The Welfare Effects of Trade with Labour Market Risk*

Omid Mousavi
University of Melbourne

Lawrence Uren
University of Melbourne

June 26, 2017

Abstract

This paper shows that moving from autarky to free trade may reduce welfare in a small open economy. We deviate from a baseline trade model in two dimensions. First, workers search for jobs in a frictional labour market. As a result workers face employment risk. Second, workers are risk averse and financial markets are incomplete. Our key finding is that if trade results in small changes in relative prices, that moving from autarky to free trade may reduce welfare. The effects are driven by how changes in relative prices affect income risk that workers face in the labour market.

*The authors would like to thank Reshad Ahsan, Chris Edmond, Thijs van Rens and Juan Carlos Gozzi and seminar participants at the Australian National University, the Univeristy of Warwick and the National Bank of Poland for helpful comments. All remaining errors are ours.

1 Introduction

The standard welfare effects of trade for a small open economy are well known.[1] The consensus is that international trade is welfare improving but there are distributional issues associated with trade. In other words, there are winners and losers from trade but if lump-sum transfers are available then the losers can be adequately compensated so that everyone wins. The importance of these distributional issues has recently been highlighted by Autor, Dorn and Hanson (2013). They find that regions more exposed to trade with China (as determined by their initial composition of production) tend to have worse labour market outcomes over the period from 1990-2007.

Our standard economic theory also highlights why redistribution may be difficult to achieve. The political process may limit the use of lump-sum transfers or alternative methods of redistribution. Alternatively, and also plausibly, there are informational issues that may hinder redistribution. Governments may be unaware of which individuals are left worse off by trade. Furthermore, transfers to mitigate welfare losses associated with trade may weaken incentives.

This paper shows that in the presence of labour market risk, moving from autarky to free trade may reduce welfare even if lump-sum transfers are feasible. That is, simply put, free trade may result in an aggregate welfare loss for a small open economy. We study a variant of a Ricardo-Viner model. In our model, labour is attached to a sector while capital responds to endogenous saving decisions and can be allocated across sectors. To generate our result we deviate from a standard trade model in two dimensions. First, we assume that labour markets are frictional. This implies that workers are exposed to employment risk due to the stochastic arrival of job destruction and job finding shocks. Our second deviation from a baseline model is that workers are risk averse and financial markets are incomplete.

In Section 2, to provide the main intuition of our result, we analyse a simple endowment economy. Individuals have preferences over two goods and receive a stochastic endowment. If endowment risk is absent, then moving from autarky to free trade will always be welfare improving. That is, in a free trade equilibrium workers can be compensated so as to be at least as well off as under autarky and typically some workers will be better off. The same is true if endowment risk exists but workers are risk neutral or equivalently if markets are complete. However if endowment risk exists *and* if risk averse workers lack access to complete markets, then moving from autarky to trade may reduce welfare. That is, even if lump-sum transfers are available it may not be possible to compensate individuals without a positive inflow of resources into the economy.

[1] See Dixit and Norman (1980) for a discussion of gains from trade in a traditional framework or Arkolakis, Costinot and Rodriguez-Clare (2012) in new trade models.

The potential welfare loss arises from the interaction of uncertainty with risk aversion. The combination of risk aversion and uncertainty implies that sectors will vary in their insurance properties. In general, one sector will have relatively high covariance between marginal utility of workers within a sector and the size of the endowment. This sector provides relatively good insurance properties in the sense that it provides a relatively high purchasing power when marginal utility is high. Furthermore, a change in relative prices due to trade affects the real income of workers in both sectors. Workers in the sector in which relative output price increases gain and workers in the sector in which the relative price decreases lose from trade. However, the size of the transfers required to compensate individuals for a gain or loss from trade are dependent upon the insurance properties of a sector. If workers in a sector with good insurance properties gains from a price increase, they may compensate workers in the sector that lose with a relatively large transfer of resources. Hence, gains from trade exist. On the other hand, if workers in a sector with good insurance properties face a decline in relative prices they will require a relatively large transfer to compensate them and, at least for small price changes, gains from trade will not exist.

The model presented in Section 2 is a stylised model that captures the key mechanism in a simple setting. In Section 3 we move on to consider a dynamic endowment economy. In this economy workers transition between productive and unproductive states at a stochastic rate. These transitions add risk to a workers income. Our results mirror those of the static endowment economy. With risk neutral workers or in the absence of risk, opening up to trade always increases welfare. When workers are subject to income risk *and* are risk averse, we provide a set of conditions under which marginal changes in relative prices due to international trade will lower aggregate welfare in the economy.

In Section 4 to examine the quantitative relevance of our theory, we develop a dynamic general equilibrium model. This model features incomplete markets with labour market frictions in a framework similar to Krusell, Mukoyama and Şahin (2010). Our model differs in a number of aspects. Most importantly, we consider an economy with two sectors in which international trade can alter relative prices. Workers participate in a frictional labour market and hence are subject to uncertainty regarding the timing of both job loss and job finding. Furthermore, they participate in an incomplete financial market by making a consumption-saving decision. Participation in this market can provide a degree of self-insurance and reduce the negative consequences of job loss but the absence of complete markets implies that some risk remains. This combination of features imply that risk averse workers are subject to labour market risk. On the production side of the economy we follow Moen (1997) and Acemoglu and Shimer (1999). Firms search for workers in a directed search environment. They create vacancies, set wages and purchase capital before matching with a worker to produce output. As a result, changes in the relative price brought about by international trade can also lead to a more productive allocation of resources as both vacancies and capital per worker respond to price signals. So even though workers are immobile across sectors, the level of employment and capital can vary within a sector in response to international trade.

This dynamic trade model has similar forces at work as our static model of the economy. Two standard sources of gains from trade are available. First, as relative prices change the economy can use more intensively factors of production in the sector that produces more valuable output. Second, consumers are able to substitute their consumption towards products that have become relatively cheaper. Hence, the scope of our result is concerned with welfare implications associated with changes in prices that allow a more efficient allocation of resources in production or in consumption. These are the traditional gains from trade. Of course, there are other possible sources via which an economy may gain from trade. Trade may facilitate the transfer of technology across borders, widen the variety of goods consumed, and increase the degree of product market competition.[2] Our paper is silent on the benefits of trade along these dimensions.

In Section 4 and 5 we calibrate the model and undertake a numerical analysis to examine the impact of moving from autarky to trade for a small open economy. We select parameters to match a series of targets based upon long run features of the macroeconomy and upon observed differences in labour market outcomes by educational groups in the US. We then examine the impact upon welfare of a change in relative prices due to opening to trade. If there is a small change in relative prices (of the order of magnitude of about 3 per cent) that favours the skilled sector, we find that there are no gains from trade available. If relative price changes from trade are larger or favour the less skilled sector we find that the gains from trade exist. We also note that the gains from trade are asymmetric. That is, a rise in the relative price that favours the skilled sector will provide a smaller benefit than a price change of similar magnitude that favours the less skilled.

Our main result is reminiscent of Newbery and Stiglitz (1984) who highlight that free trade may reduce welfare. In their work, trade integration raises product price volatility. If individuals are risk averse and markets incomplete this increase in price volatility may reduce welfare. Our paper has a similar message - in the absence of complete markets free trade may reduce welfare - but it differs in important respects. In our model, there is no aggregate uncertainty but rather risk is idiosyncratic and associated with labour market outcomes due to search frictions. Second, our result does not rely upon the rather special assumptions that are used in Newbery and Stiglitz (1984). In fact, we believe that the deviations from a baseline trade model that we consider - frictional labour markets

[2] See Broda and Weinstein (2006), Grossman and Helpman (1991) and Feenstra and Weinstein (2010) for a discuss of the gains from trade from increased variety, technological spillovers, and increased competition.

and incomplete financial markets - are modifications that many would find palatable.

Also related is the work of Dixit (1989) and Ranjan (2016). Dixit (1989) discusses welfare improving trade policy in an economy with one risky and one safe sector. He does not, however, outline conditions under which a move from autarky to free trade will reduce welfare, which is the focus of our paper. Ranjan (2016) examines a setting with risk averse workers and how trade may worsen the allocation of productive resources in a directed search setting with risk averse workers. He builds upon the work of Acemoglu and Shimer (1999), who find in a directed search setting that firms may respond to greater labour market risk by offering lower quality that jobs that are easier for workers to find. This has a welfare cost since it distorts the allocation of resources away from the optimal allocation. Our work differs in emphasising that trade may reduce welfare even if resources are optimally allocated across sectors or if the allocation of productive resources is unchanged by trade integration. That is, the potential negative welfare effects associated with trade are present even if changes in the production side of the economy are absent.

There is a growing literature on the role of search frictions in international trade. Early examples are Davidson, Martin and Matusz (1987) and Davidson, Martin and Matusz (1988) and more recently, Helpman and Itskhoki (2010) and Helpman, Itskhoki and Redding (2010). These papers have focused upon search frictions in environments with risk neutral workers. Finally a number of papers examine the welfare costs of trade in settings in which workers are immobile or partially immobile across regions. Examples include Artuç, Chaudhuri and McLaren (2010), Dix-Carniero (2014) and Guren, Hémous and Olsen (2015). These papers focus upon the distributional issues associated with trade, while our paper emphasises that the welfare losses to individuals harmed by trade may outweigh the benefits to individuals that gain from trade.

In Section 2 we outline a simple one period model in which the key mechanism is highlighted. In Section 3 we analyse a dynamic endowment economy. In Section 4 we develop a production economy in which workers can reduce risk by participating in financial markets. Section 5 calibrates the model to match a set of reasonable targets observed in the data. Section 6 presents a numerical exercise that shows that for small price changes, it is possible that no aggregate gains from trade arise. Section 7 concludes.

2 A Static Endowment Economy with Uncertainty

We build a simple one-period, two-sector model of trade to develop the underlying intution of our result. Sectors are denoted by $i \in \{1, 2\}$ and workers are assigned to a specific sector and each sector produces a distinct good. There is a mass of workers normalised to size one with π_i workers in sector i. Individuals in sector i receive a stochastic amount of good i output that will be denoted by the random variable X_i with a particular realisation denoted by x_i. Realisations across individuals within a sector are i.i.d. To keep matters simple assume $X_i = \bar{x}_i$ with probability h_i and $X_i = \underline{x}_i$ with probability $1 - h_i$. Assume without loss of generality that $\bar{x}_i > \underline{x}_i$.

Each worker combines output from each sector into a composite consumption good described by a CES aggregator. Explicitly,

$$C = \left((1-\alpha)^{\frac{1}{\eta}} c_1^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_2^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{1}$$

and assume each worker is an expected utility maximiser with identical preferences over this composite consumption good that are described by a utility function, $U(C)$. Our focus will be upon an economy in which workers are risk averse so that $U'(C) > 0$ and $U''(C) < 0$.

In this static model workers consume all of their income. This implies the following budget constraint for a worker in sector i

$$p_i x_i = p_1 c_1 + p_2 c_2,$$

where p_i is the price associated with good i. With a risky endowment an individual's composite consumption is a random variable. Denote C_i as the random variable describing composite consumption of an individual in sector i. As is standard, there is an ideal price index, P, that describes the expenditure required to purchase one unit of C. Explicitly,

$$P = \left((1-\alpha) p_1^{1-\eta} + \alpha p_2^{1-\eta} \right)^{\frac{1}{1-\eta}} . \tag{2}$$

To complete the model we specify how prices are determined. We consider two cases. First, in autarky individuals in this economy trade amongst themselves. Prices adjust to ensure domestic supply equals domestic demand. Second, we consider a trade equilibrium for a small open economy in which prices are exogenous. In both cases, we normalise $p_2 = 1$.

Our welfare comparisons are based on the concept of compensating variation. In both the autarky and the free trade equilibrium we can calculate the expected utility of an individual in each sector. We can also calculate the size of the transfer required to provide an individual in the trade equilibrium with their expected utility in autarky. We then aggregate over individuals to calculate the net transfer required in a trade equilibrium to provide all individuals with their initial autarky utility. We describe trade as welfare improving if the size of the net transfer is negative, implying that all individuals can be compensated from the change in trade and resources are left over. If net transfers are positive, then trade reduces welfare.

In this setting we have two initial propositions.

Proposition 1. *In the absence of labour market risk, that is if $\bar{x}_i = \underline{x}_i$ or if $h_i = 1$ for all i then, a move from autarky to free trade is welfare improving in the sense that net transfers required to fully compsenate workers are non-positive.*

This result is not surprising. It is our standard result that trade is welfare improving for a small open economy. In an economy with certainty, changes in relative prices affect real wages of workers; some workers gain while others lose. However, in this endowment economy the total resources available are at least as much as in autarky. As a result the government is able to redistribute between individuals to ensure all workers are compensated from the change in real wages without requiring a net positive transfer of resources into the economy.

Proposition 2. *In a model with risk neutral workers the movement from autarky to free trade is welfare improving in the sense that net transfers required to fully compensate workers are non-positive.*

These two propositions demonstrate that this endowment economy satisfies standard trade results. In the absence of labour market risk or with risk neutral workers, the movement from autarky to free trade is welfare improving in the sense that workers can be compensated by using lump-sum transfers without requiring additional resources. Note that these propositions do not rely upon CES preferences. Our final result in this Section demonstrates that under certain conditions that movement from autarky to free trade will reduce welfare.

Proposition 3. *In an endowment economy with risk averse workers if in autarky,*

$$-\frac{cov[U'(C_1) \cdot X_1]}{E[U'(C_1)]E[X_1]} + \frac{cov[U'(C_2) \cdot X_2]}{E[U'(C_2)]E[X_2]} > 0$$

then a positive net transfer is required to compensate workers when moving from autarky to trade if there is a marginal increase in the relative price of p_1 and a negative net transfer is required to compensate workers if there is a marginal decrease in the relative price of p_1.

If in autarky,

$$-\frac{cov[U'(C_1) \cdot X_1]}{E[U'(C_1)]E[X_1]} + \frac{cov[U'(C_2) \cdot X_2]}{E[U'(C_2)]E[X_2]} < 0$$

then a negative net transfer is required to compensate workers when moving from autarky to trade if there is a marginal increase in the relative price of p_1 *and a positive net transfer is required to compensate workers if there is a marginal increase in the relative price of* p_1.

This proposition outlines under what conditions a net inflow of resources will be needed to compensate individuals following a small change in relative prices due to a movement from autarky to a free trade equilibrium. As relative prices change the real income of some workers increases while it decreases for others. In a world with risk neutral workers lump sum transfers can compensate individuals for these changes in real income without an inflow of resources into the economy (Proposition 2). In a world with risk aversion and uncertainty, sectors vary in how the marginal utility of workers within a sector covaries with the quantity of the endowment. From an asset pricing perspective, an asset that pays a relatively high return when marginal utility is high is particularly valuable since it provides good insurance properties.[3] Similarly, in this economy, a sector that provides, on average, a relatively high endowment when the marginal utility is high provides good insurance properties. As international trade changes relative prices it also changes the real value of output produced in different sectors of the economy. If a change in relative price increases the value of output in a sector that provides relatively good insurance properties then lump sum transfers exist such that all workers may be compensated without an inflow of net resources into the economy. On the other hand, if a small change in relative prices increases the value of output in a sector that has poor insurance properties then lump sum transfers to compensate individuals will require a net inflow of resources into the economy.

For example, if

$$\frac{cov[U'(C_2) \cdot X_2]}{E[U'(C_2)]E[X_2]} > \frac{cov[U'(C_1) \cdot X_1]}{E[U'(C_1)]E[X_1]}$$

then sector 2 has relatively good insurance properties in the sense that it has, on average, a relatively high endowment when the marginal utility of consumption is high. A marginal increase in the relative price of p_1 raises the real value of the endowment of workers in sector 1 and reduces the value of the endowment of workers in sector 2. This benefits workers in the sector that has poor insurance properties and harms workers in the sector that has good insurance properties. Our proposition states that compensation for a small relative price increase in sector 1 output requires a net inflow of resources into the economy.

[3]See Chapter 5 of Gollier (2001) or Chapter 10 of Blanchard and Fischer (1989) for a textbook exposition of asset pricing literature where the value of an asset depends upon the joint distribution of returns and marginal utility.

Also note that in a setting with risk neutrality (or complete financial markets) or with no uncertainty

$$-\frac{cov[U'(C_1)\cdot X_1]}{E[U'(C_1)]E[X_1]}+\frac{cov[U'(C_2)\cdot X_2]}{E[U'(C_2)]E[X_2]}=0$$

which implies that a marginal change in the relative price can be compensated without positive transfers as implied by our first two propositions.

The above illustrates that in the presence of risk aversion and uncertainty, integration of a small closed economy into a world trade equilibrium can result in welfare losses. To the best of our knowledge, this result is new to the literature.[4] Of course, if the government was able to condition transfers upon realised endowments then it could act to ensure that international trade improves welfare by replicating a complete markets outcome. Although this may seem reasonable in a simple endowment economy, we find it less compelling in a dynamic production economy where conditional transfers may generate moral hazard.

A natural question is whether the gains from trade in a global economy are positive? The key point to note here is that the direction of trade is based upon comparative advantage but, at least for small price changes, the gains from trade are determined by the relative sectoral covariance of marginal utility with the endowment. To the extent that the direction and the gains from trade are determined by different forces, it's certaintly possible that trade integration can reduce global welfare. Moreover, if countries are allowed to differ in their distribution of endowments, then it is possible to construct a two-country model in which both countries lose from trade integration.

This static model is stylised along several dimensions. First, the economy is an endowment economy so that one of the traditional gains from trade of reallocating factors of production to the sector in which the relative price increases is absent. However, potential gains from trade still exist from consumption reallocation. In particular, it is possible to shift consumption goods to the country where their relative marginal product is highest. Second, the static model abstracts from mechanisms that may help reduce the impact of uncertainty and risk, such as self-insurance. To examine the quantitative relevance of the welfare effects associated with trade in a model with labour market risk we construct a dynamic model in Section 3 and a production economy with self-insurance in Section 4.

[4] As discussed in the introduced the closest result we can find is Dixit (1989). He identifies a welfare improving policy for an economy in which one sector is safe and one is risky. He does not consider risk in more than one sector or identify conditions under which trade is welfare decreasing.

3 A Dynamic Endowment Economy with Labour Market Risk

The previous section examined a static endowment economy with risk aversion. This section investigates a dynamic endowment economy. We retain the assumption that workers are allocated to a specific sector denoted $i \in \{1, 2\}$. Again, π_i determines the proportion of workers assigned to sector i. Time is continuous. We denote the employment state of a worker as $j \in \{e, u\}$ where e indicates an employed worker and u indicates an unemployed worker. An employed worker in sector i produces x_i units of sector i output per unit of time. If unemployed they produce zero output. Output is traded and we denote the equilibrium price of sector i output as p_i and we normalise the price of sector 2 output so $p_2 = 1$ in all equilibria.

Workers in sector i transition from unemployment to employment at an exogenous rate of h_i and from employment to unemployment at a rate δ that is constant across sectors.[5] To evaluate a setting without financial markets assume that workers are unable to lend or borrow and must consume their wage income.

The expected utility of a worker is

$$E_0 \int_0^\infty e^{-\rho t} u(C)\, dt$$

where C is a CES aggregator that depends upon the consumption of goods produced by both sectors, denoted c_1 and c_2 as described in equation (1). There is a corresponding ideal price index, P, given by equation (2). The income of a worker in sector i is $w_i = p_i x_i$ if employed and $w_i = 0$ if unemployed and the real wage becomes w_i/P. Implicitly there are no firms in this economy and the workers randomly transition between a productive and unproductive state. In autarky, prices are set so that for each good demand equals supply. In a free trade equilibrium, prices are exogenous.

Define $V^a_{i,e}$ the expected utility an employed worker in sector i under autarky and $V^a_{i,u}$ the expected utility of an unemployed worker in sector i in autarky. In a steady state equilibrium these values are defined recursively below

$$\rho V^a_{i,e} = u(w_i/P) + \delta(V^a_{i,u} - V^a_{i,e})$$
$$\rho V^a_{i,u} = u(0) + h_i(V^a_{i,e} - V^a_{i,u})$$

[5] We allow h_i to vary across sectors with δ constant. This is not essential to the results that follow.

which can be solved as

$$V^a_{i,e} = \frac{\delta \cdot u(0) + (h_i + \rho) \cdot u(w_i/P)}{\rho(\delta + h_i + \rho)}$$
$$V^a_{i,u} = \frac{(\delta + \rho) \cdot u(0) + h_i \cdot u(w_i/P)}{\rho(\delta + h_i + \rho)}.$$

We consider integrating this closed economy into a global economy. The resulting trade equilibrium will alter relative prices and this alters wages and the nominal expenditure required to consume one unit of the composite consumption good. We keep fixed the labour market transition rates.[6] We relax this assumption below. We assume that trade integration is (i) unexpected and (ii) permanent. We then evaluate the welfare effects of trade using a concept similar to the compensating variation introduced in our static model. In particular, we allow the government to costlessly implement a system of transfers. The transfer size is dependent upon an individual's specific sector *and* upon their employment status when the economy transitions from autarky to free trade. We do not allow transfers to vary as an individual transitions between employment states.[7] Formally, let $\tau_{i,j}$ describe the transfer to a worker in sector i in employment state $j \in \{e, u\}$ when the economy is opened to trade. Define $V^t_{i,j}$ denote the expected utility of a worker in sector i and employment state $j \in \{e, u\}$ in a trade equilibrium. Explicitly,

$$V^t_{i,e}(\tau) = \frac{\delta \cdot u(\tau) + (h_i + \rho) \cdot u(w'_i/P' + \tau)}{\rho(\delta + h_i + \rho)}$$
$$V^t_{i,u}(\tau) = \frac{(\delta + \rho) \cdot u(\tau) + h_i \cdot u(w'_i/P' + \tau)}{\rho(\delta + h_i + \rho)}$$

where the w'_i and P' reflect wages and prices that a worker receives in the trade equilibrium and τ is the size of the transfer they receive, that depends in turn, upon their sector and employment status when trade integration occurs. Our welfare criterion is an extension of the standard static compensating variation.[8] That is, for a worker in sector i in employment state j, we seek a transfer τ_{ij} such that $V^t_{ij}(\tau_{ij}) = V^a_{ij}$. We then sum over workers to calculate the net transfer required to compensate all individuals.

A number of results arise. First, if workers are risk neutral trade is always welfare improving. That is, it is possible to compensate all individuals so that their expected utility is unchanged from trade

[6] A standard Ricardian model has two sources of gains from trade. First, trade allows goods to be shifted to countries in which the relative marginal utility of that good is high. Second, productive resources can be shifted to areas in which the marginal product is relatively high. In our endowment economy, by keeping the sectoral allocation of labour and the labour market transition rates fixed we are shutting down potential gains from trade arising due to a more efficient allocation of resources in the global economy. The first mechanism is still active in our model.

[7] Doing so, would allow the government to implement state-dependent insurance and trade would always be welfare increasing.

[8] To avoid technical difficulties associated with one-off transfers in a continuous time model, we focus upon allowing workers to be compensated by transfers over time. This would be equivalent to a setting in which the government transferred assets between individuals and these assets paid a fixed rate or return indefinitely.

integration and still have resources left over. The intuition for this result is straightforward. The real value of output produced in the economy increases as relative prices vary. With risk neutrality, expected utility is essentially transferable between individuals. Hence, the increase in real resources allows the government to redistribute in a welfare improving manner.

Second, with risk averse workers a change in relative prices brought about by trade may reduce welfare. This is captured in the following proposition.

Proposition 4. *In a dynamic endowment economy with risk averse workers, if in autarky*

$$\left(\frac{(h_1+\rho)u'(p_1x_1/P)}{\delta u'(0)+(h_1+\rho)u'(p_1x_1/P)}+\frac{\delta u'(p_1x_1/P)}{(\delta+\rho)u'(0)+h_1u'(p_1x_1/P)}\right)>$$
$$\left(\frac{(h_2+\rho)u'(x_2/P)}{\delta u'(0)+(h_2+\rho)u'(x_2/P)}+\frac{\delta u'(x_2/P)}{(\delta+\rho)u'(0)+h_2u'(x_2/P)}\right)$$

then a negative net transfer is required to compensate workers when moving from autarky to trade if there is a marginal increase in the relative price p_1 *and a positive net transfer is required to compensate workers if there is a marginal decrease in the relative price* p_1*.*

This result is proved in the Appendix. It relies upon examining the size of transfer required to compensate individuals in different sectoral and employment states for a marginal change in relative prices and then aggregating over individuals. This condition differs from the static case examined in the previous section. Rather than depending upon a static covariance between endowment and marginal utility, this condition depends upon the sectoral transition rates and the marginal utilities when employed and unemployed in different sectors.

An interesting case is one in which workers become arbitrarily patient ($\rho \to 0$). Our key necessary condition for a small increase in p_1 to yield a welfare improvement becomes

$$\frac{h_1/\delta+1}{h_1/\delta+u'(0)/u'(p_1x_1/P)}>\frac{h_2/\delta+1}{h_2/\delta+u'(0)/u'(x_2/P)}.$$

With infinitely patient workers, the welfare effects of trade for a small price change depends upon two factors. First, an increase in the price of the sector with lower frictions is more likely to raise welfare.[9] Second, welfare is more likely to improve if price changes favour the sector in which the ratio of marginal utility of unemployed to the marginal utility of the employed is relatively low.

Our theoretical result above relates to the case of small price changes. We now turn our attention to the effect of large price changes in this dynamic endowment model. A numerical analysis suggests that welfare gains may be non-existent even for large price changes. To proceed we outline a set

[9] As is standard in this literature, the ratio of h_j/δ is inversely related to the size of search frictions in sector j.

of parameters that we consider reasonable. For these parameters we calculate the net aggregate transfer required to compensate individuals for price changes of various magnitudes.

We divide workers into sectors on the basis of their educational attainment. Sector 1 contains workers with characteristics similar to college graduates and sector 2 employs workers with characteristics similar to high school graduates. The Current Population Survey suggests that workers with a college degree have an unemployment rate of 2.8 per cent compared to a 5.9 per cent unemployment rate of high school graduates. Furthermore, the ratio of wages of workers with high school degrees relative to college degrees is approximately 0.56 in the US data.

Parameters are selected to match these facts. The transition rates between employment states in each sector are selected to match the sectoral unemployment rates and $\pi_1 = 0.36$ which is close to the proportion of college graduates in the USA economy according to the CPS. The discount rate is set to a standard value of 0.05 and we normalise $x_2 = 1$ and set $x_1 = 2.2$ so that the wage differences between sectors are consistent with the ratio of wages between college and high school workers observed in the data. For our CES aggregator we set $\eta = 2$ and $\alpha = 0.5$ although our results are robust to deviations in these parameters. For our utility function we consider CARA utility with $U(C) = -\frac{e^{-\gamma C}}{\gamma}$ and consider a range of values for γ. When $\gamma = 0.01$ our preferences are close to risk neutral, at least over the relevant range. We also consider values of $\gamma = 1$ and 5.

The results are presented in Figure 1. When preferences are close to risk neutral the standard result attains; trade is essentially welfare improving for all but the smallest price changes. For the case in which $\gamma = 1$ there are no gains from trade if the relative price of the skilled sector increases by up to 8 per cent. For the case in which $\gamma = 5$, even large increases in the price of the skilled sector lower welfare.

4 A Dynamic Production Economy with Labour Market Risk

Here, we add two new elements to our previous analysis. First we allow workers to self-insure themselves against income risk by participating in an incomplete financial market. Second, we move beyond an endowment economy and consider a production economy. Time is continuous. There are two types of agents: workers and firms. These agents interact in three markets. First, workers and firms match in a frictional labour market, in the tradition of Mortensen-Pissarides. Second, individuals trade an annuity in a financial market, in the tradition of Blanchard-Yaari. Finally, goods are traded in a competitive goods market.

As before, there are two sectors denoted by $i \in \{1, 2\}$ and workers are allocated to a specific sector that produces a specific good. The mass of workers in sector i is denoted π_i and let $\pi_1 + \pi_2 = 1$. Workers die at a rate of d and are replaced by an inflow of newly born workers who enter the economy unemployed with zero assets and attached to the same sector as the individual they replace. Unemployed workers search for jobs in a frictional labour market and employed workers produce output and earn a wage w_i. Workers face labour market risk in the form of stochastic job finding and separation that occur at a rate h_i and δ_i, respectively. All workers make a consumption-saving decision. They are able to reduce labour market risk by participating in an incomplete financial market in which they are able to trade an annuity that offers a fixed rate of return r.

There is a large mass of firms able to enter either sector by creating a vacancy. To do so, they purchase an amount of sector i output as capital, k_i, and commence searching for workers. We adopt a directed search paradigm with firms posting wages and workers directing their search as in Moen (1997) and Acemoglu and Shimer (1999). The number of job matches formed per unit of time in sector i is determined by a matching function, $m_i(u_i, v_i)$, that features constant returns to scale in u_i and v_i. Once a worker is employed, output is produced with the amount of output, $f_i(k_i)$, dependent on the level of capital stock.

In a closed economy, the total demand equals the supply of goods and this determines the equilibrium interest rates and relative prices across sectors. Also, the net wealth held by workers will equal the value of firms. In a trade equilibrium, we consider a small open economy so that the interest rate and the relative price between sectors are exogenous. The demand for good i may differ from supply and that net domestic wealth may differ from the value of domestic firms due to either international borrowing or lending.

We commence the analysis by solving an individual worker's problem in an arbitrary sector. As a result, we temporarily suppress sectoral subscripts. We show how it is possible to aggregate worker behaviour and then solve for the endogenous worker transition rates and prices.

4.1 Worker's Problem

Workers discount future utility at a rate of ρ. They receive utility from consuming both goods which are combined into an aggregate consumption good denoted C using a constant elasticity of substitution aggregator. Let c_i denote the consumption of sector i output and p_i the corresponding price. Individuals have CARA utility with respect to the consumption aggregator with parameter γ determining the degree of absolute risk aversion. The employment state of a worker is denoted

$j \in \{e, u\}$ where $j = e$ indicates an employed worker and $j = u$ indicates an unemployed worker. Workers lose a job at a Poisson arrival rate of δ and find a job at a Poisson arrival rate of h. Although the rate of job loss remains exogenous, we will endogenise the rate of job finding when discussing firm entry. Workers are able to lend or borrow using an annuity, a, that offers a fixed rate of return equal to r. Formally, workers face the following consumption-saving problem:

$$\max_{\{c_1,c_2\}} E_0 \int_0^\infty -e^{-(\rho+d)t} e^{-\gamma C}\, dt$$

subject to

$$C = \left((1-\alpha)^{\frac{1}{\eta}} c_1^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_2^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

$$\dot{a} = ra + y_j - c_1 p_1 - c_2 p_2$$

In this setting workers face stochastic transitions between employment and unemployment so wage income equals $y_e = w$ and we will assume that workers receive no wage income or benefits when unemployed so $y_u = 0$. To proceed we first solve the intratemporal problem of allocating expenditure between c_1 and c_2 at a given point in time and then tackle the intertemporal problem of allocating expenditure over time. As is well known the solution to the intratemporal problem implies that for a given level of expenditure X,

$$c_1 = (1-\alpha) \left(\frac{p_1}{P}\right)^{-\eta} \frac{X}{P}$$

$$c_2 = \alpha \left(\frac{p_2}{P}\right)^{-\eta} \frac{X}{P}$$

where P is an ideal price index given by (2). Using the solution to the intratemporal CES problem, the worker's problem can be restated as:

$$\max E_0 \int_0^\infty -e^{-(\rho+d)t} e^{-\gamma C}\, dt$$

subject to

$$\dot{a} = ra + y_j - PC$$

and an associated transversality condition. The value function associated with a worker in employment state $j \in \{e, u\}$ and asset level a is denoted $V_j(a)$ and is defined recursively below,

$$\rho V_j(a) = \max_c \left(u(c) + (ra + y_j - PC) V_j'(a) + \phi_{jj'} (V_{j'}(a) - V_j(a)) + d(0 - V_j(a)) \right)$$

where $\phi_{jj'}$ is the transition rate from employment state j to state j'. That is, $\phi_{ue} = h$ and $\phi_{eu} = \delta$. Following Wang (2007) and Uren (2017), a guess and verify approach can be used to confirm that the solution is given by,

$$V_j(a) = -\frac{exp\left(-\frac{\gamma}{P}\left(ra + b_j + P\left(\frac{\rho+d-r}{\gamma r}\right)\right)\right)}{r}. \qquad (3)$$

The solution to the intertemporal consumption problem is given by

$$C = \frac{1}{P}\left(ra + b_j + P\left(\frac{\rho + d - r}{\gamma r}\right)\right) \tag{4}$$

where (b_e, b_u) solve:

$$he^{-\gamma b_e/P} = \left(h + \frac{\gamma r}{P}(y_u - b_u)\right)e^{-\gamma b_u/P} \tag{5}$$

$$\delta e^{-\gamma b_u/P} = \left(\delta + \frac{\gamma r}{P}(w - b_e)\right)e^{-\gamma b_e/P}. \tag{6}$$

With CARA utility, consumption can be broken into three components. First, workers consume all of their interest income. Second, consumption depends upon a component that reflects expected labour income flows over time captured by b_e and b_u. Finally, consumption also reflects the difference between the rate of time preference and the market discount rate.

A useful result that arises due to the CARA utility specification is that

$$\begin{aligned} \dot{a} &= ra + y_j - PC \\ &= y_j - b_j - P\left(\frac{\rho + d - r}{\gamma r}\right) \end{aligned}$$

where $j \in \{e, u\}$. This implies that the level of savings depends upon employment status but is independent asset holdings. Uren (2017) shows how to use this property in a one sector economy to derive a steady state distribution of assets.[10] A similar result applies in this setting and the equilibrium level of assets of workers in sector i is:

$$A_i = \frac{1}{d}\left((\pi_i - u_i)(y_{ie} - b_{ie}) + u_i(y_{iu} - b_{iu}) + \pi_i P\left(\frac{r - \rho - d}{\gamma r}\right)\right) \tag{7}$$

where u_i is the level of unemployment in sector i and the notation x_{ij} represents a variable, x, for a worker in sector i and employment state j. The aggregate level of assets held in this economy becomes,

$$A = \sum_i A_i.$$

This environment captures the key elements of our earlier models. Most importantly, workers are risk averse and face labour market uncertainty in the form of employment shocks. Unlike before, workers can partially self-insure themselves against employment risk by accumulating assets during good times and running down assets while unemployed.

The above results outline the solution to a consumption-saving problem of workers subject to exogenous wages and labour market transition rates. We now move on to endogenising job finding rates and wages by discussing firm behaviour.

[10] The death of individuals in this economy is required to attain a steady state distribution since assets for an individual are expected to drift upwards at a constant rate over time.

4.2 Firm's Problem

The labour market is frictional. To create a vacancy a firm in sector i purchases an amount of sector i output as capital, k_i, to be used in the production process.[11] The amount of output produced by a worker-firm match in sector i with capital k_i is $f_i(k_i)$. Workers and firms are matched according to a constant returns to scale matching function $m_i(u_i, v_i)$ that has the standard properties[12] such that the job finding rate of workers and the worker finding rate of firms in sector i can be expressed as functions $h_i(\theta_i)$ and $q_i(\theta_i)$ where θ_i is the vacancy-unemployment rate in sector i.

Once a job match is created, the firm earns a revenue of $p_i f_i(k_i)$ and a wage w_i is paid to workers. A job match is destroyed exogenously at a rate δ or if a worker dies at a rate of d. In either case, the capital investment is lost. The interest rate that firms pay on their capital debt is r_f. We define the value of a filled position to a firm in sector i with capital k_i and wage of w_i as $F_i(k_i, w_i)$ and the value of a vacancy that has a posted wage of w_i as $N_i(k_i, w_i)$. Using standard arguments these values can be represented as

$$r_f F_i(k_i, w_i) = p_i f_i(k_i) - w_i + (\delta_i + d)(-F_i(k_i, w_i))$$
$$r_f N_i(k_i, w_i) = q_i(\theta_i)(F_i(k_i, w_i) - N_i(k_i, w_i))$$

We assume that a large number of firms are prepared to create vacancies so that the value of a vacancy equals the cost of vacancy creation. Hence, $N_i(k_i, w_i) = p_i k_i$. Following Moen (1997) and Acemoglu and Shimer (1999) we assume a directed search environment in which firms post wages to attract workers. Competition between firms ensures that in equilibrium the wages, capital and level of vacancy creation maximise the value of unemployment subject to a zero profit condition. Formally,

$$\max_{\{k_i, w_i, \theta_i\}} V_{u,i}(a)$$

subject to $q_i(\theta_i)(F_i(k_i, w_i) - p_i k_i) = r_f N_i(k_i, w_i)$.

The following first order conditions pin down sectoral wages, w_i, capital per firm, k_i and the

[11] Capital in sector i is created from sector i output. We follow this convention so that changes in relative prices do not affect optimal investment decisions within a sector although this assumption would be easy to alter.

[12] Explicitly, $m_i(u_i, v_i)$ is increasing in each of its arguments and features constant returns to scale.

vacancy-unemployment rate, θ_i:

$$f_i'(k_i) = \frac{q(\theta_i) + r_f}{q_i(\theta_i)}(r_f + \delta_i + d) \tag{8}$$

$$p_i k_i = \frac{q(\theta_i)}{q(\theta_i) + r_f} \cdot \frac{p_i f_i(k_i) - w_i}{r + \delta_i + d} \tag{9}$$

$$\frac{\partial V_u(a; w, \theta)/\partial w}{\partial V_u(a; w, \theta)/\partial \theta} = \frac{q(\theta) F_w}{q'(\theta)(F_i - N_i)} \tag{10}$$

The first equation reflects an optimal capital intensity choice. As frictions disappear ($q(\theta_i) \to \infty$) this equation corresponds to the standard marginal benefit equals the marginal cost of capital. The second equation is a restatement of the free entry condition. The final equation arises since the marginal rate at which unemployed workers trade off higher wages against a higher probability of job finding should correspond to the rate at which firms are able to trade off higher wages against the probability of a worker finding employment given the zero profit condition.

4.3 Labour Market Equilibrium

Unemployment in sector i evolves according to the following differential equation,

$$\dot{u}_i = (\delta + d)(\pi_i - u_i) - h_i(\theta_i) u_i$$

so steady state unemployment in sector i is

$$u_i = \frac{(\delta + d)\pi_i}{\delta + d + h_i(\theta_i)}. \tag{11}$$

4.4 Equilibrium

We now turn to discuss equilibrium in the goods and the financial market. We consider a Blanchard-Yaari economy with actuarially fair annuities the rate of return that is provided to workers is higher than the rate of return that firms are able to provide.[13] In particular,

$$r = r_f + d. \tag{12}$$

Note that r_f is the interest rate at which firms discount profits. The value of firms in the closed economy will equal the value of assets so in the absence of death individuals would earn an interest rate of r_f on their capital holdings. However, with finite lifetimes and an annuity market, individuals

[13] This structure allows us a tractable way of redistributing assets of workers who die. Finite worker lifetimes are required for a steady state asset distribution to exist when workers have CARA utility.

die at rate d and their assets are redistributed to households in the form of higher interest payments. As a result households earn an actuarially fair interest rate of $r_f + d$.

As before consider two different equilibrium concepts in the goods market. In a closed economy the goods market clear in both sectors. With $p_2 = 1$ available as a normalisation, this allows us to determine the relative price of goods as well as the equilibrium interest rate. When we move to a small open economy we take the relative price of goods and the interest rate as exogenous. In this case the domestic demand may differ from domestic supply due to trade. Furthermore, the exogenous interest rate allows the economy to lend and borrow on international financial markets so that the level of domestic wealth held by households may vary from the equilibrium value of firms.

In a closed economy the aggregate demand for the good produced in each sector equals the supply. Define C_i the aggregate consumption of workers in sector i. Using our solution to the worker's decision problem (4) and aggregating across workers implies,

$$C_i = \frac{1}{P}\left(rA_i + (\pi_i - u_i)b_{ie} + u_i b_{iu}\right) + \pi\left(\frac{\rho + d - r}{\gamma r}\right) \tag{13}$$

It follows that,

$$(\pi_1 - u_1)f_1(k_1) = (1-\alpha)\left(\frac{p_1}{P}\right)^{-\eta} C_1 + (1-\alpha)\left(\frac{p_1}{P}\right)^{-\eta} C_2 + k_1 h_1(\theta_1)u_1 \tag{14}$$

$$(\pi_2 - u_2)f_2(k_2) = \alpha\left(\frac{p_2}{P}\right)^{-\eta} C_1 + \alpha\left(\frac{p_2}{P}\right)^{-\eta} C_2 + k_2 h_2(\theta_2)u_2 \tag{15}$$

so that the output produced in each sector equals consumption plus investment. Combining (14) and (15) and rearranging allows us to express the equilibrium relative price ratio,

$$\frac{(\pi_1 - u_1)f_1(k_1) - k_1 h_1(\theta_1)u_1}{(\pi_2 - u_2)f_2(k_2) - k_2 h_2(\theta_2)u_2} = \frac{1-\alpha}{\alpha}\cdot\left(\frac{p_1}{p_2}\right)^{-\eta}. \tag{16}$$

In an open economy the relative prices of goods as well as the equilibrium interest rate are both exogenous. This implies that the output produced by firms will vary from the level of consumption and investment by workers. In our experiments in Section 5 we examine how welfare is affected by deviations in relative prices from autarky while holding interest rates fixed.

In equilibrium we seek to find at the sectoral level the consumption choices of individuals in different states (b_{ie}, b_{iu}), aggregate asset holdings, A_i, the level of wages w_i, vacancies v_i, unemployment u_i and capital k_i, for each sector. These are determined by equations (5) through (11). In a closed economy we seek to find the relative price of goods and the equilibrium interest rate. These are determined by equations (12), (14) and (15). These endogenous variables are consistent with optimal behaviour by workers and firms, and with market clearing in financial and product markets. In the open economy relative prices and interest rates are exogenous so we solve equations (5) through (12) given p_1 and r_f noting that the normalisation $p_2 = 1$ is available.

5 Calibration and Results

We use this dynamic model to examine the quantitative implications of our theory. There are a number of potential interpretations that could be given to the sectors. Plausibly, a sector could be associated with a geographical area or with an industry. In these interpretations different regions or industries could be differentially exposed to price changes as a result of international trade. However, in our quantitative analysis we focus upon sectors as being associated with educational attainment. We view this as being consistent with a large volume of international trade literature that highlights countries differ in the ratio of skilled to unskilled workers and that the opening of an economy to trade will affect economic outcomes by altering relative product prices that vary by skill intensity in production.[14]

To be concrete, sector 1 will be associated with high skilled workers and sector 2 will be associated with low-skilled workers. In our calibration, parameters will be set so that high skilled workers have characteristics similar to workers with a college education or higher. Low skilled workers will have characteristics similar to workers with high school degrees or less.

We calibrate the model at an annual frequency. Some of the aspects of our calibration follow Uren (2017). We set d, the arrival rate of death to equal 0.025. This implies that the average life expectancy of a worker in our economy is 40 years. The average tenure of a worker in the middle of their career is approximately ten years (Farber (2008)), so we set $\delta = 0.1$. The relative size of these two education groups in the US suggests that $\pi_1 = 0.36$ and $\pi_2 = 0.64$ based on the 2010 Current Population Survey.

We assume that the matching function is Cobb-Douglas, $M(u_i, v_i) = \mu u_i^{\alpha_m} v_i^{1-\alpha_m}$, for $i \in \{1, 2\}$. We maintain a constant matching efficiency and elasticity of matching with respect to unemployment across sectors. We set $\alpha_m = 0.5$ which is within the range of estimates reported by Petrongolo and Pissarides (2001).[15] We assume a production of the form $f(k_i) = x_i k_i^{\alpha_k}$ for $i \in \{1, 2\}$. The

[14] Early studies such as Lawrence and Slaughter (1993) find small effects of trade on the skill premium. This evidence has been re-evaluated by Krugman (2008) and Ebenstein, Harrison, McMillan and Phillips (2014), among others.

[15] They obtain a range of estimated values from 0.12 to 0.81 for the elasticity of matching function with respect

labour market is imperfectly competitive. But with the production and matching technology and the wage formation mechanism we adopt, the labour share equals $1 - \alpha_k$. Hence we set $\alpha_k = 0.4$ and normalise $x_2 = 1$ while x_1 is calibrated to match differences in labour market outcomes across sectors. We also set the output of unemployed workers $y_u = 0$.

The elasticity of substitution between skilled and unskilled sectors is set so $\eta = 2$. This follows the empirical literature that relates the skill premium, which is defined as the ratio of college to high school wages, to the relative college to high school labour supply over time in the US. A widely used elasticity in the literature is 1.4 that is estimated by Katz and Murphy (1992). However, Acemoglu and Autor (2011) extend their work to more recent data and estimate a larger value of 2.5. Hence, our choice of $\eta = 2$ is within the reasonable range of the reported empirical values for this parameter.

We assume that the share of unskilled sector goods in the aggregate consumption is $\alpha = 0.6$. Finally, we set the coefficient of absolute risk aversion, $\gamma = 6$. This parameter implies a coefficient of relative risk aversion of 6.6 and 6.3 for employed skilled and unskilled workers when evaluated at the mean level of assets. Although this implies a large value for the coefficient of relative risk aversion, it is within the range of plausible parameter values.[16] In the Appendix we investigate how our results are altered by changes in these pre-specified parameters.

This leaves us with the following parameters to be determined: (ρ, x_1, μ). That is, the time discount rate, production efficiency in the skilled sector, and the efficiency of the matching function are to be calibrated to match empirical facts. We choose these parameters to minimise the sum of squared percentage deviation of the model implied variables from a set of empirically observed moments. We target a ratio of unskilled to skilled workers' wage of 0.56 that corresponds to the mean wages of workers with some college or less education to the workers with at least college education from the 2010 Current Population Survey. We target unemployment rates in the skilled and unskilled sectors of 2.8% and 5.8% respectively which is the average unemployment rates for the defined groups over 1995-2015. Finally, we target investment to GDP ratio of 0.2 and a real interest rate of 5% for firms.

to unemployment using various methods. Our choice for this parameter is in the middle of the range of parameters observed in the literature. Shimer (2005), for instance, sets this parameter equals to 0.72 which is close to the top of the estimates reported in the Petrongolo and Pissarides (2001), and Borowczyk-Martins, Jolivet and Postel-Vinay (2013) estimate this parameter to be 0.3 which is close to the bottom range of estimates reported by Petrongolo and Pissarides (2001).

[16] Kimball, Sahm, and Shapiro (2008) use survey responses to hypothetical income lotteries in the Health and Retirement Study to estimate that the coefficient of relative risk aversion has a median of 6.3 and a mean of 8.2 (see table 4) of their paper.

Table 1: **Parameter values**

Parameter	Description	Value
Pre-specified Parameters		
d	Rate of death	0.025
δ	Job destruction rate	0.1
π	Size of a sector 1 & 2	$\{0.36, 0.64\}$
α_m	Matching function elasticity with respect to u	0.5
y_u	Unemployment production	0
α_k	Elasticity of output with respect to capital	0.4
α	Unskilled sector's share in CES consumption	0.6
γ	Degree of Risk Aversion	6
η	CES elasticity	2
x_2	Sector 2 (education less than college degree) productivity	1
p_2	Price of sector 2's good	1
Calibrated Parameters		
x_1	Sector 1 (education at least college degree) productivity	2.3
μ	Match efficiency	1.06
ρ	Time discount rate	-0.04

Table 1 summarises the complete list of pre-specified parameters along with the endogenously calibrated parameter values in this section.[17] The result of the calibration is presented in Table 2. Overall, the model matches the targeted moments quite well.

Table 2: **Steady State Results**

Targets	**Model**	**Data**
Unskilled to skilled wage ratio	0.47	0.56
Skilled sector unemployment rate (%)	3.5	2.8
Unskilled sector unemployment rate (%)	4.2	5.8
Risk free interest rate (%)	5.5	5
Investment-gdp ratio	0.24	0.2

We take our baseline calibration and examine the welfare effects of changes in relative prices that arise from moving from autarky into a trade equilibrium. Our method is as follows: our baseline calibration solves for the equilibrium of an economy in autarky. We then consider a permanent and unanticipated change in relative prices due to trade but hold the real interest rate fixed. We solve for the steady state of an economy as international trade changes the relative price of goods in the economy and allow the capital, vacancy decisions and wages of firms to instantly adjust to price changes.

[17] Note that since the calibrated model is overidentified, it is difficult to assign each parameter to a specific moment. However, x_1 is the main parameter that is responsible for the relative wages of sectors and μ helps pin down the sectoral unemployment rates.

Figures 2, 3, and 4 map out the changes in wages, unemployment and capital per worker in response to changes in relative prices. Unsurprisingly, an increase in the relative price of output produced by skilled labour leads to an increase in the real wage of skilled labour and a decrease in the real wage of unskilled labour. Almost all of this change arises due to changes in relative output prices since there are only modest changes in capital intensity in each sector. There are also changes in the unemployment rate, with unemployment increasing in the unskilled sector and decreasing in the skilled sector.

As before, we examine welfare by considering the compensating variation. We calculate the change in real assets needed to keep an individual indifferent between a trade and autarky equilibrium. In particular, we take an individual with real asset level a/P and employment status $j \in \{e, u\}$ in sector i in autarky. We ask what level of real wealth, a'/P', would this person need to achieve the same level of expected lifetime utility in a trade equilibrium in which relative prices were given by p_1' and corresponding ideal price index P'. This change in real asset levels $(a'/P' - a/P)$ is the compensating variation for this particular individual. We sum over all individuals to calculate an aggregate compensating variation.[18] If this value is negative, then we can compensate all workers and still have resources remaining. Conversely, if positive then resources must be imported to compensate individuals for the change in relative prices.

The calculation of the individual compensating variation is easily done since we have an explicit solution for the value function given by (3). Let superscripts a and t define the equilibrium values in the autarky and the trade equilibrium, respectively (ie. $V_j^a(a)$ and $V_j^t(a)$ are the values associated with employment status j, asset level a, in an autarky and trade equilibrium). We then equate $V_j^a(a) = V_j^t(a')$ for any given a to find the implied a' that yields the same expected utility under our new trade prices. With our assumption that interest rates are unchanged the compensating variation for a worker in sector i becomes,

$$\frac{a'}{P'} - \frac{a}{P} = \frac{1}{r}\left(\frac{b_{ij}^a}{P} - \frac{b_{ij}^t}{P'}\right)$$

Note that with unchanged interest rates the intial level of assets is unrelated to the compensating variation. It therefore follows that the magnitude of the aggregate compensating variation does not depend on the distribution of wealth.[19]

[18]There are actually two reasonable aggregations to consider. First, we could compensate only the workers that are alive when trade integration occurs. Second, we can compensate not only living workers but also workers who are born into a trade equilibrium rather than in autarky. The quantitative difference between these two concepts is small, so we focus upon the setting of compensating only workers alive when integration occurs.

[19]If interest rates adjust the compensating variation does depend upon an individual's asset level but calculating the net compensating variation remains feasible since we have an expression for both the value function and for the steady state joint distribution of employment states and assets. See Uren (2017) for a more detailed description of the steady state distribution of assets.

The effect of changes in relative prices upon aggregate welfare is displayed in Figure 5. The key result is that for small increase in the relative price of output produced by skilled workers, the aggregate compensating variation is positive. This implies that the economy needs a positive inflow of resources to compensate for opening up to trade. For this calibration, a small price change is approximately equal to an increase in p_1 of about 3.4 per cent. To put this into some perspective, Amiti, Dai, Feenstra and Romalis (2017) estimate that China's entry into the WTO reduced the US manufacturing price index by around 7.6 per cent. The key thing to note is, in contrast to standard trade models, that the inclusion of risk aversion and employment risk implies that small changes in relative prices may lead to a welfare loss.

The breakdown of compensating variation required by sector is provided in Figure 6. Unsurprisingly, the sector that features a rise in relative price gains from trade and the workers in the sector that face a fall in relative prices lose if uncompensated.

Ranjan (2016) identifies that changes in prices due to trade may lead to a less efficient allocation of resources in environments in which workers are risk averse. If a change in relative prices increases the level of risk in a labour market, firms may respond by offering, as described by Acemoglu and Shimer (1999), market insurance. In our setting, these potential changes in resource allocation are not driving our results. In our baseline calibration we allow capital and vacancies to adjust. Alternatively, we can consider how welfare changes assuming that firms do not adjust capital or vacancies but rather only adjust wages to ensure the zero profit condition is satisfied. The compensating variation for each sector are shown in Figure 6 and demonstrate that the welfare consequences of trade are not driven by changes in resource allocation in response to trade.

6 Conclusion

Standard trade theory suggests that opening up to trade is welfare improving. We deviate from a standard model in two ways. First, we assume frictions exist in the labour market. This prevents workers from moving across sectors as well as generating labour market risk within sectors. Second, we assume that financial markets are incomplete. Workers have access to annuities that pay a constant rate of return over time but are not able to perfectly insure themselves against labour market shocks. In this setting, the process of opening a small open economy to international trade can reduce welfare. In our quantitative exercise we find that for small changes in relative prices that favour workers in the skilled sector that there are essentially no gains from trade. Gains from trade exist if changes in relative prices favour the unskilled, or if there are large changes in prices that favour the skilled.

Our work also highlights the importance of the proper functioning of credit markets in generating potential gains from trade. In our dynamic model without asset markets, for large enough values of risk aversion it was essentially impossible for an increase in the price of output of the skilled sector to increase welfare. In contrast, when workers had similar degrees of risk aversion but had access to a Blanchard-Yaari financial market, the potential for welfare gains to arise from trade occurred for relatively small price changes.

Our quantitative model has some special features. In particular, there is no natural borrowing constraint. With CARA preferences we allow individuals to borrow a large amount and repay debt by permitting negative levels of consumption. In this sense financial markets are not complete but they may provide a greater degree of insurance than what we may expect in reality. We suspect that if individuals had CRRA utility and were subject to a natural borrowing constraint that individuals would be less able to self-insure and that the loss in welfare would exist for larger price changes. In reality, we also expect that the degree of access to financial markets may vary by sector. High skilled workers with greater education may be better able to access financial markets. This is at least consistent with the work of Kaplan, Violante and Weidner (2014) who find that hand-to-mouth consumers have less education. In such a setting, we would expect that range over which an increase in the price of output intensive in skilled labour creates a welfare loss to increase.

It would also be interesting to consider the role of economic policy in this environment. For example, what impact do labour market or trade policies have in mitigating the potential negative effects of trade. Standard labour market policies such as unemployment insurance would have some impact upon the welfare gains from trade and it would be of interest to see how they interact. We leave these issues as areas for future research.

7 Appendix

7.1 Proof of Proposition 1

The results in this Section modify the discussion in Dixit and Norman (1980) to our particular setting. Without uncertainty let the endowment of an individual in sector 1 and sector 2 equal x_1 and x_2 respectively. The equilibrium consumption of worker from sector i of good j in autarky and trade equilibrium is denoted c_{ij}^a and c_{ij}^t respectively. The amount of expenditure a worker can undertake in autarky is $p^a x_1$ if employed sector 1 or x_2 if employed in sector 2. In a trade equilibrium $p^t x_1$ in sector 1 or x_2 in sector is the amount of expenditure a worker can take.

Consider the following transfer system to workers:

$$
\begin{aligned}
T_1 &= (p^t - p^a)c_{11}^a - (p^t x_1 - p^a x_1) \\
T_2 &= (p^t - p^a)c_{21}^a
\end{aligned}
$$

to compensate a move from autarky to a trade equilibrium. These transfers are feasible and allow individuals to purchase their original autarky bundles at the new trade prices.

The relevant budget constraints are:

$$
\begin{aligned}
p^a c_{11}^a + c_{12}^a &= p_1^a x_1 \\
p^a c_{21}^a + c_{22}^a &= x_2 \\
p^t c_{11}^t + c_{12}^t &= p_1^t x_1 \\
p^t c_{21}^t + c_{22}^t &= x_2
\end{aligned}
$$

And in our closed economy we know that the total endowment equals total consumption for each good.

$$
\begin{aligned}
\pi x_1 &= \pi c_{11}^a + (1-\pi)c_{21}^a \\
(1-\pi)x_2 &= \pi c_{12}^a + (1-\pi)c_{22}^a
\end{aligned}
$$

Are the transfers feasible without net resources into the economy?

$$
\begin{aligned}
\pi T_1 + (1-\pi)T_2 &= \pi((p^t - p^a)c_{11}^a - (p^t x_1 - p^a x_1)) + (1-\pi)(p^t - p^a)c_{21}^a \\
&= \pi((p^t - p^a)c_{11}^a + (c_{12}^a - c_{12}^a) - (p^t x_1 - p^a x_1)) + (1-\pi)((p^t - p^a)c_{21}^a + (c_{22}^a - c_{22}^a)) \\
&= \pi((p^t c_{11}^a + c_{12}^a - p^t x_1) - (p^a c_{11}^a + c_{12}^a - p^a x_1)) + (1-\pi)((p^t c_{21}^a + c_{22}^a - x_2) \\
&- (p^a c_{12}^a + c_{22}^a - x_2)) \\
&= \pi(p^t c_{11}^a + c_{12}^a - p^t x_1) + (1-\pi)(p^t c_{21}^a + c_{22}^a - x_2)
\end{aligned}
$$

The budget constraint for the economy allows us to set the above equal to zero. So this transfer system is feasible.

The final thing to check is that the original bundles consumed are now affordable to workers under the new trade prices. First in sector 1, note their total expenditure after transfer is

$$
\begin{aligned}
p^t x_1 + (p^t - p^a)c_{11}^a - (p^t x_1 - p^a x_1) &= (p^t - p^a)c_{11}^a + p^a x_1 \\
&= p^t c_{11}^a + c_{12}^a + \underbrace{(p^a x_1 - p^a c_{11}^a - c_{12}^a)}_{0}
\end{aligned}
$$

which confirms that the autarky bundle is affordable under trade prices and for the second sector we have a similar calculation.

$$x_2 + (p^t - p^a)c_{21}^a = x_2 + p^t c_{12}^a + c_{22}^a - p^a c_{12}^a - c_{22}^a$$
$$= p^t c_{21}^a + c_{22}^a$$

So the transfer is feasible and allows workers to consume their autarky consumption bundle.

7.2 Proof of Proposition 2

In the non-stochastic setting the transfer required was:

$$T_1 = (p_1^t - p_1^a)c_{11}^a - (w_1^t - w_1^a)$$
$$T_2 = (p_1^t - p_1^a)c_{21}^a$$

In the case with uncertainty the corresponding transfer is

$$T_1 = (p^t - p^a)Ec_{11}^a - E(p^t x_1 - p^a x_1)$$
$$T_2 = (p^t - p^a)Ec_{21}^a$$

where Ec_{ij}^a is the expected consumption of good j by consumer from sector i in the autarky equilibrium and x_i^t or x_i^a is output in trade or autarky equilibrium for a worker in sector i.

Two steps - first, show that this generates the correct level of utility and second, show that this set of transfers is feasible.

Let's think about feasibility. We have budget constraints that apply:

$$p^a c_{11}^a + c_{12}^a = p_1^a x_1$$
$$p^a c_{21}^a + c_{22}^a = x_2$$
$$p^t c_{11}^t + c_{12}^t = p_1^t x_1$$
$$p^t c_{21}^t + c_{22}^t = x_2$$

and similar equations for workers with low level of output except that $y's$ replaces the $x's$. They imply that if we take expectations

$$E(p^a c_{11}^a + c_{12}^a) = Ep^a x_1$$
$$E(p^a c_{21}^a + c_{22}^a) = Ex_2$$
$$E(p^t c_{11}^t + c_{12}^t) = Ep^t x_1$$
$$E(p^t c_{21}^t + c_{22}^t) = Ex_2$$

and for our aggregate economy,

$$\pi E(x_1) = \pi E(c_{11}^a) + (1-\pi)E(c_{21}^a)$$
$$\pi E(x_2) = \pi E(c_{12}^a) + (1-\pi)E(c_{22}^a)$$

The transfer scheme implies that in aggregate,

$$\begin{aligned}
\pi T_1 + (1-\pi)T_2 &= \pi((p^t - p^a)Ec_{11}^a - E(p^t x_1 - p^a x_1)) + (1-\pi)(p^t - p^a)Ec_{21}^a \\
&= \pi((p^t Ec_{11}^a + Ec_{12}^a - Ep^t x_1) - (Ep^a c_{11}^a + Ec_{12}^a - Ep^a x_1)) \\
&+ (1-\pi)((p^t Ec_{12}^a + Ec_{22}^a - Ex_2) - (p^a Ec_{21}^a + Ec_{22}^a - Ex_2)) \\
&= \pi(Ep^t c_{11}^a - Ep^t x_1) + (1-\pi)Ep^t c_{12}^a + \pi Ec_{12}^a + (1-\pi)(Ec_{22}^a - Ex_2)
\end{aligned}$$

which when rearranged implies that the net transfer equals zero since the feasibility constraints are satisfied.

The second step is to show that this transfer mechanism yields equal utility as in autarky. The expected budget constraint that an individual in sector 1 faces in a trade equilibrium with transfer is:

$$\begin{aligned}
&Ep^t x_1 + (p^t - p^a)Ec_{11}^a - E(p^t x_1 - p^a x_1) \\
&= p^t Ec_{11}^a - p^a Ec_{11}^a + Ep^a x_1 \\
&= p^t Ec_{11}^a + Ec_{12}^a - Ep^a c_{11}^a - Ec_{12}^a + E(p^a)x_1 \\
&= p^t Ec_{11}^a + Ec_{12}^a
\end{aligned}$$

which implies that in expectation, the autarky bundle can be purchased under this transfer system. For an individual in sector 2:

$$\begin{aligned}
&Ex_2 + (p^t - p^a)Ec_{21}^a \\
&= p^t Ec_{21}^a + Ec_{22}^a - p^a Ec_{21}^a - Ec_{22}^a + Ex_2 \\
&= p^t Ec_{21}^a + Ec_{22}^a
\end{aligned}$$

which implies in expectation that the autarky bundle is feasible for sector 2 individuals given the transfer.

7.3 Proof of Proposition 3

There is trade within an economy and that defines prices p_1 and p_2 such that markets clear. This requires that

$$\frac{1-\alpha}{\alpha}\left(\frac{p_1}{p_2}\right)^{-\eta} = \frac{\pi_1(h_1\bar{x}_1 + (1-h_1)\underline{x}_1)}{(1-\pi)(h_2\bar{x}_2 + (1-h_2)\underline{x}_2)} = \frac{\pi E[X_1]}{(1-\pi)E[X_2]}$$
$$P = ((1-\alpha)p_1^{1-\eta} + \alpha)^{1/(1-\eta)}$$

Further we can find the compensating variation to make sure that an individual is indifferent. To do so, let

$$U_j = h_j U(p_j\bar{x}_j/P + \tau_j) + (1-h_j)U(p_j\underline{x}_j/P + \tau_j)$$

define the amount of transfer, τ_j, required to return this person back to their autarky level of utility (defined as U_j). The implicit function theorem implies,

$$h_j U'(p_j\bar{x}_j/P)\frac{d(p_j x_j/P)}{dp_1}dp_1 + (1-h_j)U'(p_j\underline{x}_j/P)\frac{d(p_j\underline{x}_j/P)}{dp_1}dp_1$$
$$+ \left(h_j U'(p_j\bar{x}_j/P) + (1-h_j)U'(p_j\underline{x}_j/P)\right)d\tau_j = 0$$
$$\rightarrow \frac{d\tau_j}{dp_1} = -\frac{h_j U'(p_j\bar{x}_j/P)\frac{d(p_j\bar{x}_j/P)}{dp_1} + (1-h_j)U'(p_j\underline{x}_j/P)\frac{d(p_j\underline{x}_j/P)}{dp_1}}{h_j U'(p_j\bar{x}_j/P) + (1-h_j)U'(p_j\underline{x}_j/P)}$$

And evaluating the derivatives of real wage with respect to p_1 we find,

$$\frac{dp_1\bar{x}_1/P}{dp_1} = \alpha\bar{x}_1((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$
$$\frac{dp_1\underline{x}_1/P}{dp_1} = \alpha\underline{x}_1((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$
$$\frac{dp_2\bar{x}_2/P}{dp_1} = -(1-\alpha)p_1^{-\eta}\bar{x}_2((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$
$$\frac{dp_2\underline{x}_2/P}{dp_1} = -(1-\alpha)p_1^{-\eta}\underline{x}_2((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$

A number of special cases arise. If $h_j = 1$ then we find

$$\frac{d\tau_j}{dp_1} = \frac{dp_j\bar{x}_j/P}{dp_1}$$

and this implies that the aggregate transfer is

$$\pi\frac{dp_1\bar{x}_1/P}{dp_1} + (1-\pi)\frac{dp_2\bar{x}_2/P}{dp_1} = (\pi\alpha\bar{x}_1 - (1-\pi)(1-\alpha)p_1^{-\eta}\bar{x}_2)((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$
$$= (\pi\alpha\bar{x}_1 - (1-\pi)(1-\alpha)\frac{\alpha}{1-\alpha}\left(\frac{\pi\bar{x}_1}{(1-\pi)\bar{x}_2}\right)\bar{x}_2)((1-\alpha)p_1^{1-\eta} + \alpha)^{-(2-\eta)/(1-\eta)}$$
$$= 0$$

where we use $\pi = \pi_1$ to simplify notation. We should be able to do the same thing with risk neutral case. Finally, in a setting with both risk aversion and luck the aggregate transfer required to compensate individuals is

$$\pi \frac{d\tau_1}{dp_1} + (1-\pi)\frac{d\tau_2}{dp_1}$$

We can express this as:

$$-\pi \frac{h_1 U'(p_1\bar{x}_1/P)\frac{d(p_1\bar{x}_1/P)}{dp_1} + (1-h_1)U'(p_1\underline{x}_1/P)\frac{d(p_1\underline{x}_1/P)}{dp_1}}{h_1 U'(p_1\bar{x}_1/P) + (1-h_1)U'(p_1\underline{x}_1/P)}$$
$$-(1-\pi)\frac{h_2 U'(p_2\bar{x}_2/P)\frac{d(p_2\bar{x}_2/P)}{dp_1} + (1-h_2)U'(p_2\underline{x}_2/P)\frac{d(p_2\underline{x}_2/P)}{dp_1}}{h_2 U'(p_2\bar{x}_2/P) + (1-h_2)U'(p_2\underline{x}_2/P)}$$

Let's look at the first term,

$$-\pi \frac{(h_1 U'(p_1\bar{x}_1/P)\alpha\bar{x}_1 + (1-h_1)U'(p_1\underline{x}_1/P)\alpha\underline{x}_1)((1-\alpha)p_1^{1-\eta}+\alpha)^{-(2-\eta)/1-\eta}}{h_1 U'(p_1\bar{x}_1/P) + (1-h_1)U'(p_1\underline{x}_1/P)}$$

and the second term,

$$-(1-\pi)\frac{(-(1-\alpha)p_1^{-\eta}\bar{x}_2 h_2 U'(p_2\bar{x}_2/P) - (1-\alpha)p_1^{-\eta}\underline{x}_2(1-h_2)U'(p_2\underline{x}_2/P))((1-\alpha)p_1^{1-\eta}+\alpha)^{-(2-\eta)/1-\eta}}{h_2 U'(p_2\bar{x}_2/P) + (1-h_2)U'(p_2\underline{x}_2/P)}$$

We should be able to say when the sum is equal to or greater than or less than zero. Let's factor out $((1-\alpha)p_1^{1-\eta}+\alpha)^{-(2-\eta)/1-\eta}$. Then we find,

$$-\pi \frac{(h_1 U'(p_1\bar{x}_1/P)\alpha\bar{x}_1 + (1-h_1)U'(p_1\underline{x}_1/P)\alpha\underline{x}_1)}{h_1 U'(p_1\bar{x}_1/P) + (1-h_1)U'(p_1\underline{x}_1/P)} + (1-\pi)\frac{(1-\alpha)p_1^{-\eta}(\bar{x}_2 h_2 U'(p_2\bar{x}_2/P) + \underline{x}_2(1-h_2)U'(p_2\underline{x}_2/P))}{h_2 U'(p_2\bar{x}_2/P) + (1-h_2)U'(p_2\underline{x}_2/P)}$$

Here we can see with risk neutral agents that this simplifies to

$$\begin{aligned} &-\pi(h_1\alpha\bar{x}_1 + (1-h_1\alpha\underline{x}_1)) + (1-\pi)(1-\alpha)p_1^{-\eta}(\bar{x}_2 h_2 + \underline{x}_2(1-h_2)) \\ &= -\pi\alpha E[X_1] + (1-\pi)(1-\alpha)\frac{\alpha}{1-\alpha}\frac{\pi E[X_1]}{(1-\pi)E[X_2]}(\bar{x}_2 h_2 + \underline{x}_2(1-h_2)) = 0 \end{aligned}$$

If risk aversion exists then using $p_2 = 1$ and our equilibrium relative price, the above becomes

$$-\pi \frac{(h_1 U'(p_1\bar{x}_1/P)\alpha\bar{x}_1 + (1-h_1)U'(p_1\underline{x}_1/P)\alpha\underline{x}_1)}{h_1 U'(p_1\bar{x}_1/P) + (1-h_1)U'(p_1\underline{x}_1/P)} + (1-\pi)\frac{\alpha\frac{\pi E[X_1]}{E[X_2]}(\bar{x}_2 h_2 U'(p_2\bar{x}_2/P) + \underline{x}_2(1-h_2)U'(p_2\underline{x}_2/P))}{h_2 U'(p_2\bar{x}_2/P) + (1-h_2)U'(p_2\underline{x}_2/P)}$$

The sign of this depends upon the sign of

$$-\pi \frac{(h_1 U'(p_1\bar{x}_1/P)\bar{x}_1 + (1-h_1)U'(p_1\underline{x}_1/P)\underline{x}_1)}{h_1 U'(p_1\bar{x}_1/P) + (1-h_1)U'(p_1\underline{x}_1/P)} + (1-\pi)\frac{\frac{\pi E[X_1]}{(1-\pi)E[X_2]}(\bar{x}_2 h_2 U'(p_2\bar{x}_2/P) + \underline{x}_2(1-h_2)U'(p_2\underline{x}_2/P))}{h_2 U'(p_2\bar{x}_2/P) + (1-h_2)U'(p_2\underline{x}_2/P)}$$

The sign of the above is the same as the sign of,

$$-\frac{E[U_1'(C)\cdot X_1]}{E[U_1'(C)]E[X_1]} + \frac{E[U_2'(C)\cdot X_2]}{E[U_2'(C)]E[X_2]}$$

which reveals the sign of $\sum_j \pi_j \frac{d\tau_j}{dp_1}$ and generates our result.

Proof of Proposition 4

Proposition 4 follows a similar direct approach of Proposition 3. The value of an employed worker in sector i

$$\frac{\delta u(\tau_{i,e}) + (h_i + \rho)u(w_i'/P' + \tau_{i,e})}{\rho(\delta + h_i + \rho)}$$

Applying the implicit function theorem at $p = p_a$ and $\tau = 0$ implies that for an employed worker in sector i that utility is constant if

$$\frac{d\tau_{i,e}}{dp_1}|_{p=p^a,\tau=0} = -\frac{(\rho + h_i)u'(w_i/P)\frac{d(w_i/P)}{dp_1}}{(\rho + h_i)u'(w_i/P) + \delta u'(0)}$$

and for the unemployed

$$\frac{d\tau_{i,u}}{dp_1}|_{p=p^a,\tau=0} = -\frac{h_i u'(w_i/P)\frac{d(w_i/P)}{dp_1}}{h_i u'(w_i/P) + (\delta + \rho)u'(0)}$$

and the aggregate transfer for a marginal change in relative prices

$$\pi\left(\frac{d\tau_{1,e}}{dp}(1 - u_1) + \frac{d\tau_{1,u}}{dp}u_1\right) + (1 - \pi)\left(\frac{d\tau_{2,e}}{dp_1}(1 - u_2) + \frac{d\tau_{2,u}}{dp_1}u_2\right)$$

Note that given our CES structure the ideal price index P is given by (2) and the fact that $w_1 = p_1 x_1$ and $w_2 = x_2$ using our normalisation implies,

$$\begin{aligned}
\frac{d(w_1/P)}{dp_1} &= \alpha(\alpha + (1-\alpha)p_1^{1-\eta})^{-\frac{2-\eta}{1-\eta}}x_1 \\
\frac{d(w_2/P)}{dp_1} &= -(1-\alpha)p_1^{-\eta}(\alpha + (1-\alpha)p_1^{1-\eta})^{-\frac{2-\eta}{1-\eta}}x_2
\end{aligned}$$

Evaluating we find that the sign of the aggregate transfer required to compensate everyone for an increase in p_1 is the same as the sign of

$$\begin{aligned}
&\pi\alpha x_1\left(-\frac{(\rho + h_1)u'(w_1/P)}{(\rho + h_1)u'(w_1/P) + \delta u'(0)}(1 - u_1) - \frac{h_1 u'(w_1/P)}{h_1 u'(w_1/P) + (\delta + \rho)u'(0)}u_1\right) + \\
&(1-\pi)(1-\alpha)x_2 p_1^{-\eta}\left(\frac{(\rho + h_2)u'(w_2/P)}{(\rho + h_2)u'(w_2/P) + \delta u'(0)}(1 - u_2) + \frac{h_2 u'(w_2/P)}{h_2 u'(w_2/P) + (\delta + \rho)u'(0)}u_2\right)
\end{aligned}$$

The equilibrium price ratio in autarky is

$$p_1^{-\eta} = \frac{\alpha}{1-\alpha}\cdot\frac{\pi}{1-\pi}\cdot\frac{(1-u_1)x_1}{(1-u_2)x_2}$$

Using this in the above implies that the sign of the aggregate transfer equals the sign of the following:

$$\begin{aligned}
&\left(-\frac{(\rho + h_1)u'(w_1/P)}{(\rho + h_1)u'(w_1/P) + \delta u'(0)} - \frac{h_1 u'(w_1/P)}{h_1 u'(w_1/P) + (\delta + \rho)u'(0)}\cdot\frac{u_1}{1 - u_1}\right) + \\
&\left(\frac{(\rho + h_2)u'(w_2/P)}{(\rho + h_2)u'(w_2/P) + \delta u'(0)} + \frac{h_2 u'(w_2/P)}{h_2 u'(w_2/P) + (\delta + \rho)u'(0)}\cdot\frac{u_2}{1 - u_2}\right)
\end{aligned}$$

Using that $u_i = \delta/(\delta + h_i)$ we find the sign of the aggregate transfer is

$$\left(-\frac{(\rho + h_i)u'(w_i/P)}{(\rho + h_i)u'(w_i/P) + \delta u'(0)} - \frac{\delta u'(w_i/P)}{h_i u'(w_i/P) + (\delta + \rho)u'(0)}\right) + \left(\frac{(\rho + h_2)u'(w_2/P)}{(\rho + h_2)u'(w_2/P) + \delta u'(0)} + \frac{\delta u'(w_2/P)}{h_2 u'(w_2/P) + (\delta + \rho)u'(0)}\right)$$

7.4 Robustness Checks

In this section we examine the sensitivity of our results to different parameter configurations, i.e γ, α, α_m and η. In deriving the results, we re-calibrate the model by varying the values of each single parameters as specified in the table and hold fixed the other parameters to their baseline calibrated values.

Table 3 reports the results. The first row of the table reports the result of the baseline calibration. We showed that with the calibrated parameters, i.e $\gamma = 6, \alpha = 0.6, \alpha_m = 0.5, \eta = 2$, the required size of price deviation for obtaining negative welfare loss following trade openness was 3.4%. In other words, if opening up to trade increases the price of goods produced by skilled workers by 3.4%, relative to the autarky equilibrium, the economy needs a positive transfer of resources to compensate all individuals. The second to fourth rows of the table, in addition, show the sensitivity of our results to the variation in the degree of risk aversion, CES elasticity, elasticity of matching function with respect to unemployment, and share of unskilled workers in the CES consumption.

Table 3: **Sensitivity Analysis**

Parameter	**Max p_1 increase that generates welfare loss**
Baseline	3.4 %
$\gamma = 5$	2.03 %
$\gamma = 7$	4.48 %
$\eta = 1.5$	3.87 %
$\eta = 2.5$	1.9 %
$\alpha_m = 0.3$	1.86 %
$\alpha_m = 0.7$	4.21 %
$\alpha = 0.5$	2.05 %

In the second panel, we investigate the effect of changing the risk aversion parameter, γ, by one unit around its baseline value. The required price deviation for welfare loss increases with the risk aversion. Increasing the risk aversion in the model by one unit, $\gamma = 7$, increases the required size of deviation in price of goods produced in the skilled sector for generating the negative welfare by about one percentage point.

In the third row, we experiment varying the elasticity of CES consumption aggregator. We first set this parameter to $\eta = 1.5$ which is closed to the findings of the Katz and Murphy (1992). We then increase this value to 2.5 which follows from the more recent study by Acemoglu and Autor (2011). The result of this experiment shows that with an increase in the elasticity of consumption aggregator, the required deviation in the price of goods produced by skilled worker for generating the negative welfare declines.

Next, we experiment the sensitivity of our results with changing the elasticity of matching function with respect to unemployment. We re-calibrate the model and report the results for $\alpha_m = 0.3$ and $\alpha_m = 0.7$. The former follows from the work by Borowczyk-Martins, Jolivet and Postel-Vinay (2013) and the latter is close to the value used by Shimer (2005). We find that with increase in the elasticity of matching function with respect to unemployment, the required deviation in price of goods in the skilled sector for generating the welfare loss increases following trade openness.

Finally we examine the sensitivity of results to changes in the relative share of goods produced by unskilled workers, α. With $\alpha = 0.5$, which implies equal shares of workers types in the aggregate consumption, the required price increase for generating negative welfare decreases to 2.05%.

References

Acemoglu, Daron & David H. Autor. 2011. Skill, Tasks and Technologies: Implications for Employment and Earnings. In *Handbook of Labor Economics, Vol 4B*, ed. David Card & Orley Ashenfelter. Elsevier chapter 12, pp. 1043–1171.

Acemoglu, Daron & Robert Shimer. 1999. "Efficient Unemployment Insurance." *The Journal of Political Economy* 107(5):893–928.

Amiti, Mary, Mi Dai, Robert C. Feenstra & John Romalis. 2017. "How Did China's WTO Entry Benefit U.S. Consumers?" *NBER Working Paper 23487* .

Arkolakis, Costas, Arnaud Costinot & Andres Rodriguez-Clare. 2012. "New Trade Models, Same Old Gains?" *The American Economic Review* 102(1):94–130.

Artuç, Erhan, Shubham Chaudhuri & John McLaren. 2010. "Trade Shocks and Labor Adjustment: A Structural Empirical Approach." *The American Economic Review* 100(3):1008–1045.

Autor, David H., David Dorn & Gordon H. Hanson. 2013. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review* 103(6):2121–2168.

Blanchard, Olivier & Stanley Fischer. 1989. *Lectures in Macroeconomics.* MIT Press.

Borowczyk-Martins, Daniel, Gregory Jolivet & Fabien Postel-Vinay. 2013. "Accounting for endogeneity in matching function estimation." *Review of Economic Dynamics* 16:440–451.

Broda, Christian & David E. Weinstein. 2006. "Globalization and the Gains from Variety." *The Quarterly Journal of Economics* 121(2):541–585.

Davidson, Carl, Lawrence Martin & Steven Matusz. 1987. "Search, Unemployment, and the Production of Jobs." *The Economic Journal* 97(388):857–876.

Davidson, Carl, Lawrence Martin & Steven Matusz. 1988. "The Structure of Simple General Equilibrium Models with Frictional Unemployment." *The Journal of Political Economy* 96(6):1267–1293.

Dix-Carneiro, Rafael. 2014. "Trade Liberalization and Labor Market Dynamics." *Econometrica* 82(3):825–885.

Dixit, Avinash. 1989. "Trade and Insurance with Imperfectly Observed Outcomes." *The Quarterly Journal of Economics* 104(1):195–203.

Dixit, Avinash & Victor Norman. 1980. *Theory of International Trade.* Cambridge University Press.

Evenstein, Avraham, Ann Harrison, Margaret McMillan & Shannon Phillips. 2014. "Estimating the Impact of Trade and Offshoring on American Workers Using the Current Population Surveys." *The Review of Economics and Statistics* 46(4):581–595.

Farber, Henry S. 2008. "Employment Insecurity: The Decline in Worker-Firm Attachment in the United States." *Working Paper, Princeton University, Industrial Relations Section* .

Feenstra, Robert C. & David E. Weinstein. 2010. "Globalization, Markups and U.S. Welfare." *NBER Working Paper 15749* .

Gollier, Christian. 2001. *The Economics of Risk and Time.* MIT Press.

Grossman, Gene M. & Elhanan Helpman. 1991. *Innovation and Growth in the Global Economy.* MIT Press.

Guren, Adam, David Hémous & Morten Olsen. 2015. "Trade Dynamics with sector-specific human capital." *Journal of International Economics* 97:126–147.

Helpman, Elhanan & Oleg Itskhoki. 2010. "Labour Market Rigidities, Trade and Unemployment." *The Review of Economic Studies* 77:1100–1137.

Helpman, Elhanan, Oleg Itskhoki & Stephen Redding. 2010. "Inequality and Unemployment in a Global Economy." *Econometrica* 78(4):1239–1283.

Kaplan, Greg, Giovanni L. Violante & Justin Weidner. 2014. "The Wealthy Hand-to-Mouth." *Brookings Papers on Economic Activity* pp. 77–138.

Katz, Lawrence F. & Kevin M. Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *The Quarterly Journal of Economics* 107(1):35–78.

Kimball, Miles S., Claudia R. Sahm & Matthew D. Shapiro. 2008. "Imputing Risk Tolerances From Survey Responses." *Journal of the American Statistical Association* 103(483):1028–1038.

Krugman, Paul. 2008. "Trade and Wages, Reconsidered." *Brookings Papers on Economic Activity* pp. 103–137.

Krusell, Per, Toshihiko Mukoyama & Ayşegül Şahin. 2010. "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations." *Review of Economic Studies* 77(4):1477–1507.

Lawrence, Robert Z. & Matthew J. Slaughter. 1993. "International Trade and American Wages in the 1980s: Giant Sucking Sound or Small Hiccup?" *Brookings Papers on Economic Activity: Microeconomics* pp. 161–226.

Moen, Espen R. 1997. "Competitive Search Equilibrium." *The Journal of Political Economy* 105(2):385–411.

Newbery, David M. G & Joseph E. Stiglitz. 1984. "Pareto Inferior Trade." *Review of Economic Studies* 51(1):1–12.

Petrongolo, Barbara & Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39(2):390–431.

Ranjan, Priya. 2016. "Globalization and risk averse workers: The roles of labour market and trade policies." *Journal of International Economics* 103:64–79.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *The American Economic Review* 95(1):25–49.

Uren, Lawrence. 2017. "The Redistributive Role of Unemployment Benefits." mimeo, University of Melbourne.

Wang, Neng. 2007. "An Equilibrium Model of Wealth Distribution." *Journal of Monetary Economics* 54(7):1882–1904.

Figures

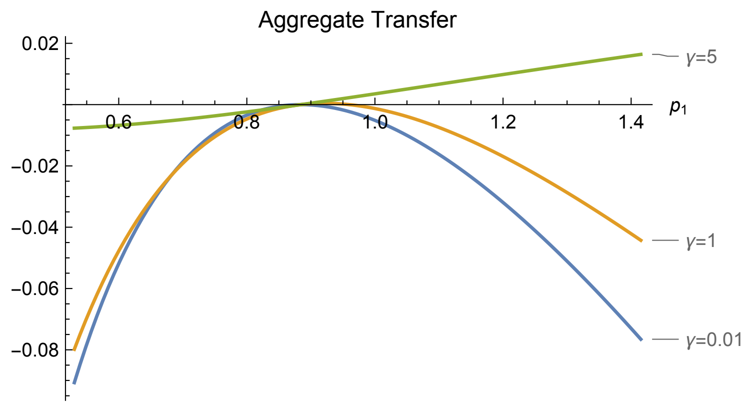

Figure 1: Aggregate transfer required to compensate workers when moving from autarky to a price of p_1 for varying values of γ

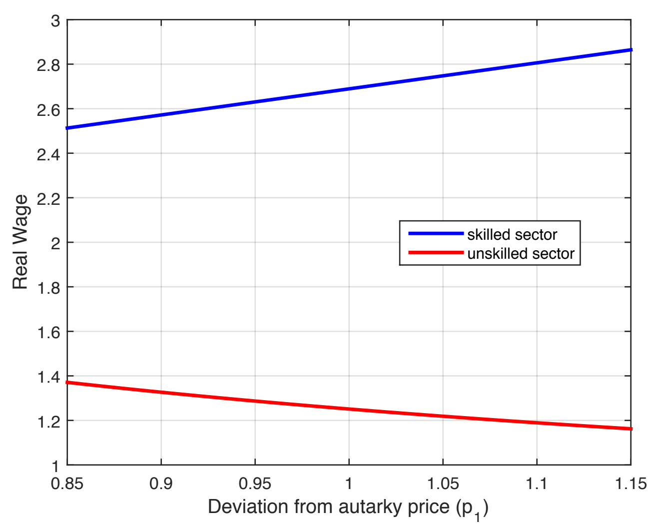

Figure 2: Response of real wages by sector to relative price changes

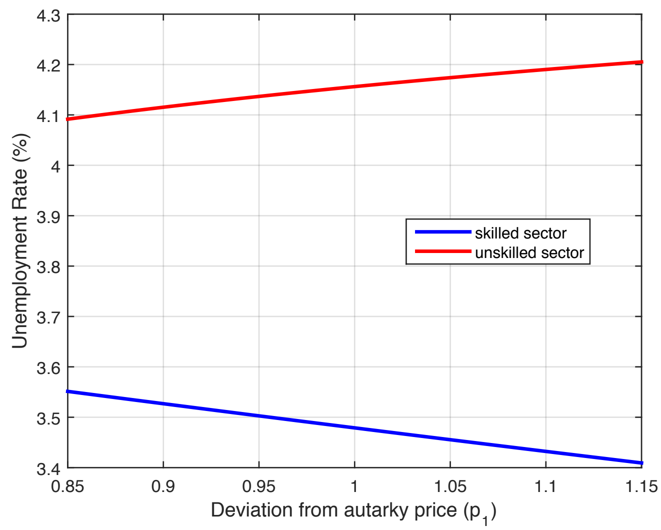

Figure 3: Response of unemployment by sector to relative price changes

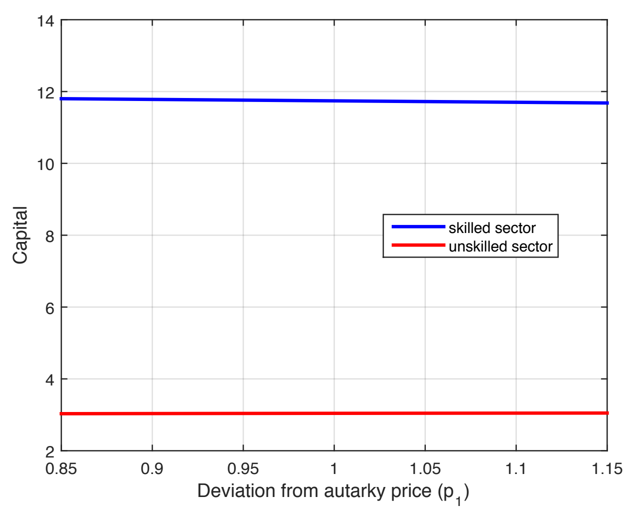

Figure 4: Response of capital per worker by sector to relative price changes

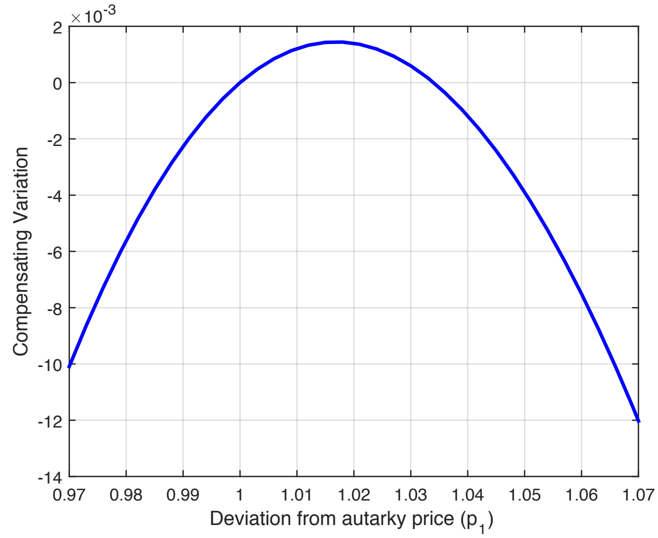

Figure 5: Compensating variation to relative price changes

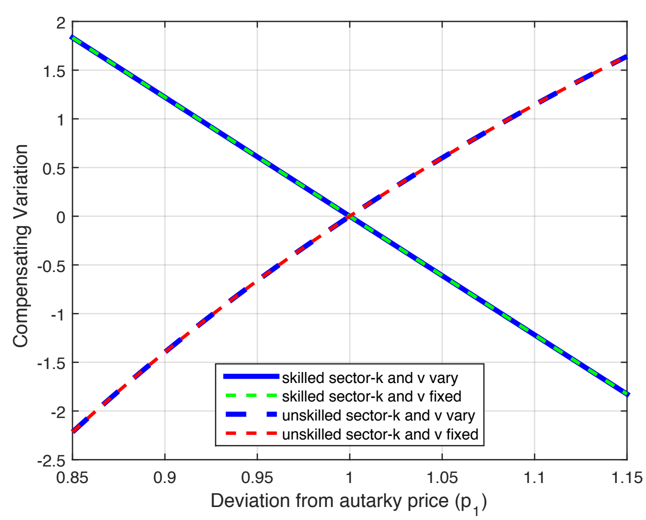

Figure 6: Compensating variation by sector to a relative price change