

# The Missing Risk Premium in Exchange Rates

Magnus Dahlquist and Julien Penasse\*

First draft: December 2016

This draft: May 2017

## Abstract

It is well known that the interest rate differential (the forward premium) predicts currency returns. However, we find that the real exchange rate, not the interest rate differential, is the main predictor of currency returns at longer horizons. We relate this finding to other puzzling features of currency markets, namely that the real exchange rate contemporaneously appreciates with the interest rate differential and that the positive relationship between currency risk premia and the interest rate differential reverses over longer horizons. Models in which the currency risk premium depends on the interest rate differential and a missing risk premium, capturing deviations from the purchasing power parity, can rationalize these observations.

**JEL classifications:** E43, F31, G15.

**Keywords:** Currency returns, forward premium puzzle, present-value model, real exchange rates, uncovered interest rate parity.

---

\*We have benefited from the comments of Patrick Augustin, Philippe Bacchetta, Mike Chernov, Charles Engel, Benjamin Holcblat, Peter Hördahl, Martin Lettau, Roberto Marfè, Tyler Muir, Valeri Sokolovski, Adrien Verdelhan, and Irina Zviadadze, of participants in seminars at the London Business School, Lund University, Norges Bank Investment Management, the Stockholm School of Economics, the University of St. Gallen, and the University of Luxembourg, and of participants in the CEPR-CityU-BIS Conference on Exchange Rate Models. Magnus Dahlquist gratefully acknowledges support from the Jan Wallander and Tom Hedelius Foundation. Dahlquist: Stockholm School of Economics and CEPR; e-mail: magnus.dahlquist@hhs.se. Penasse: University of Luxembourg; e-mail: julien.penasse@uni.lu.

We ask whether two assumptions about exchange rates are sufficient to quantify movements in exchange rates. The first assumption is that currency risk premia depend on the difference between foreign and domestic interest rates. The second assumption is that real exchange rates are stationary, so that the prices of foreign goods and services cannot forever deviate from the prices of domestic goods and services. We find that a present-value model with these two assumptions cannot quantify how much real exchange rates move over time. We argue that to quantitatively match the movements of real exchange rates in the data, currency risk premia must be more volatile and depend on an additional component, which is related to the real exchange rate.

It has been widely documented that a currency tends to appreciate in the short run when its nominal interest rates exceed foreign interest rates. This is in sharp contrast to uncovered interest rate parity (UIP), which states that the exchange rate should instead depreciate to compensate for any difference in interest rates. A common interpretation of the deviations from UIP is that currency risk premia are positively correlated with the interest rate differential, so that currencies with higher interest rates appear riskier to investors (Fama, 1984). In the past, economic models have had difficulties generating such currency risk premia and the literature refers to this as the UIP (or forward premium) puzzle. However, evidence suggests that deviations from UIP are less severe at longer horizons (see, e.g., Chinn and Meredith, 2004; Bacchetta and Van Wincoop, 2010; Boudoukh et al., 2016). Engel (2016) documents that the relationship between currency risk premia and the interest rate differential actually reverses over longer horizons, so that currencies with higher interest rates appear safer in the long run.

It seems natural to assume that foreign and domestic prices cannot deviate in the long run, i.e., that purchasing power parity (PPP) holds. This assumption allows us, within a present-value model, to decompose the current real exchange rate in terms of future cash flows and returns. More precisely, the real exchange rate equals the sum of all future interest rate differentials minus the sum of all future returns. In asset pricing terms, the real exchange rate is the relative price of two currencies and PPP excludes bubble-type fluctuations in this

price. Therefore, if the foreign currency is relatively weak today (i.e., the price is low), it must be that future interest rate differentials will be low (i.e., expected future cash flows are low), that future returns will be high, or both.

While deviations from UIP relate interest rate differentials to short-term returns, PPP ties the real exchange rate to the infinite sum of future short-term returns. Taken together, these two assumptions describe the formation of expectations about future changes in exchange rates and interest rates. An increase in the real exchange rate today should predict an equal increase in future interest rate differentials. As future interest rate differentials are highly predictable from the current interest rate differential, we should observe a near-perfect correlation between real exchange rates and interest rate differentials. However, this correlation is weak in the data. Often, it is even of the wrong sign: an increase in the interest rate differential today predicts a higher return tomorrow, but tends to come with a higher exchange rate today, which points toward a lower return tomorrow.

This suggests that although interest rate differentials covary with currency risk premia, they alone cannot pin down the time variation in currency risk premia. We use long-term PPP to extract the component of currency risk premia that is necessary to explain real exchange rate movements. We label this missing component “PPP,” and without loss of generality, we describe it as a risk premium. We assume that the interest rate differential and the PPP risk premium follow autoregressive processes. This allows us to identify the missing risk premium component using standard regression methods.

We find that accounting for this missing risk premium raises the R-squared in predictive regressions for monthly returns by about 30% on a portfolio of currencies. The PPP risk premium is highly persistent, more persistent than the risk premium related to the interest rate differential, and negatively correlated with the real exchange rate. The R-squared increases with the forecasting horizon, from 3% at the one-month horizon to about 60% at the ten-year horizon. Accounting for the PPP risk premium improves return predictability in an out-of-sample evaluation. Finally, we find that it accounts for most of the real exchange rate movements. For the currency portfolio, PPP shocks account for 132.4% of the exchange

rate innovations, the remainder being attributed to shocks in interest rate differentials (9.5% is due to pure cash flow shocks and 49.8% is due to risk premia), inflation rates (9.5%), and covariance terms (−93.2%).

Our findings help us understand Engel’s (2016) evidence that the positive relationship between currency risk premia and the interest rate differential reverses over the horizon. The interest rate differential and the real exchange rate have opposite effects. A positive interest rate differential shock raises future returns but also comes with an immediate appreciation of the real exchange rate. The net effect is an increase in expected return in the short run. However, because the real exchange rate is more persistent than the interest rate differential, this leads to a decrease in expected return in the long run. Hence, the currency appears riskier in the short run and safer in the long run.

Our empirical results shed light on the desirable properties of currency risk premia that asset pricing models must accommodate: (i) currency risk premia must depend on at least two sources of risk; (ii) these two risk sources must be negatively correlated; and (iii) one risk source must be much more persistent than the other. Interestingly, Engel (2016) reaches similar conclusions in a model based on liquidity risk. His model can accommodate the reversal in the relationship between currency risk premia and the interest rate differential. However, he argues that leading representative-agent models are unable to generate a similar relationship. Guided by our empirical results, we show that such a model can yield the above properties, and also generate a positive contemporaneous relationship between real exchange rates and interest rate differentials. We conclude that leading currency models should not only aim to explain deviations from UIP but also reproduce various moments of the real exchange rate.

The insight that return predictability and cash-flow predictability are best studied jointly originates from the literature on the behavior of the aggregate stock market (Campbell and Shiller, 1988; Fama and French, 1988; Cochrane, 2008). Just as the real exchange rate reflects future interest rate differentials and future currency returns, the dividend–price ratio reflects future dividend growth rates and future stock returns. van Binsbergen and

Koijen (2010) and Rytchkov (2012) use this to filter out expected dividend growth rates and expected stock returns in a present-value system. They find that the expected stock return and the expected dividend growth rate are both persistent, but that the expected stock return is much more persistent than the expected dividend growth rate. Unlike dividend growth rates, interest rate differentials are quite persistent and well approximated by a simple autoregressive process. The interest rate differential also tends to predict future currency returns. We nevertheless find that the component of expected returns that relates to PPP movements is more persistent than the one that relates to the interest rate differential. In this dimension, the similarity between the persistence of the real exchange rate (for studying currency returns) and of the dividend yield (for studying stock returns) is striking. Put differently, highly persistent risk premia can explain long-term deviations from PPP and long-term movements in the dividend yield.

Several applications of the present-value model have been used to understand exchange rates (Froot and Ramadorai, 2005; Engel and West, 2005, 2010). Recently, Filipe and Maio (2016) and Balduzzi and Chiang (2017) have used long-term PPP and vector autoregressive (VAR) models to derive variance decompositions of the real exchange rate. Balduzzi and Chiang (2017), in particular, use such a model to construct tests of whether the real exchange rate predicts returns. Our focus is different, in that we set the conditions that a model must meet to obtain currency return predictability, rather than strictly testing predictability. Asness et al. (2013) and Menkhoff et al. (2017) have built trading strategies based on currency value. However, these strategies invest according to relative currency values, i.e., they consider cross-sectional predictability rather than time-series predictability as we do.<sup>1</sup>

Our work relates to several other literatures. First, there is a vast literature on tests of real exchange rate stationarity and PPP (Rogoff, 1996; Burstein and Gopinath, 2014). It is notoriously difficult to test for stationarity and the evidence is mixed for real exchange rates. We do not formally test for real exchange rate stationarity, but assume it throughout the

---

<sup>1</sup>Relatedly, Jordà and Taylor (2012) and Barroso and Santa-Clara (2015) consider the diversification benefits of incorporating currency value into other currency trading strategies.

analysis. Eichenbaum et al. (2017) also maintain the assumption that the real exchange rate is stationary. They show that mean reversion in the real exchange rate arises overwhelmingly through changes in nominal exchange rates, rather than through differences in inflation rates. This is consistent with our interpretation that changes in the real exchange rate reflect return shocks. Interestingly, Lustig et al. (2016) argue that no-arbitrage conditions and the similar returns on domestic and foreign long-term bonds expressed in the same currency suggest that even the nominal exchange rate is stationary.

Second, a large literature documents predictors of currency returns beyond the interest rate differential (see Rossi, 2013, for a survey). As we do, Jordà and Taylor (2012), Boudoukh et al. (2016), Filipe and Maio (2016), and Balduzzi and Chiang (2017) use the real exchange rate to forecast exchange rates. Recent examples of other predictors include implied volatility (Chernov et al., 2016), volatility and variance risk premia (Della Corte et al., 2016; Londono and Zhou, 2017), quanto forward prices (Kremens and Martin, 2017), and net foreign assets (Gourinchas and Rey, 2007; Della Corte et al., 2012). There is also evidence of time-series momentum in currency returns (Burnside et al., 2011; Menkhoff et al., 2012; Moskowitz et al., 2012). Common to these additional predictors and to the time-series momentum evidence is that the persistence of the component complementing the interest rate differential is lower than that of the PPP risk premium or the real exchange rate. While being interesting predictors in themselves, with their lower persistence it would be challenging to generate the observed reverse relationship between the interest rate differential and future currency risk premia over longer horizons.

Third, our paper relates to the literature on representative-agent rational expectations models in open economies, such as the habit models (Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2017), the long-run risk models (Colacito and Croce, 2011, 2013; Bansal and Shaliastovich, 2013; Colacito et al., 2017), and the rare disaster model (Farhi and Gabaix, 2016). Most of these models explain deviations from UIP using a single risk source. Engel (2016) presents a long-run risk model with two risk sources, but argues that this model is unable to reproduce the changing sign between interest rates and future returns. We show

that our assumptions regarding risk premia can be rationalized as a long-run risk model. We discuss the necessary conditions that leading asset pricing models must meet to solve Engel (2016)’s puzzle, and argue that researchers should target additional moments related to the behavior of the real exchange rate.

The remainder of this paper proceeds as follow. The next section discusses a simple model in which currency risk premia depend only on interest rate differentials. Section II extends this model by allowing for an additional risk source. In Section III we introduce our data and provide preliminary evidence on the behavior of currency risk premia and exchange rates. Section IV estimates our main model and comments on its ability to reproduce key observations. Section V discusses how our findings matter for the building of asset pricing models and presents a representative-agent model that corresponds to the dynamics of our empirical reduced-form model. Section VI offers conclusions. An Online Appendix provides supporting details.

## I. Currency returns and the real exchange rate

Consider a zero-cost strategy that borrows in dollars and invests in a foreign currency. By definition, the log excess return on this strategy is the log exchange rate appreciation plus the interest rate differential:

$$r_{t+1} = s_{t+1} - s_t + i_t^* - i_t, \tag{1}$$

where  $s_t$  is the nominal exchange rate in dollars per unit of foreign currency at date  $t$ , and  $i_t$  and  $i_t^*$  are the US and foreign interest rates between  $t$  and  $t + 1$ , respectively. We refer to  $r_{t+1}$  as a currency return and the conditional expectation of it,  $E_t(r_{t+1})$ , as the expected currency return or the currency risk premium.

Uncovered interest rate parity (UIP) states that the nominal exchange rate should, in expectation, move to compensate exactly for any difference in interest rates and that the currency risk premium should be zero, i.e.,  $E_t(s_{t+1} - s_t) = -(i_t^* - i_t)$  and  $E_t(r_{t+1}) = 0$ .<sup>2</sup> This

---

<sup>2</sup>UIP is actually a statement of the expected level (not log) of excess return, which means that we abstract

is typically evaluated by regressing the change in the exchange rate on the lagged interest rate differential (Fama, 1984):

$$s_{t+1} - s_t = \alpha - \beta(i_t^* - i_t) + \varepsilon_{t+1}, \quad (2)$$

where  $\varepsilon_{t+1}$  is an error term that is assumed to be uncorrelated with all available information at date  $t$  (in particular, uncorrelated with the interest rate differential). UIP implies that  $\beta = 1$ , which is routinely rejected in the data. Estimates of the  $\beta$  coefficient are less than one and often negative. A negative  $\beta$  means that a currency with a relatively high interest rate tends to appreciate against the dollar, while UIP implies that it should instead depreciate against the dollar. This is often referred to as the UIP (or forward premium) puzzle, as it was challenging for earlier models to qualitatively and quantitatively match the estimated  $\beta$  coefficient.

Add the interest rate differential,  $i_t^* - i_t$ , to both sides of (2) and recall the return definition, (1), to obtain:

$$r_{t+1} = \alpha + (1 - \beta)(i_t^* - i_t) + \varepsilon_{t+1}. \quad (3)$$

The general interpretation of  $\beta < 1$  in the literature is that currency risk premia are time varying and positively correlated with interest rate differentials. A number of papers study the properties of asset pricing models that can generate a time-varying currency risk premium. It is common to choose model parameters so that the currency risk premium is perfectly correlated with the interest rate differential (see, e.g., Backus et al., 2001; Verdelhan, 2010; Farhi and Gabaix, 2016). We relax that assumption in Section II.

Express (1) in terms of real variables:

$$r_{t+1} = q_{t+1} - q_t + (i_t^* - \pi_{t+1}^*) - (i_t - \pi_{t+1}), \quad (4)$$

---

from a Jensen's inequality term. Here, the important deviation from UIP is a time-varying currency risk premium, not a non-zero but a constant currency risk premium.



where  $q_t = s_t + p_t^* - p_t$  is the real exchange rate, and  $\pi_{t+1} = p_{t+1} - p_t$  and  $\pi_{t+1}^* = p_{t+1}^* - p_t^*$  are the US and foreign inflation rates between  $t$  and  $t + 1$ , respectively. Rewrite (4) in terms of the real exchange rate, iterate forward, and take conditional expectations (Engel and West, 2005):

$$q_t - \mu_q = \sum_{j=1}^{\infty} \mathbb{E}_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^{\infty} \mathbb{E}_t(\pi_{t+j}^* - \pi_{t+j}) - \sum_{j=1}^{\infty} \mathbb{E}_t(r_{t+j}). \quad (5)$$

To derive (5), we assume that PPP holds in the long run. The PPP restriction plays a similar role as do no-bubble conditions in present-value models of stock returns (see, e.g., Campbell and Shiller, 1988; van Binsbergen and Koijen, 2010). This amounts to assuming that  $\lim_{j \rightarrow \infty} \mathbb{E}_t(q_{t+j}) = \mathbb{E}(q_t) = \mu_q$ .

For simplicity, assume that the interest rate differential follows an AR(1) process and that the inflation differential is unpredictable:

$$i_{t+1}^* - i_{t+1} = (1 - \rho_i)\mu_i + \rho_i(i_t^* - i_t) + \varepsilon_{t+1}^i, \quad (6)$$

$$\pi_{t+1}^* - \pi_{t+1} = \mu_\pi + \varepsilon_{t+1}^\pi, \quad (7)$$

where the shocks  $\varepsilon_{t+1}^i$  and  $\varepsilon_{t+1}^\pi$  are independently and identically distributed (IID) over time. Together with (3), these assumptions lead to the following expression for the real exchange rate (see Appendix A):

$$q_t - \mu_q = \beta \frac{i_t^* - i_t - \mu_i}{1 - \rho_i}. \quad (8)$$

Comparing (5) and (8), we see that rather than being correlated with future interest rate differentials minus future returns, the real exchange rate is simply correlated with the current interest rate differential. This is in contrast with the literature on equity return predictability (e.g., van Binsbergen and Koijen, 2010), where valuation ratios depend on future returns and cash flows. In this simple model, both future returns and future cash flows are proportional to the current interest rate differential. The proportional relationship between future returns and the interest rate differential is given by (3). The proportional relationship between future interest rate differentials and the current interest rate differential follows from our AR(1)

assumption for the interest rate differential in (6). This assumption implies that future interest rate differentials are perfectly correlated with the current interest rate differential. Also note that expectations of inflation differentials are constant in (7) and therefore do not affect the real exchange rate in (8).<sup>3</sup>

Unless  $\beta = 0$ , these assumptions imply that the real exchange rate is perfectly correlated with the interest rate differential. This perfect correlation does not depend on whether or not UIP holds. If UIP holds, then  $\beta = 1$  and the real exchange rate equals the present value of expected future interest rate differentials. Hence, the real exchange rate moves only because of “fundamentals” (or cash flows). The foreign currency is expensive ( $q_t > 0$ ) when the interest rate in the foreign country is higher than in the domestic country ( $i_t^* - i_t > 0$ ). When  $\beta \neq 1$ , the interest rate differential predicts future returns. This means that the real exchange rate also moves because of expected future returns (or discount rates). When  $\beta < 0$ , the discount rate effect dominates so that the foreign currency appears weak when its relative interest rate is high. That is,  $\beta$  captures the sensitivity of the real exchange rate with respect to both cash flows and discount rate shocks.

We make different assumptions regarding cash flow expectations than in the stock market literature because cash flows are far more predictable in a currency investment than in a stock investment. Table I indicates that nominal interest rate differentials are highly persistent. In contrast, the dividend growth rate is notoriously difficult to predict (e.g., Cochrane, 2008). This motivates the filtering approach of van Binsbergen and Koijen (2010) to extract dividend expectations from the dividend-price ratio. The model we present in Section II does not require this complication as expectations of the interest rate differential are easier to pin down than expectations of the dividend growth rate. We prefer to model interest rate differentials in nominal terms, as nominal interest rate differentials are far more persistent than real interest rate differentials. Inflation differentials exhibit low persistence (see Table I), which motivates our assumption (7). Irrespectively, as is apparent in (8), only highly

---

<sup>3</sup>From (8), the real exchange rate follows a stationary AR(1) process. Our assumption (7) regarding the inflation differential implies that the price differential,  $p_t - p_t^*$ , follows a random walk, which in turn implies that the nominal exchange rate,  $s_t = q_t + p_t - p_t^*$ , is non-stationary.

persistent variables can realistically exhibit strong correlations with the real exchange rate.

We later demonstrate that the correlation between the real exchange rate and the interest rate differential is fairly weak; in addition, this correlation is often of the wrong sign. As noted earlier, interest rate differentials tend to predict future currency returns and exchange rate changes positively, suggesting a  $\beta < 0$ . However, regressions of the real exchange rate on the interest rate differential tend to yield positive coefficients, suggesting a  $\beta > 0$ . This empirical tension between current exchange rates, interest rate differentials, and future returns requires that the expected currency return depends on an additional risk premium, as we discuss next.

## II. The missing risk premium

We hypothesize a two-factor structure for expected currency returns. The first factor correlates with the interest rate differential. The second factor is determined by the long-term PPP restriction (5). As this second factor will capture the mean reversion of the real exchange rate, we label it PPP. Section V presents a long-run risk model that corresponds to the reduced-form dynamics that we assume here. We alter (3) so that the error term consists of a mean-zero term,  $\eta_t$ , and a pure return shock,  $\varepsilon_{t+1}^r$ :

$$r_{t+1} = \alpha + (1 - \beta)(i_t^* - i_t) + \eta_t + \varepsilon_{t+1}^r. \quad (9)$$

We maintain assumptions (7) and (6), and further assume that  $\eta_t$  follows an AR(1) process:

$$\eta_{t+1} = \rho_\eta \eta_t + \varepsilon_{t+1}^\eta, \quad (10)$$

where the shock  $\varepsilon_{t+1}^\eta$  is assumed to be IID over time. That  $\eta_t$  has a zero mean ensures identifiability. As  $\eta_t$  predicts currency returns, we interpret this variable as a risk premium, although other forces such as (rational) inattention or liquidity premia could justify the

predictability. In practice many variables have been shown to predict exchange rates (Rossi, 2013). However, to keep it simple, we assume that two factors are sufficient to characterize currency risk premia. The interest rate differential will be especially important in the short term and  $\eta_t$  will be especially important in the long term. To make another analogy with the stock market, a large number of variables can predict returns beyond valuation ratios, but the dividend-price ratio is the natural variable to look at to understand returns and asset prices. The other variables are typically less persistent than valuation ratios, and therefore predict returns over shorter horizons (Cochrane, 2011).

The model features three shocks, namely, the interest rate differential shock,  $\varepsilon_t^i$ , the inflation differential shock,  $\varepsilon_t^\pi$ , and the PPP shock,  $\varepsilon_t^\eta$ . We have assumed that these shocks are IID over time with zero means but allow them to be correlated with the following covariance matrix:

$$\text{Var} \left( \begin{bmatrix} \varepsilon_t^i \\ \varepsilon_t^\pi \\ \varepsilon_t^\eta \end{bmatrix} \right) = \begin{bmatrix} \sigma_i^2 & \sigma_{i\pi} & \sigma_{\eta i} \\ \sigma_{i\pi} & \sigma_\pi^2 & \sigma_{\eta\pi} \\ \sigma_{\eta i} & \sigma_{\eta\pi} & \sigma_\eta^2 \end{bmatrix}. \quad (11)$$

The corresponding correlations between the shocks are denoted  $\rho_{i\pi}$ ,  $\rho_{\eta i}$ , and  $\rho_{\eta\pi}$ .

We further assume that  $\alpha = \beta\mu_i - \mu_\pi$ . We show in Appendix A that this assumption is necessary for the real exchange rate to be stationary.

To obtain a new expression for the real exchange rate (see Appendix A), use assumptions (6), (7), and (10) together with the return (9) to obtain:

$$q_t - \mu_q = \beta \frac{i_t^* - i_t - \mu_i}{1 - \rho_i} - \frac{\eta_t}{1 - \rho_\eta}. \quad (12)$$

Hence, our two-factor structure for expected returns implies that the same two factors drive the level of the real exchange rate. We emphasize that there is no error term in (12). For the purpose of estimation, this relationship effectively binds the predictions of future returns and the interest rate differential so that long-term PPP holds.

This relationship also implies that one of the shocks is redundant. We choose to express return shocks in terms of the three primitive shocks: the interest rate differential shock,  $\varepsilon_{t+1}^i$ ,

the inflation differential shock,  $\varepsilon_{t+1}^\pi$ , and the PPP shock,  $\varepsilon_{t+1}^\eta$ . Using (12), the change in the real exchange rate is:

$$q_{t+1} - q_t = \beta\mu_i - \beta(i_t^* - i_t) + \eta_t + \beta\frac{\varepsilon_{t+1}^i}{1 - \rho_i} - \frac{\varepsilon_{t+1}^\eta}{1 - \rho_\eta}. \quad (13)$$

Likewise, using (4) and (7), the return is:

$$r_{t+1} = \beta\mu_i - \mu_\pi + (1 - \beta)(i_t^* - i_t) + \eta_t + \beta\frac{\varepsilon_{t+1}^i}{1 - \rho_i} - \varepsilon_{t+1}^\pi - \frac{\varepsilon_{t+1}^\eta}{1 - \rho_\eta}. \quad (14)$$

It follows that the return shock can be expressed as:

$$\varepsilon_{t+1}^r = r_{t+1} - E_t(r_{t+1}) = \beta\frac{\varepsilon_{t+1}^i}{1 - \rho_i} - \varepsilon_{t+1}^\pi - \frac{\varepsilon_{t+1}^\eta}{1 - \rho_\eta}. \quad (15)$$

The unexpected currency return is thus a weighted sum of the three primitive shocks. Note that as the inflation differential is unpredictable (it has no persistence), it does not appear in the long-run expression for the real exchange rate in (12). However, inflation shocks matter in the short run and affect currency returns in (15).

Similarly, (12) suggests that the missing risk premium,  $\eta_t$ , is a function of the interest rate differential and the real exchange rate:

$$\eta_t = (1 - \rho_\eta) \left[ \beta\frac{i_t^* - i_t - \mu_i}{1 - \rho_i} - (q_t - \mu_q) \right]. \quad (16)$$

Inserting (16) into (9), we find that:

$$r_{t+1} = \beta\mu_i \left( \frac{\rho_\eta - \rho_i}{1 - \rho_i} \right) - \mu_\pi + \left( 1 - \beta\frac{\rho_\eta - \rho_i}{1 - \rho_i} \right) (i_t^* - i_t) - (1 - \rho_\eta)(q_t - \mu_q) + \varepsilon_{t+1}^r. \quad (17)$$

This suggests that currency return predictions should include not only the interest rate differential but also the real exchange rate. Univariate regressions ignore the long-run restriction implied by PPP and may therefore yield potentially biased estimates of  $\beta$ .

Our assumptions regarding the dynamics of the interest rate differential, inflation dif-

ferential, and PPP risk premium are rather simplistic. They correspond to a sparse VAR with off-diagonal coefficients in the autocorrelation matrix set to zero, and with the inflation differential autocorrelation coefficient set to zero. In the Online Appendix, we repeat our analysis in the context of a full VAR model. We show that this implies that, as long as it is persistent, the inflation differential should predict currency returns. In the data, we find some evidence that the inflation differential is persistent, although it is much less so than the interest rate differential or the PPP risk premium. Consequently, an increase in the inflation differential lowers future returns at a monthly horizon, but vanishes at an annual horizon. Hence, the inflation differential matters little for long-horizon returns, implying that estimates of the PPP risk premium do not strongly depend on inflation differential dynamics. We verify this empirically and present the results in the Online Appendix.

### III. Data and preliminaries

#### A. Data

We retrieve daily data on spot and one-month forward exchange rates from Barclays Bank International and Reuters (via Datastream). Our sample covers nine countries (currencies): Australia (AUD), Canada (CAD), Germany (EUR), Japan (JPY), New Zealand (NZD), Norway (NOK), Sweden (SEK), Switzerland (CHF), and the UK (GBP). Our sample covers the period from January 1976 to December 2015 for seven currencies (all except AUD and NZD), and starts in January 1985 for AUD and NZD.<sup>4</sup> We also construct an equally weighted portfolio of the seven countries, referred to simply as “Portfolio,” which covers the full sample period.<sup>5</sup> We assume the US dollar (USD) to be the domestic currency and express all seven currencies in USD per unit of the foreign currency. Hence, an increase in the exchange rate of a given currency implies the appreciation of the foreign currency and the depreciation of the

---

<sup>4</sup>For the Japanese yen up to 1978 we use data obtained from the *Financial Times* as in Hsieh (1984).

<sup>5</sup>We choose not to include AUD and NZD in this portfolio to avoid complications related to the inclusion of these currencies in 1985 (i.e., introducing currencies within the sample requires demeaning of series and complicates the interpretation of out-of-sample exercises).

USD. As one-month interbank rates are not available for all countries during this period, we compute implied one-month interest rate differentials assuming covered interest-rate parity:  $i_t^* - i_t = s_t - f_t$ , where  $s_t$  and  $f_t$  denote the log spot and forward exchange rates, respectively. Log excess returns for a US investor going long in a foreign currency are computed as  $r_{t+1} = s_{t+1} - s_t + i_t^* - i_t$ . Log real exchange rates are computed as  $q_t = s_t + p_t^* - p_t$ , where  $p_t^*$  and  $p_t$  are consumer price indices obtained from the OECD. Log inflation differentials are computed as  $\pi_t^* - \pi_t = (p_t^* - p_{t-1}^*) - (p_t - p_{t-1})$ . The statistical agencies in Australia and New Zealand release price indices on a quarterly basis, while our dataset is sampled monthly. To avoid introducing future information into the econometrician's information set, we choose not to interpolate inflation rates for these two currencies, and instead update price indices at the end of each quarter.

Table I reports means, standard deviations, and first-order autocorrelations for monthly returns, real exchange rates, interest rate differentials, and inflation differentials for the nine currencies individually as well as the portfolio of all seven currencies. Returns are on average low but quite volatile, with the monthly standard deviation ranging from 2% to 3.5% (corresponding to an annual volatility of 7% to 12%). Unsurprisingly, returns exhibit little autocorrelation. Both the real exchange rates and the interest rate differentials are persistent, although first-order autocorrelations are much higher for the real exchange rates than for the interest rate differentials.

### *B. Fama regressions*

The first panel of Table II reports regressions of the future currency return on the current interest rate differential as in (3). This type of regression is referred to as a Fama regression (Fama, 1984). As usually found in the literature, the estimated slope coefficients are, except for Sweden, negative, indicating that a currency with a higher interest rate than the US interest rate tends to appreciate in value. (Recall that UIP predicts a slope coefficient of one, so that the interest rate differential is equaled by the currency depreciation.)

We argue that regressions of this kind may suffer from an omitted variable bias once one

recognizes that the real exchange rate must be stationary. The second panel of Table II therefore reports regressions in which we control for the lagged real exchange rate. Formally, we run the following regression:

$$r_{t+1} = a + (1 - b)(i_t^* - i_t) + cq_t + u_{t+1}. \quad (18)$$

We now find stronger evidence that interest rate differentials negatively predict future exchange rates (i.e., the  $b$  coefficients become more negative). Importantly, the coefficients associated with the real exchange rate are negative for all countries. When the real exchange rate is high (i.e., the foreign currency is expensive with respect to the dollar), currency returns tend to be lower. A one-standard-deviation increase in the real exchange rate lowers the next-month return by 0.26%. This effect is about the same as for a one-standard-deviation change in the interest rate differential.

What does this tell us about risk premia? The R-squared from the Fama regression of the currency portfolio is only 2.2%, as reported in the last column of Table II. However, this seemingly low R-squared implies that the risk premium for holding a foreign deposit is quite volatile. The R-squared tells us what fraction of currency return *variance* is predictable. The ratio of expected return *standard deviation* to return standard deviation is  $\sqrt{2.2\%} \approx 14.8\%$ , which suggests a volatile risk premium, as originally argued in Fama (1984). Adding the real exchange rate to the right-hand side raises the R-squared to 2.9%, a 32% increase. This suggests an even more volatile risk premium, with the predictable variation accounting for 17.0% of the currency return standard deviation.

While the volatility of the risk premium is large, the estimates are imprecise and one may worry about statistical significance. In addition, both predictors are highly persistent. Predictive regressions inherit the near-unit-root properties of the right-hand-side variable (Stambaugh, 1999). Innovations for both right-hand-side variables are positively correlated with return innovations, leading to biases. In the currency portfolio, the correlation between the residuals in (18) and the interest rate differential innovations is 12.7%. Table II indicates that the  $b$  estimate is typically negative. The Stambaugh (1999) bias implies that we



underestimate predictability from interest rate differentials (i.e.,  $b$  should be more negative). The correlation between the residuals in (18) and real exchange rate innovations is 98.6%. Hence,  $c$  is also biased downward, but in that case we may overestimate the predictability from the real exchange rate.

Should we accept the null hypothesis that  $c = 0$  and reject that the real exchange rate predicts currency returns? Equation (17) suggests that  $\rho_\eta = c + 1$ . All point estimates of  $c$  coefficients are negative but close to zero, suggesting that  $\rho_\eta$  is slightly lower than one. Under the null hypothesis that  $c = 0$ ,  $\rho_\eta = 1$ , so that both  $\eta_t$  and the real exchange rate have unit roots. A more powerful null hypothesis would be conditioned on long-term PPP being true (i.e., conditioned on  $\rho_\eta < 1$ ). Cochrane (2008) makes this point in the context of predicting stock market returns by the dividend yield. If the dividend yield is stationary, it must predict future dividend growth, future returns, or both. Likewise, if the real exchange rate is stationary, it must predict future interest rate differentials, future returns, or both. Balduzzi and Chiang (2017) apply this idea to exchange rates and reject the null hypothesis that the real exchange rate does not predict currency returns.

Perhaps the best way to gauge the importance of the real exchange rate is to consider long-horizon predictions. The third and fourth panels of Table II report results of the predictive regression of one-year returns. The R-squared values are now 13.4% and 25.4% for the currency portfolio. The results indicate that the persistence of the interest rate differential and the real exchange rate strengthens the predictability over the investment horizon. We further illustrate this point in Figure 1, which plots the negative of the real exchange rate against subsequent five-year currency returns. The correlation is high between the two variables. The real exchange rate could fluctuate over time to reflect changing expectations of the long-term differences in interest rates across currencies. However, Figure 1 suggests that future currency returns offset changes in the real exchange rate. This pattern is common in predictive regressions of stock returns on various valuation ratios, where valuation ratios predict future returns rather than future cash flows. For example, Cochrane (2011) presents a similar figure, relating the dividend-price ratio and subsequent stock returns. Interest-

ingly, in his presidential address, Cochrane (2011) mentions exchange rate predictability, but only by interest rate differentials, while interest rate differentials mostly capture information regarding short-term currency returns. Figure 1 does not account for the information contained in current interest rate differentials. Long-term PPP tells us that the real exchange rate, once purged from expectations from future interest rate differentials, should capture long-term currency returns, not the interest rate differential. We return to this point in the context of our present-value model in Section IV.D.

### *C. Real exchange rates and interest rate differentials*

If currency risk premia are well described by the Fama regression and if real exchange rates are stationary, then real exchange rates must be perfectly correlated with the expected future interest rate differentials. We assume that interest rate differentials follow an AR(1) process, which in turn implies that expectations regarding future interest rate differentials are perfectly correlated with the current interest rate differential; see (8). In addition, this correlation must have the same sign as the  $\beta$  coefficient.

Table III runs contemporaneous regressions of the real exchange rates on interest rate differentials. The error term in this regression is perfectly correlated with  $\eta_t$ , and is likely serially correlated. We therefore use a Cochrane-Orcutt estimation method, specifying an AR(1) process for the error term. If the simple present-value model of Section I is true, the R-squared values should be high and the slope coefficients should be negative. Table III shows that the estimated slope coefficients are positive in all countries except Canada. Although the slope coefficients are statistically significant in some cases, the R-squared values are unambiguously small.

## IV. Empirical results

### A. Estimation

Our present-value model features twelve parameters:  $\mu_q$ ,  $\mu_i$ ,  $\mu_\pi$ ,  $\rho_i$ ,  $\sigma_i$ ,  $\sigma_\pi$ ,  $\beta$ ,  $\rho_\eta$ ,  $\sigma_\eta$ ,  $\rho_{i\pi}$ ,  $\rho_{\eta i}$ , and  $\rho_{\eta\pi}$ . Parameters for the dynamics of interest rate and inflation differentials in (6) and (7) relate only to observed variables and are therefore straightforward to estimate. Five parameters depend on the unobserved risk premium,  $\eta_t$ :  $\beta$ ,  $\rho_\eta$ ,  $\sigma_\eta$ ,  $\rho_{\eta i}$ , and  $\rho_{\eta\pi}$ . As our model features only one latent variable, we can estimate these parameters using the following predictive regression:

$$r_{t+1} = a + (1 - b)(i_t^* - i_t) + cq_t + u_{t+1}. \quad (19)$$

Matching coefficients with (17) gives:

$$\hat{\beta} = \hat{b} \frac{1 - \hat{\rho}_i}{\hat{\rho}_\eta - \hat{\rho}_i}, \quad (20)$$

$$\hat{\rho}_\eta = \hat{c} + 1, \quad (21)$$

where a “hat” denotes an estimated parameter. Equation (16) then gives estimates of  $\eta_t$ . Finally, (10) gives  $\text{Var}(\eta_t) = \sigma_\eta^2 / (1 - \rho_\eta^2)$ , which we use to obtain an estimate of  $\sigma_\eta$ . We estimate the correlations  $\rho_{\eta i}$  and  $\rho_{\eta\pi}$  from the residuals.

### B. Estimation results

Table IV presents the estimated model parameters for the seven currencies in the sample and for the currency portfolio. Bootstrapped standard errors are reported in parentheses.

We documented in Section III.B, in line with the literature, that the  $\beta$  coefficients in the Fama regressions are mostly negative, indicating that high-interest currencies tend to have higher future returns. This is the UIP (or forward premium) puzzle discussed earlier. Table IV reports  $\beta$  coefficients that are even more negative than the slope coefficients in Table II. Hence, controlling for PPP deviations intensifies the puzzle.

Table IV also highlights the volatility and persistence of the two risk premia. Interest rate differential shocks tend to be more volatile than PPP shocks (i.e.,  $\sigma_i > \sigma_\eta$ ). However, the PPP risk premium is always more persistent than the interest rate differential (i.e.,  $\rho_\eta > \rho_i$ ). This difference is economically large and significant. For example, Table IV reports that the estimate of  $\rho_i$  for the currency portfolio is 0.878, corresponding to a half-life of about 5.3 months for the average interest rate differential. The estimate of  $\rho_\eta$  is 0.982, corresponding to a half-life of 38.6 months for the PPP risk premium. This is broadly consistent with previous estimates of deviations from PPP (see, e.g., Rogoff, 1996; Burstein and Gopinath, 2014).

Finally, the last row in Table IV reports R-squared values for predictive regressions of future currency returns on interest rate differentials and PPP deviations,  $\hat{\eta}_t$ , obtained from our model. The R-squared values in Table IV are only slightly lower than those reported in Table II. This is because the model identifies PPP deviations from the predictive regression, while imposing that the constant term in this regression equals  $\beta\mu_i - \mu_\pi$  (unreported  $F$ -tests indicate that this restriction is never rejected at usual significance levels). It is again noteworthy that our model has greater explanatory power than do the unconstrained Fama regressions. Hence, the information contained in PPP deviations is meaningful for predicting currency returns.

### *C. Why does the real exchange rate appreciate with the interest rate differential?*

Another salient result of Table IV is that the PPP risk premium and the interest rate differential shock are negatively correlated, the correlation being  $-0.53$  for the currency portfolio. An increase in the domestic interest rate (or a decrease in the foreign interest rate) corresponds to a decrease in the PPP risk premium. This is useful to understand why the real exchange rate appreciates contemporaneously with the interest rate differential, as noted in Table III. When inflation differentials are unpredictable, the covariance between

the real exchange rate and the interest rate differential,  $\text{Cov}(q_t, i_t^* - i_t)$ , is given by:

$$\text{Cov}\left(\mathbb{E}_t \sum_{j=1}^{\infty} (i_{t+j-1}^* - i_{t+j-1}) - \mathbb{E}_t \sum_{j=1}^{\infty} r_{t+j}, i_t^* - i_t\right). \quad (22)$$

From (12), this can be simplified to:

$$\frac{\beta \text{Var}(i_t^* - i_t)}{1 - \rho_i} - \frac{\text{Cov}(\eta_t, i_t^* - i_t)}{1 - \rho_\eta}, \quad (23)$$

and its sign depends on the coefficient  $\beta$  and the covariance between the PPP premium  $\eta_t$  and the interest rate differential. Estimates of  $\beta$  are often negative, which calls for a compensating force to obtain a positive covariance between the real exchange rate and the interest rate differential. This compensating force manifests itself in the negative correlation between the PPP premium and the interest rate differential ( $\rho_{\eta i} < 0$ ). Economically, the short-run effect of a increase in the domestic interest rate is an increase in future expected return (i.e.,  $\beta < 0$ ), but less than if the PPP risk premium is left constant. As interest rate differentials revert to their means faster than do PPP deviations (i.e.,  $\rho_i < \rho_\eta$ ), the long-term effect is a decrease in future expected currency returns as increases in interest rate differentials are associated with real exchange rate appreciations. Hence, PPP deviations can rationalize Engel's (2016) finding that deviations from UIP change direction at long horizons. We return to this point in Section IV.E.

#### *D. The PPP risk premium predicts currency returns*

We now further assess the ability of our present-value model to predict currency returns.

**Out-of-sample predictions** Table V shows that the increased ability of predictability regressions including the real exchange rate holds out of sample. We report the Campbell and Thompson (2008) out-of-sample  $R_{OS}^2$  for Fama regressions, unconstrained bivariate regressions that expand the Fama regression with the lagged real exchange rate, and predictions based on our present-value model. We also report Sharpe ratios for a simple mean-variance

strategy exploiting return predictability. For each currency, we divide the data into two roughly equal sample periods. We use the first half of the sample (i.e., observations up to and including 1995) as a training period, and predict returns recursively over an expanding window from 1996 onwards. We compare the predictions obtained from the three models with predictions based on the average of past returns. All predictions are formed to capture the situation of a forecaster in real time.

Campbell and Thompson's (2008)  $R_{OS}^2$  measures the proportional reduction in mean squared prediction error (MSPE) for a given model relative to a benchmark of no predictability in which predictions are based on the sample average of past returns. The statistic is computed over the out-of-sample period  $\underline{t}, \dots, T$  as:

$$R_{OS}^2 = 1 - \frac{\sum_{t=\underline{t}}^T (r_t - \hat{r}_t)^2}{\sum_{t=\underline{t}}^T (r_t - \bar{r}_t)^2}, \quad (24)$$

where  $\hat{r}_t$  and  $\bar{r}_t$  are predictions obtained by the model and predictions based on the historical average, respectively. A positive  $R_{OS}^2$  implies that the MSPE of the predictive regression is lower than the MSPE of the historical average return. We assess statistical significance using the Clark and West (2007) adjusted statistic to test the null hypothesis that the MSPE of the historical average is less than or equal to that of the predictive regression, against the alternative hypothesis that the MSPE of the historical average is greater than that of the predictive regression.

The top panel of Table V reports out-of-sample R-squared values for the seven currencies of our sample and for the portfolio, using Fama regressions, bivariate regressions, and our present-value model. Overall, the R-squared values are positive for most countries, Australia, Japan, and New Zealand being the exceptions.<sup>6</sup> Predictions that use only information from lagged interest rate differentials produce relatively low R-squared values for individual currencies. However, we find a significant improvement in the currency portfolio. For the portfolio, the prediction is based on the average interest rate differential. These predictions

---

<sup>6</sup>Recall that consumer price indices are not available at the monthly frequency for AUD and NZD, which introduces noise into the measurement of the real exchange rate for these currencies.

beat the historical average benchmark, with an R-squared of 1.4%. This in line with evidence presented by Lustig et al. (2014), who document that a strategy that goes long in all foreign currencies when the average foreign interest rate is above the US interest rate and otherwise short in all foreign currencies, exhibits high risk-adjusted returns.

Conditioning on the real exchange rate improves predictions. The R-squared is now 2.1% for the currency portfolio, both in the bivariate specification and in the present-value model. The predictions are significantly better than the historical average for most currencies.

How large is an R-squared of 2.1% for the monthly frequency? Campbell and Thompson (2008) show that R-squared values of this magnitude can translate into high Sharpe ratios through market timing. With this idea in mind, we construct a simple strategy based on an investor with mean-variance preferences and a one-month investment horizon. The investor allocates a fraction,  $\omega_t$ , of his or her wealth each month, so as to maximize expected portfolio excess return minus  $\gamma/2$  times the portfolio variance, where  $\gamma$  is the coefficient of relative risk aversion. The investor therefore chooses to invest  $\omega_t = \frac{E_t r_{t+1}}{\gamma \text{Var}_t(r_{t+1})}$ . Figure 2 depicts the cumulative excess return on an initial one dollar invested in three strategies, assuming  $\gamma = 3$ . The solid line shows the return of the present-value market-timing strategy, which is compared with the return of a strategy that invests in the equally weighted currency portfolio (dotted line) or of a mean-variance strategy conditioned only on the interest rate differential (dashed line). We report annualized Sharpe ratios in Table V, together with  $p$ -values for the null of zero Sharpe ratio. Note that ex post Sharpe ratios depend on the correlation between the portfolio weights,  $\omega_t$ , and realized returns,  $r_{t+1}$ , but are unaffected by leverage and therefore do not depend on the level of risk aversion. In our sample period, the Sharpe ratio of an investment in the currency portfolio is close to zero. In comparison, the annualized Sharpe ratio of the strategy when conditioned on the interest rate differential is 0.35, while the annualized Sharpe ratio of the strategy based on the present-value model is 0.46. This seems large, considering that the strategy uses only time-series information.

**Long-horizon predictability** As interest rate differentials and PPP deviations are highly persistent variables, it is natural to ask whether predictability increases with the forecasting horizon. To determine whether this is so, we predict cumulative currency returns with our model. Importantly, we predict long-horizon returns using one-month parameter estimates. We compare these predictions with those of Fama regressions, letting the coefficients change with the horizon. We do so to account for the potential change in sign of the predictive slope coefficient over long horizons. Our approach is conservative, as outperformance of the present-value model cannot come from the misspecification of Fama regressions over longer horizons.

The top panel of Figure 3 depicts R-squared values for horizons of one month to ten years (120 months) for the currency portfolio. R-squared values for the present-value model increase monotonically with the horizon. The predictive power over long horizons is impressive: the R-squared rises above 60% at the ten-year horizon. In contrast, R-squared values for the Fama regressions increase to about 20% for horizons of 31 months, and then decline steadily for longer horizons.

We gauge statistical significance by regressing realized returns on the predictions (without a constant). The bottom panel of Figure 3 shows  $t$ -statistics based on the Hansen and Hodrick (1980) correction for overlapping observations. For the present-value model,  $t$ -statistics are above three for all horizons. In contrast, the predictability of the interest rate differentials decays slowly and becomes insignificant at traditional levels for horizons greater than 40 months.

*E. Why does the relationship between currency risk premia and the interest rate differential reverse over longer horizons?*

Engel (2016) documents that the positive relationship between currency risk premia and the interest rate differential reverses over longer horizons. For short horizons, a relatively high foreign interest rate is associated with a relatively large currency premium (i.e., the usual UIP deviations that we reproduced in Section III.B). This implies that currencies with high



interest rates appear riskier to investors. This relationship reverses for longer horizons and currency risk premia becomes negatively correlated with the interest rate differential. Hence, currencies with high interest rates appear relatively safer. Does our present-value model in Section II generate such a reversal? The model’s implied expected return is given by:

$$\hat{E}_t(r_{t+1+j}) = (1 - \hat{\beta})\hat{\rho}_i^j(i_t^* - i_t) + \hat{\rho}^j\hat{\eta}_t, \quad (25)$$

where we use a “hat” on the expectation to emphasize that it is based on estimates from our model, rather than based on directly observable variables. Panel (a) in Figure 4 shows, for the currency portfolio, the slope coefficients and 90% confidence interval of the following regression:

$$\hat{E}_t(r_{t+1+j}) = a + b(i_t^* - i_t) + u_{t+1+j}. \quad (26)$$

The figure depicts a positive slope coefficient of about two for monthly horizons. In line with our previous results, this indicates that currency risk premia are positively correlated with the interest rate differential (corresponding to the negative  $\beta$  in the Fama regressions). However, the slope coefficient weakens with the horizon and becomes negative after 18 months. This result is strikingly similar to Figure 2 in Engel (2016), although the model we use to generate expected returns differs from the one he considers.

Panel (b) of Figure 4 sheds light on why our model can replicate the changing relationship between currency risk premia and the interest rate differential. We plot the covariance between currency risk premia and the interest rate differential, which mirrors the slope coefficient in Panel (a). We also plot two components of this covariance. The first component equals  $(1 - \hat{\beta})\hat{\rho}_i^j\text{Var}(i_t^* - i_t)$  and captures the covariance attributable to the interest rate differential. This component is always positive as  $\hat{\rho}_i > 0$  and decays towards zero as the horizon increases. The second component equals  $\hat{\rho}_\eta^j\text{Cov}(i_t^* - i_t, \hat{\eta}_t)$  and captures the covariance between the interest rate differential and the PPP risk premium. This component is negative and it also decays towards zero as the horizon increases. (Recall that Table IV reported an estimated correlation between interest rate differential shocks and PPP shocks of  $-0.53$ .)

As the PPP component of expected returns is much more persistent than the interest rate component (i.e.,  $\hat{\rho}_\eta > \hat{\rho}_i$ ), the negative PPP effect eventually dominates the positive interest rate effect on currency risk premia.

Our empirical results therefore suggest that not only should asset pricing models feature two risk premia, but also that these two risk premia should be negatively correlated, and that the PPP risk premium should be more persistent than the interest rate differential. These conditions are necessary to reproduce Engel's (2016) finding in our present-value model. Interestingly, these conditions are related, but quantitatively distinct, from the required conditions to obtain a positive covariance between the real exchange rate and interest rate differentials. For  $t \rightarrow \infty$ , Engel's finding can be expressed as:

$$\text{Cov}(\mathbb{E}_t \sum_{j=1}^{\infty} r_{t+j}, i_t^* - i_t) < 0. \quad (27)$$

From (22), we have that:

$$\text{Cov}(q_t, i_t^* - i_t) = \text{Cov}(\mathbb{E}_t \sum_{j=1}^{\infty} (i_{t+j-1}^* - i_{t+j-1}), i_t^* - i_t) - \text{Cov}(\mathbb{E}_t \sum_{j=1}^{\infty} r_{t+j}, i_t^* - i_t), \quad (28)$$

which is positive if

$$\text{Cov}(\mathbb{E}_t \sum_{j=1}^{\infty} r_{t+j}, i_t^* - i_t) < \text{Cov}(\mathbb{E}_t \sum_{j=1}^{\infty} (i_{t+j-1}^* - i_{t+j-1}), i_t^* - i_t). \quad (29)$$

We recognize Engel's result (27) on the left-hand side of (29). As this term is negative, a sufficient condition for (29) to hold is that its right-hand side is non-negative. This term depends on the long-term autocorrelations of interest rate differentials. It is positive when interest rate differentials follow an AR(1) process, as our present-value model assumes, but note that this needs not be the case in the data. In general, both condition (27) and our finding of a positive covariance between the real exchange rate and the interest rate differential may imply or be implied by the other. Empirical researchers may find the latter

more appealing as real exchange rates and interest rate differentials are observable, while currency risk premia need to be constructed by the researcher. In any case, we recommend that calibrated exchange rate models match both moments, in additions to other moments of the real exchange rate.

*F. Why does the real exchange rate move over time?*

Our analysis so far shows that a PPP risk premium is required to understand fluctuations in the real exchange rate and shows that the premium indeed contains useful information regarding future currency returns. We next gauge the relative importance of PPP versus interest rate differentials to understand fluctuations in real exchange rates and returns. Recall that the real exchange rate can be expressed as an infinite sum of future interest rate differentials minus an infinite sum of future PPP deviations; see (12). We derive variance decompositions of both the real exchange rate and unexpected currency returns. As is common in the asset pricing literature, we decompose a price—the real exchange rate—in terms of cash flow shocks and discount rate shocks. The objective is to understand the magnitude of each component. To make an analogy with the stock market literature, if the real exchange rate behaves like a scaled price ratio such as the dividend yield, we expect discount rate shocks to explain most of its variation. We can rewrite (12) as

$$q_t = \frac{i_t^* - i_t}{1 - \rho_i} - (1 - \beta) \frac{i_t^* - i_t}{1 - \rho_i} - \frac{\eta_t}{1 - \rho_\eta} + \mu_q. \quad (30)$$

The first term on the right-hand side of (30) gives the present value of future interest rate differentials. This corresponds to the cash flow component of the real exchange rate. As shown in Section I, if UIP holds, the real exchange moves only to correct future interest rate differentials. The second and third terms correspond to the discount rate components stemming from interest rate differentials and PPP deviations, respectively. Taking the variance

of (30) gives:

$$\begin{aligned}
\text{Var}(q_t) &= A^2 \text{Var}(i_t^* - i_t) + B^2 \text{Var}(i_t^* - i_t) + C^2 \text{Var}(\eta_t) \\
&\quad - 2 A B \text{Var}(i_t^* - i_t) - 2 A C \text{Cov}(i_t^* - i_t, \eta_t) \\
&\quad + 2 B C \text{Cov}(i_t^* - i_t, \eta_t),
\end{aligned} \tag{31}$$

where  $A = \frac{1}{1-\rho_i}$ ,  $B = \frac{1-\beta}{1-\rho_i}$ , and  $C = \frac{1}{1-\rho_\eta}$ . The first term measures the variation in the real exchange rate due to cash flow shocks. The second and third terms represent the variation in the real exchange rate due to the two discount rate shocks. The remaining terms correspond to the covariation between these three components. On the second line, we collect the terms reflecting the covariance between expected future cash flows and the risk premium. The third line corresponds to the covariance between the two risk premium components (i.e., the interest rate differential and PPP). It is also interesting to decompose unexpected returns. The variance of (15) is:

$$\begin{aligned}
\text{Var}(\varepsilon_t^r) &= A^2 \text{Var}(\varepsilon_t^i) + \text{Var}(\varepsilon_t^\pi) + B^2 \text{Var}(\varepsilon_t^i) + C^2 \text{Var}(\varepsilon_t^\eta) \\
&\quad - 2 A B \text{Var}(\varepsilon_t^i) - 2 A C \text{Cov}(\varepsilon_t^i, \varepsilon_t^\eta) + 2 B \text{Cov}(\varepsilon_t^i, \varepsilon_t^\pi) + 2 C \text{Cov}(\varepsilon_t^\eta, \varepsilon_t^\pi) \\
&\quad + 2 B C \text{Cov}(\varepsilon_t^i, \varepsilon_t^\eta) \\
&\quad - 2 A \text{Cov}(\varepsilon_t^i, \varepsilon_t^\pi).
\end{aligned} \tag{32}$$

The first two terms capture the shocks of interest rate and inflation differentials (i.e., cash flow shocks). The next two terms capture interest rate differential and PPP shocks (i.e., discount rate shocks). The remaining terms capture covariances. The second line collects the covariances between cash flow (i.e., interest rate and inflation differentials) and discount rate (i.e., interest rate differential and PPP) shocks. The third and fourth lines capture the covariances between pure discount rate shocks and pure cash flow shocks, respectively.

Table VI presents the resulting decompositions. Panel A presents the results for the real exchange rate and Panel B presents the results for unexpected returns. In both cases, we

standardize each component by the variance of the real exchange rate or the variance of unexpected returns so that the decompositions sum to 100%.

The results reported in Panel A suggest that, in all countries, cash flow shocks never account for more than 5% of real exchange rate movements. Discount rates account for more than 100% of the real exchange rate variation. The bulk of this variation stems from PPP, while the share attributable to interest rate differential variation is never above 25%. Covariance terms are relatively small and generally negative. These results do not come as a surprise: there is mounting evidence that discount rate shocks dominate asset price fluctuations (Cochrane, 2011); Filipe and Maio (2016) and Balduzzi and Chiang (2017) document similar reduced-form evidence for the real exchange rate.

Panel B reports similar results for the decomposition of unexpected returns. Cash flow shocks are again small. Unexpected inflation shocks never represent more than 5% of unexpected currency returns. The share of discount rate shocks is often well above 100%, which is compensated for by relatively larger negative covariance terms. PPP shocks again account for most of the unexpected returns, although the share attributable to interest rate differential shocks is now higher.

## V. Implications for asset pricing models

We have highlighted three key observations that have been considered separately in the literature, namely that the real exchange rate appreciates with the interest rate differential, that the real exchange rate predicts currency returns, and that the positive relationship between currency risk premia and the interest rate differential reverses over longer horizons. We next discuss the implications of our empirical findings in the context of complete market asset pricing models.

We first outline the implications under no-arbitrage conditions. We then link our reduced-form, present-value approach to the long-run risk model of Bansal and Yaron (2004), and in particular its application to foreign exchange markets (e.g., Bansal and Shaliastovich, 2013;

Backus et al., 2013). We choose this class of models to illustrate our findings because of its popularity and because Engel (2016) shows that extent versions of the model struggle to reconcile the empirical reversal above. Our empirical findings suggest a slightly different dynamics for consumption, which we discuss below.<sup>7</sup>

Under no-arbitrage conditions, there exists a stochastic discount factor (SDF) that prices any payoff. Let  $m_t$  and  $m_t^*$  denote the log SDFs in the domestic and foreign countries, respectively. We assume that markets are complete so the SDFs are unique; we further assume that the log SDFs are normally distributed. As discussed in Backus et al. (2001), a change in the exchange rate can then be written as the difference between the two SDFs:

$$s_{t+1} - s_t = m_{t+1}^* - m_{t+1}, \quad (33)$$

Moreover, the domestic and foreign interest rates are given by:

$$i_t = -E_t(m_{t+1}) - \frac{1}{2}\text{Var}_t(m_{t+1}), \quad (34)$$

$$i_t^* = -E_t(m_{t+1}^*) - \frac{1}{2}\text{Var}_t(m_{t+1}^*), \quad (35)$$

so the interest rate differential can be written:

$$i_t^* - i_t = E_t(m_{t+1}) - E_t(m_{t+1}^*) + \frac{1}{2} [\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*)]. \quad (36)$$

From (33), the expected change in the exchange rate is:

$$E_t(s_{t+1} - s_t) = E_t(m_{t+1}^*) - E_t(m_{t+1}). \quad (37)$$

---

<sup>7</sup>Bacchetta and Van Wincoop (2010) propose a model with infrequent portfolio rebalancing that accounts for the reversal of the relationship between currency risk premia and the interest rate differential; Engel (2016) and Valchev (2016) propose models with a liquidity premium and convenience yields, respectively, to explain the reversal; Itskhoki and Mukhin (2017) propose a dynamic general equilibrium model with a financial sector and UIP shocks to produce the reversal.

Substituting the expressions for the interest rate differential gives:

$$E_t(s_{t+1} - s_t) = i_t - i_t^* + \frac{1}{2} [\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*)], \quad (38)$$

or, equivalently,

$$E_t(r_{t+1}) = \frac{1}{2} [\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*)]. \quad (39)$$

The expected log excess return on a currency thus equals half the difference in the variances of the SDFs. UIP implies that this term equals zero. We next link these no-arbitrage implications to specific models of the SDFs. For simplicity, we abstract from inflation, so there is no difference between real and nominal SDFs.

A common assumption in the literature is that the two components  $E_t(m_{t+1}^*) - E_t(m_{t+1})$  and  $\text{Var}_t(m_{t+1}) - \text{Var}_t(m_{t+1}^*)$  are driven by the same shocks. For example, Verdelhan (2010) considers the habit formation model of Campbell and Cochrane (1999) in an international setting, and provides an argument for UIP deviations. In his model, both components are driven by the difference in the log surplus consumption ratio, which generates a perfect correlation between currency risk premia and interest rate differentials.

We argue that multiple sources of risk premia are necessary to generate a disconnect between currency risk premia and interest rate differentials. In addition, these sources must be negatively correlated and have with different persistence. We illustrate our point in an international version of the long-run risk model. As is common in this model class, we consider a two-country economy in which consumption growth contains a small and persistent component (its “long-run risk”), and in which both consumption growth  $\Delta c_t = \ln(C_t/C_{t-1})$  and the long-run risk component  $x_t$  exhibit stochastic volatility:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \epsilon_{t+1}^c, \quad (40)$$

$$x_{t+1} = \rho_x x_t + \varsigma_t \epsilon_{t+1}^x, \quad (41)$$

$$\sigma_{t+1}^2 = (1 - \rho_\sigma) \sigma_t^2 + \rho_\sigma \sigma_t^2 + \sigma_\sigma \epsilon_{t+1}^\sigma + \phi_\sigma \epsilon_{t+1}^\varsigma, \quad (42)$$

$$\varsigma_{t+1}^2 = (1 - \rho_\varsigma) \varsigma_t^2 + \rho_\varsigma \varsigma_t^2 + \sigma_\varsigma \epsilon_{t+1}^\varsigma + \phi_\varsigma \epsilon_{t+1}^\sigma. \quad (43)$$

We assume that all shocks are IID standard normal and that all  $\rho$  coefficients are between 0 and 1. We assume that all parameters are identical in the domestic and foreign countries, and that countries only differ in their volatility processes (indicated by \* in the foreign country). These dynamics are very similar to the models described in Backus et al. (2013) and Engel (2016). Our only departure is that we allow the volatilities to be cross-correlated. This assumption is sufficient to reproduce the observations that we document.

We assume that both countries are populated by representative agents with recursive preferences as in Epstein and Zin (1989):

$$V_t = \left[ (1 - \delta)C_t^{1-1/\psi} + \delta E_t(V_{t+1}^{1-\gamma})^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \quad (44)$$

where  $\delta$  is the time discount factor,  $\gamma \neq 1$  is the relative risk aversion, and  $\psi$  is the elasticity of intertemporal substitution. We consider a special case where  $\psi = 1$ , which generates similar dynamics for currency risk premia and interest rate differentials as our present-value model.<sup>8</sup>

We show in Appendix B that the domestic SDF is given by:

$$m_{t+1} = -\frac{(1-\gamma)^2}{2}\sigma_t^2 - \frac{(1-\gamma)^2}{2}\omega_x^2\varsigma_t^2 - \gamma\sigma_t\epsilon_{t+1}^c + (1-\gamma)\omega_x\varsigma_t\epsilon_{t+1}^\varsigma + \Xi_{t+1}, \quad (45)$$

where  $\omega_x = \delta/(1 - \delta\rho_x)$  and  $\Xi_{t+1}$  groups terms that are not time-varying or that do not affect both the conditional means and variances of the SDF. The discount factor that price assets in the foreign currency is analogous, but depends on the foreign realizations of the consumption volatilities.

Using (36) and (39), the currency risk premium and the interest rate differential are given

---

<sup>8</sup>Engel (2016) shows that a similar model obtains with common dynamics but where the representative agent in the foreign country has a different risk aversion than the domestic agent.



by:

$$\mathbb{E}_t(r_{t+1}) = A_{r\sigma}(\sigma_t^2 - \sigma_t^{*2}) + A_{r\zeta}(\zeta_t^2 - \zeta_t^{*2}), \quad (46)$$

$$i_{t+1}^* - i_{t+1} = A_{i\sigma}(\sigma_t^2 - \sigma_t^{*2}), \quad (47)$$

where  $A_{r\sigma} = \gamma^2/2$ ,  $A_{r\zeta} = (\gamma - 1)^2\omega_x^2/2$ , and  $A_{i\sigma} = (2\gamma - 1)/2$ . Note that all  $A$  coefficient are positive provided that the risk aversion is large enough ( $\gamma > 1/2$ ). Also note that the currency risk premium can be expressed as:

$$\mathbb{E}_t(r_{t+1}) = \frac{A_{r\sigma}}{A_{i\sigma}}(i_{t+1}^* - i_{t+1}) + A_{r\zeta}(\zeta_t^2 - \zeta_t^{*2}). \quad (48)$$

For  $\gamma > 1/2$ , the model delivers a positive partial correlation between the currency risk premium and the interest rate differential. Hence, it qualitatively provides a solution to the UIP deviations, as previously noted in the literature.<sup>9</sup> The currency risk premium also depends on a second term, a “missing risk premium.” The model therefore corresponds to the dynamics of our present-value model.

We next show that this model can yield a positive covariance between the real exchange rate and the interest rate differential, as well as Engel’s (2016) findings. The deviation of the real exchange rate from its long-run mean is given by:

$$\begin{aligned} q_t - \mu_q &= \sum_{j=1}^{\infty} \mathbb{E}_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^{\infty} \mathbb{E}_t(r_{t+j}) \\ &= (A_{i\sigma} - A_{r\sigma}) \frac{\sigma_t^2 - \sigma_t^{*2}}{1 - \rho_\sigma} - A_{r\zeta} \frac{(\zeta_t^2 - \zeta_t^{*2})}{1 - \rho_\zeta}. \end{aligned} \quad (49)$$

It is straightforward to see that  $A_{i\sigma} - A_{r\sigma} < 0$ . That is, the real exchange rate depreciates when domestic consumption volatility increases more than foreign consumption volatility.

---

<sup>9</sup>See, e.g., Bansal and Shaliastovich (2013), Colacito and Croce (2013), and Backus et al. (2013). Additional explanations for deviations from UIP include expectation errors (Froot and Frankel, 1989), partially segmented financial markets (Maggiore and Gabaix, 2015), and infrequent portfolio decisions (Bacchetta and Van Wincoop, 2010). Engel (2014) provides an extensive review of foreign exchange rate determination and deviations from UIP.

Equivalently, as the consumption volatility differential is perfectly correlated with the interest rate differential, the partial correlation between the real exchange rate and the interest rate differential is negative. This happens in our present-value model when the  $\beta$  coefficient is sufficiently negative. The covariance between the real exchange rate and the interest rate differential in (22) is given by:

$$A_{i\sigma}(A_{i\sigma} - A_{r\sigma}) \frac{\text{Var}(\sigma_t^2 - \sigma_t^{*2})}{1 - \rho_\sigma} - A_{r\varsigma} A_{i\sigma} \frac{\text{Cov}(\sigma_t^2 - \sigma_t^{*2}, \varsigma_t^2 - \varsigma_t^{*2})}{1 - \rho_\varsigma}. \quad (50)$$

When the risk aversion is large enough, the first term is negative, while the sign of the second term depends on the covariance between the volatilities. A necessary condition to obtain a positive covariance between the real exchange rate and the interest rate differential is therefore a negative covariance between the underlying volatility shocks that affect the economies. We also see that the relative magnitude of the two terms depends on the relative persistence of the volatility shocks. When  $\rho_\varsigma > \rho_\sigma$ , the covariance term will be proportionally larger, all else equal, which helps obtaining a positive covariance, as we find in the data.

As we discuss in Section IV.E, Engel (2016) shows that the covariance between currency risk premia and the interest rate differential reverses over the horizon. One way to express this is to consider the following covariances:

$$\text{Cov}(E_t r_{t+1}, i_t^* - i_t) = A_{r\sigma} A_{i\sigma} \text{Var}(\sigma_t^2 - \sigma_t^{*2}) + A_{r\varsigma} A_{i\sigma} \text{Cov}(\sigma_t^2 - \sigma_t^{*2}, \varsigma_t^2 - \varsigma_t^{*2}), \quad (51)$$

$$\text{Cov}(E_t \sum_{j=1}^{\infty} r_{t+j}, i_t^* - i_t) = A_{r\sigma} A_{i\sigma} \frac{\text{Var}(\sigma_t^2 - \sigma_t^{*2})}{1 - \rho_\sigma} + A_{r\varsigma} A_{i\sigma} \frac{\text{Cov}(\sigma_t^2 - \sigma_t^{*2}, \varsigma_t^2 - \varsigma_t^{*2})}{1 - \rho_\varsigma}. \quad (52)$$

Engel (2016) finds empirically that the first covariance is positive and the second covariance is negative, and notes that a baseline long-run risk model cannot accommodate the second covariance. This is because he follows the extant literature and assumes that volatility shocks are uncorrelated, so that  $\text{Cov}(\sigma_t^2 - \sigma_t^{*2}, \varsigma_t^2 - \varsigma_t^{*2}) = 0$ . When this covariance is negative, both (51) and (52) may be positive or negative. In the context of our present-value model, we have shown that the two risk premium components must be negatively correlated, and

that the PPP risk premium must be more persistent than the interest rate differential to accommodate the reversal in the covariance. In the context of the long-run risk model here, we need  $\text{Cov}(\sigma_t^2 - \sigma_t^{*2}, \zeta_t^2 - \zeta_t^{*2})$  to be negative, but small enough to accommodate a positive (51). For (52) to be negative, we need the variance term to be multiplied by a smaller amount than the covariance term. This requires  $\rho_\zeta > \rho_\sigma$ . Remarkably, as previously noted in Section IV.E, these two conditions resemble the conditions needed to obtain a positive covariance between the real exchange rate and the interest rate differential.

## VI. Conclusion

We ask whether two maintained assumptions about exchange rates are sufficient to quantify movements in exchange rates. The first assumption is that expected currency returns depend on the difference between foreign and domestic interest rates. The second assumption is that the real exchange rate is stationary, so that the prices of foreign goods and services cannot forever deviate from the prices of domestic goods and services. We find that a present-value model with these two assumptions cannot quantify how much the real exchange rate moves over time. We extract the component of the real exchange rate that complements the interest rate differential. This component captures deviations from purchasing power parity and increases the R-squared in predictive regressions of currency returns by more than 30%. The predictability holds in an out-of-sample evaluation and increases substantially with the investment horizon. Our empirical results shed light on the desirable properties of currency risk premia that asset pricing models must accommodate and, moreover, help us pose challenges for standard asset pricing models.

## Appendix A. The real exchange rate

This appendix provides a derivation of (8) and (12). Start with the return definition:

$$r_{t+1} = q_{t+1} - q_t - (\pi_{t+1}^* - \pi_{t+1}) + (i_t^* - i_t). \quad (\text{A1})$$

Rewrite it in terms of the real exchange rate:

$$q_t = (i_t^* - i_t) - (\pi_{t+1}^* - \pi_{t+1}) - r_{t+1} + q_{t+1}. \quad (\text{A2})$$

Iterate forward and take conditional expectations:

$$q_t = \sum_{j=1}^T \text{E}_t[(i_{t+j-1}^* - i_{t+j-1}) - (\pi_{t+j}^* - \pi_{t+j}) - r_{t+j}] + q_{t+T}. \quad (\text{A3})$$

Let  $T \rightarrow \infty$  and assume that long-run PPP holds,  $\lim_{j \rightarrow \infty} \text{E}_t(q_{t+j}) = \mu_q$ , giving:

$$q_t - \mu_q = \sum_{j=1}^{\infty} \text{E}_t[(i_{t+j-1}^* - i_{t+j-1}) - (\pi_{t+j}^* - \pi_{t+j}) - r_{t+j}]. \quad (\text{A4})$$

This corresponds to (5); then insert (7), (6), (9), and (10) to obtain:

$$\begin{aligned} q_t - \mu_q &= \sum_{j=1}^{\infty} \text{E}_t[(i_{t+j-1}^* - i_{t+j-1}) - (\pi_{t+j}^* - \pi_{t+j}) - \alpha - (1 - \beta)(i_{t+j-1}^* - i_{t+j-1}) - \eta_{t+j-1} - \varepsilon_{t+j}^r] \\ &= \sum_{j=1}^{\infty} \text{E}_t[\beta(i_{t+j-1}^* - i_{t+j-1}) - \mu_\pi - \varepsilon_{t+j}^\pi - \alpha - \eta_{t+j-1} - \varepsilon_{t+j}^r] \\ &= \sum_{j=1}^{\infty} \text{E}_t[\beta(i_{t+j-1}^* - i_{t+j-1} - \mu_i) + \beta\mu_i - \mu_\pi - \alpha - \eta_{t+j-1}]. \end{aligned} \quad (\text{A5})$$

Note that  $E(q_t) = \mu_q$  requires that  $\alpha = \beta\mu_i - \mu_\pi$ , which is a condition considered in this paper. Finally, use that for a given process  $y_{t+1} - a = b(y_t - a) + e_{t+1}$ ,  $E_t(y_{t+j}) = a + b^j(y_t - a)$ :

$$\begin{aligned} q_t - \mu_q &= \sum_{j=1}^{\infty} [\beta\rho_i^{j-1}(i_t^* - i_t - \mu_i) - \rho_\eta^{j-1}\eta_t] \\ &= \beta\frac{i_t^* - i_t - \mu_i}{1 - \rho_i} - \frac{\eta_t}{1 - \rho_\eta}, \end{aligned} \tag{A6}$$

which corresponds to (12) in the main text. A special case is when there is no complementing risk premium  $\eta_t$ , which corresponds to (8) in the main text.

## Appendix B. A long-run risk model

This appendix discusses the long-run risk model that produces (45). We consider a two-country exchange economy. Both countries are populated by representative agents with recursive preferences given by (44). To focus on growth dynamics, we normalize the value function (44) by the consumption level; let  $vc_t = \ln(V_t/C_t)$ . When  $\psi \rightarrow 1$ , the log of the normalized value function equals (Hansen et al., 2008):

$$vc_t = \frac{\delta}{1 - \gamma} \ln \left\{ E_t \left[ e^{(1-\gamma)(vc_{t+1} + \Delta c_{t+1})} \right] \right\}. \tag{B1}$$

Assuming log-normality, we can simplify this to:

$$vc_t = \delta \left[ E_t (vc_{t+1} + \Delta c_{t+1}) + \frac{(1 - \gamma)}{2} \text{Var}_t (vc_{t+1} + \Delta c_{t+1}) \right]. \tag{B2}$$

The log stochastic discount factor (SDF), using our log-normality assumption, is

$$\begin{aligned} m_{t+1} &= \log \delta - \Delta c_{t+1} + (1 - \gamma) [vc_{t+1} + \Delta c_{t+1} - E_t(vc_{t+1} + \Delta c_{t+1})] \\ &\quad - \frac{(1 - \gamma)^2}{2} \text{Var}_t(vc_{t+1} + \Delta c_{t+1}). \end{aligned} \tag{B3}$$

Recall that we assume the dynamics for consumption in (40)–(43).

Conjecture the value function:

$$vc_t = \omega_x x_t + \omega_\sigma \sigma_t^2 + \omega_\varsigma \varsigma_t^2 + \text{constant terms}, \quad (\text{B4})$$

where we ignore constant terms from now on. Write:

$$\begin{aligned} vc_{t+1} + \Delta c_{t+1} &= \omega_x (\rho_x x_t + \varsigma_t \epsilon_{t+1}^x) + \omega_\sigma (\rho_\sigma \sigma_t^2 + \sigma_\sigma \epsilon_{t+1}^\sigma + \phi_\sigma \epsilon_{t+1}^\varsigma) \\ &\quad + \omega_\varsigma (\rho_\varsigma \varsigma_t^2 + \sigma_\varsigma \epsilon_{t+1}^\varsigma + \phi_\varsigma \epsilon_{t+1}^\sigma) + x_t + \sigma_t \epsilon_{t+1}^c, \end{aligned} \quad (\text{B5})$$

and compute:

$$E_t(vc_{t+1} + \Delta c_{t+1}) = (1 + \omega_x \rho_x) x_t + \omega_\sigma \rho_\sigma \sigma_t^2 + \omega_\varsigma \rho_\varsigma \varsigma_t^2 \quad (\text{B6})$$

$$\text{Var}_t(vc_{t+1} + \Delta c_{t+1}) = \sigma_t^2 + \omega_x^2 \varsigma_t^2. \quad (\text{B7})$$

Substituting back into (B2) gives:

$$vc_t = \delta \left[ (1 + \omega_x \rho_x) x_t + \omega_\sigma \rho_\sigma \sigma_t^2 + \omega_\varsigma \rho_\varsigma \varsigma_t^2 + \frac{(1 - \gamma)}{2} (\sigma_t^2 + \omega_x^2 \varsigma_t^2) \right]. \quad (\text{B8})$$

We then solve for the value-function parameters by matching coefficients:

$$\omega_x = \frac{\delta}{1 - \delta \rho_x}, \quad \omega_\sigma = \frac{(1 - \gamma)}{2} \frac{\delta}{1 - \delta \rho_\sigma}, \quad \omega_\varsigma = \frac{(1 - \gamma)}{2} \frac{\omega_x^2 \delta}{1 - \delta \rho_\varsigma}.$$

We then obtain an operational expression for the log SDF:

$$\begin{aligned} m_{t+1} &= -\frac{(1 - \gamma)^2}{2} \sigma_t^2 - \frac{(1 - \gamma)^2}{2} \omega_x^2 \varsigma_t^2 - \gamma \sigma_t \epsilon_{t+1}^c + (1 - \gamma) \omega_x \varsigma_t \epsilon_{t+1}^\varsigma \\ &\quad - x_t + (1 - \gamma) (\omega_\sigma \sigma_\sigma \epsilon_{t+1}^\sigma + \omega_\sigma \phi_\sigma \epsilon_{t+1}^\varsigma + \omega_\varsigma \sigma_\varsigma \epsilon_{t+1}^\varsigma + \omega_\varsigma \phi_\varsigma \epsilon_{t+1}^\sigma), \end{aligned} \quad (\text{B9})$$

which corresponds to (45) in the main text.

## REFERENCES

- Asness, Clifford S., Tobias J. Moskowitz, and Lasse Heje Pedersen, 2013, Value and momentum everywhere, *Journal of Finance* 68, 929–985.
- Bacchetta, Philippe, and Eric Van Wincoop, 2010, Infrequent portfolio decisions: A solution to the forward discount puzzle, *American Economic Review* 100, 870–904.
- Backus, David K., Silverio Foresi, and Chris I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* 56, 279–304.
- Backus, David K., Federico Gavazzoni, Chris Telmer, and Stanley E. Zin, 2013, Monetary policy and the uncovered interest rate parity puzzle, NBER working paper.
- Balduzzi, Pierluigi, and I-Hsuan Ethan Chiang, 2017, Real exchange rates and currency risk premia, Working paper.
- Bansal, Ravi, and Ivan Shaliastovich, 2013, A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies* 26, 1–44.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Barroso, Pedro, and Pedro Santa-Clara, 2015, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis* 50, 1037–1056.
- Binsbergen, Jules H., and Ralph S. J. Koijen, 2010, Predictive regressions: A present-value approach, *Journal of Finance* 65, 1439–1471.
- Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw, 2016, New evidence on the forward premium puzzle, *Journal of Financial and Quantitative Analysis* 51, 875–897.
- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, 2011, Carry trade and momentum in currency markets, *Annual Review of Financial Economics* 3, 511–535.
- Burstein, Ariel, and Gita Gopinath, 2014, International prices and exchange rates, in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics, Volume 4*, 391–451 (Elsevier, Amsterdam).
- Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.
- Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.
- Campbell, John Y., and Samuel B. Thompson, 2008, Predicting excess stock returns out of sample: Can anything beat the historical average?, *Review of Financial Studies* 21, 1509–1531.
- Chernov, Mikhail, Jeremy Graveline, and Irina Zviadadze, 2016, Crash risk in currency returns, forthcoming in *Journal of Financial and Quantitative Analysis*.
- Chinn, Menzie D., and Guy Meredith, 2004, Monetary policy and long-horizon uncovered interest parity, *IMF Staff Papers* 51, 409–430.

- Clark, Todd E., and Kenneth D. West, 2007, Approximately normal tests for equal predictive accuracy in nested models, *Journal of Econometrics* 138, 291–311.
- Cochrane, John H., 2008, The dog that did not bark: A defense of return predictability, *Review of Financial Studies* 21, 1533–1575.
- Cochrane, John H., 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.
- Colacito, Riccardo, and Mariano M. Croce, 2011, Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–181.
- Colacito, Riccardo, and Mariano M. Croce, 2013, International asset pricing with recursive preferences, *Journal of Finance* 68, 2651–2686.
- Colacito, Riccardo, Mariano M. Croce, Federico Gavazzoni, and Robert C. Ready, 2017, Currency risk factors in a recursive multi-country economy, Working paper.
- Della Corte, Pasquale, Tarun Ramadorai, and Lucio Sarno, 2016, Volatility risk premia and exchange rate predictability, *Journal of Financial Economics* 120, 21–40.
- Della Corte, Pasquale, Lucio Sarno, and Giulia Sestieri, 2012, The predictive information content of external imbalances for exchange rate returns: How much is it worth?, *Review of Economics and Statistics* 94, 100–115.
- Eichenbaum, Martin, Benjamin K. Johannsen, and Sergio Rebelo, 2017, Monetary policy and the predictability of nominal exchange rates, Working paper.
- Engel, Charles, 2014, Exchange rates and interest parity, in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics, Volume 4*, 453–522 (Elsevier, Amsterdam).
- Engel, Charles, 2016, Exchange rates, interest rates, and the risk premium, *American Economic Review* 106, 436–474.
- Engel, Charles, and Kenneth D. West, 2005, Exchange rates and fundamentals, *Journal of Political Economy* 113, 485–517.
- Engel, Charles, and Kenneth D. West, 2010, Global interest rates, currency returns, and the real value of the dollar, *American Economic Review, Papers & Proceedings* 100, 562–567.
- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Fama, Eugene F., 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–338.
- Fama, Eugene F., and Kenneth R. French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.
- Farhi, Emmanuel, and Xavier Gabaix, 2016, Rare disasters and exchange rates, *Quarterly Journal of Economics* 131, 1–52.
- Filipe, Sara Ferreira, and Paulo Maio, 2016, What drives exchange rates? Reassessing



- currency return predictability, Working paper.
- Froot, Kenneth A., and Jeffrey A. Frankel, 1989, Forward discount bias: Is it an exchange risk premium?, *Quarterly Journal of Economics* 104, 139–161.
- Froot, Kenneth A., and Tarun Ramadorai, 2005, Currency returns, intrinsic value, and institutional-investor flows, *Journal of Finance* 60, 1535–1566.
- Gourinchas, Pierre-Olivier, and Hélène Rey, 2007, International financial adjustment, *Journal of Political Economy* 115, 665–703.
- Hansen, Lars Peter, John C. Heaton, and Nan Li, 2008, Consumption strikes back? Measuring long-run risk, *Journal of Political Economy* 116, 260–302.
- Hansen, Lars Peter, and Robert J. Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, *Journal of Political Economy* 88, 829–853.
- Heyerdahl-Larsen, Christian, 2014, Asset prices and real exchange rates with deep habits, *Review of Financial Studies* 27, 3280–3317.
- Hsieh, David A., 1984, Tests of rational expectations and no risk premium in forward exchange markets, *Journal of International Economics* 17, 173–184.
- Itskhoki, Oleg, and Dmitry Mukhin, 2017, Exchange rate disconnect in general equilibrium, NBER working paper.
- Jordà, Òscar, and Alan M. Taylor, 2012, The carry trade and fundamentals: Nothing to fear but FEER itself, *Journal of International Economics* 88, 74–90.
- Kremens, Lukas, and Ian Martin, 2017, The quanto theory of exchange rates, Working paper.
- Londono, Juan M., and Hao Zhou, 2017, Variance risk premiums and the forward premium puzzle, *Journal of Financial Economics* 124, 415–440.
- Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2014, Countercyclical currency risk premia, *Journal of Financial Economics* 111, 527–553.
- Lustig, Hanno, Andreas Stathopoulos, and Adrien Verdelhan, 2016, Nominal exchange rate stationarity and long-term bond returns, Working paper.
- Maggiore, Matteo, and Xavier Gabaix, 2015, International liquidity and exchange rate dynamics, *Quarterly Journal of Economics* 130, 1369–1420.
- Menkhoff, Lukas, Lucio Sarno, Maik Schmelming, and Andreas Schrimpf, 2012, Currency momentum strategies, *Journal of Financial Economics* 106, 660–684.
- Menkhoff, Lukas, Lucio Sarno, Maik Schmelming, and Andreas Schrimpf, 2017, Currency value, *Review of Financial Studies* 30, 416–441.
- Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.
- Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

- Rogoff, Kenneth S., 1996, The purchasing power parity puzzle, *Journal of Economic Literature* 34, 647–668.
- Rossi, Barbara, 2013, Exchange rate predictability, *Journal of Economic Literature* 51, 1063–1119.
- Rytchkov, Oleg, 2012, Filtering out expected dividends and expected returns, *Quarterly Journal of Finance* 2, 1250012.
- Stambaugh, Robert F., 1999, Predictive regressions, *Journal of Financial Economics* 54, 375–421.
- Stathopoulos, Andreas, 2017, Asset prices and risk sharing in open economies, *Review of Financial Studies* 30, 363–415.
- Valchev, Rosen, 2016, Bond convenience yields and exchange rate dynamics, Working paper.
- Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, *Journal of Finance* 65, 123–146.
- White, Halbert L., 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48, 817–838.

**Table I: Summary statistics**

		AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
Sample begins:		85.01	76.02	76.02	76.02	85.01	76.02	76.02	76.02	76.02	76.02
Sample ends:		15.12	15.12	15.12	15.12	15.12	15.12	15.12	15.12	15.12	15.12
$r_t$	Mean (%)	0.23	-0.00	-0.03	-0.05	0.45	0.09	0.01	-0.03	0.08	0.01
	S.D. (%)	3.48	2.01	3.20	3.32	3.64	3.09	3.19	3.51	3.00	2.40
	AC(1)	0.07	-0.05	0.02	0.07	0.00	0.03	0.08	0.01	0.09	0.05
$q_t - \mu_q$	Mean (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	S.D. (%)	19.23	11.97	15.63	18.71	19.61	13.30	18.85	16.49	11.35	11.68
	AC(1)	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.98	0.96	0.98
$i_t^* - i_t$	Mean (%)	0.26	0.07	-0.11	-0.24	0.35	0.19	0.15	-0.23	0.15	-0.00
	S.D. (%)	0.23	0.14	0.24	0.24	0.37	0.29	0.31	0.28	0.22	0.19
	AC(1)	0.89	0.80	0.88	0.88	0.87	0.81	0.76	0.87	0.89	0.88
$\pi_t^* - \pi_t$	Mean (%)	0.07	-0.00	-0.12	-0.18	0.07	0.03	0.03	-0.16	0.06	-0.05
	S.D. (%)	0.63	0.34	0.40	0.50	0.81	0.52	0.53	0.40	0.53	0.29
	AC(1)	-0.17	0.02	0.16	0.11	-0.11	0.17	0.18	0.25	0.09	0.25

The table presents means, standard deviations, and first-order autocorrelations for key variables:  $r_t$  is the log excess return for a US investor going long in a foreign currency;  $q_t$  is the log real exchange rate in US dollars per unit of foreign currency;  $i_t^* - i_t$  is the difference between foreign and US interest rates; and  $\pi_t^* - \pi_t$  is the difference between foreign and US inflation rates. Results are reported for Australia (AUD), Canada (CAD), Germany (EUR), Japan (JPY), New Zealand (NZD), Norway (NOK), Sweden (SEK), Switzerland (CHF), the UK (GBP), and an equally weighted average of the seven currencies with full coverage (“Portfolio”). The sample period is February 1976 to December 2015, except for AUD and NZD, which begin in January 1985.

**Table II: Predicting currency returns**

	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
One-month horizon										
$r_{t+1} = a + (1 - b)(i_t^* - i_t) + u_{t+1}$										
$b$	-0.913	-0.918	-0.514	-1.516	-0.965	-0.081	0.526	-0.790	-1.380	-0.921
(s.e.)	(0.650)	(0.561)	(0.777)	(0.596)	(0.471)	(0.695)	(0.750)	(0.713)	(0.790)	(0.779)
$R^2$	0.016	0.019	0.013	0.033	0.039	0.011	0.002	0.020	0.032	0.022
$r_{t+1} = a + (1 - b)(i_t^* - i_t) + cq_t + u_{t+1}$										
$b$	-1.104	-1.477	-0.693	-1.524	-0.782	-0.098	0.228	-1.296	-1.362	-1.103
(s.e.)	(0.677)	(0.660)	(0.770)	(0.599)	(0.468)	(0.680)	(0.785)	(0.744)	(0.794)	(0.750)
$c$	-0.016	-0.020	-0.018	-0.014	-0.017	-0.017	-0.013	-0.027	-0.030	-0.018
(s.e.)	(0.009)	(0.008)	(0.010)	(0.009)	(0.010)	(0.012)	(0.009)	(0.012)	(0.016)	(0.011)
$R^2$	0.023	0.031	0.021	0.040	0.047	0.016	0.007	0.035	0.045	0.029
One-year horizon										
$r_{t,t+12} = a + (1 - b)(i_t^* - i_t) + u_{t,t+12}$										
$b$	-14.961	-4.523	-13.125	-22.948	-12.923	-9.738	-6.421	-16.547	-14.549	-18.344
(s.e.)	(8.794)	(6.422)	(7.425)	(7.165)	(5.724)	(5.314)	(5.610)	(6.431)	(6.507)	(6.918)
$R^2$	0.085	0.012	0.071	0.175	0.131	0.069	0.029	0.134	0.092	0.134
$r_{t,t+12} = a + (1 - b)(i_t^* - i_t) + cq_t + u_{t,t+12}$										
$b$	-17.684	-9.945	-16.086	-23.659	-10.189	-9.853	-12.261	-22.801	-14.096	-21.366
(s.e.)	(8.338)	(6.255)	(6.984)	(6.677)	(5.427)	(5.018)	(5.461)	(5.996)	(5.686)	(6.485)
$c$	-0.217	-0.197	-0.292	-0.247	-0.256	-0.299	-0.268	-0.345	-0.471	-0.301
(s.e.)	(0.105)	(0.091)	(0.114)	(0.097)	(0.110)	(0.129)	(0.110)	(0.108)	(0.124)	(0.114)
$R^2$	0.191	0.110	0.194	0.282	0.252	0.170	0.144	0.295	0.307	0.254

This table presents the results of predictive regressions of the log excess currency return,  $r_{t+1}$ , on the lagged interest rate differential,  $i_t^* - i_t$ , and the lagged real exchange rate,  $q_t$ . The top panel presents results of predictions at the one-month horizon. The bottom panel presents results of predictions of one-year cumulative returns,  $r_{t,t+12} = \sum_{k=1}^{12} r_{t+k}$ . One-month horizon regressions are reported with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to three lags, in parentheses. One-year regressions are reported with Hansen and Hodrick (1980) corrected standard errors with twelve lags. The regressions include constant terms, though they are not reported.

**Table III: Real exchange rates and interest rate differentials**

	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
	$q_t = a + b(i_t^* - i_t) + u_t$									
$b$	1.49	-1.37	3.27	1.83	-0.79	2.68	2.71	2.67	1.07	4.68
(s.e.)	(3.82)	(1.03)	(1.64)	(1.50)	(1.12)	(0.99)	(0.71)	(1.49)	(2.06)	(1.48)
$R^2$	0.002	0.004	0.014	0.004	0.002	0.025	0.032	0.011	0.001	0.032

This table presents slope coefficients in regressions of the real exchange rates,  $q_t$ , on the interest rate differentials,  $i_t^* - i_t$ . We use a Cochrane–Orcutt procedure to account for autocorrelation of the error term. White (1980) standard errors, accounting for conditional heteroscedasticity, are reported in parentheses. The regressions include constant terms, though they are not reported.

**Table IV: Model estimation**

	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
$\rho_i$	0.893	0.804	0.881	0.883	0.866	0.806	0.757	0.874	0.886	0.878
(s.e.)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)
$\sigma_i$	0.104	0.084	0.113	0.113	0.181	0.174	0.201	0.136	0.104	0.089
(s.e.)	(0.03)	(0.01)	(0.02)	(0.01)	(0.05)	(0.03)	(0.04)	(0.02)	(0.01)	(0.01)
$\sigma_\pi$	0.626	0.338	0.403	0.501	0.781	0.520	0.531	0.401	0.530	0.291
(s.e.)	(0.06)	(0.03)	(0.03)	(0.04)	(0.16)	(0.04)	(0.04)	(0.03)	(0.05)	(0.02)
$\beta$	-1.295	-1.646	-0.820	-1.731	-0.894	-0.107	0.241	-1.658	-1.847	-1.292
(s.e.)	(3.53)	(0.94)	(1.10)	(1.51)	(3.44)	(0.63)	(0.59)	(2.99)	(2.45)	(1.08)
$\rho_\eta$	0.984	0.980	0.982	0.986	0.983	0.983	0.987	0.973	0.970	0.982
(s.e.)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.02)	(0.01)
$\sigma_\eta$	0.055	0.050	0.056	0.045	0.060	0.040	0.038	0.115	0.086	0.041
(s.e.)	(0.29)	(0.03)	(0.06)	(0.11)	(0.59)	(0.04)	(0.04)	(0.55)	(0.21)	(0.05)
$\rho_{i\pi}$	0.042	0.011	0.100	0.096	0.056	0.079	0.092	0.080	0.050	0.083
(s.e.)	(0.06)	(0.06)	(0.04)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.07)	(0.05)
$\rho_{\eta i}$	-0.394	-0.314	-0.362	-0.517	-0.303	-0.203	-0.135	-0.551	-0.537	-0.526
(s.e.)	(0.35)	(0.19)	(0.26)	(0.26)	(0.28)	(0.17)	(0.15)	(0.29)	(0.31)	(0.26)
$\rho_{\eta\pi}$	-0.139	-0.147	-0.109	-0.194	-0.250	-0.101	-0.144	-0.099	-0.147	-0.086
(s.e.)	(0.06)	(0.05)	(0.05)	(0.05)	(0.10)	(0.05)	(0.05)	(0.05)	(0.06)	(0.04)
$R^2$	0.023	0.030	0.020	0.040	0.045	0.015	0.006	0.035	0.045	0.029

This table presents estimates of the present-value model given by (6), (7), (9), (10), and (12). The parameters  $\rho_i$ ,  $\sigma_i$ , and  $\sigma_\pi$  relate to the dynamics of interest rate and inflation differentials and are estimated by OLS. The elasticity of the expected return with respect to the interest rate differential,  $\beta$ , and the persistence of the PPP risk premium,  $\rho_\eta$ , is estimated by regressing the currency return on the past interest rate differential and the past real exchange rate, and then by matching the coefficient to the restricted counterpart in (17). The remaining parameters are estimated by filtering out estimates of the latent  $\eta_t$  based on (10) and by computing the covariance matrix of the estimated residuals. Standard errors, obtained in a wild bootstrap, are reported in parentheses. The last row presents R-squared values for a predictive regression of currency returns on interest rate differentials and the extracted PPP risk premium,  $\hat{\eta}_t$ .

**Table V: Out-of-sample predictability**

	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
Out-of-sample predictive $R^2$										
Fama	-0.002	0.000	0.008	-0.036	-0.023	0.008	0.003	0.007	0.005	0.014
	[0.46]	[0.21]	[0.11]	[0.44]	[0.94]	[0.06]	[0.19]	[0.11]	[0.15]	[0.03]
Bivariate	0.001	0.001	0.015	-0.036	-0.014	0.011	0.006	0.022	0.021	0.021
	[0.21]	[0.17]	[0.04]	[0.34]	[0.40]	[0.04]	[0.06]	[0.01]	[0.03]	[0.01]
Present value	-0.005	0.001	0.014	-0.030	-0.020	0.011	0.006	0.023	0.021	0.021
	[0.25]	[0.14]	[0.04]	[0.28]	[0.34]	[0.04]	[0.14]	[0.01]	[0.02]	[0.01]
Sharpe ratio										
Fama	0.165	0.239	0.222	-0.026	-0.021	0.211	0.014	0.207	0.147	0.352
	[0.54]	[0.29]	[0.31]	[0.91]	[0.95]	[0.44]	[0.95]	[0.37]	[0.44]	[0.09]
Bivariate	0.262	0.215	0.352	0.023	0.129	0.316	0.197	0.534	0.377	0.455
	[0.29]	[0.33]	[0.11]	[0.92]	[0.67]	[0.21]	[0.38]	[0.02]	[0.01]	[0.04]
Present value	0.139	0.228	0.353	0.062	0.051	0.331	0.156	0.512	0.385	0.463
	[0.55]	[0.31]	[0.12]	[0.79]	[0.85]	[0.18]	[0.51]	[0.02]	[0.01]	[0.04]

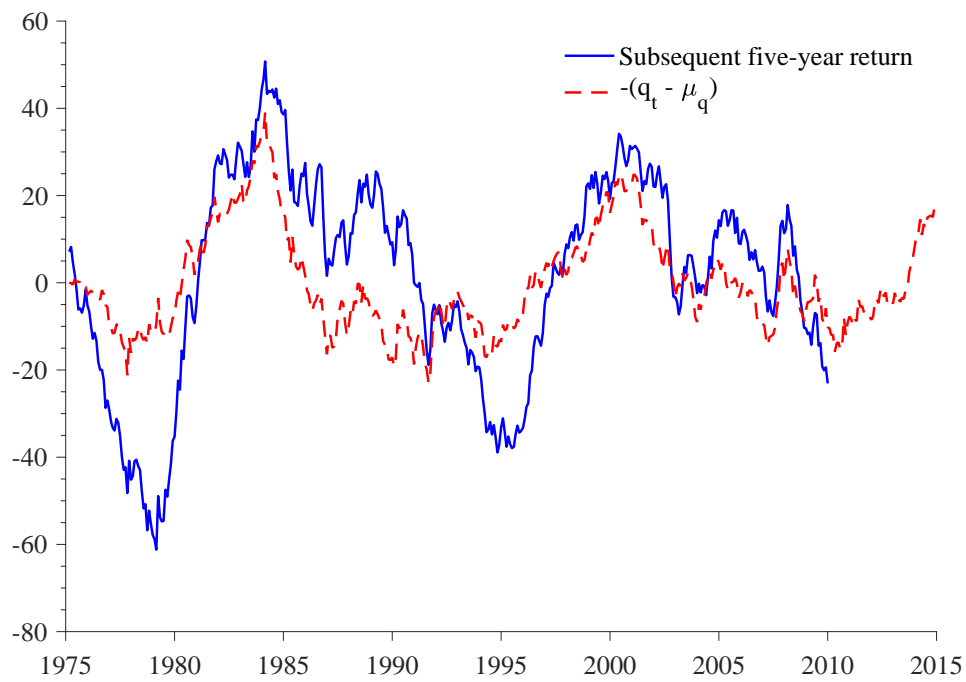
The top panel presents Campbell and Thompson (2008) out-of-sample  $R^2_{OS}$  for Fama regressions, unconstrained bivariate regressions that expand the Fama regression with the lagged real exchange rate, and predictions based on the present-value model. The out-of-sample exercise is performed for the 1996–2015 period. In brackets we report the  $p$ -values for a Clark and West (2007) test of the null hypothesis of equal predictive accuracy as a benchmark based on the historical average. The bottom panel reports annualized Sharpe ratios for a market-timing strategy based on an investor with mean-variance preferences and a one-month investment horizon. We report in brackets  $p$ -values for the null of a zero Sharpe ratio. Estimates of the mean returns and variances are computed using the method of moments. The standard errors for the Sharpe ratios are computed using the delta method with a Newey and West (1987) covariance matrix, accounting for conditional heteroscedasticity and serial correlation up to three lags.

**Table VI: Variance decompositions**

	AUD	CAD	EUR	JPY	NZD	NOK	SEK	CHF	GBP	Portfolio
Panel A: Real exchange rates										
Cash flows: interest rates	1.3	0.4	1.7	1.2	1.9	1.3	0.4	1.8	3.0	1.7
Risk premium: interest rates	6.6	2.6	5.5	9.0	6.9	1.6	0.3	12.9	24.6	8.9
Risk premium: PPP	106.3	107.7	104.2	103.8	96.5	100.1	98.8	119.0	109.7	108.3
Covariance: CF and RP	0.8	3.3	0.5	-2.3	-9.5	-2.0	4.2	4.8	-6.4	0.9
Covariance: RP	-14.9	-13.9	-11.9	-11.8	4.1	-0.9	-3.7	-38.5	-30.8	-19.8
Panel B: Unexpected returns										
Cash flows: interest rates	7.9	4.7	9.0	8.8	14.4	8.6	6.7	9.8	9.8	9.5
Cash flows: inflation	3.3	2.9	1.6	2.4	4.8	2.9	2.8	1.3	3.3	1.5
Risk premium: interest rates	41.8	33.0	29.8	65.5	51.9	10.5	3.9	69.5	79.2	49.8
Risk premium: PPP	119.9	113.4	113.6	139.4	116.9	101.7	100.1	141.6	142.8	132.4
Covariance: CF and RP	-16.6	-15.6	-11.2	-16.5	-39.9	-9.6	-7.4	-12.3	-20.3	-7.2
Covariance: RP	-55.8	-38.4	-42.1	-98.8	-47.2	-13.3	-5.3	-109.4	-114.2	-85.4
Covariance: CF	-0.4	-0.1	-0.8	-0.9	-0.9	-0.8	-0.8	-0.6	-0.6	-0.6

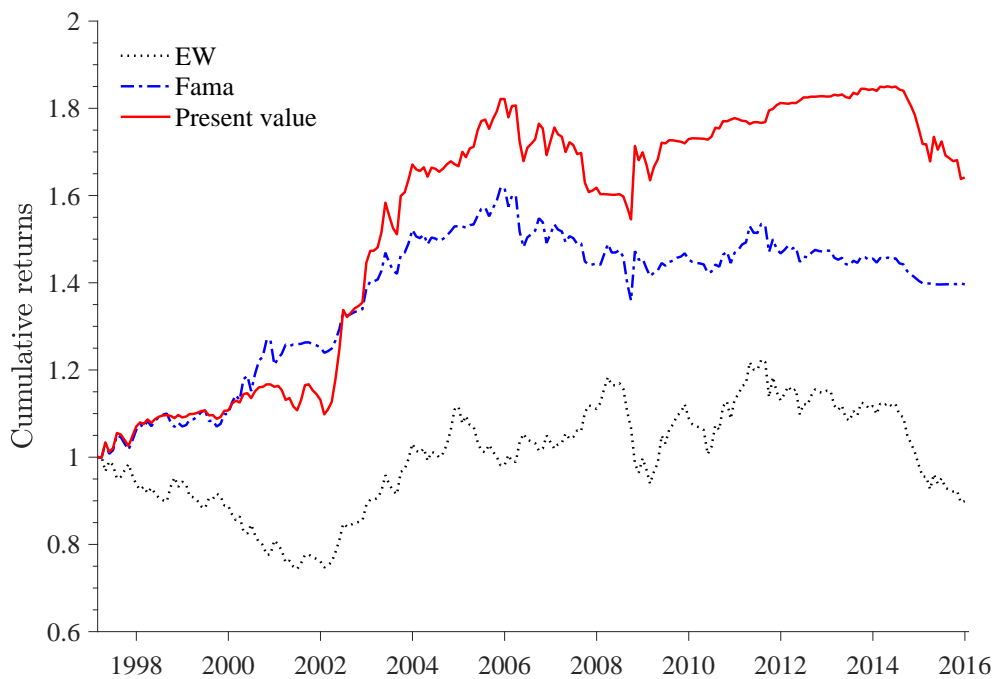
This table presents variance decompositions of the real exchange rates and the unexpected returns in terms of cash flow components (i.e., interest rate differentials and, for unexpected returns, inflation differentials as well) and risk premium components (i.e., interest rate differentials and PPP deviations). CF and RP refer to cash flows and risk premium, respectively.





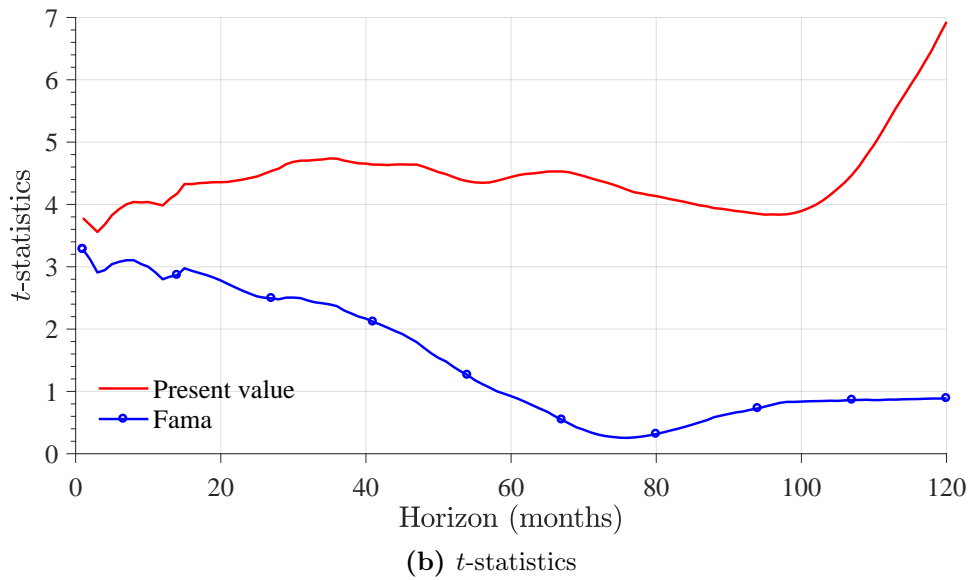
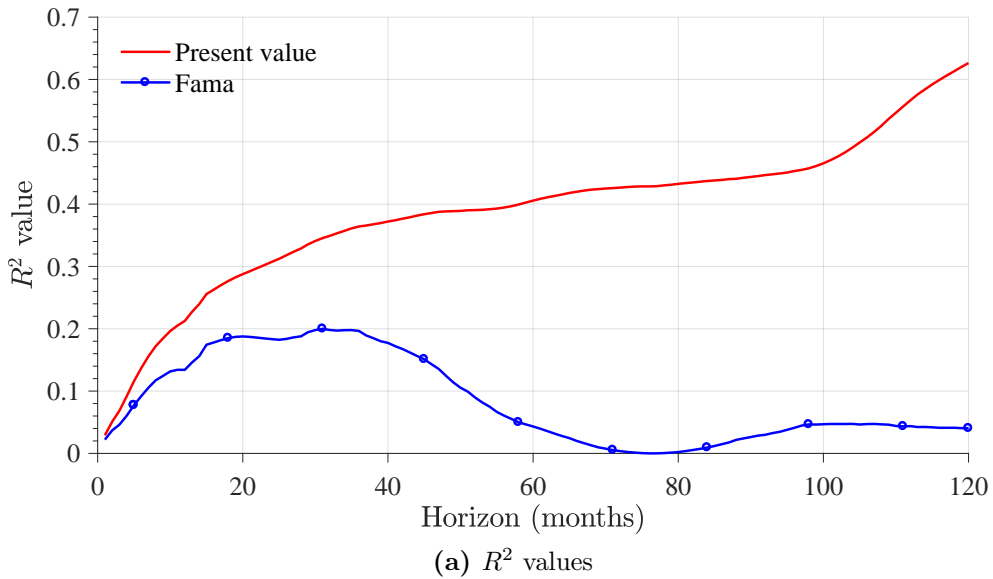
**Figure 1: Real exchange rates and subsequent five-year currency returns**

The figure shows the negative of the log real exchange rate (expressed in dollars per unit of foreign currency) and subsequent five-year returns for the currency portfolio.



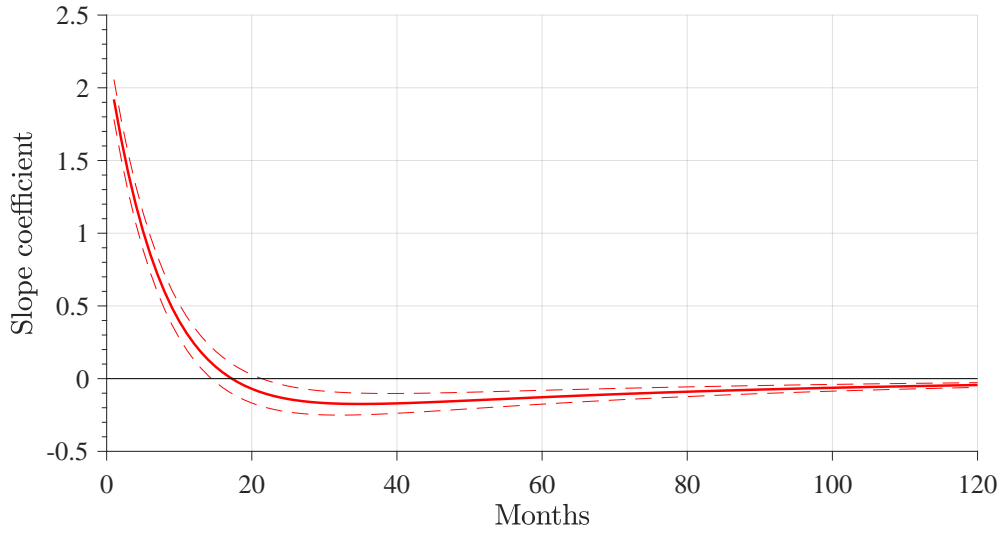
**Figure 2: Out-of-sample cumulative returns**

The figure shows cumulative returns of an initial one dollar invested in an equally weighted currency portfolio (EW) and in the market-timing strategies of a mean-variance investor who predicts based on the interest rate differential (Fama) and using the present-value model (Present value).

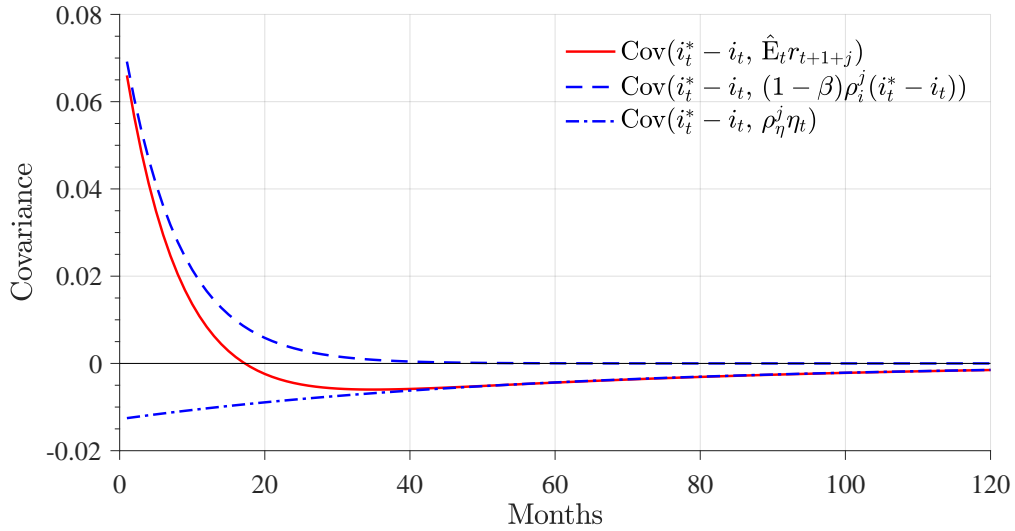


**Figure 3: Long-horizon predictability (portfolio)**

Panel (a) shows  $R$ -squared values for predictions of overlapping cumulative currency excess returns based on the present-value model and on Fama regressions, for horizons of one to 120 months using the currency portfolio. Present-value predictions exploit information from both the real exchange rate and interest rate differentials, while Fama regressions use only information from interest rate differentials. The Fama coefficients are allowed to change with the horizon. Panel (b) shows  $t$ -statistics for regressions of realized returns on the predictions (without a constant), based on the Hansen and Hodrick (1980) correction for overlapping observations (the number of lags equals the horizon).



(a) Slope coefficients



(b) Covariances

**Figure 4: Long-horizon predictability and interest rate differentials (portfolio)**

Panel (a) shows the slope coefficients and 90% confidence interval of the regression  $\hat{E}_t(r_{t+1+j}) = a + b(i_t^* - i_t) + u_{t+1+j}$ . Panel (b) decomposes the covariance into the interest rate differential and PPP components.

Online Appendix for  
“The Missing Risk Premium in Exchange Rates”

Magnus Dahlquist      Julien Penasse\*

May 2017

---

\*Stockholm School of Economics and CEPR; e-mail: [magnus.dahlquist@hhs.se](mailto:magnus.dahlquist@hhs.se). University of Luxembourg; e-mail: [julien.penasse@uni.lu](mailto:julien.penasse@uni.lu).

## A vector autoregressive model

In this online appendix we consider more general dynamics of the interest rate differential, inflation differential, and PPP risk premium. Let  $z_t = [i_t^* - i_t, \pi_t^* - \pi_t, \eta_t]'$ . We now assume a vector autoregressive (VAR) model for  $z_t$ :

$$z_{t+1} - A_0 = A_1(z_t - A_0) + \epsilon_{t+1}, \quad (\text{OA.1})$$

where  $A_0 = [\mu_i, \mu_\pi, 0]'$ ,  $A_1$  is a  $3 \times 3$  matrix of parameters, and  $\epsilon_{t+1}$  is the vector of mean-zero shocks to the three variables. We also assume that all the roots of the  $A_1$  matrix are outside the unit circle. The VAR model (OA.1) effectively replaces (7), (6), and (10) in the main text.

Recall the currency return expressed in real terms (4), the real exchange rate (5), and the augmented regression (9) in the main text, all reproduced below:

$$r_{t+1} = q_{t+1} - q_t + (i_t^* - \pi_{t+1}^*) - (i_t - \pi_{t+1}), \quad (\text{OA.2})$$

$$q_t - \mu_q = \sum_{j=1}^{\infty} E_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^{\infty} E_t(\pi_{t+j}^* - \pi_{t+j}) - \sum_{j=1}^{\infty} E_t(r_{t+j}), \quad (\text{OA.3})$$

$$r_{t+1} = \alpha + (1 - \beta)(i_t^* - i_t) + \eta_t + \epsilon_{t+1}^r. \quad (\text{OA.4})$$

Let  $\iota_i$ ,  $\iota_\pi$ , and  $\iota_\eta$  be  $1 \times 3$  vectors that select the first, second, and third rows in a given vector/matrix, respectively (e.g.,  $\iota_i z_t = i_t^* - i_t$ ). Substitute (OA.4) into (OA.3) and rewrite the real exchange rate in terms of  $z_t$  using (OA.1):

$$q_t - \mu_q = \sum_{j=1}^{\infty} E_t(\beta \iota_i z_{t+j-1} - \iota_\pi A_1 z_{t+j-1} - \iota_\eta z_{t+j-1} - \alpha). \quad (\text{OA.5})$$

The VAR model (OA.1) implies  $z_t$ -predictions given by  $E_t(z_{t+j} - A_0) = A_1^j(z_t - A_0)$ . This means that:

$$\sum_{j=1}^{\infty} E_t(z_{t+j} - A_0) = (I - A_1)^{-1}(z_t - A_0), \quad (\text{OA.6})$$

where  $I$  is the  $3 \times 3$  identity matrix. Substituting these into (OA.5) gives:

$$q_t - \mu_q = (\beta\iota_i - \iota_\pi A_1 - \iota_\eta)(I - A_1)^{-1}(z_t - A_0), \quad (\text{OA.7})$$

which generalizes (12) in the main text under the condition that  $(\beta\iota_i - \iota_\pi)A_0 = \alpha$ . It follows that the change in the real exchange rate is:

$$q_{t+1} - q_t = \beta\mu_i - \beta(i_t^* - i_t) + \iota_\pi A_1(z_t - A_0) + \eta_t + (\beta\iota_i - \iota_\pi A_1 - \iota_\eta)(I - A_1)^{-1}\epsilon_{t+1}. \quad (\text{OA.8})$$

It also follows that the currency return is:

$$r_{t+1} = \beta\mu_i - \mu_\pi + (1 - \beta)(i_t^* - i_t) + \eta_t + \epsilon_{t+1}^r, \quad (\text{OA.9})$$

where:

$$\epsilon_{t+1}^r = [(\beta\iota_i - \iota_\pi A_1 - \iota_\eta)(I - A_1)^{-1} - \iota_\pi] \epsilon_{t+1}. \quad (\text{OA.10})$$

Equations (OA.8), (OA.9), and (OA.10) generalize (13), (14), and (15) in the main text.

The PPP risk premium in  $z_t$  is not observed. To estimate the model, we convert  $z_t$  into observable variables. Define:

$$\left( \begin{bmatrix} i_t^* - i_t \\ \pi_t^* - \pi_t \\ q_t \end{bmatrix} - \begin{bmatrix} \mu_i \\ \mu_\pi \\ \mu_q \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (\beta\iota_i - \iota_\pi A_1 - \iota_\eta)(I - A_1)^{-1} \end{bmatrix} \left( \begin{bmatrix} i_t^* - i_t \\ \pi_t^* - \pi_t \\ \eta_t \end{bmatrix} - \begin{bmatrix} \mu_i \\ \mu_\pi \\ 0 \end{bmatrix} \right),$$

or more compactly:

$$y_t - P_0 = P_1(z_t - A_0). \quad (\text{OA.11})$$

With  $P_1$  invertible, we can express  $z_t$  in terms of the observable  $y_t$ :  $z_t - A_0 = P_1^{-1}(y_t - P_0)$ , and then rewrite (OA.1) as:

$$y_{t+1} - P_0 = P_1 A_1 P_1^{-1}(y_t - P_0) + P_1 \epsilon_{t+1}. \quad (\text{OA.12})$$

We can therefore estimate the model from the reduced-form parameters of the following VAR model:

$$y_{t+1} - \Phi_0 = \Phi_1(y_t - \Phi_0) + u_{t+1}, \quad (\text{OA.13})$$

and solve  $\hat{\Phi}_1 = P_1 A_1 P_1^{-1}$  to find the parameters in  $A_1$ . We can finally recover the PPP risk premium as:

$$\eta_t = \iota_\eta P_1^{-1}(y_t - P_0). \quad (\text{OA.14})$$

A special case arises when  $A_1$  is diagonal:

$$(I - A_1)^{-1} = \begin{bmatrix} \frac{1}{1-\rho_i} & 0 & 0 \\ 0 & \frac{1}{1-\rho_\pi} & 0 \\ 0 & 0 & \frac{1}{1-\rho_\eta} \end{bmatrix},$$

so that (OA.7) equals:

$$q_t - \mu_q = \beta \frac{i_t^* - i_t - \mu_i}{1 - \rho_i} - \frac{\rho_\pi}{1 - \rho_\pi} (\pi_t^* - \pi_t - \mu_\pi) - \frac{\eta_t}{1 - \rho_\eta}. \quad (\text{OA.15})$$

This, in turn, implies that the currency return is given by:

$$\begin{aligned} r_{t+1} = & \beta \mu_i \left( \frac{\rho_\eta - \rho_i}{1 - \rho_i} \right) - \mu_\pi \left[ 1 - \frac{\rho_\pi}{1 - \rho_\pi} (1 - \rho_\eta) \right] + \left( 1 - \beta \frac{\rho_\eta - \rho_i}{1 - \rho_i} \right) (i_t^* - i_t) \\ & - \frac{\rho_\pi}{1 - \rho_\pi} (1 - \rho_\eta) (\pi_t^* - \pi_t) + (1 - \rho_\eta) (q_t - \mu_q) + \epsilon_{t+1}^r. \end{aligned} \quad (\text{OA.16})$$

This equation resembles (17) in the main text, but now includes terms related to the inflation differential. It suggests that the currency return should be predictable, not only from the interest rate differential and the real exchange rate, but also from the inflation differential. Given a positive first-order autocorrelation coefficient for the inflation differential, we expect the predictability to have a negative sign and to be proportional to the persistence of the inflation differential. Note that when  $\rho_\pi = 0$ , (OA.16) equals (17) and the expected return does not depend on the inflation differential.



Table OA.I presents the results of regressing currency returns on interest rate differentials, inflation differentials, and real exchange rates for horizons of one and twelve months. Recall that consumer price indices are only available on a quarterly basis for AUD and NZD, likely leading to spurious negative persistence in interest rate differentials. As the persistence in the inflation differential now is key for predicting the currency return, we do not present estimates for AUD and NZD. Table I in the main text presents first-order autocorrelations between 0.02 and 0.25 for the other currencies.

At the monthly horizon, the coefficient of the inflation differential has, as expected, a negative sign for all currencies except JPY, but the coefficients are not precisely estimated and are significant only for EUR and the portfolio of currencies. For these two cases, the coefficients seem too negative. For example, Table OA.I reports  $\hat{c} \approx -0.78$  for the portfolio of currencies, which suggests an implausible persistence for the inflation differential. Matching the coefficients in (OA.16) gives  $\hat{\rho}_\pi = -\hat{c}/(1 - \hat{\rho}_\eta - \hat{c}) \approx 0.97$ , where  $\hat{\rho}_\eta$  is obtained as described in the main text. This corresponds to extremely persistent inflation differential dynamics and points towards model misspecification and/or statistical noise. At the annual horizon, the coefficients are never significant, which indicates that the effect of the inflation differential on the currency return is short-lived. This is consistent with the low persistence in the inflation differential.

As the diagonal VAR is inconclusive, we present estimates of a full VAR model for the portfolio of currencies in Table OA.II. Consistently with the baseline specification in the main text, the interest rate differential is well approximated by an AR(1) process and the real exchange rate is strongly autocorrelated and positively correlated with the lagged interest rate differential. The inflation differential is not only positively autocorrelated, but also significantly predicted by the lagged interest rate differential (positively) and by the lagged real exchange rate (negatively). The degree of persistence is small, with an R-squared less than 10%. We nevertheless explore how our assumption of a nonpersistent inflation differential affects estimates of  $\eta_t$ . Figure OA.I plots estimates from the baseline model and the full VAR model for the portfolio of currencies. The estimates track each

other with only minor differences. This confirms our conjecture that the inflation differential does not materially matter for currency risk premia. In untabulated results, we find similar results for individual currencies.

## REFERENCES

- Hansen, Lars Peter, and Robert J. Hodrick, 1980, Forward exchange rates as optimal predictors of future spot rates: An econometric analysis, *Journal of Political Economy* 88, 829–853.
- Newey, Whitney K., and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.
- White, Halbert L., 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48, 817–838.

**Table OA.I: Predicting currency returns**

	CAD	EUR	JPY	NOK	SEK	CHF	GBP	Portfolio
One-month horizon								
	$r_{t+1} = a + (1 - b)(i_t^* - i_t) + c(\pi_t^* - \pi_t) + dq_t + u_{t+1}$							
$b$	-1.555	-1.099	-1.434	-0.213	0.019	-1.316	-1.454	-1.360
(s.e.)	(0.683)	(0.773)	(0.633)	(0.692)	(0.823)	(0.730)	(0.807)	(0.764)
$c$	-0.226	-0.844	0.221	-0.527	-0.503	-0.055	-0.219	-0.775
(s.e.)	(0.263)	(0.376)	(0.356)	(0.322)	(0.310)	(0.462)	(0.261)	(0.407)
$d$	-0.020	-0.021	-0.014	-0.019	-0.013	-0.028	-0.030	-0.021
(s.e.)	(0.008)	(0.010)	(0.009)	(0.012)	(0.008)	(0.011)	(0.016)	(0.011)
$R^2$	0.033	0.031	0.041	0.023	0.014	0.035	0.046	0.038
One-year horizon								
	$r_{t,t+12} = a + (1 - b)(i_t^* - i_t) + c(\pi_t^* - \pi_t) + dq_t + u_{t,t+12}$							
$b$	-9.534	-16.026	-23.781	-10.113	-12.009	-23.039	-14.010	-21.534
(s.e.)	(6.240)	(6.923)	(6.675)	(4.995)	(5.394)	(5.900)	(5.694)	(6.496)
$c$	1.198	0.123	-0.294	-1.177	0.605	-0.639	0.205	-0.503
(s.e.)	(1.126)	(1.313)	(0.941)	(1.209)	(1.234)	(1.574)	(0.875)	(1.555)
$d$	-0.196	-0.292	-0.247	-0.304	-0.267	-0.347	-0.471	-0.303
(s.e.)	(0.090)	(0.114)	(0.097)	(0.129)	(0.110)	(0.108)	(0.124)	(0.114)
$R^2$	0.113	0.194	0.282	0.172	0.144	0.295	0.308	0.255

This table presents the results of predictive regressions of the log excess currency return,  $r_{t+1}$ , on the lagged interest rate differential,  $i_t^* - i_t$ , the lagged inflation differential,  $\pi_t^* - \pi_t$ , and the lagged real exchange rate,  $q_t$ . The top panel presents the results of predictions at the one-month horizon. The bottom panel presents the results of predictions of one-year cumulative returns,  $r_{t,t+12} = \sum_{k=1}^{12} r_{t+k}$ . One-month horizon regressions are reported with [Newey and West \(1987\)](#) standard errors, accounting for conditional heteroscedasticity and serial correlation up to three lags, in parentheses. One-year regressions are reported with [Hansen and Hodrick \(1980\)](#) corrected standard errors with twelve lags. The regressions include constant terms, though they are not reported.

**Table OA.II: VAR estimation results**

Dep. variable $\longrightarrow$	$i_{t+1}^* - i_{t+1}$	$\pi_{t+1}^* - \pi_{t+1}$	$q_{t+1}$
VAR estimation			
$i_t^* - i_t$	0.882	0.234	1.594
(s.e.)	(0.035)	(0.080)	(0.710)
$\pi_t^* - \pi_t$	-0.003	0.209	-0.566
(s.e.)	(0.014)	(0.063)	(0.441)
$q_t$	-0.000	-0.004	0.975
(s.e.)	(0.000)	(0.001)	(0.011)
$R^2$	0.770	0.094	0.959
Standard deviations and correlations of residuals			
	$i^* - i$	$\pi^* - \pi$	$q$
$i^* - i$	0.089	0.083	0.206
$\pi^* - \pi$	—	0.276	0.076
$q$	—	—	2.360

This table presents estimation results of the VAR model for the portfolio of currencies:

$$y_{t+1} - \Phi_0 = \Phi_1(y_t - \Phi_0) + u_{t+1}.$$

The top panel presents estimates of parameters in  $\Phi_1$  with [White \(1980\)](#) standard errors, accounting for conditional heteroscedasticity, in parentheses. The bottom panel presents estimates of the standard deviations of the residuals (in percentage terms) on the diagonal and the correlations of the residuals off the diagonal.

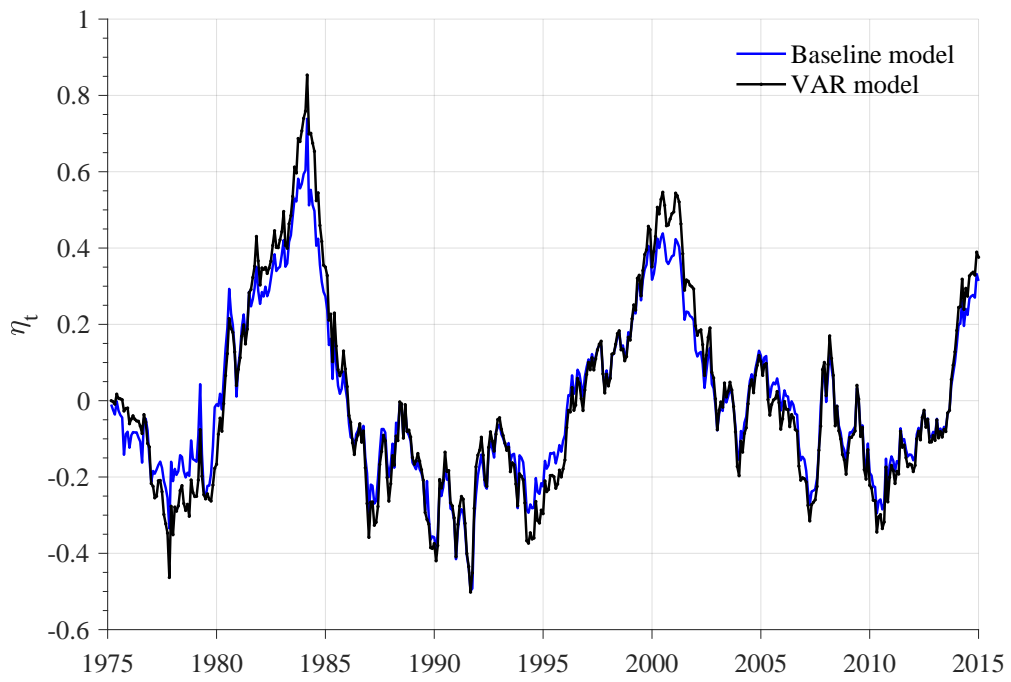


Figure OA.I: PPP risk premium in the baseline and VAR models