# Firm Dynamics with Frictional Product and Labor Markets*

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**Abstract**

This study analyzes the joint dynamics of prices, output, employment and wages across firms. We develop a dynamic equilibrium model of heterogeneous firms who compete for workers and customers in frictional labor and product markets. Prices and wages are dispersed across firms, reflecting differences in firm size, firm productivity and demand. Firm-specific productivity and demand shocks have distinct implications for the firms' employment, output and price adjustments. Using panel data on prices and output for German manufacturing firms, we calibrate the model to evaluate its implications for firm dynamics and for the cross-sectional dispersion of prices, wages and labor productivity. We use the model to examine the effects of product market deregulation on the aggregate economy.

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# 1 Introduction

Heterogeneity of firms is important for the understanding of labor market flows and for wage inequality: Firms which differ in size, age or productivity create and destroy jobs at different rates and they pay different wages (e.g. Davis et al. (2006), Haltiwanger et al. (2013), Lentz and Mortensen (2010)). This motivates a large literature on the role of firms in the labor market, much of which builds on the seminal contributions of Hopenhayn (1992) and Hopenhayn and Rogerson (1993), augmented by richer labor market features.[1] Realistic size distributions and growth dynamics require permanent and transitory productivity differentials which are typically based on the firms' revenues, thus reflecting both price and quantity components. This is a reasonable theoretical shortcut, given that most datasets have no separate information on firm-level prices and output.

Recent empirical evidence points at a prominent role of firm-specific demand for firm growth. Using U.S. data on narrowly defined industries that permit a distinction between price and quantity, Foster et al. (2008) examine the separate contributions of demand and productivity for firm performance, finding that demand variation is the dominant driver of firm growth and firm survival.[2] While there are no significant productivity differences across firms of different ages, younger firms charge lower prices than incumbents which suggests that those firms attempt to build a customer base (relationship capital).[3]

This paper aims at better understanding the respective roles of firm-specific demand and productivity for labor market dynamics. To this end, we develop an equilibrium model of firm dynamics with (i) product and labor market frictions, (ii) wage and price dispersion, and (iii) separate roles for demand and productivity shocks. In a quantitative analysis, we examine how well this model captures several features of the cross-sectional dynamics of prices, wages, labor productivity, and employment for German manufacturing firms. We also use the model to explore the effects of product market deregulation on the labor market and on other macroeconomic outcomes.

In Section 2, we build an equilibrium model in which heterogeneous firms compete for workers

[1] Extensions of the Hopenhayn-Rogerson framework can address a variety of issues, such as international trade (Melitz (2003)), industry dynamics (Ericson and Pakes (1995)), labor-market policy (Alvarez and Veracierto (2001)), and cross-country productivity differences (Restuccia and Rogerson (2008)). Search and matching frictions in the labor market have been introduced into such settings by, e.g., Smith (1999), Acemoglu and Hawkins (2014), Elsby and Michaels (2013) and Kaas and Kircher (2015).

[2] The quinquennial manufacturing census data they use does not permit them to study the dynamics of firms over time, or labor flows and wages, which are of particular interest in this paper.

[3] This idea motivates Foster et al. (2016) to build a structural econometric model of firm dynamics in which product demand stochastically adjusts slowly as firms actively expend resources to build a customer base. They find that demand gaps across firms are substantial and are bridged only slowly over time: in a given industry, a firm that is five years old or younger is expected to sell only 41 percent of the output of a firm that is more than fifteen years old. As they stress, the plight of young firms hinges on the coexistence of high technological efficiency and low demand, which is an indication of an inefficient allocation of customers across firms.

in a frictional labor market and simultaneously compete for buyers in a frictional product market. Productivity and demand are firm-specific state variables, and idiosyncratic shocks to both variables have distinct implications for the employment, wage, output and price policies of a firm. Frictions in labor and product markets imply that growing firms need to build a workforce as well as a customer base over time. This sluggish adjustment gives rise to dispersion of wages and prices across firms: firms with a higher desire to grow offer higher wages to recruit more workers and they offer lower discount prices to attract new customers. With these ingredients, our theory combines the competitive-search models of Gourio and Rudanko (2014) and Kaas and Kircher (2015), and it introduces firm-specific demand shocks besides the usual productivity shocks into this combined framework.

Different from the standard Diamond-Mortensen-Pissarides framework, firms in our model employ multiple workers and operate under decreasing returns. Next to labor market frictions, we also introduce a search-and-matching process in the product market which is meant to capture the observation that firms spend substantial time and resources for sales and marketing activities in order to attract customers.[4] As in Gourio and Rudanko (2014), firms adjust slowly to productivity shocks since customer acquisition is costly and time-consuming. Different from their contribution, we address the role of demand for firm growth and we examine the labor market spillovers.

In Section 3, we calibrate the model to account for the joint dynamics of output and prices at the nine-digit product level in an administrative panel of manufacturing firms in Germany, which also contains information on employment, working hours and wages. The data shows that the the overall dispersion of annual price growth is substantial, and that it correlates negatively with output growth across firms, which suggests a dominant role of supply (cost) shocks for firm dynamics. Nonetheless, price increases go together with employment expansions, which also points at a prominent role of demand for firm growth. The calibrated model matches several cross-sectional features of the dynamics of prices, quantities and employment. Also in line, with the data, the model generates a negative correlation between firm-level prices and quantity labor productivity, which implies that quantity labor productivity is more dispersed than revenue labor productivity. Moreover, larger firms have higher quantity productivity, lower prices, and they pay higher wages. However, compared to the data, the model generates too little dispersion of prices, wages and productivity.[5]

In a policy experiment we examine the implications of product market deregulation, which we model as a cut in the startup cost of new firms. As new firms enter the economy, the price level falls and output increases, partly because the new firms are relatively more productive than existing firms. However, the surge of young productive firms also triggers an increase of wages which brings about a temporary fall in aggregate employment because existing firms lay

[4] In the US, marketing expenditure are as high as 7.7% of GDP (Arkolakis, 2010).

[5] Relatedly, Felbermayr et al. (2014) also examine patterns of wage dispersion in Germany using a quantitative two-country version of Kaas and Kircher (2015).

off some of their workers. Also in the transition, the policy leads to a substantial increase of price and productivity dispersion across firms. In the long run, however, there are substantial gains in aggregate productivity and output and modest employment gains.

Related to our work are several recent contributions that introduce product market search frictions into macroeconomic models. In the presence of such frictions, Bai et al. (2012) and Michaillat and Saez (2015) argue that aggregate demand shocks play a more prominent role than aggregate technology shocks. Kaplan and Menzio (2016), Petrosky-Nadeau and Wasmer (2015) and Den Haan (2013) combine frictions in product and labor markets, introducing new amplification mechanisms for business-cycle dynamics. Unlike our paper, none of these contributions addresses firm heterogeneity and the role of firm-specific demand.

Our work also relates to an empirical literature which investigates the dispersion of firm-level prices and productivity. While Abbott (1991) and Foster et al. (2008) document dispersion of producer prices in specific industries, Carlsson and Skans (2012) and Carlsson et al. (2014) use Swedish firm-level data for the manufacturing sector showing that unit labor costs are transmitted less than one-to-one to output prices and that much of the variation in output prices remains unexplained by productivity shocks. Furthermore, they find that employment responds negligibly to productivity shocks, while permanent demand shocks are the main driving force of employment adjustment. Smeets and Warzynski (2013) and De Loecker (2011) analyze the price effects of international trade on the basis of Danish and Belgian (resp.) manufacturing firms. Kugler and Verhoogen (2011) use Colombian firm data to document a positive relationship between plant size and prices, which contrasts with the negative correlation between firm size and prices reported by Foster et al. (2008) and Roberts and Supina (1996). Our findings, which are also based on rather homogeneous products, are consistent with the latter results.

# 2 The Model

In this section we build a canonical model that describes the dynamics of firms in the presence of frictions in product and labor markets. In the product market, firms compete for buyers via costly sales activities and by offering discounts on their products, which helps to build a customer base. In the labor market, firms build up a workforce by spending resources on recruitment and by offering long-term contracts to new hires.

There is a representative household which owns all the firms. The household comprises a fixed stock of workers and it sends buyers to purchase goods. A worker can be either employed at a firm or unemployed; likewise, a buyer can be either attached to a firm or unattached. Search in both markets is competitive: Firms post employment contracts to attract job seekers, and they offer discounts on product prices to attract new customers. In both markets, firms trade off higher matching rates against more profitable offers. Firms face idiosyncratic demand and

productivity shocks. In the event of adverse shocks, a firm may find it optimal to layoff workers; it may also decide not to serve some of its previous customers.
We describe a stationary equilibrium in which search values of buyers and workers are constant over time, while individual firms' employment and sales grow and shrink, depending on their idiosyncratic productivity and demand shocks. We then establish a welfare theorem which permits a tractable equilibrium characterization by a social planning problem.

## 2.1 The Environment

**Goods and preferences.** The representative household consumes the goods produced by firms, as well as a separate numeraire good. Utility of the household is $\sum_t \beta^t[e_t + u(C_t)]$, where $e_t$ is consumption of the numeraire good, $u$ is a concave utility function and $\beta$ is the discount factor. $C_t = \int y_t(f)c_t(f)d\mu_t(f)$ is a consumption aggregator which integrates over the measure $\mu_t$ of active firms $f$ in period $t$ at which the household buys $c_t(f)$ units of output. $y_t(f)$ is an idiosyncratic (firm-specific) taste parameter which reflects, for instance, local preferences or quality differences between firms and over time. This preference specification captures the idea that goods within an industry are close substitutes and $C$ stands for the consumption of the industry output. We abstract from imperfect substitutability across industries or from industry-specific taste shocks.[6] In a stationary equilibrium, $C_t = C$ is a constant, as is the marginal rate of substitution between the consumption aggregator and the numeraire, $u'(C)$. All prices, wages and costs defined below are expressed in units of the numeraire good.
**Workers and customers.** There is a constant stock $\bar{L}$ of workers who are members of the household. A worker can be either employed at a firm or unemployed. An unemployed worker produces $b$ units of the numeraire good. The household also has a large number of potential buyers that are either attached to the customer base of a firm or that search for purchases elsewhere in the goods market.[7] Any active buyer (shopping or searching) imposes a cost $c$ on the household; once matched to a firm, the buyer can buy up to one unit of the good produced by the firm per period. Attached customers or employed workers do not search.
**Technology.** A firm with $L$ workers produces $xF(L)$ units of its output good. $F$ is increasing and concave and it satisfies the Inada condition $F'(L) \to 0$ for $L \to \infty$. $x$ is the firm's idiosyncratic productivity. With this reduced-form modeling of a firm's production technology, changes of $x$ stand for any type of supply-side shocks, such as technology changes or price changes of factor inputs besides labor.
**Demand.** A firm with $B$ customers can sell up to $B$ units of output. Every customer of the

[6] Such features could easily be introduced into our framework if $u$ aggregates the consumption from different industries. In fact, with symmetric Dixit-Stiglitz preferences and in the absence of industry-specific shocks, our framework is isomorphic to such a more general economy.

[7] A "buyer" can also be interpreted as a unit of shopping time that can be used to either buy a good from a previously known seller or to search for a purchase elsewhere.

firm wishes to buy one unit of the firm's good, as long as the unit price together with the shopping cost $c$ is smaller than the marginal rate of substitution between the firm's good and the numeraire good, which is $u'(C)y$ where $y$ is the firm-specific demand state. Because the good is non-storable, the firm is naturally constrained by $B \leq xF(L)$ in any period. If that inequality is strict, the firm wastes some of its output.

**Shocks.** Both $x$ and $y$ follow a joint Markov process on a finite state space. We write $z = (x, y) \in Z$ and denote $\pi(z_+|z)$ the transition probability from $z$ to $z_+$. For a firm of age $a$, we write $z^a = (z_0, \ldots, z_a)$ for the shock history from the entry period (firm age zero) up to the current period (firm age $a$), where $z_k = (x_k, y_k)$, $k = 0, \ldots, a$. $\pi^a(z^a)$ denotes the unconditional probability of that history event.

**Recruitment and sales activities.** For recruitment and sales, the firm spends $r(R, L)$ and $s(S, L)$ respectively, where $R$ and $S$ measure recruitment and sales effort and $L$ is the size of the firm's workforce before it matches with new workers and customers. Such recruitment and sales costs represent pecuniary costs as well as opportunity costs of managers and staff assigned to these non-productive activities. Both $r$ and $s$ are increasing and convex in their respective first arguments. They are non-increasing in the size of the workforce $L$ to capture scale effects.

**Labor market search.** Search in the labor market is directed. Recruiting firms offer long-term contracts to new hires. They are matched with unemployed workers in submarkets that differ by the offered contract values. In a given submarket, a firm hires $m(\lambda) \leq \lambda$ workers per unit of recruitment effort, where $\lambda$ measures unemployed workers per unit of recruitment effort in the submarket, and $m$ is a strictly increasing and concave function. Hence, $m(\lambda)/\lambda$ is the probability that an unemployed worker finds a job in this submarket, which decreases in $\lambda$. An employment contract specifies wage payments and separation probabilities contingent on realizations of firm-specific shocks. We write $\mathcal{C}^a = (w^a(z^k), \delta^a_w(z^{k+1}))_{k \geq a}$ for the employment contract of a worker who is hired by a firm of age $a$. $w^a(z^k)$ is the wage that the worker earns when the firm has age $k \geq a$, conditional on the shock history $z^k$ and conditional on staying employed at this firm. $\delta^a_w(z^{k+1})$ is the probability to separate from the firm in event history $z^{k+1}$ with $k + 1 > a$.

**Product market search.** Search in the product market is also directed, but firms cannot commit to long-term contracts.[8] Instead, firms that aim to expand the customer base offer discount prices $p$ to new customers. In all subsequent periods, attached customers con-

[8]The assumption that firms offer long-term contracts to workers though not to customers is intended to reflect realistic features of worker-firm and customer-firm relationships. Although long-term contracts with customers are common in some industries, they tend to be rather short. For German manufacturing firms, Stahl (2010) finds that although 50% of sales are undertaken in written contracts, the average contract duration is just 9 months. With an annual calibration, the absence of price commitment seems a plausible abstraction. Nonetheless, in subsection 2.4 we consider the case where firms offer long-term contracts and do not price discriminate among customers.

tinue purchasing at this firm, but anticipate that the firm charges the reservation price $p^R$ which makes the buyer exactly indifferent between buying at this firm or remaining inactive. Unattached buyers and selling firms are matched in submarkets that differ by the buyers' match values. For each unit of sales effort, the firm attracts $q(\varphi) \leq \varphi$ new customers, where $\varphi$ is the measure of unattached buyers per unit of sales effort in the submarket, and $q$ is an increasing and concave function. An unattached buyer searching for purchases is successful with probability $q(\varphi)/\varphi$, which is a decreasing function of $\varphi$.

**Entry, separations and exit.** New firms can enter the economy at cost $K > 0$ with zero workforce and zero customer base. They draw an initial productivity and demand state $(x, y)$ from probability distribution $\pi^0$. Any existing firm, depending on its supply and demand shocks, separates from workers according to the contractual commitments. Separated workers can search for jobs in the same period. The firm may also decide not to serve some of its attached customers who then leave the firm's customer base. Workers quit the job into unemployment with exogenous probability $\bar{\delta}_w$, and buyers leave the customer pool of a firm with exogenous probability $\bar{\delta}_b$. This implies that the actual customer churn rate is bounded below by $\delta_b \geq \bar{\delta}_b$. Likewise, the contractual state-contingent worker separation rates are bounded below by $\delta_w \geq \bar{\delta}_w$. At the end of the period, after firms produce, sell and pay workers, any firm exits with probability $\delta$ in which case all its workers enter the unemployment pool and all its customers become unattached.

## 2.2 Competitive Search Equilibrium

We describe a stationary equilibrium in which search values of workers and customers, as well as the distributions of workers and customers across firm types, are constant over time. Any firm's policy only depends on the idiosyncratic shock history $z^a$ where $a$ is the firm's age. Hence we identify the different firm types with $z^a$.

### 2.2.1 Workers

Let $U$ denote the value of an unemployed worker and let $W(\mathcal{C}^a, z^k)$ denote the value of an employed worker in contract $\mathcal{C}^a$ at firm $z^k$ with $k \geq a$. Those values represent the marginal contribution of the worker to the representative household's utility. Unemployed workers observe the offered contracts $\mathcal{C}^a$ at firm types $z^a$ and the corresponding market tightness $\lambda$ in the submarkets in which value-equivalent contracts are traded. The unemployed worker's Bellman equation is

$$U = \max_{(W(\mathcal{C}^a, z^a), \lambda)} \frac{m(\lambda)}{\lambda} W(\mathcal{C}^a, z^a) + \Big(1 - \frac{m(\lambda)}{\lambda}\Big)[b + \beta U] \, , \tag{1}$$

where maximization is over all active submarkets $(W(\mathcal{C}^a, z^a), \lambda)$. With probability $m(\lambda)/\lambda$, the worker finds employment in which case the continuation value is $W(\mathcal{C}^a, z^a)$. With the

counter-probability, the worker earns unemployment income $b$ and stays unemployed to the next period.
The employment value $W(\mathcal{C}^a, z^k)$ satisfies the Bellman equation

$$W(\mathcal{C}^a, z^k) = w^a(z^k) + \beta(1-\delta)\mathbb{E}_{z^k} W'(\mathcal{C}^a, z^{k+1}) + \beta\delta U \ . \tag{2}$$

This worker earns $w^a(z^k)$ in the current period. At the end of the period, the firm exits with probability $\delta$ in which case the worker becomes unemployed. With the counter-probability, the worker stays employed to the next period which yields continuation value $W'(\mathcal{C}^a, z^{k+1})$ where the prime indicates the employment value *before* the firm separates from workers after demand and productivity shocks are realized:

$$W'(\mathcal{C}^a, z^k) = [1 - \delta_w^a(z^k)]W(\mathcal{C}^a, z^k) + \delta_w^a(z^k)U \ . \tag{3}$$

With contractual separation probability $\delta_w^a(z^k)$, the worker leaves the firm and can search for employment in the same period (continuation utility $U$). Otherwise the worker stays employed with continuation utility $W(\mathcal{C}^a, z^k)$.
It is convenient to define the option value of search in submarket $(W, \lambda)$ by

$$\rho(W, \lambda) \equiv \frac{m(\lambda)}{\lambda}\Big(W - b - \beta U\Big) \ .$$

Then, the flow utility value of unemployment satisfies

$$(1-\beta)U = b + \rho^* \ , \tag{4}$$

where $\rho^* \equiv \max \rho(W(\mathcal{C}^a, z^a), \lambda)$ is the maximal search value. It follows that any contract that attracts unemployed workers (i.e., $\lambda > 0$) yields the same search value $\rho^*$.

### 2.2.2 Customers

The household can send arbitrarily many buyers to the goods market at shopping cost $c$ per buyer. Hence the marginal contribution of any buyer to the household's utility must be zero. Searching buyers observe unit discount prices $p$ offered by firms of different types. Buyers and firms are matched in submarkets that yield identical match values $u'(C)y_a - p$ to the buyer. Since the firm charges the reservation price for all attached customers in subsequent periods, the continuation value beyond the matching period is zero. Let $\varphi$ denote buyer-to-sales-effort ratio in such a submarket with matching probability $q(\varphi)/\varphi$. The expected gain from searching must equal the search cost:

$$c = \max_{(p, y_a, \varphi)} \frac{q(\varphi)}{\varphi}\Big[u'(C)y_a - p\Big] \ , \tag{5}$$

where maximization is over all active submarkets $(p, y_a, \varphi)$. Any discount price that attracts new customers (i.e., $\varphi > 0$) yields the same search value $c$. It follows that discount prices are linked to market tightness $\varphi$ via

$$p = u'(C)y_a - \frac{c\varphi}{q(\varphi)} \ .$$

Reservation prices $p^R$ charged on existing customers make the buyer indifferent between buying at this price after incurring the shopping cost $c$, or remaining inactive. Hence,

$$p^R = u'(C)y_a - c \ .$$

### 2.2.3 Firms

A firm of type $z^a$ takes as given the workers hired in earlier periods, $L^\tau$, $\tau = 0, \ldots, a-1$, together with their respective contracts $\mathcal{C}^\tau$.[9] It also takes as given the existing stock of the customer base $B_-$. Hence the firm's state vector is $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$. Let $J_a(\sigma)$ denote the value of the firm at the beginning of the period. The firm chooses recruitment policy $(\lambda, R, \mathcal{C}^a)$ and sales policy $(\delta_b, \varphi, S, p, p^R)$ to solve the problem

$$J_a(\sigma) = \max_{(\lambda,R,\mathcal{C}^a),(\delta_b,\varphi,S,p,p^R)} \Big\{ p^R B_-(1-\delta_b) + pq(\varphi)S - W - r(R, L_0) - s(S, L_0) + \beta(1-\delta)\mathbb{E}J_{a+1}(\sigma_+) \Big\} \tag{6}$$

subject to

$$\sigma_+ = [(L^{\tau+}, \mathcal{C}^\tau)_{\tau=0}^{a}, B, z^{a+1}] \ , \ \mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k\geq a} \ , \ \delta_w^a(.) \geq \bar{\delta}_w \ , \tag{7}$$

$$L^{\tau+} = (1 - \delta_w^\tau(z^a))L^\tau \ , \ \tau = 0, \ldots, a-1 \ , \ L^{a+} = m(\lambda)R \ , \ L_0 = \sum_{\tau=0}^{a-1} L^{\tau+} \ , \tag{8}$$

$$W = \sum_{\tau=0}^{a} w^\tau(z^a)L^{\tau+} \ , \tag{9}$$

$$B = B_-(1 - \delta_b) + q(\varphi)S \ , \ \delta_b \geq \bar{\delta}_b \ , \tag{10}$$

$$B \leq xF(L) \ , \ L = \sum_{\tau=0}^{a} L^{\tau+} \ , \tag{11}$$

$$\rho^* = \rho(W(\mathcal{C}^a, z^a), \lambda) \ \text{ if } \ \lambda > 0 \ , \tag{12}$$

$$p = u'(C)y_a - \frac{c\varphi}{q(\varphi)} \ \text{ if } \ \varphi > 0 \ , \ p^R = u'(C)y_a - c \ . \tag{13}$$

The firm's problem (6) is to maximize revenue from sales to existing and new customers minus expenditures for wages, sales and recruitment plus the expected continuation profit. The firm

[9] Without loss of generality, all workers hired by a firm of a given type are hired in the same contract, which is an optimal policy of the firm (see Kaas and Kircher (2015) for a formal argument).

is committed to separation rates $\delta_w^\tau(z^a)$, $\tau < a$, for workers hired in the past. For workers hired in this period, it is free to commit to any future separation rates, $\delta_w^a(z^k) \geq \bar{\delta}_w$. Together with wages $w^a(.)$, they define the contract $\mathcal{C}^a$ offered to new hires. Equations (8) say how employment in different worker cohorts evolves over time. $L_0$ is the firm's workforce *before hiring* which affects recruitment and sales costs. Equation (9) states the total wage bill of the firm. (10) says how the firm's customer stock evolves. Because the firm is not committed in the product market, it decides customer separations $\delta_b \geq \bar{\delta}_b$ (if required) freely. Condition (11) says that the firm cannot sell more than what it produces. Regarding wage contract offers to new hires $\mathcal{C}^a$, as well as discount price offers $p$ to new customers, the firm respects the search incentives of workers and customers, as expressed by constraints (12) and (13). That is, to attract more workers per recruitment effort (higher $\lambda$), the firm needs to offer a more attractive employment contract. Likewise, to attract more customers per sales effort (higher $\varphi$), the firm needs to offer a lower discount price. The last equation in (13) says that the firm optimally charges the reservation price $p^R$ on existing customers.

### 2.2.4 Equilibrium

We can express all firm policy functions defined above to depend on the firm's history $z^a$, ignoring the dependence on pre-committed contracts and worker cohorts. This is feasible because those firm state variables evolve endogenously as functions of the firm's past shocks and policies. Hence, all firm policies (in stationary equilibrium) are functions of the idiosyncratic state history. For a firm of type $z^a$, we write $\lambda(z^a)$ and $R(z^a)$ for the recruitment policy, $\varphi(z^a)$ and $S(z^a)$ for the sales policy, and so on.[10] We also define

$$L(z^a) = \sum_{\tau=0}^{a} L^\tau(z^a) \ , \tag{14}$$

$$B(z^a) = B(z^{a-1})[1 - \delta_b(z^a)] + q(\varphi(z^a))S(z^a) \ , \tag{15}$$

for the stocks of workers and customers in firm history $z^a$, where $L^\tau(z^a) = L^\tau(z^{a-1})[1 - \delta_w^\tau(z^a)]$ if $a > \tau$, $L^a(z^a) = m(\lambda(z^a))R(z^a)$, and $B(z^{-1}) = 0$. Further, there are

$$N(z^a) = N_0(1-\delta)^a \pi^a(z^a) \tag{16}$$

firms of type $z^a$ when $N_0$ is the mass of entrant firms in any period. We are now ready to define the stationary equilibrium.

**Definition:** *A stationary competitive search equilibrium is a list of value functions $U$, $W$, $W'$, $J_a$, firm policies $\lambda$, $R$, $\varphi$, $S$, $\delta_b$, $\mathcal{C}^a = (w^a(.), \delta_w^a(.))$, $(L^\tau)_{\tau=0}^a$, $L$, $B$, $p$, $p^R$ which are all functions of the firm type $z^a$, entrant firms $N_0$, aggregate consumption $C$ and a search value $\rho^*$ such that*

[10] With abuse of notation, we do not index those functions by the firm's age.

*(a) Workers' value functions $U$, $W$, $W'$ and the search value $\rho^*$ describe optimal search behavior, equations (1)–(4).*

*(b) Buyers search optimally, equation (5), and aggregate consumption is given by*

$$C = \sum_{z^a} y_a N(z^a) B(z^a) \; . \tag{17}$$

*(c) Firms' value functions $J_a$ and policy functions solve problem (6)–(13), and $L(.)$, $B(.)$ and $N(.)$ evolve according to (14), (15) and (16).*

*(d) Firm entry is optimal. That is, $N_0 > 0$ and*

$$K = \sum_{z^0} \pi^0(z^0) J_0(0, z^0) \; . \tag{18}$$

*(e) Aggregate resource feasibility:*

$$\bar{L} = \sum_{z^a} N(z^a) \Big\{ L(z^a) + [\lambda(z^a) - m(\lambda(z^a))] R(z^a) \Big\} \; . \tag{19}$$

Aggregate resource feasibility (e) requires that any worker either belongs to the workforce $L(z^a)$ at one of $N(z^a)$ firms of type $z^a$ or that the worker is searching for a job in the same submarket as this firm and does not find a job: Precisely, $\lambda(z^a)R(z^a)$ workers are searching for employment per firm of type $z^a$, and share $1 - m(\lambda(z^a))/\lambda(z^a)$ of those workers are not successful and hence remain unemployed.

We can verify that the aggregate resource constraint for the numeraire good is satisfied in a stationary equilibrium. The budget constraint of the representative household is

$$\begin{aligned}
&\sum_{z^a} N(z^a) \Big[ p^R(z^a) B(z^{a-1})[1 - \delta_b(z^a)] + p(z^a) q(\varphi(z^a)) S(z^a) \Big] + e \\
&= \sum_{z^a} N(z^a) \Big[ \pi(z^a) + \sum_{\tau \le a} L^\tau(z^a) w^\tau(z^a) \Big] + b \Big[ \bar{L} - \sum_{z^a} N(z^a) L(z^a) \Big] \\
&\qquad - K N_0 - c \sum_{z^a} N(z^a) \Big\{ B(z^a) + [\varphi(z^a) - q(\varphi(z^a))] S(z^a) \Big\} \; .
\end{aligned}$$

The left-hand side expresses the household's consumption expenditures for the differentiated goods and for the numeraire $e$. The right-hand side gives the household's income which includes wage and profit income at all firm types $z^a$ plus income from home production net of expenditures for the creation of new firms and for shopping. Shopping costs are paid both for searching and for attached buyers. Profit income of firm $z^a$ is

$$\begin{aligned}
\pi(z^a) = {} & p^R(z^a) B(z^{a-1})[1 - \delta_b(z^a)] + p(z^a) q(\varphi(z^a)) S(z^a) - \sum_{\tau \le a} L^\tau(z^a) w^\tau(z^a) \\
& - r(R(z^a), L_0(z^a)) - s(S(z^a), L_0(z^a)) \; .
\end{aligned}$$

Rearranging shows that the household's consumption of the numeraire good[11] is identical to the home production of the numeraire good net of the costs for recruitment, sales, firm entry, and shopping which are all paid in the numeraire good:

$$e = b\Big[\bar{L} - \sum_{z^a} N(z^a)L(z^a)\Big] - \sum_{z^a} N(z^a)\Big[r(R(z^a), L_0(z^a)) + s(S(z^a), L_0(z^a))\Big]$$
$$-KN_0 - c\sum_{z^a} N(z^a)\Big\{B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big\} .$$

## 2.3 Social Optimum and Firm Policies

A stationary competitive search equilibrium is identical to the solution of a social planning problem which maximizes the utility of the representative household, starting from the given initial distribution of customers and workers across firms. The social planner is subject to search frictions in product and labor markets, decides firms' recruitment and sales efforts, and assigns workers and customers into submarkets which differ by the characteristics of the searching firms. In the Appendix we formally define the planner's problem and show that it permits a rather simple recursive formulation at the level of individual firms. Let $\rho$ denote the multiplier on the aggregate resource condition (19) which is a binding constraint for the planner. For a given firm in a given period, the planner takes as given the firm's stocks of workers and customers, $L_-$ and $B_-$, as well as the current productivity and demand state $z = (x, y)$. Write $G(L_-, B_-, z)$ for the social value of a firm, i.e., the contribution of the firm to the representative household's utility. It satisfies the recursive equation

$$G(L_-, B_-, z) = \max_{(\lambda,R,\delta_w),(\varphi,S,\delta_b)} \Big\{u'(C)yB - bL - r(R, L_-(1-\delta_w)) - s(S, L_-(1-\delta_w))$$
$$- \rho[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E}_z G(L, B, z_+)\Big\} , \tag{20}$$

subject to

$$L = L_-(1-\delta_w) + m(\lambda)R , \; B = B_-(1-\delta_b) + q(\varphi)S ,$$
$$B \leq xF(L) , \; \delta_w \geq \bar{\delta}_w , \; \delta_b \geq \bar{\delta}_b .$$

The flow surplus of the firm includes the marginal utility value of sales, $u'(C)yB$, net of the opportunity cost of employment, $bL$, net of recruitment and sales costs, $r(.)$ and $s(.)$, net of shopping costs and net of the social costs for the workers who are linked to this firm in the given period. Regarding the latter, there are $L$ workers employed at the firm, and $(\lambda - m(\lambda))R$ unemployed workers who search for employment at this firm and do not find a job. Any of

[11] If $e < 0$, the household produces $-e$ units of the numeraire good which, together with unemployment income and net of shopping costs is identical to the firms' expenditures on entry, recruitment and sales.

those workers can neither work nor search for jobs elsewhere in the economy and hence impose a social cost equal to the multiplier $\rho$. Shopping costs are incurred by the $B$ customers buying at this firm, but also by $(\varphi - q(\varphi))S$ unsuccessful customers who search for purchases in the same submarket as this firm. The planner neither needs to commit to separation rates, nor is there a need to discriminate between workers hired at different points in time (see the Appendix for details).

The Inada condition $F'(\infty) = 0$ and standard dynamic programming techniques imply that problem (20) has a solution $G : [0, L^{\max}] \times [0, B^{\max}] \times Z \to \mathbb{R}$ which is continuous in $(L_-, B_-) \in [0, L^{\max}] \times [0, B^{\max}]$ for some appropriately specified upper bounds $L^{\max}$ and $B^{\max}$.

Because $G(0, 0, z)$ denotes the social firm value upon entry, socially optimal entry requires that

$$K = \sum_z \pi^0(z) G(0, 0, z) \ . \tag{21}$$

We show that a joint solution of (20) and (21) together with the resource constraint (19) indeed gives rise to a stationary planning solution. Moreover we prove a welfare theorem: the stationary planning solution corresponds to a stationary competitive search equilibrium with identical firm policies and where the social multiplier on the resource constraint coincides with the equilibrium search value of workers: $\rho = \rho^*$.[12]

**Proposition 1** *Suppose that $(\rho, G, N_0, C)$ solves the recursive social planning problem (20) together with (21), aggregate consumption (17) and the aggregate resource constraint (19), where $N(z^a)$ is defined by (16), and $L(z^a)$ and $B(z^a)$ are recursively defined by iterating over the policy functions of problem (20). Then:*

*(a) The firm policies solve the sequential social planning problem which maximizes the discounted household's utility, starting from the initial distribution $(N(z^a), L(z^a), B(z^a))_{z^a}$.*

*(b) $(\rho, G, N_0, C)$ corresponds to a stationary competitive search equilibrium with identical firm policies and search value $\rho^* = \rho$.*

The welfare theorem (b) extends well-known efficiency results for competitive search economies (cf. Moen (1997)) to a setting with two-sided market frictions and firms with multiple workers and multiple buyers. Kaas and Kircher (2015) prove a similar result for multi-worker firms in an environment without product market frictions (and without demand shocks). The main intuition for efficiency is that private search values of workers and customers in the competitive

[12] Assuming alternatively that the household has a fixed stock of buyers (or shopping time) $\bar{B}$, rather than a large potential stock of buyers, does not change this characterization of a stationary equilibrium by a planning solution. The only difference is that $c$ stands for the multiplier on a resource constraint for buyers, defined similarly as (19), and in a stationary equilibrium the buyers' search value $c^*$ coincides with the social value $c$. This alternative formulation makes a difference, however, for comparative statics or for the analysis of aggregate shocks.

search equilibrium reflect their social values; by either posting long-term contracts (to workers) or discounts (to customers), firms fully internalize all congestion externalities of search. They also internalize the trade-off between costly search effort and higher matching rates. We elaborate on this trade-off in the next paragraph. Long-term contingent contracts further implement the socially efficient worker separation rates.

The socially optimal recruitment and sales policies permit a straightforward characterization which link the effort of search (recruitment and sales expenditures) to the matching rates, resembling earlier findings of Kaas and Kircher (2015) and Gourio and Rudanko (2014). Write $\gamma$ for the multiplier on the constraint $B \leq xF(L)$. Then, the first-order conditions for $R$, $\lambda$, $S$, and $\varphi$, for positive recruitment and sales activities, are:

$$-r_1' - \rho\lambda + [\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma xF' - b]m = 0 \ , \tag{22}$$

$$-\rho R + [\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma xF' - b]m'R = 0 \ , \tag{23}$$

$$-s_1' - c\varphi + [\beta(1-\delta)\mathbb{E}G_2'(+) + (yu'(C) - \gamma)]q = 0 \ , \tag{24}$$

$$-cS + [\beta(1-\delta)\mathbb{E}G_2'(+) + (yu'(C) - \gamma)]q'S = 0 \ . \tag{25}$$

Here $\mathbb{E}G_i'(+)$, $i = 1, 2$, denotes the derivative of $\mathbb{E}G(L, B, z_+)$ with respect to the first/second argument. Equations (22) and (23) can be combined to obtain

$$r_1'(R, L_-(1-\delta_w)) = \rho\Big[\frac{m(\lambda)}{m'(\lambda)} - \lambda\Big] \ . \tag{26}$$

Similarly, (24) and (25) yield a relation between $\varphi$ and $S$:

$$s_1'(S, L_-(1-\delta_w)) = c\Big[\frac{q(\varphi)}{q'(\varphi)} - \varphi\Big] \ . \tag{27}$$

Condition (26) says that across firms (of a given size) recruitment effort and matching rates are positively related: if the planner wishes that a firm grows faster, this is achieved by spending more on recruitment (higher $R$) but also by assigning more workers to find employment at this type of firm. In the decentralization with competitive search, faster growing firms spend more on recruitment and they offer higher salaries, attracting more workers (cf. Kaas and Kircher (2015)). Condition (27) expresses a similar relation in the product market: firms that spend more on sales also have lower discount prices (cf. Gourio and Rudanko (2014)).

Another straightforward insight of the first-order conditions of problem (20) is that the firm does not recruit workers and fire workers at the same time, i.e. $R > 0$ and $\delta_w > \bar{\delta}_w$ are mutually exclusive. To see this formally, it follows from (22) that $R > 0$ requires that

$$\rho < \frac{m(\lambda)}{\lambda}[\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma xF' - b] \ .$$

For $\delta_w > \bar{\delta}_w$, the first-order condition is

$$\rho = \beta(1-\delta)\mathbb{E}G_1'(+) + \gamma xF' - b - (r_2' + s_2') \geq \beta(1-\delta)\mathbb{E}G_1'(+) + \gamma xF' - b \ ,$$

where the inequality follows since $r$ and $s$ are non-increasing in employment. Because of $m(\lambda)/\lambda \leq 1$, the two conditions are mutually exclusive. By a similar argument, the firm does not reject existing customers and attract new buyers simultaneously. It may however be possible that the firm hires new workers and rejects customers at the same time (e.g., in response to a strong positive demand shock). Conversely, it is conceivable that the firm fires workers and attracts new customers at the same time (e.g., in response to a strong positive productivity shock).

## 2.4 Revenue, Prices and Wages

Productivity and demand shocks impact the joint dynamics of the firms' revenue (total sales value) and employment in distinct ways. While the firms' employment dynamics $L(z^a)$ follows directly from the solution of the social planning problem, the sales dynamics requires the calculation of equilibrium prices in the decentralized competitive search equilibrium. Using (13), the revenue of firm $z^a$ is

$$\begin{aligned} Re(z^a) &\equiv p^R(z^a)B(z^{a-1})(1-\delta_b(z^a)) + p(z^a)q(\varphi(z^a))S(z^a) \\ &= u'(C)y_a B(z^a) - c\Big[B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big] \ . \end{aligned}$$

Note that both prices $p$ and $p^R$ are increasing in firm-specific demand $y$. Via (27), there is however a countervailing effect of $y$ on the discount price $p$: if a firm experiences a positive demand shock, it increases sales effort $S$ and it wants to attract more buyers per unit of effort which is achieved by a lower discount price ($q(\varphi)/\varphi$ falls). The firm's (average) price is $P(z^a) \equiv Re(z^a)/B(z^a)$, because $B(z^a)$ is the quantity of output units sold.
Other decentralizations without price discrimination (albeit with commitment) are also possible. Suppose for example that each firm charges the same price $p(z^a)$ for all its customers who also know that they are separated from firms with identical probability $\delta_b(z^a)$. Then optimal search requires that

$$c = \frac{q(\varphi(z^a))}{\varphi(z^a)} Q(z^a) \ ,$$

where the value of a customer $Q(z^a)$ buying from firm $z^a$ satisfies the Bellman equation.

$$Q(z^a) = u'(C)y_a - p(z^a) + \beta(1-\delta)\mathbb{E}_{z^a}\Big([1-\delta_b(z^{a+1})][Q(z^{a+1}) - c]\Big) \ .$$

Given firm policies $\varphi(z^a)$ and $\delta_b(z^a)$, these two equations can be directly solved for non-discriminatory prices $p(z^a)$ and for the firms' revenue $Re(z^a) = p(z^a)B(z^a)$.
We can also solve for wages in the competitive search equilibrium. As in the social planning problem specified in the previous subsection, separation rates for all workers in a firm are assumed identical:[13] $\delta_w^\tau(z^a) = \delta_w(z^a)$. Furthermore, we consider a particular decentralization

[13] Other implementations are feasible as long as total separations at the firm are unchanged.

in which each firm pays the same wage to all its workers, i.e. $w^\tau(z^a) = w(z^a)$ for all $\tau \leq a$. In this case, worker values $W$ and $W'$ do not depend on the particular contract $\mathcal{C}^a$ and can therefore be written $W(z^a)$ and $W'(z^a)$, so that (1)–(4) become

$$\begin{aligned}
U &= \frac{m(\lambda(z^a))}{\lambda(z^a)} W(z^a) + \Big(1 - \frac{m(\lambda(z^a))}{\lambda(z^a)}\Big)[b + \beta U] \ , \\
W(z^a) &= w(z^a) + \beta(1-\delta)\mathbb{E}_{z^a} W'(z^{a+1}) + \beta\delta U \ , \\
W'(z^a) &= [1 - \delta_w(z^a)]W(z^a) + \delta_w(z^a) U \ , \\
U &= b + \rho + \beta U \ .
\end{aligned}$$

These equations can be solved for the worker surplus

$$W(z^a) - U = \rho\Big[\frac{\lambda(z^a) - m(\lambda(z^a))}{m(\lambda(z^a))}\Big] \equiv S^w(z^a) \ ,$$

and for wages:

$$w(z^a) = b + \rho + S^w(z^a) - \beta(1-\delta)\mathbb{E}_{z^a}\Big([1 - \delta_w(z^{a+1})]S^w(z^{a+1})\Big) \ . \tag{28}$$

Alternative wage schedules are feasible as well since the model does not pin down individual wage profiles. For example, it is conceivable that firms offer flat wage contracts to all workers, thus paying different wages to workers hired at different points in time; see Appendix C in Kaas and Kircher (2015) for the procedure how to calculate wages in that case. Another (less plausible) possibility is the absence of commitment: here the firm pays a hiring bonus upfront and (identical) reservation wages afterwards to all workers.

# 3 Quantitative Evaluation

In this section we quantitatively examine the role of supply and demand for the dynamics of firms and for the dispersion of prices, wages, and productivity. We describe the data and measurement first. We then calibrate the model to match certain features of the dynamics of German manufacturing firms, and use the calibrated model to examine the impact of product market deregulation on output, employment, prices and productivity.

## 3.1 Data

This section describes the data and how we treat them. Further details and results are contained in Appendix B. We use the German administrative firm-level data *Amtliche Firmendaten für Deutschland* (AFiD) which are provided by the Research Data Centers of the Federal and State Statistical Offices. Officially collected by the statistical offices based in each of the federal states, the data are a result of several combined statistics whereby all establishments with operations in the country are required by law to report them.

Depending on the underlying statistics, the data are provided in different panels. We work with the *AFiD-Panel Industriebetriebe* covering establishments in manufacturing, mining and quarrying in the years 2005–2007.[14] For establishments with at least 20 employees, the panel includes total annual sales value, total employment, wages and hours worked in the reporting year.[15] We merge this panel with the *AFiD-Module Produkte* that has recordings on quantities and sales values for nine-digit products that the establishment produced in a given year. For around 20 percent of the establishments in the year 2006, we can further match the panel with the *AFiD-Module Verdienste* which is a matched employer–employee dataset containing detailed information for a subsample of the establishment's workers, such as wage, working hours, age, education, gender, and job tenure.[16] For our analysis of wage dispersion, we consider this subsample of establishments.

The original merged dataset is unbalanced and has 375,016 establishment-product-years.[17] We clean the data to remove products with missing or invalid sales and quantity information and establishments with missing employment or wage information, less than 20 employees, or which are not active throughout the full year (see Appendix B for details). The remainder of the observations have positive sales values, quantities, valid units of measurement, employment, wage information and valid sectors. Establishments with missing reported working hours are automatically dropped only when statistics for hourly wages or hourly productivity are calculated. Since pricing decisions are likely made at the firm level, and in order to eliminate products traded between establishments in the same firm, we remove all establishments that belong to multi-establishment firms. We refer to these single-establishment firms as "firms" in the following text, keeping in mind that these firms represent the portion of smaller and medium-sized firms in German manufacturing. The final sample has 161,747 firm-product-years.

We measure the (average annual) price of a product by its sales value divided by the quantity sold. This price can reflect a quality component of the product which may differ across firms producing the same nine-digit product in the same year, or across time in the same firm. For our analysis of firm dynamics, i.e. year-to-year changes in price and quantity, we presume that quality adjustments within the same firm are rather unimportant. This leads us to

[14]The actual panel covers the much longer horizon 1995-2014. We are currently extending this analysis for the complete time period. This extension allows us to address business-cycle features of several cross-sectional statistics in the data and to relate them to model responses to aggregate productivity or demand shocks.

[15]The annual sales value is the sum of sales values reported in quarterly production surveys. Employment, wages and hours are obtained as averages from monthly surveys. We only consider establishments which are active throughout the full year.

[16]For the longer period 1995–2014, matched employer-employee modules are also available for the years 2001, 2010 and 2014.

[17]This corresponds to 135,074 establishment-years.

consider the *full sample*, defined precisely in Appendix B, for our analysis of firm dynamics. For cross-sectional price and productivity variation, however, quality differences between firms should be a bigger concern. Therefore we also consider a *homogeneous sample* which is based on those products which are measured in physical units of weight, length, area, or volume, whereas we remove all products which are measured in other units such as "items" or "pairs". The underlying hypothesis is that products measured in physical units have a lower degree of processing, so that quality differences are less relevant.[18] For further details about the two samples, the removal of outliers, and formal definitions of all variables described below, see Appendix B.

### 3.1.1 Firm Dynamics

This analysis is based on the *full sample*. We track a firm's product portfolio between any two years, and if the product portfolio changes we replace the missing product price with the quantity-weighted average price of this product obtained from other firms in the sample. Based on price and quantity information in any two consecutive years, we calculate a firm-level Paasche price index $P$, referred to as the firm's *price*). The firm's *quantity* $Q$ (output) equals the total sales value divided by this price index. We consider two measures of labor productivity: *Quantity labor productivity* ($\text{QLP}_E$, $\text{QLP}_H$), obtained by dividing quantity by employment or hours, and *revenue labor productivity* ($\text{RLP}_E$, $\text{RLP}_H$), obtained by dividing revenue by employment or hours.
Growth rates are expressed as log annual changes of the respective variables. Summary statistics of the growth rates of the main variables (after controlling for industry, region and year), are presented in 4. Price growth is quite dispersed across firms, more than employment, hours or wage growth, but less dispersed than quantity or revenue productivity growth. Price growth correlates negatively with output growth and with the growth of QLP. In a variance decomposition shown in Table 5 we find that price variation accounts for 50% of the variation in sales growth or in revenue productivity growth. Figure 5 shows kernel densities of the various log growth rates. Consistent with e.g. Carlsson and Skans (2012), there is a substantial spike around zero price growth.

### 3.1.2 Price, Wage and Productivity Dispersion

This analysis is based on the *homogeneous sample* as defined above. We define a firm-specific price index $\widetilde{P}$ as the firm's total sales value relative to the hypothetical sales value had the

[18] To give examples, the *homogeneous sample* includes products "1720 32 144: Fabric of synthetic fibers (with more than 85% synthetic) for curtains (measured in $m^2$)" and "2112 30 200: Cigarette paper, not in the form of booklets, husks, or rolls less than 5 cm broad (measured in $t$)", whereas it does not include "1740 24 300: Sleeping bags (measured in 'items')" and "2513 60 550: Gloves made of vulcanized rubber for housework usage (measured in 'pairs')".

firm sold its products at the (quantity-weighted) average market prices.[19] The firm-specific price index is a measure of the relative expensiveness of the firm relative to other firms selling the same products. As before, we define revenue labor productivity ($\mathrm{RLP}_E$, $\mathrm{RLP}_H$) as the total sales value divided by employment or hours. Quantity labor productivity is total sales value evaluated at average market prices divided by employment or hours, denoted $\mathrm{QLP}_E$ ($\mathrm{QLP}_H$).[20] Hence revenue productivity is the product between quantity productivity and the firm-specific price index (see Appendix B and Table 7 for a variance decomposition). Lastly, using detailed worker-level information in the year 2006, we define a firm-specific wage index $\widetilde{W}$ which expresses the ratio between the firm's actual wage bill and the hypothetical wage bill had the firm paid "average" (predicted) wages based on the observable worker characteristics. Then cross-sectional variation of the firm's hourly wage can be decomposed into the firm-specific wage index and into a predicted hourly wage that reflects worker observables (see Table 7).

All these variables are expressed in logs. Table 6 presents summary statistics conditional on industry, region and year. Consistent with the findings of Foster et al. (2008) for TFP dispersion, quantity labor productivity is more dispersed than revenue labor productivity, which is accounted for by the fact that prices correlated negative with quantity productivity. Figure 6 shows kernel density plots of the firm-specific price index, revenue and quantity productivity, both unweighted and weighted by employer size.

## 3.2 Parameterization

We calibrate the model at annual frequency and hence set the discount factor to $\beta = 0.96$. The production function is Cobb-Douglas $F(L) = L^\alpha$ where $\alpha = 0.7$ gives rise to a labor income share of about 70 percent. As Felbermayr et al. (2014), we set the firm exit rate to $\delta = 0.05$, and the exogenous worker separation rate to $\bar{\delta}_w = 0.02$ so that the total separation rate in equilibrium is around 7 percent.[21] The exogenous customer separation rate is set to $\bar{\delta}_b = 0.43$; this corresponds to the finding of Stahl (2010) that regular customers account for 57% of the annual sales in German manufacturing firms. This choice implies rather high customer turnover so that no firm in equilibrium voluntarily decides to separate from customers.

[19] This procedure is analogous to the construction of household-level price indices in Kaplan and Menzio (2015).

[20] In the previous subsection we measure *the growth of* QLP by deflating revenue by the firm's prices. Here we define QLP *in a given year* on the basis of average market prices for the firm's products. Therefore, the growth of the latter measure between any two years also reflects a change in average market prices. Appendix B.4.2 discusses results for a decomposition of a firm's RLP growth for the *homogeneous sample* into three components: (i) quantity labor productivity growth at fixed average prices, (ii) average price growth, (iii) firm-specific price growth.

[21] These targets are based on Fuchs and Weyh (2010) who measure plant-level job creation and destruction rates from the IAB Establishment History Panel for the period 2000–2006.

For recruitment and sales costs we adopt the constant-returns specifications $r(R,L) = \frac{r_0}{3}\left(\frac{R}{L}\right)^2 R$ and $s(S,L) = \frac{s_0}{3}\left(\frac{S}{L}\right)^2 S$ which are cubic functions of recruitment and sales effort; division by employment size makes sure that larger firms with proportionally higher recruitment and sales effort incur the same unit costs (cf. Merz and Yashiv (2007)). As in Kaas and Kircher (2015), convex recruitment costs give rise to sluggish employment adjustment with some variation in job-filling rates and wages across firms. Similarly in the product market, convex sales costs are responsible for variation in discount prices across firms. The scale parameters $r_0$ and $s_0$ are set to match plausible shares of spending on recruitment and advertising; specifically we target recruitment (advertising) expenditures to be 1 (2.5) percent of GDP as in Christiano et al. (2013) (Arseneau and Chugh (2007)). For the labor market matching function, we choose the Cobb-Douglas form $m(\lambda) = m_0\lambda^{0.5}$. The elasticity of 0.5 is a standard value (e.g. Petrongolo and Pissarides (2001)) and the scale parameter $m_0$ is set to match a stationary unemployment rate of 7 percent. In the absence of suitable empirical studies, we follow Arseneau and Chugh (2007) and Mathä and Pierrard (2011) and adopt the same Cobb-Douglas functional form $q(\varphi) = q_0\varphi^{0.5}$ in the product market, and we set parameter $q_0$ such that the average matching probability of a shopper is 50 percent. This reflects that consumers visit on average 1.1–3 stores for a given purchase (cf. Lehmann and Van der Linden (2010)), so that roughly half of all visits do not result in a match.[22]

The unemployment income parameter is set to 60 percent of the average wage to capture the unemployment replacement rate in Germany (cf. Krebs and Scheffel (2013)). According to the time use survey of the German Statistical Office, the average person in Germany spends 46 minutes shopping per day (including transit time) and he/she spends five hours on market and non-market work (excluding shopping). This leads us to set $c$ to equal 15 percent of the average wage.

The utility function has constant elasticity, $u(C) = \frac{u_0}{1-\sigma}C^{1-\sigma}$ with $\sigma \geq 0$. Lacking comparable estimates for Germany, we set $\sigma = 2/3$ so that the elasticity of industry demand corresponds to the mean estimate for U.S. manufacturing industries of Chang et al. (2009). This parameter does not affect any steady state outcomes of our model, it only matters for the analysis of aggregate changes. The marginal valuation of a good in the model equals $yu'(C)$ in units of the numeraire good. As the unit of measurement is arbitrary, we normalize the average value of $yu'(C)$ to unity by setting the mean value of the demand shock to $y = \bar{y} = 1$ and adjusting $u_0$ accordingly.

Firm productivity is $x = \bar{x}x'$ where $\bar{x} \in \{\bar{x}^s, \bar{x}^l\}$ is a permanent component which differs between two firm size classes: small firms with less than 50 employees and larger firms.[23] $\bar{x} = \bar{x}^i$ is drawn upon entry with probability $\pi^i$ and constant over time. Demand shocks $y$

[22] For both matching functions, we make sure that the matching rates of workers and shoppers ($m(\lambda)/\lambda$ and $q(\varphi)/\varphi$ resp.) do not exceed one; that is we set $m(\lambda) = \min(\lambda, m_0\lambda^{0.5})$ and $q(\varphi) = \min(\varphi, q_0\varphi^{0.5})$.

[23] This parsimonious specification clearly cannot generate a realistic firm-size distribution; instead we aim to describe *firm dynamics* and yet analyze the implications of firm size for several model outcomes.

and transitory productivity shocks $x'$ follow AR(1) processes $\ln(y_{t+1}) = \rho^y \ln(y_t) + \sigma^y \varepsilon^y_{t+1}$ and $\ln(x'_{t+1}) = \rho^x \ln(x'_t) + \sigma^x \varepsilon^x_{t+1}$, with standard normally distributed $\varepsilon^x$ and $\varepsilon^y$. The autocorrelations $\rho^y$ and $\rho^x$ and standard deviations $\sigma^y$ and $\sigma^x$ are decisive for the dynamics of firm-level prices and output. We choose the following calibration targets to pin down these four parameters:[24] (i) the standard deviation of log price growth (0.114); (ii) the standard deviation of log output growth (0.174); (iii) the correlation of log price growth and log output growth (-0.382); (iv) the fraction of firms where price growth is within the interval $[-0.02, +0.02]$ (43%). The entry cost parameter $K$ (and thereby the endogenous variable $\rho$) is set so that the average firm employs 50 workers.[25] Table 1 summarizes all parameter choices.

Table 1: Parameter choices

| Parameter | Value | Explanation/Target |
|---|---|---|
| $\beta$ | 0.96 | Annual interest rate 4% |
| $\alpha$ | 0.7 | Labor income share |
| $\delta$ | 0.05 | Firm exit rate (Fuchs and Weyh (2010)) |
| $\bar{\delta}_w$ | 0.02 | Worker separation rate 7% |
| $\bar{\delta}_b$ | 0.43 | Customer retention rate 57% |
| $r_0$ | 0.334 | Recruitment costs 1% of output |
| $s_0$ | 293.6 | Sales costs 2.5% of output |
| $m_0$ | 0.593 | Unemployment rate 7% |
| $q_0$ | 1.423 | Customer matching rate 50% |
| $b$ | 0.113 | Unemployment income 60% of average wage |
| $c$ | 0.070 | Shopping costs 15% of average wage |
| $K$ | 34.15 | Entry cost (average firm size = 50) |
| $(\bar{x}^s, \pi^s)$ | (0.8,0.84) | Firm and employment shares for firms with $<$ 50 workers |
| $(\bar{x}^l, \pi^l)$ | (1.5,0.16) | Firm and employment shares for firms with $\geq$ 50 workers |
| $\sigma^x$ | 0.109 | Standard deviation of quantity growth |
| $\sigma^y$ | 0.045 | Standard deviation of price growth |
| $\rho^x$ | -0.238 | Correlation price and quantity growth |
| $\rho^y$ | 0.070 | Log price growth within $[-0.02, 0.02]$ |

[24] All these data moments are residuals after controlling for year, industry and region (cf. Table 2).

[25] Parameter $K$ cannot be identified independently of the average values of productivity $\bar{x}^s$ and $\bar{x}^l$ because firm-level value functions are linearly homogeneous in the vector $(x, b, r_0, s_0^{-2}, \rho, K)$ (see problem (20), together with the assumed functional forms), so that all firm-level policies are independent of scaling transformations.

## 3.3 Cross-Sectional Features

Table 2: Firm dynamics: Data and model

| | Data | Model |
|---|---|---|
| $\sigma(\hat{P})$ | 0.114 | 0.081 |
| $\sigma(\hat{Q})$ | 0.174 | 0.207 |
| $\rho(\hat{P},\hat{Q})$ | -0.382 | -0.207 |
| Share $\hat{P} \in [-0.02, 0.02]$ | 0.43 | 0.53 |
| $\sigma(\hat{E})$ | 0.082 | 0.077 |
| $\rho(\hat{P},\hat{E})$ | 0.026 | 0.311 |
| $\rho(\hat{Q},\hat{E})$ | 0.249 | 0.396 |
| $\sigma(\hat{w})$ | 0.094 | 0.056 |
| $\rho(\hat{P},\hat{w})$ | 0.006 | -0.143 |

Note: Standard deviations ($\sigma$) and correlations ($\rho$) for various firm growth rates. Data statistics are based on residuals after controlling for (two-digit) industry, region and year. $\hat{w}$ is the log change of the average hourly wage.

Table 2 shows how the model fits the cross-sectional features of firm dynamics. Although the calibration targets (the first four rows) are not hit exactly, the model presents a reasonably close approximation to the data. The other rows of the table show that the model accounts rather well for the volatility and cross-correlations of employment growth. Regarding wage growth, the model generates less dispersion and weak correlations with price growth.

Table 3 shows the dispersion of quantity and revenue productivity, prices and wages. All dispersion measures in the model are smaller than in the data which may reflect additional heterogeneity that is not adequately captured by the model. As in the data, firm-level prices correlate negatively with quantity productivity so that quantity productivity is (slightly) less dispersed than revenue productivity. More productive firms pay higher wages and they sell their products at lower prices, both in the data and in the model. The correlation between the firm-specific price and wage indices is weakly negative.

Figure 1 compares quantity and revenue productivity as well as the firm-specific price and wage index between small firms (up to 50 employees) and larger firms (more than 50 employees). In both productivity measures, larger firms are more productive, but the gap is bigger in terms of quantity productivity. The difference is accounted for by firm-level prices which are lower at larger firms. The right graph in the figure shows that the model accounts for these differences rather well although the gaps in productivity and prices are considerably smaller in the model,

Table 3: Dispersion: Data and model

| | Data | Model |
|---|---|---|
| $\sigma(\text{RLP}_E)$ | 0.688 | 0.127 |
| $\sigma(\text{QLP}_E)$ | 0.991 | 0.133 |
| $\sigma(\widetilde{P})$ | 0.624 | 0.058 |
| $\sigma(\widetilde{W})$ | 0.183 | 0.036 |
| $\rho(\text{QLP}_E, \widetilde{P})$ | -0.726 | -0.328 |
| $\rho(\text{QLP}_E, \widetilde{W})$ | 0.321 | 0.437 |
| $\rho(\widetilde{W}, \widetilde{P})$ | -0.041 | -0.118 |

Note: Standard deviations ($\sigma$) and correlations ($\rho$) for various dispersion measures. Data statistics are based on residuals after controlling for (two-digit) industry, region and year.

which is in line with Table 3.


(a) Data (b) Model

Figure 1: Differences in means between firms with more than 50 and less than 50 workers.
Note: Data statistics are average residuals after controlling for industry, region and year.

Figure 2 illustrates the growth path of a new firm with mean productivity and demand in the absence of shocks. It takes about 15–20 years for an entrant firm to reach its long-run optimal size. Young firms charge high discounts, but prices converge much faster than output or employment. Compared with the benchmark firm, we also show the growth path of a more productive firm (higher $x$) and of a firm with a higher valuation of its product (higher $y$). Both these firms reach higher employment and revenue levels. Higher productivity is not reflected in the firm's price but only in its output, whereas higher demand induces the firm to raise both output and price.

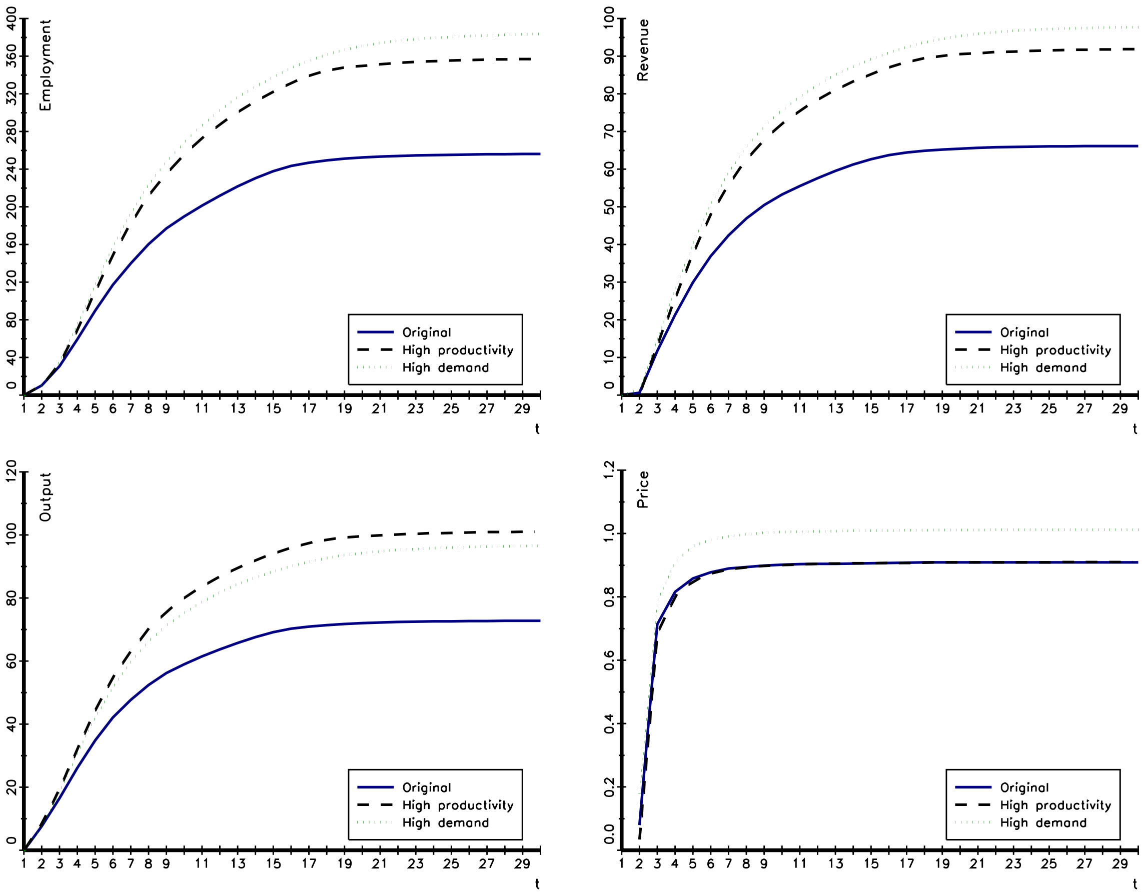


Figure 2: Firm growth.

### 3.4 Product Market Deregulation

During the last two decades, many European countries implemented reforms to deregulate their product markets. In Germany, such changes show up in a decline of the OECD's index of product market regulation as well as in a reduction of markups since the mid 1990s (see Felbermayr et al. (2014)). To examine the impact of such a reform, we consider the model response to an unexpected cut in entry costs by ten percent.[26] Figure 3 shows the transition after the reform to the new stationary equilibrium. Unsurprisingly, the policy leads to a surge

[26]To compute the transition path from the previous to the new steady state, we need to iterate over worker search values $\rho_t$ and marginal utilities $u'(C_t)$ such that the aggregate resource constraint (19) and the aggregate consumption equation (17) hold in all transition periods.

of entry so that the number of firms increases by more than ten percent. At the same time, wages increase substantially which induces existing firms to separate from workers, leading to a temporary decline of employment, followed by (rather minor) employment gains five to ten years after the policy change. Over these years, output increases by four percent, and the average price declines by about three percent. The policy change further brings about sizeable gains in quantity labor productivity (evaluated at pre-reform prices). Revenue productivity also increases after the reform, albeit much less than quantity productivity due to the decline in the general price level. Despite the productivity gains, the policy triggers a large temporary increase in price and productivity dispersion across firms, since new firms with few workers and few customers have especially high labor productivity and offer large price discounts. After the transition period, however, quantity productivity is slightly less dispersed than before the reform.

# 4 Conclusions

To be written.

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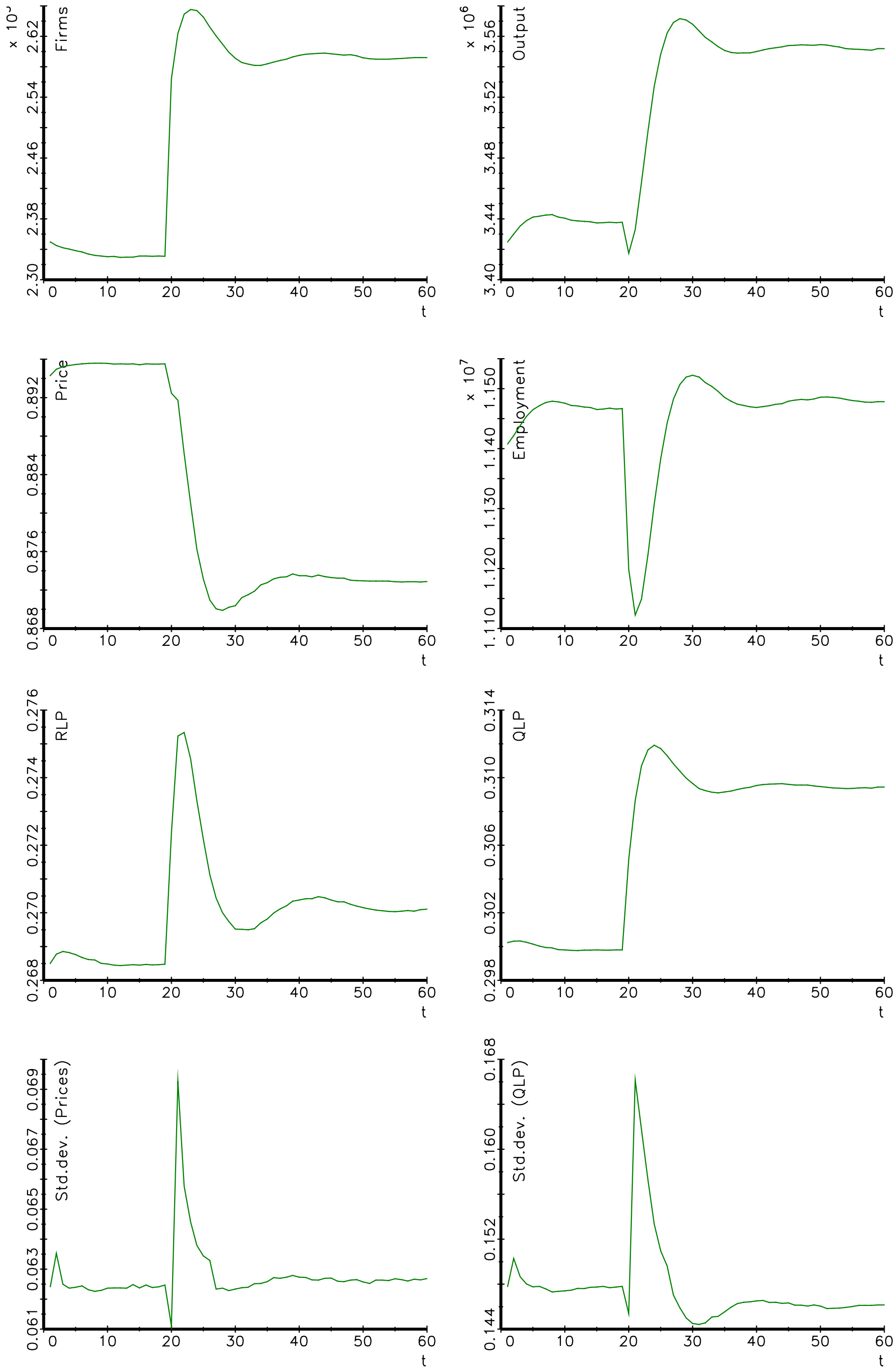


Figure 3: Response to a 10 percent cut in entry costs in period $t = 20$.

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# Appendices

## A Proofs

**Proof of Proposition 1:**
Part (a).
Consider first the sequential planning problem to maximize the discounted household utility for a given initial distribution of workers and customers among heterogeneous firms. For any time $t$ and any firm's age $a$, write $z_{a,t} = (x_{a,t}, y_{a,t})$ for the firm's productivity and demand state, and write $z^{a,t} = (z_{0,t-a}, z_{1,t-a+1}, \ldots, z_{a,t})$ for the idiosyncratic state history. At time $t = 0$, the planner takes as given the initial firm distribution $(N(z^{a-1,-1}), L(z^{a-1,-1}), B(z^{a-1,-1}))_{a\geq 1, z^{a-1,-1}}$. The planner decides for all periods $t \geq 0$ and state-contingent firm histories $z^{a,t}$ the firm

policies $\lambda(z^{a,t})$, $R(z^{a,t})$, $\varphi(z^{a,t})$, $S(z^{a,t})$, $\delta_w(z^{a,t})$, $\delta_b(z^{a,t})$, as well as entrant firms $N_t$ so as to maximize discounted household utility

$$\sum_{t\geq 0}\beta^t\Bigg\{u\Big(\sum_{z^{a,t}}N(z^{a,t})y_{a,t}B(z^{a,t})\Big)+b\bar{L}-KN_t$$
$$-\sum_{z^{a,t}}N(z^{a,t})\Bigg[bL(z^{a,t})+r(R(z^{a,t}),L_0(z^{a,t}))+s(S(z^{a,t}),L_0(z^{a,t}))$$
$$+c\Big(B(z^{a,t})+[\varphi(z^{a,t})-q(\varphi(z^{a,t}))]S(z^{a,t})\Big)\Bigg]\Bigg\}\ .$$

subject to

$$\begin{aligned}L(z^{a,t})&=L(z^{a-1,t-1})[1-\delta_w(z^{a,t})]+m(\lambda(z^{a,t}))R(z^{a,t})\ ,\\ B(z^{a,t})&=B(z^{a-1,t-1})[1-\delta_b(z^{a,t})]+q(\varphi(z^{a,t}))S(z^{a,t})\ ,\\ L_0(z^{a,t})&=L(z^{a-1,t-1})[1-\delta_w(z^{a,t})]\ ,\\ N(z^{a,t})&=(1-\delta)\pi(z_{a,t}|z_{a-1,t-1})N(z^{a-1,t-1})\ ,\end{aligned}$$

for $t\geq 0$ and $a\geq 1$,

$$\begin{aligned}L(z^{0,t})&=m(\lambda(z^{0,t}))R(z^{0,t})\ ,\ B(z^{0,t})=q(\varphi(z^{0,t}))S(z^{0,t})\ ,\\ N(z^{0,t})&=\pi^0(z^{0,t})N_t\ ,\end{aligned}$$

for $t\geq 0$,

$$B(z^{a,t})\leq x_{a,t}F(L(z^{a,t}))\ ,\ \delta_w(z^{a,t})\geq\bar{\delta}_w\ ,\ \delta_b(z^{a,t})\geq\bar{\delta}_b$$

for $t\geq 0$ and $a\geq 0$, and subject to the resource constraint for all $t\geq 0$,

$$\bar{L}\geq\sum_{z^{a,t}}N(z^{a,t})\Big[L(z^{a,t})+\Big(\lambda(z^{a,t})-m(\lambda(z^{a,t}))\Big)R(z^{a,t})\Big]\ . \tag{29}$$

Write $\beta^t\rho_t$ for the multiplier on constraint (29). The Lagrange function of the planning problem is

$$\mathcal{L}=\sum_{t\geq 0}\beta^t\Bigg\{u\Big(\sum_{z^{a,t}}N(z^{a,t})y_{a,t}B(z^{a,t})\Big)-KN_t \tag{30}$$
$$-\sum_{z^{a,t}}N(z^{a,t})\Bigg[bL(z^{a,t})+r(R(z^{a,t}),L_0(z^{a,t}))+s(S(z^{a,t}),L_0(z^{a,t}))$$
$$+c\Big(B(z^{a,t})+[\varphi(z^{a,t})-q(\varphi(z^{a,t}))]S(z^{a,t})\Big)+\rho_t\Big(L(z^{a,t})+[\lambda(z^{a,t})-m(\lambda(z^{a,t}))]R(z^{a,t})\Big)\Bigg]\Bigg\}\ .$$

The derivative of the Lagrangian with respect to firm $z^{a,t}$'s output $B(z^{a,t})$ is

$$\frac{d\mathcal{L}}{dB(z^{a,t})} = \beta^t N(z^{a,t})\Big[u'(C_t)y_{a,t} - c\Big] \ ,$$

with aggregate consumption $C_t \equiv \sum_{z^{a,t}} N(z^{a,t})y_{a,t}B(z^{a,t})$. Therefore, the number of firms of type $z^{a,t}$, $N(z^{a,t})$, enters *linearly* all first-order conditions of the sequential planning problem with respect to firm-level policies, namely $B(.)$, $L(.)$, $\lambda(.)$, $\varphi(.)$, $S(.)$, $R(.)$. The number of firm types is thus irrelevant for the planner's firm-level policies which solve the firm-level problem, defined recursively for a given (bounded) sequence $(\rho_t, C_t)_{t\geq 0}$ by

$$\begin{aligned} G_t(L_-, B_-, z) = \max_{(\lambda,R,\delta_w),(\varphi,S,\delta_b)} \Big\{ & u'(C_t)yB - bL - r(R, L_-(1-\delta_w)) - s(S, L_-(1-\delta_w)) \\ & - \rho_t[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E}_z G_{t+1}(L, B, z_+) \Big\} \ , \end{aligned} \tag{31}$$

subject to

$$\begin{aligned} & L = L_-(1-\delta_w) + m(\lambda)R \ , \ B = B_-(1-\delta_b) + q(\varphi)S \ , \\ & B \leq xF(L) \ , \ \delta_w \geq \bar{\delta}_w \ , \ \delta_b \geq \bar{\delta}_b \ . \end{aligned}$$

A solution $(G_t)_{t\geq 0}$ for this problem exists with functions $G_t : [0, L^{\max}] \times [0, B^{\max}] \times Z \to \mathbb{R}$ for appropriately define upper bounds $L^{\max}$ and $B^{\max}$. The proof of this assertion follows the same lines as in Lemma A.4, part (a), of Kaas and Kircher (2015).

To prove part (a) of Proposition 1, suppose that $(\rho, G, N_0, C)$ solves the recursive social planning problem (20) together with (21), aggregate consumption (17) and the aggregate resource constraint (19) are satisfied. Then, for the constant sequences $\rho_t = \rho$ and $C_t = C$, value functions $G_t = G$ for all $t \geq 0$ also solves problem (31). If $N_t = N_0$ for all $t$, the resource constraint (29) is satisfied in all periods $t$ because the distribution of firm types and the distribution of workers across firms is stationary: $N(z^{a,t}) = N(z^a)$, $L(z^{a,t}) = L(z^a)$, provided that $(N(z^a), L(z^a), B(z^a))$ is the initial firm distribution. Because individual firm policies solve problem (31), they also maximize the Lagrange function (30). Condition (21) further says that $K = \sum_z \pi^0(z)G_t(0, 0, z)$. On the other hand, the first-order condition of (30) with respect to $N_t$ is

$$\begin{aligned} 0 = & -\beta^t K + \sum_{a\geq 0} \beta^{t+a}(1-\delta)^a \pi(z^a) \Bigg\{ u'(C)y_a B(z^a) - bL(z^a) - r(R(z^a), L_0(z^a)) - s(S(z^a), L_0(z^a)) \\ & - c\Big(B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big) - \rho\Big(L(z^a) + [\lambda(z^a) - m(\lambda(z^a))]R(z^a)\Big) \Bigg\} \\ & = \beta^t\Big[ - K + \sum_z \pi^0(z^0)G_t(0, 0, z^0)\Big] \ . \end{aligned}$$

Condition (21) then implies that $N_t = N_0$, $t \geq 0$, solve the Lagrange problem. Since aggregate resource feasibility is satisfied, these firm policies solve the sequential planning problem where the multiplier on (29) is $\beta^t \rho$. No other feasible allocation dominates the one defined by the individual firm's problem; for a formal argument, see the proof of part (b) of Lemma A.4 in Kaas and Kircher (2015).

Part (b).

Consider $(\rho, G, N_0, C)$ where $G$ solves the recursive social planning problem (20) together with (21). Further aggregate consumption is (17) and the aggregate resource constraint (19) is satisfied when $L(z^a)$, $B(z^a)$ etc. are defined by the policy functions of problem (20). Define candidate equilibrium contracts $\mathcal{C}^{a*} = (w^{a*}(z^k), \delta_w^{a*}(z^{k+1})_{k \geq a}$ with separation rates $\delta_w^{a*}(z^k) \equiv \delta_w(z^k)$ from the policy functions of problem (20) (hence, separations are independent of the tenure in the firm). Equilibrium wages can be defined such that all workers within the firm earn the same: $w^{a*}(z^k) = w(z^k)$, where $w(z^k)$ is defined as in (28). As in Section 2.2.3, define the generic state vector of the firm as $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$, and let $G_a(\sigma)$ denote the *social* value of firm type $z^a$, assuming that the firm takes as given previous worker cohorts $L^\tau$ and the precommitted separation rates as specified in contracts $\mathcal{C}^\tau$, $\tau < a$. For the contracts $(\mathcal{C}^{\tau *})_{\tau=0}^{a-1}$ in the candidate equilibrium (and the corresponding worker cohorts $L^{\tau *}$) write $\sigma^*$ for the firm's state vector.

The recursive problem to maximize social firm value is

$$G_a(\sigma) = \max_{(\lambda, R, \mathcal{C}^a), (\varphi, S, \delta_b)} \Big\{ u'(C) y_a B - bL - r(R, L_0) - s(S, L_0) \\ - \rho[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E} G_{a+1}(\sigma_+) \Big\} , \tag{32}$$

subject to (7), (8), (10) and (11). Wages in contracts $\mathcal{C}^\tau$ are clearly irrelevant for that problem. The same policies that solve problem (20), and in particular contracts $\mathcal{C}^{a*}$ for all $a \geq 0$, also solve problem (32). The only difference between those two problems is that the firm is precommitted to separation rates for existing workers in the latter but not in the former problem. But since policies for the latter problem are time consistent, both problems have the same solutions. Hence it remains to show that those policies not only solve problem (32) but that they also maximize the *private* value of the firm, as specified in the recursive problem (6)–(13), provided that $\rho^* = \rho$.

Substitution of (13) shows that

$$u'(C) y_a B - c[B + (\varphi - q(\varphi))S] = p^R B_-(1 - \delta_b) + pq(\varphi) S \ .$$

Hence, the left-hand side of that term in problem (32) can be replaced by the right-hand side together with constraint (13). Further, we can write the social labor costs

$$bL + \rho[L + (\lambda - m(\lambda))R] = (b + \rho)L_0 + [b + \rho \frac{\lambda}{m(\lambda)}] m(\lambda) R \ . \tag{33}$$

Given the precommitted contracts $\mathcal{C}^{\tau*}$, $\tau < a$, the first term can be written

$$\begin{aligned}(b+\rho)L_0 &= \sum_{\tau=0}^{a-1}[1-\delta_w^{\tau*}(z^a)]L^\tau \cdot (b+\rho) \\ &= \sum_{\tau=0}^{a-1}[1-\delta_w^{\tau*}(z^a)]L^\tau \Big[w^{\tau*}(z^a) - [W(\mathcal{C}^{\tau*}, z^a) - U] + \beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U]\Big] \\ &= -\sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U] + \sum_{\tau=0}^{a-1} L^{\tau+} w^{\tau*}(z^a) + \beta(1-\delta)\mathbb{E}\sum_{\tau=0}^{a-1} L^{\tau+}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U] \ .\end{aligned}$$

For any contract $\mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k\geq a}$ offered to new hires $m(\lambda)R = L^{a+}$, the second term in (33) can be written

$$\begin{aligned}[b+\rho\frac{\lambda}{m(\lambda)}]m(\lambda)R &= [W(\mathcal{C}^a, z^a) - \beta U]m(\lambda)R \\ &= w^a(z^a)L^{a+} + \beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^a, z^{a+1}) - U]L^{a+} \ .\end{aligned}$$

Substituting those expressions into (32) at $\sigma = \sigma^*$ shows

$$\begin{aligned}G_a(\sigma^*) = \max_{(\lambda,R,\mathcal{C}^a),(\varphi,S,p,p^R,\delta_b)} \Bigg\{ & p^R B_-(1-\delta_b) + pq(\varphi)S - W \\ & + \sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U] - r(R, L_0) - s(S, L_0) \\ & + \beta(1-\delta)\mathbb{E}\Big\{G_{a+1}(\sigma_+^*) - \sum_{\tau=0}^{a-1} L^{\tau+}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U] - L^{a+}[W'(\mathcal{C}^a, z^{a+1}) - U]\Big\}\Bigg\} \ ,\end{aligned} \tag{34}$$

where maximization is subject to (8)–(13) with $\sigma_+^* = [(L^\tau, \mathcal{C}^{\tau*})_{\tau=0}^{a-1}, (L^{a+}, \mathcal{C}^a), B, z^{a+1}]$. In this maximization problem, the term $\sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U]$ is predetermined and thus not subject to the maximization. Therefore, we can define the private firm value

$$J_a(\sigma) \equiv G_a(\sigma) - \sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^\tau, z^a) - U] \ ,$$

i.e. the difference between the social value of firm $z^a$ and the surplus of the existing workers. Then problem (34) (at given state vector $\sigma^*$) is equivalent to problem (6). In particular, the firm policies $\lambda$, $R$, $\varphi$, $S$, $p$ and $p^R$ and $\mathcal{C}^{a*}$ that solve (34) also solve (6). Moreover, because of $G(0,0,z) = J_0(0,z)$, socially optimal entry (21) implies the equilibrium condition (18). Since resource constraints are satisfied, the stationary planning solution gives rise to a stationary competitive search equilibrium. $\square$

# B Data

## B.1 Further Details

Starting from the original dataset with 375,016 establishment-product-years, we undertake a step-by-step cleaning of the data as follows. Dropped are products that are produced in subcontracts (tenth digit "2", 16,518); establishments with missing employment (1,043) or less than 20 employees (27,417); and all products whose sales value is zero or missing (16,214). Another 20 observations are dropped because of zero or missing wage information. We lose 82,113 observations by deleting products without quantity information, e.g., services such as cleaning or repair which are measured in Euros. Because only quantities of products measured in the same unit can be compared, one product measured in different units across establishments or years is dropped. Lastly, we drop 4,667 observations corresponding to those establishments that are not active throughout the year, and 65,276 observations are lost by removing multi-establishment firms.

Next we describe further details about the construction of the *full* and *homogeneous samples*. Reportage of any statistics is meaningful provided that the products that remain in either sample contribute significantly to the total sales value of the firm, as in the original uncleaned sample. This is why we keep both in the *full sample* and in the *homogeneous sample* only those firm-year observations for which the sample sales value is at least 50 percent of the total sales value of the firms, as in Foster et al. (2008). We further follow their procedure and adjust proportionally the sales values (and quantities) of the firm's products so that the sales value in the cleaned sample equals the total sales value before removing any goods from the analysis.[27]

*Full sample.* The full sample is restricted to firms whose products with valid quantity information, in any given year, make at least 50% of the given firm's total sales value. Across firms, the sales share of these products is 98.8% at the 25th percentile and 100% at the 50th percentile, which reflects that most of the firms in the data produce only one good. We drop as outliers price, quantity, employment, and wage growth rates that are beyond the 2nd and 98th percentiles. After these adjustments, the full sample has 143,073 firm-product-years from 58,467 firm-years and 13,155 product-years. On average, in the full sample, a given nine-digit product is produced by 10.87 firms whereas the median product is produced by four firms. Firms in the full sample produce an average of 2.45 products with about 71.2% of the firms producing one or two products. The average size of firms is 105.09 employees whereas the median firm has 55.43 employees.

*Homogeneous sample (Dispersion).* To construct the homogeneous sample, we start at the cleaned sample *before* truncating it to obtain the full sample. We then drop all products which are not measured in physical units (length, area, volume or weight) and those which are

[27] The underlying assumption is that the firm's production technology is as if it produces the sample products in the same proportion as any other products with its labor input.

produced by up to five firms, in order to be able to compute a meaningful average price for each product. Across firms producing at least one of those products, the valid products contribute 91.45% of the total sales value at the 25th percentile and 100% at the 50th percentile. Again we restrict the sample to firms whose valid products make at least 50% of the total sales value. We drop as outliers all log firm-specific price indices and log quantity labor productivity (as defined below) beyond the 2nd and 98th percentiles to arrive at 82,118 firm-product-years (33,174 firm-years and 3,668 product-years). Averaged over the years 2005–2007, firms produce 2.48 products, and 72.52% of the firms produce one or two products. A given product is on average produced by 22.39 firms, whereas the median product is produced by ten firms. Averaged over the three years, the average size of the firms is 100.62 employees with the median firms employing 54.31 workers.

*Homogeneous sample (Dynamics).* This sample is related to the one used to study price, productivity and wage dispersion, and is needed to complete the analysis by relating productivity and sales growth with the growth of the firm-specific price, the growth of the average price, and the growth of quantity. To achieve this, we restrict ourselves to the goods measured in physical units of length, area, volume or weight. We note that the sales share of the goods that remain is 99.7% at the 25th percentile and 100% at the median. Restricting the analysis to those goods whose contribution to the firm's total sales value is at least 50% and dropping as outliers price, quantity, employment, and wage growth rates beyond the 2nd and 98th percentiles yields 93,604 firm-product-years (36,261 firm-years and 8,018 product-years). A firm in this sample produces 2.58 products, with the median firm producing one product. About 70.89% of all firms on average produce one or two products whereas a given product is on average produced by 11.67 firms with the median product produced by 4 firms. A firm in the sample employs 100.62 employees while the median firm has 54.14 workers, each as average of the yearly averages of the three years of the panel.

## B.2 Empirical Methodology

Here we describe how we construct our growth and dispersion measures.

Let $I$ denote the set of firms in the data panel, $J$ the set of products, and $T$ the set of years covered by the panel. Let $J_0 \subset J$ be the set of products in either the *full sample* or the *homogeneous sample* described above. Let $\mathcal{S} \subset I \times J_0 \times T$ be the firm-product-year sample under consideration. For each observation $(i, j, t) \in \mathcal{S}$, calculate the unit price $P_{ijt}$ as sales $S_{ijt}$ divided by quantity $Q_{ijt}$. Define the quantity-weighted average price of good $j$ at time $t$ as

$$\overline{P}_{jt} = \frac{\sum_i P_{ijt} Q_{ijt}}{\sum_i Q_{ijt}} \ .$$

Denote by $S_{it} = \sum_{j \in J} S_{ijt}$ the total sales value of firm $i$ in year $t$ (including *all* products) and define adjusted quantities $\widetilde{Q}_{ijt} = Q_{ijt} \frac{S_{it}}{\sum_{j \in J_0} S_{ijt}}$ so that $S_{it} = \sum_{j \in J_0} P_{ijt} \widetilde{Q}_{ijt}$. This adjustment is a valid modification of the data if the goods belonging to sample $J_0$ are sufficiently representative for the set of goods that this firm produces so that a firm's price and output policies are well proxied by goods $j \in J_0$. In order to simplify notation, we denote with $Q_{ijt}$ these adjusted quantities in the following.

### B.2.1 Measuring Price and Output Growth (Full Sample)

We begin by splitting sales growth into a price and a quantity component. We replace a firm's product price with the average market price of that good whenever the firm's product portfolio changes between two periods (so that a firm's price for some of its products is not available in one of the periods). That is, for any $(i, t, t+1)$ and $j \in J_0$ define

$$\widehat{P}_{ij\tau} = \begin{cases} P_{ij\tau} & \text{if } Q_{ij\tau} > 0 \ , \\ \overline{P}_{j\tau} & \text{else.} \end{cases} \quad , \ \tau = t, t+1 \ .$$

Define the firm's Paasche price index by $P_{i,t,t} = 1$ and

$$P_{i,t+1,t} = \frac{\sum_j \widehat{P}_{i,j,t+1} Q_{i,j,t+1}}{\sum_j \widehat{P}_{ijt} Q_{i,j,t+1}} \ .$$

Define the firm's quantity (real output) by

$$Q_{i,t,t} = \frac{S_{it}}{P_{i,t,t}} = \sum_j \widehat{P}_{ijt} Q_{ijt} \ , \qquad Q_{i,t+1,t} = \frac{S_{i,t+1}}{P_{i,t+1,t}} = \sum_j \widehat{P}_{i,j,t} Q_{i,j,t+1} \ .$$

Therefore sales growth is split into a price and quantity component:

$$\frac{S_{i,t+1}}{S_{i,t}} = \frac{P_{i,t+1,t}}{P_{i,t,t}} \frac{Q_{i,t+1,t}}{Q_{i,t,t}} \ . \tag{35}$$

Write $\widehat{X}_t = \ln(X_t / X_{t-1})$ for the log growth rate of variable $X$. Therefore

$$\widehat{S}_{i,t} = \widehat{P}_{i,t} + \widehat{Q}_{i,t} \ .$$

Define the wage (per employee) $w_{i,t} = W_{i,t}/E_{i,t}$, the hourly wage $wh_{i,t} = W_{i,t}/H_{i,t}$, quantity labor productivity $\text{QLP}_{E;i,t,t+k} = Q_{i,t,t+k}/E_{it}$ and hourly quantity labor productivity $\text{QLP}_{H;i,t,t+k} = Q_{i,t,t+k}/H_{it}$, for $k = 0, 1$, revenue labor productivity $\text{RLP}_{L;it} = S_{it}/E_{it}$ and hourly revenue labor productivity $\text{RLP}_{H;it} = S_{it}/H_{it}$, where $E_{it}$ and $H_{it}$ are employment and working hours. This leads to the decompositions

$$\widehat{\text{RLP}}_{E;i,t} = \widehat{P}_{i,t} + \widehat{\text{QLP}}_{E;i,t} \ , \ \widehat{\text{RLP}}_{H;i,t} = \widehat{P}_{i,t} + \widehat{\text{QLP}}_{H;i,t} \ .$$

### B.2.2 Measuring Price and Productivity Dispersion (Homogeneous Sample)

Start with the set of firms $i \in I$ producing goods $j \in J_0$ in the homogeneous sample, and define the *firm-specific price index*:

$$\widetilde{P}_{it} = \frac{\sum_{j\in J_0} P_{ijt}Q_{ijt}}{\sum_{j\in J_0} \overline{P}_{jt}Q_{ijt}} \ , \tag{36}$$

where as before $\overline{P}_{jt}$ is the quantity-weighted average price. The denominator expresses the firm's hypothetical sales value if the firm would charge the average price for all its products. $\widetilde{P}_{it} - 1$ measures the "expensiveness" of firm $i$ relative to other firms.
Define revenue and quantity productivity:

$$\mathrm{RLP}_{E;it} = \frac{\sum_{j\in J_0} Q_{ijt}P_{ijt}}{E_{it}}\ , \quad \mathrm{RLP}_{H;it} = \frac{\sum_{j\in J_0} Q_{ijt}P_{ijt}}{H_{it}}\ ,$$

$$\mathrm{QLP}_{E;it} = \frac{\sum_{j\in J_0} Q_{ijt}\overline{P}_{jt}}{E_{it}}\ , \quad \mathrm{QLP}_{H;it} = \frac{\sum_{j\in J_0} Q_{ijt}\overline{P}_{jt}}{H_{it}}\ .$$

Revenue labor productivity can be split into quantity productivity and the firm's price level, as defined in (36):

$$\mathrm{RLP}_{E;it} = \mathrm{QLP}_{E;it} \cdot \widetilde{P}_{it}\ ,\ \mathrm{RLP}_{H;it} = \mathrm{QLP}_{H;it} \cdot \widetilde{P}_{it}\ . \tag{37}$$

To describe firm dynamics in the *homogeneous sample*, there is another decomposition which allows to split sales growth into a component reflecting growth of the firm-specific price index, average prices, and output growth. Define

$$\overline{Q}_{it} = \sum_{j\in J_0} \overline{P}_{jt}Q_{ijt}$$

for firm $i$'s output measured in average date $t$ prices. Total sales of firm $i$ are therefore

$$S_{it} = \widetilde{P}_{it}\overline{Q}_{it}\ ,$$

the product between firm $i$'s price index and its output.
We can decompose sales growth as

$$\widehat{S}_{i,t+1} = \frac{S_{i,t+1}}{S_{it}} = \underbrace{\frac{\widetilde{P}_{i,t+1}}{\widetilde{P}_{i,t}}}_{=\widehat{\widetilde{P}}_{i,t+1}} \cdot \frac{\overline{Q}_{i,t+1}}{\overline{Q}_{it}}\ .$$

The latter part can be split into

$$\frac{\overline{Q}_{i,t+1}}{\overline{Q}_{it}} = \underbrace{\frac{\sum_{j\in J_0} \overline{P}_{jt}Q_{ij,t+1}}{\sum_{j\in J_0} \overline{P}_{jt}Q_{ijt}}}_{=\widehat{Q}_{i,t+1}} \cdot \underbrace{\frac{\sum_{j\in J_0} \overline{P}_{j,t+1}Q_{ij,t+1}}{\sum_{j\in J_0} \overline{P}_{jt}Q_{ij,t+1}}}_{=\widehat{\overline{P}}_{i,t+1}}\ .$$

Hence, sales growth is

$$\widehat{S}_{i,t+1} = \widehat{\widetilde{P}}_{i,t+1} \cdot \widehat{\overline{P}}_{i,t+1} \cdot \widehat{Q}_{i,t+1} \ .$$

The first two components are price growth: the first is the change of firm $i$'s price index and the second is the general growth of prices that firm $i$ produces. The last component is firm $i$'s quantity growth.

Note the difference to the decomposition (35) that we undertake for the full sample. There we measure firm $i$'s quantity growth evaluated at firm $i$'s prices in period $t$. Here $\widehat{Q}_{i,t+1}$ is measured with average goods prices in period $t$. Clearly, for firms producing one good both measures are identical.

### B.2.3 Measuring Wage Dispersion

In the module *Verdienste* we have data on workers for establishments $i \in I^V$ which includes the service sector and hence is distinct from set $I$. When we relate wage dispersion with price, quantity and productivity, we look at the intersection of firms contained in $I$ and $I^V$, labeled $I_\cap = I \cap I^V$.

For all $i \in I^V$ and a (sub)sample of workers $k \in K(i)$, we have information on hourly wages $w_k$ and working hours $h_k$, together with other worker and firm characteristics. We estimate four wage regressions (based on workers $k \in K(i)$ in all establishments $i \in I^V$), separately for males and females located in East and West German states:

$$\ln(w_k) = \beta_0 + \beta_1 X_k + \varepsilon_k \ ,$$

where vector $X_k$ contains a cubic of age, a cubic of tenure, education dummies, sector union dummy, firm-union dummy, apprentice, temporary contract, old-age part-time, minijob. We drop all workers for which education, age or tenure is missing. Then we remove outliers where the standardized residuals have absolute value greater than five, and we run the regressions again with the remaining observations.

Write $\overline{w}_k = e^{\beta_0 + \beta_1 X_k}$ for the predicted hourly wage of worker $k$ (i.e. the part of the wage explained by worker observables). Define for each firm $i \in I^V$ the firm-specific wage index which expresses how much more (or less) firm $i$ pays to its workers relative to what observationally equivalent workers earn on average:

$$\widetilde{W}_i = \frac{\sum_{k \in K(i)} w_k h_k}{\sum_{k \in K(i)} \overline{w}_k h_k} \ .$$

This index is analogous to the firm-specific price index $\widetilde{P}_i$ defined before. Note that

$$\widetilde{W}_i - 1 = \frac{\sum_{k \in K(i)} (e^{\varepsilon_k} - 1) \overline{w}_k h_k}{\sum_{k \in K(i)} \overline{w}_k h_k} \ ,$$

which shows that the firm's wage index (minus one) is an earnings-weighted average of the wage residuals ($e^{\varepsilon_k} - 1 \approx \varepsilon_k$). Note however that this firm-specific wage index captures both a direct firm effect on wages but also any unobserved worker characteristics that correlate between different workers in the same firm. If those unobserved characteristics were uncorrelated across workers in the same firm, they should not matter much (and they should be negligible if the firm is sufficiently large). But if those characteristics correlate (positive sorting *between workers in the same firm*), then they matter for the overall index. This issue cannot be addressed with our data which have no panel dimension on the worker side.
Define the firm's average hourly wage

$$WH_i = \frac{\sum_{k \in K(i)} w_k h_k}{\sum_{k \in K(i)} h_k} .$$

The firm's hourly wage can be decomposed into the wage predicted by worker observables $\overline{WH}_i$ and into the firm-specific wage index:

$$WH_i = \overline{WH}_i \cdot \widetilde{W}_i \ , \ \text{with } \overline{WH}_i \equiv \frac{\sum_{k \in K(i)} \overline{w}_k h_k}{\sum_{k \in K(i)} h_k} . \tag{38}$$

This decomposition says that a firm pays higher wages either because it employs more valuable workers (based on worker observables) or because its wage is higher than in other firms (which may be due to the firm's wage policy but may also reflect correlated unobservable worker characteristics). Note that this decomposition is similar to the decomposition (37) of revenue labor productivity into quantity labor productivity and the firm's price index. See Figure 7 for kernel density plots (unweighted and weighted by firm size) for these three measures.

## B.3 Findings

This section describes detailed statistics for the various measures defined above.

### B.3.1 Firm Dynamics

Table 4 shows statistics on firm growth rates. Price growth is less dispersed than sales growth, quantity growth and labor productivity growth (however measured) whose standard deviations are all between 0.16 and 0.18. Price growth is more dispersed than employment (hours) growth and more dispersed than wage per employee (per hour) growth. The standard deviation of quantity labor productivity growth is in general slightly larger than the standard deviation of revenue labor productivity growth.
The table further reveals that price growth shows no strong correlations with the growth rates of employment and hours. In contrast, it correlates negatively with quantity growth which itself correlates positively with employment and hours. These results which are conditional

Table 4: Summary statistics for growth dynamics

| | $\widehat{S}$ | $\widehat{P}$ | $\widehat{Q}$ | $\widehat{E}$ | $\widehat{H}$ | $\widehat{W/E}$ | $\widehat{W/H}$ | $\widehat{\text{QLP}_E}$ | $\widehat{\text{QLP}_H}$ | $\widehat{\text{RLP}_E}$ | $\widehat{\text{RLP}_H}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\widehat{S}$ | 1.000 | | | | | | | | | | |
| $\widehat{P}$ | 0.283 | 1.000 | | | | | | | | | |
| $\widehat{Q}$ | 0.778 | -0.382 | 1.000 | | | | | | | | |
| $\widehat{E}$ | 0.275 | 0.026 | 0.249 | 1.000 | | | | | | | |
| $\widehat{H}$ | 0.291 | 0.028 | 0.262 | 0.605 | 1.000 | | | | | | |
| $\widehat{W/E}$ | 0.097 | 0.021 | 0.079 | -0.289 | 0.051 | 1.000 | | | | | |
| $\widehat{W/H}$ | -0.008 | 0.006 | -0.014 | -0.044 | -0.611 | 0.471 | 1.000 | | | | |
| $\widehat{\text{QLP}_E}$ | 0.653 | -0.397 | 0.889 | -0.222 | -0.006 | 0.217 | 0.006 | 1.000 | | | |
| $\widehat{\text{QLP}_H}$ | 0.567 | -0.398 | 0.809 | -0.095 | -0.352 | 0.048 | 0.358 | 0.874 | 1.000 | | |
| $\widehat{\text{RLP}_E}$ | 0.880 | 0.274 | 0.667 | -0.214 | 0.013 | 0.243 | 0.012 | 0.774 | 0.633 | 1.000 | |
| $\widehat{\text{RLP}_H}$ | 0.793 | 0.261 | 0.584 | -0.083 | -0.350 | 0.069 | 0.381 | 0.638 | 0.781 | 0.862 | 1.000 |
| Std. dev. | 0.168 | 0.114 | 0.174 | 0.082 | 0.107 | 0.078 | 0.094 | 0.173 | 0.175 | 0.166 | 0.167 |

Notes: Each variable is expressed in logs. $\widehat{S}$ is sales growth, $\widehat{P}$ is price growth, $\widehat{Q}$ is quantity growth, $\widehat{E}$ is employment growth, $\widehat{H}$ is hours growth, $\widehat{W/E}$ is wage per employee growth, $\widehat{W/H}$ is wage per hour growth, ($\widehat{\text{QLP}_H}$) $\widehat{\text{QLP}_E}$ is (hourly) quantity labor productivity growth and ($\widehat{\text{RLP}_H}$) $\widehat{\text{RLP}_E}$ is (hourly) revenue labor productivity growth. All statistics are based on residuals after controlling for two-digit industry, four German regions, year, and interaction terms.

on industry, region and year, are similar for unconditional variables (see Tables 8 and 9 for unweighted and employment-weighted statistics).

Based on these statistics, we conduct variance decompositions of sales growth into price growth and quantity growth. Likewise, we decompose the variance of revenue labor productivity growth into a component due to price growth and one due to quantity labor productivity growth (see Table 5).

In all decompositions, the price variation amounts to roughly 50% of the variance in sales growth and in revenue productivity growth, whereas quantity variation is about twice as large. The negative covariance term between price and quantity growth reduces the overall variance.

To explore further the observation that employment growth correlates more strongly with quantity growth than with price growth, we sort firms into bins of log quantity growth and log price growth. For each bin, we calculate the employment-weighted average log employment growth rate. Using locally weighted regressions the resulting relationships are smoothed and shown in Figure 4. The figure shows that for firms that record positive growth in either price or quantity, they also expand employment, although the responsiveness to quantity growth is much stronger which is little surprising given that firms need to adjust their factor inputs to produce more output. On the other hand, employment responds much weaker to nega-

Table 5: Variance decompositions

| $Var(\widehat{S})$ | $Var(\widehat{P})$ | $Var(\widehat{Q})$ | $2Cov(\widehat{P},\widehat{Q})$ |
|---|---|---|---|
| 0.0251 | 0.0118 | 0.0258 | -0.0125 |
| $Var(\widehat{RLP_E})$ | $Var(\widehat{P})$ | $Var(\widehat{QLP_E})$ | $2Cov(\widehat{P},\widehat{QLP_E})$ |
| 0.0230 | 0.0118 | 0.0242 | -0.0130 |
| $Var(\widehat{RLP_H})$ | $Var(\widehat{P})$ | $Var(\widehat{QLP_H})$ | $2Cov(\widehat{P},\widehat{QLP_H})$ |
| 0.0243 | 0.0118 | 0.0257 | -0.0132 |

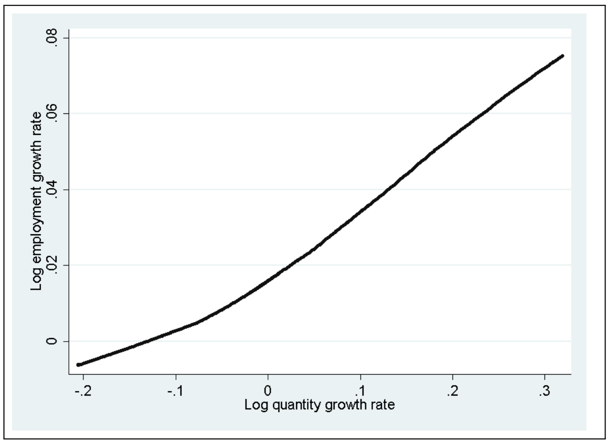
Notes: Each variable is expressed in logs. Estimates are weighted by the average of employment of the firm for all its years of observation.

tive quantity growth. Indeed, on average firms cut employment only when quantity growth is smaller than minus 11 percent. In contrast, when firms cut prices, they do not reduce employment on average which may be an indication of labor hoarding in periods of temporary low demand (to which firms respond by cutting prices though not employment). Firms may also be subject to favorable cost (productivity) shocks which may lead to price cuts in combination with employment expansions.

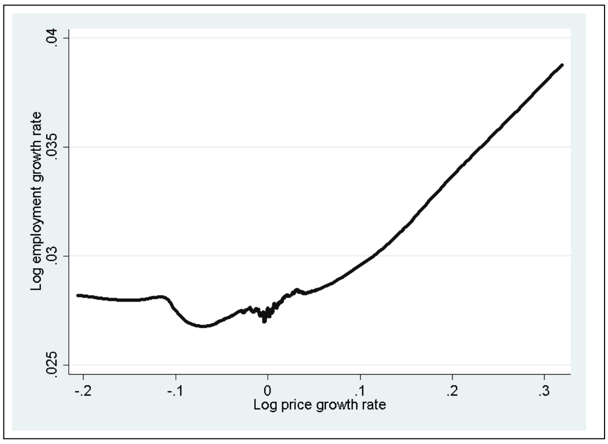
### B.3.2 Price, Wage and Productivity Dispersion

Table 6 summarizes statistics for the firm-specific price index, quantity and revenue labor productivity, wage per hour, and the firm-specific wage index, again conditional on industry, region and year. Prices are more dispersed than wage per hour or the firm-specific wage index. For either measure (per hour or per employee), quantity labor productivity is more dispersed than revenue labor productivity. The table also shows that prices are weakly negatively correlatedwith wage per hour, with the wage index $\widetilde{W}$ and with both measures of revenue labor productivity.Furthermore, prices show a strong negative correlation with both measures of quantity labor productivity. This is the main reason why quantity labor productivity is considerably more dispersed than revenue labor productivity, similar to the results of Foster et al. (2008) about TFP dispersion.

In Table 7 we present variance decompositions of revenue labor productivity (per hour or per employee). Most variation of revenue productivity comes from quantity productivity, but price dispersion also amounts to over 70% of the variation in revenue productivity. The negative covariance between prices and quantity productivity dampens overall revenue dispersion. The last column of the table decomposes the variance of average hourly wages between firms. 33% of that variation comes from observable worker effects, 43% is due to variation in the firm-




(a) Employment growth vs quantity growth




(b) Employment growth vs price growth

Figure 4: Nonlinear relations between employment growth and price growth, and between employment growth and quantity growth.

specific wage index, and the rest is accounted for by a positive covariance between the two measures.

Table 6: Summary statistics for price, productivity and wage dispersion

| | $\text{RLP}_E$ | $\text{QLP}_E$ | $\text{RLP}_H$ | $\text{QLP}_H$ | $\widetilde{P}$ | $W/H$ | $\widetilde{W}$ |
|---|---|---|---|---|---|---|---|
| $\text{RLP}_E$ | 1.000 | | | | | | |
| $\text{QLP}_E$ | 0.781 | 1.000 | | | | | |
| $\text{RLP}_H$ | 0.957 | 0.743 | 1.000 | | | | |
| $\text{QLP}_H$ | 0.738 | 0.978 | 0.763 | 1.000 | | | |
| $\widetilde{P}$ | -0.138 | -0.726 | -0.125 | -0.737 | 1.000 | | |
| $W/H$ | 0.444 | 0.333 | 0.538 | 0.392 | -0.039 | 1.000 | |
| $\widetilde{W}$ | 0.409 | 0.321 | 0.408 | 0.315 | -0.041 | 0.601 | 1.000 |
| Std. dev. | 0.688 | 0.991 | 0.645 | 0.948 | 0.624 | 0.293 | 0.183 |

Notes: Each variable is expressed in logs. $\text{QLP}_E$, $\text{QLP}_H$ and $\text{RLP}_E$, $\text{RLP}_H$ denote quantity labor productivity and revenue labor productivity respectively. $\widetilde{P}$, $W/H$, and $\widetilde{W}$ denote firm-specific price index, wage per hour and the firm-specific wage index, respectively. Statistics for the latter variable are based on the smaller sample using the module *Verdienste* in the year 2006. All statistics are based on residuals after controlling for two-digit industry, four German regions, year, and interaction terms.

Table 7: Variance decompositions

| $Var(RLP_E)$ | $Var(\widetilde{P})$ | $Var(QLP_E)$ | $2Cov(\widetilde{P}, QLP_E)$ |
|---|---|---|---|
| 0.484 | 0.358 | 0.974 | -0.898 |
| $Var(RLP_H)$ | $Var(\widetilde{P})$ | $Var(QLP_H)$ | $2Cov(\widetilde{P}, QLP_H)$ |
| 0.439 | 0.358 | 0.903 | -0.822 |
| $Var(WH)$ | $Var(\widetilde{W})$ | $Var(\overline{WH})$ | $2Cov(\widetilde{W}, \overline{WH})$ |
| 0.081 | 0.035 | 0.027 | 0.019 |

Notes: Each variable is expressed in logs. Estimates are weighted by the average of employment of the firm for all its years of observation.

## B.4 Additional Tables

### B.4.1 Firm Dynamics

Table 8: Summary statistics for growth dynamics (unweighted)

| | $\widehat{S}$ | $\widehat{P}$ | $\widehat{Q}$ | $\widehat{E}$ | $\widehat{H}$ | $\widehat{W/E}$ | $\widehat{W/H}$ | $\widehat{\mathrm{QLP}_E}$ | $\widehat{\mathrm{QLP}_H}$ | $\widehat{\mathrm{RLP}_E}$ | $\widehat{\mathrm{RLP}_H}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\widehat{S}$ | 1.000 | | | | | | | | | | |
| $\widehat{P}$ | 0.285 | 1.000 | | | | | | | | | |
| $\widehat{Q}$ | 0.784 | -0.371 | 1.000 | | | | | | | | |
| $\widehat{E}$ | 0.286 | 0.028 | 0.259 | 1.000 | | | | | | | |
| $\widehat{H}$ | 0.305 | 0.030 | 0.276 | 0.609 | 1.000 | | | | | | |
| $\widehat{W/E}$ | 0.109 | 0.024 | 0.089 | -0.280 | 0.061 | 1.000 | | | | | |
| $\widehat{W/H}$ | -0.003 | 0.007 | -0.01 | -0.036 | -0.601 | 0.473 | 1.000 | | | | |
| $\widehat{\mathrm{QLP}_E}$ | 0.658 | -0.389 | 0.890 | -0.210 | 0.007 | 0.223 | 0.006 | 1.000 | | | |
| $\widehat{\mathrm{QLP}_H}$ | 0.570 | -0.393 | 0.810 | -0.085 | -0.338 | 0.055 | 0.357 | 0.875 | 1.000 | | |
| $\widehat{\mathrm{RLP}_E}$ | 0.882 | 0.278 | 0.674 | -0.199 | 0.028 | 0.250 | 0.014 | 0.777 | 0.636 | 1.000 | |
| $\widehat{\mathrm{RLP}_H}$ | 0.796 | 0.266 | 0.590 | -0.07 | -0.333 | 0.078 | 0.380 | 0.641 | 0.781 | 0.865 | 1.000 |
| Std. dev. | 0.172 | 0.115 | 0.177 | 0.083 | 0.107 | 0.078 | 0.094 | 0.175 | 0.177 | 0.168 | 0.169 |

Notes:

Table 9: Summary statistics for growth dynamics (weighted)

| | $\widehat{S}$ | $\widehat{P}$ | $\widehat{Q}$ | $\widehat{E}$ | $\widehat{H}$ | $\widehat{W/E}$ | $\widehat{W/H}$ | $\widehat{\mathrm{QLP}_E}$ | $\widehat{\mathrm{QLP}_H}$ | $\widehat{\mathrm{RLP}_E}$ | $\widehat{\mathrm{RLP}_H}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\widehat{S}$ | 1.000 | | | | | | | | | | |
| $\widehat{P}$ | 0.323 | 1.000 | | | | | | | | | |
| $\widehat{Q}$ | 0.767 | -0.359 | 1.000 | | | | | | | | |
| $\widehat{E}$ | 0.326 | 0.039 | 0.296 | 1.000 | | | | | | | |
| $\widehat{H}$ | 0.325 | 0.043 | 0.292 | 0.631 | 1.000 | | | | | | |
| $\widehat{W/E}$ | 0.109 | 0.034 | 0.084 | -0.221 | 0.042 | 1.000 | | | | | |
| $\widehat{W/H}$ | -0.013 | 0.008 | -0.02 | -0.036 | -0.596 | 0.502 | 1.000 | | | | |
| $\widehat{\mathrm{QLP}_E}$ | 0.638 | -0.389 | 0.893 | -0.167 | 0.011 | 0.190 | -0.004 | 1.000 | | | |
| $\widehat{\mathrm{QLP}_H}$ | 0.551 | -0.381 | 0.804 | -0.083 | -0.332 | 0.058 | 0.351 | 0.877 | 1.000 | | |
| $\widehat{\mathrm{RLP}_E}$ | 0.890 | 0.320 | 0.660 | -0.143 | 0.042 | 0.221 | 0.003 | 0.749 | 0.622 | 1.000 | |
| $\widehat{\mathrm{RLP}_H}$ | 0.797 | 0.302 | 0.580 | -0.057 | -0.312 | 0.085 | 0.368 | 0.631 | 0.766 | 0.870 | 1.000 |
| Std. dev. | 0.159 | 0.109 | 0.160 | 0.073 | 0.100 | 0.065 | 0.088 | 0.155 | 0.160 | 0.152 | 0.156 |

Notes:

### B.4.2 Firm Dynamics in the Homogeneous Sample

Table 10 shows that the growth rate of the firm-specific price index and of the average price are negatively correlated and each more dispersed than the growth rate of the Paasche price index. The table also shows that the firm-specific price growth is more negatively correlated (-0.234) with quantity growth than is the average price, (-0.018).

Table 10: Summary statistics for growth dynamics in the homogeneous sample

| | $\widehat{S}$ | $\widehat{\widetilde{P}}$ | $\widehat{\overline{P}}$ | $\widehat{Q}$ | $\widehat{RLP}$ | $\widehat{RHP}$ |
|---|---|---|---|---|---|---|
| $\widehat{S}$ | 1.000 | | | | | |
| $\widehat{\widetilde{P}}$ | 0.070 | 1.000 | | | | |
| $\widehat{\overline{P}}$ | 0.055 | -0.816 | 1.000 | | | |
| $\widehat{Q}$ | 0.782 | -0.234 | -0.018 | 1.000 | | |
| $\widehat{RLP}$ | 0.836 | 0.067 | 0.053 | 0.637 | 1.000 | |
| $\widehat{RHP}$ | 0.736 | 0.071 | 0.053 | 0.549 | 0.829 | 1.000 |
| Std. dev. | 0.142 | 0.194 | 0.169 | 0.155 | 0.143 | 0.146 |

Notes: All statistics are based on residuals after controlling for two-digit industry, four German regions, year, and interaction terms.

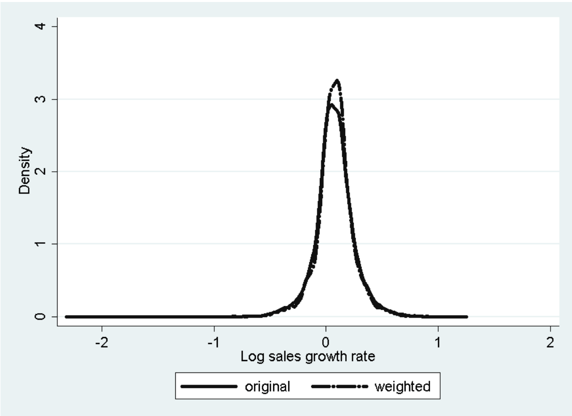
Table 11: Variance decomposition

| | $Var(\widehat{\widetilde{P}})$ | $Var(\widehat{\overline{P}})$ | $Var(\widehat{Q})$ | $2Cov(\widehat{\widetilde{P}},\widehat{\overline{P}})$ | $2Cov(\widehat{\widetilde{P}},\widehat{Q})$ | $2Cov(\widehat{\overline{P}},\widehat{Q})$ |
|---|---|---|---|---|---|---|
| $Var(\widehat{S}) = 0.017$ | 0.030 | 0.026 | 0.020 | -0.048 | -0.010 | -0.001 |

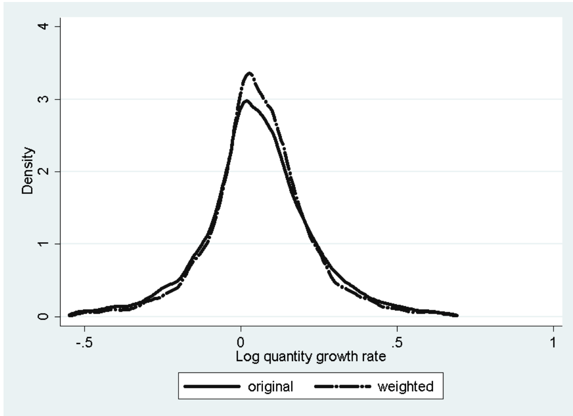
| | $Var(\widehat{\widetilde{P}})$ | $Var(\widehat{\overline{P}})$ | $Var(\widehat{Q})$ | $Var(\widehat{E})$ | $2Cov(\widehat{\widetilde{P}},\widehat{\overline{P}})$ | $2Cov(\widehat{\widetilde{P}},\widehat{Q})$ | $2Cov(\widehat{\widetilde{P}},\widehat{E})$ | $2Cov(\widehat{\overline{P}},\widehat{Q})$ | $2Cov(\widehat{\overline{P}},\widehat{E})$ | $2Cov(\widehat{Q},\widehat{E})$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $Var(\widehat{RLP}) = 0.017$ | 0.030 | 0.026 | 0.020 | 0.005 | -0.048 | -0.010 | -0.000 | -0.001 | -0.000 | -0.006 |

Notes: Each variable is expressed in logs. Estimates are weighted by the average of employment of the firm for all its years of observation.

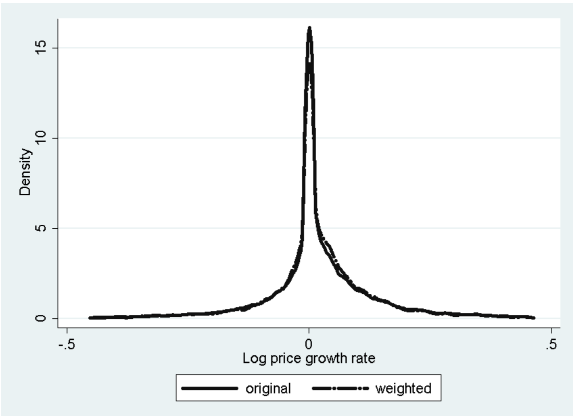
## B.5 Kernel Density Plots



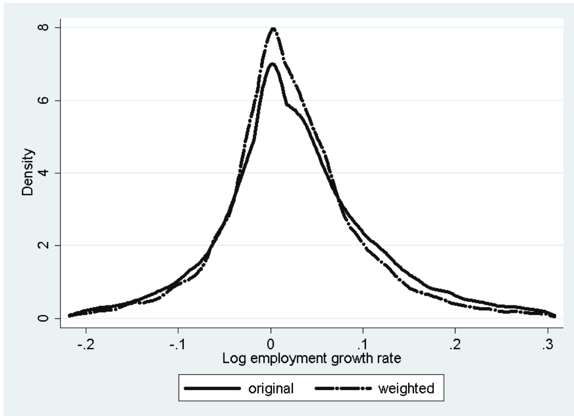

(a) Sales growth



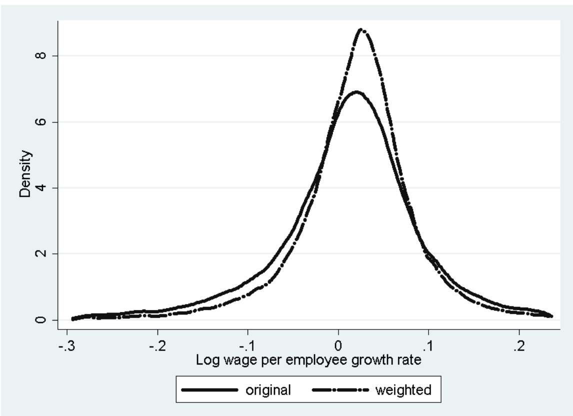

(b) Quantity growth



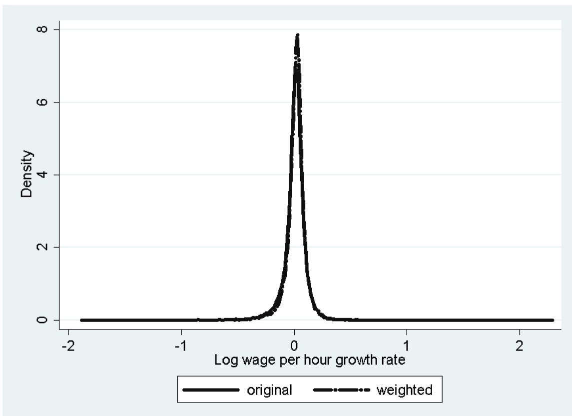

(c) Price growth




(d) Employment growth

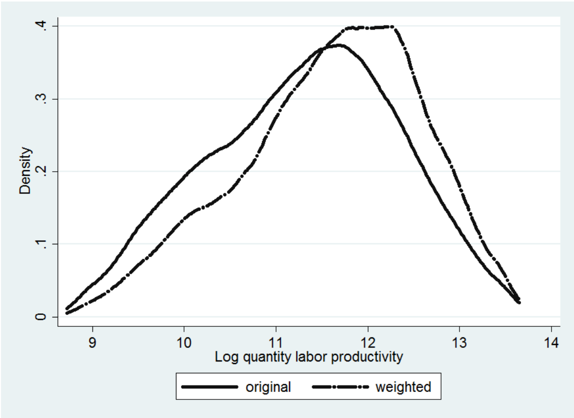



(e) Wage per employee growth

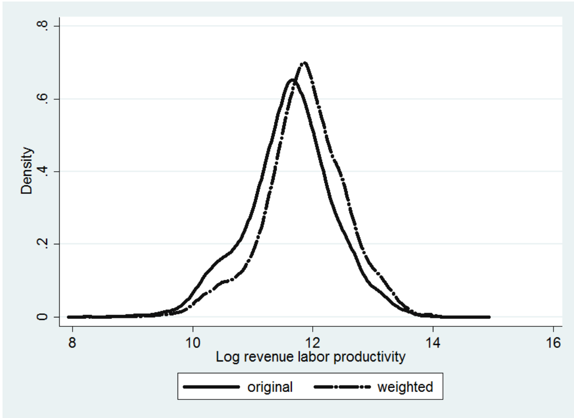



(f) Wage per hour growth

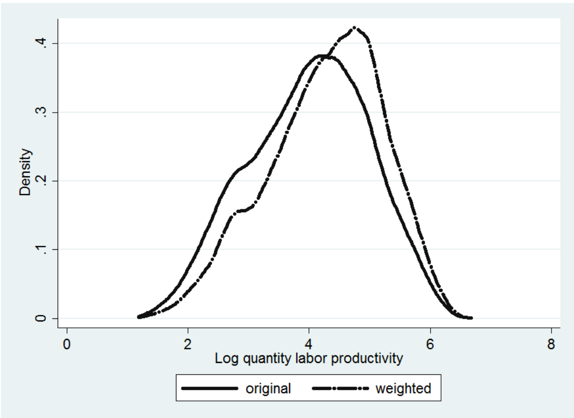
Figure 5: Distribution of growth rates in the *full sample*



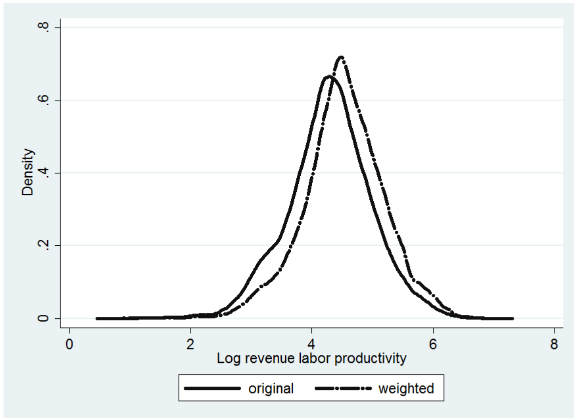

(a) Quantity labor productivity



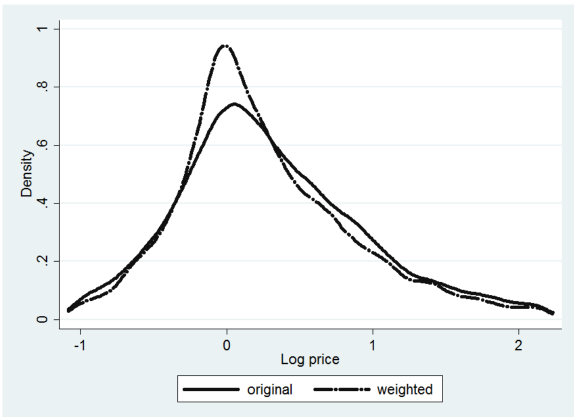

(b) Revenue labor productivity




(c) Hourly quantity labor productivity

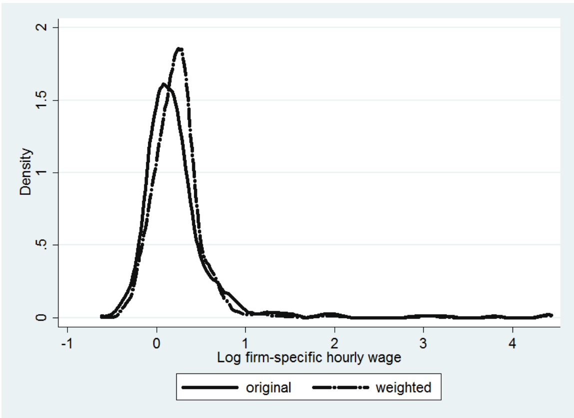



(d) Hourly revenue labor productivity

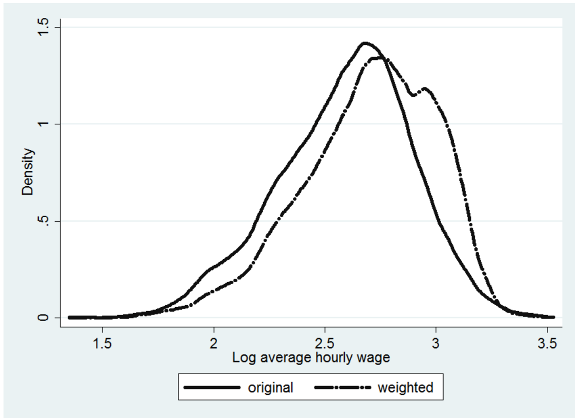



(e) Firm-specific price index

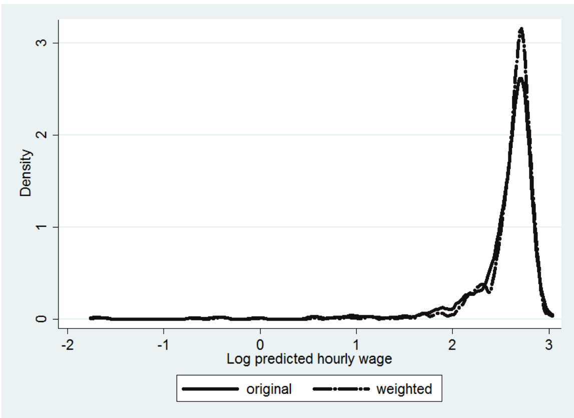
Figure 6: Quantity and revenue (hourly) labor productivity and firm-specific price index in the *homogeneous sample*




(a) Firm-specific wage index




(b) Average hourly wage




(c) Predicted hourly wage

Figure 7: Wage distributions including module *Verdienste* in the *homogeneous sample*