Debauchery and Original Sin:

The Currency Composition of Sovereign Debt

Preliminary

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Abstract

This paper quantitatively investigates the currency composition of sovereign debt in the presence of two types of limited enforcement frictions arising from a government's monetary and debt policy: strategic currency debasement and default on sovereign debt. Local currency debt has better state contingency than foreign currency debt in the sense that its real value can be changed by a government's monetary policy, thus acting as a better consumption hedge against income shocks. However, this higher degree of state contingency for local currency debt provides a government with more temptation to deviate from disciplined monetary policy, thus restricting borrowing in local currency more than in foreign currency. The two financial frictions related to the two limited enforcement problems combine to generate an endogenous debt frontier for local and foreign currency debts. Our model predicts that a less disciplined country in terms of monetary policy borrows mainly in foreign currency, as the country faces a much tighter borrowing limit for local currency debt than for the foreign currency debt. Our model accounts for the surge in local currency borrowings by emerging economies in the recent decade and the "Mystery of Original Sin" by Eichengreen, Hausmann, Panizza (2004).

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1 Introduction

"Original Sin" in the international finance literature refers to a situation in which most emerging economy central governments are not able to borrow abroad in their own currency. This concept, first introduced by Eichengreen and Hausmann (1999), is still a prevailing phenomenon for a number of emerging economies, even though the recent studies by Du and Schreger (2016 a,b), and Arslanalp and Tsuda (2014) find that the ability of emerging markets to borrow abroad in their own currency has improved in the last decade.[3]

We study the currency composition of sovereign debt in the presence of two types of limited enforcement frictions arising from a government's monetary and debt policy: strategic currency debasement and default on sovereign debt. We build a dynamic general equilibrium model of a small open economy to quantitatively investigate the implications of these two different enforcement frictions for a government's debt portfolio choice. In particular, we focus on how these two frictions combine to constrain borrowing limits for local and foreign currency debt.

The temptation to debase or "debauch" the currency leads markets to restrict lending to local-currency debt for some sovereign borrowers. This temptation has been understood by economists for many years, though the literature lacks a full model of the dynamic contracting problem in a setting of debasement and default. Indeed, Keynes (1919) asserted that "Lenin is said to have declared that the best way to destroy the capitalist system is to debauch the currency." Keynes made this point in the context of the debate over debt forgiveness after the First World War – countries could effectively renege on debt by debauching the currency.[4]

Our setting is a standard small open economy model with stochastic endowment shocks, extended to allow a benevolent sovereign government to borrow in both local and foreign currency. Strategic debasement, by reducing the real value of debt through infation, is punished by an "Original Sin" regime in which the country is restricted to borrow only in foreign currency, and default is punished by permanent autarky. Risk neutral foreign investors in international financial markets are willing to lend to the sovereign government any amount, whether in local or foreign currency, as long as they are guaranteed an expected return of the gross risk-free rate R^* prevailing in the international financial markets. Since the real value of repayment for local currency debt can change depending on the inflation rate (currency depreciation rate), the foreign investors who lend in local currency offer a contract which specifies an inflation rate at each state of the world. We consider an optimal self-enforcing contract which maximizes utility of the

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[3] For example, Du and Schreger find that the cross-country mean of the share of external government debt in local currency has increased to around 60% for a sample of 14 developing countriesThe countries in the sample are Brazil, Colombia, Hungary, Indonesia, Israel, South Korea, Malaysia, Mexico, Peru, Poland, Russia, South Africa, Thailand and Turkey.

[4] See (White & Schule, 2009) for a discussion of the context of Keynes's famous statement.

representative household in the small open economy and which prevents the government from breaching the contract (i.e., satisfying the enforcement constraints).

Our model predicts that the optimal contract for local currency debt allows the government to inflate away a certain fraction of local currency debt in times of bad income shocks but asks for currency appreciation in times of good shocks as a compensation for the bad times. Hence, local currency debt in our model smooths consumption of the economy better than foreign currency debt, acting like a state-contingent asset. However, due to the limited enforcement constraint arising from a government's temptation to inflate away local currency debt, the borrowing limit for local currency is endogenously constrained, thus restricting the degree of consumption smoothing function of local currency debt. On the other hand, the enforcement constraint arising from the option to fully default on its debt mainly determines the endogenous borrowing limit for foreign currency debt. With the interaction of two enforcement frictions, our model generates a debt frontier for local and foreign currency debt, inside of which the equilibrium is supported without violating the enforcement constraints.

Quantitative results show that the country with more disciplined monetary policy can borrow more in both foreign and local currency, and that the country borrows mainly in local currency as it provides a better consumption hedge. The country with less disciplined monetary policy wants to borrow more in local currency, but is restricted to borrow mainly in foreign currency due to the enforcement constraints. Thus, our model can account for both "Original Sin" phenomenon for the emerging economies with less disciplined monetary policy and a recent surge in local currency borrowing by those with more disciplined monetary policy.

The term "original sin" has been applied in the literature to countries that are unable to borrow in their own currency because empirically there seems to be very little link between the share of external debt denominated in local currency and variables such as the volatility of inflation or the size of the country's total external liabilities that perhaps should determine how much the country can borrow in local currency. We make the point that the relationship between these endogenous variables and the currency composition of debt is not straightforward. When there is lack of commitment to repay, there is a tension between the wishes of the borrowers – who, for example, may wish to have high levels of local-currency debt as a channel for smoothing consumption – and lenders who may be reluctant to lend a portfolio heavily weighted toward local-currency debt to precisely those borrowers that most desire such a portfolio. For example, as the overall external debt of a country increases, the borrower may prefer a portfolio more weighted toward local-currency debt, because the borrower can use inflation to make debt repayment more state-contingent. But the lender may be less likely to offer a portfolio with a large amount of local-currency debt in such a scenario because the temptation to deviate from the terms of the debt contract may be too high. The currency composition of debt and variables such as the volatility of inflation or the total debt/GDP ratio are all

endogenous. They depend on parameters such as the patience and degree of risk aversion of borrowers, the cost of default and the borrower's cost of inflation. The model shows that there is no simple monotonic relationship among these variables, so it is perhaps not surprising that empirically there is no clear-cut link between the currency composition of the external portfolio and endogenous macroeconomic variables.

1.1. Related Literature

Our work builds on the intuition from the classical argument which attributes the predominance of foreign currency debt in international financial markets to a lack of monetary credibility. A government's strategic debasement to inflate away the real value of debt can pose a significant obstacle to issuing local currency debt (Calvo, 1978; Kydland and Prescott, 1977.)

Bohn (1990) builds a model in which governments can only commit to repayment of nominal sums, and have an incentive to inflate away debt. In Bohn's set-up, some domestic-currency debt is sustainable because the government bears some exogenous cost to inflation. In more recent work, Ottonello and Perez (2016) study the currency composition of sovereign debt in a dynamic general equilibrium model of a small open economy with a government with limited commitment to monetary and debt policy. As in Bohn (1990), the government faces an exogenous cost of inflation. Ottonello and Perez provide a quantitative analysis of the optimal monetary policy with local-currency debt. In both models, the original-sin regime – in which governments can borrow only in foreign currency – arises only as the special case in which the cost of inflation is zero. In practice, there must be a fairly high cost of inflation internally to underpin realistic levels of domestic currency borrowing in these models. These models also do not incorporate any possibility of outright default, which plays an important role in limiting the size of sovereign debt.

Du et al (2016) also study the currency composition of sovereign debt in a two period New-Keynesian model to show how credibility of monetary policy affects the currency composition of sovereign debt. In their model, as in Bohn and Ottonello-Perez, local-currency debt is sustainable even when governments cannot commit to a monetary policy because there is an internal cost to inflation. However, in their model, the costs arise endogenously due to sticky-price distortions. In that model, the sovereign government randomly inherits or not the ability to follow through on commitments in the second period. Since ex ante there is some probability governments will keep their word, the equilibrium can maintain more domestic-currency debt.

In our model, lenders recognize that the sovereign borrower has an incentive to inflate away the debt, and that this option to inflate is more valuable to the borrower when, for example, it is suffering from low output or has high debt obligations. The lender and the sovereign sign a contract – perhaps an implicit one – that allows for more inflation in circumstances such as this. In that sense, inflation is akin to "excusable

default" as in Grossman and van Huyck (1988). That paper presents a static model of debt (that is, debt is acquired a period in advance, but can be used only as working capital. It completely depreciatres after one period so itcannot be accumulated, nor can it be used to smooth consumption) in which the two parties agree to a contract that specifies debt repayment in each state of the world. If the borrowing country abrogates the contract, they fall into complete autarky. Grossman and van Huyck (1993) present a version of that model in which the debt is contracted in nominal terms, but the real repayment is determined by the inflation rate of the government. That paper is a step in the direction of our model, but differs in that the model is static and there is actually no debt. Instead, there is an agreement by the sovereign makes an agreement with risk-neutral "lenders" to receive a state-contingent payoff one period hence, which could be negative, and which has a mean of zero. The contract is written in such a way that the actual payoff is determiend by the rate of inflation chosen by the sovereign, and the penalty for violating the terms of the contract is complete autarky. There is no original sin regime in which the country can borrow in foreign currency. Moreover, Grossman and van Huyck (1993) do not consider a portfolio choice problem between local and foreign currency debts as in our paper, only focusing on the implications of debasement risk on local currency debt.

Our work draws on, and is closely related to models with optinal dynamic contracts in the presence of commitment problems. Atkeson (1991), Kehoe and Levine (1993), Alvarez and Jermann (2000), and Bai and Zhang (2010) are the closest analogs. These studies show that constrained borrowing limits arising from the limited enforcement problems can cause significant distortions to allocations of an economy.

Our model differs from this last set of papers in that, in our setting, there is not a full set of state-contingent claims traded internationally. Instead, our starting point resembles Eaton and Gersovitz (1981), Arellano (2008), and Aguiar and Gopinath (2006) in that we assume that only bonds that are nominally non-state-contingent can be traded. As in those papers, we do not derive this limitation endogenously, and instead appeal to the real-world observation that sovereign debt typically is not explicitly state contingent. However, our paper is unique in that it recognizes the two ways in which the debt repayments may be state contingent – because of debasement and outright default. Thus, our model shares some of the features of both strands of literature – optimal contracts but with debt that has some, but not full, state contingency. Debt denominated in home currency can be supported because of the threat of falling into the original sin regime in which all debt is denominated in foreign currency. And, foreign-currency debt can be supported in the original sin regime because of the threat of autarky.

Finally, we note that our model does share the characteristic of papers mentioned previously that governments also bear an exogenous cost to inflation. However, in contrast to the earlier work, that cost is not needed to account for why some countries can borrow in domestic-currency debt. Just the threat of falling into the orignal sin regime is sufficient to allow for some domestic-currency borrowing. That

exogenous cost of inflation is necessary in our model to nail down the nominal interest rate. Borrowers and lenders primarily are concerned with the real return on loans. The nominal interest rate would not matter per se, but is determined by the borrower's desire to avoid the exogenous inflation costs.

1.2 "Mystery of Original Sin" Revisited

Eichengreen, et al. (2004), and Hausmann and Panizza (2003) find weak empirical support for the idea that the level of development, institutional quality, or monetary credibility is correlated with Original Sin. These studies find that only the absolute size of the economy proxied by its GDP is robustly correlated with Original Sin. They call their finding the mystery of original sin and claim that the original sin problem of emerging market economies is exogenous to a country's economic fundamentals – it is rather related to the structure of the international financial system.

In this subsection, we replicate the findings of Eichengreen, et al. (2004) on an updated data set. They estimate a Tobit regression in which the dependent variable is $OSIN_i$, defined as $1 - \frac{Securities\ issued\ by\ country\ i\ in\ currency\ i}{Securities\ issued\ by\ country\ i}$, which measures the degree of Original Sin for country i. The main explanatory variables are the GDP per capita as a proxy for the level of development of country i; average inflation as a proxy for monetary credibility; GDP for the size of a country; and a country group dummy variable that indicates whether country i belongs to the financial center or Europe or not. Both $OSIN_i$ and $x_i's$ are period averages of country i. They find that after controlling for country grouping, only country size proxied by GDP is robustly correlated with a country's ability to borrow in local currency, refuting hypotheses that the emerging economies' weak financial system or lack of monetary credibility account for the Original Sin phenomenon.

We re-examine the Eichengreen, et al.'s (2004) finding with updated data from Arslanalp and Tsuda (2014), which provides a data set for external sovereign debt denominated in local currency for 23 emerging economies from 2004Q1 to 2015Q4.[5] We use the share of external local currency debt in total external debt as a dependent variable (LC Share) but use the same explanatory variables (GDP per capita, average inflation, GDP, and a dummy for European countries) as in Eichengreen, et al. (2004).[6]

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[5] The countries in the sample are Argentina, Brazil, Bulgaria, Chile, China, Colombia, Egypt, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, and Ukraine.
[6] The data source for these explanatory variables is the World Bank.

Table 1: LC Share and GDP

	LC Share		
Log_GDP_PC	-.098 (-1.18)	-.039 (-0.45)	-.043 (-0.54)
Log_GDP			.081 (2.02)*
Europe		-.179 (-1.59)	-.118 (-1.09)
Constant	1.25 (1.70)	.801 (1.07)	-1.35 (-1.07)

Absolute value of *t* statistics in parentheses.
* Significant at 10%

Eichengreen, et al. (2004) use GDP per capita as a proxy for the level of development of a country as the quality of institutions is considered to be highly correlated with GDP per capita. They find that GDP per capita alone is highly positively correlated with the degree of Original Sin, but after controlling for country grouping and country size, the estimated coefficient is not statistically significant even at the 5 percent level. Table 1 reports our regression on the updated data. It is a double censored (from 0 to 1) Tobit regression with explanatory variables of GDP per capita (measured in logs and denoted by Log_GDP_PC in the table), GDP, and a dummy for European countries. Consistent with Eichengreen, et al. (2004), only the size of a country proxied by GDP is statistically significant.

Table2: LC Share and Inflation

	LC Share					
Average Inflation	-.014 (-0.78)	-.008 (-0.50)				
Std Inflation			-.051(-1.41)	-.028 (-0.74)		
Max Inflation					-.009 (-1.53)	-.006 (-0.97)
Europe		-.191(-1.85)*		-.167 (-1.51)		-.164 (-1.53)
Constant	.465 (4.07)	.509(4.66)***	.513 (4.94)***	.521 (5.25)***	.490*** (5.77)	.517*** (6.25)

Absolute value of *t* statistics in parentheses.
* Significant at 10%
** Significant at 5%
*** Significant at 1%

Eichengreen, et al. (2004) also investigates the cross-country correlation between Original Sin and monetary credibility by regressing $OSIN_i$ on inflation-related variables. In our regression, we use three different proxies of monetary credibility: the average of inflation, the standard deviation of inflation, and the maximum inflation rate for the sample period (2004-2015). Table 2 reports the result. Even though coefficients on all three different proxies for monetary credibility are negative, all of them are statistically insignificant, thus consistent with their findings that the inflation does not explain Original Sin once the country group is controlled for. Figure 1 plots LC share and inflation variables for each country and shows

that inflation variables do not have much predictive power on which country can borrow more in local currency[7].

Table3: LC Share and all Macro variables

	LC Share		
Log_GDP_PC	-.097 (-1.11)	-.0961 (-1.04)	-.110 (-1.21)
Log_GDP	.091 (2.31)**	.077 (1.97)*	.077 (2.00)*
Average Inflation	-.023 (-1.33)		
Std Inflation		-.045 (-1.10)	
Max Inflation			-.009 (-1.43)
Europe	-.055 (-0.49)	-.037 (-0.29)	-.026 (-0.22)
Constant	-1.041 (-0.84)	-.710 (-0.52)	-.591 (-0.45)

Absolute value of *t* statistics in parentheses.
* Significant at 10%
** Significant at 5%

Table 3 shows the results of multivariate regression with all the main explanatory variables included as in Eichengreen, et al.(2004). Consistent with their finding, the country size is the only explanatory variable which is statistically significant. Even though emerging economies' ability to borrow in local currency has improved greatly in recent decades, our finding suggests that "the mystery of Original Sin" seems to still exist today. In the next section, however, we will provide a structural model to explain the recent surge in local currency borrowings by emerging economies and why we observe the mystery of Original sin.

The rest of the paper is organized as follows. Section 2 describes the model environment and defines the recursive equilibrium of the economy. Section 3 presents main quantitative results of the model, and Section 4 concludes.

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[7] India's LC share has been around 99% over the entire sample period, so that it can be considered as an outlier for our sample. Without India's data, however, the main regression results are robust.

Figure 1: LC Share and Inflation
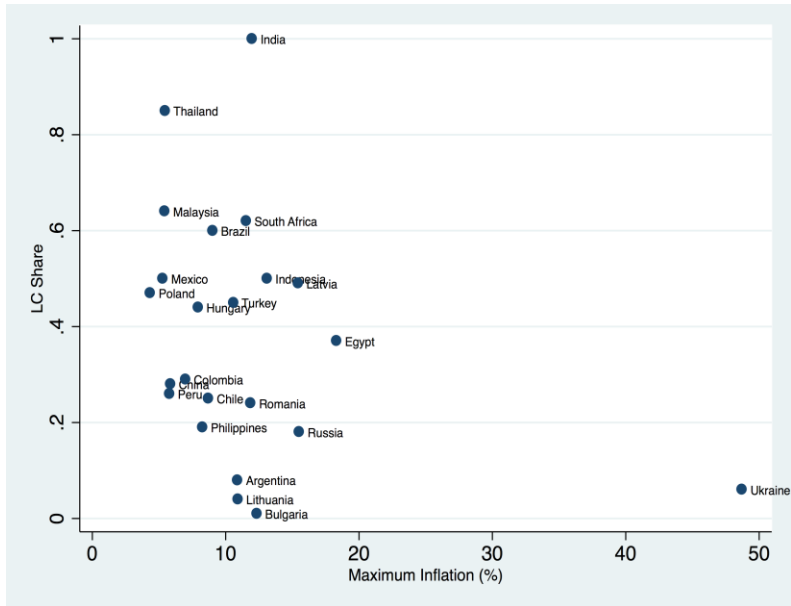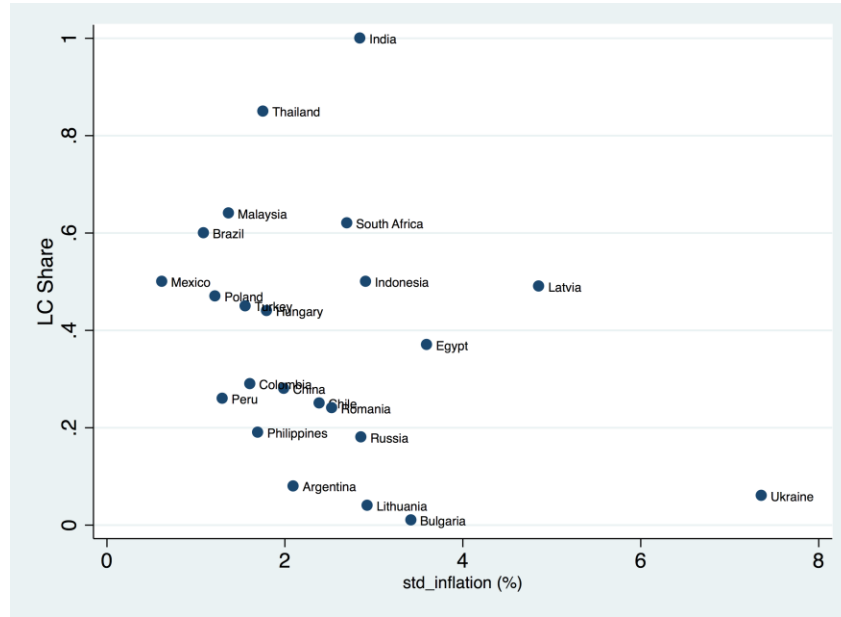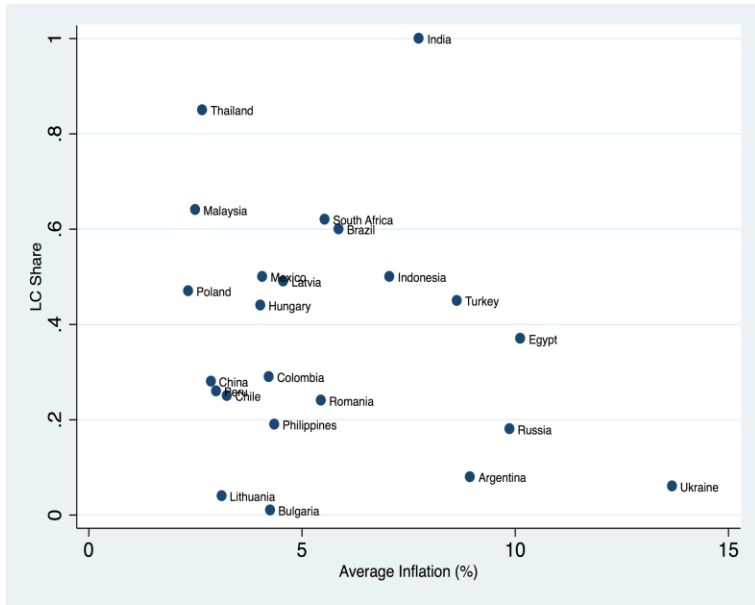
2. The Model Economy

We consider a standard small open economy model, extended to allow a government to borrow in both local and foreign currency from foreign lenders in international financial markets. The representative household receives stochastic endowment shocks every period and has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - C(\pi_t - \bar{\pi})] \tag{1}$$

where β denotes the time discount factor, c_t consumption, π_t a gross inflation rate at period t (i.e., ($\frac{P_t}{P_{t-1}}$), and $\bar{\pi}$ a target inflation rate of the country. The period utility function $u(.)$ is strictly increasing and strictly concave, and satisfies the standard Inada conditions. Following Barro and Gordon (1983), we introduce the cost of inflation in the form of utility loss $C(\pi_t - \bar{\pi})$, which is assumed to be symmetric around the target inflation rate $\bar{\pi}$; any deviation in inflation rates from the target inflation rate incurs utility loss. The sovereign government is benevolent and makes borrowing, default, and debasement decisions so as to maximize social welfare of this economy.

There is one tradable consumption good, denoted by y_t, in this economy. The income shock y_t has a compact support and follows a Markov process with a transition function $\Pr\left(y_{t+1} \mid y_t\right)$. The history of the income shock is denoted by s^t. Let P_t and P_t^* be the price of the consumption good in Home (i.e., the small open economy) and Foreign country, respectively. The budget constraint in nominal terms is given by

$$P_t c_t + S_t P_t^* b_{t+1}^{for} + P_t b_{t+1}^{loc} = P_t y_t + R^* S_{t-1} P_{t-1}^* b_t^{for} + \frac{i_t P_{t-1} b_t^{loc}}{\pi_t},$$

where S_t is the exchange rate, $b_t^{for} < 0$ foreign currency debt, $b_t^{loc} < 0$ local currency debt, i_t a gross interest rate on local currency debt, R^* a constant gross risk-free rate prevailing in the international financial market.[8] We assume that the law of one price holds and the foreign price P_t^* is normalized to be

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[8] As we investigate the currency composition of two types of sovereign debts, we don't allow the government to accumulate assets. For the local currency asset, it is not well justified to assume that the sovereign government buys its home currency bond issued from foreign lenders. For the foreign currency asset, the no accumulation constraint is not binding in the simulation.

one, so that $P_t = P_t^* S_t = S_t$. Then the budget constraint for the economy, conditional that the sovereign government is rolling over its debt by following the terms of contract, is given in real terms by

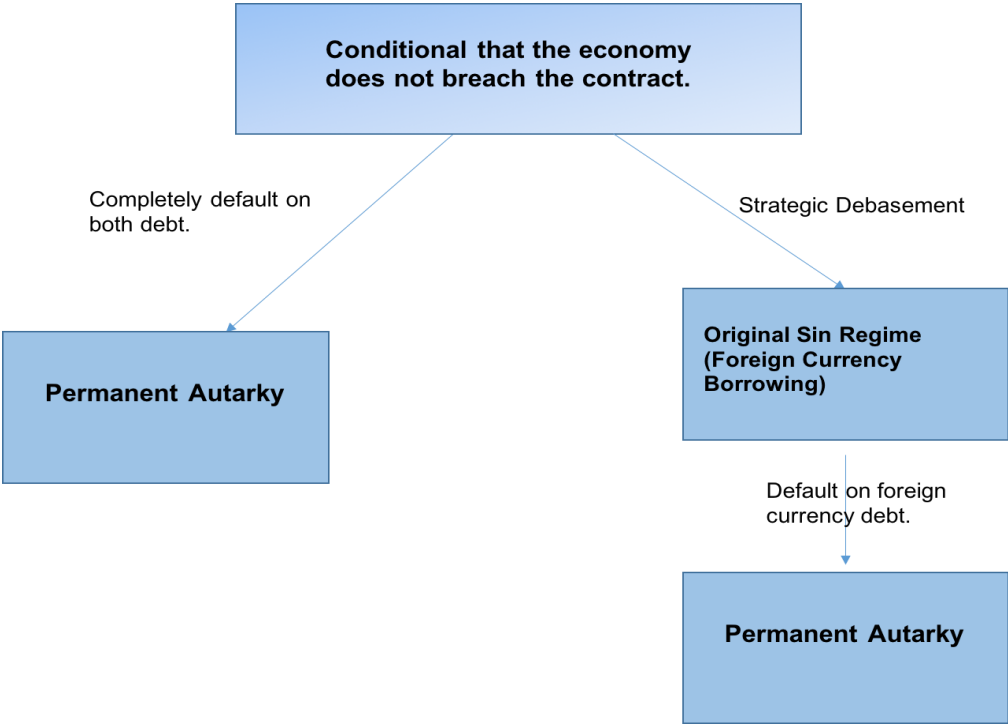$$c_t + b_{t+1}^{for} + b_{t+1}^{loc} = y_t + R^* b_t^{for} + \frac{i_t b_t^{loc}}{\pi_t} \tag{2}$$

When the government does not breach the contract, it solves a portfolio problem between local and foreign currency debt to maximize social welfare of the economy. b_t^{for} and b_t^{loc} are non-contingent bonds in nominal terms, but depending on the inflation rate π_t, the real value of the local currency debt b_t^{loc} / π_t can be state-contingent.

The government can breach the debt contract in the following two ways: First, the government can fully default on its debt denominated in both local and foreign currency simultaneously[9]. Second, the government can debase its currency more than required in the contract for local currency debt, the terms of which will be specified in detail later. Thus our model features two types of enforcement (commitment) frictions arising from a government's monetary and debt policy: strategic default and debasement. This is a novel feature of our model and we quantitatively study how these two frictions affect the currency composition of sovereign debt.

When the government fully defaults on its debts, the economy enters permanent autarky during which it loses access to international financial markets and suffers from a drop in income. When the government breaches the contract by debasing its currency, the country is restricted to borrow only in foreign currency, thus entering the "Original Sin" regime. When the government in this regime defaults on its foreign currency debt, the economy also enters permanent autarky. Figure 2 summarizes the two different types of breaches of the debt contract and their consequences.

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[9] Selective default on a certain type of debt is not allowed in our model, consistent with practices in sovereign debt markets. See Broner et. al. (2010) for theoretical study on this problem.

Figure 2: Two Types of Breaches of Contract

Conditional that the economy does not breach the contract.

Completely default on both debt.

Strategic Debasement

Permanent Autarky

Original Sin Regime (Foreign Currency Borrowing)

Default on foreign currency debt.

Permanent Autarky

Local Currency Debt Contract

Foreign lenders in competitive international financial markets are risk-neutral and have deep pockets. There are two types of lenders: lenders who lend in local currency and those who lend in foreign currency. Both are willing to lend to the sovereign government any amount, whether in local or foreign currency, as long as they are guaranteed an expected return of the gross risk-free rate R^* prevailing in the international financial markets. Even if the local currency debt is non-contingent in nominal terms with a gross interest rate i_{t+1}, depending on the government's choice on the inflation rate π_{t+1} (or equivalently depreciation rate), the real value of repayment $\dfrac{i_{t+1}b_{t+1}^{loc}}{\pi_{t+1}}$ can differ. We consider the following recursive contract for the local currency debt, which consists of two components: a nominal gross interest rate i_{t+1} and state contingent inflation rates in the next period π_{t+1}.

$$i_{t+1} = \Pi(b_{t+1}^{for}, b_{t+1}^{loc}, y_t) \tag{3}$$

$$\pi_{t+1} = \Pi(b_{t+1}^{for}, b_{t+1}^{loc}, y_t, y_{t+1}) \tag{4}$$

When the sovereign government borrows b_{t+1}^{for} and b_{t+1}^{loc} in foreign and local currency in period t, the contract charges a nominal gross interest rate i_{t+1} on the local currency debt b_{t+1}^{loc}. Moreover, the contract asks for a certain inflation (depreciation) rate depending on the realization of y_{t+1} in period t + 1.

Since the foreign investors who lend in local currency must be guaranteed an expected return of a gross risk-free rate R^* for the local currency debt, we have the following zero-profit condition on the contract:

$$R^* = \sum_{y_{t+1}} Pr(y_{t+1} \mid y_t) \frac{i_{t+1}}{\pi_{t+1}(b_{t+1}^{for}, b_{t+1}^{loc}, y_t, y_{t+1})} \tag{5}$$

Note that there is y_t as well as y_{t+1} in $\Pi(\;)$ because of the persistent income shock process $\Pr(y_{t+1}\mid y_t)$ in eq (5).

On the other hand, the foreign lenders charge the gross risk-free rate R^* on the foreign currency debt as typical of a standard small open economy model featuring a non-contingent debt. From now on, s_t denotes the vector of state variables at period t, which consists of ($b_t^{for}, b_t^{loc}, y_{t-1}, y_t$).

Value of Debasement

Due to the limited commitment (enforcement) of monetary policy, the sovereign government can debase its currency at any time by choosing a higher inflation rate than $\pi_t(s_t)$ in the contract to inflate away a certain fraction of local currency debt. When the government breaches the contract by debasing its currency, the country is restricted to borrowing *only* in foreign currency thereafter as a punishment. That is, the country enters the regime of "Original Sin" or foreign currency borrowing. Note that the foreign investors who lend in foreign currency do not incur any losses even when the government inflates away the local currency debt; they continue to lend to the government in foreign currency after the debasement.

The value of debasement is given by

$$V^{debase}(b_t^{for}, b_t^{loc}, y_{t-1}, y_t) = \max_{\pi_t \neq \pi_t(s_t), b_{t+1}^{for}} [u(c_t) - C(\pi_t - \bar{\pi})] + \beta E_t V^{for}(b_{t+1}^{for}, y_{t+1}) \quad (6)$$

subject to the budget constraint:

$$c_t + b_{t+1}^{for} = y_t + R^* b_t^{for} + \frac{i_t b_t^{loc}}{\pi_t}$$

$V^{for}(\;)$ denotes the value of borrowing in foreign currency after the debasement.

Value of Foreign Currency Borrowing (Original Sin Regime)

The value of foreign currency borrowing is given by

$$V^{for}(b_t^{for}, y_t) = \max_{b_{t+1}^{for}} u(c_t) + \beta E_t V^{for}(b_{t+1}^{for}, y_{t+1}) \tag{7}$$

subject to the following constraints:

$$c_t + b_{t+1}^{for} = y_t + R^* b_t^{for}$$

$$V^{for}(b_{t+1}^{for}, y_{t+1}) \geq V^{def}(y_{t+1}) \quad \text{for all } y_{t+1} \tag{8}$$

Equation (8) is the enforcement constraint related to the government's default decision and requires that the continuation value of foreign currency borrowing be equal to or higher than the value of default in any possible future contingencies. Note that this enforcement constraint determines an endogenous debt limit for the foreign currency borrowing that can be supported in equilibrium for the Original Sin regime. $V^{def}(y_{t+1})$ denotes the value of default when the government chooses to default on its debt, whether in local or (and) foreign currency.

Value of Default

Upon default, the economy enters permanent autarky during which the economy loses access to the international financial market, and the economy suffers a drop in income. The value of default is given by

$$V^{def}(y_t) = u(c_t) + \beta E_t V^{def}(y_{t+1}) \tag{9}$$

$$c_t = h(y_t) \tag{10}$$

,where $h(y_t) < y_t$. $h(y_t)$ represents a decrease in income associated with default, which is consistent with empirical findings in the sovereign debt literature.

Original Problem under the Optimal Contract

The contract is optimal in the sense that it maximizes utility of the representative household in the small open economy. Moreover, the contract is self-enforcing in the sense that the government under this contract does not have any incentive to breach the contract. The original problem under the optimal self-enforcing contract is given by:

$$\max_{\{c_t, b_{t+1}^{for}, b_{t+1}^{loc}, \pi_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t(s^t)) - C(\pi_t(s^t) - \bar{\pi})] \quad (11)$$

subject to (1) the budget constraint, (2) the enforcement constraint, (3) the expected zero profit condition for the lenders.

$$c_t(s^t) + b_{t+1}^{for}(s^t) + b_{t+1}^{loc}(s^t) = y_t(s^t) + R^* b_t^{for}(s^t) + b_t^{loc}(s^t) \frac{i_t(s^t)}{\pi_t(s^t)} \quad (12)$$

$$E_t \sum_{s=t}^{\infty} \beta^{s-t}[u(c_s) - C(\pi_s - \bar{\pi})] \geq \max\{V_{def}(y_t), V_{debase}(b_t^{for}, b_t^{loc}, y_{t-1}, y_t)\} \ for \ any \ y_t \quad (13)$$

$$R^* = \sum_{y_{t+1}} Pr(y_{t+1} \mid y_t) \frac{i_{t+1}}{\pi_{t+1}(b_{t+1}^{for}, b_{t+1}^{loc}, y_t, y_{t+1})} \quad (14)$$

Then, an equilibrium in this model is a sequence of inflation rates and interest rates on local currency debt $\pi(s^t)$ and $i_t(s^t)$ in the contract and allocations $\{c(s^t), b_t^{for}(s^t), b_t^{loc}(s^t)\}$ such that the contract and the allocations solve the maximization problem subject to the budget constraint (equation (12)), the enforcement constraint (equation (13)), and the lender's expected return condition (equation (14)).

Note that the enforcement constraint eq (13) has two value functions on the right hand side: the values of debasement and default. These enforcement constraints come from two different types of limited commitment problems regarding the government's monetary and debt policy. These two enforcement

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constraints combine to generate an endogenous debt frontier for local and foreign currency debt, thus determining the currency composition of sovereign debt.

Recursive Formulation of the Original Problem

Since the enforcement constraint equation (13) has expected values of future variables, we cannot use the standard recursive Bellman equation, as pointed out first by the classical paper by Kyland and Prescott (1977). This is a common problem shared with the economic models dealing with time-inconsistent government policy. However, our original problem can be recast recursively following Atkeson (1991), which uses the solution techniques of Abreu et al (1990) and is extended by Bai and Zhang (2010).

Before the income shock is realized at period t, the optimal contract chooses a nominal interest rate i_t and an inflation rate π_t (i.e., currency depreciation rate) for each state for the period t that maximizes the expected sum of value functions V^cs.

$$W(b_t^{for}, b_t^{loc}, y_{t-1}) = \max_{\pi(s_t)} \sum_{y_t} Pr(y_t \mid y_{t-1}) V^c(b_t^{for}, b_t^{loc}, y_{t-1}, y_t; \pi(s^t)) \qquad (15)$$

subject to the lender's expected zero profit condition:

$$R^* = \sum_{y_{t+1}} Pr(y_t \mid y_{t-1}) \frac{i_t}{\pi_t(b_t^{for}, b_t^{loc}, y_{t-1}, y_t)}$$

After the income shock is realized at period t, taking $\pi(s^t)$ as given, the government solves the following value function:

$$V^C(b_t^{for}, b_t^{loc}, y_{t-1}, y_t; \pi(s_t)) = \max_{b_{t+1}^{for}, b_{t+1}^{loc}} [u(c_t) - C(\pi_t - \bar{\pi})] + \beta W(b_{t+1}^{for}, b_{t+1}^{loc}, y_t) \qquad (16)$$

$$c_t + b_{t+1}^{for} + b_{t+1}^{loc} = y_t + R^* b_t^{for} + \frac{i_t b_t^{loc}}{\pi(s^t)} \qquad (17)$$

$$V^C(b_{t+1}^{for}, b_{t+1}^{loc}, y_t, y_{t+1}; \pi(s_{t+1})) \geq \max\{V^{debase}(b_{t+1}^{for}, b_{t+1}^{loc}, y_t, y_{t+1}), V^{def}(y_{t+1})\} \quad \textit{for all } y_{t+1}^{'s}$$

$$(18)$$

Following Atkeson (1991) and Bai and Zhang (2010), we solve the above problem iteratively starting with sufficiently high initial values W_0 and V_0, where the subscript denotes the number of iterations. At each iteration, the domain D_0 of W_0 and V_0 is updated such that it solves the maximization problems of equations ((15), (16)) subject to the budget and the enforcement constraints eq ((17), (18)). The sequences of $\{W_n\}$, $\{V_n\}$, and $\{D_n\}$ are decreasing, finally converging to W,V, and D, respectively. Then, we have combinations of (b^{loc}, b^{for}) in D that satisfy the budget and the enforcement constraint.

For π_t and i_t, we have the following first order conditions.

$$[\pi(s_t^i)]:\ u'(s_t^i, y_t^i)i_t b_t^{loc} + C'(\pi(s_t^i))\pi(s_t^i)^2 = u'(s_t^j, y_t^j)i_t b_t^{loc} + C'(\pi(s_t^j))\pi(s_t^j)^2\ for\ i \neq j$$

(19)

$$[i_t]:\ \sum_{y_t^i}\Pr(y_t^i \mid y_{t-1})C'(\pi_t(s_t^i))\pi(s_t^i) = 0,$$

(20)

where i denotes an income shock y^i, and $y^i < y^j\ for\ i < j$.

At optimum, the first term on the left hand side is the benefit of a marginal increase in an inflation rate: an increase in inflation rates leads to a decrease in the real value of local currency debt, thus increasing consumption at the state of (s_t^i, y_t^i). Note that the first term on the left hand side has b_t^{loc}; the more local currency debt the economy holds at period t, the higher the marginal benefit of an increase in inflation rates is. The second term on the left hand side is the marginal cost of the increase in inflation rates. If there is an increase in inflation rates (i.e., depreciation) at the state (s_t^i, y_t^i), the zero profit condition for the foreign lenders (eq ((21)) requires a decrease in inflation rates (i.e., appreciation) at other states (s_t^j, y_t^j) to compensate for the loss incurred to the lenders at state (s_t^i, y_t^i). The right hand side is the marginal cost associated with the appreciation of the currency. That is, the optimal contract equates the marginal benefit and cost across states.

The first order condition with respect to i_t shows that at optimum, the nominal interest rate i_t on the local currency debt is chosen to minimize the expected sum of costs of inflation across states. Note that

with a symmetric cost of inflation around the target inflation rate $\bar{\pi}$, the marginal cost at $\pi_t < \bar{\pi}$ is negative.

The following proposition and corollary characterize the state-contingent nature of local currency debt in our model.

Proposition 1: *Suppose that there is no enforcement constraint (eq(18)) and that there is no cost of inflation (i.e., $C\left(\pi_t - \bar{\pi}\right) = 0$ for all π_t). Then the optimal contract for the interior solutions is such that*

$$c(s_t^i, y_t^i) = c(s_t^j, y_t^j) \quad for \quad any \quad i \neq j. \tag{22}$$

Proof: See the Appendix

This proposition shows that local currency debt has characteristics similar to the Arrow-securities in the complete markets model. Without any financial frictions and cost of inflation, the local currency debt completely smooths consumption of the representative household across states.

Corollary 1: *Suppose that $y_t^i \leq y_t^j$. Then, under the same conditions as the proposition 1, π_t on the optimal contract is such that*

$$\pi(s_t^i, y_t^i)) \geq \pi(s_t^j, y_t^j) \tag{23}$$

The corollary shows that without any frictions, the optimal contract for the local currency debt allows the government to depreciate its currency in times of bad income shocks but asks for currency appreciation in times of good income shocks as a compensation to the investors for the bad times. Thus, compared to the foreign currency debt, local currency debt under the optimal contract is a better instrument for consumption hedging against the income shock, especially when there is no cost of inflation.

Debt Frontier

Let $\underline{b}^{loc}$ be the maximum debt amount for local currency debt supported in equilibrium, which is given by

$$\underline{b}^{loc} \equiv \min_{y_{t+1}|\Pr(y_{t+1}|y_t)>0} \{b^{loc} : V^c(0,b^{loc},y_t,y_{t+1}) \geq \max\{V^{debase}(0,b^{loc},y_t,y_{t+1}),V^{def}(y_{t+1})\} \;\; for \;\; y_t^{'s}$$

(24)

That is, $\underline{b}^{loc}$ is the maximum borrowing limit for local currency debt without any borrowing in foreign currency (i.e., $b^{for} = 0$) that can be supported in equilibrium without violating the enforcement constraints. Note that b denotes bond holdings, not debt.

Then the **debt frontier**, which is a function of b^{loc}, is defined as the following:

$$\underline{b}^{for}(b^{loc}) \equiv \min_{y_{t+1}|\Pr(y_{t+1}|y_t)>0} \{b^{for} : V^c(b^{for},b^{loc},y_t,y_{t+1}) \geq \max\{V^{debase}(b^{for},b^{loc},y_t,y_{t+1}),V^{def}(y_{t+1})\}\}$$
$$, for \;\; \underline{b}^{loc} \leq b^{loc} < 0 \;\; and \;\; all \;\; y_t^{'s}$$

(25)

That is, $\underline{b}^{for}(b^{loc})$ is the maximum amount of borrowing in foreign currency, given that the economy chooses to borrow b^{loc} in local currency. Any more borrowing than $\underline{b}^{for}(b^{loc})$ violates the enforcement constraints, thus not supported in equilibrium.

Definition 1: *If $\underline{b}^{loc} = 0$ and $\underline{b}^{for}(0) > 0$ in equilibrium, the model economy is in "Original Sin" Regime.*

When the economy is not able to borrow any amount in local currency under the contract, we have that $\underline{b}^{loc} = 0$. In this case, the value of contract reduces to the value of foreign currency borrowing (that is, "Original Sin" Regime).

Proposition 2: *Suppose that the cost of the inflation $C(\pi_t;\xi)$ is differentiable and strictly increasing in π_t, and that $C(\pi_t;\xi)$ is strictly increasing and convex in ξ for any π_t. Then $\underline{b}^{loc,H} \leq \underline{b}^{loc,L}$ for $\xi^H > \xi^L$.*

That is, a country with a higher cost of inflation has a more relaxed debt limit for local currency debt.

Proof: See the Appendix

Proposition 2 shows that the degree of monetary credibility represented by the cost of inflation parameter ξ determines the borrowing limit for local currency debt. This is consistent with the recent empirical findings by Du et al (2016) which show that in the recent decades, even developing countries with more disciplined monetary policy have managed to borrow more in local currency, which departs from the trend of "Original Sin" in the 70's and 80's.

Proposition 3: *Suppose that there is no cost of inflation (i.e., $C(\pi_t;\xi)=0$ for any ξ). Then the equilibrium interest rate on local currency debt i_{t+1} is indeterminate.*

The proof is straightforward and is from the lender's expected zero profit condition eq (26). With no cost of inflation, the real interest rate on local currency debt $\frac{i_{t+1}}{\pi_t+1}$ only matters for the equilibrium allocations.

Proposition 4: *Under the same conditions in the proposition 2, If $\xi \to \infty$ then $\pi_t\left(s^t\right)=1$. Moreover, the currency composition between foreign and local currency debts is indeterminate.*

When the cost of inflation is infinite, the foreign currency debt becomes the same as the local currency debt, so the currency composition between two types of debts is indeterminate.

3 Quantitative Results

Table 4: Parameters

Parameter			
γ	2	Risk aversion	Literature
β	0.96	Time Discount Factor	Time Period is a year
r_f	4%	Risk Free Rate	Literature
ϵ	0.04	Std of Income	Volatility of income
ξ	different values	Cost of Inflation	Volatility of inflation
μ	1	Mean Income	-
λ	3%	Default Cost	Benjamin and Wright (2009)
ρ	0.7	Persistence of Income Shock	-

3.1 Parameters and Functional Forms

In this section, we solve the model numerically and simulate it to investigate quantitative implications of the two limited commitment frictions for the currency composition of sovereign debt.

Table 4 reports the parameters used for the benchmark calibration. A period is a year. We use the standard CRRA utility function $\dfrac{c^{1-\gamma}-1}{1-\gamma}$ and set γ to be 2, which is standard in the literature. The time discount factor β is set to be 0.96.

- The income shock takes on two values, y_H and y_L, where $y_H = \mu + \varepsilon$ and $y_L = \mu - \varepsilon$. μ is the mean income and ε the standard deviation of the income shock. The mean income μ is normalized to be one, and ε is set to be 4%. As a benchmark case, the persistence of the income shock is assumed to be $\Pr(y_H \mid y_H) = \Pr(y_L \mid y_L)$.

- We use the quadratic cost of inflation given by

$$C(\pi) = \xi(\pi - 1)^2, \tag{27}$$

which implies that the target inflation rate $\bar{\pi}$ is normalized to be one.

- The cost of default during autarky is in the form of a drop in income:

$$h(y_t) = (1 - \lambda) y_t \tag{28}$$

As with other studies in the sovereign debt literature, we assume that the economy suffers from a drop in income during autarky. As a benchmark value, we set λ to be 0.03 from Benjamin and Wright (2009).

3.2   Model Moments

Table 5: Model Moments

	$\xi = 0.005$	$\xi = 0.105$
b^{loc} (% of GDP)	31.2%	50.1%
Average Total Debt (% of GDP)	63.65%	62.70%
Average LC Debt (% of GDP)	13.77%	34.47%
Average LC Share (%)	22.92%	58.45%
Corr (GDP, Total Debt)	-0.41	-0.43
Corr (GDP, LC Share)	0.22	0.40
Corr (GDP, inflation rate)	-0.85	-0.84
Std (inflation rate)	13.57%	2.287%

Table 5 compares the model moments of debt and inflation for the cases of low ($\xi = 0.005$) and high ($\xi = 0.105$) cost of inflation. To get the statistics in Table 5, we simulate the model 5000 times and the first 1000 simulated data points are removed to rule out any effects of initial conditions.

For the high cost of inflation, the maximum local currency debt limit is 0.501, whereas for the low cost of inflation, the maximum local currency debt limit is 0.312. That is, a high cost of inflation is associated with a more relaxed borrowing limit for the local currency debt. The total debt – the sum of local and foreign currency debts in real terms – is on average not much different in the two cases. However, the average local currency debt and local currency share in total debt shows a significant difference: the economy with a high cost of inflation borrows on average more (34.47% of its GDP) than that with a low cost of inflation (13.77% of its GDP). Moreover, the local currency debt shares for the high and low cost

inflation cases are 58.45% and 22.92%, respectively. As the local currency debt limit with a high cost of inflation is more relaxed than that with a low cost of inflation, the economy with a high cost of inflation tends to borrow more in local currency. It must be, however, noted that too much local currency debt is not preferred as it incurs the cost of inflation. For both cases, the correlation between GDP and inflation is negative at around -0.85, consistent with the Corollary 1: the optimal contract asks for depreciation in bad income times but asks for appreciation in good income times. However, the economy with a low cost of inflation uses monetary policy more actively to use the local currency debt as a consumption-hedging device. The last row of the table reports volatilities of inflation for the two cases.

3.3. Debt Frontiers for Different Costs of Inflation

Figure 3: Debt Frontiers with Different Costs of Inflation ($\xi = 0.005\ and\ \xi = 0.0105$)
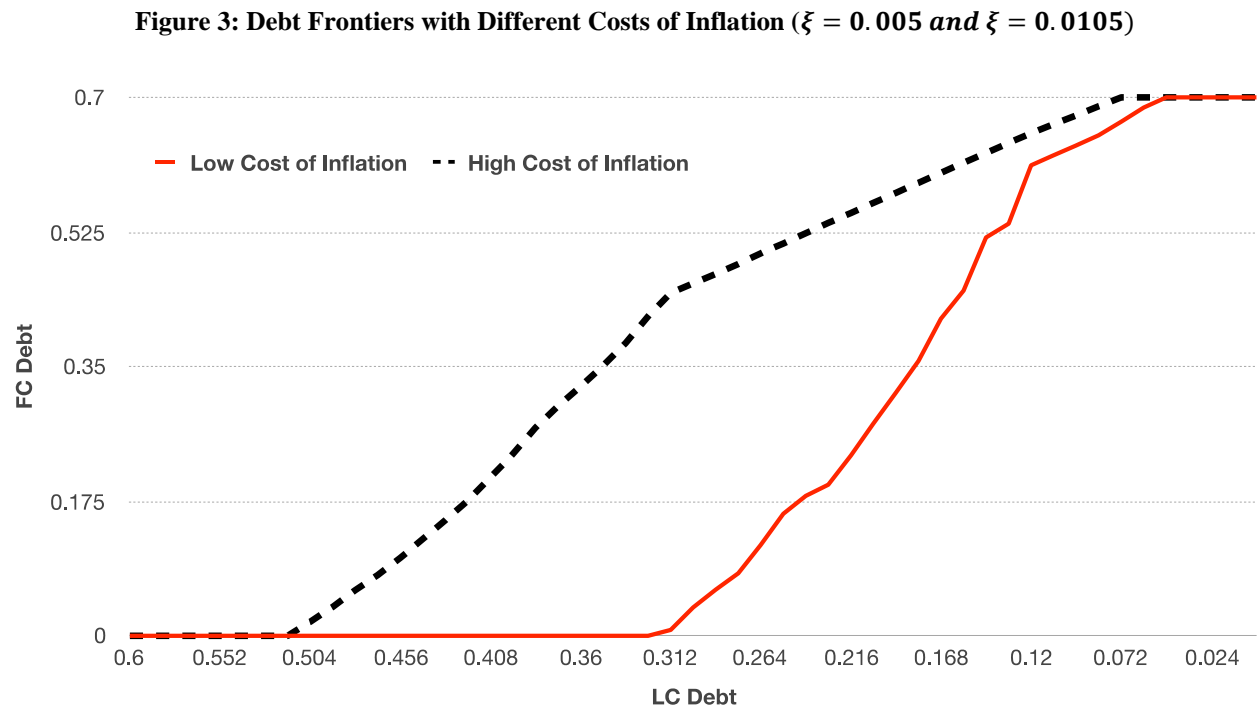
Figure 3 plots two debt frontiers for two different values of cost of inflation ($\boldsymbol{\xi = 0.005}$ **and** $\boldsymbol{\xi = 0.0105}$). The debt frontier shows the maximum debt limits for both types of debts supported in equilibrium without violating the enforcement constraints. For the case of the high cost of inflation, the debt frontier is a dashed black line, and for the case of the low cost of inflation, the debt frontier is a red line. The region under the frontier is feasible combinations of local and foreign currency debts that the government can choose without violating the enforcement constraints. Note that the region for the high cost of inflation is strictly larger and covers that for the low cost of inflation. That is, the country with more disciplined

monetary policy is able to borrow more in both local and foreign currency debts. However, the debt frontier for the low cost of inflation is more restricted along the dimension of local currency debt.
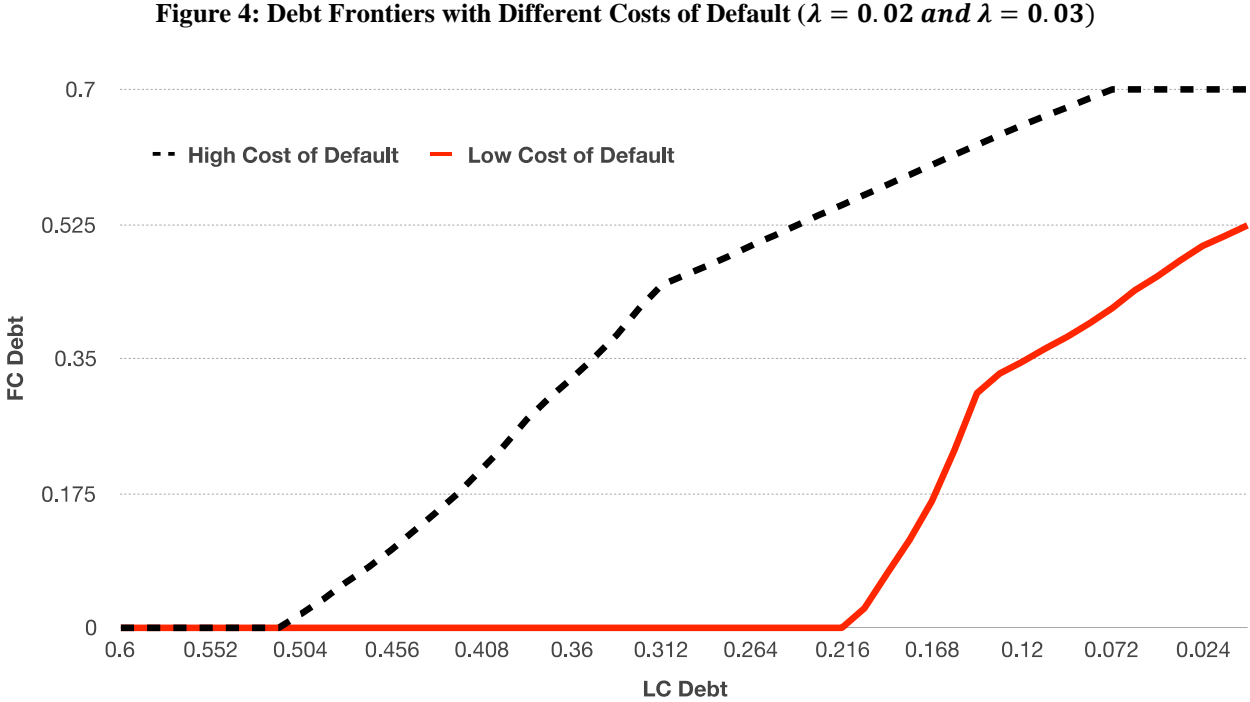
3.4. Debt Frontiers for Different Costs of Default

Figure 4: Debt Frontiers with Different Costs of Default ($\lambda = 0.02 \ and \ \lambda = 0.03$)

Figure 4 plots two debt frontiers for two different values of cost of default ($\lambda = \mathbf{0.02} \textbf{ and } \lambda = \mathbf{0.03}$) with $\xi = \mathbf{0.0105}$ and other benchmark parameters. For the low cost of default, the borrowing limits for local can foreign currency debt (solid red line) are much tighter than those for the high cost of default.

3.5   Further Numerical Results

This section conducts a sensitivity analysis with respect to several key parameters to investigate the effects of changes in the key parameters on the optimal composition of sovereign debt. The main finding of this section is that even if the cost of inflation is the most important determinant for emerging economies' ability to borrow in local currency debt, there is *no* clear-cut link between the currency composition of external sovereign debt and inflation related variables. Both the currency composition of debt and inflation related variables are endogenous, and depending on changes in exogenous variables or different parameters, we can have either a positive or negative relationship between these variables.

Costs of Inflation

Figure 5: Sensitivity Analysis w.r.t Cost of Inflation
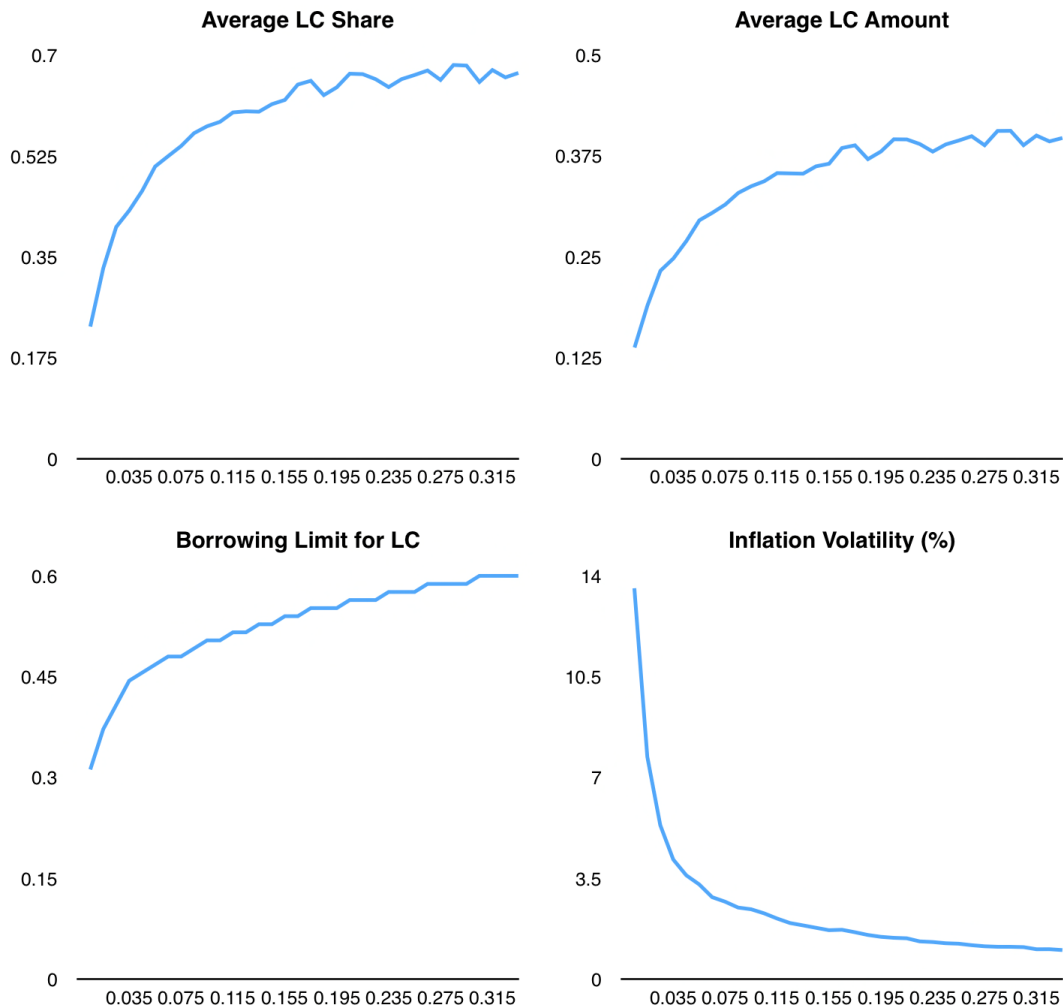
Figure 5 plots average share of local currency debt, average local currency debt amounts, borrowing limits for the local currency debt, and volatilities of inflation for different values of ξ from 0.005 through 0.335. As the cost of inflation increases, the economy can borrow more in local currency as shown in the increase in the borrowing limit for local currency debt (the left panel in the bottom). Accordingly, the average LC share and local currency debt amount increase. Moreover, the volatility of inflation decreases as the cost of inflation increases.

The predictions of the model are consistent with two empirical facts regarding "Original Sin" phenomenon: First, the emerging economies which suffered high inflation volatility during the 80s and 90s borrowed mostly in foreign currency. Second, the emerging economies which have gotten increasingly more disciplined in monetary policy are more likely to borrow in local currency during the last decade. That is, our model can predict both "Original Sin" phenomenon for the emerging economies in the 80s and 90s and a recent surge in local currency borrowings for the emerging economies with more disciplined monetary policy.

Discount Factor

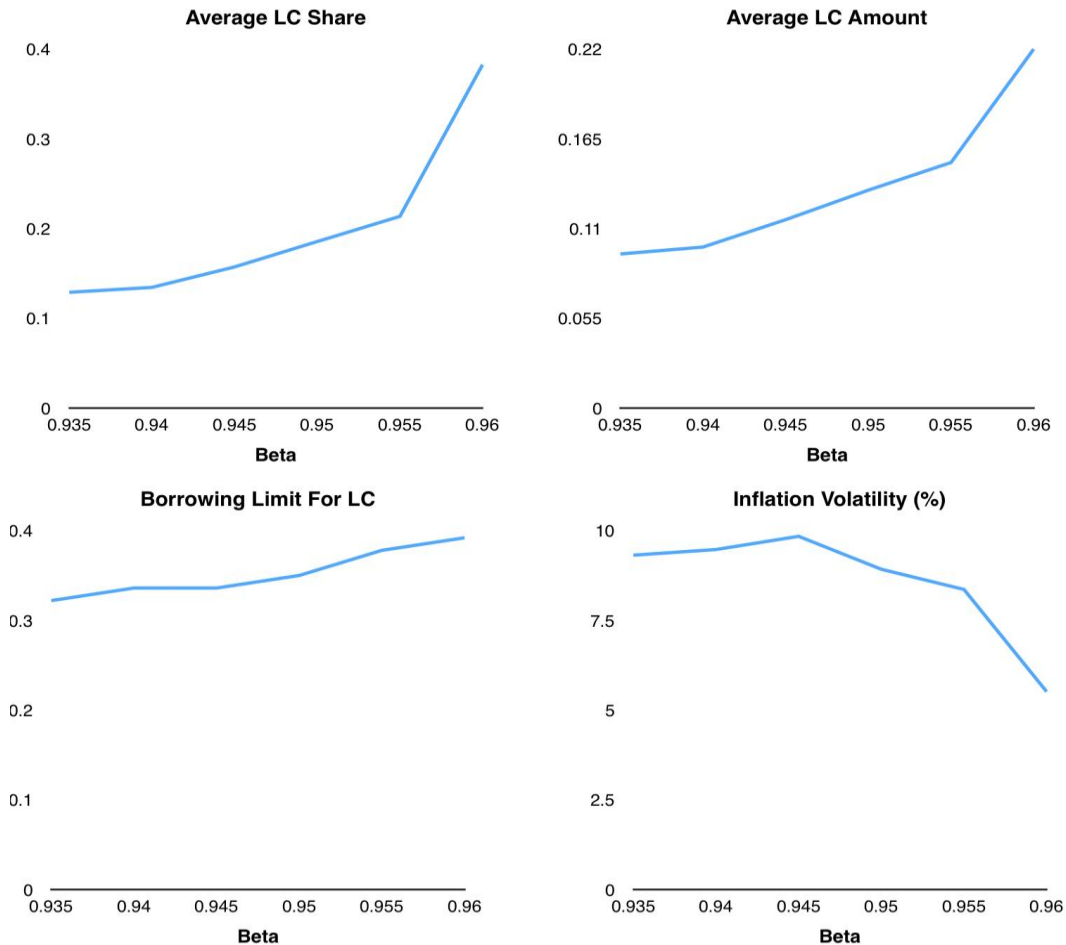**Figure 6: Sensitivity Analysis w.r.t Discount Factor**

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Figure 6 shows a sensitivity analysis with respect to different values of the discount factor, β, from 0.935 to 0.96. As the household in the economy gets more patient, it gets the less tempted to debase its currency or default on its debt. Accordingly, the economy can borrow more in local currency, thus borrowing more in local currency. However, the volatility of inflation decreases as the discount factor increases.
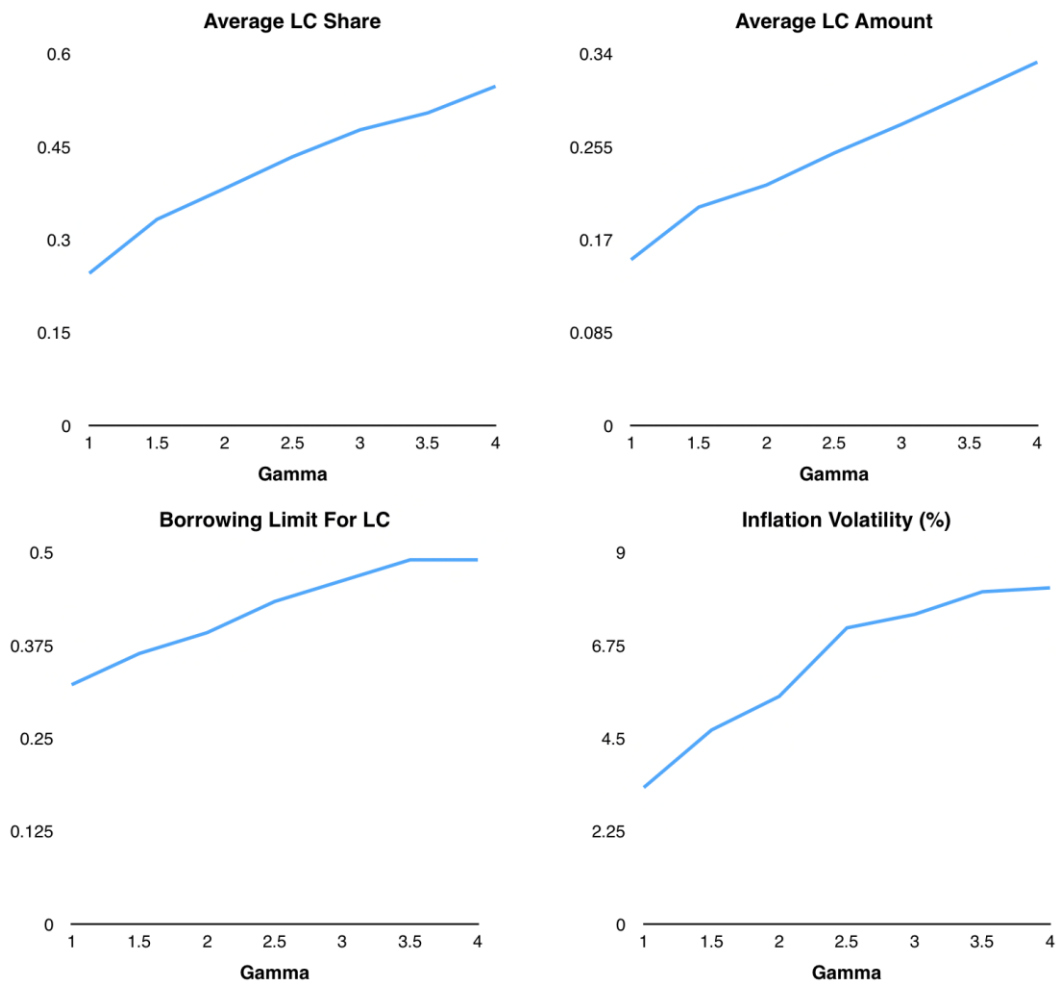
Risk Aversion

Figure 7: Sensitivity Analysis w.r.t Gamma

Figure 7 shows a sensitivity analysis with respect to different values of γ. As the household gets more risk averse, they value consumption smoothing more, and thus prefer local currency debt. Hence, the borrowing limit for the local currency debt increases with γ. Even though the local currency share of debt increases, the inflation volatility increases, which is different from the predictions for the cases of increasing

ξ and β. As the household gets more risk-averse, the economy borrows more in local currency and actively conducts monetary policy to smooth consumption, taking advantage of state-contingent nature of local currency debt.

Output Cost of Default

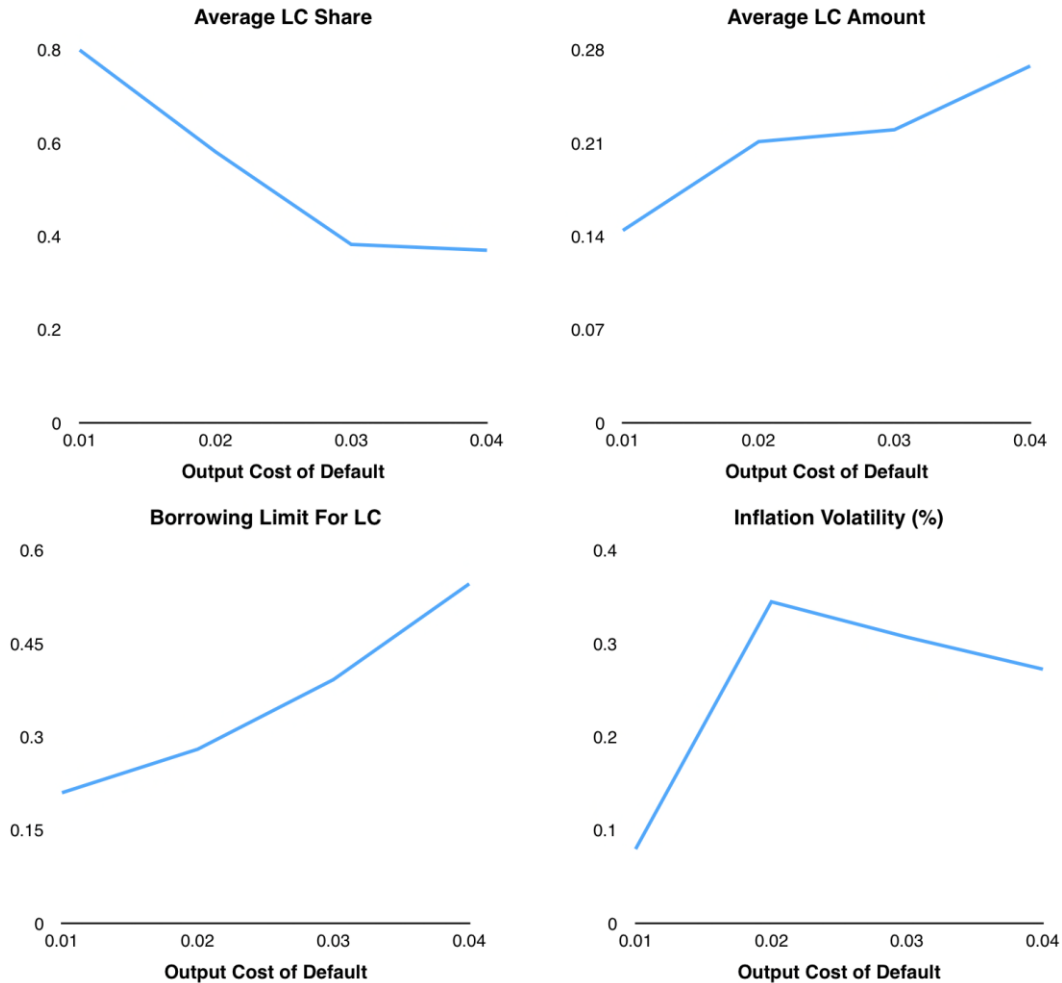**Figure 8: Sensitivity Analysis w.r.t Output Cost of Default**

Figure 8 shows a sensitivity analysis with respect to different output cost parameters, λ. As the output cost of default increases, the borrowing limit for local currency increases, and the economy borrows more in local currency. However, the average local currency share decreases with λ. The debt frontier enlarges as the output cost of default increases; however as shown in the figure 2, compared to the local currency debt, the borrowing limit for *foreign* currency debt gets more relaxed. The economy tends to borrow more in foreign currency than in local currency, even if the absolute amount of local currency debt increases with λ.

Persistence of Income Shock

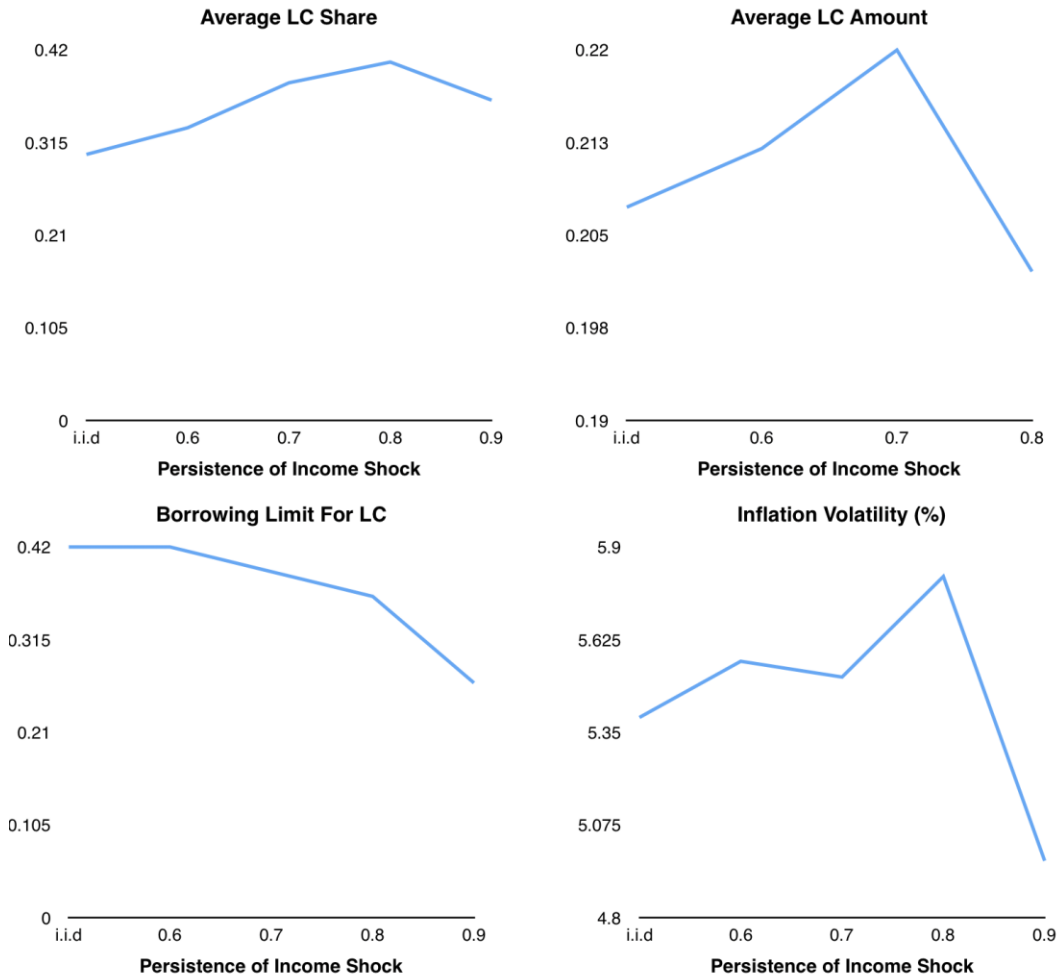**Figure 9: Sensitivity Analysis w.r.t Persistence of Income Shock**

Figure 9 shows a sensitivity analysis with respect to different degrees of persistence of the income shock process. As the income shock gets more persistent, the value of debasement/default increases; when a good income shock hits the economy, the good income shock is expected to persist for a long period of time, thus increasing the value of breaching the contract. This is reflected in the decrease in the borrowing limit for the local currency debt with an increase in the degree of persistence. However, the average local currency share and amount of local currency debt shows a non-monotonic shape in the degrees of persistence.

Income Variance

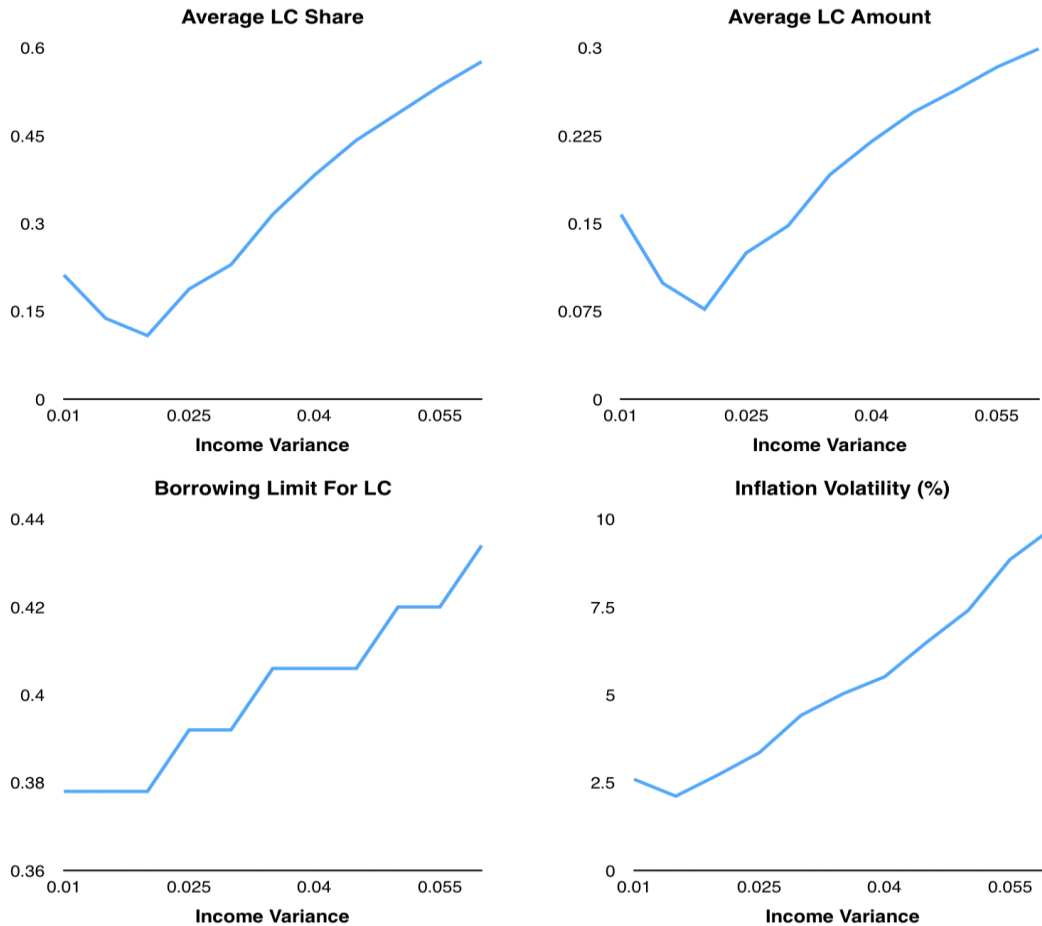**Figure 10: Sensitivity Analysis w.r.t Income Variance**

Figure 10 shows a sensitivity analysis with respect to different income variances. As the income variance increases, the value of breaching the contract decreases; the value of debasement and default decrease because the economy has less efficient consumption smoothing vehicle in the "Original Sin" regime and permanent autarky. As the income variance increase, the borrowing limit for local currency debt increases. The average local currency share and amount of local currency debt shows a U-shape in the income variance. The inflation volatility follows the shape of the average local currency share as the income variance increases.

3.6   Why We Still Have Mystery of Original Sin ?

In section 1.2, we find that there is still a "mystery of original sin". The regression results show that there are no meaningful economic regressors except for the absolute size of a country to account for Original Sin of emerging economies. However, the findings in the previous sensitivity analysis show why we might have weak empirical support for the hypothesis that monetary credibility and weak institutions of emerging economies are the main cause of Original Sin.

Table 6: Correlation between LC share and Inflation Volatility

Parameters	Correlation (LC share, Inflation Volatility)
Cost of Inflation	negative
Discount Factor	negative
Risk Aversion	positive
Output Cost of Default	non-monotonic
Persistence of Income Shock	positive
Volatility of Income Shock	positive

Table 6 summarizes the correlations between LC share and inflation volatility in simulations with respect to changes in key parameters in our model. With respect to the change in parameters for cost of inflation and the discount factor, the LC share and inflation volatility move in the opposite direction. On the other hand, with respect to the change in parameters for the degree of risk-aversion, and persistence and volatility of income shock, the LC share and inflation volatility move in the same direction. With respect to the change in the parameter for the output cost of default, we do not see any monotonic relationship between the two variables.

Figures 9 and 10 shows two scatterplots of LC share and volatility of inflation and illustrate this point more clearly. The red lines in the left panel of Figure 9 show movements in (LC share, volatility of inflation) pairs for two different costs of inflation as the income variance increases, fixing other parameters at the benchmark values. The pair moves toward northeast as the income variance increases. The blue dashed line shows movements in (LC share and volatility of inflation) pairs as the cost of inflation parameter ξ increases, fixing other parameters at the benchmark values. The pair moves toward northwest as the cost of inflation increases. The red line in the right panel of Figure 10 tracks movements in (LC share and volatility of inflation) pairs as the degree of risk-aversion increases. The pair in red line moves toward northwest as the degree of risk-aversion increases. The blue dashed line in the right panel is the same as that in the left panel.

Figure 11: Scatterplots of Volatility of Inflation and LC Share
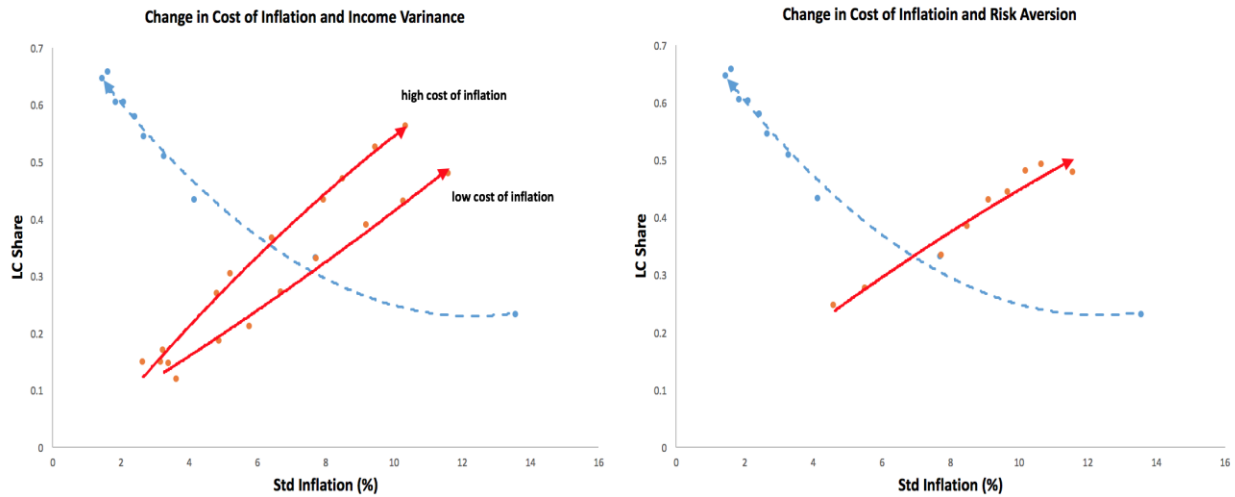

Figure 11 shows that even though emerging economies have been able to borrow more in local currency due to more disciplined monetary policy in the last decade,[10] we can still have weak empirical support for the hypothesis that lack of monetary credibility is the root cause of Original Sin.

Our model predicts that the monetary credibility associated with the cost of inflation is the most important determinant for emerging economies' ability to borrow in local currency debt, but both inflation and the local currency share of external sovereign debt are endogenous variables, so that for countries with different characteristics (e.g., different degree of patience, risk-averseness, etc.) we can observe a non-monotonic relationship between inflation and LC share in the data as shown in the Figure 1.

3.7 An Example Estimated by Simulated Method of Moments

As we have noted, Figure 1 shows that there is very little discernible relationship between the inflation performance of emerging market sovereign debtors and the share of external debt deonominated in domestic currency. In this section, we undertake an examination of two of those countries, Colombia and Indonesia. These countries are at comparable levels of development. We see from the figure that a larger

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[10] See Ebeke and Fouejieu (2015).

share of Indonesia's debt is denominated in domestic currency, but its overall inflation performance is worse than Colombia's. Table 7 presents some summary statistics for the two countries. All values in the table are averages over the sample period from 2004-2015. Our objective in this section is to illustrate how our model can account for this pattern of external debt.

Table7: Summary Statistics for Colombia and Indonesia

	Colombia	Indonesia
LC_Share	29.26%	50.02%
GDP per Capita	$6292.47	$3,094.62
GDP (Current US Dollars)	$262,852,748,129	$635,014,986,077
Average Inflation	4.23	7.06
Std inflation	1.60	2.91
Max inflation	6.99	13.11
Inflation Targeting (in the last decade or before)	Yes	Yes

Specifically, our objective is to see how our model can explain the local-currency shares of external sovereign debt – 29.26% in Colombia, 50.02% in Indonesia – while simultaneously allowing Indonesia to have a higher standard deviation of inflation (2.90% versus 1.60% in Colombia, on an annual basis.) Our approach is to calibrate some standard parameters used in our model, such as the rate of time preference and the risk free rate of interest. We calibrate the cost of default (as a percentage of income) as well. Two other parameters – the mean and persistence of output – are estimated for each of these two countries directly from the data.[11] Table 8 lists these parameter values. That leaves us with two parameters – the cost of inflation and the coefficient of risk aversion – that we vary in order to explain the two moments (domestic-currency share of external debt and standard deviation of inflation.

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[11] We estimate using annual GDP data from 1960-2015 from the World Bank. We HP filtered the series with a smoothing parameter of 400 (annual frequency) and estimate the AR(1) process using the cyclical components of each series.

Table 8: Parameters

Common Parameters

Parameters	Values	Description
β	0.96	Time Discount factor
r_f	0.04	Risk Free Rate
λ	3.0%	Output Cost of Default
$\overline{\pi}$	1	Mean Inflation (Normalization)

Parameters Estimated Directly from the Data

Parameters	Values		Description
	Colombia	Indonesia	
ρ	0.79	0.76	Persistence of GDP
σ_ϵ	0.018	0.03	Volatility of GDP

We use simulated method of moments (SMM) to estimate the remaining two parameters. We undertake this exercise to illustrate how our model might be consistent with the "mystery of original sin" – the lack of relationship between the local-currency share of external foreign debt and measures of economic performance – and do not intend these estimates to be taken as earnest measures of the utility cost of inflation or the degree of risk aversion in each country. Instead, we show how the determinants of orignal sin may be hidden in deep parameters. There may be many factors – especially political – that affect both the country's perceived cost of inflation, and the risk aversion of households (or of the government acting on their behalf when it issues foreign debt.)

Our SMM finds the values of ξ (the inflation cost) and γ (the coefficient of relative risk aversion) that minimizes for each country:

$$\left(\frac{LC^{sim} - LC^{dat}}{LC^{dat}} \right)^2 + \left(\frac{sdinf^{sim} - sdinf^{dat}}{sdinf^{dat}} \right)^2$$

Here, LC^{sim} and $sdinf^{sim}$ refer to the mean local-currency share of external sovereign debt and the standard deviation of inflation, respectively, generated from simulations of the model. The same variables with the *dat* superscript refer to those moments in the data.

Table 9: Parameters Estimated from SMM

Parameters	Estimates		Taget Moments
	Colombia	Indonesia	
ξ (Cost of Inflation)	0.028	0.078	LC Share (50.02% in Indonesia, 29.26% in Colombia)
γ (Risk Aversion)	2.30	2.88	Std of Inflation (2.91% in Indonesia, 1.60% in Colombia)

Table 9 lists the parameters estimated from the simulated method of moment. The table shows that in the estimated model, Indonesia has a higher cost of inflation and is more risk-averse than Colombia. Recall, however, that Indonesia's volatility of inflation is also higher than Colombia (2.91% in Indonesia vs. 1.60% in Colombia), and that Indonesia's LC share is higher than Colombia in the data (50.02% in Indonesia vs. 29.26% in Colombia). This fact has so far puzzled economists as this seems to suggest that debasement risk has little to do with the ability of emerging economies to borrow in local currency. However, our estimated parameters for the two countries show why we observe this puzzling facts called "Mystery of Original Sin". Indonesia is more credible in terms of monetary policy so it can borrow more in local currency than Colombia. But Indonesia is more risk averse, so that it uses the monetary policy more actively to smooth its consumption as shown in the sensitivity analysis in the previous section.

4   Conclusions

This paper quantitatively investigates the currency composition of sovereign debt in the presence of two types of limited enforcement problems arising from a government's monetary and debt policy: strategic currency debasement and default on sovereign debt. Local currency debt has better state contingency than foreign currency debt in the sense that its real value can be changed by a government's monetary policy, thus acting as a better consumption hedge against income shocks. However, this higher degree of state contingency for local currency debt provides a government with more temptation to deviate from disciplined monetary policy, thus restricting borrowing in local currency more than in foreign currency. The two financial frictions related to the two limited enforcement problems combine to generate an endogenous debt frontier for local and foreign currency debt. Our model predicts that a less disciplined country in terms of monetary policy borrows mainly in foreign currency, as the country faces a much tighter borrowing limit for the local currency debt than for the foreign currency debt. The prediction of our model is consistent with the "Original Sin" phenomenon and can also account for a surge in local currency borrowing by emerging economies in the recent decades.

Appendix 1 : Proof of the Propositions

Proposition 1: *Suppose that there is no enforcement constraint (eq(18)) and that there is no cost of inflation (i.e.,* $C(\pi_t) = 0$ *for all* π_t *). Assume that the optimal debt levels and inflation rates* $\pi_t(s^t)$ *on the contract are interior. Then the optimal contract is such that*

$$u'(c(s_t^i, y_t^i)) = u'(c(s_t^j, y_t^j)) \quad for \quad any \quad i \neq j. \tag{1}$$

Proof] The Envelope condition for V^c with respect to $\pi(s)$ is given by

$$V_\pi^c = -u'(c(s))\frac{i_t b_t^{loc}}{\pi(s)^2} \quad for \; all \; s$$

The Lagrangean for the equation (15) is given by

$$L = \max{}_{\pi(s)} \sum_{y_s} Pr(y_s \mid y_{t-1})V^c(b_t^{for}, b_t^{loc}, y_{t-1}, y_t; \pi(y_t)) + \lambda\{\sum_{y_s} Pr(y_s \mid y_{t-1})\frac{i_t}{\pi(y_s)} - R^*\}$$

The first order condition w.r.t $\pi(s_i)$ is given by

$$Pr(y_{si} \mid y_{t-1})V_\pi^c(s_i) - \lambda Pr(y_{si} \mid y_{t-1})\frac{1}{\pi(y_{si})^2} = 0$$

$$V_\pi^c(s_i)\pi(y_{s_i})^2 = \lambda = V_\pi^c(s_j)\pi(s_j)^2$$

Combining the first order conditions and the envelope condition, we have:

$$u'(c(s_i))i_t b_t^{loc} = u'(c(s_j))i_t b_t^{loc}$$

Since $b_t^{loc} \neq 0$, we have the following:

$$u'(c(s_t^i, y_t^i)) = u'(c(s_t^j, y_t^j)) \quad for \quad any \quad i \neq j$$

Proposition 2: *Suppose that the cost of the inflation $C(\pi_t; \xi)$ is differentiable and strictly increasing in π_t, and that $C(\pi_t; \xi)$ is strictly increasing and convex in ξ for any π_t. Then $\underline{b}^{loc,H} \leq \underline{b}^{loc,L}$ for $\xi^H > \xi^L$.*

Proof: The Envelope conditions for the values of debasement and contract w.r.t ξ are identical with $-C_\xi(\pi_t)$, where $C_\xi(.)$ denotes a partial derivative w.r.t ξ. However, the inflation rate for the case of debasement π^{debase} must be higher than that of the contract, as the government does not follows the inflation rate in the contract π_t^*. Thus, we have that $V_\xi^{debase}(0, b^{loc}, y_t, y_{t+1}) = -C_\xi(\pi_t^{debase}) < -C_\xi(\pi_t^*) = V_\xi^{con}(0, b^{loc}, y_t, y_{t+1})$ for all b^{loc}, y_t, y_{t+1}. That is, an increase in ξ leads to a decrease in both value functions V^{debase} and V^{con}, but V^{debase} decreases more than V^{con}. Recall that $\underline{b}^{loc,L}$ is defined as follows:

$$\underline{b}^{loc,L} \equiv min\{b^{loc} : V_L^{con}(0, b^{loc,L}, y_t, y_{t+1}) \geq max\{V_L^{debase}(0, b^{loc,L}, y_t, y_{t+1}), V^{def}(y_{t+1})\}\} \text{ for all } (y_t, y_{t+1})'$$

(2)

,where subscript L denotes the value function associated with the cost of inflation $C(\pi_t; \xi^L)$. If we increase ξ at ξ^L, the difference between the right and left hand side $V_L^{con} - max\{V_L^{debase}(0, b^{loc,L}, y_t, y_{t+1}), V^{def}(y_{t+1})\}$ is increasing or stays the same. Since both V^{con} and V^{debase} are strictly increasing in b^{loc}, there exists $b^{loc} \leq \underline{b}^{loc,L}$ that satisfies the inequality ((2)) for $\xi^H > \xi^L$.

Proposition 4: *Under the same conditions in the proposition 2, If $\xi \to \infty$ then $\pi_t(s^t) = 1$. Moreover, the currency composition between foreign and local currency debts is indeterminate.*

Proof: The proof is straightforward. With the infinite marginal cost of inflation, any deviation of inflation (π_t) from one incurs infinitely high costs of inflation. With the equilibrium inflation being one for any local currency debt, the nominal interest rate on local currency debt i_{t+1} becomes $R*$ from the lender's expected zero profit condition. Then, the local and foreign currency debt become identical, so we have indeterminate currency composition in equilibrium.

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Additional Figures

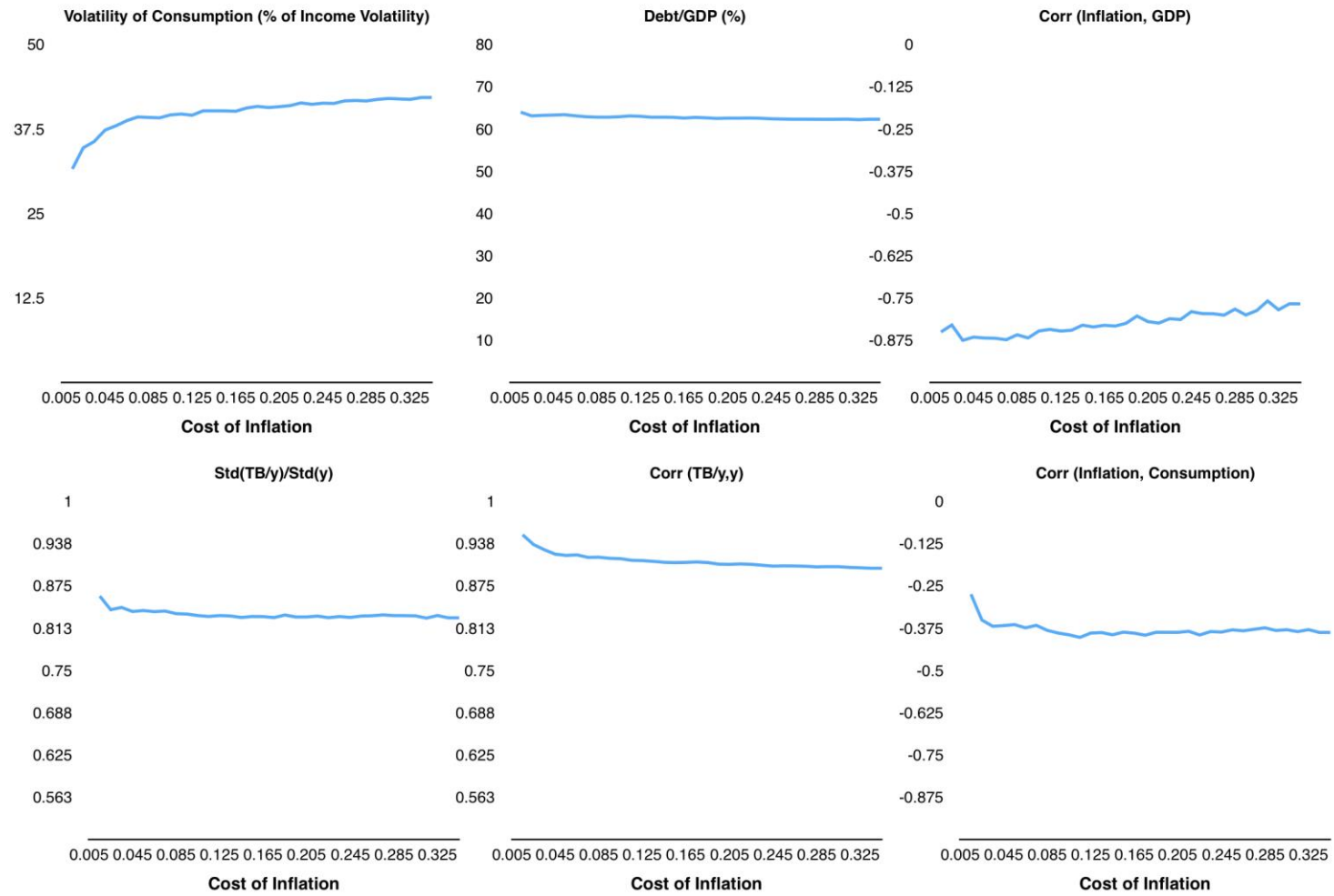**Figure 5: Sensitivity Analysis w.r.t Cost of Inflation**

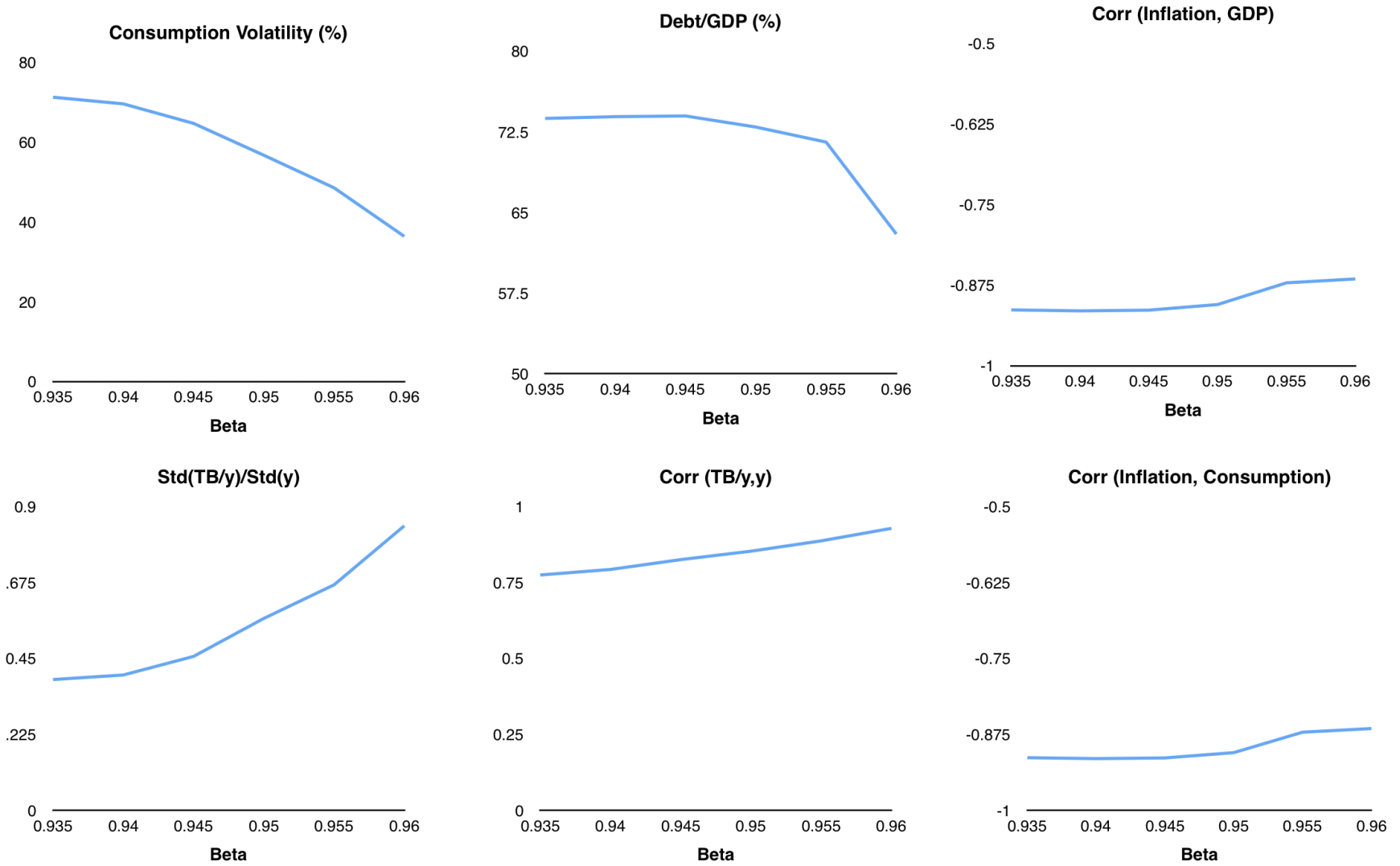**Figure 6: Sensitivity Analysis w.r.t Discount Factor**
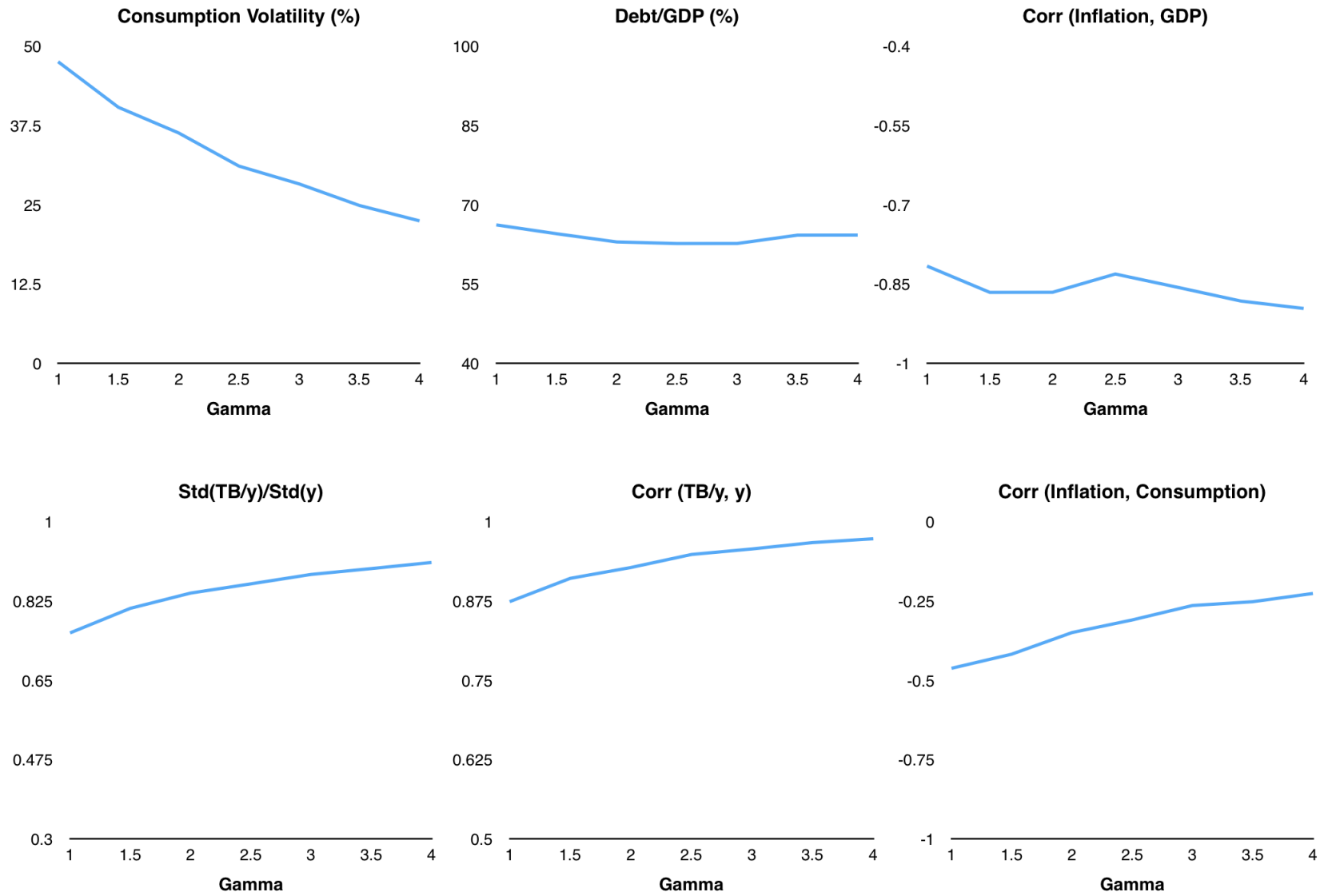
Figure 7: Sensitivity Analysis w.r.t Risk-Averseness

Figure 8: Sensitivity Analysis w.r.t Output Cost of Default

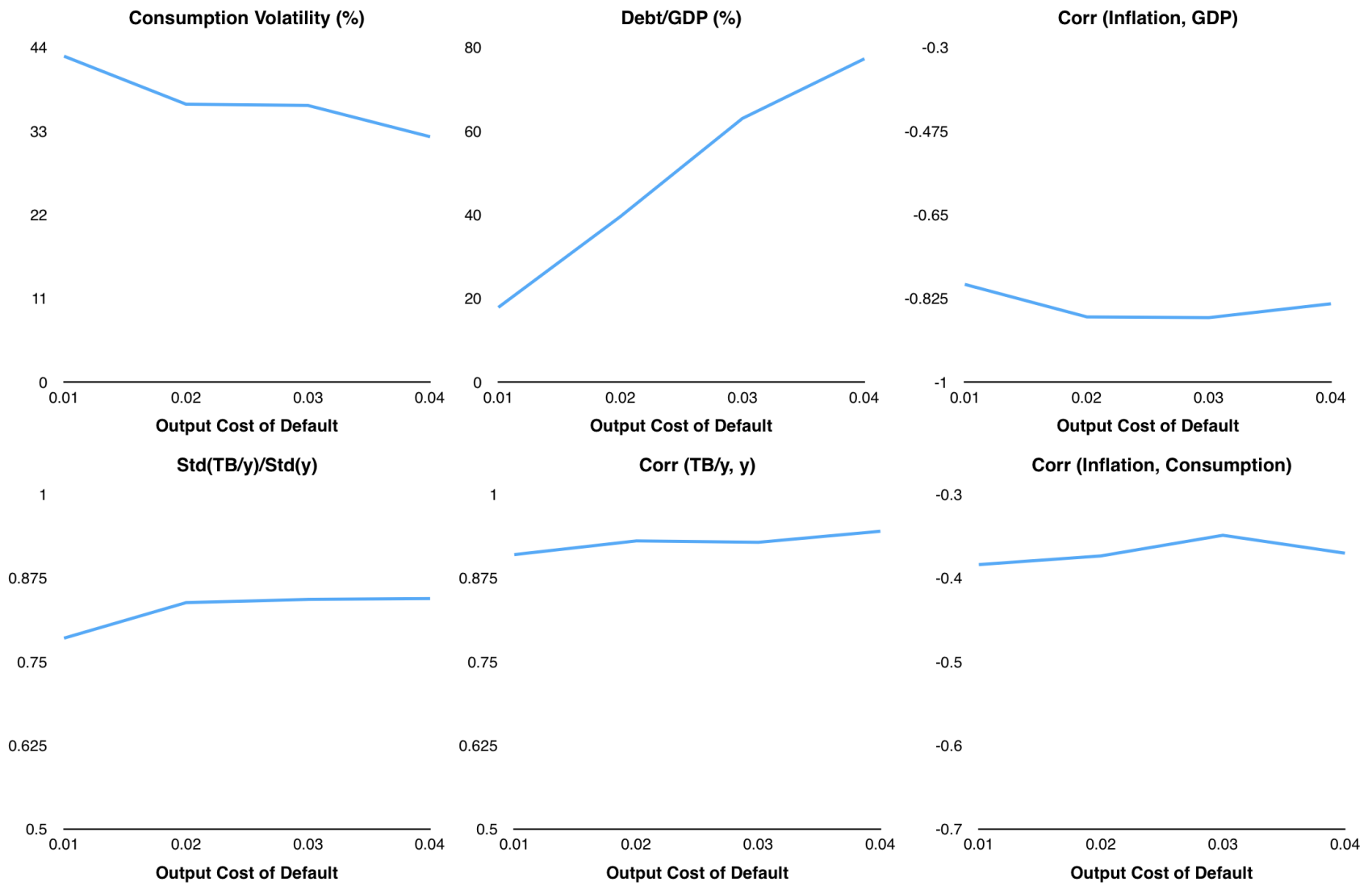| Consumption Volatility (%)	Debt/GDP (%)	Corr (Inflation, GDP)
Std(TB/y)/Std(y)	Corr (TB/y, y)	Corr (Inflation, Consumption)

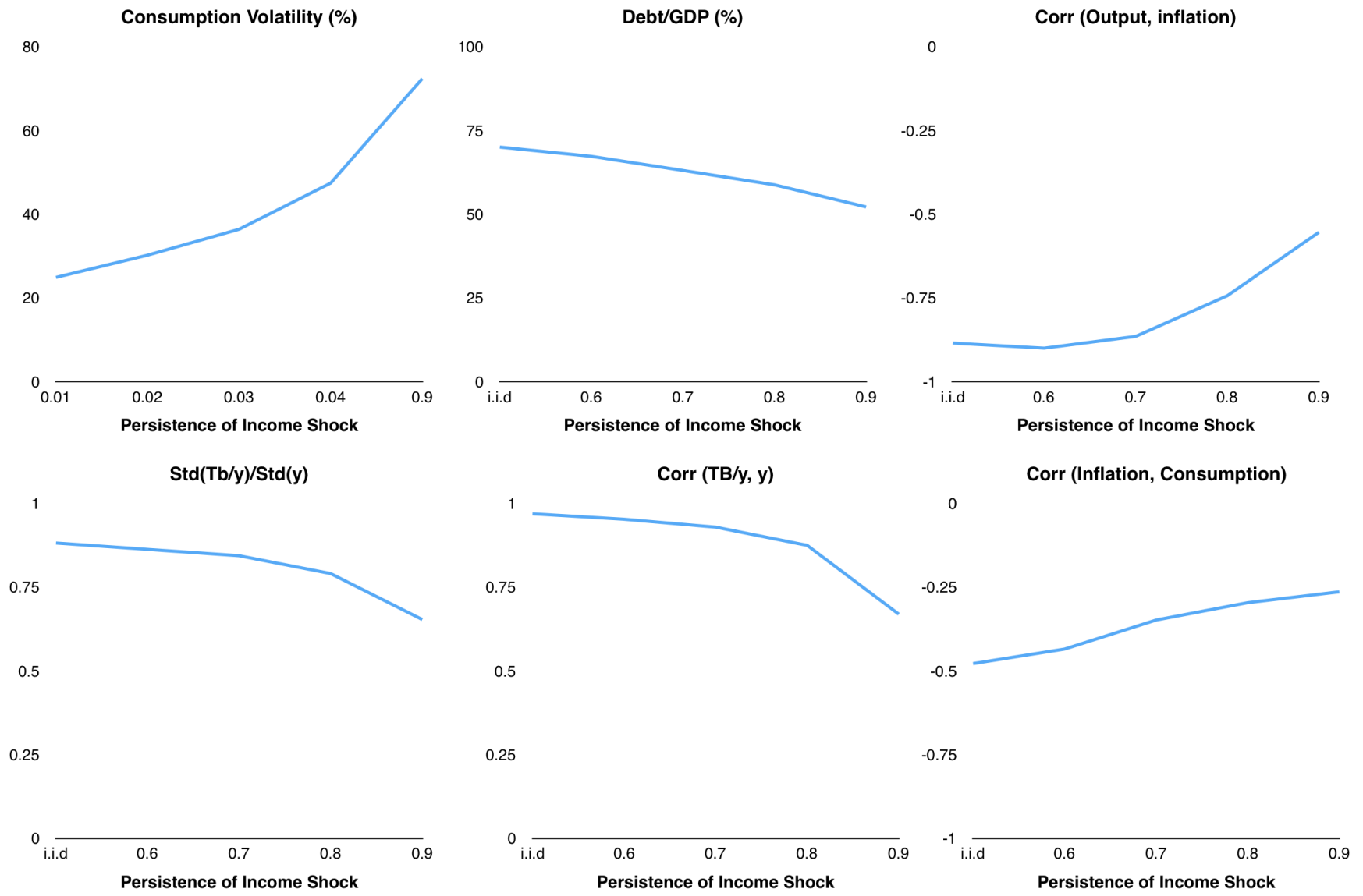**Figure 9: Sensitivity Analysis w.r.t Persistence of Income Shock**

Figure 10: Sensitivity Analysis w.r.t Variance of Income Shock

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