Learning About Commodity Cycles and Saving-Investment Dynamics in a Commodity-Exporting Economy*

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Abstract

Despite sustained high levels of commodity prices, the current account balances of several commodity-exporting countries have deteriorated in recent years. This phenomenon is examined quantitatively using a small open economy model with a commodity-producing sector under the assumption that agents have imperfect information and learn about the persistence of commodity price shocks. A central prediction of the model is that during a persistent commodity price increase, agents believe at first that this increase is temporary but eventually revise their expectations upward as they are surprised by higher than forecasted commodity price levels. Domestic investment therefore expands in a gradual way driven by investment in the commodity sector, while domestic savings decrease such that the current account declines over time. The model is estimated with data for Chile, and the results show that through the above mechanism the model explains several stylized facts regarding the recent evolution of the Chilean economy, including part of its step-wise current account reversal since the mid-2000s. Additionally, we use the model to analyze the effects of a persistent commodity price shock under alternative monetary and fiscal policies.

Keywords: Commodity Prices; Commodity-Exporting Economies; Savings, Investment, and the Current Account; Imperfect Information and Learning.

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1 Introduction

Commodity prices have surged over the past decade, interrupted only briefly by the global financial crisis. Most commodity-exporting countries have thus experienced record or near-record terms of trade during a prolonged period of time. Despite this fact, many commodity exporters have accumulated significant current account deficits that have become an important policy concern, due to the risk of a painful adjustment in the face of a sudden stop in capital flows. Some countries such as Brazil, Canada, Chile and Peru have even experienced a current account reversal from positive balances of several percentage points of GDP in the mid-2000s into deficits of more than 2% of GDP in recent years. In Chile, a major copper producer, the reversal has been especially large, up to around 8% of GDP (see Figure 1).

What explains this surprising pattern of current account dynamics in several commodity-exporting economies during the latest commodity price boom? The observed dynamics seem at odds with the response of the current account to a positive terms of trade shock that may be expected according to standard macroeconomic theory, which predicts that temporary income gains would be mostly saved by rational agents while permanent income gains would be immediately and entirely spent. That theory, which was formalized after the early 1980s, is well known as the intertemporal approach to the current account (see Obstfeld, 1982; Svensson and Razin, 1983; Obstfeld and Rogoff, 1995; Mendoza, 1995).[1] However, Obstfeld (1982) and Svensson and Razin (1983) also elegantly explained that the effect of a terms of trade shock on the trade balance should depend on the *perceived* persistence of the shock. The latter points towards a possibly important role for imperfect information on the persistence of terms of trade changes and learning behavior by economic agents.

In this paper we therefore examine the relevance of learning under imperfect information on the persistence of commodity price shocks for saving-investment dynamics in a dynamic stochastic general equilibrium (DSGE) model for a commodity-exporting small open economy. Unlike most existing models of this type, our framework supposes that agents cannot perfectly distinguish between persistent and transitory movements in the price of the exported economy. Instead, agents learn over time about the true persistence of the shock. Technically, agents use optimal filtering to update their inference on the persistence of the shock based on past forecasting errors. We provide evidence that supports the practical relevance of this hypothesis by analyzing actual revisions of forecasts made by expert forecasters.

The model predicts that, if the persistence of a commodity price shock turns out to be high, then higher expected returns of capital especially in the commodity sector trigger an investment boom that has important spillover effects on investment in other sectors and the rest of the economy. The positive but lagged response of commodity investment, which is subject to

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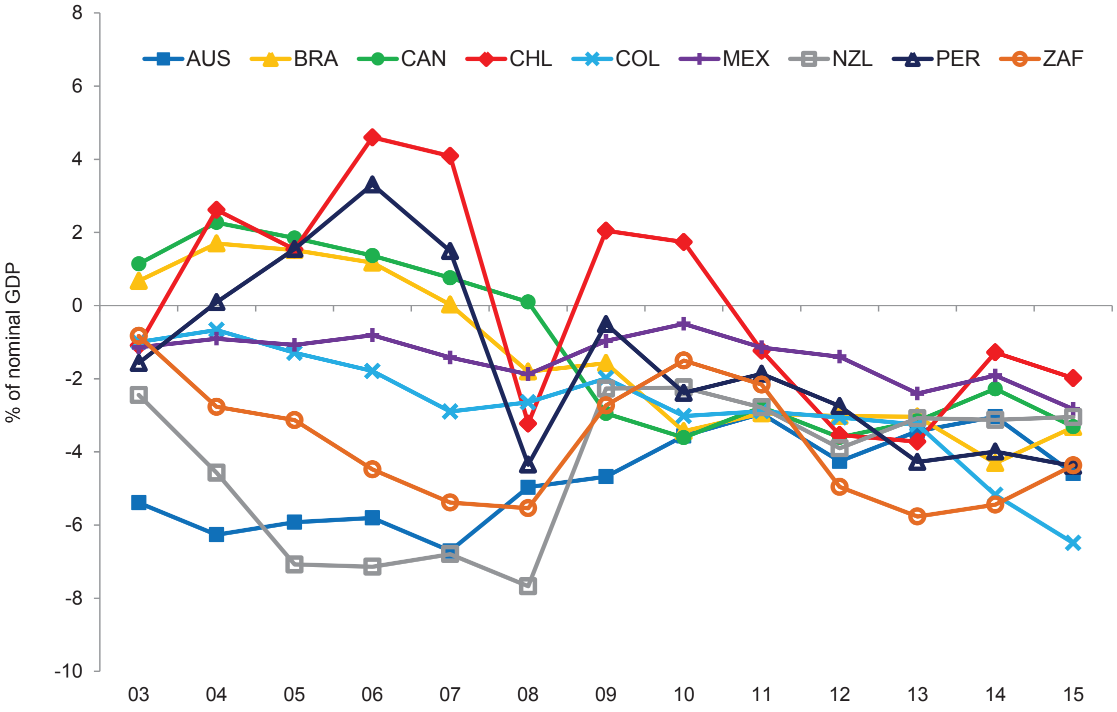
[1] The Obstfeld-Razin-Svenson effect challenged the earlier known Harberger-Laursen-Metzler effect, according to which higher terms of trade lead to an improvement of the trade balance if the marginal propensity to consume is lower than one, see Harberger (1950) and Laursen and Metzler (1950).

Figure 1: Current account balances of major commodity-exporting countries, 2003-2015.



Notes: This figure shows the annual current account balances as a percentage of GDP from 2003-2015 of Australia (AUS), Brazil (BRA), Canada (CAN), Chile (CHL), Colombia (COL), Mexico (MEX), New Zealand (NZL), Peru (PER) and South Africa (ZAF). Data source: International Monetary Fund (IMF), World Economic Outlook Database, April 2016. The numbers for Brazil, Chile, Colombia and South Africa for 2015 are IMF estimates.

adjustment costs and time to build frictions in our model, is also key to the eventual appearance of a current account deficit after an initial surplus due to a predicted increase in savings in the early phase of the commodity price surge (while people think it is short-lived).

We conduct a formal Bayesian estimation of the model using quarterly Chilean data to assess the quantitative significance of the above mechanism. The results show that a relatively large fraction of recent investment dynamics and the evolution of the current account balance in Chile are directly explained by the latest commodity price surge through that mechanism, in addition to exceptionally relaxed external financing conditions and historically low international interest rates. Hence, our results suggest that changes in agents' perceptions on the persistence of the recent commodity price cycle and its interaction with investment in the commodity sector are validated empirically as additional factors, along with low external interest rates, to explain recent current account dynamics in commodity-exporting economies.

Previous studies used simpler models to examine the key drivers of the current account in commodity-exporting countries. For example, Medina, Munro, and Soto (2008) estimate a DSGE model modified for Chile and New Zealand and show that the main factors that account for current account fluctuations of both countries are investment-specific shocks, changes in foreign financial conditions, and variations in foreign demand. This study builds on the New

Keynesian small open economy model for Chile developed in Medina and Soto (2007a).[2]

Our first contribution to this literature is to extend the exogenous commodity supply assumed by Medina and Soto (2007a), by incorporating endogenous production decisions. Production is carried out with capital, which is subject time to build and besides there are adjustment costs in modifying investment projects, following Kydland and Prescott (1982) and Uribe and Yue (2006). The recent boom of mining investment in most commodity-exporting countries—in Chile mainly through foreign direct investment by international mining companies and capital formation of the Chilean state owned copper company (Codelco)—points towards the importance of introducing endogenous commodity investment in the model.[3] In addition, the introduction of time to build in capital accumulation next to standard adjustment costs accounts for the fact that the majority of investment projects in the mining sector are large and thus take several quarters until they mature and become productive.

The second contribution of this paper is to implement imperfect information and learning through the Kalman filter on the persistence of commodity price shocks, drawing upon studies on the role of imperfect information and optimal filtering in macroeconomic dynamics following Erceg and Levin (2003).[4] This application is motivated by evidence of gradual revisions of commodity price forecasts by chief international forecasting institutions and, in the case of Chile, by the panel of experts that determines the long-run reference price of copper that enters Chile's structural balance fiscal rule. As we document in the following section, in the initial stages of the latest commodity price cycle, the forecasters predicted that the copper price would revert relatively quickly towards its long-run mean. However, as the actual price increase turned out to be more persistent than expected, the forecasts were gradually revised upwards and the reference price was raised in a step-wise fashion. Those gradual forecast revisions are consistent with the mechanism of optimal filtering that we apply in our model.

Given the results summarized above, our model is a capable tool for interpreting gradual current account reversals and for conducting policy analysis in commodity-exporting countries. Hence, we conduct a number of policy experiments taking Chile as a case study. Those experiments show that while fiscal policy design has important general macroeconomic implications in the face of commodity price fluctuations, it does not critically affect the evolution of the current account which is mainly driven by investment and FDI in the commodity sector. Monetary policy, on the other hand, seems to be best conducted under a flexible exchange rate, because exchange rate targeting would further enhance current account deficits that may emerge under persistent commodity price shocks according to our model.

The remainder of the paper is structured as follows. Section 2 discusses some stylized facts

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[2] The model belongs to the family of New Open-Economy Macroeconomic models (see Lane, 2001, for a survey).

[3] Sachs (1981) also emphasized the role of investment in magnifying the responses of the current account and the real exchange rate in the medium term.

[4] See Céspedes and Soto (2007) for an application of imperfect information and optimal filtering to inflation dynamics in Chile.

related to saving-investment dynamics in Chile as well as the evolution of copper price forecasts over the latest commodity price cycle that we seek to match with the model. Section 3 describes the model, Section 4 discusses the estimation of the model and Section 5 presents the results. Finally, Section 6 concludes. The appendix provides some technical details on the model.

2 Stylized Facts

In this section we discuss a number of key stylized facts that we seek to match with our model. These facts relate to saving-investment dynamics in Chile over the recent commodity price cycle, as well as the evolution of copper price forecasts by professional forecasters and the panel of experts that determines the long-run reference price of copper that enters the structural balance fiscal rule applied by the Chilean government.

2.1 Copper price and current account dynamics

The Chilean economy has exhibited a large current account deficit over the past couple of years, despite relatively high levels of commodity prices and in particular the price of copper. Figure 2 plots the evolution of annual national savings, fixed capital formation and the current account balance as a percentage of nominal GDP in Chile against the average annual spot price of refined copper quoted at the London Metals Exchange over the period 2003-2015. The figure shows that the Chilean current account has moved from a deficit of around 1% of GDP in 2003 to a surplus of more than 4% of GDP in 2006 and 2007, at a time when the copper price quadrupled in value from 80 US dollar cents per metric pound in 2003 to 320 cents in 2007.

The global financial crisis generated an abrupt but short-lived fall in the price of copper, and during 2009-10 it steadily recovered.[5] Indeed, in 2010 the copper price exceeded its pre-crisis levels reaching a record of 340 cents per pound, on average. The large copper price decline triggered a fall in Chile's nominal export volumes and a significant current account deficit in 2008, and along with higher copper prices the current account moved again into surplus after the crisis. However, it is remarkable that unlike during the commodity price boom of the pre-crisis years, the current account gradually deteriorated in the post-crisis period despite the strong and sustained recovery of the copper price, reaching a deficit of more than 3% of GDP after 2011. This pattern can also be observed in Figure 1 for other countries.

These peculiar dynamics of the current account can be explained by the evolution of the gap between national savings and investment. For space considerations, we do not conduct a detailed analysis for all countries reported in Figure 1 and focus on Chile as case of study. In Chile, the current account surplus during the mid-2000s was generated by a rise in national savings while national investment remained steady at a share of GDP of roughly 21%. However, from 2008 onwards investment started to increase reaching approximately 25% of GDP in 2012. On the

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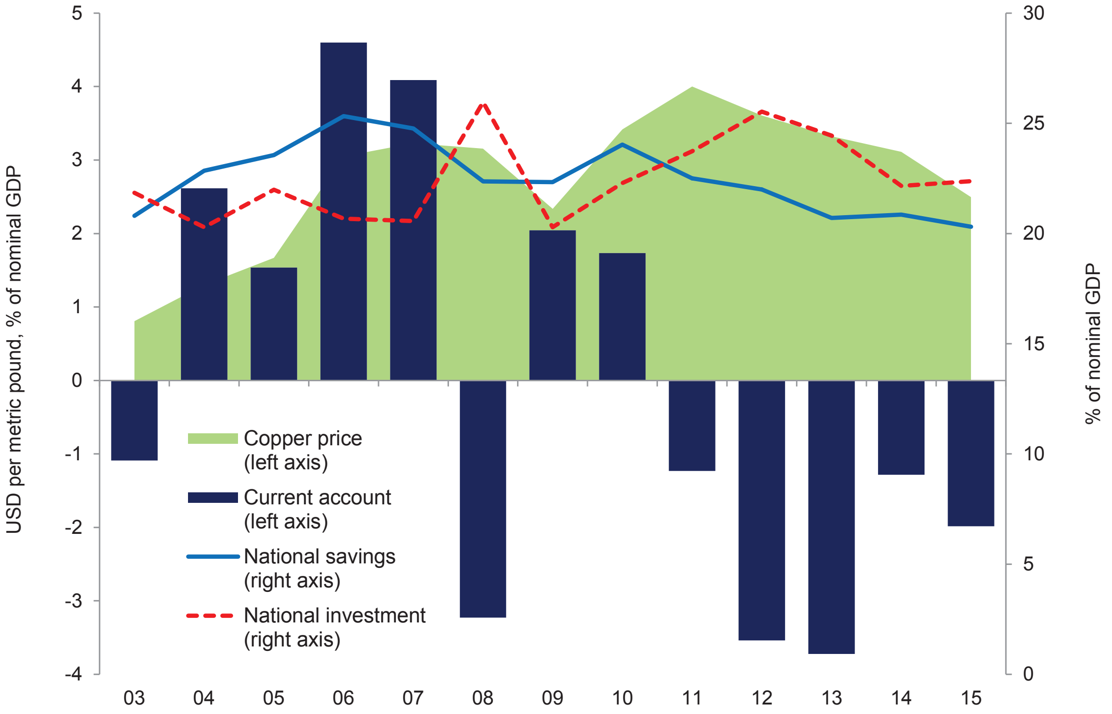
[5] At its minimum in December 2008, the copper price dropped to 140 cents per pound.

Figure 2: Savings, investment, and the current account in Chile vs. copper price, 2003-2015.



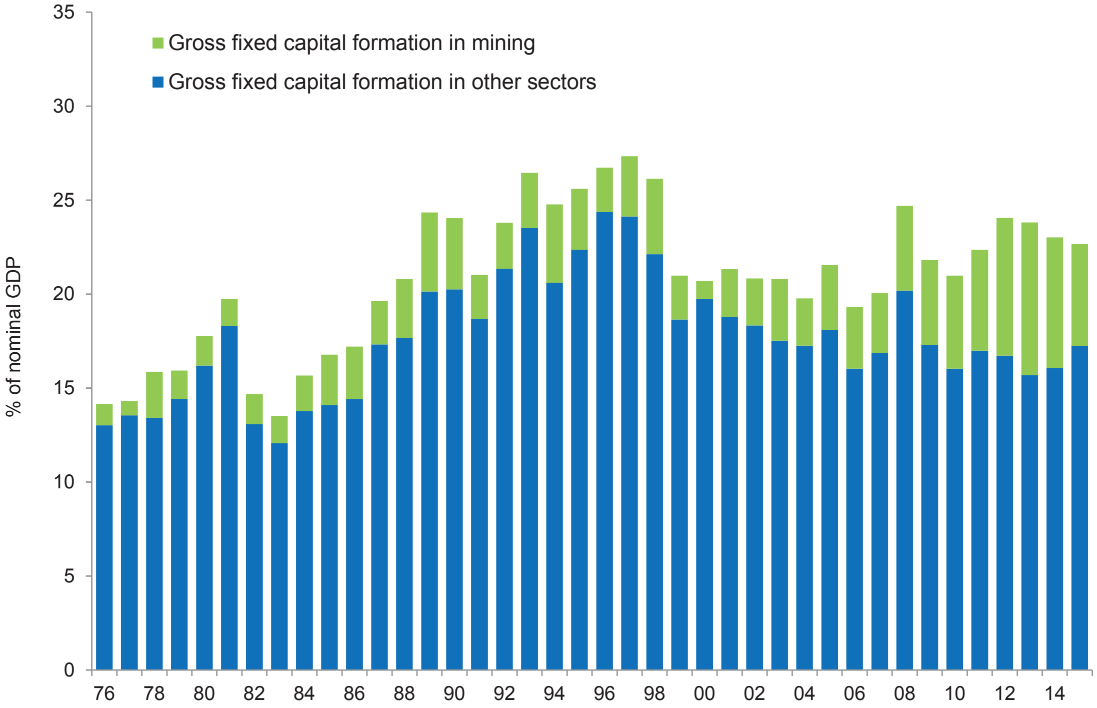
Notes: This figure shows annual national savings (solid blue line, right axis), fixed capital formation (dashed red line, right axis) and the current account balance (bars, left axis) in Chile from 2003-2015. All variables are expressed as percentages of GDP. The green shaded area corresponds to the average annual spot price of refined copper at the London Metals Exchange in US dollars per metric pound (left axis). Data source: Central Bank of Chile and International Monetary Fund, World Economic Outlook Database, April 2016.

other hand, national savings declined gradually over that period from almost 25% of GDP in 2006 to less than 22% of GDP in 2012, such that the current account moved into deficit in a step-wise fashion. Hence, while the gains from commodity exports seem to have been mostly saved by domestic agents in the mid-2000s period, this was not the case in the post-crisis period.

2.2 The role of mining investment

Most of the investment boom in Chile after 2008 has been due to a relatively large increase of investment in the mining sector of the economy. Figure 3 shows the evolution of gross fixed capital formation in mining and other sectors as a percentage of GDP over the period 1976-2015. Investment in mining was on average less than 2% of GDP from 1976 until the mid-1980s. From the second half of the 1980s until the mid-2000s, it maintained average levels of around 3% of GDP. However, starting in 2008, mining investment has increased significantly, up from around 4.5% of GDP in 2008 to more than 8% of GDP in 2013. Only very recently has investment in mining started to decrease again, reaching approximately 7% of GDP in 2014 and 5% of GDP in 2015. The figure also shows that non-mining investment in Chile has remained comparatively stable compared to its pre-2008 levels.

Figure 3: Gross fixed capital formation in mining and other sectors in Chile, 1976-2015.



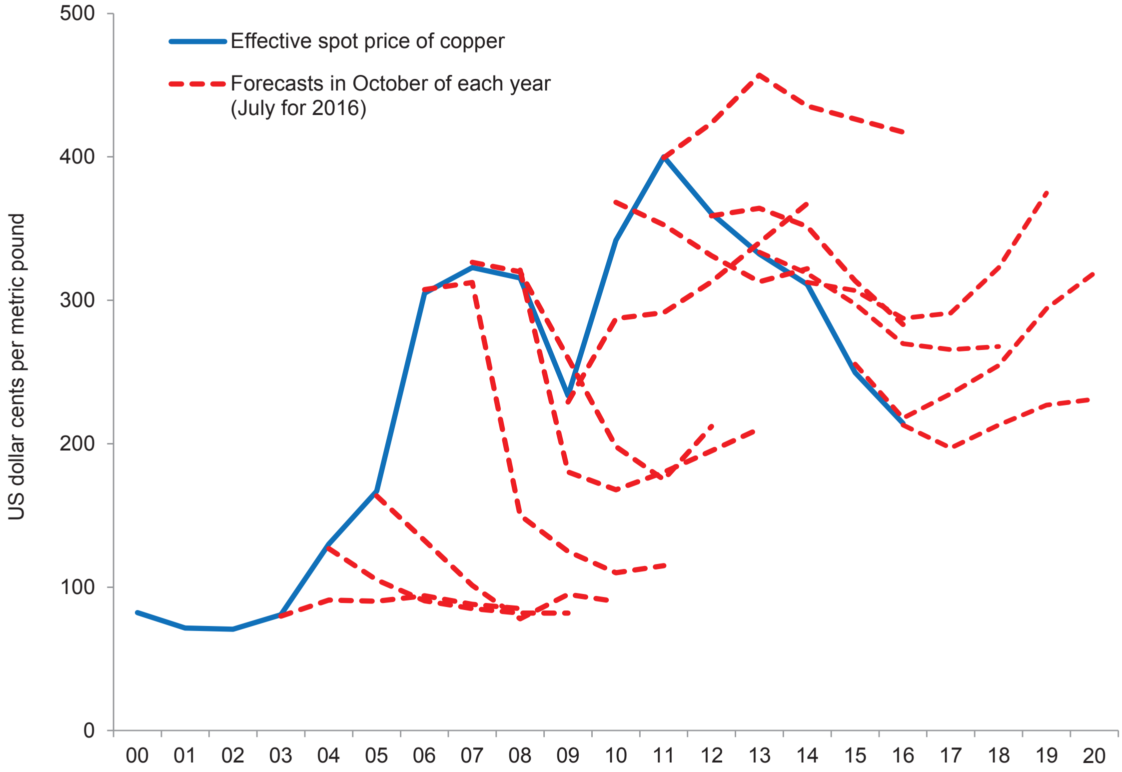
Notes: This figure shows total gross fixed capital formation in the mining sector and other sectors in Chile from 1976-2015, expressed as percentages of nominal GDP. Data sources: Chilean State Copper Commission (Cochilco) and Central Bank of Chile. The sectoral division of investment for 2015 is a preliminary estimate from the Central Bank of Chile's Monetary Policy Report (September 2016).

2.3 Copper price forecasts and forecast revisions

We now highlight the differences between the copper price forecasts made by professional forecasters of the CRU Group (reports in October of each year) prior to the global financial crisis with those made subsequently (see Figure 4). On the one hand, the copper price quadrupled in value between 2003 and 2007, from around 80 to approximately 320 US dollar cents per metric pound (annual average). However, the rapid rise of the spot price was not validated by higher forecasted prices on a medium- to long-term horizon at that time. Instead, it was considered as a transitory price increase by the professional forecasters who predicted that the spot price would return to values of around 100 cents.

Due to the crisis, the copper price fell and almost reversed the rise from 2003 to 2007, reaching a minimum of approximately 140 cents (where the annual average understates somewhat the dynamic evolution of the spot price). That decline was relatively short-lived, and after the crisis the copper price quickly recovered and even exceeded its pre-crisis levels. Interestingly, the higher post-crisis prices were also accompanied by larger forecasted prices, as part of a process of gradual forecast revisions that had started already around 2007. In the following years, the forecasted prices on a medium- to long-term horizon reached values much closer to the effective spot price. Hence, the professional forecasters seem to have incorporated a more persistent

Figure 4: Effective spot price of copper from 2000-2016 vs. forecasts by CRU Group.



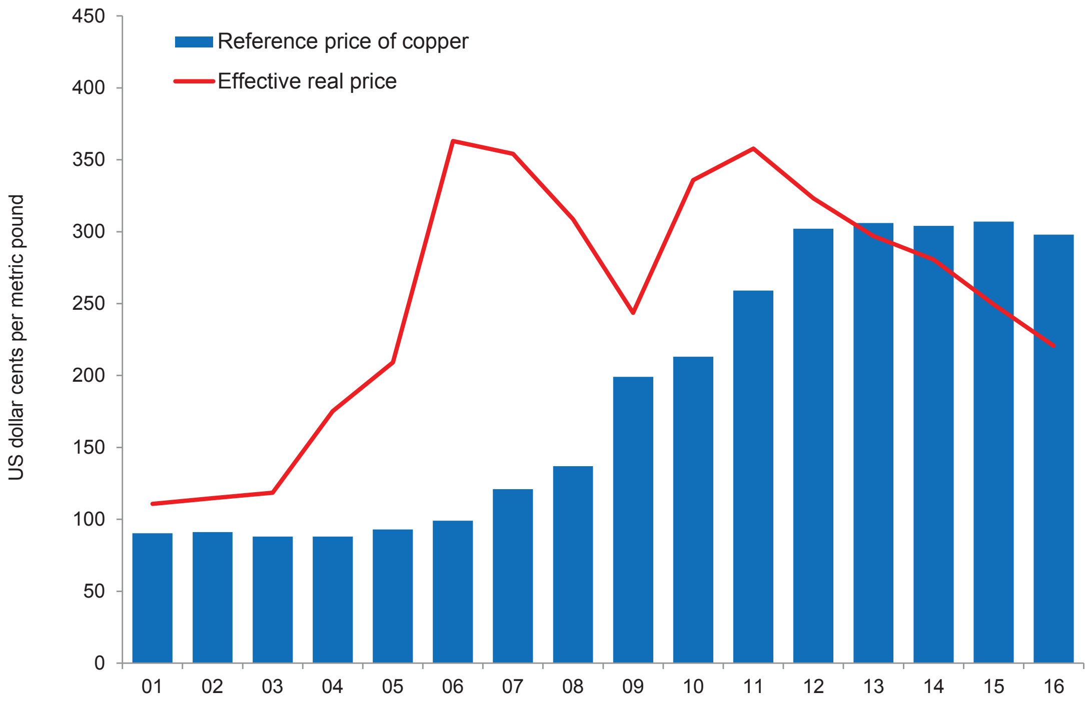
Notes: This figure shows the average annual spot price of refined copper at the London Metals Exchange in US dollar cents per metric pound from 2003-2016 (blue line) and forecasts of the spot price by professional forecasters of the CRU Group made in October of each year starting in 2003 (dashed red lines). The spot price for 2016 is the average price from January to July 2016 and the forecast for 2016 corresponds to July. Data source: Central Bank of Chile and CRU Group.

price increase in their forecasts over time, which is broadly consistent with the mechanism of imperfect information and learning behavior that we describe in our model. More recently, as the commodity cycle has turned and prices have fallen, it has taken the forecasters again several years to adjust their expectations on future prices downwards.

This interpretation of the persistence of the latest commodity price cycle by market agents is also critical to understand, in our view, the recent evolution of mining investment and the current account in Chile and other commodity exporters. More precisely, for the market it took between four and five years to adjust their expectations regarding the persistence of the commodity price boom. While expectations were revised upwards, in parallel mining investment increased significantly, which contributed to the observed deterioration of current account balances. Recently, despite the observed reversal of commodity prices, mining investment remained stable as long as price forecasts remained relatively high.

We now complement the previous analysis by examining the evolution of the reference price of copper used by the Chilean Ministry of Finance. The reference price is computed from long-

Figure 5: Government reference price of copper vs. effective real price of copper, 2001-2016.



Notes: This figure shows the average annual spot price of refined copper at the London Metals Exchange in US dollar cents per metric pound deflated by an index of external prices published by the Central Bank of Chile ("IPE") which was converted into an index with base year 2015 (red line) and the reference price of copper used by the Chilean government from 2001-2016 (bars). The spot price for 2016 is the average price from January to July 2016. Data source: Chilean Ministry of Finance and Central Bank of Chile.

run forecasts of future spot prices over a ten-year period by a panel of independent experts.[6] It is one of the main variables of the fiscal rule applied by the government, and it is therefore also one of the key variables in the determination the fiscal budget.

Figure 5 plots the evolution of the reference price, which is expressed in real terms by construction, against the effective real price of copper, for the fiscal years starting in 2001 until 2016. As the professional forecasts discussed above, the reference price gradually increased over time, from just below 100 cents per pound in 2001-2005 to more than 300 cents in 2011-2015 and only slightly less in 2016. What is remarkable is that the latest forecasts after 2011 were not too far from the spot prices in the previous years, indicating that the experts had incorporated sustained high levels of commodity prices in their forecasts by that time, similarly as the professional forecasters of the CRU Group. In addition, the reference price has only started to decrease very recently, despite the fact that effective copper prices have already been lower for some time.

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[6] The panel of experts comprises sixteen members. Each member is asked to submit forecasts of the annual (real) spot price for the following ten years (in US dollar cents of the following fiscal year, using as deflator the forecast of the US CPI by the IMF). The forecasts of each expert are averaged over time and the two most extreme values—the highest one and the lowest one—are discarded. The reference price is the average of the remaining ten-year averages.

Due to the characteristics of the Chilean fiscal rule, which adjusts the government budget by the gap between the reference price and the actual price of copper, this step-wise increase in the reference price gradually allowed for more public spending out of copper revenues, both from income of the state-owned copper company (Codelco) and from taxes on private copper producers. Hence, while the government mainly saved the additional copper revenues during the initial stages of the latest commodity cycle in the mid-2000s, which added to the overall current account surplus during that period, it decreased its savings over time, contributing to the gradual deterioration of the current account balance. Changes in perceptions on the persistence of the commodity cycle again played a key role in this process.

2.4 Summary of stylized facts

In summary, we have presented the following four simplified empirical findings related to saving-investment dynamics in the case of Chile and the evolution of copper price forecasts over the latest commodity price cycle: (i) sustained high levels of the copper price gradually led to higher forecasted prices by private agents and the government; (ii) high spot prices and higher forecasted prices were also associated with a declining current account balance, from a surplus to a deficit; (iii) the initial current account surplus was due to an increase in national savings, while the subsequent decline and eventual deficit has been due to increasing investment and insufficient national savings; (iv) the investment boom has been driven by investment in mining.

Those observations point towards the importance of changing perceptions on the persistence of the commodity price cycle, a key ingredient of our model. In addition, our stylized interpretation of the data suggests the hypothesis that those changing perceptions are one of the determinants of the recent boom in mining investment in Chile, lower savings especially by the government, and the eventual deterioration of the current account balance. Of course, the observed correlations may be spurious as both mining investment and the current account may be driven by other factors than copper price shocks and expectations of future copper prices; therefore, we investigate this hypothesis in a formal structural way using the model described in the next section.

3 The Model

The starting framework is the DSGE model described in Medina and Soto (2007a). It includes non-Ricardian households, sticky prices and wages with partial indexation to past inflation, incomplete exchange rate pass-through into import prices in the short run, adjustment costs in investment, habit persistence in consumption, and a structural balance fiscal rule to describe fiscal policy in Chile. On the production side, the model incorporates a domestic tradable goods sector and a commodity-exporting sector that is owned in part by the government and in part by foreign agents.[7]

We add to this model two features. First, the main sector that exports the commodity, i.e. copper, expands as new capital is allocated into the sector. The standard law of accumulation of capital is modified by adding time to build in capital accumulation and adjustment costs in investment, following Kydland and Prescott (1982) and Uribe and Yue (2006).[8] Second, the model departs from the pure rational expectations solution by including imperfect information on the persistence of commodity price shocks and learning behavior by economic agents. Specifically, imperfect information and learning is modelled along the lines of Erceg and Levin (2003). Those two extensions are described in detail below, but the description of the basic framework is deliberately brief and we refer to Medina and Soto (2007a) for details.[9,10]

3.1 Households

There is a continuum of households indexed by $j \in [0,1]$. A fraction λ of those households are non-Ricardian ones without access to the capital market. These households receive no profits and do not save, and thus consume entirely their disposable wage income. The remaining households are Ricardian ones that do have access to the capital market and make intertemporal consumption and savings decisions in a forward-looking manner.

Households of the Ricardian type maximize the present value of expected utility at time t:

$$\max E_t \sum_{i=0}^{\infty} \beta^i \zeta_{C,t+i} \left[\log(C_{t+i}^R(j) - hC_{t+i-1}^R) - \psi \frac{l_{t+i}(j)^{1+\sigma_L}}{1+\sigma_L} \right], \qquad j \in (1-\lambda, 1],$$

subject to the period-by-period budget constraint

$$\begin{aligned} P_{C,t}C_t^R(j) + E_t\{d_{t,t+1}D_{t+1}(j)\} + \frac{B_t(j)}{r_t} + \frac{\varepsilon_t B_{P,t}^*(j)}{r_t^*\Theta_t} = \\ W_t(j)l_t(j) + \Xi_t(j) - TAXN_t(j) + D_t(j) + B_{t-1}(j) + \varepsilon_t B_{P,t-1}^*(j), \end{aligned}$$

where $C_t^R(j)$ is consumption of household j and C_t^R is aggregate consumption of Ricardian households, respectively, while $l_t(j)$ is household j's labor effort (in hours). The variable $\zeta_{C,t}$ is a preference shock to the households' subjective discount factor. Further, $P_{C,t}$ is the aggregate consumer price index (CPI), $W_t(j)$ is the nominal wage set by the household, $\Xi_t(j)$ collects

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[7] Pieschacón (2012) also studies how commodity price shocks transmit differently in the cases of Mexico and Norway. Her model differs from the model of Medina and Soto (2007a) in the following dimensions: (i) it adds non-tradable goods, (ii) it includes a government goods basket in the consumption basket, (iii) it assumes only Ricardian agents, and (iv) the micro-foundations of the fiscal rule in the model are less developed.

[8] While time to build as in Kydland and Prescott (1982) introduces lags between the initiation of investment projects and effective production, it tends to imply "jumpy"dynamics of investment that are inconsistent with the data. We therefore allow for adjustment costs in investment projects in addition to time to build, following Uribe and Yue (2006), to better match investment dynamics.

[9] We make some smaller additional modifications to the basic framework of Medina and Soto (2007a) to be able to match more recent Chilean data, such as the inclusion of food consumption to be able to match the 2008-2009 food price increase and a shock to the stock of capital to resemble the 2010 earthquake in Chile.

[10] Appendix C contains the complete list of model equations.

payouts by firms, $TAXN_t(j)$ are lump-sum tax payments to the government, ε_t is the nominal exchange rate (units of domestic currency to buy one unit of foreign currency), and $d_{t,t+1}$ is the period t price of one-period domestic contingent bonds, $D_t(j)$, normalized by the probability of the occurrence of the state. The variable r_t denotes the gross interest rate on a non-contingent domestic bond denominated in domestic currency, $B_t(j)$, whereas r_t^* is the (exogenous) interest rate on a non-contingent foreign bond denominated in foreign currency, $B_{P,t}^*(j)$. The term $\Theta(\cdot)$ is a premium paid by domestic agents on top of the foreign interest rate.[11]

Following Erceg, Henderson, and Levin (2000), each household is a monopolistic supplier of a differentiated labor service. These labor services are bundled by a set of perfectly competitive labor packers that hire labor and combine it into an aggregate labor service unit used as an input in production of domestic intermediate varieties. Cost minimization of labor packers yields the demand for each type of labor as a function of relative wages and aggregate labor demand by firms. There are wage rigidities in the spirit of Calvo (1983). In each period, a household faces a probability $(1 - \phi_L)$ of being able to reoptimize its nominal wage. The households that can reoptimize at time t will maximize the expected discounted future stream of labor income net of the disutility from work, subject to the labor demand constraint. All those that cannot reoptimize at time t set their wages according to a weighted average of past CPI inflation and the inflation target set by the central bank. Once a household has set its wage, it must supply any quantity of labor service demanded at that wage.

Households of the non-Ricardian type consume their disposable wage income each period:

$$P_{C,t}C_t^{NR}(j) = W_t l_t(j) - TAXN_t(j), \qquad j \in [0, \lambda].$$

For simplicity, it is assumed that non-Ricardian households set a wage equivalent to the average wage set by Ricardian households. As a consequence, the supply of labor by non-Ricardian households coincides with the average labor supply of Ricardian households.

The households' consumption bundle is a constant elasticity of substitution (CES) composite of a core consumption bundle, $C_{Z,t}(j)$, food consumption, $C_{A,t}(j)$, and oil consumption, $C_{O,t}(j)$. Core consumption is a CES composite of final domestic goods, $C_{H,t}(j)$, and imported goods, $C_{F,t}(j)$. Food consumption is a similar composite of domestic and imported goods but subject to an exogenous shock $(\zeta_{A,t})$ to capture deviations of food price inflation from core inflation. Households minimize the costs of the different bundles, which yields standard Dixit-Stiglitz type demand functions for the individual components as well as expressions for the headline CPI, food prices, and the core CPI excluding oil and food (given oil prices).

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[11] The premium is a function of the aggregate (private, $B_{P,t}^*$, plus government, $B_{G,t}^*$) net foreign bond position relative to nominal GDP ($BY_t = \varepsilon_t B_t^*/P_{Y,t}Y_t$), i.e. $\Theta_t = \bar{\Theta}\exp[-\varrho(BY_t - \overline{BY}) + \zeta_{\Theta,t}/\bar{\zeta}_\Theta - 1]$, where $\varrho > 0$ to ensure stationarity of net foreign bonds and where $\zeta_{\Theta,t}$ is a shock to the premium (throughout, bars indicate deterministic steady state values).

3.2 Domestic goods

In the domestic goods sector, there is a continuum of firms that produce differentiated varieties of intermediate tradable goods using labor, capital and oil as inputs. They have monopoly power over the varieties they produce and adjust prices infrequently. These firms sell their varieties to competitive assemblers that produce final domestic goods that are sold in the domestic and foreign market. Another set of competitive firms produces the capital goods used in intermediate goods production. All firms in this sector are owned by Ricardian households.

A representative capital goods producer rents capital goods to domestic intermediate goods producing firms. It decides how much capital to accumulate each period, assembling investment goods, I_t, with a CES technology that combines final domestic goods, $I_{H,t}$, and imported goods, $I_{F,t}$. The optimal composition of investment is determined through cost minimization. The firm may adjust investment to produce new capital goods, K_t, in each period but there are convex costs of adjusting investment, $\Phi(\cdot)$, following Christiano, Eichenbaum, and Evans (2005). The firm chooses the level of investment and its stock of capital to maximize the present value to households of expected profits (rental returns on capital net of the cost of investment):

$$\max E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}[Z_{t+i} \exp(\zeta_{K,t+i}) K_{t+i-1} - P_{I,t+i} I_{t+i}],$$

subject to the law of motion of capital

$$K_t = (1-\delta)\exp(\zeta_{K,t}) K_{t-1} + [1 - \Phi(I_t/I_{t-1})]\zeta_{I,t} I_t,$$

where $\Lambda_{t,t+i}$ is the stochastic discount factor for nominal payoffs from the Ricardian household's problem and Z_t is the rental price of capital.[12] The variable $\zeta_{I,t}$ is an investment-specific shock that alters the rate at which investment is transformed into productive capital (see Greenwood, Hercowitz, and Krusell, 2000), while $\zeta_{K,t}$ is an i.i.d. shock to the stock of capital that captures physical destruction of capital due to natural disasters such as an earthquake.

There is a large set of firms that use a CES technology to assemble final domestic goods from domestic intermediate varieties. A quantity $Y_{H,t}$ of those goods is sold domestically and a quantity $Y^*_{H,t}$ is sold abroad. The assemblers demand intermediate goods of variety z_H for domestic sale, $Y_{H,t}(z_H)$, and intermediate goods for foreign sale, $Y^*_{H,t}(z_H)$. Input cost minimization yields the typical Dixit-Stiglitz demand functions for each variety.

Intermediate goods producers decide on the most efficient combination of labor, capital and

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[12] The stochastic discount factor satisfies $\Lambda_{t,t+i} = \beta^i(\zeta_{C,t+i}/\zeta_{C,t})(C^R_t - hC^R_{t-1})/(C^R_{t+i} - hC^R_{t+i-1})(P_{C,t}/P_{C,t+i})$. Further, the function $\Phi(\cdot)$ satisfies $\Phi(1+g_Y) = \Phi'(1+g_Y) = 0$ and $\Phi''(1+g_Y) = \mu_S > 0$, where g_Y is the steady state (balanced) growth rate of the economy.

oil (i.e. to minimize input costs given factor prices). The available technology is as follows:

$$\mathbf{Y}_{H,t}(z_H) = a_{H,t}\left[{\alpha_H}^{1/\omega_H} V_{H,t}(z_H)^{1-1/\omega H} + (1-\alpha_H)^{1/\omega_H} O_{H,t}(z_H)^{1-1/\omega H}\right]^{\frac{\omega_H}{\omega_H - 1}},$$

where $V_{H,t}$ denotes value added produced out of labor and capital, while $O_{H,t}(z_H)$ is the amount of oil used as intermediate input and $a_{H,t}$ represents a stationary productivity shock common to all firms.[13] Value added is generated through a Cobb-Douglas function:

$$V_{H,t}(z_H) = [T_t l_t(z_H)]^{\eta_H}[\exp(\zeta_{K,t}) K_{t-1}(z_H)]^{1-\eta_H},$$

where $l_t(z_H)$ is the amount of labor utilized, T_t is a stochastic trend in labor productivity, and $K_{t-1}(z_H)$ is the amount of capital rented at the beginning of period t.[14]

The intermediate producers have monopoly power and set their prices separately in the domestic market, $P_{H,t}(z_H)$, and the foreign market, $P^*_{H,t}(z_H)$, maximizing profits subject to the corresponding demand constraints. Prices are set in a staggered way, following Calvo (1983). In every period, the probability that a firm receives a signal for optimally adjusting its price for the domestic market is $1 - \phi_{H_D}$, and the probability of optimally adjusting its price for the foreign market is $1 - \phi_{H_F}$. If a firm does not receive a signal, it updates its price according to a weighted average of past changes of aggregate producer prices ($P_{H,t}$ or $P^*_{H,t}$) and steady state domestic or foreign CPI inflation. Let $MC_{H,t}$ denote the marginal cost of producing variety z_H. When a firm can reoptimize its price for the domestic market or foreign market, it solves

$$\max E_t \sum_{i=0}^{\infty} \phi^i_{H_D} \Lambda_{t,t+i}[\Gamma^i_{H_D,t} P_{H,t}(z_H) - MC_{H,t+i}] Y_{H,t+i}(z_H),$$

subject to the final domestic goods producer's demand $Y_{H,t}(z_H) = [P_{H,t}(z_H)/P_{H,t}]^{-\epsilon_H} \mathrm{Y}_{H,t}$. Analogously, when a firm can reoptimize its price for the foreign market, it solves

$$\max E_t \sum_{i=0}^{\infty} \phi^i_{H_F} \Lambda_{t,t+i}[\Gamma^i_{H_F,t} \varepsilon_{t+i} P^*_{H,t}(z_H) - MC_{H,t+i}] Y^*_{H,t+i}(z_H),$$

subject to the demand constraint $Y^*_{H,t}(z_H) = [P^*_{H,t}(z_H)/P^*_{H,t}]^{-\epsilon_H} Y^*_{H,t}$.[15]

3.3 Imported goods

In the imported goods sector, there is a continuum of retail firms that repackage a homogeneous good bought from abroad into differentiated imported varieties through a brand naming tech-

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[13] By market clearing, it holds that $\mathbf{Y}_{H,t}(z_H) = Y_{H,t}(z_H) + Y^*_{H,t}(z_H)$.

[14] The productivity trend evolves according to the process $T_t/T_{t-1} = \zeta_{T,t} = (1+g_Y)^{1-\rho_T} \zeta^{\rho_T}_{T,t-1} \exp(\varepsilon_{T,t})$.

[15] For $i > 1$, the passive price updating rules are $\Gamma^i_{H_D,t} = \Gamma^{i-1}_{H_D,t} \pi^{\chi_{H_D}}_{H,t+i-1} \bar{\pi}^{1-\chi_{H_D}}$ for domestically sold goods and $\Gamma^i_{H_F,t} = \Gamma^{i-1}_{H_F,t} \pi^{*\chi_{H_F}}_{H,t+i-1} \bar{\pi}^{*1-\chi_{H_F}}$ for goods sold abroad, where $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, $\pi^*_{H,t} = P^*_{H,t}/P^*_{H,t-1}$ and $\bar{\pi}^*$ denotes steady state foreign CPI inflation. For $i = 0$, we have $\Gamma^0_{H_D,t} = \Gamma^0_{H_F,t} = 1$.

nology. Those firms have monopoly power in the domestic retailing of their particular variety and set prices infrequently. They sell their varieties to competitive assemblers that produce final imported goods that are bought by households and firms. As in the case of the domestic goods sector, all firms in the imported goods sector are owned by Ricardian households.

There is a large set of firms that use a CES technology to assemble final imported goods, $Y_{F,t}$, from imported varieties. Demand for a particular imported variety, $Y_{F,t}(z_F)$, is determined through minimization of costs, which yields the Dixit-Stiglitz demand functions for each variety.

Imported goods retailers buy a homogeneous good from abroad at the price $P^*_{F,t}$, which is then differentiated into a particular variety and sold domestically to assemblers of final imported goods. It takes one unit of the homogenous foreign good to produce a unit of retail output. Each importing firm has monopoly power, and adjusts the domestic price of its variety in a staggered way, following Calvo (1983). Each period, a firm optimally adjusts its price with probability $1 - \phi_F$. If a firm does not receive a signal, it updates its price according to a weighted average of past changes of aggregate producer prices ($P_{F,t}$) and steady state CPI inflation. When a firm can reoptimize its price, it solves

$$\max E_t \sum_{i=0}^{\infty} \phi_F^i \Lambda_{t,t+i} [\Gamma^i_{F,t} P_{F,t}(z_F) - \varepsilon_{t+i} P^*_{F,t+i}] Y_{F,t+i}(z_F),$$

subject to the final imported goods producer's demand $Y_{F,t}(z_H) = [P_{F,t}(z_F)/P_{F,t}]^{-\epsilon_F} \mathrm{Y}_{F,t}$. [16]

3.4 Commodity goods

We extend the model of Medina and Soto (2007a) by endogenizing commodity production. As in Medina and Soto (2007a), there is a representative firm in the commodity sector (S) that produces a homogeneous commodity good. The entire production is exported. A fraction χ of the assets of that firm is owned by the government and the remaining fraction is owned by foreign investors. The revenue generated in the commodity sector is shared accordingly, but the government levies taxes on the profits that accrue to foreign investors.

The firm in sector S uses capital specific to that sector, $K_{S,t}$, to produce commodity goods, $Y_{S,t}$. Production evolves along the balanced growth path of the economy, but we admit transitory deviations from that growth path due to sectoral technology shocks, $a_{S,t}$. Specifically, commodity production satisfies

$$Y_{S,t} = a_{S,t} F^S(T_t, K_{S,t-1}). \tag{1}$$

The function $F^S(\cdot)$ is homogeneous of degree one in its arguments and has diminishing returns to capital additions. While we focus on capital-intensive commodity production and abstract from other inputs such as labor and land for simplicity, the shock $A_{S,t}$ can be interpreted to capture

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[16] The passive price updating rule is $\Gamma^i_{F,t} = \Gamma^{i-1}_{F,t} \pi^{\chi_F}_{F,t+i-1} \bar{\pi}^{1-\chi_F}$ for $i > 1$ and $\Gamma^0_{F,t} = 1$ for $i = 1$.

any variations in such additional inputs.[17] We also allow for a fixed transfer to households to capture eventual labor remunerations or other fixed costs (see below).

Let $P^*_{S,t}$ denote the international price of the commodity good and let $P_{S,t} = \varepsilon_t P^*_{S,t}$ be its domestic price, which the firm takes as given. Gross profits of the firm are given by

$$\Pi_{S,t} = P_{S,t} Y_{S,t} - P_{C,t} T_t \kappa_S,$$

where $P_{C,t} T_t \kappa_S$ is a fixed cost of production that grows at the same rate as nominal output. We assume that this fixed cost is a lump-sum transfer to Ricardian households. The cash flow of the firm is $CF_{S,t} = \Pi_{S,t} - P_{I_S,t} I_{S,t}$, where $P_{I_S,t} I_{S,t}$ is the firm's investment. The objective of the firm is to maximize the present real value of its expected cash flow:

$$\max E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}(S) \frac{CF_{S,t+i}}{P_{C,t+i}},$$

where $\Lambda_{t,t+i}(S)$ denotes the stochastic discount factor relevant to the firm. This discount factor is taken to be identical to the one of the households, i.e. $\Lambda_{t,t+i}(S) = \Lambda_{t,t+i}$.[18]

Notice that the definition of profits does not have any importance for firm's optimality; however, it is a key determinant of the tax base, which is needed to characterize the Chilean fiscal rule (see next section for details). The stock of capital in sector S is augmented through investment projects, $X_{S,t}$. Following Uribe and Yue (2006), there are adjustment costs in investment and time to build in the installation of capital à la Kydland and Prescott (1982). In particular, the firm can start new investment projects in each period but at a cost that is convex: the larger the change in investment the larger the implied cost. In addition, new investment projects take $n \geq 1$ periods to mature. Collecting these assumptions results in the following law of motion of capital:

$$K_{S,t} = (1 - \delta_S) K_{S,t-1} + [1 - \Phi_S(X_{S,t-n+1}/X_{S,t-n})] X_{S,t-n+1}. \tag{2}$$

The function $\Phi_S(\cdot)$ is analogous to the Christiano, Eichenbaum, and Evans (2005) style flow adjustment cost function from the law of motion of capital used in the domestic goods sector, and satisfies $\Phi_S(1 + g_Y) = \Phi'_S(1 + g_Y) = 0$ and $\Phi'_S(1 + g_Y) = \mu_{I_S} > 0$. A similar specification of the law of motion of capital is employed in Uribe and Yue (2006). The effective flow of

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[17] For instance, we could take the Cobb-Douglas production function $Y_{S,t} = \tilde{F}^S(T_t l_S, T_t \digamma_{S,t}, K_{S,t-1}) = (T_t l_S)^{\eta_{l_S}} (T_t \digamma_{S,t})^{\eta_{\digamma_S}} K_{S,t-1}^{1-\eta_{l_S}-\eta_{\digamma_S}}$, where l_S would be a fixed input of labor and $\digamma_{S,t}$ would capture variations in other factors such as the mineral content of land. Defining $\eta_S = \eta_{l_S} + \eta_{\digamma_S}$, we obtain $Y_{S,t} = a_{S,t} T_t^{\eta_S} K_{S,t-1}^{1-\eta_S}$, which is a representation of (1) with $a_{S,t} = l_S^{\eta_{l_S}} \digamma_{S,t}^{\eta_{\digamma_S}}$ and with $F^S(T_t, K_{S,t-1}) = T_t^{\eta_S} K_{S,t-1}^{1-\eta_S}$. Under those assumptions, total factor productivity $a_{S,t}$ is a function of labor and other factors subsumed in $\digamma_{S,t}$.

[18] The relation $\Lambda_{t,t+i}(S) = \Lambda_{t,t+i}$ holds, as we assume, if the government has a stochastic discount factor equivalent to the one of the households and if foreign investors have access to domestic currency bonds.

investment in period t is given by

$$I_{S,t} = \sum_{j=0}^{n-1} \varphi_j X_{S,t-j}, \tag{3}$$

where φ_j denotes the fraction of projects initiated in period $t-j$ that is financed in period t, with $\sum_{j=0}^{n-1} \varphi_j = 1$. We will assume that $\varphi_0 = \varphi_1 = \ldots = \varphi_{n-1}$, as in Kydland and Prescott (1982) i.e. the cost of a project is spread equally over the horizon of its installation. In the extreme when $n = 1$, we obtain the familiar law of motion $K_{S,t} = (1-\delta_S)K_{S,t-1} + [1-\Phi_S(I_{S,t}/I_{S,t-1})]I_{S,t}$.

The firm's first-order optimality conditions are as follows:

$$\begin{aligned}
K_{S,t} \quad &: \quad \frac{Q_{S,t}}{P_{C,t}} = E_t \left\{ \Lambda_{t,t+1} \left[\frac{Q_{S,t+1}}{P_{C,t+1}}(1-\delta_S) + \frac{P_{S,t+1}A_{S,t+1}F^S_{K_S}(T_{t+1}, K_{S,t})}{P_{C,t+1}} \right] \right\}, \\
X_{S,t} \quad &: \quad \varphi_0 \frac{P_{I_S,t}}{P_{C,t}} + \varphi_1 E_t \left\{ \Lambda_{t,t+1} \frac{P_{I_S,t+1}}{P_{C,t+1}} \right\} + \cdots + \varphi_{n-1} E_t \left\{ \Lambda_{t,t+n-1} \frac{P_{I_S,t+n-1}}{P_{C,t+n-1}} \right\} \\
&\qquad = E_t \left\{ \begin{array}{c} \Lambda_{t,t+n-1} \frac{Q_{S,t+n-1}}{P_{C,t+n-1}} [1 - \Phi_S(X_{S,t}/X_{S,t-1}) - \Phi'_S(X_{S,t}/X_{S,t-1})X_{S,t}/X_{S,t-1}] \\ + \Lambda_{t,t+n} \frac{Q_{S,t+n}}{P_{C,t+n}} \Phi'_S(X_{S,t+1}/X_{S,t})(X_{S,t+1}/X_{S,t})^2 \end{array} \right\},
\end{aligned}$$

where $F^S_{K_S}(\cdot)$ is the derivative of the production function in (1) with respect to capital. These two conditions jointly determine the evolution of investment projects and the mark-to-market value of capital, $Q_{S,t}$, in sector S. The law of motion (2) determines the evolution of the stock of capital and (3) determines the effective flow of investment in this sector.

The investment good that is required to build the stock of capital in sector S is a CES bundle of final domestic goods, $I_{H,t}(S)$, and imported goods, $I_{F,t}(S)$:

$$I_{S,t} = \left[\gamma_{I_S}^{1/\eta_{I_S}} I_{H,t}(S)^{1-1/\eta_{I_S}} + (1-\gamma_{I_S})^{1/\eta_{I_S}} I_{F,t}(S)^{1-1/\eta_{I_S}} \right]^{\frac{\eta_{I_S}}{\eta_{I_S}-1}}. \tag{4}$$

The optimal composition of investment is determined through cost minimization. In each period, given the effective flow of investment, the firm minimizes $P_{I_S,t}I_{S,t} = P_{H,t}I_{H,t}(S) + P_{F,t}I_{F,t}(S)$ subject to (4), which yields the following demands for investment inputs originating in sector S:

$$I_{H,t}(S) = \gamma_{I_S}(P_{H,t}/P_{I_S,t})^{-\eta_{I_S}} I_{S,t}, \qquad I_{F,t}(S) = (1-\gamma_{I_S})(P_{F,t}/P_{I_S,t})^{-\eta_{I_S}} I_{S,t}.$$

3.5 Fiscal and monetary policy

A share χ of the cash flow that is generated in sector S goes directly to the government, and the government also levies taxes at a fixed rate τ_S on the profits—net of depreciation—that accrue to foreign investors. The budget constraint of the government is therefore as follows:

$$P_{G,t}G_t + \frac{\varepsilon_t B^*_{G,t}}{r^*_t \Theta_t} = \varepsilon_t B^*_{G,t-1} + \tau_t P_{Y,t} Y_t + \chi CF_{S,t} + \tau_S(1-\chi)(\Pi_{S,t} - \delta_S Q_{S,t} K_{S,t-1}),$$

where $P_{G,t}G_t$ denotes nominal government consumption expenditure, $B^*_{G,t}$ is the government net foreign asset position, and τ_t are lump-sum taxes from households net of transfers (as a share of nominal GDP, $P_{Y,t}Y_t$). Note that the government net asset position is assumed to be completely denominated in foreign currency, as in Medina and Soto (2007a). In addition, government consumption is characterized by complete home bias, i.e. $G_t = G_{H,t}$ and $P_{G,t} = P_{H,t}$.

Government expenditure follows a structural balance fiscal rule analogous to the one described in Medina and Soto (2007a):

$$\frac{P_{G,t}G_t}{P_{Y,t}Y_t} = \left[\begin{array}{c} \left(1 - \frac{1}{r^*_{t-1}\Theta_{t-1}}\right)\frac{\varepsilon_t B^*_{G,t-1}}{P_{Y,t}Y_t} + \tau_t \frac{P_{Y,t}\bar{Y}}{P_{Y,t}Y_t} + \chi \frac{CF_{S,t}}{P_{Y,t}Y_t} \\ +\tau_S(1-\chi)\frac{\Pi_{S,t} - \delta_S Q_{S,t} K_{S,t-1}}{P_{Y,t}Y_t} - \frac{VC_t}{P_{Y,t}Y_t} - \bar{s}_B \end{array}\right] \frac{P_{G,t}\zeta_{G,t}T_t}{P_{Y,t}Y_t},$$

where $VC_t = [\chi + \tau_S(1-\chi)]\varepsilon_t(P^*_t - \tilde{P}^*_t)Y_{S,t}$ is the cyclical adjustment of the rule that depends crucially on the difference between the effective commodity price, P^*_t, and the long-run reference price, $\tilde{P}^*_t$, which is calculated as the forecast of the effective commodity price averaged over a 10 years horizon. In addition, $\bar{Y}$ stands for potential real GDP, which for simplicity is taken to be equal to steady state output, and the parameter $\bar{s}_B$ is the structural balance target. The variable $\zeta_{G,t}$ is a shock capturing deviations of government expenditure from the fiscal rule.

Monetary policy is conducted through a simple Taylor-type feedback rule for the nominal interest rate, which is a slightly modified version of the one presented in Medina and Soto (2007a). In particular, while the latter assume that the central bank responds entirely to deviations of core CPI inflation from target and of output growth from potential growth, we allow for a partial response to headline CPI inflation to capture possible concerns by the central bank on oil and food price inflation. Hence, the monetary policy rule is specified as follows:

$$\frac{r_t}{\bar{r}} = \left(\frac{r_{t-1}}{\bar{r}}\right)^{\psi_r} \left[\left(\frac{\pi_{Z,t}}{\bar{\pi}}\right)^{\psi_\pi \psi_{\pi_Z}} \left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_\pi(1-\psi_{\pi_Z})} \left(\frac{Y_t/Y_{t-1}}{T_t/T_{t-1}}\right)^{\psi_Y}\right]^{1-\psi_r} \exp(\zeta_{m,t}),$$

where $\pi_{Z,t}$ and π_t are core and headline CPI inflation, respectively, Y_t is real GDP, and $\zeta_{m,t}$ is an i.i.d. shock that captures deviations of the interest rate from the monetary policy rule.

3.6 Rest of the world

Foreign agents demand the commodity good and the final domestic good. They supply oil and the homogeneous good that is bought by importing firms. Foreign demand for the commodity good is assumed to be completely elastic at its international price, $P^*_{S,t}$. Likewise, foreign supply of oil is assumed to be completely elastic at any given price, $P^*_{O,t}$. The real exchange rate is defined as the domestic currency price of a foreign price index, $\varepsilon_t P^*_t$, relative to the domestic CPI. The domestic economy is assumed to be small relative to the rest of the world. As a consequence, the price of the homogeneous foreign good, $P^*_{F,t}$, coincides with the foreign price index.[19] Foreign demand for the final domestic good depends on its relative price abroad, $P^*_{H,t}/P^*_t$, and foreign aggregate demand, Y^*_t , according to the demand function $Y^*_{H,t} = \zeta^*(P^*_{H,t}/P^*_t)^{-\eta^*}Y^*_t$.

3.7 Aggregate equilibrium

The market clearing condition for each variety of domestic goods is

$$\mathbf{Y}_{H,t}(z_H) = [P_{H,t}(z_H)/P_{H,t}]^{-\epsilon_H}Y_{H,t} + [P^*_{H,t}(z_H)/P^*_{H,t}]^{-\epsilon_H}Y^*_{H,t},$$

where $Y_{H,t} = C_{H,t} + I_{H,t} + I_{H,t}(S) + G_{H,t}$. In the labor market, labor demand by intermediate goods producers equals labor supply: $\int_0^1 l_t(z_H)dz_H = l_t$, where the aggregate labor service unit is given by

$$l_t = \left[\int_0^1 l_t(j)^{1-1/\varepsilon_L}dj\right]^{\frac{\varepsilon_L-1}{\varepsilon_L}}.$$

Nominal GDP satisfies $P_{Y,t}Y_t = P_{C,t}C_t + P_{I,t}I_t + P_{I_S,t}I_{S,t} + P_{G,t}G_t + P_{X,t}X_t - P_{M,t}M_t$, where $P_{X,t}X_t = \varepsilon_t(P^*_{H,t}Y^*_{H,t} + P^*_{S,t}Y_{S,t})$ and $P_{M,t}M_t = \varepsilon_t[P^*_{F,t}Y_{F,t} + P^*_{O,t}(C_{O,t} + O_{H,t})]$ are nominal exports and imports, respectively, with $Y_{F,t} = C_{F,t} + I_{F,t} + I_{F,t}(S)$. Real GDP is defined as $Y_t = C_t + I_t + I_{S,t} + G_t + X_t - M_t$. Substituting out aggregate profits in the budget constraint of the households and combining the latter with the budget constraint of the government yields the following expression for the evolution of aggregate net foreign bonds:

$$\frac{\varepsilon_t B^*_t}{r^*_t\Theta_t} = P_{X,t}X_t - P_{M,t}M_t + \varepsilon_t B^*_{t-1} - (1-\chi)CF_{S,t} + \tau_S(1-\chi)(\Pi_{S,t} - \delta_S Q_{S,t}K_{S,t-1}).$$

The terms on the right-hand side are net exports, net interest receipts minus the cash flow from the commodity sector that accrues to foreign investors, and transfers from foreigners due to taxes on profits net of the mark-to-market value of capital depreciation in the commodity sector. Finally, the current account balance is equivalent to the quarter-on-quarter change in the international investment position of the country (relative to nominal GDP):

$$CAY_t = \frac{1}{P_{Y,t}Y_t}\left[\frac{\varepsilon_t B^*_t}{r^*_t\Theta_t} - \frac{\varepsilon_t B^*_{t-1}}{r^*_{t-1}\Theta_{t-1}}\right] - (1-\chi)\frac{Q_{S,t}(K_{S,t} - K_{S,t-1})}{P_{Y,t}Y_t}.$$

Abstracting from the exogenous process of the commodity price, which is described below, there are fourteen shocks in the model: preferences ($\zeta_{C,t}$), neutral technology ($a_{H,t}$), productivity growth ($\zeta_{T,t}$), investment-specific technology ($\zeta_{I,t}$), capital destruction ($\zeta_{K,t}$), commodity-specific technology ($a_{S,t}$), fiscal policy ($\zeta_{G,t}$), monetary policy ($\zeta_{m,t}$), stationary foreign demand ($y^*_t = Y^*_t/T_t$), foreign inflation ($\pi^*_t = P^*_t/P^*_{t-1}$), food prices ($\zeta_{A,t}$), the real oil price ($p^*_{O,t} = P^*_{O,t}/P^*_t$), the foreign interest rate (r^*_t), and the country premium ($\zeta_{\Theta,t}$), All of those

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[19] We drop a shock to the ratio $P^*_{F,t}/P^*_t$ that is present in the model of Medina and Soto (2007a) because this shock was not identified in our estimations.

shocks are assumed to follow autoregressive processes of order one in logs, AR(1), except the monetary policy shock and the capital destruction shock, which are i.i.d. processes in levels.

3.8 Imperfect information and learning about commodity price shocks

Let $p^*_{S,t} = P^*_{S,t}/P^*_t$ be the real international commodity price and let $\breve{p}^*_{S,t}$ denote the log deviation of $p^*_{S,t}$ from its steady state value, i.e. $\breve{p}^*_{S,t} = \log(p^*_{S,t}/\bar{p}^*_S)$. We introduce imperfect information on the persistence of commodity price shocks in the model, assuming that $\breve{p}^*_{S,t}$ has two different components: a transitory component, a_t, and a persistent component, b_t. The dynamics of the persistent component are interpreted to capture changes in economic "fundamentals", whereas the transitory component captures any remaining commodity price changes or "noise". Hence, the real commodity price is assumed to evolve according to the following statistical process:

$$\breve{p}^*_{S,t} = a_t + b_t, \qquad b_t = \rho b_{t-1} + u_t, \qquad a_t \sim N(0, \sigma_a^2), \qquad u_t \sim N(0, \sigma_u^2), \tag{5}$$

for $t = 1, \ldots, T$. Throughout, we assume that $\rho \in [0, 1)$. For $\rho = 1$, this model would be identical to the standard local level model (see Harvey, Koopman, and Shephard, 2004). However, we impose the additional constraint that $\rho < 1$ to avoid non-stationary dynamics of the parts of the variables that are affected by commodity price changes.

We assume that the effective real commodity price is fully observable to all agents but its individual components are not. Instead, following Erceg and Levin (2003), the agents infer the unobserved components in an optimal linear way using the Kalman filter, according to which

$$\hat{b}_t = \rho \hat{b}_{t-1} + K_t \rho^{-1} (\breve{p}^*_{S,t} - \rho \hat{b}_{t-1}), \qquad \hat{a}_t = \breve{p}^*_{S,t} - \hat{b}_t.$$

The variables $\hat{a}_t$ and $\hat{b}_t$ are the optimal linear inferences of a_t and b_t with the information available at time t, i.e. $\hat{a}_t = E[a_t | p^*_{S,1}, p^*_{S,2}, p^*_{S,3}, \ldots, p^*_{S,t}]$ and $\hat{b}_t = E[b_t | p^*_{S,1}, p^*_{S,2}, p^*_{S,3}, \ldots, p^*_{S,t}]$. Note that the current inference $\hat{b}_t$ and the forecasts $\hat{b}_{t+h} = \rho^h \hat{b}_t$ for horizon $h = 1, 2, 3, \ldots$ are adjusted at a rate $K_t \rho^{-1}$ with the prediction errors from the previous period's forecast, i.e. $\breve{p}^*_{S,t} - \rho \hat{b}_{t-1}$, where K_t is the Kalman gain parameter. For $t \to \infty$, the latter will be constant, i.e. $K_t \to K$. This fact will be exploited in the estimation of the model.

4 Estimation

The model is solved by a log-linear approximation around the deterministic steady state.[20] We then estimate the model by Bayesian methods, as described in An and Schorfheide (2007). We now briefly describe the estimation strategy, and how we incorporate imperfect information in the estimation, before discussing the details of the procedure (data, priors, etc.).

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[20] A detailed description of the solution under learning is provided in Appendix A.

Let x_t denote the model variables and let Y_t be a vector of observed variables of the econometrician. The log-linear solution of the model has the following state-space representation:

$$Y_t = Hx_t + v_t, \qquad x_t = D\hat{a}_t + E\hat{b}_t + Fx_{t-1} + G\varepsilon_t, \qquad v_t \sim N(0, \Sigma_v), \qquad \varepsilon_t \sim N(0, \Sigma_\varepsilon),$$

for $t = 1, \ldots, T$. The variables $\hat{a}_t$ and $\hat{b}_t$ are defined as above. The vector ε_t represents the innovations to the exogenous processes listed above and v_t is a vector of observation errors. Further, let θ collect the structural parameters of the model, let $P(\theta)$ be a prior density on those parameters, and let $L(Y^T|\theta)$ denote the likelihood function for the observed data, $Y^T = [Y_1, \ldots, Y_T]'$. The Kalman filter is applied to evaluate $L(Y^T|\theta)$ and the posterior density, $P(\theta|Y^T) = L(Y^T|\theta)$ $\times P(\theta)/\int L(Y^T|\theta)P(\theta)d\theta$, is evaluated with the Metropolis-Hastings algorithm.[21]

4.1 Commodity price decomposition

We treat the inferred persistent component of the real copper price (which we use as a proxy for the commodity price in the model) computed by the Kalman filter as an econometrically observable variable together with the actual real copper price in the estimation of the model. To do this, we need to assign values to the parameters ρ, σ_u and σ_a from (5), and run the Kalman filter prior to the estimation of the model. We assume that the Kalman gain parameter associated with (5) is constant over the estimation sample, i.e. $K_t = K$, as is standard in the literature; see Erceg and Levin (2003) and Céspedes and Soto (2007).[22] We calibrate the value of K to 0.15 to roughly match a learning horizon of around four to five years as indicated by the actual revisions of the forecasts from Figures 4 and 5. Given K and ρ we obtain the signal-to-noise ratio (i.e. σ_u/σ_a) from the Kalman filter recursions.[23] The parameters ρ and σ_u are thus estimated by constrained maximum likelihood, and an estimate for σ_a is obtained residually from the signal-to-noise ratio implied by those estimates.[24]

The estimation in this preliminary step is based on quarterly data from 1960Q1-2012Q4 for the real price of copper, i.e. the nominal price of refined copper quoted at the London Metal Exchange deflated by an index of external prices. The latter is constructed from an external price index published by the Central Bank of Chile ("IPE") from 1986Q1 onwards, which is extrapolated backwards using the quarterly growth rate of the seasonally adjusted US CPI for all urban consumers and all items. As shown in Table 1, the maximum likelihood estimates imply an AR(1) parameter of the persistent component of about 0.98, a quarterly standard deviation

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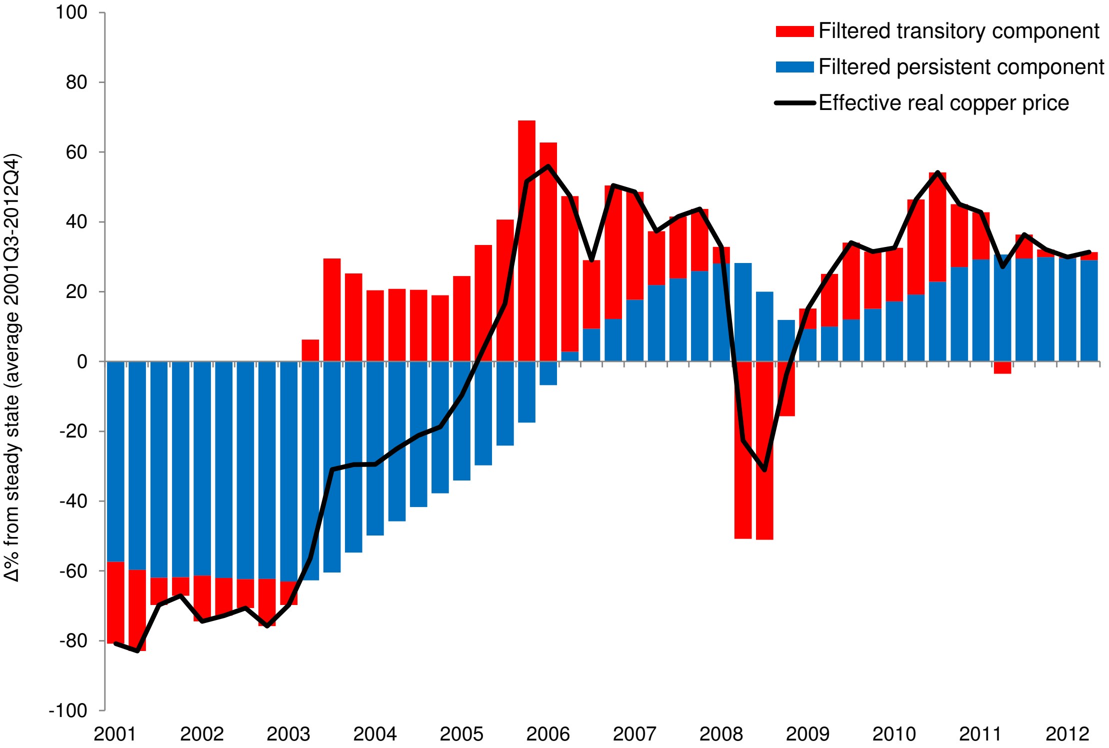
[21] We use version 4.3.3 of the Dynare toolbox for MATLAB in the computations.

[22] The assumption of a constant Kalman gain seems justified, as the sample used for the prior evaluation of the Kalman filter starts much earlier than the sample used for the estimation of the remaining parameters.

[23] The derivation of the ratio σ_u/σ_a is provided in Appendix B.

[24] We have also explored an alternative way of putting a value on K, i.e. estimating it based on (5) and data for the real spot price and the long-run reference price of copper from Figure 5. The reference price has the advantage that it is a forecast in real terms and therefore closely related to (5), but since it is only reported once per year the results of this exercise should be interpreted with caution. The estimates imply a signal-to-noise ratio (σ_u/σ_a) of around one fourth, similar as in the baseline case where this ratio is around one fifth, which yields a value for K of about 0.2 which is not significantly different from the baseline value of 0.15.

Figure 6: Historical decomposition of the real price of copper into a persistent component and a transitory component computed from the Kalman filter.



Notes: This figure shows the estimated decomposition of the quarterly deviations of the real price of copper from its sample average (2001Q3-2012Q4) into a persistent component and a transitory component computed from the Kalman filter. The results are based on constrained maximum likelihood estimates as described in the text.

of innovations to the persistent component of approximately 4%, and a standard deviation of innovations to the transitory component of about 20%. The implied signal-to-noise ratio is around one fifth. Those pre-estimated parameters are included as calibrated parameters in the Bayesian estimation of the remaining parameters of the model afterwards.

Figure 6 shows the resulting Kalman filter decomposition of the real copper price over our sample period 2001Q3-2012Q4, in deviation from its average value over that period, into a persistent component and a transitory one. The persistent component starts out in the negative region, reflecting the slump of commodity prices during the 1980s and 1990s. The increase in the copper price towards the mid-2000s is partially assigned to transitory shocks, while the contribution of the persistent component was still negative at that time. However, the persistent component gradually increases over time, as prices maintained their high levels during the mid-2000s, such that the price increases are gradually inferred to be more persistent, where the transitory component captures most of the effect of the global financial crisis.

4.2 Observed variables

We use quarterly data for Chile covering the period 2001Q3-2012Q4 for the Bayesian estimation of the model.[25] The observed variables include real GDP (Y_t), real copper mining output as a proxy of commodity production ($Y_{S,t}$), private non-durable consumption (C_t), total investment ($I_t + I_{S,t}$), government consumption (G_t), consumer price inflation ($\pi_{C,t}$), the inflation rate of a price index excluding food and energy prices computed by the Central Bank ("IPCSAE") as a proxy for core inflation ($\pi_{Z,t}$), the short-term central bank target rate (r_t), the real exchange rate (rer_t), the current account balance expressed as a ratio of nominal GDP (CAY_t), an indicator of export-weighted real GDP of Chile's main trading partners constructed by the Central Bank as a proxy for foreign demand ($Y_{F,t}$), the inflation rate of the external price index constructed by the Central Bank as a proxy for foreign inflation (π_t^*), the London Interbank Offered Rate as a proxy for the foreign interest rate (r_t^*), the J.P. Morgan Emerging Market Bond Index (EMBI) spread for Chile as a proxy for the country premium (Θ_t), the price of West Texas Intermediate crude oil in dollars per barrel deflated by the external price index ($P_{O,t}^*/P_t^*$), and the London Metals Exchange price of refined copper in dollars per metric pound deflated by the external price index ($P_{S,t}^*/P_t^*$).[26] As discussed above, we also treat the inferred persistent component of the real copper price computed by the Kalman filter ($\hat{b}_t$) as an observed variable. Finally, we include an indicator variable for $z_{K,t}$ that takes the value of -3 in the first quarter of 2010 and 0 otherwise. The latter reflects official government estimates of the economic effects of the large earthquake that occurred off the coast of central Chile on February 27, 2010, which destroyed around 3 percent of the economy's capital stock.[27]

A number of additional transformations are made. In the case of GDP, private consumption, investment, and government consumption, we take the first difference of their natural logarithms. The inflation rates are the log first differences of the underlying indices. Copper output, foreign output, the real exchange rate, and the oil and copper prices are transformed into natural logarithms. The interest rate series are measured as logs of gross quarterly effective rates. We stationarize copper output and foreign output by fitting linear trends over the sample period by least squares, while the remaining variables are demeaned using their average sample values.

4.3 Calibrated parameters

The calibrated parameters include some steady state values and parameters that are chosen to match sample averages or long-run ratios for the Chilean economy, as well as a number of parameters that are not identified or only weakly identified. In addition, we also fix the

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[25] While the Central Bank of Chile switched from partial to full inflation targeting under a flexible exchange rate already at the end of 1999, it has used an overnight interest rate as policy instrument since July 2001, which we use as the starting point of our sample.

[26] The data is publicly available from the Central Bank of Chile's SIETE database, see `http://si3.bcentral.cl/Siete/secure/cuadros/home.aspx`.

[27] The 2010 earthquake is assumed to not have affected the capital stock in the mining sector, as mining activity is concentrated in northern Chile while this earthquake mainly impacted in south-central Chile.

Table 1: Calibrated parameters.

Parameter	Value	Description
Steady state values		
g_Y	2.5%	balanced growth path (net rate, annual basis)
$\bar{\pi}$	3%	st. state inflation target (net rate, annual basis)
$\bar{\pi}^*$	3%	st. state foreign inflation rate (net rate, annual basis)
$\bar{r}^*$	4.5%	st. state foreign interest rate (net rate, annual basis)
$\bar{\Theta}$	1.3%	st. state country premium (net rate, annual basis)
$\bar{s}_B$	0%	st. state structural fiscal balance target
$\bar{g}$	0.055	st. state government consumption (stationary level)
$\bar{a}_S$	0.081	st. state productivity in commodity sector
$\bar{y}^*$	1	st. state foreign demand (stationary level)
$\bar{p}_O$	1	st. state real international price of oil (dom. currency)
$\bar{p}_S$	1	st. state real international commodity price (dom. currency)
Non-Ricardian households		
λ	0.5	share of non-Ricardian households
Ricardian households		
β	0.999	subjective discount factor (quarterly basis)
σ_L	1	inverse Frisch elasticity of labor supply
ψ	15.622	disutility of labor parameter
α_a	0.19	share of food consumption in total consumption
α_o	0.03	share of oil consumption in total consumption
α_c	0.78	share of core consumption in total consumption
γ_c	0.74	share of domestic goods in core and food consumption
Non-commodity sectors		
δ	0.01	depreciation rate (quarterly basis), sector H
α_H	0.99	share of non-oil inputs in production, sector H
η_H	0.66	share of labor in Cobb-Douglas value added, sector H
ϵ_L	11	elast. of substitution among labor varieties, sector H
ϵ_H	11	elast. of substitution among domestic varieties, sector H
ϵ_F	11	elast. of substitution among imported varieties, sector F
γ_I	0.64	share of domestic goods in investment, sector H
Commodity sector		
χ	0.31	government ownership of assets, sector S
τ_S	0.35	tax rate on foreign profits, sector S
κ_S	0.009	fixed cost of production, sector S
$1-\eta_S$	0.314	capital elasticity of production, sector S
δ_S	0.032	depreciation rate (quarterly basis), sector S
γ_{IS}	0.59	share of domestic goods in investment, sector S
n	6	periods of time to build (quarters), sector S
ϕ_j	0.167	financing profile of investment projects, sector S
Foreign economy		
ζ^*	0.101	import share of foreign economy
Exogenous processes		
ρ_{y^*}, σ_{y^*}	0.866, 0.0062	AR(1) coef. & innov. s.d., foreign demand shock
ρ_{i^*}, σ_{i^*}	0.961, 0.0011	AR(1) coef. & innov. s.d., foreign interest rate shock
$\rho_{\pi^*}, \sigma_{\pi^*}$	0.366, 0.0272	AR(1) coef. & innov. s.d., foreign inflation shock
$\rho_{p_O^*}, \sigma_{p_O^*}$	0.893, 0.1393	AR(1) coef. & innov. s.d., international oil price shock
ρ, σ_u	0.979, 0.0375	AR(1) coef. & innov. s.d., persistent commodity price shock
σ_a	0.2032	innov. s.d., transitory commodity price shock
σ_{z_k}	0.0044	innov. s.d., capital destruction shock

parameters of the observable exogenous processes prior to the actual estimation of the model. Table 1 presents the list of calibrated parameters.

We assume a steady state labor productivity growth rate (g_Y) of 2.5% on an annual basis, which is consistent with an observed average growth rate of real GDP of approximately 4.5% over the sample period, given an average labor force growth of around 2%. The steady state inflation target is set to the Central Bank's CPI inflation target of 3%. The average foreign inflation rate is set to the same value, so that the nominal exchange rate exhibits no trend appreciation or depreciation. The household's subjective discount factor (β) is approximately equal to 0.999 to match a steady state real interest rate of 2.8%, in line with existing estimates of the neutral real interest rate for Chile, see Fuentes and Gredig (2008). The steady state nominal foreign interest rate is set to 4.5%, also in line with Fuentes and Gredig (2008). The average country premium (Θ) consistent with these choices is around 130 basis points. The fiscal balance target ratio ($\bar{s}_B$) is set to 0, consistent with the fact that the fiscal rule started out with a structural fiscal surplus of 1% of GDP in 2001, whereas the Chilean government has later targeted a deficit of 1%. The steady state values of a number of exogenous variables (foreign output and the relative international prices of oil and copper) are normalized to one. The value of steady state government consumption is chosen to match a nominal government consumption-to-GDP ratio of 11%, and the steady state value of productivity in the commodity sector is chosen to match an average share of mining-based GDP in total GDP of around 13% over the sample period. The foreign import share (ζ^*) is chosen to match a current account-to-GDP ratio of about -1.8%, as in Medina and Soto (2007a) and Medina, Munro, and Soto (2008).

Further, the disutility of labor parameter (ψ) is chosen to normalize the steady state real wage conveniently to one. The share of non-Ricardian households is calibrated to 0.5, following Medina and Soto (2007a) and Céspedes, Fornero, and Galí (2011). The shares of food and oil in the household's consumption basket are chosen to match their weights in the Chilean CPI basket, i.e. 19% and 3%, respectively, such that the weight of core consumption is 78%. The share of domestic goods in core and food consumption is set to 74%, to match an average value of imported consumption goods relative to total goods consumption of 26% over the sample period. The labor supply elasticity (σ_L) is set to one and the quarterly depreciation rate of capital in the domestic goods sector (δ) is set to 1%, in line with the literature (see, for instance, Adolfson, Laséen, Lindé, and Villani, 2008). Following Medina and Soto (2007a), the labor share in the Cobb-Douglas production function of value added in the domestic goods sector (η_H) is set to 0.66, the share of non-oil inputs in production (α_H) is set to 0.99, and the elasticities of substitution among labor varieties, domestic and imported varieties (ε_L, ε_H and ε_F) are set to 11. The latter imply steady state wage and price markups of 10%. The home bias in investment in the domestic goods sector (i.e. the share of home goods in investment, γ_I) is set to 54% to match the average share of construction in total investment over the sample period, as the remaining part of investment (machinery and equipment) is mainly imported.

Turning to the commodity sector, we calibrate the share of government ownership in this sector (χ) to 0.31, consistent with the average share of production of the state-owned copper mining company (Codelco) relative to total copper production since 2001. The tax rate on foreign profits (τ_S) is set to 0.35, which is the flat rate tax on foreign companies in Chile. The fixed costs in production parameter (κ_S) is chosen to match a labor share in total value added of about 14%, according to recent data. The elasticity of production with respect to capital ($1-\eta_S$) is set to 0.31, in order to obtain a share of physical capital to quarterly output in the commodity sector of 12. This number is in line with available data on the value of financial assets over total sales of Codelco. The quarterly depreciation rate of capital in the commodity sector (δ) is approximately 3%, chosen to match an average investment-to-output ratio in Chile's mining sector of about 4% from 2001-2012. The home bias in investment in the commodity sector is set to 0.59, consistent with available data on the share of construction in total investment in the mining sector. The horizon of time to build is set to $n = 6$ quarters, consistent with the average duration of investment projects by private mining companies according to data from a regular survey of the Chilean Corporation of Capital Goods (CBC). Following Kydland and Prescott (1982), the financing profile of projects (ϕ_j) is set to $1/n$.

We also fix the parameters of the observable exogenous processes. In addition to the parameters for the process of the real international commodity price discussed above, this concerns the processes for foreign demand, the foreign interest rate, foreign inflation, the real international oil price, and the capital destruction shock. The parameters of those processes are estimated by constrained maximum likelihood, and the results are also documented in Table 1.

Finally, we use observation errors on all observed endogenous variables except the domestic nominal interest rate. We calibrate the variance of those errors to 10% of the variance of the respective variable. Given this value, the measurement errors capture some of the high-frequency noise in the data, while the model explains approximately 90% of the variance of the data.

4.4 Prior distributions and posterior estimates

We specify independent univariate prior distributions for the estimated structural parameters as documented in Table 2. The types of the priors are chosen according to the domain on which the individual parameters are defined, while the means and standard deviations of the priors are selected according to our beliefs on plausible regions for the parameters.

For the consumption habit parameter (h), we select a beta prior with mean 0.7 and standard deviation 0.1, implying some positive degree of habits. For the elasticity of the country premium with respect to the economy's net foreign bond position (ϱ), we use an inverse gamma prior with mean 0.01 and an infinite standard deviation. For the elasticities of substitution between domestic and foreign goods (η_C, η_I, η_{I_S}, and η^*) and the degrees of substitution for oil in the domestic goods production technology and the consumption basket (ω_H and ω_C), we set inverse

Table 2: Prior and posterior distributions.

Parameter	Description	Prior			Posterior				
		dist.	mean	s.d.	mean	mode	s.d.	5%	95%
Households									
h	Habit formation	B	0.7	0.1	0.893	0.862	0.0284	0.8468	0.9389
ω_C	EoS oil and core cons.	IG	1	Inf	0.389	0.385	0.1091	0.2237	0.5512
η_C	EoS H and F core cons.	IG	1	Inf	1.477	1.575	0.7059	0.3414	2.4797
Wages									
ϕ_L	Calvo prob. wages	B	0.75	0.1	0.966	0.975	0.0128	0.9471	0.9874
ξ_L	Indexation wages	B	0.5	0.2	0.785	0.710	0.1191	0.6102	0.9672
Prices									
ϕ_{H_D}	Calvo prob. dom. prices	B	0.75	0.1	0.630	0.626	0.0595	0.5355	0.7272
ϕ_{H_F}	Calvo prob. exp. prices	B	0.75	0.1	0.901	0.890	0.0451	0.8381	0.9689
ϕ_F	Calvo prob. imp. prices	B	0.75	0.1	0.507	0.504	0.0501	0.4236	0.5882
ξ_{H_D}	Indexation dom. prices	B	0.5	0.2	0.260	0.268	0.1406	0.0401	0.4654
ξ_{H_F}	Indexation exp. prices	B	0.5	0.2	0.430	0.346	0.1919	0.1124	0.7379
ξ_F	Indexation imp. prices	B	0.5	0.2	0.414	0.399	0.1475	0.1701	0.6568
Production									
ω_H	EoS oil, other inputs	IG	1	Inf	0.393	0.460	0.1264	0.2124	0.5648
Investment									
η_I	EoS H and F inv., non-S	IG	1	Inf	1.345	0.497	0.9417	0.2511	2.8218
η_{I_S}	EoS H and F inv., S	IG	1	Inf	0.868	0.695	0.6367	0.2340	1.6080
μ_S	Inv. adjustm. cost, non-S	G	2	0.5	1.844	1.533	0.4220	1.1542	2.5062
μ_{I_S}	Inv. adjustm. cost, S	G	3	0.5	2.864	3.078	0.5169	2.0249	3.7079
For. economy									
η_F	Price elast. for. demand	IG	1	Inf	0.273	0.239	0.0549	0.1891	0.3559
ϱ	Country prem. debt elast.	IG	0.01	Inf	0.008	0.009	0.0016	0.0051	0.0104
Mon. policy									
ψ_r	Interest rate smoothing	B	0.75	0.1	0.818	0.828	0.0233	0.7797	0.8556
ψ_Y	Int. feedb. GDP growth	N	0.125	0.05	0.101	0.116	0.0439	0.0302	0.1735
ψ_π	Int. feedb. inflation	N	1.5	0.1	1.700	1.733	0.0793	1.5702	1.8314
ψ_{π_Z}	Feedb. weight core infl.	B	0.5	0.2	0.740	0.656	0.1282	0.5456	0.9442
AR(1) coef.									
ρ_{a_H}	Neutral technology shock	B	0.75	0.1	0.783	0.785	0.0865	0.6452	0.9243
ρ_{ζ_T}	Productivity growth shock	B	0.75	0.1	0.768	0.718	0.0601	0.6755	0.8680
ρ_{ζ_C}	Preference shock	B	0.75	0.1	0.757	0.715	0.0732	0.6423	0.8764
ρ_{ζ_I}	Inv.-specif. techn. shock	B	0.75	0.1	0.619	0.627	0.0724	0.5034	0.7404
ρ_{ζ_G}	Fiscal policy shock	B	0.75	0.1	0.870	0.878	0.0390	0.8084	0.9338
ρ_{ζ_Θ}	Country premium shock	B	0.75	0.1	0.955	0.958	0.0198	0.9269	0.9854
ρ_{ζ_A}	Food price shock	B	0.75	0.1	0.781	0.0825	0.806	0.6566	0.9175
ρ_{a_S}	Comm.-specif. techn. sh.	B	0.75	0.1	0.959	0.960	0.0173	0.9324	0.9855
Innov. s.d.									
u_{a_H}	Neutral technology shock	IG	0.005	Inf	0.008	0.008	0.0016	0.0051	0.0103
u_{ζ_T}	Productivity growth shock	IG	0.005	Inf	0.011	0.012	0.0024	0.0070	0.0149
u_{ζ_C}	Preference shock	IG	0.005	Inf	0.047	0.036	0.0131	0.0271	0.0668
u_{ζ_I}	Inv.-specif. techn. shock	IG	0.005	Inf	0.047	0.040	0.0115	0.0292	0.0649
u_{ζ_G}	Fiscal policy shock	IG	0.005	Inf	0.038	0.036	0.0046	0.0304	0.0452
u_{ζ_m}	Monetary policy shock	IG	0.005	Inf	0.002	0.002	0.0002	0.0014	0.0020
u_{ζ_Θ}	Country premium shock	IG	0.005	Inf	0.001	0.001	0.0001	0.0008	0.0012
u_{ζ_A}	Food price shock	IG	0.005	Inf	0.014	0.013	0.0033	0.0082	0.0189
u_{a_S}	Comm.-specif. techn. sh.	IG	0.005	Inf	0.034	0.034	0.0035	0.0287	0.0401

Notes: The prior distributions are: beta distribution (B) on the open interval $(0, 1)$, inverse gamma distribution (IG) on $\mathbb{R}^+$, gamma distribution (G) on $\mathbb{R}_0^+$, normal distribution (N) on $\mathbb{R}$. The posterior distribution is computed from a Metropolis-Hastings chain of 1,000,000 draws with an average acceptance rate of approximately 25%, dropping the first 500,000 draws for convergence. The posterior mode is the maximum of the posterior distribution obtained from the Metropolis-Hastings algorithm. The reported interval is the 90% highest posterior density credible interval.

gamma priors around a unitary degree of substitution with an infinite standard deviation. For the elasticities of the investment adjustment cost functions in the domestic goods sector (μ_I) and the commodity sector (μ_{I_S}), we use gamma priors with mean 2 and 3, respectively, and standard deviation 0.5. These priors reflect our beliefs that, due to the scale of the associated projects, commodity investment is more difficult and sluggish to adjust than investment in other sectors of the economy. For the slopes of the various price and wage Phillips curves (ϕ_{H_D}, ϕ_{H_F}, ϕ_F, and ϕ_L), we use beta priors with mean 0.75 and standard deviation 0.1. This prior implies that, on average, prices and wages are optimally adjusted once a year. For the shares of past inflation in the passive price and wage updating rules (ξ_{H_D}, ξ_{H_F}, ξ_F, and ξ_L), we use rather uninformative beta priors centered at 0.5 with standard deviation 0.2.

Our priors on the parameters in the monetary policy rule are as follows. The interest rate smoothing parameter (ψ_r) obtains a beta prior with mean 0.75 and 0.1, implying a considerable degree of inertia in the nominal interest rate. The feedbacks from inflation and quarterly output growth (ψ_π and ψ_Y) obtain normal priors with mean 1.5 and 0.5/4, respectively, and standard deviations 0.1 and 0.05. The parameter controlling the weight of core CPI inflation in the inflation response (ψ_{π_Z}) obtains a beta prior with mean 0.5 and standard deviation 0.2. Hence, while we specify relatively tight priors on the standard parameters of the monetary policy rule, our prior on the latter non-standard parameter is significantly less informative. The quarterly standard deviation of i.i.d. shocks to the monetary policy rule obtains an inverse gamma prior with mean 0.5% and an infinite standard deviation.

We specify beta priors with mean 0.75 and standard deviation 0.1 for the AR(1) parameters of the remaining unobservable exogenous processes, implying that the half-life of those shocks is on average less than one year. The priors of the standard deviations of the associated innovations are of the inverse gamma type, with mean 0.5% and an infinite standard deviation.

To obtain the posterior distribution of the parameters, we generate a chain of 1,000,000 draws using the Metropolis-Hastings algorithm and drop the first 500,000 draws for convergence.[28] Based on the remaining draws, the mode and the mean of the posterior as well as posterior probability intervals are computed. The estimates are presented in Table 2. For space reasons, we do not discuss those estimates in more detail.

5 Results

In this section we present selected results from the estimated model. We first discuss the effects of commodity price shocks, highlighting the main transmission mechanisms of interest through impulse response analysis. We then present historical decompositions over the period 2001Q3-2012Q4 of investment, GDP and the current account in Chile, emphasizing the role of (inferred)

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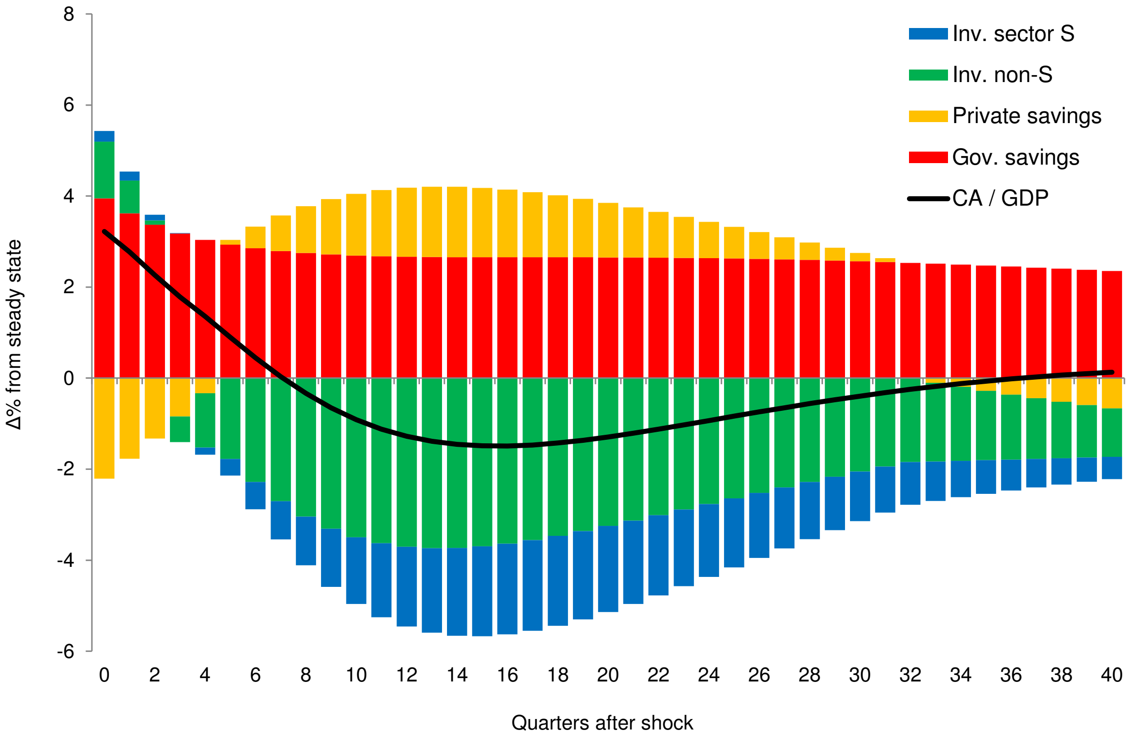
[28] The mode of the posterior kernel that is used to initialize the algorithm is obtained through the numerical optimization routine developed by Sims (1999).

Figure 7: Impulse responses of savings, investment, and the current account balance (% of GDP) to a persistent commodity price shock of 50 percent.



Notes: This figure shows the estimated impulse response functions of investment in the commodity sector, non-commodity investment, private savings, government savings (government balance), and the current account balance to a persistent commodity price shock of 50 percent in the baseline model with endogenous commodity production and imperfect information on the persistence of the commodity price. All variables are expressed as percentages of nominal GDP in absolute deviations from steady state.

transitory and persistent commodity price shocks in comparison to other shocks. We then use the model to analyze the implications of alternative monetary and fiscal policies for macroeconomic dynamics under a persistent commodity price shock.[29]

5.1 Saving-investment dynamics under a commodity price shock

Figure 7 shows the impulse responses of commodity investment, non-commodity investment, private savings, government savings, and the current account to a persistent commodity price shock, simulated under the assumption of incomplete information on the persistence of the shock.[30] The size of the shock is 50 percent, which roughly corresponds to the observed increase of the real price of copper towards the mid-2000s with respect to its long-run mean. The shock generates an initial current account surplus of around 3% of GDP followed by a slowly declining and eventually negative balance reaching almost -2% of GDP. The initial current account surplus is associated with a significant increase in government savings, which then gradually

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[29] All results are computed using the posterior mode obtained from the Metropolis-Hastings algorithm.

[30] Government savings (SG) is defined as the government balance, while private savings (SP) is computed as the difference of total investment (IT) and the sum of government and external savings (SE, i.e. minus the current account balance): $SP = IT - SG - SE$.

diminish over time. The shock also generates a step-wise increase in investment both in the mining sector and in other sectors of the economy, which in total reach almost 6% of GDP, contributing to the eventual current account deficit.

5.2 The role of incomplete information and learning

We now analyze the role of the key mechanisms of the model for the model's dynamics. Figure 8 shows the simulated responses of selected variables to a persistent commodity price shock under incomplete and full information, as well as a transitory (i.i.d.) shock under full information. The size of both shocks is again 50% on impact. The first row of the figure shows the responses of the actual commodity price together with the filtered persistent and transitory components. According to the model with incomplete information, when faced with the higher commodity price, the agents believe at first that the price increases are mostly transitory because the transitory component is more volatile than the persistent one. However, the agents revise their beliefs over time as they observe that the effective price remains high, attributing a larger weight to the persistent component. In this way, the short-term responses of macroeconomic variables to the persistent shock fall in between the corresponding responses to persistent and transitory shocks in the full information case. For instance, while consumption and investment do almost not react to a transitory shock, they increase over time due to a persistent shock, but less so under imperfect information where their responses are more sluggish.[31] Hence, given their changing perception of the persistence of the shock, agents gradually increase consumption and investment over time, which slowly raises domestic demand and real GDP. The presence of imperfect information is critical for this result, as the increase in domestic spending and GDP would occur more instantaneously in the full information case. The steady increase in domestic demand leads to a gradual real exchange rate appreciation and higher inflation over the medium-term horizon, with the associated monetary policy response. Interestingly, the current account deficit would materialize much more instantaneously in the full information case, while the transitory commodity price increase is mainly saved especially by the government.

Note that the model's dynamics in the full information case are broadly consistent with standard theory on saving-investment behavior in small open economies, as in Obstfeld and Rogoff (1995), although nominal and real rigidities of course add some additional sluggishness. However, the presence of imperfect information and learning à la Erceg and Levin (2003) seems useful to reproduce the empirical observations on current account dynamics in Chile and other commodity-exporting economies that were discussed in the Introduction and Section 2.

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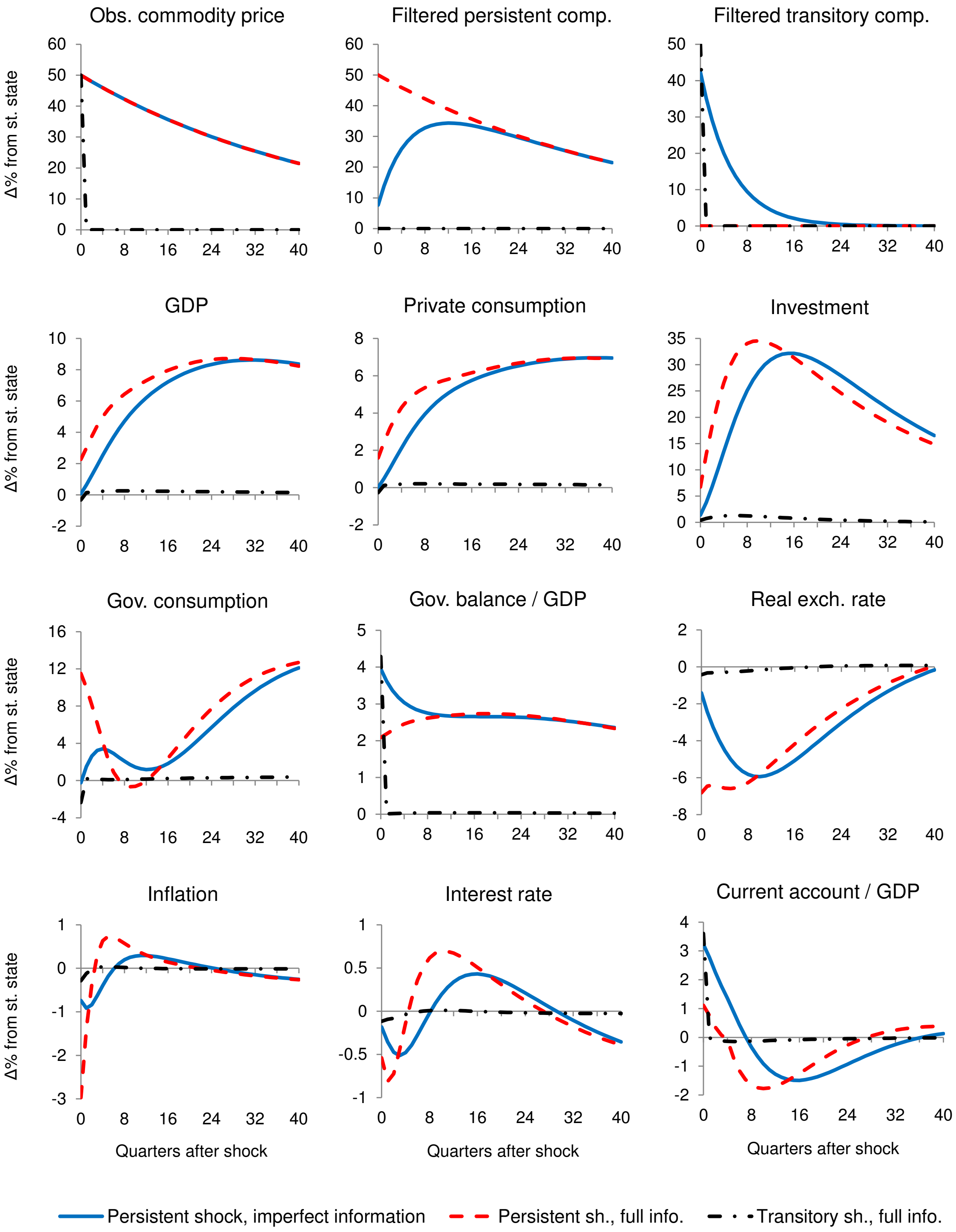
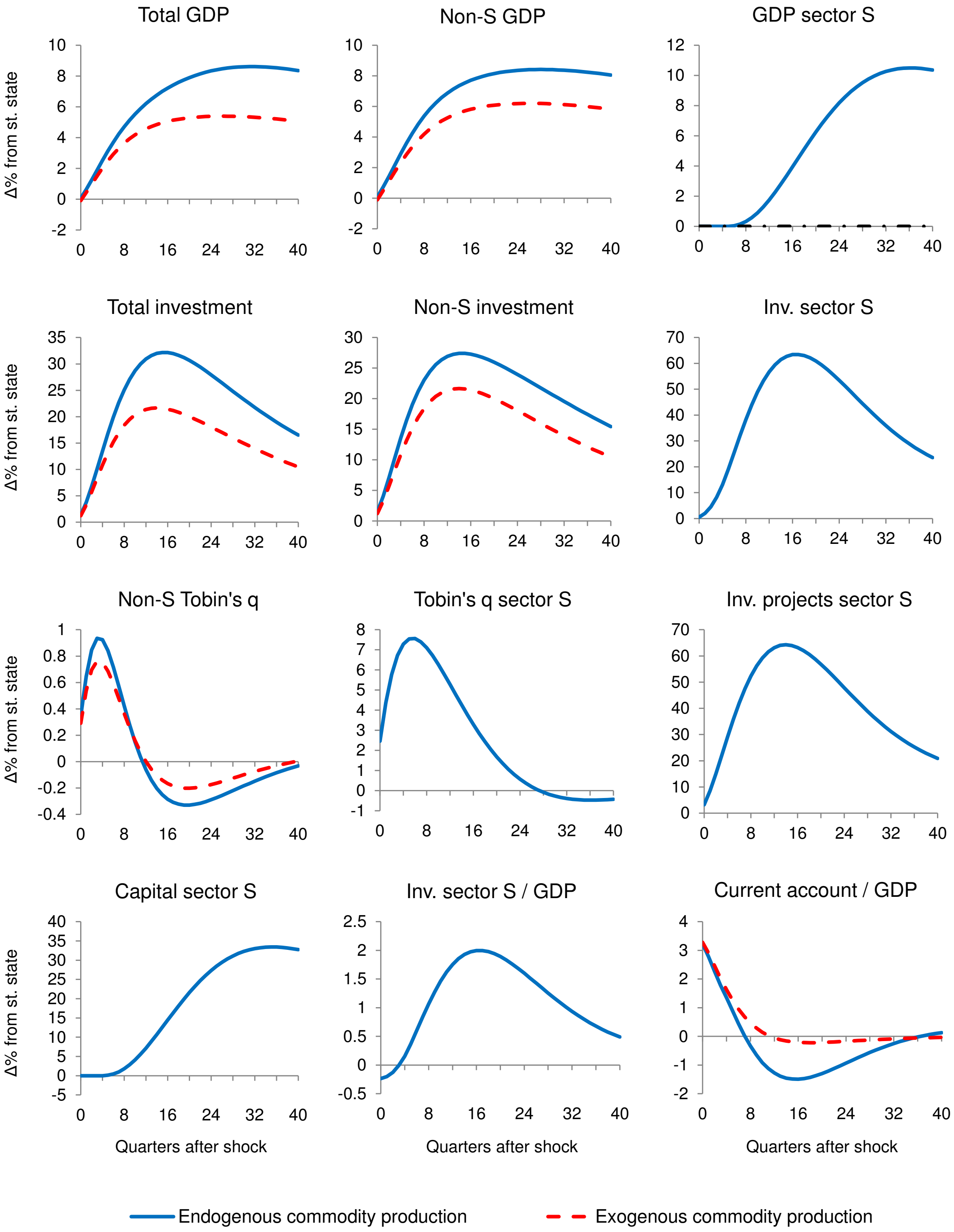
[31] This is a well-known type of finding of the literature on learning; see for instance Milani (2007).

Figure 8: Impulse responses to persistent and transitory commodity price shocks, imperfect vs. full information.



Notes: This figure shows the estimated impulse response functions of selected variables to persistent (u_t) and transitory (a_t) commodity price shocks of 50 percent. The solid blue lines correspond to a persistent shock in the baseline model with imperfect information on the persistence of the commodity price. The dashed red lines correspond to a persistent shock under full information. The dash-dotted black lines correspond to a transitory shock under full information. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance and the government balance as percentages of nominal GDP which are measured in absolute deviations from steady state. Inflation and the interest rate are expressed in annualized terms.

Figure 9: Impulse responses to a persistent commodity price shock, endogenous vs. exogenous commodity production.



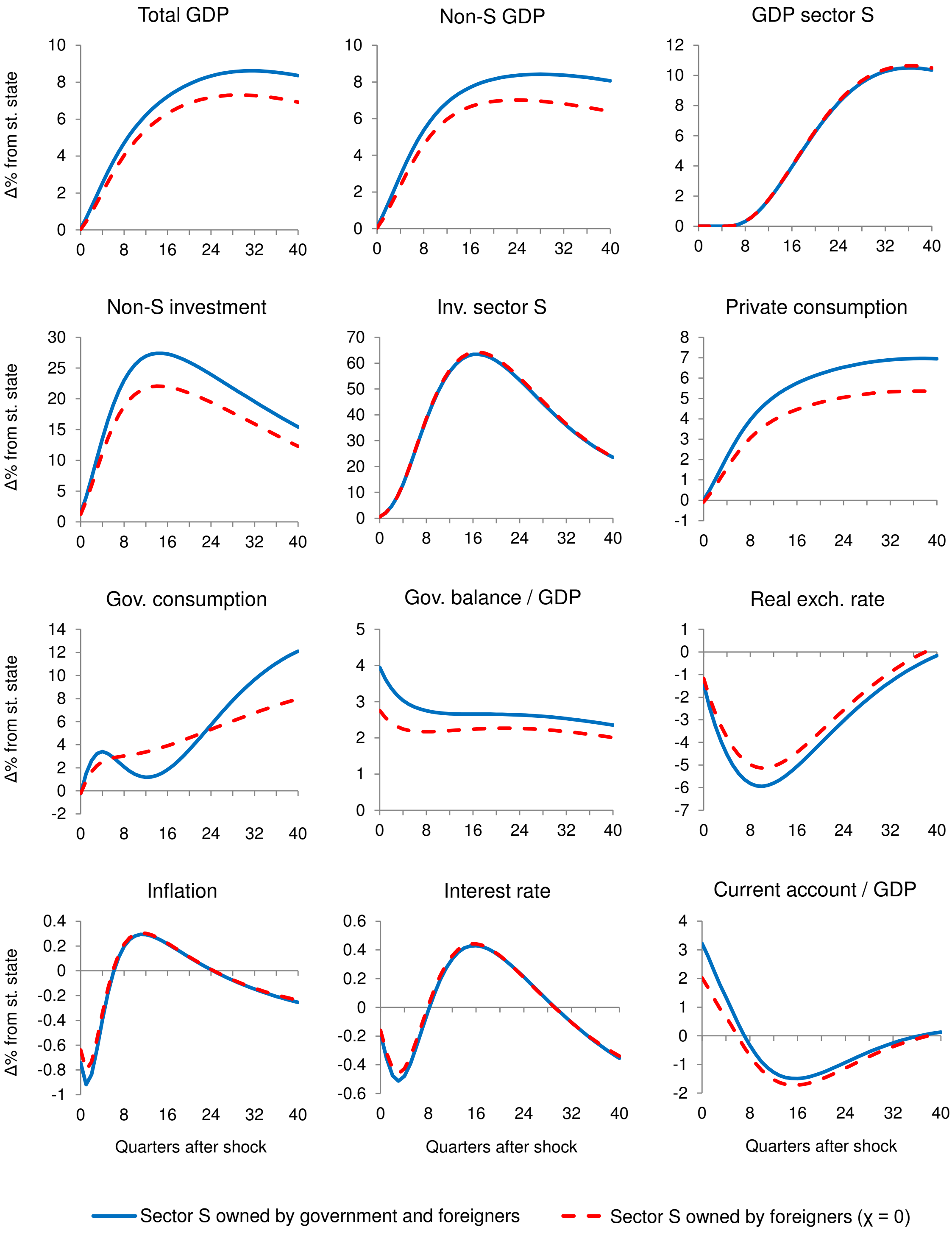
Notes: This figure shows the estimated impulse response functions of selected variables to a persistent commodity price shock of 50 percent. The solid blue lines correspond to the baseline model with endogenous commodity production and imperfect information on the persistence of the commodity price. The dashed red lines correspond to the model with exogenous commodity production and no investment in the commodity sector, with the calibrated and estimated parameters as in the baseline model. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance and investment in sector S as percentages of nominal GDP which are measured in absolute deviations from steady state.

5.3 The role of commodity investment

To analyze the role of commodity investment, Figure 9 shows the responses of investment-related variables (Tobin's q, investment projects, effective investment, etc.) to the persistent commodity price shock, and compares those responses to the case where commodity production is exogenous such that there are no endogenous movements in commodity investment. The shock is seen to have a significant effect on investment in the commodity sector, which increases by more than 60 percent above its steady state value, but other types of investment also increase. Note that despite a larger increase in the sectoral Tobin's q, the increase in commodity investment is slower than in the rest of the economy due to the presence of time to build and investment adjustment costs, which makes the firms in this sector invest in a more cautious way under uncertainty on the duration of the shock. Hence, other types of investment are not crowded out by the presence of commodity investment; instead, other investment turns out to be stimulated by the demand for domestic goods to produce the investment goods used in the commodity sector. As a result of the higher investment and its spillovers to other components of domestic demand such as private consumption, the response of total GDP is around one third larger in the case of endogenous commodity production, while the increase in commodity production after investment projects have matured further enhances the overall GDP response. Regarding the current account balance, the endogenous response of investment in the commodity sector is the main factor behind the eventual current account deficit; without commodity investment, the current account would return to a balanced position after the initial surplus. Hence, just like imperfect information and learning on the persistence of commodity price shocks, the presence of commodity investment seems critical to understand recent current account developments in commodity-exporting countries such as Chile.

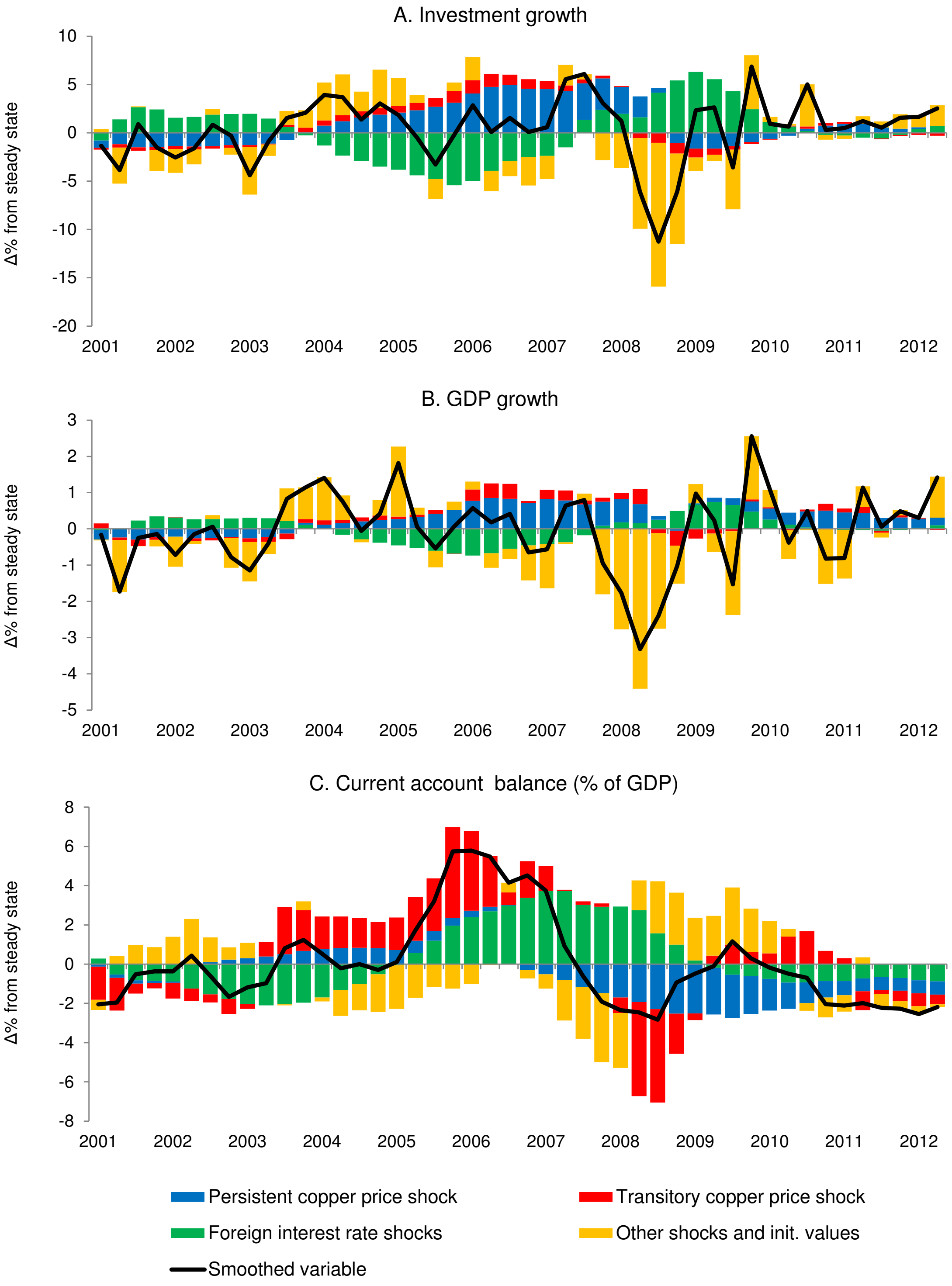
To highlight the role of FDI in the commodity sector, Figure 10 compares the responses to the persistent commodity price shock in the baseline case when the firms in sector S are owned in part by the government (share $\chi = 0.31$) and in part by foreign investors (share $1-\chi = 0.69$), and the case when the firms are fully owned by foreign investors and government ownership is nil ($\chi = 0$). Hence, in the last case, all investment in sector S is conducted through FDI and all profits generated in this sector that are not reinvested are repatriated. The results show that under complete foreign ownership of sector S the initial increase in the government balance due to the commodity price shock is smaller, because the government then receives less direct income from this sector, now entirely out of profit taxes on foreign companies. The shock thus generates a smaller wealth effect on domestic households, such that private consumption increases by less than in the baseline case, decreasing demand for consumption goods and therefore also non-commodity production and investment. Overall, this exercise points towards the importance of FDI in the commodity sector for the evolution of the current account balance, as the peak

Figure 10: Impulse responses to a persistent commodity price shock, positive vs. zero government share in sector S.



Notes: This figure shows the estimated impulse response functions of selected variables to a persistent commodity price shock of 50 percent. The solid blue lines correspond to the baseline model with a positive government ownership of assets in the commodity sector ($\chi > 0$) and imperfect information on the persistence of the commodity price. The dashed red lines correspond to the model with zero government share in the commodity sector ($\chi = 0$), with the remaining calibrated and estimated parameters as in the baseline model. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance and the government balance as percentages of nominal GDP which are measured in absolute deviations from steady state.

Figure 11: Historical decomposition of investment growth, GDP growth and the current account balance in Chile.



Notes: This figure shows the estimated contributions to the quarterly growth rates of real investment (chart A), real GDP (chart B) and the current account balance as a percentage of nominal GDP (chart C) from 2001Q3-2012Q4 of shocks to the persistent and transitory components of the real price of copper computed from the Kalman filter, foreign interest rate shocks (including shocks to the country spread) and other shocks plus the initial values of the state variables. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance which is measured in absolute deviations from steady state.

current account deficit is larger than in the baseline case.[32]

5.4 Historical decomposition of investment, GDP, and the current account

With respect to the experience of the Chilean economy during the recent commodity price cycle, assuming that the copper price is characterized by both persistent and transitory movements, according to our model agents would initially think that the increase in the copper price towards the mid-2000s has been due to transitory shocks. However, they would gradually update their perceptions on the persistence of the underlying shock over time as they observe that price remain high, attributing more weight to persistent price increases (see Figure 7). The latter is associated with an initial increase in savings by the government while private savings would increase with some lag, but after some time domestic savings would start to decrease consistent with expectations of more persistent income gains. This evolution is paralleled by a gradual increase in investment both in the mining sector and in other sectors of the economy, generating a declining current account balance (see Figures 8 to 10).

We now assess the consequences of this mechanism for the observed fluctuations of output, investment and the current account in Chile. Figure 11 shows the historical decompositions of real investment growth, real GDP growth and the current account balance as a percentage of GDP, all smoothed and in deviations from their steady state values. We report the contributions of innovations to the copper price and the foreign interest rate (including shocks to the country spread), and sum up the contributions of any other shocks. The results show that a large fraction of the observed investment boom and GDP growth until the 2008-2009 crisis can be attributed to the inferred persistent increase in the copper price. The current account surplus of the mid-2000s can be attributed to the inferred transitory component, whereas the subsequent reversal is mainly explained by the inferred persistent component. However, declining foreign interest rates are also relevant and explain about half of the initial deficit and somewhat more than half in the last quarters of the sample as the investment boom has faded off.

5.5 Dynamics under alternative monetary and fiscal policies

The results presented so far are conditional on the monetary and fiscal policy framework in place in Chile, which consists of a flexible inflation targeting monetary regime with a floating exchange rate and a structural balance rule for fiscal policy. What would be the implications of alternative types of monetary and fiscal policy behavior for the effects of a persistent commodity price shock? For instance, on the monetary side it is sometimes argued that a (strong) exchange rate appreciation may be responsible for current account imbalances by reducing demand for exports and stimulating demand for imports, which may be a reason for monetary authorities in commodity-exporting countries to try to limit exchange rate fluctuations. On the fiscal side,

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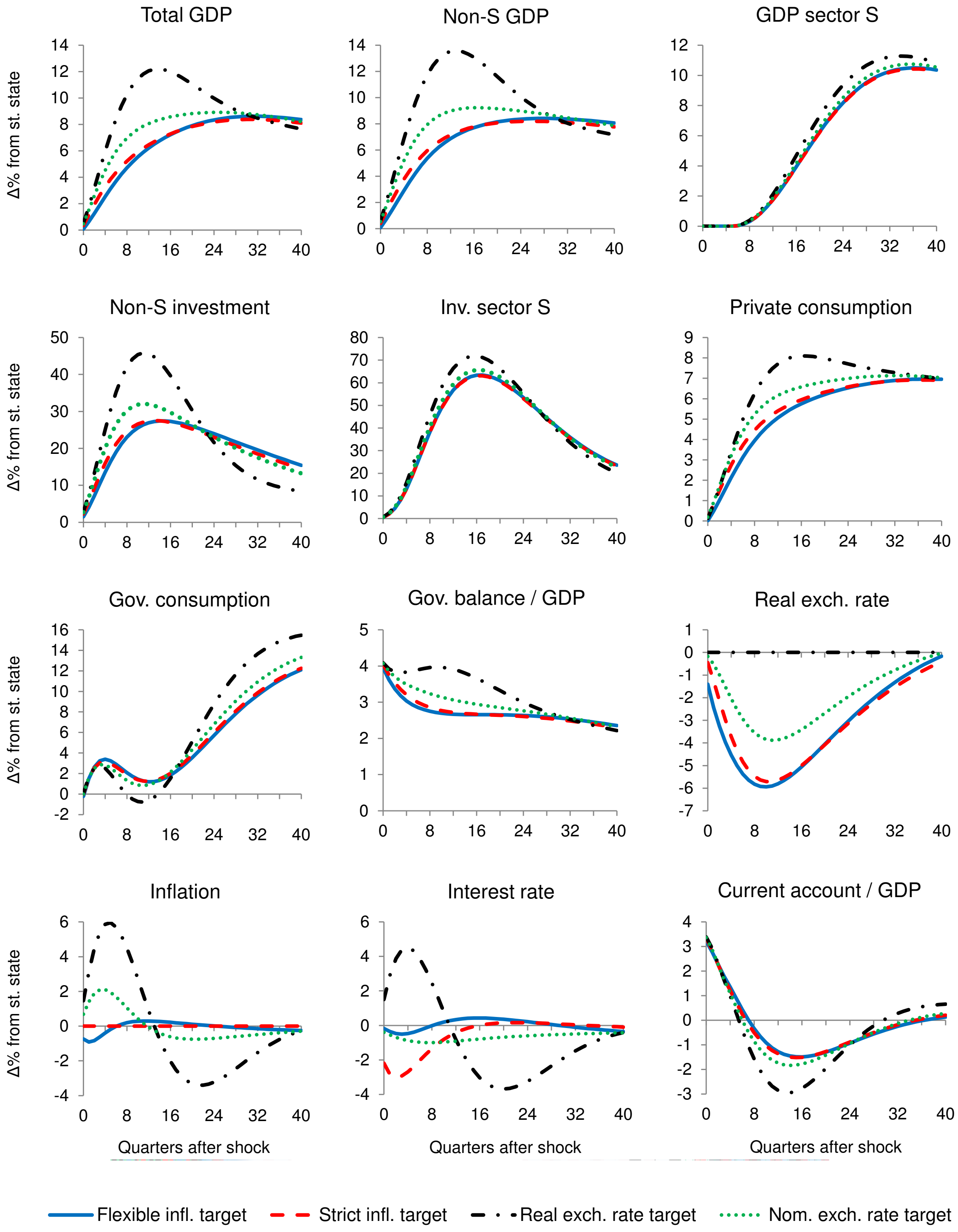
[32] These results are broadly in line with Medina and Soto (2007b), who analyze the implications of different fiscal rules for the transmission of commodity price shocks in Chile.

Figure 12: Impulse responses to a persistent commodity price shock, different monetary policies.



Notes: This figure shows the estimated impulse response functions of selected variables to a persistent commodity price shock of 50 percent. The solid blue lines correspond to the baseline model with a flexible inflation targeting monetary rule and imperfect information on the persistence of the commodity price. The dashed red lines correspond to the model with a strict inflation target ($\pi_t = \bar{\pi}$ for all t), the dash-dotted black lines to a real exchange rate target ($rer_t = \overline{rer}$ for all t), and the dotted green lines to a nominal exchange rate target ($\pi_{\varepsilon,t} = \bar{\pi}_{\varepsilon}$ for all t), with the calibrated and estimated parameters as in the baseline model. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance and the government balance as percentages of nominal GDP which are measured in absolute deviations from steady state.

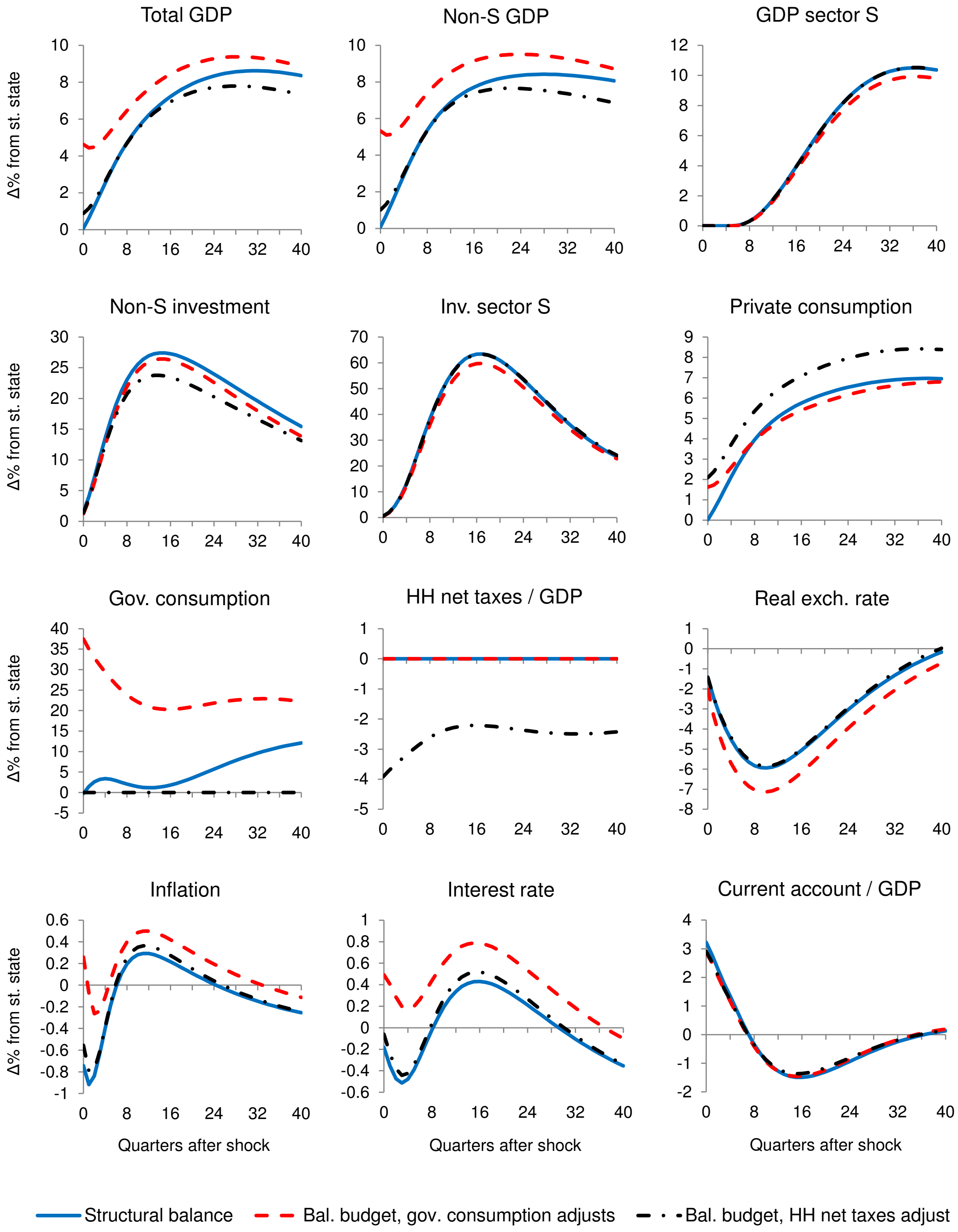
a structural balance rule that allows for additional government spending under persistent commodity price shocks may add to current account imbalances, and it therefore seems interesting to analyze saving-investment dynamics under different fiscal rules.

We first analyze alternative monetary policies. Figure 12 compares the flexible inflation targeting regime from the baseline model with real and nominal exchange rate targets (i.e. $rer_t = \overline{rer}$ and $\pi_{\varepsilon,t} = \bar{\pi}_\varepsilon$ for all t), as well as a strict inflation targeting rule where inflation is fully stabilized at its steady state level in each period (i.e. $\pi_t = \bar{\pi}$ for all t). We analyze those extreme cases since the actual conduct of monetary policy in any small open economy can most likely be classified as some combination of these extremes. The results show that a real exchange rate target implies the largest differences in the dynamics due to the persistent commodity price shock compared to flexible inflation targeting, while a strict inflation target has quantitatively similar implications for the dynamics of real variables and the current account as a flexible target. The nominal exchange rate target implies dynamics that fall in between the remaining cases. The real exchange rate and real interest rate responses are key to understand those differences: to keep the real exchange rate fixed, the central bank needs to generate a reduction in the real interest rate, but this generates higher inflation that would imply an additional real appreciation and the central bank therefore raises the nominal interest rate initially before cutting it. Under the nominal exchange rate target, the central bank is not concerned about prices and only needs to reduce the nominal interest rate to a sufficient extent. The negative real interest rate response under both exchange rate policies generate a larger expansion of the economy and a larger peak current account deficit than under the inflation targeting policies.

Turning to fiscal policy, Figure 13 compares the effects of the persistent commodity price shock under the baseline structural balance rule with two types of balanced budget policies: one where government consumption is the fiscal instrument that adjusts to keep the government balance unchanged, and one where household taxes net of transfers are the fiscal instrument. The balanced budget policy with consumption expenditure adjustment implies a larger expansion of real GDP, higher inflation, a stronger interest rate response, a larger real appreciation and, initially, a larger expansion of private consumption due to the response of consumption by non-Ricardian households to higher labor income. However, consumption by Ricardian households and investment is crowded out, which explains why the current account response hardly changes although government savings are nil. The balanced budget policy with net tax adjustment, on the other hand, implies smaller changes in the responses of real GDP, inflation, the interest rate and the real exchange rate, but private consumption shows a much stronger response because the reduction in net taxes directly raises non-Ricardian consumption. The latter also generates a somewhat larger crowding-out effect on investment. However, the current account response is again very similar to the baseline case.

Interestingly, while non-commodity GDP and investment are quite sensitive to the monetary and fiscal policy responses to the shock, the responses of real GDP and investment in sector S

Figure 13: Impulse responses to a persistent commodity price shock, different fiscal policies.



Notes: This figure shows the estimated impulse response functions of selected variables to a persistent commodity price shock of 50 percent. The solid blue lines correspond to the baseline model with a structural balance fiscal rule and imperfect information on the persistence of the commodity price. The dashed red lines and the dash-dotted black lines correspond to the model with a balanced budget rule with government consumption and household net taxes, respectively, as fiscal instruments, with the calibrated and estimated parameters as in the baseline model. The results are based on the posterior mode computed from the Metropolis-Hastings algorithm. All variables are expressed as relative percentage deviations from their steady state values, except the current account balance and household net taxes as percentages of nominal GDP which are measured in absolute deviations from steady state.

are much less sensitive to those policy responses. The reason is that commodity investment and production are primarily driven by (the persistent component) of the international commodity price, while the alternative policies—through different real interest rate and real exchange rate responses—mainly affect the economy-wide stochastic discount factor and the conversion of the international commodity price into domestic currency terms, but those indirect effects are less relevant for investment decisions in this sector.

Overall, if current account stabilization was a policy objective in commodity-exporting economies, then the above exercises show that while fiscal policy design has important general macroeconomic implications in the face of commodity price fluctuations, it does not critically affect the evolution of the current account which is mainly driven by investment and FDI in the commodity sector. Monetary policy, on the other hand, seems to be best conducted under a flexible exchange rate, because exchange rate targeting would enhance current account deficits that emerge under persistent commodity price shocks according to our model.

6 Conclusions

This paper deployed two additional mechanisms to a small open economy model with a commodity export-oriented sector. First, we hypothesized that economic agents face imperfect information and they learn on the persistence of commodity price shocks. Second, we conceived that the investment process in the commodity sector matures slowly and has important spillovers to the rest of the economy. The essence of our results relies on both mechanisms: the first is responsible for delaying the response of the economy to commodity price shocks, whereas the second makes investment projects feasible if the expected price in the near medium term is high. Thus, these two features jointly trigger the prediction that a current account reversal (from a surplus into a deficit) can arise *gradually* in a commodity-exporting economy as an endogenous response of the economy to a persistent increase in commodity prices.

We have also presented the following four stylized facts related to the evolution of copper price forecasts over the latest commodity price cycle and saving-investment dynamics in the case of Chile, which were also paralleled in other commodity-exporting economies. First, sustained high levels of the copper price gradually led to higher forecasted prices by private agents and the government. Second, high spot prices and higher forecasted prices were also associated with a declining current account balance, from a surplus to a deficit. Third, the initial current account surplus was due to an increase in national savings, while the subsequent decline and eventual deficit has been due to increasing investment and insufficient national savings. Fourth, the investment boom has been driven by investment and FDI in mining.

Based on the estimation of parameters of the model using Chilean quarterly data from 2001Q3-2012Q4, we were able to conduct several quantitative experiments. One of the key predictions of the model is that during a lasting increase in commodity prices, agents believe at first that the price increase is mostly temporary but revise their expectations over time as they observe that commodity prices remain high, and thus gradually decrease savings and increase investment especially in the commodity sector. Hence, this mechanism is apt to explain all of the above facts and, in particular, a significant fraction of the step-wise deterioration of the Chilean current account since the mid-2000s. Our results thereby permit a novel interpretation of current account developments in commodity-exporting economies, in comparison to previous related work which has pointed towards investment-specific shocks, changes in foreign financial conditions, and variations in foreign demand (see Medina, Munro, and Soto, 2008).

Our analysis allows to draw a number of conclusions for macroeconomic policy in commodity-exporting countries. First, a gradual current account deterioration in a commodity-exporting country in times of a persistent commodity price boom is not necessarily a sign of emerging macroeconomic imbalances, because it may be a natural by-product of investment in the commodity sector that enhances the productive capacity of the economy. Further, current account deficits that are primarily due to FDI in the commodity sector may be less worrisome to policy-makers than other types of external savings deficits, for instance due to large portfolio inflows, as FDI is often regarded as a more stable source of funding (see for instance Albuquerque, 2003; Goldstein and Razin, 2006).[33] Finally, due to the dominant role of investment and FDI in the commodity sector, there seems to be limited scope for conventional monetary and fiscal policy in commodity-exporting countries to affect the evolution of the current account in the face of persistent commodity price shocks. In particular, our model predicts that alternative fiscal policies and rules do not critically affect the evolution of the current account, while an inflation targeting monetary regime under a flexible exchange rate is preferred over an exchange rate target if current account stabilization is a policy concern.

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[33] In addition, several studies suggest that FDI may have positive effects on long-term economic growth through technology adoption by host countries. See Borensztein, De Gregorio, and Lee (1998), Javorcik (2004), Haskel, Pereira, and Slaughter (2007), Li and Liu (2005), and Markusen and Venables (1999).

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A Implementing Learning about Exogenous Shocks

This appendix explains in detail our implementation of imperfect information and learning about exogenous shocks, following Erceg and Levin (2003). Consider a log-linearized model:

$$x_t = AE_t\{x_{t+1}\} + Bx_{t-1} + Cz_t, \qquad t = 1, \ldots, T, \tag{A.1}$$

where z_t is the exogenous variable that drives the model. The latter has two components, a transitory component, a_t, and persistent component, b_t:

$$z_t = a_t + b_t. \tag{A.2}$$

The persistent component follows an AR(1) process:

$$b_t = \rho b_{t-1} + u_t, \qquad u_t \sim N(0, \sigma_u^2). \tag{A.3}$$

The transitory component is white noise:

$$a_t \sim N(0, \sigma_a^2). \tag{A.4}$$

The variables a_t and b_t are not observable. The agents infer those unobserved components in an optimal linear way using the Kalman filter, as follows. Let $\mathcal{Z}^t = \{z_1, z_2, z_3, \ldots, z_t\}$ and define $\hat{b}_t = E[b_t|\mathcal{Z}^t]$, i.e. the inference regarding the persistent component of z_t upon observing current and past values of z_t. The (steady state) Kalman filter yields

$$\hat{b}_t = \rho\hat{b}_{t-1} + K\rho^{-1}(z_t - \rho\hat{b}_{t-1}). \tag{A.5}$$

Notice that $\rho\hat{b}_{t-1}$ is the forecast of b_t and $\hat{b}_t$ conditional on the information available at time $t-1$, i.e. $E[b_t|\mathcal{Z}^{t-1}] = E[\hat{b}_t|\mathcal{Z}^{t-1}] = \rho\hat{b}_{t-1}$ with forecast error $\hat{u}_t = \hat{b}_t - \rho\hat{b}_{t-1}$. Now define $\hat{a}_t = E[a_t|\mathcal{Z}^t]$, i.e. the inference of the transitory component of z_t with the information available at time t, which satisfies

$$\hat{a}_t = z_t - \hat{b}_t. \tag{A.6}$$

Notice that $E[\hat{a}_t|\mathcal{Z}^{t-1}] = E[z_t|\mathcal{Z}^{t-1}] - E[\hat{b}_t|\mathcal{Z}^{t-1}] = E[b_t|\mathcal{Z}^{t-1}] - \rho\hat{b}_{t-1} = 0$. Analogously, we have that $E[\hat{a}_{t+1}|\mathcal{Z}^t] = 0$, such that $\hat{a}_t$ can be interpreted as an exogenous disturbance with conditional mean equal to zero. The model (A.1) can therefore be represented as follows:

$$x_t = AE_t\{x_{t+1}\} + Bx_{t-1} + C\hat{a}_t + C\hat{b}_t, \tag{A.7}$$

with

$$\hat{b}_{t+1} = \rho\hat{b}_t + \hat{u}_{t+1}, \tag{A.8}$$

and with (A.2)-(A.6). There is one exogenous variable, z_t, and two “perceived” exogenous variables, $\hat{a}_t$ and $\hat{b}_t$. However, (A.7)-(A.8) can be solved independently from (A.2)-(A.6).

Hence, the following algorithm can be used to simulate the response of the endogenous variables in x_t to shocks to z_t:

1. Solve (A.7)-(A.8) to obtain the decision rules

$$x_t = D\hat{a}_t + E\hat{b}_t + Fx_{t-1}. \tag{A.9}$$

2. Generate a sequence for z_t using (A.2)-(A.4).
3. Generate a sequence for $\hat{b}_t$ using (A.5).
4. Generate a sequence for $\hat{a}_t$ using (A.6).
5. Evaluate (A.9) using the sequences for $\hat{b}_t$ and $\hat{a}_t$ generated in steps 3 and 4.

B Derivation of the Signal-to-Noise Ratio in the Model for $p^*_{S,t}$

This appendix describes the derivation of the signal-to-noise ratio in the unobserved components model for the real international copper price described in Section 3. For convenience, we repeat this model here:

$$\begin{aligned} \log(p^*_{S,t}/\bar{p}^*_S) &= b_t + a_t, & a_t &\sim N(0,\sigma_a^2), \\ b_t &= \rho b_{t-1} + u_t, & u_t &\sim N(0,\sigma_u^2), \qquad t = 1,\ldots,T. \end{aligned}$$

From the Kalman filter recursions, the steady state value of the Kalman gain is as follows:

$$K = \rho\Sigma(\Sigma + \sigma_a^2)^{-1},$$

where Σ, the steady state value of the conditional variance of the unobserved state, satisfies

$$\Sigma = \rho^2\left[\Sigma - \frac{\Sigma^2}{\Sigma + \sigma_a^2}\right] + \sigma_u^2. \tag{B.1}$$

The above equation can be rewritten as follows:

$$\left(\frac{\sigma_u}{\sigma_a}\right)^2 = \frac{\Sigma}{\sigma_a^2} - \frac{\rho^2}{1 + \frac{\sigma_a^2}{\Sigma}} = \frac{\left(\Sigma/\sigma_a^2\right)^2 + \left(\Sigma/\sigma_a^2\right)\left(1-\rho^2\right)}{1 + \Sigma/\sigma_a^2}. \tag{B.2}$$

Rewriting (B.1) in terms of the ratio Σ/σ_a^2 yields

$$\frac{\Sigma}{\sigma_a^2} = \left(\rho/K - 1\right)^{-1}. \tag{B.3}$$

By (B.2) and (B.3), given values of K and ρ, the signal-to-noise ratio satisfies

$$\frac{\sigma_u}{\sigma_a} = \sqrt{\frac{\left(\rho/K-1\right)^{-2} + \left(\rho/K-1\right)^{-1}\left(1-\rho^2\right)}{1 + \left(\rho/K-1\right)^{-1}}}. \tag{B.4}$$

C Model Equations

This appendix contains the complete set of equations from the stationary version of the model. The variables of the model in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock T_t. We need to to transform these variables to have a stationary version of the model. To do this, with the exceptions we enumerate below, lowercase variables denote the uppercase variable divided by T_t, e.g. $c_t \equiv C_t/T_t$. One exception is the marginal utility of consumption Λ_t that is multiplied by T_t, i.e. $\lambda_t \equiv \Lambda_t T_t$, for it decreases along the balanced growth path. Another exception is the nominal wage W_t that is divided by the consumption price index and by T_t, i.e. $w_t \equiv W_t/(P_{C,t}T_t)$. We further define stationary ratios of nominal values with respect to nominal GDP through the ending Y_t, e.g. $BY_t \equiv \varepsilon_t B_t^*/(P_{Y,t}Y_t)$.

Among the variables that were not defined in the main text, $\Omega_{t,t+s}$ is the households' stochastic discount factor for real payoffs, $\pi_{C,t}$ is gross consumer price inflation (similarly for $\pi_{H_D,t}$ and $\pi^*_{H_F,t}$), $\pi_{\varepsilon,t}$ is the gross rate of change of the nominal exchange rate, rer_t is the real exchange rate, $\tilde{\mu}_t$ is the wedge due to the presence of sticky wages of the marginal disutility of labor with respect to the allocation under flexible wages, $\mathcal{F}_{H_D,t}$, $\mathcal{F}_{H_F,t}$, $\mathcal{F}_{F,t}$ and $\mathcal{F}_{W,t}$ are recursive terms from the price and wage setting conditions, $\tilde{p}_{H_D,t}$, $\tilde{p}^*_{H_F,t}$, $\tilde{p}_{F,t}$ and $\tilde{w}_t$ are optimal prices and wages, q_t is the shadow price of capital in the domestic goods sector, and tax_t denotes real lump-sum taxes. In addition, we define a number of ratios with respect to nominal GDP: government debt and expenditure (BGY_t and GY_t), effective and structural fiscal income (FIY_t and $\widetilde{FIY}_t$), effective and potential tax revenues ($TAXY_t$ and $\widetilde{TAXY}_t$), effective and structural income in the commodity sector (YSY_t and $\widetilde{YSY}_t$), net exports (NXY_t), and net rents from the commodity sector that are disimbursed to foreign investors ($RENY_t$).

The rational expectations equilibrium under incomplete information of the stationary version of the model is then the set of sequences

$$\begin{aligned}
&\{\lambda_t, \Omega_{t,t+s}, c_t, c_t^R, c_t^{NR}, c_{Z,t}, c_{A,t}, c_{O,t}, c_{H,t}, c_{F,t}, p_{Z,t}, p_{A,t}, p_{O,t}, p_{H_D,t}, p^*_{H_F,t}, p_{F,t}, \tilde{p}_{H_D,t},\\
&\tilde{p}^*_{H_F,t}, \tilde{p}_{F,t}, p_{G,t}, p_{I,t}, p_{I_S,t}, p_{M,t}, p_{X,t}, p_{Y,t}, p^*_{S,t}, \tilde{p}_{S,t}, r_t, rer_t, \pi_{C,t}, \pi_{Z,t}, \pi_{\varepsilon,t}, \pi_{H_D,t}, \pi^*_{H_F,t},\\
&\mathcal{F}_{H_D,t}, \mathcal{F}_{H_F,t}, \mathcal{F}_{F,t}, \mathcal{F}_{W,t}, mc_{H,t}, y_t, y_{H,t}, y_{F,t}, y^*_{H,t}, \mathcal{Y}_{H,t}, y_{S,t}, o_{H,t}, v_{H,t}, z_t, l_t, w_t, \tilde{w}_t, \tilde{\mu}_t, \Theta_t,\\
&k_t, k_{S,t}, i_t, i_{H,t}, i_{F,t}, i_{S,t}, i_{H,t}(S), i_{F,t}(S), x_{S,t}, \Phi_t, \Phi'_t, \Phi_{S,t}, \Phi'_{S,t}, q_t, q_{S,t}, x_t, m_t, g_t, g_{H,t}, g_{F,t},\\
&tax_t, BY_t, BGY_t, GY_t, FIY_t, \widetilde{FIY}_t, TAXY_t, \widetilde{TAXY}_t, YSY_t, \widetilde{YSY}_t, NXY_t, RENY_t, CAY_t\}_{t=0}^{\infty},
\end{aligned}$$

such that for given initial values and exogenous sequences

$$\{\zeta_{C,t}, \zeta_{T,t}, \zeta_{I,t}, \zeta_{K,t}, \zeta_{G,t}, \zeta_{m,t}, \zeta_{A,t}, \zeta_{\Theta,t}, a_{H,t}, a_{S,t}, r_t^*, \pi_t^*, y_t^*, p^*_{O,t}, \hat{a}_t, \hat{b}_t\}_{t=0}^{\infty},$$

the following conditions are satisfied.[34]

C.1 Households

- Marginal utility of Ricardian consumption:

$$\lambda_t = (c_t^R - hc_{t-1}^R/\zeta_{T,t})^{-1}. \tag{C.1}$$

- Marginal disutility of labor:

$$\psi l_t^{\sigma_L} = \lambda_t w_t/\tilde{\mu}_t. \tag{C.2}$$

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[34] Due to full indexation to past and steady state inflation, the model exhibits no price and wage dispersion up to a first-order approximation. As we only consider this type of approximation in this paper, we therefore ignore the price and wage dispersion terms that would in principle enter in the non-linear model.

- Discount factor between t and $t+s$:

$$\Omega_{t,t+s} = \beta^s E_t\{(\zeta_{C,t+s}/\zeta_{C,t})(\lambda_{t+s}/\lambda_t)\prod_{j=1}^{s}\zeta_{T,t+j}^{-1}\}. \tag{C.3}$$

- First-order condition for domestic bonds:

$$\Omega_{t,t+1} = r_t^{-1}E_t\{\pi_{C,t+1}\}. \tag{C.4}$$

- First-order condition for foreign bonds:

$$\Omega_{t,t+1} = (r_t^*\Theta_t)^{-1}E_t\{\pi_{C,t+1}/\pi_{\varepsilon,t+1}\}. \tag{C.5}$$

- Country premium:

$$\Theta_t = \Theta\exp[-\varrho(BY_t - \overline{BY}) + (\zeta_{\Theta,t} - \bar{\zeta}_\Theta)/\bar{\zeta}_\Theta]. \tag{C.6}$$

- Non-Ricardian consumption:

$$c_t^{NR} = \lambda(w_t l_t - tax_t). \tag{C.7}$$

- Consumption across households:

$$c_t = (1-\lambda)c_t^R + \lambda c_t^{NR}. \tag{C.8}$$

- Real price of consumption (numeraire $p_{C,t}=1$):

$$1 = \alpha_C p_{Z,t}^{1-\omega_C} + \alpha_A p_{A,t}^{1-\omega_C} + \alpha_O p_{O,t}^{1-\omega_C}. \tag{C.9}$$

- Core consumption:

$$c_{Z,t} = \alpha_C p_{Z,t}^{-\omega_C} c_t. \tag{C.10}$$

- Food consumption:

$$c_{A,t} = \alpha_A p_{A,t}^{-\omega_C} c_t. \tag{C.11}$$

- Oil consumption:

$$c_{O,t} = \alpha_O p_{O,t}^{-\omega_C} c_t. \tag{C.12}$$

- Real price of core consumption:

$$p_{Z,t} = [\gamma_C p_{H_D,t}^{1-\eta_C} + (1-\gamma_C)p_{F,t}^{1-\eta_C}]^{1/(1-\eta_C)}. \tag{C.13}$$

- Core consumption price inflation:

$$\pi_{Z,t} = (p_{Z,t}/p_{Z,t-1})\pi_{C,t}. \tag{C.14}$$

- Consumption of home goods:

$$c_{H,t} = (\alpha_C + \alpha_A)\gamma_C(p_{H_D,t}/p_{Z,t})^{-\eta_C} p_{Z,t}^{-\omega_C} c_t. \tag{C.15}$$

- Consumption of foreign goods:

$$c_{F,t} = (\alpha_C + \alpha_A)(1 - \gamma_C)(p_{F,t}/p_{Z,t})^{-\eta_C} p_{Z,t}^{-\omega_C} c_t. \tag{C.16}$$

- Real price of food consumption:

$$p_{A,t} = [\gamma_C p_{H,t}^{1-\eta_C} + (1 - \gamma_C) p_{F,t}^{1-\eta_C}]^{1/(1-\eta_C)} \zeta_{A,t}. \tag{C.17}$$

- Wage setting, first recursive term:

$$\begin{aligned} \mathcal{F}_{W,t} \;\; = \;\; & \lambda_t \zeta_{C,t} w_t l_t \left(\tilde{w}_t / w_t\right)^{1-\epsilon_L} (\epsilon_L - 1)/\epsilon_L \\ & + \beta \phi_L E_t \{[(\tilde{w}_t / \tilde{w}_{t+1})\, \bar{\pi}^{1-\chi_L} \pi_{C,t}^{\chi_L} \pi_{C,t+1}^{-1}]^{1-\epsilon_L} \mathcal{F}_{W,t+1}\}. \end{aligned} \tag{C.18}$$

- Wage setting, second recursive term:

$$\begin{aligned} \mathcal{F}_{W,t} \;\; = \;\; & \lambda_t \zeta_{C,t} (w_t / \tilde{\mu}_t) l_t \left(\tilde{w}_t / w_t\right)^{-\epsilon_L(1+\sigma_L)} \\ & + \beta \phi_L E_t \{[(\tilde{w}_t / \tilde{w}_{t+1})\, \bar{\pi}^{1-\chi_L} \pi_{C,t}^{\chi_L} \pi_{C,t+1}^{-1}]^{-\epsilon_L(1+\sigma_L)} \mathcal{F}_{W,t+1}\}. \end{aligned} \tag{C.19}$$

- Wage aggregation:

$$w_t^{-\epsilon_L} = (1 - \phi_L) \tilde{w}_t^{-\epsilon_L} + \phi_L (\bar{\pi}^{1-\chi_L} \pi_{C,t-1}^{\chi_L} \pi_{C,t}^{-1} w_{t-1})^{-\epsilon_L}. \tag{C.20}$$

C.2 Domestic goods

- Law of motion for capital:

$$k_t = (1 - \delta) \exp(\zeta_{K,t}) k_{t-1} \zeta_{T,t}^{-1} + (1 - \Phi_t) \zeta_{I,t} i_t. \tag{C.21}$$

- Investment adjustment costs:

$$\Phi_t = (\mu/2)[(i_t / i_{t-1}) \zeta_{T,t} - (1 + g_Y)]^2. \tag{C.22}$$

- Investment adjustment costs, first derivative:

$$\Phi_t' = \mu[(i_t / i_{t-1}) \zeta_{T,t} - (1 + g_Y)]. \tag{C.23}$$

- First-order condition for investment:

$$p_{I,t} = q_t[1 - \Phi_t - (i_t / i_{t-1}) \zeta_{T,t} \Phi_t'] \zeta_{I,t} + E_t\{\Omega_{t,t+1} q_{t+1} [(i_{t+1} / i_t) \zeta_{T,t+1}]^2 \Phi_{t+1}' \zeta_{I,t+1}\}. \tag{C.24}$$

- Shadow price of capital:

$$q_t = E_t\{\Omega_{t,t+1}[z_{t+1} + q_{t+1}(1 - \delta)]\}. \tag{C.25}$$

- Investment of home goods:

$$i_{H,t} = \gamma_I (p_{H_D,t} / p_{I,t})^{-\eta_I} i_t. \tag{C.26}$$

- Investment of foreign goods:
$$i_{F,t} = (1-\gamma_I)(p_{F,t}/p_{I,t})^{-\eta_I} i_t. \tag{C.27}$$

- Real price of investment:
$$p_{I,t} = [\gamma_I p_{H_D,t}^{1-\eta_I} + (1-\gamma_I) p_{F,t}^{1-\eta_I}]^{1/(1-\eta_I)}. \tag{C.28}$$

- Production technology:
$$y_{H,t} = a_{H,t}[{\alpha_H}^{1/\omega_H} {v_{H,t}}^{(\omega_H-1)/\omega_H} + (1-\alpha_H)^{1/\omega_H} o_{H,t}^{(\omega_H-1)/\omega_H}]^{\omega_H/(\omega_H-1)}. \tag{C.29}$$

- Value added:
$$v_{H,t} = l_t^{\eta_H}[\exp(\zeta_{K,t}) k_{t-1} \zeta_{T,t}^{-1}]^{1-\eta_H}. \tag{C.30}$$

- Labor/capital relative input demand:
$$w_t/z_t = [\eta_H/(1-\eta_H)] \exp(\zeta_{K,t}) k_{t-1} \zeta_{T,t}^{-1}/l_t. \tag{C.31}$$

- Labor/oil relative input demand:
$$w_t/p_{O,t} = \eta_H[\alpha_H/(1-\alpha_H)]^{1/\omega_H} v_{H,t}^{(\omega_H-1)/\omega_H} o_{H,t}^{1/\omega_H}/l_t. \tag{C.32}$$

- Marginal cost of home goods:
$$mc_{H,t} = (w_t l_t + z_t \exp(\zeta_{K,t}) k_{t-1} \zeta_{T,t}^{-1} + p_{O,t} o_{H,t})/y_{H,t}. \tag{C.33}$$

- Domestic price setting, first recursive term:
$$\begin{aligned} \mathcal{F}_{H_D,t} &= \tilde{p}_{H_D,t}^{-\epsilon_H} p_{H_D}^{-1} mc_{H,t}[c_{H,t} + i_{H,t} + i_{H,t}(S) + g_{H,t}] \\ &\quad +\phi_{H_D} E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\tilde{p}_{H_D,t}/\tilde{p}_{H_D,t+1})\bar{\pi}^{1-\chi_{H_D}} \pi_{H_D,t}^{\chi_{H_D}} \pi_{H_D,t+1}^{-1}]^{-\epsilon_H} \\ (\pi_{H_D,t+1}/\pi_{C,t+1})\mathcal{F}_{H_D,t+1} \end{array} \right\}. \end{aligned} \tag{C.34}$$

- Domestic price setting, second recursive term:
$$\begin{aligned} \mathcal{F}_{H_D,t} &= \tilde{p}_{H_D,t}^{1-\epsilon_H}[c_{H,t} + i_{H,t} + i_{H,t}(S) + g_{H,t}](\epsilon_H - 1)/\epsilon_H \\ &\quad +\phi_{H_D} E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\tilde{p}_{H_D,t}/\tilde{p}_{H_D,t+1})\bar{\pi}^{1-\chi_{H_D}} \pi_{H_D,t}^{\chi_{H_D}} \pi_{H_D,t+1}^{-1}]^{1-\epsilon_H} \\ (\pi_{H_D,t+1}/\pi_{C,t+1})\mathcal{F}_{H_D,t+1} \end{array} \right\}. \end{aligned} \tag{C.35}$$

- Domestic price aggregation:
$$1 = (1-\phi_{H_D})\tilde{p}_{H_D,t}^{1-\epsilon_H} + \phi_{H_D}(\bar{\pi}^{1-\chi_{H_D}} \pi_{H_D,t-1}^{\chi_{H_D}} \pi_{H_D,t}^{-1})^{1-\epsilon_H}. \tag{C.36}$$

- Domestic price inflation:
$$\pi_{H_D,t} = (p_{H_D,t}/p_{H_D,t-1})\pi_{C,t}. \tag{C.37}$$

- Export price setting, first recursive term:

$$\begin{aligned} \mathcal{F}_{H_F,t} &= (\tilde{p}^*_{H_F,t})^{-\epsilon_H}(p^*_{H_F})^{-1} mc_{H,t} y^*_{H,t} \\ &\quad + \phi_{H_F} E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\pi^*)^{1-\chi_{H_F}}(\pi^*_{H_F,t})^{\chi_{H_F}}(\pi^*_{H_F,t+1})^{-1}]^{-\epsilon_H} \\ (\tilde{p}^*_{H_F,t}/\tilde{p}^*_{H_F,t+1})^{-\epsilon_H}(\pi^*_{H_F,t+1}/\pi^*_{t+1})\mathcal{F}_{H_F,t+1} \end{array} \right\}. \end{aligned} \tag{C.38}$$

- Export price setting, second recursive term:

$$\begin{aligned} \mathcal{F}_{H_F,t} &= (\tilde{p}^*_{H_F,t})^{1-\epsilon_H} rer_t y^*_{H,t}(\epsilon_H - 1)/\epsilon_H \\ &\quad + \phi_{H_F} E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\pi^*)^{1-\chi_{H_F}}(\pi^*_{H_F,t})^{\chi_{H_F}}(\pi^*_{H_F,t+1})^{-1}]^{1-\epsilon_H} \\ (\tilde{p}^*_{H_F,t}/\tilde{p}^*_{H_F,t+1})^{1-\epsilon_H}(\pi^*_{H_F,t+1}/\pi^*_{t+1})\mathcal{F}_{H_F,t+1} \end{array} \right\}. \end{aligned} \tag{C.39}$$

- Export price aggregation:

$$1 = (1 - \phi_{H_F})\,(\tilde{p}^*_{H_F,t})^{1-\epsilon_H} + \phi_{H_F}[(\pi^*)^{1-\chi_{H_F}}(\pi^*_{H_F,t-1})^{\chi_{H_F}}(\pi^*_{H_F,t})^{-1}]^{1-\epsilon_H}. \tag{C.40}$$

- Export price inflation:

$$\pi^*_{H_F,t} = (p^*_{H_F,t}/p^*_{H_F,t-1})\pi^*_t. \tag{C.41}$$

C.3 Imported goods

- Import price setting, first recursive term:

$$\begin{aligned} \mathcal{F}_{F,t} &= \tilde{p}_{F,t}^{-\epsilon_F} p_F^{-1} rer_t y_{F,t} \\ &\quad + \phi_F E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\tilde{p}_{F,t}/\tilde{p}_{F,t+1})\bar{\pi}^{1-\chi_F}\pi_{F,t}^{\chi_F}\pi_{F,t+1}^{-1}]^{-\epsilon_F} \\ (\pi_{F,t+1}/\pi_{C,t+1})\mathcal{F}_{F,t+1} \end{array} \right\}. \end{aligned} \tag{C.42}$$

- Import price setting, second recursive term:

$$\begin{aligned} \mathcal{F}_{F,t} &= \tilde{p}_{F,t}^{1-\epsilon_F} y_{F,t}(\epsilon_F - 1)/\epsilon_F \\ &\quad + \phi_F E_t \left\{ \begin{array}{c} \Omega_{t,t+1}\zeta_{T,t+1}[(\tilde{p}_{F,t}/\tilde{p}_{F,t+1})\bar{\pi}^{1-\chi_F}\pi_{F,t}^{\chi_F}\pi_{F,t+1}^{-1}]^{1-\epsilon_F} \\ (\pi_{F,t+1}/\pi_{C,t+1})\mathcal{F}_{F,t+1} \end{array} \right\}. \end{aligned} \tag{C.43}$$

- Import price aggregation:

$$1 = (1 - \phi_F)\,\tilde{p}_{F,t}^{1-\epsilon_F} + \phi_F(\bar{\pi}^{1-\chi_F}\pi_{F,t-1}^{\chi_{H_F}}\pi_{F,t}^{-1})^{1-\epsilon_F}. \tag{C.44}$$

- Import price inflation:

$$\pi_{F,t} = (p_{F,t}/p_{F,t-1})\pi_{C,t}. \tag{C.45}$$

C.4 Commodity sector

- Production technology:

$$y_{S,t} = a_{S,t}(k_{S,t-1}\zeta_{T,t}^{-1})^{1-\eta_S}. \tag{C.46}$$

- Law of motion for capital:

$$k_{S,t} = (1-\delta_S)k_{S,t-1}\zeta_{T,t}^{-1} + (1-\Phi_{S,t-n+1})x_{S,t-n+1}\prod_{j=0}^{n-2}\zeta_{T,t-j}^{-1}. \tag{C.47}$$

- Investment adjustment costs:

$$\Phi_{S,t} = (\mu_{I_S}/2)[(x_{S,t}/x_{S,t-1})\zeta_{T,t} - (1+g_Y)]^2. \tag{C.48}$$

- Investment adjustment costs, first derivative:

$$\Phi'_{S,t} = \mu_{I_S}[(x_{S,t}/x_{S,t-1})\zeta_{T,t} - (1+g_Y)]. \tag{C.49}$$

- First-order condition for investment projects:

$$\begin{aligned}&\sum_{j=0}^{n-1}\varphi_j E_t\{\Omega_{t,t+j}p_{I_S,t+j}\}\\ &\quad = E_t\left\{\begin{array}{c}\Omega_{t,t+n-1}q_{S,t+n-1}\left[1-\Phi_{S,t}-(x_{S,t}/x_{S,t-1})\,\zeta_{T,t}\Phi'_{S,t}\right]\zeta_{I_S,t}\\ +\Omega_{t,t+n}q_{S,t+n}[(x_{S,t+1}/x_{S,t})\zeta_{T,t+1}]^2\Phi'_{S,t+1}\zeta_{I_S,t+1}\end{array}\right\}.\end{aligned} \tag{C.50}$$

- Shadow price of capital:

$$q_{S,t} = E_t\{\Omega_{t,t+1}[q_{S,t+1}(1-\delta_S) + p_{S,t+1}A_{S,t+1}(k_{S,t}\zeta_{T,t+1}^{-1})^{1-\eta_S}]\}. \tag{C.51}$$

- Law of motion for investment:

$$i_{S,t} = \sum_{j=0}^{n-1}\varphi_j x_{S,t-j}\prod_{h=0}^{j-1}\zeta_{T,t-h}^{-1}. \tag{C.52}$$

- Investment of home goods:

$$i_{H,t}(S) = \gamma_{I_S}(p_{H_D,t}/p_{I_S,t})^{-\eta_{I_S}}i_{S,t}. \tag{C.53}$$

- Investment of foreign goods:

$$i_{F,t}(S) = (1-\gamma_{I_S})(p_{F,t}/p_{I_S,t})^{-\eta_{I_S}}i_{S,t}. \tag{C.54}$$

- Real price of investment:

$$p_{I_S,t} = [\gamma_{I_S}p_{H_D,t}^{1-\eta_{I_S}} + (1-\gamma_{I_S})p_{F,t}^{1-\eta_{I_S}}]^{1/(1-\eta_{I_S})}. \tag{C.55}$$

C.5 Fiscal and monetary policy

- Government net asset position (% of GDP):

$$BGY_t(r_t^*\Theta_t)^{-1} = BGY_{t-1}(\pi_{\varepsilon,t}/\pi_{C,t})(p_{Y,t-1}/p_{Y,t})(y_{t-1}/y_t)\zeta_{T,t}^{-1} + FIY_t - GY_t. \tag{C.56}$$

- Fiscal income:

$$\begin{aligned} FIY_t &= TAXY_t + \chi[YSY_t - \kappa_S(p_{Y,t}y_t)^{-1} - p_{I_S,t}i_{S,t}(p_{Y,t}y_t)^{-1}] \\ &\quad +\tau_S(1-\chi)[YSY_t - \kappa_S(p_{Y,t}y_t)^{-1} - \delta_S q_{S,t}k_{S,t-1}(\zeta_{T,t}p_{Y,t}y_t)^{-1}]. \end{aligned} \tag{C.57}$$

- Income tax:

$$TAXY_t = tax_t(p_{Y,t}y_t)^{-1}. \tag{C.58}$$

- Income from commodity sector:

$$YSY_t = p_{S,t}y_{S,t}(p_{Y,t}y_t)^{-1}. \tag{C.59}$$

- Structural fiscal income:

$$\begin{aligned} \widetilde{FIY}_t &= \widetilde{TAXY}_t + \chi[\widetilde{YSY}_t - \kappa_S(p_{Y,t}y_t)^{-1} - p_{I_S,t}i_{S,t}(p_{Y,t}y_t)^{-1}] \\ &\quad +\tau_S(1-\chi)[\widetilde{YSY}_t - \kappa_S(p_{Y,t}y_t)^{-1} - \delta_S q_{S,t}k_{S,t-1}(\zeta_{T,t}p_{Y,t}y_t)^{-1}]. \end{aligned} \tag{C.60}$$

- Structural income tax:

$$\widetilde{TAXY}_t = TAXY_t(\bar{y}/y_t). \tag{C.61}$$

- Structural income from commodity sector:

$$\widetilde{YSY}_t = \tilde{p}_{S,t}y_{S,t}(p_{Y,t}y_t)^{-1}. \tag{C.62}$$

- Long-run commodity price:

$$\log(\tilde{p}_{S,t}/\bar{p}_S) - \log(rer_t/\overline{rer}) = (\rho/40)(1-\rho^{40})(1-\rho)^{-1}\hat{b}_t. \tag{C.63}$$

- Fiscal rule for expenditure:

$$GY_t = \left[\begin{array}{c} (1-(r^*_{t-1})^{-1}\Theta^{-1}_{t-1})BGY_{t-1}(\pi_{\varepsilon,t}/\pi_{C,t})(p_{Y,t-1}/p_{Y,t})(y_{t-1}/y_t)\zeta^{-1}_{T,t} \\ +\widetilde{FIY}_t - \bar{s}_B \end{array} \right] \frac{p_{G,t}\zeta_{G,t}}{p_{Y,t}y_t}. \tag{C.64}$$

- Fiscal instrument (expenditure adjusts to rule):

$$TAXY_t = \overline{TAXY}. \tag{C.65}$$

- Government consumption of home goods:

$$g_{H,t} = g_t. \tag{C.66}$$

- Government consumption of foreign goods:

$$g_{F,t} = 0. \tag{C.67}$$

- Real price of government consumption:

$$p_{G,t} = p_{H,t}. \tag{C.68}$$

- Monetary policy rule:

$$r_t/\bar{r} = (r_t/\bar{r})^{\psi_r}\left[(\pi_{Z,t}/\bar{\pi})^{\psi_\pi \psi_{\pi Z}}(\pi_{C,t}/\bar{\pi})^{\psi_\pi\left(1-\psi_{\pi Z}\right)}(y_t/y_{t-1})^{\psi_Y}\right]^{1-\psi_r}\exp(\zeta_{m,t}). \tag{C.69}$$

C.6 Rest of the world

- Real exchange rate:

$$rer_t/rer_{t-1} = \pi_{\varepsilon,t}\pi^*_t\pi_{C,t}. \tag{C.70}$$

- Law of one price, commodity good:

$$p_{S,t} = rer_t p^*_{S,t}. \tag{C.71}$$

- Law of one price, oil:

$$p_{O,t} = rer_t p^*_{O,t}. \tag{C.72}$$

- Foreign demand for home goods:

$$y^*_{H,t} = \zeta^*(p^*_{H_F,t})^{-\eta^*}y^*_t. \tag{C.73}$$

C.7 Aggregate equilibrium

- Internal equilibrium:

$$\mathbf{y}_{H,t} = c_{H,t} + i_{H,t} + i_{H,t}(S) + g_{H,t} + y^*_{H,t}. \tag{C.74}$$

- External equilibrium:

$$y_{F,t} = c_{F,t} + i_{F,t} + i_{F,t}(S) + g_{F,t}. \tag{C.75}$$

- Exports quantum:

$$x_t = y^*_{H,t} + y_{S,t}. \tag{C.76}$$

- Imports quantum:

$$m_t = y_{F,t} + c_{O,t} + o_{H,t}. \tag{C.77}$$

- Relative price of exports:

$$p_{X,t}x_t = rer_t p^*_{H_F,t}y^*_{H,t} + p_{S,t}y_{S,t}. \tag{C.78}$$

- Relative price of imports:

$$p_{M,t}m_t = p_{F,t}y_{F,t} + p_{O,t}(c_{O,t} + o_{H,t}). \tag{C.79}$$

- Real GDP:

$$y_t = c + i_t + i_{S,t} + g_t + x_t - m_t. \tag{C.80}$$

- GDP deflator:

$$p_{Y,t}y_t = p_{C,t}c_t + p_{I,t}i_t + p_{I_S,t}i_{S,t} + p_{G,t}g_t + p_{X,t}x_t - p_{M,t}m_t. \tag{C.81}$$

- Net foreign asset position (% of GDP):

$$BY_t(r_t^*\Theta_t)^{-1} = BY_{t-1}(\pi_{\varepsilon,t}/\pi_{C,t})(p_{Y,t-1}/p_{Y,t})(y_{t-1}/y_t)\zeta_{T,t}^{-1} + NXY_t + RENY_t. \tag{C.82}$$

- Net exports:

$$NXY_t = (p_{X,t}x_t - p_{M,t}m_t)(p_{Y,t}y_t)^{-1}. \tag{C.83}$$

- Net rents:

$$\begin{aligned} RENY_t &= (1-\chi)[p_{I_S,t}i_{S,t}(p_{Y,t}y_t)^{-1} - \tau_S\delta_S q_{S,t}k_{S,t-1}(\zeta_{T,t}p_{Y,t}y_t)^{-1}] \\ &\quad -(1-\chi)(1-\tau_S)[YSY_t - \kappa_S(p_{Y,t}y_t)^{-1}]. \end{aligned} \tag{C.84}$$

- Current account:

$$\begin{aligned} CAY_t &= BY_t(r_t^*\Theta_t)^{-1} - BY_{t-1}(r_{t-1}^*\Theta_{t-1})^{-1}(\pi_{\varepsilon,t}/\pi_{C,t})(p_{Y,t-1}/p_{Y,t})(y_{t-1}/y_t)\zeta_{T,t}^{-1} \\ &\quad -(1-\chi)q_{S,t}(k_{S,t} - k_{S,t-1}\zeta_{T,t}^{-1})(p_{Y,t}y_t)^{-1}. \end{aligned} \tag{C.85}$$

C.8 Imperfect information and learning about commodity price shocks

- Actual (log) real commodity price:

$$\log(p_{S,t}^*/\bar{p}_S^*) = \hat{a}_t + \hat{b}_t. \tag{C.86}$$