

Default Risk, Sectoral Reallocation, and Persistent Recessions*

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Abstract

Sovereign debt crises are associated with large and persistent declines in economic activity, disproportionately so for nontradable sectors. This paper documents these patterns using Spanish data and builds a two sector dynamic quantitative model of sovereign default with capital accumulation. Recessions are very persistent in the model and more pronounced for nontraded sectors because of default risk. An adverse domestic shock raises default risk and limits capital inflows which restrict the ability of the economy to exploit investment opportunities. The economy responds by reducing investment, reallocating capital towards the traded sector, and reducing tradable consumption to support the repayment of debt. Real exchange rates depreciate, as a reflection of the scarcity of traded goods. We find that these mechanisms are quantitatively important for rationalizing the experience of Spain during the recent debt crisis.

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1 Introduction

The recent sovereign debt crisis in Southern European countries was accompanied by a large and persistent decline in aggregate output. Similar to the case of debt crises in emerging markets, the decline in output was more pronounced for less traded sectors. Using sectoral data from Spain we document large differential output performance across sectors, with less traded sectors experiencing much larger declines in production. We build a dynamic model of sovereign default risk, capital accumulation, and two sectors that rationalizes the large and persistent decline in aggregate output that is more pronounced for nontraded production during a sovereign debt crisis.

The main mechanism in our model that rationalizes the dynamics of sectoral and aggregate output during the crisis is the rise in sovereign default risk. Default risk amplifies and makes the recession more persistent, especially for the non-traded sector. Low aggregate shocks increase default risk and make international financial conditions tight with higher bond spreads. The economy responds by reducing investment dramatically, not only because of the low productivity, but also to smooth tradable consumption. Tradable production, however, decreases much less than nontradable production because the economy reallocates inputs toward the traded sector to support the international debt repayment at higher interest rate spreads. The tight financial conditions persist and slow down the recovery. Real exchange rates depreciates as a reflection of the scarcity of traded goods.

Using two-digit sectoral data for Spain, we document sizable and robust differential performance across sectors during the debt crisis correlated with *tradedness*. Using input-output tables, we define a continuous measure for tradedness by the ratio of exports relative to gross output which features a large variation across sectors ranging from 0 to 50%. We find that the sectoral output decline from the peak of 2007 to the trough of 2013 is larger for sectors that are less traded. Within manufacturing, the peak to trough decline is about 30% for sectors with zero tradedness, and about 0 for those with 50% tradedness. We also correlate annual growth rates with the sovereign bond spread, a measure of the severity of the debt crisis. We find that a 1% increase in the bond spread is associated with an average decline of 3% annual growth rate for sectors with zero tradedness and actually an increase in annual growth of about 1.5% for sectors with 50% tradedness.

We build a dynamic small open economy model with capital accumulation and two sectors producing tradable and nontradable goods. International debt is unenforceable and the economy can default on its debt. The interest rate on debt carries a spread that compensates for endogenous default risk. Consumption and investment are produced with a bundle of traded and nontraded goods and the international debt is denominated in tradable goods.

Aggregate capital accumulates over time with investment decisions and is allocated across sectors such that the value marginal product of capital is equalized. The economy is subject to aggregate productivity shocks that affect both sectors symmetrically.

In this framework default risk restricts capital inflows and limits the ability of the economy to smooth consumption and to effectively exploit productive domestic investment opportunities. How indebted the economy is matters for consumption and investment decisions as well as the allocation of inputs across sectors. When debt is high, more resources are allocated for tradable production to support debt repayment. High indebtedness is also associated with low investment and low consumption because the tight financial conditions arising from default risk are more binding when debt is high and because the economy shifts resources away from investment towards consumption. This decline in investment depresses future capital and output.

We analyze the impulse response functions to declines in aggregate productivity that affect equally both sectors. A decline in aggregate productivity of 0.5% results in an increase in bond spreads of about 0.3% and a decline in aggregate output of about 0.7%. The decline in aggregate output is very persistent. By period 15 after the impact period, the shock has largely recovered yet aggregate output continues to be about 0.4% below trend. The responses are markedly different across sectors. Nontraded production falls over 1% on impact while traded production is almost unchanged. The decline in tradable consumption however is more than twice the decline in nontradable consumption, leading to a real exchange rate depreciation.

To tease out how default risk shapes these impulse responses, we compare them against a standard two sector reference model without default risk. The impulse responses in this reference model are quite different from those in the benchmark default model. The decline in aggregate output is more muted and it recovers rapidly after the shock. The sectoral decline across sectors is very similar with traded production actually declining a bit more than nontraded production. The reason is that with perfect financial markets and no default risk, the economy can borrow more when it experiences low shocks and smooth consumption. Well functioning financial markets in the reference model prevent the large and persistent decline in investment as well as the real exchange rate depreciation present in the benchmark default model.

We conduct an event analysis to compare our model implications directly to Spanish data. We focus on the peak to trough performance from 2007 to 2013. We feed in the sequence of shocks such that the model replicates the 9.6% decline in aggregate output observed in Spain. We then compare the implications of the model against data for bond spreads, sectoral

output, and real exchange rate depreciations. We use the sectoral data to construct the data counterparts of tradable and nontradable output for Spain. We find that the model predicts an increase in spreads of 3% close to the 2.7% increase observed in the data. The model predicts declines of 10% and 6.8% for nontraded and traded sectors, very close to the data counterparts of 10% and 6.4%. The model also predicts as in the data a real exchange rate depreciation. The magnitude of the depreciation in the model of 2.4% on average is however higher than the one observed in the data of 1.1%. The predictions of our model with default stand in contrast to those predicted by the reference model with no default, which predicts a more muted recession that is larger for the traded sector and no change in bond spreads or the real exchange rate.

Finally, we also use our model to examine forecasts about the persistence of the recession. We extend the event such that the shocks in the model after 2013 recover following their Markov chains. We find that our model predicts a very slow recovery. By 2040 our model predicts that aggregate output will have closed only half of the gap from trend.

Literature Our paper is closely related to the literature studying the boom-bust cycle and sectoral differential responses, for example Schneider and Tornell (2004), Kehoe and Ruhl (2009), Pratap and Urrutia (2012), and de Ferra (2016). First, in terms of empirical findings, it has been documented that during crisis, real exchange rate depreciates and nontradable sectors suffer a bigger decline than the tradable sectors, see Schneider and Tornell (2004) for a review. Schnedier and Tornell present these stylized facts with an event study for the boom-bust cycles of 11 countries from 1980 to 1999. Kehoe and Ruhl (2009) and Pratap and Urrutia (2012) confirm these stylized facts for Mexico 1994 crisis and the recent paper de Ferra (2016) reports similar facts for Italy 2012. Our empirical contribution confirms these findings for Spain during the European debt crisis with disaggregated data and with a continuous measure for tradability.

In terms of theory, both Kehoe and Ruhl (2009) and Pratap and Urrutia (2012) focus on the effect of sudden stops on aggregate total factor productivity and exchange rate depreciations. Kehoe and Ruhl model sudden stops as an unexpected halt in foreign capital flows. Pratap and Urrutia have a working capital model and model sudden stops as an exogenous interest rate shock. Hence, in their paper sudden stop is exogenous while aggregate total factor productivity is endogenous. In our paper productivity shocks generate endogenous sudden stops and interest rate fluctuations arising from default risk. De Ferra (2016), as in our work, has endogenous default risk. This paper, however, abstracts from capital accumulation and the persistence of recessions, and emphasizes the endogenous fiscal policies during

the recession.

Our finding that default risk generates amplification and persistent effects echoes the classic ideas that financial frictions amplify cycles by Bernanke et al. (1999) and Kiyotaki and Moore (1997). Mendoza (2010) quantifies these ideas for the international context with financial frictions arising from exogenously imposed collateral constraints. Our work share a similar conclusion to Mendoza's in that financial frictions amplify the shocks and lead to a slow recovery. Our model differs from his work in that our financial friction arise from endogenous default risk which give rise to endogenous bond spreads.

Our model extends the standard sovereign default model, as in Arellano (2008) and Aguiar and Gopinath (2006), with two sectors and capital. A few papers have also considered the connection between default risk in the context of a multi-sector framework. Na et al. (2014) study a government's tradeoff between devaluation and default in a sovereign default model with sticky wages. Asonuma (2016) documents the link between real exchange rate and sovereign default in a pure-exchange sovereign default model. Most works in sovereign default literature abstract from capital, except for Bai and Zhang (2012) and Gordon and Guerrón-Quintana (2013). Gorton and Guerron-Quintana focus on the effect of capital on sovereign spreads and Bai and Zhang pay attention to the effect of default risk on international risk sharing. In contrast, we emphasize the amplification during international business cycles and the differential sectoral responses.

2 Spanish Sectoral Data

We document the large decline in production that Spain experienced during the debt crisis. From the peak of 2007 to the trough in 2013, aggregate real GDP declined in absolute terms by 8%.¹ In this section we document that the decline in output was not homogenous across sectors and is correlated with the tradeness of the sector.

To document these facts, we use two-digit sectoral real gross value added data for Spain from Eurostat to construct sectoral output series from 2000 to 2014. We define the *tradedness* of the sector by the ratio of exports of goods and services to total use gross output using the 2011 input-output table from Eurostat. Tradedness is a continuous variable that proxies for how tradable each two-digit sector is.

We find large variation in the tradeness of sectors across two digit sectors ranging from 50% for water transport to zero for services such as education and health. Inside manufac-

¹The decline is much larger when it is computed relative to trend. From 2000 until 2007 GDP in Spain grew 3% annually. Hence, from 2007 to 2013 the decline in GDP relative to a 3% trend is more than 20%.

turing, we also find large variation in the tradedness of two-digit sectors ranging from 50% for manufacture of motor vehicles to about 2% for repair and installation of machinery.

To assess the contraction of sectoral output during the crisis, we compute the growth rate of real value added of each sector. We construct two growth series for each sector. The first series measures growth as the peak to trough percentage change in value added from 2007 to 2013. The second is an annual growth rate. We construct these series relative to corresponding sector average to filter out sector specific growth rate trends. In the appendix we report the details of the tradedness and peak to trough declines across all two-digit sectors.

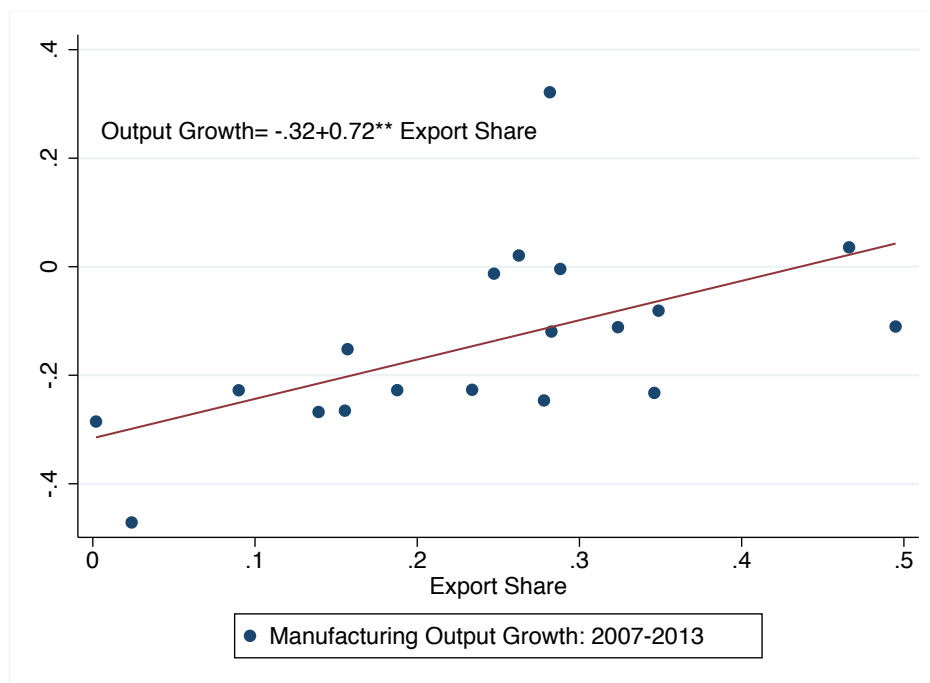
We start analyzing the relation between tradedness and the output performance during the debt crisis by focusing on the manufacturing sector and the peak to trough growth rate. The peak to trough growth is -14% on average across these sectors but varies considerably. Sectors such as manufacturing of machinery contracted by about 8%, while sectors such as repair and installations of machinery declined by 47%. In Figure 1 we illustrate the relation between the tradedness of sectors and the peak to trough decline in sectoral output for two-digit sectors within manufacturing. The figure shows that sectors with low tradedness experienced a much larger decline in output relative to sectors with high tradedness. In the first column of Table 1 we report the estimates of a cross section linear regression of peak to trough growth on tradedness for the two-digit manufacturing data. The coefficient on tradedness of 0.72 indicates that the decline in output is 36% larger for sectors with 0 tradedness relative to 50% tradedness over 6 years from 2007 to 2013.

Table 1: Output Growth and Tradedness during the Crisis

	Peak to Trough Manufacturing	Peak to Trough All	Annual Growth Manufacturing	Annual Growth All
Tradedness	0.72**	0.28**		
Tradedness \times Spread			0.09***	0.04***
Spread			-0.03***	-0.02***
Sector Fixed Effects	No	No	Yes	Yes
Adj. R^2	27	5	15	14
No. observations	19	62	266	770

This table reports linear regressions of two-digit real value added growth. The growth variable in the first and second columns is computed as the growth from the peak in 2007 to the trough in 2013 and reported relative to the mean 6 year growth of the sector from 2000 to 2015. The third and fourth columns, growth is computed as annual changes with data from 2000 to 2015. Tradedness is a time invariant measure of the export share for each sector. Spread is the time series of the government spread. All regressions contain a constant.

Figure 1: Tradedness and Output Decline in Manufacturing



In the second column of Table 1, we report results for the cross section regression of the peak to trough growth rate on tradedness across all two-digit sectors. As in the case for manufacturing sectors only, more traded sectors experienced a smaller decline in output from 2007 and 2013. The coefficient on tradedness is significant and positive, although the magnitude is smaller than for manufacturing sectors alone. The smaller coefficient arises because across all sectors of the economy, there is a large fraction of sectors with very small tradedness such as services and construction with varying performance during the crisis.

In the third and fourth column of Table 1, we consider a panel dataset with our second measure for sectoral growth which is the annual output growth. We regress this variable on a time varying measure of the crisis given by the government spread and on an interaction variable of the government spread times the sector tradedness. This interaction variable allows us to recover the differential effect of the crisis on sectors based on their tradability. The third column shows results for two-digit manufacturing sectors and the fourth column shows results for all two-digit sectors.

In these regressions, the coefficients on the government spread are negative and significant while the coefficients on the interaction terms are positive. The sign of these coefficients indicates that sector growth declines in periods of high government spreads and that the

decline is smaller in sectors that are more traded. The magnitude of the coefficients for the manufacturing regression indicate that a 1% increase in the government spread is associated with an average growth rate decline of 3% for sectors with zero tradedness. The growth for sectors with high tradedness of equal 0.5 is actually positive and equal to 1.5% ($-0.03 + 0.09 \times 0.5$). The coefficients in the regression with all sectors are similar, although the magnitudes are a bit smaller.

In summary, the large decline of aggregate output that Spain experienced during the recent debt crisis was not homogenous across different sectors of the economy. Less traded sectors experienced a more severe downturn than more traded sectors. This empirical fact is present across all sectors of the economy and as well as within the manufacturing sector.

3 Model

We consider a two sector dynamic small open economy model with a sovereign government that can default on its debt and capital accumulation. The model extends the one sector framework of Bai and Zhang (2012) and Gordon and Guerrón-Quintana (2013), that study sovereign default in an environment with capital accumulation, to consider two sectors. The two sectors produce tradable and nontradable goods that are used for consumption and investment purposes. The government is benevolent and trades one period bonds with international risk neutral lenders. International debt is unenforceable and the government can default on it. The costs of default consists of temporary exclusion from financial markets and reduction in productivity. We consider the problem of a government who directly chooses allocations. Below we show that this problem can be decentralized with appropriate choice of taxes.

Firms in each sector produce tradable and nontradable goods, y_{Tt} and y_{Nt} , using capital with decreasing returns to scale technology and productivity z_t .

$$y_{Tt} = z_t k_{Tt}^{\alpha_T} \tag{1}$$

$$y_{Nt} = z_t k_{Nt}^{\alpha_N}. \tag{2}$$

Productivity is subject to shocks that follow a first order Markov process with transition matrix $\pi(z_t, z_{t-1})$. These are the only aggregate shocks in the model.

The small open economy starts each period with the aggregate capital stock k_t which is distributed for production across the two sectors after the shocks are realized such that $k_t = k_{Tt} + k_{Nt}$. Capital depreciates each period at rate δ and accumulates with investment

x_t subject to adjustment costs $\Psi(k_{t+1}, k_t)$. The law of motion for capital is

$$k_{t+1} = (1 - \delta)k_t + x_t - \Psi_T(k_{t+1}, k_t). \quad (3)$$

Investment goods are produced by investment producers using a bundle of tradable x_{Tt} and nontradable x_{Nt} goods with a CES production function with parameter η

$$x_t = \left[(1 - \theta)x_{Tt}^{\frac{\eta-1}{\eta}} + \theta x_{Nt}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \quad (4)$$

Households are identical and have preferences over discounted expected lifetime stream of consumption c_t as follows

$$E \sum \beta^t u(c_t) \quad (5)$$

where β is their rate of time preferences. Consumption c_t is also a CES bundle of tradable c_{Tt} and non-tradable c_{Nt} goods

$$c_t = \left[(1 - \theta)c_{Tt}^{\frac{\eta-1}{\eta}} + \theta c_{Nt}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \quad (6)$$

The government trades international discount bonds denominated in tradable goods with risk neutral lenders that discount the future at the international interest rate R . Each period the government starts with debt b_t and decides whether to repay the debt or default. If the government repays the debt, then it can borrow b_{t+1} at price q_t . The price is a function that compensates lenders for the expected loss from default.

Traded goods produced by the small open economy, y_{Tt} , and new borrowing, $q_t b_{t+1}$, are used for consumption and investment purposes as well as for paying back the debt. The traded goods budget constraint is

$$c_{Tt} + x_{Tt} = y_{Tt} + q_t b_{t+1} - b_t. \quad (7)$$

Nontraded goods produced by the small open economy, y_{Nt} , are used for consumption and investment

$$c_{Nt} + x_{Nt} = y_{Nt}. \quad (8)$$

What about labor? Right now given parameters labor can be interpreted as inelastically supplied and constant per sector.

3.1 Recursive Formulation

The aggregate states of the small open economy are the exogenous productivity shock z and the endogenous states of capital k and debt b . Let s be the state of the economy given by $s = (b, k, z)$.

Let $V(s)$ be the value of the benevolent government with state s after shocks are realized. The government chooses to repay the debt $d = 0$ or default $d = 1$ to maximize its value

$$V(s) = \max_{d=\{0,1\}} \{dV^d(s) + (1-d)V^n(k, z)\}. \quad (9)$$

where $V^n(s)$ is the value of repayment and $V^d(k, z)$ is the value of default. The value of default depends on capital and productivity because debt is eliminated with default.

Conditional on repaying the debt, the government chooses the allocation of capital between tradable and nontradable sectors $\{k_T, k_N\}$, tradable and nontradable consumption and investment $\{c_T, c_N, x_T, x_N\}$, and new borrowing b' to maximize its value

$$V^n(s) = \max_{\{k_N, k_T, c_T, c_N, x_T, x_N, b'\}} u(c) + \beta EV(s') \quad (10)$$

subject to the tradable and nontradable budget constraints (7) and (8), the constraint on capital $k = k_T + k_N$, the accumulation of capital (3), and the consumption and investment aggregators (6) and (4). The government understands that the following period it has the option to default when choosing its debt.

If the government chooses to default, its debt obligations b are eliminated from the budget constraint but it is excluded from international bond markets temporarily and the economy suffers productivity losses. Every period after default the government faces a probability λ that it will reenter financial markets and productivity costs will be lifted. Upon reentry, the government starts with zero debt. Conditional on default, the government also chooses the allocation of capital between tradable and nontradable sectors $\{k_T, k_N\}$, and tradable and nontradable consumption and investment $\{c_T, c_N, x_T, x_N\}$ to maximize its value

$$V^d(k, z) = \max_{\{k_N, k_T, c_T, c_N, x_T, x_N\}} u(c) + \beta E [\lambda V(0, k', z') + (1-\lambda)V^d(k', z')] \quad (11)$$

subject to the constraint on capital $k = k_T + k_N$, the accumulation of capital (3), and the consumption and investment aggregators (6) and (4). The tradable and nontradable budget

constraints during default are

$$c_T + x_T = z^d k_T^{\alpha_T} \quad (12)$$

$$c_N + x_N = z^d k_N^{\alpha_N} \quad (13)$$

where $z^d \leq z$ reflecting the productivity costs of default.

This problem gives rise to policy functions for default $d(s)$, the allocation of capital $\{k_T(s), k_N(s)\}$, tradable and nontradable consumption and investment $\{c_T(s), c_N(s), x_T(s), x_N(s)\}$, and borrowing $b'(s)$.

International lenders are risk neutral and discount at the international interest rate R . The bond price function $q(b', k', z)$ compensates international lenders for default risk. It is a function that depends on the choice of borrowing b' and capital k' because the default decision the following period $d(b', k', z')$ depends on both endogenous states. It also depends on the shock z because of the Markov structure of shocks. The bond price function satisfies

$$q(b', k', z) = \frac{E_{z', z} (1 - d'(b', k', z'))}{R} \quad (14)$$

The government takes as given the bond price function in its recursive problem, but internalizes that different choices of b' and k' map into different bond prices. We define the *spread* as the inverse of the bond price relative to the risk free rate $spr = 1/q - R$.

We now define the equilibrium of this economy.

Recursive equilibrium. Given state $s = (b, k, z)$, the recursive Markov equilibrium consist of policy functions for default $d(s)$, the allocation of capital $\{k_T(s), k_N(s)\}$, tradable and nontradable consumption and investment $\{c_T(s), c_N(s), x_T(s), x_N(s)\}$, and borrowing $b'(s)$; value functions $\{V(s), V^d(s), V^n(k, z)\}$, and the bond price function $q(b', k', z)$ such that: (i) the policy and value functions for the government satisfy its optimization problem and (ii) the government bond price schedule satisfies equation (14).

3.2 Prices and Real Exchange Rate

We show in the appendix that the allocations from the centralized problem described above can be decentralized in an economy with the appropriate choices of capital taxes. The decentralized environment consists of competitive traded and nontraded firms that rent sectoral capital from the households. Identical households decide on investment, and consumption of traded and non-traded goods. Households buy investment goods from competitive in-

vestment producers that choose the mix of traded and nontraded investment inputs. The government of this economy makes all the international borrowing, decide on default, and transfers the receipts of its international operations to households with lump sum transfers.

In this decentralized economy, we show that the relative price of nontraded goods to traded goods denoted by p_N determines many of the sectoral allocations as well as the real exchange rate for this economy. Households choose the ratio of the marginal utility of nontraded relative to traded consumption to equal p_N . Investment producers choose the ratio of the marginal product of nontraded relative to traded investment goods to equal p_N .

$$p_N(s) = \frac{(1 - \theta)}{\theta} \left(\frac{c_T(s)}{c_N(s)} \right)^{-1/\eta} = \frac{(1 - \theta)}{\theta} \left(\frac{x_T(s)}{x_N(s)} \right)^{-1/\eta} \quad (15)$$

The sectoral allocation of capital across traded and nontraded firms also depend on the relative price. Firms rent capital from households such that the marginal product equals the domestic rental rate R^d . This implies that in equilibrium the ratio of marginal products across sectors also equals the relative price of nontraded goods

$$p_N(s) = \frac{\alpha_T k_T(s)^{\alpha_T - 1}}{\alpha_N k_N(s)^{\alpha_N - 1}} \quad (16)$$

The common CES functions for aggregate consumption and investment imply that the ratio of traded to nontraded consumption equals the ratio of traded to nontraded investment goods inputs and that the price index for aggregate consumption p_c equals the price index for aggregate investment p_x . This aggregate price index is a function of the relative price of nontraded goods given by

$$p_C(s) = p_X(s) = [(1 - \theta) + \theta p_N(s)^{1-\eta}]^{\frac{1}{1-\eta}}$$

where we have expressed the index relative to the price of traded goods P_T . The real exchange rate in this environment, e_C , is then the inverse of the consumption price index. We can easily derive this relation by imposing the law of one price for traded goods such that $P_T = \varepsilon P^*$ where ε is the nominal exchange rate and P^* is the international price.

$$e_C(s) = \frac{\varepsilon P^*}{P_C} = \frac{P_T}{P_C} = \frac{1}{p_C(s)} \quad (17)$$

These relations imply a depreciation of the real exchange rate, an increase in e_C , translates into reductions in the ratio of traded to nontraded consumption and investment and increases in the ratio traded to nontraded capital allocation.

In our two sector model, we define GDP in terms of tradable goods as

$$GDP_t = y_{Tt} + p_{Nt}y_{Nt}$$

Following standard accounting practices, we will also define real GDP using constant prices as $GDP_t = y_{Tt} + \bar{p}_N y_{Nt}$ where \bar{p}_N is the average nontraded relative price.

4 Quantitative Results

4.1 Parametrization

We use a CRRA utility function $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for the consumers. The productivity loss after default takes a similar form as in Chatterjee and Eyigungor (2012), $z_d = z - \max\{\chi_1 z + \chi_2 z^2, 0\}$. We adopt a standard quadratic capital adjustment cost function $\Psi(k_{t+1}, k_t) = \phi \left(\frac{k_{t+1} - k_t}{k_t} \right)^2 k_t$. Finally, the productivity shocks z_t follows an AR(1) process

$$\log(z_t) = \rho \log(z_{t-1}) + \sigma_z \varepsilon_t,$$

where ε has a standard Normal distribution.

There are two sets of parameters. The first is taken directly from the literature while we calibrate the second to match stylized facts related to sovereign default.

The first set of parameters includes $\{\sigma, \alpha_T, \alpha_N, \eta, \theta, \delta, R, \rho, \sigma_z\}$. We set the risk aversion σ to 2 and the yearly, risk-free rate R to 4 percent. We take the capital shares and elasticity of substitution between tradable and nontradable from Mendoza (1995). The capital share in the tradable sector α_T is 0.57, while that of the nontradable sector is $\alpha_N = 0.66$. The elasticity η is 0.74. We choose the share of nontradable goods in the CES bundle, θ , to be 0.6. The capital depreciation rate δ is standard, 7 percent annually. The return parameter λ is chosen to be 0.25 so that defaulting countries are excluded from financial markets for 4 years, consistent with Gelos et al. (2011). We pick the shock persistence ρ and volatility σ_z to be 0.9 and 0.0075 respectively, consistent with standard business cycle literature.

The second set of parameters includes the discount factor β , productivity loss parameters (χ_1, χ_2) , and the capital adjustment cost scale ϕ . These jointly match the following moments: the mean spread of 2 percent, the volatility of spread of 2 percent, the relative volatility of investment to GDP about 2, and the relative consumption volatility about 0.9.

Table 2: Parameter Values

Parameters	Value	Targets
<i>Assigned Parameters</i>		
Capital shares	$\alpha_T=0.57$ $\alpha_N=0.66$	Mendoza (1995)
Non-tradable share	$\alpha=0.6$	Mendoza (1995)
Elasticity of substitution	$\eta=0.74$	Mendoza (1995)
Probability of re-entry	$\lambda=0.25$	Gelos et al. (2011)
Gross risk-free rate	$R=104\%$	RBC literature
Depreciation rate	$\delta = 7\%$	RBC literature
Risk aversion	$\sigma = 2$	RBC literature
Productivity process	$\rho = 0.9, \sigma_z = 0.0075$	RBC literature
<i>Moment-Matching Parameters</i>		
Discount factor	$\beta = 0.82$	mean(spread) = 2%
Penalty parameters	$\chi_1=-0.71$ $\chi_2=0.73$	vol(spread) = 2% vol(C) / vol(Y) = 0.9
Capital adjustment cost	$\phi=1.3$	vol(I) / vol(Y) = 2

4.2 Default Risk and Decision Rules

Before describing the model time series, we illustrate the model mechanisms by describing how default risk limits capital flows to the economy. We also discuss how the choices of consumption, investment, and the sectoral allocation of capital vary with the level of debt of the economy.

As it is typical in dynamic sovereign default models, default risk restricts capital inflows to the economy. In our model, given a shock z , each combinations of levels of borrowing and capital choices are associated with a different bond price, encoded in the bond price function $q(b', k', z)$. In the left panel of Figure 4.2 we plot the *spread schedule*, $spr(b', k', z) = 1/q(b', k', z) - R$ as a function of borrowing. Spreads increase in the borrowing level b' . The schedule is also tighter for a smaller capital choice k' and when productivity z is low. As explained in detail in Gordon and Guerrón-Quintana (2013), lower capital and/or lower productivity are associated with a lower debt repayment capacity, which increases default risk today. The bond price schedule also encodes a “Laffer curve” of borrowing and a maximum amount of capital inflow. In the right panel, we plot the *capital inflows schedule* $q(b', k', z)b'$. Capital inflows are restricted by default risk and bound by the peak of the Laffer curve.

Next we describe the decision rules as a function of the economy’s level of debt at the start of the period, b . In Figure 3, we plot the economy’s choices, holding constant the level

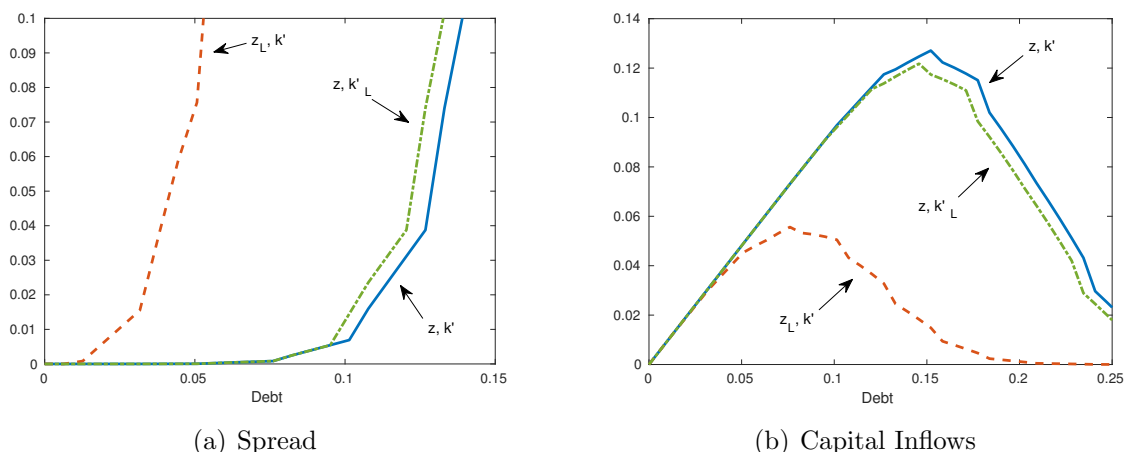


Figure 2: Spread and Capital Inflows Schedules

of capital k and shock z at their mean levels, while we vary b , which we normalize by the mean level of aggregate output. The panels in the figure feature two different regions. When debt is low enough, the economy repays the debt and the policy rules vary with b . When debt is high enough, $b \geq 0.2$, the economy defaults and the policy rules no longer vary with b .

Panel (a) plots the allocation of capital $\{k_T, k_N\}$ which translates directly into sectoral output. As debt increases, more capital is allocated to the tradable sector to support repaying the increasing debt. For high enough debt levels the economy defaults and the allocation of capital reverts back towards a larger nontraded sector because the economy no longer services the foreign, traded-denominated debt.

In Panel (b), we plot the choices of tradable and nontradable consumption $\{c_T, c_N\}$ as well as aggregate consumption as functions of debt. Consumption of each of the two goods falls with debt. Tradable consumption falls despite an increasing traded output because more of the economy's traded resources are devoted to debt repayment. Nontraded consumption falls too, because of the shift in resources away from nontraded sector that arises as debt increases. When debt is high enough and the economy defaults, the consumption of the two goods settle at levels similar to those at moderate levels of debt.

Panel (c) contains the choices for tradable and nontradable investment $\{x_T, x_N\}$. The use of traded goods for investment falls with debt because net capital inflows $qb' - b$ are more restricted when debt is high. As illustrated by the capital inflows schedule 4.2, in our model the economy faces limits on the extent to which the traded input can flow into the economy to exploit investment opportunities. These restrictions in capital flows are more binding for investment when the economy has to pay large levels of debt. The use of nontraded goods

for investment falls with debt also, because of the lower nontraded output, due to sectoral shift towards traded good production.

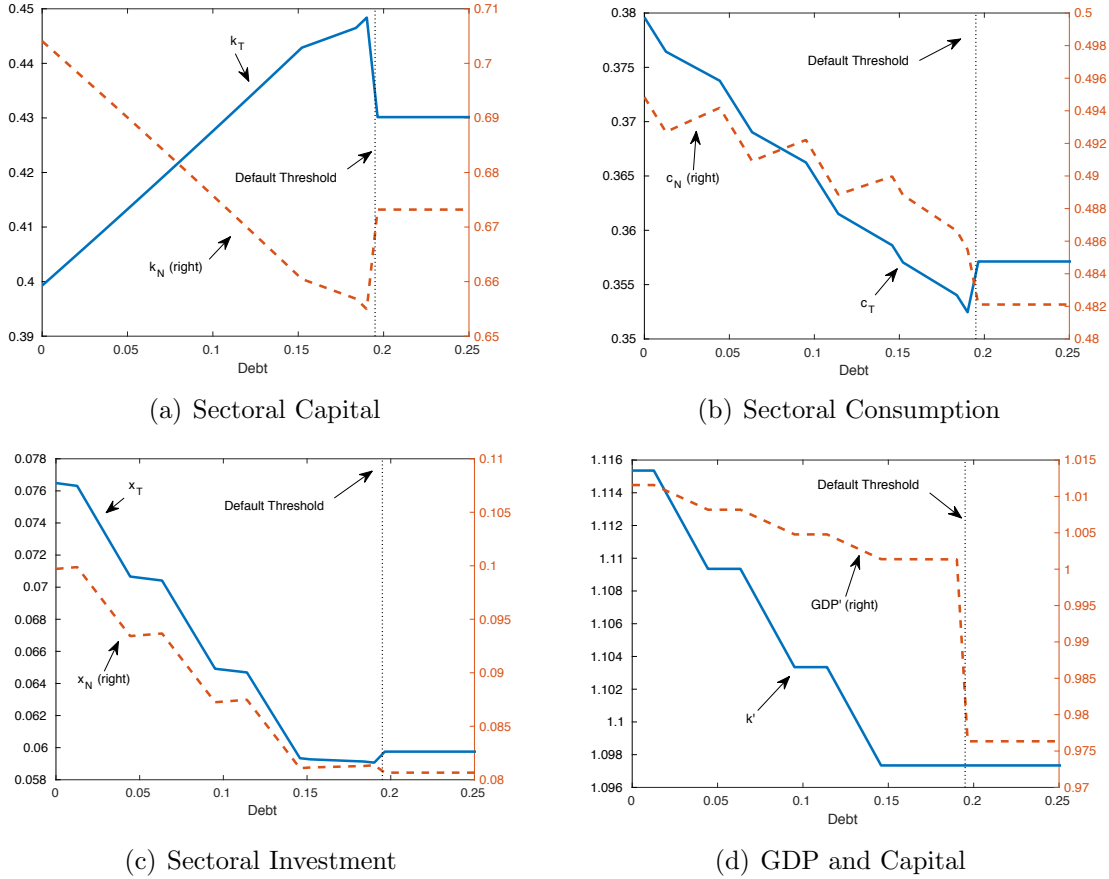


Figure 3: Policy Rules as Function of Debt

The decline in investment, of course, lowers the aggregate level of capital and output in the next period. Panel (d) plots the resulting capital for the next period k' as well as GDP in the following period when z' is kept again at its mean. A large debt today lowers capital and output tomorrow, because of the decline in investment. For example, as debt increases from 10% to 20% of output, GDP next period falls by about 5%.

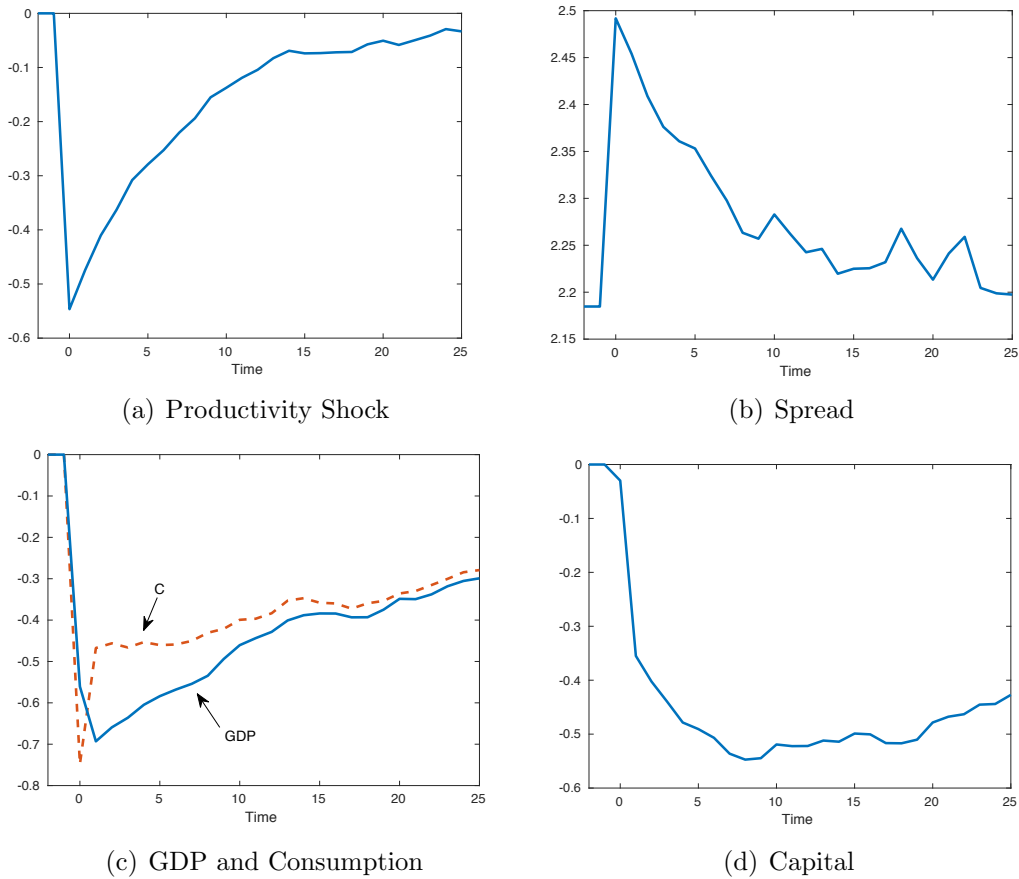
4.3 Impulse Responses: Benchmark

We now describe the time series dynamics of our model by presenting impulse response functions of aggregates to a negative productivity shock.

We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate 3000 paths for the model for 350 periods. From periods 1 to 300, the

aggregate shock follow its underlying Markov chains so that the cross-sectional distribution of debt and capital converges to the limiting distribution of endogenous states. In period 301, the *impact period*, normalized to 0 in the plots, we reduce all histories' productivity shock by the same amount. From period 301 on, the productivity shocks follow the conditional Markov chain. The impulse responses plot the average, across the 3000 paths, of the variables from period 299 to 325, conditional on the economy not defaulting.

Figure 4: Impulse response functions to a decline in productivity



In Figure 4 we plot the impulse responses to productivity declines for the productivity shock z , the bond spread, real GDP, aggregate consumption and capital. Panel (a) shows that average productivity falls a bit over 0.55% which corresponds to about half of one standard deviation of the shock. After the impact period, the shock follows its Markov chain and, by period 15 it recovers by more than 75%, to about 0.1% below the average level. In these IRFs, we are conditioning on the histories without default; the mean productivity including the paths with default is negligibly different, about 0.03% lower than the plot here.

In Panel (b) we plot the path for the bond spread. The spread increases from 2.2% to about 2.5% on impact following the decline in productivity. Low productivity increases the probability that the economy with default. The spread rises to compensate for such default risk. The spread largely recovers by period 15. In Panel (c) we plot the responses for real GDP and aggregate consumption. Real GDP falls on impact by the same magnitude as the shock, about 0.55%, because capital is predetermined. In the period after the impact, GDP declines further to about 0.7% below average, because investment also contracts due to the low productivity. GDP starts to recover two periods after the impact period, but only very slowly. By period 15, GDP continues to be depressed, about 0.4% below average. Aggregate consumption falls sharply on impact, not only because production is depressed but also because of the high borrowing interest rate spreads. Consumption contracts on impact more than production because the economy experiences net capital outflows due to the restricted capital inflow schedule and associated high interest rates spreads. Consumption recovers after adjusting the debt but remains depressed thereafter. By period 15, consumption is almost as depressed as production.

In Panel (d) we plot the impulse response for aggregate capital. It falls substantially for about 8 periods after the initial shock, to more than 0.5% below its average. Afterwards, capital recovers but only very slowly. After 25 periods, capital continues to be quite depressed, more than 0.4% below the mean.

The large endogenous persistence that our model generates is due to two reasons: First, the capital stock reacts slowly to productivity shocks because of adjustment costs. This effect is present in standard small open economy models with capital adjustment costs. Second, the financial frictions that arise in the model because of default risk also make recessions more persistent. Default risk severely limits the ability of the economy to smooth fluctuations in consumption and exploit investment opportunities. When productivity is low, financial frictions tighten. The economy then reduces investment to support consumption. These financial frictions effectively act like an additional adjustment cost, one that makes recessions more persistent².

To illustrate the effect of default risk on aggregate capital dynamics, it is useful to analyze the first order conditions for capital and borrowing from the model.³ As in standard in models with multiple assets the allocation of capital and borrowing is such that the expected return on capital, denoted by $R^K(s')$ weighted by the marginal utility equals the expected return of

²These results are consistent with the findings in Reinhart and Rogoff (2009), that recessions accompanied by financial crises are followed by slower and more modest recoveries.

³In deriving these expressions we assume that value function and bond price function are differentiable.

borrowing, denoted by $R^K(s')$

$$Eu'_{c_T}(s')R^K(s') = Eu'_{c_T}(s')R^B(s') \quad (18)$$

The return on capital equals the marginal benefit of capital tomorrow in terms of tradables relative to the marginal benefit of capital today. The marginal benefit of capital equals the marginal product of capital plus the un-depreciated capital minus the adjustments costs. The marginal cost of capital is one plus the adjustment costs today as well as the effect capital directly has on the bond price

$$R^K(s') = \frac{p'_X(s')(z(s')\alpha_T k_T^{\alpha_T-1} + 1 - \delta - \Psi'_2)}{p_X(1 + \Psi_1) - (1 - \theta)\frac{dq}{db'}b'} \quad (19)$$

The return on borrowing equates the marginal cost of servicing debt tomorrow, which only occurs in the no default states, relative to the benefit of borrowing, which equals the bond price q minus the reduction in the price due to additional borrowing $\frac{dq}{db'} < 0$.

$$R^B(s') = \frac{(1 - d'(s'))}{q + \frac{dq}{db'}b'}. \quad (20)$$

In times of high spreads, q is low and the sensitivity of the price with respect to borrowing is large, as shown in the spread curves in Figure 4.2, both of which increase the return on borrowing $R^B(s')$.

The response of capital to a low productivity shock shown in Panel (d) can be understood as a response to low expected return on capital $R^K(s')$ because of low productivity and also as a response to a high return on borrowing $R^B(s')$. When productivity is low, spreads increase, which pushes up the return on borrowing $R^B(s')$. Capital then decreases because of the large borrowing costs, which are encoded not only in the high spread but also in the slope of the bond price. The large and persistent decline in capital leads to a persistent decline in aggregate output and consumption.

We now turn to the impulse responses of sector-specific production and consumption, as well as of the real exchange rate to the decline in aggregate productivity. Panel (a) in Figure 5 plots the responses of traded and nontraded production. The model generates a large differential response to the shock across the traded and nontraded sector. On impact, traded production increases about 0.2% while nontraded production declines over 1%. Recall that the traded and nontraded sectors are subject to a common productivity shock. The reason why nontraded goods decline by more is that capital inputs are reallocated to the traded sector, to support the payment of the external debt which carries higher bond spreads. After

the impact period, traded production declines slightly more than nontraded production and both sectors recover at an equal rate.

In Panel (b) we plot the sectoral consumption paths. Traded consumption declines on impact about 1.1%, much more than nontraded consumption which declines about 0.5%. Nontraded consumption declines less than production because the use of nontraded goods for investment, x_N , decline more, such that nontraded consumption is smoothed. Traded consumption declines despite the increase in production and large decline in investment x_T because of the large debt repayment at high bond spreads. The relative decline in traded and nontraded investment is equal to the decline in relative consumption. $x_T/x_N = c_T/c_N$, as this ratio only depends on the real exchange rate.

Panel (c) plots the real exchange rate, as defined in (17). The real exchange rate depreciates on impact about 0.5%. The depreciation occurs because of the increase in bond spreads and tight international borrowing conditions that push down the relative price of nontraded goods. The depreciation is short lived and the exchange rate reverts back to a slightly more appreciated level thereafter. Real exchange rate depreciations during debt crises are a robust feature documented for emerging markets.⁴

In summary, the impulse responses to low productivity shocks show that our model generates persistent recessions accompanied by sluggish recovery in investment and tightening of international borrowing conditions, manifested in high bond spreads. The responses across sectors are different, with larger declines in nontraded than traded production.

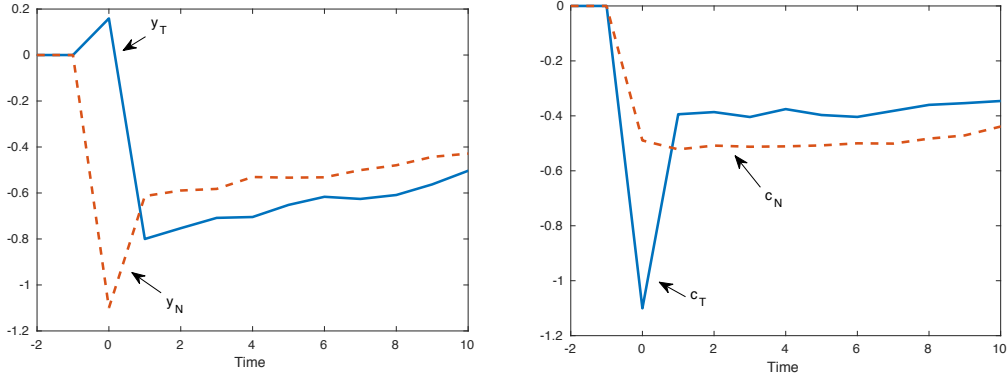
4.4 Impulse Responses: Reference Model with No Default

We now assess the endogenous amplification that our model generates by comparing its dynamics to a *No Default* reference model. The reference model we consider is a standard two-sector small open economy that trades bonds in international markets. Unlike in our benchmark model, here the bonds are enforceable. We find that our benchmark model with default risk provides large endogenous amplification of shock and contrasting implications for the evolution of sectoral production. The no default model differs in that it generates larger declines in tradables relative to nontradable, which is at odds with the data.

The reference model is a version of our model but without default risk. We close this small open economy reference model and ensure stationarity following Schmitt-Grohé and

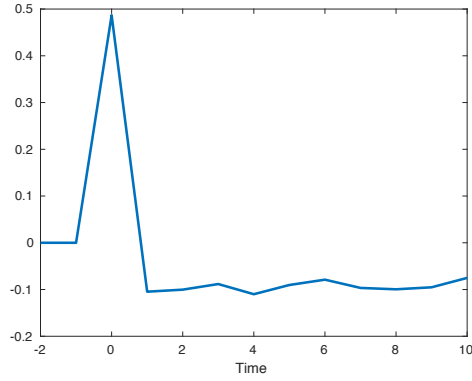
⁴Na et al. (2014) document that sovereign debt crises are accompanied by large devaluations in emerging markets.

Figure 5: Impulse response functions to a decline in the productivity



(a) Sectoral Output

(b) Sectoral Consumption



(c) Real Exchange Rate

Uribe (2003), with a price debt elasticity formulation such that

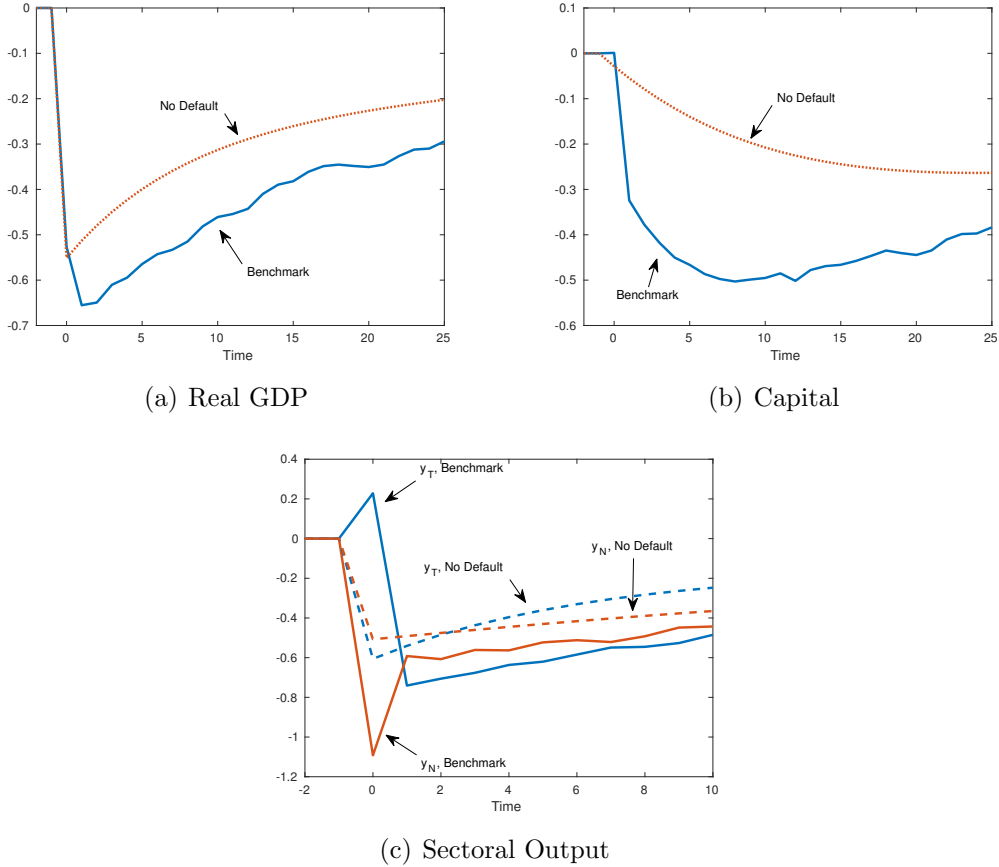
$$q(b') = 1/(R + \phi_B[\exp(b' - \bar{b}) - 1]).$$

We hold all the parameters in this reference model constant at their values in the benchmark. The additional parameters controlling the price debt elasticity $\{\bar{b}, \phi_B\}$ are chosen such that the mean level of debt in the reference model equals to the level in the benchmark and such that the elasticity of bond prices is minuscule.⁵

In Figure 6 we compare the impulse response functions for real GDP, aggregate capital, and traded and nontraded production of the no-default reference model to our benchmark model. As seen in Panels (a) and (b) the no default model has a much more muted and less persistent recession than our benchmark model. GDP in the benchmark model falls

⁵We solve the reference *no default* model using Dynare.

Figure 6: Impulse response functions to a decline in productivity: Benchmark and No Default Model



about 40% more than in the reference model. Capital falls much less and quite slower in the reference model than in the benchmark. By 5 periods after impact, the fall in capital in the benchmark is 5 times smaller than in the reference model.

Panel (c) contains the impulse responses for traded and nontraded output. In contrast to the benchmark model, in the reference no-default model traded output falls more on impact than nontraded output, about 50% more. The smaller decline in traded production in reference model reflects the less frictional financial markets available in this environment. When low productivity hits, the economy expands borrowing at low interest rates to smooth traded consumption and investment. The availability of traded goods from financial markets allows capital to be allocated away from the traded sector towards the support of the nontraded sector. In contrast, in our benchmark model with default risk, low productivity shocks restrict the availability of international borrowing which generates the opposite sectoral responses.

4.5 Event analysis for Spain

We now compare the quantitative implications of the model to Spanish data. We are interested in quantifying our model against the peak to trough data in Spain from 2007 to 2013.⁶ We analyze GDP, tradable and nontradable output, and bond spreads during the event.

We construct data series for tradable and nontradable output based on the sectoral evidence in Section 2. We define tradable output as the sum of the value added output for two-digit sectors that have an export share of greater than or equal to 10%, which corresponds to about the median share across 62 two digit sectors. Nontradable output is the sum of the output for the rest (those under 10%). We deflate the nominal output series with the GDP deflator and detrend the annual time series for GDP, tradable and nontradable output by logging the series and filtering with the Hodrick-Prescott filter, using a smoothing parameter of 100.

We conduct the numerical experiment in the model with a similar procedure as the impulse responses. We simulated 3000 paths for 300 periods. We use the resulting limiting distribution of capital, debt, and productivity shocks in period 300 as the initial condition for the event. We then feed in a path of shocks such that the conditional mean aggregate output of the model reproduces the path of Spanish GDP from 2004 to 2013.⁷ The shocks during the event are parallel deviations across all the 3000 paths. For reference, we also conduct the event study for the no default reference model, where in each period we feed in the average productivity shocks recovered from the benchmark model.

We focus on the peak-to-trough dynamics of GDP, the government spread, as well as tradable and nontradable output. Table 3 reports the difference in the time series between 2007 and 2013, with the corresponding levels of these two years for the data and the models. In the data, GDP declines by 9.6 percent, varying from 5.5 percent above trend to 4.4 percent below trend. The nontradable output has a similar magnitude of decline in this period, while tradable declines much less, only 6 percent. In 2013, nontradable output is about 6 percent below trend and tradable output is only 1.4 percent below trend. As for most crises, Spain spread increases, climbing from an initial value of almost 0, and reaching a final value of 2.7 percent in 2013.

By construction, our benchmark model aims to match the dynamics of GDP during this episode and thus generates the sizable decline of GDP as in the data. The government spread increases by 3 percent, rising from a level close to zero. The model also successfully generates the observed decline in both tradable and non-tradable output, 6.8 percent and

⁶GDP reaches a trough in 2013, however the spread peaks in 2012, with a value of 4.2 percent.

⁷We choose 2004 as a start year because Spain GDP in that year is close to the trend.

10.1 percent respectively. In addition, the benchmark model matches the levels in the peak, 3.3 for tradable and 4.3 for nontradable. We overshoot the decline in the tradable output, -3.5 for the model but -1.4 for the data. Nonetheless, as in the data, the nontradable sector suffers more than the tradable one.

To explore the role of default risk, we look at the no-default model. Due to the TFP shock, the no-default model exhibits a bust as well, but with a smaller decline in GDP, about -6.8 percent. It's obvious that this model cannot produce any movement in spread. Most importantly, opposite to the data and the benchmark model, the tradable sector has a larger decline than the non-tradable sector, -7.5 percent versus -6.2 percent. The reason is because the nontradable sector has a smaller boom in 2007, 3.7 percent, and a smaller bust in 2013 in the no-default model, only -2.4 percent.

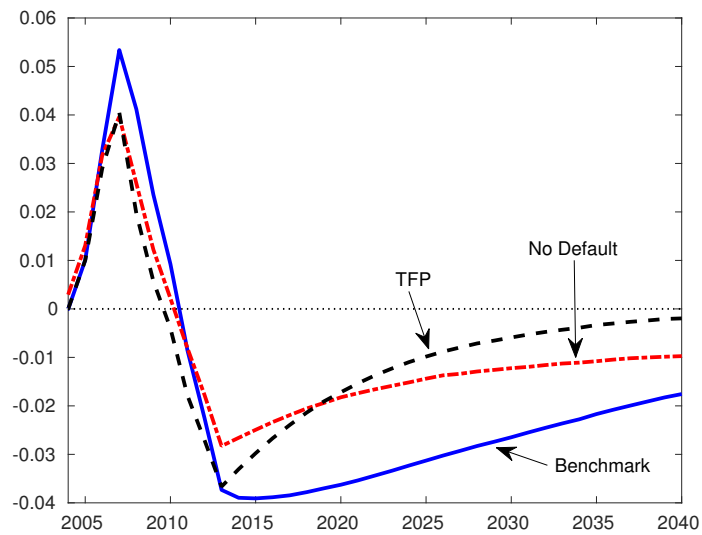
We also look at the implications of model for the real exchange rate. In the data, the real exchange rate in 2007 is around the trend. It depreciates by 2.5 percent in 2008, then appreciates and comes back to the trend by 2013. The average depreciation relative to 2007 is about 1.1 percent. Our benchmark model has a similar pattern, about zero in 2007, reaching the peak in 2008 about 2.8 percent, and appreciating afterwards. The average depreciation relative to 2007 in the benchmark model is 2.4 percent. In contrast, the no-default model implies an almost zero average depreciation.

Finally, we are interested in the persistence of the recession. After 2013, we let TFP follow the underlying AR(1) shock process from the model. As show in Figure 7, the productivity returns to the mean by 2040. With capital, the non-default model generates slower recovery than TFP, it is still 1 percent below the trend in 2040. With financial frictions, the benchmark model takes longer to build up capital and generates an even slower recover, still 2 percent below the trend in 2040.

Table 3: Spain from 2007 to 2013

	GDP	Spread	Tradable Output	Nontradable Output
<i>2007-2013 % Difference</i>				
Data	-9.6	2.7	-6.4	-10.0
Benchmark Model	-9.1	3.0	-6.8	-10.1
No-Default Model	-6.8	0	-7.5	-6.2
<i>2007 % Levels</i>				
Data	5.5	0.0	4.9	4.3
Benchmark Model	5.3	0.7	3.3	4.3
No-Default Model	4.0	0.0	4.4	3.7
<i>2013 % Levels</i>				
Data	-4.4	2.7	-1.4	-5.8
Benchmark Model	-3.7	3.8	-3.5	-3.9
No-Default Model	-2.8	0.0	-3.2	-2.4

Figure 7: GDP Event Dynamics: Benchmark and No Default Models



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A Computation

We compute the sovereign's problem of Section 3.1 using value function iteration. For periods with financial markets access, the state space is given by (z, k, b) while during default it is (z, k) . We discretize the AR(1) process for the z shock using 21 equally-spaced grid points over ± 3 standard deviations of the ergodic distribution $\mathcal{LN}(0, \sigma^2/\sqrt{1-\rho^2})$. For the bonds we use a grid with 72 equally-spaced points on $b \in [0, 0.45]$ and for capital we use a 72 points, equally-spaced grid on $k \in [2.25, 3.25]$. The capital is expressed in units of the CES composite while debt is in terms of tradable commodity units. For reference, the mean GDP level, expressed in tradable units is 2.35, out of which $y_T = 1.0$ is the mean output level in the tradable goods sector and the rest is contributed by $\bar{p}_N y_N$.

The sovereign makes investment/asset decisions b' and k' for next period (k' only, if in default). We restrict these choice variables to be on the grid. Then, for every (z, k, b) point in the state space and for every possible asset choices (k', b') we solve for the domestic allocation:

1. Solve for the quantity of capital employed in the tradables sector, k_T , as the root of equation

$$\frac{1-\alpha}{\alpha} \left(\frac{zk_T^\theta + q(b', z)b' - b}{z(k - k_T)^\gamma} \right)^{-1/\eta} = \frac{\gamma(k - k_T)^{\gamma-1}}{\theta k_T^{\theta-1}}$$

2. Compute the mix of inputs in the CES composite, for consumption and investment

$$\frac{c_T}{c_N} = \frac{x_T}{x_N} = \frac{zk_T^\theta + q(b', z)b' - b}{z(k - k_T)^\gamma}$$

3. Use the level of investment to determine the levels of x_T and x_N , given their ratio found in the previous step (a system of 2 equations in 2 unknowns)

$$k' - (1 - \delta)k + \Psi(k', k) = I = \left[(1 - \alpha)x_T^{(\eta-1)/\eta} + \alpha x_N^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}$$

4. Use the resource constraint in each sector to determine c_T and c_N as residuals, and compute the CES aggregate C .

During default, the same equations characterize the domestic allocation, with the additional constraint that $b' = 0$ and TFP is $z_d \leq z$.

We use a one-loop algorithm, in which we update the value functions and the bond price schedule each iteration. We stop when changes in the all value functions and the bond price schedule do not exceed $1.0\text{e-}5$.

B Spanish Sectoral Data

Table 4: Tradedness and Growth for Two-digits Sectors in Spain

Sector Name	Export Share	07-13 Growth
Water transport	50.4	-21.1
Manufacture of motor vehicles, trailers and semi-trailers	49.5	-11.0
Manufacture of other transport equipment	46.6	3.6
Air transport	44.2	10.9
Manufacture of machinery and equipment n.e.c.	34.9	-8.1
Manufacture of electrical equipment	34.6	-23.3
Manufacture of basic pharmaceutical products and pharmaceutical preparations	32.4	-11.1
Manufacture of textiles, wearing apparel, leather and related products	28.8	-0.4
Manufacture of chemicals and chemical products	28.3	-12.0
Manufacture of basic metals	28.2	32.2
Manufacture of coke and refined petroleum products	27.8	-24.6
Manufacture of rubber and plastic products	26.3	2.1
Manufacture of paper and paper products	24.7	-1.3
Computer programming, consultancy, and information service activities	24.2	-10.6
Manufacture of other non-metallic mineral products	23.4	-22.7
Manufacture of fabricated metal products, except machinery and equipment	18.8	-22.7
Crop and animal production, hunting and related service activities	18.3	1.4
Manufacture of computer, electronic and optical products	15.7	-15.2
Manufacture of wood and of products of wood and cork, except furniture	15.5	-26.5
Travel agency, tour operator reservation service and related activities	14.3	-32.8
Manufacture of food products; beverages and tobacco products	13.9	-26.8
Fishing and aquaculture	13.1	19.0
Wholesale trade, except of motor vehicles and motorcycles	13.1	-16.8
Architectural and engineering activities; technical testing and analysis	13.1	-47.1
Security and investigation, service and landscape, administrative and support activities	11.9	-24.4
Sewerage, waste management, remediation activities	11.2	-7.3
Forestry and logging	10.8	-10.7
Publishing activities	10.7	-23.6
Wholesale and retail trade and repair of motor vehicles and motorcycles	9.6	-21.1
Advertising and market research	9.6	-3.9
Financial service activities, except insurance and pension funding	9.0	-52.2
Manufacture of furniture; other manufacturing	9.0	-22.8
Repair of computers and personal and household goods	9.0	-14.4
Warehousing and support activities for transportation	7.1	-9.0
Legal and accounting activities; activities of head offices; management consultancy	6.9	-18.9
Activities auxiliary to financial services and insurance activities	5.7	-27.1
Telecommunications	5.1	-7.1
Retail trade, except of motor vehicles and motorcycles	4.4	-13.2
Scientific research and development	4.3	4.9
Insurance, reinsurance and pension funding, except compulsory social security	3.2	-36.8
Motion picture, video, television programme production	3.1	-20.5
Repair and installation of machinery and equipment	2.4	-47.1
Sports activities and amusement and recreation activities	1.9	-7.9
Postal and courier activities	1.9	-25.3
Rental and leasing activities	1.6	-31.0
Creative, arts and entertainment activities; libraries, archives, museums	1.2	-17.4
Other personal service activities	0.5	-3.4
Printing and reproduction of recorded media	0.2	-28.5
Human health activities	0.1	-12.7
Other professional, scientific and technical activities; veterinary activities	0.1	-20.8
Water collection, treatment and supply	0.0	0.3
Imputed rents of owner-occupied dwellings	0.0	-9.5
Employment activities	0.0	-13.9
Residential care activities and social work activities without accommodation	0.0	-6.2
Activities of membership organizations	0.0	-0.1
Average	14.2	-14.5

C Decentralization

In this appendix we describe the decentralization of the benchmark model. We start characterizing the optimality conditions of the centralized model described in the paper. The state of the country is given by $S = (h, s)$ with $s = (z, k, b)$ and h denoting which regime the country is in, $h = 0$ normal regime and $h = 1$ default regime.

Characterization of the centralized problem The solution of the centralized model includes allocations $\{c_T, c_N, x_T, x_N, k_T, k', I, b'\}$, all as a function of the state S . They satisfy the resources constraint (1), (7), (8), the capital accumulation (3), and the following equations

$$\frac{c_T(S)}{c_N(S)} = \frac{x_T(S)}{x_N(S)} \quad (21)$$

$$\frac{(1-\alpha)}{\alpha} \left(\frac{c_T(S)}{c_N(S)} \right)^{-\frac{1}{\eta}} = \frac{f'(k - k_T(S))}{f'(k_T(S))} \quad (22)$$

$$I(S) = \left[(1-\alpha)x_T(S)^{\frac{\eta-1}{\eta}} + \alpha x_N(S)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (23)$$

$$\begin{aligned} u_{c_T}(0, s) \left\{ \left(\frac{x_T(0, s)}{I(0, s)} \right)^{\frac{1}{\eta}} [1 + \Psi_1(k'(0, s), k)] - (1-\alpha) \frac{dq(b', k', s)}{dk'} b'(0, s) \right\} = \\ \beta E \left\{ (1 - d(0, s')) u_{c_T}(0, s') \left(\frac{x_T(0, s')}{I(0, s')} \right)^{\frac{1}{\eta}} [z' f'(k'_T(0, s')) + 1 - \delta - \Psi_2(k''(0, s'), k'(1, s))] \right\} \\ + \beta E \left\{ d(0, s') u_{c_T}(1, s') \left(\frac{x_T(1, s')}{I(1, s')} \right)^{\frac{1}{\eta}} [h(z') f'(k'_T(1, s')) + 1 - \delta - \Psi_2(k''(1, s'), k'(0, s))] \right\} \end{aligned} \quad (24)$$

$$\begin{aligned} u_{c_T}(1, s) \left\{ \left(\frac{x_T(1, s)}{I(1, s)} \right)^{\frac{1}{\eta}} [1 + \Psi_1(k'(1, s), k)] \right\} = \\ \beta E \left\{ \lambda u_{c_T}(0, s') \left(\frac{x_T(0, s')}{I(0, s')} \right)^{\frac{1}{\eta}} [z' f'(k'_T(0, s')) + 1 - \delta - \Psi_2(k''(0, s'), k'(1, s))] \right\} \\ + \beta E \left\{ (1 - \lambda) u_{c_T}(1, s') \left(\frac{x_T(1, s')}{I(1, s')} \right)^{\frac{1}{\eta}} [h(z') f'(k'_T(1, s')) + 1 - \delta - \Psi_2(k''_d(0, s'), k'(1, s))] \right\}, \end{aligned} \quad (25)$$

and the optimal condition for the debt choice if not default.

C.1 Decentralized model

We assume the government imposes capital control and only itself can choose international borrowing and lending. The government makes default decision. The government also imposes capital tax τ^k and lump sum transfer T to implement the centralized allocations. The private sectors choose consumption and investment.

Consumer's problem Consumers take as given the government's policy functions $\tau^k(S)$, $T(S)$, $d(S)$, and $b'(S)$, market prices, and dividend as given and choose tradable and non-tradable consumption and investment to maximize their lifetime value,

$$W(S) = \max_{c_T, c_N, k', I} u \left(\left[(1 - \alpha)c_T^{\frac{\eta-1}{\eta}} + \alpha c_N^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) + \beta EW(S')$$

subject to the budget constraint

$$c_T + P_N(s)c_N + P_I(s)I = R(s)k + \Pi(s) - T(s),$$

the capital accumulation (3), and the law of motion of the economy $S' = G(S)$. In particular, $h' = 0$ if $d(S) = 0$ and $h' = 1$ if $d(S) = 1$.

We can characterize the consumer's problem as

$$\frac{u_{c_N}}{u_{c_T}} = P_N(S),$$

$$u_{c_T} P_I(S) [(1 - \tau^k(S)) + \Psi_1(k', k)] = \beta E u_{c_T}(S') P_I(S') [R(S') + 1 - \delta - \Psi_2(k''(S'), k')].$$

Tradable firm's problem All firms are competitive. Thus tradable firms rent k_T to equate marginal product of capital with the rental cost,

$$\hat{z}(S) f'(k_T) = R(S).$$

Note that the firm's productivity depends on the aggregate state in particular the government's default policy. When the government is in the normal regime $h = 0$ and choose not defaulting $d(S) = 0$, $\hat{z}(S) = z$; otherwise $\hat{z}(S) = h(z)$.

Nontradable firm's problem Nontradable firms rent k_N to equate marginal product of capital with the rental cost,

$$P_N(S) \hat{z}(S) f'(k_N) = R(S).$$

Investment firm's problem Investment firms purchase tradable goods x_T and nontradable goods x_N to assemble investment goods I . They solve the following problems,

$$\max_{x_T, x_N, I} P_I(S)I - x_T - P_N(S)x_N$$

subject to

$$I \leq \left[(1 - \alpha)x_T^{\frac{\eta-1}{\eta}} + \alpha x_N^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

The first order conditions are given by

$$(1 - \alpha)P_I(S)I^{\frac{1}{\eta}}x_T^{-\frac{1}{\eta}} = 1,$$

$$\alpha P_I(S)I^{\frac{1}{\eta}}x_N^{-\frac{1}{\eta}} = P_N(S).$$

Market clearing conditions

$$c_N + x_N = z f(k_N), \tag{26}$$

$$c_T + x_T = z f(k_T) - T(S), \tag{27}$$

$$k = k_N + k_T. \tag{28}$$

Characterization of private sector equilibrium The private sector equilibrium includes allocations $\{c_T, c_N, x_T, x_N, k_T, k', I\}$, prices $\{P_N, P_I, R\}$ and dividend income Π , all as a function of the state S . They satisfy the market clearing conditions (26-28) and the following conditions

$$\frac{c_T(S)}{c_N(S)} = \frac{x_T(S)}{x_N(S)}, \tag{29}$$

$$\frac{(1 - \alpha)}{\alpha} \left(\frac{c_T(S)}{c_N(S)} \right)^{-\frac{1}{\eta}} = \frac{f'(k - k_T(S))}{f'(k_T(S))}, \tag{30}$$

$$I(S) = \left[(1 - \alpha)x_T(S)^{\frac{\eta-1}{\eta}} + \alpha x_N(S)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{31}$$

$$u_{c_T}(S) \left(\frac{x_T(S)}{I(S)} \right)^{\frac{1}{\eta}} [1 + \Psi_1(k'(S), k) - \tau^k(S)] =$$

$$\beta E u_{c_T}(S') \left(\frac{x_T(S')}{I(S')} \right)^{\frac{1}{\eta}} [\hat{z}(S') f'(k'_T(S')) + 1 - \delta - \Psi_2(k''(S'), k'(S))], \tag{32}$$

$$P_N(S) = \frac{u_{c_N}(S)}{u_{c_T}(S)} = \frac{(1-\alpha)}{\alpha} \left(\frac{c_T(S)}{c_N(S)} \right)^{-\frac{1}{\eta}}, \quad (33)$$

$$P_I(S) = (1-\alpha)^{-1} \frac{1}{1-\alpha} \left(\frac{x_T(S)}{I(S)} \right)^{\frac{1}{\eta}}. \quad (34)$$

Government's problem We assume only the government can borrow or default internationally. The government's policy includes d , b' , τ^k , T . The government chooses whether or not to default

$$V(s) = \max_{d \in \{0,1\}} \{dV^d(z, k) + (1-d)V^n(s)\}. \quad (35)$$

If not default, the government solves the following problem

$$V^n(s) = \max_{\{b', k', c_T, c_N, x_T, x_N, k_T, \tau^k, I\}} u \left(\left[(1-\alpha_c)c_T^{\frac{\eta-1}{\eta}} + \alpha_c c_N^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) + \beta EV(s') \quad (36)$$

subject to the resources constraints (7) and (8), the investment production (31), the capital accumulation (3), and the implementability conditions for $S = (0, s)$ from the private economy (29), (30), the Euler equation (32), and the tax implementability

$$\tau^k = \frac{dq(b', k', s)}{dk'} b'(1-\alpha) \left(\frac{I}{x_T} \right)^{\frac{1}{\eta}}. \quad (37)$$

If the government defaults, the capital tax and lump-sum transfer both equal to zero, $\tau^k(1, s) = 0$ and $T(1, s) = 0$. It solves the following problem,

$$V^d(z, k) = \max_{c_T, c_N, k, k_T, x_N, x_T, k'} u \left(\left[(1-\alpha)c_T^{\frac{\eta-1}{\eta}} + \alpha c_N^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \right) + \beta E [\lambda V(z', k', 0) + (1-\lambda)V^d(z', k')] \quad (38)$$

subject to the resources constraints (12), the investment production (31), the capital accumulation (3), and the implementability conditions for $S = (1, s)$ from the private economy, (29), (30), the Euler equation (32) with $\tau^k(1, s) = 0$.

C.2 Equivalence

Proposition 1. *With tax function $\tau^k(0, s)$ from equation (37) and $\tau^k(1, s) = 0$, the allocations in the decentralized equilibrium in Section C.1 solve the centralized problem in Section 3.*

Proof. Let's first consider the unconstrained problem of the government, i.e. the problem

of (35), (36), and 38) without the implementability conditions. It is easy to see that the optimization coincides with the centralized problem. In particular, the solution can be characterized with equations of (21-25). These are the same as the implementability conditions under the tax system of τ^k . Thus the constrained problem with implementability conditions are the same as the unconstrained problem, and the same as the centralized problem. Q.E.D.