

Commitment vs. Flexibility with Costly Verification*

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Abstract

We introduce costly verification into a general delegation framework. A principal faces an agent who is better informed about the efficient action but biased towards higher actions. An audit verifies the agent's information, but is costly. The principal chooses a permissible action set as a function of the audit decision and result. We show that if the audit cost is small enough, a threshold with an escape clause (TEC) is optimal: the agent can select any action up to a threshold, or request audit and the efficient action if the threshold is sufficiently binding. For higher audit costs, the principal may instead prefer auditing only intermediate actions. However, if the principal cannot commit to inefficient allocations following the audit decision and result, TEC is always optimal. Our results provide a theoretical foundation for the use of TEC in practice, including in capital budgeting in organizations, fiscal policy, and consumption-savings problems.

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1 Introduction

How to optimally delegate decision making is central to a variety of applications. In organizations, headquarters delegate investment decisions to division managers who have superior information about project benefits but also a desire for a larger empire. When designing fiscal policy institutions, society delegates spending decisions to a government which can better assess current social needs but tends to overweigh the value of present spending. In a self-control context, an individual delegates consumption to his ex-post self who learns from taste shocks but may be tempted to overconsume and undersave.

The delegation problem was first formally analyzed by [Holmström \(1977, 1984\)](#) and has since been studied by an extensive literature (e.g., [Melumad and Shibano, 1991](#); [Amador, Werning and Angeletos, 2006](#); [Alonso and Matouschek, 2008](#); [Amador and Bagwell, 2013](#)). The canonical model considers a principal who faces a better informed but biased agent, as in the above applications. Transfers between the parties are infeasible, so the principal simply chooses an allowable set of actions from which the agent can select.¹ Optimal delegation reflects a fundamental tradeoff between commitment and flexibility: on the one hand, commitment is valuable to limit biased decisions by the agent; on the other hand, flexibility is valuable as the efficient action depends on the agent’s private information, and only with discretion can the agent react to this information. A main insight from the literature is that, under weak conditions, this commitment-versus-flexibility tradeoff is resolved by threshold delegation. Specifically, if the agent is biased towards higher actions, the principal optimally allows him to select any action up to a threshold.

Threshold rules are indeed common in applications. However, real-world rules also typically feature an “escape clause.” As reported in survey studies on capital budgeting (e.g., [Ross, 1986](#); [Taggart, 1987](#)) and discussed in [Harris and Raviv \(1996\)](#), division managers in organizations are given a budgetary limit but can ask for a revision of the budget when in need. Managers must provide project documentation for headquarters to review and approve an increase in the division’s capital allocation. Likewise, governments are constrained by deficit limits but legal procedures exist to break these limits under exceptional circumstances. In fact, [Budina et al. \(2012\)](#) find that formal escape clause provisions are included in fiscal rules in many countries, with more recently introduced

¹In various applications, like those described above, transfers are ruled out because of institutional reasons or ethical considerations. See [Alonso and Matouschek \(2008\)](#) for a discussion and other examples where transfers are not feasible.

rules defining clearer trigger points.² As for savings policies, most pension plans impose minimum savings requirements, but these do not apply if an individual can document special conditions.³ Real-world rules make use of escape clauses because agents' private information can often be verified, albeit at a cost. Once an agent's information is ascertained, the appropriate action by the agent can be selected.⁴

In this paper, we introduce costly state verification, as in the seminal work of [Townsend \(1979\)](#), into a general delegation framework. Our main goal is to explore the conditions under which a threshold with an escape clause (TEC), as we observe in practice, is optimal. We define TEC as a rule in which an agent can freely select any action up to a threshold, and if the threshold is sufficiently binding, he can request an audit to be assigned the efficient action by triggering the escape clause.

Costly verification has been shown to play an important role in other contexts, most notably in models of financial contracting and tax collection (e.g., [Townsend, 1979](#); [Gale and Hellwig, 1985](#); [Border and Sobel, 1987](#); [Mookherjee and Png, 1989](#)) but also more recently in various allocation problems (e.g., [Ben-Porath, Dekel and Lipman, 2014](#); [Erlanson and Kleiner, 2015](#)). Existing work, however, focuses on environments in which the objectives of a principal and an agent are in complete disagreement, an assumption that is at odds with the subject of delegation theory. In a delegation problem like the one we study, flexibility is valuable precisely because the principal and agent are in partial agreement, namely the agent's bias is not "extreme." We find that this distinction has key implications for an agent's incentives to be audited and, in turn, for the optimal rule for a principal. Intuitively, an agent with an extreme bias towards higher actions would only pursue an audit to increase his action, whereas one with a moderate bias may pursue an audit to increase or *decrease* his action.

²For instance, until 2010 Germany had in place an escape clause allowing for temporary deviations from its fiscal rule in case of a "distortion of the macroeconomic equilibrium." Since 2010, the clause applies more precisely to "natural disasters or unusual emergency situation which are outside government control and have major impact on the financial position of the government." Parliament must approve the use of the escape clause and an amortization plan for reducing the accumulated deviation. See [Budina et al. \(2012\)](#), p. 42.

³For example, retirement plan distributions are not subject to early withdrawal penalties in a number of circumstances pre-specified by the plan. See <https://www.irs.gov/retirement-plans/plan-participant-employee/retirement-topics-tax-on-early-distributions>.

⁴Other applications include international trade agreements and price delegation in firms. [Beshkar and Bond \(2016\)](#) examine the properties of an optimal trade agreement within the class of caps with escape clauses and provide examples of agreements that take this structure. [Lo et al. \(2016\)](#) study the pricing flexibility afforded to sales people, who are allowed to unilaterally offer their customers discounts up to a certain percentage off the list price but must request approval from a supervisor for larger discounts.

We examine a general principal-agent model with no transfers in which the agent is biased towards higher spending relative to the principal. The agent’s private information, or *type*, concerns the value of spending; a higher agent type corresponds to a higher marginal value of spending for both the principal and the agent. We expand this delegation model by allowing the principal to audit the agent. The principal incurs an additive cost if she conducts an audit, which may also be partially born by the agent. An audit verifies the agent’s type perfectly.⁵

For most of our analysis, we assume that the principal can fully commit to a delegation rule. The problem can be viewed in three steps: first, the principal chooses a mapping from the agent’s audit decision and result to a set of allowable spending; second, the agent decides whether to seek audit, in which case the principal verifies his type; third, the agent chooses a spending level from the allowable set. Formally, a delegation rule is a pair of schedules specifying, for each agent type, whether he is audited or not and his spending level.⁶ A delegation rule is optimal if it maximizes the principal’s expected welfare subject to the incentive compatibility constraint that each agent type prefer his audit assignment and spending level to those of any other type. Specifically, each agent type must prefer his allocation to that of any other type who is not prescribed audit; deviations to types who are audited can be trivially deterred as the principal can punish the agent when the audit reveals that he has deviated.

Our first main result shows that if the cost of audit is sufficiently small, TEC is optimal. Importantly, we also show that auditing all agent types is never optimal; hence, no matter how small the audit cost is, an optimal rule prescribes no audit for some types. The intuition why TEC is optimal is that auditing an upper region of agent types not only allows the principal to improve the spending allocation for these types, but is also an efficient means of imposing discipline on lower agent types who are not audited: these types select from a set of lower spending levels and cannot mimic a higher type who is audited. In fact, the proof of this result rests on showing that any rule with decreasing auditing — prescribing audit for a set of agent types and no audit for a set of higher types — can be dominated. Decreasing auditing is expensive for the principal because it requires incentivizing types in the audit region to seek audit rather than mimic a higher type in a no-audit region above them, and this in turn requires inducing significant overspending in the no-audit region. We show that when the audit

⁵See [Section 6](#) for a discussion of imperfect verification.

⁶We restrict attention to deterministic auditing in our analysis. The case of random auditing is discussed in [Section 6](#).

cost is small enough, a perturbation that audits all types in the decreasing auditing region increases the principal's welfare.

TEC however may not be optimal when the audit cost is relatively higher. Our second main result shows that auditing only an intermediate set of types can yield the principal higher welfare relative to not auditing any type as well as relative to using a TEC rule. The main reason why auditing only intermediate types can dominate not auditing any type is that an intermediate audit region serves to discipline types in the no-audit region below. The main reason why auditing only intermediate types can dominate TEC is that it allows the principal to save on audit costs. We show that these benefits can outweigh the cost of overspending that is needed to incentivize intermediate types to be audited. Thus, when the audit cost is not low (and not high) enough, a rule that involves decreasing auditing can be optimal.

An implication of our construction is that high commitment power from the principal's side is needed whenever decreasing auditing is induced. Consider the aforementioned delegation rule where the principal only audits an intermediate set of types. To implement this rule, the principal commits to an allocation that may be inefficient ex post, following the audit decision and result. In particular, the rule may assign an inefficient spending level after an audit is conducted and the agent's type is verified, both in the case that the agent's seeking audit is "on path" as well as when this audit is part of a deviation. Moreover, the rule may induce an allocation after the agent decides not to seek audit that is inefficient conditional on no audit, i.e. when ignoring the incentives of audited types. What happens if the principal is unable to commit ex ante to these ex-post inefficient allocations?

Our third main result characterizes the optimal rule when the principal's commitment power is limited. In terms of the three-step timing described previously, limited commitment means that the principal now revises the agent's allowable spending set following the agent's audit decision and result. We show that under limited commitment, TEC is optimal whenever auditing is optimal. Indeed, we prove that any incentive compatible rule must have weakly increasing auditing everywhere. The reason is that inducing decreasing auditing requires incentivizing audited types not to deviate and choose a higher spending level in a no-audit region above them, and under limited commitment it also requires incentivizing non-audited types not to seek an audit that guarantees them efficient spending. When unable to fully commit to a rule ex ante, the principal cannot implement the spending levels that would be required to make these deviations unattrac-

tive, and thus decreasing auditing is not feasible. We obtain that in an environment with limited commitment, the delegation rules that we observe in practice coincide with the predictions of the theory.

Related literature. Our paper is related to two literatures. First, we contribute to the literature on optimal delegation and self control, starting with [Holmström \(1977, 1984\)](#). [Melumad and Shibano \(1991\)](#) and [Alonso and Matouschek \(2008\)](#) study delegation under quadratic preferences; [Amador, Werning and Angeletos \(2006\)](#) analyze a model of consumption with hyperbolic preferences; and [Amador and Bagwell \(2013\)](#) consider a general framework which we take as our baseline.⁷ As in this literature, we study a principal-agent environment with no transfers in which the agent is better informed about the efficient action but biased relative to the principal. In contrast to this literature, we allow the principal to verify the agent’s information at a cost. By introducing this additional tool, we are able to explore the conditions under which a threshold with an escape clause is optimal, and how these conditions depend on the extent of the principal’s commitment power.⁸

Second, we contribute to the literature on costly verification, starting with [Townsend \(1979\)](#). Both that paper and others that followed it, including [Gale and Hellwig \(1985\)](#), [Border and Sobel \(1987\)](#), and [Mookherjee and Png \(1989\)](#), analyze settings with transfers, which we rule out. More recently, [Ben-Porath, Dekel and Lipman \(2014\)](#) and [Erlanson and Kleiner \(2015\)](#) consider costly verification in one-good and collective allocation problems without transfers.⁹ Our main departure from this literature (in addition to other differences specific to each paper) is that we study a delegation setting in which we allow for different degrees of bias by the agent relative to the principal. This is also the main distinction with respect to [Harris and Raviv \(1996\)](#), which analyzes costly verification in a delegation model in which the agent always benefits from higher actions.¹⁰ The

⁷See also [Athey, Atkeson and Kehoe \(2005\)](#), [Ambrus and Egorov \(2013, 2015\)](#), and [Halac and Yared \(2014, 2015\)](#).

⁸We study the effects of the principal not being able to commit to not changing the agent’s allowable spending set following the audit decision and result. A different question that a literature on auditing has investigated concerns a principal’s ability to commit to an audit strategy; see, e.g., [Reinganum and Wilde \(1986\)](#), [Banks \(1989\)](#), and [Chatterjee, Morton and Mukherji \(2002\)](#).

⁹See also [Glazer and Rubinstein \(2004, 2006\)](#) and [Mylovannov and Zapechelnyuk \(2014\)](#), which use different verification technologies. More broadly, there is a literature on mechanism design and implementation with evidence, including [Green and Laffont \(1986\)](#), [Bull and Watson \(2007\)](#), [Deneckere and Severinov \(2008\)](#), [Ben-Porath and Lipman \(2012\)](#), and [Kartik and Tercieux \(2012\)](#).

¹⁰In their model, the agent’s marginal utility from a higher action depends on his type but is always positive. Their model also differs from ours in other respects: there are only three agent types, the agent

results in [Harris and Raviv \(1996\)](#) are consistent with our benchmark finding that TEC is optimal for any audit cost if the agent’s bias is extreme. As previously discussed, our interest is in understanding optimal delegation and auditing when the agent’s bias is not extreme: the agent’s most preferred action is higher than the principal’s but not necessarily the highest possible action.¹¹ The principal as a result faces a tradeoff between commitment and flexibility, which introduces new conceptual issues into our mechanism design problem. In fact, the agent’s bias not being extreme implies that decreasing auditing is sometimes optimal in our setting, a feature that does not emerge in this related work.

2 Model

Our baseline model of delegation is the same general principal-agent environment of [Amador and Bagwell \(2013\)](#), where we focus on the case in which the agent’s bias is towards higher actions. We extend this delegation model by allowing for costly state verification, following [Townsend \(1979\)](#).

2.1 Environment

There are a principal and an agent. The state is $\gamma \in \Gamma \equiv [\underline{\gamma}, \bar{\gamma}]$ for $\underline{\gamma} > 0$, with continuous density $f(\gamma) > 0$ for all γ . The corresponding distribution function is $F(\gamma)$. The level of spending is denoted by $\pi \in [\underline{\pi}, \bar{\pi}]$.

The principal’s welfare is $U_P(\gamma, \pi)$, twice continuously differentiable with $\frac{\partial^2 U_P(\gamma, \pi)}{\partial \pi^2} < 0$. We assume that the principal’s optimum, $\pi_P(\gamma) \equiv \arg \max_{\pi} U_P(\gamma, \pi)$, is interior, and refer to it as the *efficient* level of spending. We impose the following single-crossing condition:

$$\frac{\partial^2 U_P(\gamma, \pi)}{\partial \gamma \partial \pi} > 0. \tag{1}$$

Thus, the efficient level of spending is increasing in the state: $\pi'_P(\gamma) > 0$.

The agent’s welfare is $U_A(\gamma, \pi) = \gamma\pi + b(\pi)$, with $b(\pi)$ twice continuously differentiable and $b''(\pi) < 0$. We assume that the agent’s optimum, $\pi_A(\gamma) \equiv \arg \max_{\pi} U_A(\gamma, \pi)$,

receives a non-contingent transfer from the principal, and the principal can choose to audit the agent with an interior probability. [Harris and Raviv \(1998\)](#) extend the setting in [Harris and Raviv \(1996\)](#) to study the allocation of capital not to one project but across multiple projects.

¹¹Hence, the agent may agree or disagree with the principal when she prefers a lower action, whereas in other work the agent would always disagree with the principal about lowering the action.

is interior, and refer to it as the *flexible* level of spending. Note that the agent’s welfare satisfies the single-crossing condition $\frac{\partial^2 U_A(\gamma, \pi)}{\partial \gamma \partial \pi} > 0$.¹² We study the case in which the agent is biased towards higher spending:

$$\frac{\partial U_A(\gamma, \pi)}{\partial \pi} > \frac{\partial U_P(\gamma, \pi)}{\partial \pi}. \quad (2)$$

Thus, conditional on the state, the flexible level of spending always exceeds the efficient level: $\pi_A(\gamma) > \pi_P(\gamma)$ for all $\gamma \in \Gamma$.

The state γ is private information to the agent, i.e. the agent’s *type*. The principal can conduct an audit to perfectly verify γ by paying an additive cost $\phi > 0$. The agent’s cost of audit is $\alpha\phi$ for $\alpha \in [0, 1]$. This formulation allows us to cover situations in which the agent pays no audit cost ($\alpha = 0$) as well as situations in which he pays a cost no larger than the principal’s ($\alpha \in (0, 1]$).¹³

By featuring both a bias and private information by the agent, our environment gives rise to a tradeoff between commitment and flexibility. If the agent were not biased relative to the principal, the principal could implement the efficient level of spending by providing full flexibility to the agent (who would in this case choose $\pi_A(\gamma) = \pi_P(\gamma)$). Similarly, if the state γ were not the agent’s private information, the principal could implement the efficient level of spending by committing the agent to a fully contingent spending plan. In the presence of both a bias and private information, however, the principal cannot implement efficient spending $\pi_P(\gamma)$ for all γ without audit, and she faces a non-trivial tradeoff between commitment and flexibility.

Special cases. As noted in Amador and Bagwell (2013), the model of delegation described above encompasses specific cases commonly studied in the literature. One example is the case of quadratic preferences, examined by Melumad and Shibano (1991) and Alonso and Matouschek (2008). Under quadratic preferences, the principal’s welfare is $-\frac{(\gamma-\pi)^2}{2}$ and the agent’s welfare is $-\frac{(\gamma+\beta-\pi)^2}{2}$, for some $\beta > 0$ representing the agent’s bias. This formulation is equivalent to letting $U_P(\gamma, \pi) = \gamma\pi + b(\pi) - \beta\pi$ and $U_A(\gamma, \pi) = \gamma\pi + b(\pi)$ for $b(\pi) = \beta\pi - \frac{\pi^2}{2}$, and is therefore a special case of our model. We will use the quadratic preferences case to illustrate some of our results.

¹²For both the principal and the agent’s preferences, we will refer to “single-crossing” as the (stronger) supermodularity condition that we have assumed these preferences satisfy.

¹³One can also allow for the agent to pay a higher audit cost than the principal’s (namely let $\alpha > 1$). Our main results continue to hold in this case if the agent’s bias is sufficiently large.

Another example is the model of consumption under hyperbolic preferences, analyzed by [Amador, Werning and Angeletos \(2006\)](#) and [Halac and Yared \(2014, 2015\)](#). The principal's welfare in this case is $\gamma u(c) + w(y - c)$ and the agent's welfare is $\gamma u(c) + \beta w(y - c)$, where u and w are utility functions, c and y represent consumption and exogenous income respectively, and $\beta \in (0, 1)$ captures the degree of present bias by the agent. This formulation is equivalent to letting $U_P(\gamma, \pi) = \gamma\pi + \frac{1}{\beta}b(\pi)$ and $U_A(\gamma, \pi) = \gamma\pi + b(\pi)$ with $\pi = u(c)$ and $b(\pi) = \beta w(y - u^{-1}(\pi))$, and is thus also encompassed by our model.

2.2 Timing

The order of events is as follows:

1. The principal sets a rule, which maps an audit decision and result into an allowable spending set Π .
2. The agent chooses whether or not to seek audit, $a \in \{0, 1\}$, and the principal verifies his type γ if $a = 1$.
3. The agent chooses a spending level π from the allowable set Π .

The above timing assumes that the agent learns his type γ before the principal sets a rule in Step 1. Our analysis is unchanged if instead the agent learns his type after the rule has been set, i.e. at the beginning of Step 2.

2.3 Delegation Rules

Given the game form described above, we can analyze the principal's problem as that of choosing a delegation rule M which consists of a pair of schedules $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$, specifying an audit decision and spending level for each type γ . The principal chooses a rule M to maximize her expected welfare:

$$\max_{\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}} (U_P(\gamma, \pi(\gamma)) - a(\gamma)\phi) f(\gamma) d\gamma \quad (3)$$

subject to

$$U_A(\gamma, \pi(\gamma)) - a(\gamma)\alpha\phi \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\hat{\gamma}) = 0. \quad (4)$$

The objective (3) is the principal's expected welfare under a given rule, taking into account the additive audit costs. The constraint (4) is an incentive compatibility (or truth-telling) constraint: it guarantees that an agent of type γ prefers his assigned audit decision and spending level, $a(\gamma)$ and $\pi(\gamma)$, to a different allocation $a(\hat{\gamma})$ and $\pi(\hat{\gamma})$ for some type $\hat{\gamma}$ who is not audited (that is, with $a(\hat{\gamma}) = 0$). Note that it is sufficient to consider deviations to non-audited types: since a deviation in which an agent of type γ mimics an audited type $\hat{\gamma}$ would be detected by the principal (as an audit reveals the true type) and the principal can arbitrarily punish the agent when she learns that he has deviated (off path), we do not need to consider such a deviation.¹⁴

We also note that the formulation above does not rule out mixed strategies by the agent. If the agent were willing to mix over audit and no audit or over two spending levels, he would be indifferent over these allocations, and thus the principal can select one of these that maximizes her expected welfare.¹⁵ In fact, building on this observation, we can show that our results are not limited to the game form in Section 2.2 but continue to hold when allowing for any indirect mechanism specifying a message space for the agent and a deterministic allocation function to which the principal commits. Such a mechanism induces a game in which the agent sends a message, is either audited or not as a function of the message, and is assigned a spending level as a function of the message and audit result. Appendix B shows that a version of the Revelation Principle in terms of payoffs holds in our setting, implying that to study the optimal deterministic mechanism for the principal, it is without loss to restrict attention to deterministic direct mechanisms (i.e. where the message space coincides with the agent's type space) that induce truthful reporting by the agent, as considered in program (3)-(4) above.

Because there is a continuum of types, it is possible that the problem in (3)-(4) admit multiple solutions that are identical everywhere except for a countable set of types. As a means of selecting the optimum in such a situation, we say that a rule M is *optimal* if it solves (3)-(4) and there is no other solution \tilde{M} , with associated audit and spending schedules $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$, such that

$$U_P(\gamma, \tilde{\pi}(\gamma)) - \tilde{a}(\gamma)\phi \geq U_P(\gamma, \pi(\gamma)) - a(\gamma)\phi \quad (5)$$

¹⁴The principal can punish a deviation of a type γ in which he mimics a type $\hat{\gamma} \neq \gamma$ with $a(\hat{\gamma}) = 1$ by assigning following audit some spending level $\pi(\hat{\gamma}, \gamma)$ such that $U_A(\gamma, \pi(\hat{\gamma}, \gamma)) \leq U_A(\gamma, \pi(\gamma))$. It is clear that such a spending level exists; in fact, setting $\pi(\hat{\gamma}, \gamma) = \pi(\gamma)$ would be a sufficient punishment.

¹⁵While this selection relaxes the principal's problem, it is not used under the optimal rule described in our main result in Proposition 3, which induces a unique best response by the agent. Hence, the result does not rely on selection of equilibria of the game in Section 2.2.

for all γ and strictly for some $\gamma \in \Gamma$.¹⁶

3 No Verification Benchmark

Before analyzing the optimal delegation rule with verification, we review the results of the literature by considering the optimal rule in the absence of verification. Suppose the principal faces the constraint that $a(\gamma) = 0$ for all γ .¹⁷ The problem in (3)-(4) subject to this additional constraint is studied by [Amador and Bagwell \(2013\)](#). To solve this problem, they make the following [Assumption 1](#) on the distribution of γ ; we extend this assumption to any truncation from above, with support $[\underline{\gamma}, \bar{\gamma}']$ for $\bar{\gamma}' \leq \bar{\gamma}$, density $f(\gamma)/F(\bar{\gamma}')$, and distribution function $F(\gamma)/F(\bar{\gamma}')$:

Assumption 1. *Take the distribution of γ truncated from above by $\bar{\gamma}' \leq \bar{\gamma}$. For each such truncated distribution, there exists γ^* such that for $\kappa \equiv \inf_{\{\gamma, \pi\}} \left(\frac{\partial^2 U_P(\gamma, \pi)/\partial^2 \pi}{b''(\pi)} \right)$,*

(i) $\kappa F(\gamma) - \frac{\partial U_P(\gamma, \pi_A(\gamma))}{\partial \pi} f(\gamma)$ is nondecreasing for all $\gamma \in [\underline{\gamma}, \gamma^*]$, and

(ii) $(\gamma - \gamma^*) \kappa \geq \int_{\gamma}^{\bar{\gamma}'} \frac{\partial U_P(\tilde{\gamma}, \pi_A(\gamma^*))}{\partial \pi} \frac{f(\tilde{\gamma})}{1-F(\tilde{\gamma})} d\tilde{\gamma}$ for all $\gamma \in [\gamma^*, \bar{\gamma}']$, with equality at γ^* .

One can verify that for the special cases typically studied in the literature, such as those with quadratic or hyperbolic preferences, [Assumption 1](#) is satisfied under commonly used distribution functions, including exponential, log-normal, and any nondecreasing density.¹⁸ Given [Assumption 1](#), the results in [Amador and Bagwell \(2013\)](#) yield:

Proposition 1 (no verification). *Take the distribution of γ truncated from above by $\bar{\gamma}' \leq \bar{\gamma}$. If the principal is constrained to $a(\gamma) = 0$ for all $\gamma \in [\underline{\gamma}, \bar{\gamma}']$, an optimal rule is a threshold $\gamma^* < \bar{\gamma}'$ such that*

$$\pi(\gamma) = \min \{ \pi_A(\gamma), \pi_A(\gamma^*) \} \quad \text{for } \gamma \in [\underline{\gamma}, \bar{\gamma}'].$$

Under no verification, an optimal rule is a threshold γ^* such that all types $\gamma \leq \gamma^*$ spend at their flexible level and all types $\gamma > \gamma^*$ are bunched at the flexible spending level

¹⁶Although multiple solutions can in principle continue to exist under this condition, this criterion turns out to be sufficient for our characterization.

¹⁷In this case, the constraint (4) becomes $U_A(\gamma, \pi(\gamma)) \geq U_A(\gamma, \pi(\hat{\gamma}))$ for all $\gamma, \hat{\gamma}$.

¹⁸We note also that [Assumption 1](#) on the original, non-truncated distribution implies that the assumption is satisfied for all truncations from above if the conditions in Proposition 2 of [Amador and Bagwell \(2013\)](#) hold.

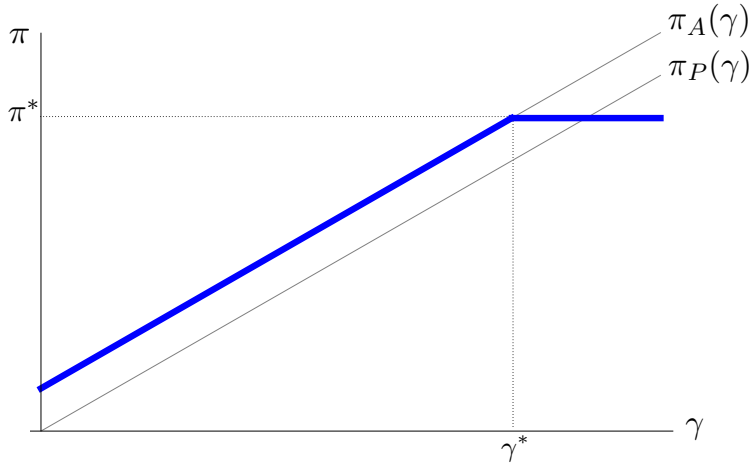


Figure 1: An optimal rule under no verification. The figure is drawn for the quadratic preferences case (see Section 2.1), where we let $\underline{\gamma} = 0.5$, $\bar{\gamma} = 1.5$, $\beta = 0.12$, and $F(\gamma)$ uniform.

of γ^* . The principal can implement this rule by setting a spending limit $\pi^* = \pi_A(\gamma^*)$ and allowing the agent to choose any spending level up to this limit.

Figure 1 illustrates an optimal rule with no verification for the case of quadratic preferences. The level of spending is on the vertical axis and the agent’s type on the horizontal axis. In this simple example, both efficient and flexible spending are increasing linear functions of the state γ , and flexible spending exceeds efficient spending by a constant amount representing the agent’s bias. The rule characterized in Proposition 1 specifies a spending level that coincides with the agent’s flexible level for $\gamma \leq \gamma^*$ and equals $\pi_A(\gamma^*)$ for $\gamma > \gamma^*$.

A key insight behind the result in Proposition 1 is that “holes” are suboptimal. More precisely, the principal can always improve upon a rule as that depicted in Figure 2, which does not allow the agent to choose a spending level $\pi \in [\pi_L, \pi_H]$, for some $\underline{\pi} < \pi_L < \pi_H < \bar{\pi}'$, but allows the agent to choose spending immediately below π_L and immediately above π_H . The hole $[\pi_L, \pi_H]$ implies that an agent of type γ for whom $\pi_A(\gamma) \in (\pi_L, \pi_H)$ is not allowed to spend at his flexible level. Such an agent spends at the lower limit of the hole $\pi_L < \pi_A(\gamma)$ if his type is relatively low, but he spends at the upper limit of the hole $\pi_H > \pi_A(\gamma)$ if his type is higher. Assumption 1 implies that if the principal removes the hole and allows full flexibility over $[\pi_L, \pi_H]$, the benefit of reducing overspending for the types that bunch at π_H would outweigh the (potential) cost of increasing spending for the types that bunch at π_L . Therefore, the principal is better off by closing the hole.

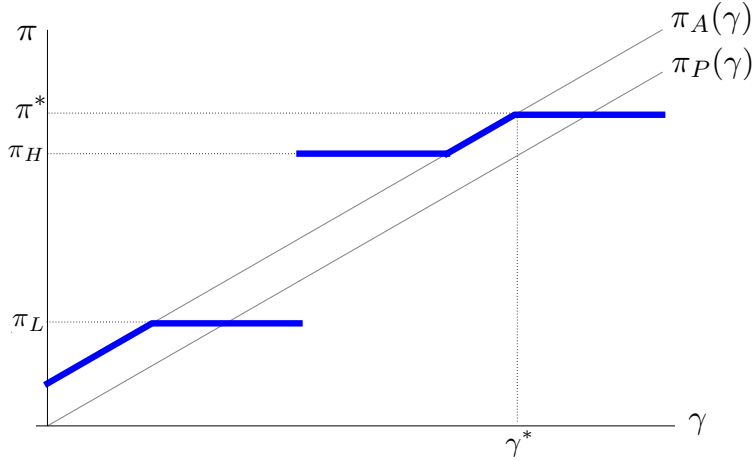


Figure 2: A rule without verification with a hole $[\pi_L, \pi_H]$. Parameters are the same as in Figure 1.

4 Optimal Rule

We now turn to the study of optimal delegation when costly verification is possible. The following class of rules will play a central role in our analysis:

Definition 1. A rule is a *threshold with an escape clause (TEC)* if it consists of $\{\gamma^*, \gamma^{**}\}$ with $\gamma^* < \gamma^{**}$ and $\underline{\gamma} < \gamma^{**} < \bar{\gamma}$ such that

- (i) (*threshold*) If $\gamma \leq \gamma^{**}$, $a(\gamma) = 0$ and $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$, and
- (ii) (*escape clause*) if $\gamma > \gamma^{**}$, $a(\gamma) = 1$ and $\pi(\gamma) = \pi_P(\gamma)$.

Figure 3 illustrates a TEC rule using the quadratic preferences example. Under TEC, types $\gamma \leq \gamma^*$ are not audited and spend at their flexible level, types $\gamma \in (\gamma^*, \gamma^{**}]$ are not audited and are bunched at the flexible spending level of γ^* , and types $\gamma > \gamma^{**}$ are audited and are assigned their efficient spending level. This rule therefore adds an escape clause to the threshold rule that we described in the previous section. In particular, the principal can implement TEC by allowing the agent to choose any spending level up to a limit $\pi^* = \pi_A(\gamma^*)$ or request audit by triggering an escape clause. When the agent is audited, he is assigned his efficient spending level provided that it is above a specified level $\pi^{**} = \pi_P(\gamma^{**})$ (and is otherwise punished).

An important feature of TEC is that the audit function $a(\gamma)$ is weakly increasing, that is, there is no decreasing auditing:

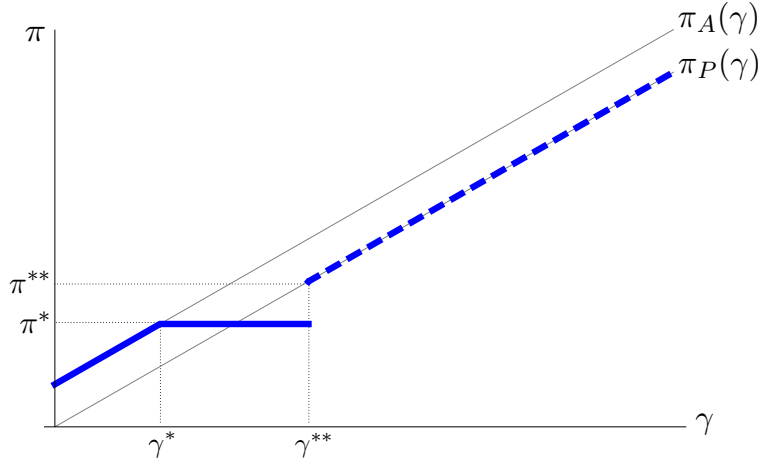


Figure 3: A TEC rule. Parameters are the same as in Figure 1, with $\phi = 0.008$ and $\alpha = 0$. The solid line depicts the allocation of non-audited types; the dashed line corresponds to audited types.

Definition 2. A rule features decreasing auditing at γ' if for all $\varepsilon > 0$ arbitrarily small, either (i) $a(\gamma') < a(\gamma' - \varepsilon)$ or (ii) $a(\gamma') > a(\gamma' + \varepsilon)$. A rule features weakly increasing auditing at γ' if neither (i) nor (ii) holds.

Note that we will refer to decreasing/increasing auditing in the strict sense, and we will clarify whenever we use decreasing/increasing auditing in the weak sense. Figure 4 depicts an example of a rule with decreasing auditing. This rule specifies audit only for types between two interior cutoffs, γ_L and $\gamma_H > \gamma_L$. Types above and below this audit region are not audited, and hence the rule features decreasing auditing at γ_H . We will return to this example in Section 4.3.

Another feature of TEC is that it specifies audit for some agent types but not for all, i.e. the principal conducts an audit only when the agent triggers the escape clause. We begin by showing in Section 4.1 that inducing no audit for some types is in fact a property of any optimal rule. As a result, we show that to identify conditions under which TEC is optimal, it is sufficient to find conditions under which decreasing auditing is suboptimal. We study a simple extreme bias case in Section 4.2 and provide a characterization for our general setting in Section 4.3.

4.1 No Auditing All

A possibility we must rule out to establish the optimality of TEC is that of auditing all agent types. The next lemma shows this is never optimal for the principal:

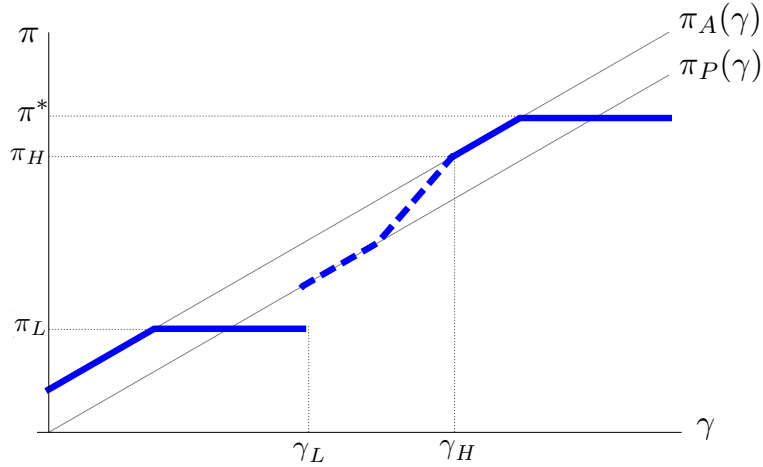


Figure 4: A rule with decreasing auditing. Parameters are the same as in Figure 3. The solid line depicts the allocation of non-audited types; the dashed line corresponds to audited types.

Lemma 1. *A rule with $a(\gamma) = 1$ for all $\gamma \in \Gamma$ is not optimal.*

The logic is simple. Suppose that a rule that audits all types is optimal. Such a rule must trivially assign efficient spending to all types. Now consider a perturbation in which the principal allows the agent to choose $\pi_P(\underline{\gamma})$ without audit. Under the perturbed rule, a set of types $[\underline{\gamma}, \gamma']$, for $\gamma' \geq \underline{\gamma}$, will prefer $\pi_P(\underline{\gamma})$ over being audited and assigned efficient spending. Moreover, since the agent is biased towards higher spending and pays an audit cost no larger than the principal's, it must be that the principal is strictly better off by not auditing these types. Hence, we find that incentivizing low types to not overspend is cheaper than auditing them, and thus auditing all types cannot be optimal.

Given Lemma 1, we can establish:

Corollary 1. *If an optimal rule features auditing that is weakly increasing everywhere, then TEC is optimal.*

Since auditing all agent types is suboptimal, an optimal rule with auditing that is weakly increasing everywhere must feature a no-audit region followed by an audit region, i.e. there must be a type γ^{**} such that $a(\gamma) = 0$ for $\gamma < \gamma^{**}$ and $a(\gamma) = 1$ for $\gamma > \gamma^{**}$. Conditional on the agent's type being in the no-audit region, an optimal rule is a threshold $\gamma^* < \gamma^{**}$ (by Proposition 1), and conditional on the agent's type being in the audit region, an optimal rule assigns efficient spending to all types. To prove Corollary 1, we show that the rule that results from optimizing over each region separately is incentive compatible,

and thus optimal, over the whole set of types. Specifically, we show that an optimal rule conditional on no-audit sets a maximum allowable spending level $\pi_A(\gamma^*) \leq \pi_P(\gamma^{**})$, and by optimality of γ^{**} the principal prefers to pay the cost of auditing type $\gamma > \gamma^{**}$ to assign him $\pi_P(\gamma)$ rather than bunch him at $\pi_A(\gamma^*)$. Since the agent is biased towards higher spending and pays an audit cost no larger than the principal's, it follows that types $\gamma > \gamma^{**}$ also prefer to be audited rather than deviating to $\pi_A(\gamma^*)$. Therefore, the resulting rule is incentive compatible and thus optimal, and it is TEC.

4.2 Extreme Bias

Suppose $b(\pi) = 0$ for all $\pi \in [\underline{\pi}, \bar{\pi}]$, so that the agent's welfare is simply $U_A(\gamma, \pi) = \gamma\pi$. We call this an extreme bias case because the agent always prefers higher levels of spending: the agent's flexible spending level is $\pi_A(\gamma) = \bar{\pi}$ for all $\gamma \in \Gamma$.¹⁹ This is analogous to what is assumed in other models of costly verification, including the seminal work of [Townsend \(1979\)](#) and more recent contributions such as [Ben-Porath, Dekel and Lipman \(2014\)](#). It is also the assumption that is maintained in the delegation model of [Harris and Raviv \(1996, 1998\)](#).

An extreme bias implies that if the agent is not audited, he will choose the highest allowable level of spending, regardless of his type. Moreover, the agent will seek an audit only if that allows him to spend *more* than under no audit. As a result, the analysis is significantly simplified. The only incentive compatible rule for an agent with an extreme bias involves bunching all non-audited types at one spending level; that is, flexibility has no value in this setting. Furthermore, any agent type that is audited must be assigned a higher spending level than that at which non-audited types are bunched. These observations yield:

Proposition 2 (extreme bias). *Suppose $b(\pi) = 0$ for all $\pi \in [\underline{\pi}, \bar{\pi}]$. Then if auditing is optimal, TEC is optimal.*

When the agent's bias relative to the principal is extreme and auditing some types is optimal, an optimal rule is TEC, with non-audited types $\gamma \leq \gamma^{**}$ bunched and awarded no flexibility and audited types $\gamma > \gamma^{**}$ spending at their efficient level. The optimality of TEC follows from the optimality of weakly increasing auditing. Suppose for the purpose of contradiction that an optimal rule featured decreasing auditing. Take γ' to be a marginal

¹⁹As assumed in [Section 2.1](#), we are primarily interested in the case in which $\pi_A(\gamma)$ is interior rather than a corner; however, we find it is instructive to study this corner case first.

non-audited type splitting an audit region and a higher no-audit region, i.e. with $a(\gamma') = 0$ and $a(\gamma' - \varepsilon) = 1$ for $\varepsilon > 0$ arbitrarily small. Let $\pi_A(\gamma^*)$ be the level of spending at which non-audited types are bunched. The optimality of auditing $\gamma' - \varepsilon$ implies

$$U_P(\gamma' - \varepsilon, \pi(\gamma' - \varepsilon)) - U_P(\gamma' - \varepsilon, \pi_A(\gamma^*)) \geq \phi, \quad (6)$$

where, as noted, incentive compatibility requires $\pi(\gamma' - \varepsilon) \geq \pi_A(\gamma^*)$, and since $\phi > 0$, (6) yields $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$. The optimality of not auditing γ' then implies

$$U_P(\gamma', \pi(\gamma' - \varepsilon)) - U_P(\gamma', \pi_A(\gamma^*)) \leq \phi. \quad (7)$$

However, (6) and (7) together with $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$ violate the single-crossing condition (1), yielding a contradiction. Intuitively, the principal can improve upon a rule with decreasing auditing by auditing a higher agent type instead of a lower type, as the marginal benefit of letting the higher type spend more is higher. Note that such a perturbation is always incentive compatible for the agent because all non-audited types are bunched at the same spending level $\pi_A(\gamma^*)$, which (by incentive compatibility) is lower than the spending level assigned to any audited type. This feature is of course due to the agent's bias being extreme.

4.3 Optimal Rule with Verification

We next study the optimal rule with verification in our general setting in which the agent's bias is not extreme. To this end, it is useful to consider a relaxed version of the problem in (3)-(4), in which we assume that the agent pays no audit cost ($\alpha = 0$):

$$\max_{\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}} (U_P(\gamma, \pi(\gamma)) - a(\gamma)\phi) f(\gamma) d\gamma \quad (8)$$

subject to

$$U_A(\gamma, \pi(\gamma)) \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\hat{\gamma}) = 0. \quad (9)$$

Since the original incentive compatibility constraint (4) is tighter than the relaxed constraint (9), if a solution to (8)-(9) satisfies (4), then it is also a solution to the problem in (3)-(4). Furthermore, we can show that if a solution to (8)-(9) is TEC, then it will indeed satisfy (4), implying:

Lemma 2. *If a TEC rule is a solution to (8)-(9), then it is also a solution to (3)-(4).*

To verify that a TEC rule $\{\gamma^*, \gamma^{**}\}$ that solves (8)-(9) satisfies the original constraint (4), we must check that an agent of type $\gamma > \gamma^{**}$ would prefer to pay the audit cost $\alpha\phi$ and spend at his efficient level $\pi_P(\gamma)$ rather than pay no audit cost and choose the threshold flexible spending level $\pi_A(\gamma^*)$. Now TEC being a solution to (8)-(9) implies that the principal prefers auditing such an agent type γ to assign him $\pi_P(\gamma)$ rather than bunching this type at $\pi_A(\gamma^*)$, where $\pi_A(\gamma^*) \leq \pi_P(\gamma)$ for all $\gamma > \gamma^{**}$. Since the agent is biased towards higher spending and pays an audit cost no larger than the principal's, the optimality of auditing γ for the principal therefore yields that auditing γ is incentive compatible for the agent. This is the logic behind Lemma 2, and it implies that in order to prove the optimality of TEC, it is without loss to focus on the relaxed problem in (8)-(9).²⁰ We thus analyze this problem for the remainder of this section.

The following two lemmas establish useful properties of any solution:

Lemma 3. *If a solution to (8)-(9) prescribes audit for type γ , it has $\pi_P(\gamma) \leq \pi(\gamma) \leq \pi_A(\gamma)$. If (9) does not bind for γ , then $\pi(\gamma) = \pi_P(\gamma)$.*

Lemma 4. *In any solution to (8)-(9), $\pi(\gamma)$ is weakly increasing.*

Lemma 3 states that if a type γ is audited, his assigned spending level is (weakly) between his efficient level and his flexible level. The argument is straightforward. If assigned spending for type γ is either below efficient or above flexible, then either increasing or decreasing this spending, respectively, makes the principal better off and is incentive compatible for the agent. Since the principal maximizes her expected welfare subject to incentive compatibility, if an audited type's incentive compatibility constraint is slack, the principal assigns this type efficient spending.

Lemma 4 shows that the principal assigns a spending level that is weakly increasing in the agent's type γ . When comparing two agent types that are not audited, the result naturally follows from incentive compatibility: a type γ cannot be assigned higher spending than a higher type $\gamma' > \gamma$, as at least one of them would have an incentive to deviate given that preferences satisfy single-crossing. When comparing two agent types such that (at least) one of them is audited, the result follows from optimality: if a type

²⁰We maintain our optimality condition (5), so that to prove the optimality of TEC, it is sufficient to show that TEC solves (8)-(9) and no other solution provides the principal weakly larger welfare from each type γ and strictly larger from some type γ .

γ is assigned higher spending than a higher type $\gamma' > \gamma$, the principal can improve welfare by swapping these types' spending levels and audit assignments, and if incentive compatibility was initially satisfied, it will continue to be satisfied after the swap, given single-crossing.

To show the optimality of TEC, we must rule out decreasing auditing, namely a situation in which a set of types is audited and a set of higher types is not audited. Using the two lemmas above, we show that a rule with decreasing auditing must induce significant overspending, limiting the welfare that this rule can provide to the principal:

Lemma 5. *Suppose a solution to (8)-(9) features decreasing auditing at $\gamma' < \bar{\gamma}$. Then the solution satisfies*

$$\frac{\int_{\gamma'}^{\bar{\gamma}} [U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))] f(\gamma) d\gamma}{1 - F(\gamma')} \geq \eta(\gamma') \quad (10)$$

for

$$\eta(\gamma') = \frac{\int_{\gamma'}^{\min\{\pi_P^{-1}(\pi_A(\gamma')), \bar{\gamma}\}} [U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi_A(\gamma))] f(\gamma) d\gamma}{1 - F(\gamma')} > 0. \quad (11)$$

This lemma shows that under a rule featuring decreasing auditing at $\gamma' < \bar{\gamma}$, the principal's expected welfare in the region above γ' is strictly bounded away from that achieved under efficient spending. The reason is that such a rule must induce strict overspending by a positive mass of types $\gamma \geq \gamma'$. To see the intuition, let $a(\gamma') = 1$ and thus $a(\gamma' + \varepsilon) = 0$ for $\varepsilon > 0$ arbitrarily small. Note that the principal must incentivize types in the audit region below γ' to seek audit rather than deviate and mimic a type in the no-audit region above γ' . By Lemma 4, all types above γ' spend more than types below γ' , and by Lemma 3, audited types γ spend no more than their flexible amount $\pi_A(\gamma)$. Thus, for types in the audit region below γ' not to deviate to the no-audit region above γ' , we must have $\pi(\gamma' + \varepsilon) \geq \pi_A(\gamma')$. Given that by Lemma 4 all types $\gamma > \gamma'$ spend above $\pi(\gamma' + \varepsilon)$, it follows that all types $\gamma \in (\gamma', \min\{\pi_P^{-1}(\pi_A(\gamma')), \bar{\gamma}\})$ spend above $\pi_A(\gamma') > \pi_P(\gamma)$, yielding the bound in (11).

The properties shown in Lemma 3-Lemma 5 are satisfied in the examples of Figure 3 and Figure 4. Importantly, Lemma 5 shows that the principal's expected welfare under a rule featuring decreasing auditing is bounded away from efficient welfare by a bound that is independent of the audit cost ϕ . This allows us to establish our first main result:

Proposition 3 (low audit cost). *Let $\bar{\phi} \equiv \min_{\gamma \in \Gamma} \eta(\gamma) > 0$. If $\phi < \bar{\phi}$ and auditing is optimal, TEC is optimal.*

Recall that by [Corollary 1](#), if a rule with auditing that is weakly increasing everywhere is optimal, then TEC is optimal. The proof of [Proposition 3](#) therefore rests on showing that, for any audit cost $\phi < \bar{\phi}$, an optimal rule induces weakly increasing auditing everywhere, i.e. decreasing auditing is suboptimal. To see why this must be true, suppose by contradiction that an optimal rule induces decreasing auditing at some point, and let γ^{**} be the lowest audited type under this rule. We show that the principal can improve upon such a rule by performing a global perturbation: in the perturbed rule, the principal audits all types $\gamma \geq \gamma^{**}$ and assigns them efficient spending, while solving for an optimal rule without verification for types $\gamma < \gamma^{**}$. By [Proposition 1](#), an optimal rule for the no-audit region is a threshold $\gamma^* < \gamma^{**}$, and since $\pi_A(\gamma^*) \leq \pi_P(\gamma^{**})$ (by optimality of γ^*) and $\alpha = 0$, it is easy to verify that the perturbed rule is incentive compatible.

To show that the perturbation strictly raises the principal's welfare, note first that expected welfare conditional on $\gamma < \gamma^{**}$ weakly increases because it is now maximized subject to fewer constraints: under the perturbed rule, types $\gamma < \gamma^{**}$ cannot mimic a type $\hat{\gamma} \geq \gamma^{**}$. Thus, all we need to show is that expected welfare conditional on $\gamma \geq \gamma^{**}$ increases strictly, namely that the (allocative) benefit of auditing these types is strictly greater than the additional auditing cost the principal incurs. Because audited types are assigned efficient spending, the benefit of auditing $\gamma \geq \gamma^{**}$ is weakly positive for all such types. Moreover, note that by the contradiction assumption, there exists a type above γ^{**} at which the original rule features decreasing auditing. Thus, if $\gamma' < \bar{\gamma}$ is the lowest such type, [Lemma 5](#) implies that the benefit of auditing types $\gamma \geq \gamma^{**}$ is bounded from below by $(1 - F(\gamma'))\eta(\gamma')$, where $\eta(\cdot)$ is defined in [\(11\)](#). The claim then follows in this case from the fact that, given $\phi < \bar{\phi}$, the additional cost of auditing types $\gamma \geq \gamma^{**}$ is strictly smaller than $(1 - F(\gamma'))\bar{\phi} = (1 - F(\gamma')) \min_{\gamma \in \Gamma} \eta(\gamma)$, and hence strictly smaller than the benefit of auditing these types. If the lowest type above γ^{**} at which the original rule features decreasing auditing is $\gamma' = \bar{\gamma}$, an analogous argument applies, as in this case the original rule induces strict overspending by $\bar{\gamma}$ and the benefit of auditing this type is no smaller than $\bar{\phi}$.

[Proposition 3](#) implies a positive result: if the principal's cost of audit ϕ is low enough, a TEC rule as we observe in practice is optimal. But what happens if ϕ is higher? Our next result shows that there exist environments and audit costs for which TEC is

suboptimal, even though the principal benefits from auditing some agent types:

Proposition 4 (intermediate audit cost). *There exist $\{U_P, b, f, \phi, \alpha\}$ such that auditing is optimal but TEC is not.*

To prove this result, we construct examples in which auditing only an intermediate range of types $[\gamma_L, \gamma_H]$ dominates both not auditing any type as well as using TEC. The main reason why auditing only intermediate types can dominate not auditing any type is that an intermediate audit region imposes discipline on the no-audit region below. That is, even when the audit cost is high enough that the principal would not benefit from auditing types in $[\gamma_L, \gamma_H]$ only to improve their allocation relative to flexible spending, she may benefit from auditing these types to discipline lower types: with the intermediate audit region, types $\gamma < \gamma_L$ can no longer mimic types in $[\gamma_L, \gamma_H]$. On the other hand, the main reason why auditing only intermediate types can dominate auditing with a TEC rule is that it allows the principal to save on audit costs. Specifically, with intermediate auditing, the principal may be able to impose discipline on types $\gamma < \gamma_L$ without prescribing audit for types $\gamma > \gamma_H$ as she would under a TEC rule; this will be the case if γ_L has no incentive to deviate to mimic a type as high as γ_H . In such a situation, intermediate auditing allows the principal to save on the cost of auditing types above γ_H .

These arguments yield that a rule with decreasing auditing as that depicted in [Figure 4](#) can dominate a no-audit rule as that in [Figure 1](#) and a TEC rule as that in [Figure 3](#), provided that the cost of audit ϕ is not low (nor high) enough. We emphasize that [Proposition 4](#) does not rely on non-uniformity of the principal's objective across types or any other sort of asymmetry; in fact, we prove the result in [Appendix A](#) by constructing examples as those depicted in our figures, with quadratic preferences and a uniform distribution of types.

An interesting implication of our construction is that the principal must have strong commitment power to implement a rule that features decreasing auditing. In particular, take the rule depicted in [Figure 4](#). The principal assigns spending strictly above the efficient level to some agent types $\gamma \in [\gamma_L, \gamma_H]$ who are audited, that is for whom the principal verifies the true type. By doing this, the principal incentivizes those types to be audited: if they were instead assigned efficient spending following audit, they would not seek an audit in the first place. The principal must be committed to allowing this inefficient spending despite her learning the true type of the agent after the audit is conducted. Strong commitment power from the principal is also required to incentivize types $\gamma < \gamma_L$ sufficiently close to γ_L to not seek audit. In the example of [Figure 4](#),

these types are punished by the principal if they seek audit, even though ex post, once the audit is conducted, both the principal and the agent would strictly prefer efficient spending to punishment. Without the threat of punishment, the principal may not be able to prevent an agent of type $\gamma < \gamma_L$ sufficiently close to γ_L from seeking audit, as an efficient allocation following audit would allow this agent to increase his spending.

In practice, principals may not have sufficient commitment power to implement allocations that are inefficient ex post, following an audit. Can headquarters commit to an inefficient budget after verifying the benefits of a manager’s investment project? We explore the implications of limited commitment power in the next section.

5 Limited Commitment

We study a setting in which the principal has limited commitment power. The order of events is as follows:

1. The principal sets a rule, which maps an audit decision and result into an allowable spending set Π .
2. The agent chooses whether or not to seek audit, $a \in \{0, 1\}$, and the principal verifies his type γ if $a = 1$.
3. The principal revises the allowable spending set Π to Π' .
4. The agent chooses a spending level π from the allowable set Π' .

The first two steps are the same as those in our environment of [Section 2](#) with full commitment power. What is new is Step 3: after observing the agent’s audit decision and the result of the audit if one is conducted, the principal now revises the allowable spending set for the agent. Note that this is a rather mild form of limited commitment. In particular, in Step 2 we maintain the assumption that the principal is able to commit to an audit plan, so the agent’s type is verified if and only if the agent requests audit, and in Step 4 we maintain the assumption that the principal is able to commit to allowing the agent to choose freely any spending level from the allowable spending set.²¹ Our problem

²¹As noted in [fn. 8](#), there is a literature that studies auditing when the principal cannot commit to an audit strategy. In many of the applications of our problem, however, we find that there are often institutions ensuring that principals cannot deny an audit once it has been requested. In this sense, the agent can always choose to *trigger* an audit. Lack of commitment in this respect would change the nature of our problem, and so we leave its analysis for future work.

is therefore still one of delegation rather than cheap talk. The only assumption that we relax is about the principal's commitment to not changing the allowable spending set following the audit decision and result; we believe lack of commitment in this respect often shapes delegation rules in the real world.²²

Limited commitment on the side of the principal matters for two reasons. First, conditional on an audit, the principal verifies the agent's true type γ and must necessarily assign the agent the efficient spending level $\pi_P(\gamma)$. This is true both when the agent's seeking audit is on path as well as when this audit decision is part of a deviation. As such, the agent can always choose to be audited to guarantee himself the efficient level of spending. Importantly, this means that all agent types who are not audited must weakly prefer their allocation under no audit to being audited and receiving efficient spending. A second implication of limited commitment is that conditional on no audit, the principal chooses an allocation that is optimal for the non-audited types. More precisely, when the agent chooses not to seek audit, the principal assigns spending taking into account the distribution of non-audited types and ignoring the incentives of audited types.

Limited commitment as a result implies certain conditions that any incentive compatible rule must satisfy. In what follows, we restrict attention to strategies that specify piecewise continuous mappings $\{a(\gamma), \pi(\gamma)\}$.

Lemma 6. *Under limited commitment, any incentive compatible rule must satisfy:*

(i) *If there is decreasing auditing at γ_H , then*

$$U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha\phi = U_A(\gamma_H, \pi(\gamma_H)), \quad (12)$$

where $\pi(\gamma_H) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$ if $a(\gamma_H) = 1$. Moreover,

$$\pi(\gamma_H) > \pi_A(\gamma_H). \quad (13)$$

(ii) *If there is increasing auditing at γ_L , then*

$$U_A(\gamma_L, \pi_P(\gamma_L)) - \alpha\phi = U_A(\gamma_L, \pi(\gamma_L)), \quad (14)$$

where $\pi(\gamma_L) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon)$ if $a(\gamma_L) = 1$.

²²It is worth noting that our results in this section are not limited to the exact game described above; analogous to our claims in [Section 2.3](#), our findings can be extended to variations of this game that allow messages between principal and agent (while maintaining our assumptions on the principal's limited commitment). We also emphasize that throughout this section, we maintain our optimality condition (5), so that a rule is optimal if it maximizes the principal's expected welfare and no other rule provides weakly larger welfare from each type γ and strictly larger welfare from some type γ .

Part (i) shows that if γ_H splits an audit region from a higher no-audit region, then γ_H must be indifferent between being audited and spending at the efficient level versus not being audited and spending at $\pi(\gamma_H)$ as allowed in the no-audit region above this type. Likewise, part (ii) shows that if γ_L splits a no-audit region from a higher audit region, then γ_L must be indifferent between being audited and spending at the efficient level versus not being audited and spending at $\pi(\gamma_L)$ as allowed in the no-audit region below this type. This result follows from the fact that a principal with limited commitment power assigns efficient spending whenever the agent seeks an audit, both on and off path. Therefore, if there is a point at which an audit region either ends or starts, the marginal audited type at such point must weakly prefer audit with efficient spending to no audit, and the marginal non-audited type must weakly prefer no audit to audit with efficient spending. The marginal type must thus be indifferent.

Lemma 6 also shows that for type γ_H as defined in the lemma, an incentive compatible rule must set $\pi(\gamma_H) > \pi_A(\gamma_H)$. This is required to make γ_H indifferent between audit and no audit: if this inequality is not satisfied, the marginal audited type would instead prefer to deviate and not seek an audit.

For the remainder of our analysis, we require:

Assumption 2. *If*

$$R(\gamma, \pi_H) \equiv U_A(\gamma, \pi_P(\gamma)) - \alpha\phi - U_A(\gamma, \pi_H) \geq 0$$

for $\pi_H > \pi_P(\gamma)$, then

$$R(\gamma', \pi_H) > 0 \text{ for all } \gamma' < \gamma.$$

This is a single-crossing property: we assume that if a type γ weakly prefers audit with efficient spending $\pi_P(\gamma)$ to no audit with a higher spending level $\pi_H > \pi_P(\gamma)$, then any lower type $\gamma' < \gamma$ strictly prefers audit with efficient spending $\pi_P(\gamma')$ to no audit with the higher spending level π_H .²³ A sufficient condition for this assumption is that if $R(\gamma, \pi_H) \geq 0$ for some $\gamma \in \Gamma$ and $\pi_H > \pi_P(\gamma)$, then $U_A(\gamma, \pi_P(\gamma))$ be convex in γ for all $\gamma \in \Gamma$; it can be established that in this case $U_A(\gamma, \pi_P(\gamma))$ is convex somewhere, and a sufficient condition is that it be convex everywhere. This convexity assumption is in fact

²³Our single-crossing conditions on preferences imply that if a type γ weakly prefers audit with efficient spending $\pi_P(\gamma)$ to no audit with a lower spending level $\pi_L < \pi_P(\gamma)$, then any higher type $\gamma' > \gamma$ strictly prefers audit with efficient spending $\pi_P(\gamma')$ to no audit with the lower spending level π_L . **Assumption 2** requires that this property be maintained in the opposite direction as well.

satisfied in the cases commonly studied in the literature, such as those with quadratic preferences or with hyperbolic preferences under common parameterizations.²⁴

Given [Assumption 2](#), we obtain:

Proposition 5 (limited commitment). *Under limited commitment, any incentive compatible rule features weakly increasing auditing everywhere. Moreover, if auditing is optimal, TEC is optimal.*

Under limited commitment, decreasing auditing is not incentive compatible for the principal. As we discussed in [Section 4.3](#), decreasing auditing requires that the principal commit to allowing the agent to spend at a level that is inefficient ex post, following the agent’s audit decision and result. Without this commitment, the principal cannot induce decreasing auditing, and hence any incentive compatible rule must feature weakly increasing auditing at all types $\gamma \in \Gamma$. Analogous arguments to those behind [Lemma 1](#) and [Corollary 1](#) in our full-commitment environment then imply that if auditing some agent types is optimal, a TEC rule is optimal.

A sketch of the proof of [Proposition 5](#) is as follows. Suppose by contradiction that there is an incentive compatible rule that induces decreasing auditing, with γ_H being a type splitting an audit region from a higher no-audit region. Given limited commitment, audited types immediately below γ_H are assigned efficient spending, and types γ immediately above γ_H spend at a level $\pi_H > \pi_A(\gamma)$ that makes γ_H indifferent between audit and no audit (cf. [Lemma 6](#)). This means that types immediately above γ_H must be strictly overspending, in fact spending above their flexible level. The heart of the proof is showing that the principal cannot commit to allowing such overspending.

It is clear that conditional on the agent not seeking an audit, the principal would like to reduce the overspending by types immediately above γ_H . Reducing this overspending is ex post incentive compatible for these types: having chosen no audit, types $\gamma > \gamma_H$ would prefer $\pi_A(\gamma)$ to $\pi_H > \pi_A(\gamma)$. Hence, the only reason the principal would not reduce the overspending above γ_H once the agent chooses no audit is if doing so would violate incentive compatibility for some other non-audited type. Such a non-audited type must be below γ_H ; specifically, there must exist a type $\gamma_L < \gamma_H$ who is not audited and is exactly indifferent between his assigned spending level, call it π_L , and the spending level $\pi_H > \pi_L$. In fact, because of single-crossing, this type must be the marginal type

²⁴For example, in the hyperbolic preferences case described in [Section 2.1](#), $U_A(\gamma, \pi_P(\gamma))$ will be convex in γ if the utility functions for present and future consumption are the same and either exponential or CRRA with a coefficient weakly greater than 1.

right below the audit region that ends at γ_H , i.e. the rule must induce audit for types $\gamma \in [\gamma_L, \gamma_H]$ and no audit for types immediately below and above this set. An example is the rule depicted in [Figure 4](#).

Now if the principal induces such an interior audit region $[\gamma_L, \gamma_H]$, then by [Lemma 6](#) type γ_L must be indifferent between no audit with spending π_L and audit with efficient spending. Since we have defined γ_L as being indifferent between spending at π_L and spending at π_H under no audit, by transitivity, we obtain that γ_L must be indifferent between no audit with spending π_H and audit with efficient spending. However, recall that type γ_H is also indifferent between no audit with spending π_H and audit with efficient spending. Hence, by [Assumption 2](#), $\gamma_L < \gamma_H$ cannot hold,²⁵ and we must have $\gamma_L = \gamma_H$. This means that the principal audits a single type at this point who is indifferent between audit with efficient spending, no audit with higher spending at π_H , and no audit with lower spending at π_L . Conditional on no audit, this is thus an allocation in which the agent faces a hole $[\pi_L, \pi_H]$, namely he is not allowed to choose spending in this set but can choose spending immediately below and above this set. But our analysis in [Section 3](#) shows that such a hole is suboptimal conditional on no verification; hence, following no audit, the principal would have a strict incentive to close the hole. This shows that a rule with decreasing auditing cannot be incentive compatible when the principal has limited commitment power, allowing us to establish that TEC is optimal in this case.

It is worth pointing out that while TEC is optimal both when the principal has full commitment power and a low audit cost (as shown in [Proposition 3](#)) as well as when she has limited commitment power (as shown in [Proposition 5](#)), the specific details of an optimal TEC rule vary with each case. Under full commitment, an optimal TEC rule $\{\gamma^*, \gamma^{**}\}$ is such that the principal prefers to audit types $\gamma > \gamma^{**}$ to assign them efficient spending rather than bunch them at $\pi_A(\gamma^*)$ without audit, whereas the opposite is true for types $\gamma \in [\gamma^*, \gamma^{**}]$. Hence, the principal is indifferent between auditing and not auditing the threshold type γ^{**} ; graphically, in [Figure 3](#), the increase in assigned spending at γ^{**} exactly compensates the principal for the cost ϕ of auditing this type. In contrast, under limited commitment, it is the agent who is indifferent at γ^{**} : as implied by [Lemma 6](#), type γ^{**} must be indifferent between being audited and assigned efficient spending versus not being audited and assigned $\pi_A(\gamma^*)$, and thus any increase in assigned spending at γ^{**} must exactly compensate this type for his audit cost $\alpha\phi$.

²⁵If $\gamma_L < \gamma_H$, the indifference of type γ_H between audit with efficient spending and no audit with spending π_H would imply that γ_L strictly prefers audit with efficient spending to no audit with spending π_H , a contradiction.

6 Conclusion

This paper has studied the tradeoff between commitment and flexibility in the presence of costly verification. We have examined a general delegation problem in which a principal delegates decision making to an agent who has superior information about the efficient action but is biased towards higher actions. A novel element of our framework is that the principal can verify the agent's private information by conducting an audit. Because audits are costly, the principal wishes to use this technology selectively, and in a way that supplements delegation and improves her commitment-versus-flexibility tradeoff.

Our results provide insight into how the principal achieves this by designing an optimal delegation rule. We have shown that if the cost of audit is small enough, an optimal rule is a threshold with an escape clause (TEC), allowing the agent to freely select any action up to a threshold or to request audit and the efficient action if the threshold is sufficiently binding. If the cost of audit is higher, the principal may instead prefer to prescribe audit only for intermediate actions, still imposing some discipline on the agent but saving on audit costs. Yet, we find that the optimality of TEC is recovered under mild limitations to the principal's commitment power: if the principal is unable to commit to not changing the agent's permissible action set following the audit decision and result, TEC is optimal for any audit cost for which auditing is optimal.

A main contribution of our paper is to provide a theoretical foundation for the use of TEC in practice. As discussed in the Introduction, there is a variety of applications where delegation rules make use of audits and often take the form of TEC, including in capital budgeting in organizations, fiscal policy, and consumption-savings problems. More broadly, our framework may help inform the empirical analysis of real-world rules. Data on delegation policies and the way verification is used is increasingly available and offers an opportunity to explore the structure of these rules in more detail. For example, data on fiscal rules around the world may be used to study how delegation varies with the institutional and macroeconomic context, which may affect both the cost of auditing a government and the importance of flexibility in responding to shocks.

Lastly, by uncovering a new set of issues that arise when audits are introduced to a setting in which both commitment and flexibility are valuable, our paper opens the door for further work that can help understand the optimal joint design of delegation and verification. We have focused on a simple model that emphasizes the main forces at play but abstracts from other potentially relevant aspects, for instance associated with more complex verification technologies. We close by discussing some possible extensions

and variations of our work.

Random auditing. Our analysis restricted attention to deterministic auditing, namely, we assumed that the principal’s rule assigns $a(\gamma) \in \{0, 1\}$ to each agent type γ . More generally, one could allow for mechanisms in which the principal randomizes over the audit assignment, choosing a probability of audit for each type. In our game form of [Section 2.2](#), random audits would be implemented by letting the agent choose in Step 2 not between audit and no-audit but rather between lotteries over audit. The literature on financial contracting and tax collection finds that random audits can yield qualitatively different results compared to deterministic audits; see [Border and Sobel \(1987\)](#) and [Mookherjee and Png \(1989\)](#).

While the study of random audits in delegation would be an interesting extension of our work, we emphasize two points. First, as noted in the aforementioned papers, an analysis of optimal rules with random audits requires imposing a bound on the extent to which an agent can be punished following an audit. The reason is that, otherwise, the efficient allocation can be approached with a rule that audits all agent types with very low probability and arbitrarily punishes the agent when the audit verifies that he has deviated — such a rule would prevent deviations with a deadweight loss that approaches zero as the probability of audit approaches zero.²⁶ This possibility not only yields rather implausible predictions, but also generates the problem that an optimal rule in general will fail to exist unless a bound is imposed.

Random audits therefore demand taking a stance on how (and why) punishments are bounded. One possibility is to consider some form of limited commitment by the principal, as we have done in [Section 5](#). However, that takes us to our second point: implementing random audits requires high commitment power from the principal. When the decision is simply over audit or no-audit, commitment to the audit policy would in principle be facilitated by the fact that the principal’s execution of the agent’s audit/no-audit request can be easily monitored. But checking that the principal implements a specific non-degenerate lottery is more difficult, as it requires monitoring of the randomization itself rather than its outcome. The difficulty to commit to randomized mechanisms may be an obstacle to their implementation in applications.

²⁶In our specific game form, a rule that approaches the efficient allocation would be implemented by inducing each agent type to choose a different lottery over audit, so the agent’s choice perfectly reveals his type and allows the principal to assign efficient spending following no audit.

Imperfect auditing. Another simplifying assumption of our setting is that the principal verifies the agent’s type perfectly when she conducts an audit. An alternative would be to consider imperfect audits, namely audits that provide only imperfect information about the agent’s type. For example, in the context of capital budgeting in organizations, headquarters may review information about the benefits of a project that a manager advocates, but the available documentation may be incomplete and fail to reveal the full merits of the project.

A simple specification that may be possible to accommodate within our framework is when an imperfect audit either verifies the agent’s type perfectly or provides no information (i.e., when there are no “false” audit results). Provided that available punishments are unbounded, the principal would be able to prevent, at no cost, any deviation in which an agent type mimics another type who is audited, as is true in our problem with perfect audits. Yet, a difference introduced by imperfect audits is that the principal may not observe the agent’s type and thus may not be able to assign a type-dependent spending level following audit; the principal’s rule must specify a spending allocation for the case of audit and no information. Allowing for imperfect audits that may produce false results would naturally introduce further issues, as now punishing an agent type for mimicking another type who is audited would require imposing punishments on path.

How imperfect is an imperfect audit? At one extreme, if audits are sufficiently accurate, we conjecture that our (qualitative) results would remain valid (where the definition of TEC would be adjusted to account for the issues discussed above). At the other extreme, if audits are sufficiently inaccurate, they would become equivalent to money burning, and the results of the literature on when money burning is used in an optimal delegation rule would then apply (see [Amador, Werning and Angeletos, 2006](#); [Amador and Bagwell, 2013](#); [Ambrus and Egorov, 2015](#)). More generally, it would be of interest to explore the role of audits in delegation away from these two extremes.

Audit costs. We have assumed that audit costs are both type-independent and exogenous. An extension of our problem could explore the effects of type-dependent audit costs: the principal’s cost of auditing the agent’s private information may be increasing in his type, for example because more evidence is needed to verify larger projects benefits, or one may take the view that audit costs are actually lower for extreme types, as these states are more “visible.” One possible difficulty is that monotonicity of the spending allocation (as shown in [Lemma 4](#)) may fail to hold if audit costs increase very rapidly

with the agent's type. But if the audit cost function is such that the principal would still prefer to swap the audit and spending allocations of two types γ and $\gamma' > \gamma$ whenever type γ has higher spending than γ' , monotonicity will be satisfied and our analysis could be extended to allow for type-dependent audit costs.

Another variation would be to endogenize α , so that the principal can affect the agent's cost of audit. In our problem with full commitment power, the principal would optimally set $\alpha = 0$, as a zero cost of audit for the agent maximally relaxes the agent's incentive compatibility constraint (4). Things are less straightforward in the setting of Section 5 where the principal has limited commitment power: here the principal may want to set a strictly positive audit cost for the agent in order to limit the set of agent types that may want to demand audit and efficient spending.

Transfers. Our focus has been on a canonical delegation problem in which transfers between the principal and the agent are not feasible. There are various ways in which transfers could be introduced in our framework and used to alter the feasibility and cost of inducing different allocations. Transfers could be contingent on the agent's audit decision and/or the result of the audit; moreover, the principal could offer different allowable spending sets for the agent to choose from and specify transfers associated with each set. These questions are beyond the scope of our paper and so we leave them for future research.

A Appendix: Proofs

A.1 Proof of Proposition 1

The claim follows from Proposition 1(a) in Amador and Bagwell (2013, p. 1551).

A.2 Proof of Lemma 1

Suppose by contradiction that a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ with $a(\gamma) = 1$ for all $\gamma \in \Gamma$ is optimal. Since the incentive compatibility constraint (4) is trivially satisfied under this rule, it must be that $\pi(\gamma) = \pi_P(\gamma)$ for all $\gamma \in \Gamma$. Define $\gamma' \in [\underline{\gamma}, \bar{\gamma}]$ as the solution to

$$U_A(\gamma', \pi_P(\gamma')) - \alpha\phi = U_A(\gamma', \pi_P(\underline{\gamma})) \quad (15)$$

if such a solution exists and $\gamma' = \bar{\gamma}$ otherwise. Consider now a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ with $\tilde{a}(\gamma) = 0$, $\tilde{\pi}(\gamma) = \pi_P(\underline{\gamma})$ for $\gamma \leq \gamma'$, and $\tilde{a}(\gamma) = a(\gamma)$, $\tilde{\pi}(\gamma) = \pi(\gamma)$ for $\gamma > \gamma'$. By single-crossing and the definition of γ' in (15), the perturbed rule satisfies the incentive compatibility constraint (4). Conditional on $\gamma > \gamma'$, this rule yields the same expected welfare to the principal and the agent as the original rule. However, conditional on $\gamma \leq \gamma'$, the perturbed rule yields the agent a higher welfare than the original one, since, by (15),

$$U_A(\gamma, \pi_P(\gamma)) - \alpha\phi \leq U_A(\gamma, \pi_P(\underline{\gamma})) \quad (16)$$

for all $\gamma \leq \gamma'$. Moreover, note that (2) implies

$$U_A(\gamma, \pi_P(\gamma)) - U_A(\gamma, \pi_P(\underline{\gamma})) > U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi_P(\underline{\gamma}))$$

for all $\gamma > \underline{\gamma}$, and hence, using (16) and the fact that $\alpha \in [0, 1]$,

$$U_P(\gamma, \pi_P(\gamma)) - \phi < U_P(\gamma, \pi_P(\underline{\gamma}))$$

for all $\gamma \leq \gamma'$. Conditional on $\gamma \leq \gamma'$, the principal is therefore strictly better off under the perturbed rule than under the original rule. It follows that the perturbed rule with no audit below γ' strictly dominates the original rule, contradicting the optimality of a rule that audits all types.

A.3 Proof of Corollary 1

Suppose an optimal rule features auditing which is weakly increasing everywhere. By Lemma 1, $a(\gamma) = 0$ for some $\gamma \in \Gamma$, and hence this rule must feature a no-audit region followed by an audit region. That is, the principal solves (3)-(4) by choosing a threshold γ^{**} such that $a(\gamma) = 0$ for $\gamma < \gamma^{**}$ and $a(\gamma) = 1$ for $\gamma > \gamma^{**}$, and a spending allocation $\pi(\gamma)$ for each $\gamma \in \Gamma$.

Now consider a relaxed version of this problem in which the principal chooses an optimal allocation in the no-audit and audit regions separately, ignoring the incentives of types in one region to deviate to the other region. Taking the no-audit region to be $[\underline{\gamma}, \gamma^{**}]$, it follows from Proposition 1 that an optimal allocation is a threshold $\gamma^* < \gamma^{**}$ such that $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ for each $\gamma \in [\underline{\gamma}, \gamma^{**}]$. For the audit region $(\gamma^{**}, \bar{\gamma}]$, since incentive compatibility is trivially satisfied, an optimal allocation assigns $\pi_P(\gamma)$ to each $\gamma \in (\gamma^{**}, \bar{\gamma}]$. Note that the resulting rule for the whole set Γ is TEC. Moreover,

because this rule solves a relaxed problem, it is sufficient to show that it is incentive compatible over the whole set Γ to prove its optimality in the original problem.

To show incentive compatibility, note first that incentive compatibility within each region is guaranteed by construction. Furthermore, since, as explained in [Section 2.3](#), no type would have incentives to deviate to mimic a different type which is audited, incentive compatibility is satisfied for all $\gamma \in [\underline{\gamma}, \gamma^{**}]$. All is left to be shown is that no type $\gamma \in (\gamma^{**}, \bar{\gamma}]$ has incentives to deviate to mimic a type $\hat{\gamma} \in [\underline{\gamma}, \gamma^{**}]$:

$$U_A(\gamma, \pi_P(\gamma)) - \alpha\phi \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma > \gamma^{**}, \hat{\gamma} \leq \gamma^{**}.$$

The single-crossing condition in U_A implies that a sufficient condition for the above inequality to hold is

$$U_A(\gamma, \pi_P(\gamma)) - \alpha\phi \geq U_A(\gamma, \pi_A(\gamma^*)) \text{ for all } \gamma > \gamma^{**}. \quad (17)$$

Now note that optimality of γ^{**} for the principal implies

$$U_P(\gamma, \pi_P(\gamma)) - \phi \geq U_P(\gamma, \pi_A(\gamma^*)) \text{ for all } \gamma > \gamma^{**}. \quad (18)$$

Given the agent's bias (2) and $\alpha \in [0, 1]$, (18) implies (17) if $\pi_P(\gamma) \geq \pi_A(\gamma^*)$ for all $\gamma > \gamma^{**}$, or equivalently since $\pi'_P(\gamma) > 0$, if

$$\pi_P(\gamma^{**}) \geq \pi_A(\gamma^*). \quad (19)$$

We prove that the TEC rule that we constructed satisfies (19). The optimal threshold γ^* in the no-audit region solves

$$\max_{\gamma^*} \left\{ \int_{\underline{\gamma}}^{\gamma^*} U_P(\gamma, \pi_A(\gamma)) f(\gamma) d\gamma + \int_{\gamma^*}^{\gamma^{**}} U_P(\gamma, \pi_A(\gamma^*)) f(\gamma) d\gamma \right\}.$$

The first-order condition yields

$$\int_{\gamma^*}^{\gamma^{**}} \frac{\partial U_P(\gamma, \pi_A(\gamma^*))}{\partial \pi_A(\gamma^*)} \pi'_A(\gamma^*) f(\gamma) d\gamma = 0.$$

Note that $\pi'_A(\gamma^*) > 0$, $\frac{\partial U_P(\gamma, \pi_A(\gamma^*))}{\partial \pi_A(\gamma^*)} < 0$ if $\pi_P(\gamma) < \pi_A(\gamma^*)$, and $\frac{\partial U_P(\gamma, \pi_A(\gamma^*))}{\partial \pi_A(\gamma^*)} > 0$ if $\pi_P(\gamma) > \pi_A(\gamma^*)$. Hence, the first-order condition requires $\pi_P(\gamma) > \pi_A(\gamma^*)$ for some

$\gamma \in [\gamma^*, \gamma^{**}]$, implying that (19) must hold.

A.4 Proof of Proposition 2

Assume $b(\pi) = 0$ for all $\pi \in [\underline{\pi}, \bar{\pi}]$. Suppose by contradiction that an optimal rule specifies $a(\gamma) = 1$ for some $\gamma \in \Gamma$ but TEC is not optimal. By Lemma 1, $a(\gamma) = 0$ for some $\gamma \in \Gamma$. Moreover, it follows from the incentive compatibility constraint (4) and $b(\cdot) = 0$ that all types γ with $a(\gamma) = 0$ are bunched at the same level of spending, and, letting such level be $\pi_A(\gamma^*)$ for some γ^* , any type γ with $a(\gamma) = 1$ must be assigned $\pi(\gamma) \geq \pi_A(\gamma^*)$. It is then immediate that if an optimal rule features auditing which is weakly increasing everywhere, it must be TEC, and hence by the contradiction assumption the optimal rule under consideration must feature decreasing auditing. We proceed by showing that an optimal rule cannot feature decreasing auditing at any $\gamma' \in \Gamma$.

Consider first the case in which $a(\gamma') = 0$, $a(\gamma' - \varepsilon) = 1$ for some $\gamma' \in \Gamma$ and $\varepsilon > 0$ arbitrarily small. As shown in the text, the optimality of auditing type $\gamma' - \varepsilon$ implies (6) and $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$, whereas the optimality of not auditing γ' implies (7). However, the two equations together with $\pi(\gamma' - \varepsilon) > \pi_A(\gamma^*)$ violate the single-crossing condition (1). Contradiction.

Consider next the case in which $a(\gamma') = 1$, $a(\gamma' + \varepsilon) = 0$ for some $\gamma' \in \Gamma$ and $\varepsilon > 0$ arbitrarily small. Analogous arguments to those above apply to this case and yield a contradiction.

A.5 Proof of Lemma 2

Suppose TEC is a solution to (8)-(9) with associated cutoffs γ^* and γ^{**} . Note that any rule satisfying constraint (4) will satisfy constraint (9). Hence, (8)-(9) is a relaxed version of (3)-(4), implying that any solution to (8)-(9) that satisfies (4) will also be a solution to (3)-(4). It follows that to prove the claim, all we need to show is that the TEC rule that solves (8)-(9) will satisfy constraint (4). It is immediate that for any γ with $a(\gamma) = 0$, (9) being satisfied implies that (4) will be satisfied. Now consider γ with $a(\gamma) = 1$. Optimality of auditing type γ under a TEC rule that solves (8)-(9) implies

$$U_P(\gamma, \pi_P(\gamma)) - \phi \geq U_P(\gamma, \pi_A(\gamma^*)), \quad (20)$$

since a perturbation that assigns no audit and spending level $\pi_A(\gamma^*)$ to a type $\gamma > \gamma^{**}$ is incentive compatible. Note that by the arguments in the proof of [Corollary 1](#), a TEC rule that solves (8)-(9) satisfies $\pi_P(\gamma) \geq \pi_A(\gamma^*)$ for all $\gamma > \gamma^{**}$. Hence, combining (20) with (2) and the fact that $\alpha \in [0, 1]$ implies

$$U_A(\gamma, \pi_P(\gamma)) - \alpha\phi \geq U_A(\gamma, \pi_A(\gamma^*)).$$

It follows that (4) is satisfied for type γ with $a(\gamma) = 1$.

A.6 Proof of [Lemma 3](#)

Suppose a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ solving (8)-(9) specifies $a(\gamma') = 1$ for some type $\gamma' \in \Gamma$.

To prove that the rule specifies $\pi(\gamma') \leq \pi_A(\gamma')$, suppose by contradiction that $\pi(\gamma') > \pi_A(\gamma')$. Consider a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma') = 1$ and $\tilde{\pi}(\gamma') = \pi_A(\gamma')$ while keeping the allocation unchanged for all $\gamma \neq \gamma'$. This perturbation strictly increases the principal's welfare conditional on γ' , leaves the principal's welfare conditional on $\gamma \neq \gamma'$ unchanged, and is incentive compatible for the agent.

Similarly, to prove that the rule specifies $\pi(\gamma') \geq \pi_P(\gamma')$, suppose by contradiction that $\pi(\gamma') < \pi_P(\gamma')$. Consider a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma') = 1$ and $\tilde{\pi}(\gamma') = \pi_P(\gamma')$ while keeping the allocation unchanged for all $\gamma \neq \gamma'$. This perturbation strictly increases the principal's welfare conditional on γ' , leaves the principal's welfare conditional on $\gamma \neq \gamma'$ unchanged, and is incentive compatible for the agent.

Finally, we prove that the rule must specify $\pi(\gamma') = \pi_P(\gamma')$ if (9) does not bind for γ' . Suppose by contradiction that (9) does not bind for γ' and $\pi(\gamma') \neq \pi_P(\gamma')$. By the claim above, $\pi(\gamma') \geq \pi_P(\gamma')$, and thus the rule must set $\pi(\gamma') > \pi_P(\gamma')$. But then a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma') = 1$ and $\tilde{\pi}(\gamma') = \pi(\gamma') - \varepsilon$ for $\varepsilon > 0$ arbitrarily small, while keeping the allocation unchanged for all $\gamma \neq \gamma'$, strictly increases the principal's welfare conditional on γ' , leaves the principal's welfare conditional on $\gamma \neq \gamma'$ unchanged, and is incentive compatible for the agent.

A.7 Proof of [Lemma 4](#)

Suppose by contradiction that a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ that solves (8)-(9) specifies $\pi(\gamma') > \pi(\gamma'')$ for some $\gamma' < \gamma''$. We consider four cases separately.

Case 1. Suppose $a(\gamma') = a(\gamma'') = 0$. Then (9) for γ' and γ'' requires

$$\begin{aligned} U_A(\gamma', \pi(\gamma')) &\geq U_A(\gamma', \pi(\gamma'')), \\ U_A(\gamma'', \pi(\gamma'')) &\geq U_A(\gamma'', \pi(\gamma')), \end{aligned}$$

which together imply

$$U_A(\gamma', \pi(\gamma')) - U_A(\gamma', \pi(\gamma'')) \geq U_A(\gamma'', \pi(\gamma')) - U_A(\gamma'', \pi(\gamma'')). \quad (21)$$

However, given $\gamma' < \gamma''$ and $\pi(\gamma') > \pi(\gamma'')$, (21) violates the single-crossing condition in U_A . Contradiction.

Case 2. Suppose $a(\gamma') = a(\gamma'') = 1$. By Lemma 3, $\pi(\gamma'') \geq \pi_P(\gamma'')$, and thus $\pi(\gamma') > \pi(\gamma'')$ implies $\pi(\gamma') > \pi_P(\gamma'') > \pi_P(\gamma')$. Using Lemma 3 again, it then follows that (9) binds for γ' , that is, there exists $\hat{\gamma} \in \Gamma$ with $a(\hat{\gamma}) = 0$ such that

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\hat{\gamma})). \quad (22)$$

Furthermore, note that we must have $\pi(\hat{\gamma}) \geq \pi(\gamma')$, since $\pi(\gamma') \leq \pi_A(\gamma')$ and U_A is strictly concave. Incentive compatibility for γ'' requires

$$U_A(\gamma'', \pi(\gamma'')) \geq U_A(\gamma'', \pi(\hat{\gamma})),$$

which, combined with the observation that

$$\pi(\gamma'') < \pi(\gamma') \leq \pi_A(\gamma') < \pi_A(\gamma''), \quad (23)$$

implies

$$U_A(\gamma'', \pi(\gamma')) > U_A(\gamma'', \pi(\hat{\gamma})). \quad (24)$$

Combining (22) and (24) yields

$$U_A(\gamma', \pi(\hat{\gamma})) - U_A(\gamma', \pi(\gamma')) > U_A(\gamma'', \pi(\hat{\gamma})) - U_A(\gamma'', \pi(\gamma')). \quad (25)$$

However, given $\gamma' < \gamma''$ and $\pi(\hat{\gamma}) \geq \pi(\gamma')$, (25) violates the single-crossing condition in U_A . Contradiction.

Case 3. Suppose $a(\gamma') = 1$ and $a(\gamma'') = 0$. Note that (23) must hold. Then consider a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma'') = 1$ and $\tilde{\pi}(\gamma'') = \pi(\gamma')$ while leaving the allocation for types $\gamma \neq \gamma''$ unchanged. Since incentive compatibility was initially satisfied and $\gamma' < \gamma''$ while (23) holds, this perturbation is incentive compatible. Optimality of the original rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ therefore requires that this perturbation do not strictly increase the principal's welfare, which requires

$$U_P(\gamma'', \pi(\gamma'')) \geq U_P(\gamma'', \pi(\gamma')) - \phi.$$

The single-crossing condition in U_P then implies

$$U_P(\gamma', \pi(\gamma'')) > U_P(\gamma', \pi(\gamma')) - \phi. \quad (26)$$

Now consider a different perturbed rule $\{\hat{a}(\gamma), \hat{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\hat{a}(\gamma') = 0$ and $\hat{\pi}(\gamma') = \pi(\gamma'')$ while leaving the allocation for types $\gamma \neq \gamma'$ unchanged. Equation (26) implies that this perturbation would strictly increase the principal's welfare. Hence, optimality of the original rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ requires that this perturbation violate incentive compatibility, that is, there must exist $\hat{\gamma} \in \Gamma$ with $a(\hat{\gamma}) = 0$ such that

$$U_A(\gamma', \pi(\hat{\gamma})) > U_A(\gamma', \pi(\gamma'')). \quad (27)$$

Note that since $\pi(\gamma'') < \pi_A(\gamma')$, we must have $\pi(\hat{\gamma}) > \pi(\gamma'')$. Moreover, by incentive compatibility being satisfied under the original rule, we have

$$U_A(\gamma'', \pi(\gamma'')) \geq U_A(\gamma'', \pi(\hat{\gamma})).$$

Combining this equation with (27) yields

$$U_A(\gamma', \pi(\hat{\gamma})) - U_A(\gamma', \pi(\gamma'')) > U_A(\gamma'', \pi(\hat{\gamma})) - U_A(\gamma'', \pi(\gamma'')). \quad (28)$$

However, given $\gamma' < \gamma''$ and $\pi(\hat{\gamma}) > \pi(\gamma'')$, (28) violates the single-crossing condition in U_A . Contradiction.

Case 4. Suppose $a(\gamma') = 0$ and $a(\gamma'') = 1$. By Lemma 3, $\pi(\gamma'') \leq \pi_A(\gamma'')$, and hence given $\pi(\gamma') > \pi(\gamma'')$, incentive compatibility for type γ'' requires $\pi(\gamma') > \pi_A(\gamma'')$. Consider a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma') = 1$ and $\tilde{\pi}(\gamma') = \pi(\gamma'')$

while leaving the allocation for types $\gamma \neq \gamma'$ unchanged. Since the original rule satisfies incentive compatibility for γ'' , single-crossing implies that this perturbation is incentive compatible for γ' . Optimality of the original rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ then requires that this perturbation do not strictly increase the principal's welfare, which requires

$$U_P(\gamma', \pi(\gamma')) \geq U_P(\gamma', \pi(\gamma'')) - \phi.$$

The single-crossing condition in U_P then implies

$$U_P(\gamma'', \pi(\gamma')) > U_P(\gamma'', \pi(\gamma'')) - \phi. \quad (29)$$

Now consider a different perturbed rule $\{\hat{a}(\gamma), \hat{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\hat{a}(\gamma'') = 0$ and $\hat{\pi}(\gamma'') = \pi(\gamma')$ while leaving the allocation for types $\gamma \neq \gamma''$ unchanged. Equation (29) implies that such a perturbation would strictly increase the principal's welfare. Hence, optimality of the original rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ requires that this perturbation violate incentive compatibility, that is, there must exist $\hat{\gamma} \in \Gamma$ with $a(\hat{\gamma}) = 0$ such that

$$U_A(\gamma'', \pi(\hat{\gamma})) > U_A(\gamma'', \pi(\gamma')). \quad (30)$$

Note that since $\pi(\gamma') > \pi_A(\gamma'')$, we must have $\pi(\hat{\gamma}) < \pi(\gamma')$. Moreover, by incentive compatibility being satisfied under the original rule, we have

$$U_A(\gamma', \pi(\gamma')) \geq U_A(\gamma', \pi(\hat{\gamma})).$$

Combining this equation with (30) yields

$$U_A(\gamma', \pi(\gamma')) - U_A(\gamma', \pi(\hat{\gamma})) > U_A(\gamma'', \pi(\gamma')) - U_A(\gamma'', \pi(\hat{\gamma})). \quad (31)$$

However, given $\gamma' < \gamma''$ and $\pi(\hat{\gamma}) < \pi(\gamma')$, (31) violates the single-crossing condition in U_A . Contradiction.

A.8 Proof of Lemma 5

Suppose a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ solves (8)-(9) and features decreasing auditing at some $\gamma' \in \Gamma$ with $a(\gamma') = 1$. Then $a(\gamma' + \varepsilon) = 0$ for some $\varepsilon > 0$ arbitrarily small. Suppose it were the case that $\pi(\gamma' + \varepsilon) = \pi(\gamma')$. Then optimality of this rule would be violated, as

a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma') = 0$ and $\tilde{\pi}(\gamma') = \pi(\gamma')$ while keeping the allocation unchanged for $\gamma \neq \gamma'$ would be incentive compatible and strictly increase the principal's welfare (recall $\phi > 0$). It follows that $\pi(\gamma' + \varepsilon) \neq \pi(\gamma')$, and hence by [Lemma 4](#), $\pi(\gamma' + \varepsilon) > \pi(\gamma')$. Moreover, by [Lemma 3](#), $\pi(\gamma') \leq \pi_A(\gamma')$, and thus incentive compatibility for γ' would be violated if it were the case that $\pi_A(\gamma') \geq \pi(\gamma' + \varepsilon) > \pi(\gamma')$. It therefore follows that

$$\pi(\gamma' + \varepsilon) > \pi_A(\gamma') \quad (32)$$

for $\varepsilon > 0$ arbitrarily small. [Lemma 4](#) then implies $\pi(\gamma) > \pi_A(\gamma')$ for $\gamma \in (\gamma', \gamma'')$, $\gamma'' \equiv \min\{\pi_P^{-1}(\pi_A(\gamma')), \bar{\gamma}\}$, which implies

$$\int_{\gamma'}^{\gamma''} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma < \int_{\gamma'}^{\gamma''} U_P(\gamma, \pi_A(\gamma')) f(\gamma) d\gamma. \quad (33)$$

Moreover, by definition,

$$\int_{\gamma''}^{\bar{\gamma}} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma \leq \int_{\gamma''}^{\bar{\gamma}} U_P(\gamma, \pi_P(\gamma)) f(\gamma) d\gamma. \quad (34)$$

Combining [\(33\)](#) and [\(34\)](#), and taking into account that $1 - F(\gamma') > 0$, yields [\(10\)](#).

Suppose next that a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ solves [\(8\)](#)-[\(9\)](#) and features decreasing auditing at some $\gamma' \in \Gamma$ with $a(\gamma') = 0$. Then $a(\gamma' - \varepsilon) = 1$ for some $\varepsilon > 0$ arbitrarily small and arguments analogous to those above can be used to establish [\(10\)](#).

A.9 Proof of [Proposition 3](#)

The arguments in the proofs of [Lemma 1](#) and [Corollary 1](#) apply to the relaxed problem, implying that if a solution to [\(8\)](#)-[\(9\)](#) involves auditing some type $\gamma \in \Gamma$, this solution is either a TEC rule or a rule that features decreasing auditing at some $\gamma' \in \Gamma$. To prove the optimality of TEC for $\phi < \bar{\phi}$, we thus proceed by showing that for any such audit cost a rule featuring decreasing auditing cannot be a solution to [\(8\)](#)-[\(9\)](#).

Suppose a rule $\{a(\gamma), \pi(\gamma)\}_{\gamma \in \Gamma}$ solves [\(8\)](#)-[\(9\)](#) and features decreasing auditing. Denote by γ^{**} the infimum of the lowest audit region under this rule. Now consider a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ which sets $\tilde{a}(\gamma) = 0$ for $\gamma < \gamma^{**}$, $\tilde{a}(\gamma^{**}) = a(\gamma^{**})$, and $\tilde{a}(\gamma) = 1$ for $\gamma > \gamma^{**}$. If $\tilde{a}(\gamma) = 0$, let $\tilde{\pi}(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ for γ^* as defined in [Proposition 1](#) under $\bar{\gamma}' = \gamma^{**}$. If $\tilde{a}(\gamma) = 1$, let $\tilde{\pi}(\gamma) = \pi_P(\gamma)$. By the arguments in the proof of [Corollary 1](#), this rule is incentive compatible for types prescribed no audit and sets

$\pi_A(\gamma^*) \leq \pi_P(\gamma^{**})$. Moreover, given this inequality and the fact that $\alpha = 0$, it follows that the rule is also incentive compatible for types prescribed audit. We now show that this rule strictly increases the principal's expected welfare for $\phi < \bar{\phi}$, contradicting the optimality of the original rule. Denote by γ' the lowest type above γ^{**} featuring decreasing auditing in the original rule. Then the change in the principal's expected welfare from using the perturbed rule instead of the original rule is

$$\begin{aligned} & \int_{\underline{\gamma}}^{\gamma^{**}} [U_P(\gamma, \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}) - U_P(\gamma, \pi(\gamma))] f(\gamma) d\gamma \\ & + \int_{\gamma^{**}}^{\bar{\gamma}} [U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))] f(\gamma) d\gamma \\ & - \int_{\gamma'}^{\bar{\gamma}} [\phi(1 - a(\gamma))] f(\gamma) d\gamma. \end{aligned} \quad (35)$$

Note that since all types above γ^{**} are audited, the principal's welfare conditional on the agent's type being in the no-audit region of the perturbed rule is optimized subject to fewer incentive compatibility constraints in this rule compared to the original rule. Hence, the first term in (35) is weakly positive.

To evaluate the second and third terms in (35), suppose first that $\gamma' < \bar{\gamma}$. Then by Lemma 5, the second term in (35) satisfies

$$\begin{aligned} \int_{\gamma^{**}}^{\bar{\gamma}} [U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))] f(\gamma) d\gamma & \geq \int_{\gamma'}^{\bar{\gamma}} [U_P(\gamma, \pi_P(\gamma)) - U_P(\gamma, \pi(\gamma))] f(\gamma) d\gamma \\ & \geq (1 - F(\gamma')) \eta(\gamma'). \end{aligned} \quad (36)$$

Moreover, the third term in (35) satisfies

$$\begin{aligned} - \int_{\gamma'}^{\bar{\gamma}} [\phi(1 - a(\gamma))] f(\gamma) d\gamma & > -(1 - F(\gamma')) \bar{\phi} \\ & = -(1 - F(\gamma')) \min_{\gamma \in \Gamma} \eta(\gamma). \end{aligned} \quad (37)$$

Together, (36) and (37) imply that the perturbation strictly increases welfare.

Suppose next that $\gamma' = \bar{\gamma}$. Analogous arguments to those above imply that the perturbation makes the principal weakly better off conditional on $\gamma < \bar{\gamma}$. To evaluate the change in welfare conditional on $\gamma = \bar{\gamma}$, note that decreasing auditing in this case implies $a(\bar{\gamma}) = 0$ and $a(\bar{\gamma} - \varepsilon) = 1$ for $\varepsilon > 0$ arbitrarily small. Analogous arguments to those in

the proof of [Lemma 5](#) then imply $\pi(\bar{\gamma}) \geq \pi_A(\bar{\gamma})$. Moreover, [\(11\)](#) implies

$$\eta(\bar{\gamma}) = \lim_{\gamma \rightarrow \bar{\gamma}} \eta(\gamma) = U_P(\bar{\gamma}, \pi_P(\bar{\gamma})) - U_P(\bar{\gamma}, \pi_A(\bar{\gamma})) \geq \bar{\phi} > \phi, \quad (38)$$

where we have appealed to the definition of $\bar{\phi}$. It thus follows from [\(38\)](#) that the perturbation strictly increases the principal's welfare conditional on $\gamma = \bar{\gamma}$.

A.10 Proof of [Proposition 4](#)

Consider the following *quadratic-uniform setting*: preferences satisfy $U_P(\gamma, \pi) = \gamma\pi - \pi^2/2$ and $U_A(\gamma, \pi) = (\gamma + \beta)\pi - \pi^2/2$ for $\beta > 0$, and $f(\gamma) = 1$ for all $\gamma \in \Gamma$. In this setting, the efficient and flexible spending levels are given by $\pi_P(\gamma) = \gamma$ and $\pi_A(\gamma) = \gamma + \beta$ respectively. Let $\alpha = 0$, so that the agent pays no audit cost.

We first establish that in this setting, if the audit cost ϕ is high enough, TEC is suboptimal, as it is dominated by a rule without verification.

Lemma 7. *Consider the quadratic-uniform setting with $\alpha = 0$. If $\phi > \beta^2/2$, then TEC is not optimal.*

Proof. Take the quadratic-uniform setting with $\alpha = 0$ and $\phi > \beta^2/2$. Consider the following problem:

$$\max_{\{\underline{\gamma}^*, \gamma^{**}\}} \left\{ \int_{\underline{\gamma}}^{\gamma^*} U_P(\gamma, \pi_A(\gamma)) f(\gamma) d\gamma + \int_{\gamma^*}^{\gamma^{**}} U_P(\gamma, \pi_A(\gamma^*)) f(\gamma) d\gamma \right. \\ \left. + \int_{\gamma^{**}}^{\bar{\gamma}} (U_P(\gamma, \pi_P(\gamma)) - \phi) f(\gamma) d\gamma \right\}. \quad (39)$$

Note that the solution to this program coincides with a rule without verification if it sets $\gamma^{**} = \bar{\gamma}$, and it coincides with a rule that audits all types if it sets $\gamma^{**} = \underline{\gamma}$. By the definition of TEC, a necessary condition for a TEC rule to be optimal is that the solution to program [\(39\)](#) specify $\underline{\gamma} < \gamma^{**} < \bar{\gamma}$. We show that this cannot be satisfied when $\phi > \beta^2/2$.

The first-order condition for γ^* , given our assumptions on preferences and the distribution of γ , implies

$$\gamma^* = \max \left\{ \frac{\underline{\gamma} + \gamma^{**}}{2} - \beta, \gamma^{**} - 2\beta \right\}, \quad (40)$$

where we have taken into account the fact that γ^* may be lower than $\underline{\gamma}$. If the solution

to (39) sets γ^{**} strictly interior, then the first-order condition for γ^{**} implies

$$-\gamma^{**}(\gamma^* + \beta) + \frac{(\gamma^* + \beta)^2}{2} + \frac{\gamma^{**2}}{2} = \phi.$$

Substituting with (40) and rearranging terms yields

$$\frac{\left(\gamma^{**} - \max\left\{\frac{\gamma + \gamma^{**}}{2} - \beta, \gamma^{**} - 2\beta\right\} - \beta\right)^2}{2} = \phi. \quad (41)$$

Note that if $\gamma^* \geq \underline{\gamma}$, (41) implies $\phi = \beta^2/2$, contradicting the assumption that $\phi > \beta^2/2$. Therefore,

$$\gamma^* < \underline{\gamma}, \quad (42)$$

and thus (41) implies

$$\gamma^{**} = \underline{\gamma} + 2\sqrt{2\phi}.$$

Substituting back into (40), we obtain

$$\gamma^* = \underline{\gamma} + \sqrt{2\phi} - \beta. \quad (43)$$

However, combined with (42), equation (43) implies $\phi < \beta^2/2$, contradicting the assumption that $\phi > \beta^2/2$. Therefore, the solution to (39) cannot set γ^{**} strictly interior when $\phi > \beta^2/2$. \square

Given this lemma, we prove the proposition by showing that there exists $\phi > \beta^2/2$ under which a rule with auditing is optimal.

Lemma 8. *Consider the quadratic-uniform setting with $\alpha = 0$. If $\beta^2/2 < \phi < 2\beta^2/3$ and $6\beta < \bar{\gamma} - \underline{\gamma}$, then a rule with auditing is optimal.*

Proof. Take the quadratic-uniform setting with $\alpha = 0$, $\beta^2/2 < \phi < 2\beta^2/3$, and $6\beta < \bar{\gamma} - \underline{\gamma}$. An optimal rule without verification sets $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$, where using (40) (with $\gamma^{**} = \bar{\gamma}$) and the fact that $\bar{\gamma} - 2\beta > \underline{\gamma} + 4\beta > \underline{\gamma}$, we have

$$\gamma^* = \bar{\gamma} - 2\beta.$$

We construct a perturbed rule $\{\tilde{a}(\gamma), \tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$ that features auditing and yields the principal strictly higher expected welfare than this optimal rule without verification. For any

given $\gamma_H < \gamma^*$, define γ_L as the solution to

$$U_A(\gamma_L, \gamma_L - \beta) = U_A(\gamma_L, \pi_A(\gamma_H)),$$

which after some algebra yields

$$\gamma_L = \gamma_H - 2\beta. \quad (44)$$

Take $\gamma_H < \gamma^*$ sufficiently close to γ^* so that γ_L satisfies $\gamma_L - 2\beta > \underline{\gamma}$ (note that the assumption that $6\beta < \bar{\gamma} - \underline{\gamma}$ ensures that such a γ_H exists). Type γ_L is defined so that he is indifferent between the flexible spending level of γ_H and the optimal spending limit under no verification for a distribution truncated at γ_L (which is given by $\pi_A(\gamma_L - 2\beta) = \gamma_L - \beta$). Now construct the perturbed rule as follows: if $\gamma < \gamma_L - 2\beta$ or $\gamma > \gamma_H$, then $\tilde{a}(\gamma) = 0$ and $\tilde{\pi}(\gamma) = \pi(\gamma)$; if $\gamma \in [\gamma_L - 2\beta, \gamma_L)$, then $\tilde{a}(\gamma) = 0$ and $\tilde{\pi}(\gamma) = \gamma_L - \beta$; and if $\gamma \in [\gamma_L, \gamma_H]$, then $\tilde{a}(\gamma) = 1$ and $\tilde{\pi}(\gamma)$ satisfies

$$U_A(\gamma, \tilde{\pi}(\gamma)) = U_A(\gamma, \pi_A(\gamma_H)),$$

which after some algebra yields

$$\tilde{\pi}(\gamma) = 2\gamma - \gamma_H + \beta.$$

Note that given the definition of γ_L , this rule is incentive compatible. The perturbation only changes the principal's welfare for types $\gamma \in [\gamma_L - 2\beta, \gamma_H]$. The change in welfare is equal to

$$\begin{aligned} & \int_{\gamma_L - 2\beta}^{\gamma_L} [U_P(\gamma, \gamma_L - \beta) - U_P(\gamma, \gamma + \beta)] f(\gamma) d\gamma \\ & + \int_{\gamma_L}^{\gamma_H} [U_P(\gamma, 2\gamma - \gamma_H + \beta) - \phi - U_P(\gamma, \gamma + \beta)] f(\gamma) d\gamma. \end{aligned}$$

After some algebra and substitution of (44), using our assumptions on preferences and the distribution of γ , this simplifies to

$$- \int_{\gamma_H - 4\beta}^{\gamma_H - 2\beta} \frac{(\gamma - \gamma_H + 3\beta)^2}{2} d\gamma - \int_{\gamma_H - 2\beta}^{\gamma_H} \frac{(\gamma_H - \gamma - \beta)^2}{2} d\gamma - \int_{\gamma_H - 2\beta}^{\gamma_H} \phi d\gamma + \int_{\gamma_H - 4\beta}^{\gamma_H} \frac{\beta^2}{2} d\gamma.$$

Simplifying further yields that the change in welfare is equal to

$$\frac{4}{3}\beta^3 - 2\beta\phi > 0,$$

where the inequality follows from the assumption that $\phi < 2\beta^2/3$. Therefore, the perturbed rule with auditing strictly increases the principal's expected welfare relative to no verification. \square

It follows from [Lemma 7](#) and [Lemma 8](#) that in a quadratic-uniform setting with $\alpha = 0$, $\beta^2/2 < \phi < 2\beta^2/3$, and $6\beta < \bar{\gamma} - \underline{\gamma}$, auditing is optimal but TEC is not.

A.11 Proof of [Lemma 6](#)

Part (i). Suppose an incentive compatible rule induces decreasing auditing at γ_H . Consider first the case in which $a(\gamma_H) = 0$ and thus $a(\gamma_H - \varepsilon) = 1$ for $\varepsilon > 0$ arbitrarily small. Incentive compatibility for type γ_H requires

$$U_A(\gamma_H, \pi(\gamma_H)) \geq U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha\phi, \quad (45)$$

since γ_H can choose to be audited and guarantee himself the efficient level of spending. Incentive compatibility for type $\gamma_H - \varepsilon$ requires

$$U_A(\gamma_H - \varepsilon, \pi_P(\gamma_H - \varepsilon)) - \alpha\phi \geq U_A(\gamma_H - \varepsilon, \pi(\gamma_H)), \quad (46)$$

since $\gamma_H - \varepsilon$ can choose not to be audited and spend at $\pi(\gamma_H)$. Given the continuity of U_A and π_P in their respective arguments, we can take the limit of both sides of (46) as ε approaches 0 to obtain

$$U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha\phi \geq U_A(\gamma_H, \pi(\gamma_H)). \quad (47)$$

Combining (45) and (47) yields (12).

Consider next the case in which $a(\gamma_H) = 1$ and thus $a(\gamma_H + \varepsilon) = 0$ for $\varepsilon > 0$ arbitrarily small. Analogous arguments to those above imply the following incentive compatibility constraints for γ_H and $\gamma_H + \varepsilon$, respectively:

$$U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha\phi \geq U_A(\gamma_H, \pi(\gamma_H + \varepsilon)), \quad (48)$$

$$U_A(\gamma_H + \varepsilon, \pi(\gamma_H + \varepsilon)) \geq U_A(\gamma_H + \varepsilon, \pi_P(\gamma_H + \varepsilon)) - \alpha\phi. \quad (49)$$

Since the rule is piecewise continuous, $\lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$ exists and can be defined as $\pi(\gamma_H)$. Taking the limit of both sides of (48) and (49) as ε goes to 0 yields (47) and (45), and combining these two inequalities yields (12).

To complete the proof of part (i), we show that $\pi(\gamma_H) > \pi_A(\gamma_H)$ must hold. Note that by (12), either $\pi(\gamma_H) > \pi_A(\gamma_H)$ or $\pi(\gamma_H) \leq \pi_P(\gamma_H)$. For the purpose of contradiction, suppose it were the case that $\pi(\gamma_H) \leq \pi_P(\gamma_H)$. Consider the incentive compatibility constraint of type $\gamma_H - \varepsilon$ for $\varepsilon > 0$ arbitrarily small. Take first the case in which $a(\gamma_H - \varepsilon) = 1$. Then $\gamma_H - \varepsilon$ must weakly prefer audit to no audit, which requires

$$U_A(\gamma_H - \varepsilon, \pi_P(\gamma_H - \varepsilon)) - \alpha\phi \geq U_A(\gamma_H - \varepsilon, \pi(\gamma_H)). \quad (50)$$

Since $\pi_P(\gamma_H - \varepsilon) < \pi_P(\gamma_H) < \pi_A(\gamma_H - \varepsilon)$, (50) implies

$$U_A(\gamma_H - \varepsilon, \pi_P(\gamma_H)) - \alpha\phi > U_A(\gamma_H - \varepsilon, \pi(\gamma_H)). \quad (51)$$

Combining (12) and (51) yields

$$U_A(\gamma_H - \varepsilon, \pi_P(\gamma_H)) - U_A(\gamma_H - \varepsilon, \pi(\gamma_H)) > U_A(\gamma_H, \pi_P(\gamma_H)) - U_A(\gamma_H, \pi(\gamma_H)).$$

Given $\pi(\gamma_H) \leq \pi_P(\gamma_H)$, this inequality violates the single-crossing condition in U_A , thus yielding a contradiction.

Consider next the case in which $a(\gamma_H - \varepsilon) = 0$. Given decreasing auditing at γ_H , in this case we must have $a(\gamma_H) = 1$ and $a(\gamma_H + \varepsilon) = 0$ for $\varepsilon > 0$ arbitrarily small. Moreover, given our definition of $\pi(\gamma_H)$, $\pi(\gamma_H) \leq \pi_P(\gamma_H)$ implies $\lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon) \leq \pi_P(\gamma_H)$. By incentive compatibility, type γ_H must weakly prefer audit to no audit, which requires

$$U_A(\gamma_H, \pi_P(\gamma_H)) - \alpha\phi \geq U_A(\gamma_H, \pi(\gamma_H + \varepsilon)), \quad (52)$$

whereas type $\gamma_H + \varepsilon$ must weakly prefer no audit to audit, which requires

$$U_A(\gamma_H + \varepsilon, \pi(\gamma_H + \varepsilon)) \geq U_A(\gamma_H + \varepsilon, \pi_P(\gamma_H + \varepsilon)) - \alpha\phi. \quad (53)$$

Combining (52) and (53) and using the fact that $\pi_A(\gamma_H) > \pi_P(\gamma_H + \varepsilon) > \pi_P(\gamma_H)$ yields

$$U_A(\gamma_H, \pi_P(\gamma_H + \varepsilon)) - U_A(\gamma_H, \pi(\gamma_H + \varepsilon)) > U_A(\gamma_H + \varepsilon, \pi_P(\gamma_H + \varepsilon)) - U_A(\gamma_H + \varepsilon, \pi(\gamma_H + \varepsilon)).$$

Since $\pi(\gamma_H + \varepsilon) \leq \pi_P(\gamma_H) \leq \pi_P(\gamma_H + \varepsilon)$ for ε approaching 0, this inequality violates the single-crossing condition in U_A , thus yielding again a contradiction.

Therefore, we obtain that $\pi(\gamma_H) \leq \pi_P(\gamma_H)$ cannot hold and we must thus have $\pi(\gamma_H) > \pi_A(\gamma_H)$.

Part (ii). Suppose an incentive compatible rule induces increasing auditing at γ_L . Then analogous arguments to those used to prove part (i) can be applied to show that (14) must hold at γ_L . Since the steps are analogous, we omit the details.

A.12 Proof of Proposition 5

To prove this result, we first establish the following lemmas.

Lemma 9. *Under limited commitment, if an incentive compatible rule features increasing auditing at γ_L , then*

$$\pi(\gamma_L) \leq \pi_P(\gamma_L), \quad (54)$$

where $\pi(\gamma_L) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon)$ if $a(\gamma_L) = 1$.

Proof. Suppose an incentive compatible rule features increasing auditing at γ_L . By equation (14) in Lemma 6, either $\pi(\gamma_L) > \pi_A(\gamma_L)$ or $\pi(\gamma_L) \leq \pi_P(\gamma_L)$. For the purpose of contradiction, suppose $\pi(\gamma_L) > \pi_A(\gamma_L)$ holds. Take first the case in which $a(\gamma_L) = 1$, so that $a(\gamma_L - \varepsilon) = 0$ for $\varepsilon > 0$ arbitrarily small and, given our definition of $\pi(\gamma_L)$, $\lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon) > \pi_A(\gamma_L)$. By incentive compatibility, type $\gamma_L - \varepsilon$ must weakly prefer no audit to audit, which requires

$$U_A(\gamma_L - \varepsilon, \pi(\gamma_L - \varepsilon)) \geq U_A(\gamma_L - \varepsilon, \pi_P(\gamma_L - \varepsilon)) - \alpha\phi. \quad (55)$$

However, (14) and (55) together with the fact that $\pi(\gamma_L) > \pi_A(\gamma_L)$ imply that Assumption 2 is violated, yielding a contradiction.

Consider next the case in which $a(\gamma_L) = 0$, so that $a(\gamma_L + \varepsilon) = 1$ for $\varepsilon > 0$ arbitrarily small. By incentive compatibility, type $\gamma_L + \varepsilon$ must weakly prefer audit to no audit, which requires

$$U_A(\gamma_L + \varepsilon, \pi_P(\gamma_L + \varepsilon)) - \alpha\phi \geq U_A(\gamma_L + \varepsilon, \pi(\gamma_L)). \quad (56)$$

Note that in this case, $\pi(\gamma_L) > \pi_A(\gamma_L) > \pi_P(\gamma_L + \varepsilon)$ requires $\pi(\gamma_L) > \pi_A(\gamma_L + \varepsilon)$. However, (14) and (56) together with $\pi(\gamma_L) > \pi_A(\gamma_L + \varepsilon)$ imply that Assumption 2 is violated, yielding again a contradiction.

Therefore, we obtain that $\pi(\gamma_L) > \pi_A(\gamma_L)$ cannot hold and we must thus have $\pi(\gamma_L) \leq \pi_P(\gamma_L)$. \square

Lemma 10. *Under limited commitment, if an incentive compatible rule features decreasing auditing at γ_H , then there exists $\gamma' \leq \gamma_H$ satisfying*

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H)) \quad (57)$$

for $\pi(\gamma') < \pi_A(\gamma')$, $\pi(\gamma_H) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$ if $a(\gamma_H) = 1$, and either $a(\gamma') = 0$, or $a(\gamma') = 1$, $a(\gamma' - \varepsilon) = 0$ and $\pi(\gamma') \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma' - \varepsilon)$ for $\varepsilon > 0$ arbitrarily small.

Proof. Suppose an incentive compatible rule features decreasing auditing at γ_H . By condition (13) in Lemma 6, $\pi(\gamma_H) > \pi_A(\gamma_H)$. Consider the problem of the principal after the audit decision $a(\gamma)$ has been made and the audit result (in case of audit) has been obtained:

$$\max_{\{\pi(\gamma)\}_{\gamma \in \Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma \quad (58)$$

subject to

$$\pi(\gamma) = \pi_P(\gamma) \text{ if } a(\gamma) = 1, \quad (59)$$

$$U_A(\gamma, \pi(\gamma)) \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\gamma) = a(\hat{\gamma}) = 0. \quad (60)$$

This program takes into account that the principal will assign the efficient spending level to any agent type who chooses to be audited, and she will ignore the incentives of audited types when deciding the spending allocation of types who choose not to be audited. We now consider the optimal level of $\pi(\gamma_H)$ given decreasing auditing at γ_H and the conditions that are necessary for the principal to choose $\pi(\gamma_H) > \pi_A(\gamma_H)$.

Step 1. Consider the spending allocation conditional on no audit. Note that analogous arguments to those used in the proof of Lemma 4 imply that $\pi(\gamma)$ must be weakly increasing for non-audited types γ . For each non-audited type γ , denote by $\underline{\pi}(\gamma)$ the spending level closest to $\pi_A(\gamma)$ from below in the allowable spending set for non-audited types (i.e., among all spending levels assigned to types who choose no audit). Analogously, denote by $\bar{\pi}(\gamma)$ the closest spending level to $\pi_A(\gamma)$ from above in the allowable spending set for non-audited types. Clearly, if $\pi_A(\gamma)$ is in this allowable spending set, then $\pi_A(\gamma) =$

$\underline{\pi}(\gamma) = \bar{\pi}(\gamma)$. The incentive compatibility constraint (60) together with the concavity of U_A require that if $a(\gamma) = 0$, then

$$\pi(\gamma) = \arg \max_{\pi \in \{\underline{\pi}(\gamma), \bar{\pi}(\gamma)\}} U_A(\gamma, \pi). \quad (61)$$

Step 2. As noted, given decreasing auditing at γ_H , the rule must set $\pi(\gamma_H) > \pi_A(\gamma_H)$. We show that as a result, the rule must induce $a(\gamma) = 0$ and $\pi(\gamma) = \pi(\gamma_H)$ for all types $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$. To see why, note first that by (61) and the single-crossing condition in U_A , any type $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ who is not audited necessarily chooses spending $\pi(\gamma) = \pi(\gamma_H)$. Therefore, it is sufficient to show that any type $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ must have $a(\gamma) = 0$. Suppose by contradiction that this were not the case. Then incentive compatibility for a type $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$ with $a(\gamma) = 1$ requires that this type weakly prefer audit to no audit, which requires

$$U_A(\gamma, \pi_P(\gamma)) - \alpha\phi \geq U_A(\gamma, \pi(\gamma_H)). \quad (62)$$

However, (12) and (62) together with the fact that $\gamma > \gamma_H$ and $\pi(\gamma_H) > \pi_A(\gamma)$ violate [Assumption 2](#). The claim therefore follows.

Step 3. We show that in an incentive compatible rule, constraint (60) cannot be uniformly slack for all $\gamma \leq \gamma_H$ and $\hat{\gamma} = \gamma_H$, where recall $\pi(\gamma_H) > \pi_A(\gamma_H)$ by decreasing auditing at γ_H . Suppose by contradiction that this is true. Note that from Step 2, (60) is then uniformly slack for all $\gamma \leq \gamma_H$ and $\hat{\gamma} \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$, where $a(\hat{\gamma}) = 0$ for all such $\hat{\gamma}$. Now consider the following perturbation $\{\tilde{\pi}(\gamma)\}_{\gamma \in \Gamma}$: for $\varepsilon > 0$ arbitrarily small and all $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H) - \varepsilon)]$, set $\tilde{\pi}(\gamma) = \pi(\gamma_H) - \varepsilon$; for all $\gamma \in (\pi_A^{-1}(\pi(\gamma_H) - \varepsilon), \pi_A^{-1}(\pi(\gamma_H)))$, set $\tilde{\pi}(\gamma) = \pi_A(\gamma)$; and for all other types leave the spending allocation unchanged. This perturbation strictly increases the principal's welfare as it reduces overspending by types $\gamma \in (\gamma_H, \pi_A^{-1}(\pi(\gamma_H)))$. Moreover, since (by the contradiction assumption) (60) was uniformly slack before the perturbation for all $\gamma \leq \gamma_H$, it is still satisfied after the perturbation, and incentive compatibility for all types $\gamma \geq \gamma_H$ is guaranteed as the perturbation satisfies (61). Therefore, we obtain that if (60) is uniformly slack for all $\gamma \leq \gamma_H$ and $\hat{\gamma} = \gamma_H$, the principal can strictly improve upon the original rule by reducing $\pi(\gamma_H)$ after the audit decision has been made, and hence the original rule violates incentive compatibility for the principal. The claim follows.

Step 4. By Step 3, in any incentive compatible rule with decreasing auditing at γ_H , there exists $\gamma' \leq \gamma_H$ satisfying (57). Moreover, since decreasing auditing at γ_H implies $\pi(\gamma_H) > \pi_A(\gamma_H) \geq \pi_A(\gamma')$, this requires $\pi(\gamma') < \pi_A(\gamma')$. This proves the lemma. \square

Lemma 11. *Under limited commitment, if an incentive compatible rule features decreasing auditing at γ_H , then there exists $\gamma_L \leq \gamma_H$ at which the rule features increasing auditing. Moreover, $a(\gamma) = 1$ for all $\gamma \in (\gamma_L, \gamma_H)$ and*

$$U_A(\gamma_L, \pi(\gamma_L)) = U_A(\gamma_L, \pi(\gamma_H)) \quad (63)$$

for $\pi(\gamma_L) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_L - \varepsilon)$ if $a(\gamma_L) = 1$ and $\pi(\gamma_H) \equiv \lim_{\varepsilon \downarrow 0} \pi(\gamma_H + \varepsilon)$ if $a(\gamma_H) = 1$.

Proof. Suppose an incentive compatible rule features decreasing auditing at γ_H . By Lemma 10, there exists a type $\gamma' \leq \gamma_H$ satisfying (57) either with $a(\gamma') = 0$ or at which there is increasing auditing. We can establish that such a type is unique. Suppose by contradiction that there are two types, $\gamma'' \leq \gamma_H$ and $\gamma' < \gamma''$, satisfying the condition in Lemma 10. Then

$$U_A(\gamma'', \pi(\gamma'')) = U_A(\gamma'', \pi(\gamma_H)), \quad (64)$$

$$U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H)). \quad (65)$$

Incentive compatibility requires

$$U_A(\gamma'', \pi(\gamma'')) \geq U_A(\gamma'', \pi(\gamma')), \quad (66)$$

$$U_A(\gamma', \pi(\gamma')) \geq U_A(\gamma', \pi(\gamma'')). \quad (67)$$

Combining (64)-(67) yields

$$U_A(\gamma', \pi(\gamma_H)) - U_A(\gamma', \pi(\gamma'')) \geq U_A(\gamma'', \pi(\gamma_H)) - U_A(\gamma'', \pi(\gamma'')).$$

Since $\gamma' < \gamma''$ and $\pi(\gamma_H) > \pi_A(\gamma_H) \geq \pi_A(\gamma'') > \pi(\gamma'')$ by decreasing auditing at γ_H and Lemma 10, this inequality violates the single-crossing condition in U_A , yielding a contradiction. Therefore, there exists a unique type below γ_H for which (57) holds, and denoting this type by γ_L yields (63).

Next, we show that $a(\gamma) = 1$ for all $\gamma \in (\gamma_L, \gamma_H)$. Note first that a spending level $\pi \in (\pi(\gamma_L), \pi(\gamma_H))$ cannot be allowed by the rule under no audit, since otherwise type γ_L

would have a strict incentive to deviate to such a spending level. Consider the relevant case in which $\gamma_L < \gamma_H$ and suppose by contradiction that $a(\gamma) = 0$ for some type $\gamma \in (\gamma_L, \gamma_H)$. Let γ' denote the highest such type γ . Since, as noted, spending levels strictly between $\pi(\gamma_L) < \pi_A(\gamma')$ and $\pi(\gamma_H) > \pi_A(\gamma')$ are not allowed, it follows from (63) and $\gamma' > \gamma_L$ that the rule must set $\pi(\gamma') = \pi(\gamma_H)$. Moreover, since by construction the rule features increasing auditing at γ' , condition (14) in Lemma 6 implies

$$U_A(\gamma', \pi_P(\gamma')) - \alpha\phi = U_A(\gamma', \pi(\gamma')) = U_A(\gamma', \pi(\gamma_H)). \quad (68)$$

However, given (12) and (13), equation (68) violates Assumption 2. It follows that $a(\gamma) = 1$ for all $\gamma \in (\gamma_L, \gamma_H)$. \square

We can now prove the proposition. We begin by ruling out decreasing auditing. Suppose by contradiction that an incentive compatible rule features decreasing auditing at some $\gamma_H \in \Gamma$. By Lemma 11, there must exist a type $\gamma_L \leq \gamma_H$ satisfying the conditions in the lemma. We proceed in two steps.

Step 1. Suppose $\gamma_L < \gamma_H$. Then it follows from (14) and (63) that

$$U_A(\gamma_L, \pi_P(\gamma_L)) - \alpha\phi = U_A(\gamma_L, \pi(\gamma_H)). \quad (69)$$

However, (12) and (69) together with the fact that $\gamma_L < \gamma_H$ and $\pi(\gamma_H) > \pi_A(\gamma_H)$ (by (13)) imply that Assumption 2 is violated. Contradiction.

Step 2. By Step 1, any incentive compatible rule with decreasing auditing must have $\gamma_L = \gamma_H$ at each point γ_H at which there is decreasing auditing. Now consider the principal's problem (58)-(60). Let $\bar{\gamma}' \leq \bar{\gamma}$ be the highest non-audited type. Since the types with decreasing auditing are atomistic and the rule is piecewise continuous, following a decision of no audit the principal solves

$$\max_{\{\pi(\gamma)\}_{\gamma \in \Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}'} U_P(\gamma, \pi(\gamma)) f(\gamma) d\gamma$$

subject to

$$U_A(\gamma, \pi(\gamma)) \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\gamma) = a(\hat{\gamma}) = 0.$$

By [Proposition 1](#), the solution assigns $\pi(\gamma) = \min\{\pi_A(\gamma), \pi_A(\gamma^*)\}$ for $\gamma \in [\underline{\gamma}, \bar{\gamma}']$ and some $\gamma^* < \bar{\gamma}'$. However, in this case, conditions [\(13\)](#) and [\(54\)](#) (which require $\pi(\gamma_H) > \pi_A(\gamma_H)$ and $\pi(\gamma_L) \leq \pi_P(\gamma_L)$ respectively) cannot be satisfied at a point $\gamma_H \in [\underline{\gamma}, \bar{\gamma}']$ at which there is decreasing auditing and thus $\gamma_L = \gamma_H$. Contradiction.

The claims above show that under limited commitment, any incentive compatible rule features weakly increasing auditing everywhere. Analogous arguments to those in the proofs of [Lemma 1](#) and [Corollary 1](#) can then be applied to show that auditing all types is suboptimal and TEC is thus optimal if an optimal rule features auditing that is weakly increasing everywhere. Hence, under limited commitment, if auditing is optimal, TEC is optimal.

B Appendix: Other Mechanisms

In this Appendix, we show that our results under full commitment are not limited to the game form in [Section 2.2](#) but apply more generally when allowing for any indirect mechanism specifying a message space for the agent and a deterministic allocation function to which the principal commits. Specifically, we prove that a Revelation Principle in terms of payoffs holds in our setting, implying that to study the optimal deterministic mechanism for the principal, it is without loss to restrict attention to deterministic direct mechanisms in which the agent reports his type truthfully, as in program [\(3\)](#)-[\(4\)](#).

The usual version of the Revelation Principle cannot be directly applied to our problem because we consider a game with verification and limit attention to deterministic allocations. We note that [Townsend \(1988\)](#) provides an extension of the Revelation Principle to a class of models with verification and [Strausz \(2003\)](#) provides an extension to a setting with deterministic mechanisms and one agent. Our results below build on this work, particularly the latter.

Consider a general problem in which the principal must select a deterministic allocation specifying whether the agent is audited or not, $a \in A \equiv \{0, 1\}$, and the spending level that the agent is assigned, $\pi \in [\underline{\pi}, \bar{\pi}]$. A mechanism (S, a, π) consists of a message space for the agent S , an audit function $a : S \rightarrow A$ that commits the principal to implement the audit assignment $a(s)$ when the agent sends message s , and a spending function $\pi : S \times R \rightarrow \pi \in [\underline{\pi}, \bar{\pi}]$ that commits the principal to implement the spending level $\pi(s, r)$ when the agent sends message s and the audit result is r . Without loss given our assumption that an audit verifies the agent's type perfectly, we let $r = a(s)\gamma$, that

is, the audit result is equal to the agent's type γ if an audit is conducted and it is equal to 0 if an audit is not conducted (recall $\underline{\gamma} > 0$).

Given a mechanism (S, a, π) , the agent chooses a message. The agent's reporting strategy $\mu(\gamma) : \Gamma \rightarrow S$ selects the message s with probability $\mu(s|\gamma)$. A mechanism is a direct mechanism if $S = \Gamma$.

Proposition 6. *Consider an equilibrium of a game induced by a deterministic indirect mechanism (S, a, π) . There exists a deterministic direct mechanism, (Γ, a, π) , that induces an equilibrium with truthful revelation yielding the principal a weakly larger expected welfare than that in the equilibrium under the indirect mechanism. Hence, an optimal deterministic mechanism for the principal solves program (3)-(4).*

Proof. Consider a deterministic mechanism (S, a, π) and equilibrium reporting strategies $\mu(\gamma)$ for each type $\gamma \in \Gamma$. For each γ , let S_γ be the set of messages that the agent sends with positive probability, i.e. $S_\gamma = \{s | \mu(s|\gamma) > 0\}$. Since $\mu(\gamma)$ is an equilibrium strategy, it satisfies

$$U_A(\gamma, \pi(s, a(s)\gamma)) - a(s)\alpha\phi \geq U_A(\gamma, \pi(\hat{s}, a(\hat{s})\gamma)) - a(\hat{s})\alpha\phi \text{ for all } s \in S_\gamma, \hat{s} \in S, \quad (70)$$

$$U_A(\gamma, \pi(s, a(s)\gamma)) - a(s)\alpha\phi = U_A(\gamma, \pi(\hat{s}, a(\hat{s})\gamma)) - a(\hat{s})\alpha\phi \text{ for all } s, \hat{s} \in S_\gamma. \quad (71)$$

Define the set $S_{\gamma,P}$ as the set of messages that, given the allocation specified by the mechanism, yield the principal the highest welfare from type γ among the messages that are sent with positive probability under this type's reporting strategy. That is, $S_{\gamma,P} = \{s \in S_\gamma | U_P(\gamma, \pi(s, a(s)\gamma)) - a(s)\phi = \max_{z \in S_\gamma} \{U_P(\gamma, \pi(z, a(z)\gamma)) - a(z)\phi\}\}$. Then construct a direct mechanism (Γ, a, π) specifying: for a given $s \in S_{\gamma,P}$ (arbitrarily chosen if $|S_{\gamma,P}| > 1$), $a(\gamma) = a(s)$, $\pi(\gamma, r) = \pi(s, a(s)\gamma)$ if $r \in \{0, \gamma\}$, and $\pi(\gamma, r) = \pi(s, \hat{\gamma})$ if $r = \hat{\gamma} \in \Gamma$, $\hat{\gamma} \neq \gamma$. By construction, given this direct mechanism, it is an optimal strategy for each type γ to report his type truthfully. This equilibrium yields each type γ the same welfare as the equilibrium under the original indirect mechanism and it yields the principal weakly larger expected welfare than that equilibrium. The latter follows from the fact that the principal receives weakly larger welfare conditional on any type γ in the equilibrium of the direct mechanism with truthful revelation. The principal's expected welfare is the same in the two equilibria if $S_\gamma = S_{\gamma,P}$ for all γ and is strictly larger in the equilibrium of the direct mechanism with truthful revelation if $S_\gamma \neq S_{\gamma,P}$ for some γ . This proves the first part of the proposition.

We now prove the second part, namely that an optimal deterministic mechanism solves program (3)-(4). By the result just established, we can restrict attention to direct mechanisms in which the agent reports his type truthfully. With some abuse of notation, let $\pi(\gamma) \equiv \pi(\gamma, a(\gamma)\gamma)$. The principal's problem is

$$\max_{\{a(\gamma), \pi(\gamma), \pi(\hat{\gamma}, \gamma)\}_{\gamma, \hat{\gamma} \in \Gamma}} \int_{\underline{\gamma}}^{\bar{\gamma}} (U_P(\gamma, \pi(\gamma)) - a(\gamma)\phi) f(\gamma) d\gamma \quad (72)$$

subject to

$$U_A(\gamma, \pi(\gamma)) - a(\gamma)\alpha\phi \geq U_A(\gamma, \pi(\hat{\gamma})) \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\hat{\gamma}) = 0, \quad (73)$$

$$U_A(\gamma, \pi(\gamma)) - a(\gamma)\alpha\phi \geq U_A(\gamma, \pi(\hat{\gamma}, \gamma)) - \alpha\phi \text{ for all } \gamma, \hat{\gamma} \text{ for which } a(\hat{\gamma}) = 1. \quad (74)$$

The only difference between this program and program (3)-(4) is that the incentive compatibility constraint (74) is absent in (3)-(4). However, the principal can trivially prevent a deviation of a type γ in which he mimics a type $\hat{\gamma}$ with $a(\hat{\gamma}) = 1$: as this type is audited following the deviation, the principal can verify that he has deviated and punish him by assigning a spending level $\pi(\hat{\gamma}, \gamma)$ such that $U_A(\gamma, \pi(\hat{\gamma}, \gamma)) \leq U_A(\gamma, \pi(\gamma))$ for $\gamma \neq \hat{\gamma}$ (where it is clear that such a spending level $\pi(\hat{\gamma}, \gamma)$ exists, and in fact $\pi(\hat{\gamma}, \gamma) = \pi(\gamma)$ would be a sufficient punishment). Therefore, the principal can satisfy (74) at no cost, and hence the solution to this program coincides with the solution to (72)-(73), which is equivalent to program (3)-(4). \square

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