Replicating Private Equity with Value Investing, Homemade Leverage, and Hold-to-Maturity Accounting

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ABSTRACT

Private equity funds tend to select small firms with low EBITDA multiples. Public equities with these characteristics have high risk-adjusted returns after controlling for common factors. Hold-to-maturity accounting of portfolio net asset value eliminates the majority of measured risk. A passive portfolio of small, low EBITDA multiple stocks with modest leverage and hold-to-maturity accounting produces an unconditional return distribution that is highly consistent with that of the pre-fee aggregate private equity index. The passive replicating strategy represents an economically large improvement in risk- and liquidity-adjusted returns over direct allocations to private equity funds, which charge estimated fees of 3.5% to 5% annually.

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The professional and active management of private equity investments is widely believed to have many unique advantages over passive portfolios of publicly traded equities. Jensen (1986) and Kaplan (1989) are among the first to argue and provide evidence that leveraged buyout (LBOs) firms create value by providing management with strong incentives to improve operations. Specialized knowledge (Leland and Pyle (1977)), monitoring (Diamond (1984)), and access to credit markets (Ivashina and Kovner (2011)) are a few ways in which intermediated investing may provide advantages over a non-intermediated strategy. To the extent that these are material advantages in equity investing, the pre-fee returns on an aggregate private equity index are expected to outperform a passively managed portfolio comprised of otherwise similar public investments. This paper investigates whether an outside investor can replicate the risks and returns of a diversified private equity allocation with passive investments in public equities using similar investment selection, holding periods, leverage, and the calculation of portfolio net asset value under a hold-to-maturity accounting scheme.

Most research evaluating the performance of private equity investments investigates whether the end investor earns risk-adjusted returns net-of-fees. Estimates of unearned mean returns (alpha) vary across papers mostly based on time periods and datasets, as risk-adjusting methodologies are highly similar across papers, typically relying on a version of the public market equivalent (PME) introduced in Kaplan and Schoar (1995). Recently, Harris, Jenkinson, and Kaplan (2014) compare the private equity return data across various commercial datasets and find that the returns from newer datasets from Cambridge Associates, Burgiss, and Prequin are generally consistent with each other, while the returns from the earlier Venture Economics dataset appear to be biased downwards, affecting interpretations from early research that studied the Venture Economics dataset. The recent papers relying on newer datasets tend to find evidence suggesting that private equity investments reliably outperform their public market alternatives after-fees, indicating large outperformance gross of fees (e.g. Robinson and Sensoy

(2013) and Harris, Jenkinson, and Kaplan (2014)). Consistent with the interpretation of these recent papers, evidence that institutional investors find the risk/return profile of private equity highly attractive comes from the success of recent fund raising efforts with estimates of private equity buying power (including generous credit terms) exceeding $2 trillion near the end of the sample period (Bain & Co (2015)).[1]

Figure 1 displays the cumulative value of $1 invested in the Cambridge Associates (CA) Private Equity (PE) Index as reported net-of-fees (after-fee), this same index estimated *pre-fees* with fees measured two different ways, along with a portfolio that levers the aggregate US public stock market two times in the top panel, while the bottom panel plots the quarterly drawdown series for the private equity index and the levered market portfolio. The figure illustrates that the after-fee terminal wealth of the PE index is roughly similar to that of the US public equity index levered 2x. More interesting are the plots of the *pre-fee* PE compounded returns. Both of the *pre-fee* private equity return series represents highly attractive investments compared to a portfolio that is invested in the aggregate US stock market and levered two times to mimic the typical leverage of private equity investments. The levered market portfolio comes close to delivering the mean after-fee returns of the private equity index, but is far from keeping pace with the pre-fee returns, highlighting the remarkable success of private equity investments in increasing wealth over this sample period. The other distinguishing feature of the return properties is that the measured risks of the PE indices are considerably lower than those of the public market investments.

Another stream of the literature has shown that inferences about private equity performance can be altered after refining the public equity benchmarks to reflect that private

[1] Bain & Co. (2015) report that at the end of 2014, PE funds have over $1.2 trillion of "dry powder" (committed, but non-deployed capital) that is likely to be levered at least two times.

equity investments are concentrated among firms with specific characteristics like relatively small firm size and high book-to-market equity that are themselves known to be associated with better return-for-risk profiles than typical stocks (e.g. Phalippou (2014) and Ang, Chen, Goetzmann, and Phalippou (2014)). Figure 1 suggests that this is a critical step in trying to reproducing the high *pre-fee* returns of PE over this sample as the aggregate public stock index levered 2x falls far short.

The approach in this paper is to identify the key elements of the private equity investment process that can be executed passively and to assemble these components into a PE replicating strategy. While there are many active elements to the PE process, there are several distinct components to the process that can be executed passively. Specifically, these are asset selection, long holding periods, the use of financial leverage, and the use of conservative estimates portfolio net asset value.

To study the asset selection by private equity funds, I assemble a dataset of public-to-private transactions sponsored by financial buyers, similar to the approach used by Axelson, Jenkinson, Strömberg, and Weisbach (2013). A selection model finds that private equity investors consistently tend to target relatively small firms (e.g. Phalippou (2014)), and firms that would generally be characterized as "value" stocks. Firms with higher book-to-market equity, lower EBITDA (earnings before interest, taxes, and depreciation) multiples, and lower net equity issuance (i.e. repurchasers as opposed to issuers) are more likely to be buyout targets, with EBITDA multiples tending to dominate the explanatory power of book-to-market. Interestingly, a firm's market beta is not a reliable predictor of whether a firm is selected for a going-private transaction after controlling for other characteristics. In fact, the average pre-transaction market beta for the public-to-private firms is 1.

The literature on the cross section of expected stock returns suggests that portfolios comprised of firms with the characteristics shown to be associated with the highest buyout likelihood should be expected to have high returns in this sample period (1996 through 2016). There is strong empirical evidence that value firms earn high stock returns (Rosenberg, Reid, and Lanstein (1985), Fama and French (1992), and Loughran and Wellman (2011)). These papers empirically link realized excess equity returns to a firm's ratio of book equity, *BE*, to market equity, *ME*. Interestingly, I find that the operating cash flow (EBITDA) multiple is a more powerful variable than *BE*/*ME* for sourcing a value premium in stocks over this sample period producing a larger spread in returns and driving out the statistical significance of *BE*/*ME* in Fama-MacBeth (1973) return regressions. Another strong predictor of a firm being a buyout target is that firm's recent net equity issuance, which Daniel and Titman (2006) show is a strong predictor of future stock returns.

An *unlevered* portfolio of public equities formed on the basis of their likelihood of being selected for a buyout (PE-Selected) realizes a mean return nearly as high as the *pre-fee* PE index, such that modest leverage will allow an investor to match the mean return. However, the *measured* risks of this PE-Selected portfolio are considerably higher than those estimated directly from private equity investments, and will be higher still with leverage. The two remaining passive components of the PE investment process – long holding periods and conservative portfolio marking – are important for understanding these measured risk differences.

Long holding periods are required to execute the active components of the PE process. It turns out that long holding periods are also important for reproducing the distortion in measured risks that can arise from conservative portfolio marking. A portfolio that is rebalanced monthly offers little discretion in portfolio marking, as transactions that force accurate marking-to-market occur frequently. However, with long holding periods, conservative portfolio marking can dramatically distort measured risks. To investigate how long holding periods and conservatism in the reporting process can combine to alter inferences about risks, two different accounting schemes are used to report portfolio net asset values from which periodic returns are calculated. The first is the traditional market-value based rule where all holdings are reported at their closing price. Portfolios comprised of stocks with market betas averaging 1, with portfolio leverage of 2x, have measured portfolio betas near 2 under the market-value based accounting rule. The second accounting scheme is based on a hold-to-maturity rule, whereby securities that are intended to be held for long periods of time are measured at cost until they are sold. Over periods where security valuations are increasing on average, this accounting scheme appears to provide a conservative estimate of portfolio value and therefore can overstate leverage. However, an additional feature of this accounting scheme is that it significantly distorts portfolio risk measures by recognizing the profits and losses on the underlying holdings only at the time of sale. Consequently, portfolios with highly statistically significant measured betas near 2 under the market-value reporting rule have measured beta that are statistically indistinguishable from zero under the hold-to-maturity reporting rule. This suggests that the long holding periods of private equity portfolios, combined with conservativism in measuring asset values can effectively eliminate a majority of the measured risk over this sample period.

The final key passive element of the PE investment process is leverage. In the spirit of Modigliani and Miller (1958), an outside investor interested in the levered equity return of a firm that has chosen too little leverage can manufacture a return levered to the investor's desired level on their own using a brokerage margin account. A modest amount of portfolio leverage through a brokerage margin account is efficient because the debt is essentially riskfree due to over-collateralization and high frequency marking-to-market, allowing for borrowing rates close to the riskfree rate. This so-called homemade leverage will not manufacture the incentive, tax effects, and costs of financial distress that increased leverage at the firm-level may produce, but will significantly alter the risk and return properties of the underlying equity. A prototypical private equity transaction increases a firm's leverage, measured as the ratio of market debt to firm value, from around 30 percent to 70 percent (Axelson, Jenkinson, Strömberg, and Weisbach (2013)). An outside investor would need to select a portfolio of comparable pre-transaction stocks and invest slightly more than two times their equity capital in this portfolio to match the post-transaction levered equity return, which is expected to essentially double the underlying market beta and standard deviation of the underlying stock.

The results demonstrate that the high mean returns of pre-fee PE investments illustrated in Figure 1 can be obtained relatively easily with a wide variety of specific execution choices within each of the key passive components of the PE process. This suggests that the overall argument – an investment strategy that replicates the passive components of the PE investment process matches the high realized *pre-fee* returns of PE investments and can explain the low measured risks of PE investments – is fairly robust. An additional benefit of such a replicating strategy is that indirect estimates of PE risks from the well-marked replicating portfolio comprised of liquid public equities are likely to provide more accurate risk assessments than estimates made directly from illiquid and conservatively marked PE investments. The risk estimates provided in this paper are considerably higher than those commonly used to describe PE investments. Moreover, the risk estimates provided here are generally consistent with estimates from secondary PE transactions and with LBO bond prices during the 2008 financial crisis, while an investor relying on risk estimates made directly from PE investments would find these market transaction prices to be highly anomalous.

The remainder of the paper is organized as follows. Section I studies the asset selection tendencies of private equity investors. Section II evaluates the return properties of portfolios comprised of firms with the characteristics shown to be associated with the highest buyout likelihood. Section III develops a simple strategy for replicating the risks and returns of a broad portfolio of private equity investments with firms with similar characteristics to those selected by private equity investors, similar holding periods, similar portfolio leverage, and hold-to-maturity accounting for portfolio net asset value. Section IV offers a discussion of the robustness of the main results and some implications for asset allocation. Finally, Section V concludes the paper.

I. Asset Selection by Private Equity Funds

There is little empirical evidence on the specific asset characteristics that private equity funds select for their portfolios. Based on aggregates of activity, it appears that private equity investments are not evenly distributed throughout the economy, suggesting that they target specific asset types (Kaplan and Strömberg (2009)). The highly limited data availability on the financials and governance of private firms is a major obstacle to knowing which asset characteristics are associated with private equity asset selection. The approach in this paper follows Axelson, Jenkinson, Strömberg, and Weisbach (2013) whereby the subsample of public equities that have been taken private is studied, recognizing that the investments in private firms are not completely representative of the full sample. For example, the public targets in this sample are likely to be considerably larger than the private targets that are excluded. Phalippou (2014) reports that 95% of buyout investments fall in the Fama-French small-cap index. The sample of buyouts of public firms allows the pre-transaction financial characteristics to be collected from Compustat and CRSP.

The data on public targets taken private by private equity firms come from the Thompson Reuters Merger and Acquisition database where the acquiring firm is identified as a financial buyer and the transaction results in at least 80% ownership of the target firm over the period 1983 to 2014. This results in 711 firms that can be linked to CRSP and Compustat. The sample size is in line with the U.S. sample of 694 deals identified by Axelson, Jenkinson, Strömberg, and Weisbach (2013) over the period 1980 to 2008.

Table 1 reports results from regressions explaining which firm characteristics are associated with private equity buyouts from 1984 to 2014. Both ordinary least squares (OLS) and logistic regressions of a binary “PE-selected” variable on firm characteristics are reported (OLS in Panel A and logistic in Panel B). All of the specifications include year fixed effects. The reported OLS coefficients are multiplied by 100 to improve readability. The OLS *t*-statistics are calculated with standard errors clustered by firm. The firm characteristics are firm size, proxied by either equity market capitalization (ME) or total revenues (sales); EBITDA multiple (M_{EBITDA}); market beta; profitability measured as the ratio of EBITDA to sales; market leverage ratio measured as long-term debt to the sum of long-term debt and ME; the three-year net equity issuance variable (ISS) described in Daniel and Titman (2006); and the book-to-market equity ratio (BE/ME). The EBITDA multiple is calculated as the firm enterprise value divided by EBITDA, so long as EBITDA exceeds $1 million. Firms that do not satisfy the minimum EBITDA requirement at the time of portfolio formation are excluded from the analysis. Additionally, to be consistent with other research that relies on EBITDA multiples, financial firms identified as those with SIC codes between 6000 and 6999 are excluded from the analysis. The firm enterprise value is the sum of the market value of equity from CRSP (price per share multiplied by shares outstanding) and the book value of long-term debt from Compustat less cash and marketable securities from Compustat.[2] The firm characteristics are all assumed to be known at the time of the event. The event time is measured as the announcement date. Stock market variables (ME) are assumed to be known with no delay. Accounting variables are assumed to be known with a three-month delay.

The regressions indicate that among the public firms taken private, the selected investments are relatively small firms as proxied by either ME or sales, with these variables being highly statistically significant in all specifications. The selected firms tend to have relatively low recent net equity issuance, indicating that the selected firms tend to be

[2] Adjusting the enterprise value calculation for excess cash (defined as cash above 2% of sales) or skipping the subtraction of cash altogether has virtually no quantitative effect on the results.

repurchasing their own shares. The negative coefficient on profitability suggests that the selected firms are not highly profitable, although these regressions condition on firms having EBITDA over $1M. The coefficient on profitability is positive without this condition (results not reported). Additionally, the selected firms tend to be value firms. BE/ME is positively associated with the event (firms with high BE/ME are considered value firms) and M_{EBITDA} is negatively associated with the event. When both variables are included in the same specification, M_{EBITDA} tends to eliminate the statistical reliability of BE/ME. Interestingly, market beta and leverage are not reliable predictors of the PE selection. The leverage result is consistent with Axelson, Jenkinson, Strömberg, and Weisbach (2013) who find that the leverage choice by the PE fund for the target firm is unrelated to the target firm's leverage and the industry average leverage ratio at the time of the transaction, seemingly determined by aggregate credit market conditions.

To investigate the stability of these statistical relations over time, the logistic regressions are estimated over two sub-periods. Panel A of Table 2 reports results from 1984 to 1999, and Panel B reports results from 2000 to 2014. Overall, the results from this analysis are qualitatively similar to the full sample analysis, suggesting that the asset characteristics that attract PE investors are reasonably stable across the entire sample period. Within each half of the sample, there is a tendency for the PE-selected firms to be relatively small, value firms, with low net equity issuance, and modest profitability. The time series stability in the relation between characteristics and PE-selection will be useful in constructing replicating portfolios that rely only on information available at the time of portfolio formation.

II. Returns to Investing in PE-Selected and Value Stocks

The literature studying the cross-section of stock returns typically measures a value premium from the time series mean of a long-short portfolio that is long stocks in the top third of the book-to-market equity (BE/ME) distribution and is short stocks from the bottom third of this distribution (Fama and French (1993)). The firms with high BE/ME are considered value stocks, while the firms with low BE/ME are considered growth stocks. Firms identified as being value stocks earn relatively high returns and are sometimes referred to as being distressed (Fama and French (1996)).

A common metric for identifying value stocks in practice is the EBITDA multiple, M_{EBITDA}. This multiple represents the price per unit of operating income available to the capital providers of the firm (i.e. debt- and equity-holders). To the extent that debt is priced consistently across firms, sorting stocks on their firm's M_{EBITDA} provides an alternative means to sourcing a value premium in stocks, which according to the regressions reported in the previous section more accurately reflect the selection method of financial buyers.

A simple rational asset pricing model suggests several reasons why a firm may have a low valuation multiple. From the perspective of the perpetuity value formula, the EBITDA multiple can be rewritten as:

$$M_{EBITDA} = \frac{\gamma}{R - g}$$

where γ is the fraction of EBITDA that is converted into cash flow, g is the constant cash flow growth rate, and R is the discount rate. Value firms may have relatively low multiples because of low conversion rates of EBITDA to cash flow, perhaps due to high capital expenditure needs; and/or relatively low cash flow growth rates; and finally due to high discount rates presumably because of highly systematic risk exposures of the underlying cash flows. The premise behind value investing is that low multiples occur through an additional discount rate channel, namely non-systematic risk exposures, or market mispricing, which is expected to translate into positive risk-adjusted returns.

Table 3 summarizes returns for five portfolios formed on M_{EBITDA}. The portfolios are formed monthly based on information assumed to be known at the beginning of the month. Equity market values are assumed to be known with no delay. Debt market values are assumed to equal their book values and to be known with a reporting delay of three months. Similarly, all other accounting data are assumed to be known with a three month reporting delay.

Table 3 confirms the basic premise behind value investing with EBITDA multiples. There is a strong monotonic relation in the realized excess returns across portfolios formed on M_{EBITDA} over the period 1986 to 2016. Portfolios comprised of low multiple stocks (i.e. the bottom quintile of all CRSP stocks with annual EBITDA in excess of $1M ranked on the basis of M_{EBITDA}) have high excess returns, averaging 18% per year for the equal-weight portfolio and 13% for the value-weight portfolio, while portfolios comprised of the high multiple stocks (top quintile) have average excess returns of 6.4% and 7.9% for equal- and value-weight portfolios, respectively. Over this same period, the excess return on the value-weight market portfolio is 8.7%. The annualized volatility is reasonably similar across portfolios, such that Sharpe ratios share the same pattern as the excess returns.

Additionally, Table 3 shows that systematic risk exposure does not explain this pattern. The unexplained mean excess return (or alpha), as measured by the intercept from a time series regression of the portfolio excess returns onto the zero-investment portfolio returns suggested by either the Sharpe (1964) and Lintner (1965) capital asset pricing model (CAPM), the Fama and French (1993) three-factor model (FF3), the Fama and French (2014) five-factor model (FF5), or the FF5 plus a momentum factor (Carhart (1997)), UMD, shares the same strong monotonic relation across M_{EBITDA} portfolios over this period.[3] A long-short portfolio that is constructed by being long low M_{EBITDA} stocks and short high M_{EBITDA} stocks earns an alpha of 1% per month (t-statistic = 4.9) against the CAPM when stocks are equally weighted in the portfolio. Value-weighting produces smaller alphas for the long-short portfolio, but they remain reliably positive, with a monthly alpha of 45 basis points (t-statistic = 2.1) against the CAPM. The Fama and

[3] Factor returns are from Ken French's website.

French five-factor model includes a factor called RMW, which is long stocks with robust profitability and short stocks with weak profitability, and CMA, which is long low investment stocks and short high investment firms. Fama and French (2014) find that these factors weaken the statistical power of HML in explaining the cross section of returns. For the portfolios formed on M_{EBITDA}, HML remains statistically significant after including these factors. Additionally, with regressions that include these additional factors result in economically large and highly reliable intercepts for the low M_{EBITDA} portfolios using both equal weights and value weights.

These results are qualitatively similar to those reported in Loughran and Wellman (2011) who also examine excess and abnormal returns to portfolios formed based on sorts of EBITDA multiples. The results here are slightly stronger primarily due to the use of quarterly updates to EBITDA rather than annual updates and a more recent sample period. These results serve to confirm the premise that sourcing a value premium via EBITDA multiples is very effective over the sample period where I am considering the performance of private equity returns.

The PE-selection analysis in the previous section also identifies several additional characteristics that are reliably associated with public equities taken private by financial buyers. Consequently, I also rank stocks on their predicted likelihood for being “selected.” Specifically, each year, an expanding dataset including only information available at that point in time is used to estimate the PE-selection model. Stocks are sorted on their fitted values, with the top quintile of stocks being viewed as the most similar to those being selected. Table 4 summarizes the excess and abnormal returns to five portfolios formed on sorts of the predicted PE-selection model.

The portfolio comprised of stocks most similar to PE-selected stocks (i.e. the top quintile of predicted PE-selection) have high excess returns, averaging 18% per year for the equal-weight portfolio and 14% for the value-weight portfolio, while portfolios comprised of the high multiple stocks (top quintile) have average excess returns of 7.1% and 7.6% for equal- and value-weight portfolios, respectively. Again, the Sharpe ratios share the same pattern as the excess returns. The equal-weighted portfolio of stocks most similar to the PE-selected stocks has a Sharpe ratio of 0.90, which is highly similar to the 0.88 Sharpe ratio of the low EBITDA multiple portfolio. The time series correlation between these two portfolios is 0.975.

In light of the success of M_{EBITDA} in producing a large spread in returns and abnormal returns, it is interesting to investigate the statistical power of this characteristic in explaining the cross section of stock returns in the presence of other characteristics known to be reliable explanatory variables. In particular, I am interested in regressions that include the book-to-market equity ratio and the net equity issuance a firm has done over the past three years, the latter of which Daniel and Titman (2006) have shown to be a highly reliable explanatory variable in cross sectional monthly return regressions. Table 5 reports the results from Fama-MacBeth (1973) regressions of monthly excess returns, R_t, on various stock characteristics known at the beginning of the period, X_{t-1}. The independent variables include Beta, ln(ME), ln(BE/ME), ISS, and ln(M_{EBITDA}), where Beta is the estimated slope coefficient from a regression using the past 60 months of excess stock returns (requiring at least 36 valid returns) onto the excess return on the VW market portfolio with 2% Winsorisation, ME is the equity market capitalization, BE/ME is the book-to-market equity ratio, ISS is the three-year net equity issuance measure from Daniel and Titman (2006), and M_{EBITDA} is as defined earlier. The regressions confirm the findings of prior research that the premium earned for market beta is not statistically reliable and that size, book-to-market, and net issuance are associated with statistically reliable premia in this sample. The regressions also find that M_{EBITDA} is associated with a statistically large premia and that in regressions that include both BE/ME and M_{EBITDA}, only M_{EBITDA} is statistically distinguishable from zero. These regressions suggest that the EBITDA multiple is a powerful variable for sourcing a value premium in stocks during this sample period,[4] and that several of the most reliable stock characteristics of the PE-selection strategy have tended to be associated with high subsequent excess returns.

III. Replicating the Return Distribution of Private Equity Investments

Many of the characteristics that predict whether a public equity is selected for a buyout transaction (PE-selected) are associated with high subsequent stock returns. This section explores whether this relation can be used to create a passive replicating portfolio to match the risk and return properties of a diversified portfolio of private equity investments.

A. Comparing return properties of private and public equities

The returns that an outside investor receives from an allocation to private equity are proxied by the U.S. Private Equity Index from Cambridge Associates. The Cambridge Associates' Private Investments database is collected from over 1,700 institutional fund managers, covering 5,700 funds, and includes the quarterly net return to investors. Quarterly returns are calculated from the unaudited quarterly financial statements and the audited annual financial statements prepared by the fund managers for their limited partners (i.e. outside investors). The quarterly return series begins in 1986Q2 and extends to 2016Q4. Harris, Jenkinson, and Kaplan (2014) compare the private equity return data across various commercial datasets and find that the Cambridge Associates (CA) returns are consistent with those from Burgiss and Prequin, while the returns from Venture Economics appear to be biased downwards.

[4] Further evidence in support of this conclusion is provided from a regression of a M_{EBITDA} factor, proxied as a value-weight long-short portfolio that is long stocks M_{EBITDA} in the bottom quintile of M_{EBITDA} and short stocks in the top quintile, on the Fama-French three factors. To be consistent with the construction of the Fama-French factors, this M_{EBITDA} factor includes financial firms, which are excluded from all other analyses presented in this paper. The intercept from this regression is statistically positive (t-statistic = 2.0), while the intercept from a regression of HML on the remaining two Fama-French factors and this M_{EBITDA} factor is statistically indistinguishable from zero, 0.0016 with a t-statistic = 1.3.

Therefore, the choice of a specific proxy among the newer datasets (e.g. CA, Burgiss, and Prequin) should not have a meaningful impact on inferences. The Yale Endowment buyout portfolio is used as an additional proxy to represent the returns achieved by a specific investor widely considered to have an enviable track record of managing a diversified PE allocation. The Yale buyout returns are available annually from 1997 through 2014, measured on an academic calendar beginning in July prior to the reporting year and ending in June of the reporting year.[5]

The premise being investigated is whether active PE investments earn average returns beyond what can be obtained in strategies that passively mimic the key elements of the PE investment process, as opposed to how the returns are shared between the investor and the fund manager. Therefore, it is most natural to replicate the *pre-fee* return to private equity. I estimate pre-fee returns in two different ways. In the first method, I treat the observed net-of-fee time series as if it represented the return of a representative fund that charges a 1% fixed fee plus a 20% performance fee on returns above an annualized hurdle rate of 8% when at its high water mark, both payable quarterly.[6] The second method is calculated similarly, except that it does not impose any hurdle rate for the performance fee to be earned, leading to a more aggressive fee estimate. The difference between the mean pre-fee and net-of-fee returns represents an approximation of the all-in fee paid by investors.

Table 6 reports a summary of the quarterly and annual returns to private equity, both pre-fees and net-of-fees, along with various portfolios consisting of public equities. Panel A reports annualized statistics measured from quarterly returns over the period 1996Q2 through 2016Q4, Panel B summarizes annual returns over the same time period, and Panel C summarizes annual

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[5] The annual returns for the Yale Endowment buyout portfolio come from the HBS case study, "Yale University Investments Office: February 2015," by Josh Lerner.

[6] Jurek and Stafford (2015) apply a similar calculation to hedge fund return indices. The 1% fixed fee is chosen to reflect the tendency for private equity funds to charge a 2% fixed fee over an investment period, which is typically one-half of the life of the fund (see for example, Metrick and Yasuda (2010)).

returns corresponding to the academic calendar over the period 1997 through 2014, corresponding to the availability of the Yale buyout portfolio returns.

The portfolios of public equities include the value-weight CRSP stock index, this same index levered two times (2x), paying brokerage interest at the 1-month US Treasury bill rate, an unlevered value stock portfolio based on stocks in the lowest quintile of EBITDA multiples, and the unlevered PE-selected portfolio based on stocks in the highest quintile of predicted buyout likelihood.[7]

There are several striking features of the private equity return properties. First, the implied all-in fee paid by investors is 3.5% to 5% per year, which is economically large. As a point of reference, the realized market risk premium over this period is just under 8%. The lower range of these fee estimates are in-line with practitioner estimates of all-in PE fees. For example, Cliffwater LLC (2010), a provider of advisory services to institutional investors in alternative assets, estimates that the all-in fee for a typical PE fund is 3.7%, suggesting that the fee calculation that excludes the performance hurdle rate is likely to overstate mean pre-fee PE returns.

Second, the geometric mean pre-fee returns are 17% to 18% per year. Given the significant difference in measured volatility between the private and public equities, the geometric means are probably the most appropriate for comparing average returns across portfolios. These estimates of mean pre-fee returns are considerably higher than those earned with the aggregate index of public equities levered 2x, which realized a geometric mean of 13.4% over this period. Interestingly, the *unlevered* portfolios of firms shown to be the most similar to the firms targeted for buyout transactions, labeled Value and PE-Selected, have

[7] The construction of these portfolios is similar to those reported in Tables 4 and 5, with the added restriction that share price must be over $5 at the time of portfolio formation.

geometric mean returns nearly as high as the pre-fee PE returns. The value portfolio has a geometric mean return of 16.7% and the PE-selected portfolio has geometric mean return of 15.6%.

Third, the reported private equity returns have relatively little measured risk. The net-of-fee quarterly returns (i.e. the raw data) have an annualized volatility of 9.4% over a period where the market return volatility is 17%. Similarly, the measured CAPM beta from the net-of-fee quarterly excess returns is 0.4. The low measured volatility and beta result in economically large Sharpe ratios and CAPM risk-adjusted returns. The measured risks of the *unlevered* Value and PE-Selected portfolios are also considerably higher with annualized standard deviations of 22% and 21%, respectively. In addition, these public equity portfolios have much larger drawdowns during the 2008 financial crisis, slightly worse that the VW stock market index.

Comparing the measured risks of the PE portfolios across Panels A (quarterly returns) and B (annual returns) reveals that the annual PE returns have higher measured risks than the quarterly returns. This is consistent with return smoothing, which will generally induce some positive autocorrelation in returns leading to higher risk estimates from returns measured at a longer frequency. The variance ratios (annual to quarterly) are over 2.0 for the PE return series with p-values of 0.00, while the variance ratios for the portfolios of public equities are statistically indistinguishable from 1.0.

Finally, Panel C summarizes annual returns over the period where the Yale buyout portfolio returns are available. Consistent with the perception that Yale's buyout portfolio has performed well, this portfolio realized a mean annual return of 18.5% with measured volatility of 19%, producing a Sharpe ratio of 0.78.[8] Note that these returns are measured after-fees. The

[8] The correlation in annual returns between the Yale Endowment buyout portfolio and the Cambridge Associates Private Equity Index is 0.85.

unlevered public stock portfolios based on Value and PE-Selection characteristics have highly similar returns in terms of mean and standard deviation, producing Sharpe ratios over this sample of 0.90 and 0.78, respectively. Interestingly, the worst realized annual returns are highly similar between the Value, PE-Selected, and Yale buyout portfolios over this sample, all roughly -25%.

B. The effect of long holding periods on mean returns and measured risks

The results so far suggest that to match the mean pre-fee PE return with portfolios of public equities, it is important to select public equites with similar characteristics to those targeted in buyout transactions, roughly consistent with the results in Phalippou (2014).[9] With this focus, matching the mean return with modest leverage seems quite feasible. However, matching the measured risks looks to be considerably more daunting. To the extent that the risk properties of private equity have been distorted through a discretionary marking-to-market process, the monthly rebalanced portfolios will be unable to replicate this distortion. The relatively frequent rebalancing of publicly traded securities offers little discretion in marking-to-market.

A key element of the private equity investment process is relatively long holding periods, typically extending several years. Long holding periods are necessary to implement and realize the various active elements of the PE investment process. At the same time, there appear to be some material costs associated with long holding periods that will, at least partially, offset these active management benefits. Two costs related to extended holding periods that will be explored here are (1) the reduced mean returns for value portfolios, as firms do not necessarily remain value firms for long periods and (2) the possibility of increased distortions in measured risks

[9] These monthly rebalanced strategies are similar in spirit to one described by Chingono and Rasmussen (2015).

from conservatively marked portfolios. Evaluating the first of these is straightforward, while evaluating the second effect requires modeling a conservative portfolio marking scheme.

Given the nature of the investments, return smoothing is a concern. Private investments require considerable discretion in marking the portfolio net asset value and this process is likely to destroy the covariance structure in returns and lead to downward biased estimates of risk, particularly over economically benign periods. Evidence from hedge fund returns suggests that unconditional (Asness, Krail, and Liew (2001), Getmansky, Lo, and Makarov (2004)) and conditional (Bollen and Pool (2008, 2009)) return smoothing, due to asset illiquidity and discretion in marking portfolio NAVs (Cassar and Gerakos (2011), Cao et al. (2013)), creates a significant downward bias in the measured risks in these portfolios. Jurek and Stafford (2015) demonstrate that over the period from 1996 to 2014, return smoothing in just two key months (August 1998 and October 2008) is sufficient to statistically obscure the exposure of aggregate hedge fund indices to downside market risks. An investor relying on the accuracy of reported returns infers that average pre-fee hedge fund alphas are 6% to 10% per year, while an investor who is skeptical of the accuracy of reported returns cannot statistically reject the presence of downside market risks and pre-fee alpha estimates of zero. The challenge is likely to be greater with a portfolio comprised entirely of private investments.

B.1. A Portfolio Marking Rule based on Portfolio Cash Flows

A portfolio consisting of long positions of liquidly traded securities like publicly traded equities is typically marked-to-market value based on the day's closing prices of each underlying position. The equity capital is determined as the residual of the total portfolio asset value net of any borrowing. Under this market value based accounting system, the equity capital evolves over time by cumulating the daily profits and losses for the underlying securities based on daily changes in market values, net of interest expenses. Portfolio transactions to include or eliminate positions do not alter the equity capital since the cash flow associated with these transactions is assumed to occur precisely at the market value. For example, selling 100 shares of a stock at $20 per share, reduces the value of the stock holdings by $2,000 and increases the cash (or reduces the borrowing) by the same $2,000, leaving the equity value unaltered.

To investigate the scope for discretion in marking the portfolio net asset value when positions are held for long periods of time, I consider a simple hold-to-maturity accounting scheme that measures positions at their purchase price until they are sold. This is similar to the accounting rules for large U.S. banks for positions that are classified as "hold-to-maturity" investments. Under this accounting scheme, daily fluctuations in the prices of underlying investments do not impact the daily portfolio net asset value. Instead, the daily portfolio net asset value changes based primarily on the cumulative profit and loss of positions at the time of liquidation. Interest expenses and dividends will have a small periodic effect on the portfolio book value, but the majority of the variation in the portfolio net asset value will occur when the portfolio is rebalanced to eliminate positions. Selling 100 shares of a stock at $20 per share, that were originally purchased for $15 per share, reduces the book value of the stock holdings by $1,500 and increases the cash (or reduces the borrowing) by the transaction value of $2,000, thereby increasing the equity value by $500. Over a long time period, where the terminal portfolio book value is near its market value, the mean returns under these two accounting schemes will be similar, but the book value accounting scheme will alter the timing of the portfolio profits and losses, thereby distorting the covariance structure in returns relative to portfolio returns that make use of the mark value based accounting scheme, as in factor model regressions.

B.2. Results on the Impact of Extended Holding Periods

Table 7 reports return statistics for both the Value and PE-Selected portfolio strategies that differ in their holding period. All portfolios are rebalanced monthly, but are initially constructed by adding 2% of the possible investments each month so that the portfolio is fully seasoned by the time the comparison with the private equity returns begins in 1986. The considered holding periods are monthly, quarterly, annual, 2-years, 3-years, and 4 years. The reported statistics include the geometric and arithmetic annualized mean, standard deviation, Sharpe ratio, CAPM beta measured as the sum of the coefficients from the contemporaneous market excess return and two lagged values, the CAPM intercept and *t*-statistic, and the minimum drawdown. Results are reported under each of the two accounting schemes described above, labeled as Mark-to-Market and Hold-to-Maturity.

Panel A of Table 7 shows results for *unlevered* value portfolios formed on the basis of low EBITDA multiples under both accounting schemes. It is important to note that the actual transactions are identical under the two accounting schemes, resulting in identical terminal market values with the assumption that the portfolios are liquidated at market value at the end of the sample. Panel B shows these results for value portfolios levered 1.5x, Panel C reports results for *unlevered* PE-Selected portfolios, and Panel D reports results for PE-Selected portfolios levered 1.5x.

Figure 2 illustrates the properties of the unlevered Value and PE-Selected portfolios. The mean returns to both the Value and PE-Selected are monotonically decreasing in holding period beyond the quarterly horizon, consistent with the notion that as the value premium is realized and firms cease to be classified as value stocks, the value premium is no longer earned. It is interesting that this pattern holds in the PE-Selected portfolios too. By construction, the geometric mean returns are identical across accounting schemes. As consequence of the decay in mean returns with holding period is that the estimated CAPM alpha is highly statistically significant at shorter holding periods up to two-years, and becomes statistically indistinguishable from zero at longer holding periods, highlighting that long holding periods are costly for both the Value and PE-Selected portfolios.

Measured risks behave quite differently across accounting schemes. The standard deviation, CAPM beta, and minimum drawdown are relatively stable across holding periods for both the Value and PE-Selected portfolios under the mark-to-market accounting scheme, but tend to fall with holding period under the hold-to-maturity accounting scheme. For example, the CAPM beta for the value portfolio ranges from 1.07 to 1.14 across holding periods with mark-to-market accounting, but falls from 1.18 to 0.05 as the holding period increases from monthly to 4-years. Interestingly, even the worst drawdown becomes meaningfully different across accounting schemes as the holding period extends, with hold-to-maturity accounting revealing roughly half the risk at 2-years and only 6% of the risk at 4-years. These results demonstrate the potential for highly distorted risk measures when portfolio returns are marked conservatively and holding periods are long and mechanical. There is further scope for distortion in measured risks arising from discretion in choosing when to exit a position.

C. Replicating Private Equity Returns

The remaining empirical analysis in this section is to assemble the key components of the passive PE investment process into feasible replicating portfolios. These passive replicating portfolios can be used estimate the likely risks of private equity investments and to demonstrate how significantly these measured risks can be distorted over this sample period.

C.1. Constructing a Replicating Portfolio

At the end of each month from 1981 to 2015, all publicly traded firms listed on CRSP are sorted based on their EBITDA multiple and their predicted likelihood of being selected for a buyout transaction, as calculated earlier. Firms with M_{EBITDA} in the bottom quintile of the monthly distribution and stock prices above $5 are selected to be included in the Value replicating portfolio, while firms with in the top quintile of PE-selection likelihood are selected to be included in the PE-Selected replicating portfolio.

A constant target portfolio leverage, L, of 2.0x is assumed based on the tendency for private equity transactions to increase the leverage of the underlying firms selected. Here, leverage is applied to the portfolio through a margin account. According to Axelson, Jenkinson, Strömberg, and Weisbach (2013), the typical publicly traded firm has a market debt-to-value ratio of roughly 30% (Assets/Equity = 1.43), while this ratio is increased to nearly 70% (Assets/Equity = 3.33) as the result of a private equity transaction. An outside investor holding the pre-LBO equity, but wanting the post-LBO levered return would need to apply portfolio leverage, measured as portfolio assets divided by portfolio equity capital, of 2.3x = 3.33 / 1.43. Borrowed funds are assumed to pay the one-month U.S. Treasury bill yield.

As argued earlier, a crucial element of the PE investment process is a multi-year holding period, providing some scope for discretion in the calculation of the portfolio net asset value. Individual investments are held in the replicating portfolio for at least six months, but are sold in the event that they have realized annualized returns of 50% or if they have been held for three years. This return hurdle triggers nearly half of the positions to be liquidated prior to the three year default holding period.

Due to the long-term holding periods and the buy-and-hold strategy, portfolio weights for newly added positions are determined each month by calculating a target number of holdings as the sum of current positions plus the number of additions less the number of firms exiting the portfolio. The current equity capital times target portfolio leverage divided by the target number of holdings determines the amount that is equally invested in each new addition. This results in the realized portfolio leverage varying somewhat over time.

C.2. Summarizing the Returns

Table 8 reports summary statistics for the returns on the two replicating portfolios, under both the market value and hold-to-maturity accounting schemes. Panel A reports results for portfolios that are marked-to-market value based on month-end security prices as reported in

CRSP. Panel B reports results under the portfolio hold-to-maturity accounting scheme described above, with portfolio values primarily being updated when positions are liquidated.

Table 8 shows that the Value Replicating Strategy earns an annualized geometric mean return of 18.6% over the sample period, while the PE-Selected Replicating Strategy earns an annualized geometric mean return of 20.0%. Figure 3 displays the cumulative value of $1 invested in the CA Private Equity Index pre-fees based on the most aggressive fee calculation (i.e. no performance hurdle rate), along with the Value Replicating Portfolio under both mark-to-market and hold-to-maturity accounting schemes, while the bottom panel plots the corresponding quarterly drawdown series for the private equity index and the replicating portfolios.

The risks of both replicating portfolios are extreme. Both replicating strategies experience massive drawdowns during the financial crisis of 2008, with both portfolios losing more than 90% of their value relative to their historical peak valuation. This appears to be consistent with secondary market transaction prices of private equity investments at the time. For example, in February 2009, Forbes describes the gap between market transaction values and the asset values reported by some of the private equity firms in the Harvard University endowment, managed by Harvard Management Company (HMC).[10]

> Mendillo did move quickly to deal with the private equity portfolio. One of her first moves at HMC, which she initiated before the markets started to fall in earnest, was to sell between $1 billion to $1.5 billion of Harvard's private equity assets in one of the biggest such sales ever attempted. The high bids on such assets have recently been 60 cents on the dollar, says Cogent Capital, an investment bank that advised Harvard on the sale. Cogent says the big discounts are due to "unrealistic pricing levels at which funds continued to hold their investments" and "fantasy valuations."
>
> …
>
> So what are Harvard's private equity stakes worth? Most private equity investors like Harvard have been waiting for their money managers to finish marking down their assets following a brutal 2008. It is a slow process that lags the public markets by as much as 180 days, says William Frieske, a performance consultant at Northern Trust, which administers endowment accounts.
>
> But one clue to what may be coming can be found in Harvard's own portfolio. It owns units of Conversus Capital, a publicly traded vehicle that holds slices of 210 private equity funds. Conversus has cut its net

[10] http://www.forbes.com/2009/02/20/harvard-endowment-failed-business_harvard.html.

asset value by 21% since last summer to make a "best estimate." Yet stock investors think things are a lot worse. Conversus shares have fallen 67% since June 30 and are trading at a 62% discount to the net asset value.

The estimated market betas of the replicating portfolios are 1.8 and 2.0, which is in line with what is expected for portfolios targeting an initial leverage of 2x and maintaining positions over a period of generally increasing asset values. The massive drawdowns in 2008 for the replicating portfolios are striking. The drawdown based on reported returns for private equity is only -25%. If the replicating portfolios are properly reflecting the risks, how can discretion in marking-to-market reduce the appearance of the risks so dramatically? The results in Panel B suggest a possibility. Panel B repeats the analysis shown in Panel A for the same two replicating portfolios, but under the hold-to-maturity accounting scheme. In other words, all transactions and portfolio holdings are identical under each accounting scheme, only the calculation of the portfolio net asset value is different. Consequently, the annualized geometric mean returns are identical under this accounting scheme.[11] More interestingly, the annualized volatility, the worst drawdown, and the market beta indicate that these portfolios have very little risk relative to what is estimated from the well-marked portfolios. The market betas for both replicating portfolios are statistically indistinguishable from zero, whereas they were essentially 2.0 with market value accounting. Remarkably, the worst drawdowns for the replicating portfolios with book value accounting are only -27% and -32%, whereas the identical portfolios with market value accounting exceeded -90%. As a consequence of the highly biased risk properties of these portfolios due to the book value accounting scheme applied to portfolios with long holding periods, the resulting measures of risk-adjusted returns suggest highly significant unearned returns, or annualized alpha, averaging 14% for both strategies.

[11] The portfolio is liquidated at market values at the end of the sample so that the actual terminal portfolio values are identical in Panels A and B. This results in identical geometric mean returns across the two accounting schemes.

These results clearly do not establish that private equity returns have distorted risks because of discretion in the marking of the net asset value, but do demonstrate that discretion in marking the portfolio net asset value can eliminate most of the measured risk. The hold-to-maturity accounting scheme used here to proxy for conservative portfolio marking is a legitimate portfolio marking rule that is supported by market regulators for firms that are identified as systematically important for financial stability.

Taken at face value, the results in this section push against the view that private equity adds value relative to what can be earned in public markets. Popular stories suggest that private equity investors add value through operating improvements, preferred access to financial leverage, and improved monitoring and governance. These stories most clearly map into private equity investments delivering higher mean returns than similarly selected public equities held with similar amounts of leverage, but this does not appear to be the case *before fees*. After paying fees, which are estimated to be 3.5% to 5% per year, investors who agree that the risk-match between the private equity index and the two replicating portfolios is appropriate are considerably underperforming the feasible alternative of investing in similar passive replicating portfolios.

IV. Discussion

The potential for highly smoothed returns is the critical feature of the data that will govern how the results are interpreted. Smoothed returns hinder traditional risk adjustment, allowing for highly competing beliefs about private equity performance to be sustained. The private equity investment process – here viewed to be the combination of a specific stock selection criterion, long holding periods, conservative net asset value accounting, leverage, and active management at the portfolio companies – can mostly be reproduced with a passive portfolio strategy. The element that cannot be well reproduced is the active management at the portfolio companies, which necessitates the long holding periods. Concurrently, effective return smoothing, whether intentional or simply consequential, requires relatively long holding periods. This section evaluates how the various elements of the private equity structure contribute to performance, beginning with the robustness of the risk-match of the replicating portfolio strategy.

A. Have the Risks been Properly Matched?

Investors may not view the characteristic-matched and leverage-matched replicating portfolio to appropriately describe the risks of diversified private equity allocations. There will always be some uncertainty around this question. The empirical strategy in the previous section is to select stocks with characteristics shown to be predictive of buyout likelihood and to apply portfolio leverage to reproduce the increased corporate leverage typical in a private equity transaction. Rather than focus on the subtleties of precise factor exposures, the main investigation here is on whether the market betas of the replicating portfolios are plausible.

A reasonable starting point is that the equity market beta for a random firm is 1.0, such that if it is held in a portfolio with 2x leverage, the resulting equity component of the portfolio will have a market beta of 2.0. The average market beta for the sample of public firms taken private over the period 1984 to 2014 at the time of the transaction is 1.1.[12] According to Axelson, Jenkinson, Strömberg, and Weisbach (2013) the average corporate leverage more than doubles as a consequence of a private equity transaction. Together, these facts suggest that the replicating portfolio betas reported in Table 8, which range from 1.8 to 2.0, are likely to be consistent with those of actual private equity investors. These estimates stand in striking contrast to the market betas estimated directly from the Cambridge Associates Private Equity Index, which range from 0.4 to 0.7 depending on whether quarterly or annual returns are used. Moreover, accurate

[12] This is roughly consistent with Frazzini and Pedersen (2013) who find the average beta for firms taken private to be 0.94 using a different procedure for estimating market beta.

estimates of the underlying covariance-based risk measures cannot be recovered with the traditional method of including lagged values of factor returns (Scholes and Williams (1977)). For example, adding six lags of quarterly market excess returns to the excess return regressions and summing the slope coefficients produces a beta estimate of only 0.8. Interestingly, the lower volatility of the smoothed returns associated with the hold-to-maturity accounting scheme suggests that there is strong long-run mean reversion in the portfolio returns over this sample period, as volatility estimates from a smoothed independent and identically distributed series are higher than those from the unsmoothed series.

Another perspective on the market betas for private equity investments can be gleaned from the market betas of PE-backed debt that was relatively actively traded during the financial crisis of 2008. Specifically, I identify all US corporate bonds in the Dealscan Facilities database issued prior to 2008, where the primary purpose is "LBO." Transaction prices and dates for these bonds are collected from Trace. For the sample of LBO-issued bonds with at least one transaction in each time period July 1, 2008 to August 31, 2008 and September 15, 2008 to March 31, 2009, I determine the median pre-crisis bond price and date and the low bond price and date during the crisis period, along with the corresponding S&P 500 Index return. There are 361 LBO-issued bonds with valid returns. The average bond return into the depths of the financial crisis is -39%, while the average calendar-time matched S&P 500 return is -29%. The mean difference of -9.8% has a *t*-statistic of -10.6. The ratio of the mean bond return to the mean market return provides a rough estimate of the PE-backed bond market beta, which equals 1.3. As the junior most claim in the capital structure, the equity of these firms is surely riskier than the associated bonds, consistent with the estimated replicating portfolio market betas, and importantly, highly inconsistent with beta estimates far below 2.

B. Implications for Investors in Private Equity Funds

Two Misconceptions:

There are two claimed benefits of the private equity investment process that outside investors commonly promote that appear to be incorrect. These are (1) the long-term corporate debt used to increase leverage at portfolio companies provides the outside equity investors access to an advantaged form of leverage that allows them to avoid the economic costs associated with margin calls; and (2) holding illiquid assets allows long horizon outside equity investors to earn an illiquidity premium. The key challenge to both of these views is that they should show up in returns, but do not.

The first story involves both lenders and outside equity investors who presumably have a proper understanding of the high systematic risks of private equity. The lenders offer attractive long-term debt terms to portfolio companies because they forecast that loss rates will be relatively low.[13] The outside equity investors want large allocations to value stocks, but view the possibility of margin calls to be economically costly.

Margin calls are viewed to be economically costly because this investment is being considered in isolation. However, these exposures rarely make up the entirety of the investors wealth. Investors in private equity funds are typically well diversified, holding large public stock and bond portfolios in addition to their private equity allocations. Consider an endowment fund with a 20% allocation to private equity and an 80% allocation with a traditional mix of 60% public stocks and 40% bonds (i.e. 20% private equity, 48% public equity, and 32% bonds). The arguments and evidence in this paper suggest that the 20% private equity allocation can be viewed as a levered value stock allocation with systematic risks equivalent to 40% public stocks

[13] There is some evidence supporting this view. The Private Equity Council (2010) finds that during the 2008-2009 recession, PE-backed firms had a default rate of 2.8%, while similar firms had a default rate of 6.2%.

and -20% bonds, resulting in an overall endowment portfolio allocation equivalent to 88% stocks and 12% bonds. Such a portfolio will not be at risk of a margin call. This suggests that the economic costs of margin calls on well-marked portfolios can be effectively avoided for most private equity investors who have the institutional flexibility to manage exposures from the perspective of the overall portfolio.

The notion that one earns an illiquidity premium from private equity investments is explicitly noted by some private equity investors. For example, the Harvard Management Company (HMC) 2013 Annual Report states: "When we invest in private equity, we lock up Harvard's money for multiple years. In exchange for that lock-up we expect to earn returns over time that are in excess of the public markets – an "illiquidity premium."[14] While these investments are in fact illiquid, they do not appear to earn an illiquidity premium, as demonstrated by the passive liquid replicating portfolios having similarly high mean returns.

Underappreciated Costs of the Private Equity Investment Process:

A potentially large cost of the private equity investment process arises from conservativism in the reporting of net asset values. Institutions with large allocations are likely either to have significantly more risk than they realize and/or to have a meaningful internal agency conflict. Conditional on allocating any capital to private equity, investors typically have large allocations (Lerner, Schoar, and Wang (2008)).

To reconcile the large allocation with a proper understanding that the systematic risks are high and that the returns are somewhat lower than what can be earned passively with improved liquidity, it is possible that there are significant institutional constraints that lead sophisticated investment entities to prefer "hidden" or smoothed risks. For example, underfunded pensions

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[14] In 2013, the Harvard endowment allocated 16% of its total portfolio to private equity.

need investments that are expected to deliver high returns, but well-marked portfolios that indicate high risks may not be institutionally feasible. Conservative portfolio marking may allow such allocations to occur. The fee for this service is economically large, including the 3.5% to 5% average all-in management fee, as well as the substantially reduced liquidity of the direct PE investments relative to the risk-matched passive portfolio.

Under the alternative view that the market risk of private equity is falsely viewed to be low (say 0.7, as would be estimated from the annual returns in Table 6) when it is actually high (say 1.5 or 2.0, as argued above), it is likely that the high returns would appear to be compensation for illiquidity. Such an error in the risk assessment of private equity could produce large over-allocations to private equity by some investors who would consequently bear considerably more market risk than appreciated.

Consider again the HMC private equity allocation. In 2013, the Harvard endowment allocated 16% of its total portfolio to private equity, while benchmarking itself against a 60% stock and 40% bond portfolio. In terms of the overall portfolio benchmark market beta, the private equity allocation contributes roughly 1.5 / .6 = 2.5 times as much market exposure, such that the 16% private equity allocation is equivalent to 40% of the total market risk budget.

V. Conclusion

The private equity investment process essentially combines a value stock investment strategy, leverage, long holding periods, conservative portfolio marking, and active investment management. A passive portfolio strategy can effectively mimic each element except for the activism and can usefully be well-marked to accurately measure portfolio risks relative to common investment benchmarks.

I show that private equity investors tend to select relatively small firms with value stock characteristics, including low EBITDA multiples. The EBITDA multiple is a highly effective variable for sourcing a value premium over this sample period. Consequently, buy-and-hold portfolios comprised of firms with characteristics similar to those chosen by private equity funds earn high risk-adjusted returns, exceeding the mean pre-fee private equity return when target portfolio leverage is chosen to match the levered returns accruing inside private equity funds. The measured risks of replicating portfolios are high under traditional market-value based net asset value reporting, with market betas near 2 and the worst time series drawdown in 2008, near -90%. I demonstrate that a hold-to-maturity accounting scheme for measuring portfolio net asset value used to mimic the discretion available to private equity fund managers effectively eliminates the majority of measured risks for the replicating portfolios during this sample period. The results indicate that sophisticated institutional investors appear to significantly overpay for the active portfolio management services associated with private equity investments.

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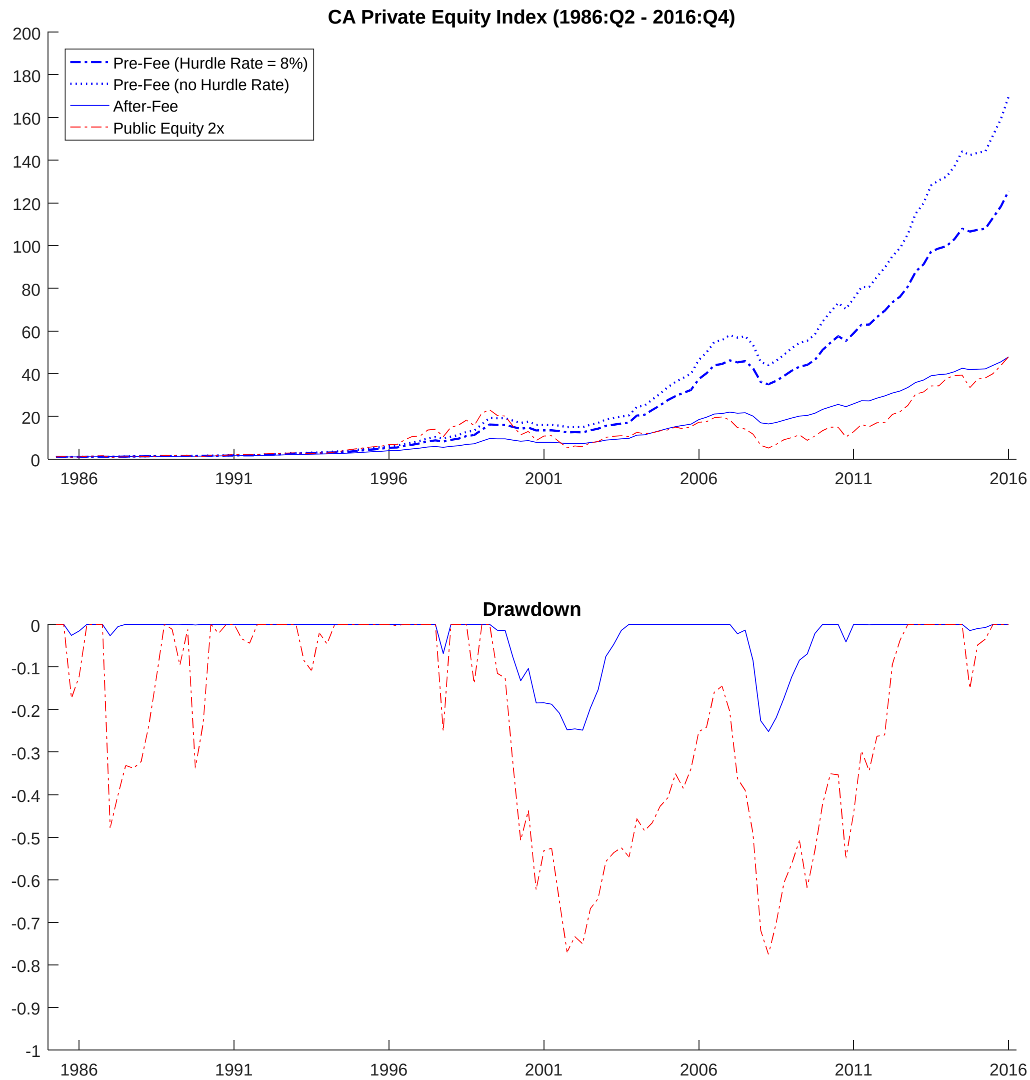

Figure 1. Realized risks and returns of the CA Private Equity Index (1986Q2 to 2016Q4).
The top panel plot the cumulative value of $1 invested in the Cambridge Associates (CA) Private Equity Index net-of-fees (after-fee), and two pre-fee versions of this index, along with a portfolio that levers the aggregate US public stock market two times. The pre-fee private equity returns are calculated by assuming that the observed net-of-fee time series represents the return of a fund that charges a 1% fixed fee plus a 20% performance fee on returns above an annualized hurdle rate of either 8% (Hurdle Rate = 8%) or 0% (no Hurdle Rate) when at its high water mark, both payable quarterly. The bottom panel plots the corresponding quarterly drawdown series for the private equity index and the levered market portfolio. Drawdown is measured as the percentage change in the current index level relative to its prior maximum.

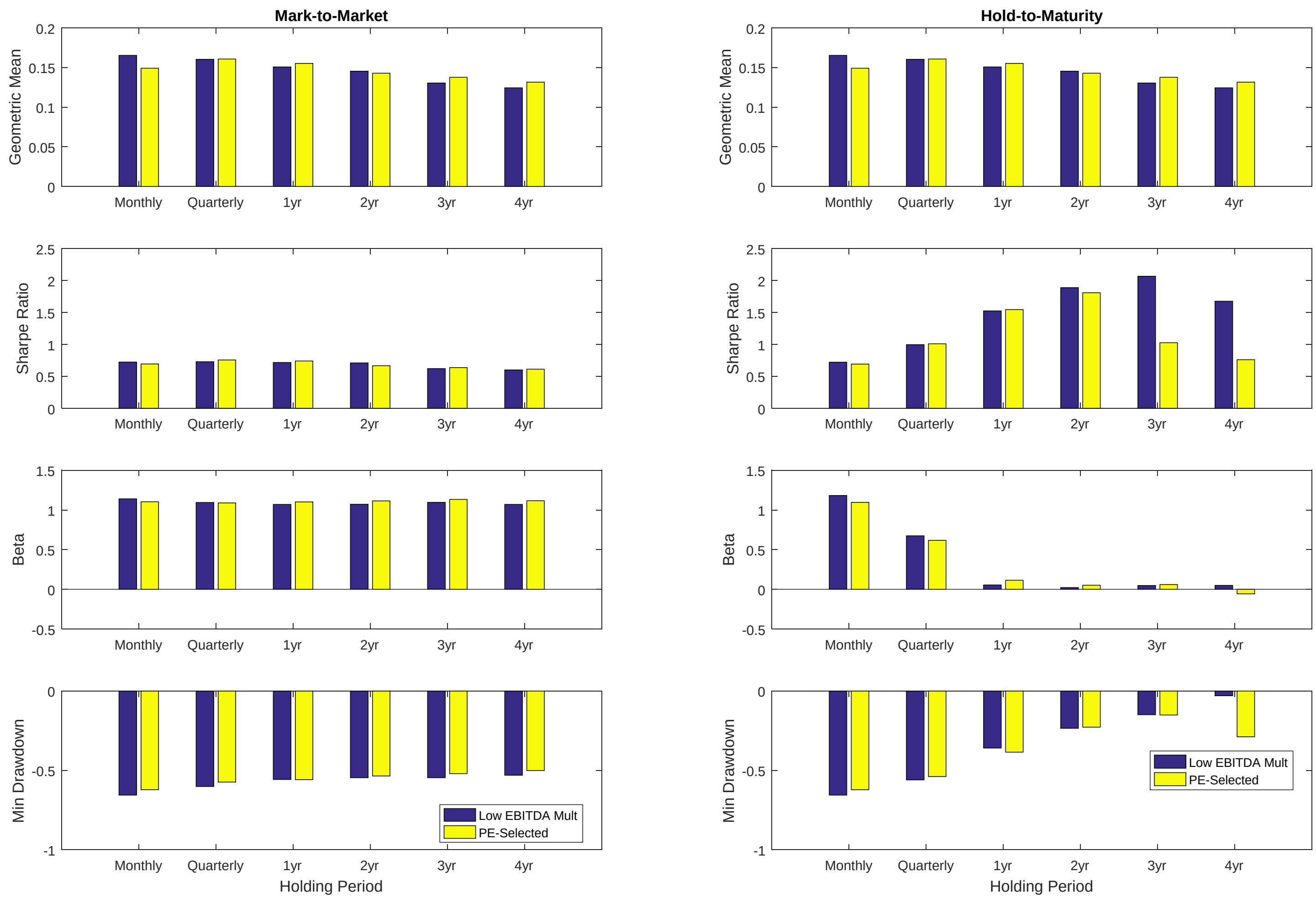

Figure 2. Properties of Unlevered Replicating Portfolios with Various Holding Periods.
This figure displays summary statistics for unlevered replicating portfolios with various holding periods. The graphs displayed in the left column reflect mark-to-market accounting, while the graphs in the right column reflect hold-to-maturity accounting. The Sharpe ratio is calculated as the mean annualized excess return divided by the annualized standard deviation. Beta is the slope coefficient from a regression of portfolio excess returns on the market excess return and alpha is the intercept from this regression. Min Drawdown is the minimum index value relative to its previous maximum value in percent.

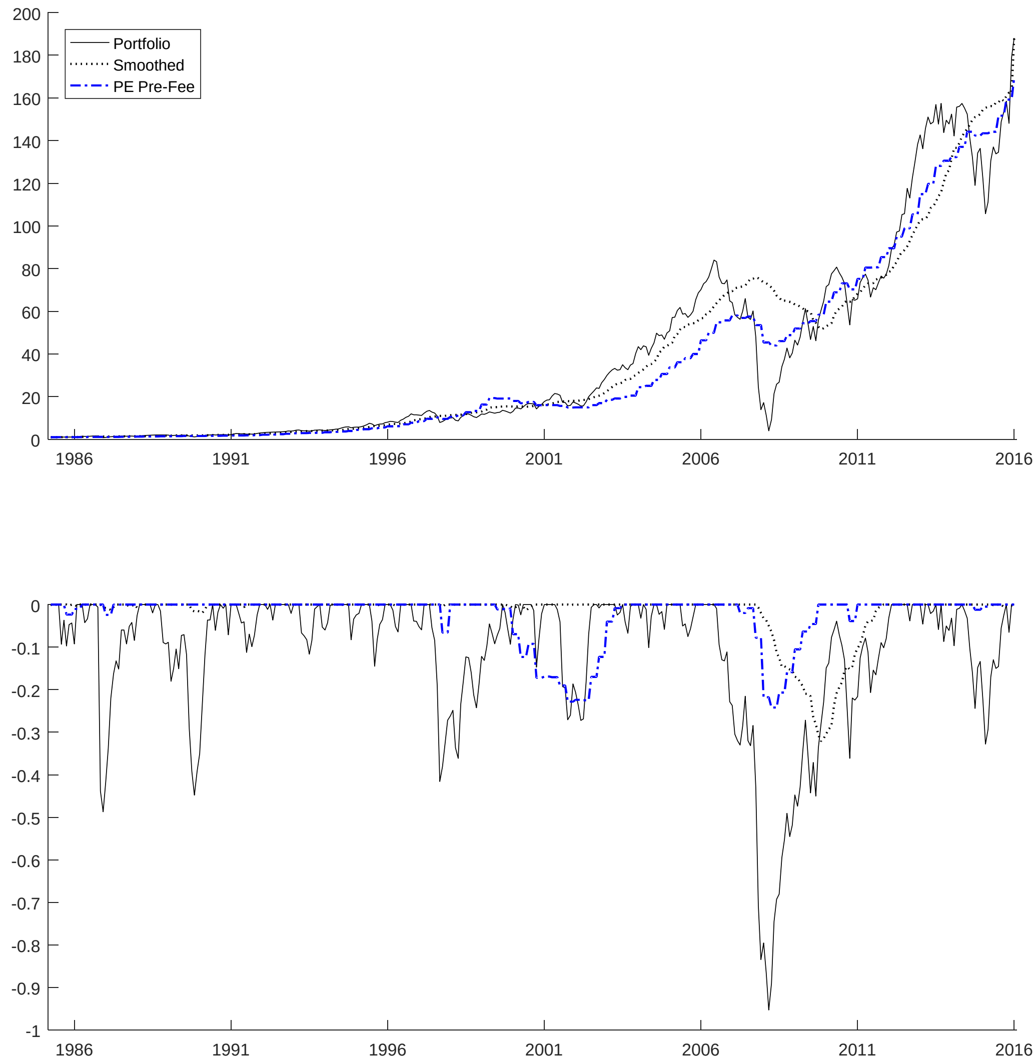

Figure 3. Realized risks and returns of the replicating portfolio (1986Q2 to 2016Q4).
The top panel plot the cumulative value of $1 invested in the Cambridge Associates (CA) Private Equity Index pre-fees, along with the replicating portfolio based on low EBITDA multiple stocks with mark-to-market accounting (Portfolio) and hold-to-maturity accounting (Smoothed). The bottom panel plots the corresponding quarterly drawdown series for the private equity index and the replicating portfolios.

Table I
Regressions Explaining the Selection of Public Equities taken Private (1984-2014)

This table reports the results of regressions of a binary variable indicating a public equity was taken private on various lagged firm characteristics. Panel A reports results from ordinary least squares (OLS) regressions with all coefficients multiplied by 100. Panel B reports results from Logistic regressions. The EBITDA multiple for each firm is calculated as the ratio of firm enterprise value to earnings before interest, taxes, depreciation, and amortization. The firm enterprise value is the sum of long term debt and the market value of equity less cash and marketable securities. Equity market values are assumed to be reported with no delay, while accounting information (long term debt, cash, and EBITDA) are assumed to be reported with a three month delay. Beta is the estimated slope coefficient from a regression using the past 60 months of excess returns (requiring at least 36 valid returns) with a 2% Winsorisation, ME is the market capitalization, BE/ME is the book-to-market equity ratio, ISS is the three-year net equity issuance measure from Daniel and Titman (2006). Profit is the ratio of annual EBITDA to annual Sales. The leverage ratio, D/V, is calculated by dividing long-term debt by the sum of long-term debt and ME. The time period is 1984 to 2014. All specifications include year fixed effects. The OLS regression standard errors are clustered by firm with *t*-statistics reported in parentheses.

Panel A: OLS Regressions

Regression Number	Beta	ln(ME)	ln(Sales)	ln(BE/ME)	ISS	ln(M_{EBITDA})	ln(Profit)	D/V
1	-0.14							
	(-2.75)							
2		-0.18						
		(-10.59)						
3			-0.09					
			(-4.93)					
4				0.38				
				(8.02)				
5					-0.50			
					(-4.41)			
6						-0.46		
						(-7.79)		
7							-1.62	
							(-6.78)	
8								0.40
								(2.18)
9	-0.04	-0.14		0.07	-0.44	-0.32	-1.08	-0.02
	(-0.71)	(-7.55)		(1.16)	(-3.68)	(-4.25)	(-4.15)	(-0.10)
10	-0.04		-0.12	0.12	-0.43	-0.41	-1.76	0.19
	(-0.72)		(-6.34)	(1.86)	(-3.56)	(-5.38)	(-6.89)	(0.89)

Table I - continued

Panel B: Logistic Regressions

Regression Number	Beta	ln(ME)	ln(Sales)	ln(BE/ME)	ISS	ln(M_{EBITDA})	ln(Profit)	D/V
1	-0.17							
	(-2.58)							
2		-0.24						
		(-10.01)						
3			-0.10					
			(-4.23)					
4				0.46				
				(8.62)				
5					-0.73			
					(-5.04)			
6						-0.61		
						(-8.43)		
7							-2.46	
							(-5.62)	
8								0.46
								(2.43)
9	-0.04	-0.17		0.06	-0.59	-0.38	-1.60	0.04
	(-0.61)	(-6.31)		(0.87)	(-4.09)	(-4.70)	(-3.56)	(0.22)
10	-0.04		-0.15	0.12	-0.58	-0.50	-2.54	0.28
	(-0.64)		(-5.39)	(1.61)	(-4.00)	(-6.03)	(-5.54)	(1.30)

Table II
Sub-Sample Logistic Regressions Explaining the Selection of Public Equities taken Private

This table reports the results of logistic regressions of a binary variable indicating a public equity was taken private on various lagged firm characteristics. Panel A reports results for the period 1984 to 1999. Panel B reports results from the period 2000 to 2014. Beta is the estimated slope coefficient from a regression using the past 60 months of excess returns (requiring at least 36 valid returns), ME is the market capitalization, BE/ME is the book-to-market equity ratio, ISS is the three-year net equity issuance measure from Daniel and Titman (2006), and M_{EBITDA} is as defined in Table I. Profit is the ratio of annual EBITDA to annual Sales. The leverage ratio, D/V, is calculated by dividing long-term debt by the sum of long-term debt and ME. All specifications include year fixed effects with *t*-statistics reported in parentheses.

Panel A: Logistic regressions (1984 to 1999)

Regression Number	Beta	ln(ME)	ln(Sales)	ln(BE/ME)	ISS	ln(M_{EBITDA})	ln(Profit)	D/V
1	-0.11							
	(-0.94)							
2		-0.15						
		(-4.18)						
3			-0.05					
			(-1.22)					
4				0.31				
				(3.37)				
5					-0.71			
					(-3.29)			
6						-0.73		
						(-6.12)		
7							-2.13	
							(-3.18)	
8								-0.03
								(-0.08)
9	0.05	-0.10		-0.12	-0.57	-0.71	-2.08	-0.17
	(0.42)	(-2.46)		(-1.07)	(-2.59)	(-4.89)	(-2.92)	(-0.54)
10	0.05		-0.09	-0.10	-0.56	-0.79	-2.63	-0.03
	(0.39)		(-2.13)	(-0.88)	(-2.53)	(-5.43)	(-3.68)	(-0.09)

Table II - continued

Panel B: Logistic regressions (2000 to 2014)

Regression Number	Beta	ln(ME)	ln(Sales)	ln(BE/ME)	ISS	ln(M_{EBITDA})	ln(Profit)	D/V
1	-0.20							
	(-2.46)							
2		-0.30						
		(-9.60)						
3			-0.15					
			(-4.54)					
4				0.54				
				(8.33)				
5					-0.74			
					(-3.83)			
6						-0.54		
						(-5.96)		
7							-2.71	
							(-4.66)	
8								0.81
								(3.32)
9	-0.11	-0.22		0.14	-0.59	-0.22	-1.31	0.26
	(-1.28)	(-6.21)		(1.54)	(-3.06)	(-2.26)	(-2.26)	(0.97)
10	-0.11		-0.19	0.22	-0.59	-0.37	-2.55	0.57
	(-1.29)		(-5.33)	(2.42)	(-3.01)	(-3.64)	(-4.24)	(2.01)

Table III
Average excess and abnormal returns for portfolios formed on EBITDA multiples (1986-2016)

Each month from January 1986 to December 2016, five portfolios are formed from sorts of EBITDA multiples for CRSP stocks. Panel A reports results for equal-weight portfolios and Panel B reports results for value-weight portfolios. Returns are measured in excess of the one-month Treasury bill rate. The EBITDA multiple for each firm is calculated as the ratio of firm enterprise value to earnings before interest, taxes, depreciation, and amortization. The firm enterprise value is the sum of long term debt and the market value of equity less cash and marketable securities. Equity market values are assumed to be reported with no delay, while accounting information (long term debt, cash, and EBITDA) are assumed to be reported with a three month delay. The annualized excess return is calculated as the average monthly excess return times 12. Annualized standard deviation is calculated as the monthly standard deviation times the square root of 12. The Sharpe ratio is calculated by dividing the annualized excess return by the annualized standard deviation. The CAPM alpha is the intercept (times 100) from a time series regression of the monthly portfolio excess return on the CRSP value-weight market portfolio excess return. The Fama-French three-factor alpha (FF 3-factor alpha) is the intercept (times 100) from a time series regression of the monthly portfolio excess return on the CRSP value-weight market portfolio excess return, SMB, and HML. The Fama-French five-factor model adds RMW and CMA and the final specification adds UMD.

Panel A: Equal-weights	Low	2	3	4	High	L-H
Annualized Excess Return	0.182	0.145	0.108	0.081	0.064	0.118
Annualized Standard Deviation	0.206	0.182	0.171	0.176	0.222	0.128
Sharpe Ratio	0.88	0.80	0.63	0.46	0.29	0.93
CAPM alpha (%)	0.87	0.59	0.29	0.05	-0.09	0.96
t-statistic	(3.61)	(2.96)	(1.59)	(0.28)	(-0.31)	(4.93)
FF 3-factor alpha (%)	0.65	0.41	0.14	-0.04	-0.11	0.77
t-statistic	(3.82)	(3.12)	(1.33)	(-0.38)	(-0.67)	(5.22)
FF 5-factor alpha (%)	0.90	0.60	0.30	0.13	0.21	0.69
t-statistic	(6.35)	(6.17)	(4.33)	(1.86)	(2.89)	(4.94)
FF 5-factor plus UMD alpha (%)	1.08	0.73	0.37	0.18	0.23	0.85
t-statistic	(9.65)	(9.75)	(5.99)	(2.66)	(3.18)	(7.27)

Panel B: Value-weights	Low	2	3	4	High	L-H
Annualized Excess Return	0.133	0.098	0.086	0.073	0.079	0.054
Annualized Standard Deviation	0.177	0.147	0.143	0.150	0.189	0.141
Sharpe Ratio	0.75	0.67	0.60	0.49	0.42	0.38
CAPM alpha (%)	0.43	0.22	0.11	-0.03	-0.02	0.45
t-statistic	(2.56)	(1.81)	(1.03)	(-0.24)	(-0.08)	(2.09)
FF 3-factor alpha (%)	0.36	0.17	0.07	-0.01	0.12	0.24
t-statistic	(2.22)	(1.40)	(0.68)	(-0.08)	(0.80)	(1.39)
FF 5-factor alpha (%)	0.53	0.27	0.13	0.07	0.37	0.16
t-statistic	(3.91)	(2.69)	(1.70)	(0.94)	(4.14)	(0.92)
FF 5-factor plus UMD alpha (%)	0.65	0.35	0.17	0.08	0.33	0.32
t-statistic	(5.16)	(3.69)	(2.22)	(1.07)	(3.72)	(2.02)

Table IV
Average excess and abnormal returns for portfolios formed on Predicted PE-Selection (1986-2016)

Each month from January 1986 to December 2016, five portfolios are formed from sorts of predicted values from a PE-Selection model estimated for CRSP stocks. The PE-Selection model is a regression of a binary variable indicating a public equity was taken private on various firm characteristics. The predictive variables are the ln market capitalization, net equity issuance, ln M_{EBITDA}, and ln Profit, as defined in Table 1. The Panel A reports results for equal-weight portfolios and Panel B reports results for value-weight portfolios. Returns are measured in excess of the one-month Treasury bill rate. The annualized excess return is calculated as the average monthly excess return times 12. Annualized standard deviation is calculated as the monthly standard deviation times the square root of 12. The Sharpe ratio is calculated by dividing the annualized excess return by the annualized standard deviation. The CAPM alpha is the intercept (times 100) from a time series regression of the monthly portfolio excess return on the CRSP value-weight market portfolio excess return. The Fama-French three-factor alpha (FF 3-factor alpha) is the intercept (times 100) from a time series regression of the monthly portfolio excess return on the CRSP value-weight market portfolio excess return, SMB, and HML. The Fama-French five-factor model adds RMW and CMA and the final specification adds UMD.

Panel A: Equal-weights	Low	2	3	4	High	H-L
Annualized Excess Return	0.071	0.100	0.112	0.138	0.177	0.106
Annualized Standard Deviation	0.196	0.173	0.182	0.191	0.198	0.127
Sharpe Ratio	0.36	0.58	0.62	0.72	0.90	0.84
CAPM alpha (%)	-0.02	0.20	0.29	0.51	0.89	0.92
t-statistic	(-0.10)	(1.13)	(1.50)	(2.35)	(3.68)	(4.77)
FF 3-factor alpha (%)	-0.03	0.10	0.14	0.31	0.68	0.71
t-statistic	(-0.20)	(0.89)	(1.18)	(2.38)	(4.03)	(4.40)
FF 5-factor alpha (%)	0.25	0.28	0.31	0.50	0.90	0.65
t-statistic	(3.15)	(3.98)	(4.12)	(5.26)	(6.34)	(4.08)
FF 5-factor plus UMD alpha (%)	0.25	0.34	0.40	0.61	1.05	0.80
t-statistic	(3.19)	(5.44)	(6.40)	(7.96)	(8.79)	(5.64)

Panel B: Value-weights	Low	2	3	4	High	H-L
Annualized Excess Return	0.076	0.098	0.105	0.123	0.144	0.069
Annualized Standard Deviation	0.151	0.147	0.170	0.185	0.206	0.146
Sharpe Ratio	0.50	0.67	0.62	0.67	0.70	0.47
CAPM alpha (%)	0.03	0.18	0.18	0.31	0.47	0.44
t-statistic	(0.24)	(1.70)	(1.24)	(1.78)	(2.16)	(2.00)
FF 3-factor alpha (%)	0.13	0.13	0.07	0.15	0.26	0.13
t-statistic	(1.11)	(1.32)	(0.57)	(1.11)	(1.61)	(0.86)
FF 5-factor alpha (%)	0.34	0.21	0.19	0.32	0.46	0.12
t-statistic	(6.53)	(2.85)	(2.09)	(2.97)	(3.43)	(0.77)
FF 5-factor plus UMD alpha (%)	0.32	0.25	0.27	0.42	0.61	0.29
t-statistic	(6.21)	(3.41)	(3.35)	(4.44)	(5.32)	(2.22)

Table V
Fama–MacBeth Regressions of Monthly Returns on Stock Characteristics (1986-2016)

This table reports the results of Fama–MacBeth regressions of monthly returns on lagged stock characteristics. Beta is the estimated slope coefficient from a regression using the past 60 months of excess returns (requiring at least 36 valid returns) with a 2% Winsorisation. ME is the equity market capitalization, BE/ME is the book-to-market equity ratio, ISS is the three-year net equity issuance measure from Daniel and Titman (2006), and M_{EBITDA} is as defined in Table I. Profit is the ratio of annual EBITDA to annual Sales. The leverage ratio, D/V, is calculated by dividing long-term debt by the sum of long-term debt and ME. The time period is January 1986 to December 2016, with *t*-statistics reported in parentheses.

Regression Number	Intercept	Beta	ln(ME)	ln(BE/ME)	ISS	ln(M_{EBITDA})	ln(Profit)	D/V
1	0.85	0.13						
	(4.94)	(0.78)						
2	2.60		-0.12					
	(4.37)		(-3.46)					
3	1.15			0.28				
	(4.14)			(3.35)				
4	1.03				-0.56			
	(3.90)				(-4.29)			
5	2.15					-0.53		
	(6.86)					(-5.71)		
6	0.86						-0.07	
	(3.60)						(-1.02)	
7	0.96							0.20
	(3.60)							(0.69)
8	2.05			0.10		-0.47		
	(7.18)			(1.29)		(-5.45)		
9	2.97	0.23	-0.10	0.00	-0.47	-0.45	-0.03	-0.25
	(6.62)	(1.56)	(-2.69)	(-0.02)	(-5.06)	(-5.65)	(-0.44)	(-1.18)

Table VI
Summary of Private Equity and Capital Market Benchmark Returns

This table reports summary statistics for private equity index returns and various capital market benchmark portfolios, reported in percentage. Excess returns are measured in excess of the one-month U.S. Treasury bill return (riskfree rate). Quarterly net-of-fee returns come from the Cambridge Associates Private Equity Index and cover the period 1986Q2 to 2016Q4. The pre-fee returns are calculated assuming that the observed net-of-fee time series represents the return of a representative fund that charges a 1% flat fee plus a 20% performance fee when at its high water mark, both payable quarterly. The public market returns are those of the value-weight CRSP market index and this index levered two times (2x) paying interest at the riskfree rate. The Value Stock Portfolio contains firms with the lowest 20% of EBITDA multiples as defined earlier. The PE-Selected portfolio contains firms ranked on their predicted value from a PE-Selection model as described earlier. Panel A reports summary statistics from the quarterly returns. Panel B reports summary statistics from the annual returns, which are compounded from the quarterly returns. Panel C reports summary statistics from annual returns measured from July of the previous calendar year through June of the current year (Academic Year). The Yale Endowment Buyout Portfolio return is available annually from 1987 through 2014 based on the Academic Year. The Sharpe ratio is calculated as the mean annualized excess return divided by the annualized standard deviation. Beta is the slope coefficient from a regression of portfolio excess returns on the market excess return and alpha is the intercept from this regression. Drawdown is the minimum index value relative to its previous maximum value in percent. The Variance Ratio measures the ratio of annualized quarterly return variance to annual return variance and the p-value corresponds to an F-test of the equality of variances.

	1-mo US T-bill Return	CRSP VW Market Return	CRSP VW Market Return (L=2x)	Net-of-Fee Private Equity Index	Pre-Fee Private Equity Index (8% Hurdle)	Pre-Fee Private Equity Index (No Hurdle)	Unlevered Value Stock Portfolio (Low M_{EBITDA})	Unlevered PE-Selected Stock Portfolio	Yale Endowment Buyout Portfolio
Panel A: Annualized quarterly returns (1986Q2 – 2016Q4)									
Geometric:	3.29	9.92	13.41	13.41	17.02	18.17	16.73	15.61	
Mean:	3.26	10.98	18.69	13.20	16.56	17.61	18.13	16.95	
Excess:	0.00	7.72	15.44	9.94	13.31	14.35	14.88	13.69	
Std Dev:	1.26	16.67	33.34	9.27	10.57	10.80	21.94	21.00	
Sharpe:	0.00	0.46	0.46	1.07	1.26	1.33	0.68	0.65	
Beta:	0.00	1.00	2.00	0.40	0.43	0.44	1.07	1.00	
Alpha:	0.00	0.00	0.00	6.89	9.97	10.94	6.58	6.00	
Drawdown:	0.00	-45.08	-77.54	-25.19	-24.19	-24.19	-56.39	-55.67	
Panel B: Annual returns (1986 – 2016)									
Mean:	3.29	11.31	19.56	14.08	17.88	19.09	18.79	17.72	
Excess:	0.00	8.02	16.27	10.78	14.59	15.80	15.49	14.43	
Std Dev:	2.55	17.43	35.56	13.14	15.29	15.82	22.24	22.41	
Sharpe:	0.00	0.46	0.46	0.82	0.95	1.00	0.70	0.64	
Beta:	0.00	1.00	2.04	0.56	0.61	0.63	0.87	0.86	
Alpha:	0.00	0.00	-0.05	6.33	9.69	10.73	8.51	7.54	
Min Ret:	0.00	-36.75	-64.69	-22.61	-21.78	-21.78	-39.28	-38.32	
Variance Ratio:	4.08	1.09	1.14	2.01	2.09	2.15	1.03	1.14	
p-value:	0.00	0.35	0.30	0.00	0.00	0.00	0.44	0.30	
Panel C: Academic-year annual returns (1987 – 2014)									
Mean:	3.58	11.13	18.07	14.76	18.80	20.04	19.27	17.30	18.47
Excess:	0.00	7.55	14.49	11.18	15.22	16.46	15.69	13.72	14.89
Std Dev:	2.48	15.27	32.02	14.58	17.16	17.76	17.40	17.56	19.01
Sharpe:	0.00	0.49	0.45	0.77	0.89	0.93	0.90	0.78	0.78
Beta:	0.00	1.00	2.10	0.74	0.84	0.87	0.58	0.60	0.90
Alpha:	0.00	0.00	-1.35	5.58	8.88	9.88	11.28	9.19	8.12
Min Ret:	0.00	-25.20	-51.07	-20.85	-20.00	-20.00	-21.28	-24.47	-25.90

Table VII
Return Properties of Replicating Portfolios with Various Holding Periods

This table reports summary statistics of replicating portfolios with various holding periods under both mark-to-market and hold-to-maturity accounting. The considered holding periods are monthly, quarterly, annual, two-years, three-years, and four-years. Panel A displays results for unlevered portfolios containing stocks with EBITDA multiples in the bottom quintile of publicly traded firms listed in CRSP (value stock portfolio). Panel B reports results for portfolios that are levered 1.5x, meaning that they represent 150% of invested capital in the value stock portfolio and -50% invested in the riskfree asset (one-month US Treasury Bills). Panel C displays results for unlevered portfolios containing stocks with predicted to be PE-Selected based on a PE-Selection model in the top quintile of publicly traded firms listed in CRSP (PE-Selected portfolio). Panel D reports results for portfolios that are levered 1.5x, meaning that they represent 150% of invested capital in the PE-Selected stock portfolio and -50% invested in the riskfree asset (one-month US Treasury Bills). The holding period is the time that individual stocks remain in the portfolio. The Sharpe ratio is the annualized mean excess return divided by the annualized standard deviation. The drawdown is measured each month as the current price level measured as a percentage of the historical maximum price level up to that month. Beta corresponds to the slope from the regression of monthly portfolio excess returns on the corresponding excess return on the value-weight market portfolio, Alpha is the intercept from this regression, and *t*-statistic (reported in parentheses) corresponds to the estimated monthly alpha. The time period is 1986Q2 to 2016Q4.

	Monthly	Quarterly	1yr	2yr	3yr	4yr
Panel A: Low EBITDA Multiple Portfolios Unlevered:						
		Mark-to-Market				
Geometric mean	0.165	0.160	0.151	0.145	0.131	0.124
Mean	0.174	0.167	0.157	0.151	0.139	0.132
Excess return	0.141	0.134	0.124	0.118	0.106	0.100
Standard deviation	0.195	0.185	0.173	0.167	0.171	0.166
Sharpe ratio	0.725	0.728	0.717	0.709	0.621	0.599
Beta	1.140	1.095	1.071	1.072	1.096	1.070
Min drawdown	-0.656	-0.602	-0.557	-0.546	-0.547	-0.531
Alpha	0.052	0.049	0.040	0.034	0.020	0.015
t-statistic	(2.59)	(2.67)	(2.40)	(2.21)	(1.32)	(1.04)
		Hold-to-Maturity				
Geometric mean	0.165	0.160	0.151	0.145	0.131	0.124
Mean	0.174	0.158	0.144	0.138	0.125	0.119
Excess return	0.141	0.125	0.111	0.106	0.092	0.087
Standard deviation	0.196	0.126	0.073	0.056	0.044	0.052
Sharpe ratio	0.722	0.996	1.524	1.889	2.067	1.675
Beta	1.182	0.674	0.055	0.023	0.048	0.051
Min drawdown	-0.656	-0.560	-0.359	-0.235	-0.149	-0.030
Alpha	0.047	0.072	0.104	0.101	0.085	0.080
t-statistic	(2.31)	(3.94)	(7.88)	(10.13)	(10.90)	(8.73)
Panel B: Low EBITDA Multiple Portfolios Levered 1.5x						
		Mark-to-Market				
Geometric mean	0.205	0.204	0.195	0.187	0.167	0.158
Mean	0.238	0.230	0.218	0.209	0.192	0.180
Excess return	0.205	0.197	0.185	0.176	0.159	0.148
Standard deviation	0.302	0.284	0.274	0.263	0.264	0.249
Sharpe ratio	0.680	0.693	0.678	0.670	0.603	0.593
Beta	1.746	1.650	1.629	1.631	1.645	1.578
Min drawdown	-0.858	-0.799	-0.785	-0.788	-0.785	-0.752
Alpha	0.069	0.069	0.057	0.048	0.030	0.023
t-statistic	(2.15)	(2.40)	(2.11)	(1.89)	(1.23)	(1.05)
		Hold-to-Maturity				
Geometric mean	0.205	0.204	0.195	0.187	0.167	0.158
Mean	0.239	0.207	0.185	0.177	0.158	0.151
Excess return	0.206	0.174	0.153	0.144	0.125	0.118
Standard deviation	0.304	0.193	0.108	0.081	0.064	0.074
Sharpe ratio	0.678	0.904	1.413	1.779	1.965	1.590
Beta	1.804	1.039	0.080	0.023	0.063	0.067
Min drawdown	-0.858	-0.761	-0.530	-0.368	-0.235	-0.092
Alpha	0.062	0.092	0.142	0.138	0.116	0.108
t-statistic	(1.91)	(3.29)	(7.27)	(9.55)	(10.29)	(8.21)

	Monthly	Quarterly	1yr	2yr	3yr	4yr
Panel C: PE-Selected Portfolios Unlevered:						
		Mark-to-Market				
Geometric mean	0.149	0.161	0.155	0.143	0.138	0.132
Mean	0.156	0.166	0.161	0.151	0.147	0.141
Excess return	0.124	0.133	0.128	0.119	0.114	0.109
Standard deviation	0.178	0.176	0.173	0.178	0.180	0.177
Sharpe ratio	0.695	0.756	0.741	0.667	0.637	0.612
Beta	1.103	1.089	1.102	1.116	1.133	1.116
Min drawdown	-0.622	-0.574	-0.558	-0.535	-0.521	-0.501
Alpha	0.038	0.049	0.043	0.032	0.027	0.022
t-statistic	(2.00)	(2.59)	(2.46)	(1.86)	(1.55)	(1.32)
		Hold-to-Maturity				
Geometric mean	0.149	0.161	0.155	0.143	0.138	0.132
Mean	0.157	0.158	0.148	0.136	0.134	0.131
Excess return	0.124	0.125	0.115	0.103	0.101	0.098
Standard deviation	0.179	0.124	0.075	0.057	0.098	0.129
Sharpe ratio	0.692	1.010	1.546	1.808	1.026	0.761
Beta	1.096	0.617	0.114	0.053	0.062	-0.057
Min drawdown	-0.622	-0.539	-0.385	-0.227	-0.151	-0.288
Alpha	0.038	0.076	0.105	0.098	0.095	0.101
t-statistic	(1.93)	(4.12)	(7.72)	(9.36)	(5.24)	(4.24)
Panel D: PE-Selected Portfolios Levered 1.5x						
		Mark-to-Market				
Geometric mean	0.186	0.206	0.201	0.184	0.178	0.168
Mean	0.212	0.227	0.222	0.210	0.204	0.193
Excess return	0.179	0.194	0.189	0.177	0.172	0.161
Standard deviation	0.272	0.269	0.268	0.269	0.270	0.259
Sharpe ratio	0.659	0.723	0.707	0.658	0.636	0.621
Beta	1.684	1.635	1.672	1.667	1.683	1.624
Min drawdown	-0.814	-0.765	-0.783	-0.768	-0.749	-0.709
Alpha	0.049	0.068	0.060	0.048	0.041	0.035
t-statistic	(1.66)	(2.35)	(2.21)	(1.84)	(1.60)	(1.45)
		Hold-to-Maturity				
Geometric mean	0.186	0.206	0.201	0.184	0.178	0.168
Mean	0.213	0.207	0.190	0.174	0.175	0.179
Excess return	0.180	0.174	0.158	0.141	0.142	0.146
Standard deviation	0.274	0.188	0.110	0.084	0.164	0.280
Sharpe ratio	0.657	0.927	1.431	1.680	0.865	0.520
Beta	1.667	0.945	0.169	0.071	0.092	-0.133
Min drawdown	-0.813	-0.732	-0.568	-0.355	-0.304	-0.560
Alpha	0.049	0.100	0.143	0.134	0.133	0.154
t-statistic	(1.63)	(3.55)	(7.09)	(8.70)	(4.38)	(2.97)

Table VIII
Return Summary for Replicating Portfolios

This table reports summary statistics and the results of regressions of monthly portfolio excess returns. Panel A displays results under market value accounting and Panel B reports results under book value accounting. Two different replicating portfolios are constructed based on PE-Selected (top quintile of predicted value from PE-Selection model) and based on value stocks (bottom quintile of EBITDA multiples). Both replicating strategies are buy-and-hold portfolios that, each month, select all stocks in the relevant quintile of the monthly distribution (top quintile for predicted PE-Selection and low quintile for EBITDA multiples). Both portfolios target leverage of 2x, defined as the ratio of the current market value of portfolio holdings divided by the current equity capital of the portfolio. All stocks remain in the portfolio for a minimum of six months and up to three years. Both replicating strategies sell individual positions that have realized an annualized holding period return in excess of 50%. The Sharpe ratio is the annualized mean excess return divided by the annualized standard deviation. The drawdown is measured each month as the current price level measured as a percentage of the historical maximum price level up to that month. Alpha corresponds to the intercept from the regression of monthly portfolio excess returns on the corresponding excess return on the value-weight market portfolio, Beta is the slope coefficient from this regression, and R^2 is the R-squared. M_{EBITDA} is as defined in Table I. The time period is 1986Q2 to 2016Q4. *t*-statistics are reported in parentheses.

Panel A: Market value accounting scheme for portfolio net asset value

	PE-Selected Replicating Strategy			Value Replicating Strategy		
Geometric mean	20.0%			18.6%		
Annualized mean return	25.0%			26.5%		
Annualized standard deviation	36.8%			46.3%		
Sharpe ratio	0.68			0.57		
Minimum drawdown	-91.3%			-95.3%		
Average portfolio leverage	1.76x			1.80x		
Terminal discount from MV	0.0%			0.0%		
CAPM regression	Alpha	Beta	R^2	Alpha	Beta	R^2
Coefficient	0.0071	1.77	0.55	0.0070	1.99	0.44
t-statistic	(1.89)	(21.16)		(1.33)	(16.88)	

Panel B: Hold-to-maturity accounting scheme for portfolio net asset value

	PE-Selected Replicating Strategy			Value Replicating Strategy		
Geometric mean	20.0%			18.6%		
Annualized mean return	18.6%			17.3%		
Annualized standard deviation	7.0%			6.3%		
Sharpe ratio	2.66			2.76		
Minimum drawdown	-26.7%			-32.1%		
Average portfolio leverage	1.69x			1.72x		
Terminal discount from MV	0.0%			0.0%		
CAPM regression	Alpha	Beta	R^2	Alpha	Beta	R^2
Coefficient	0.0127	0.01	0.00	0.0119	-0.03	0.00
t-statistic	(11.99)	(0.44)		(12.52)	(-1.22)	