

A Macroeconomic Model with Financially Constrained Producers and Intermediaries *

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Abstract

We propose a model that can simultaneously capture the sharp and persistent drop in macro-economic aggregates and the sharp change in credit spreads observed in the U.S. during the Great Recession. We use the model to evaluate the quantitative effects of macro-prudential policy. The model features borrower-entrepreneurs who produce output financed with long-term debt issued to financial intermediaries and their own equity. Intermediaries fund these loans combining deposits and equity. Savers provide funding to banks and to the government. Both entrepreneurs and intermediaries have limited liability. The government issues debt to finance budget deficits and to pay for bank bailouts. Intermediaries are subject to a regulatory capital constraint. Financial recessions, triggered by low aggregate and dispersed idiosyncratic productivity shocks result in financial crises with elevated loan defaults and more frequent intermediary insolvencies. Output, balance sheet, and price reactions are substantially more severe and persistent than in non-financial recession. Policies that limit intermediary leverage redistribute wealth from savers to equity owners of producers and banks. The benefits of lower intermediary leverage for financial and macro-economic stability are offset by the costs from more constrained firms who produce less output.

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1 Introduction

The financial crisis and Great Recession of 2007-09 underscored the importance of the financial system for the broader economy. Borrower default rates, bank insolvencies, government bailouts, and credit spreads all spiked while real interest rates were very low. The disruptions in financial intermediation fed back on the real economy. Consumption, investment, and output all fell substantially and persistently.

These events have caused economists to revisit the role of the financial sector in models of the macro economy. Building on early work that emphasized the importance of credit markets in amplifying business cycle shocks,¹ a second generation of models has added nonlinear dynamics and a richer financial sector.² While a lot of progress has been made in understanding how financial intermediaries affect asset prices and macroeconomic performance, an important remaining challenge is to deliver a quantitatively successful model that can capture the dynamics of financial intermediary capital, asset prices, and the real economy during normal times and credit crises. Such a model requires a government, so that possible crisis responses can be studied, and explicit and implicit government guarantees to the financial sector can be incorporated. Our paper aims to make progress on this important agenda. It delivers a calibrated model that matches key features of the U.S. macroeconomy and asset prices in a model with an explicit financial sector. In addition, it makes four methodological contributions.

First, we separate out the role of producers and banks. Most of the existing literature, as exemplified by the seminal Brunnermeier and Sannikov (2014) paper, combines the roles of financial intermediaries and producers (“experts”). This setup assumes frictionless interaction between banks and borrowers and focuses on the interaction between experts and saving households. It implicitly assumes that financial intermediaries hold equity claims in productive firms. In reality, financial intermediaries make corporate loans and hold corporate bonds which are debt claims.³ These debt contracts are subject to default risk of the borrowers. Intermediaries

¹E.g., Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997).

²E.g., Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012, 2013, 2014), Gârleanu and Pedersen (2011), Adrian and Boyarchenko (2012), Maggiori (2013), and Moreira and Savov (2016).

³It is well understood that debt-like contracts arise in order to reduce the cost of gathering information and to mitigate principal-agent problems. See the costly state verification models in the tradition of Townsend (1979) and Gale and Hellwig (1985), and the work on the information insensitivity of debt by Dang, Gorton, and Holmstrom (2015). Our debt is non-state contingent which confers the advantage that loan defaults induce

help to optimally allocate risk between borrowers and savers, and their risk-bearing capacity becomes a key state variable. Both firms and banks choose leverage optimally, and the model generates the large differences observed between non-financial and financial leverage. The separation of producers and intermediaries activates a second financial accelerator, in addition to the traditional financial accelerator mechanism. Losses on corporate loans reduce intermediary net worth, make them effectively more risk averse, and reduce their ability to extend loans that producers rely on to make investments.

The second contribution is to introduce the possibility of default for financial intermediaries, with the government guaranteeing bank debt for savers (deposit insurance). The combination of limited liability and government guarantees affect banks' risk taking incentives, creating scope for regulation that limits bank leverage.⁴ We model a Basel-style regulatory capital requirement that limits intermediary liabilities to a certain fraction of their assets. The minimum regulatory capital that banks must hold is the key macro-prudential policy parameter. Our calibration suggests a welfare-maximizing bank equity capital requirement of 10%. Banks optimally trade off the costs and benefits of default. Our equilibrium features a fraction of banks defaulting, consistent with the data.⁵ In case of bank default, the government steps in, liquidates the bank's assets and makes whole their creditors. By allowing for the possibility of bank insolvencies, our model can help explain how a corporate default wave can trigger financial fragility. Bailouts are financed by government debt issuance, which must ultimately be paid back through higher taxes. The government's ability to postpone the cost of financial crises until after the crisis is a realistic feature of the model.

The third methodological contribution is to endogenize the interest rate on safe debt. Most models in the intermediation literature keep the interest rate on safe assets (deposits or government debt) constant. With risk averse savers and endogenous safe asset rates, the dynamics

losses for the intermediaries. Costly state verification models also justify the existence of financial intermediaries who avoid the duplication of verification costs, as in Williamson (1987), Krasa and Villamil (1992), Diamond (1984). Recent work by Klimenko, Pfeil, Rochet, and Nicolo (2016), Rampini and Viswanathan (2017), and Gale and Gottardi (2017) also models intermediaries separately from producers. While there are several differences in model setup, the main difference is that their focus is qualitative while ours is quantitative.

⁴Excessive risk-taking in the presence of deposit insurance or government guaranteed has long been recognized as an important rationale for bank regulation. See Kareken and Wallace (1978), Van den Heuvel (2008), or Farhi and Tirole (2012). Others justify the presence of bank leverage or net worth constraints by the ability of banks to divert cash flows, as in Gertler, Kiyotaki, and Queralto (2012) for example.

⁵As Reinhart and Rogoff (2009) and Jorda, Schularick, and Taylor (2014) make clear, financial intermediaries frequently become insolvent. When they do, their creditors (mostly depositors) are bailed out by the government.

of the model change substantially. In a crisis, intermediaries contract the size of their balance sheet, reducing the supply of safe debt in the economy. Simultaneously, risk averse depositors with precautionary savings motives increase their demand for safe assets. To clear the market, the equilibrium real interest rates must fall sharply. The low cost of debt allows the intermediaries to recapitalize quickly, dampening the effect of the crisis. Put differently, the endogenous price response of safe debt short-circuits the amplification mechanism that arises in a balance sheet recession in partial equilibrium models that hold the interest rate fixed. A mitigating effect is the expansion in the supply of government debt that arises as a result of counter-cyclical government spending.

The fourth methodological contribution is that the model features long-term debt. Intermediaries perform the traditional role of maturity transformation. Most models in the intermediary literature feature no default on corporate loans.⁶ Those that feature default employ short-term debt, abstracting from a key source of risk of banking.

What results is a rich and quantitatively relevant framework of the interaction between four balance sheets: those of borrower-entrepreneurs, financial intermediaries, saving households, and the government, featuring occasionally binding borrowing constraints for both borrower-entrepreneurs and for intermediaries, and bankruptcy of both borrowers and intermediaries. The model generates amplification whereby aggregate shocks not only directly affect production and investment, but also affect the financial and non-financial sectors' leverage. Tighter financial constraints on banks reduce the fragility of the financial sector, but also limit the availability of credit to firms which hurts investment and output, beyond the effects familiar from standard accelerator models.

The model quantitatively matches the maturity, default risk, and loss-given default of corporate debt. It generates a large and volatile credit spread, again matching the data. The endogenous price of credit risk dynamics amplify the dynamics in the quantity of credit risk. Intermediary wealth fluctuations are behind this resolution of the credit spread puzzle.⁷ We

⁶For example, Crdia and Woodford (2008), Goodfriend and McCallum (2007), Meh and Moran (2010), and Christiano, Motto, and Rostagno (2014). A few exceptions are Gertler and Kiyotaki (2010), Angeloni and Faia (2013), Hirakata, Sudo, and Ueda (2013), Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), and Gete (2016).

⁷On the credit spread puzzle in quantitative models, see for example Chen (2010). The model generates substantial fluctuations in risk premia which can be traced back to the dynamics of the intermediary stochastic discount factor. A new empirical literature in finance emphasizes the importance of intermediary wealth and

use the model to study the differences between regular non-financial recessions and financial recessions, which are recessions that coincide with credit crisis.

Our main exercise is to investigate the quantitative effects of macro-prudential policies for financial stability, the size of the economy, economic stability, fiscal stability, and economy-wide welfare. Our model belongs to the class of models where incomplete markets and borrowing constraints create room for macro-prudential policy intervention.⁸ We find that while macro-prudential policies improve financial stability, they also shrink the size of the economy. They further make intermediaries effectively more risk averse and limit their willingness to absorb aggregate risk. On net, a reduction in maximum bank leverage has large redistributive consequences, shifting wealth from savers to borrowers, who are the equity owners of firms and banks. It has modest positive effects on aggregate welfare. We further explore the effect of a procyclical capital requirement that is conditional on the aggregate uncertainty state of the economy. We find such a time-varying requirement to be optimal in the sense that it allows a larger financial sector and improves macroeconomic risk sharing. Out of the policies we evaluate, it is the only one that creates sufficient welfare gains to allow implementation of a Pareto improving transfer scheme. Our model offers quantitative answers to these important policy questions.

Our paper provides a state-of-the-art solution technique. The model has two exogenous and persistent sources of aggregate risk. Standard TFP shocks hit the production function. In addition, shocks to the cross-sectional dispersion of idiosyncratic firm productivity govern credit risk. The model also has five endogenous aggregate state variables: the capital stock, corporate debt stock, intermediary net worth, household wealth, and the government debt stock. To solve this complex problem, we provide a nonlinear global solution method, called policy time iteration, which is a variant of the parameterized expectations approach. Policy

leverage for asset prices. See Adrian, Etula, and Muir (2014), He, Kelly, and Manela (2017), and Adrian, Moench, and Shin (2017).

⁸Other models in this class are Lorenzoni (2008), Mendoza (2010), Korinek (2012), Bianchi and Mendoza (2013), Bianchi and Mendoza (2015), Guerrieri and Lorenzoni (2015), Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015), and . Farhi and Werning (2016) study macroprudential policy in a model with demand externalities. A different branch of the literature instead studies the interactions between conventional and unconventional monetary policy and financial intermediation. See Gertler and Karadi (2011), Angeloni and Faia (2013), Drechsler, Savov, and Schnabl (2017b, 2017a), Crdia and Woodford (2016), Begenau (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016), Elenev (2016), Begenau and Landvoigt (2017), and De Fiore, Hoerova, and Uhlig (2017).

functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of nonlinear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities.⁹

The rest of the paper is organized as follows. Section 2 discusses the model setup. Section 3 presents the calibration. Section 4 contains the main results. Section 5 uses the model to study various macro-prudential policies. Section 6 concludes. All model derivations and some details on the calibration are relegated to the appendix.

2 The Model

2.1 Preferences, Technology, Timing

Preferences The model features a government and two groups of households: borrower-entrepreneurs (denoted by superscript B) and savers (denoted by S). Savers are more patient than borrower-entrepreneurs, implying for the discount factors that $\beta_B < \beta_S$. All agents have Epstein-Zin preferences over utility streams $\{u_t^j\}_{t=0}^\infty$ with intertemporal elasticity of substitution ν_j and risk aversion σ^j

$$U_t^j = \left\{ (1 - \beta) (u_t^j)^{1-1/\nu_j} + \beta_j (E_t [(U_{t+1}^j)^{1-\sigma_j}])^{\frac{1-1/\nu_j}{1-\sigma_j}} \right\}^{\frac{1}{1-1/\nu_j}}, \quad (1)$$

for $j = B, S$. Agents derive utility from consumption of the economy's sole good, such that $u_t^j = C_t^j$, for $j = B, S$.

⁹One output of this research project will be a set of computer code which will be made publicly available. Discussions with the research department at three different Central Banks indicate that there is a demand for this type of output. Our method improves on existing methods which compute two non-stochastic steady states: one steady state when the constraint never binds and one where it always binds, and then linearizes the solution around both of these states. In this approach, agents inside the model do not take into account the fact that borrowing constraints may become binding in the future due to future shock realizations. As a result, the approach ignores agents' precautionary savings motives related to future switches between "regimes" with and without binding constraints. While the piecewise-linear solution may prove sufficiently accurate in some contexts, it remains an open question whether it offers an appropriate solution to models with substantial risk and higher risk aversion, designed to match not only macroeconomic quantities but also asset prices (risk premia). See Guerrieri and Iacoviello (2015) for a nice discussion on these issues.

Production and Intermediation Technology Borrower-entrepreneurs are the only households that invest in risky equity, both of the non-financial and the financial (intermediary) sector.¹⁰

Borrower-entrepreneurs own the productive capital stock of the economy.¹¹ They operate its production technology of the form

$$Y_t = Z_t^A K_t^{(1-\alpha)} L_t^\alpha, \quad (2)$$

where K_t is capital, L_t is labor, and Z_t^A is total factor productivity (TFP). We assume that TFP fluctuations follow an AR(1) process; Z^A has mean one.

In addition to the technology for producing consumption goods, borrower-entrepreneurs also have access to a technology that can turn consumption into capital goods subject to adjustment costs.

Borrower-entrepreneurs and savers are endowed with \bar{L}^B and \bar{L}^S units of labor, respectively. We assume that both types of households supply their labor endowment inelastically.

Intermediaries are profit-maximizing firms that extend loans to non-financial firms, and they fund these loans through equity capital and deposits that they issue to savers.

These are the two additional financial assets in the economy: one risky long-term bond that borrower-entrepreneurs can issue to intermediaries (corporate loans), and one short-term risk free bond that intermediaries can issue to savers (deposits).

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Aggregate and idiosyncratic productivity shocks for borrower-entrepreneurs are realized. Idiosyncratic profit shocks for intermediaries are realized. Production occurs.
2. Borrower-entrepreneurs with low idiosyncratic productivity realizations default. Intermediaries assume ownership of bankrupt firms.

¹⁰Our model can be viewed as one of limited stock market participation. There is ample evidence for lack of participation and for high concentration in stock market wealth in the data.

¹¹In the absence of frictions on equity issuance, this is equivalent to assuming that borrower households own the equity capital of producing firms, which in turn hold the capital stock.

3. Individual intermediaries decide whether to declare bankruptcy. The government liquidates bankrupt intermediaries. If intermediary assets are insufficient to cover the amount owed to depositors, the government provides the shortfall (deposit insurance).
4. All agents solve their consumption and portfolio choice problems. Markets clear. Households consume.

Figure 1: Overview of Balance Sheets of Model Agents

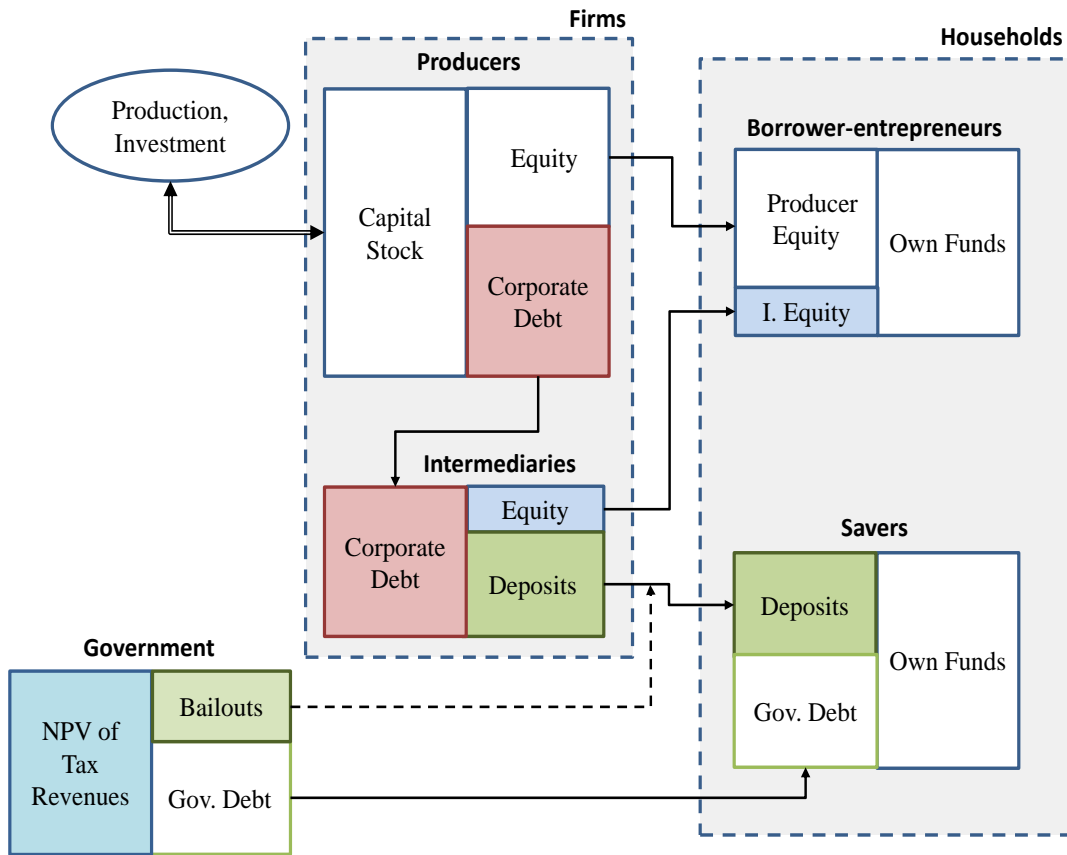


Figure 1 illustrates the balance sheets of the model's agents and their interactions. Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves, including the bankruptcy decisions of borrowers and intermediaries. We now describe each of the three types of maximization problems and the government problem in detail.

2.2 Savers

Savers can invest in one-period risk free bonds (deposits and government debt) that trade at price q_t^f . They inelastically supply their unit of labor \bar{L}^S and earn wage w_t^S . Entering with wealth W_t^S , the saver's problem is to choose consumption C_t^S and short-term bonds B_t^S to maximize life-time utility U_t^S in (1), subject to the budget constraint:

$$C_t^S + (q_t^f + \tau^D r_t^f) B_t^S \leq W_t^S + (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + O_t^S \quad (3)$$

and a short-sale constraints on bond holdings:

$$B_t^S \geq 0, \quad (4)$$

where saver wealth is simply given by the face value of last period's bond purchases $W_t^S = B_{t-1}^S$. The budget constraint (3) shows that savers use beginning-of-period wealth, after-tax labor income, transfer income from the government ($G_t^{T,S}$), transfer income from bankruptcy proceedings (O_t^S) to be defined below, to pay for consumption, and purchases of short-term bonds. Savers are taxed on interest rate income at the time they purchase the bonds at rate τ^D .¹²

2.3 Borrower-Entrepreneurs

There is a unit-mass of identical borrower-entrepreneurs indexed by i . The households form a collective (“family”) that provides insurance against idiosyncratic shocks.

Each entrepreneur has access to a technology that creates consumption goods $Y_{i,t}$ from capital $K_{i,t}$ and labor $L_{i,t}$. At the beginning of the period, each entrepreneur receives an idiosyncratic productivity shock $\omega_{i,t} \sim F_{\omega,t}$. The shocks are uncorrelated across entrepreneurs and time. However, the *distribution* of the ω -shocks varies over time; specifically, the cross-sectional dispersion of the shocks, $\sigma_{\omega,t}$, follows a first-order Markov process. Output depends

¹²We define the risk-free interest rate as the yield on risk free bonds, $r_t^f = 1/q_t^f - 1$. Further, we think of the interest income as being realized at the time they purchase the bonds with a fixed one period yield.

on aggregate productivity Z_t^A and idiosyncratic productivity $\omega_{i,t}$:

$$Y_{i,t} = \omega_{i,t} Z_t^A K_{i,t}^{1-\alpha} L_{i,t}^\alpha.$$

While each individual entrepreneur manages her own production, the family manages the allocation of production inputs and consumption and issues debt to intermediaries. The cross-sectional standard deviation of the idiosyncratic productivity shocks, $\sigma_{\omega,t}$ fluctuates over time, and constitutes the second exogenous source of aggregate risk in the model. We refer to it as the uncertainty shock.¹³

The corporate debt is long-term, modeled as perpetuity bonds. Bond coupon payments decline geometrically, $\{1, \delta, \delta^2, \dots\}$, where δ captures the duration of the bond. We introduce a “face value” $F = \frac{\theta}{1-\delta}$, a fixed fraction θ of all repayments for each bond issued. Per definition, interest payments are the remainder $\frac{1-\theta}{1-\delta}$.

At the beginning of the period, the family jointly holds K_t^B units of capital, and has A_t^B bonds outstanding. In addition, producers jointly hire their own labor and the labor of savers, denoted by L_t^j , with $j = B, S$. During production, the labor inputs of the two types are combined into aggregate labor:

$$L_t = (L_t^B)^{1-\gamma_S} (L_t^S)^{\gamma_S}.$$

Before idiosyncratic productivity shocks are realized, each producer is given the same amount of capital and labor for production, such that $K_{i,t} = K_t^B$ and $L_{i,t} = L_t$. Further, each producer is responsible for repaying the coupon on an equal share of the total debt, $A_{i,t} = A_t^B$.

The individual profit of producer i is therefore given by

$$\pi_{i,t} = \omega_{i,t} Z_t^A (K_t^B)^{1-\alpha} L_t^\alpha - \sum_j w_t^j L_t^j - A_t^B. \quad (5)$$

After production, each producer who achieves a sufficiently high profit, $\pi_{i,t} \geq \underline{\pi}$, returns this profit to the family, where $\underline{\pi}$ is a parameter. Further, capital depreciates during production

¹³Christiano, Motto, and Rostagno (2014) introduce a similar “risk” shock and argue it is an important driver of business cycle dynamics. Jermann and Quadrini (2012) study how financial shocks affect balance sheet variables. Alfaro, Bloom, and Lin (2016) study how financial frictions amplify the effect of uncertainty shocks on investment and hiring.

by fraction δ_K , and individual members with profit above the threshold return the depreciated capital after production. Producers with $\pi_{i,t} < \underline{\pi}$ default on the share of debt they were allocated. The debt is erased, and the intermediary takes ownership of the bankrupt firm, including its share of the capital stock. The intermediary liquidates the bankrupt firms' capital, seizes their output, and pays their wage bill. The remaining funds are the intermediary's recovery value. In return for production, each family member receives the same amount of consumption goods $C_{i,t} = C_t^B$.

From (5), it immediately follows that there exists a cutoff productivity shock

$$\omega_t^* = \frac{\underline{\pi} + \sum_{j=B,S} w_t^j L_t^j + A_t^B}{Z_t^A (K_t^B)^{1-\alpha} (L_t)^\alpha}, \quad (6)$$

such that all entrepreneurs receiving productivity shocks below this cutoff default on their debt.

Using the threshold level ω_t^* , we define $\Omega_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $\Omega_K(\omega_t^*)$ to be the average productivity of the firms that do not default:

$$\Omega_A(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*], \quad (7)$$

$$\Omega_K(\omega_t^*) = \Pr[\omega_{i,t} \geq \omega_t^*] \mathbb{E}[\omega_{i,t} | \omega_{i,t} \geq \omega_t^*]. \quad (8)$$

After making a coupon payment of 1 per unit of remaining outstanding debt, the amount of outstanding debt declines to $\delta \Omega_A(\omega_t^*) A_t^B$.

The total profit of the producers' business is subject to a corporate profit tax with rate τ_Π^B . The profit for tax purposes is defined as sales revenue net of labor expenses, and capital depreciation and interest payments of non-bankrupt producers:¹⁴

$$\Pi_t^{B,\tau} = \Omega_K(\omega_t^*) Z_t^A (K_t^B)^{1-\alpha} (L_t)^\alpha - \Omega_A(\omega_t^*) \left(\sum_j w_t^j L_t^j + \delta_K p_t K_t^B + (1 - \theta) A_t^B \right).$$

The fact that interest expenditure $(1 - \theta) A_t^B$ and capital depreciation $\delta_K p_t K_t^B$ are deducted from taxable profit creates a “tax shield” and hence a preference for debt funding.

In addition to producing consumption goods, producers jointly create capital goods from

¹⁴Aggregate producer profit is the integral over the idiosyncratic profit (5) of non-defaulting producers, net of capital depreciation expenses and adding back principal payments θA_t^B which are not tax deductible.

consumption goods. In order to create X_t new capital units, the required input of consumption goods is

$$X_t + \Psi(X_t/K_t^B)K_t^B, \quad (9)$$

with adjustment cost function $\Psi(\cdot)$ which satisfies $\Psi''(\cdot) > 0$, $\Psi(\mu_G + \delta_K) = 0$, and $\Psi'(\mu_G + \delta_K) = 0$.

Borrower-entrepreneurs further own all equity shares of the economy's banking sector. Each period, they receive an effective dividend D_t^I from intermediaries, to be defined below in equation (17).

The borrower-entrepreneur family's problem is to choose consumption C_t^B , capital for next period K_{t+1}^B , new debt A_{t+1}^B , investment X_t and labor inputs L_t^j to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$\begin{aligned} & C_t^B + X_t + \Psi(X_t/K_t^B)K_t^B + \Omega_A(\omega_t^*)A_t^B(1 + \delta q_t^m) + p_t K_{t+1}^B + \Omega_A(\omega_t^*) \sum_{j=B,S} w_t^j L_t^j + \tau_{\Pi}^B \Pi_t^{B,\tau} \\ & \leq \Omega_K(\omega_t^*) Z_t^A (K_t^B)^{1-\alpha} (L_t)^\alpha + (1 - \tau_t^B) w_t^B \bar{L}^B + p_t (X_t + \Omega_A(\omega_t^*) (1 - \delta_K) K_t^B) \\ & \quad + D_t^I + q_t^m A_{t+1}^B + G_t^{T,B} + O_t^B, \end{aligned} \quad (10)$$

and a leverage constraint:

$$FA_{t+1}^B \leq \Phi p_t (1 - (1 - \tau_{\Pi}^B) \delta_K) \Omega_A(\omega_t^*) K_t^B. \quad (11)$$

The borrower household uses output, after-tax labor income, sales of old (K_t^B) and newly produced (X_t) capital units, the dividend from the intermediation sector (D_t^I), new debt raised ($q_t^m A_{t+1}^B$), where q_t^m is the price of one bond in terms of the consumption good, transfer income from the government ($G_t^{T,B}$), and transfer income from bankruptcy proceedings (O_t^B) to be defined below. These resources are used to pay for consumption, investment including adjustment costs, debt service, new capital purchases, wages, and corporate taxes.

The borrowing constraint in (11) caps the face value of debt at the end of the period, FA_{t+1}^B , to a fraction of the market value of the available capital units after default and depreciation, $p_t (1 - (1 - \tau_{\Pi}^B) \delta_K) \Omega_A(\omega_t^*) K_t^B$, where Φ is the maximum leverage ratio. With such a constraint,

declines in capital prices (in bad times) tighten borrowing constraints. The constraint (11) imposes a hard upper bound on borrower leverage. In addition, costly defaults of individual borrowers who received bad idiosyncratic shocks, endogenously limit the optimal leverage of borrowers. Borrowers take into account that each marginal unit of debt issued in t increases costly defaults in $t + 1$. Therefore, for a high enough maximum leverage ratio Φ , constraint (11) will never be binding.

2.4 Intermediaries

2.4.1 Setup

Intermediaries are financial firms (“banks”) fully owned by borrowers that buy long-term risky debt issued by producers, and use this corporate debt as collateral to issue short-term debt to savers. Bank debt is guaranteed by the government (deposit insurance) and therefore risk-free. There are two important frictions in the banking sector:

1. Moving funds into or out of banks is costly, i.e. paying a (positive or negative) dividend d_t^I is subject to a cost $\Sigma(d_t^I)$ that is convex in deviation of d_t^I from a target level.
2. Banks receive idiosyncratic profit shocks ϵ_t^I , realized at the time of dividend payouts, and can decide to default on their liabilities. The shocks are i.i.d. across banks and time with $E(\epsilon_t^I) = 0$ and c.d.f. F_ϵ .¹⁵

Together, these assumptions mean that the total cost of paying out dividend d_t^I is $d_t^I + \Sigma(d_t^I)$ for the intermediary. The net dividend received by the shareholders is $d_t^I - \epsilon_t^I$. Intermediaries maximize the present value of net dividend payments to their shareholders, by choosing the dividend d_t^I , and by choosing a portfolio loans to producing firms (A_t^I) and short-term bonds (B_t^I).

Loans are modeled as bonds aggregating the debt of producers. The coupon payment on performing loans in the current period is $A_t^I \Omega_A(\omega_t^*)$. For firms that default and enter into foreclosure, banks repossess these firms, including this period’s output, as collateral. Banks

¹⁵The idiosyncratic shocks to bank profitability may reflect unmodeled heterogeneity in bank portfolios, including that resulting from lending to consumers. Technically, the assumption guarantees that there is always a fraction of banks which defaults.

must pay the wages owed by the defaulting firms, a senior claim. Payments on defaulted bonds are:

$$M_t = (1 - \zeta) \left[(1 - \Omega_A(\omega_t^*)) (1 - \delta_K) p_t K_t^B + (1 - \Omega_K(\omega_t^*)) (K_t^B)^{1-\alpha} L_t^\alpha \right] - (1 - \Omega_A(\omega_t^*)) \sum_j w_t^j L_t^j, \quad (12)$$

where ζ is the fraction of capital value and output destroyed in bankruptcy.

Thus, the total (performing and defaulting) payoff per unit of the bond is $\Omega_A(\omega_t^*) + M_t/A_t^B$. The price per unit of the bond is q_t^m .

At the beginning of the period, after aggregate and idiosyncratic shocks are realized and a fraction $1 - \Omega_A(\omega_t^*)$ of firms has defaulted, the wealth (net worth) of an intermediary is:

$$W_t^I = \Omega_A(\omega_t^*) (1 + \delta q_t^m) A_t^I + M_t + B_{t-1}^I. \quad (13)$$

Each intermediary optimally decides on bankruptcy, conditional on the realization of W_t^I and the idiosyncratic shock ϵ_t^I . Bankrupt intermediaries are liquidated by the government, which redeems deposits at par value. Immediately thereafter, borrower households replace all bankrupt intermediaries with new banks that receive initial equity equal to the non-defaulting banks, W_t^I . This ensures that at the time of the dividend payout and portfolio decisions, all intermediaries have identical wealth and face identical decision problems.

In addition to making loans, intermediaries can trade in short-term bonds with savers and the government. They are allowed to take a short position in these bonds, using their loans to borrower-entrepreneurs as collateral. Intermediary debt is subject to a leverage constraint:

$$-q_t^f B_t^I \leq \xi q_t^m A_{t+1}^I. \quad (14)$$

A negative position in the short-term bond is akin to intermediaries issuing deposits. The negative position in the short-term bond must be collateralized by the market value of intermediaries' holdings of long-term loan bonds. The parameter ξ determines how useful loans are as collateral. The constraint (14) is a Basel-style regulatory capital constraint. The parameter ξ is the key macro-prudential policy parameter in the paper.

Intermediaries are subject to corporate profit taxes at rate τ_{Π}^I . Their profit for tax purposes is defined as the net interest income on their loan business:

$$\Pi_t^I = (1 - \theta)\Omega_A(\omega_t^*)A_t^I + r_t^f B_t^I.$$

They further need to pay a deposit insurance fee (κ) to the government that is proportional to the amount of short-term bonds they issue.

2.4.2 Recursive Intermediary Problem

Denote by \mathcal{S}_t^I the vector of aggregate state variables exogenous to the problem of the individual intermediary. At the end of the period, all intermediaries face the same optimization problem:¹⁶

$$V_t^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} d_t^I + \mathbb{E}_t [\mathcal{M}_{t,t+1}^B \max \{V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0\}]. \quad (15)$$

subject to the budget constraint

$$d_t^I + \Sigma(d_t^I) + q_t^m A_{t+1}^I + (q_t^f + \mathbb{I}_{\{B_t^I < 0\}} \kappa) B_t^I + \tau_{\Pi}^I \Pi_t^I \leq W_t^I, \quad (16)$$

the collateral constraint (14), and the definition of wealth (13). The continuation value in the objective function (15) reflects that the value of the bank in case of default is zero. Intermediaries discount future payoffs by $\mathcal{M}_{t,t+1}^B$, which is the stochastic discount factor of borrowers, their equity holders.

Since the idiosyncratic shocks are independent of the aggregate state of the economy, an individual bank's probability of continuing (i.e. not defaulting) conditional on the aggregate

¹⁶At the time intermediaries decide on dividend payout and portfolio, they have already decided whether or not to default. If they defaulted, they were replaced by new intermediaries with identical wealth W_t^I , and the idiosyncratic shocks of these intermediaries were absorbed by the government. If they did not default, the contemporaneous idiosyncratic shock ϵ_t^I is no longer relevant for the optimization problem. Hence we do not need to distinguish between individual and aggregate intermediary wealth. We can define an intermediary value function V^I that does not condition on the idiosyncratic shock.

state, but before realization of the idiosyncratic shock is¹⁷

$$\text{Prob}(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I > 0) = \text{Prob}(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)) = F_\epsilon(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)).$$

Hence we can express the intermediary problem as

$$V_t^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} d_t^I + \text{E}_t \left[\mathcal{M}_{t,t+1}^B F_\epsilon(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)) \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right].$$

The conditional expectation

$$\epsilon_t^{I,-} = \text{E}_\epsilon(\epsilon | \epsilon \leq V^I(W_t^I, \mathcal{S}_t^I)).$$

is the expected idiosyncratic loss conditional on not defaulting.

2.4.3 Aggregation and Government Bailouts

The aggregate net dividend to borrowers from the banking sector is

$$\begin{aligned} D_t^I &= \underbrace{F_{\epsilon,t}(d_t^I - \epsilon_t^{I,-})}_{\text{Dividend of non-defaulters}} + \underbrace{(1 - F_{\epsilon,t})(d_t^I - W_t^I)}_{\text{Dividend of defaulters net of initial equity}} \\ &= d_t^I - F_{\epsilon,t} \epsilon_t^{I,-} - (1 - F_{\epsilon,t}) W_t^I. \end{aligned} \quad (17)$$

Defaulting intermediaries are liquidated by the government. During the bankruptcy process, a fraction ζ of the asset value of a bank is lost. Hence the aggregate bailout payment of the government is

$$\text{bailout}_t = (1 - F_{\epsilon,t}) \left[\epsilon_t^{I,+} - W_t^I + \zeta(\Omega_A(\omega_t^*)(1 + \delta q_t^m) A_t^I + M_t) \right]. \quad (18)$$

The conditional expectation

$$\epsilon_t^{I,+} = \text{E}_\epsilon(\epsilon | \epsilon \geq V^I(W_t^I, \mathcal{S}_t^I)).$$

¹⁷By the law of large numbers, this is also the aggregate survival rate of intermediaries, i.e. $1 - F_\epsilon(V^I(W_t^I, \mathcal{S}_t^I))$ is the intermediary default rate.

is the expected idiosyncratic loss of defaulting intermediaries.

2.4.4 Aggregate Bankruptcy Costs

Default of producing firms and intermediaries causes bankruptcy losses. When firms default, a fraction ζ of their capital value and output is lost to banks, see equation (12). When banks default, a fraction ζ of their asset value is lost to the government, see equation (18). We assume that only a fraction η of this total loss from bankruptcy is a deadweight loss to society while the remainder is rebated to the households in proportion to their population shares; these are the O_t^i terms in the budget constraints:

$$\begin{aligned} \sum_{i=B,S} O_t^i = & \zeta(1-\eta) \left[(1-\Omega_A(\omega_t^*)) (1-\delta_K) p_t K_t^B + (1-\Omega_K(\omega_t^*)) (K_t^B)^{1-\alpha} L_t^\alpha \right] \\ & + \zeta(1-\eta) (1-F_{\epsilon,t}) \left[\Omega_A(\omega_t^*) (1+\delta q_t^m) A_t^I + M_t \right]. \end{aligned}$$

This can be interpreted as income payments to the actors involved in bankruptcy cases. We avoid the strong assumption that all bankruptcy costs are deadweight losses to society.

2.5 Government

The actions of the government are determined via fiscal rules: taxation, spending, bailout, and debt issuance policies. Government tax revenues, T_t , are labor income tax, corporate profit tax, deposit insurance fee receipts, and deposit income tax:

$$T_t = \sum_{j=B,S} \tau_t^j w_t^j L_t^j + \tau_{\Pi}^B \Pi_t^B + \tau_{\Pi}^I \Pi_t^I + \tau^D r_t^f B_t^S - \mathbf{I}_{\{B_t^I < 0\}} \kappa B_t^I$$

Government expenditures, G_t are the sum of exogenous government spending, G_t^o , transfer spending G_t^T , and financial sector bailouts:

$$G_t = G_t^o + \sum_{j=B,S} G_t^{T,j} + \text{bailout}_t.$$

The aggregate bailout payment is defined in equation (18).

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \quad (19)$$

We impose a transversality condition on government debt:

$$\lim_{u \rightarrow \infty} E_t \left[\tilde{\mathcal{M}}_{t,t+u}^S B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^S$ is the SDF of the saver.¹⁸ Because of its unique ability to tax, the government can spread out the cost of default waves and financial sector rescue operations over time.

Government policy parameters are $\Theta_t = (\tau_t^i, \tau_{\Pi}^i, G_t^o, G_t^{T,i}, \kappa, \Phi, \xi)$. The deposit insurance fee κ and the capital requirement ξ in equation (14) can be thought of as macro-prudential policy tools.

2.6 Equilibrium

Given a sequence of aggregate productivity shocks $\{Z_t^A\}$, idiosyncratic productivity shocks $\{\omega_{t,i}\}_{i \in B}$, and idiosyncratic intermediary profit shocks $\{\epsilon_{t,i}\}_{i \in I}$, and given a government policy Θ_t , a competitive equilibrium is an allocation $\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}$ for borrower-entrepreneurs, $\{C_t^S, B_t^S\}$ for savers, $\{d_t^I, A_{t+1}^I, B_t^I\}$ for intermediaries, and a price vector $\{p_t, q_t^m, q_t^f, w_t^B, w_t^S\}$, such that given the prices, borrower-entrepreneurs and savers maximize life-time utility, intermediaries maximize shareholder value, the government satisfies its budget constraint, and markets clear.

¹⁸We show below that the risk averse saver is the marginal agent for short-term risk-free debt. In the numerical work below, we keep the ratio of government debt to GDP contained between \underline{b}^G and \overline{b}^G by decreasing taxes convexly when the debt-to-GDP threatens to fall below \underline{b}^G and raising taxes convexly when debt-to-GDP threatens to exceed \overline{b}^G .

The market clearing conditions are:

$$\text{Risk-free bonds: } B_t^G = B_t^S + B_t^I \quad (20)$$

$$\text{Loans: } A_{t+1}^B = A_{t+1}^I \quad (21)$$

$$\text{Capital: } K_{t+1}^B = (1 - \delta_K)K_t^B + X_t \quad (22)$$

$$\text{Labor: } L_t^j = \bar{L}^j \text{ for all } j = B, S \quad (23)$$

$$\begin{aligned} \text{Consumption: } Y_t = & C_t^B + C_t^S + G_t^o + X_t + K_t^B \Psi(X_t/K_t^B) \\ & + \eta \zeta [(1 - \Omega_A(\omega_t^*))(1 - \delta_K)p_t K_t^B + (1 - \Omega_K(\omega_t^*))(K_t^B)^{1-\alpha} L_t^\alpha] \\ & + \eta \zeta (1 - F_{\epsilon,t}) [\Omega_A(\omega_t^*)(1 + \delta q_t^m)A_t^I + M_t] \end{aligned}$$

The last equation is the economy's resource constraint. It states that total output (GDP) equals the sum of aggregate consumption, discretionary government spending, investment, and aggregate resource losses due to bankruptcies.

2.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t , we must take a stance on how to weigh the two households, borrowers and savers. We propose two different measures of aggregate welfare. First, we compute an ex-post utilitarian social welfare function summing value functions of the agents

$$\mathcal{W}_t(\cdot; \Theta_t) = V_t^B + V_t^S,$$

where the $V^j(\cdot)$ functions are the value functions defined in the appendix. The value functions already incorporate the mass of agents of each type (population shares ℓ^i).¹⁹

Secondly, we compute an ex-ante measure of welfare based on compensating variation. Consider the equilibrium of two different economies $k = 0, 1$, characterized by policy vectors Θ^0

¹⁹Equivalently, we could first express the value functions per capita by scaling them by their population weights, and then calculating a population-weighted average of the per capita value functions.

and Θ^1 , and denote expected lifetime utility at time 0 for agent j in economy k by

$$\bar{V}^{j,k} = E_0[V_1^j(\cdot; \Theta^k)].$$

Similarly, denote the time-0 price of the consumption stream of agent j in economy k by

$$\bar{P}^{j,k} = E_0 \left[\sum_{t=0}^{\infty} \mathcal{M}_{t,t+1}^{j,k} C_{t+1}^{j,k} \right],$$

where $\mathcal{M}_{t,t+1}^{j,k}$ is the SDF of agent j in economy k . We can then compute the percentage welfare gain for agent j from existing in economy Θ^1 relative to economy Θ^0 , in expectation, as:

$$\Delta \bar{V}^j(0 \rightarrow 1) = \frac{\bar{V}^{j,1}}{\bar{V}^{j,0}} - 1.$$

Since the value functions are expressed in consumption units, we can multiply these welfare gains with the time-0 prices of consumption streams in the Θ^0 economy and add up

$$\bar{W}(0 \rightarrow 1) = \Delta \bar{V}^B(0 \rightarrow 1) \bar{P}^{B,0} + \Delta \bar{V}^S(0 \rightarrow 1) \bar{P}^{S,0}.$$

This measure is the minimum one-time wealth transfer into the Θ^0 economy (the benchmark) required to make agents at least as well off as in the Θ^1 economy (the alternative). If this number is positive, it means that a transfer scheme could be implemented (taking as given state prices in the benchmark) to make the alternative economy a Pareto improvement. If this number is negative, it means that such a scheme cannot be implemented because it would require a bigger transfer to one agent than the other is willing to give up.

3 Calibration

The model is calibrated at annual frequency. The parameters of the model and their targets are summarized in Table 1.

Table 1: Calibration

Par	Description	Value	Target
Exogenous Shocks			
ρ_A	persistence TFP	0.7	AC(1) HP-detr GDP 53-14 of 0.55
σ_A	innov. vol. TFP	2.0%	Vol HP-detr GDP 53-14 of 2.56%
$\sigma_{\omega,L}$	low uncertainty	0.095	Avg. corporate default rate of 2%
$\sigma_{\omega,H}$	high uncertainty	0.175	Avg. IQR firm-level productivity (Bloom et al. (2012))
$p_{LL}^{\omega}, p_{HH}^{\omega}$	transition prob	{0.91, 0.80}	Bloom et al. (2012)
Production, Population, Labor Income Shares			
ψ	marginal adjustment cost	2	Vol. investment-to-GDP ratio 53-14 of 1.58%
α	labor share in prod. fct.	0.71	Labor share of output of 2/3
δ_K	capital depreciation rate	8%	Investment-to-capital ratio, 53-14
ℓ^i	pop. shares $i \in \{S, B\}$	{69,31}%	Population shares SCF 95-13
γ^i	inc. shares $i \in \{S, B\}$	{60,40}%	Labor inc. shares SCF 95-13
Corporate loans and Intermediation			
δ	average life loan pool	0.937	Duration fcn. in App. B.1
θ	principal fraction	0.582	Duration fcn. in App. B.1
ζ	Losses in bankruptcy	0.6	Corporate loan and bond severities 81-15 of 44%
η	% bankr. loss is DWL	0.2	Bris, Welch, and Zhu (2006)
Φ	maximum LTV ratio	0.45	Vol. of non-fin sector debt-to-GDP 53-14 of 5.17%
π	profit default threshold	0.04	FoF non-fin sector leverage 85-14 of 37%
σ_{ϵ}	cross-sect. dispersion ϵ_t^I	0.025	FDIC failure rate of deposit. inst. of 0.5%
σ^I	marg. dividend payout cost	5	FoF fin sector leverage 85-14 of 93%
Preferences			
β^B	time discount factor B	0.931	Capital-to-GDP ratio 53-14 of 2.24
$\sigma^B = \sigma^S$	risk aversion B & S	1	Standard value
$\nu^B = \nu^S$	IES B & S	1	Log utility
β^S	time discount factor S	0.982	Mean risk-free rate 76-14 of 2.2%
Government Policy			
G^o	discr. spending	17.17%	BEA discr. spending to GDP 53-14 of 17.58%
G^T	transfer spending	2.42%	BEA transfer spending to GDP 53-14 of 3.18%
τ	labor income tax rate	29.5%	BEA pers. tax rev. to GDP 53-14 of 17.30%
$\tau_{\Pi}^B = \tau_{\Pi}^I$	corporate tax rate	21.7%	BEA corp. tax rev. to GDP 53-14 of 3.41%
τ^D	interest rate income tax rate	13.2%	tax code; see text
b_o	cyclicality discr. spending	-2.5	slope log discr. sp./GDP on GDP growth
b_T	cyclicality transfer spending	-25	slope log transfer sp./GDP on GDP growth
b_{τ}	cyclicality lab. inc. tax	2	slope log discr. sp./GDP on GDP growth
κ	deposit insurance fee	0.001	Deposit insurance fee 97-06
ξ	max. intermediary leverage	0.94	Basel II reg. capital charge for C&I loans

Aggregate Productivity Following the macro-economics literature, the TFP process Z_t^A follows an AR(1) in logs with persistence parameter ρ_A and innovation volatility σ^A . Because TFP is persistent, it becomes a state variable. We discretize Z_t^A into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points and the transition probabilities between them to match the volatility and persistence of HP-detrended GDP. The latter is endogenously determined but heavily influenced by TFP. Consistent with the model, our measurement of GDP excludes net exports and government investment. We define the GDP deflator correspondingly. Observed real per capita HP-detrended GDP has a volatility of 2.53% and its persistence is 0.55. The model generates a volatility of 2.43% and a persistence of 0.55.

Idiosyncratic Productivity We calibrate the firm-level productivity risk directly to the micro evidence. We normalize the mean of idiosyncratic productivity at $\mu_\omega = 1$. We let the cross-sectional standard deviation of idiosyncratic productivity shocks $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ are the second source of aggregate risk. Fluctuations in $\sigma_{t,\omega}$ govern aggregate corporate credit risk since high levels of $\sigma_{t,\omega}$ cause a larger left tail of low-productivity firms that will default in equilibrium. We refer to periods in the high $\sigma_{t,\omega}$ state as *high uncertainty* periods. We set $(\sigma_{L,\omega}, \sigma_{H,\omega}) = (0.095, 0.175)$. The value for $\sigma_{L,\omega}$ targets the unconditional mean corporate default rate. The model-implied average default rate of 2.2% is similar to the data.²⁰

The high value, $\sigma_{H,\omega}$, is chosen to match the time-series standard deviation of the cross-sectional interquartile range of firm productivity, which is 4.9% according to Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) (their Table 6).

The transition probabilities from the low to the high uncertainty state of 9% and from the high to the low state of 20% are also taken directly from Bloom et al. (2012).²¹ The model

²⁰We look at two sources of data: corporate loans and corporate bonds. From the Flow of Funds, we obtain delinquency and charge-off rates on Commercial and Industrial loans and Commercial Real Estate loans by U.S. Commercial Banks for the period 1991-2015. The average delinquency rate is 3.1%. The second source of data is Standard & Poors' default rates on publicly-rated corporate bonds for 1981-2014. The average default rate is 1.5%; 0.1% on investment-grade bonds and 4.1% on high-yield bonds. The model is in between these two values.

²¹They estimate a two-state Markov chain for the cross-sectional standard deviation of establishment-level productivity using annual data for 1972-2010 from the Census of Manufactures and Annual Survey of Manufactures. We annualize their quarterly transition probability matrix.

spends 31% of periods in the high uncertainty regime. Like in Bloom et al., our uncertainty process is independent of the first-moment shocks. About 10% of periods feature both high uncertainty and low TFP realizations. We will refer to those periods as financial recessions or financial crises. Using a long time series for the U.S., Reinhart and Rogoff (2009) find the same 10% frequency of financial crises.

Production Adjustment costs are quadratic. We set the marginal adjustment cost parameter $\psi = 2$ in order to match the observed volatility of the ratio of investment to GDP, X/Y , of 1.58%. The model generates a value of 1.56%. The adjustment costs on average amount to a tiny 0.04% of GDP. We set the parameter α in the Cobb-Douglas production function equal to 0.71, which yields an overall labor income share of 65%, the standard value in the business cycle literature. We choose δ_K to match an annual depreciation of capital of 8% to match the investment-to-output ratio of 18% observed in the data.

Population and Labor Income Shares To pin down the population shares of our two different types of households we turn to the Survey of Consumer Finance (SCF). We define savers as those households who hold a low share of their wealth in the form of risky assets. In particular, we compute for each household in the survey the share of assets, excluding all real estate, held in stocks or private business equity, considering both direct and indirect holdings of stock. Using this definition of the risky share, we then calculate the fraction of households whose risky share is less than one percent.²² This amounts to 69% of SCF households. The remaining 31% of households have a nontrivial risky asset share, corresponding to the borrowers in our model.

The labor income share of savers in the SCF is 60%. The income share of the borrower-entrepreneurs is the remaining 40%. The income shares determine the Cobb-Douglas parameters γ_B and γ_S . By virtue of the calibration, the model matches basic aspects of the observed income distribution.

Corporate Loans In the model, a corporate loan is a geometric bond. The issuer of one unit of the bond at time t promises to pay 1 at time $t + 1$, δ at time $t + 2$, δ^2 at time $t + 3$, and so

²²We use all survey waves from 1995 until 2013 and average across them.

on. Given that the present value of all payments ($1/(1 - \delta)$) can be thought of as the sum of a principal (share θ) and an interest component (share $1 - \theta$), we define the book value of the debt as $F = \theta/(1 - \delta)$. This book value of debt is used in the firm’s collateral constraint. We set $\delta = 0.937$ and $\theta = 0.582$ ($F = 9.238$) to match the observed duration of corporate bonds. Appendix B.1 contains the details. The model’s corporate loans have a duration of 6.8 years on average.

As in standard trade-off theory, corporate debt enjoys a tax shield but incurs costs of distress. We set the $\zeta = 0.6$ to match the observed average severity rate of 44% on bonds rated by S&P and Moody’s rated during 1985-2004. The model produces a similar unconditional loss-given default of 43%. Combined with the average default rate, this LGD number implies a loss rate on corporate loans of 1.0%. Our baseline model generates a modest quantity of corporate default risk, consistent with the data.

A fraction η of the cost of distress to intermediaries is a deadweight loss to the economy. The remainder $1 - \eta$ is transfer income that enters in the budget constraint of the agents. We set $\eta = 0.2$ based on evidence in Bris, Welch, and Zhu (2006).

Borrowers can obtain a loan with principal value up to a fraction Φ of the market value of their assets. We set the maximum LTV ratio parameter $\Phi = 0.45$. This value is just large enough so that the LTV constraint never binds during expansions and non-financial recessions. In the simulation of the benchmark model, the borrower’s LTV constraint binds in 3% of financial recessions. The LTV constraint limits corporate borrowing as a fraction of the market value of capital. We set Φ to match the volatility of corporate debt-to-GDP of the non-financial sector, which is 5.2% in the data and 4.3% in the model.

We set the profit default threshold to $\underline{\pi} = 0.04$ to target non-financial leverage. The higher this threshold, the more firms will default on average for a given level of firm debt. Since defaults are costly to the borrower family, borrower leverage is decreasing in $\underline{\pi}$. The model generates a ratio of borrower book debt-to-assets of 36%. In the Flow of Funds data, the average ratio of loans and debt securities of the nonfinancial corporate and nonfinancial non-corporate businesses to their non-financial assets is 37%.

Intermediary and Macro-prudential Policy Parameters The intermediary payout shocks are distributed Gaussian with mean zero. The cross-sectional standard deviation $\sigma_\epsilon = \text{Var}(\epsilon_t^I)^{0.5}$ governs the average intermediary failure rate. The benchmark model generates an average failure rate of intermediaries of 0.54%, which is exactly the asset-weighted failure rate of depository institutions in the FDIC data.

We adopt the following functional form for the dividend payout cost of intermediaries

$$\Sigma(d_t^I) = \frac{\sigma_I}{2}(d_t^I - \bar{d})^2,$$

The marginal dividend payout cost for intermediaries is set to $\sigma_I = 5$ to match the average credit spread. The higher the payout cost, the more costly it becomes for intermediaries to deviate from their dividend target \bar{d} .²³ A higher adjustment cost causes a higher risk premium in the corporate loan rate and thus increases the credit spread. We define the credit spread in the data as a 80.6%-19.4% weighted average²⁴ of the Moody's Aaa and Baa yields and subtract the one-year constant maturity Treasury rate. The mean spread over the 1953-2015 period is 2.08%, while the model generates a mean spread of 2.05%.

We can interpret the intermediary borrowing constraint parameters, ξ , as a regulatory capital constraint set by the government. Under Basel II and III, corporate loans and bonds have a risk weight that depends on their credit quality. The risk weight on commercial and industrial bank loans with 2.5 year maturity ranges from 13% for AAA, 54% for BBB-, 125% for B+, to 325% for CCC. A blended regulatory capital requirement of 6% (8% times a blended risk weight of 75%) seems appropriate. This implies that $\xi = 0.94$. This is the key parameter we vary in our macro-prudential policy experiments.

We set the deposit insurance fee parameter κ to 8.4 basis points. To compute this number, we divide the total assessment revenue reported by the FDIC for 2016, \$10 billion, by the total short-term debt of U.S. chartered financial institutions from the Flow of Funds, \$11,849 billion.²⁵

²³We set the target to the dividend level in the deterministic steady state of the model.

²⁴To determine the portfolio weights on the Aaa versus Baa grade bonds, we use market values of the amounts outstanding from Barclays.

²⁵Our measure of short-term debt includes both insured and uninsured deposits. Therefore, the insurance fee we calculate is smaller than the fee banks pay per dollar of insured deposits only, which would amount to 14.2 bps.

Preference Parameters Preference parameters affect many equilibrium quantities and prices simultaneously, and are harder to pin down directly by data. For simplicity, we assume that both borrowers and savers have log utility: $\sigma_B = \nu_B = 1$ and $\sigma_S = \nu_S = 1$. The subjective time discount factor of borrowers $\beta_B = 0.931$ targets the capital-to-GDP ratio, an obvious target as it governs borrowers desire to accumulate wealth. The capital-to-output ratio is 2.25 in the model, and 2.24 in the data.²⁶

The time discount factor of the saver disproportionately affects the mean of the short-term interest rate. We set $\beta_S = 0.982$ to generate a low average real rate of interest of 2.2%.

Government Parameters To add quantitative realism to the model, we match both the unconditional average and the cyclical properties of discretionary spending, transfer spending, labor income tax revenue, and corporate income tax revenue.

Discretionary and transfer spending as a fraction of GDP are modeled as follows: $G_t^i/Y_t = G^i \exp\{b_i(g_t - \bar{g})\}$, $i = o, T$. The scalars G^o and G^T are set to match the observed average discretionary spending to GDP of 17.58% in the 1953-2014 NIPA data, and transfer spending to GDP of 3.18%, respectively.²⁷ We set $b_o = -2.5$ and $b_T = -25$ in order to match the slope in a regression of log spending to GDP on GDP growth and a constant. We match these slopes: -0.75 and -7.26 in the model versus -0.71 and -7.14 in the 1953-2014 data.

Similarly, we model the labor income tax rate as $\tau_t = \tau \exp\{b_\tau(g_t - \bar{g})\}$. We set the tax rate $\tau = 29.5\%$ in order to match observed average income tax revenue to GDP of 17.3%.²⁸ The model generates an average of 18.6%. We set the sensitivity of the tax rate to aggregate productivity growth $b_\tau = 2$ to match the observed sensitivity of log income tax revenue to GDP

²⁶Consistent with our definition of GDP, we include the residential and commercial real estate stock in the total capital stock.

²⁷We divide by $\exp\{b_i/2\sigma_g^2/(1 - \rho_g^2)(b_i - 1)\}$, a Jensen correction, to ensure that average spending means match the targets.

²⁸We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

to GDP growth. The regression slope of log income tax revenue to GDP on GDP growth and a constant produces similar pro-cyclicality: 0.86 in the model and 0.70 in the data.

Fourth, we set the corporate tax rate that both financial and non-financial corporations pay to a constant $\tau_{\Pi} = 20\%$ to match observed corporate tax revenues of 3.41% of GDP. The model generates an average of 3.62%. The tax shield of debt and depreciation that firms and banks enjoy in the model substantially reduces the effective tax rate corporations pay, both in the model and in the data. We set the tax rate on financial income for savers (interest on short-term debt) equal to $\tau_D = 13.2\%$, 2/3 of the corporate tax rate.²⁹

Government debt to GDP averages 60% of GDP in a long simulation of the benchmark model. While it fluctuates meaningfully over prolonged periods of time (standard deviation of 49%), the government debt to GDP ratio remains stationary.³⁰

4 Results

Before discussing the main results on macro-prudential policy, we study the behavior of key macro-economic and financial variables. They capture important features of the data and lend credibility to the policy experiments that are to follow. Specifically, we report means and standard deviations from a long simulation of the model (10,000 years), as well as averages conditional on being in a good state (positive TFP growth and low uncertainty, i.e. $\sigma_{\omega,L}$), non-financial recession (negative TFP growth, low uncertainty), and financial recession (negative TFP growth and high uncertainty $\sigma_{\omega,H}$).

²⁹This calculation reflects that short term debt in the model also includes government debt, 50-65% of which is held by foreigners who do not pay US tax.

³⁰In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates τ_t when debt-to-GDP falls below $\underline{b^G} = 0.1$ –the profligacy region– and by gradually increasing personal tax rates when debt-to-GDP exceed $\overline{b^G} = 1.2$ –the austerity region. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -0.1. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 150%. Our simulations never reach the -10% and +150% debt/GDP states. The simulation spends 24.4% of the time in the profligacy and 15% of the time in the austerity region. The fraction of time spent in these regions has no effect on the overall resources of the economy.

Table 2: Unconditional Macroeconomic Quantity Moments

	Data			Model		
	stdev	output corr.	AC	stdev	output corr.	AC
GDP	2.53%	1.00	0.55	2.43%	1.00	0.55
CONS	1.75%	0.88	0.42	2.43%	0.87	0.60
X/Y	1.58%	0.73	0.57	1.56%	0.55	0.18
X/K	0.82%	0.63	0.72	0.81%	0.60	0.21

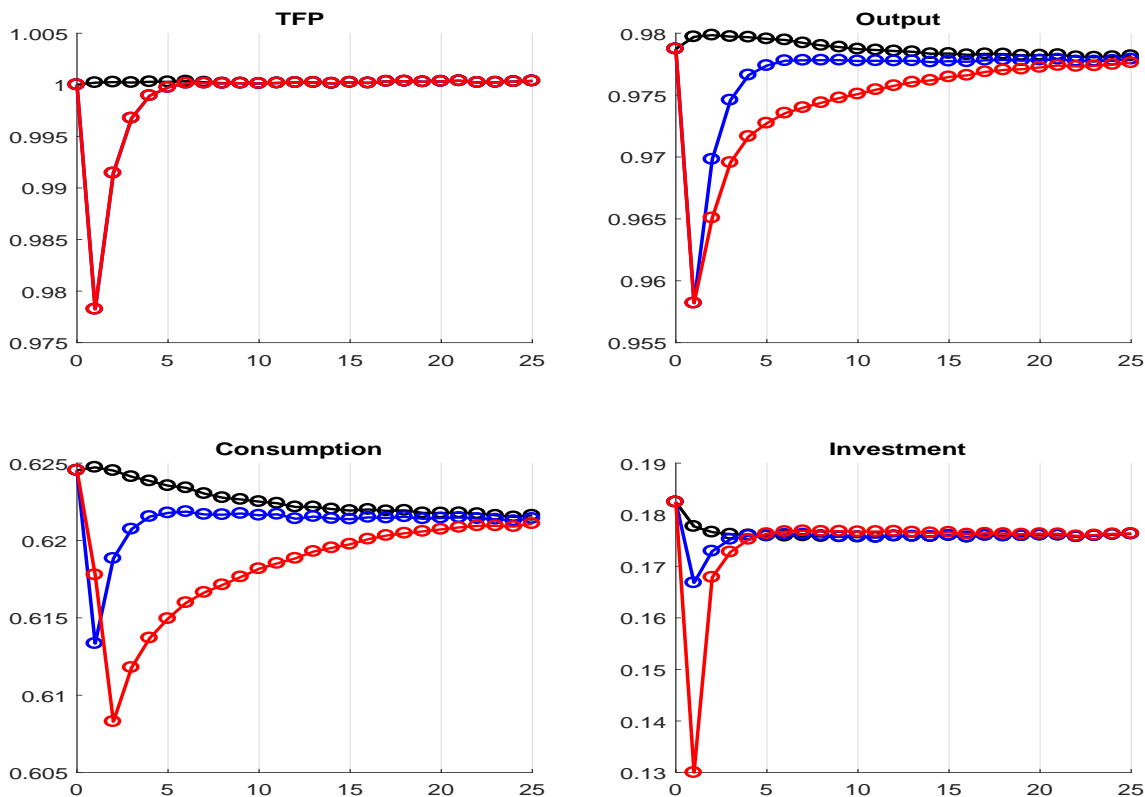
4.1 Macro Quantities

Table 2 reports the standard deviation of aggregate quantities, their correlation with GDP, and their autocorrelation. Moments in the data are computed from HP-detrended log series. Moments in the model are by assumption stationary, and are also computed from log series of the simulation. The model matches the volatility of GDP and its autocorrelation. The model further matches the volatility of the investment to GDP ratio (0.82% vs. 0.81%). The investment/GDP ratio and investment rate display modest pro-cyclicality in both data and model. Investment rates are insufficiently persistent in the model. The model overstates consumption volatility. While in the data log aggregate consumption is clearly less volatile than log output (1.75% vs. 2.53%), these volatilities are identical in the model. Consumption in the model exhibits the right cyclicalities, but is slightly too persistent relative to the data. We will further discuss the source of the consumption volatility in the model below.

We present impulse-response graphs to explore the behavior of macro-economic quantities conditional on the state of the economy. We start off the model in year 0 in the average TFP state (the middle of the five points on the TFP grid) and in the low uncertainty state ($\sigma_{\omega,L}$). The five endogenous state variables are at their ergodic averages. In period 1, the model undergoes a change to a lower TFP grid point. In one case (red line), the recession is accompanied by a switch to the high uncertainty state ($\sigma_{\omega,H}$); a financial recession. In the second case, the economy remains in the low uncertainty state; a non-financial recession (blue line). From period 2 onwards, the two exogenous state variables follow their stochastic laws of motion. For comparison, we also show a series that does not undergo any shock in period 1 but where the exogenous states stochastically mean revert from the high-TFP state in period 0 (black line). For each of the three scenarios, we simulate 10,000 sample paths of 25 years and average across them. Figure 2 plots the macro-economic quantities. The top left panel is

for the productivity level Z^A . By construction, it falls by the same amount in financial and non-financial recessions; a 2% drop. Productivity then gradually mean reverts over the next decade. The black line shows how productivity would have evolved absent a shock in period 1.

Figure 2: Financial vs. Non-financial Recessions: Macro Quantities



The graphs show the average path of the economy through a recession episode which starts at time 1. In period 0, the economy is in the average TFP state. The recession is either accompanied by high uncertainty (high σ_ω), a financial recession plotted in red, or low uncertainty (low σ_ω), a non-financial recession) plotted in blue. From period 2 onwards, the economy evolves according to its regular probability laws. The black line plots the dynamics of the economy absent any shock in period 1. We obtain the three lines via a Monte Carlo simulation of 10,000 paths of 25 periods, and averaging across these paths. **Blue line:** non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

The other three panels show impulse-responses for output, consumption, and investment. The percentage drop in output is larger than that in productivity. In the initial period of the shock, the drop in output is the same when the economy is additionally hit by an uncertainty shock (red line) as if it is not (blue line). This has to be the case because capital is a state variable, labor is supplied inelastically, and productivity is identical. In financial recessions,

the economy suffers from a second period of decline in consumption, despite the rebound in productivity. Output remains lower for longer in a financial recession. The added persistence resembles the slow recovery that typically follows a financial crisis. The bottom right panel shows a 28% drop in investment in financial recessions but only a modest drop in non-financial recessions. Despite the bounce back in period 2, investment remains depressed for a prolonged period of time. Aggregate consumption partially offsets the initial decline in investment in a financial recession: the initial drop in consumption is smaller than in a non-financial recession. The low rate of return on savings induces the saver to consume more in a financial crisis.³¹ Consumption drops subsequently and remains below the non-financial recession level for the remaining periods, as the capital stock remains depressed.

4.2 Balance Sheet Variables

Next, we turn to the key balance sheet variables in Table 3. The first two columns report the unconditional mean and volatility. The last three columns report conditional averages in expansions, non-financial recessions, and financial recessions, respectively.

Non-financial Corporate Sector The first panel focuses on the non-financial corporate sector. Rows 1 and 2 display the market value of assets ($p_t K_t^B$) and the market value of liabilities ($q_t^m A_t^B$), both scaled by GDP. Their difference is the market value of firm equity scaled by GDP. Their ratio is the market leverage ratio (row 4). Book leverage, defined as the book value of debt to the book value of assets in row 5, is 35.16%, matching the low observed corporate leverage in the data. Entrepreneurs own substantially more than half of their firms in the form of corporate equity (64.84%). Total credit (in book values) to non-financial firms amounts to 79.1% of GDP. Firms delever in financial recessions, hence the fall in book leverage. The mild drop in the market price of firm assets mitigates the counter-cyclical of market leverage. Indeed, Tobin's q (the variable p in row 16) falls by 2.6% from expansions to financial recessions.

³¹Since output in the first period is by construction identical for both types of recessions, where do the approximately 2.5% of output go in a financial recession that are not reflected in consumption and investment? Bankruptcies of firms and banks spike in financial recessions and cause deadweight losses that account for the remaining output.

Borrowers default when their profits fall below the default threshold $\underline{\pi}$. This is more likely when the cross-sectional distribution of idiosyncratic productivity shocks widens, as the mass of firms with productivity shocks below the threshold ω_t^* increases. The model generates average corporate default and loss rates of 2.25% (row 7) and 0.96% points (row 9), respectively, implying an average loss-given-default rate of 43.09% (row 8). All these numbers are in line with the data. Default and loss rates are 6-7 times higher in financial recessions (5.5% and 2.31%) than in non-financial recessions and expansions (about 0.9% and 0.4% in both). The model generates the right amount of corporate credit risk, on average, and generates the strong cyclicity in the quantity of risk observed in the data.³²

Most of the time, firms stay away from the leverage constraint because entrepreneurs are risk averse and take into account the costs of bankruptcy when making their leverage choices. However, borrowers are constrained in some of the financial recessions (3% of the time, row 6). These are the worst crisis episodes in the simulation. More generally, firms reduce their reliance on debt financing in financial recessions, as they get close to a binding constraint and debt funding becomes more costly due to a higher rate of costly bankruptcies. As a result of the higher cost of debt funding, firms cut their borrowing from the financial sector, and do not pursue the investment projects they would otherwise undertake. Relative to expansions, output falls by 4.5% and investment by 20% in financial recessions.

Intermediaries Intermediary leverage is 97.13% on average in book values (row 11 of Table 3) and 93.3% in market values. The average ratio of total intermediary debt-to-assets in the data for 1985-2014 is 90.7%, close to the intermediaries in our model.³³ Intermediaries choose to be so highly levered for a number of reasons. Like the corporate firms, they are owned

³²In the 1991 recession, the delinquency rate spiked at 8.2% and the charge-off rate at 2.2%. For the 2007-09 crisis, the respective numbers are 6.8% and 2.7%. These are far above the unconditional averages of 3.1% and 0.7% cited in footnote 20. Similarly, during the 2001 recession, the default rate on high-yield bonds was 9.9%, far above the 1981-2014 average of 4.1%.

³³Krishnamurthy and Vissing-Jorgensen (2015) identify a group of financial institutions as net suppliers of safe, liquid assets. This group contains U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations, Money market mutual funds, GSEs, Agency- and GSE-backed mortgage pools, Issuers of ABS, and REITs. The group of excluded financial institutions are Insurance Companies, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. We note that intermediaries in the model only hold loans as assets, while intermediaries in the data hold other, riskier assets such as derivatives in addition that require greater regulatory equity capital.

Table 3: Balance Sheet Variables and Prices

	Unconditional		Expansions	Non-fin Rec.	Fin Rec.
	mean	stdev	mean	mean	mean
Borrower					
1. Mkt Val of Capital / Y	2.250	0.043	2.265	2.273	2.199
2. Mkt Val of Corp Debt / Y	0.806	0.047	0.816	0.804	0.780
3. Book val corp debt / Y	0.791	0.045	0.792	0.802	0.788
4. Market corp leverage	35.82%	1.94%	36.03%	35.38%	35.48%
5. Book corp leverage	35.16%	1.82%	35.32%	34.92%	34.90%
6. Fraction leverage constr binds	0.32%	5.65%	0.00%	0.00%	2.91%
7. Default rate	2.25%	2.07%	0.85%	0.91%	5.50%
8. Loss-given-default rate	43.09%	3.23%	43.95%	42.60%	41.69%
9. Loss Rate	0.96%	0.89%	0.38%	0.39%	2.31%
Intermediary					
10. Mkt fin leverage	93.30%	3.16%	93.24%	93.57%	92.92%
11. Book fin leverage	97.13%	4.46%	98.37%	97.66%	92.24%
12. Fraction leverage constr binds	61.30%	48.71%	31.50%	89.70%	91.05%
13. Bankruptcies	0.54%	1.12%	0.10%	0.81%	2.23%
14. Dividends	0.52%	1.57%	1.14%	0.06%	-1.26%
Saver					
15. Deposits / Y	0.769	0.059	0.780	0.784	0.728
16. Government Debt / Y	0.602	0.498	0.578	0.692	0.648
Prices					
17. Tobin's q	1.000	0.017	1.010	0.990	0.974
18. Risk-free rate	2.19%	2.86%	2.45%	4.10%	0.26%
19. Corporate bond rate	4.24%	0.20%	4.13%	4.40%	4.55%
20. Credit spread	2.05%	2.94%	1.68%	0.30%	4.28%
21. Excess return on corp. bonds	1.09%	3.44%	1.87%	-0.15%	-0.49%

by impatient shareholders. They also enjoy a tax shield. They earn a large spread (2.05%, row 20) between the rate on corporate loans (4.24%, row 19) and the short-term deposit rate (2.19%, row 18). They bear the interest rate risk associated with the maturity transformation they perform, as well as the credit risk on the loans. Given the low (but realistically calibrated) average loan loss rate and their equity issuance cost, they choose to take up substantial leverage to reach their desired risk-return combination.

Adrian, Boyarchenko, and Shin (2015) show that book leverage is pro-cyclical while market leverage is counter-cyclical both for commercial banks and for broker-dealers. Our model generates this pattern, at least directionally. Market leverage increases to 93.57% in non-financial recessions. Book leverage, in contrast, falls from 98% in expansions to 92.24% in financial recessions. Why does book leverage decline in financial recessions? Intermediaries suffer losses on their credit portfolio in the worst states of the world (row 21). At the same time risk is high. Low prices (high yields, row 19) of corporate loans reflect the higher default risk and the higher credit risk premium. This reduces the market value of intermediary assets.

A lower value of bank assets in turn tightens the regulatory capital constraint. The intermediary leverage constraint binds in 91.05% of the financial crises compared to 61.3% unconditionally and 31.5% in expansions. When binding, intermediaries must reduce liabilities to meet capital requirements, as measured by deposits to GDP in row 15, which explains the drop in book leverage. Given the low cost of deposit funding in a financial crises (0.26%) and the high credit spreads they earn in those states of the world (4.28%), intermediaries make high profits going forward. They would like to raise more deposits and increase corporate lending but their constraint prevents them from doing so. Intermediaries raise new equity to partially offset the credit losses (negative dividends of 1.26% of GDP, line 14), but this is costly due to the dividend adjustment cost.

In contrast, the credit spread is much lower at 0.3% in non-financial recessions, making lending less attractive, all else equal. These recessions resemble standard TFP-induced recessions in real business cycle models: as productivity is temporarily low, depositors want to borrow against future income to smooth consumption. In addition, the supply of government debt goes up due to increased government spending and lower labor income tax revenue. As a result, the risk free rate has to rise to 4.1% to clear the market for short-term debt. At the same

time, low productivity reduces corporate loan demand, with both factors resulting in the low credit spread of 0.3% and a drop in bank profits. In response, banks lower dividend payments to 0.06% of GDP, relative to 1.14% in expansions. The lower profits reduce bank equity and cause a binding leverage constraint in 89.7% all non-financial recessions, compared to 31.5% of expansions.

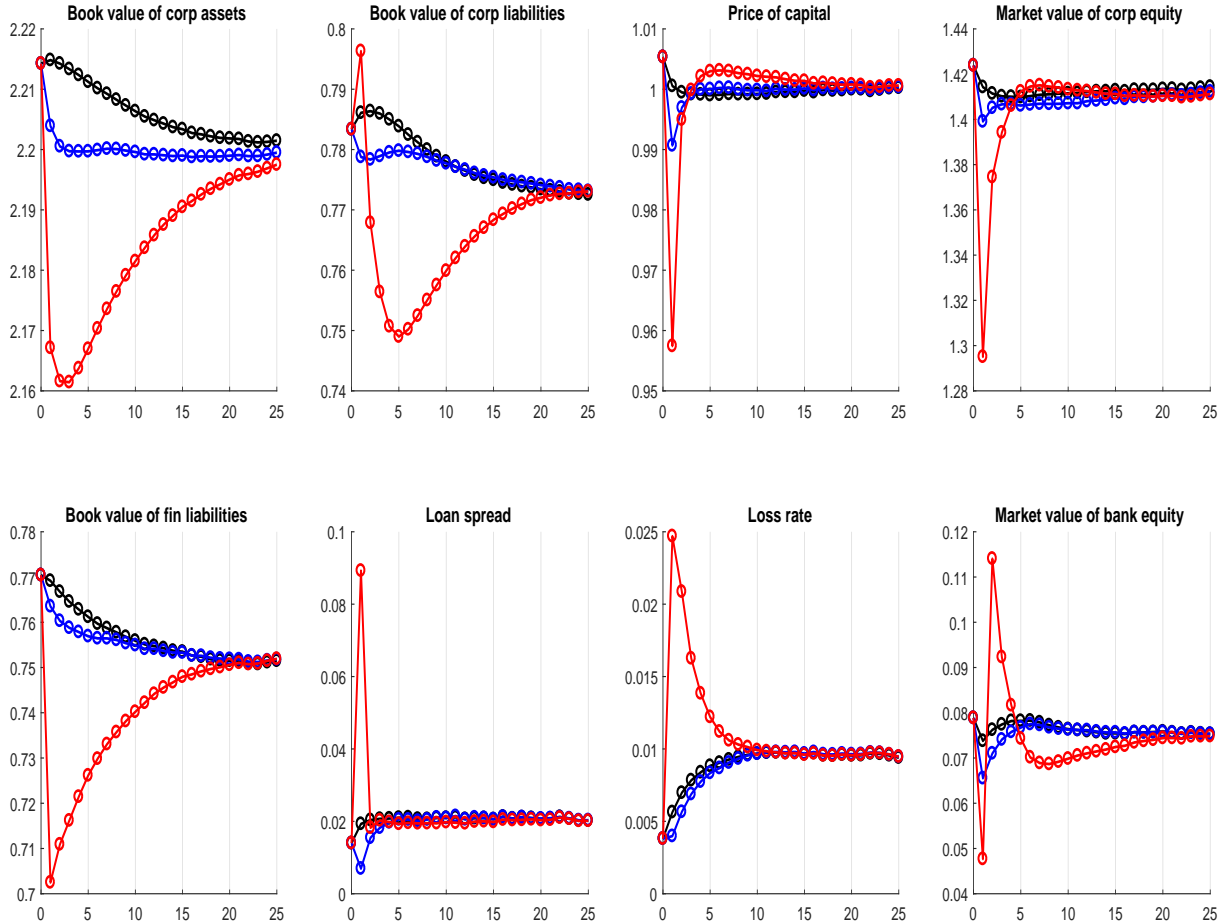
In summary, while banks are roughly equally likely to be constrained in financial and non-financial recessions, the reasons for the bindingness of the constraint are fundamentally different. In financial recessions, banks suffer large credit losses and are forced to shrink, delever and issue equity. However, banks earn high risk premia on corporate bonds in these period and would like to expand lending, but cannot do so due to the cost of raising equity. In non-financial recessions, banking becomes unprofitable due to the shrinking net interest margin, which slowly depletes equity. Banks have no desire to expand lending, but exhaust their borrowing constraint to avoid raising costly equity.

The size of the intermediary sector, relative to GDP, shrinks in financial recessions. Both book and market values of intermediary assets shrink about 3-5% relative to their levels in expansions (rows 2 and 3). At the same time, their liabilities shrink from 78% of GDP in expansions to 72% of GDP in financial recessions (row 15). Since GDP itself falls, bank liabilities fall by 11%.

In the equilibrium of our model, 0.54% of banks are insolvent in a typical year (row 13). Intermediary failures are concentrated in financial crisis, when 2.23% of banks are insolvent, 22 times as many as in expansions and 2.8 times as many as in non-financial recessions. In those periods, the government steps in, makes whole the depositors (short-term creditors), and takes over the assets of the banks at their market value. The failed banks are replaced with an equal number of new banks that are seeded with at the average capital level by the bank equity holders (through a costly equity issuance). Deposit insurance lowers the cost of debt funding and provides banks with a risk shifting motive vis-a-vis the government. However, because of the cost of equity issuance, bank owners try to avoid low intermediary net worth states. The balance of these two factors generates rare financial disasters when a non-trivial fraction of the banking system is insolvent.

Figure 3 show the impulse-response functions for assets and liabilities of both non-financial

Figure 3: Financial vs. Non-financial Recessions: Balance Sheet Variables Intermediaries



Blue line: non-financial recession, **Red line:** financial recession, **Black line:** no shocks.

firms and banks. The top row reports book values of corporate assets (capital) and liabilities (loans). Corporations shrink on both sides of their balance sheet during financial recessions. The market value of corporate equity, with both assets and liabilities valued at market prices, drops sharply in financial recessions due to the sharp drop in Tobin's q (top row, right hand side). The book value of corporate liabilities is also the book value of financial assets. In the first graph on the bottom row, we can see that banks' liabilities (book value) fall by even more than their assets during financial recession; simply put, banks delever during these periods. Because of large losses on their loan portfolio (bottom row, third graph), banks charge large credit spreads (bottom row, second graph). The spike in the loan spread in the first period of a financial recession is driven by a large drop in the risk free rate. Even though financial

book leverage declines, market leverage increases in crises, or equivalently, the market value of financial equity initially declines (bottom row, fourth graph). This is because the market value of liabilities rises more sharply than the market value of assets. Due to the large spread earned by banks in the first year, bank equity recovers and overshoots in the second year of the crises. However, high credit losses persist and bank equity declines once more in years 3-5 of the crisis. From there, banks slowly rebuild their equity capital and expand their loan business as loss rates return to normal levels. In the simulation, this process takes close to 20 years.

Savers Risk averse savers only hold safe debt, provided both by the intermediaries and the government. On average, these two sources of safe assets account for 77% (row 15) and 60% of GDP (row 15). In our model, as in the data, the government's tax revenues are pro-cyclical and its expenses counter-cyclical. The supply of safe debt by intermediaries is strongly pro-cyclical. In financial recessions, intermediaries have to delever due to their equity losses. This delevering requires savers to increase consumption and reduce savings. To induce savers to dissave, a large drop in the real interest rate is required – on average the real interest rate is close to 4 percentage points lower in a financial recession compared to a non-financial recession. The magnitude of this drop depends on savers' EIS, everything else equal. If savers' EIS is high, they are more willing to increase consumption in crises, and the interest will fall by less. A smaller reduction in the interest rate limits the benefit to borrowers and intermediaries that arises from low interest rates.³⁴

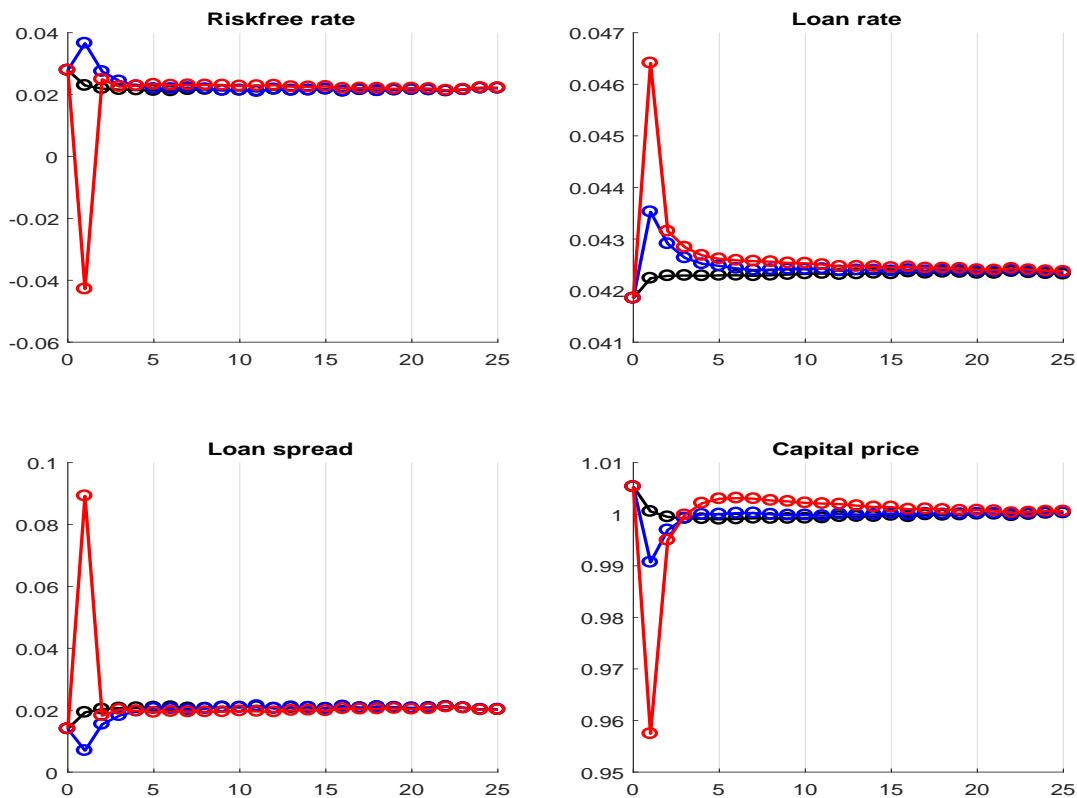
Prices Real interest rates on safe debt are 2.2% on average and have a volatility of 2.86% (row 18). Both are reasonable numbers, especially for production-based asset pricing model which typically struggle with these moments. Financial recessions see declines in collateral values (row 17), low real interest rates (row 18), high corporate credit spreads (row 20), and high expected excess returns on corporate loans (row 21). All of these are important features of real-life financial crises.

One important quantitative success of the model is its ability to generate a high unconditional

³⁴For example, when $\nu_S = 60 \gg 1$, the risk-free interest rate volatility approaches zero. Intermediaries no longer benefit from low, even negative interest rates in crises. The absence of cheap funding in crises makes them more reluctant to take on more leverage in the first case.

credit spread while matching the observed amount of default risk. The credit spread is also highly volatile (2.94% standard deviation) and more than twice as high in financial recessions than in expansions. The rise in the credit spread in financial recessions to 4.28% reflects not only the increase in the default risk but also an increase in the credit risk premium. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical “shadow SDF” for the intermediary sector. We discuss this further in the next section.

Figure 4: Financial vs. Non-financial Recessions: Prices



Blue line: non-financial recession **Red line:** financial recession.

Figure 4 shows the impulse-responses in the bust experiment for the interest rates, the credit spread, and the price of capital. In the first period of a financial recession following a boom, the real risk-free rate turns sharply negative and the credit spread blows out to 9%. Financial recessions are periods of high credit risk and credit risk premia, both of which enter in the credit spread. Non-financial recessions are characterized by a rise in the risk-free rate and a

decline in the credit spread, and a much smaller decline in the price of capital.

4.3 Consumption and Welfare

Table 4 reports the moments of consumption for both households, as well as each agent’s value function, and aggregate welfare. For intermediaries, we report the inverse of their marginal value of wealth instead of consumption. The intermediary SDF is given by

$$\mathcal{M}_{t,t+1}^I = \mathcal{M}_{t,t+1}^B \left(\frac{1 + \sigma^I(d_{t+1}^I - \bar{d})}{1 + \sigma^I(d_t^I - \bar{d})} \right)^{-1} F_{\epsilon,t+1},$$

where $\mathcal{M}_{t,t+1}^B$ is the borrower SDF, $F_{\epsilon,t+1}$ is the probability of intermediary failure in $t + 1$, and $\frac{1}{1 + \sigma^I(d_t^I - \bar{d})}$ is the marginal value of wealth to intermediaries in t . We can think of the inverse of this marginal value,

$$c_t^I \equiv 1 + \sigma^I(d_t^I - \bar{d})$$

as a measure of “intermediary consumption” for the purpose of understanding the contribution of intermediaries to asset pricing and risk sharing.

Using this definition of intermediary consumption, the intermediary has by far the most volatile consumption growth (14.5%), followed by the saver (4.08%), and the borrower (3.12%). Intermediaries end up absorbing a disproportionate fraction of the aggregate risk in the economy. In financial recessions, borrowers and intermediaries suffer large drops in their consumption, while saver consumption remain unchanged. Financial recessions destroy aggregate wealth (higher DWL from bankruptcy), but they also have important redistributive implications. Specifically, financial recessions redistribute wealth from borrowers to savers.

The third panel reports moments related to aggregate welfare. Overall welfare, the population weighted average of the two households’ value functions measured in consumption equivalent units, is highest in expansions and lowest in financial recessions. The welfare difference between expansions and financial recessions for savers is small at 0.4%. However, borrowers have 1.7% lower welfare in financial recessions. The market value of intermediaries shrinks by 30%, going from expansion to financial recession. The large reduction in the market value of intermediary equity is consistent with the experience in the Great Recession.

Table 4: Consumption and Welfare

	Unconditional		Expansions	Non-fin Rec.	Fin Rec.
	mean	stdev	mean	mean	mean
Consumption					
Consumption, B	0.285	0.011	0.292	0.282	0.268
Consumption, S	0.336	0.014	0.337	0.326	0.337
Consumption, I	0.999	0.076	1.028	0.977	0.916
Consumption growth					
Consumption, B	0.00%	3.12%	1.14%	-1.06%	-2.73%
Consumption, S	0.00%	4.08%	0.41%	-1.55%	-0.68%
Consumption, I	0.00%	14.45%	1.46%	-1.62%	-5.01%
Welfare					
Aggregate welfare	0.620	0.003	0.622	0.619	0.616
Value function, B	0.285	0.005	0.287	0.283	0.282
Value function, S	0.336	0.003	0.336	0.336	0.335
Value function, I	0.075	0.024	0.083	0.066	0.064
DWL/GDP	0.008	0.007	0.003	0.004	0.020
Marginal utility ratios					
log(MU B / MU S)	1.315	0.052	1.297	1.290	1.368

The last panel reports the ratio of marginal utilities between borrowers and savers. In a complete markets model this ratio would be constant over time. Our model is an incomplete markets model featuring imperfect risk sharing; the marginal utility ratio displays high volatility of 5%. For example, the borrower has 30% higher marginal utility than the saver in expansions, but 37% higher marginal utility in financial recessions.

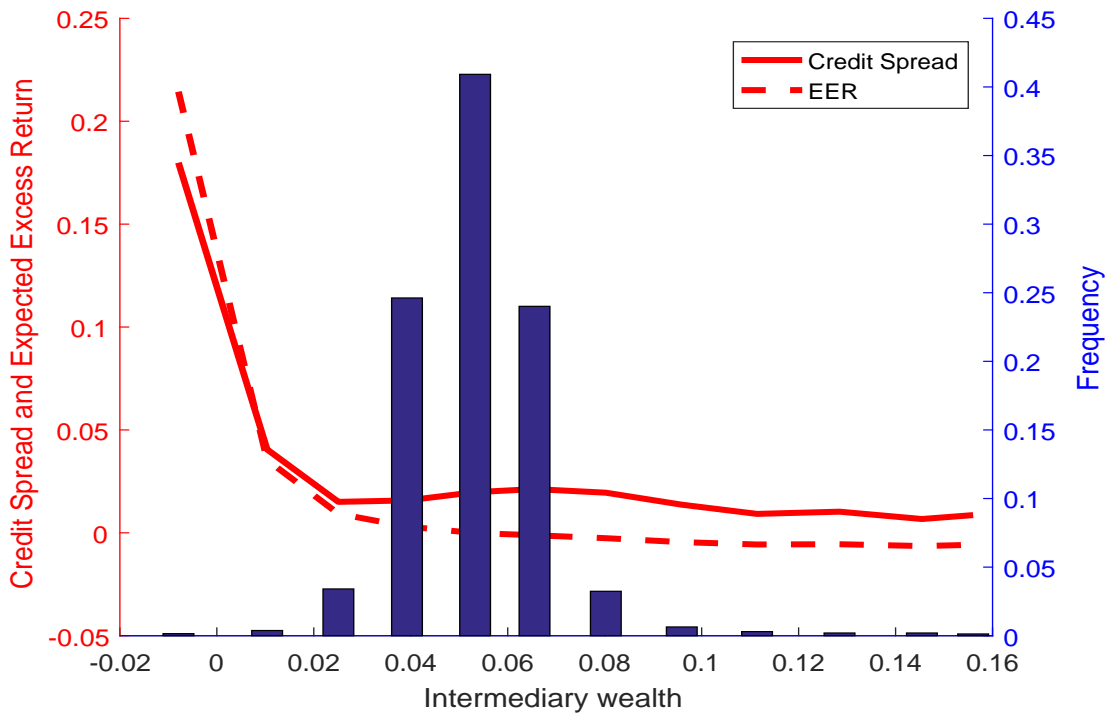
4.4 Credit Spread and Risk Premium

Figure 5 shows the histogram of intermediary wealth plotted against two different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield r_t^m on corporate bonds and the risk-free rate, where we compute the bond yield as $r_t^m = \log\left(\frac{1}{q_t^m} + \delta\right)$.³⁵ Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.

³⁵This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream $(1, \delta, \delta^2, \dots)$ occurring in the future.

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) on corporate loans/bonds earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary’s stochastic discount factor with the corporate bond’s excess return, and an additional component that reflects the tightness of the intermediary’s leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by the constrained intermediary. The market risk free rate is lower than the “shadow” risk free rate implied by the intermediary SDF. When intermediary wealth is relatively high, the leverage constraint is not binding and the EER is approximately zero in this region of the state space. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during financial crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis episodes when intermediary wealth reaches zero or drops below zero, the EER reaches 20 percent.

Figure 5: The Credit Spread and the Financial Intermediary Wealth Share



Solid line: credit spread; **dashed line:** expected excess return

5 Macro-prudential Policy

We use our calibrated model to investigate the effects of macro-prudential policy choices. Our main experiment is a variation in the intermediaries' leverage constraint. In the benchmark model, intermediaries can borrow 94 cents against every dollar in assets ($\xi = .94$). We explore tighter constraints ($\xi = .75, \xi = .80, \xi = .85, \xi = 0.90$), as well as looser constraints ($\xi = .97$). We further explore a time-varying capital requirement conditional on the uncertainty state, $\sigma_{\omega,t}$, with a tighter requirement during low and a looser requirement during high uncertainty periods ($\xi = \{.93, .95\}$). Our third macro-prudential policy experiment is to charge intermediaries $\kappa = 1.0\%$ for deposit insurance, a much higher tax on leverage than in the benchmark (0.084%). Tables 5 and 6 show the results. Table 6 reports results in percentage deviation from the benchmark.

Changing maximum intermediary leverage Rows 10 and 11 of Table 5 show that a policy that constrains bank leverage is indeed successful at bringing down that leverage. Banks reduce the size of their assets, both in book and market value terms (rows 2 and 3) and the size of their liabilities (row 16). On net, intermediary equity increases sharply as ξ is lowered (row 14). With intermediaries better capitalized, financial fragility falls. Intermediary sector insolvencies (row 13) drop rapidly from 0.54% to 0.01% first (at $\xi = .90$) and further to 0% for all tighter capital requirements. Interestingly, with tighter regulation, intermediaries' constraints bind *less* frequently (row 12). Intermediaries become more cautious when they are farther away from insolvency, since the option to default (limited liability) is farther out-of-the-money. In sum, tighter regulation leads to a safer intermediary sector, but also to a smaller one.

The firm value of intermediaries, V^I (row 15), increases by less than intermediary capital W^I (row 14). We can interpret the ratio of both, the value of banks per dollar of bank capital, in row 15A as the gross franchise value of banks. If the value of banks was simply equal to the difference between the market value of assets and liabilities, this ratio would be equal to one.³⁶ A ratio greater than unity reflects the additional franchise value of banking. We can see that the franchise value of intermediaries declines as regulation is tightened. At lower levels of ξ , banks

³⁶Savers are the marginal agents in the market for risk free debt. Savers' SDF effectively determines the market value of bank debt, which is greater than the internal "shadow" value of debt to banks.

require more capital to fund a given amount of loans. Bank equity capital is more costly than debt funding due to (i) tax advantages of debt, (ii) deposit insurance, (iii) dividend adjustment costs, (iv) differences in patience between borrowers and lenders. Therefore, as intermediaries are forced to fund each dollar of loans with a greater proportion of equity capital, the value created for bank shareholders per dollar of capital invested declines. While in the benchmark, the gross franchise value of intermediaries is 1.34, this value declines to 1.04 at $\xi = .75$.

The increased safety of the financial sector is also reflected in lower corporate default, loss-given-default, and loss rates for non-financial firms as ξ falls (rows 7-9). Firms choose to reduce leverage (rows 4-5) and their LTV constraints never bind (row 6). Firms reluctance to undertake more leverage despite the safer environment may be understood from the higher interest rates and spreads they face on debt (rows 21 and 22). When intermediary capacity shrinks (with lower ξ), the reward for providing intermediation services increases. Higher financial sector net interest income is evidenced by a higher credit spread in the face of lower loss loan rates.

A first key effect of tighter macro-prudential policy is that the economy's output shrinks (row 29), and the corporate and financial sectors become less levered and fragile. The capital stock shrinks sizeably (row 30). The reduction in output arises because firms are smaller and borrow less from a smaller intermediary sector, since debt finance became more costly. Even though GDP shrinks, aggregate consumption increases slightly (row 31) as the capital requirement is tightened. Lower DWL from fewer firm and bank failures offset the decrease in production (row 28).

A second key effect of tighter capital regulation is that it effectively makes banks more risk averse. Maintaining a larger equity buffer means that intermediaries need to incur larger (in absolute value) deviations from the dividend target. At lower ξ , the dividend adjustment cost makes reacting to economic fluctuations by varying dividends more expensive, relative to the benchmark. Therefore, intermediaries react to fluctuations by adjusting assets and debt instead of equity (dividends), as can be seen in rows 34-36 of table 6: lower ξ increases the volatility of asset and debt growth, but reduces the volatility of dividend growth. When going from the benchmark to slightly tighter regulation, the reduced ability of intermediaries to absorb aggregate risk thus spills over to higher consumption volatility for both households (rows 38-40). However, the overall effect on macroeconomic volatility is non-monotonic: tighter

Table 5: Macroprudential Policy

	Bench ($\xi = .94$)	$\xi = .75$	$\xi = .80$	$\xi = .85$	$\xi = .90$	$\xi = .97$	$\xi = \{.93, .95\}$	$\kappa = .01$
Borrowers								
1. Mkt value capital/GDP	2.250	2.161	2.174	2.189	2.215	2.298	2.267	2.183
2. Mkt value corp debt/GDP	0.806	0.603	0.618	0.646	0.717	0.894	0.846	0.632
3. Book val corp debt/GDP	0.791	0.617	0.628	0.652	0.716	0.847	0.824	0.640
4. Market corp leverage	0.358	0.279	0.284	0.295	0.324	0.389	0.373	0.289
5. Book corp leverage	0.352	0.285	0.289	0.298	0.323	0.368	0.363	0.293
6. Fraction LTV constr binds	0.32%	0.00%	0.00%	0.00%	0.00%	3.94%	0.43%	0.00%
7. Default rate	2.25%	1.58%	1.62%	1.69%	1.92%	2.47%	2.38%	1.65%
8. Loss-given-default rate	43.09%	30.36%	31.14%	33.04%	38.05%	46.09%	45.11%	32.03%
9. Loss Rate	0.96%	0.44%	0.47%	0.53%	0.72%	1.14%	1.06%	0.50%
Intermediaries								
10. Mkt fin leverage	0.933	0.737	0.786	0.836	0.889	0.970	0.935	0.939
11. Book fin leverage	0.971	0.738	0.791	0.846	0.910	1.043	0.981	0.948
12. Fraction intermed constr binds	61.30%	30.91%	42.61%	43.53%	60.38%	96.03%	81.43%	92.18%
13. Bankruptcies	0.54%	0.00%	0.00%	0.00%	0.01%	9.13%	1.13%	1.55%
14. Wealth I	0.056	0.162	0.135	0.109	0.083	0.027	0.056	0.044
15. Value function, I	0.075	0.168	0.146	0.130	0.110	0.046	0.066	0.057
15A. Value/Wealth I	1.339	1.038	1.078	1.195	1.338	1.717	1.176	1.271
Savers								
16. Deposits/GDP	0.769	0.455	0.497	0.553	0.653	0.884	0.808	0.606
17. Government debt/GDP	0.602	0.109	0.109	0.111	0.129	1.348	1.146	0.236
18. Wage S	0.539	-1.09%	-0.88%	-0.69%	-0.43%	+0.71%	+0.19%	-0.77%
Prices								
19. Tobin's q	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20. Risk-free rate	2.19%	2.36%	2.29%	2.26%	2.25%	2.19%	2.25%	2.18%
21. Corporate bond rate	4.24%	4.66%	4.60%	4.53%	4.42%	3.91%	4.16%	4.56%
22. Credit spread	2.05%	2.30%	2.31%	2.27%	2.16%	1.72%	1.91%	2.37%
23. Excess return on corp. bonds	1.09%	1.90%	1.82%	1.70%	1.44%	0.62%	0.89%	1.94%

regulation diminishes the intermediary's willingness to absorb aggregate risk, but at the same time it makes financial crises less severe. Volatility of investment and consumption peaks at $\xi = .85$. For even tighter regulation, volatility declines as the difference between financial and non-financial recessions becomes smaller. At a capital requirement of 25% ($\xi = .75$), volatilities are lower than in the benchmark. Interestingly, looser regulation than benchmark *also* raises macroeconomic volatility. The economy with $\xi = .97$ experiences severe financial recessions more frequently, as the indicated by the higher default rates of firms and banks.³⁷

In summary, tightening the capital requirement has two key effects: (1) it shrinks the economy and lowers leverage of firms and banks, reducing macroeconomic risk; and (2), it effectively makes banks more risk averse, reducing their willingness to absorb aggregate risk and causing higher macroeconomic volatility, holding constant risk. The net effect is that population weighted aggregate welfare (row 24) is maximized at $\xi = .90$, just 4 percentage points below the benchmark, for, leading to a welfare gain of 35 bps. An even lower capital requirement increases volatility without additional benefits from fewer defaults.

The small aggregate gain masks large heterogeneity in gains and losses among borrowers savers. Tighter regulation redistributes wealth from savers to borrowers: it both reduces the supply of safe assets and makes it less reliable. As debt finance becomes more expensive, borrowers rely more on equity finance and a larger share of firm earnings accrues to them. At $\xi = .90$, borrower consumption is 3.4% higher than in the benchmark and saver consumption is 2.7% lower (row 32 and 33), leading to welfare gains and losses of similar size (row 26 and 27). Looser capital requirements have the opposite distributional effect and increase saver wealth at the expense of borrowers. However, DWL from firm and bank failures are so large at $\xi = .97$ that consumption of both agents is lower than in the benchmark. The increased financial fragility makes both agents worse off.

None of the variations of ξ allow a Pareto improving transfer scheme (row 25), which means that welfare gains to borrowers are not sufficient to compensate savers for their losses.³⁸

³⁷At this low capital requirement of 3%, intermediaries only keep a small equity buffer and are much closer to their default threshold. Higher likelihood of bankruptcy causes them to optimally absorb less aggregate risk.

³⁸Savers are more patient than borrowers and require greater compensation for the same permanent reduction in consumption.

Table 6: Macroprudential Policy: Macro and Welfare

	Bench	$\xi = .75$	$\xi = .80$	$\xi = .85$	$\xi = .90$	$\xi = .97$	$\xi = \{.93, .95\}$	$\kappa = .01$
Welfare								
24. Pop. weighted aggregate welfare	0.620	-0.01%	+0.15%	+0.25%	+0.32%	-0.80%	-0.31%	+0.49%
25. CV residual aggregate welfare	0%	-75.08%	-62.00%	-50.25%	-28.87%	-11.12%	17.92%	-20.87%
26. Value function, B	0.285	+6.31%	+5.73%	+5.05%	+3.40%	-1.31%	-2.42%	+3.28%
27. Value function, S	0.336	-5.37%	-4.59%	-3.83%	-2.29%	-0.36%	+1.49%	-1.88%
28. DWL/GDP	0.008	-36.35%	-34.66%	-31.23%	-20.89%	+124.88%	+14.05%	-18.12%
Size of the Economy								
29. GDP	0.978	-1.6%	-1.4%	-1.1%	-0.6%	+0.9%	+0.3%	-1.2%
30. Capital stock	2.199	-5.5%	-4.7%	-3.8%	-2.2%	+3.0%	+1.1%	-4.2%
31. Aggr. Consumption	0.621	+0.0%	+0.04%	+0.06%	+0.06%	-1.39%	+0.0%	-0.1%
32. Consumption, B	0.291	+5.7%	+5.4%	+4.9%	+3.4%	-1.4%	-2.6%	+3.3%
33. Consumption, S	0.343	-4.8%	-4.5%	-4.1%	-2.7%	-1.4%	+2.2%	-2.9%
Volatility								
34. Mkt value corp debt gr	0.029	+15.2%	+59.6%	+68.7%	+45.3%	+180.3%	-10.0%	+0.5%
35. Deposits gr	0.049	-23.8%	+69.6%	+86.7%	+44.9%	+27.7%	-62.4%	-52.3%
36. Dividend gr	2.370	-53.1%	-49.7%	-31.6%	-26.9%	+6.1%	+1.9%	+3.0%
37. Investment gr	29.56%	-68.0%	-6.9%	+35.2%	+16.4%	-5.5%	-65.7%	-69.9%
38. Consumption gr	2.17%	-11.6%	+23.4%	+27.7%	+16.6%	+58.5%	-18.3%	-18.0%
39. Consumption gr, B	3.12%	-22.2%	+0.9%	+10.7%	+2.2%	+18.0%	-9.7%	-22.1%
40. Consumption gr, S	4.08%	-24.6%	+36.8%	+49.1%	+24.2%	+28.0%	-49.8%	-36.6%
41. log (MU B / MU S)	0.052	-32.7%	+13.7%	+27.7%	+12.1%	+13.2%	-34.2%	-41.4%

Numbers in columns two to eight are percentage changes relative to the benchmark.

Time-varying Capital Requirement The 7th column of tables 5 and 6 show an experiment with a capital requirement that varies conditional on the uncertainty state $\sigma_{\omega,t}$. When uncertainty is low, banks' constraint is tightened ($\xi = .93$) than in the benchmark, whereas it is loosened ($\xi = .95$) when uncertainty is high. This procyclical capital requirement causes a moderate expansion in corporate and financial leverage, leading to slightly higher loan losses (1.06% vs .96%) and substantially more frequent bank defaults (1.13% vs. .54%). However, the higher DWL are offset by a greater capital stock (+1.1%) and higher GDP (+.3%), such that aggregate consumption remains unchanged. Even though credit risk increases, the credit spread shrinks due to a smaller credit risk premium (+.89% vs. 1.09%). Since intermediaries are less constrained in financial crises, they require less compensation for carrying aggregate risk and macroeconomic volatility decreases. Risk sharing among borrowers and savers improves, as indicated by the lower volatility of the MU ratio (row 41, -34.2%). The greater financial sector distributes wealth from borrowers to savers: saver consumption increases by 2.2% and welfare by 1.5%. Borrower welfare declines by 2.4%, implying an ex-post aggregate welfare loss of 42 bps.

However, since the experiment makes the more patient savers significantly better off, it allows for Pareto improving wealth transfers. The compensating variation wealth residual (row 25) is 17% of GDP. This numbers implies that a transfer scheme making both agents exactly as well off as in the benchmark would leave the policy maker with a residual payment stream that has a present value equal to 17% of GDP.³⁹

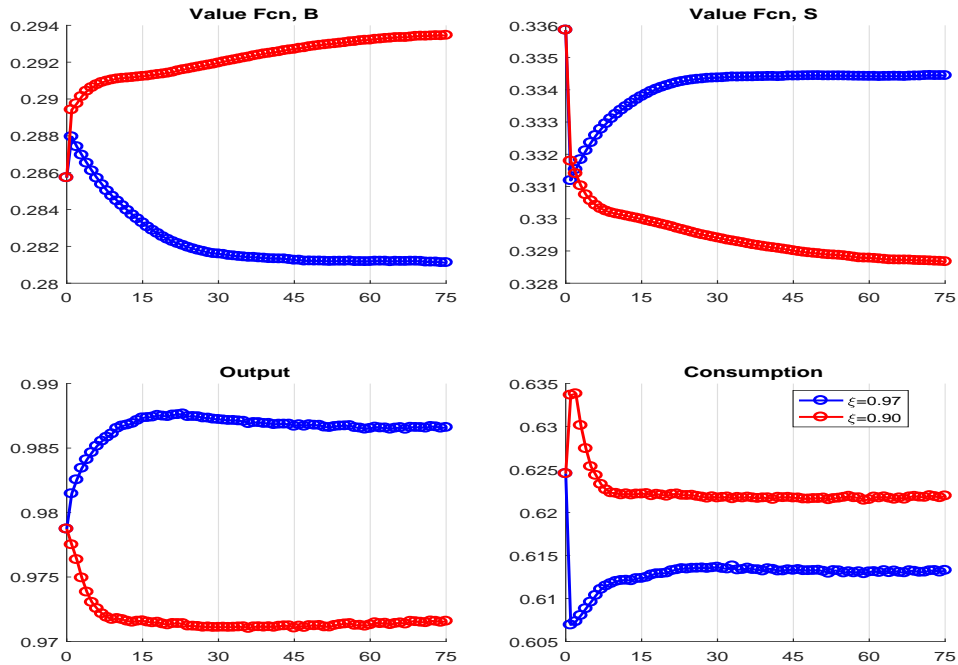
Increasing cost of deposit insurance The last column of tables 5 and 6 show the result of an experiment that increases the cost of deposit insurance κ from 0.08% to 1% per unit of deposit. While this is a direct tax on bank leverage, the incidence of this tax in equilibrium falls on firms and savers. Firms bear most of the cost through a significantly higher credit risk premium, paying a 2.39% credit spread (vs. 2.05%) despite a reduction in the loan loss rate from .96% to .5%. As a result, equilibrium firm leverage is much lower. Contrary to the

³⁹To interpret the number, consider the following thought experiment. In the $\xi = \{.93, .95\}$ economy, the policy maker could levy a tax on savers that reduces their utility to the level of the benchmark. The present value of the tax revenue would be 26% of benchmark GDP. The revenue could be used to pay a subsidy to borrowers that makes them exactly as well off as in the benchmark; the present value of this subsidy would be 9% of GDP. The remaining revenue has a present value of 17% of GDP and constitutes the Pareto improvement.

presumed intention of the policy, banks lever up due to the high excess returns they earn on loans and become more fragile (1.13% bank failures). The capital stock shrinks by 4.2% and GDP by 1.2%, which is mostly offset by smaller DWL from corporate defaults. Overall, the policy redistributes wealth from savers to borrowers, as the overall supply of risk free debt shrinks. The policy leads to the largest ex-post aggregate welfare gain of .49%. However, since it makes savers significantly worse off, it does not allow for Pareto improving transfers.

Policy transitions The tables above only compare the ergodic distributions of economies with different policy parameters. How would an unanticipated policy change to a tighter or looser capital requirement affect output, consumption, and the welfare of borrowers and savers in the short term?

Figure 6: Transition Dynamics After Change in Capital Requirement



Blue line: lower capital requirement; **red line:** higher capital requirement

Figure 6 plots the evolution of these variables after a policy change from the benchmark to either a higher ($\xi = .9$) or a lower ($\xi = .97$) capital requirement. In the long run, output, consumption, and agent welfare converge to their ergodic means in tables 5 and 6. In the short

run, consumption “overshoot” in both cases. Tightening the capital requirement by 4 p.p. leads a contraction in GDP as investment drops. But lower investment also causes a consumption boom in the short run as the economy transitions to a permanently lower capital stock.

6 Conclusion

We provide the first calibrated macro-economic model which features intermediaries who extend long-term defaultable loans to firms producing output and raise deposits from risk averse savers, and in which both banks and firms can default. The model incorporates a rich set of fiscal policy rules, including deposit insurance, and endogenizes the risk-free interest rate.

Like in the standard accelerator model, shocks to the macro-economy affect entrepreneurial net worth. Since firm borrowing is constrained by net worth, macro-economic shocks are amplified by tighter borrowing constraints. Unlike the original models, ours features risk averse infinitely-lived entrepreneurs and long-term defaultable debt. A second financial accelerator arises from explicitly modeling the financial intermediaries’ balance sheet as separate from that of the entrepreneur-borrowers and saving households. Intermediaries are subject to regulatory capital constraints. Macro-economic shocks that lead to binding intermediary borrowing constraints amplify the shocks through their direct effect on intermediaries’ net worth and the indirect effect on borrowers to whom the intermediaries lend.

We explore the dynamics of quantities and prices in this setting and compare them to U.S. data, with a focus on understanding differences between financial and non-financial recessions. Our main application studies macro-prudential policy that imposes restrictions on bank leverage. While such policies reduce credit risk and bank fragility, they increase the cost of debt funding for firms and intermediaries, hampering investment and shrinking the size of the economy. They further redistribute wealth from savers to bank and firm equity holders. Our experiments show that the incidence of policies designed to limit the riskiness of the financial sector may fall on other sectors of the economy.

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A Model Appendix

A.1 Borrower-entrepreneur problem

A.1.1 Technology

The exogenous laws of motion for the TFP level Z_t^A is (lower case letters denote logs):

$$\log Z_t^A = (1 - \rho_A)z^A + \rho_A \log Z_{t-1}^A + \epsilon_t^A \quad \epsilon_t^A \sim iid \mathcal{N}(0, \sigma^A)$$

Denote $\mu_{ZA} = e^{z^A + \frac{(\sigma^A)^2}{2(1-\rho_A^2)}}$.

Idiosyncratic productivity of borrower-entrepreneur i at date t is denoted

$$\omega_{i,t} \sim iid \text{Gamma}(\gamma_{0,t}, \gamma_{1,t}),$$

where the parameters $\gamma_{0,t}$ and $\gamma_{1,t}$ are chosen such that

$$\begin{aligned} \mathbb{E}(\omega_{i,t}) &= 1, \\ \text{Var}(\omega_{i,t}) &= \sigma_{\omega,t}^2. \end{aligned}$$

Individual output is

$$Y_{i,t} = \omega_{i,t} Z_t^A K_t^{1-\alpha} L_t^\alpha.$$

Aggregate production is

$$Y_t = \int_{\Omega} Y_{i,t} dF(\omega_i) = \int_{\Omega} \omega dF(\omega) Z_t^A K_t^{1-\alpha} (L_t)^\alpha = Z_t^A K_t^{1-\alpha} (L_t)^\alpha.$$

Individual producer profit is

$$\pi_{i,t} = Y_{i,t} - \sum_j w^j L^j - A_t.$$

Therefore, the default cutoff at $\pi_{i,t} = 0$ is

$$\omega_t^* = \frac{\pi + \sum_j w_t^j L_t^j + A_t}{Y_t}.$$

A.1.2 Preliminaries

We start by defining some preliminaries.

Borrower Defaults

$$\begin{aligned}\Omega_A(\omega_t^*) &= 1 - F_{\omega,t}(\omega_t^*) \\ \Omega_K(\omega_t^*) &= \int_{\omega_t^*}^{\infty} \omega dF_{\omega,t}(\omega)\end{aligned}$$

where $F_{\omega,t}(\cdot)$ is the CDF of $\omega_{i,t}$.

It is useful to compute the derivatives of $\Omega_K(\cdot)$ and $\Omega_A(\cdot)$:

$$\begin{aligned}\frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_{\omega}(\omega) d\omega = -\omega_t^* f_{\omega}(\omega_t^*), \\ \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} &= \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_{\omega}(\omega) d\omega = -f_{\omega}(\omega_t^*),\end{aligned}$$

where $f_{\omega}(\cdot)$ is the p.d.f. of $\omega_{i,t}$.

Capital Adjustment Cost

$$\Psi(X_t, K_t^B) = \frac{\psi}{2} \left(\frac{X_t}{K_t^B} - \delta_K \right)^2 K_t^B.$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) = \psi \left(\frac{X_t}{K_t^B} - \delta_K \right) \tag{24}$$

$$\Psi_K(X_t, K_t^B) = -\frac{\psi}{2} \left(\left(\frac{X_t}{K_t^B} \right)^2 - \delta_K^2 \right) \tag{25}$$

Dividend Adjustment Cost

$$\Sigma(d_t^I) = \frac{\sigma^I}{2} (d_t^I - \bar{d})^2.$$

The derivative is

$$\Sigma'(d_t^I) = \sigma^I (d_t^I - \bar{d}).$$

A.1.3 Optimization Problem

We consider the producers's problem in the current period after aggregate TFP and idiosyncratic productivity shocks have been realized.

Let $\mathcal{S}_t^B = (Z_t^A, \sigma_{\omega,t}, W_t^I, W_t^S, B_{t-1}^G)$ represent state variables exogenous to the borrower-entrepreneur's decision.

Then the borrower problem is

$$V^B(K_t^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_{t+1}^B, 1, X_t, A_{t+1}^B, L_t^j\}} \left\{ (1 - \beta_B) (C_t^B)^{1-1/\nu} + \right. \\ \left. + \beta_B \mathbb{E}_t \left[(V^B(K_{t+1}^B, A_{t+1}^B, \mathcal{S}_{t+1}^B))^{1-\sigma_B} \right]^{\frac{1-1/\nu}{1-\sigma_B}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^B = (1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) Y_t + (1 - \tau_t^B) w_t^B \bar{L}^B + G_t^{T,B} + p_t [X_t + \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) K_t^B] \\ + q_t^m A_{t+1}^B - \Omega_A(\omega_t^*) A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) \\ - p_t K_{t+1}^B - X_t - \Psi(X_t, K_t^B) - (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) \sum_{j=B,S} w_t^j L_t^j + D_t^I \quad (26)$$

$$F A_t^B \leq \Phi p_t \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) K_t^B, \quad (27)$$

where we have define after-tax depreciation $\tilde{\delta}_K = (1 - \tau_{\Pi}^B) \delta_K$.

Denote the value function and the partial derivatives of the value function as:

$$V_t^B \equiv V(K_t^B, A_t^B, \mathcal{S}_t^B), \\ V_{A,t}^B \equiv \frac{\partial V(K_t^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B}, \\ V_{K,t}^B \equiv \frac{\partial V(K_t^B, A_t^B, \mathcal{S}_t^B)}{\partial K_t^B}.$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathbb{E}_t \left[(V^B(K_{t+1}^B, A_{t+1}^B, \mathcal{S}_{t+1}^B))^{1-\sigma_B} \right]^{\frac{1}{1-\sigma_B}}.$$

Marginal Cost of Default Before deriving optimality conditions, it is useful to compute the marginal consumption loss due to an increased default threshold ω_t^*

$$\begin{aligned}
\frac{\partial C_t^B}{\partial \omega_t^*} &= \frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} (1 - \tau_{\Pi}^B) Y_t \\
&+ \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} \left[(1 - \tilde{\delta}_K) p_t K_t^B - A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) - (1 - \tau_{\Pi}^B) \sum_j w_t^j L_t^j \right] \\
&= -f_{\omega}(\omega_t^*) \left[(1 - \tau_{\Pi}^B) \omega_t^* Y_t + (1 - \tilde{\delta}_K) p_t K_t^B - A_t^B (1 - (1 - \theta) \tau_{\Pi}^B + \delta q_t^m) - (1 - \tau_{\Pi}^B) \sum_j w_t^j L_t^j \right] \\
&= -f_{\omega}(\omega_t^*) Y_t \underbrace{\left[\frac{(1 - \tilde{\delta}_K) p_t K_t^B - A_t^B (\theta \tau_{\Pi}^B + \delta q_t^m)}{Y_t} \right]}_{=\mathcal{F}_t} \\
&= -f_{\omega}(\omega_t^*) Y_t \mathcal{F}_t.
\end{aligned}$$

The function \mathcal{F}_t has an intuitive interpretation as the marginal loss, expressed in consumption units per unit of aggregate output, to producers from an increase in the default threshold. The first term is the loss of capital due to defaulting members. The second term represents gains due to debt erased in foreclosure.

A.1.4 First-order conditions

Loans The FOC for loans A_{t+1}^B is:

$$\begin{aligned}
q_t^m \frac{(u_t^B)^{1-1/\nu}}{C_t^B} (1 - \beta_B) (V_t^B)^{1/\nu} &= \\
\lambda_t^B F - \beta_B \mathbb{E}_t [(V_{t+1}^B)^{-\sigma_B} V_{A,t+1}^B] (C E_t^B)^{\sigma_B - 1/\nu} (V_t^B)^{1/\nu} &
\end{aligned} \tag{28}$$

where λ_t^B is the Lagrange multiplier on the constraint in (27).

Capital Similarly, the FOC for new capital K_{t+1}^B is:

$$\begin{aligned}
p_t \frac{(1 - \beta_B) (V_t^B)^{1/\nu} (u_t^B)^{1-1/\nu}}{C_t^B} &= \\
\beta_B \mathbb{E}_t [(V_{t+1}^B)^{-\sigma_B} V_{K,t+1}^B] (C E_t^B)^{\sigma_B - 1/\nu} (V_t^B)^{1/\nu} &
\end{aligned} \tag{29}$$

Investment The FOC for investment X_t is:

$$[1 + \Psi_X(X_t^B, K_t^B) - p_t] \frac{(1 - \beta_B) (U_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B} = 0,$$

which simplifies to

$$1 + \Psi_X(X_t^B, K_t^B) = p_t. \quad (30)$$

Labor Inputs Defining $\gamma_B = 1 - \gamma_I - \gamma_S$, aggregate labor input is

$$L_t = \prod_{j=B,I,S} (L_t^j)^{\gamma_j}.$$

We further compute

$$\frac{\partial \omega_t^*}{\partial L_t^j} = \left(\frac{w_t^j}{Y_t} - \omega_t^* \frac{\text{MPL}_t^j}{Y_t} \right),$$

defining the marginal product of labor of type j as

$$\text{MPL}_t^j = \alpha \gamma_j Z_t^A \frac{L_t}{L_t^j} \left(\frac{K_t^B}{L_t} \right)^{1-\alpha}.$$

The FOC for labor input L_t^j is then

$$\frac{(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B} \left[(1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) \text{MPL}_t^j - (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) w_t^j + \frac{\partial \omega_t^*}{\partial L_t^j} \frac{\partial C_t^B}{\partial \omega_t^*} \right] = 0,$$

which yields

$$(1 - \tau_{\Pi}^B) \Omega_K(\omega_t^*) \text{MPL}_t^j = (1 - \tau_{\Pi}^B) \Omega_A(\omega_t^*) w_t^j + f_{\omega}(\omega_t^*) (w_t^j - \omega_t^* \text{MPL}_t^j) \mathcal{F}_t. \quad (31)$$

A.1.5 Marginal Values of State Variables and SDF

Loans Taking the derivative of the value function with respect to A_t^B gives:

$$\begin{aligned} V_{A,t}^B &= \left[- (1 - (1 - \theta) \tau_{\Pi}^I + \delta q_t^m) \Omega_A(\omega_t^*) + \frac{\partial \omega_t^*}{\partial A_t^B} \frac{\partial C_t^B}{\partial \omega_t^*} \right] \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B} \\ &= - \left[(1 - (1 - \theta) \tau_{\Pi}^I + \delta q_t^m) \Omega_A(\omega_t^*) + f_{\omega}(\omega_t^*) \mathcal{F}_t \right] \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B}, \end{aligned} \quad (32)$$

where we used the fact that $\frac{\partial \omega_t^*}{\partial A_t^B} = \frac{1}{Y_t}$.

Capital Taking the derivative of the value function with respect to K_t^B gives:

$$\begin{aligned} V_{K,t}^B &= \left[p_t \Omega_A(\omega_t^*) (1 - (1 - \tau_{\Pi}^B) \delta_K) + (1 - \tau_{\Pi}^B) (1 - \alpha) \Omega_K(\omega_t^*) Z_t^A \left(\frac{K_t^B}{L_t} \right)^{-\alpha} - \Psi_K(X_t^B, K_t^B) + \frac{\partial C_t^B}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^B} \right. \\ &\quad \left. + \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K) \left(\Omega_A(\omega_t^*) + K_t^B \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} \frac{\partial \omega_t^*}{\partial K_t^B} \right) \right] \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B}. \end{aligned}$$

Taking the derivative

$$\frac{\partial \omega_t^*}{\partial K_t^B} = -\frac{\omega_t^*}{Y_t} (1 - \alpha) Z_t^A \left(\frac{K_t^B}{L_t} \right)^{-\alpha},$$

we get

$$\begin{aligned} V_{K,t}^B = & \left\{ p_t \Omega_A(\omega_t^*) (1 - \tilde{\delta}_K) \left(1 + \Phi \tilde{\lambda}_t^B \right) + (1 - \tau_{\Pi}^B) (1 - \alpha) \Omega_K(\omega_t^*) Z_t^A \left(\frac{K_t^B}{L_t} \right)^{-\alpha} - \Psi_K(X_t^B, K_t^B) \right. \\ & \left. + (1 - \alpha) f_\omega(\omega_t^*) \omega_t^* \left[Z_t^A \left(\frac{K_t^B}{L_t} \right)^{-\alpha} \mathcal{F}_t + \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K) \right] \right\} \frac{(1 - \beta_B) (u_t^B)^{1-1/\nu} (V_t^B)^{1/\nu}}{C_t^B}. \end{aligned} \quad (33)$$

SDF We can define the stochastic discount factor (SDF) from t to $t + 1$ of borrowers:

$$\mathcal{M}_{t,t+1}^B = \beta_B \left(\frac{C_{t+1}^B}{C_t^B} \right)^{-1/\nu_B} \left(\frac{V_{t+1}^B}{C E_t^B} \right)^{1/\nu_B - \sigma_B}. \quad (34)$$

A.1.6 Euler Equations

Loans Substituting in for $V_{A,t+1}^B$ in (28) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + E_t \left\{ \mathcal{M}_{t,t+1}^B \left[\Omega_A(\omega_{t+1}^*) \left(1 - (1 - \theta) \tau_{\Pi} + \delta q_{t+1}^m \right) + f_\omega(\omega_{t+1}^*) \mathcal{F}_{t+1} \right] \right\}. \quad (35)$$

Capital Substituting in for $V_{K,t+1}^B$ and using the SDF expression, we get the recursion:

$$\begin{aligned} p_t = E_t \left[\mathcal{M}_{t,t+1}^B \left\{ p_{t+1} \Omega_A(\omega_{t+1}^*) (1 - \tilde{\delta}_K) \left(1 + \Phi \tilde{\lambda}_{t+1}^B \right) + (1 - \tau_{\Pi}) (1 - \alpha) \Omega_K(\omega_{t+1}^*) Z_{t+1}^A \left(\frac{K_{t+1}^B}{L_{t+1}} \right)^{-\alpha} \right. \right. \\ \left. \left. - \Psi_K(X_{t+1}^B, K_{t+1}^B) + (1 - \alpha) f_\omega(\omega_{t+1}^*) \omega_{t+1}^* \left(Z_{t+1}^A \left(\frac{K_{t+1}^B}{L_{t+1}} \right)^{-\alpha} \mathcal{F}_{t+1} + (1 - \tilde{\delta}_K) \Phi \tilde{\lambda}_{t+1}^B p_{t+1} \right) \right\} \right]. \end{aligned} \quad (36)$$

A.2 Intermediaries

A.2.1 Statement of stationary problem

Wealth W_t^I is the wealth of all intermediaries after firm and intermediary bankruptcies and recapitalization of defaulting intermediaries by borrowers.

At the end of each period, all intermediaries face the following optimization problem over dividend payout and portfolio composition (see equation (15) in the main text):

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} d_t^I + E_t \left[\mathcal{M}_{t,t+1}^B F_{\epsilon,t+1} \left(V_{t+1}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right] \quad (37)$$

subject to:

$$W_t^I \geq d_t^I + \Sigma(d_t^I) + q_t^m A_{t+1}^I + (q_t^f + \tau^\Pi r_t^f + \mathbf{1}_{\{B_t^I < 0\}} \kappa) B_t^I, \quad (38)$$

$$W_{t+1}^I = \left[\left(\tilde{M}_{t+1} + \Omega_A(\omega_{t+1}^*) \delta q_{t+1}^m \right) A_{t+1}^I + B_t^I \right], \quad (39)$$

$$q_t^f B_t^I \geq -\xi q_t^m A_{t+1}^I, \quad (40)$$

$$A_{t+1}^I \geq 0, \quad (41)$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \quad (42)$$

For the evolution of intermediary wealth in (39), we have defined the total after-tax payoff per bond

$$\tilde{M}_{t+1} = (1 - (1 - \theta)\tau_\Pi^I)\Omega_A(\omega_{t+1}^*) + M_{t+1}/A_{t+1}^B,$$

where M_{t+1} is the total recovery value of bankrupt borrower firms seized by intermediaries, as defined in (12).

A.2.2 First-order conditions

Dividend Payout To take the FOC for dividends d_t^I , eliminate B_t^I by substituting the budget constraint into the transition law for wealth to get

$$W_{t+1}^I = (\tilde{M}_{t+1} + \delta\Omega_A(\omega_{t+1}^*)q_{t+1}^m)A_{t+1}^I + \frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau^\Pi r_t^f - \kappa}, \quad (43)$$

and for the leverage constraint

$$-\frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau^\Pi r_t^f - \kappa} q_t^f \leq \xi q_t^m A_{t+1}^I. \quad (44)$$

Now we can differentiate the objective function with respect to d_t^I

$$\frac{1}{1 + \Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[q_t^f \lambda_t^I + \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^B \frac{\partial}{\partial W_{t+1}^I} \left(F_{\epsilon,t+1} \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right) \right\} \right],$$

where λ_t^I denotes the Lagrange multiplier on the leverage constraint.

To compute the derivative in the expectation, rewrite the expression as

$$F_{\epsilon,t+1} \left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) = F_{\epsilon,t} V_t^I(W_t^I, \mathcal{S}_t^I) - \int_{-\infty}^{V_t^I(W_t^I, \mathcal{S}_t^I)} \epsilon dF_\epsilon(\epsilon).$$

Differentiating with respect to W_t^I gives (by application of Leibniz' rule)

$$V_t^I V_{W,t}^I f_{\epsilon,t} + V_{W,t}^I F_{\epsilon,t} - V_t^I V_{W,t}^I f_{\epsilon,t} = V_{W,t}^I F_{\epsilon,t}.$$

Substituting in this result, the FOC becomes

$$\frac{1}{1 + \Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[q_t^f \lambda_t^I + \text{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right].$$

Loans Using the same approach as for the dividend payout FOC, the FOC for loans A_{t+1}^I is

$$\begin{aligned} \frac{q_t^m}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[q_t^f \lambda_t^I + \text{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right] \\ = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[\xi q_t^m \lambda_t^I + \text{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \left(\tilde{M}_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m \right) \right\} \right]. \end{aligned}$$

Noting that the LHS is equal to the RHS of the dividend FOC above, this can be written more compactly as

$$\frac{1}{1 + \Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[\xi q_t^m \lambda_t^I + \text{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \left(\tilde{M}_{t+1} + \delta \Omega_A(\omega_{t+1}^*) q_{t+1}^m \right) \right\} \right].$$

A.2.3 Marginal value of wealth and SDF

First take the envelope condition

$$V_{W,t}^I = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa} \left[q_t^f \lambda_t^I + \text{E}_t \left\{ \mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1} \right\} \right].$$

Combining this with the FOC for dividends above yields

$$V_{W,t}^I = \frac{1}{1 + \Sigma'(d_t^I)}. \quad (45)$$

We can define a stochastic discount factor for intermediaries as

$$\mathcal{M}_{t,t+1}^I = \mathcal{M}_{t,t+1}^B \frac{1 + \Sigma'(d_t^I)}{1 + \Sigma'(d_{t+1}^I)} F_{\epsilon,t+1}. \quad (46)$$

A.2.4 Euler Equations

Using the definition of the SDF $\mathcal{M}_{t,t+1}^I$ above, we can write the FOC for dividend payout and new loans more compactly as:

$$q_t^f + \tau^\Pi r_t^f - \kappa = q_t^f \tilde{\lambda}_t^I + \text{E}_t \left[\mathcal{M}_{t,t+1}^I \right], \quad (47)$$

$$q_t^m = \xi \tilde{\lambda}_t^I q_t^m + \text{E}_t \left[\mathcal{M}_{t,t+1}^I \left(\tilde{M}_{t+1} + \delta q_{t+1}^m \Omega_A(\omega_{t+1}^*) \right) \right], \quad (48)$$

where $\tilde{\lambda}_t^I$ is the original multiplier λ_t^I divided by the marginal value of wealth.

A.3 Savers

A.3.1 Statement of stationary problem

Let $\mathcal{S}_t^S = (Z_t^A, \sigma_{\omega,t}, K_t^B, A_t^B, W_t^I, B_{t-1}^G)$ be the saver's state vector capturing all exogenous state variables. The problem of the saver is:

$$V^S(W_t^S, \mathcal{S}_t^S) = \max_{\{C_t^S, B_t^S\}} \left\{ (1 - \beta_S) [C_t^S]^{1-1/\nu} + \beta_S \mathbb{E}_t \left[\left(\tilde{V}^S(W_{t+1}^S, \mathcal{S}_{t+1}^S) \right)^{1-\sigma_S} \right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^S = (1 - \tau_t^S) w_t^S \bar{L}^S + G_t^{T,S} + W_t^S - q_t^f B_t^S \quad (49)$$

$$W_{t+1}^S = B_t^S \quad (50)$$

$$B_t^S \geq 0 \quad (51)$$

$$\mathcal{S}_{t+1}^S = h(\mathcal{S}_t^S) \quad (52)$$

As before, we will drop the arguments of the value function and denote the marginal value of wealth as:

$$\begin{aligned} V_t^S &\equiv V_t^S(W_t^S, \mathcal{S}_t^S), \\ V_{W,t}^S &\equiv \frac{\partial V_t^S(W_t^S, \mathcal{S}_t^S)}{\partial W_t^S}, \end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE_t^S = \mathbb{E}_t \left[(V^S(W_t^S, \mathcal{S}_t^S))^{1-\sigma_S} \right].$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu} = \lambda_t^S + \beta_S \mathbb{E}_t [(V_{t+1}^S)^{-\sigma_S} V_{W,t+1}^S] (CE_t^S)^{\sigma_S - 1/\nu} (V_t^S)^{1/\nu} \quad (53)$$

where λ_t^S is the Lagrange multiplier on the no-borrowing constraint (51).

A.3.3 Marginal Values of State Variables and SDF

The marginal value of saver wealth is:

$$V_{W,t}^S = (C_t^S)^{-1/\nu} (1 - \beta_S) (V_t^S)^{1/\nu}, \quad (54)$$

Defining the SDF in the same fashion as we did for borrowers, we get:

$$\mathcal{M}_{t,t+1}^S = \beta_S \left(\frac{V_{t+1}^S}{CE_t^S} \right)^{1/\nu_S - \sigma_S} \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-1/\nu_S}.$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (53) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^S + E_t [\mathcal{M}_{t,t+1}^S] \quad (55)$$

where $\tilde{\lambda}_t^S$ is the original multiplier λ_t^S divided by the marginal value of wealth.

A.4 Equilibrium

The optimality conditions describing the problem are (26), (30), (35), (36), and (31) for borrowers, (38), (47), and (48) for intermediaries, and (49) and (55) for depositors. We add complementary slackness conditions for the constraints (27) for borrowers, (40) and (41) for intermediaries, and (51) for depositors. Together with the market clearing conditions (20), (21), (22), and (23) these equations fully characterize the economy.

B Calibration Appendix

B.1 Long-term corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by $F = \frac{\theta}{1-\delta}$, a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for δ and θ .

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merrill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices⁴⁰ we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices⁴¹ we obtain a time series of option-adjusted spreads over the Treasury yield curve.

⁴⁰Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively

⁴¹They are named C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields r_t . Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter δ as

$$P^G(r_t) = \frac{1}{1+r_t} [1 + \delta P^G(r_t)]$$

Solving for $P^G(y_t)$, we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units X of the geometric bond with parameter δ one needs to sell to hedge one unit of plain vanilla bond P^c against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta, X} \sum_{t=1997.1}^{2015.12} [P^c(r_t) - X P^G(r_t; \delta)]^2$$

We estimate $\delta = 0.937$ and $X = 12.9$, yielding an average pricing error of only 0.41%. This value for δ implies a time series of durations $D_t = -\frac{1}{P_t^G} \frac{dP_t^G}{dr_t}$ with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it D_t years from now. The principal of this loan is just the price of the corresponding D_t maturity zero-coupon bond $\frac{1}{(1+r_t)^{D_t}}$

We set the "principal" F of one unit of the geometric bond to be some fraction θ of the undiscounted sum of all its cash flows $\frac{\theta}{1-\delta}$, where

$$\theta = \frac{1}{N} \sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get $\theta = 0.582$ and $F = 9.18$.

C Computational Solution

The computational solution of the model is implemented using what Judd (1998) calls “time iteration” on the system of equations that characterizes the equilibrium of the economy defined in appendix section A.4. Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of non-linear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities. The general solution approach for heterogeneous agent models with incomplete markets and portfolio constraints that we employ in this paper is well described by Kubler and Schmedders (2003). They show that there exist stationary equilibria in this class of models when all exogenous state variables follow Markov chains, as is the case in our model as well.

The procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** The state space consists of
 - two exogenous state variables $[Z_t^A, \sigma_{\omega_t}]$, and
 - five endogenous state variables $[K_t^B, A_t^B, W_t^I, W_t^S, B_t^G]$.

Denote the sets of values these state variables can take as \mathcal{S}_x and \mathcal{S}_n respectively. The aggregate state space is $\mathcal{S} = \mathcal{S}_x \times \mathcal{S}_n$. There are two sets of unknown functions of the state variables that need to be computed. The first set of unknown functions $\mathcal{C}_P : \mathcal{S} \rightarrow \mathcal{P} \subseteq \mathcal{R}^N$ determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents’ choice variables, and the Lagrange multipliers on the portfolio constraints. There is an equal number of these unknown functions and nonlinear functional equations. The second set of functions $\mathcal{C}_T : \mathcal{S} \times \mathcal{S}_x \rightarrow \mathcal{S}_n$ determines the next-period endogenous state as a function of the endogenous state in the current period and the next-period realization of shocks. To approximate the unknown functions, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents’ budget constraints, conditional on any three other state variables. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding, and we test the accuracy of the approximation by computing relative Euler equation errors.

2. **Iteratively solve for the unknown functions.** Given an initial guess $\mathcal{C}^0 = \{\mathcal{C}_P^0, \mathcal{C}_T^0\}$ at each point in the discretized state space compute tomorrow’s optimal policies as functions of tomorrow’s states. Then, compute expectation terms in the equilibrium conditions using quadrature methods. Next, solve the system of nonlinear equations for the current-period optimal policies. This defines the value of the next iterate \mathcal{C}_P^1 at the given point. Using it, compute the next iterate of the transition approximation \mathcal{C}_T^1 at the given point.

Once \mathcal{C}_1 has been computed at every point on the grid, repeat until convergence in the sup norm.

The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Judd, Kubler, and Schmedders (2002) show how Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. For example, consider the saver’s Euler Equation for risk-free bonds and constraint:

$$\begin{aligned} q_t^f &= \tilde{\lambda}_t^S + E_t [\mathcal{M}_{t,t+1}^S] \\ 0 &\leq B_t^S \end{aligned}$$

Now define an auxiliary variable x_t and two functions of this variable, such that $\tilde{\lambda}_t^{S,+} = \max\{0, x_t\}^3$ and $\tilde{\lambda}_t^{S,-} = \max\{0, -x_t\}^3$. Then the two equations above can be written as equalities:

$$\begin{aligned} q_t^f &= \tilde{\lambda}_t^{S,+} + E_t [\mathcal{M}_{t,t+1}^S] \\ 0 &= B_t^S - \tilde{\lambda}_t^{S,-} \end{aligned}$$

The solution variable for the nonlinear equation solver corresponding to the constraint is x_t .

The nonlinear equation solver needs to compute the Jacobian of the system at each step. Numerical central-difference (forward-difference) approximation of the Jacobian can be inaccurate and is computationally costly because it requires $2N + 1$ ($N + 1$) evaluations of the system, whereas analytically computed Jacobians are exact and require only one evaluation. We follow Elenev (2016) in pre-computing expectations, which simplifies the nonlinear system such that its Jacobian can be computed analytically. This greatly speeds up calculations.

3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a “burn-in” phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed.

In a long simulation, errors in the nonlinear equations are low. Table 7 reports the median error, the 95th percentile of the error distribution, the 99th, and 100th percentiles.

Table 7: Computational Errors

	Percentile				
	50th	75th	95th	99th	Max
(35)	0.0004	0.0009	0.0019	0.0033	0.0316
(36)	0.0003	0.0005	0.0011	0.0017	0.0051
(31), <i>B</i>	0.0001	0.0002	0.0002	0.0003	0.0004
(31), <i>S</i>	0.0001	0.0002	0.0002	0.0003	0.0004
(47)	0.0038	0.0079	0.0140	0.0180	0.1302
(48)	0.0042	0.0091	0.0185	0.0212	0.1389
(55)	0.0007	0.0014	0.0026	0.0036	0.0119
(40)	0.0005	0.0011	0.0027	0.0048	0.1069
(51)	0.0055	0.0080	0.0181	0.0288	0.0783
(20)	0.0005	0.0006	0.0007	0.0009	0.0369
(41)	0.0002	0.0006	0.0010	0.0015	0.0079
(27)	0.0041	0.0065	0.0137	0.0228	0.0581

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the benchmark model. The 12 equations define policy functions. They are a subset of the 21 equations that define the equilibrium.