Sovereign Default Risk and Firm Heterogeneity*

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Abstract

This paper studies the recessionary effects of sovereign default risk using firm-level data and a model of sovereign debt with firm heterogeneity. Our environment features a two-way feedback loop. Low output decreases the tax revenues of the government and raises the risk that it will default on its debt. The associated increase in sovereign interest rate spreads, in turn, raises the interest rates paid by firms, which further depresses their production. Importantly, these effects are not homogeneous across firms, as interest rate hikes have more severe consequences for firms that are in need of borrowing. Our approach consists of using these cross-sectional implications of the model, together with micro data, to measure the effects that sovereign risk has on real economic activity. In an application to Italy, we find that the progressive heightening of sovereign risk during the recent crisis was responsible for 50% of the observed decline in output.

Keywords: Sovereign debt crises, Firm heterogeneity, Financial frictions.

JEL codes: F34, E44, G12, G15

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1 Introduction

During the recent debt crisis, countries in Southern Europe have experienced sharp increases in sovereign interest rate spreads, a tightening in private financial conditions with increases in firms' borrowing rates, and large declines in economic activity. These dynamics are typical of sovereign debt crises in emerging markets. Researchers have proposed two different explanations for these events. One explanation is that these patterns are induced by the deterioration in real economic activity that increases default risk for both firms and the government.[1] In this view, the sovereign debt crisis is a reflection, rather than the cause, of the problems that originate in the private sector. A contrasting explanation for these patterns is that the sovereign debt crisis is the culprit and disrupts economic activity and financial conditions for firms.[2]

Sorting out these explanations is an important open question in macroeconomics and is of relevance for policymakers dealing with sovereign debt crises. For example, quantifying the recessionary effects of sovereign risk is essential for evaluating the fiscal austerity measures that were implemented by Southern European countries in the past few years. Despite its importance, disentangling to what extent a sovereign debt crisis contributes to poor economic conditions and to what extent it merely reflects them is challenging. The difficulty arises because both views have similar implications for the behavior of aggregate time series; they both imply a negative comovement between interest rates and aggregate economic activity. In this paper, we address this question by building a model of sovereign debt crises with heterogeneous firms and by combining its cross-sectional implications with firm-level data to measure the *pass-through* of sovereign risk on the economy.

In our framework, both the government and the firms can default on their debt obligations. The private sector affects the government through tax revenues, and the government affects the private sector because the borrowing rates of firms depend on sovereign interest rate spreads. Firms that use debt more heavily are more severely affected by the increase in borrowing rates, and thus sovereign risk has a differential impact on firms. Our approach consists of using these cross-sectional implications of the model, along with detailed firm-level data during the Italian debt crisis, to indirectly measure the effect of sovereign risk on economic activity. We find that the recessionary effects of sovereign risk are of first-order importance, explaining roughly 50% of the output decline in Italy during the crisis.

Firms in the model are heterogeneous in their productivity and financing needs, and they

[1] The sovereign default literature focuses on how fluctuations in domestic output affect sovereign interest rate spreads. See Aguiar and Gopinath (2006) and Arellano (2008), among others.

[2] Neumeyer and Perri (2005), Uribe and Yue (2006), and Corsetti, Kuester, Meier, and Müller (2013) develop models in which changes in sovereign spreads have an impact on the real economy via their effect on firms' borrowing rates.

produce a final good using capital and labor. Firms finance part of their operations with debt that they can default on, and they face interest rates that compensate lenders for their default risk. Following the work of Neumeyer and Perri (2005) and Corsetti et al. (2013), we also assume that the interest rates at which firms borrow depend on the government interest rate spreads according to a flexible reduced-form specification. The government receives fluctuating tax revenues and borrows to finance valuable public goods. The government can default on its debt obligations, and its interest rate spreads compensate lenders for this default risk. It also internalizes that its borrowing affects interest rate spreads and hence output. The economic environment is perturbed by two types of aggregate shocks: a shock that moves the productivity process of firms and a shock to the value of default for the government. This latter shock generates variation in sovereign risk that is orthogonal to aggregate productivity, and it can be interpreted as capturing time variation in the enforcement of sovereign debt.[3]

As in Eaton and Gersovitz (1981), the endogenous risk of a government default in our model responds to changes in the state of the economy. A key feature in our environment is that this variation in sovereign default risk feeds back into real economic activity through its impact on the interest rates that firms face. Thus, our model features a two-way feedback loop. An increase in sovereign spreads leads to an increase in firm interest rates, which depresses firms' production. As output and tax revenues fall, the government's risk of default increases, which further raises sovereign spreads.

A contribution of this paper is to use firm-level data to empirically discipline the feedback from the government to the private sector. To understand why cross-sectional moments are useful for this purpose, suppose that the government in our economy suddenly becomes at risk of a default. Interest rate spreads on government bonds increase because lenders demand compensation for the heightened default risk. When the pass-through is sizable, the interest rates at which firms borrow increase as well and depress firms' production. Importantly, however, the implications of these higher borrowing rates are not homogeneous in the population of firms, because they are more damaging to the performance of firms with large borrowing needs. Therefore, when the propagation of sovereign risk is quantitatively sizable, we should observe a large differential effect in performance across firms, depending on whether they need loans or not.

We apply our framework to Italian data during the 2005-2012 period. We use balance sheets data for many privately and publicly held Italian firms from the ORBIS-AMADEUS database. We measure the differential performance of firms by running regressions of firm sales growth on aggregate and firm-specific variables. The coefficient of interest is the inter-

[3] As discussed in the quantitative literature of sovereign debt, variation in the default value gives the model flexibility to fit sizable government spreads; see Arellano (2008) and Chatterjee and Eyigungor (2012).

action between an indicator of firm leverage and interest rate spreads on Italian government bonds. We find that highly levered firms experienced a larger contraction in their sales growth during periods of high Italian government spreads relative to less levered firms. Specifically, an increase of 100 basis points in sovereign spreads is associated with a decline in sales growth of 1.7% for firms with leverage at the 75^{th} percentile and a decline of 1.2% for firms with leverage at the 25^{th} percentile. Importantly, in assessing these effects, we control for the behavior of aggregate productivity, firm-specific fixed effects, and time-varying firm characteristics.[4] The parametrization of the model ensures that we replicate this elasticity in simulations, as well as other empirical targets that summarize the behavior of sovereign and firm spreads, firm productivity, and their leverage. The estimated model has a good fit with the targeted moments, and it captures additional aggregate and firm level statistics.

The main quantitative experiment consists in using the parameterized model to measure the recessionary effects of sovereign risk. To this end, we feed into the model the aggregate productivity series measured in the data, and we retrieve the path for the enforcement shocks that guarantees that the model-implied sovereign spreads match their behavior in the data. As an overidentifying restriction of our theory, we show that the model replicates the decline in observed output and over 80% of the increase in the average spreads of Italian firms. We then feed the same sequence of shocks into a nested version of the model in which sovereign spreads do not feed back into real economic activity. We find that firm interest rate spreads in this counterfactual barely move, and output falls by about 50% less than in our benchmark specification. We conclude that the pass-through from sovereign risk to the private sector is essential for rationalizing the severe output declines observed in Italy.

Our results also shed light on the recent debate on the negative effects of public debt for the private sector. In a second counterfactual, we examine how different public borrowing policies affect economic activity. We find that a 10% reduction in new borrowing by the government would have dampened the associated output decline by about 30%. Our model, hence, implies sizable overhang effects of public debt on the private sector.

Related Literature. Our paper combines elements of the sovereign default literature with the literature on the impact of financial imperfections on firms. We also contribute to the growing literature that combines structural models with micro data to infer aggregate elasticities.

A number of papers in the sovereign default literature emphasize the connections between sovereign default and the private sector through financial intermediation. Mendoza

[4]We also show that this differential effect is robust to various measures for firm leverage as well as to additional controls such as the volatility index VIX, European stock price indexes, Italian banks' stock market indexes, and the interaction of these aggregate variables with firm leverage.

and Yue (2012) study a sovereign default model in which firms lose access to external financing conditional on a government default, and they show that such a mechanism can generate substantial output costs of a sovereign default. Similar dynamics are present in the quantitative models of Sosa Padilla (2013) and Perez (2015), and in the more stylized frameworks of Fahri and Tirole (2014) and Gennaioli, Martin, and Rossi (2014). We share with these papers the emphasis on financial intermediation, but we depart from their analysis in focusing on the feedback that is outside actual default events: in our model, an increase in sovereign risk (even when the government keeps repaying) propagates to the real sector because of its impact on the interest rates paid by firms. This is an important distinction because we are studying the Italian debt crisis, which is characterized by rising sovereign spreads but no actual default.

These recessionary effects of sovereign risk have been previously studied in the literature. Neumeyer and Perri (2005) consider a model in which exogenous movements in sovereign interest rates are perfectly passed on to firm interest rates, and they argue that these interest rate fluctuations account for a substantial fraction of the volatility of output in emerging markets. Bocola (2016) models in detail financial intermediaries and shows how increases in sovereign risk worsen intermediaries' balance sheets, which in turn tightens domestic credit and increases domestic interest rates. He estimates his structural model using data from Italy and finds that this channel is important for rationalizing the output movements.[5] We share with these papers the focus on measuring the aggregate effects of sovereign risk pass-through, but in contrast with these papers, we tackle this question in a framework in which sovereign default risk is endogenous.

An important difference in our approach relative to all of the above-mentioned papers is that we use firm-level data and a model with firm heterogeneity to empirically discipline the recessionary effects of sovereign risk. Our emphasis on informing macro-elasticities with firm-level data is shared by a number of recent papers. Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2015), for example, use European firm-level data to measure how declines in real interest rates affect aggregate productivity through the misallocation of inputs across firms. Gopinath and Neiman (2014) use Argentinian firm-level data to quantify the effects of trade adjustment on aggregate productivity. Arellano, Bai, and Kehoe (2016) use U.S. firm-level data to measure the effects of volatility shocks at the firm level on aggregates during the Great Recession. We apply a similar set of tools to study the macroeconomic effects of sovereign debt crises.

Our heterogeneous firm model builds on the literature of firm dynamics with financial

[5]Hebert and Schreger (2016) presents a less structural approach to measure the macroeconomic implications of sovereign risk. The authors exploit the rulings in the case *Republic of Argentina v. NML Capital* as exogenous variation, and document large negative effects of sovereign risk on Argentinian stock returns.

frictions. Cooley and Quadrini (2001) develop a model of heterogeneous firms with incomplete financial markets and default risk. They explore its implications for the dynamics of firm investment growth and exit while abstracting from aggregate fluctuations. Kahn and Thomas (2013) focus on aggregate fluctuations in a model with heterogeneous firms facing financial frictions and financial shocks. In their work, shocks to the collateral constraint can generate long-lasting recessions. Buera and Moll (2015), Buera, Kaboski, and Shin (2011), Arellano, Bai, and Zhang (2012), and Midrigan and Xu (2014) also develop models with firm heterogeneity and financial frictions and compare the misallocation costs across economies with varying degrees of financial development. In contrast to these papers, we focus on the interaction between firm default risk and sovereign default risk. Our paper shares this emphasis with the recent work by de Ferra (2016) and Kaas, Mellert, and Scholl (2016).

2 Model

We consider a dynamic economy with heterogeneous firms and a government. Firms differ in their productivity and their financing needs. They produce a homogeneous good using capital and labor, and they borrow from lenders to finance a portion of their input costs. The government receives a fraction of output as tax revenues, and it borrows from lenders to finance public goods and service its outstanding debt. Both the firms and the government can default on their debt.

The economy is perturbed by two aggregate shocks. The first shock, p_t, is an aggregate shock to the firms' productivity. The second shock, ν_t, affects the utility of the government in case of a default, and it controls the enforcement of sovereign debt.

The timing of events within the period is as follows. In the beginning of the period, the aggregate shocks are realized. The government chooses whether to default and how much to borrow, while firms make production and borrowing choices. At the end of the period, the idiosyncratic shocks to firms are realized, and firms choose whether to default or not.

Firms. A measure one of heterogeneous firms produce output in this economy. Each firm i combines capital $k_{i,t}$ and labor $\ell_{i,t}$ in order to produce output $y_{i,t}$ using a decreasing return to scale technology. Production is affected by firm-specific productivity shocks, $z_{i,t}$.[6] The

[6] Our empirical approach to measure $z_{i,t}$ in the data does not separate pure changes in firms' productivity from changes in the demand for the firms' product. Thus, it is more appropriate to think about $z_{i,t}$ as capturing random shocks to both demand and productivity at the firm level. For simplicity, we will keep referring to $z_{i,t}$ as "firm productivity" throughout the paper.

output produced by firm i at time t is then

$$y_{i,t} = z_{i,t}^{1-\eta}(\ell_{i,t}^{\alpha}k_{i,t}^{1-\alpha})^{\eta}. \tag{1}$$

Firms' productivity is affected by an aggregate and an idiosyncratic component. We model the aggregate shock following the literature on disaster risk (Gourio, 2012): every period, there is a probability p_t that a firm's productivity declines by μ. This probability is common across firms, and it is drawn every period from a distribution $\Pi^p(p)$. The idiosyncratic shock is persistent with coefficient ρ_z, and we denote by σ_z the standard deviation of productivity innovations. The process for firms' productivity is

$$\begin{aligned} \log z_{i,t} &= \rho_z \log z_{i,t-1} - I_{i,t}\mu + \sigma_z \varepsilon_{i,t}, \\ Pr(I_{i,t}=1) &= p_t, \end{aligned} \tag{2}$$

where $I_{i,t}$ is a Bernoulli random variable that takes the value of 1 if firm i at time t receives the decline in productivity of μ, with p_t being the probability of such an event. The innovation $\varepsilon_{i,t}$ follows a standard normal random process. Firms also face an additional idiosyncratic cost shock $\xi_{i,t}$, which is drawn from a distribution $\Pi^{\xi}(\xi)$.

Note that the aggregate shock p_t affects not only the average productivity in this economy but also higher moments such as standard deviation and skewness. As we will discuss in the quantitative section of the paper, this specification allows us to capture in a parsimonious way the time variation in the cross-sectional distribution of firms' productivity that we document in the data.

In the beginning of the period, firms choose capital $k_{i,t}$ and labor $\ell_{i,t}$ before their idiosyncratic shocks are realized. We introduce heterogeneity in the borrowing needs of firms by assuming that they face a working capital constraint requiring them to pay λ_i fraction of their input costs before production takes place, which is a time-invariant firm-specific attribute. Firms borrow their working capital needs by issuing a defaultable debt $b_{i,t}$ at price $q_{i,t}$. Accordingly, we have

$$q_{i,t}b_{i,t} = \lambda_i(r_k k_{i,t} + w\ell_{i,t}), \tag{3}$$

where r_k and w are factor prices that are taken as given by the firms and are assumed to be constant.[7]

At the end of the period, the idiosyncratic shocks are realized, production takes place,

[7] We abstract from variation in wages and rental rates of capital given the focus in our quantitative analysis on a short event period and to maintain computational tractability.

and firms decide to default on their debt $d_{i,t} = 1$ or repay it $d_{i,t} = 0$. Assuming that a firm repays its debt, it also repays the remainder of the input costs and the cost $\xi_{i,t}$. In this case, the profits of the firm

$$\pi_{i,t} = y_{i,t} - (1 - \lambda_i)(w\ell_{i,t} + r_k k_{i,t}) - b_{i,t} - \xi_{i,t}. \tag{4}$$

are distributed back to shareholders as equity payouts which are required to be non-negative, $\pi_{i,t} \geq 0$.[8]

Firms will choose to default on their debt if their value conditional on repayment is negative. When firms default, they exit and their resources from production are used to pay for all of its costs. Any shortfall in resources for firms' costs is paid by the government with a transfer f such that

$$f_{i,t} = \max\{d_{i,t}[(1 - \lambda_i)(w\ell_{i,t} + r_k k_{i,t}) + \xi_{i,t} - y_{i,t}], 0\}. \tag{5}$$

Upon default firms' shareholders and lenders obtain a payout of zero. Defaulting firms are replaced by new entrant firms with the same idiosyncratic state. This assumption guarantees that the mass of firms is constant in the model.

Government. The government decides the level of public goods G_t to provide to its citizens. It finances public spending by collecting a constant fraction τ of aggregate output Y_t as tax revenues and by borrowing from financial intermediaries short-term loans B_{t+1} at price q_t^g. Letting B_t denote the outstanding debt of the government, and F_t any costs from defaulting firms, we can write the government budget constraint as

$$B_t + G_t = q_t^g B_{t+1} + \tau Y_t - F_t. \tag{6}$$

Every period the government chooses (G_t, B_{t+1}) and decides whether to default on its outstanding debt.[9] Default induces a reduction in outstanding debt to R and a utility cost ν_t. During default the government budget constraint is as in (6) but with $B_t = R$, and the government's period utility is reduced by an exogenous shock ν_t, which follows the stochastic process

$$\nu_t = \bar{\nu} + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_{\nu,t},$$

[8]The assumption that firms' payouts need to be positive is equivalent to assuming that firms cannot issue new equity, and it stands for the large equity issuance costs measured in the corporate finance literature. See Hennessy and Whited (2005).

[9]While we assume that the government acts strategically, we could equally conduct our analysis by assuming that the government follows some reduced form fiscal rule for debt and default (Bi and Traum, 2012; Lorenzoni and Werning, 2014).

with $\epsilon \sim \mathcal{N}(0, 1)$.

The ν_t shock generates fluctuations in the value of default for the government. The quantitative literature of sovereign debt emphasizes that random fluctuations in default values are necessary to fit data on government spreads in emerging and advanced economies (Arellano, 2008; Bocola and Dovis, 2016). In most of the quantitative literature, fluctuations in default values are generated by assuming convexity of the output costs of default. In our model, as in Aguiar and Amador (2013), such fluctuations are directly induced by ν_t shocks. This shock generates variation in sovereign risk that is orthogonal to the economic fundamentals of the country, and it proxies for changes in bailouts expectations, contagion from other countries, and other factors that have played a role during the European debt crisis (Bahaj, 2013).

The government's objective is to maximize the present discounted value of the utility derived from public goods net of any default costs,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u_g(G_t) - D_t \nu_t), \tag{7}$$

where $D_t = 1$ when the government defaults and zero otherwise.

Financial intermediaries. The government's bond prices compensate financial intermediaries for the losses suffered in case of default. Intermediaries discount the future at the international interest rate r. The government's bond price is given by

$$q_t^g = \mathbb{E}_t \left[\frac{1 - D_{t+1}(1 - \frac{R}{B_{t+1}})}{1 + r} \right]. \tag{8}$$

Interest rate spreads on government bonds are defined as the difference in the yield of the bond relative to the risk-free rate,

$$s_t^g = \frac{1}{q_t^g} - (1 + r). \tag{9}$$

Similarly, firms' bond prices compensate lenders for the losses in case of default. Following Neumeyer and Perri (2005) and Corsetti et al. (2013), we assume that the interest rates faced by firms also depend on the interest rate spreads on government debt, with such effect being controlled by the parameter γ. The bond price for firms satisfies

$$q_{i,t} = \mathbb{E}_t \left[\frac{1 - d_{i,t}}{1 + \gamma s_t^g} \right]. \tag{10}$$

When $\gamma > 0$, an increase in the government's interest rate spreads leads to depressed bond prices for firms and thus to higher borrowing costs. We view the reduced-form relation in equation (10) as standing in for the detrimental effects that a decline in the price of government securities has on the balance sheet of financial intermediaries, and through this channel, on the borrowing costs of nonfinancial firms.[10] Bocola (2016), for example, develops a model in which financial intermediaries hold long-term risky government bonds on their balance sheets and are subject to occasionally binding leverage constraints. A decline in the price of government debt tightens the leverage constraints of financial intermediaries, which in turn increases the borrowing costs of nonfinancial firms.

2.1 Recursive problems

We now describe the recursive problems for firms and the government and define the equilibrium. We consider a Markov equilibrium where the government takes into account that its policy choices affect the private economy.

In the beginning of the period, the aggregate state of the economy includes the aggregate shocks for default costs and productivity, $s = \{\nu, p\}$, the distribution of firms' productivity and financing needs Λ, and government debt B. Given these state variables $\{\nu, p, \Lambda, B\}$, the government chooses its default policy D and new borrowing B'. These choices result in a government spread that is a function of aggregate shocks, the distribution of firms, and the borrowing level B', $s^g = s^g(\nu, p, \Lambda, B')$. Let these end-of-period variables be $X = \{\nu, p, \Lambda, B'\}$.

The firms' problem depends on the aggregate states and on the government policy B' because their borrowing rates are affected by the government's bond spreads. The recursive structure of the problems makes it admissible to set X as the aggregate state in the firms' problem. Firms also make choices that depend on their idiosyncratic state, which consists of their lag productivity z_{-1} and their time-invariant financing needs λ. The firms' idiosyncratic and aggregate states are $\{z_{-1}, \lambda, X\}$.

Firms' recursive problem. In the beginning of the period, before the idiosyncratic shocks are realized, firms choose inputs (k, ℓ) and borrowing b. At the end of the period, firms observe the realization of their idiosyncratic shocks to productivity z and cost ξ and decide whether to repay or default $d = \{0, 1\}$. The firms' bond price schedule depends on their choices as well as their idiosyncratic state and aggregate state, $q(b, k, \ell, z_{-1}, \lambda, X)$. The firm value,

[10] This assumption is consistent with the findings in Acharya, Eisert, Eufinger, and Hirsch (2014), Bottero, Lenzu, and Mezzanotti (2015), and Baskaya and Kalemli-Ozcan (2016), who show empirically that sovereign default risk negatively affects the conditions of domestic banks.

denoted by $v(z_{-1}, \lambda, X)$, is given by the following problem

$$v(z_{-1}, \lambda, X) = \max_{b,k,\ell} \mathbb{E} \max_{d=\{0,1\}} \left\{ \left[\pi(b, k, \ell, z, \xi, \lambda, X) + \frac{1}{1+r} v(z, \lambda, X') \right] [1-d] + 0 \times d \right\} \tag{11}$$

subject to their financing requirement (3), the equity payouts (4) required to be nonnegative, $\pi \geq 0$, the firms' bond price schedule (10), the spread function that maps aggregate states to the government spread

$$s^g(X) = H_S(X), \tag{12}$$

and the evolution of the aggregate states $X = \{\nu, p, \Lambda, B'\}$.

The evolution of shocks $\{\nu, p\}$ in the aggregate state X is given by the shocks' Markov structure. The evolution of the distribution of firms over idiosyncratic states $\{z, \lambda\}$ only depends on the productivity shock p because of the assumption that the measure of firms is constant, and defaulting firms are replaced with new firms that have identical idiosyncratic states. The transition for the distribution of firms satisfies

$$\Lambda'(z, \lambda) = H_\Lambda(\Lambda(z_{-1}, \lambda), p) = H_\Lambda(X). \tag{13}$$

The evolution of government borrowing from B' to B'', which is indexed by the next period's shocks $\{\nu', p'\}$ and firm distribution Λ', is given by

$$B''(v', p', \Lambda', B') \quad = \quad H_B(X, H_\Lambda(X)) = H_B(X). \tag{14}$$

This problem gives decision rules for firms' demand for capital $k(z_{-1}, \lambda, X)$, labor $\ell(z_{-1}, \lambda, X)$, and borrowing $b(z_{-1}, \lambda, X)$, which are decided before idiosyncratic shocks are realized, as well as default $d(z_{-1}, z, \xi, \lambda, X)$, which is decided after idiosyncratic shocks are realized.

We now define a private equilibrium given government policies.

Private equilibrium. Given a spread function $s^g(X) = H_s(X)$ and an evolution of government borrowing $B''(v', p', \Lambda', B') = H_B(X)$, the private recursive equilibrium consists of policy and value functions of firms $k(z_{-1}, \lambda, X)$, $\ell(z_{-1}, \lambda, X)$, $b(z_{-1}, \lambda, X)$, $d(z_{-1}, z, \xi, \lambda, X)$, and $v(z_{-1}, \lambda, X)$, the bond price schedule for firms $q(b, k, \ell, z_{-1}, \lambda, X)$, and the transition function for the distribution of firms $H_\Lambda(\Lambda(z_{-1}, \lambda), p)$, such that: (*i*) the policy and value functions of firms satisfy their optimization problem; (*ii*) the bond price schedule satisfies equation (10); and (*iii*) the evolution of the distribution of firms is consistent with the equilibrium behavior of firms.

Government's recursive problem. We now describe the government's recursive problem. The government chooses its policies taking as given the private sector equilibrium that determines the tax revenue for the government as well as any costs incurred by the government from defaulting firms. Let $T(\nu, p, \Lambda, B')$ be the tax revenue schedule given by

$$T(\nu, p, \Lambda, B') = \sum_{z_{-1},\lambda} \Lambda(z_{-1}, \lambda)\mathbb{E}_{z,\xi}\{\tau y(z, z_{-1}, \lambda, X) - f(z, \xi, z_{-1}, \lambda, X)]\}, \tag{15}$$

where $\tau y(z, z_{-1}, \lambda, X)$ is the tax revenue contribution of each firm with state (z_{-1}, λ, X) and productivity shock z, and $f(z, \xi, z_{-1}, \lambda, X)$ is the cost incurred by the government to pay inputs from each defaulting firm with state (z_{-1}, λ, X) and shocks (z, ξ) as defined in equation (5).[11]

The recursive problem of the government follows the quantitative sovereign default literature. The government can choose to default any period. Let $W(\nu, p, B, \Lambda)$ be the value of the option to default. After default, the debt B is reduced to R and the government pays the default cost ν. The value of the option to default is then

$$W(\nu, p, B, \Lambda) = \max_{D=\{0,1\}}\{(1 - D)V(\nu, p, B, \Lambda) + D\ (V(\nu, p, R, \Lambda) - \nu)\}, \tag{16}$$

where $D = 1$ in default and 0 otherwise, and $V(\nu, p, B, \Lambda)$ is the value of repaying debt B and given by

$$V(\nu, p, B, \Lambda) = \max_{B'} u_g(G) + \beta\mathbb{E}\, W(\nu', p', B', \Lambda'),$$

subject to its budget constraint

$$G + B \leq T(\nu, p, \Lambda, B') + q^g(\nu, p, \Lambda, B')B',$$

the tax revenue schedule (15), and the government bond price schedule

$$q^g(\nu, p, \Lambda, B') = \mathbb{E}_t \left[\frac{1 - D(\nu', p', \Lambda', B')(1 - \frac{R}{B'})}{1 + r}\right]. \tag{17}$$

The bond price takes into account that for every unit of borrowing B' today, lenders get tomorrow one unit in states of no default $D' = 0$ and the recovery rate R/B' in states of default $D' = 1$.[12]

[11] In all our simulations, $f(z, \xi, z_{-1}, \lambda, X)$ will be close to zero. Therefore, we can think about $T(\nu, p, \Lambda, B')$ as a fixed proportion of output in practice.

[12] We assume directly that the recovery value is exogenous, and in the moment-matching exercise, chose R to match historical recovery rates. In models of endogenous recoveries from bargaining games between lenders and the government, such as Yue (2010) and Benjamin and Wright (2014), the empirical recoveries

This problem gives the government decision rules for default $D(\nu, p, B, \Lambda)$, borrowing $B'(\nu, p, B, \Lambda)$, and public consumption $G(\nu, p, B, \Lambda)$. We can now define the recursive equilibrium of this economy.

Recursive equilibrium. The recursive Markov equilibrium consists of government policy functions for default $D(\nu, p, B, \Lambda)$, borrowing $B'(\nu, p, B, \Lambda)$, public consumption $G(\nu, p, B, \Lambda)$, value functions $V(\nu, p, B, \Lambda)$ and $W(\nu, p, B, \Lambda)$, and the bond price schedule $q^g(\nu, p, \Lambda, B')$ such that: (i) the policy and value functions for the government satisfy its optimization problem; (ii) the government bond price schedule satisfies equation (17); (iii) the private equilibrium is satisfied; and (iv) the functions H_B and H_S are consistent with the government policies.

Our framework features a two-way feedback loop between the government and the private sector decision problem. To understand this feedback, it is useful to study the optimal borrowing choice B' for the government. As in standard models of sovereign debt, this choice trades off standard consumption smoothing and front-loading incentives with increases in default risk. In standard models, the government takes into account that higher default risk today depresses the price at which the government borrows. In our setup, the government also internalizes that higher default risk reduces its tax revenues. We can illustrate this point by assuming that the bond price schedule and the repayment value function are differentiable. The optimal borrowing choice for the government then satisfies

$$u_g'(G)\left[q^g + \frac{\partial q^g}{\partial B'}B' + \frac{\partial T}{\partial B'}\right] = \beta E[u_g'(G')|D' = 0]. \tag{18}$$

This Euler equation resembles those that arise in models of sovereign default, as in Arellano (2008). In those models, as is the case here, the Euler condition equates the marginal gain in utility today from borrowing to the marginal reduction in utility from repaying tomorrow, taking into account two factors. First, the marginal cost of repaying the debt is relevant only in states where the government chooses to repay, namely, when $D' = 0$. Second, the government takes into account that the bond price depends on the quantity of debt issued, as captured in the second term in square brackets. Given that incentives to repay decline with debt, bond prices are decreasing in borrowing levels $\frac{\partial q^g}{\partial B'} < 0$. The new effect in our framework is the third term in the square brackets and encodes a negative effect that the government's borrowing has on tax revenues. As borrowing B' increases, the government spread increases and passes through to the borrowing costs of firms, which in turn depresses production and tax revenues. This additional cost for government borrowing reduces the

are obtained from varying bargaining parameters.

incentives to borrow. The extent of pass-through is crucial in determining the power of this mechanism. In the next section, we discuss how we use firm-level data to determine its strength.

3 Measuring the propagation of sovereign risk

We are interested in using the model to measure the recessionary effects of sovereign default risk. This effect is governed by several structural parameters in the model, most notably the parameter γ that controls how the financing costs of firms are affected by sovereign interest rate spreads. Before turning to the quantitative analysis, we now discuss in more detail the mechanisms that govern the propagation of sovereign risk to economic activity in the model and why firm-level data may be used to empirically discipline this mechanism.

To this end, we work with a simplified version of the model in which firms are not subject to the risk of default. This simplified model results from the same problem as in equation (11) but with an additional assumption that firms can access equity freely and do not face a nonnegative equity payout requirement. In this case, one can show that firms optimally choose capital and labor in fixed proportion, and that the demand of capital by firm i is

$$k_{i,t} = (M\eta)^{\frac{1}{1-\eta}} r_{i,t}^{\frac{1}{\eta-1}} \{\mathbb{E}[z_{i,t}^{1-\eta}|z_{i,t-1}, p_t]\}^{\frac{1}{1-\eta}},$$

where $M = \left(\frac{r_k\alpha}{w(1-\alpha)}\right)^{\alpha\eta}$ and $r_{i,t} = [1+\gamma s_t^g\lambda_i]\frac{1}{1-\alpha}r_k$ is the rental price of capital. Firms demand less capital when they expect productivity to be low and when their borrowing rate $r_{i,t}$ is high.

Using the production function in equation (1), we can express the output of firm i as

$$y_{i,t} = M z_{i,t}^{(1-\eta)} \eta^{\frac{\eta}{1-\eta}} \left\{[1+\gamma s_t^g\lambda_i]\frac{1}{\alpha}r_k\right\}^{\frac{\eta}{\eta-1}} \left\{z_{i,t-1}^{\rho(1-\eta)}\left[e^{\frac{[(1-\eta)\sigma]^2}{2}}[1+p_t(e^{-(1-\eta)\mu}-1)]\right]\right\}^{\frac{\eta}{1-\eta}}.$$

Taking logs of the above expression, and using the approximation $\log(1+x) \approx x$, we then have

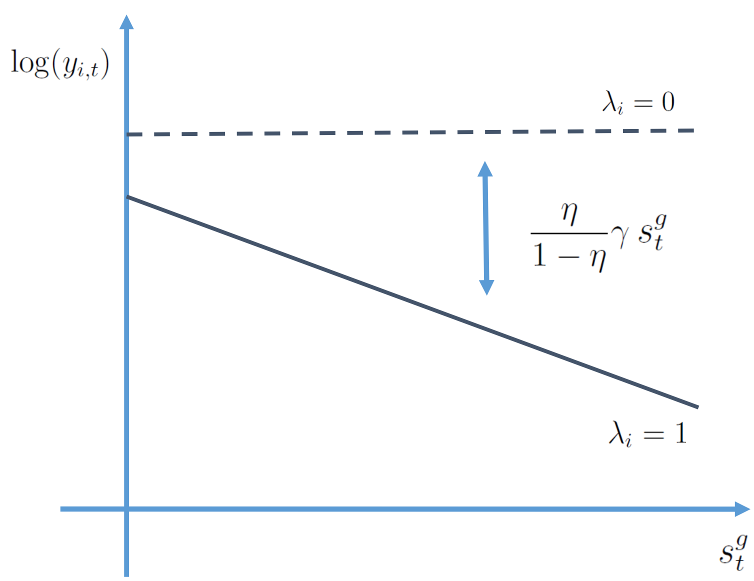
$$\log(y_{i,t}) \approx \overline{y}_i + \rho\log(z_{i,t-1}) + \left[e^{-(1-\eta)\mu}-1\right]\frac{\eta}{1-\eta}p_t + (1-\eta)(\sigma_z\varepsilon_{i,t} - I_i\mu) - \frac{\lambda_i\eta}{1-\eta}\gamma s_t^g, \quad (19)$$

where $\overline{y}_i$ is a convolution of the model's parameters. Equation (19) is the policy functions for firm-level output $y_{i,t} = y(z_t, z_{t-1}, \lambda_i, X_t)$, where $X_t = (\nu_t, p_t, \Lambda_t, B_{t+1})$. Note that $\{\nu_t, B_t\}$ and Λ_t influence the firms' production choices only to the extent that they affect s_t^g.

A key feature of the above equation is that an increase in sovereign risk has adverse effects on firms' production. When $\gamma > 0$, an increase in $s^g(X_t)$ translates into higher borrowing costs for the firms, depressing their production. Importantly, the equation shows that these effects are heterogeneous across firms. Indeed, an increase in borrowing costs is more harmful for the firms that need to finance a larger share of their inputs (high λ firms). This property, in turn, implies that changes in the cross section of firms' output are informative about γ, the key parameter governing the propagation of sovereign risk to output in our model.

Figure 1 describes the logic of our argument. In the figure, we plot firm-level output as a function of sovereign interest rate spreads for two types of firms: a firm that does not need to finance its input costs in advance ($\lambda_i = 0$) and a firm that needs to finance all of its inputs in advance ($\lambda_i = 1$). Suppose now that s_t^g increases, holding fixed aggregate productivity p_t and the firms' idiosyncratic productivity.[13] If $\gamma > 0$, the increase in sovereign spreads is passed on to private sector interest rates, and the firm with $\lambda_i = 1$ will reduce on its factors' demand and decrease production. The production choices of the firm with $\lambda_i = 0$, instead, are not affected by this increase in s_t^g. If $\gamma \approx 0$, instead, the relative performance of the two types of firms would not change as sovereign risk increases. Because of this property, one can indirectly infer γ by studying how the relative performance of firms with high and low borrowing needs evolves during a period of heightened sovereign risk.

Figure 1: Sovereign risk and the cross section of firms' output



In what follows, we build on this insight and we incorporate in the empirical targets the coefficients of a more sophisticated version of equation (19) that we estimate using firm-level data. In view of this discussion, the main empirical target will be the coefficient on the interaction between sovereign interest rate spreads and an indicator of firms' leverage,

[13]This could be accomplished in our model by a reduction in ν_t or an increase in B_{t+1}.

our proxy for the financing needs of firms. Importantly, the nature of our dataset does not allow us to identify a causal relation between sovereign risk and firms' economic conditions. We will nonetheless perform a battery of controls for factors that might potentially bias the sensitivity of firms' performance to sovereign risk based on their financing need. In what follows, we describe in details the factors that might bias the estimation of equation (19), and discuss how we control for them.

First, as seen in equation (19), productivity shocks have no differential effects on firms' output once we condition on s_t^g. This restriction is not necessary for our approach, and it will typically not be true in the full model described in the previous section where firms can default on their debt. In fact, when firms are subject to the risk of default, p_t will typically have differential effects on firms based on their financing needs.[14] Not controlling for these effects in our firm-level regressions leads to a typical omitted variable bias, as p_t might impact sovereign spreads and at the same time have differential effects on firms' performance. In the empirical analysis, we control for this effect by incorporating in the reduced-form regression an interaction between p_t and our indicator of firms' financing needs.

A second challenge is that we do not have a direct measure for λ_i. In our application, we proxy this variable in the data with the ratio of firms' short-term debt to assets. This is clearly a choice variable for firms, introducing the possibility that selection and endogeneity might affect the estimated coefficients. For example, suppose that firms that are highly levered are also less profitable, and hence more sensitive to downturns. Then, finding that highly levered firms decrease drastically their production in periods of high sovereign stress might not necessarily be an indication that financial frictions are important. We control for this issue in three ways. First, as in the indirect inference procedure, we reproduce in model simulations the same exact specification that we estimate in the data. Second, we add in the reduced form regression firms' specific fixed effects which control for selection based on unobserved time-invariant firm characteristics. Third, we conduct extensive robustness analysis in Section 7. Specifically, we document that the interaction coefficient between sovereign spreads and firm leverage in the reduced form regression is robust to the inclusion of several time-varying determinants of firm leverage commonly used in the literature, and to alternative ways of proxying for λ_i.

The third challenge is that our model might be omitting some aggregate factors other than productivity that could affect the interest rate spreads of the government and that might have different effects on firms based on their leverage. Omitting these factors might affect the interaction coefficient between sovereign spreads and firms' leverage in the firm-level

[14] For example, a negative shock to aggregate productivity will increase firms' default risk and hence their interest rates. This would imply that firms with a high λ_i will need to cut their factors' demand by more than firms with low λ_i.

regression, inducing us to overstate or understate the propagation of sovereign risk. While formally incorporating additional aggregate shocks in the model is challenging because of the complexity of its numerical solution, we will nonetheless check the sensitivity of our results to the inclusion of several aggregate factors in our firm-level regressions. All of these robustness checks are reported in Section 7.

4 Data and model parametrization

We conduct our empirical analysis using Italian data. This section discusses data sources and the parametrization of the model. We denote by $\theta = [\theta_1, \theta_2, \theta_3]$ the vector of structural parameters. The parameters $\theta_1 = [\eta, \alpha, \gamma, r_k, \tau, \rho_\nu]$ are set outside the model based on other studies. We use Italian firm-level data to estimate the parameters governing the productivity process for firms, $\theta_2 = [\rho_z, \sigma_z, \mu, \Pi^p(p)]$. Finally, we chose the parameters in $\theta_3 = [\Pi^\xi(\xi), \Pi^\lambda(\lambda), \bar{\nu}, \sigma_\nu, \beta, R, \gamma]$ so that our model matches a set of aggregate and firm-level moments for the Italian economy. We start by describing our data and then discuss the parametrization of $\theta = [\theta_1, \theta_2, \theta_3]$ and indicators of model fit.

4.1 Data

The firm-level data for Italy are obtained from AMADEUS. This dataset provides balance sheet information for public and privately held firms. We have data for the period 2004-2012, restricting our attention to a balanced panel of firms that consists of 178,810 firm-year observations. The sample mostly comprises small and medium sized firms. In the Appendix we describe the construction of our sample of firms.

We use these data for two different purposes. First, we estimate the productivity process $z_{i,t}$ at the firm level and use these series to set the parameters θ_2. Second, we compute a set of moments that is used in the parametrization of θ_3. The variables that we use in this second step are: firm sales growth defined as the ratio of the change in sales relative to the previous year to the average sales in those two years, $\Delta y_{i,t} = (\text{sale}_{i,t} - \text{sale}_{i,t-1})/0.5(\text{sale}_{i,t} + \text{sale}_{i,t-1}))$; firm leverage defined as the ratio of short-term debt to total assets; firm profit ratio defined as the ratio of profits to sales; and firm interest ratio defined as the ratio of interest payments to assets.

We also obtain aggregate time series for the Italian economy. Annual time series for real GDP and public debt are obtained from the *OECD Stat* database from 1980 to 2015. We also obtain an aggregate times series for interest rate spreads on government securities and corporate debt. Interest rate spreads on government securities are the difference between the 10-year yields on the Italian government's bonds and their German counterpart, and the data come from the OECD. The interest rate spreads on corporate debt are the difference in average corporate yields relative to German government yields and are obtained from Gilchrist and Mojon (2016).

4.2 Parametrization of θ_1

We first describe how we select the parameters in θ_1. The parameters of the production function controlling the labor share and the decreasing returns to scale, α and η, are set to 0.67 and 0.85. These are common values in the literature. The rental rate on capital, r_k, is set to 15%, reflecting the annual interest rate plus the depreciation rate. For the utility of public consumption, we use a CRRA utility, $u_g(G) = \frac{G^{1-\sigma}}{1-\sigma}$, with $\sigma = 2$. In terms of taxes, we choose $\tau = 0.2$ to match the average government consumption to output ratio in Italy. Finally, we also set the persistence of the enforcement shock ρ_ν to 0.5, and as we describe below, perform a sensitivity analysis on it.

4.3 Parametrization of θ_2

We now describe the parametrization of θ_2. We estimate the stochastic process for revenue total factor productivity $z_{i,t}$ by estimating the production function of firms at the sector level s. Specifically, we estimate

$$\log(y_{i,t}) = \beta_t(s) + \beta_1(s)\log(\ell_{i,t}) + \beta_2(s)\log(k_{i,t}) + \epsilon_{i,t}, \tag{20}$$

where $y_{i,t}$ is the value added of firm i at time t, $\ell_{i,t}$ is the labor input, and $k_{i,t}$ is the capital. We measure labor input using the wage bill, and we measure capital using the book value of fixed assets. We let $\beta_{s,t}$ be an industry-specific time effect. Industry level s is defined at the two-digit NACE level. We scale value added and the wage bill with a value-added deflator constructed for each two-digit NACE industry using national accounts data from Eurostat. The book value of fixed assets is deflated using the producer price index of Italian domestic investment goods obtained from the FRED database.

We estimate equation (20) using the two-step generalized method of moments implementation of Levinsohn and Petrin (2003) developed in Wooldridge (2009) (see Appendix A). Given the estimates for the coefficients in equation (20), we can compute for each firm in our panel the implied (log) of productivity,

$$\hat{z}_{i,t} = \log(y_{i,t}) - [\beta_{0,t}(s) + \beta_1(s)\log(\ell_{i,t}) + \beta_2(s)\log(k_{i,t})]$$

We then use demeaned firm-level productivity, $z_{i,t} = \hat{z}_{i,t} - (1/T)\sum_{t=1}^{T} \hat{z}_{i,t}$, to estimate the parameters of the productivity process described in equation (2). The short time dimension of our data makes it challenging to estimate the persistence parameter ρ_z. For this reason, we fix ρ_z to 0.9, in line with the results in Foster, Haltiwanger, and Syverson (2008), which applied a similar methodology to a longer panel for the U.S. economy. We also set μ to 0.3, which corresponds to the 5^{th} percentile of the panel data for $z_{i,t}$, as this helps the identification of the probability p_t. Indeed, given ρ_z and μ, we can average across firms' productivity and construct an empirical counterpart to p_t as follows:

$$p_t = -\max\left\{\frac{\overline{z}_t - \rho_z \overline{z}_{t-1}}{\mu}, 0\right\},$$

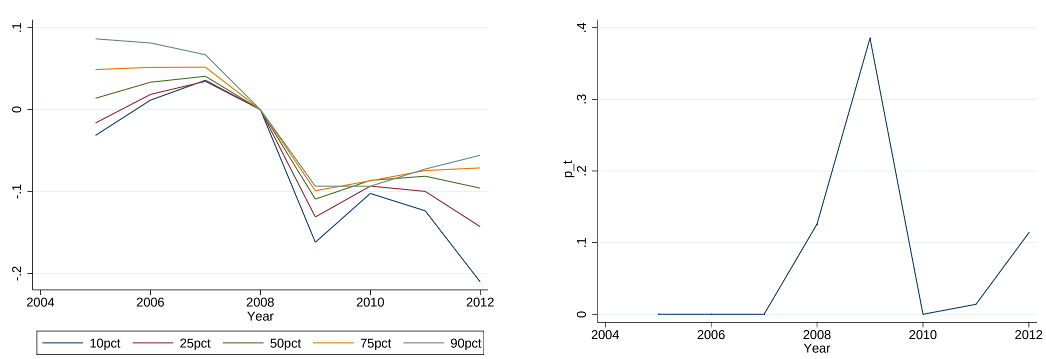
where $\overline{z}_t$ denotes the cross-sectional mean of firms' productivity at time t.

With the time series for p_t at hand, we can next estimate the standard deviation σ_z. Specifically, we compute for every period t in the sample

$$(\hat{\sigma}_z)_t^2 = \text{var}_t[z_{i,t} - \rho_z z_{i,t-1}] - [\mu^2 p_t(1-p_t)].$$

Under our productivity process, the right-hand side of this expression provides an estimate for σ_z. We thus set σ_z to the time average of $\hat{\sigma}_t$. This resulting estimate for the standard deviation of firm productivity innovations is $\sigma_z = 0.074$, which is in line with estimates in Foster, Haltiwanger, and Syverson (2008) for the U.S. economy.

Figure 2: The cross section of firms' productivity in Italy, 2005-2012



(a) Firm Productivity Percentiles (b) Aggregate Productivity Shock

The plots in Figure 2 illustrate the behavior of firms' productivity in our sample. The left panel reports percentiles of the cross-sectional distribution of $z_{i,t}$ for each year t. We have normalized each percentile series to be zero in 2008. We see that average productivity fell sharply in 2008-2009 and recovered little after that.[15] The figure also shows that the decline in productivity was more pronounced for the left tail of the distribution. A similar fattening of the left tail of the productivity distribution can be observed in 2012. Our model fits these distributional dynamics with time variation in p_t. The right panel of the figure plots the p_t process that we recover from the data. Consistent with the distributional plot, p_t displays a sharp increase in 2008-2009 and a somewhat smaller increase in 2012.

We discretize the aggregate and idiosyncratic productivity shocks and approximate their continuous distribution using transitions matrices. Firm-level productivity is discretized into a two-point distribution, $\{z_{\text{low}}, z_{\text{high}}\}$, using Tauchen (1986) criteria. The transition matrix between low and high productivity depends on the aggregate shock p_t. The process for p_t is i.i.d., and it is discretized in three values, $\{p_{\text{low}}, p_{\text{medium}}, p_{\text{high}}\}$. The associated values and respective probabilities, $\{\pi^p_{\text{low}}, \pi^p_{\text{medium}}, \pi^p_{\text{high}}\}$, are chosen from the constructed empirical p_t series.

4.4 Parametrization of θ_3

We now discuss the parametrization of the remaining parameters θ_3. We select these parameters so that our model matches a set of empirical targets. In view of the discussion in Section 3, we start by measuring the elasticity of firms' output to sovereign interest rate spreads as a function of firms' leverage. This will be the empirical target that disciplines the pass-through of sovereign risk to real economic activity in the model. We also describe the other empirical targets, the parameter estimates, and indicators of model fit.

4.4.1 Firms' sales and sovereign risk

We estimate an empirical counterpart to equation (19) using firm-level data. We run regressions of firm economic performance on aggregate and firm-level time series. Our benchmark regression specification is

$$\Delta y_{i,t} = a_i + a_1 s^g_t + a_2 s^g_t \times \text{lev}_{i,t} + a_3 p_t + a_4 p_t \times \text{lev}_{i,t} + a_5 \text{lev}_{i,t} + \eta_{i,t}, \tag{21}$$

where $\Delta y_{i,t}$ denotes the growth rate of sales for firm i at time t, $\text{lev}_{i,t}$ is the leverage for firm i at time t, s^g_t is the interest rate spreads on Italian government bonds, and p_t is our retrieved productivity shock. The regression includes firm leverage as a control and also firm-specific fixed effects. We estimate the regression with standard errors clustered two ways, across t

[15]In our estimates, the average decline in TFP between 2008 and 2009 is of the order of 10%. This is consistent with OECD data (*Productivity and ULC by main economic activity*) showing a decline in value added per hours worked of 7.1% in manufacturing during the same period.

and i. The coefficient of interest for the differential responses of firms to sovereign spreads is a_2. As discussed in Section 3, we expect this coefficient to be negative. Firms with high leverage are expected be more sensitive to fluctuations in sovereign spreads. We also allow for differential responses of firms to the productivity shock p_t based on leverage.

Table 1: Sales growth, government spreads, and leverage

	Benchmark (1)	Spread Only (2)	Productivity Only (3)
s_t^g	-0.82^{***}	-1.45^{**}	
	(0.18)	(0.70)	
$s_t^g \times \text{lev}_{i,t}$	-1.70^{***}	-2.19^{***}	
	(0.32)	(0.78)	
p_t	-0.66^{***}		-0.67^{***}
	(0.03)		(0.03)
$p_t \times \text{lev}_{i,t}$	0.11^{***}		0.03
	(0.04)		(0.08)
$\text{lev}_{i,t}$	27.90^{***}	40.91^{***}	28.95^{***}
	(4.31)	(13.29)	(3.83)
Firm fixed effects	yes	yes	yes
Adjusted R^2	16	5	15

This table reports the regression of firms sales growth $\Delta y_{i,t}$ on sovereign spreads s_t^g, aggregate productivity measured by disaster risk p_t, firm leverage $\text{lev}_{i,t}$, interactions of firm leverage with spreads and productivity, and firm-specific fixed effects. Standard errors are clustered two ways, across t and i. See the Appendix for the definition of variables.

Column (1) of Table 1 reports the results for our benchmark specification. Periods during which interest rate spreads on government debt are high are associated, on average, with poor economic performance in firms: an increase of 100 basis points in interest rate spreads is associated with a decline of 1.45% in the sales growth rate of the firm with average leverage in the sample of 37% (1.7% $\times$ 0.37 + 0.82%). Importantly, the association between firms' growth and government spreads is stronger for firms with higher leverage: the interaction coefficient a_2 is significantly negative. In terms of magnitudes, sales growth falls by 0.54% more for firms with leverage equal to 52% (the 75^{th} percentile of the leverage distribution) relative to firms with leverage equal to 20% (the 25^{th} percentile) for an increase of 100 basis points in the spreads.

Periods of low aggregate productivity, which correspond to high p_t, are also associated with lower sales growth for firms: a 1% increase in the probability of a large productivity drop is associated with a decline of 0.66% in firms' growth. The coefficient a_4 on the interaction of firms' leverage with productivity is small and significantly positive. This means that the high leverage firms decline their sales growth by less when the probability of low productivity is high. The interaction coefficient between productivity and leverage is small in magnitude, and as we will see in the robustness analysis, it is not stable across specifications.

In columns (2) and (3), we report simplified versions of equation (21) where the right-hand-side variables are exclusively the government interest rate spread s_t^g or productivity p_t and its interaction with firms' leverage. The results are similar to those in the benchmark specification.

In Section 7 we report the robustness analysis for this result. We show that these results are robust to alternative definitions of leverage, adding additional firm controls, and adding other aggregate time series including indexes for volatility, VIX, the European stock market, and Italian bank stocks. Under each of these specifications, the interaction of government spreads and firm leverage continues to be significantly negative.

As discussed in Section 3, the differential response of firms' output to an increase in interest rate spreads (conditional on aggregate productivity) provides information on the parameter γ, which governs the strength of the pass-through of sovereign risk on private sector interest rates. Therefore, we include the estimated coefficient a_2 as a target in the moment-matching exercise.

4.4.2 Method of simulated moments

The parameters in $\theta_3 = [\Pi^{\xi}(\xi), \Pi^{\lambda}(\lambda), \bar{\nu}, \sigma_\nu, \rho_\nu, \beta, R, \gamma]$ are estimated using the method of simulated moments. The empirical moments that we target include both aggregate and firm-level statistics. Among the aggregate statistics, we include the mean and standard deviation of government interest rate spreads, the mean of the short-term public debt-to-output ratio, and the average recovery rates historically observed during debt crises and documented in Cruces and Trebesch (2013). Among the firm-level moments, we target the sample average of firms' profits over sales, the 25^{th} and 75^{th} percentile in the distribution of firm leverage, the average spreads on corporate debt, and the estimated coefficient a_2 in regression (21).

We solve the model using global methods. The algorithm embeds the recursive problem of domestic firms inside the recursive problem of the government. The complication arises because both sets of agents can default, and their respective bond prices reflect their default probability. The Appendix provides a detailed description of the algorithm.

We discretize the distribution for λ, ξ, and ν as follows. We consider a two-point distribution for $\lambda = \{\lambda_{\text{low}}, \lambda_{\text{high}}\}$ and assign equal probabilities to these two values. The stochastic process for firms' fixed cost is assumed to be Gaussian, with parameter $\xi \sim \mathcal{N}(\bar{\xi}, \sigma_\xi)$, and discretized into 500 points. Finally, the ν shocks are discretized into 50 points. For given $\{\bar{\xi}, \sigma_\xi, \bar{\nu}, \sigma_\nu, \rho_\nu\}$, the resulting discretized values and transition matrices follow Tauchen's (1986) method.

Given the model policy functions, we then compute moments by simulating a long realization ($T = 5000$) from the model, and computing sample statistics on the simulated data. As for the model-implied value for a_2, we estimate in the model the same exact panel regression as we have done in the data, equation (21), and store the coefficient on the interaction between government spreads and firms' leverage. In the model simulations, we define sales growth and firm leverage as in the data, namely, $\Delta y_{i,t} = (y_{i,t} - y_{i,t-1})/0.5(y_{i,t} + y_{i,t-1})$ and $\text{lev}_{i,t} = b_{i,t}/k_{i,t}$. The parameters in θ_3 are then chosen so that our model matches the empirical targets. Specifically, we construct a weighted distance between the moments in the model and their corresponding counterparts in the data, and choose the θ_3 that minimizes such distance. Table 2 summarizes all values for the parameters $\theta = \{\theta_1, \theta_2, \theta_3\}$.

Table 2: Parameter Values

Assigned Parameters θ_1		
Labor share	$\alpha = 0.67$	National accounts Italy
Markup	$\eta = 0.85$	Basu and Fernald (1997)
Return to capital	$r_k = 0.15$	Annual interest plus depreciation
Utility curvature	$\sigma = 2$	Standard business cycle model
Tax rate	$\tau = 0.2$	Public consumption to output Italy
Persistence enforcement shock	$\rho_\nu = 0.5$	Perform sensitivity
Productivity Parameters θ_2		
Firm persistence z	$\rho_z = 0.9$	Foster, Haltiwanger, and Syverson (2008)
Firm volatility z	$\sigma_z = 0.074$	AMADEUS dataset, Italy
Aggregate p shocks	$p_t = \{0, 0.12, 0.37\}$	AMADEUS dataset, Italy
	$\pi^p = \{0.6, 0.33, 0.07\}$	AMADEUS dataset, Italy
Productivity decline	$\mu = 0.3$	AMADEUS dataset, Italy
Parameters from Moment Matching θ_3		*Italian Data*
Volatility revenue shock	$\sigma_\xi = 0.3$	Mean firm spread
Mean revenue shock	$\bar{\xi} = 3.75$	Mean firm profit/sale
Financing requirement	$\{\lambda_1 = 0.4, \lambda_2 = 1.0\}$	Firm leverage 25^{th} and 75^{th} percentiles
Mean enforcement shock	$\bar{\nu} = 1.0$	Short public debt / output
Volatility enforcement shock	$\sigma_\nu = 0.3$	Mean govt spread
Government discount factor	$\beta = 0.88$	Volatility govt spread
Debt recovery	$R = 0.13$	Cruces and Trebesch (2013)
Pass-through coefficient	$\gamma = 0.4$	Regression coefficient

While all parameters are chosen simultaneously in our procedure, we can give a heuristic description of the moments in the data that are particularly important for determining specific parameters. The mean and standard deviation of the fixed cost distribution have tight implications for firms' profits and their likelihood of default. As such, these two parameters are mostly informed by the average firms' profit-to-sale ratio and the firms' average interest rate spread. The two points of the firms' financing needs distribution, λ_{low} and λ_{high}, are informed from the distribution of firms' leverage in the data, specifically the 25^{th} and 75^{th} percentiles. The mean and standard deviation of the enforcement shock ν_t have a strong connection with the mean and standard deviation of sovereign spreads. The discount rate of the government is informed by the mean government spread and the debt-to-output ratio of the government. The recovery parameter R in the model controls the average debt recovery rate. Finally, as we have discussed previously, the parameter γ has implications for a_2 in the firm-level regression (21).

Table 3 reports the target moments in the model and the data. Overall, the model matches the sample statistics well. The model average and standard deviation of interest rate spreads on government debt of 1.9% and 0.9% are similar to the data. The model also generates an average debt-to-output ratio and recovery rates that are similar to those in the data. The model does a good job in terms of firm-level moments too. The model average firms' profit-to-asset ratio of 2.5% is close to the 3% in the data; the 25^{th} and 75^{th} percentiles of the firm leverage distribution of 20% and 50% are exactly matched. The firms' spread of 1.8% in the model is similar to, although a bit lower than the data counterpart of 2.5%. Importantly, the model matches the interaction coefficient from the firm-level regressions in equation (21). In the model, a 1% increase in the government spread is associated with a lower growth rate of -0.57% for firms with leverage at the 75^{th} percentile relative to those at the 25^{th} percentile.

Table 3: Target Moments in Model and Data

	Data	Model
Govt spread mean	1.8	1.9
Govt spread volatility	1.1	0.9
Short debt/output	25	28
Recovery	60	54
Firms profits/sales	2.5	3
Firms leverage 25 and 75 pct	[20,50]	[20,50]
Firms spread	2.3	1.8
Interaction coeff$\times[\overline{\lambda}_{75,t} - \overline{\lambda}_{25,t}]$	-0.54	-0.57

In Table 4, we also report some additional implications of our model for aggregate and firm-level moments and compare them with the data. In terms of aggregate series, the table

Table 4: Other Moments in Model and Data

	Data	Model
Aggregate Moments		
Corr (Firm spread, Govt spread)	82	98
Corr (Output, Govt spread)	-54	-79
Firm Moments		
Sales Growth (St. Dev.)	26	30
Leverage (St. Dev.)	21	14
Profits/Assets (St. Dev.)	8	2
Interest/Assets (St. Dev.)	1	0.4
Corr (Leverage, Profits/Assets)	-18	-6
Corr (Leverage, Interest/Assets)	40	96
Corr (Profits/Assets, Sales Growth)	28	14

shows that government spreads are positively correlated with the average firm spreads and negatively correlated with output in both the model and the data. The magnitudes of these correlations, however, are stronger in the model than in the data. In terms of firm-level statistics, the table shows that the model generates standard deviations of sales growth and leverage that are similar to the data. The standard deviations of the ratio of profits and interest payments to assets are sizable in the model, but smaller than they are in the data. In terms of correlations, the model generates a negative correlation between leverage and profit to assets, and positive correlations of leverage and interest payments to assets, and profit to assets and sales growth. The firm-level data feature similar correlations, although the magnitudes here are also smaller.

5 Model Mechanisms

This section analyzes our model mechanisms. We start by presenting the decision rules of firms and of the government under the benchmark parametrization, and compare them to a version of the model in which all parameters are kept the same, with the exception that $\gamma = 0$. This comparison highlights the differences between our environment with pass-through of sovereign risk to the private sector and the one typically studied in the literature with no pass-through. We then present impulse response functions of key macroeconomic variables to the two aggregate shocks, productivity and enforcement.

5.1 Firms' decision rules

We start by analyzing the decision rules of firms as a function of government borrowing. Doing so isolates the effects of the government borrowing policy on the private sector. Recall that an increase in government borrowing in our model, as in standard sovereign default models, increases the sovereign interest rate spreads because of a higher likelihood of default.

The top left panel of Figure 3 plots firms' choice of capital $k(z_{-1}, \lambda, \nu, p, \Lambda, B')$ as a function of government borrowing B'. In our model, firms choose capital k and labor ℓ in proportion, and hence it is sufficient to discuss only the former. We set the idiosyncratic shock to z_{high}, the enforcement shock ν to the median level, the aggregate productivity shock to the first level $p = p_{\text{low}}$, and the distribution of firms Λ to the one that arises after a sufficiently long sequence of ν and p at those levels. The solid line reports the capital choices for firms with low financing needs ($\lambda = \lambda_{\text{low}}$) and the circled line for firms with high financing needs ($\lambda = \lambda_{\text{high}}$).

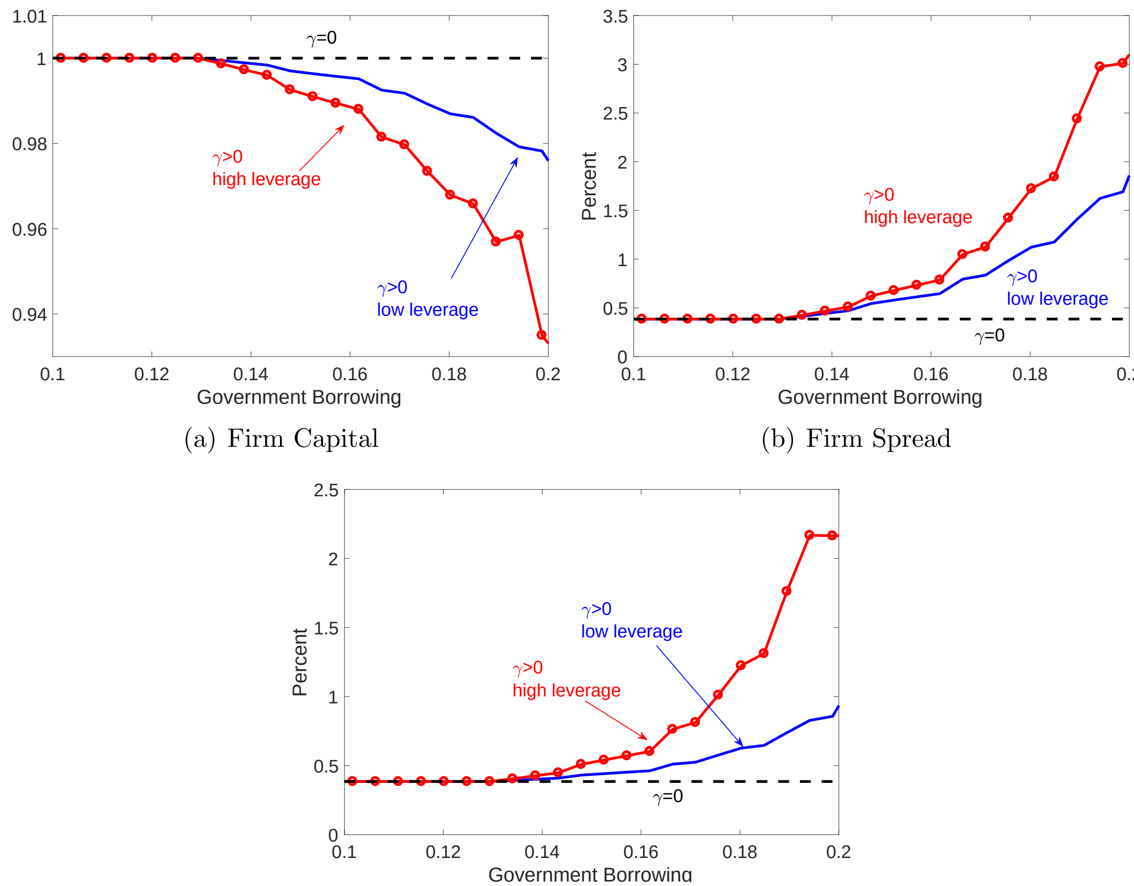
Consider first the benchmark parametrization with pass-through, labeled $\gamma > 0$. The figure shows that firms choose smaller levels of capital as B' increases, with the effects being more pronounced for firms that have higher financing needs. This latter result confirms that the mechanisms emphasized in Section 3 operate in the full version of our model.

The crowding out of private sector investment arises because the increase in government borrowing leads to higher sovereign interest rate spreads, which are then passed on to firms and depress their demand for inputs. This can be seen in the top-right panel of Figure 3, where we plot the firm interest rate spread, defined as $1/q(b^*, k^*, \ell^*, z_{-1}, \lambda, B') - 1$, as a function of B'.[16] We can see that private sector spreads of both types of firms increase in government borrowing. This increase in the borrowing costs of firms arises for two reasons. First, the default-free component of the firms' bond price, represented by the denominator in equation (10), increases with sovereign spreads and hence with B'. Second, the default probabilities of the firms also increase with B'. This second effect can be seen in the bottom panel of Figure 3, which plots the equilibrium default probability of firms as a function of B'. Such an increase in firms' default risk arises because higher government borrowing and spreads reduce firms' profits and increase their leverage, placing them at higher risk of a default.

Importantly, this second effect makes interest rate spreads of highly levered firms more sensitive to B' relative to those of less levered firms because the former are more at risk of a default when sovereign spreads rise. Thus, the larger decline in production of highly levered firms results from higher firm interest rates due to pass-through and also due to the more

[16] These are equilibrium firm spreads because they are evaluated at the firms' optimal choices of debt, capital, and labor (b^*, k^*, ℓ^*) given the state. These optimal choices vary with B'.

Figure 3: Firms' decisions as function of government borrowing



(a) Firm Capital

(b) Firm Spread

(c) Firm Default Probability

elevated risk of default relative to less levered firms.

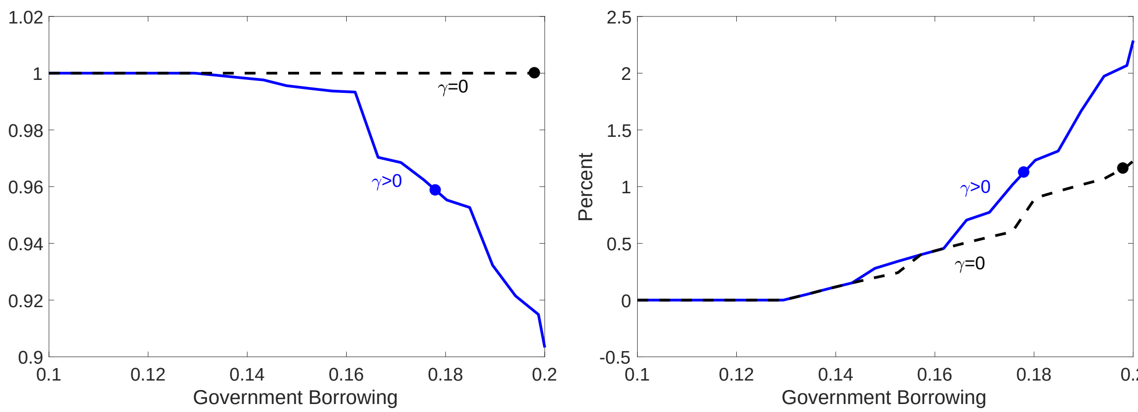
Now consider the firm decision rules with no pass-through, $\gamma = 0$. In stark contrast to our benchmark model, firms' decisions in this case are independent of government policies and also do not vary with their leverage. As we explore below, the extent of pass-through also has important implications for the government's borrowing and default policies.

5.2 Government spread and tax schedules

We now describe the government spread and tax schedules, as well as its borrowing policy. The government spread schedule is the inverse of the bond price schedule relative to the risk free rate, $s^g(\nu, p, \Lambda, B') = 1/q^g(\nu, p, \Lambda, B') - (1 + r)$, and maps borrowing choices B' into government spreads given the aggregate shocks and the distribution of firms. The tax schedule $T(\nu, p, \Lambda, B')$ is defined in equation (15) and describes how tax revenues vary with the borrowing choices of the government B'. We present these schedules for our benchmark parametrization and for the version of the model without pass-through.

The solid lines in Figure 4 plot the spread and tax schedules as a function of government borrowing B' in the benchmark parametrization. The dashed lines are the schedules in a model with no pass-through, $\gamma = 0$. We set the other states as we have done in the private sector decision rules, namely ν to the median level, the productivity shock at p_{low}, and the distribution of firms Λ to the one that arises after a sufficiently long sequence of ν and p at those levels.

Figure 4: Government Tax and Spread Schedule



(a) Tax Schedule (b) Spread Schedule

The left panel in the figure shows that tax revenues fall with government borrowing in the benchmark parametrization. As explained earlier, an increase in government borrowing raises the interest rate at which firms borrow and depresses their production. The reduction in firms' output implies a reduction in the tax revenues that the government collects. In contrast, in a model without pass-through, tax revenues do not depend on the borrowing behavior of the government.

The right panel in the figure shows that spreads increase with government borrowing. Higher borrowing is associated with a higher likelihood of a government default next period, and hence interest rate spreads rise to compensate for the losses that lenders face in a default. This relation between borrowing and spreads is standard in the sovereign default literature. In our framework, however, these schedules are influenced by the crowding out of public debt on private sector production. This can be seen by comparing the schedules in the benchmark parametrization with those of a model without pass-through. The feedback from government policies to the private sector implies a tighter spread schedule relative to the case of no pass-through. The reason is that large borrowing today B' implies a reduction in firms' output and in the tax revenues of the government. Because these effects are persistent, output is forecasted to be depressed in the future, and this raises the probability that the government will default next period.

Faced with these spreads and tax schedules, the government makes its choice of borrowing B'. We illustrate the optimal B' chosen by the government with a dot in the figure when B equals its ergodic mean in the benchmark economy. As the figure shows, in the benchmark parametrization the government chooses smaller borrowing levels than in the model without pass-through. The main reason, as illustrated in equation (18), is that the government internalizes that larger borrowing has an extra cost as it reduces tax revenues, and this extra cost disciplines its borrowing behavior.

The tax schedule also illustrates that our model features a sovereign debt overhang effect, in that higher borrowing by the government has negative effects on output. Recall that tax revenues in our model are a constant fraction of aggregate output, and hence the tax revenue schedule in Figure 4 also encodes the negative elasticity of aggregate output with respect to public borrowing. Increases in government borrowing depress output, and these effects are nonlinear. The figure shows that when borrowing increases from very low levels, aggregate output does not change, but after borrowing crosses a threshold, output decreases rapidly. The magnitudes of the sovereign debt overhang effect depend on the borrowing choices and the aggregate state of the economy.

5.3 Impulse response functions

Having presented some key features of the firms' and the government decision process, we now study the aggregate behavior of this economy by presenting impulse response functions to the two aggregate shocks in the model, enforcement and productivity. Here too, we compare the impulse response functions in our benchmark parametrization to those in a model without pass-through. We consider the responses of aggregate output and of interest rate spreads for the government and firms. The firm spread is the average spread across all firms in the economy.

We construct the impulse response functions in our nonlinear model following Koop, Pesaran, and Potter (1996). Specifically, we simulate $25,000$ paths for the model for $1,000$ periods. From periods 1 to 499, the aggregate shocks follow their underlying Markov chains. In period 500, the impact period, we feed in the shock of interest. For the impulse responses to the enforcement shock, we consider a value for ν equal to one standard deviation below the mean. This corresponds to a decline in the enforcement of government debt. For the impulse responses to the productivity shock, we set $p = p_{\text{high}}$ in period 500, which corresponds to an increase in the probability of a large decline in firms' productivity. From period 501 on, the aggregate shocks follow the conditional Markov chains. The impulse responses plot the average, across the 25,000 paths, of the variables from period 499 to 510. The shocks and output are reported in percentage deviations from their value at $t = 499$, while the spreads are in percentage points and reported as a difference from their value at $t = 499$.

Figure 5: Impulse response functions to a decline in the enforcement of public debt

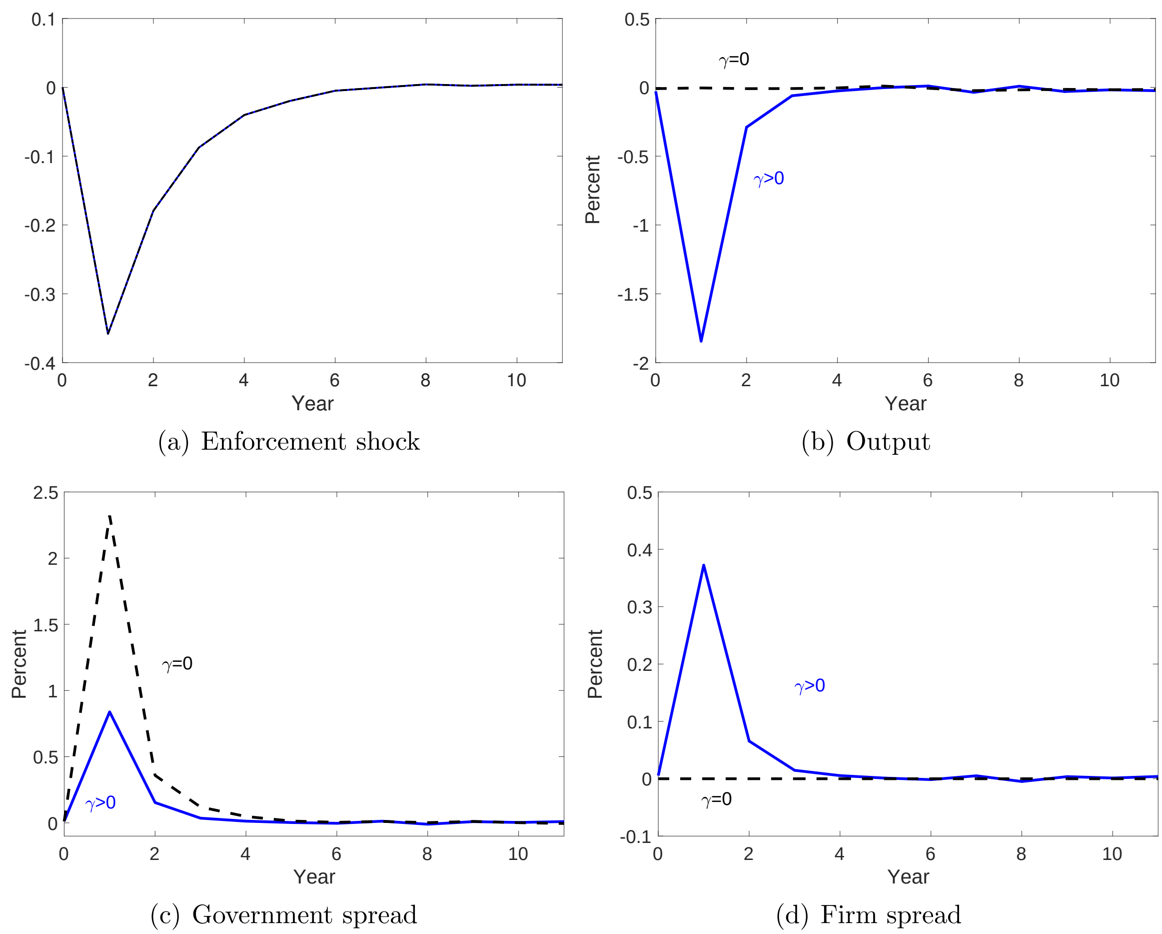
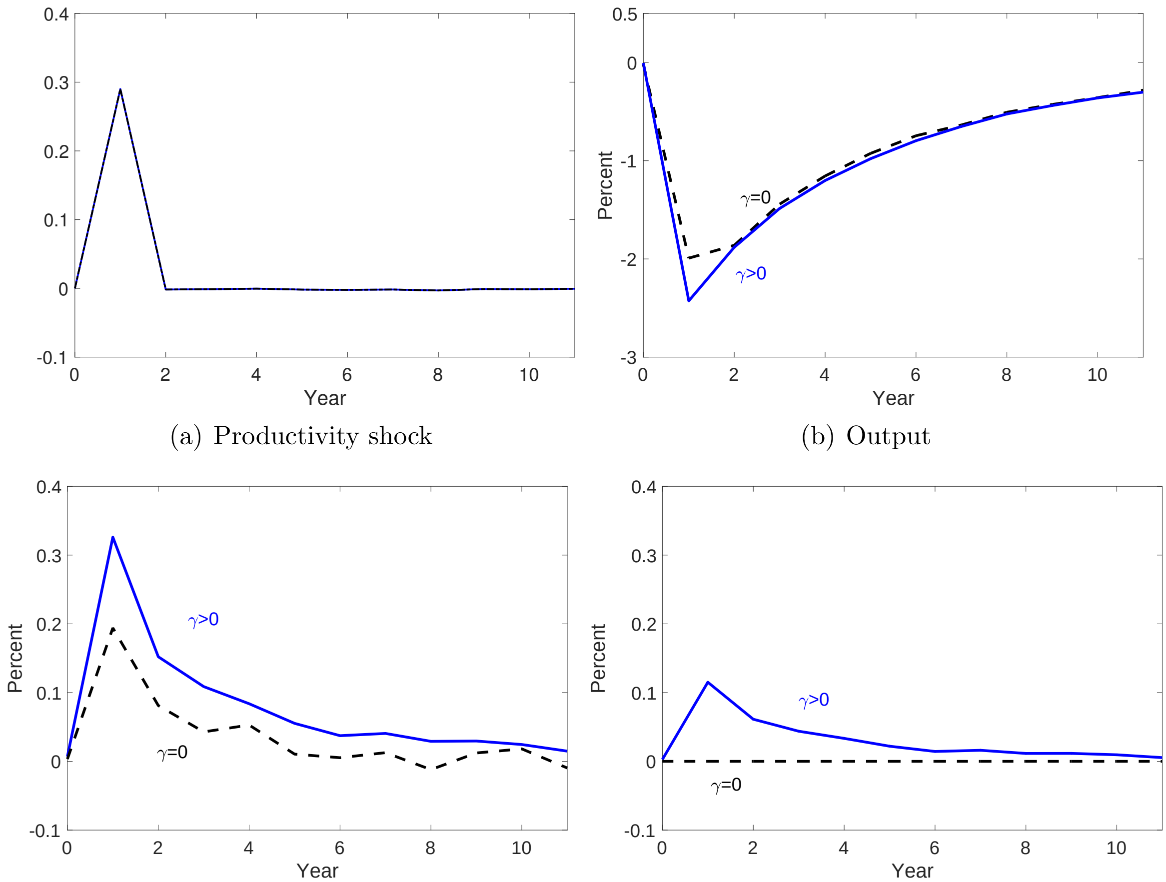


Figure 5 plots the responses to a decline in enforcement. The shock falls on impact and returns back to the mean in period 8. Consider first the benchmark parametrization represented by solid lines. Aggregate output declines by 1.8%, government spreads increase by almost 1%, and the average firm spreads rise by 0.4%. The persistence of the variables mirrors the persistence of shocks which return back to their mean in about five periods. The impulse responses in the model without pass-through, indicated by the dotted lines, are quite different. Government spreads rise more than in the benchmark, almost to 2.5%, yet the spreads in the private sector and aggregate output do not respond.

The benchmark model contains a two-way feedback loop between public and private choices, which amplifies fluctuations in output and spreads. The impulse responses to the enforcement shock starkly illustrate this interdependence. A low enforcement shock tightens the government spread schedule and increases spreads in equilibrium. The increase in government spreads increases firm spreads, and it leads to a reduction in aggregate output and higher private default risk. Low aggregate output lowers tax revenues, which in turn further tightens the government spread schedule. The magnitude of this interdependence is controlled by the parameter γ, as illustrated in these impulse responses.

Figure 6: Impulse response functions to a decline in aggregate productivity



(a) Productivity shock

(b) Output

(c) Government spread

(d) Firm spread

Figure 6 plots the impulse responses to productivity shocks in the benchmark model and in the model without pass-through. The probability of a large productivity drop increases from about 9% to about 37% for only the impact period. In the benchmark model, aggregate output declines by 2.5% on impact, government spreads increase by 0.3%, and firm spreads rise by about 0.1%. These responses are persistent because although the negative productivity shock is i.i.d., its effect on the distribution of firm productivity is persistent. The impulse responses in the model without pass-through are qualitatively similar, although the response on spreads is muted. Aggregate output falls by about 2%, government spreads rise by 0.2%, and firm spreads barely change.

The two-way feedback loop between the public and the private sector is also active in response to productivity shocks. A low productivity shock lowers aggregate output and tax revenues, which in turn increases government spreads. High government spreads translate into higher firm spreads and lower aggregate output, which further reduces tax revenues. In the model with $\gamma = 0$, productivity shocks still affect the government because of their effect on tax revenues, but it does not feed back into the private sector.

6 Quantitative Experiments

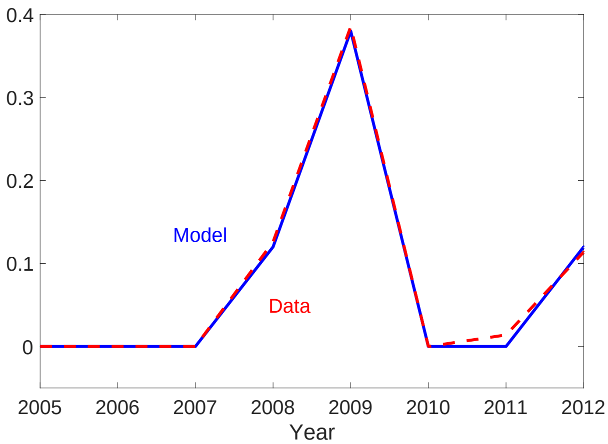
This section uses our model to study the evolution of the Italian economy from 2005 to 2012. We quantitatively compare the predictions for aggregate output, government spreads, and firm spreads of our benchmark model to the data. We then perform a counterfactual exercise to measure the contribution of sovereign risk pass-through on the evolution of these variables. We also conduct an additional counterfactual where we analyze the effects on aggregates from reducing government borrowing and provide a measure of the government borrowing multiplier. Finally, we conduct a variance decomposition analysis.

6.1 The propagation of sovereign risk in the Italian debt crisis

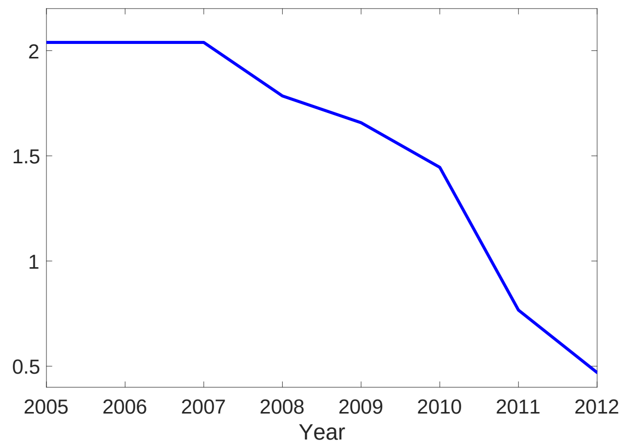
We now use the calibrated model to measure the propagation of sovereign risk to the real economy during the Italian debt crisis. Our counterfactual experiment proceeds in two steps. In the first step, we construct a time path for the two aggregate shocks, productivity and enforcement. In the second step, we feed these two time series into the calibrated model and into the restricted version of the model without pass-through. By comparing the behavior of endogenous variables across these two parametrizations of the model, we are able to net out the effects of the sovereign crisis on the variables of interest.

Regarding the first step, we have constructed in Section 4.3 a time series for p_t using firm-level data. We do not, however, have an empirical counterpart for the enforcement shock. Hence, we use the structure of the model to measure it as follows. We start the model with $p = p_{\text{low}}$, ν at the median level and choose the level of government B and distribution of firms Λ to be the ones that arise after a sufficiently long sequence of ν and p at those levels. We then feed into the model the aggregate productivity series, and we choose the sequence of ν_t shocks that guarantees that interest rate spreads on government debt in the model match, as closely as possible, those observed in the data. Figure 7 plots the time paths for the productivity and enforcement shocks. We can verify that the model calls for a progressive deterioration of enforcement after 2008 in order to reproduce the rise in sovereign interest rate spreads observed in the sample.

Figure 7: Productivity and enforcement shocks in the event study



(a) Productivity shock



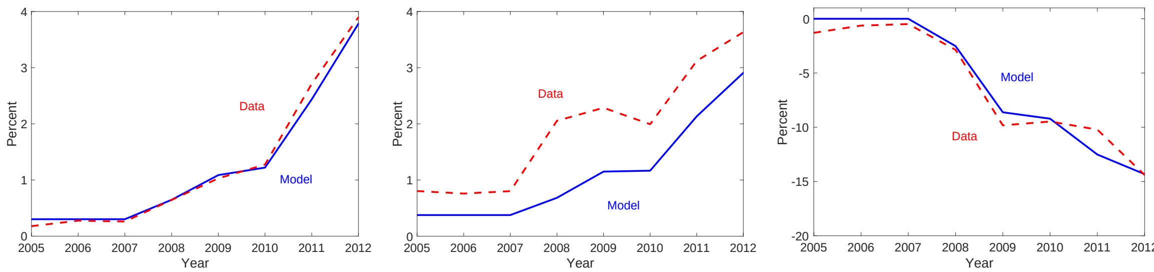
(b) Enforcement shock

Having obtained the sequence of structural shocks, we can next proceed with the second step of the counterfactual. We feed into the model the sequence of shocks in Figure 7 and track the behavior of government spreads, average firm spreads, and aggregate output for two different parametrizations of the model: the benchmark and the model without pass-through. Figure 8 reports the results of this experiment. Starting from the top panels, we can verify that the benchmark parametrization tracks the behavior of these three variables in the event. This is not surprising for government interest rate spreads because we have chosen the sequence of ν_t shocks to reproduce the path of this variable in the sample. What is surprising, instead, is the fact that the model-implied series for firm spreads and output closely track the ones in the data. This latter result indicates the good out-of- sample fit for our model, as we did not include indicators of volatility of firm spreads and aggregate output in the estimation.

The bottom panels in Figure 8 report the behavior of these three variables in the $\gamma = 0$ parametrization. We wish to highlight two main results. First, in absence of a sovereign risk pass-through, the recession in Italy would have continued to be sizable but substantially milder. The borrowing costs of firms would not have been affected by the debt crisis, and firms' production would have been less depressed. By 2012, output would have been about 7% below trend, whereas in the data it was 14%. Our measurement strategy therefore suggests a strong propagation of sovereign risk to real economic activity.

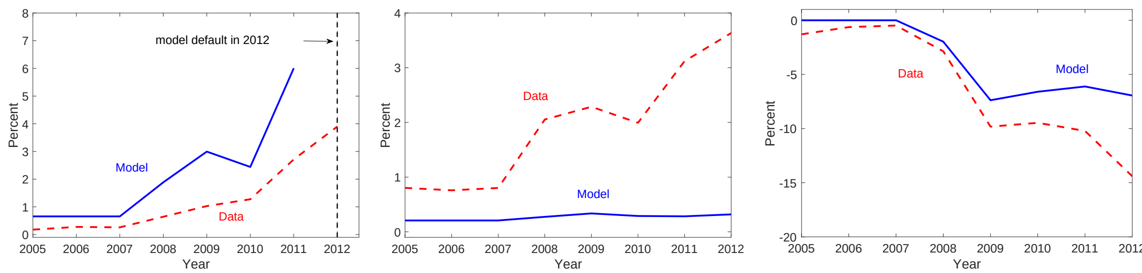
Figure 8: Event study: interest rate spreads and output

Benchmark parametrization



(a) Government spread (b) Firm spread (c) Government spread

No pass-through ($\gamma = 0$)



(d) Government spread (e) Firm spread (f) Government spread

The second result is that, in the absence of a sovereign risk pass-through, the Italian government would have been more at risk of default. In this counterfactual, sovereign spreads reach 6% in 2011 and the government defaults on its debt in 2012. This path arises because of the different endogenous behavior of fiscal policy. When $\gamma > 0$, the government internalizes that high interest rate spreads lead to lower output and lower fiscal revenues, a force that curbs borrowing and sovereign spreads. When $\gamma = 0$, tax revenues are instead independent of the behavior of interest rate spreads. The government thus has fewer incentives to restrict spending, and the more lax fiscal policy implies a higher risk of default relative to the benchmark parametrization.

The top panel in Table 5 summarizes these results. We report the change in government spreads, firms spreads, and output during the period 2007-2012 in the data, in the benchmark model, and in the model with no pass-through. Recall that the benchmark model is set to mirror the increase in government spreads of around 3.5% during this event. The model then predicts an increase in firm spreads of 2.5%, very similar to the increase observed in the data of 2.8%. From this increase, about 1.1% is due to an increase in the default probability of firms and 1.4% reflects the higher borrowing cost arising from the increase in government spreads. In terms of output, the model predicts a decline comparable to the data of 14%. Turning to the model with no pass-through, we see from the table that the responses are more muted. Output declines by 7%, government spreads increase by about 4% (computed from 2008 until 2011), and private spreads increase by only 0.1% in this model. This counterfactual confirms that 51% of the output decline is explained by the pass-through that sovereign risk has on real economic activity. This mechanism also accounts for most of the increase in the firm spreads.

6.2 Additional counterfactuals

We now perform an additional experiment to analyze the effects of reducing government borrowing during the event. By doing so, we measure the importance of sovereign debt overhang effects on the Italian economy during the episode. Specifically, we fit the benchmark model with the aggregate shocks in Figure 7, but force the government to issue 10% less debt every period relative to the path of borrowing in the benchmark.

The lower panel in Table 5 reports the changes in aggregate variables during the event. Less government borrowing during the event dampens the increases in government and firms' spreads as well as the reduction in output, although the changes in these variables are still quite sizable. With 10% less borrowing, government and firms' spreads would increase by 1.5% and 1.7%, respectively, and the output fall would be close to 10%. Comparing these changes with those in the benchmark model, we can see that a 1% decline in new borrowing leads to a 0.46% increase in output $((14.3 - 9.7)/10 = 0.46)$. Of course, in our model this reduction in borrowing is not optimal for the government because it implies a severe reduction in public goods. Nevertheless, our analysis highlights that increases in government expenditures financed by borrowing carry substantial negative effects driven by a sovereign risk pass-through.[17]

Next we perform a variance decomposition analysis for the aggregate variables of interest in the benchmark model. To this end, we evaluate the model predictions for the event when we feed into the model only one shock series at a time, p_t or ν_t, holding constant the other shock. The last two rows in Table 5 report the change in these variables when only one of the shocks is operative.

These results show that productivity shocks p_t generate much smaller increases in interest rate spreads than the benchmark for both the government and firms.[18] We can also see that

[17]Our model abstracts from potential positive effects of government expenditures multipliers as in House, Proebsting, and Tesar (2017). They use a two country DSGE model with nominal rigidities and financial frictions to show that austerity measures during the Great Recession was detrimental for countries in southern Europe in the context of the zero lower bound.

[18]This result underscores the importance of time variation in default costs for capturing the dynamics of

the decline in output induced by productivity shocks of 7.6% is about 53% of the decline in the benchmark. The output decline in this model is only slightly larger than the one predicted in the model with no pass-through because only minor variations in government interest rate spreads feed back to the private sector.

The results with only enforcement shocks ν_t show that these shocks generate sizable increases in government and firms spreads, although a bit less than 50% than in the benchmark. These shocks also generate sizable decreases in output of 6.5%, which equals 44% of the decline in the benchmark model. Hence, shocks to enforcement contribute substantially to the increase in interest rate spreads of the government and, because of their effect on the borrowing rates paid by firms, they have a sizable recessionary impact on the economy.

Table 5: The Italian debt crisis, 2007-2012

	Government spreads	Firm spreads	Output
Data	3.6	2.8	-13.9
Benchmark model	3.5	2.5	-14.3
No pass-through	4.1	0.1	-7.0
Other counterfactuals			
Less govt borrowing	1.5	1.7	-9.7
Benchmark only p shocks	0.4	0.4	-7.6
Benchmark only ν shocks	1.6	1.2	-6.3

Output is reported as the percentage deviation between 2012 and 2007. Spreads are reported as the difference between their 2012 and 2007 values. Spreads are measured in percentage points.

7 Robustness of Empirical Findings

In this section, we provide a robustness analysis of our empirical finding that more levered firms have lower sales growth rates when government spreads are high. Although we do not interpret these findings as causal, a main concern for teasing out such a correlation is the endogeneity of firm leverage and government spreads.

In Table 6, we address the endogeneity of leverage by providing results for alternative specifications with various definitions for firm leverage and with additional firm controls and interactions. All specifications include firm fixed effects. Column 2 considers a broader definition of firm leverage where broad $\text{lev}_{i,t}$ is total liabilities over total assets for firm i at time t. The interaction coefficient continues to be negative and significant. Economically,

interest rate spreads as in Arellano (2008) and Chatterjee and Eyigungor (2012).

this coefficient is stronger: firms' growth rates are 0.9 lower for firms with leverage at the 75^{th} percentile relative to the 25^{th} percentile when government spreads are 1% higher.

A major concern for the endogeneity of leverage is reverse causality. For example we might expect firms to lever more if their growth prospects are good, and such an effect to be differential during sovereign debt crises. In fact in our sample the correlation between firm growth and leverage is positive and equal to 0.10. To address this concern, the specification in column 3 measures leverage as the average for each firm. Hence, given our inclusion of firm fixed effects, the variation in growth rates for each firm is uncorrelated with the measure of leverage. The coefficient of interest in this specification of -1.5 is significant and very similar to the benchmark regression.

A related concern is that the differential effects we find occur for spurious reasons. A third factor is leading firms to both lever more and be more sensitive to movements in government spreads. Following Titman and Wessels (1988), the classic reference that studies the determinants of firm leverage, we control here for the main firm factors that correlate with leverage, namely, profitability, size, collateral abundance, and volatility. In our sample of firms, consistent with the findings in Titman and Wessels (1988), we find that firm leverage correlates negatively with profitability, size, and volatility, and positively with collateral abundance. Controlling for these firm characteristics can alleviate the concerns that, for example, less profitable firms are both more levered and suffer a deeper reduction in sales during the sovereign debt crisis.

In columns 4 and 5 of Table 6, we report results when we control for these four firm specific determinants. We define profitability by the ratio of firm income to sales, size is the log of firm total assets, collateral abundance is proxied with the ratio of financial assets to total assets, and volatility is the standard deviation sales growth in the time series. The specification in column 4 includes only the firm specific controls, and the specification in column 5 adds interactions of each of these firm specific variables with the government spread. These additional controls, as well as the interactions of these variables with the government spread, are significant in the regressions of firm growth. The correlation of firm growth and government spreads, however, continue to vary as a function of firm leverage, with highly levered firms decreasing their growth more during the sovereign debt crisis. In fact, the interaction coefficient between leverage and government spread increases by more than 50% in the specification of column 5, which contains the most number of controls.

A different concern in our main result is that we have omitted aggregate factors that affect both government spreads and firms' performance differentially based on firm leverage. For example, global financial shocks might impact government spreads in Italy and affect the production of firms that rely heavily on debt financing. Table 7 presents robustness results

Table 6: Robustness I: Government spreads, leverage, and growth

	Benchmark	Broad Leverage	Mean Leverage	More Firm Controls	More Firm Controls + Interactions
$s_t^g \times \text{lev}_{i,t}$	-1.70^{***}			-2.18^{***}	-2.61^{***}
$p_t \times \text{lev}_{i,t}$	-0.11^{***}			0.05	0.05
$s_t^g \times \text{broad lev}_{i,t}$		-2.87^{***}			
$p_t \times \text{broad lev}_{i,t}$		0.30^{**}			
$s_t^g \times \overline{\text{lev}}_i$			-1.54^{***}		
$p_t \times \overline{\text{lev}}_i$			-0.13^{***}		
$s_t^g \times \text{profitability}_{i,t}$					-5.69^{***}
$s_t^g \times \text{size}_{i,t}$					21.49^{***}
$s_t^g \times \text{collateral}_{i,t}$					-0.94^{***}
$s_t^g \times \text{volatility}_{i,t}$					-6.28^{***}
s_t^g	-0.82^{***}	0.53	-1.02^{***}	-0.39^{***}	5.20^{***}
p_t	-0.66^{***}	-0.82^{**}	-0.61^{***}	-0.58^{***}	-0.58^{***}
Firm controls	yes	yes	yes	yes	yes
Adjusted R^2	16	16	14	27	27

This table reports regressions of firms' sales growth on sovereign spreads s_t^g, aggregate productivity measured by disaster risk p_t, firm leverage measured by short debt relative to assets $\text{lev}_{i,t}$, total liabilities relative to assets broad $\text{lev}_{i,t}$, firm average short debt relative to assets $\overline{\text{lev}}_i$, firm interactions of firm leverage with spreads and productivity, firm-specific fixed effects, firm profitability measured by the ratio of net income to sales $\text{profitability}_{i,t}$, firm size as measured by the log of assets $\text{size}_{i,t}$, firm collateral abundance as the ratio of financial assets to total assets $\text{collateral}_{i,t}$, firms volatility as the standard deviation of sales growth $\text{volatility}_{i,t}$, and interactions of these variables with sovereign spreads. Standard errors are clustered two ways, across t and i. See the Appendix for the definition of variables.

for alternative specifications of the benchmark regression with additional aggregate time series interacted with firm leverage. Columns 2-4 of the table add the U.S. volatility index (CBOE S&P 500 Volatility Index), European index stock growth (EURO STOXX 50), and Italian banks index stock growth (FTSE Italia All-Share Banks), respectively, as additional controls by themselves and also interacted with firm leverage. The coefficient of interest of the interaction of firm leverage with government spreads remains significant and is stable across specifications. Finally column 5 reports the results when we add time fixed effects to the benchmark specification. Recall that by adding time fixed effects, we are controlling for the mean effect on firms' sales growth of any aggregate shocks. The interaction coefficient between firm leverage and government spreads continues to be significant, and the level is similar to the benchmark result.

Table 7: Robustness II: Government spreads, leverage, and growth

	Benchmark	VIX	EU Stocks	Italian Banks Stocks	Time FE
$s_t^g \times \text{lev}_{i,t}$	-1.70^{***}	-2.02^{***}	-1.66^{***}	-1.22^{***}	-1.83^{***}
$p_t \times \text{lev}_{i,t}$	-0.11^{***}	0.16^{***}	0.22^{***}	0.16^{***}	0.08
$\text{VIX}_t \times \text{lev}_{i,t}$		-0.19			
$\text{EU stck}_t \times \text{lev}_{i,t}$			12.17^{**}		
$\text{IT Bank stck}_t \times \text{lev}_{i,t}$				6.32	
s_t^g	-0.82^{***}	-0.55^{**}	-0.85^{***}	-1.16^{***}	
p_t^g	-0.66^{***}	-0.72^{***}	-0.74^{***}	-0.71^{***}	
VIX_t		0.19^{*}			
EU stck_t			-8.47^{**}		
IT Bank stck_t				-4.39^{*}	
Firm controls	yes	yes	yes	yes	yes
Adjusted R^2	16	16	16	16	24

This table reports the regression of firms sales growth on aggregate times series for sovereign spreads s_t^g, aggregate productivity measured by disaster risk p_t, the U.S. volatility index, European stock index growth, Italian banks index stock growth, as well as firm leverage $\text{lev}_{i,t}$ and interactions of firm leverage with each of the aggregate series and firm-specific fixed effects. Standard errors are clustered two ways, across t and i. See the Appendix for the definition of variables.

8 Conclusion

We have developed a framework that combines firm-level data and a structural model of sovereign debt with firm heterogeneity to study the recessionary interplay between the government and the private sector during sovereign debt crises. We showed that mapping the cross-sectional implications of the model to firm-level data can be useful in measuring the effects of sovereign risk on the private economy. In our application, we find that the pass-through of sovereign risk on the private economy is sizable, accounting for about half of the observed decline in output during the Italian debt crisis.

Our approach could be generalized along other dimensions. For example, one could introduce into the model other sources of firm heterogeneity e.g., export intensity, reliance on foreign currency debt, or proximity to the government sector, and exploit the cross-sectional variation that is present in firm-level datasets to discriminate among different theories of the costs of sovereign defaults.

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Appendix to "Sovereign Default Risk and Firm Heterogeneity"

by Cristina Arellano, Yan Bai, and Luigi Bocola

A Firm-level data and production function estimation

The construction of the dataset and the estimation of the production function follows closely the work of Gopinath et al. (2015). Here we report some basic information, and we refer the reader to that paper for additional details.

A.1 Construction of the dataset

We use firm-level data on Italian firms from Amadeus, accessed online through Wharton Research Data Services (WRDS). The data set has detailed balance sheet information for public and privately held firms, and use only the unconsolidated data. We focus on firms with NACE codes ranging from 05 to 43. This covers firms operating in mining, manufacturing, utilities and construction. In our sample, 80% of the firms belong to the manufacturing sector.

We clean our data sets in a series of steps. First, we control for basic reporting mistakes by dropping firm-year observations with the following characteristics

- Missing, zero or negative values for total assets, tangible fixed assets, operating revenue, sales, number of employees, material costs and wage bill.
- Negative values for intangible fixed assets.
- Negative values for value added where value added is calculated as the difference between operating revenue and material costs.

Second, we drop firms that have missing values for the variables of interest over the 2005-2012 period. As a result of these steps, we are left with a balanced panel of firms that contains 178,810 firm-year observations over the 2005-2012 period.

A.2 Production function estimation

We estimate the production function in equation (20) separately for each two-digits industry s. Our estimation use the methodology developed in Levinsohn and Petrin (2003) and

Wooldridge (2009) to deal with the endogeneity of labor choices with respect to productivity, and it uses capital and the costs of materials as instruments in a GMM approach. The variables used in estimations are nominal value added, the wage bill, the costs of materials used in production, and the book value of total assets as a proxy for capital. We use the value added deflator at two-digits industry level from Eurostat to deflate value added, the wage bill and the costs of material. The book value of total assets is deflated using the Producer price index of domestic investment goods obtained from FRED. This steps of the procedure gives us estimated for $(\beta_1(s), \beta_2(s))$ for each industry s. Given these estimates, we can calculate the productivity of firm i in industry s as follows

$$\log(z_{i,t}) = \log(y_{i,t}) - [\beta_{0,t}(s) + \beta_1(s)\log(\ell_{i,t}) + \beta_2(s)\log(k_{i,t})].$$

A.3 Definitions of firm variables

GROWTH: Ratio of change in sales relative to the previous year to the average sales in those years (growth_t=(opre_t-opre_(t-1))/0.5(opre_t+opre_(t-1))).

LEVERAGE: Ratio of total short term debt (loan+cred) relative to total assets (toas). We then winsorize the ratio at the 1st and the 99th percentiles.

PROFITS/ASSETS: Ratio of profits (plbt) to assets (toas). We then winsorize the ratio at the 1st and the 99th percentiles.

INTEREST/ASSETS: Ratio of interest payments (inte) to assets (toas). We then winsorize the ratio at the 1st and the 99th percentiles.

BROAD LEVERAGE: Ratio of total liabilities (li) relative to total assets (toas). We then winsorize the ratio at the 1st and the 99th percentiles.

PROFITABILITY: Ratio of profit (plbt) to sales (opre). We then winsorize the ratio at the 1st and the 99th percentiles.

SIZE: Log of total assets (toas).

COLLATERAL: Ratio of financial assets, defined as total assets minus fixed assets, (toas-fias) to total assets (toas).

B Computational Algorithm

To compute the model, we first discretize the shock processes and state variables. We then solve the model with value function iteration. Each iteration takes three steps and uses the value functions, government's decision rule, and bond price schedule from the previous iteration. The first step finds each firm's optimal decision rules and generates an aggregate tax schedule by summing up firms' output. The second step solves the government's optimal default and borrowing decision. The third step updates the government's bond price schedule. We end the iteration when the value functions and bond price schedule in the last two iterations are close enough or when the maximum iteration is reached.

We discretize the aggregate and idiosyncratic productivity shocks and approximate their continuous distribution using transitions matrices. The idiosyncratic process p_t is discretized in three values, $\{p_{\text{low}}, p_{\text{medium}}, p_{\text{high}}\}$, with the probability $\pi^p = \{\pi^p_{\text{low}}, \pi^p_{\text{medium}}, \pi^p_{\text{high}}\}$. Firm-level productivity is discretized into a two-point distribution, $\{z_{\text{low}}, z_{\text{high}}\}$, with transition matrix $\pi^z(z|z_{-1}, p)$ using Tauchen's (1986) method. We choose two values for the financing requirement $\lambda = \{\lambda_{\text{low}}, \lambda_{\text{high}}\}$ with equal probability. With the uncorrelated two-point z and λ distributions and the equal probability of λ, we only need to record the fraction of firms with z_{low} to capture the whole distribution of firms. We abuse the notation and use Λ to denote the fraction of firms with z_{low}. The stochastic process for firms' fixed cost has the cumulative distribution $\Phi(\xi)$ and probability distribution $\phi(\xi)$. We also use Tauchen's method to discretize it into 500 points with probability matrix $\pi^\xi(\xi)$. The ν shocks are discretized into 50 points with transition matrix $\pi^\nu(\nu|\nu_{-1})$ using Tauchen's (1986) method. Finally, we discretize the government's borrowing B into 120 points.

Let $S = (\nu, p, \Lambda)$ be the state including the exogenous shocks and the probability of firms with z_{low} at the beginning of the period. Let $\pi^{\nu p}(\nu, p|\nu_{-1}) = \pi^p(p)\pi^\nu(\nu|\nu_{-1})$. Let $x = (z_{-1}, \lambda, S, B')$ denote a firm's state variable.

We start with a guess of two arrays for the value functions for the government $W(S, B)$ and for the firm $V^f(x)$, the government's borrowing function $H^b(S, B)$, and the government's bond price schedule $q(S, B')$.

In the nth iteration, taking as given the functions of W_n, V^f_n, H^b_n, and q_n, we first compute the spread function

$$s^g_n(S, B') = \frac{1}{q_n(S, B')} - (1 + r),$$

where r is the risk-free rate. We then solve the model in the following three steps.

- Step 1. Firm optimization

  Firms decide on default d, capital k, labor ℓ, and borrowing b. Firms choose k and ℓ in proportion to the relative rental prices. Also, it is easy to show that given the constraint that $\pi \geq 0$, firms default only when their budget set is empty. Hence, default follows a cutoff rule based on the stochastic cost ξ. Let ξ^* be the maximum cost for which the firm will not default, which is defined by

$$\xi^*(z, \lambda, k, b) = Mz^{1-\eta}k^{\eta} - (1-\lambda)\frac{r_k}{1-\alpha}k - b, \tag{A.1}$$

where M is a constant equal to $\left(\frac{r_k \alpha}{w(1-\alpha)}\right)^{\alpha\eta}$. With this default cutoff at hand, for each firm x, we find two unknowns, the capital k and debt b, to satisfy the following two equations:

$$\frac{\lambda r_k}{1-\alpha}\left[1 + \gamma s_n^g(S, B')\right] = \frac{b}{k}\sum_z \pi^z(z|z_{-1}, p)\Phi\left[\xi^*(z, \lambda, k, b)\right] \tag{A.2}$$

$$\begin{aligned} &\eta M k^{\eta-1}\sum_z \pi^z(z|z_{-1}, p)\left\{z^{1-\eta}G(k, b, z, \lambda; S, B')\right\} \\ &= \frac{r_k}{1-\alpha}\left[(1-\lambda)\sum_z \pi^z(z|z_{-1}, p)\left\{G(k, b, z, \lambda; S, B')\right\} + \lambda\omega(k, b, \lambda; S, B')\right], \end{aligned} \tag{A.3}$$

where

$$G(k, b, z, \lambda; S, B') = \Phi(\xi^*) + \phi(\xi^*)\left\{EV^f(z, \lambda, S, B') + \frac{\omega(k, b, \lambda; S, B')b}{1 + \gamma s_n^g(S, B')}\right\}$$

$$\omega(k, b, \lambda; S, B') = (1 + \gamma s_n^g(S, B'))\frac{\sum_z \pi^z(z|z_{-1}, p)\left[\Phi(\xi^*) + \phi(\xi^*)EV^f(z, \lambda, S, B')\right]}{\sum_z \pi^z(z|z_{-1}, p)\left[\Phi(\xi^*) - \phi(\xi^*)b\right]},$$

$$EV^f(z, \lambda, S, B') = \frac{1}{1+r}\sum_{\nu', p'} \pi^{\nu p}(\nu', p'|\nu)V_n^f(z, \lambda; S', H_n^b(S', B')),$$

$$\Lambda'(\Lambda, p) = \Lambda\pi^z(z_{low}|z_{low}, p) + (1-\Lambda)\pi^z(z_{low}|z_{high}, p). \tag{A.4}$$

Let the optimal decision rules of the firm be $k^*(x)$ and $b^*(x)$. We can compute the optimal output as

$$y(z; x) = Mz^{1-\eta}\left[k^*(x)\right]^{\eta}$$

and the optimal profit as

$$\Omega(z, \xi; x) = y(z; x) - (1-\lambda)\frac{r_k}{1-\alpha}k^*(x) - b^*(x) - \xi.$$

Let the government transfer for the firm be

$$m(z,\xi;x) = I_{\xi>\xi^*} \max\left\{0, -\Omega(z,\xi;x) - b^*(x)\right\}.$$

We can update the firm's value function as follows:

$$V^f_{n+1}(x) = \sum_z \pi^z(z|z_{-1},p)\left\{\sum_{\xi\leq\xi^*} \pi^\xi(\xi)\Omega(z,\xi;x) + \Phi(\xi^*)EV^f(z,\lambda,S,B')\right\},$$

with ξ^* given by equation (A.1) evaluated at the optimal $k^*(x)$ and $b^*(x)$.

Aggregating up, we get the aggregate tax schedule:

$$\begin{aligned} T(S,B') = {} & \frac{\Lambda}{2}\sum_{\lambda,z,\xi} \pi^z(z|z_{low},p)\pi^\xi(\xi)\left[\tau y(z;z_{low},\lambda,S,B') - m(z,\xi;z_{low},\lambda,S,B')\right] \\ & + \frac{(1-\Lambda)}{2}\sum_{\lambda,z,\xi} \pi^z(z|z_{high},p)\pi^\xi(\xi)\left[\tau y(z;z_{high},\lambda,S,B') - m(z,\xi;z_{high},\lambda,S,B')\right]. \end{aligned}$$

- Step 2. Government optimization

  We solve the government's problem with the grid search method. For each state (S,B), we first check whether B is feasible. If it is not feasible, that is,

  $$B > \max_{B'} \left\{T(S,B') + q_n(S,B')B'\right\},$$

  we set $D_{n+1}(S,B) = 1$.

  If B is feasible, we solve the following problem:

  $$V(S,B) = \max_{B'} u_g\left[T(S,B') + q(S,B')B' - B\right] + \beta\sum_{\nu',p'} \pi^{\nu p}(\nu',p'|\nu)W_n(S',B'),$$

  with $\Lambda'(\Lambda,p)$ given by equation (A.4). Let the optimal decision be $B^{*'}(S,B)$. We set $D_{n+1}(S,B) = 0$ if $V(S,R) - \nu \leq V(S,B)$, and $D_{n+1}(S,B) = 1$ otherwise.

  We then update the value function and borrowing function of the government. If $D_{n+1}(S,B) = 0$, we set the value function $W_{n+1}(S,B) = V(S,B)$ and the borrowing function $H^b_{n+1}(S,B) = B^{*'}(S,B)$. If $D_{n+1}(S,B) = 1$, we set the value function $W_{n+1}(S,B) = V(S,R) - \nu$ and the borrowing function $H^b_{n+1}(S,B) = B^{*'}(S,R)$.

- Step 3. For each (S, B'), the updated bond price schedule is given by

$$q_{n+1}(S, B') = \frac{1}{1+r} \sum_{\nu', p'} \pi^{\nu p}(\nu', p'|\nu) \left[1 - D_{n+1}(S', B') \left(1 - \frac{R}{B'}\right)\right].$$

We compute the distance between (W_n, q_n, V_n^f) and $(W_{n+1}, q_{n+1}, V_{n+1}^f)$. We finish the iteration if those functions are close enough or if the number of iterations reaches 500; otherwise we go back to step 1 for the $n + 1^{th}$ iteration.

C Sensitivity Analysis

In the benchmark model, we set the persistence of the enforcement shock to 0.5. This section reports sensitivity results for our model with a higher persistence of $\rho_\nu = .8$, while keeping all other parameters as in Table 2. Table A-1 reproduces the moments in Tables 3 and 4. These results show only minor changes in the statistics. Notably, all else equal higher persistence for enforcement shocks decrease the average government spread, and lower the interaction coefficient of the panel regression.

These comparative static results show that the main results and message of the paper are unchanged with higher ρ_ν.

Table A-1: Sensitivity ρ_ν: Moments in Data and Models

	Data	Benchmark	Higher ρ_ν
Govt spread mean	1.8	1.9	1.6
Govt spread volatility	1.1	0.9	1.0
Short debt/output	25	28	25
Recovery	0.6	0.54	0.75
Firms profits	2.5	3	3
Firms leverage 25 and 75 pct	[0.2,0.5]	[0.2,0.5]	[0.2,0.5]
Firms spread	2.3	1.8	1.6
Interaction coeff$\times[\overline{\lambda}_{75,t} - \overline{\lambda}_{25,t}]$	-0.54	-0.57	-0.68
Other Moments			
Aggregate Moments			
Corr (Firm spread, Govt spread)	82	98	99
Corr (Output, Govt spread)	-54	-79	-82
Firm Moments			
Sales Growth (St. Dev.)	26	30	28
Leverage (St. Dev.)	21	14	14
Profits/Assets (St. Dev.)	8	2	2
Interest/Assets (St. Dev.)	1	0.4	0.4
Corr (Leverage, Profits/Assets)	-18	-6	-4
Corr (Leverage, Interest/Assets)	40	96	9
Corr (Profits/Assets, Sales Growth)	28	14	14

Table A-2: Sensitivity ρ_ν: The Italian debt crisis, 2007-2012

	Government spreads	Firm spreads	Output
Data	3.6	2.8	-13.9
Benchmark			
Model	3.5	2.5	-14.3
No pass-through	4.1	0.1	-7.0
Higher ρ_ν			
Model	2.6	2.0	-12.0
No pass-through	2.8	0.1	-7.0

Output is reported as the percentage deviation between 2012 and 2007. Spreads are reported as the difference between their 2012 and 2007 values. Spreads are measured in percentage points.