Currency Manipulation*

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Abstract

We propose a novel, risk-based transmission mechanism for the effects of currency manipulation: policies that systematically induce a country's currency to appreciate in bad times, lower its risk premium in international markets and, as a result, lower the country's risk-free interest rate and increase domestic capital accumulation and wages. Currency manipulations by large countries also have external effects on foreign interest rates and capital accumulation. Applying this logic to policies that lower the variance of the bilateral exchange rate relative to some target country ("currency stabilization"), we find that a small economy stabilizing its exchange rate relative to a large economy increases domestic capital accumulation and wages. The size of this effect increases with the size of the target economy, offering a potential explanation why the vast majority of currency stabilizations in the data are to the US dollar, the currency of the largest economy in the world. A large economy (such as China) stabilizing its exchange rate relative to a larger economy (such as the US) diverts capital accumulation from the target country to itself, increasing domestic wages, while decreasing wages in the target country.

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Differences in real interest rates across developed economies are large and persistent; some countries have lower real interest rates than others for decades rather than years. These long-lasting differences in interest rates correlate with differences in capital-output ratios across countries, and account for the majority of excess returns on the carry trade, a trading strategy where international investors borrow in low interest rate currencies, such as the Japanese Yen, and lend in high interest rate currencies, such as the New Zealand dollar (Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2015).

A growing literature studying these "unconditional" differences in currency returns argues that they may be attributable to heterogeneity in the stochastic properties of exchange rates: currencies with low interest rates pay lower returns because they tend to appreciate in bad times and depreciate in good times, providing a hedge to international investors and making them a safer investment (Lustig and Verdelhan, 2007; Menkhoff et al., 2013). This literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates, ranging from differences in country size (Martin, 2012; Hassan, 2013) and financial development (Maggiori, 2013) to trade centrality (Richmond, 2015) and differential resilience to disaster risk (Farhi and Gabaix, 2015).[1] The common theme across these papers is that whatever makes countries different from each other results in differential sensitivities of their exchange rates to various shocks, such that some currencies tend to appreciate systematically in "bad" states of the world (when the price of traded goods is high). Currencies with this property then pay lower expected returns and have lower risk-free interest rates.

In this paper, we argue that this risk-based view of differences in currency returns provides a novel way of thinking about the effects of currency manipulation: interventions in currency markets that change the stochastic properties of exchange rates should also change the expected returns on currencies and other assets. In particular, policies that induce a country's currency to appreciate in bad times should lower domestic interest rates, lower the cost of capital for the production of non-traded goods, and, as a result, increase capital accumulation. Moreover, if these interventions are large enough, that is, if the country manipulating its exchange rate is large relative to the world, its policies will affect interest rates and capital accumulation in other countries, potentially diverting capital accumulation from other countries to itself. Policies that change the variances and covariances of exchange rates should thus, via their effect on interest

[1] Other papers in this literature have studied heterogeneity in the volatility of shocks affecting the nontraded sector (Tran, 2013), factor endowments (Ready, Roussanov, and Ward, 2013; Powers, 2015), risk aversion in combination with country size (Govillot, Rey, and Gourinchas, 2010), and differences in exposure to long-run risk (Colacito et al., 2016).

rates and asset returns, affect the allocation of capital across countries.

After making this argument in its most general form, we illustrate its implications with an application to currency stabilization. Table 1 shows that 88% of countries (representing 47% of world GDP) stabilize their currency relative to some target country (Reinhart and Rogoff, 2004). Such policies specify a target currency (two thirds of them the US dollar) and set an upper bound for the volatility of the real or nominal exchange rate relative to that target country. A conventional "hard" peg may set this volatility to zero, while "soft" pegs (including moving bands, crawling pegs, stabilized arrangements, and managed floats), may officially or unofficially specify a band of allowable fluctuations around some mean. The common feature of all of these policies is that they manipulate the variances and covariances of exchange rates by changing the states of the world and the extent to which they appreciate and depreciate, without necessarily manipulating their level.

Table 1: 2010 Exchange Rate Arrangements based on Reinhart and Rogoff (2004, 2011)

	% of Countries	% of GDP
Panel A	*Exchange rate arrangement*	
Floating	3%	34%
Stabilized	88%	47%
soft peg	47%	32%
hard peg	41%	15%
Currency union	9%	19%
Panel B	*Target currencies of stabilizations*	
Dollar	67%	80%
Euro	27%	19%

Notes: Classification of exchange rate regimes as of 2010 according to Reinhart and Rogoff (2004, 2011). All data are available on Carmen Reinhart's website at www.carmenreinhart.com/data/browse-by-topic/topics/11.

We analyze the effects of such currency stabilizations on interest rates, capital accumulation, and wages within a generic model of exchange rate determination. In the model, households consume a bundle of a freely traded good and a country-specific nontraded good. The nontraded good is produced using capital and labor as inputs. In equilibrium, the real exchange rate fluctuates in response to country-specific (supply) shocks to the productivity in the production of nontraded goods across countries, (demand) shocks to preferences, and (monetary) shocks to the inflation rates of national currencies.

As a stand-in for the various potential sources of heterogeneity in the stochastic properties of countries' exchange rates studied in the literature mentioned above, we add heterogeneity in country size to this canonical setup, as in Hassan (2013). That is, we assume that all shocks are common within countries and some countries account for a larger share of world GDP than others. This heterogeneity in country size generates differences in the stochastic properties of countries' exchange rates, where the currencies of larger countries tend to appreciate in "bad" times: households react to supply, demand, and monetary shocks by shipping traded goods across countries in an effort to share risk across borders. However, shocks that affect larger countries are harder to diversify internationally. For example, when a country has a low per capita output of nontraded goods, its consumption bundle becomes relatively more expensive and its real exchange rate appreciates. To compensate for the shortfall of nontraded goods, the country imports additional traded goods from the rest of the world. However, a low output of nontraded goods in a large country simultaneously triggers a rise in the world market price of traded goods, while a low output of nontraded goods in a small country does not. As a consequence, currencies of large countries tend to appreciate when the world market price of traded goods is high, offering a hedge against world-wide consumption risk. Because of these hedging properties, the currencies of large countries pay lower expected returns and have lower risk-free interest rates. Lower interest rates in turn lower the cost of capital in these countries, prompting them to install higher capital-output ratios and pay higher wages in equilibrium (Hassan et al., 2015).

Within this economic environment, we study the positive and normative effects of a class of policies that lower the variance of one "stabilizing" country's real exchange rate relative to a "target" country's currency, while leaving the mean of the real exchange rate unaffected. We largely focus our discussion on policies that stabilize the *real* exchange rate but also generalize our main results to nominal stabilizations in a setting where money is not neutral.

To stabilize its exchange rate, the stabilizing country's government alters the state-contingent plan of imports and exports of traded goods. In particular, when the target country appreciates, it matches this appreciation by reducing traded goods consumption and thus raising domestic marginal utility and the price of domestic consumption. Similarly, when the stabilizing country suffers a shock that increases domestic marginal utility that would ordinarily result in an appreciation, it imports additional traded goods to lower domestic marginal utility. The stabilizing country's government implements these policies using monetary policy if money is not neutral or, more generally, by imposing a set of state-contingent taxes financed by using an independent source of wealth ("currency reserves").

We first consider the case in which the stabilizing country is small and thus only affects its own price of consumption. A small country that imposes a stabilization of its exchange rate relative to a larger country inherits the stochastic properties of the larger country's exchange rate: the stabilized exchange rate now tends to appreciate when the price of traded goods in world markets is high, making the stabilizing country's currency a better hedge against consumption risk, lowering its risk-free interest rate and the expected return on its currency. Similarly, investments in the stabilizing country's capital stock now become more valuable, increasing its capital-output ratio and raising wages within the country.

To sustain the stabilization, the stabilizing country ships additional traded goods to the rest of the world in states of the world when the target country appreciates. If the target country is large, then these tend to be states when the shadow price of traded goods is high. As a result, stabilizing relative to a larger target country generates an insurance premium, which lowers the cost of implementing the stabilization. If the target country is sufficiently large, this insurance premium may be so large that the stabilization generates positive revenues and the stabilizing country accumulates, rather than depletes, reserves.

However, this revenue-generating effect of currency stabilizations relative to larger countries diminishes when the stabilizing country itself becomes larger, because the stabilization exaggerates the variation in the stabilizing country's own demand for traded goods, increasing its price-impact: in states of the world in which the stabilizing country has high marginal utility, and would ordinarily appreciate relative to the target country, it must import even more traded goods than it would have in the absence of the stabilization to prevent appreciation. When the stabilizing country is large enough to affect the equilibrium price of traded goods, the stabilization induces an unfavorable change in the state-contingent prices of traded goods. The larger the stabilizing country, the more reserves are required to maintain the policy.

Our model also allows us to solve for the effects of the stabilization on the target country: a country that becomes the target of a stabilization imposed by a country that is large enough to affect world prices (or the target of multiple stabilizations imposed by a non-zero measure of small countries) experiences a rise in its risk-free interest rate, a decrease in its capital-output ratio, and a decrease in wages. The reason is that, to sustain its stabilization, the stabilizing country supplies additional traded goods to the world market whenever the target country appreciates. This activity dampens the impact of the target country's shocks on the shadow price of traded goods, reducing their spill-over to the world market. The lower this impact, the lower the co-movement between the price of traded goods and the target country's exchange rate. Hence, the currency of the target country becomes a less attractive hedge for international investors, raising its risk-free interest rate.

In various robustness checks we show that this broad set of conclusions arises regardless of whether variation in exchange rates are driven primarily by supply, demand, or monetary shocks; and regardless of whether financial markets are complete or segmented within countries. Moreover, we show that in the presence of sticky prices, stabilizations of the nominal exchange rate, implemented with monetary policy, have the same positive implications as the stabilizations of the real exchange rate discussed above.

We also examine the welfare effects of currency stabilizations for a special case of our model, where markets are complete and exchange rates vary exclusively as a result of supply shocks. In this simpler model, currency stabilizations tend to be welfare decreasing for the stabilizing country because any gains in revenues from the stabilization and from the increase in capital accumulation are outweighed by the adverse effect of an increase in the volatility of consumption. Conversely, becoming the target of a stabilization reduces one's variance of consumption, resulting in a net welfare increase, despite the detrimental effects on the target country's capital stock. However, these welfare results may reverse depending on valuation effects. For example, imposing a stabilization may increase the stabilizing country's welfare if, at announcement of the policy, the country exhibits a large home-bias towards domestic rather than foreign bonds or towards stocks in its own rather than foreign firms. Upon announcement of the exchange rate stabilization, domestic stocks and bonds become more valuable in world markets, resulting in a positive wealth effect if households in the stabilizing country own a disproportionate share of these assets.

Taken together, we believe our results provide a novel way of thinking about currency manipulation in a world in which risk-premia affect the level of interest rates. First, by manipulating exchange rates, policymakers may be able to manipulate the allocation of capital across countries. Second, although currency stabilization does not generally appear optimal under standard welfare measures, our model shows that policymakers might have a motive to engage in it if their objective is to increase wages, increase capital accumulation, or raise revenue. For example, we might think of political reasons why policymakers might have an interest in raising wages or of externalities that may make it optimal to increase capital accumulation. Third, whatever the motive for stabilizing, stabilizations relative to larger countries appear to be cheaper to implement and more impactful on all dimensions than those to smaller countries, offering a potential explanation for the fact that almost all stabilizations in the data are relative to the euro or dollar. Fourth, the costs of stabilizing are increasing in the size of the country implementing the policy, offering a potential explanation why most large developed countries do not stabilize their exchange rates. Finally, our model speaks to the external effects of stabilizations on the target country, providing a meaningful notion of what it means to be at the center of the world's monetary system: countries that stabilize relative to a common target divert capital accumulation from the target while dampening the effects of shocks emanating from the target on the world economy.[2]

This latter point also offers an interesting perspective on the large public debate over the Chinese exchange rate regime: U.S. policymakers have often voiced concern that China may be undervaluing its exchange rate and that this undervaluation may be bad for U.S. workers and good for Chinese workers. The official Chinese response to these allegations has been that China is merely stabilizing the exchange rate and not systematically distorting its level. The implication of our analysis is that even if this assertion is accurate, the mere fact that China is stabilizing its currency to the dollar may divert capital accumulation from the U.S. to China, a policy that is likely to be bad for U.S. workers.

We make two main caveats to our interpretation. First, although we use differences in country size as the only mechanism generating differences in interest rates in our model, we do so mainly for parsimony. Variations of the model where differences in interest rates also arise as a result of differences in trade centrality, financial development, or some of the other microfoundations mentioned above, should yield similar results and interpretations—with the United States typically emerging as the most systemic country for the world economy. Second, in our model, currency manipulation transmits itself only through its effects on trade flows. That is, the frictions that allow the government to manipulate exchange rates are between countries. We do not consider richer frictions, where currency manipulation could also affect allocations within countries, such as the amount of labor supplied or the distribution of wealth across households.

A large literature studies the effects of monetary stabilization and exchange rate pegs in the presence of nominal frictions.[3] Closely related are Kollmann (2002) and Bergin and Corsetti

[2]In this sense, our paper also relates to a growing literature that argues for a special role of the US dollar in world financial markets. See for example Gourinchas and Rey (2007), Lustig et al. (2011), Maggiori (2013), and Miranda-Agrippino and Rey (2015).

[3]One strand of the literature analyzes optimal monetary policy in small open economies with fixed exchange rates (Kollmann, 2002; Parrado and Velasco, 2002; Gali and Monacelli, 2005), while another deals with the choice of the exchange rate regime in the presence of nominal rigidities (Helpman and Razin, 1987; Bacchetta and van Wincoop, 2000; Devereux and Engel, 2003; Corsetti, Dedola, and Leduc, 2010; Schmitt-Grohé and Uribe, 2012; Bergin and Corsetti, 2015).

(2015), where currency pegs affect markups and the level of capital accumulation through their effects on nominal rigidities. Another, largely empirical, literature investigates the effects currency stabilizations on the level of trade flows.[4] We add to this literature in two ways. First, we study a novel effect of currency stabilization on risk premia that operates even in a frictionless economy in which money is neutral. Second, we are able to study how the (internal) effects of currency stabilization vary with the choice of the target currency and how these policies affect the target country.

More broadly, our paper also relates to a large literature on capital controls.[5] Similar to the work by Costinot, Lorenzoni, and Werning (2014), who argue that capital controls may be thought of as a manipulation of intertemporal prices, we show that currency stabilizations, and other policies altering the stochastic properties of exchange rates, may be thought of as a manipulation of state-contingent prices. The key difference between the two concepts is that capital controls affect allocations through market power and rents, while currency manipulation affects allocations through risk premia, even when the country manipulating its exchange rate has no effect on world market prices. In addition, our work shows that, in contrast to capital controls, currency stabilizations may be harder to rationalize as optimal policies within a frictionless neoclassical model.

Finally, as mentioned above, our paper relates to a growing empirical literature that argues that "unconditional" differences in currency returns may be attributable to heterogeneity in the stochastic properties of exchange rates.[6] The theoretical side of this literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates. We add to this literature by showing that this class of model implies that exchange rate manipulations affect allocations through their effect on currency risk premia.

The remainder of this paper is structured as follows: section 1 outlines the effects of currency manipulation on risk premia in their most general form. Section 2 analyzes the effects of stabilizations of the real exchange rate in the context of a simple international real business cycle model. Section 3 generalizes the results from this analysis to stabilizations of the nominal exchange rate when prices are sticky or markets are segmented. Section 4 considers more general economic environments where exchange rates are driven by monetary or preference shocks.

[4] Hooper and Kohlhagen (1978) find that exchange rate stabilization has little effect on trade volume, whereas studies by Kenen and Rodrik (1986) and Frankel and Rose (2002) suggest the gains to trade volume are substantial.

[5] See for example Calvo and Mendoza (2000), Jeanne and Korinek (2010), Bianchi (2011), Farhi and Werning (2012), Schmitt-Grohé and Uribe (2012), Farhi and Werning (2013), and Korinek (2013).

[6] See for example Lustig and Verdelhan (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Lustig et al. (2011), Menkhoff et al. (2012), David, Henriksen, and Simonovska (2016), and Verdelhan (2015).

1 Reduced Form Model of Exchange Rates

We begin by deriving the main insights of our analysis in their most general form. Consider a class of models in which the utility of a representative household in each country n depends on its consumption of a final good that consists of a country-specific nontraded good and a freely traded good. In this class of models, we may write the price of the final good in country n in reduced form as

$$p^n = a\lambda_T - bx^n, \tag{1}$$

where p^n is the log of the number of traded goods required to purchase one unit of the final good in country n, λ_T is the log shadow price traded goods in the world market, b is a constant greater than zero, and $x^n \sim N(0, \sigma_x^2)$ is a normally distributed shock to the log price of consumption in country n. We may think of this shock interchangeably as the effect of a country-specific supply, demand, or monetary shock; in other words, it is a stand-in for any factor that affects the price of consumption in one country more than in others. The higher x^n, the lower is the price of domestic consumption.

If households can share risk by shipping traded goods between countries, these country-specific shocks will be reflected in the equilibrium shadow price of traded goods in the world. Thus, if many countries have adverse shocks, the shadow price of traded goods will be high in the world, and vice versa. In the model we derive below, this relationship is linear with

$$\lambda_T = -\sum_n w^n x^n, \tag{2}$$

where the weights $w^n \geq 0$ may differ across countries.

The real exchange rate between two countries is the relative price of their respective final goods. The log real exchange is thus

$$s^{f,h} = p^f - p^h.$$

The risk-based view of differences in currency returns applies some elementary asset pricing to this expression. Using the Euler equation of an international investor, one can show that the log expected return to borrowing in country h and to lending in country f is

$$r^f + \Delta\mathbb{E}s^{f,h} - r^h = cov\left(\lambda_T, p^h - p^f\right) = b\left(w^h - w^f\right)\sigma_x^2, \tag{3}$$

where r^n is the risk-free interest rate in country n. [7] This statement means that a currency that tends to appreciate when the shadow price of traded goods is high pays a lower expected return and, if $\Delta\mathbb{E}s^{f,h} = 0$, also has a lower risk-free interest rate. Currencies that appreciate in bad times provide a hedge against world-wide consumption risk and must pay lower returns in equilibrium. From inspecting (2), it is clear that these "safe" currencies must be those that have a relatively large w^n. These are the currencies of countries whose shocks spill over to world markets more than the shocks of other countries and move λ_T whenever they move the price of domestic consumption.

This line of argument (equations (1)-(3)) is the main ingredient of risk-based models of unconditional differences in interest rates across countries, where different approaches model differences in w^n as the result of heterogeneity in country size, the volatility of shocks, trade centrality, financial development, factor endowments, etc.

We make a simple point relative to this literature: if there is merit to this risk-based view of currency returns, policies that alter the covariance between a country's exchange rate and the shadow price of traded goods can alter interest rates, currency returns, and the allocation of capital across countries. In particular, a country that adopts policies that increase the price of domestic consumption in states of the world where λ_T is high, can lower its risk-free interest rate relative to all other countries in the world.

As an example, consider a "manipulating" country (indexed by m) that levies a state-contingent tax on domestic consumption of traded goods that is proportional to the realization of x^t in some target country t, such that

$$p^m = a\lambda_T - bx^m - cx^t.$$

If the target country's shock affects the world price of traded goods, that is if $w^t > 0$, the tax increases the covariance between p^m and λ_T by $cw^t\sigma_x^2$. As a result, it lowers country m's interest rate relative to all other countries in the world. The larger w^t, the larger is this effect. In this sense, currency manipulations that hone in on the shocks affecting the most systemic countries in the world are most impactful.

In addition, and this will become clear when we move to our fully specified model, the state-contingent tax also impacts country m's state-contingent plan of shipping traded goods to and from the world. Specifically, a tax that increases the price of consumption when x^t is low induces

[7] Where $\Delta\mathbb{E}s^{f,h}$ is defined as the logarithm of the ratio of the countries' expected real price changes. See Appendix A for a formal derivation.

shipments of traded goods from country m to the rest of the world in those states. If the country that manipulates its exchange rate is itself large in the sense that its actions affect the world-market price of traded goods, its manipulation reduces the target country's weight w^t in (2). That is, it dampens the extent to which the target country's shock spills over to other countries. As a consequence, the covariance between p^t and λ_T falls, increasing the interest rate in the target country. A state-contingent tax of the form above thus *raises* the interest rate in the target country, while lowering it in the country that manipulates its exchange rate.

If interest rates play a role in allocating capital across countries (as it is the case in our fully specified model), manipulations of the stochastic properties of exchange rates can thus divert capital accumulation from the target country of the manipulation to the country that conducts the manipulation, and, more broadly, alter the equilibrium allocation of capital across countries.

The remainder of this paper fleshes out this argument in the context of a general equilibrium model of exchange rate determination and applies it to one of the most pervasive policies in international financial markets: currency stabilization.

2 Stabilizing the Real Exchange Rate

We begin by studying the effect of stabilizations of the real exchange rate in an environment where markets are complete, money is neutral, and the allocation of capital across countries, as well as the stochastic properties of real exchange rates, are determined solely as a function of productivity shocks (Backus and Smith, 1993). Within this canonical model, one country, labeled the "stabilizing" country, deviates from the competitive equilibrium by stabilizing its exchange rate with respect to a "target" country.

The following sections then generalize the findings from this analysis by considering stabilizations of the nominal exchange rate, monetary frictions, and preference shocks.

2.1 Economic Environment

There are two discrete time periods, $t = 1, 2$. There exists a unit measure of households $i \in [0, 1]$, partitioned into three subsets Θ^n of measure θ^n. Each subset represents the constituent households of a country. We label these countries $n = \{p, t, o\}$ for the "stabilizing", "target", and "outside" country, respectively. Households make an investment decision in the first period. All consumption occurs in the second period.

Households derive utility from consuming a consumption index composed of a country-specific nontraded good, $C_{N,2}$, and a traded good $C_{T,2}$ where

$$C_2(i) = C_{T,2}(i)^{\tau}\; C_{N,2}(i)^{1-\tau}$$

and $\tau \in (0,1)$. Each household exhibits constant relative risk aversion according to

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(C_2(i))^{1-\gamma}\right], \tag{4}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

At the start of the first period, each household receives one traded good and one unit of a capital good. Traded goods can be stored for consumption in the second period and are freely shipped internationally. Capital goods can only be freely shipped in the first period when they are invested for use in the production of nontraded goods in the second period.

Households produce their country-specific nontraded good using a Cobb-Douglas production technology that employs capital and labor. Each household supplies one unit of labor inelastically and purchases capital in international markets in the first period. The per capita output of nontraded goods is

$$Y^n_{N,2} = \exp(\eta^n)\,(K^n)^{\nu}$$

where $0 < \nu < 1$ is the capital share in production, K^n is the *per capita* stock of capital in country n and η^n is a country-specific productivity shock realized at the start of the second period,

$$\eta^n \sim N\left(-\frac{1}{2}\sigma^2_N, \sigma^2_N\right).$$

At the end of the first period, a complete set of Arrow-Debreu securities is traded. Throughout we use the traded consumption good as the numéraire, such that all prices and returns are accounted for in the same units. To simplify the derivation, we also assume that households receive a country-specific transfer, κ^n, before trading begins, which decentralizes the allocation corresponding to the Social Planner's problem with unit Pareto weights.

Currency Stabilization The stabilizing country's government has the ability to levy a state-contingent tax on the domestic price of Arrow-Debreu securities paying traded goods, $Z(\omega)$, and can pay a lump-sum transfer of traded goods to each household in its country, $\bar{Z}$. The government's sole policy objective is to decrease fluctuations of its country's log real exchange rate with the target country by a fraction $\zeta \in (0,1]$ relative to the freely-floating regime, without distorting the conditional mean of the log real exchange rate. Denoting the real exchange rate that would arise under a free floating exchange rate regime with an asterisk, we can write the policy objectives as

$$var\left(s^{t,p}\right) = (1-\zeta)^2 var\left(s^{t,p*}\right) \tag{P1}$$

and

$$\mathbb{E}\left[s^{t,p}|\{K^n\}\right] = \mathbb{E}_1\left[s^{t,p*}|\{K^n\}\right]. \tag{P2}$$

We refer to $\zeta \in (0,1]$ as a stabilized real exchange rate and $\zeta = 1$ as a "hard" peg.

The government thus has two policy instruments to achieve two objectives, using the state contingent tax to archive (P1) and the lump-sum transfer to simultaneously achieve (P2). We can write the per capita cost of implementing this stabilization policy as

$$\Delta Res = \bar{Z} - \int \left(Z(\omega) - 1\right) Q(\omega) \left(P^p(\omega)C^p(\omega) - P^p_N(\omega)Y^p_N(\omega)\right) d\omega, \tag{5}$$

where $Q(\omega)$ is the price of a state-contingent security that pays one traded good in state ω, $P^p(\omega)C^p(\omega)$ is the number of traded goods needed to finance domestic consumption in state ω, and $P^p_N(\omega)Y^p_N(\omega)$ is the value, again in terms of traded goods, of domestic nontraded goods production. The cost of the tax is simply the cost of the lump-sum transfer, less the net revenues from the state-contingent tax.

To focus on stabilizations of the exchange rate that do not distort the mean, we begin by assuming that the government finances this policy using an independent supply of traded goods (currency reserves) that absorbs any surpluses or deficits generated by the taxation scheme (ΔRes). We show below that under a range of relevant parameters, the cost of currency stabilization is negative, such that the policy is implementable even if the government has no access to currency reserves. We also show that all the main insights of our analysis below continue to hold if for some reason the government's implementation of the policy is only partially credible. When we analyze the welfare effects of exchange rate stabilization in section 2.5, we assume that ΔRes is fully borne by the households in the stabilizing country, and instead allow the stabilization to distort the mean (and thus violate (P2)).

The market clearing conditions for traded, nontraded, and capital goods are

$$\int_{i\in[0,1]} C_{T,2}(i,\omega)di = 1 + \theta^p \Delta Res, \tag{6}$$

$$\int_{i\in\theta^n} C_{N,2}(i,\omega)di = \theta^n Y^n_{N,2}(\omega), \tag{7}$$

and

$$\sum_n \theta^n K^n = 1. \tag{8}$$

The economy is in an equilibrium when all households maximize utility taking prices and taxes as given, firms maximize profits, and goods markets clear.

Because all households within a given country are identical and consumption only occurs in the second period, we write their consumption bundle as (C^n_T, C^n_N) and henceforth drop the household index i as well as the time subscript t.

2.2 Solving the Model

Appendix D derives the conditions of optimality characterizing the equilibrium allocation. The first-order conditions with respect to C^n_T equate the shadow price of traded consumption across the target and outside countries

$$\left(C^n(\omega)\right)^{1-\gamma}\left(C^n_T(\omega)\right)^{-1} = \Lambda_T(\omega),\ n = o, t. \tag{9}$$

In the stabilizing country, the state-contingent tax that implements the currency stabilization appears as a wedge on that shadow price

$$\left(C^p(\omega)\right)^{1-\gamma}\left(C^p_T(\omega)\right)^{-1} = Z(\omega)\Lambda_T(\omega). \tag{10}$$

In all countries, marginal utilities with respect to $C^n_{N,2}$ define the shadow prices of nontraded goods

$$(1-\tau)\left(C^n(\omega)\right)^{1-\gamma}\left(C^n_N(\omega)\right)^{-1} = \Lambda^n_N(\omega). \tag{11}$$

In addition, we derive households' optimal demand for capital by taking first-order conditions with respect to K^n. Using the fact that competitive markets imply $P^n_N(\omega) = \Lambda^n_N(\omega)/\Lambda_T(\omega)$, we get

$$K^n = \frac{\nu}{\Psi_T q_1}\mathbb{E}\left[\Lambda^n_N Y^n_N\right], \tag{12}$$

where q_1 is the equilibrium price of one unit of capital and $\Psi_T = \mathbb{E}\left[\Lambda_T(\omega)\right]$ is the shadow price of a traded good in the first period, prior to the realization of shocks. This Euler equation defines the level of capital accumulation in country n as a function of first-period prices and the expected (utility) value of its nontraded goods, $\mathbb{E}[\Lambda_N^n Y_N^n]$. This latter term will differ across countries and reflect any precautionary motives for capital accumulation, including those that arise as a function of the stochastic properties of the country's exchange rate.

Importantly, this condition holds in all countries, including the stabilizing country, because the stabilizing government's intervention alters the state-contingent valuation of nontraded output and nontraded consumption in an off-setting way. In equilibrium, two effects cancel such that (12) holds in all countries (see Appendix D for a formal derivation).

Finally, the (redundant) first-order conditions with respect to the consumption index C^n pin down the shadow prices of overall consumption in each country

$$(C^n(\omega))^{-\gamma} = \Lambda^n(\omega). \tag{13}$$

The real exchange rate between two countries h and f equals the ratio of these shadow prices,

$$S^{f,h}(\omega) = \Lambda^f(\omega)/\Lambda^h(\omega).$$

In equilibrium, the resource constraints (6)-(8) and the conditions of optimality (9)-(12) jointly determine define the endogenous variables $\left\{C_{N,2}^n, C_{T,2}^n, K^n, \Lambda_{N,2}^n\right\}_{n\in\{p,t,o\}}, \Lambda_{T,2}$ and q_1. To study the model in closed form, we log-linearize it around the deterministic solution — the point at which the variances of shocks are zero ($\sigma_{N,n} = 0$) and all firms have a capital stock that is fixed at the deterministic steady state level. That is, we study the *incentives* to accumulate different levels of capital across countries, while holding the capital stock fixed. For simplicity we ignore the feedback effect of differential capital accumulation on the size of risk premia. Appendix Q shows that the propositions hold when we allow for endogenous capital accumulation and account for these feedback effects. Throughout, lowercase variables continue to refer to natural logs.

2.3 The Freely Floating Regime

We begin by showing that, in the absence of currency manipulation, the model predicts that large countries should have lower real interest rates (Hassan, 2013) and accumulate higher capital-output ratios (Hassan, Mertens, and Zhang, 2015). If $\zeta = 0$, equilibrium consumption of traded goods is given by

$$c_T^{n*} = \frac{(1-\tau)(\gamma-1)}{(1-\tau)+\gamma\tau}\left(\bar{y}_N - y_N^n\right),$$

where $\bar{y}_N = \sum_n \theta^n y_N^n$ is the average log per-capita output of nontraded goods across countries. The expression shows that households use shipments of traded goods to insure themselves against shocks to the output of nontraded goods. If $\gamma > 1$, households receive additional traded goods whenever they have a lower-than-average output of nontraded goods, and vice versa.[8]

This risk-sharing behavior generates a shadow price of traded goods of the form given in (2),

$$\lambda_T^* = -(\gamma - 1)(1 - \tau) \sum_n \theta^n y_N^n, \tag{14}$$

where each country's weight is proportional to its size: shocks to the productivity of larger countries affect a larger measure of households and thus tend to spill over to the rest of the world in the form of higher shadow prices of traded goods. If $\gamma > 1$, the shadow price of traded goods falls with the average output of nontraded goods across countries. Thus, λ_T tends to be low in "good" states of the world when countries experience positive productivity shocks in their nontraded sectors.

The real exchange rate between two arbitrary countries f and h is

$$s^{f,h*} = \lambda^{f*} - \lambda^{h*} = \frac{\gamma(1-\tau)}{(1-\tau) + \gamma\tau} \left(y_N^h - y_N^f \right).$$

The currency of the country with lower per-capita output of nontraded goods appreciates because its final consumption bundle is expensive relative to that in other countries.

Inspecting λ_T^* and $s^{f,h*}$ shows that currencies of larger countries are "systemic" in the sense that they tend to appreciate when the shadow price of traded goods is high: whenever a country suffers a low productivity shock and its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is (note that $s^{f,h*}$ is independent of θ). However, a shock to a larger country has a larger impact on the rest of the world. It immediately follows from the first equality in (3) that larger countries have a lower risk-free rate.

$$r^{f*} + \Delta \mathbb{E} s^{f,h*} - r^{h*} = cov\left(\lambda_T^*, p^{h*} - p^{f*}\right) = \frac{(\gamma-1)\gamma(1-\tau)^2}{1 + (\gamma - 1)\tau} \left(\theta^h - \theta^f\right) \sigma_N^2 \tag{15}$$

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, we can rearrange the Euler equation for capital accumulation (12)

[8] Condition $\gamma > 1$ ensures that the cross-partial of marginal utility from traded consumption with respect to the nontraded good is negative, that is, the relative price of a country's nontraded good falls when its supply increases (see Coeurdacier (2009) for a detailed discussion).

and derive an expression that links difference in capital to differences in interest rates[9]

$$k^{f*} - k^{h*} = \frac{\gamma}{\tau(\gamma-1)^2}\left(r^{h*} - \Delta\mathbb{E}s^{f,h*} - r^{f*}\right) \tag{16}$$

It is efficient to accumulate more capital in the larger country because a larger capital stock in larger country represents a good hedge against global consumption risk. Households around the world fear states of the world in which the large country receives a low output from its nontraded sector, because larger countries transmit these shocks to the rest of the world through a higher shadow price of traded consumption. Although households cannot affect the realization of productivity shocks, they can partially insure themselves against low output in the nontraded sector of large countries by accumulating more capital in these countries. This raises expected output in the nontraded sector and dampens the negative effects of a low productivity shock.

2.4 Positive Effects of Currency Stabilization

Under freely floating exchange rates, larger countries thus have lower risk-free rates and higher capital-output ratios. With this result in mind, we now analyze how a country can influence interest rates and the allocation of capital by stabilizing its currency.

While the policy objectives (P1) and (P2) with $\zeta < 1$ can in principle be achieved with a range of different non-linear policies, such as intervening only in response to shocks smaller or larger than some critical value, we focus our discussion on the unique linear policy that entails a proportional intervention in each state. The advantage of focusing on this case is simply that it preserves the Gaussian structure of the problem and thus lends itself to closed-form solutions. In section 2.6 we discuss issues that arise when the government cannot credibly commit to stabilizing shocks larger or smaller than some critical value and show that our main conclusions do not change in that case.

The following lemma characterizes the unique linear form of state contingent taxes that implements the exchange rate stabilization:

Lemma 1

A tax on all assets paying off consumption goods in the stabilizing country of the form

$$z(\omega) = \zeta\frac{1-\tau}{\tau}\left(y_N^p - y_N^t\right)$$

[9]For a derivation, see Appendix F.

implements a real exchange rate stabilization of strength ζ.

The cost of the stabilization, ΔRes, equals the change in the world-market cost traded goods consumed by households in the stabilizing country,

$$\Delta Res = \int Q(\omega) C_T^p(\omega) d\omega - \int Q^*(\omega) C_T^{p*}(\omega) d\omega. \tag{17}$$

Proof. See Appendix G. ■

To build intuition for the effects of this state-contingent tax, it is useful to solve for the change in the equilibrium consumption of traded goods by domestic households relative to the freely floating regime:

$$c_T^p - c_T^{p*} = \zeta \frac{(1-\tau)(1-\theta^p)}{\tau\left((1-\tau)+\gamma\tau\right)} \left(y_N^t - y_N^p\right). \tag{18}$$

When the target country receives a relatively bad productivity shock ($y_N^t < y_N^p$), its price of consumption appreciates. To mirror this increase, the stabilizing country raises taxes on traded goods, reduces its consumption of traded goods relative to the freely floating regime, and thus raises its own marginal utility. Conversely, when the stabilizing country receives a relatively bad shock, its price of consumption would ordinarily increase. To offset this increase and prevent its currency from appreciating, the government subsidizes imports of traded goods, resulting in even higher imports of traded goods than under the freely floating regime.

We start by analyzing the effect of this stabilization policy on allocations, prices, and currency reserves in the stabilizing country. Afterwards, we analyze its impact on prices and quantities in the target country.

2.4.1 Internal Effects of Currency Stabilization

The most obvious effect of currency stabilization is that the price level in the stabilizing country becomes more correlated with price level in the target country.

$$\lambda^p = \lambda^{p*} + (1-\theta^p)\zeta \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau} \left(y_N^p - y_N^t\right)$$

Similar to the intervention considered in (1), the real exchange rate stabilization increases the weight of the target country's shock in the stabilizing country's price level, while also decreasing the weight of its own shock. That is, λ^p starts behaving more like λ^t. If $\theta^t > \theta^p$, the stabilization policy makes the stabilizing country appreciate in bad times, that is, it increases the covariance between the stabilizing country's price level, λ^p, and the shadow price of traded goods, λ_T. As a result, a risk-free asset that pays one unit of the stabilizing country's consumption bundle with certainty becomes a better hedge against consumption risk, increasing its value in the world market, and lowering the stabilizing country's risk-free interest rate.

Similarly, the stabilization also increases the effect of the target country's shock on the world-market value of the stabilizing country's output of nontraded goods

$$p_N^p + y_N^p = (p_N^{p*} + y_N^{p*}) + \zeta \frac{(1-\tau)\left(\theta^p + (\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)} \left(y_N^p - y_N^t\right),$$

increasing its covariance with the shadow price of traded goods, and thus increasing the value of capital installed in the stabilizing country.

Proposition 1

If $\gamma > 1$, a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to the target country.

Proof. The interest rate differential with respect to the target country is

$$r^p + \Delta\mathbb{E}s^{p,t} - r^t = r^{p*} + \Delta\mathbb{E}s^{p,t*} - r^{t*} - \zeta \frac{\gamma(1-\tau)^2\left((\theta^t - \theta^p)(\gamma-1)\tau + 2\theta^p(1-\zeta)\right)}{\tau(1+(\gamma-1)\tau)}\sigma_N^2.$$

See Appendix H for details and the corresponding proof for capital accumulation, which requires that the target country be sufficiently large. ■

Aside from these effects on interest rates and capital accumulation, the stabilization policy affects the level of currency reserves. From (17), we already know that the cost of implementing the stabilization is simply the cost of altering the state-contingent purchases of traded goods in world markets. Moreover, we also know that the stabilization induces the stabilizing country to sell additional traded goods in response to adverse productivity shocks in the target country, and to buy additional traded goods in response to adverse productivity shocks at home. If the target country is larger than the stabilizing country, traded goods are more expensive in the states in which it sells than in the states in which it buys. In this case, the stabilization induces the stabilizing country to provide insurance to the world market, pocketing an insurance premium.

Proposition 2

If $\gamma > 1$ and the stabilizing country is small, $\theta^p = 0$, then the cost of stabilization globally decreases with the size of the target country and locally increases with the size of the stabilizing *country. Additionally, the cost of stabilization (ΔRes) is negative if and only if*

$$\theta^t > \frac{\zeta + (\gamma - 1)\tau}{(\gamma - 1)^2 \tau^2}.$$

Proof. See Appendix I. ■

If the target country is sufficiently large relative to the stabilizing country and risk aversion is sufficiently high, this insurance premium can be so large that the stabilization generates revenues for the government, accumulating rather than depleting currency reserves.

When the stabilizing country itself is large ($\theta^p > 0$), its purchases and sales of traded goods also affect the equilibrium shadow price of traded goods, λ_T. This price impact generally increases the cost of stabilization. The reason is that stabilization effectively increases the volatility of shipments of traded goods to the rest of the world. In states where the stabilizing country has a bad productivity shock, it imports more traded goods than it ordinarily would have. In states where the target country has bad productivity shock, it exports more than it ordinarily would have. The more price impact the stabilizing country has, the more costly it therefore is to maintain the stabilization.

2.4.2 External effects of currency stabilization

When the stabilizing country is large ($\theta^p > 0$), the exchange rate stabilization affects traded consumption and prices in the rest of the world. The shadow price of traded goods is

$$\lambda_T = -(1 - \tau)(\gamma - 1)\bar{y}_N + \frac{\theta^p(1 - \tau)}{\tau}\zeta\left(y_N^t - y_N^p\right).$$

The second term on the right hand side shows that if the stabilizing country is large, stabilization dampens the effect of the target country's shocks on the shadow price of traded goods, reducing the extent to which its shocks spill over to the rest of the world, and making it *less* systemic. As a result, the currency stabilization decreases the covariance between the target country's real exchange rate and λ_T. This increases the target country's interest rate and lowers capital accumulation.

Proposition 3

If $\gamma > 1$, a country that becomes the target of a stabilization of any strength $\zeta > 0$ imposed by a large country experiences a rise in its risk-free interest rate, a fall in capital accumulation, and *a fall in average wages relative to all other countries.*

Proof. The interest rate differential between the target and outside country is

$$r^t + \Delta \mathbb{E} s^{t,o} - r^o = \left(r^{t*} + \Delta \mathbb{E} s^{t,o*} - r^{o*} \right) + \zeta \frac{\theta^p (1-\tau)^2 \gamma}{\tau \left(1 + (\gamma - 1)\tau\right)} \sigma_N^2.$$

See Appendix J for details and the corresponding proof for capital accumulation. ■

In this sense, a currency stabilization can divert capital from the target country to the stabilizing country even though it has no effect on the level of the real exchange rate. This finding is particularly interesting because it sheds new light on recent public controversies, for example between Chinese and U.S. officials, which usually focuses on the idea that an under-valuation of the Chinese real exchange rate favors Chinese workers at the expense of U.S. workers. By contrast our results suggest, that even a currency stabilization that manipulates the variance but not the level of the real exchange rate can have this effect.

2.5 Welfare and the Rationale for Stabilization

Next, we study the welfare effects of currency stabilization. So far we have defined a currency stabilization as reducing the variance of the log real exchange rate (P1) while not distorting its level (P2). Achieving both objectives simultaneously requires that the government has the ability to add or subtract resources from the economy by accumulating or depleting currency reserves. For the purposes of assessing the welfare effects of currency stabilization, we now drop the objective (P2) and assume that instead the government rebates the cost of stabilizing the exchange rate (ΔRes) back to households using a lump-sum tax. That is, households in the stabilizing country directly bear the financial cost stabilizing the exchange rate, shifting the level of their traded consumption in all states of the world, and thus also affecting the level of their real exchange rate.

Proposition 4

If $\gamma > 1$ and if households in the stabilizing country directly bear the cost of implementing stabilization (ΔRes) and if all households own the claims to consumption that de-centralize the allocation of consumption under freely floating exchange rates at the time of the announcement of the stabilization policy

1. *a country that imposes an exchange rate stabilization decreases the welfare of its households.*

2. *a country that becomes the target of an exchange rate stabilization imposed by a smaller country with positive mass will see the volatility of its households' consumption decrease. Expected utility in the target country increases as a result of the stabilization.*

Proof. See Appendix K. ■

We have already seen that a stabilization relative to a larger country can increase the level of consumption by increasing capital accumulation and by generating revenues (a negative $\Delta\, Res$). However, stabilization also increases the variance of consumption because the stabilizing country effectively provides insurance to the world market against shocks that affect the target country. This increase in the volatility of consumption reduces expected utility. The first statement in the proposition above shows that, if markets are complete, the latter effect dominates and the exchange rate stabilization decreases the welfare of households within the stabilizing country.

When thinking about a small country imposing a currency stabilization, this result follows almost directly from the first theorem of welfare economics: under freely floating exchange rates and complete markets, the stabilizing country already provides the Pareto-optimal amount of insurance to other countries. Because small countries have no effect on world prices, and there are no frictions to heal, increasing this provision of insurance does not increase welfare. For large countries stabilizing their currency, the calculation is similar. However, as already discussed above, the stabilization policy additionally manipulates state-contingent prices in the wrong direction, which increases the cost of the stabilization, and decreases expected utility even more than for a small country.

On the flip side, currency stabilization by a large country decreases the volatility of consumption in the target country because it dampens the effect of the target country's shock on the shadow price of traded goods. The second statement in the proposition above shows that this positive effect of insurance provision outweighs the detrimental effect on the level of consumption that results from diverted capital accumulation, such that, on balance, stabilizations increase welfare in the target country.

In contrast to the positive results outlined above, these welfare results do not generalize easily to models with frictions and should therefore be interpreted with some caution. Importantly, they may be reversed by simple valuation effects if we deviate from the assumption in Proposition 4 that households and governments are not strategic in the assets they hold prior to announcement. For example, consider a decentralization of the model where the only assets that can be traded internationally are risk-free bonds paying one unit of each of the countries' consumption. Because there are three shocks in our economy, there exists a unique mixture of the three bonds that enables households in the stabilizing country to afford exactly $P^{p*}(\omega)C^{p*}(\omega)$ in each state under the freely floating exchange rate regime—the mixture we assume them to hold in our derivation above. However, if households exhibit "home bias" and hold more of their domestic bond than specified in this mixture at the time when the stabilization policy is announced, then they experience a valuation gain because the value of their bond increases as the stabilizing country's interest rate falls relative to that of the other countries. We show in Appendix L that this valuation gain by itself can outweigh the adverse welfare effects of announcing a currency stabilization. In other words, a government that, for some reason outside the model, observes home bias in its households' portfolio holdings may find currency stabilization to be a welfare-increasing policy, even if markets are complete.

Maybe more relevant in practice than these welfare considerations, we may also think of our model as providing a political economy rationalization for the numerous stabilizations relative the US dollar observed in the data: a large literature argues that policymakers trying to win elections have an interest in raising wages (if the median voter is a worker); and often prefer generating revenue through central bank or currency board operations to direct taxation (even if these are distortionary), because these operations are less visible to the public and easier to control. Currency stabilizations relative to the largest economy in the world achieve both of these objectives, and may thus be politically attractive. For example, a stabilization relative to the largest economy in the world may be optimal if policymakers in a stabilizing country maximize a function of the form

$$EU^n + \mu_1 K^n - \mu_2 \Delta Res,$$

where μ_1 and μ_2 are constants that may reflect the political influence of workers, externalities from capital accumulation, or a motive for generating revenues in a way that avoids direct taxation of households or firms.

2.6 Partially Credible Stabilizations and Floating Bands

A major issue in the study of policies that manipulate the first moment of exchange rates (under- or over-valuations), is the depletion of reserves and the credibility of such manipulations in the face of potential speculative attacks (Krugman, 1979; Garber and Svensson, 1995). By contrast, we have already shown that stabilizations of the real exchange rates relative to a large target country may generate, rather than deplete, reserves. This consequence may assuage some potential concerns about the credibility of this manipulation of the second moment of exchange rates. Nevertheless, it is worth considering the effects of only partially credible stabilizations. Suppose the government, either by choice or necessity, abandons the stabilization in a subset of states $\Omega_{-s} \subset \Omega$ (where Ω is the set of all possible states).

Assuming that the government continues to stabilize state-by-state within $\Omega_s = \Omega \backslash \Omega_{-s}$, and that this (limited) stabilization continues to leave the mean of the real exchange rate undistorted (that is, partition of Ω into Ω_s and Ω_{-s} is symmetric around the mean), we can show that[10]

$$var(s^{p,t}) = \left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta)^2 + \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) var\left[s^{*p,t}|\Omega_{-s}\right] < var(s^{p,t*}).$$

and

$$r^p + \Delta\mathbb{E}[s^{p,t}] - r^t = -\left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta) - \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) cov\left[\lambda_T, s^{*p,t}|\Omega_s\right].$$

In contrast with partially credible manipulations of the level of the real exchange rate, partially credible manipulations of its variance are still effective. They reduce the variance of the real exchange rate and affect interest rates and other outcomes in the same way as characterized above—only less so than a fully credible stabilization. In this sense, we may simply think of partially credible stabilizations as "weaker" credible stabilizations.

Additionally, the two expressions above directly describe the effects of a variety of non-linear stabilization policies, such as floating bands, policies that allow a freely floating exchange rate between some upper and lower limit and intervene state-by-state only when the real exchange rate departs this band.

2.7 Stabilization Relative to a Basket of Currencies

Similarly, our analysis above extends directly to stabilizations relative to a basket of currencies. Consider a country that wishes to stabilize its real exchange rate with the basket

$$p^b = (1-w)p^t + wp^o$$

where w is the basket's weight on the outside country and $1-w$ the weight on target country. Using (3) it is then easy to show that stabilizing relative to a basket of currencies has effects akin to a stabilization relative to a (hypothetical) country with a weighted average size of the

[10] See Appendix M for a formal derivation.

basket's constituents.

$$r^p + \Delta\mathbb{E}s^{p,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) - \zeta\frac{\gamma(1-\tau)^2\left((\bar{\theta}-\theta^p)(\gamma-1)\tau + \theta^p(2-w-2\bar{w}\zeta)\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

where $\bar{\theta} = w\theta^t + (1-w)\theta^o$ is the weighted average size of the basket's constituents and $\bar{w} = 1-(1-w)w$ is a positive constant less than one.

Although clearly a less effective means of lowering domestic interest rates, stabilizing relative to a basket rather than to the largest economy in the world decreases the impact the stabilization has on world-market prices, reducing the stabilizing country's price impact. For a large country, stabilizing relative to a basket may thus be cheaper to implement, providing a potential rationale for China's recent moves towards targeting a basket of currencies rather than the US dollar.

3 Nominal Stabilization and Monetary Policy

In the previous section, we have characterized the internal and external effects of a stabilization of the *real* exchange rate implemented with state-contingent taxes (or capital controls). In the data, most governments instead appear to implement stabilizations of the nominal exchange rate using monetary policy. In this section, we study the relationship between real and nominal stabilizations and show how monetary policy can serve to implement such stabilizations in the presence of nominal rigidities.

To this end, we extend our model by assuming that the prices of traded goods are rigid in local currency. A large body of empirical work documents that traded goods are indeed often rigid in local currency, resulting in wedges in the prices of traded goods across borders, that is, failures in the law of one price (Mussa, 1986; Engel, 1999; Cavallo et al., 2014).

Parallel to our definition of a real exchange rate stabilization above, we define a stabilization of the *nominal* exchange rate of strength $\tilde{\zeta}$ as a set of policies that decreases the variance of the log nominal exchange rate between the stabilizing and target countries, $var\left(\tilde{s}^{t,p}\right) = (1-\tilde{\zeta})^2 var\left(\tilde{s}^{t,p*}\right)$, while keeping the conditional mean of the log nominal exchange rate unchanged, $\mathbb{E}\left[\tilde{s}^{t,p}|\{K^n\}\right] = \mathbb{E}\left[\tilde{s}^{t,p*}|\{K^n\}\right]$. Throughout this section, tildes denote nominal prices, where, $\tilde{s}^{t,p}$ is the log nominal exchange rate between the currencies of countries t and p.

If prices are rigid, monetary policy can affect real allocations. Assume that all consumption goods consumed in a given country must be purchased using the domestic currency,

$$\tilde{P}_T^n P^n C^n = M^n$$

where $\tilde{P}_T^n$ is the price of one unit of the traded good in terms of the currency of country n and M^n is the supply of the domestic currency that can be changed at the discretion of the country's central bank. In addition, households in each country trade state-contingent securities with their central bank that pay off in units of their own currency, so that they cannot use financial markets to hedge against inflation. The central banks in turn trade a complete set of state contingent securities with each other, completing international financial markets except for households' susceptibility to domestic inflation (Alvarez et al., 2002). If all central banks adjust their respective M^n to neutralize the effects of the nominal rigidity in their own country, we thus recover the same allocation that arises under freely floating exchange rates in the frictionless model above.[11]

While the central banks of the target and outside countries follow this neutralizing policy, the central bank of the stabilizing country deviates and instead uses its control of M^n to stabilize its real exchange rate relative to the target country by driving a wedge between the shadow price of traded goods at home and abroad.

Proposition 5

If the price of the traded good is rigid in terms of the stabilizing country's currency,

1. *a nominal stabilization implements a real stabilization of equal strength $\zeta = \tilde{\zeta}$*
2. *the central bank of the stabilizing country implements a real and nominal exchange rate stabilization of strength ζ by setting the log money supply,*

$$m^p = \frac{(\gamma - 1)(1 - \tau)}{1 + (\gamma - 1)\tau} \left(\bar{y}_N - y_N^p\right) + \zeta \frac{(1 - \theta^p)(1 - \tau)}{\tau(1 + (\gamma - 1)\tau)} \left(y_N^t - y_N^p\right) - \log[\tau]$$

Proof. See Appendix N. ■

The first part of the proposition follows directly from the nature of the monetary friction. Because the price of the traded good is fully rigid, the price of consumption in terms of traded goods (the real price level, P^n) always moves in the same direction as the price of consumption in domestic currency (the nominal price level $\tilde{P}_T^n P^n$). As a result, the nominal exchange rate moves in lock-step with the real exchange rate.

[11] See Appendix N for formal details of the extension of the model in section 2.

The second part of the proposition is also intuitive: when the target country appreciates ($y_N^t < y_N^p$), the central bank in the stabilizing country decreases the money supply to increase the value of the domestic currency and match the nominal appreciation. This reduction in the money supply then has the same real effects as a state-contingent tax that drives a wedge between the price of traded goods at home and abroad: it raises the real price of traded goods relative to nontraded goods in the stabilizing country, prompting domestic households to consume fewer traded goods whenever the target country appreciates. The stabilizing country thus again exports additional traded goods whenever the target country appreciates and vice versa.

If households need money to buy consumption goods and prices are sufficiently sticky to give the central bank some leverage over real allocations, we thus conclude that stabilizations of the real exchange rate can be implemented with a simple rule that commits the central bank's control of money supply to enforce a nominal stabilization. That is, even if prices are only partially rigid, a nominal peg, where the central bank commits to exchanging currency at a pre-determined rate, implements some real exchange rate stabilization, entailing all the effects on real allocations discussed in the previous section.

4 Segmented Markets and Preference Shocks

So far, we have based our analysis of currency stabilization on a conventional international real business cycle model, where productivity shocks are the only drivers of variation in real exchange rates (Backus and Smith, 1993). Although an important benchmark, this framework has a number of well-known empirical shortcomings. First, it predicts a perfectly negative correlation between appreciations of the real exchange rate and aggregate consumption growth — a currency appreciates when the country's aggregate consumption falls. Second, the model predicts that consumption should be more correlated across countries than output, whereas the opposite is true in the data (Backus, Kehoe, and Kydland, 1994). Third, real exchange rates and terms of trade seem much too volatile to be rationalized exclusively by real (productivity) shocks alone (Chari, Kehoe, and McGrattan, 2002). As a result, many authors have argued for incomplete markets models that allow for an effect of monetary shocks on equilibrium real exchange rates, or models with demand shocks.

In this section, we argue that the intuition and all positive results from our analysis of exchange rate stabilizations in section 2 continue to hold in a more general class of models where real exchange rates fluctuate in response to inflation shocks, market incompleteness, and preference shocks.

We again depart from the model in section 2, where prices are fully flexible. We now assume that households in each country experience preference shocks as in Pavlova and Rigobon (2007)

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[\left(\exp(\chi^n)C_2(i)\right)^{1-\gamma}\right], \tag{19}$$

where χ^n is a common shock to households' demand for consumption goods in country n,

$$\chi^n \sim N\left(-\frac{1}{2}\sigma_\chi^2, \sigma_\chi^2\right).$$

Motivated by the fact that many households in the US and in other developed economies may own savings accounts or bonds but do not own stocks, foreign bonds, or other, more sophisticated financial instruments that could hedge their portfolios against inflation (Giannetti and Koskinen, 2010; Nechio, 2010), we also allow for segmented markets: a measure $1-\phi$ of "inactive" households within each country are excluded from international financial markets. The remaining measure ϕ of households within each country continue to trade a complete set of state-contingent securities, as in our frictionless model. Label these households as "active". Inactive households do not receive endowments or own any claims to productive assets. Instead, they own only a nominal bond that pays off one unit of the country's nominal consumer price index. We can write this payment to inactive households as $P_2^n e^{-\mu^n}$, where μ^n is a shock to the growth rate of the nominal price of one unit of traded goods in the currency of country n,

$$\mu^n \sim N\left(-\frac{1}{2}\tilde{\sigma}^2, \tilde{\sigma}^2\right).$$

Active households own all productive assets within the country and are short the nominal bonds owned by inactive households. They maximize their utility (4) subject to the constraint

$$\begin{aligned}&\int Q(\omega)\left(P_2^n(\omega)C_2^n(\omega) + \frac{1-\phi}{\phi}P_2^n(\omega)e^{-\mu^n}\right)d\omega \\ \leq&\frac{1}{\phi}\left(1+q_1-q_1K^n+\int Q(\omega)P_{N,2}^n(\omega)\exp(\eta^n)\left(K^n\right)^\nu d\omega+\kappa^n\right)\end{aligned}$$

where $(1-\phi)/\phi$ is the number of inactive households per active household in each country and endowments are adjusted by a factor $1/\phi$ because active households now own proportionally more productive assets per capita; κ^n denotes the transfer that de-centralizes the allocation corresponding to the social planner's problem with unit Pareto weights under freely floating exchange rates. Inactive households maximize (4) subject to the constraint

$$\hat{C}_{T,2}(i) + P^n_{N,2}\hat{C}_{N,2}(i) \leq P^n_2(\omega)e^{-\mu^n}, \tag{20}$$

where hats to denote the consumption of inactive households. The inactive household's problem is thus simply to split the real payoff of their nominal bond (after paying the inflation tax) between traded and nontraded consumption. As before, the government of the stabilizing country stabilizes its exchange rate with the target country using state-contingent taxes.

The punch-line is that currency stabilization in this much richer model of exchange rate determination works in the same way as in our simple model with productivity shocks. Solving the model yields

$$\begin{aligned}\lambda^p = &-\frac{(1-\phi)\gamma^2\tau}{\phi\left(\phi(1-\tau)+\gamma\tau\right)}\bar{\mu} - \frac{(1-\phi)(1-\tau)\gamma}{\phi(1-\tau)+\gamma\tau}\mu^p - \frac{\gamma\tau(\gamma-1)}{\gamma\tau+(1-\tau)\phi}\bar{\chi} - \frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau+(1-\tau)\phi}\chi^p \\ &+ (1-\theta^p)\zeta\frac{\gamma(1-\tau)(1-\phi)}{\phi(1-\tau)+\gamma\tau}\left(\mu^p-\mu^t\right) + (1-\theta^p)\,\zeta\frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau+(1-\tau)\phi}\left(\chi^p-\chi^t\right)\end{aligned}$$

and

$$\begin{aligned}\lambda_T = &-\gamma\left(\frac{1-\phi}{\phi}\right)\bar{\mu} - (\gamma-1)\bar{\chi} \\ &+ \zeta\frac{\theta^p(1-\tau)}{\gamma\tau}\left(\gamma(1-\phi)\left(\mu^t-\mu^p\right) + \phi(\gamma-1)\left(\chi^t-\chi^p\right)\right),\end{aligned}$$

where $\bar{\mu} = \sum_n \theta^n\mu^n$ and $\bar{\chi} = \sum_n \theta^n\chi^n$ are the weighted sums of inflation and preference shocks in all countries, respectively, and we suppress notation relating to productivity shocks to save space.

The first lines in both expressions show the price of country p's consumption index and the shadow price of traded goods in the freely floating regime. Both inflation and preference shocks generate a relationship between exchange rates and the shadow price of traded goods identical to that in section 2: shocks that depreciate a country's currency prompt it to export more traded goods abroad and lower λ_T in proportion to the country's size.

First consider preference shocks: preference shocks move exchange rates by shifting the level of utility derived from each unit of consumption. A high preference shock reduces the marginal utility of households' consumption, and depreciates the country's price of consumption. Again, risk-sharing with households in other countries then compels domestic households to ship traded goods to the rest of the world, transmitting part of the shock to other countries.

Inflation shocks affect exchange rates by shifting resources within a given country away from inactive households, who are excluded from financial markets (and thus are irrelevant for prices in international markets), towards active households whose marginal utilities price assets in international markets. A positive inflation shock to the price of traded goods in terms of the domestic currency thus acts as an "inflation tax" on inactive households: the higher the inflation shock, the less their nominal bonds are worth and the less these households are able to consume. Since inflation shocks have no bearing on the real resources available for consumption, this reduction of inactive households' wealth shifts resources towards the country's active households such that they receive more traded and nontraded goods, which depreciates the domestic price of consumption. At the same time, risk-sharing compels the active households to ship some of the additional traded goods to active households in other countries, thus transmitting part of the inflation shock to active households in other countries via the shadow price of traded goods.

The second line in both expressions above shows that, again, a currency stabilization makes the stabilizing country's price of consumption behave more like the target country's price, and that the stabilization lowers the weight of the target country's shock in λ_T, while simultaneously increasing the weight of the stabilizing country's shock. This change in the size of spill-overs in the shadow price of traded goods again results from the fact that the stabilizing government raises taxes on the domestic consumption of traded goods whenever the target country appreciates (see Appendix G for details). Active households in the stabilizing country thus ship additional traded goods to the rest of the world whenever the target country appreciates,

$$c^p_{T,2} - c^{p*}_{T,2} = \zeta \Xi^p_T \left(\gamma(1-\phi)\left(\mu^t - \mu^p\right) + \phi(\gamma-1)\left(\chi^t - \chi^p\right)\right), \tag{21}$$

where Ξ^p_T is a positive constant shown in Appendix O.

It follows directly that all of our positive predictions about the effects of currency stabilizations carry over to this richer model

Corollary 1

In the model with market segmentation, inflation shocks, preference shocks, and productivity shocks with $\gamma > 1$,

1. *a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to the target country*

2. *if the stabilizing country is small ($\theta^p = 0$), the cost of the stabilization decreases with the*

size of the target country.

3. *if a country becomes the target of a stabilization imposed by a large country ($\theta^p > 0$), its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.*

Proof. See Appendix P. ■

In addition to reinforcing the main insights from our analysis of the frictionless model, this richer model also improves the quantitative implications of the model along the three dimensions outlined above: the combination of market segmentation, inflation shocks, and preference shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth, lowers the correlation of aggregate consumption across countries, and increases the volatility of real and nominal exchange rates (Alvarez et al., 2002; Pavlova and Rigobon, 2007; Kollmann, 2012). All of our conclusions from section 2 thus survive in this empirically viable model of exchange rate determination.

5 Conclusion

The majority of countries in the world, accounting for just under half of world GDP, stabilize their real or nominal exchange rate relative to a target currency. Almost all of these stabilizations target the US dollar, with few exceptions that all target the euro. Although currency stabilizations are possibly the most pervasive form of currency market interventions, existing theories give relatively little guidance on the effects of such stabilizations, to what might be special about the US dollar as a target currency, and to how these stabilizations might affect the target country.

Building on a growing literature that views risk premia as the main driving force behind large and persistent differences in interest rates across developed economies, we propose a novel, risk-based, transmission mechanism for the effects of interventions in currency markets: policies that systematically induce a country's currency to appreciate in bad times, lower its risk premium in international markets, lower the country's risk-free interest rate, and increase domestic capital accumulation and wages. We show that stabilizing a country's real exchange rate relative to a larger target economy is precisely such a policy. Moreover, stabilization relative to a larger economy is cheaper than stabilization relative to a smaller economy in that it generates (rather than depletes) central bank reserves, offering a potential explanation why the vast majority of currency stabilizations in the data are to the US dollar, the currency of the largest economy in the world.

We also find that larger countries must expend more resources than smaller countries when implementing a stabilization, offering a potential explanation for the fact that larger economies tend to either stabilize relative to baskets of currencies or freely float their exchange rates. Our model further predicts that a large economy (such as China) stabilizing its exchange rate relative to a larger economy (such as the US) diverts capital accumulation from the target country to itself, increasing domestic wages, while decreasing wages in the target country—thus offering a novel perspective on the ongoing controversy about Chinese interventions in foreign exchange markets.

Importantly, we show that in the presence of rigid prices, stabilizations of the real exchange rate in our model map directly to the kinds of stabilizations of nominal exchange rates we observe in the data and can be implemented simply by announcing a set of nominal exchange rates and converting currency at these pre-announced rates.

Taken together, we believe that our paper provides a novel way of thinking about the effects of currency stabilization. Along with highlighting potential political rationales for stabilizing, we give an account of the costs and benefits of important choices for the stabilization regime, such as the choice of target country, the effects of hard pegs versus floating bands, and stabilizations relative to a single country versus a basket of currencies.

Our work leaves open at least three avenues for future research. First, careful empirical work will be needed to identify the effect of currency manipulation in the data and disentangle the effects of altered risk-premia from effects that may transmit themselves through more conventional channels, such as the facilitation of trade with the target country and the import of credibility for monetary policy. A prerequisite to making progress on these questions will be to identify (and control for) stabilizations that also involve manipulation of the mean of the real exchange rate—a contentious political issue that has not been satisfactorily resolved in the empirical literature. Second, although a large number of models have been written that argue for risk-premia as the main drivers of cross-sectional differences in interest rates, all of these papers, including our own, rely on standard preferences and thus generally imply that risk premia are quantitatively small. Recent work by Govillot et al. (2010), Colacito et al. (2016), and David et al. (2016) makes progress in this dimension by studying dynamic models with heterogeneous countries and recursive preferences. Finally, our analysis has focused exclusively on a simple problem where a single country unilaterally imposes an exchange rate stabilization, taking as given the policies of other countries. In analogy to a large literature on strategic interactions in trade policy (Bagwell and Staiger, 1999; Ossa, 2011), our prediction that exchange rate policy alters the equilibrium allocation of factors of production may also serve as the basis of a multilateral theory of strategic interactions in the choice of exchange rate regime.

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A Differences in Log Asset Returns

Because markets are financially complete, the prices outside of the stabilizing country must coincide with ratios of shadow prices. In particular, the shadow price of traded goods is the unique stochastic discount factor that prices Arrow-Debreu securities that pays one unit of the traded good in state ω in the world. We denote the shadow price of traded goods in state ω as $\Lambda_T(\omega)$ in the second period and we denote the shadow price of traded goods in the first period (prior to the realization of shocks) as Ψ_T.

To derive equation (3), we use apply the consumption Euler equation (the expectation of the stochastic discount factor times the return) to the risk-free real interest rate in country n,

$$\mathbb{E}\left[\frac{\Lambda_2^h}{\Lambda_1^h}\right] R^h = \mathbb{E}\left[\frac{\Lambda_2^f}{\Lambda_1^f}\right] R^f = 1$$

where Λ_t^n is the marginal utility of consumption in country n in period t. Since we use the traded good as the numeraire in the model, we write $\Lambda_2^n = \Lambda_T P_2^n$ and $\Lambda_1^n = \Psi_T P_1^n$. P_t^n is the number of traded goods needed to purchase a unit of consumption period t.

We perform the following calculations:

$$\begin{aligned}
& \mathbb{E}\left[\frac{\Lambda_T}{\Psi_T}\frac{P_2^h}{P_1^h}\right] R^h = \mathbb{E}\left[\frac{\Lambda_T}{\Psi_T}\frac{P_2^f}{P_1^f}\right] R^f \\
\Leftrightarrow\quad & \mathbb{E}\left[\exp\left[-\psi_T - p_1^f + \lambda_T + p_2^f + r^f\right]\right] = \mathbb{E}\left[\exp\left[-\psi_T - p_1^h + \lambda_T + p_2^h + r^h\right]\right] \\
\Leftrightarrow\quad & -\psi_T - p_1^f + \mathbb{E}\left[\lambda_T + p_2^f\right] + \frac{1}{2}\mathrm{var}\left(\lambda_T\right) + \frac{1}{2}\mathrm{var}\left(p_2^f\right) + \mathrm{cov}\left(\lambda_T, p_2^f\right) + r^f \\
& = -\psi_T - p_1^h + \mathbb{E}\left[\lambda_T + p_2^h\right] + \frac{1}{2}\mathrm{var}\left(\lambda_T\right) + \frac{1}{2}\mathrm{var}\left(p_2^h\right) + \mathrm{cov}\left(\lambda_T, p_2^h\right) + r^h
\end{aligned}$$

We normalize the period one price level in each country to one ($p_1^f = p_1^h = 0$), and cancel out ψ_T and $\mathrm{var}\left(\lambda_T\right)$ from both sides of the previous equation. The remaining variables are all second period variables. Hence, we drop the time subscripts below

$$\begin{aligned}
& \mathbb{E}\left[p^f\right] + \frac{1}{2}\mathrm{var}\left(p^f\right) + \mathrm{cov}\left(\lambda_T, p^f\right) + r^f = \mathbb{E}\left[p^h\right] + \frac{1}{2}\mathrm{var}\left(p^h\right) + \mathrm{cov}\left(\lambda_T, p^h\right) + r^h \\
\Leftrightarrow\quad & r^f + \mathbb{E}\left[p^f - p^h\right] + \frac{1}{2}\mathrm{var}\left(p^f\right) - \frac{1}{2}\mathrm{var}\left(p^h\right) - r^h = -\mathrm{cov}\left(\lambda_T, p^f - p^h\right) \\
\Leftrightarrow\quad & r^f + \log\left(\mathbb{E}\left[P^f\right] / \mathbb{E}\left[P^h\right]\right) - r^h = -\mathrm{cov}\left(\lambda_T, p^f - p^h\right)
\end{aligned}$$

We define

$$\Delta\mathbb{E}\left[s^{f,h}\right] = \log\left(\mathbb{E}\left[P^f\right]/\mathbb{E}\left[P^h\right]\right).$$

With this definition

$$r^f + \Delta\mathbb{E}\left[s^{f,h}\right] - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right).$$

B Deriving the Price Index

The cost of one unit of consumption in country n is defined as

$$P^n = \arg\min C_T^n + P_N^n C_N^n \text{ s.t. } \left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1$$

for all households i. First order conditions imply

$$C_N = \frac{1}{P_N}\frac{1-\tau}{\tau}C_T$$

We solve for C_T^n by plugging this expression into $\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1$,

$$C_T^n = \left(\frac{\tau}{1-\tau}P_N^n\right)^{1-\tau}.$$

Finally, we use this expression to derive the consumption of nontraded goods and the optimal price index,

$$P^n = \frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau(1-\tau)^{1-\tau}}$$

The total value of consumption for households in country n is

$$P^n C^n = \left(\frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau(1-\tau)^{1-\tau}}\right)\left(\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau}\right) = \frac{C_T^n}{\tau}$$

Similarly, we use the expression $P_N^n = \frac{1-\tau}{\tau}\frac{C_T^n}{C_N^n}$ to show

$$C_T^n + P_N^n C_N^n = \frac{C_T^n}{\tau} = P^n C^n$$

C Equilibrium Consumption of Inactive Households

Inactive households in country n maximize utility, defined in equation (4), in each state of the world by splitting their wealth $\exp(-\mu_t^n)P_t^n$ optimally between traded and nontraded goods. Their optimization problem can be written as maximizing (4) in each state subject to (20)

$$\max_{\hat{C}_T(i),\hat{C}_N(i)} \frac{1}{1-\gamma}\left(e^{\chi^n}\hat{C}_T\left(i\right)^{\tau}\hat{C}_N\left(i\right)^{1-\tau}\right)^{1-\gamma}$$
$$\text{s.t. } \hat{C}_T\left(i\right)+P_N^n\hat{C}_N\left(i\right)\leq \exp(-\mu^n)P^n$$

We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $\hat{C}_T\left(i\right)$ and $\hat{C}_N\left(i\right)$.

Inactive households consume an optimal mix of traded and nontraded goods given by

$$\hat{C}_T^n=\exp(-\mu^n)(\tau P_2^n),\ \hat{C}_N^n=\exp(-\mu^n)\left(\frac{(1-\tau)P^n}{P_N^n}\right)$$

where $\hat{C}_T^n$ and $\hat{C}_N^n$ are the consumption of traded and nontraded goods by inactive agents in country n, respectively.

D Equilibrium Consumption of Active Households

In this section, we set up the complete model with segmented markets, monetary shocks and preference shocks. We solve the active household's problem where each governments can impose an arbitrary state contingent taxes, $Z(\omega)$, and a lump sum transfers, $\bar{Z}$, on their domestic households. We assume the governments in the target and outside countries set their state contingent taxes, $Z(\omega)=1$, and the lump sum transfer $\bar{Z}=0$. Hence, the governments in the target and outside do not intervene in financial markets. We assume the government in the stabilizing country chooses the state contingent tax and the lump sum tax to stabilize its exchange rate with the target country.

Active households maximize expected utility,

$$U(i)=u(C_T^n(\omega),C_N^n(\omega),\chi^n)=\frac{1}{1-\gamma}\mathbb{E}\left[\left(\exp\left(\chi^n\right)C_2(i)\right)^{1-\gamma}\right]$$

subject to their budget constraint,

$$\begin{aligned}
&\int Z(\omega)Q(\omega)\left(P^n(\omega)C^n(\omega)+\frac{1-\phi}{\phi}P^n(\omega)e^{-\mu^n}\right)g(\omega)d\omega\\
\leq&\frac{1}{\phi}\left(Y^n_{T,1}+q_1-q_1K^n+\int Z(\omega)Q(\omega)P^n_N(\omega)\exp(\eta^n)\left(K^n\right)^{\nu}g(\omega)d\omega+\kappa^n+\bar{Z}\right).
\end{aligned}$$

In main setup of the paper, we assume that the government in the stabilizing country has reserves, which it uses to fund the exchange rate stabilization. We assume the transfer $\bar{Z}$ equalizes the marginal utility of wealth between the stabilizing country and the rest of the world. Given the appropriate transfer $\bar{Z}$, we let Ψ_T denote the Lagrange multiplier on the budget constraint in all countries. This assumption about $\bar{Z}$ in the stabilizing country allows us to abstract from the wealth effects from imposing the exchange rate stabilization and temporarily ignore the wealth effects of exchange rate stabilization.

We analyze the wealth effects of the exchange rate stabilization in Appendix K. We consider cases where the government does not fund the stabilization and $\bar{Z}$ simply rebates (lump sum) the tax revenues from the state contingent tax back to the households

The first order conditions of the active household's problem are

$$\begin{aligned}
\frac{\partial u(C^n_T(\omega),C^n_N(\omega),\chi^n)}{\partial C^n_T(\omega)}&=\Psi_TQ(\omega)Z(\omega)\\
\frac{\partial u(C^n_T(\omega),C^n_N(\omega),\chi^n)}{\partial C^n_N(\omega)}&=\Psi_TQ(\omega)Z(\omega)P^n_N(\omega)
\end{aligned}$$

We define $\Lambda_T(\omega)$ to be the shadow price of traded goods in the target and outside countries. Setting $Z(\omega)=1$ in the target and outside countries, we show $Q(\omega)=\Lambda_T(\omega)/\Psi_T$. Applying the definition of $\Lambda_T(\omega)$ to the first order condition with respect to traded goods yields equations (9) and (10).

Let $\Lambda^n_N(\omega)=\frac{\partial u(C^n_T(\omega),C^n_N(\omega),\chi^n)}{\partial C^n_N(\omega)}$ represent the marginal utility of nontraded consumption. Hence,

$$\Lambda^n_N(\omega)=Z(\omega)\Psi_TQ(\omega)P^n_N(\omega)\quad\Rightarrow\quad P^n_N(\omega)=\frac{\Lambda^n_N(\omega)}{Z(\omega)\Lambda_T(\omega)}$$

Plugging the right-hand side of the previous equation into the first order condition with respect to nontraded consumption gives us (11).

The first order condition with respect to capital accumulation is

$$\Psi_Tq_1=\Psi_T\int Z(\omega)Q(\omega)P^n_N(\omega)e^{\eta^n}\nu\left(K^n\right)^{\nu-1}g(\omega)d\omega$$

Using the definition of $Q(\omega)$ and $P_N^n(\omega)$, this simplifies to

$$K^n = \frac{\nu}{q_1} \int Z(\omega) \frac{\Lambda_T(\omega)}{\Psi_T} \frac{\Lambda_N^n(\omega)}{Z(\omega)\Lambda_T(\omega)} Y_N^n(\omega) g(\omega) d\omega$$

where we simplify further to derive equation (12).

E Log-linearized System of Equations

This section derives the log-linearized first order conditions for the complete model. Equation (6) defines the resource constraint for traded goods. Equation (7) defines the (three) resource constraints for nontraded goods in each country, and equation (8) defines the resource constraint for capital goods. Equations (9), (10), (11) define the three first order conditions with respect to traded consumption and the three first order conditions with respect to nontraded consumption. Finally, equation (12) defines the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. We log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$). At this point, the capital stock of all firms is fixed at a level $K^n = 1$ such that $Y_N^n = 1$ as well as $Y_{T,1} = 1$ and $C_T = 1$.

The log-linear first order conditions for the target and outside countries are

$$\begin{aligned}
(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau) c_N^n\right) - c_T^n + \log\tau &= \lambda_T \\
(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau) c_N^n\right) - c_N^n + \log(1-\tau) &= \lambda_N^n
\end{aligned}$$

The log-linear first order conditions for the stabilizing country are

$$\begin{aligned}
(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau) c_N^n\right) - c_T^n + \log\tau &= \lambda_T + z \\
(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau) c_N^n\right) - c_N^n + \log(1-\tau) &= \lambda_N^n + z
\end{aligned}$$

where z is the log-linear expression for the tax given by Lemma 1.

We log-linearize the Euler equation for capital accumulation for each country, given by equation (12)

$$\log(q_1) + k^n = \log[v] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2} var\left(\lambda_N^n + y_N^n\right).$$

Finally, the log-linear resource constraints are

$$\phi c_N^n + (1-\phi)\left(-\mu^n - \tau\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right) = \eta^n + \nu k^n = y_N^n$$

$$\sum_{n=p,t,o} \theta^n \left[\phi c_T^n + (1-\phi)\left(-\mu^n - (1-\tau)\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right)\right] = \sum_{n=p,t,o} \theta^n y_{T,1}^n = 1$$

$$\sum_{n=p,t,o} \theta^n k^n = 1$$

This set of fourteen equations allows us to solve for the following fourteen unknowns $\{k^n, c_N^n, c_T^n, \lambda_N^n\}_{n=p,t,o}$, λ_T and $\log(q_1) + \psi_T$. We can write these endogenous variables in terms of the following nine state variables $\{y_N^n, \mu^n, \chi^n\}$ for $n = p, t, o$. Note that agents only care about the total output of nontraded goods, $y_N^n = \eta^n + \nu k^n$, in each country in the second period.

In the following section, we show the solution the log-linear system of equations in the real business cycle model. We set the log state contingent tax $z = \zeta \frac{1-\tau}{\tau}(y_N^p - y_N^t)$. Lemma 1 shows this tax stabilizes the real exchange rate with strength ζ. The following six equations define traded consumption and nontraded consumption.

$$\begin{aligned}
c_T^p &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^p\right) + \frac{\zeta(1-\tau)(1-\theta^p)}{\tau\left(1+(\gamma-1)\tau\right)}\left(y_N^t - y_N^p\right) \\
c_T^t &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^t\right) + \frac{\zeta(1-\tau)\theta^p}{\tau\left(1+(\gamma-1)\tau\right)}\left(y_N^p - y_N^t\right) \\
c_T^o &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^o\right) + \frac{\zeta(1-\tau)\theta^p}{\tau\left(1+(\gamma-1)\tau\right)}\left(y_N^p - y_N^t\right) \\
c_N^n &= y_N^n \ \forall n
\end{aligned}$$

The following four equations define the shadow prices of traded and nontraded consumption.

$$\begin{aligned}
\lambda_T &= -(1-\tau)(\gamma-1)\bar{y}_N + \zeta\theta^p\frac{1-\tau}{\tau}\left(y_N^t - y_N^p\right) + \log[\tau] \\
\lambda_N^p &= -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^p + \zeta\theta^p\frac{(\gamma-1)(1-\tau)}{\tau\left(1+(\gamma-1)\tau\right)}\left(y_N^t - y_N^p\right) + \log[1-\tau] \\
\lambda_N^t &= -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^t + \zeta\theta^p\frac{(1-\tau)(\gamma-1)}{1+(\gamma-t)\tau}\left(y_N^t - y_N^p\right) + \log[1-\tau] \\
\lambda_N^o &= -\frac{(\gamma-1)^2(1-\tau)\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y_N^o + \zeta\theta^p\frac{(1-\tau)(\gamma-1)}{1+(\gamma-t)\tau}\left(y_N^t - y_N^p\right) + \log[1-\tau]
\end{aligned}$$

Finally, we use the three log-linear Euler equations, the expressions for λ_N^n and the log-linear resource constraint for capital to solve for the incentives to accumulate capital as well as the first period price of capital q_1. In the exogenous capital case with $K^n = 1$, we have $y_N^n = \eta^n + \nu k^n = \eta^n$, $\mathbb{E}[y_N^n] = 0$ and $var[y_N^n] = \sigma_N^n$.

F Incentives to Accumulate Capital

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, we can rearrange the Euler equation for capital accumulation (12) and obtain a form similar to (3): take logs of both sides of the equation, substitute $\lambda_N^n = p_N^n + \lambda_T$, and take differences across countries to obtain

$$k^{f*} - k^{h*} = \frac{1}{2} var\left(p_N^f + y_N^f\right) - \frac{1}{2} var\left(p_N^h + y_N^h\right) + cov\left(p_N^f + y_N^f - p_N^h - y_N^h, \lambda_T\right) \tag{22}$$

where we can interpret $p_N^f + y_N^f$ as the value of nontraded output in terms of traded goods, or as the payoff of a unit of stock in the nontraded sector of country f. Ignoring the two variance terms on the right hand side for the moment, this expression suggests that countries whose output increases in value when λ_T is high should accumulate more capital per capita. The solution of the model yields

$$p_N^{f*} + y_N^{f*} = \frac{(1-\tau)(\gamma-1)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^f\right).$$

It shows that differences in the payoff of stocks behave in the same way as exchange rates: when country f suffers a low productivity shock, its currency appreciates and the value of its firm's output in terms of traded goods increases. If country f is large, the same adverse productivity shock also raises λ_T, inducing a positive covariance between λ_T and the value of the firm's output.

Larger countries thus not only have lower interest rates but also have incentives to accumulate higher capital-output ratios. Solving for the variances and covariances in (22) yields

$$k^{f*} - k^{h*} = \frac{(\gamma-1)^3(1-\tau)^2\tau}{1+(\gamma-1)\tau}\left(\theta^f - \theta^h\right)\sigma_N^2.$$

To derive the link between interest rates and exchange rates in (16), combine this equation with (15).

G State Contingent Taxes and the Cost of the Peg

We solve for the form of the log-linear tax that implements the exchange rate stabilization. In the model in section 4 with monetary shocks, segmented markets and preference shocks, we assume a state contingent tax of the form

$$Z(\omega) = \left(\frac{Y_N^p}{Y_N^t}\right)^{a_1} \left(\frac{\exp\left(-\mu^t\right)}{\exp\left(-\mu^p\right)}\right)^{a_2} \left(\frac{\exp\left(\chi^p\right)}{\exp\left(\chi^t\right)}\right)^{a_3}$$

Under this assumption, the log-linear tax is

$$z = a_1\left(y_N^t - y_N^p\right) + a_2\left(-\mu^t + \mu^p\right) + a_3\left(\chi^t - \chi^p\right)$$

and the log real exchange rate is

$$\begin{aligned} s^{p,t} =& \frac{\gamma(1-\tau)}{\gamma\tau + (1-\tau)\phi\tau}\left(y_N^t - y_N^p\right) + \frac{(1-\phi)(1-\tau)}{\gamma\tau + (1-\phi)\tau}\left(\mu^t - \mu^p\right) + \frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau + (1-\tau)\phi}\left(\chi^t - \chi^p\right) \\ &+ a_1\frac{\gamma\tau\left(\phi + (1-\phi)\tau\right)}{\gamma\tau + (1-\tau)\phi}\left(y_N^p - y_N^t\right) + a_2\frac{\gamma\tau\left(\phi + \tau(1-\phi)\right)}{\gamma\tau + (1-\tau)\phi}\left(\mu^t - \mu^p\right) + a_3\frac{\gamma\tau(\tau + (1-\tau)\phi)}{\gamma\tau + \phi(1-\tau)}\left(\chi^p - \chi^t\right) \end{aligned}$$

The first line of this expression gives us the real exchange rate without intervention, $s^{p,t*}$. We choose a_1, a_2 and a_3 such that $s^{p,t} = (1-\zeta)s^{p,t*}$ individually for each shock. For example, we start by shutting down monetary shocks and preference shocks and solve

$$a_1 = \zeta\frac{(1-\tau)}{\tau\left(\tau + \phi(1-\tau)\right)}$$

We recover the expression in Lemma 1 by setting $\phi = 1$. We repeat this process to solve for a_2 and a_3. This yields,

$$a_2 = \zeta\frac{(1-\tau)(1-\phi)}{\tau(\tau + \phi(1-\tau))}, \text{ and } a_3 = \zeta\frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau(\tau + \phi(1-\tau)}.$$

We repeat the procedure for the nominal exchange rate stabilization assuming a state contingent tax of the form

$$\tilde{Z}(\omega) = \left(\frac{Y_N^p}{Y_N^t}\right)^{b_1} \left(\frac{\exp\left(-\mu^t\right)}{\exp\left(-\mu^p\right)}\right)^{b_2} \left(\frac{\exp\left(\chi^p\right)}{\exp\left(\chi^t\right)}\right)^{b_3}$$

Assuming a state contingent tax of this form, we derive the log nominal exchange rate,

$$
\begin{aligned}
\tilde{s^{p,t}} =& \frac{\gamma(1-\tau)}{\gamma\tau+(1-\tau)\phi\tau}\left(y_N^t-y_N^p\right)+\frac{\gamma(1-2\tau)-(\gamma+1)(1-\tau)\phi}{\gamma\tau+(1-\phi)\tau}\left(\mu^t-\mu^p\right)+\frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau+(1-\tau)\phi}\left(\chi^t-\chi^p\right)\\
&+b_1\frac{\gamma\tau\left(\phi+(1-\phi)\tau\right)}{\gamma\tau+(1-\tau)\phi}\left(y_N^p-y_N^t\right)+b_2\frac{\gamma\tau\left(\tau+(1-\tau)\phi\right)}{\gamma\tau+(1-\tau)\phi}\left(\mu^t-\mu^p\right)+b_3\frac{\gamma\tau(\tau+(1-\tau)\phi)}{\gamma\tau+\phi(1-\tau)}\left(\chi^p-\chi^t\right)
\end{aligned}
$$

The first line of this expression gives us the real exchange rate without intervention, $\tilde{s}^{p,t*}$. We choose b_1 and b_2 such that $\tilde{s}^{p,t}=(1-\zeta)\tilde{s}^{p,t*}$ individually for each shock. We find $b_1=a_1$, $b_3=a_3$ and

$$
b_2=\zeta\frac{\gamma(1-2\tau)-(\gamma+1)(1-\tau)\phi}{\gamma\tau\left(\phi+(1-\tau)\phi\right)}.
$$

The following lemma summarizes these results.

Lemma 2

In a more general class of models where real exchange rates fluctuate in response to inflation shocks, market segmentation and preference shocks, a tax on all assets paying off consumption goods in the stabilizing country of the form

$$
z(\omega)=\frac{\zeta(1-\tau)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(y_N^p-y_N^t\right)+\frac{(1-\tau)(1-\phi)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(\mu^p-\mu^t\right)+\frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau\left(\tau+\phi(1-\tau)\right)}\left(\chi^p-\chi^t\right)
$$

implements a real exchange rate stabilization of strength ζ.

A tax on all assets paying off consumption goods in the stabilizing country of the form

$$
\tilde{z}(\omega)=z(\omega)-\zeta\frac{\gamma\tau+\phi(1-\tau)}{\gamma\tau\left(\tau+\phi(1-\tau)\right)}\left(\mu^p-\mu^t\right)
$$

implements a nominal exchange rate stabilization of strength ζ.

To derive the cost of the stabilization, we first re-write the households' budget constraint to identify the components of the lump sum transfer, $\bar{Z}$. $\bar{Z}$ contains a lump-sum rebate of state contingent taxes, κ_{Tax}^p, and an additional transfer from government reserves, ΔRes, which equalizes the marginal utility of wealth across countries. We re-write $Z(\omega)=1+X(\omega)$ in order to look at deviations from a no-tax policy. The household in the stabilizing country faces the following budget constraint

$$
\begin{aligned}
&\int\left(1+X(\omega)\right)Q(\omega)\left(P^p(\omega)C^p(\omega)+\frac{1-\phi}{\phi}P^p(\omega)e^{-\mu^p}\right)d\omega\\
\leq&\frac{1}{\phi}\left(Y_{T,1}^p+\int\left(1+X(\omega)\right)Q(\omega)P_N^p(\omega)\exp(\eta^p)\left(K^p\right)^\nu d\omega+\kappa^p+\kappa_{Tax}^p+\Delta Res\right).
\end{aligned}
$$

Recall that κ^p is the transfer that decentralizes the Social Planner's problem with unit Pareto weights in the freely floating exchange rate economy.

This lump-sum rebate of tax revenues is

$$\kappa^p_{Tax} = \int X(\omega)Q(\omega)\left(P^p(\omega)C^p(\omega) + \frac{1-\phi}{\phi}P^p(\omega)e^{-\mu^n}\right)d\omega$$
$$-\frac{1}{\phi}\int X(\omega)Q(\omega)P^p_N(\omega)\exp(\eta^p)\left(K^p\right)^\nu d\omega$$

Subtracting the lump-sum rebate from both sides of the budget constraint and multiplying by ϕ yields

$$\int Q(\omega)\left(\phi P^p(\omega)C^p(\omega) + (1-\phi)P^p(\omega)e^{-\mu^n}\right)d\omega$$
$$\leq Y^p_{T,1} + \int Q(\omega)P^p_N(\omega)\exp(\eta^p)\left(K^p\right)^\nu d\omega + \kappa^p + \Delta Res$$

Substituting in for the market clearing condition for nontraded goods yields

$$\int Q\left(\omega\right)\left(\phi C^p_T(\omega) + (1-\phi)\hat{C}^p_T(\omega)\right)d\omega \leq Y_{T,1} + \kappa^p + \Delta Res$$

We solve for lump-sum transfer that decentralizes the Social Planner's problem, κ^p, by solving for the κ^p in the household's budget constraint in a freely floating exchange rate economy. We find that the desired lump-sum transfer is

$$\kappa^p = \int Q^*(\omega)\left(\phi C^{p*}_T(\omega) + (1-\phi)\hat{C}^{p*}_T(\omega)\right)d\omega - Y_{T,1}$$

In Lemma 1, we plug in this expression for κ^p, set $\phi = 1$ and solve for the cost of the stabilization, ΔRes in terms of the change in traded consumption by active households.

H Proof of Proposition 1

If $\gamma > 1$, a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to the target country.

We use the fact that the real price differential can be expressed as the difference in log marginal utilities. The interest rate differential between the stabilizing country and the target country is

$$
\begin{aligned}
r^p + \mathbb{E}s^{p,t} - r^t &= cov\left(\lambda_T, p^t - p^p\right) \\
&= \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \zeta \frac{(1-\tau)^2\gamma\left(2\theta^p(1-\zeta) + \left(\theta^t - \theta^p\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2,
\end{aligned}
$$

When the stabilizing country is smaller than the target country, $\theta^p < \theta^t$, this expression implies the exchange rate stabilization decreases the risk free rate in the stabilizing country relative to the risk free rate in the target country.

From equation (22), we calculate the differential incentives to accumulate capital when the stabilizing country stabilizes its real exchange rate in the real business cycle economy

$$
k^p - k^t = k^{p*} - k^{t*} + \zeta\left(\frac{(\gamma-1)^2(1-\tau)^2\left((1-2\theta^p)(1-\zeta) + (\theta^t-\theta^p)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\right)\sigma_N^2.
$$

The last term of the right hand side expression shows that incentives to accumulate capital in the stabilizing country increase relative to the target country as long as

$$
\theta^t > \theta^p + \frac{(1-2\theta^p)(1-\zeta)}{\tau(\gamma-1)}
$$

Because firms are competitive, wages are given by the marginal product of labor. $w^n = (1-\nu)\exp\left(\eta^n\right)\left(K^n\right)^\nu$. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization increases wages in the stabilizing country relative to all other countries.

I Proof of Proposition 2

If $\gamma > 1$ and the stabilizing country is small, $\theta^p = 0$, then the cost of the stabilization decreases with the size of the target country. Additionally, the cost of stabilization is negative if and only if

$$
\theta^t > \frac{\zeta + (\gamma-1)\tau}{(\gamma-1)^2\tau^2}.
$$

In Appendix D, we show that the price of a state contingent claim is

$$
Q(\omega) = \frac{\Lambda_T(\omega)}{\Psi_T}.
$$

Lemma 1 shows that the cost of the stabilization is the difference in the value of traded consumption between the free floating exchange rate economy and stabilized exchange rate economy. Because prices and consumption of traded goods are log-normally distributed, the log value of traded consumption can be written as

$$\begin{aligned} v_T &= \log \mathbb{E}\left[\exp\left(\lambda_T + c_T^p - \psi\right)\right] \\ &= \mathbb{E}\left[\lambda_T + c_T^p - \psi\right] + \frac{1}{2} var\left[\lambda_T + c_T^p - \psi\right], \end{aligned}$$

where $\psi = \mathbb{E}\lambda_T + \frac{1}{2} var\left(\lambda_T\right)$ is the log shadow price of a traded good prior to the realization of shocks. When the stabilizing country is small, we plug in equations (14) and (18). The change in the log value of traded consumption is

$$v_T - v_T^* = \frac{\left(\left(\zeta + (\gamma-1)\tau\right) - \tau^2(1-\gamma)^2\theta^t\right)(1-\tau)^2\zeta\sigma_N^2}{\tau^2\left(1+(\gamma-1)\tau\right)^2}$$

This expression is decreasing in the size of the target country, and becomes negative if and only if the target country is large enough.

We can also evaluate the change in $v_T - v_T^*$ with respect to the size of the stabilizing country at the point where the stabilizing country is small,

$$\frac{\partial(v_T - v_T^*)}{\partial\theta^p} = \zeta\frac{(\gamma-1)(1-\tau)^2\left(\theta^t + 2\zeta + 2(1+\theta^t)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2 > 0$$

Hence, the cost of the stabilization increases locally with the size of the pegging country.

J Proof of Proposition 3

If $\gamma > 1$, a country that becomes the target of a stabilization of any strength $\zeta > 0$ imposed by a large country experiences a rise in its risk-free interest rate, a fall in capital accumulation, and a fall in average wages relative to all other countries.

The interest rate differential between the target and outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = cov\left(\lambda_T, p^o - p^t\right) = \left(r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^p(1-\tau)^2\gamma}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

which implies the exchange rate stabilization increases the risk free rate in the target country relative to the risk free rate in the outside country.

Equation (22) gives us the differential incentives to accumulate capital when a country stabilizes its exchange rate in the real business cycle economy

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^p (\gamma-1)^2 (1-\tau)^2}{\left(1+(\gamma-1)\tau\right)^2} \zeta \sigma_N^2$$

The last term of the right hand side expression shows that incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.

K Proof of Welfare Results

This section calculates the welfare consequences from announcing a real exchange rate stabilization assuming households bear the full cost of maintaining the stabilization. This is different from the main body of the paper, where we assume the government in the stabilizing country provides a subsidy to households, ΔRes. In this section, we assume the lump sum transfer of goods, $\bar{Z}$, is simply equal to the tax revenues from imposing the state contingent tax, $Z(\omega)$. Hence, $\Delta Res = 0$, and the government in the stabilizing country no longer introduces any additional resources into the economy.

For simplicity, we perform this welfare calculation in the real business cycle benchmark. When $\Delta Res = 0$, the value of the household's new consumption bundle after the exchange rate stabilization must equal the value of the household's new consumption bundle prior to the exchange rate stabilization. Hence, the household's budget constraint is,

$$\int Q^p(\omega) P^p(\omega) C^p(\omega) d\omega = \int Q^{p*}(\omega) P^{p*}(\omega) C^{p*}(\omega) d\omega$$

In order to enforce this constraint, households in the stabilizing country shift the level of their consumption in each state of the world,

$$c_T^p - c_T^{p*} = \zeta \frac{(1-\tau)(1-\theta^p)}{\tau\left((1-\tau)+\gamma\tau\right)} \left(y_N^t - y_N^p\right) + \frac{(1-\theta^p)}{1+(\gamma-1)\tau} \left(\psi_T - \psi_T^p\right)$$

where ψ_T is still the marginal utility of a unit of the traded good in the first period in the target and outside countries, and ψ_T^p is the marginal utility of a unit of the traded good in the first period in the stabilizing country. Alternatively, these objects are the marginal utilities of an additional unit of wealth. Unlike the economy we studied in the main body of the paper, the marginal utility of wealth is no longer equalized across all three countries, $\Psi_T^p \neq \Psi_T$. Naturally, the marginal utility of wealth in the stabilizing country is higher than in the rest of the world whenever households in the stabilizing country consume fewer units of traded goods.

The volatility of log consumption in the stabilizing country is

$$\begin{aligned} var\left(c^p\right) &= var\left(\tau c_T^p + (1-\tau)c_N^p\right) \\ &= var\left(c^{p*}\right) + \zeta \frac{2(1-\theta^p)(1-\tau)^2\left((1-\theta^p)\zeta - 1 + \left(\theta^t - \theta^p\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2, \end{aligned}$$

which shows that the volatility of consumption in the stabilizing country increases with the size of the target country.

If

$$\theta^t > \theta^p + \frac{1-\left(1-\theta^p\right)\zeta}{(\gamma-1)\tau},$$

the exchange rate stabilization increases the volatility of consumption in the stabilizing country. A corollary of this result is that a small country ($\theta^p = 0$) that imposes a hard exchange rate stabilization always increases the volatility of consumption of its households.

We investigate changes in expected utility by examining $(1-\gamma)U(i)$. As $(1-\gamma)U(i)$ increases, utility decreases. For household i in the stabilizing country, we calculate

$$\begin{aligned} \frac{d}{d\zeta}\log\left[(1-\gamma)U(i)\right] &= \frac{d}{d\zeta}\left[(1-\gamma)\mathbb{E}\left(c^p\right) + \frac{(1-\gamma)^2}{2}var\left(c^p\right)\right] \\ &= \frac{(\gamma-1)(1-\tau)^2\left(\left(1-\left(\theta^p\right)^2\right)\zeta + \theta^p\left(1+\theta^t-\theta^p\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2 > 0. \end{aligned}$$

Hence

$$\frac{dU(i)}{d\zeta}\frac{1}{U(i)} > 0$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} < 0$, because $U(i) = \frac{1}{1-\gamma}\left(C^p\right)^{1-\gamma} < 0$. Hence, stabilizing the real exchange rate decreases utility in the stabilizing country.

The volatility of log consumption in the target country is

$$var\left(c^t\right) = var\left(c^{t*}\right) - \zeta\frac{2\theta^p(1-\tau)^2\left(1-\theta^p\zeta + \left(\theta^t-\theta^p\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2.$$

Therefore, $var\left(c^t\right)$ decreases when a country stabilizes its exchange rate relative to the target country as long as the stabilizing country is smaller, $\theta^t > \theta^p$.

Again, we examine the quantity $(1-\gamma)U(i)$. For household i in the target country, we calculate

$$\frac{d}{d\zeta}\log\left[(1-\gamma)U(i)\right] = -\frac{\theta^p(\gamma-1)^2(1-\tau)^2(\Xi_{UT1}\zeta+\Xi_{UT2})}{(1-\theta^p)\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2 < 0.$$

where Ξ_{UT1} and Ξ_{UT2} are positive constants,

$$\begin{aligned}
\Xi_{UT1} &= 2(1-\theta^p)\zeta\left(1+\theta^p\left(1+(\gamma-1)\tau\right)\right)\\
\Xi_{UT2} &= \tau(\gamma-1)\left(2-\theta^p+\theta^p\left(\left(\theta^t-\theta^p\right)\left(1+(\gamma-1)\tau\right)+\tau(\gamma-1)\right)\right)
\end{aligned}$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} > 0$ because $U(i) = \frac{1}{1-\gamma}\left(C^t\right)^{1-\gamma} < 0$. Hence, expected utility in the target country increases from the real exchange rate stabilization.

L The Welfare Consequences of Wealth Effects

In a second exercise to understand the welfare consequences of exchange rate stabilization, we perform a partial equilibrium analysis of the welfare consequences when accounting for the change in the value of the stabilizing household's asset portfolio in a small country. We decentralize the log-linearized model and show the stabilizing household's receive a positive wealth effect from the announcement of the real exchange rate stabilization. The value of the stabilizing household's portfolio increases. Households use this additional wealth to purchase traded consumption. However, the wealth effect is not large enough such that the stabilizing household's welfare increases overall. Stabilizing country households still suffer a welfare loss from announcing the real exchange rate stabilization.

We start by decentralize the household's problem in the freely floating exchange rate economy. This gives us the household's portfolio prior to announcing the exchange rate stabilization policy. We assume households can trade the real bonds of each country, which pay $P^n(\omega)$ units of the traded good in the second period in state ω. The portfolio of real bonds held by households in country n pays

$$Q_2^n(\omega) = \sum_{m=p,t,o} B_m^n \exp\left[p_T^{n*}(\omega)\right]$$

in state ω in the second period. B_m^n is the number of country m bonds held in the portfolio.

Proposition 6

In the real business cycle model, households in the freely floating exchange rate equilibrium hold long positions in their own country's bond and hold short positions on other countries' bonds,

$$B_n^n = \frac{(1-\theta^n)(\gamma-1)\tau}{\gamma} \text{ and } B_m^n = -\frac{\theta^m(\gamma-1)\tau}{\gamma} \text{ for } m \neq n.$$

Proof. We log-linearize the payoff of the bond portfolio, $Q_2^n(\omega)$,

$$q_2^n = \sum_m B_m^n \frac{1}{\tau} p^m$$

We solve for the number of bonds held in the portfolio households in country n by equating the first derivatives of the portfolio payoff, q_2^n, with the total value of consumption, $p^{n*} + c^{n*}$, with respect to the vector of endowments for households. ■

Households in the stabilizing country hold levered positions in the stabilization country's bonds. Upon announcing a real exchange rate stabilization, the value of the bonds change, which changes household wealth.

Proposition 7

In the real business cycle economy, a small country that imposes a hard real exchange rate stabilization on a larger economy increases the value of its portfolio of assets held by domestic households calculated in Proposition 6.

Proof. Let W_1^n denote the wealth of households in country n in terms of traded goods in the first period. Then,

$$W_1^n = \sum_m B_m^n V_P^m$$

where V_P^m is the value of the real bond in terms of traded goods in the first period, prior to the realization of shocks. In the deterministic steady state, $W_{1,ss}^n = V_{P,ss}^m = \frac{1}{\tau}$. Hence, the log- linear approximation of the household's wealth from announcing a hard real exchange rate stabilization ($\zeta = 1$) is

$$w_1^n + \log(\tau) = \sum_m B_m^n \left(v_P^m + \log(\tau)\right)$$

When the stabilizing country is small, the exchange rate stabilization does not affect the state contingent payoff nor the value of bonds in the target and outside countries. Announcing the exchange rate stabilization only affects the state contingent payoff of the real bond in the stabilizing country. Hence, the change in log-wealth for households in the stabilizing country is

$$(w_1^p|_{\zeta=1}) - (w_1^p|_{\zeta=0}) = W_p^p\left[(v_P^p|_{\zeta=1}) - (v_P^p|_{\zeta=0})\right] = \frac{\theta^t(\gamma-1)\gamma^2(1-\tau)^2\tau}{(1+(\gamma-1)\tau)^2}B_p^p\sigma_N^2$$

where $v_P^p = -\lambda_{T,1} + \mathbb{E}\left[\lambda_T + p^p\right] + \frac{1}{2}var\left[\lambda_T + p^p\right]$ is the log-value of the stabilizing country's bond. Households in the stabilizing country are long in domestic bonds, $B_p^p > 0$. They receive a positive wealth shock from the announcement of the real exchange rate stabilization. ■

In order to calculate the effect of this positive wealth effect on welfare, we perform the following partial equilibrium analysis. We assume the price of traded and nontraded goods remains constant and that households use all of their additional wealth to purchase additional units of traded consumption in the second period. This additional consumption is simply added onto the household's traded consumption, which was calculated earlier in this section. We continue to assume $\Delta Res = 0$.

Proposition 8

The welfare gains from the wealth effect are increasing in the households' portfolio weight on the stabilizing country's bonds. The wealth effect from a small economy announcing a hard stabilization are insufficient to raise its welfare when the initial asset position is given by 6. However, there there exists a range of more levered positions such that announcing the real exchange rate stabilization does increase the welfare of the stabilizing country's households.

Proof. Proposition 7 shows the stabilizing country households gain wealth after the announcement of the real exchange rate stabilization. The stabilizing household's welfare cannot decrease because these households always have the option of purchasing their original consumption plan and throw away the additional wealth. Household utility strictly increases from the wealth effect of announcing the real exchange rate stabilization, because households can use the additional wealth to purchase additional units of traded consumption.

We assume that households use the additional wealth to purchase traded consumption in the second period. Without the wealth effect, household traded consumption is given by,

$$c_T^p - c_T^{p*} = \zeta\frac{(1-\tau)(1-\theta^p)}{\tau\left((1-\tau)+\gamma\tau\right)}\left(y_N^t - y_N^p\right) + \frac{(1-\theta^p)}{1+(\gamma-1)\tau}\left(\psi_T - \psi_T^p\right)$$

We seek a log-linear approximation of traded consumption after accounting for the portfolio valuation effects. Given a portfolio payoff of $Q_2^p(\omega)$, the stabilizing country household spends $(1-\tau)Q_2^p(\omega)$ of the payoff on nontraded consumption and the remainder on traded consumption.

We hold the expenditure on nontraded consumption constant and assume all the additional wealth is used to purchase traded goods.

In logs, the wealth effect pays off an additional

$$\Delta q_2^{p,wealth}(\omega) = B_p^p\left[(v_P^p|_{\zeta=1}) - (v_P^p|_{\zeta=0})\right] + \psi_T - \lambda_T(\omega) - z(\omega)$$

units of traded goods to households in the stabilizing country in state ω. $\psi_T - \lambda_T(\omega)$ translates first period traded goods into second period traded goods and we have to adjust by the state contingent tax $z(\omega)$ for households in the stabilizing country.

Let $Q_2^{p,wealth}(\omega)$ denote the stabilizing household's total value of consumption in state ω after accounting for wealth effects. Then, the consumption of traded goods after accounting for wealth effects is

$$C_T^{p,wealth} = Q_2^{p,wealth}(\omega) - (1-\tau)Q_2^p(\omega)$$

where $(1-\tau)Q_2^p(\omega)$ is the expenditure on nontraded consumption after announcing the real exchange rate stabilization, but not accounting for the wealth effect. In the deterministic steady state, $Q_2^{p,wealth} = Q_2^p = \frac{1}{\tau}$. The log-linear approximation of traded consumption after accounting for wealth effects is

$$c_T^{p,wealth}(\omega) = \frac{1}{\tau}\left(\Delta q_2^{p,wealth}\right) + c_T^p(\omega)$$

We plug $c_T^{p,wealth}$ into our expressions derived in Appendix K and find that the expected utility of households in the small stabilizing country still decrease when they impose a hard exchange rate stabilization on a larger target country.

So far we showed that the change in the stabilizing household's wealth is increasing in portfolio weight B_p^p. Hence, the gain in the household welfare is increasing in B_p^p. The more of the stabilizing country's bonds the household holds, the larger the increase in the value of the household's portfolio and the more traded consumption household purchases with the additional wealth. There exists a level of wealth that allows the household to increase the level of his consumption of traded goods enough such that his expected utility increases from announcing the exchange rate stabilization. Setting the change in $\log\left[(1-\gamma)U(i)\right] = 0$, we find a cutoff weight, $\hat{W}_p^p$. ■

M Partial exchange rate stabilization

Suppose the announced stabilization is not fully credible or the government only aims to keep the exchange rate within a certain band. Partition the state space Ω into a subset in which stabilization is credible by $\Omega_s \subset \Omega$ and a part in which the government returns to a free float $\Omega_{-s} = \Omega \backslash \Omega_s$. Assume that the government stabilizes state-by-state with strength ζ within Ω_s. Further assume that the partition of Ω into Ω_s and Ω_{-s} is symmetric in the sense that it keeps the average exchange rate at its freely floating level in both parts of the partition. Note that we can use the partition to write the variance of exchange rates in the freely floating regime as

$$\begin{aligned}
\text{var}[s^{*p,t}] &= \int_\Omega \left(s^{*p,t} - E[s^{*p,t}|\{K_n\}]\right)^2 d\omega \\
&= \int_{\Omega_s} \left(s^{*p,t} - E[s^{*p,t}|\{K_n\}]\right)^2 \phi(\omega) d\omega + \int_{\Omega_{-s}} \left(s^{*p,t} - E[s^{*p,t}|\{K_n\}]\right)^2 \phi(\omega) d\omega \\
&= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[s^{*p,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*p,t}|\Omega_{-s}\right]
\end{aligned} \tag{23}$$

since the conditional mean in the two subregions of the state space are identical. By the same token, partial stabilization delivers a variance of the exchange rate of

$$\begin{aligned}
\text{var}[s^{p,t}] &= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[s^{p,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{p,t}|\Omega_{-s}\right] \\
&= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[(1-\zeta)(s^{*p,t} - E[s^{p,t}|\{K_n\}])|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*p,t}|\Omega_{-s}\right] \\
&= \text{Prob}\left[\omega \in \Omega_s\right] (1-\zeta)^2 \text{var}\left[s^{*p,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*p,t}|\Omega_{-s}\right] \\
&< \text{var}\left[s^{*p,t}\right]
\end{aligned} \tag{24}$$

With exchange rate stabilization of strength ζ, the interest rate differential becomes

$$\begin{aligned}
r^p + E[s^{p,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{p,t}\right] \\
&= -\text{cov}\left[\lambda_T, (1-\zeta)s^{*p,t}\right] \\
&= -(1-\zeta)\text{cov}\left[\lambda_T, s^{*p,t}\right]
\end{aligned}$$

The effects of partial stabilization for interest rate differentials work in the same direction. Again using the fact that the conditional means are identical, we decompose the covariance into the following terms:

$$
\begin{aligned}
r^p + E[s^{p,t}] - r^t = -\text{cov}\left[\lambda_T, s^{p,t}\right] &= -\int_{\Omega_s} \left(\lambda_T - E\left[\lambda_T|\{K_n\}\right]\right)\left(s^{p,t} - E\left[s^{p,t}|\{K_n\}\right]\right)\phi(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right]\int_{\Omega_s} \left(\lambda_T - E\left[\lambda_T|\{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right]\int_{\Omega_{-s}} \left(\lambda_T - E\left[\lambda_T|\{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right]\int_{\Omega_s} \left(\lambda_T - E\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_s\right]\left(E\left[\lambda_T|\Omega_s, \{K_n\}\right] - E\left[\lambda_T|\{K_n\}\right]\right)\int_{\Omega_s}\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right]\int_{\Omega_{-s}} \left(\lambda_T - E\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_{-s}(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right]\left(E\left[\lambda_T|\Omega_{-s}, \{K_n\}\right] - E\left[\lambda_T|\{K_n\}\right]\right)\int_{\Omega_{-s}}\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right]\int_{\Omega_s} \left(\lambda_T - E\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right]\int_{\Omega_{-s}} \left(\lambda_T - E\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{p,t} - E[s^{p,t}|\{K_n\}]\right)\phi_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{p,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{p,t}|\Omega_{-s}\right]
\end{aligned}
$$

where $\phi_s(\omega) = \dfrac{\phi(\omega)}{\text{Prob}\left[\omega \in \Omega_s\right]}$ and $\phi_{-s}(\omega) = \dfrac{\phi(\omega)}{\text{Prob}\left[\omega \in \Omega_{-s}\right]}$. The second-to-last step follows from the fact that the conditional means are identical and thus $E\left[s^{p,t} - E\left[s^{p,t}|\{K_n\}\right]|\Omega_s\right] = 0$. With partial exchange rate stabilization, we get

$$
\begin{aligned}
r^p + E[s^{p,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{p,t}\right] \\
&= -\text{Prob}\left[\omega \in \Omega_s\right]\text{cov}\left[\lambda_T, s^{p,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{p,t}|\Omega_{-s}\right] \\
&= -\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta)\text{cov}\left[\lambda_T, s^{*p,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right]\text{cov}\left[\lambda_T, s^{*p,t}|\Omega_{-s}\right]
\end{aligned}
$$

Rearranging the last equation to

$$
r^p + E[s^{p,t}] - r^t = -\text{cov}\left[\lambda_T, s^{p,t}\right] = -\text{cov}\left[\lambda_T, s^{*p,t}\right] + \text{Prob}\left[\omega \in \Omega_s\right]\zeta\text{cov}\left[\lambda_T, s^{*p,t}|\Omega_s\right],
$$

we see that the effects of partial stabilization are a milder version of currency stabilization discussed previously. In fact, partial stabilization of strength $zeta$ in a subset of the state space corresponds to currency stabilization of strength $\text{Prob}\left[\omega \in \Omega_s\right]\zeta\text{cov}\left[\lambda_T, s^{*p,t}|\Omega_s\right]$.

N Extension with Nominal Model and Rigid Prices

In this section, we extend the real business cycle model to allow for rigid prices. We let $\tilde{P}^n_T$ denote the nominal price of the traded good in country n's currency and we assume that all consumption goods consumed in a given country must be purchased using domestic currency,

$$\tilde{P}^n_T P^n C^n = \tilde{P}^n_T C^n_T + \tilde{P}^n_T P^n_N C^n_N = M^n$$

where $\tilde{P}^n_T P^n$ is the nominal price of a unit of final consumption and $\tilde{P}^n_T P^n_N$ is the nominal price of a unit of nontraded consumption. The central bank in country n sets the domestic money supply M^n.

We assume that households trade a complete set of state contingent claims with their central banks denominated in local currency. Let $\tilde{Q}^n(\omega)$ be the stochastic discount factor in the country n. Households in country n face the following budget constraint,

$$\begin{aligned}&\int \tilde{Q}^n(\omega)\left(\tilde{P}^n_T(\omega)P^n_N(\omega)C^n_N(i,\omega)+\tilde{P}^n_T(\omega)C^n_T(i,\omega)\right)g(\omega)d\omega = \\ &\int \tilde{Q}^n(\omega)\left(\tilde{P}^n_T(\omega)P^n_N(\omega)Y^n_N(\omega)+\tilde{P}^n_T(\omega)\right)g(\omega)d\omega+\kappa^n\end{aligned} \tag{25}$$

For simplicity, we set the per capita capital stock to unity and simply denote the nontraded output with Y^n_N.

We assume the nominal price of the traded good is sticky, $\tilde{P}^n_T$. This allows the central bank in each country to control the relative price of nontraded goods and the consumption of traded goods by choosing M^n.

For simplicity, we do not introduce labor supply into the model. Therefore, the quantity of goods supplied in the economy cannot adjust to changes in monetary policy. Households respond to changes in the relative price of nontraded goods by altering their consumption of traded goods. However, the total supply of traded goods in the model remains fixed at one. Hence, the monetary policy of each countries in the model cannot be independent from the others. At least one country's monetary policy must adjust to absorb any excess demand for traded goods.

We begin by deriving first order conditions for the households' problem in each country. Households maximize utility (4) subject to their budget constraint. The first order conditions with respect to traded consumption and nontraded consumption

$$\tau \left(C^n(\omega)\right)^{1-\gamma} \left(C^n_T(\omega)\right)^{-1} = \tilde{P}^n_T \tilde{Q}^n(\omega) \tag{26}$$

$$(1-\tau) \left(C^n(\omega)\right)^{1-\gamma} \left(C^n_N(\omega)\right)^{-1} = \tilde{P}^n_T \tilde{Q}^n(\omega) P^n_N(\omega) \tag{27}$$

Additionally, to remain consistent with the assumption that governments in the target and outside countries do not alter the incentives to consume traded goods, we assume that the marginal utility of traded goods in the target and outside countries are equal, $\tilde{P}^t_T \tilde{Q}^t = \tilde{P}^o_T \tilde{Q}^n$.

These six first order conditions along with the three resource constraints for nontraded goods, the resource constraint for traded goods, the money supply equation in each country and the condition that equalizes the marginal utility of traded goods across the target and outside countries comprise a system of 14 equations. These 14 equations implicitly define the following 14 variables: $\left\{C^n_T, C^n_N, \tilde{Q}^n, P^n_N\right\}_{n=p,t,o}$, M^t and M^o. The state variables are the output of nontraded goods, Y^n_N, the nominal prices of the traded goods and the money supply in the stabilizing country.

To derive closed form solutions, we log-linearize this system of equations around the deterministic steady state from the real business cycle model. In order to simplify expressions, we assume $\tilde{P}^n_T = 1$. The system of equations is,

$$\begin{aligned}
-(1-\tau)(\gamma-1)\tau c^n_N - \tau(1+(\gamma-1)\tau)c^n_T + \tau \log[\tau] &= \tau(\tilde{q}^n + \tilde{p}^n_T), \forall n \\
-(1-\tau)(\gamma-1)\tau c^p_T - \tau(1+(\gamma-1)\tau)c^n_N + (1-\tau)\log[1-\tau] &= (1-\tau)(\tilde{q}^n + \tilde{p}^n_T + p^n_N), \forall n \\
y^n_N &= c^n_N, \forall n \\
\sum_n \theta^n c^n_T &= 0 \\
\tilde{p}^n_T + c^n_T + \frac{1-\tau}{\tau}(\tilde{p}^n_T + p^n_N + c^n_T) &= \frac{1}{\tau} m^n + \frac{1-\tau}{\tau}\log[1-\tau] + \log[\tau] \\
\tilde{p}^t_T \tilde{q}^t &= \tilde{p}^o_T \tilde{q}^o
\end{aligned}$$

The consumption of traded goods in the stabilizing country is

$$c^p_T = m^p - \tilde{p}^p_T + \log[\tau]$$

We derive the monetary policy function of the stabilizing country's central bank by setting $\tilde{p}^p_T = 0$ and choosing m^p to match the consumption of traded goods given by equation (18).

O Expressions for Constants in the Incomplete Markets Model

The following constants are used to define the consumption of traded goods in the stabilizing country and the value of nontraded output in the incomplete markets model, respectively.

$$\Xi_T^p = \frac{(1-\theta^p)\left(\tau+(1-\tau)\phi\right)^2 + (1-\tau)(1-\phi)\left(\gamma\tau+(1-\tau)\phi\right)\left(1+(1-\tau)\right)(1-\tau)\phi}{\gamma\tau\left(\phi+(1-\tau)\phi\right)\left(\gamma\tau+\phi(1-\tau)\right)\left(\tau+(1-\tau)\phi\right)}$$

$$\Xi_N^p = \frac{(1-\tau)\phi\left(\theta^p\left(\tau(\gamma-\phi)+\phi\right)+(1-\theta^p)\left(\gamma-1\right)\tau\right)}{\gamma\tau(\tau(1-\phi)+\tau)(\gamma\tau+(1-\tau)\phi)}$$

P Proof of Corollary 1

We derive results for differences in interest rate and differences in the incentives to accumulate capital in an economy that is affected by productivity shocks, inflation shocks and preference shocks. We first prove results for the internal effects of a real exchange rate stabilization.

When a country stabilizes its exchange rate relative to a target country, the interest rate differential between the stabilizing country and the target is

$$\begin{aligned} r^p + \mathbb{E}s^{p,t} - r^t = & \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \\ & \zeta\frac{\gamma(1-\tau)^2\left((\theta^t-\theta^p)\tau(\gamma-\phi)+2\phi\theta^p(1-\zeta)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2 - \\ & \zeta\frac{\gamma(1-\tau)(1-\phi)^2\left((\theta^t-\theta^p)\gamma\tau+2\phi\theta^p(1-\zeta)(1-\tau)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 - \\ & \zeta\frac{\phi(1-\tau)(1-\gamma)^2\left((\theta^t-\theta^p)\gamma\tau+2\phi\theta^p(1-\zeta)(1-\tau)\right)}{\tau\gamma\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2, \end{aligned}$$

which implies the exchange rate stabilization decreases the risk free rate in the stabilizing country relative to the risk free rate in the target country as long as the target country is larger than the stabilizing country, $\theta^t > \theta^p$.

We show that the relative incentives to accumulate capital in the stabilizing country increase with the size of the target country. Hence, there exists a country size θ_{min} such that a hard exchange rate stabilization on any country larger than θ_{min} will increase the incentives to accumulate capital in the stabilizing country.

$$\frac{d}{d\theta^t}\left[k^p - k^t - \left(k^{p*} - k^{t*}\right)\right] = \frac{\zeta(\gamma-1)(1-\tau)^2\tau(\gamma-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2$$
$$+\frac{\zeta(\gamma-1)\gamma(1-\tau)\tau(\gamma-\phi)(1-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2+\frac{\zeta(\gamma-1)^3(1-\tau)\tau(\gamma-\phi)\phi^2}{\gamma\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2.$$

Because firms are competitive, wages are given by the marginal product of labor. Hence, the exchange rate stabilization increases wages in the stabilizing country relative to all other countries.

The interest rate differential between the target country and the outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}\right) + \frac{\zeta\theta^p\gamma(1-\tau)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\sigma_N^2 + \frac{\zeta\theta^p\gamma(1-\tau)^2(1-\phi)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\tilde{\sigma}^2$$
$$+\frac{\theta^p\zeta(\gamma-1)^2(1-\tau)^2\phi^2}{\gamma\tau\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2,$$

which implies the exchange rate stabilization increases the risk free rate in the target country relative to the risk free rate in the outside country.

The differential incentives to accumulate capital in the target country relative to the outside country is given by

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^p\zeta(1-\tau)^2(\gamma-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_N^2 - \frac{\theta^p\gamma\zeta(1-\tau)(\gamma-\phi)(1-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\tilde{\sigma}^2 -$$
$$\frac{\theta^p(\gamma-1)^2\zeta(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_\chi^2.$$

Incentives to accumulate capital in the target country decrease relative to the outside country. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.

Finally, we turn to the cost of the exchange rate stabilization. When the stabilizing country is small, the cost of the stabilization decreases as the target country gets larger,

$$\frac{d\log\left(\Delta Res\right)}{d\theta^t} = -\zeta\frac{\gamma(\gamma-1)(1-\tau)(1-\phi)^2(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\tilde{\sigma}^2$$
$$-\zeta\frac{(\gamma-1)(1-\tau)^2(\gamma-\phi)(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma_N^2$$
$$-\zeta\frac{(\gamma-1)^3(1-\tau)\phi(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\gamma(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma_\chi^2 < 0.$$

Hence, it is cheaper to enforce a stabilization relative to a larger country.

Q Results with Endogenous Capital Accumulation

In this section, we show that the main propositions from the paper hold when we allow for an endogenous capital stock. We assume there exists a unit of capital in the world which can be allocated across the three countries. Allowing for endogenous capital accumulation changes the expected level of consumption. It does, however, not affect the covariance of consumption across countries. Results that depend on the covariances between asset payoffs and the shadow price of traded goods are therefore not affected.

To solve for the endogenous capital stock, we start with the system of four equations that defined incentives for capital investment with an exogenous capital stock,

$$\begin{aligned} k^n &= \log[v] - \log[q_1] - \log[\psi_T] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2} var\left[\lambda_N^n + y_N^n\right] \qquad \forall n \\ 0 &= \sum_n \theta^n k^n \end{aligned}$$

We allow capital to adjust endogenously by using the solution for λ_N^n to write λ_N^n as a function of the y_N^n's and then by using $y_N^n = \eta + \nu k^n$ to write all the y_N^n's in terms of capital and a productivity shock. Afterwards, we solve the system of four equations for k^t, k^p, k^o and q_1.

The final expression for log differences in capital accumulation is a bit larger when we allow for endogenous capital accumulation. In a freely floating exchange rate economy ($\zeta = 0$), the larger country still accumulates a higher capital per capita stock

$$k^{p*} - k^{t*} = \frac{(\gamma - 1)^3 (1 - \tau)^2 \tau}{(1 + (\gamma - 1)\tau)\left(1 + (\gamma - 1)(1 - \tau)\nu + (\gamma - 1)\tau\right)} \left(\theta^p - \theta^t\right) \sigma_N^2$$

However, by comparing this expression to the one in the main body of the paper, we observe the difference in capital accumulation is *smaller* than the incentives to accumulate capital. Capital flows towards countries with higher incentives to accumulate capital and away from countries with lower incentives to accumulate capital. The final capital allocation reflects this movement.

When capital is allowed to adjust endogenously, a country that imposes a hard exchange rate stabilization on a target country equalizes the per capita capital stock across the two countries,

$$\left[k^p - k^t|_{\zeta=1}\right] = k^{p*} - k^{t*} + \frac{(\gamma - 1)^3 (1 - \tau)^2 \tau \left(\theta^t - \theta^p\right) \sigma_N^2}{(1 + (\gamma - 1)\tau)\left(1 + (\gamma - 1)(1 - \tau)\nu + (\gamma - 1)\tau\right)} = 0.$$

Hence, a smaller country increases its domestic capital by stabilizing its real exchange rate relative to a larger country.