Aggregate Effects of Collateral Constraints*

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Abstract

We structurally estimate a dynamic model with heterogeneous firms and collateral constraints. Embedding this model in a general equilibrium framework allows us to quantify the impact of financing frictions on aggregate output and welfare. The structural estimation is based on the causal effect of collateral shocks on firm level corporate investment in the United States. The estimates imply that lifting financing frictions would increase welfare by 9.4% and aggregate output by 11%. Half of this aggregate output gain is due to an increase in the aggregate stock of capital, one quarter is due to a larger aggregate labor supply, while the remaining quarter is due to a higher aggregate productivity from a better allocation of inputs across heterogeneous firms.

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There is an accumulating body of evidence showing the causal effect of financing frictions on firms' investment decisions *at the micro-level*.[1] While this literature safely rejects the null hypothesis that firms are unconstrained financially, it does not measure whether these constraints matter quantitatively. In this paper, we use a quantitative model that matches these findings to investigate the *aggregate effects* of financing frictions. We focus on a pervasive source of financing friction – collateral constraints. Our approach expands on the existing literature by (i) estimating our structural model using well-identified firm-level evidence that collateral constraints causally affect investment and (ii) nesting this model in a general equilibrium framework with heterogeneous firms to study the aggregate effect of collateral constraints. Our estimated model shows that even in a developed country like the U.S., collateral constraints can have a large effect on welfare. Compared to a counterfactual economy without financing constraints, welfare in our constrained economy is lower by 9.4%, and output by 11%. Of this ouptput loss, only a quarter can be attributed to lower aggregate TFP due to input misallocation.[2]. The remaining output loss is due to lower aggregate inputs, mostly capital. Thus, collateral constraints induce significant misallocation, but their impact on the aggregate capital stock is larger.

We estimate our structural model by targeting the sensitivity of investment to exogenous shocks to firms' real estate value. Starting with Gan (2007) and Chaney et al. (2012), a large literature documents how corporate investment responds to real estate shocks and argues that such sensitivity is evidence of financing constraints, insofar that real estate shocks are shocks to debt capacity that are uncorrelated with investment opportunity. Relying on this insight, we use this sensitivity to identify the parameter governing financing constraints in our model. The existing literature that estimates similar models (e.g., Hennessy and Whited (2007)) typically targets capital structure decisions such as the average debt to capital ratio. However, this moment is driven by many forces (e.g., trade credit, inventory, unsecured debt capacity) that may not be all

[1]See, among many others, Lamont (1997), Rauh (2006), Chaney et al. (2012), Blanchard et al. (1994) for the effect of financial frictions on investment and Benmelech et al. (2010) or Chodorow-Reich (2013) for the effect of financial frictions on employment

[2]The costs of input misallocation is the focus of Hsieh and Klenow (2009), Moll (2014), Midrigan and Xu (2014).

captured by the model. As a result, estimates of the parameters driving financial constraints will be influenced by these additional forces. In contrast to leverage, causal estimates coming from the reduced-form literature are in principal purely attributable to financing constraints. Targeting these reduced-form moments should lead to more reliable estimates of financing constraints parameters. We show that, in our data, targeting firms' leverage leads to *underestimating* the effect of financing constraints. The intuition is that the sensitivity of investment to real estate value is relatively low in the data, indicating a relatively low pledgeability of capital. Leverage is, on the other hand, relatively large empirically, so that an estimation procedure that seeks to match leverage will assume that capital is easily collateralized. This makes financing constraints less binding. At the aggregate level, when targeting leverage, the estimated aggregate output loss is only half as large as when targeting the sensitivity of investment to real estate shocks.

We start by documenting how, on a panel of U.S. firms, corporate investment and leverage respond to shocks to real estate value. Repeating earlier analysis (Chaney et al., 2012) with slightly different specifications, we find that a \$1 increase in real estate value leads to a \$0.04 increase in investment and a \$0.04 increase in financial debt. While these estimates allow to comfortably reject the null that firms are not financially constrained, they do not tell us whether these constraints matter *quantitatively* and in the *aggregate*.

To assess whether these micro-level elasticities have significant aggregate implications, we proceed in two steps. First, we set-up a structural model of firms dynamics. The model builds on the standard neo-classical model of investment with adjustment costs (Jorgenson, 1963; Lucas, 1967; Hayashi, 1982). To this standard model, we add one simple amendment. We assume that firms face a collateral constraint: the amount they can borrow every period is limited by how much tangible assets –including real estate– they own. Each period, the value of real estate assets fluctuates randomly, creating variations in the collateral constraint, thus mimicking our reduced-form empirical design.[3] We estimate this model through a Simulated Method of Moments. In addition to the

[3] While we do not explicitly micro-found the collateral constraint, it emanates naturally from limited enforcement models (Hart and Moore, 1994).

standard moments used in the structural corporate finance literature, our estimation procedure explicitly targets the sensitivity of investment to variations in local real estate prices. We show that the model manages to fit the targeted moments and some non-targeted ones precisely. It also has well-behaved comparative statics properties, which ensures a precise parameter estimation. We also show that a simple ratio of sales to capital is a good measure of financing constraints, as argued in the development literature (Hsieh and Klenow, 2009).

In a second step, the estimated model is nested in a simple general equilibrium where firms compete for customers, workers and for capital goods. We simulate two economies: one in which firms face the estimated collateral constraints, and a counterfactual economy where firms are unconstrained. We compute output and welfare loses from financing constraints by comparing the two economies. We find aggregate welfare loss from financing constraints of 9.4% and output loss of 11%. Such losses arise in part from the *misallocation* of inputs across heterogeneous producers (Hsieh and Klenow, 2009; Moll, 2014; Midrigan and Xu, 2014) and in part from a sub-optimal aggregate capital stock. While both channels matter, aggregate capital matters twice as much as misallocation. It is important to note that, in line with the macroeconomic literature, we formally quantify the *cost* of financing frictions, but not their potential *benefit*. We model collateral constraints in a reduced-form way and do not take a stance on whether the rationale behind these collateral constraints is efficient or not.

Related Literature. Our focus on collateral constraints is rooted in a large array of empirical evidence on the importance of collateral constraints. It is well documented that collateral plays a key role in financial contracting. More redeployable assets receive larger loans and loans with lower interest rates (Benmelech et al., 2005). The value of collateral affects the relative ex post bargaining power of borrowers and lenders (Benmelech and Bergman, 2008). Beyond these effects on financial contracting, collateral values also affect real outcomes at the micro-economic level: Firms with more valuable collateral invest more (Gan, 2007; Chaney et al., 2012); individuals with more valuable collateral are more likely to start up new businesses (Schmalz et al., Forthcoming;

Adelino et al., 2015). In addition, many empirical evidence point to the prevalence of real estate collateral in loan contracts (Davydenko and Franks, 2008; Calomiris et al., 2015). Our paper adds to the literature by bridging the gap between microeconomic evidence on the role of collateral constraints and the macroeconomic effect of financial frictions.

Our paper also contributes to the long-standing literature in corporate finance investigating the real effects of financing frictions. This literature has traditionally explored the effect of financing frictions on corporate investment. A key challenge is to find exogenous variations in financing capacity that are not correlated with investment opportunities. For instance, Lamont (1997) overcomes this challenge by showing that non-oil divisions of oil conglomerates increase their investment when oil prices increase. Rauh (2006) shows that firms with underfunded defined benefit plans need to make financial contributions to their pension fund, depriving them of available cash-flows and leading to reduced investment.[4][5]

Several important papers have developed a structural quantitative approach to estimate the effect of financing frictions. This literature is reviewed in Strebulaev and Whited (2012). In a seminal contribution, Hennessy and Whited (2007) use SMM to estimate a dynamic model of investment and infer the magnitude of financing costs. They find that for small firms, the estimated marginal equity flotation costs is about 10.7% of capital and bankruptcy costs 15.1%. Hennessy and Whited (2005) develop a dynamic trade-off model, which they structurally estimate to explain several empirical findings inconsistent with the static trade-off theory. Lin et al. (2011) examines the impact of the divergence between corporate insiders' control rights and cash-flow rights on firms' external finance constraints from a generalized method of moments estimation of an investment Euler equation and show that the agency problems associated with the control-ownership divergence can have a real impact on corporate financial and investment outcomes. Nikolov and Whited (2014) estimate a dynamic model of finance and investment with different

[4]See Bakke and Whited (2012) for a discussion of this identification strategy.

[5]The literature on this topic is extensive. For some important contributions, see Fazzari et al. (1988), Erickson and Whited (2000), Kaplan and Zingales (1997), Almeida and Campello (2007), Blanchard et al. (1994), Campello et al. (2010), Chaney et al. (2012), Kaplan and Zingales (2000), Peek and Rosengren (2000), Campello et al. (2011).

sources of agency conflicts between managers and shareholders to analyze the role of agency conflicts in corporate policies and investment. Our contribution to this literature is twofold. First, we include coefficient estimates from a reduced-form regression identifying the effect of collateral constraints on investment and debt as targeted moments. We show that these moments are crucial in identifying the strength of financial frictions in our data. Second, we nest our investment model into a general equilibrium model, which allows us to account for general equilibrium effects in our counterfactuals. In contrast, the literature typically only considers partial equilibrium counterfactuals. In that sense, our model is close to Gourio and Miao (2010) who focus on taxation. Compared to their paper, we focus on model estimation and the effect of financing constraints.

Finally, our paper contributes to the important macroeconomic literature on the aggregate effects of financial frictions. Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman et al. (2013) emphasize the effect of misallocation of resources across heterogeneous firms on aggregate TFP and welfare. Midrigan and Xu (2014) focus on financing frictions as a source of misallocation. They calibrate a model of establishment dynamics with financing constraints and find that financing frictions cannot explain large aggregate TFP losses from misallocation. In contrast, Moll (2014) shows that for a TFP persistence parameter in the empirically relevant range, financial frictions can matter in both the short and the long run. Buera et al. (2011) develop a quantitative framework to explain the relationship between aggregate/sector-level TFP and financial development across countries and show that financial frictions account for a substantial part of the observed cross-country differences in output per worker, aggregate TFP, sector-level relative productivity, and capital-to-output ratios. Beyond misallocation, a large literature has investigated the effects of financing friction on aggregate TFP growth and welfare. Jeong and Townsend (2007) develop a method of growth accounting based on an integrated use of transitional growth models and micro data and find that in Thailand, between 1976 and 1996, 73 percent of TFP growth is explained by occupational shifts and financial deepening. Amaral and Quintin (2010) present calibrated simulations of a model of economic development with limited enforcement and find that the average scale of production rise with the quality of enforcement. Riddick and Whited (2009) study the costly reallocation of capital across heterogeneous firms. They infer the cost of reallocation from a calibrated model and show that reallocation costs need to be strongly countercyclical to be consistent with the observed dispersion of productivity. Our contribution to this literature is that we base our quantification exercise on an estimation procedure that targets moments from a reduced-form analysis exploiting exogenous shocks to financing capacity. Second, our paper combines adjustment costs with financing frictions. Asker et al. (2014) consider the effect of adjustment costs on static misallocation measures, but their economy does not feature a financing friction. In contrast, our approach delivers interesting implications on the interaction between adjustment costs and credit frictions.

We present reduced-form evidence of the effect of collateral values on both investment and employment in Section 1. We present our formal model of firm dynamics with collateral constraints in Section 2. We structurally estimate the model using US firm level data in Section 3. Section 4 describes and implements the general equilibrium analysis. Section 5 discusses robustness and implements a policy experiment.

1 Reduced-form evidence

We estimate the investment and borrowing sensitivity to real estate value as in Chaney et al. (2012). The construction of the data is detailed in that paper. The dataset is a panel of publicly listed firms from 1993 to 2006 extracted from COMPUSTAT. We require that these firms supply information about the accounting value and cumulative depreciation of land and buildings (items ppenb, ppenli, dpacb, dpacli) in 1993. We then combine this information with office prices in the city where headquarters are located, in order to obtain a measure of the market value of firms' real estate holdings, which we normalize by the previous year property, plant and equipment. We call this measure REValue_{it} for firm i at date t. We require that this variable is available for all firms, so that we end up with a panel of 20,074 observations corresponding to 2,218 firms which are followed from 1993 until 2006 unless they drop out of the panel before (only 676 firms are still present in 2006).

We then run the following regression:

$$\frac{Y_{it}}{k_{it-1}} = \frac{\alpha + \beta.\mathrm{REValue}_{it}}{k_{it-1}} + \mathrm{Offprice}_{it} + a_i + \epsilon_{it},$$

where k_{it-1} is the lagged stock of productive capital (item ppent). $\mathrm{Offprice}_{it}$ is an index for office prices in the city where firm i's headquarters are located. This index is available from Global Real Analytics for 64 MSAs. We further add a firm fixed effect (a_i) and cluster error terms ϵ_{it} at the firm level. We are interested in β, the sensitivity of Y_{it} to real estate value. We report descriptive statistics for these variables in Table 1.

We look at two different left hand-side variables Y_{it}: capital expenditures (item capx) and net debt increase (sum of changes in long term debt – item dltt – and short term debt – item dlc). The estimated sensitivity of investment to real estate value, $\hat{\beta}$, is equal to 0.04 with a t-stat of 6.1. This can be interpreted as a \$0.04 investment response per \$1 increase in real estate value. The sensitivity of net borrowing to real estate value is also estimated at 0.04, with a t-stat of 4.5. These numbers are close to the main estimate of Chaney et al. (2012), the difference coming from the set of controls used. We opt here for a simpler specification with *fewer* controls, in order to restrict ourselves to variables available in the simulations of the model we present in the next section. This model will be estimated using the first coefficient (the investment sensitivity) as a targeted moment, while the second coefficient (the borrowing sensitivity) will serve as a non-targeted moment.

2 The model

In this section, we lay out our model of investment dynamics under collateral constraints. The economy is populated with heterogeneous, financially constrained firms, which combine capital and labor to produce differentiated goods. Those differentiated goods are then combined into a final good, consumed by a representative consumer and used as capital good.

2.1 Production technology and demand

The firm-level model is close to Hennessy and Whited (2007) in the sense that it includes a tax shield for debt and a large cost of equity issuance (in our case, infinite[6]) and Midrigan and Xu (2014) in the sense that firms face a collateral constraint. The firm's shareholder is assumed risk-neutral and has a time discount rate of r. Firm i produces output q_{it} combining capital k_{it} and efficiency units of labor l_{it} into a Cobb-Douglas production function with capital share α

$$q_{it} = F\left(e^{z_{it}}, k_{it}, l_{it}\right) = e^{z_{it}}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right), \tag{1}$$

with z_{it} the firm's log total factor productivity which is assumed to follow an AR(1) process:

$$z_{it} = \rho z_{it-1} + \epsilon_{it},$$

where we denote σ^2 the variance of the innovation ϵ_{it}. The firm faces a downward sloping demand curve with constant elasticity $\phi > 1$,

$$q_{it} = Q p_{it}^{-\phi}, \tag{2}$$

where Q is aggregate spending and will be determined in equilibrium (see Section 4).

Labor is fully flexible, and w is the wage – also determined in equilibrium. As labor is a static input, the total revenue of the firm net of labor input is

$$r\left(z_{it}; k_{it}\right) = \max_{l_{it}} p_{it} q_{it} - w l_{it} = b Q^{1-\theta} w^{-\frac{(1-\alpha)}{\alpha}\theta} e^{\frac{\theta}{\alpha} z_{it}} k_{it}^{\theta}, \tag{3}$$

with b a scaling constant and $\theta \equiv \frac{\alpha(\phi-1)}{1+\alpha(\phi-1)} < 1$.

[6]This infinite equity issuance cost simplifies the model and clarifies its exposition. We show in section 5 how the quantitative features of the model are changed when we assume a finite issuance cost within the range of the literature's estimates.

2.2 Input dynamics

Capital accumulation is subject to depreciation, time to build, and adjustment costs. At date t, gross investment i_{it} is given by

$$k_{it+1} = k_{it} + i_{it} - \delta k_t, \tag{4}$$

where δ is the depreciation rate. In period t, investing i_{it} entails a convex cost of $\frac{c}{2}\frac{i_{it}^2}{k_{it}}$. Additionally, the firm pays in period t for capital that will only be used in production in period $t+1$: this one period time to build for capital is conventional in the macro literature (Hall, 2004; Bloom, 2009) and acts as an additional adjustment cost. Introducing adjustment costs to capital is important in our estimation exercise, since they generate patterns similar to financing constraints and could thus be a natural confounding factor in our estimation procedure. For instance, adjustment costs make capital vary less than firm output, which generates a natural dispersion in capital productivities, exactly like financing constraints do (Asker et al., 2014). As we will show below, using the reduced-form moments presented in Section 1 allow us to identify both frictions separately.

We do not, however, include fixed adjustment costs to our model, a choice also made by Gourio and Kashyap (2007): our estimation targets firm-level data at an annual frequency, for which investment is not very lumpy. In our sample (described in Section 1), only 4% of the observations have an investment rate smaller than 2% of capital.[7]

2.3 Financing frictions and capital structure

The firm finances investment out of retained earnings and debt issuance to outside investors. d_{it} is net debt, so that $d_{it} < 0$ means that the firm holds cash. As is standard in the structural corporate finance literature (Hennessy and Whited, 2005), we only consider short-term debt contracts with a one period maturity. We set up the model so that debt is risk-free and pays an interest rate

[7] To compute the investment rate, we divide item capx by lagged item ppent

r[8] – determined in equilibrium in Section 4. For an amount d_{it} of debt issued at date t, the firm commits to repay $(1+r)d_{it+1}$ at date $t+1$. Finally, we also assume that the interest rate the firm receives on cash is lower than the interest rate it has to pay on its debt: if the firm has negative net debt, it receives a positive cash inflow of $-(1+(1-m)r)d_{it+1}$ with $0 < m < 1$.

Consistently with the corporate finance literature, we also assume that firm's profits net of interest payments and of capital depreciation, δk_{it}, are taxed at rate τ. As a result, debt is tax free, which creates an incentive for firms to increase their leverage. Other papers make alternative assumptions to make debt attractive to firms, either by assuming that debt holders are intrinsically more patient than shareholders, or that the shareholders seek to smooth consumption, for instance through log utility as in Midrigan and Xu (2014). Finally, note that all tax proceeds are rebated to the representative consumer – see Section 4.

The financing frictions come from the combination of two constraints. First, firms cannot issue equity, an assumption we relax in Section 5 where we instead consider a finite cost of equity issuance in line with parameter estimates from the literature. Second, firms face a collateral constraint, which emanates from limited enforcement (Hart and Moore, 1994). We follow Liu et al. (2013) and adopt the following specification for the collateral constraint:

$$(1+r)d_{it+1} \leq s\left((1-\delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\right), \tag{5}$$

The total collateral available to the creditor at the end of period $t+1$ consists of depreciated productive capital $(1-\delta)k_{it+1}$ and real estate assets with value $p_{t+1}h$. We assume $\log p_t$ to be a discretized AR(1) process. s, the fraction of the collateral value realized by creditors, captures the quality of debt enforcement, but also the extent to which collateral can be redeployed and sold.[9]

In assuming that the quantity of real estate h is the same across firms and time, we abstract

[8]While this risk-free interest rate could be time-varying, i.e. r_t, it will always be constant in our model and we thus omit the t subscript for simplicity.

[9]The formulation of the collateral using the expected future value of collateral is standard in macroeconomics. It can be justified as an optimal contract in a set-up where (1) the firm has the entire bargaining power in its relationship with creditors (2) it cannot commit not to renegotiate the debt contract at the end of period t and (3) collateral can only be seized at the end of period $t+1$.

from issues related to real estate ownership heterogeneity, which is an important limitation of this paper. In reality, we recognize that firms decision to buy or lease real estate assets can potentially depend on expected productivity, investment opportunities and financing constraints. However, we leave the analysis of how the endogeneity of real estate ownership affects current investment decisions for future research and focus this paper on measuring and aggregating financial frictions given the observed levels of real estate ownership in the data.

2.4 The optimization problem

The firm is subject to a death shock with probability d, but infinitely lived otherwise. Every period, physical capital and debt are chosen optimally to maximize a discounted sum of per period cash flows, subject to the financing constraint. The firm takes as given its productivity, local real estate prices, and forms correct expectations for future productivities and real estate prices.

Define as $V(S_{it}; X_{it})$ the value of the discounted sum of cash flows given the exogenous state variables $X_{it} = \{z_{it}, p_t\}$ and the past endogenous state variables $S_{it} = \{k_{it}, d_{it}\}$. Shareholders are assumed to be perfectly diversified so their discount rate is the same as risk-free debt r.

This value function V is the solution to the following Bellman equation,

$$\begin{cases} V(S_{it}; X_{it}) & = \max\limits_{S_{it+1}} \left\{ e(S_{it}, S_{it+1}; X_{it}) + \frac{1-d}{1+r} \mathbb{E}\left[V(S_{it+1}; X_{it+1}) | X_{it}\right] + \frac{d}{1+r}\left(k_{it+1} - (1+\tilde{r}_{it}) d_{it+1}\right)\right\} \\ \text{s.t.} & (1+r) d_{it+1} \leq s\left((1-\delta) k_{it+1} + \mathbb{E}[p_{t+1}|p_t] \times h\right) \\ & e(S_{it}, S_{it+1}; X_{it}) \geq 0 \\ \text{with:} & e(S_{it}, S_{it+1}; X_{it}) = (1-\tau)\left(r(z_{it}; k_{it}) - i_{it} - \frac{c}{2}\frac{i_{it}^2}{k_{it}} + d_{it+1} - (1+\tilde{r}_{it}) d_{it}\right) \\ & \qquad\qquad + \tau\left(\mathbf{1}_{d_{it}>0} \times r d_{it} + \delta k_{it}\right) \\ & i_{it} = k_{it+1} - (1-\delta) k_{it} \\ & \tilde{r}_{it} = r \text{ if } d_{it} > 0 \text{ and } (1-m) r \text{ if } d_{it} \leq 0 \end{cases} \tag{6}$$

where the second term in the maximand $\left(\frac{d}{1+r}\left(k_{it+1} - (1+\tilde{r}_{it}) d_{it+1}\right)\right)$ corresponds to the shareholder's payoff in case of firm death. This term avoids a bias towards borrowing. If we assume instead that bankers can recover capital when a firm exit, shareholders then have an incentive to borrow more in order to transfer value from the states of nature where they cannot consume to states where the firm survives. By assuming that shareholders receive the remaining capital when the firm exit, we ensure that this risk-shifting behavior does not drive the capital structure decisions of firms in our model.

Aggregate demand Q and the real wage w are equilibrium variables that the firms takes as given when optimizing inputs. Given the absence of aggregate uncertainty and the steady state assumption, they are fixed over time. Due to downward sloping demand, firms have an optimal scale of production. A firm initially below this level accumulates capital, but only gradually because of convex adjustment costs and time to build. Once the target scale is reached, firms replace depleted capital. Finally, spending on adjusting capital is bound by the collateral constraint. When the value of a firm's real estate assets increases, the collateral constraint is relaxed, and the firm finances more of the cost of adjusting towards its desired scale. This will generate the sensitivity of investment to real estate value that we have documented in Section 1.

3 Structural Estimation

3.1 Estimation procedure

We estimate the key parameters of the model via a Simulated Method of Moments. The entire procedure is described in detail in Appendix A. We look for the set of parameters $\hat{\mathbf{\Omega}}$ such that model-generated moments $\mathbf{m}(\hat{\mathbf{\Omega}})$ on simulated data fit a pre-determined set of data moments $\mathbf{m}$. If we could solve the model analytically, we could just invert the system of equations given by model-based moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

1. For a given set of parameters, we solve the Bellman problem (6) numerically and obtain the

policy function $S_{it+1} = (d_{it+1}, k_{it+1})$ as a function of $S_{it} = (d_{it}, k_{it})$ and exogenous variables $X_{it} = (z_{it}, p_t)$. We discretize the state space (S, X) into a grid that is as fine as possible to minimize numerical errors in the presence of hard financing constraints. This is critical: a 1-2% numerically generated error would be too large to quantify aggregate effects of this order of magnitude. Solving the model repeatedly to estimate our structural parameters would not be feasible on a conventional CPU (several hours per iteration), so we use a GPU instead (a few minutes per iteration), as described in Appendix A.1.

2. Our parameter estimates $\widehat{\boldsymbol{\Omega}}$ minimize the distance from simulated to data moments $\mathbf{m}$,

$$\widehat{\boldsymbol{\Omega}} = \arg\min_{\boldsymbol{\Omega}} \left(\mathbf{m} - \widehat{\mathbf{m}}\left(\boldsymbol{\Omega}\right)\right)' \mathbf{W} \left(\mathbf{m} - \widehat{\mathbf{m}}\left(\boldsymbol{\Omega}\right)\right),$$

where the weighting matrix $\mathbf{W}$ is the inverse of the variance-covariance matrix of data moments. Standard errors are calculated by bootstrapping. Appendix A.2 describes how we escape the many local minima present from estimating a large number of parameters.

3.2 Predefined and Estimated Parameters

The model has 14 parameters. We calibrate 9 of them using estimates from the literature or the data, and estimate the 5 remaining ones.

Predefined parameters.— Our 9 calibrated parameters are as follows. We set the capital share $\alpha = 1/3$ from Bartelsman et al. (2013) and the demand elasticity $\sigma = 5$ from Broda and Weinstein (2006) (which would lead to mark-ups of 25% in the absence of adjustment costs). Real estate prices $\log p_t$ follow a discretized AR(1) process. We estimate this AR(1) process on de-trended logged real estate prices and find a persistence 0.62 and innovation volatility 0.06. Both AR(1) processes for $\log z_t$ and $\log p_t$ are discretized using Tauchen's method. The rate of obsolescence of capital is set at $\delta = 6\%$ as in Midrigan and Xu (2014). The risk-free borrowing rate r is fixed at 3%, while the lending rate is set to $(1 - m)r = 2\%$. We fix the death rate d to 8%

which corresponds to the turnover rate of firms in our data. We set the corporate tax rate τ at 33%. Finally, we set $w = 0.03$ ($30,000) and $Q = 1$ for the estimation. They will, however, be endogenously determined in general equilibrium in our counterfactual analyses —see Section 4.

Estimated parameters.— We estimate 5 deep parameters but focus the discussion on 4 of them: The persistence ρ and innovation volatility σ of log productivity, the collateral parameter s and the adjustment cost c. The fifth parameter, the amount of real estate collateral available h, allows us to match the average ratio of real estate to capital h/k_t, and is essentially a normalization.

3.3 Data Moments

We compute the moments on the COMPUSTAT sample described in Section 1. We describe them here with a short heuristic discussion about their "identifying" power. In the next section, we discuss identification more systematically and show how simulated moments vary with parameters.

First, in the spirit of Midrigan and Xu (2014), we use the short- and long-term volatility of output to estimate the persistence and volatility of the productivity process. In our sample, the volatility of change in log sale ($\log \text{sales}_{it} - \log \text{sales}_{it-1}$, COMPUSTAT item: sale) equals 0.327. The volatility of 5-year change in log sales ($\log \text{sales}_{it} - \log \text{sales}_{it-5}$) equals 0.911. The fact that 5-year growth is less than 5 times more volatile than 1-year growth indicates mean-reversion and contributes to the identification of the persistence parameter. Targeting these two moments instead of directly matching the persistence coefficient of log sales makes our estimation less sensitive to model misspecification, e.g. for a true process with a longer memory than an AR(1).

Second, we use the autocorrelation of investment to identify adjustment costs (Bloom (2009)). For each firm in our panel we compute the ratio $\frac{i_{it}}{k_{it-1}}$ of capital expenditures (COMPUSTAT item: capx) to *lagged* capital stock (COMPUSTAT item: ppent). The correlation between $\frac{i_{it}}{k_{it-1}}$ and $\frac{i_{it-1}}{k_{it-2}}$ in our data is 0.43. Adjustment costs are needed to match this large correlation: they compel the firm to smooth its investment policy in response to a productivity shock (Asker et al., 2014). Financing frictions add to this smoothing motive.

Third, we use two alternative moments to estimate the collateral constraint parameter s. The first moment is net book leverage, a moment typically used in the literature (Hennessy and Whited, 2007; Midrigan and Xu, 2014). Book leverage is computed as financial debt (COMPUSTAT items: dlc + dltt) minus cash holdings (COMPUSTAT item: che), normalized by total assets (COMPUSTAT item: at). This definition reflects the notion that cash is equivalent to negative debt, as it is the case in our model. We obtain an average of 0.313 in our data. In our model, leverage directly identifies the collateral parameter s as higher collateral values unambiguously lead to more borrowing. Yet, as we discuss more extensively below, this moment (leverage) is not ideal to identify financing constraints for two reasons. First, from an identification standpoint, leverage may be an ambiguous moment. For instance, a firm may not be financially constrained yet choose to lever up for tax purposes. This behavior would lead to mis-attribute corporate leverage to collateral constraints (see Section 5.1 for a formal analysis of this identification problem). Second, financial leverage may be a noisy measure of a firm's indebtedness. For instance, financial debt typically includes unsecured debt, which is not part of our model (see Section 5.2 for such an extension), and which would lead to overstate the extent to which collateral can be pledged. For all these limitations of the leverage moment, we use a more direct measure of financing constraints instead, the sensitivity of investment to real estate value, computed in Section 1. Because it is also an informative and natural moment, we also look at the sensitivity of debt issuance to real estate value. We never target this second moment in our estimation, but it turns out our main model matches it very well (more on this below).

Finally, we compute the quantity of real estate held by the average firm, by taking the ratio of real estate holdings (COMPUSTAT item land + buildings) in 1993 normalized by total assets (COMPUSTAT item: at), and obtain 0.14. By adjusting h, our estimation procedure matches this moment perfectly; we view this part of the estimation as a normalization more than anything else. As a result, we omit discussion of this parameter from this point on.

3.4 Parameter Identification

This section discusses identification of the parameters of the model. In Appendix Figures C.1-C.4, we reproduce how moments vary as a function of model parameters. We also show, in Table 2, the elasticities of each moments with respect to estimated parameters – a simple transformation of the Jacobian matrix. All this analysis is about *local* identification, in the sense that we operate around our main SMM estimate for (s, c, ρ, σ) – which we discuss in detail in the next section.

We first discuss the graphical evidence. In Figures C.1-C.4, we offer visual evidence of how the different moments we use in our estimation help identify the model's parameters. To construct these figures, we first set all parameters (s, c, ρ, σ) at their estimated value, and then vary one of these parameters in partial equilibrium, i.e. holding fixed w and Q. All figures are reported using the same scale for each moment. Importantly, the comparative statics we report on these figures are *direct simulation output*: The relative smoothness of these plots gives us confidence in the robustness of our numerical procedure, which we attribute to the dense grid for capital (about 300 points), debt (29 points) and productivity (51 points) we use, as well as to the large number of simulated observations (1,000,000 firms over 10 years). See Appendix A for details.

Figure C.1 shows that the collateral parameter s influences mostly the leverage moment as well as the investment and debt sensitivities to real estate prices. This result is intuitive. Obviously, a higher s unambiguously leads to higher leverage: In our setting, the firm takes on more debt if it is allowed to. The sensitivity moments are non-monotonic with s. Intuitively, for low values of s, firms investment decisions are constrained by collateral availability: In this range of values for s, an increase in s allows firms to extract more debt and investment capacity out of a \$1 increase in collateral values. For higher values of s, however, firms become less financially constrained, so that their investment policies becomes less driven by collateral values. At the limit, when s grows close to 1, the firm becomes unconstrained and investment is no longer sensitive to fluctuations in house prices. We also see in Figure C.1 that around the SMM estimate (represented by a vertical line), both sensitivity moments are smooth and increasing functions of s. The second panel of Figure

C.1 also shows that an increase in s leads to an increase in the long-term volatility of production: when the firm is less constrained, its capital stock responds more to productivity shocks, which increases the volatility of output.

Figure C.2 shows that the adjustment cost parameter c is mostly identified by the autocorrelation of investment: Large adjustment costs lead the firm to smooth investment across time, which lead to a large autocorrelation of investment. Larger adjustment costs to capital also lead to lower short-term output volatility: Similar to financing constraints, adjustment costs prevent firms from adjusting their capital stock to productivity shocks, making output less volatile. Figures C.3 and C.4 shows that (1) the volatility of log-productivity σ has a nearly linear impact on the short-term volatility of output (2) the persistence ρ of productivity shocks strongly influences the long-term volatility of output, but has no first-order effect on short-term volatility. Combined together, these two observations are consistent with the idea that the ratio of the 1-year to 5-year output volatility allows to identify the persistence parameter ρ. Note also that the persistence of productivity shocks has a sizable positive effect on the autocorrelation of investment: Firms can afford to delay their response to productivity shocks, since these shocks are more persistent.

In Table 2, we quantify how the various simulated moments vary as a function of the estimated parameters. More precisely, we compute for each moment m_n, and each parameter ω_k, the following elasticity (Hennessy and Whited (2007)):

$$\epsilon_{n,k} = \frac{m_n^+ - m_n^-}{\omega_k^+ - \omega_k^-} \times \frac{\hat{\omega}_k}{\hat{m}_n} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{\omega}_k$ is the parameter value at the SMM estimate and $\hat{m}_n$ the corresponding value for moment n. $\hat{\omega}_k^+$ (respectively $\hat{\omega}_k^-$) is the parameter value located right above (resp. below) on the grid used to plot Figures C.1-C.4. m_n^+ (resp. m_n^-) is the corresponding moment obtained using parameter $\hat{\omega}_k^+$ (resp. $\hat{\omega}_k^-$), keeping the other parameters $\hat{\omega}_{k'}$ at their SMM estimate.

Table 2 confirms formally the results we discussed from Figure C.1-C.4.

3.5 Estimation results

We report the results of the SMM estimation in Table 3. One key contribution of the paper is to use the sensitivity of investment to real estate value as a targeted moment in this estimation. To highlight the contribution of this moment, we thus report two sets of results: One estimation where the SMM targets the mean leverage to identify financing constraints – as the existing literature does – and one set of results where the SMM instead targets the sensitivity moment. Each column corresponds to a model specification (with adjustment costs, Columns (3) and (4), and without adjustment cost, Columns (1) and (2)) and a set of targeted moments including leverage (Columns (1) and (3)) or the sensitivity of investment to house prices (Columns (2) and (4)). Column (5) corresponds to the data.

We first study the version of the model without adjustment cost ($c = 0$). There are 3 parameters to estimate: The persistence (ρ) and volatility (σ) of log-productivity, as well as the pledgeability parameter s. In Column (1) of Table 3, the SMM targets "traditional moments", i.e. the short- and long-term volatilities of log sales, and mean leverage. At the estimated parameters, the model matches all the targeted moments up to the second decimal, but does poorly on non targeted moments. The sensitivity of investment and debt to real estate value is high (three times their empirical value: 0.12 instead of 0.04 in both cases). The autocorrelation of investment is negative, instead of positive in the data, due to the absence of adjustment costs.

In Column (2), the estimation targets the sensitivity of investment to real estate prices instead of leverage. As a result, the estimated pledgeability parameter, s, is smaller than in the estimation of Column (1) (0.133 instead of 0.495). As was explicit on Figure C.1, the sensitivity of investment to real estate prices is an increasing function of s in this range of parameters: As a result, to reduce the sensitivity of investment to real estate prices relative to the one delivered by the estimation of Column (1), a smaller value for s is estimated. A lower estimated s implies a lower debt capacity, so that mean leverage in this model is much smaller, and in particular, much smaller than its empirical value (0.013 vs. 0.313 in the data). Since this model does not include adjustment costs to capital, the average autocorrelation of investment in the simulated model of Column (2) remains distant from its empirical counterpart (0.064 vs. 0.436 in the data).

We introduce these adjustment costs to capital in Columns (3) and (4). With these costs, the estimated model matches the autocorrelation of investment exactly, whether we target mean leverage (Column (3)) or the investment sensitivity coefficient (Column (4)). However, when the estimation targets the sensitivity of investment to real estate prices instead of mean leverage, we estimate a much smaller pledgeability parameter s (0.189 vs 0.422), for the same reason as mentioned in the discussion of the estimated models of Column (1) and (2). The introduction of adjustment costs to the model leads to a higher estimated pledgeability parameter (0.189 in Column (4) vs. 0.133 in Column (2)): In the presence of collateral constraints, adjustment costs to capital make investment less responsive to collateral values; as a result, to match the sensitivity of investment to real estate prices, the estimated s has to increase. With adjustment costs to capital and this sensitivity as a targeted moment (Column (4)), we are able to match perfectly not only the sensitivity of investment to real estate prices, but also the sensitivity of debt, not targeted in the estimation. The leverage ratio in the estimated model of Column (4) is larger than in the model with no adjustment costs (0.095 in Column (4) vs. 0.013 in Column (2)) – since the firm now has to pay for these adjustment costs – but it remains, however, below its empirical value (0.095 in Column (4) vs. 0.313 in the data). We do not view this discrepancy as a major source of concern. The corporate finance literature has put forth a number of determinants of leverage that are not included in our model (working capital management, moral hazard etc), but that would not necessarily interact with the *real outcomes* from the model. We thus take Column (4) as our preferred specification. We propose an extension to our model in Section 5.2, which allows us to simultaneously match the sensitivity of investment to real estate prices and mean leverage.

The calculation of standard errors is done by bootstrapping and is detailed in Appendix A. We draw 100 data samples and compute the set of targeted moments for each of these sample. We then run our SMM procedure for each one of these samples, and compute standard errors as the empirical s.d. of these parameters. To save on computing time, we estimate these 100 SMMs in parallel. Each time we solve the model with a new set of parameters, we check whether these parameters improve the matching of each one of the 100 moments. All parameters are estimated with a t-stat between 15 and 100. Such precision is not rare in SMM estimation. The collateral coefficient s is however, less precisely estimated (with a t-stat slightly above 3).

3.6 Determinants of financing constraints

In this section, we briefly discuss how firm characteristics covary with financing constraints. We use our preferred specification of Column (4), Table 3. We define a firm to be financially constrained when its capital stock is lower than 80% of its frictionless capital stock. To compute the frictionless capital stock, we solve the model using the same parameters but remove the no equity issuance constraint. We then consider various firm characteristics x, sort the simulated firms into 20 equal-sized bins of x and compute the fraction of constrained firms in each bin.[10] This methodology allows to see how, in the cross-section of firms, financing constraint covary with firm characteristics.

We report the results of this investigation in Figure 1. Panel A shows that more productive firms are more constrained: they are typically firms that experienced a positive productivity shock, but inherited a small capital stock, preventing them from growing as much as they would in the absence of collateral constraints. Panels B-E investigate the relationship between constraints and characteristics that are typically observable in firm-level data. Panel B shows a weak link between firm size and financing constraints: Larger firms are typically more productive (and therefore more constrained), but they also have more collateral (and are thus less constrained). Panel C shows that growing firms are typically more constrained, which is not surprising since they are likely to have experienced recent positive productivity shocks. Panels D shows that firms with high leverage are more likely to be constrained: Since there is no heterogeneity in s in our model, a firm with a high leverage ratio is typically a firm that experiences a large positive productivity shock and exhausts its debt capacity without being able to reach its first-best level of investment.

[10]As we do in our estimation procedure, we simulate firms over 100 years, but only use the last 10 years to compute the fraction of constrained firms, so as to make sure each firm has reached its steady-state.

Panel E shows a sharply increasing relation between the ratio of sales to capital and the fraction of constrained firms in the simulated data: This ratio captures the marginal revenue product of capital and captures the effective capital wedge firms face when optimizing investment (Hsieh and Klenow (2009)). Panel F illustrates the non-monotonic relation between the market-to-book ratio and the fraction of firms constrained: A low market-to-book ratio implies that firms have few investment opportunities and are thus less constrained; firms with a large stock of capital are close to unconstrained and as a result, have a large market-to-book ratio.

4 General Equilibrium Analysis

We now have a fully estimated model of firm behavior under financial constraints. To estimate the quantitative effect of this model on aggregate production and TFP, we embed it into a simple macro-model that accounts for general equilibrium feedbacks.

4.1 General equilibrium model

By clearing the goods and labor markets, the model endogenizes aggregate demand Q and the real wage w introduced in the firm-level model of Section 2. The model consists of the following simple assumptions.

Firms. A large number N of firms indexed by i produce intermediate inputs, in quantity q_{it}, at price p_{it}. All intermediate inputs are combined into a CES-composite final good

$$Q_t = \left(\sum_{i=1}^{N} q_{it}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}. \tag{7}$$

The final good is produced competitively. The demand for input i is thus given by $q_{it} = Q_t \left(\frac{p_{it}}{P_t} \right)^{-\phi}$, with $P_t = \left(\sum_i p_{it}^{1-\phi} \right)^{\frac{1}{1-\phi}}$. We normalize P_t to 1 and derive the demand function in equation (2).

Consumption and consumer behavior. The final good is used for (i) consumption, (ii) investment and (iii) to pay for adjustment costs. The final good market equilibrium thus writes:

$$Q_t = C_t + \text{Adj. Cost}_t + I_t \tag{8}$$

with C_t being aggregate consumption, Adj. $\text{Cost}_t = \sum_i \frac{c}{2} i_{it}^2 / k_{it}$ is the sum of all adjustment costs, and $I_t = \sum_i i_{it}$ is aggregate investment.

Consumption goes to a representative consumer that maximizes inter-temporal utility over consumption and labor supply:

$$U_s = \sum_{t \geq s} \beta^{t-s} u_t \text{ with } u_t = C_t - \bar{L}^{-\frac{1}{\epsilon}} \frac{L_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \tag{9}$$

where L_t are aggregate hours worked, $\bar{L}$ is a simple scaling constant, and ϵ is the Frisch elasticity of labor supply. With quasi-linear preferences, the Hicksian, Marshallian and Frisch elasticities of labor supply are all equal to ϵ. Labor supply is a static decision given by

$$L_t^s = \bar{L} w_t^{\epsilon}. \tag{10}$$

The consumption Euler equation ties the equilibrium interest rate r_t to the discount rate β, and so we take the interest rate $r_t = 1/\beta - 1$ as fixed throughout all counterfactuals.

Steady state assumption and equilibrium definition. We assume that the economy is in steady state. Intermediate good producers produce according to the technology described and estimated in the previous section. The log productivity shocks z_{it} that they face have no aggregate component. Given our assumption that the number of firms is large, aggregate output Q and wage w are constant over time. We are thus exactly in the case studied in Section 2.

The equilibrium (Q, w) of this economy is defined by two equations: the labor market equilibrium and the final good aggregator:

$$\bar{L}w^{\epsilon} = \sum_{i=1}^{N} l^{d}\left(\left(Q,w\right);z_{it},k_{it}\left(Q,w\right)\right) \tag{11}$$

$$Q = \sum_{i=1}^{N} p_{it}q\left(\left(Q,w\right);z_{it},k_{it}\left(Q,w\right)\right) \tag{12}$$

where $l^d(\cdot)$ is the numerically obtained labor demand function which is a function of each firm state variable and aggregate equilibrium (Q, w). Similarly $pq(\cdot)$ is the supply function, which, for each firm, associates state variables and macroeconomic conditions to its dollar sales. The equilibrium (Q, w) is the solution of these two conditions. We solve this problem by iteration, using a variant of the Newton-Raphson algorithm. We describe our methodology in detail in Appendix B.

In our quantitative exercise, we focus on the following aggregate quantities. Aggregate output Q and real wage w are direct outcomes of the algorithm. Aggregate employment is given by the supply curve: $L = \bar{L}w^{\epsilon}$. Aggregate log TFP is classically given by $\log Q - \alpha \log K - (1-\alpha)\log L$, where K, the aggregate capital stock in the steady state, is computed as the sum of capital stocks over all firms. Finally, welfare is a function of (Q, w), the aggregate capital stock K and aggregate adjustment cost

$$U = \frac{1}{1-\beta}\left(\left(Q - \delta K - \text{Adj. Cost}\right) - \frac{\bar{L}w^{1+\epsilon}}{1+\frac{1}{\epsilon}}\right).$$

4.2 The aggregate effect of financing constraints

We are now in a position to evaluate the aggregate effect of financing constraints. Compared to the firm-level model, the macroeconomic model has a few additional free parameters. Following Chetty (2012), we set the labor elasticity $\epsilon = 0.50$. We adjust $\bar{L}$ and the number of firms N so that the equilibrium parameter chosen for the estimation process ($Q = 1$ and $w = 0.03$) are actual equilibrium parameters when firm parameters are at the SMM estimate.

To measure the aggregate impact of financing constraints, we present all aggregates (output Q, wage w, TFP and welfare) in log deviations from the "unconstrained" benchmark. The appropriate way to define the unconstrained benchmark in our model is to lift the no equity issuance constraint, rather than the collateral constraint. With no equity issuance constraint, investment is unconstrained since equity is freely available to all firms and fairly priced at r. With no collateral constraint (unlimited debt), firms would raise infinite debt because it gives them a tax advantage. So strictly speaking, our unconstrained benchmark corresponds to a model with no equity issuance constraint, a collateral constraint, and all structural parameters otherwise unchanged.[11]

We first ask how the estimation method affects the aggregate effect of financing constraints. We implement this exercise in Table 4. First, we see that estimations targeting the sensitivity of investment to real estate prices (Columns (2) and (4)) generate a TFP loss twice as large as estimations targeting the leverage ratio (Columns (1) and (3)). In our preferred specification (Column (4)), we find a TFP loss of 2.7%, compared to 1.5% in Column (3). This discrepancy is at the core of our analysis: When the estimation targets the mean leverage ratio, it maps all the leverage in the data to collateralized debt in the model. This estimation thus implies a large level of pledgeability s so that the simulated model can match the high level of mean leverage in the data (0.42). This estimate is larger than actual net leverage (0.31 in the data), since in the model firms maintain some debt capacity and therefore issue less debt than they actually can. By contrast, when matching the rather low sensitivity of investment to collateral value, a moment that characterizes how real outcomes are affected by the collateral constraint, the estimated pledgeability parameter is smaller ($s = 0.189$ in Column (4)). In this estimation of Column (4), the collateral constraint is thus tighter than in the estimated model of Column (2), and as a result, losses from financing constraints are larger. In our context, the estimated TFP loss from financing constraints depends strongly on the choice of moment selected to reflect the importance of these constraints. Our paper argues that targeting the average investment response to shocks to collateral values provides more identifying power than targeting the mean leverage ratio (see the formal discussion in Section 5.1), and that as a result, we obtain larger TFP losses

[11] We show below that lifting the collateral constraint (increasing s to a large yet *finite* level) gives results similar to removing the no equity constraint.

from financing constraints.

Second, Column (4) shows that output loss from financing constraints are as large as 11%. More than half of this output loss is accounted for by a smaller aggregate stock of capital in the constrained economy ($0.192 \times 0.3 = 6.5\%$). About a quarter of this output loss comes from misallocation, since, as we discussed above, TFP in the constrained economy is lower by 3% relative to the unconstrained benchmark. These two effects combined reduce the productivity of labor, which in turn depresses labor supply. The labor supply response accounts for the remaining quarter of the overall output loss. Hence, even though misallocation is non-negligible, the total output loss from financing constraints mostly arises from aggregate under-investment: Firms are constrained, so that the representative consumer under-saves and supplies too little labor relative to the unconstrained economy. Overall, removing financing constraints has a large effect on welfare, 9.4% higher in the unconstrained relative to the actual economy. Consistent with the discussion on TFP losses, we find that the welfare loss from financing constraints is halved (5.1% in Column (3)) when using the estimated s obtained by targeting the average leverage ratio. We also see in Table 4 that adjustment costs tend to attenuate the welfare losses from financing constraints. In the presence of adjustment costs, firms smooth out investment by responding partially to productivity shocks. As a result, financing constraints bind less often. Note, however, that this effect of adjustment cost on the estimated welfare loss from financing constraints is quantitatively small.

In Figure 2, we show how these general equilibrium quantities are affected when we vary the pledgeability parameter s from 0 to 1. We start from the estimated model of Column (4), Table 3, which includes adjustment costs and target the sensitivity of investment to real estate prices. We then change s relative to its estimated value, determine the new general equilibrium of the model and compute the general equilibrium quantities reported in Table 4. As in Table 4, we report these quantities as deviations from the corresponding unconstrained benchmark. Finally, Figure 2 also reports the estimated pledgeability parameter s (vertical dark line), as well as the 95% confidence band for this parameter (light blue bar). The precision of our estimate – a standard error of 0.008 for a point estimate of 0.189 – implies that for values of s in the 95% confidence interval, aggregate effects remain close to their value reported in Table 4: The TFP loss from financing constraints vary by 0.5 percentage point, the output loss by about 2 percentage points and the capital loss by about 5 percentage points.

Overall, Figure 2 shows clearly how aggregate outcomes are affected by the pledgeability parameter s. In an economy with no pledgeability ($s = 0$) – and therefore where financing is done entirely through cash-flows – and relative to the unconstrained economy, output is smaller by about 15%, welfare by about 15% as well, employment by about 5%, capital by about 25% and aggregate TFP by about 4%. The effect of pledgeability on these aggregate quantities in general equilibrium is approximately linear. The limited response of aggregate employment to variations in s stems from the relatively small elasticity of labor supply we use. Finally, in the last panel of Figure 2, we report the cross-sectional dispersion of log MRPK ($\log p_i q_i/k_i$), the measure of distorsions used in Hsieh and Klenow (2009). Note that in the presence of adjustment costs and time-to-build in investment, this dispersion is not 0 (Asker et al., 2014). However, Figure 2 reports the dispersion of log MRPK relative to the unconstrained economy, which features the same adjustment costs and thus account for the effect of adjustment costs in the dispersion of log MRPK. When collateral cannot be pledged ($s = 0$), the dispersion in log MRPK is about 14% higher than in the unconstrained economy.

4.3 Productivity persistence and misallocation

Recent papers emphasize that the persistence of productivity shocks should reduce the aggregate effect of financing frictions (Moll, 2014; Buera et al., 2011). Intuitively, if productivity shocks are persistent, firms "grow out" of their financing constraints: productive firms are likely to remain productive and can accumulate cash holdings necessary to fund future investment. To measure this effect in our quantitative model, we start from the estimated model of Column (4), Table 3, pick alternative values for the parameter ρ, and compute the equilibrium dispersion of log MRPK ($\log p_i q_i/k_i$) (Hsieh and Klenow, 2009; Midrigan and Xu, 2014). When varying ρ, we keep $Var(z) = \sigma^2/(1-\rho^2)$ constant, varying σ^2 accordingly, as in Moll (2014).

Figure 3 shows that the amount of misallocation in equilibrium is significantly reduced when productivity shocks become very persistent. When ρ is set to 0.35 – about one third of its estimated value – the dispersion of log MRPK is more than 50% larger (0.43 vs. 0.66). At the estimated persistence (0.895 in Table 4, Column (4)), misallocation as measured through this dispersion is quite sensitive to variations in the persistence parameter.

5 Discussion

5.1 Model Identification

An important contribution of this paper is to base the estimation of a model of dynamic investment with collateral constraints on a well-identified, reduced-form moment that evaluates how real outcomes respond to shocks to collateral value – the sensitivity of investment to real estate prices. In contrast, most of the literature relies on moments related to financial leverage. Table 5 shows why our approach provides a better identification. To obtain this table, we simply simulate data from a model where firms are fully unconstrained. We then show that an estimation targeting the empirical mean leverage would fail to reject that firms are constrained; in contrast, an estimation targeting the sensitivity of investment to house prices would correctly reject this hypothesis.

More precisely, we start from the estimated model of Column (4), Table 3. We then simulate a sample of firms from these estimated parameters, but remove the no equity issuance constraint. These simulated firms are unconstrained, by definition. We then compute the following moments on this synthetic dataset: the long- and short-run volatility of log sales, the autocorrelation of investment, mean leverage, and the sensitivity of investment to real estate prices. Table 5, Column (3) show these moments. Unsurprisingly, the sensitivity of investment to real estate prices is -0.001: Firms are unconstrained, investment is efficient and unaffected by real estate shocks, which, by construction, are uncorrelated with productivity shocks.

Using this simulated sample – where the data generating process is such that firms are unconstrained – we estimate our model from Section 2 using either mean leverage (Column (1) of

Table 5) or the sensitivity of investment to real estate prices (Column (2) of Table 5) as a targeted moment. When the estimation targets leverage (Column (1)), the pledgeability parameter is in part determined to match leverage, 0.168 in the data. As a result, the estimated pledgeability parameter is low, $s = 0.436$, and in particular lower than one. This estimated s leads to wrongfully conclude that the economy suffers from substantial losses due to financing constraints: the estimated model implies, relative to the unconstrained economy, a 3.1% TFP loss, a 13.0% output loss and a 10.9% welfare loss) when the true model feature no such losses.

When we instead target the sensitivity of investment to real estate prices (Column (2)), the pledgeability parameter is estimated close to 1 ($s = 0.953$): The data used to compute the moments is such that firms are unconstrained so that their investment does not covary with real estate prices; to match this moment, the estimation has to find that the pledgeability of collateral is very high, so that firms' investment is close to its first best. As a result, the estimation based on this moment rightly concludes that there are no aggregate losses from financing constraints.

In other words, in this exercise, both models are misspecified, as they wrongly assume no equity issuance, while the firms in our synthetic dataset are free to issue equity. However, the estimated model targeting the leverage moment completely misses the fact that firms are unconstrained, while the model targeting the sensitivity of investment to real estate prices correctly infers negligible financing constraints.

Of course, our approach could also be invalidated using a similar exercise. One simply needs to find an alternative model where land-holding firms invest more following increases in house prices *relative* to firms not holding land for reasons other than collateral constraints. Finding such a model is equivalent to rejecting the identifying assumption in Chaney et al. (2012). Under their identifying assumption, however, the reduced-form moment purely arises from the existence of collateral constraints, and therefore cannot be falsified. In this sense, the point we make in this paper is generic, and goes beyond this particular reduced-form moment: A valid reduced-form moment identifying the effect of financing constraints on investment – valid in the sense that it estimates the causal effect of financing frictions under a reasonable identifying assumption –

will provide a better source of identification in the structural estimation than generic financial moments such as leverage.

5.2 Robustness: Residual leverage and costly equity issuance

In this section and Table 6, we discuss the robustness of our findings to either a setting where firms have spare debt capacity in addition to the collateralized debt that is the focus of our study, or to assuming firms are allowed to issue equity at a finite cost.

Residual leverage. A potential concern with our baseline specification is that it fails to match the mean leverage ratio (see Table 3, Panel A, column 4). The reason for this mis-match is the inherent tension in our baseline model between leverage and the sensitivity of investment to real estate prices. If one targets the leverage ratio (0.313 in our data), s has to be large (0.422), which leads to a counterfactually large investment to real estate sensitivity. If one targets the investment moment (0.04 in our data), s has to be small (0.189) and leverage is counterfactually small. We defend the choice of the investment moment in section 5.1 as a better way to detect the presence of financing constraints. In addition, leverage may be determined by a host of firm characteristics (unsecured debt capacity, trade credit, inventories etc) that we omit in our model. It is possible that once these other sources of external funding are accounted for, firms have enough debt capacity to escape financing constraints. We show here this is not the case.

We modify the baseline model and add a debt capacity $\bar{d}$ to the borrowing constraint,

$$(1+r)d_{it+1} \leq \bar{d} + s\left((1-\delta)k_{it+1} + \mathbb{E}[p_{t+1}|p_t]h\right). \tag{13}$$

This coefficient $\bar{d}$ captures un-collateralized debt capacity left out of the model.

We estimate this new model and report the results in Table 6, Column (2). The estimation targets both leverage and the sensitivity of investment to real estate prices, as well as the short- and long-term volatilities and autocorrelation of investment. To match the high level of leverage in the data, $\bar{d}$ is estimated to be high (0.45), while s remains close to our previous estimate (0.254 instead of 0.189). The productivity shock process remains similar. Interestingly, however, the aggregate impact of financing constraints (-10% welfare) is not smaller than in the model that does not fit leverage, i.e. where $\bar{d} = 0$ (-9.4% welfare). Firms do have a higher debt capacity, but this extra debt capacity is similar to free cash, i.e. cash that is not penalized in terms of returns. As a result, firms lever up more in order to minimize taxes, which is why the estimation now matches the leverage ratio almost perfectly. However, the overall borrowing constraint does not bind less because the extra debt capacity is used for tax optimization and not investment in physical capital. Overall, this simple addition to the model – an unsecured debt capacity $\bar{d}$ – allows us to match firms leverage, without changing our inference on the aggregate effects of financing constraints.

Costly equity issuance. We conclude this section by allowing for costly equity issuance. We assume a variable equity issuance cost of 15%, within the range estimated by Hennessy and Whited (2007). The results are presented in Table 6, column 3. Neither the parameter estimates nor the fit between simulated and actual moments vary much compared to our baseline specification. As firms now have the ability to issue equity, the aggregate effects of financing frictions are naturally reduced (5.8% aggregate output loss compared to 11% in our baseline specification).

5.3 Policy Experiments

In this section, we use our model to investigate the effect of an investment tax credit (ITC). We consider two types of policies. The first is a non targeted investment subsidy, where each firm in our sample receives a subsidy equal to $x \times i_{it}$, where i_{it} is the firm's investment and x is a fraction equal to 5, 10 and 15%. The second is a targeted investment subsidy, aimed only at capital poor firms, i.e. firm with a high MRPK ($\log(p_i q_i / k_i) > 0.4$). This second policy is motivated by the evidence in Figure 1 that most firms with a sales to capital ratio below 0.4 are unconstrained, while most firms above this threshold have a sub-optimally low level of capital.

In both cases, the subsidy is a linear function of investment, i.e. it becomes a tax when investment is negative. This feature avoids the emergence of short capital cycles where firms buy capital to enjoy the subsidy, and sell it the following year. Finally, this subsidy is financed via a lump-sum tax raised on household income. We make this assumption in order to focus on the effect of the ITC.

We report the results of these policy experiments in Table 7. With a non-targeted tax credit of 5% the capital stock increases by 11% and aggregate employment by about 1.4%. As a result, output rises by 4.3% and welfare by 2.9%. This large effect of the ITC occurs in our model because corporate profits are taxed at a high rate (33%), which depresses investment significantly: The ITC partially undoes the depressing effect of the corporate income tax.

Interestingly and perhaps surprisingly, the non-targeted investment subsidy increases welfare by about as much as a subsidy specifically targeted to capital poor firms, but at *a much lower cost* in terms of the aggregate amount spent to finance this subsidy. With a 15% subsidy, welfare increases by 9% for both the targeted and the non-targeted program. This increase in welfare corresponds to almost all the welfare loss from financing constraints estimated in our model (+9.4% in Table 4 column 4). However, the non-targeted subsidy requires a tax from household of about 2.4% of total output, while for the targeted subsidy, the cost of the program ends up being larger (about 4.4% of total output). The reason for this differential cost of the two subsidies is that a targeted program induces an opportunistic investment strategy: To benefit from the subsidy, firms invest little (disinvest) as long as their sales to capital ratio is below the policy threshold, and their investment rate jumps discontinuously as soon as they cross the policy threshold. Figure 4 makes clear this unintended effect of the targeted subsidy, by showing how the investment rate varies as a function of the sales to capital ratio, in both experiments. With the un-targeted subsidy, investment increases smoothly with the variable used to assign the subsidy (the sales to capital ratio); with the targeted one, investment increases sharply right at the policy threshold.

Conclusion

This paper provides a quantification of the aggregate effects of a specific source of financing frictions, collateral constraints. We build a simple dynamic general equilibrium model with heterogeneous firms and collateral constraints. To estimate this model structurally, we match not only key features of firm-level dynamics, but also a well identified reduced-form evidence that an increase in the value of a firm's collateral leads to an increase in investment. The estimated model is then used to simulate a counterfactual economy where financing frictions are lifted. Welfare increases by 9.4% and aggregate output by 11%. Quantitatively, only one quarter of these gains can be attributed to a more efficient allocation of inputs across heterogeneous firms – more productive firms are able to obtain more financing and expand – while half of these gains are due to a higher aggregate stock of capital, and the remaining quarter to a higher aggregate labor supply.

One limitation of this analysis is that the shocks to collateral value that we use to identify the effect of collateral constraints at the firm-level are exogenous in the model. Yet in equilibrium, increased investment and hiring at the local level will clearly feed back into local real estate prices. In addition, since households are not fully mobile across regions, variations in real estate prices will induce variations in wages faced by firms, which will affect their local input choices. Endogenizing the housing market and its feedback effect on local labor markets, and incorporating it into our quantitative analysis is an important step that we plan to tackle in future research.

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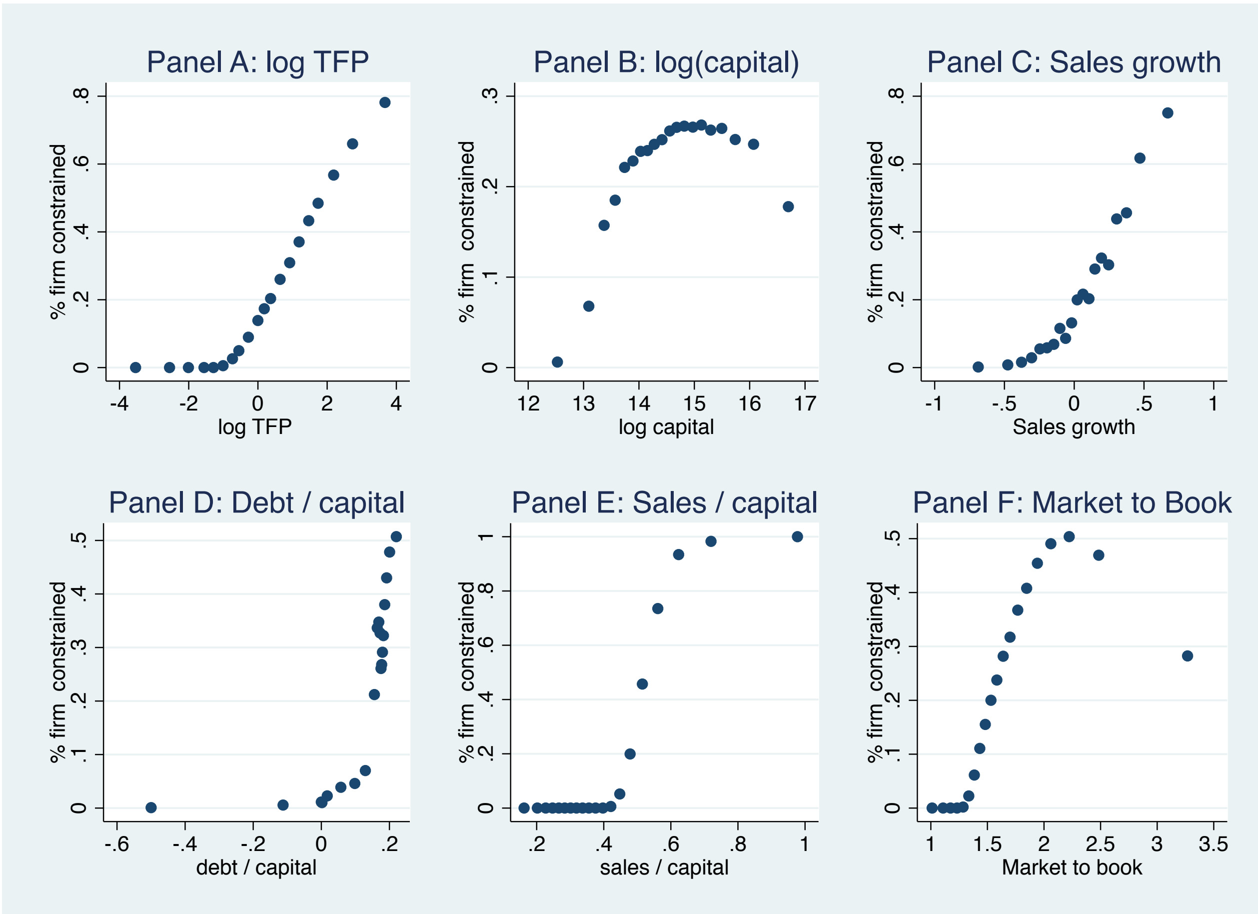
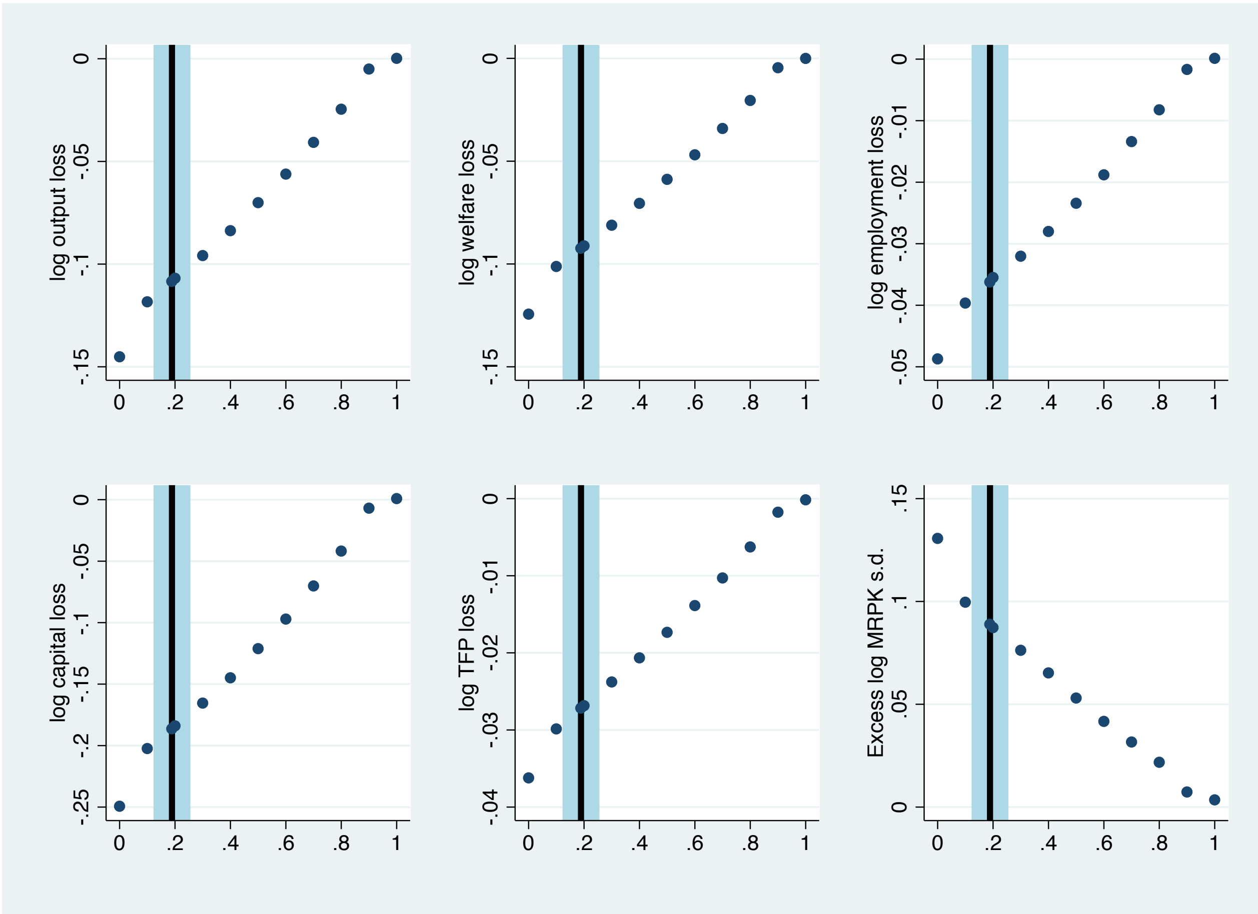
Figures

Figure 1: Financing constraints as a function of firm characteristics



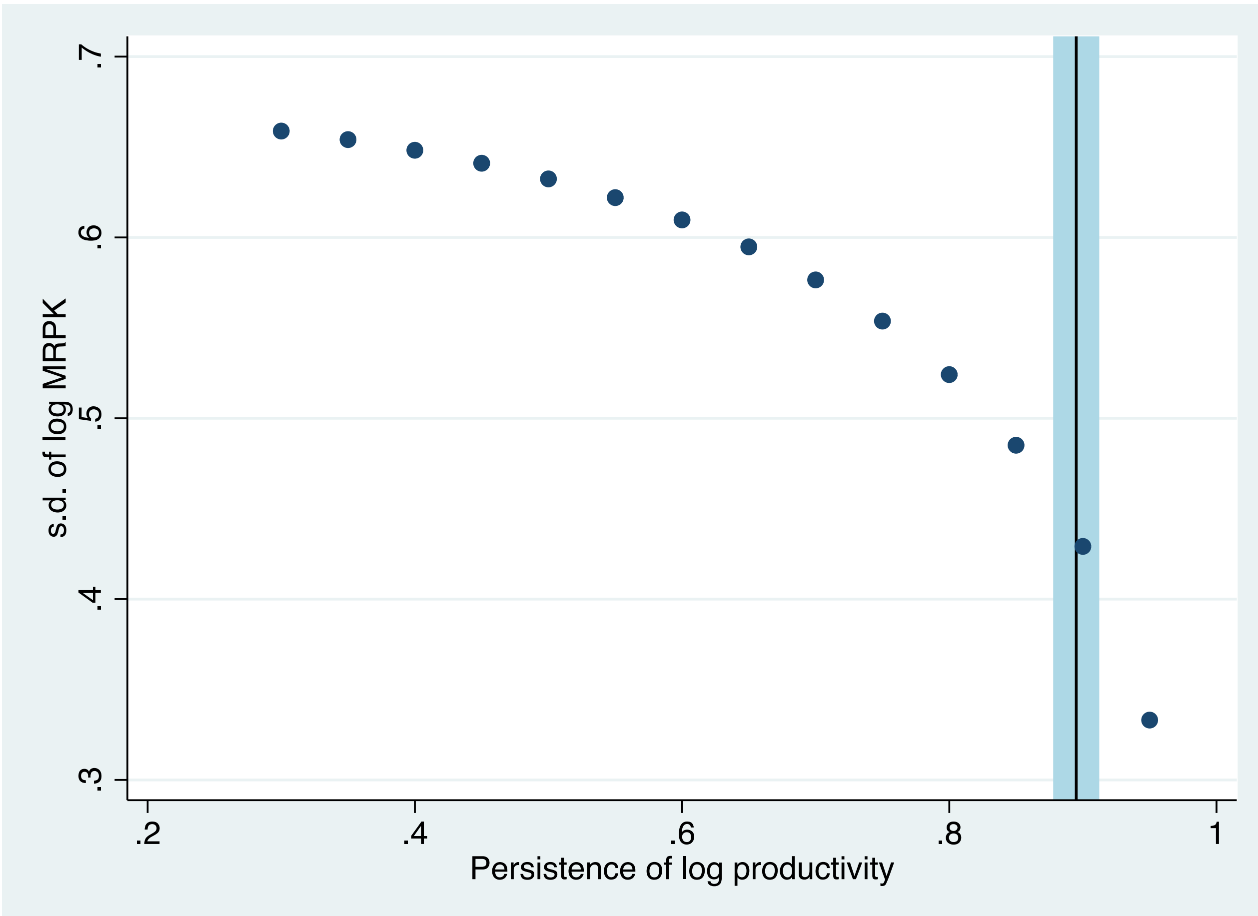
Note: This Figure shows how the extent of financing constraints covaries with firm characteristics, in the cross-section of simulated firms. We simulate a dataset of 1,000,000 firms over 10 years using parameters from our preferred specification (Table 3, Panel A, column 4). We remove the first 90 years to make sure firms are in steady state. For each characteristic x, we then sort firms into 20 quantiles of x, and for each quantile compute the average fraction of constrained firms in our simulated data. We label a firm constrained if its capital stock is less than 80% of its unconstrained capital stock. Unconstrained capital stock is computed after solving the same model, with the same parameters but without the no equity issuance constraint. The conditioning variable x is given by z (Panel A), $\log k$ (Panel B), $\log pq_t - \log pq_{t-1}$ (Panel C), $\frac{d}{k}$ (Panel D), $\frac{pq}{k}$ (Panel E), and $\frac{V}{k}$ (Panel F).

Figure 2: General equilibrium effect of pledgeability s



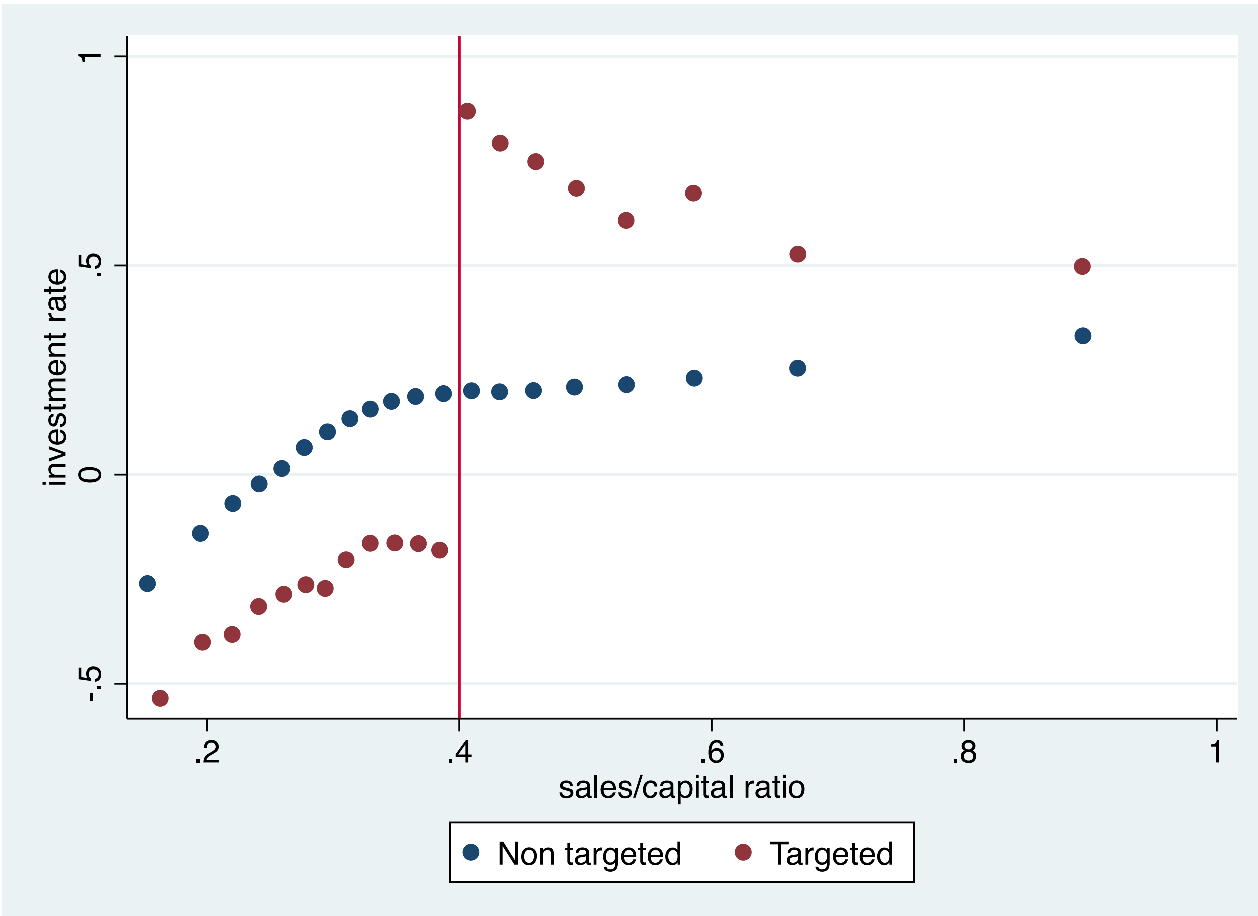
Note: This figure reports the general equilibrium effects of changing the collateral parameter s from 0 (full financial constraints) to 1 (100% of the capital stock can be pledged to lenders). We use the model with adjustment costs and estimated targeting the investment sensitivity moment (thus using the parameters reported in Table 3, Panel A, column 4). All aggregates are represented in deviation with respect to the unconstrained benchmark: For each value of s, we compute the general equilibrium of the economy populated with constrained firms, and also the GE of the economy populated by firms with the same parameters, but without the no equity issuance constraint. We then compute the log difference of output, welfare, employment, capital stock, TFP and the difference in the s.d. of log sales to capital ratio (MRPK). We then try all values of s from 0 to 1, spaced by .1. The vertical red line correspond to the SMM estimate of s (.189). Reading: When s increases from .1 to .6, the loss of log capital stock w.r.t. the unconstrained benchmark goes from -.2 to .-1.

Figure 3: Productivity persistence ρ and the dispersion of capital productivity



Note: This figure reports the effect on capital misallocation of changing the log productivity persistence ρ from 0.35 (low persistence) to .95 (high persistence). We use the model with adjustment costs and estimated targeting the investment sensitivity moment (Table 3, Panel A, column 4). Following Hsieh and Klenow (2009), we measure misallocation as the s.d. of log sales to capital ratio (MRPK).

Figure 4: Non targeted versus targeted investment subsidy



Note: This figure shows the relation between the sales to capital ratio and investment for two types of subsidies on investment – a non targeted 10% subsidy for all firms, and a targeted 10% subsidy aimed only at capital poor firms, i.e. firms with a high MRPK ($p_i q_i / k_i > 0.4$). The data is simulated using our SMM parameter estimates from Table 3 panel A column 4.

Tables

Table 1: **Summary statistics: COMPUSTAT Extract**

	Mean	s.d.	Obs
Investment$_{it}$ / k_{it-1}	.37	.42	20,074
Net borrowing / k_{it-1}	.05	.48	19,998
Real estate value$_{it}$	.77	1.27	20,074
$\frac{1}{k_{it}}$	.42	.65	20,074
Office price	.67	.21	20,074

Source: COMPUSTAT for accounting items and Global RealAnalytics for office prices. The construction of this data is described in detail in Chaney et al. (2012). The dataset is an extract of COMPUSTAT. It contains all firms present in 1993 who report accounting value and cumulative depreciation of land and buildings. These firms are then followed until they exit the sample or until 2006. We also require that office price data are available in the city where these firms have their headquarter in 1993. The variables shown are used in the two regressions presented in Section 1.

Table 2: **Elasticity of Moments with respect to Parameters**

	s.d. $\Delta \log q$	s.d. $\Delta_5 \log q$	$\frac{d_t}{k_t}$	$\beta(Inv, RE)$	$corr(\frac{i_t}{k_{t-1}}, \frac{i_{t+1}}{k_t})$	$\beta(Debt, RE)$
Pledgeability s	.077	.16	1.3	-1.3	-.044	-.99
Adjustment cost c	-.041	-.063	.34	.014	.42	.011
Volatility σ	.97	.92	-1.4	-.48	-.15	-.76
Persistence ρ	.081	1	-2.2	-.99	2	-2

Note: This table reports the elasticity of various moments with respect to the structural parameters that we estimate. First, we start with the SMM estimate $\widehat{\mathbf{\Omega}}$ of the parameters $\mathbf{\Omega}$. For each $k = 1, \cdots, 4$, we set $\omega_l = \hat{\omega}_l$ for all $l \neq k$, and vary the parameter ω_k around the estimated $\hat{\omega}_k$ in order to compute the elasticity of moments to parameters in the vicinity of the SMM estimate. For each moment m_n, we compute

$$\epsilon_{n,k} = \frac{m_n^+ - m_n^-}{\omega_k^+ - \omega_k^-} \times \frac{\hat{\omega}_k}{\hat{m}_n} \approx \frac{\partial \log(\hat{m}_n)}{\partial \log(\hat{\omega}_k)},$$

where $\hat{m}_n$ is the n^{th} data moment. m_n^+ is the moment based on data simulated with parameter $\hat{\omega}_k^+$. Likewise, m_n^- is the average of moments based on data simulated with parameters ω_k^-. ω_k^+ and ω_k^- are parameter values right above and right below the SMM estimate $\hat{\omega}_k$, when the interval of definition of ω is graded on a scale going from 0 to 10 as in Figures C.1-C.4. *Reading*: Around the SMM estimate, a 1% increase in s is associated with a 1.3% decrease in the sensitivity of investment to real estate and a 1.3% increase in leverage.

Table 3: **Parameter estimates (SMM)**

	(1) No adj cost, Lev. target	(2) No adj cost, Inv. target	(3) Adj cost, Lev. target	(4) Adj cost, Inv. target	(5) Data
Panel A: Estimated Parameters					
ρ	0.917	0.919	0.865	0.895	
	(0.011)	(0.008)	(0.008)	(0.008)	
σ	0.623	0.725	0.818	0.820	
	(0.010)	(0.017)	(0.013)	(0.012)	
s	0.495	0.133	0.422	0.189	
	(0.024)	(0.030)	(0.020)	(0.014)	
c			0.050	0.045	
			(0.003)	(0.003)	
*Panel B: Moments (targeted in **bold**)*					
Std of 1-year sales growth	**0.327**	**0.327**	**0.327**	**0.327**	0.327
Std of 5-year sales growth	**0.909**	**0.910**	**0.910**	**0.911**	0.911
Real-Estate to assets	**0.140**	**0.140**	**0.140**	**0.140**	0.140
Net debt to assets	**0.300**	0.013	**0.315**	0.095	0.313
$\beta(Inv, RE)$	0.126	**0.038**	0.082	**0.040**	0.040
Autocorrelation of investment	-0.057	0.064	**0.436**	**0.436**	0.436
$\beta(Debt, RE)$	0.124	0.037	0.084	0.038	0.039

Note: This table reports the results of our SMM estimations. The estimation procedure is described in the text and in Appendix A. Columns (1)-(4) correspond to SMMs using different sets of parameters and targeting different sets of moments. Columns (1) and (2) assume not adjustment cost ($c = 0$), while Columns (3) and (4) introduce adjustment costs to the model. Estimations reported in Columns (1) and (3) target the short- and long-term volatilities of log sales, mean leverage, and the autocorrelation of investment. Columns (2) and (4) target the sensitivity of investment to real estate prices instead of mean leverage. For each of these estimations, Panel A shows the estimated parameters, along with standard errors (obtained via bootstrapping) in parenthesis. Panel B shows the value of a set of moments, measured on simulated data (with 1,000,000 observations). Moments in bold are the ones that are targeted by the estimation. The other moments are not targeted. The last column (labeled "data") features the empirical moments.

Table 4: **Aggregate effects of collateral constraints**

	(1) No adj cost, Lev. target	(2) No adj cost, Inv. target	(3) Adj cost, Lev. target	(4) Adj cost, Inv. target
Panel A: Targeted moments				
Std of 1-year sales growth	Y	Y	Y	Y
Std of 5-year sales growth	Y	Y	Y	Y
Real-Estate to assets	Y	Y	Y	Y
Net debt to assets	Y	N	Y	N
$\beta(Inv, RE)$	N	Y	N	Y
Autocorrelation of investment	N	N	Y	Y
Panel B: Loss from financial constraint in general equilibrium				
log(TFP)	0.015	0.034	0.015	0.027
log(output)	0.081	0.160	0.061	0.110
log(wage)	0.054	0.106	0.040	0.073
log(L)	0.027	0.053	0.020	0.036
log(K)	0.157	0.296	0.107	0.192
log(welfare)	0.063	0.131	0.051	0.094

Note: This table reports the results of the general equilibrium counterfactual analysis for different SMM parameter estimates. The general equilibrium analysis is described in Section 4 and the procedure detailed in Appendix B. Columns (1)-(4) correspond to parameters from SMMs assuming different parameter restrictions and targeting different sets of moments. Columns (1) and (2) assume not adjustment cost ($c = 0$), while Columns (3) and (4) allow for them. Parameters in Columns (1) and (3) correspond to SMMs which target "classic" moments, including mean leverage, while Columns (2) and (4) target the sensitivity of investment to real estate value instead of mean leverage. For each one of these estimations, panel A simply recalls the targeted moments. Panel B reports the result of the GE counterfactual analysis. All results are shown as log deviations with respect to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – as reported in the same column, Table 3, panel A – but no constraint on equity issuance. In this unconstrained benchmark, investment reaches first best. *Reading*: In column 1 (targeted leverage, no adjustment cost), the aggregate TFP loss compared to a benchmark without financing constraints is $e^{0.015} \approx 1.5\%$.

Table 5: **Estimating a constrained model on unconstrained data**

	(1) Leverage target	(2) Investment target	(3) Unconstrained simulated model
Panel A: Parameters			
ρ	0.943	0.900	0.895
σ	0.886	0.811	0.820
s	0.436	0.953	0.189
c	0.042	0.042	0.045
Cost of equity	$+\infty$	$+\infty$	0
Panel B: Moments (matched in bold fonts)			
Std of 1-year sales growth	**0.377**	**0.366**	0.367
Std of 5-year sales growth	**1.164**	**1.178**	1.171
Real-Estate to assets	**0.133**	**0.156**	0.152
Net debt to assets	**0.167**	0.885	0.168
$\beta(Inv, RE)$	0.037	**0.003**	-0.001
Autocorrelation of investment	**0.420**	**0.431**	0.426
$\beta(Debt, RE)$	0.034	0.361	0.069
Panel C: Loss from financial constraint in general equilibrium			
log(TFP)	0.031	0.000	0
log(output)	0.130	0.002	0
log(wage)	0.086	0.001	0
log(welfare)	0.109	0.002	0

Note: This table reports the result of our SMM estimation on a synthetic dataset simulated by a model *without* financing friction. We start with our baseline parameters (Table 3 Panel A Column (4)). We remove the no equity constraint, and simulate a synthetic panel dataset of unconstrained firms. We compute various moments and report them in column 3. We then perform two SMM estimations of a model *with* no equity issuance constraint. The estimation procedure and general equilibrium analysis are described in the text and in Appendices A and B. In Column (1), we match short- and long-term log sales volatility, the autocorrelation of investment, and mean leverage. In Column (2), we match short- and long-term log sales volatility, the autocorrelation of investment, and the sensitivity of investment to real estate value. Panel A reports the estimated parameters (to be compared with the true parameters used for the simulation in Column (3)), Panel B the moments (targeted in bold, and non-targeted, to be compared with the synthetic data moments in Column (3)), and Panel C computes the implied GE losses from financial constraints by comparing output, TFP, labor and welfare with a model with the same parameters but no no equity issuance constraints (to be compared to the true losses in Column (3)).

Table 6: **Robustness: unsecured debt and costly equity issuance**

	(1) Baseline	(2) Unsecured Debt Capacity	(3) Costly Equity Issuance	(4) Data
Panel A: Estimated Parameters				
ρ	0.895	0.877	0.868	
σ	0.820	0.821	0.806	
s	0.189	0.254	0.235	
$\bar{d}$	-	0.450	-	
c	0.045	0.052	0.048	
Panel B: Targeted moments				
Std of 1-year sales growth	**0.327**	**0.327**	**0.324**	0.327
Std of 5-year sales growth	**0.911**	**0.906**	**0.920**	0.911
Real-Estate to assets	**0.140**	**0.140**	**0.145**	0.140
Net debt to assets	0.095	**0.300**	0.181	0.313
$\beta(Inv, RE)$	**0.040**	**0.040**	**0.039**	0.040
Autocorrelation of investment	**0.436**	**0.439**	**0.458**	0.436
$\beta(Debt, RE)$	0.038	0.039	0.054	0.039
Panel C: Loss from financial constraint in general equilibrium				
log(TFP)	0.027	0.027	0.013	
log(output)	0.110	0.122	0.058	
log(wage)	0.073	0.081	0.038	
log(welfare)	0.094	0.100	0.048	

Note: This table reports the SMM estimation result and GE counterfactual experiments of two alternative versions of our baseline model. In Column (2), we assume that in addition to collateralized debt, the firm has access to an extra fixed debt capacity ($\bar{d}$) as in equation (13). In column 3, we relax the zero equity constraint, and allow for costly equity issuance, with a 15% variable cost. The estimation procedure and general equilibrium analysis are described in the text and in Appendices A and B. We target the following moments: short- and long-term volatilities of log sales, the autocorrelation of investment, investment sensitivity to real estate value, and mean leverage in Column (2) only. Panel A shows the estimated parameters. Panel B shows the value of simulated moments. Moments in bold are the ones that are targeted by the estimation. Panel C reports the result of a GE counterfactual experiment. All results are shown as % losses with respect to the unconstrained benchmark. The unconstrained benchmark correspond to an equilibrium where firms face the same set of parameters as in the SMM estimate – as reported in Panel A – but no constraint on equity issuance. Column (1) recalls the results of our baseline preferred estimation for comparison. Column (2) reports the estimate of the model with fixed unsecured debt capacity targeting the 6 moments. Column (3) reports the estimate of the model with costly equity issuance (15% variable cost instead of an infinite cost), targeting the same 5 moments as in our baseline model. Column (4) reports the data moments.

Table 7: **Macro effect of an investment subsidy**

	(1)	(2)	(3)	(4)	(5)	(6)
	Untargeted			*Targeted*		
Subsidy (share of investment)	5%	10%	15%	5%	10%	15%
Aggregate subsidy (% of output)	.007	.015	.024	.027	.035	.044
Δ log Output	.043	.089	.14	.069	.093	.12
Δ log Capital	.11	.23	.36	.13	.18	.24
Δ log Labor	.014	.03	.046	.023	.031	.039
Δ log TFP	0	0	-.001	.014	.016	.017
Δ log Welfare	.029	.059	.089	.058	.074	.091

Note: This Table reports the aggregate equilibrium impact of tax subsidies. We start with the model and parameter estimates of Table 3, Column (4). To cash flows of firm i at date t, we add a tax free subsidy equal to xI_{it} where I_{it} is the investment of firm i at date t and x is a fraction equal to 5,10 and 15%. Note that this subsidy becomes a tax when the firm's investment becomes negative. This subsidy is financed by a non distortionary tax on households. In columns 1-3, the tax is not targeted. In Columns (4)-(6), the tax is targeted only towards capital-poor firms, i.e. firms with a high MRPK ($\log(p_i q_i/k_i) > 0.4$). For each one of these six policies, we compute the equilibrium and report the change in log aggregates compared to the case without subsidy. For instance, we find that giving firms a non-targeted subsidy equal to 5% of their investment leads to an increase in aggregate output of 4.3%.

APPENDIX

This Appendix contains: the method used to to solve and estimate the model (Section A), the method we use to compute the general equilibrium of our model (Section B) and the additional comparative static results in partial equilibrium designed to show that the model is well behaved around the estimate (Section C).

A Solving the model and Estimation

This Appendix details the algorithms used to solve the model and estimate it. To estimate the model, one needs to find the set of parameters such that model-generated moments fit a pre-determined set of data moments. Because our model does not have an analytic solution, we need to use indirect inference to perform the estimation. Such inference is done in two steps:

- For a given set of parameters, we need to solve the model numerically, which means solving the Bellman problem (6) and obtain the policy function $S_{t+1} = (d_{t+1}, k_{t+1})$ as a function of $S_t = (d_t, k_t)$ and exogenous variables $X_t = (z_t, p_t)$.

- We then use this resolution technique to estimate the parameters that match best a set of moments chosen from the data. We explain the methodology (Simulated Method of Moments) and the numerical algorithm that we use to implement it.

A.1 Solving the model numerically

In this section, we describe how we numerically solve the firm's problem with given parameters.

A.1.1 Grid definition

In order to solve the model numerically, we need to discretize the state space $(S; X)$. Let us start with the two exogenous variables. The log productivity process z is discretized using the standard Tauchen method on 51 grid points. Log real estate prices are also an AR(1), discretized using the Tauchen method on 11 grid points. For both variables, we set the bounds of the grid at -2.5 and 2.5 standard deviations.

Capital choice is discretized over a range going from k_{min} to k_{max}. k_{min} is the smallest level of capital chosen by a firm without adjustment costs and financing constraint. For this particular case, we can solve the capital decision analytically. In most cases, this number should serve as a lower bound because adjustment costs would prevent firms from adjusting all the way down to this level; and financial constraints would push them to keep more capital as precautionary savings. Since we did not, however, establish this result analytically, we check that k_{min} is always "far enough" from the lowest simulated value of capital. Similarly, k_{max} is the capital stock chosen by unconstrained firms, without adjustment cost, facing the highest productivity level on the grid. Again, we expect this level to be above the upper bound of capital for a constrained firm with adjustment costs. We check that this is the case in our simulations. We then form an equally spaced grid for log capital between $\log k_{min}$ and $\log k_{max}$, with increment of $\log(1+\delta/2)$. Thus, the capital grid is geometrically spaced using $(1+\delta/2)$ as the multiplying coefficient, i.e. the nth point is equal to $k_{min} \times (1+\delta/2)^n$ until k_{max}. Given that k_{min} and k_{max} are functions of productivity, the grid thus depends on the persistence ρ and volatility σ of log productivity. Larger persistence or volatility leads to wider grid. In our preferred specification, capital evolves on a grid containing 270 points. We will take this number as a reference when we later discuss grid size, bearing in mind that, in fact, the capital grid is a function of parameter values.

Finally, the debt grid d_t is defined as a function of the amount of capital k_t. This adaptive feature of the debt grid comes from the fact that the amount of debt is bounded above by a function of capital: larger firms can borrow more. We restrict future period debt d' to the $\left[-4\bar{d};\bar{d}\right]$ interval, where $\bar{d} = s\left((1-\delta)k + p_{max}h\right)$ and p_{max} is the maximum house price level. The grid interval is thus a function of the model parameters s but also ρ and σ via the grid of k. The upper bound is a natural consequence of the collateral constraint: the model imposes that it cannot be exceeded. The lower bound is somewhat arbitrary as there is in theory no upper bound as to how much cash the firm may decide to hold. We check that there is no accumulation of cash at this bound during the estimation process. Within this interval, the grid is geometrically spaced so that it is more dense when debt becomes closer to the constraint, i.e. right below $\bar{d}$. We implement this by setting the nth grid point at $\bar{d}\,(1 - 0.001 \times e^{.3n})$ until it reaches $-4\bar{d}$. Thus, the grid size for debt does not depend on parameters (in contrast to the capital grid size) and always has 29 points.

A.1.2 Bellman resolution algorithm

We solve the firm's problem using policy iteration. This algorithm is based on the fact that the value function is the solution of a fixed point problem generated by a contraction mapping.

Before starting to iterate, we compute profit flows $e(S,S';X)$ using the production and cost functions, for all possible values of S and X on the grid. We set e to "missing" when $(S,S';X)$ are such that $e<0$ – the no equity issuance constraint is violated, or when the borrowing constraint is violated. Profits are only defined when both financing constraints are satisfied.

To initiate the process, we start with the value function $V_0\,(S;X) = 1$. We then look for the policy function $(k'_0, d'_0) = P_0(S;X)$ which solves:

$$P_0(S;X) = \text{argmax}_{S'}\{e(S,S';X) + \frac{1}{1+r}\}$$

for each state of $(S;X)$. Then, we iterate the following loop (where $n \geq 1$ denotes the step in the loop):

1. Start from $(k'_{n-1}, d'_{n-1}) = P_{n-1}(S;X)$, the policy function obtained from the previous round; and $V_{n-1}\,(S;X)$, the value function obtained from the previous round. For every point $(S;X)$ on the grid, we compute the value function V_n that satisfies:

$$V_n(S;X) = e(S,P_{n-1}(S;X);X) + \frac{1-d}{1+r}\mathbb{E}_{X'}\left[V_{n-1}(P_{n-1}(S;X');X')|X\right] + \frac{d}{1+r}\left(k'_{n-1} - (1+\tilde{r}_t)d'_{n-1}\right)$$

2. We then use the new value function V_n and compute the optimal policy given this value function ($P_n(S;X)$):

$$P_n\,(S;X) = \text{argmax}_{S'}\{e(S,S';X) + \frac{1-d}{1+r}\mathbb{E}_{X'}\left[V_n(S';X')|X'\right] + \frac{d}{1+r}\left(k' - (1+\tilde{r}_t)d'\right)\}$$

3. We stop when $P_n = P_{n-1}$.

Thanks to the contraction mapping theorem, we are guaranteed to find a good approximation of the value function $V(S;X)$ and the policy function $S' = P(S;X)$ defined over the grid. The computationally costly step is the determination of the policy function in step 2 with respect to S'. This consists of $29 \times 270 \times 51 \times 11 = 4,392,630$ optimizations of vectors with $29 \times 270 = 7,830$

points. This is where parallelization achieved through a GPU accelerates the process. For the range of parameters we explore, we typically solve the model in about 2 minutes with a GPU (Nvidia K80), compared to several hours with a CPU. What prevents us from having a finer grid is the RAM of the GPU, since the computer needs to create the maximand in step 2, a $29 \times 270 \times 29 \times 270 \times 51 \times 11 \approx 34$ billion numbers array.

The above algorithm is the standard policy function iteration algorithm. We make two adjustments to adapt it to our setting. First, in order to reduce computing time, we first solve the model with a coarser grid, and then solve it again on the grid describe above. To define this coarser grid, we divide the resolution of the control (k and d) grids by two. This divides computing time by four in the first step but only gives us the value and policy functions on the coarser grid. We then re-run the algorithm on the finer grid with the "coarser" policy and value functions as starting point. Convergence occurs much more quickly.

The other adjustment is related to the treatment of missing values, which in our set-up occur when one of the two financing constraints are violated (i.e. the no-equity constraint or the collateral constraint). Without modification, the policy iteration algorithm does not behave well in the presence of missing values. This is because, for some given value functions V_{n-1}, there may exist some (S, X) for which there is no acceptable policy S'. In this case, the optimal policy function $P_n(S, X)$ is not defined everywhere on the grid (note (S_0, X_0) such states for which the policy is not defined). When this happens, the next iteration value $V_n(S; X)$ is non-defined for all (S, X), which leads with non-zero probability to states (S_0, X_0). As we iterate, missing value progressively spread to the entire grid and the algorithm is blocked. To solve this problem, we modify step 1. of the algorithm by requiring that $V_n(S; X)$ replaces $V_{n-1}(S; X)$ if and only if $V_n(S; X)$ is non missing. This prevents missing values from spreading to the entire grid of states $(S; X)$.

A.2 Estimation

We now proceed to estimate the parameters (s, c, ρ, σ) for which the model best matches a pre-defined set of moment (we experiment with different set of moments and models in the main text).

A.2.1 Estimation method: SMM

We estimate the key parameters of the model by simulated method of moments (SMM), which minimizes the distance between moments from real data and simulated data. Let us call $\mathbf{m}$ the vector of moments computed from the actual data, and let us call $\boldsymbol{\Omega}$ the moments generated by the model with parameters Ω. The SMM procdure searches the set of parameters that minimizes the weighted deviations between the actual and simulated moments,

$$(\mathbf{m} - \widehat{\mathbf{m}}(\boldsymbol{\Omega}))' \, \mathbf{W} \, (\mathbf{m} - \widehat{\mathbf{m}}(\boldsymbol{\Omega})) \tag{14}$$

We detail the various components of our implementation in the following sections.

A.2.2 Empirical moments m and Weight matrix W

The empirical moments are computed in a simple way, and the definitions are given in the main text, in Section 3.3.

The weight matrix $\mathbf{W}$ adjusts for the fact that some moments are more precisely estimated than others. It is computed as the inverse of the variance-covariance matrix of actual moments estimated by bootstrap with replacement on the actual data. To compute the elements of this matrix, we repeat 100 times the following procedure. Using our dataset, we draw, with replacement, N firms with their entire history where N is the number of firms in the sample (we use the bsample command in Stata, clustered at the firm level). We then compute the moments, and store them. Once we have performed this procedure 100 times, we compute the empirical variance-covariance matrix of the moments, and invert it.

A.2.3 Model-generated moments m

Once we have solved the model for a given set of parameter $\mathbf{\Omega}$ (Appendix A.1), we need to simulate data in order to compute the simulated moments. We simulate a balanced panel of 1,000,000 firms over 100 years, and only keep the last 10 years to ensure each firm has reached steady state. For each firm, we simulate a path of log productivities z_{it} and a path of log real estate prices p_{it}. This makes the variability of real estate prices larger than in the data, where prices only vary at the city (MSA) level. Recall however that our objective in this simulation is not to replicate the variability of the data, but ideally to estimate model-generated moments. If we had closed forms for the model, we could measure these moments without infinite precision. The problem here comes from the fact that we cannot directly compute these moments but have to "estimate" them. Ideally, we would want to generate an infinitely large simulated dataset in order to compute the model-generated moments exactly, but computational constraints make it infeasible. 100,000 firms over 100 years already generate arrays with 10m entries. Allowing real estate prices to vary at the firm-level is a way to make sure the sensitivity to prices model-generated moments are estimated as well as possible.

A.2.4 Optimization algorithm

We now have all the ingredients necessary to compute the objective function (14). In this Section, we explain how to minimize it. Since in our most preferred specification we have 5 parameters, we need to make sure that we are indeed reaching a global minimum. We do this by implementing the following two-step procedure, which follows Guvenen et al. (2014):

- We generate 1,000 quasi-random vectors of parameters $\mathbf{\Omega}$ taken from a Halton sequence. The Halton sequence is a deterministic sequence of numbers that has the property of covering the parameter space evenly. For each of these parameters, we solve the model to obtain the policy function, simulate a dataset, compute the moments and therefore the distance to data moments (14).

- We then use the lowest points (in terms of objective function) as starting points for minimization. We iterate on the following loop. We begin with parameter estimate $\hat{\mathbf{\Omega}}_{\mathbf{1}}$ for which the objective function is the lowest. We then use the Nelder-Mead method (command *fminsearch* in Matlab) to perform a local optimization starting from this point. We then compute the objective function O_1. We then move to the second lowest parameter estimate ($\hat{\boldsymbol{\Omega}}_{\mathbf{2}}$) and compute the objective function O_2. We iterate on this, and stop as soon as $O_n = 0$. Among the lowest parameters, a large fraction typically leads to the same parameters for which the objective function is equal to 0. This gives an indication that our objective function is well-behaved.

There is no general theoretical results arguing that this technique dominates other popular algorithms adapted for large dimension optimization. In our setting however, we found that the genetic algorithm and simulated annealing were much slower at converging. Also, this approach allows to "control" the smoothness of the objective function. For instance, within the lowest 20 parameters isolated after step 1., it would be worrisome if minimizations starting from each of these parameters gave inconsistent parameters. On the contrary, they tend to be very consistent. The only cases where convergence goes to alternative choice of parameters than the one we present are cases where the objective function is much bigger than zero (i.e. other local optima). Finally, the best argument in favor of our selected estimates is the well-behaved comparative statics we present in Appendix C.

A.2.5 Standard errors

We estimate our standard errors using a block-bootstrap procedure. As for the computation of the variance-covariance matrix, we start by generating $B = 100$ datasets of N firms drawn without replacement from the data, and then compute the vector of targeted moments for each dataset. To preserve the panel structure we make sure to draw firms and not observations (hence the "block" in block-bootstrapping). The result is a set of 100 vectors m_b, for each of whom we seek the vector of model parameters $\boldsymbol{\Omega}_{\mathbf{b}}$ that minimizes

$$f_b\left(\boldsymbol{\Omega}_{\mathbf{b}}\right) = \left(\mathbf{m}_{\mathbf{b}} - \widehat{\mathbf{m}}\left(\boldsymbol{\Omega}_{\mathbf{b}}\right)\right)' \mathbf{W} \left(\mathbf{m}_{\mathbf{b}} - \widehat{\mathbf{m}}\left(\boldsymbol{\Omega}_{\mathbf{b}}\right)\right). \tag{15}$$

To reduce computing time we estimate the 100 parameters $\boldsymbol{\Omega}_{\mathbf{b}}$ in parallel. We use the following algorithm. We define a new objective function as the sum of all 100 objective functions, that is

$$F\left(\Omega_1, .., \Omega_B\right) = \sum_{b=1}^{B} f_b\left(\boldsymbol{\Omega}_{\mathbf{b}}\right). \tag{16}$$

1. $\widehat{M}$ is initialized using our SMM estimate $\hat{\boldsymbol{\Omega}}$. As a result, each parameter $\boldsymbol{\Omega}_{\mathbf{b}}$ is equal to $\hat{\boldsymbol{\Omega}}$ (so they are all identical). Let b^* be the sample for which $f_b(\hat{\boldsymbol{\Omega}})$ is that highest. This corresponds to the bootstrapped sample for which the main SMM estimate fits the moments the worst.

2. We use the Nelder-Mead simplex algorithm to improve the estimate $\boldsymbol{\Omega}_{\mathbf{b}^*}$ of the least well matched sample b^*. Specifically, we use Matlab fminsearch function with the following options:

- The initial simplex Δ_{b^*} is computed using the current estimate of $\mathbf{\Omega_{b^*}}$ as an "initial guess"
- The local optimization is stopped as the soon as b^* is no longer the sample with the worst fit.
- If fminsearch reaches a maximum of 50 iterations, Δ_{b^*} is reinitialized using the best available estimate of $\mathbf{\Omega_{b^*}}$ as an "initial guess".

   We then use the outcome of this procedure to update the parameter estimate of sample b^* in the list $\widehat{M}$

3. For each vector m_b, we find in $\widehat{M}$ the vector $\mathbf{\Omega_b}$ that minimizes $f_b\left(\mathbf{\Omega_b}\right)$. We then find the new sample b^* for which the objective function f_{b^*} has the highest value.

4. If the standard deviations of $\mathbf{\Omega_b}$ have moved by less than 1% over the last 500 evaluations, and if the value of F is less than one tenth of its initial value, then the procedure stops. Otherwise, it goes back to step 2.

Standard errors of $\mathbf{\Omega}$ are estimated using the standard deviation of the $\mathbf{\Omega_b}$. The fact the value of F is divided by at least ten indicates that the dispersion of $\mathbf{\Omega_b}$ is sufficient to explain 90% of the (weighted) dispersion of $\mathbf{\Omega_b}$. To reach that point, our procedure typically takes the equivalent of 2-3 SMMs to converge, and is thus about 30 times faster than running all 100 SMMs sequentially.

B General Equilibrium Computation

In this Section, we describe how we compute the general equilibrium of an economy populated by firms whose behavior is described by the model estimated and solved in Appendix A. First, recall that this model is estimated assuming aggregate demand $Q = 1$ and aggregate wage $w = 0.03$.

The economy is described in detail in Section 4 in the main text. There is a large number of firms (a continuum in the model), each of them facing an idiosyncratic path of productivity and of real estate prices. The behavior of each of these firms is described by the dynamic model with adjustment costs, time-to-build capital, the collateral constraint and the no-equity constraint. All firms are monopolists that produce intermediate inputs combined in a CES-aggregate with elasticity of substitution ϕ. As a result ϕ measures the intensity of competition between intermediate producers ($\phi = +\infty$ means perfect competition). The final good is then consumed by a representative producer with linear utility, Frisch elasticity of labor supply ϵ and subjective discount rate r. Consumption equals production minus adjustment costs and investment. The price of the final good is normalized to 1 without loss of generality. This economy has no aggregate uncertainty and the equilibrium is uniquely described by aggregate production Q and real wage w, which are fixed over time.

Start from a set of SMM estimates $\hat{\mathbf{\Omega}}$. Our goal is to investigate the GE consequences of a change in parameter $\mathbf{\Omega}$ from its estimated value $\hat{\mathbf{\Omega}}$ to another $\mathbf{\Omega}'$. This change affects firm's behavior, hence aggregate labor demand and aggregate production. This, in turn, affects the wage and aggregate demand which, in turn, changes firm behavior. The following algorithm finds the fixed point of this problem such that: (1) aggregate production of all firms equal aggregate demand Q in firms' problems and (2) the labor market clears such that aggregate labor demand equals labor supply at prevailing wage. Our approach broadly consists of postulating a given equilibrium (Q_n, w_n), then check if aggregate labor and product supply given these values is above or below (Q_n, w_n). We then adjust (Q_{n+1}, w_{n+1}) accordingly. This approach assumes that there is a unique fixed point and that the contraction mapping theorem applies in our setting.

Formally, we proceed in three main steps:

1. Find the number of firms N and the labor supply L_0 at wage $w_0 = .03$, so that the estimated model is at equilibrium with wage $w_0 = 0.03$ and aggregate production $Q = 1$. This will become part of the structure of the economy.
   (a) Simulate the data with 100,000 firms, $w = .03$, $Q = 1$ and parameters $\hat{\mathbf{\Omega}}$.
   (b) Compute *mean* labor demand l and *mean* revenue pq.
   (c) Set $N = \frac{1}{pq}$ and $L_0 = \frac{l}{pq}$. With such parameters, the economy with N firms and labor supply parameter L_0 is at equilibrium with $w_0 = 0.03$ and $Q = 1$.
2. Change one of the parameters to its new value $\mathbf{\Omega}'$. Given this, we loop to find the new equilibrium w and Q.
   (a) Set $w_0 = 0.03$ and $Q_0 = 1$.
   (b) Initiate round number $n = 1$. Then,
      i. Solve the model with w_{n-1} and Q_{n-1} and simulate 100,000 firms.

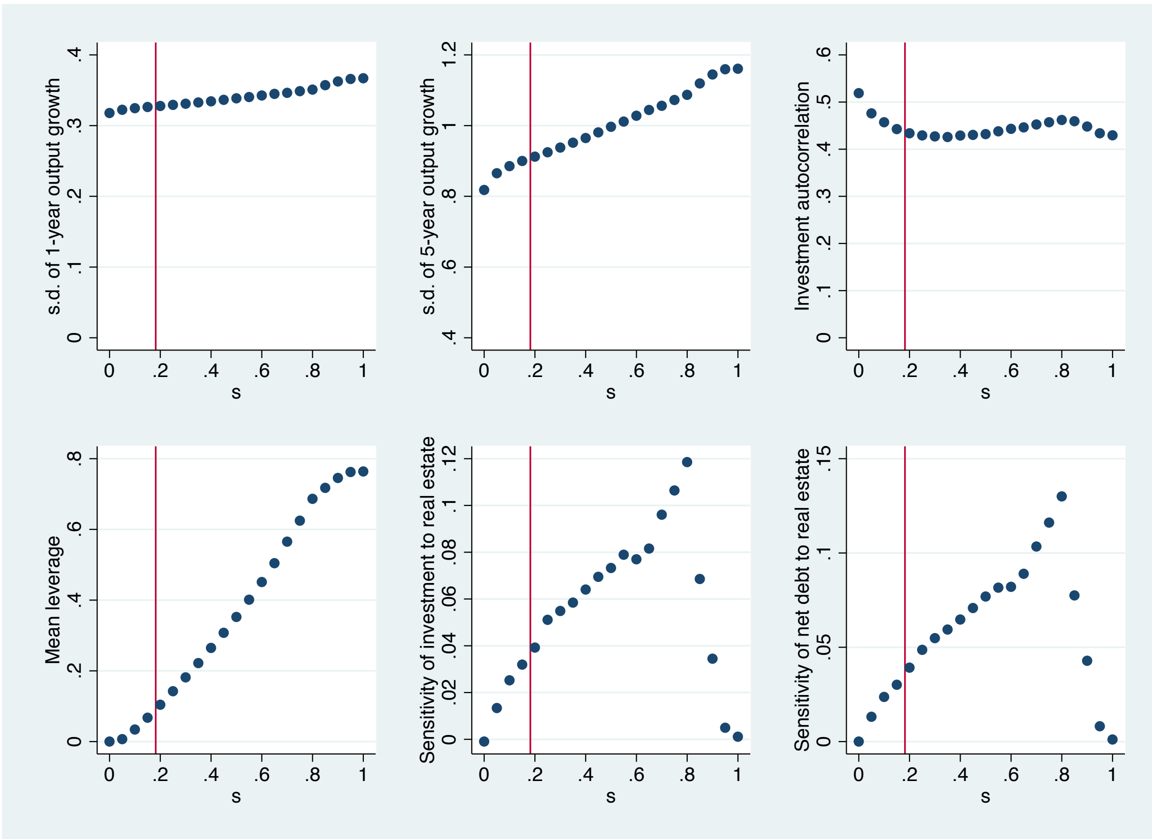
ii. Compute average revenue pq_n and average labor demand l_n and multiply both by N to obtain aggregate production Q_n^* and aggregate labor demand L_n.
iii. Compute labor market clearing wage $w_n^* = w_0(L_n/L_0)^{1/\epsilon}$
iv. Take $w_n = (w_{n-1})^\lambda (w_n^*)^{1-\lambda}$ and $Q_n = (Q_{n-1})^\lambda (Q_n^*)^{1-\lambda}$
v. go back to step (iii), until convergence in Q and w.

(c) compute aggregates:

- Q, w, $K = \sum_i k_i$, $L = \sum_i l_i$, Adj. $\text{Cost}_t = \sum_i i_{it}^2/k_{it}$.
- $\log \text{TFP} = \log Y - \alpha \log K - (1-\alpha) \log L$.
- Welfare $= (Q - \delta K - \text{Adj. Cost}) - \frac{\bar{L} w^{1+\epsilon}}{1+1/\epsilon}$

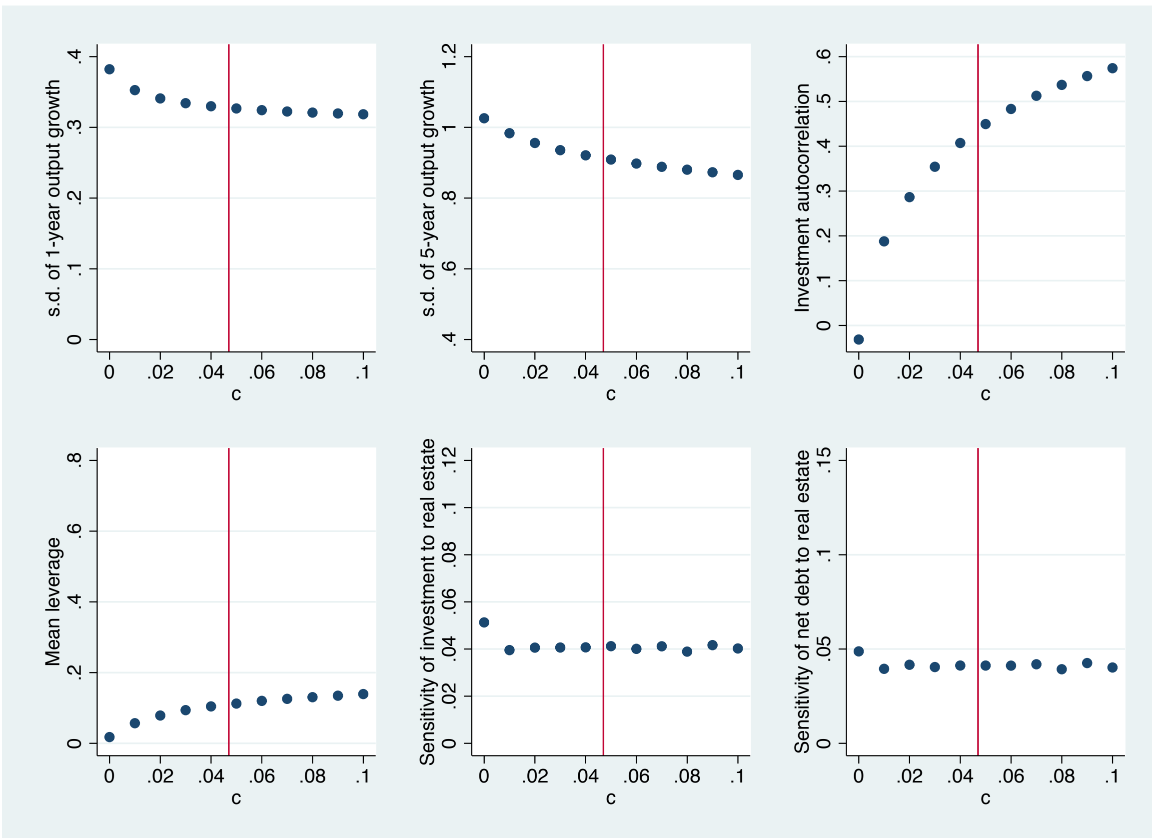
C Additional figures

Figure C.1: Sensitivity of moments to pledgeability s



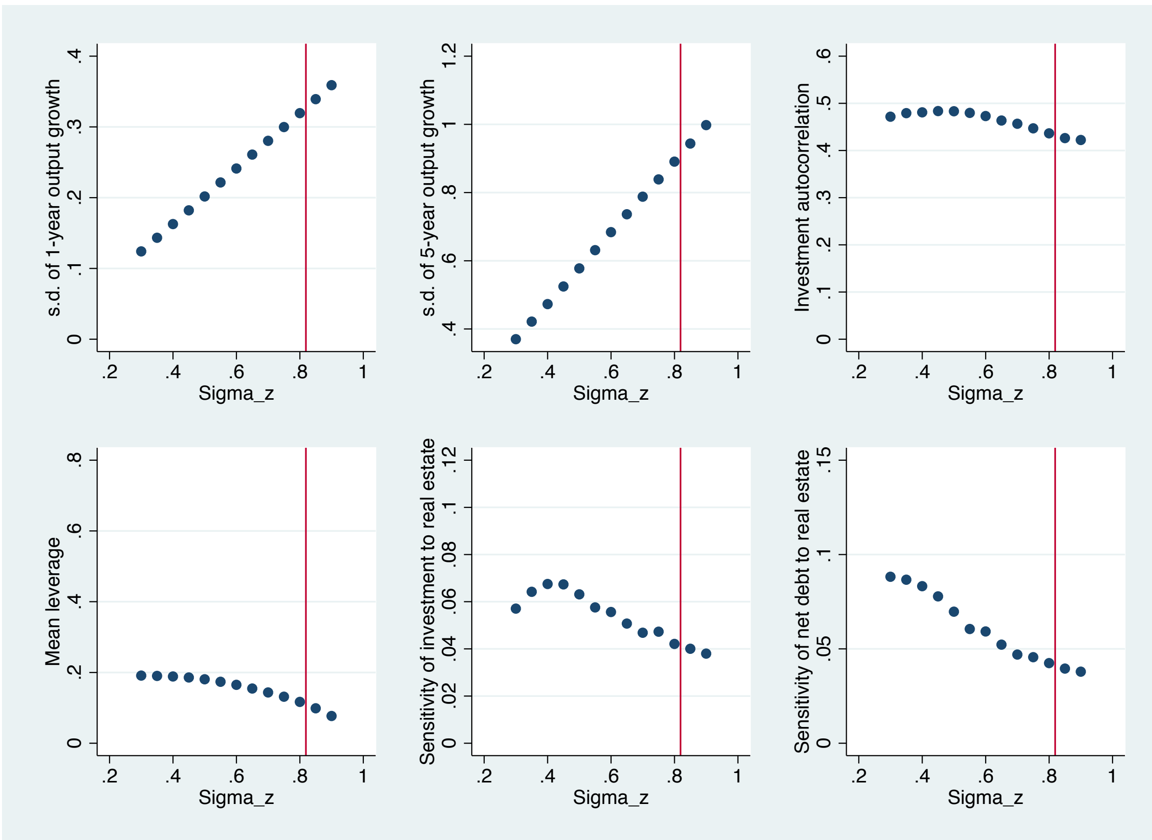
Note: In this figure, we set all estimated parameters (s, c, ρ, σ and H) at their SMM estimate in our preferred specification – as per Column (4), Table 3. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary s from 0 to 1. For each value of s that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of s.

Figure C.2: Sensitivity of moments to adjustment costs c



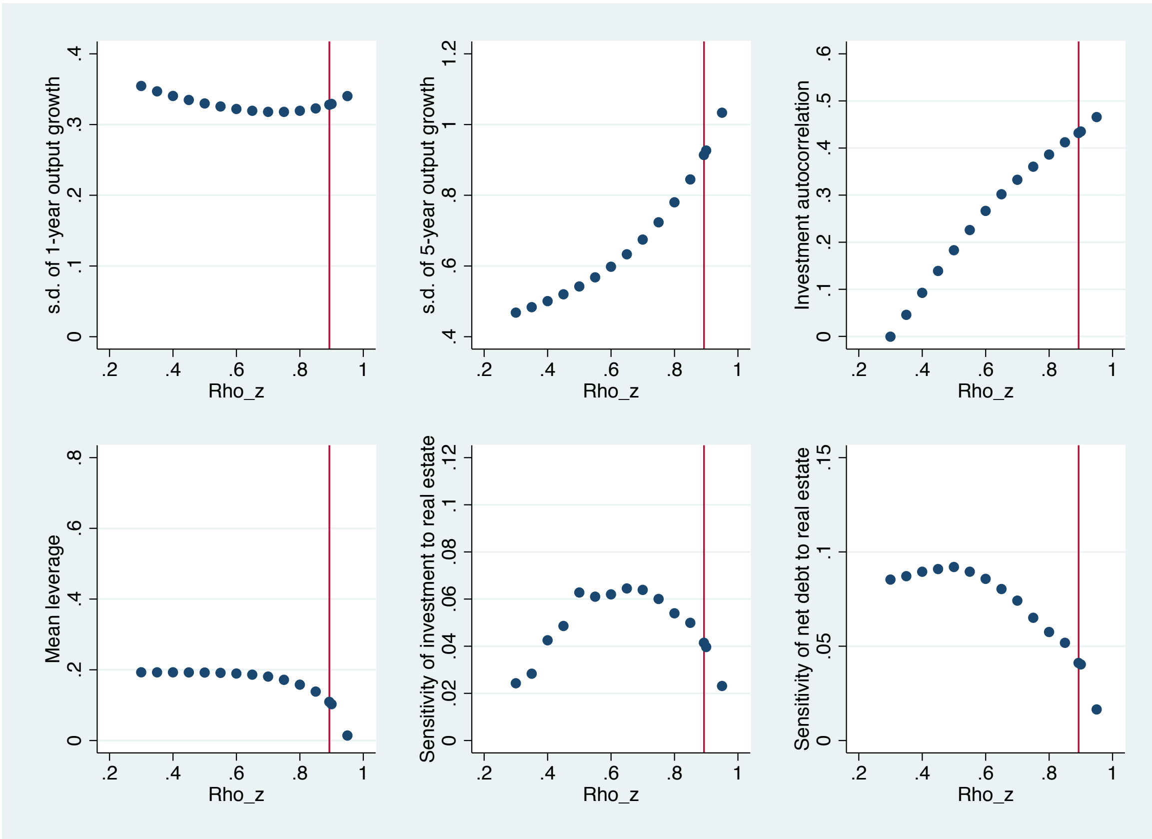
Note: In this figure, we set all estimated parameters (s, c, ρ, σ and H) at their SMM estimate in our preferred specification – as per Column (4), Table 3. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary c from 0 to .1. For each value of c that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of c.

Figure C.3: Sensitivity of moments to productivity volatility σ



Note: In this figure, we set all estimated parameters (s, c, ρ, σ and H) at their SMM estimate in our preferred specification – as per Column (4), Table 3. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary σ from 0 to 1. For each value of σ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of σ.

Figure C.4: Sensitivity of moments to productivity persistence ρ



Note: In this figure, we set all estimated parameters (s, c, ρ, σ and H) at their SMM estimate in our preferred specification – as per column 4, Table 3. We fix w and Q at their reference levels: $w = 0.03$ and $Q = 1$. We then vary ρ from 0 to 1. For each value of ρ that we choose, we solve the model, simulate the data, and compute four target moments, plus the average leverage ratio and the sensitivity of debt issuance to real estate value. Each panel corresponds to one moment. The red vertical line corresponds to the SMM estimate of ρ.