \frac{1}{\sigma}\left(\ln Q^*_{ft} + \ln \Lambda^Q_{ft}\right) + \ln D_{ft} \tag{22}$$

We thus estimate

$$\ln \widetilde{P}^*_{ft} = \alpha - \varepsilon\left(\ln Q^*_{ft} + \ln \Lambda^Q_{ft}\right) + \ln D_{ft} \tag{23}$$

Estimating (20) by OLS would yield a biased estimate of $\varepsilon = \frac{1}{\sigma}$, to the extent that Q^*_{ft}, and consequently $Q^*_{ft} + \Lambda^Q_{ft}$, may be correlated with the residual price for reasons beyond demand shocks. We thus estimate this demand function using IV methods. In particular, we use the log physical productivity shock $\ln A_{ft}$ as an instrument for the plant's output, as in Foster et al (2008, 2016) and Eslava et al (2013). In the current context, however, we explicitly recognize the multi-product character of plants by using a theory-based price index that quality-adjusts (or appeal-adjusts) our measure of output. To the extent that our approach does appropriately deal with this adjustment, $TFPQ$ obtained as a residual from this adjusted Q measure should be stripped of apparent changes in productivity related to appeal changes. This adjustment is thus crucial for the exclusion restriction to hold when using $TFPQ$ as an instrument to estimate (20).

By using $\ln A_{ft}$ as an instrument we focus on pure demand: the variability that is orthogonal to supply side shocks. The estimate that we recover for ε_{it} is an unbiased estimate of the ability of the firm to charge a different price when observing a shock to its sold quantity that is unrelated to the efficiency of its production process. Orthogonality between demand and supply shocks is also the main identifying assumption in the estimation of elasticities of substitution in research that uses data on product prices and sales (Broda and Weinstein, 2006; Redding and Weinstein 2016; and Hottman et al. 2016).

The last column of Table 2 reports obtained estimates of the elasticity of demand, $\frac{1}{\varepsilon} = \sigma$. We estimate elasticities between 1.2 and 2.1 for the different sectors. We obtain our demand shock as $\widehat{D_{ft}} = \exp\left(\widehat{\ln D_{ft}}\right)$, where the latter is the estimation residual from (20).

The lower part of Table 1 presents basic descriptive statistics for our estimates of $\ln A_{ft}$ and $\ln D_{ft}$ on a log basis. As found by Eslava et al. (2013) for an earlier period, $TFPQ$ is negatively correlated with output prices, which is intuitive to the extent that more efficient production allows charging lower prices. Both $TFPQ$ and D are positively correlated with plant size (captured in the table by output). $TFPQ$ is highly correlated with $TFPR$. By construction, our estimate for $\log D_{ft}$ captures only the part of the price effect that is uncorrelated with $TFPQ$, so the correlation between $\ln TFPQ$ and $\ln D$ is zero. Interestingly, Foster et. al. (2008,2016) find very similar correlations for prices, $TFPQ$, D, and $TFPR$ using US data for a selected number of commodity-like products.

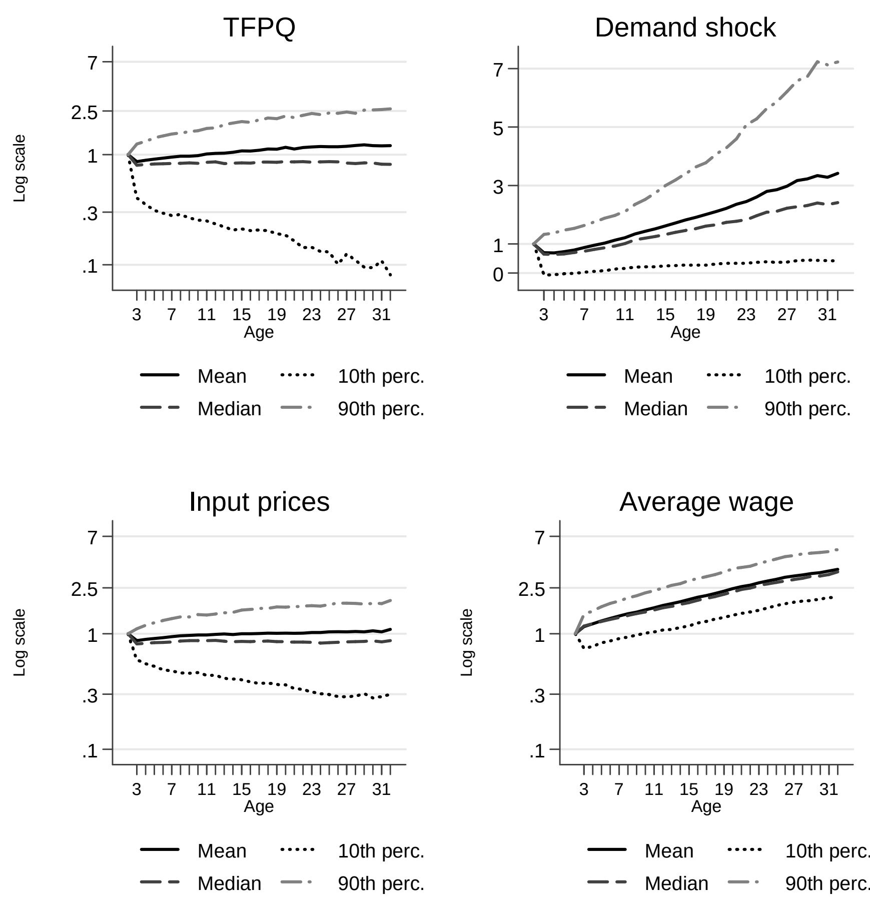
6 Results: Decomposing growth into fundamental sources

To start, the distributions of the evolution over the life cycle of these plant fundamentals, measured by $\widehat{A_{ft}}$, $\widehat{D_{ft}}$, are displayed in Figure 5, which also shows the life cycle growth of input prices. The average growth of demand shocks over the life cycle dominates that of $TFPQ$, as well as that of material input prices (notice the different scales). While, for the average plant, demand shocks grow more that three-fold over a thirty year period, compared to the level at birth, $TFPQ$ and unit prices of both input and output grow by less than 50% over the same horizon. The dominant role of demand in accounting for mean growth over the life cycle is consistent with the findings of Foster, Haltiwanger and Syverson (2016).

The interplay between output prices (Figure 4) and demand shocks is particularly interesting: with growing output over the life cycle, downward sloping demand would imply that the plant would have to charge ever shrinking prices over its life cycle, unless the appeal of f to constumers changed over time. We do not observe such fall in output prices, signaling increasing ability of the firm to sell more at given prices. By construction, this is what the life cycle growth of the demand shock, $\widehat{D_t}$, captures.

There is also considerable dispersion in the growth of both $TFPQ$ and demand shocks. The 90th-10th gap, however, is much wider for demand shocks. Moreover, even the 10th percentile grows modestly in the case of the demand shocks, while $TFPQ$ falls markedly for the 10th percentile.

Figure 5: Distribution of Fundamentals
Current to initial



Includes year and sector fixed effects

We now decompose the variance of $\frac{Q_{ft}}{Q_{f0}}$ into contributions associated with different fundamental sources, most notably $TFPQ$ and demand shocks (equation 11). We run a two stage procedure, similar to that in Hottman et al. (2016):

1. We estimate

$$\ln \frac{Q_{ft}}{Q_{f0}} = \beta_D \ln \left(\frac{D_{ft}}{D_{f0}}\right) + \beta_A \ln \left(\frac{A_{fa}}{A_{f0}}\right) + \beta_M \ln \left(\frac{pm_{ft}}{pm_{f0}}\right) + \beta_w \ln \left(\frac{w_{ft}}{w_{f0}}\right) + \upsilon_{ft} \tag{24}$$

where we include sector and time effects, so that $\upsilon_{ft} = \upsilon_t + \upsilon_s + \upsilon'_{ft}$. and coefficients β_D, β_D, β_M and β_w capture the contribution of idiosyncratic growth of demand shocks, TFPQ, material input prices and wages. As made clear by 11 $\widehat{\upsilon'_{ft}}$ captures distortions, beyond noise.

2. We then estimate the following equations:

$$\begin{aligned}
\beta_D \ln \left(\frac{D_{ft}}{D_{f0}}\right) &= \rho_D \ln \frac{Q_{ft}}{Q_{f0}} + \nu_{ft,D} \\
\beta_A \ln \left(\frac{A_{ft}}{A_{f0}}\right) &= \rho_A \ln \frac{Q_{ft}}{Q_{f0}} + \nu_{ft,A} \\
\beta_M \ln \left(\frac{pm_{ft}}{pm_{f0}}\right) &= \rho_M \ln \frac{Q_{ft}}{Q_{f0}} + \nu_{ft,M} \\
\beta_w \ln \left(\frac{w_{ft}}{w_{f0}}\right) &= \rho_w \ln \frac{Q_{ft}}{Q_{f0}} + \nu_{ft,w} \\
\widehat{\upsilon'_{ft}} &= \rho_\upsilon \ln \frac{Q_{ft}}{Q_{f0}} + \nu_{ft,\upsilon}
\end{aligned} \tag{25}$$

by the properties of OLS, $\rho_D + \rho_A + \rho_M + \rho_w + \rho_\upsilon = 1$. As with the estimation of demand and $TFPQ$ we deal with the fact that we are using P^*_{ft} rather than P_{ft} by including Λ^Q_{ft} and Λ^M_{ft} as additional factors in each stage of the decomposition.[24]

[24] In particular, we add $\ln \Lambda^Q_{ft}$ and $\ln \Lambda^M_{ft}$ as additional variables in the RHS of 24, and add two equations to the equation system 25, one where the dependent varible is $\beta_{\Lambda^Q} \ln \Lambda^Q_{ft}$ and another where it is $\beta_{\Lambda^Q} \ln \Lambda^M_{ft}$.

The first bar in Figure 6 depicts the result of this decomposition, pooling across ages, and reporting $(\rho_M + \rho_w)$ together to simplify the figure. We find that dispersion in life cycle growth is mostly explained by dispersion in fundamentals, rather than distortions and other unobserved factors. Measured fundamentals explain around 70% of the variability of output relative to birth level pooling across ages.[25] The remaining 30% is explained by the error term χ_{ft} which, from equation (11), captures distortions and other unobserved factors, such as measurement error.[26]

Concentrating on the fraction explained by measured factors, input price growth explains 7 p.p, with the remaining variability accounted for by demand shocks and $TFPQ$, with each of them accounting for approximately 30 p.p. That is, demand shocks and $TFPQ$ are both equally important drivers of life cycle growth in pooled horizons.

Both the contribution of fundamentals relative to distortions, and the relative contribution of different fundamentals within the former, vary markedly over different time horizons from birth (Figure 7, with shades representing confidence intervals).[27] Distortions (dotted line) are particularly important to explain the variance in early age growth, explaining more than 50% of the

[25] Contribution of fundamentals calculated as $\frac{0.697}{0.964} = 0.72$, since the turnover adjustment factors are simply included to control for the fact that the deflators we are using at this point include only consecutive-common-baskets of outputs/inputs. Figures 6 and 7 as well as subsequent related figures with decompositions of the variance of growth are in fact depicting the variance of the growth of Q^*_{ft} controlling for product turnover. This yields the relative contribution of fundamentals and distortions to the variance of growth of Q_{ft} after adjusting for the contribution of product turnover.

[26] In particular, $\chi_{ft} = \left(\frac{(1-\tau_{ft})^{\kappa_1} \left(1+\tau^M_{ft}\right)^{-\phi\kappa_1} \left(1+\tau^L_{ft}\right)^{-\beta\kappa_1} \delta_{ft}^{\kappa_1} \alpha_{ft}^{1+\kappa_2} \zeta_{ft}^{-\kappa_1} r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{(1-\tau_{f0})^{\kappa_1} \left(1+\tau^M_{f0}\right)^{-\phi\kappa_1} \left(1+\tau^L_{f0}\right)^{-\beta\kappa_1} \delta_{f0}^{\kappa_1} \alpha_{f0}^{1+\kappa_2} \zeta_{ft}^{-\kappa_1} r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}} \right)^{\gamma}$. The aggregate growth term $\chi_t == \kappa_t^{\gamma} \left(\frac{A_t}{A_0}\right)$ is absorbed by (calendar) time effects.

[27] To conduct the decomposition by ages, we expand equations 24 and 25 to include interactions with the different age groups. Suppose there are two mutualy excusive gruops: B and C, then we redefine the equation 24 as:

$$\ln \frac{Q_{fa}}{Q_{f0}} = \beta_0 + \beta_C X_C d_{fC} + \beta_B X_{fB} d_{fB} + \varepsilon_i \tag{26}$$

where X_{fA} is the vector of log growth of fundamentals, and $d_{fA} = 1$ if f belongs to group A (say, an age), and everything else as defined previously.

The new decomposition equation will be given by:

$$\beta_{A1} X_f d_{fA} + \beta_{B1} X_f d_{fB} = \gamma_{A1} Y_f d_{fA} + \gamma_{B1} Y_f d_{fB} + \nu_{1fA} \tag{27}$$

Figure 6: Production growth variance decomposition

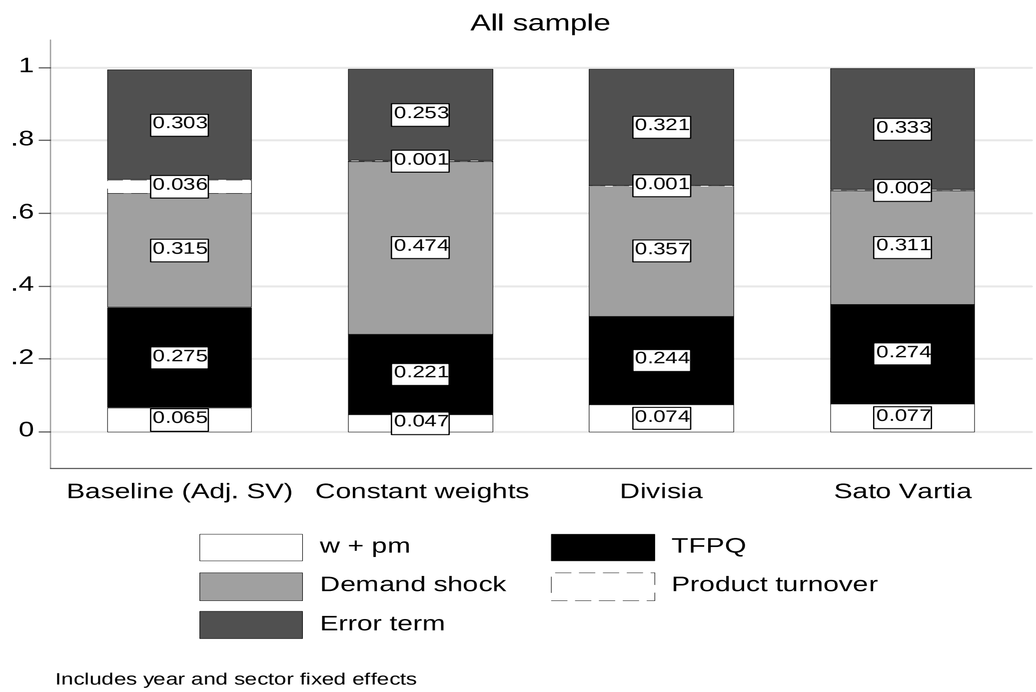
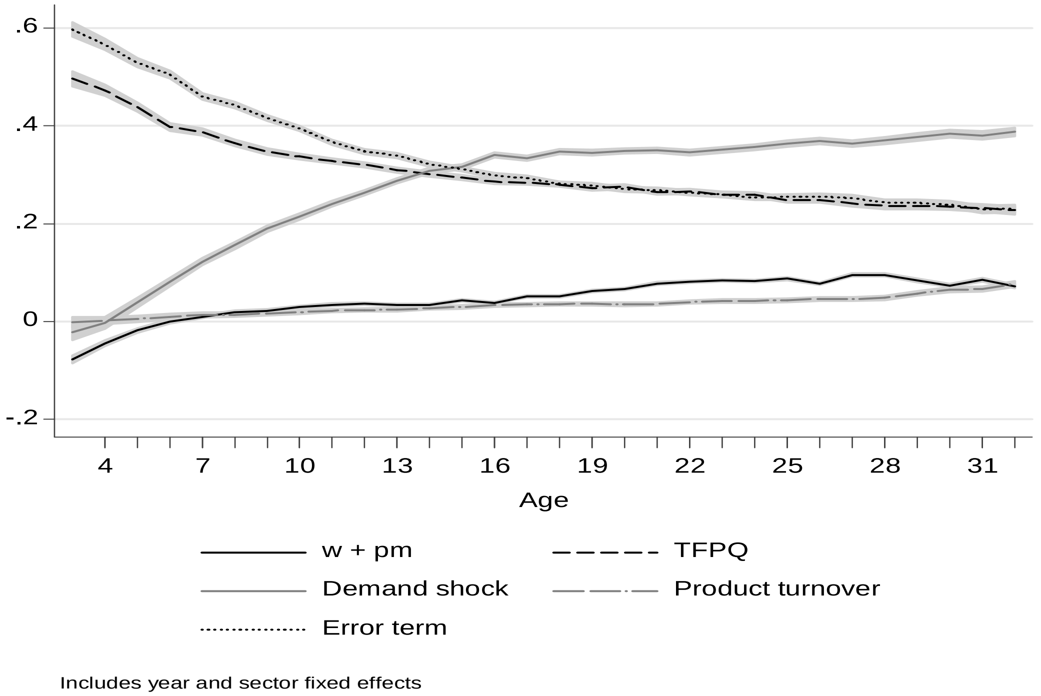


Figure 7: Production growth variance decomposition by age



variance of growth up to around age seven. But, they lose importance for longer horizons, with their contribution falling to less than 25% by around age 20. For the fraction explained by fundamentals, variance in early life growth is fully explained by $TFPQ$, but the contribution of demand grows rapidly over the life cycle, reaching around 20% by age 10 and close to 40% by age 20. Since the fraction explained by $TFPQ$ and distortions follows the opposite pattern, falling over the life-cycle, the importance of demand surpases that of $TFPQ$ and unobserved factors by around age 15.

As is probably not surprising, the growth of input prices (both for materials and employment) plays a more minor role than any of the other factors in explaining the variance of life cycle growth. Still, their contribution is 7% for the sample of pooled ages, growing from negative to almost 10% for the

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$$\varepsilon_f = \gamma_{A\varepsilon} Y_f d_{fA} + \gamma_{B\varepsilon} Y_f d_{fB} + \nu_{\varepsilon i} \tag{28}$$

Just as before $\hat{\gamma_{A1}} + \hat{\gamma_{A\varepsilon}} = \hat{\gamma_{B1}} + \hat{\gamma_{B\varepsilon}} = 1$

older ages. This is a dimension not previously explored as a driver of post entry growth. For instance, Hsieh and Klenow (2014) assume that all plants face the same input prices, an assumption under which all average revenue dispersion is atributable to distortions. Our finding that the role of life cycle input price growth is non negligible suggests that, at least in the context of life cycle growth, dispersion in average products is not solely driven by distortions. In our data, it is the growth of wages, as opposed to material input prices, that drives most of this contribution.

Figure 8 directly depicts the different terms in decomposition (11) for alternative moments of life cycle output growth. Actual output growth is also depicted. The upper left panel shows the components of the decomposition for plant with average $\frac{Q_{ft}}{Q_{f0}}$. The upper right panel, meanwhile shows each of this components for plants in the lowest decile of $\frac{Q_{ft}}{Q_{f0}}$. The two lower panels proceed similarly for the 45th to 55th percentile and the upper decile of $\frac{Q_{ft}}{Q_{f0}}$.

The black solid line represents actual outcome growth, $\frac{Q_{ft}}{Q_{f0}}$.The dotted grey line in each of the panels corresponds to $\left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa 2}$, while the dashed line adds the (generally negative) contribution of input prices by depicting $\left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_1}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}$. The solid grey line further adds the contribution of demand shocks: $\left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1}\left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_1}\left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1}\left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1}$. The difference between this solid grey line and the solid black line is the contribution of unmeasured factors, χ_{ft}, which we attribute mainly to distortions.

It is clear from Figure 8 that, as previously illustrated, both $TFPQ$ and demand shocks play crucial roles in explaining how high growth plants different from low-growth ones. But in terms of the first moment of growht, the upper left panel shows that it is especially demand that accounts for the rising average growth over the life cycle. From our analysis above, we know that it is the high growth plants that are driving the rising mean. The lower right panel of Figure 8 suggests that demand is particularly important to explain the increase in output for high growth plants.

Part of the reason for the importance in demand in explaining high growth in Figure 8 may have to do with these demand shocks being more persistent than technology. Table 3, for instance, illustrates that, though high, the degree of $TFPQ$ persistence fails to explain the extremely high persistence pervasively for both production and sales–similar to that reported for other

Figure 8: Contribution of fundamentals to production growth

Current to initial

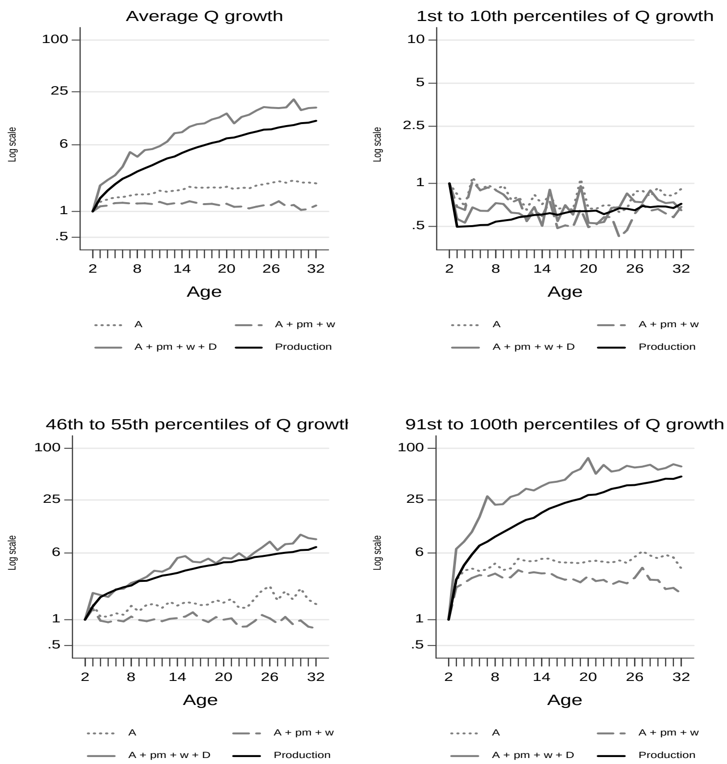


Table 3: Persistence of TFP and demand shocks vs. persistence in production and sales.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
VARIABLES	TFPQ		Demand Shock		lnQ		ln(Q*P)	
Period	**1980s**	**2000s**	**1980s**	**2000s**	**1980s**	**2000s**	**1980s**	**2000s**
Lagged TFPQ	0.857***	0.934***						
	(0.00280)	(0.00144)						
Lagged Demand Shock			0.965***	0.974***				
			(0.00122)	(0.000769)				
Lagged lnQ					0.985***	0.989***		
					(0.00101)	(0.000694)		
Lagged ln(P*Q)							0.990***	0.987***
							(0.000834)	(0.000615)
Observations	36,050	75,336	36,826	81,531	36,826	81,531	41,365	93,354
R-squared	0.817	0.887	0.981	0.983	0.972	0.967	0.979	0.971
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
p-value (H0: unit root)	0	0	0	0	0	0	0	0

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

countries. By contrast, the demand shock process does match that extreme degree of persistence. We further illustrate distinctive characteristics of demand shocks vs. TFPQ in Appendix C.

The central finding that idiosyncratic demand shocks play a key role in the average growth of output, at least as important as that of $TFPQ$ and especially for rapid growers, is in line with Foster et al's (2016) argument that consolidating a solid client basis is more central to post-entry business growth than physical efficiency gains, and their consistent results for selected US manufacturing industries. Our findings that demand shocks play a dominant role in the variance of growth over the life cycle also square well with the findings in Hottman et al (2016) pointing at demand shocks as a major determinant of sales variability in the US.

Figure 8 also sheds light on other factors such as distortions for mean growth as well as growth for other parts of the distribution. The role of other factors such as distortions is captured by the difference between the solid grey line capturing all fundamentals and the solid black line capturing actual growth. It is striking that this gap is the largest in the lower right panel for the high growth plants. This is consistent with distortions playing an especially important role in being a drag on high growth plants.

7 Robustness and extensions

7.1 Quality adjustment

All of the results discussed so far use quality-adjusted price indices calculated as explained in section 3.2. We examine now robustness of our results to two different alternative formulations of the price index. We first use a "statistical" approach based on Törnqvist indices for a constant basket of goods or, alternatively, on the divisia price index that allows that basket to change and uses average t, $t-1$ expenditure shares (see section 3.2). In each of these approaches, $TFPQ$ and demand shocks are obtained from estimating production and demand functions 18 and 20, rather than their versions adjusted for product turnover, 19 and 23, with the Q (M) calculated as revenue (materials) deflated by the respective price index in the Törnqvist or Divisia version. Product turnover is only considered as a separate factor in the growth decompostion. For comparison, we also run a version of the decomposition where $TFPQ$ and demand shocks are obtained without product adjustments but output is deflated using P^*_{ft} (the Sato-Vartia prices).

Our first alternative version of prices uses a basket of goods that is fixed over the life cycle, and constant weights for them. In particular, Törnqvist indices for the growth of prices of plant f at time t are constructed, as $\frac{P_{ft}}{P_{ft-1}} = \prod_{\Omega^f} \left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\overline{s}_{fj}}$, where Ω^f is a basket of all products produced (or materials used) by plant f at any point in which we observe f, and $\overline{s}_{fj}$ is the average share of j in that basket of products (or materials) plant f produces over the whole period. In this approach, the plant level index is initialized at $\ln P_{fA} = \sum_{\Omega^f} s_{fj} \left(\ln p_{fjA} - \ln \overline{p_{jA}}\right)$. If product j is not produced (or used as input) in years t or $t-1$ (or both), $\Delta \ln(P_{fjt})$ is inputed at the average growth of the price of that product (or input) for other plants within the sector. If no plant in the sector produces that good in t, then the average over all plants is used, independent of sector. Notice that this version of prices does not quality-adjust prices in any way, and does not take into account product turnover. Compared to this version, all versions allowing for evolving baskets of goods have the advantage of capturing evolving expenditure shares over time and therefore quality-adjusting prices, but the disadvantage of being more biased by errors from product coding and coarse aggregation, which are more likely in our context than in that of prices from scan bar codes

(Hottman et al 2016). Compared to our baseline estimation, even versions with changing baskets quality-adjust in a less precise (i.e. not exact) manner.

The second bar of Figure 6 presents results using this alternative fixed-basket, pooling observations across ages. Ignoring quality (more precisely, appeal) adjustment results in two important differences in the results using a fixed basket of goods over the life cycle, compared to our baseline, for the pooled sample (Figure 6, bars "constant weights" vs baseline). First, the error term is assigned a contribution 5 p.p. smaller in the version with a constant basket of goods, possibly reflecting the fact, mentioned above, that this version is less prone to measurement error from product coding error and aggregation. Second, demand appears twice as important as $TFPQ$. That is, ignoring quality adjustments does lead to an overestimation of the role of physical efficiency in life cycle growth. Figure 9 shows that these three features are present for different time horizons (the upper left panel of Figure 9 simply reproduces Figure 7). The fixed basket deflator, however, preserves the patterns of increasing importance of demand and decreasing importance of $TFPQ$ and the error term.

The other versions of prices (divisia and Sato Vartia) progressively move towards our baseline. In the divisia version we consider the current basket at each period t, and use average $t, t-1$ shares of sales (materials expenditure) as weights for the respective products. Divisia prices do allow for changing baskets of goods over the life cycle, but the weights given to different products are not exact, in the sense of not capturing the exact shares derived from our demand theory. Sato-Vartia prices, in turn, ignore the product turnover adjustment (λ^{Q}_{ft} and λ^{M}_{ft}, which capture turnover over pairs of years) in the estimation of $TFPQ$ and demand shocks. Not surprisingly, then, the results of decompositions based on these alternative approaches stand between those in our baseline and those in the constant weights version of prices. The divisia price index, by failing to quality-adjust prices in an exact manner, attributes more importance to demand relative to $TFPQ$ than our baseline.

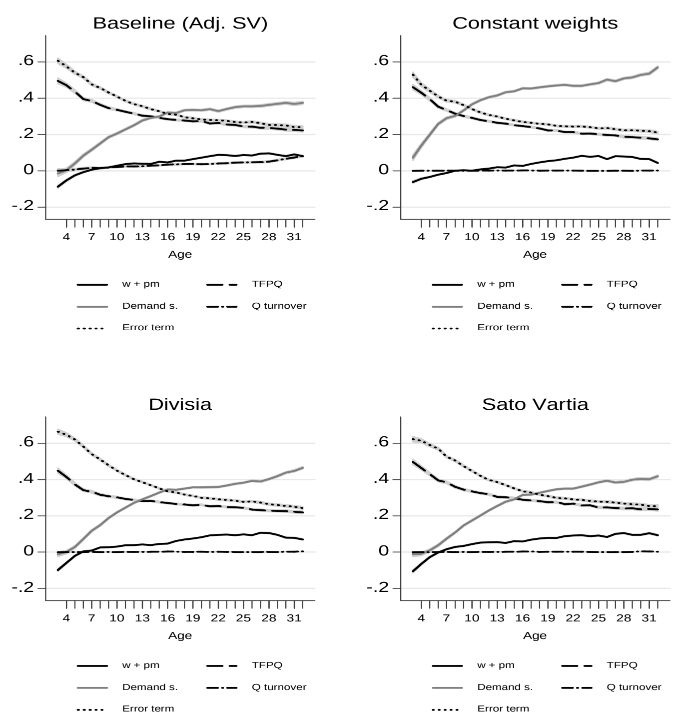
7.2 Life cycle growth: 1980s vs 2000s

Hsieh and Klenow (2014) have explained cross country differences in patterns of life cycle growth from differences in institutions that may enhance or weaken the link between market fundamentals and growth, in turn affecting market selection and incentives to invest in improving those "fundamentals".

Colombia, as many other countries in Latin America and around the

Figure 9: Production growth variance decomposition

Different price indices



Includes year and sector fixed effects

globe, undertook wide market-oriented reforms during the 1990s. These included unilateral trade openning, financial liberalization, and flexibilization of labor regulations. Eslava et al (2004, 2013, 2010) present evidence that these reforms did generate changes in business dynamics consistent with a reduction in the distortions to business incentives: allocative efficiency improved, the market selection mechanism was enhanced, capital and labor adjustment became more flexible (though in an apparently capital-biased way).

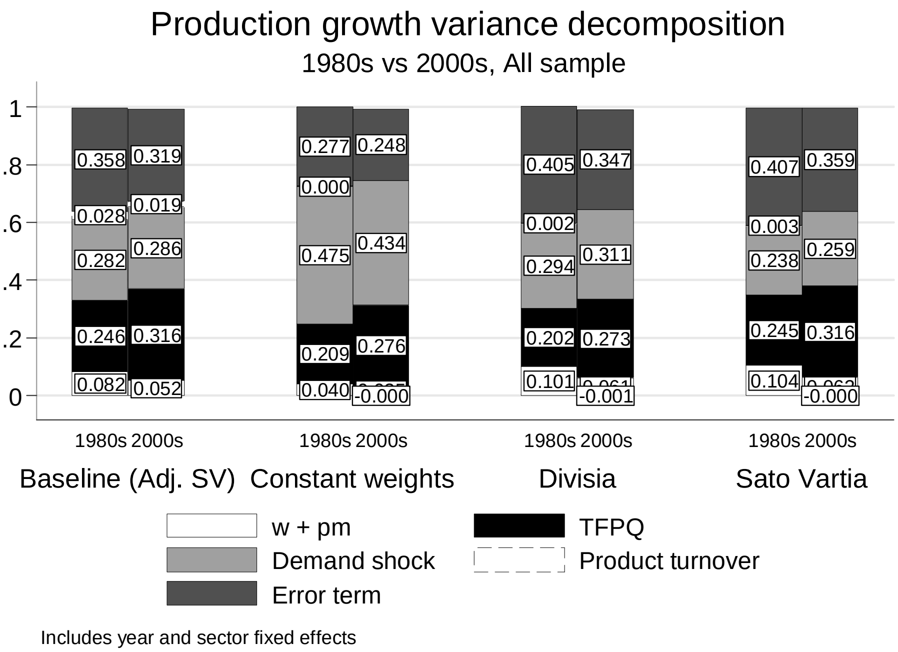
We now ask whether the contribution of our measured plant fundamentals to life cycle dynamics for Colombian manufacturing plants, and in turn whether life cycle growth seems affected by these differences. Figure 10 depicts results of our decomposition of life cycle growth separating the 1980s (actually 1982-1992) and the 2000s (2002 to 2012). The 1980s analysis continues to be constrained to plants born starting in 1970, and the 2000s analysis leaves out plants born before 1990.

We leave out the years between 1993 and 2001 for two reasons: 1) Two the extent that the comparison provides meaningful information about changes related to the reforms, it is not clear whether firms born just as the reforms are being adopted behave in their first few years as pre-reform or post-reform firms. 2) Between 1997 and 2001, the country went through its deepest recession in 70 years. The 2002-2012 period displays macroeconomic conditions that are not as widely different from those in the first decade of our sample as is the case for the years between 1993 and 2001. Still, of course, market reforms are not the only conditions that changed between the 1980s and the 2000s. We do not claim that the differences between the two decades in life cycle growth and in the contribution of fundamentals to life cycle growth are to be attributed to the reforms of the 1990s. But those reforms are clearly an important factor.

The contribution of $TFPQ$ to the life cycle output growth grows by almost 7 p.p. betwen the 2000s and the 1980s, for all horizons. This greater contribution of $TFPQ$ is offset by a composed decrease in the contribution of input prices and, interestingly, the error term, which falls by about 4 pp.

Moreover, the average plant's life cycle output growth is faster in the 2000s compared to the 1980s, markedly in the case of employment growth and timidly in the case of output (Figure 11).[28] Facts in Figure 11, and the slightly greater importance of fundamentals (by contrast to the error term)

[28]For the left panel of figure 11, we pool the two samples, and run the regression



as determinants of growth in Figure 10, square with the expectation that the 2000s in Colombia are characterized by less distortive institutions than the pre-reform decades, in light of Hsieh-Klenow's view that less distortive institutions enhance life-cycle growth.

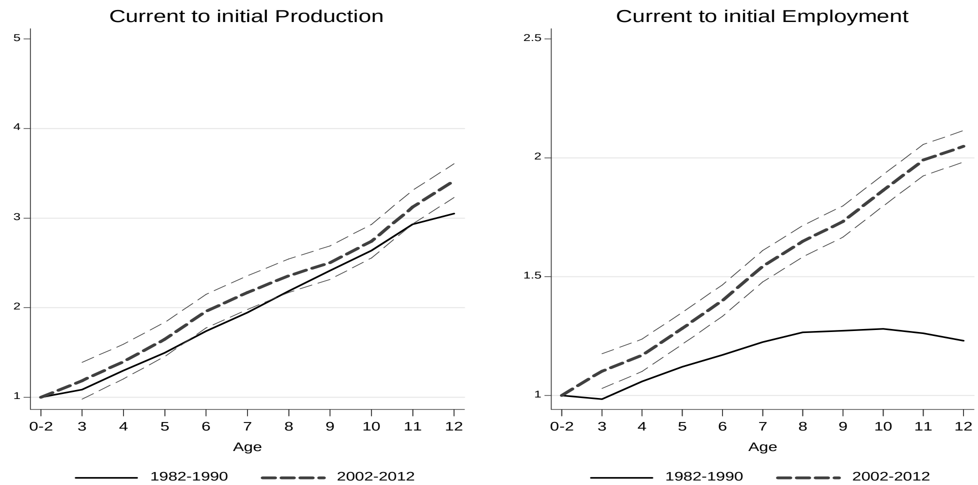
7.3 Selection

As noted in relation to Figure 1, the life cycle growth patterns that we attempt to explain are mainly driven by the growth of continuers. It is interesting to see whether fundamentals and distortions play similar roles in explaining continuer growth and exits.

$$\frac{Q_{ft}}{Q_{f0}} = \alpha_t + \alpha_s + \sum_{age=1}^{age=10} \phi_{age} d_{age,f,t} + \sum_{age=1}^{age=10} \phi_{age,post} d_{age,f,t} * dpost_t + \varepsilon_{it} \tag{29}$$

where $dpost_t = 1$ if $t \in [2002, 2012]$ and $dpost_t = 0$ otherwise. The right panel is produced in an analogous manner for $\frac{L_{ft}}{L_{f0}}$.

Figure 11: Life Cycle Growth: 1980s vs. 2000s



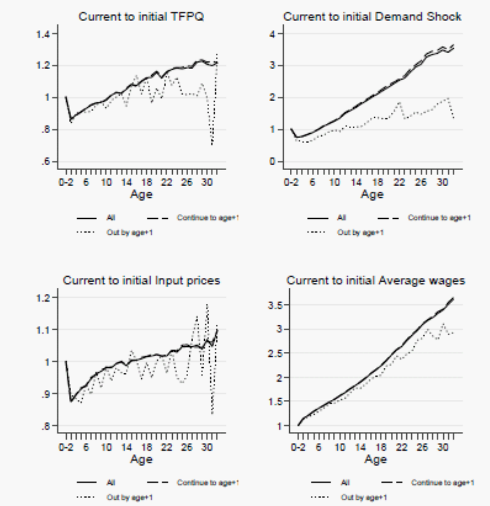
Includes year and sector fixed effects, 90% confidence interval for the difference 1980's vs. 2000's

Figure 13 illustrates average growth of fundamentals separately for plants that continue for at least one additional year and those that exit the following year. The most noteworthy difference is poorer growth in demand shocks for plants about to exit compared to those that will continue.

Figure 14 further carries our decomposition of drivers of output growth for these two groups of plants. Because exits occur at earlier ages and we know the contribution of fundamentals varies across ages, we compute the decomposition for each group at specific ages. We present three-year moving averages because the patterns for plants about to exit are noisy.[29] Figure 15 shows the average across years of the contribution of each of the fundamentals. Fundamentals still play an important role in explaining the growth of plants from birth to the moment in which they are about to exit, but $TFPQ$ plays a much more significant role, detracting from both demand and distortions.

[29]Since each point (age) in a figure for plants about to exit contains the plants that will exit at age+1, the plants included in a given line are different for each age. This explains the noisy patterns.

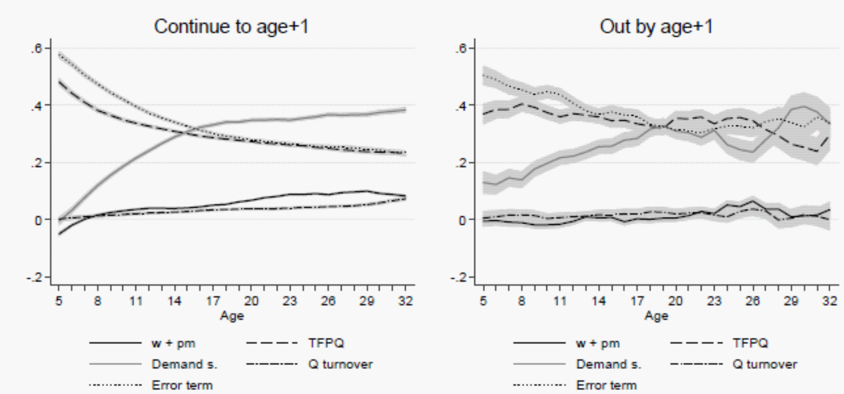
Figure 13: Life cycle growth of Fundamentals: Exiters vs. continuers



Includes year and sector fixed effects

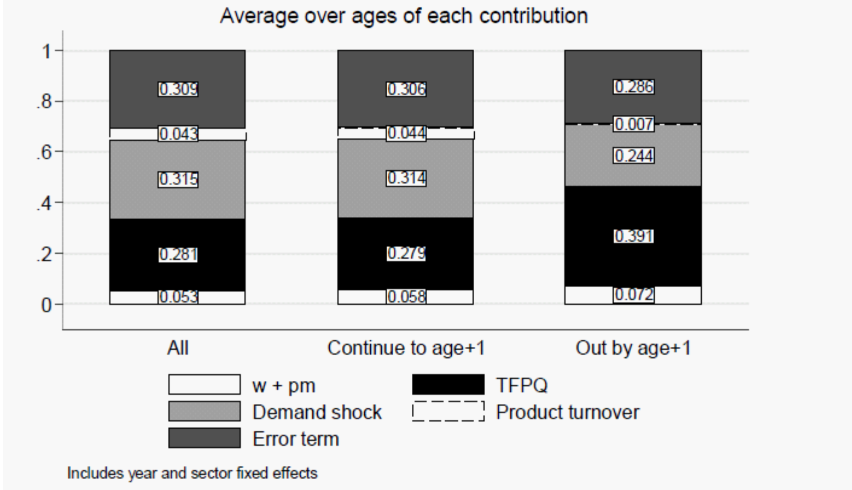
Figure 14: Production growth decomposition by age

Continuers vs. Exiters, 3 years moving average



Includes year and sector fixed effects

Figure 15: Production growth decomposition: continuers vs. exiters

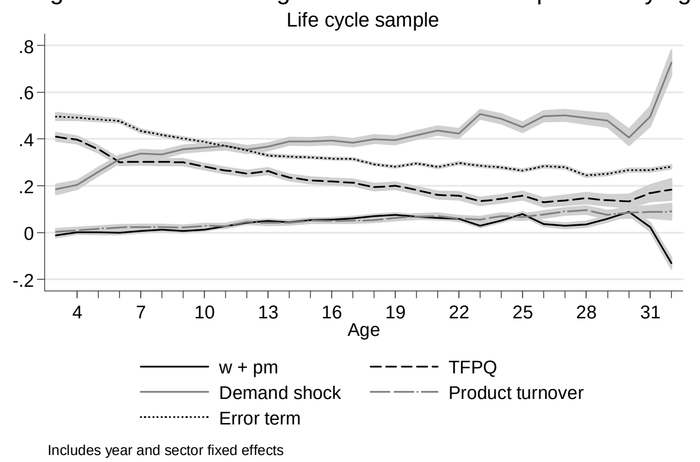


7.4 Restricted Life cycle sample

We have conducted most of our analysis characterizing and decomposing the determinants of life cycle growth where the latter reflects the growth of individual plants relative to their initial size. However, as discussed in detail in section 3.3, for some plants we don't observe them at birth but when they cross a minimal size threshold. We use what we refer to as the restricted life cycle sample to generate adjustment factors that enable our life cycle growth analysis. In this section, we confine our analysis to the restricted life cycle sample, as a robustness check. Figure 12 reproduces the decomposition by ages of Figure 7 for the restricted life cycle sample, composed of plants that we observe from birth. Those plants represent about a third of the overall sample (Table 1). The restricted life cycle sample has the advantage of allowing a precise measurement of growth from birth, both in terms of outcomes and in terms of fundamentals. This comes at the cost of biasing the sample towards firms that are born larger. We do not know ex-ante what the direction of that bias is, neither for the patterns of outcome growth nor for the contribution of fundamentals to that growth.

The basic patterns we have found are robust to focusing on the restricted life cycle sample: 1) the unexplained part of growth, which captures the role of distortions, is less important than the explained part averaging across horizons; 2) that unexplained part accounts for over 50% of variation for early ages, and becomes progressively less important for longer horizons; 3) the role

Figure 12: Production growth variance decomposition by age



of both $TFPQ$ and demand is crucial, explaining most life cycle growth; 3) $TFPQ$ is particularly important for early life growth, but quickly becomes less important than demand, whose contribution grows markedly for longer horizons. Two important quantitative differences emerge with respect to the full sample, however: 1) the measured contribution of distortions falls (from 33.6% to 29.6% in the pooled sample); and 2) the weight of demand shocks relative to $TFPQ$ grows. In particular, while $TFPQ$ and demand are equally important in the baseline decomposition pooling ages, the contribution of demand shocks is 1.5 times that of $TFPQ$ in the life cycle sample pooling ages.

8 Conclusion

We take advantage of rich microdata on Colombian manufacturing establishments to decompose growth over an establishment's life cycle into that attributable to fundamental sources of growth–physical productivity, demand shocks, and input prices–and distortions that weaken the link between those fundamentals and actual growth. We rely on a nested CES structure for preferences over products by multiproduct businesses, and data on quantities and prices for individual products for each manufacturing establishment, to decompose profitability shocks into physical productivity and demand shocks. Pooling all ages, measured fundamentals explain around 70% of the variability of output relative to birth level, with the remaining 30% explained by distortions and other unobserved factors. Demand shocks and $TFPQ$ are equally important in the explained part, while input prices play a more minor role. Distortions explain more than 50% of growth up to age seven, but their contribution falls to less than 25% by around age 20. For the fraction explained by fundamentals, early life growth is explained by $TFPQ$ with demand and input prices playing a minor role. But demand is the crucial factor in the variance of long-run growth, with a contribution that surpasses that of $TFPQ$ and unobserved factors by around age 15. In the 2000s compared to the 1980s, two decades separated by a wave of deep structural reforms, the contribution of $TFPQ$ to the variance in life cycle growth grows by around 7 percentage points, with distortions falling in importance by 4 percentage points.

In addition to providing new insights into the determinants of the variance of life cycle growth over different stages of the plant life-cycle, we also quantify the contribution of the fundamentals to average plant-level growth as well as growth at different percentiles of the plant growth rate distribution. We find that growth in plant-level demand is the dominant factor accounting for average plant-level growth as well as for high growth plants. We also find that distortions act, not surprisingly, as a drag on average plant-level growth. Interestingly, we find that this drag is especially relevant for the high growth plants that are critical for the average plant-level growth.

Our analysis focuses on the decomposition of the variance of life cycle growth into fundamentals vs. distortions and in turn on the relative contribution of different fundamentals. We regard part of our contribution as highlighting the importance of using price and quantity data at the firm level in order to conduct this decomposition. Indirect methods that use only revenue data must infer fundamentals vs. distortions less through direct measurement but instead decompose revenue productivity measures based upon strong assumptions about the structure of technology and demand.

Our findings raise a number of questions and point to important areas for future research. First, while we can directly measure fundamentals like $TFPQ$ and demand shocks with price and quantity data, we still infer distortions from indirect methods (using the terminology of Restuccia and Rogerson (2017)). Identifying the distortions that account for 50% of the variance of early life growth is one potential area of research. More generally, we are taking the evolution of $TFPQ$ and the demand shocks we measure as exogenous in our analysis. Their evolution is presumably endogenous over the life cycle and reflect investments in process innovation on the $TFPQ$ side and in developing product innovations as well as a customer base on the demand side. Our findings provide insights into the relative importance of the variance in these fundamentals but not the ultimate sources of the variance in these fundamentals. Following the insights of Restuccia and Rogerson (2017), we capture only the effect of distortions on scale given fundamentals, but do not account for the fact that the variance in the fundamentals we detect over the life cycle may reflect distortions that impact the investments that lead to this variance. Of course, we don't know that much about the life cycle dynamics of these fundamentals. Recent research on U.S. firms highlights that in the cross section (Hottman et. al. (2016)) demand factors account for a large fraction of variance in the size distribution and over the life cycle demand side factors play a dominant role (Foster et. al. (2016)). But that research, like ours, does not provide guidance about what determines the variance in these fundamentals in the U.S. Research that sheds light on the endogenous determinants of the the variance in the supply side ($TFPQ$) and demand side fundamentals should have a high priority in future research.

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10 Appendix A: measuring exact price indices

The change in prices from one period to the next is, from 7:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega_t^f} d_{fjt}^{\sigma} p_{fjt}^{1-\sigma}}{\sum_{\Omega_t^f} d_{fjt-1}^{\sigma} p_{fjt-1}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} \tag{30}$$

Recall that we are making the assumption that $\sigma_F = \sigma_J$. Defining as $\Omega^f_{t,t-1}$ the set of goods that is common to both periods, and multiplying both the numerator and the denominator by $\left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1} * \sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}\right)^{\frac{1}{1-\sigma}}$ we obtain:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega^f_t} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}} \frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\sum_{\Omega^f_{t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}} \tag{31}$$

$$= \frac{\lambda_{ft-1,t}}{\lambda_{ft,t-1}} \left(\frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}} \tag{32}$$

where $\lambda_{ft-1,\Omega^f_{t,t-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\sum_{\Omega^f_{t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}}$ and $\lambda_{ft,\Omega^f_{t,t-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}}{\sum_{\Omega^f_t} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}} \right)^{\frac{1}{1-\sigma}}$. Furthermore, since

$$s_{fjt} = \frac{p_{fjt} q_{fjt}}{R_{ft}} = \frac{p^{1-\sigma}_{fjt} \left(d^{\sigma}_{fjt} \right)}{P^{1-\sigma}_{fjt}} \tag{33}$$

we have that:

$$\lambda_{ft-1,\Omega^f_{t,t-1}} = \left(\sum_{\Omega^f_{t,t-1}} \frac{d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\sum_{\Omega^f_{t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}} \right)^{\frac{1}{1-\sigma}} = \left(\sum_{\Omega^f_{t,t-1}} s_{fjt-1} \right)^{\frac{1}{1-\sigma}}$$

That is, $\left(\lambda_{ft-1,\Omega^f_{t,t-1}}\right)^{1-\sigma}$ is the share of period $t-1$ expenditures devoted to goods that are common to both periods. Similarly, $\left(\lambda_{ft,\Omega^f_{t,t-1}}\right)^{1-\sigma}$ is the share of period t expenditure devoted to goods common to both periods.

With this, the change in prices between the two periods can be written:

$$\frac{P_{ft}}{P_{ft-1}} = \left(\frac{\sum_{\Omega^f_{t,t-1}} s_{fjt}}{\sum_{\Omega^f_{t,t-1}} s_{fjt-1}} \right)^{\frac{1}{\sigma-1}} \frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}} \tag{34}$$

where $P^*_{ft} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt} p^{1-\sigma}_{fjt}\right)^{\frac{1}{1-\sigma}}$ is a period t price index for the basket of goods common to t and $t-1$ for firm f, and $P^*_{ft-1,\Omega^f_{t,t-1}} = \left(\sum_{\Omega^f_{t,t-1}} d^{\sigma}_{fjt-1} p^{1-\sigma}_{fjt-1}\right)^{\frac{1}{1-\sigma}}$ is a period $t-1$ price index for that same basket.

Now define the Sato-Vartia growth of prices as $\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}}$ with $\omega_{fjt,t-1} = \frac{\frac{(s^*_{fjt} - s^*_{fjt-1,t})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,t}}}{\sum_{\Omega^f_{t,t-1}} \left(\frac{(s^*_{fjt} - s^*_{fjt-1,t})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,t}}\right)}$, where $s^*_{fjt} = \frac{p_{fjt} q_{fjt}}{\sum_{\Omega^f_{t,t-1}} p_{fjt} q_{fjt}}$ and $s^*_{fjt-1,\Omega^f_{t,t-1}} = \frac{p_{fjt-1} q_{fjt-1}}{\sum_{\Omega^f_{t,t-1}} p_{fjt-1} q_{fjt-1}}$ are the ratio of period t (resp. $t-1$) sales of product j for firm f to firm f's sales in period t (resp. $t-1$) of goods that belong to the basket $\Omega_{t,t-1}$.

The Marshalian demands 5, given by $q_{fjt} = d^{\sigma_F}_{ft} d^{\sigma_J}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \left(\frac{p_{fjt}}{P_{ft}}\right)^{-\sigma_J} \frac{E_t}{P_t}$, imply

$$s^*_{fjt} = \frac{d^{\sigma_F}_{ft} d^{\sigma_J}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \frac{p^{1-\sigma_J}_{fjt}}{P^{-\sigma_J}_{ft}} \frac{E_t}{P_t}}{\sum_{\Omega^f_{t,t-1}} d^{\sigma_F}_{ft} d^{\sigma_J}_{fjt} \left(\frac{P_{ft}}{P_t}\right)^{-\sigma_F} \frac{p^{1-\sigma_J}_{fjt}}{P^{-\sigma_J}_{ft}} \frac{E_t}{P_t}} = \frac{d^{\sigma_J}_{fjt} p^{1-\sigma}_{fjt}}{\left(P^*_{ft}\right)^{1-\sigma}}$$

and

$$s^*_{fjt-1,\Omega^f_{t,t-1}} = \frac{d^{\sigma_F}_{ft-1} d^{\sigma_J}_{fjt-1} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma_F} \frac{p^{1-\sigma_J}_{fjt-1}}{P^{-\sigma_J}_{ft-1}} \frac{E_{t-1}}{P_{t-1}}}{\sum_{\Omega^f_{t,t-1}} d^{\sigma_F}_{ft-1} d^{\sigma_J}_{fjt-1} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\sigma_F} \frac{p^{1-\sigma_J}_{fjt-1}}{P^{-\sigma_J}_{ft-1}} \frac{E_{t-1}}{P_{t-1}}} = \frac{d^{\sigma_J}_{fjt-1} p^{1-\sigma}_{fjt-1}}{\left(P^*_{ft-1,\Omega^f_{t,t-1}}\right)^{1-\sigma}}$$

so that $\left(\frac{p_{fjt}}{p_{fjt-1}}\right) = \left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) \left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)^{\frac{1}{1-\sigma}} \left(\frac{d_{fjt}}{d_{fjt-1}}\right)^{-\frac{\sigma}{1-\sigma}}$. Given this and the facts that $\sum_{\Omega^f_{t,t-1}} \omega_{fjt,t-1} = 1$ and $\sum_{\Omega^f_{t,t-1}} s^*_{fjt,\Omega^f_{t,t-1}} = 1$,

$$\sum_{\Omega^f_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}}$$

$$= \ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) + \frac{1}{(1-\sigma)} \sum_{\Omega^f_{t,t-1}} \frac{\frac{(s^*_{fjt} - s^*_{fjt-1,\Omega^f_{t,t-1}})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,\Omega^f_{t,t-1}}}}{\sum_{\Omega^f_{t,t-1}} \left(\frac{(s^*_{fjt} - s^*_{fjt-1,\Omega^f_{t,t-1}})}{\ln s^*_{fjt} - \ln s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)} . \ln\left(\frac{s^*_{fjt}}{s^*_{fjt-1,\Omega^f_{t,t-1}}}\right)$$

$$+ \frac{\sigma}{\sigma - 1} \sum_{\Omega^f_{t,t-1}} \omega_{fjt,t-1} \ln\left(\frac{d_{fjt}}{d_{fjt-1}}\right)$$

$$= \ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) + \frac{\sigma}{\sigma - 1} \ln\left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}}\right)$$

where $\widehat{d}^*_t = \prod_{\Omega^f_{t,t-1}} d_{fjt}^{\omega_{fjt,t-1}}$ is a weighted geometric mean of individual product appeals for the constant basket of goods. Under the assumption that this geometric mean is invariant over time ($\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} = 1$), the consecutively common good price index growth $\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right)$ can be calculated as in Sato-Vartia

$$\ln\left(\frac{P^*_{ft}}{P^*_{ft-1,\Omega^f_{t,t-1}}}\right) = \sum_{\Omega_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}}$$

As we discuss in the main text and above, our approach involves a simplifying normalization. Specifically, we assume $\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} = 1$). If $\frac{\widehat{d}^*_t}{\widehat{d}^*_{t-1}} \neq 1$, $\ln\left(\frac{P^*_t}{P^*_{t-1}}\right) = \sum_{\Omega_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}} - \frac{\sigma}{\sigma-1} \ln\left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}}\right)$. The Sato-Vartia price index $\sum_{\Omega_{t,t-1}} \ln\left(\frac{p_{fjt}}{p_{fjt-1}}\right)^{\omega_{fjt,t-1}}$ ignores the negative term $-\frac{\sigma}{\sigma-1} \ln\left(\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}}\right)$. As Redding and Weinstein (2016) indicate, it is possible that product demand shocks $\frac{d_{fjt}}{d_{fjt-1}}$ are positively correlated with the weights $\omega_{fjt,t-1}$ so that $\frac{\widehat{d}^*_{ft}}{\widehat{d}^*_{ft-1}} >$

1. Such a correlation will yield a demand bias that implies we may be overstating price inflation for the common goods produced in both $t-1$ and t. Such overstatement of price inflation implies understatement of quantity growth and therefore $TFPQ$ growth.

What are assumptions under which our normalization would hold so that there is no bias? We could assume that the $d_{fjt-1} = d_{fjt}$ for all products the firm produces in both $t-1$ and t. This would still permit entry and exit of goods. For example, suppose that d_{fjt} is a draw from a two point distribution with one value being zero and the other being a positive value. Then the firm would exhibit entry and exit of goods as good j switches from these two values. Moreover, there would be quality changes as the mix of goods changes. Alternatively, if relative product demand changes for goods produced in both $t-1$ and t keep the overall joint distribution of relative product demands and prices the same (this is in the spirit of the Hottman et. al. (2016) normalization) for goods produced by the firm in both periods then our normalization holds.

Still we are sympthetic to the arguments in Redding and Weinstein (2016) that there may be a correlation between changes in relative product demands and the Sato Vartia weights. We plan to explore ways to accommodate this in our estimation approach in future drafts of the paper.

11 Appendix B: firm problem with Cobb Douglas production function

Firm chooses X_t to solve:

$$\underset{\{X_{ft}\}}{Max} \quad \pi_{ft} = R_{ft} - C_{ft}X_{ft} = D_{ft}A_{ft}^{1-\varepsilon}X_{ft}^{\gamma(1-\varepsilon)} - C_{ft}X_{ft}$$

where $R_{ft} = P_{ft}Q_{ft}$.Optimal input demand is

$$X_{ft} = \left(\frac{\gamma\,(1-\varepsilon)\,D_{ft}A_{ft}^{1-\varepsilon}}{C_{ft}}\right)^{\frac{1}{1-\gamma(1-\varepsilon)}}$$

Suppose $X_{ft} = K_{ft}^{\frac{\alpha}{\gamma}}L_{ft}^{\frac{\beta}{\gamma}}M_{ft}^{\frac{\phi}{\gamma}}$ where K, L and M are, respectively, capital, labor and material inputs, and $\gamma = \alpha + \beta + \phi$. The firm's problem is

$$\underset{\{K_{ft},L_{ft},M_{ft}\}}{Max} \; D_{ft}A_{ft}^{1-\varepsilon}K_{ft}^{\alpha(1-\varepsilon)}L_{ft}^{\beta(1-\varepsilon)}M_{ft}^{\phi(1-\varepsilon)} - C_{ft}\left(L_{ft},K_{ft},M_{ft}\right)$$

where $C_{ft}\left(L_{ft},K_{ft},M_{ft}\right) = w_{ft}L_{ft}+r_{ft}K_{ft}+pm_{ft}M_{ft}$. w_{ft} is the wage rate, r_{ft} is the unit cost of capital, and pm_{ft} is an index of material input prices. First order conditions for L_{ft}, K_{ft}, M_{ft}, yield the following optimal demands:

$$\begin{aligned}
\frac{\beta R_{ft}}{L_{ft}} &= \frac{w_{ft}}{(1-\varepsilon)} \\
\frac{\beta K_{ft}}{\alpha L_{ft}} &= \frac{w_{ft}}{r_{ft}} \\
\frac{\phi L_{ft}}{\beta M_{ft}} &= \frac{pm_{ft}}{w_{ft}}
\end{aligned}$$

Rewriting, and replacing the last two equations into the first one:

$$\begin{aligned}
\frac{w_{ft}}{(1-\varepsilon)} &= \beta D_{ft}A_{ft}^{1-\varepsilon}\left(\frac{K_{ft}^{\alpha}L_{ft}^{\beta}M_{ft}^{\phi}}{L_{ft}^{\frac{1}{1-\varepsilon}}}\right)^{1-\varepsilon} \\
&= \beta D_{ft}A_{ft}^{1-\varepsilon}\left(\left(\frac{K_{ft}}{L_{ft}}\right)^{\alpha}\left(\frac{M_{ft}}{L_{ft}}\right)^{\phi}\frac{1}{L_{ft}^{\frac{1}{1-\varepsilon}-(\beta+\alpha+\phi)}}\right)^{1-\varepsilon} \\
L_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} &= \left(\frac{(1-\varepsilon)\,\beta D_{ft}A_{ft}^{1-\varepsilon}}{w_{ft}}\right)^{\frac{1}{(1-\varepsilon)}}\left(\frac{\alpha w_{ft}}{\beta r_{ft}}\right)^{\alpha}\left(\frac{\phi w_{ft}}{\beta pm_{ft}}\right)^{\phi} \\
L_{ft}^{\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}} &= \varpi_L\frac{D_{ft}^{\frac{1}{(1-\varepsilon)}}A_{ft}w_{ft}^{-\frac{1}{(1-\varepsilon)}+(\alpha+\phi)}}{r_{ft}^{\alpha}pm_{ft}^{\phi}}
\end{aligned}$$

where ϖ_L is a function of parameters. Proceeding similarly for capital and materials, we obtain demands $M_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} = \varpi_M\frac{D_{ft}^{\frac{1}{(1-\varepsilon)}}A_{ft}pm_{ft}^{-\frac{1}{(1-\varepsilon)}+(\alpha+\beta)}}{r_{ft}^{\alpha}w_{ft}^{\beta}}$ and $K_{ft}^{\frac{1}{(1-\varepsilon)}-\gamma} = \varpi_K\frac{D_{ft}^{\frac{1}{(1-\varepsilon)}}A_{ft}r_{ft}^{-\frac{1}{(1-\varepsilon)}+(\beta+\phi)}}{w_{ft}^{\beta}pm_{ft}^{\phi}}$.

As a consequence:

$$
\begin{aligned}
X_{ft}^{\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}} &= \varpi_L^{\frac{\beta}{\gamma}} \varpi_K^{\frac{\alpha}{\gamma}} \varpi_M^{\frac{\phi}{\gamma}} D_{ft}^{\frac{1}{1-\varepsilon}} A_{ft} \\
&\quad * \left(\frac{pm_{ft}^{-\frac{\phi}{\gamma(1-\varepsilon)}+\frac{\phi}{\gamma}(\alpha+\beta)}}{r_{ft}^{\frac{\phi}{\gamma}\alpha} w_{ft}^{\frac{\phi}{\gamma}\beta}} \frac{r_{ft}^{-\frac{\alpha}{\gamma(1-\varepsilon)}+\frac{\alpha}{\gamma}(\beta+\phi)}}{w_{ft}^{\frac{\alpha}{\gamma}\beta} pm_{ft}^{\frac{\alpha}{\gamma}\phi}} \frac{w_{ft}^{-\frac{\beta}{\gamma(1-\varepsilon)}+\frac{\beta}{\gamma}(\alpha+\phi)}}{r_{ft}^{\frac{\beta}{\gamma}\alpha} pm_{ft}^{\frac{\beta}{\gamma}\phi}} \right) \\
&= \varpi_L^{\frac{\beta}{\gamma}} \varpi_K^{\frac{\alpha}{\gamma}} \varpi_M^{\frac{\phi}{\gamma}} \left(\frac{D_{ft}^{\frac{1}{1-\varepsilon}} A_{ft}}{w_{ft}^{\frac{\beta}{\gamma(1-\varepsilon)}} pm_{ft}^{\frac{\phi}{\gamma(1-\varepsilon)}} r_{ft}^{\frac{\alpha}{\gamma(1-\varepsilon)}}} \right)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{X_{ft}}{X_{f0}} &= \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\frac{\phi}{\gamma}\kappa_1} \left(\frac{w_{ft}}{w_{f0}}\right)^{-\frac{\beta}{\gamma}\kappa_1} \kappa_t \widehat{\kappa}_{ft} && (35) \\
\frac{L_{ft}}{L_{f0}} &= \left(\frac{d_{ft}}{d_{f0}}\right)^{\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_2} \left(\frac{w_{ft}}{w_{f0}}\right)_t^{-\kappa_1+(\alpha+\phi)\kappa_2} \xi_t \xi_{ft} && (36) \\
\frac{Q_{ft}}{Q_{f0}} &= \left(\frac{d_{ft}}{d_{f0}}\right)^{\gamma\kappa_1} \left(\frac{a_{ft}}{a_{f0}}\right)^{1+\gamma\kappa_2} \left(\frac{pm_{ft}}{pm_{f0}}\right)^{-\phi\kappa_1} \left(\frac{w_{ft}}{w_{f0}}\right)^{-\beta\kappa_1} \chi_t \chi_{ft} && (37)
\end{aligned}
$$

where $\kappa_1 = \frac{1}{1-\gamma(1-\varepsilon)}$; $\kappa_2 = (1-\varepsilon)\kappa_1$; $\kappa_t = \left(\frac{D_t}{D_0}\right)^{\kappa_1} \left(\frac{A_t}{A_0}\right)^{\kappa_2} \left(\frac{C_t}{C_0}\right)^{-\kappa_1}$ captures aggregate growth between birth and age t, and ;$\widehat{\kappa}_{ft} = \frac{\delta_{ft}^{\kappa_1} \alpha_{ft}^{\kappa_2} \zeta_{ft}^{-\kappa_1} r_{ft}^{\frac{-\alpha\kappa_1}{\gamma}}}{\delta_{f0}^{\kappa_1} \alpha_{f0}^{\kappa_2} \zeta_{ft}^{-\kappa_1} r_{f0}^{\frac{-\alpha\kappa_1}{\gamma}}}$ captures residual variation from noise in fundamentals not observed by the firm at the time of choosing its scale in each period, as well as from unobserved user cost of capital. ξ_t, ξ_{ft}, χ_t and χ_{ft} are analogous residuals for the specific cases of employment and output. In particular: $\chi_t == \kappa_t^{\gamma} \left(\frac{A_t}{A_0}\right)$ and $\chi_{ft} = \widehat{\kappa}_{ft}^{\gamma} \frac{\alpha_t}{\alpha_0}$.[30] Notice also that $C_{ft} = w_{ft}^{\frac{\beta}{\gamma}} pm_{ft}^{\frac{\phi}{\gamma}} r_{ft}^{\frac{\alpha}{\gamma}}$.

12 Appendix C: demand shocks vs TFPQ

We have taken fundamentals as given, but noted that our results should help guide future work, both theoretical and empirical, about the specific drivers

[30] $\xi_{ft} = \kappa_{ft} \left(\frac{r_{ft}}{r_{f0}}\right)^{-\alpha\frac{1-\gamma(1-\varepsilon)}{(1-\varepsilon)}}$

Table A1: TFPQ, demand shocks and Innovation by Type of Innovation

VARIABLES	TFPQ	Demand Shock
Dummy product innovation	-0.0107	0.535***
	(0.0227)	(0.0279)
Dummy process innovation	0.0127	0.320***
	(0.0242)	(0.0299)
Dummy organizational innovation	-0.0199	0.202***
	(0.0336)	(0.0425)
Number product innovation	0.000244	0.00556***
	(0.000451)	(0.000841)
Number process innovation	0.00833**	0.0352***
	(0.00372)	(0.00651)
Number organizational innovation	0.0234*	0.0171
	(0.0136)	(0.0193)
Constant	2.043***	5.331***
	(0.0202)	(0.0313)
Observations	41,053	44,528
R-squared	0.155	0.627
Sector FE	Yes	Yes
Time FE	Yes	Yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

of measured productivity. To further understand the nature of $TFPQ$ vs. demand shock, and potential mechanisms through which businesses accumulate each of them, we have studied the relationship between these fundamentals and reported innovation efforts. The Colombian Manufacturing Survey can be merged with the Innovation Survey at the level of the firm (tax ID). Firms report number of innovations by type, defined by categories named "product", "process", and "organizational" innovation. They also report innovation expenditures, unfortunately not broken down in the same categories.

We regress plant $TFPQ$ and demand shocks on the parent firm's reports of having obtained at least one innovation, and on numbers of innovation by type. $TFPQ$ increases with process and organization innovation, but only with sufficiently large numbers of both. Demand shocks positively correlate with innovations of any type, incrementally with their number except for organizational innovations. Both fundamentals are positively correlated with innovation expenditures.