The Political Economy of Debt and Entitlements*

Laurent Bouton† Alessandro Lizzeri‡ Nicola Persico§

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Abstract

This paper presents a dynamic political-economic model of government obligations. Its focus is on the interplay between debt and entitlements. In our model, both are tools by which temporarily powerful groups can extract resources from groups that will be powerful in the future. Debt transfers resources across periods; entitlements directly target the future allocation of resources. We prove five main results. First, debt and entitlement are strategic substitutes in the sense that constraining debt increases entitlements (and vice versa). Second, if entitlements are unconstrained, it is sometimes beneficial not to constrain debt (even in the absence of shocks that require smoothing). Third, if debt is unconstrained, it is beneficial to limit entitlements but not to eliminate them. Fourth, debt and entitlements respond in opposite ways to political instability and, in contrast with prior literature, political instability may even reduce debt when entitlements are endogenous. Finally, we identify two possible explanations for the joint growth of debt and entitlements.

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†Department of Economics, Georgetown University, and ECARES, Université libre de Bruxelles. E-mail: boutonllj@gmail.com

‡Department of Economics, NYU. E-mail: alessandro.lizzeri@nyu.edu

§Kellogg School of Management, Northwestern University. E-mail: nicola@nicolapersico.com

1 Introduction

Increasingly, governments struggle to meet their fiscal obligations. Recent examples include Greece and Puerto Rico, as well as some US states (e.g., Illinois) and cities (e.g., Detroit and San Bernardino).

Fiscal stress is usually blamed on government debt. Of course, a large debt burden and the consequent heavy interest rate payments reduce government's capacity to absorb fiscal shocks. But in addition to debt, governments have other spending obligations that, like interest payments, are difficult to compress, most notably spending on pensions and health care. Because these programs can be difficult to renegotiate, in the US they are referred to as entitlements. In this paper, we shall call *entitlements* those spending programs that cannot easily be renegotiated. By this definition, a large portion of the social expenditure in many countries is an entitlement.[1]

Entitlements can be difficult to renegotiate for two main reasons. The first is that sometimes they are protected by law, even through bankruptcy.[2] In the vast majority of US states, legal constraints are such that "changing future benefits for current employees is extremely difficult" (Munnell and Quinby, 2012). In addition to the law, veto powers and status-quo bias built into the political system also help prevent renegotiation without the beneficiaries' consent (more on this in Section 9.4 below). This is not to say that pensions cannot be changed: in Italy, for instance, pensions were restructured in a way that may be interpreted as default, while there has been no default on debt. However, when pensions are revised, it is almost never the case that benefits of current retirees are cut.

Entitlements are an important determinant of fiscal sustainability in many jurisdictions because they are large (on a flow basis, larger than interest payments on debt) and because by definition they cannot easily be compressed. Sometimes entitlements are practically the *sole determinant* of fiscal sustainability: this is the case for some US states where debt is strictly capped by state constitutions. Entitlements have grown rapidly since the

[1] Our definition is consistent with Schick (2009, p. 2-3): "An entitlement is a provision of law that establishes a legal right to public funds. [...] The law usually sets forth eligibility requirements and either a schedule of payments or a formula by which the payments are computed. Social security, unemployment compensation, family allowances and disability payments are entitlements because they accord particular classes of the population rights to money from the public treasury. [...] The government (or the social security fund) is obligated to pay the amount to which recipients are entitled whether or not sufficient funds have been set aside for this purpose in the budget. In many countries, a permanent appropriation finances social security and various other entitlement programmes. But even when the entitlement is financed by annual appropriations, the government must provide the benefits mandated by law." For US federal spending, entitlements can be defined precisely as government spending that is *mandatory* –that is, not subject to appropriation.

[2] In the case of the San Bernardino bankruptcy, for example, state law has fully protected government pensions (thus making them senior even to debt). See http://www.sacbee.com/news/business/article20770341.html and http://www.nytimes.com/2015/07/24/business/dealbook/government-pension-cuts-tangled-in-patchwork-of-legal-rulings.html

1960s and have overtaken discretionary spending. Figure 1 displays this evolution for the US federal government, . The figure suggests that entitlement programs must crowd out other types of government spending such as, perhaps, infrastructure and R&D spending.[3,4] Figure 2 depicts a similar pattern in the growth of entitlements across OECD countries.

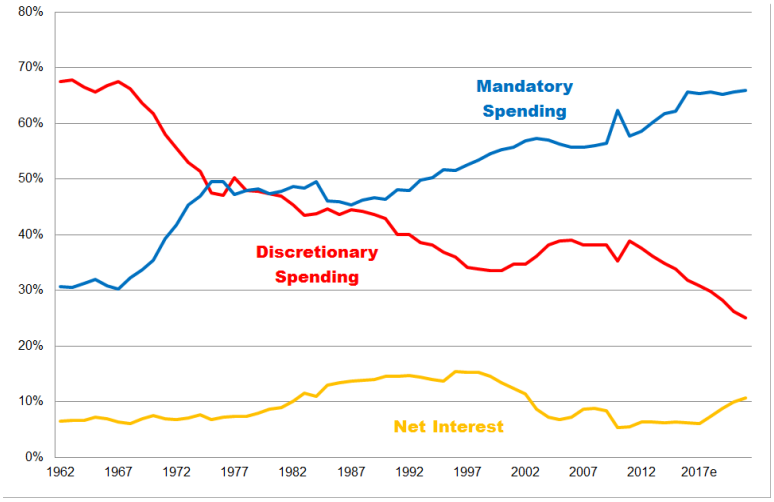


Figure 1: Types of US Federal Government Spending (as a Share of Total Government Spending). Data Source: US Office of Management and Budget, Budget of the United States Government, Fiscal Year 2015: Historical Tables, 2014, pp. 168-169, Table 8.4.

Pioneering academics have long pointed out that debt and entitlements should be recognized, from an accounting perspective, as a combined burden (see Kotlikoff and Burns, 2004). On occasion, this accounting fungibility has even been made explicit through policy choices.[5] Accordingly, the European Commission has recently developed a forward-looking measure of fiscal sustainability, the "intertemporal net worth," that captures government obligations much more broadly than simple measures of debt.[6] But despite this recogni-

[3] See, e.g., Steuerle (2014) and David Crane: "New California Taxes Pay for Pensions, Not Students." Bloomberg, April 23 2012.

[4] Another way to illustrate this evolution is to look at the Steuerle and Roeper index of Fiscal Democracy that measures the percentage of (projected) revenues **not** claimed by permanent programs currently in place. In the US, this index dropped from 65% in 1962 to a range between 0 and 20 percent in the period 1998-2012; it is forecast to stay in this range through 2022, and there is no expectation of improvement in the more distant future. See Steuerle (2014). Evans, Kotlikoff, and Phillips (2012) provide another measure of fiscal sustainability –the so-called *duration to game over*. In the case of the US, this measure also points to the high (or even unsustainable) fiscal burden of entitlement programs.

[5] "Chile performed such a greased swap in 1980. [...] It took back pension promises from workers in exchange for government bonds plus a kicker. Argentina, Bolivia, and Hungary have forcefully confiscated private pension assets (largely government bonds), swapping them for larger future pension commitments." Newsweek, January 15 2015.

[6] Intertemporal Net Worth is based on the total discounted sum of future primary balances under current policies and current net worth. This measure suggests that assessing the fiscal burden via debt only and ignoring entitlements not only severely understates the problem but can also bring about misleading

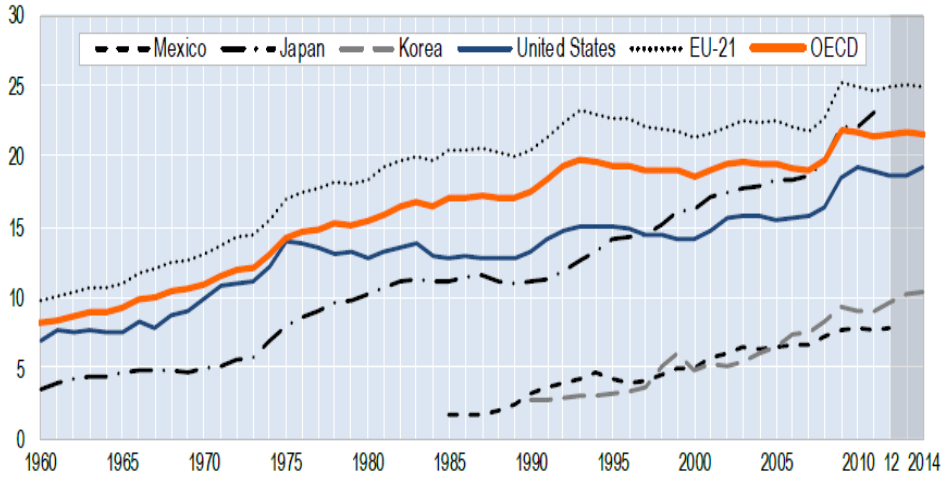


Figure 2: Public social expenditure as a percent of GDP in 1960-2014. Source: OECD (2014).

tion in the policy world and at the level of measurement, there is no academic work that studies the joint determination of debt and entitlements.

In this paper we provide a politico-economic model in which debt and entitlement levels are *jointly determined* in equilibrium. Because we aim to keep our analysis comparable with the previous literature, our starting point is a slight variant on the canonical politico-economic model by Alesina and Tabellini (1990), which focuses on debt only. We then allow our agents to set entitlements as well as debt. The resulting model allows us to study the interplay between debt and entitlements and thus to investigate the *political economy of government obligations.*

The key ingredients of the model are the following. In each period, a legislature determines spending on a public good as well as private goods for two groups. Political power changes over time, as for instance in an intergenerational setting. The currently powerful groups (we call these "group A") can use debt to transfer future resources to finance current consumption. In addition, they can set entitlements–i.e., "pre-commit" some fraction of future resources to a desired allocation–by setting up laws and institutions that protect such allocations. Thus, both debt and entitlements are tools for temporarily powerful groups to extract resources from groups that will be more powerful in the future (we call these "group B"). However, these tools have different effects that determine the way in which they are used in equilibrium.

Our results are the following. We first show that debt and entitlement are strategic substitutes in the sense that constraining debt increases entitlements (and vice versa).

inferences about the relative burden across countries. For instance, according to some of these measures the EU country in the best state of fiscal health is Italy, despite its extremely high level of government debt.

This result has direct implications for the evaluation of policies implementing debt limits–e.g., the so-called balanced budget rules that have long existed in many US states–and more generally, for the fiscal rules that are increasingly prevalent across many countries.[7] These policies may have the unintended (and hard-to-measure) consequence of increasing entitlements. This could in turn dramatically reduce the effect of those limits on total government obligations. We argue that our analysis has implications for the empirical literature studying the effect of fiscal rules (Poterba 1996, Alesina and Perotti 1999, Badinger and Reuter 2015).

We then explore the (utilitarian) social welfare effects of constraining entitlements. We show that, if debt is unconstrained, it is beneficial to limit entitlements but not to eliminate them. Entitlements are socially useful because they allow consumption smoothing for group A. Yet, because in equilibrium group A extracts too much from group B, it is socially beneficial to reduce group A's extractive ability, implying that a limit to entitlements is beneficial. Regarding debt, we show that in the presence of endogenous entitlements, a novel effect arises; this effect implies the possibility that balanced-budgets requirements may be inefficient even in the absence of any tax-smoothing motive driven by economic shocks.

We then explore how equilibrium levels of debt and entitlements respond to changes in the persistence of political power, or its opposite, government instability. This exploration is motivated by prior literature that provides compelling reasons to think that debt decreases as political power becomes more persistent.[8] This result can be reproduced in our environment when we do not allow for entitlements (or more generally, if entitlements are present but exogenous). However, we show that predictions become more nuanced if entitlements are endogenous. Specifically, we show that debt and entitlements move in opposite directions when persistence increases. Furthermore, debt sometimes increases with persistence–and even more surprisingly, the total fiscal burden (debt plus entitlements) may increase with persistence. This is a rich set of empirical implications that could in principle be tested in cross-country data, as was done for the Alesina-Tabellini model by Ozler and Tabellini (1991), Crain and Tollison (1993), Franzese (2002), and Lambertini (2003).

Finally, we study potential factors that may explain the joint growth of debt and entitlements in the past decades. In most OECD countries, fiscal pressure has been ratcheted up by the *simultaneous growth* in debt and entitlements. That these two aggregates should grow together is intriguing, because it is natural to think of debt and entitlements as *substitute* tools of intergenerational redistribution. Yet under some conditions, our model predicts that debt and entitlements are comonotonic, meaning that as some underlying

[7] The number of countries with at least one fiscal rule has grown from five in 1990 to 80 in 2012 (see Schaechter et al. 2012).

[8] The logic is that debt is issued by the currently politically powerful only when they fear losing power in the future (see the seminal work of Alesina and Tabellini 1990).

features of the environment change, debt and entitlements move together. For instance, we show that an increase in polarization of political parties, such as the one that has taken place in the US since the late 1970s, may lead to an increase in both debt and entitlements. Another example focuses on the effect of changes in idiosyncratic risk and inequalities.

2 Related Literature

Some of the papers in this literature explain debt as the outcome of a struggle between different groups in the population who want to gain more control over resources. The reason debt is accumulated is that the group that is in power today may not be in power tomorrow, and debt is a way to take advantage of this temporary power. For instance, Cukierman and Meltzer (1989) and Song, Storesletten, and Zilibotti (2012) argue that debt is a tool used to redistribute resources across generations. Persson and Svensson (1989), Alesina and Tabellini (1990), and Tabellini and Alesina (1990) argue that debt represents a way to tie the hands of future governments that will have different preferences from the current one. In Tabellini and Alesina (1990), voters choose the composition of public spending in an environment where the median voter theorem applies. If the median voter remains the same in both periods, the equilibrium involves budget balance. If the median voter tomorrow has different preferences, the current median voter may choose to run a budget deficit to take advantage of his temporary power and tie the hands of the future government. The equilibrium may also involve a budget surplus because there is an "insurance" component that links the two periods as well: a surplus tends to equalize the median voter's utility in the two periods. Tabellini and Alesina (1990) detail conditions such that deficits will be incurred and show that increased polarization leads to larger deficits.

Browning (1975) and Boadway and Wildasin (1989) have studied voting models of pensions in which age is the only dimension of heterogeneity. Conde-Ruiz and Galasso (2005) study a two-dimensional voting model in which pensions coexist with a welfare state. Thus they allow for voting on both intragenerational and intergenerational redistribution. They argue that pensions are particularly stable because the elderly are a relatively homogeneous voting group, and the pension system is supported by a broad coalition including the low-income young.

Tabellini (1991) also illustrates how debt and social security differ as distributional instruments in an overlapping generations environment. In contrast with our model, the main force concerns the difference in default between the two instruments.

Battaglini and Coate (2008) present a dynamic model of taxation and debt where a rich policy space is considered within a legislative bargaining environment. Velasco (1996) discusses a model where government resources are "common property" with which interest groups can finance their own consumption. Deficits arise in his model because of

a dynamic "common pools" problem. Lizzeri (1999) presents a model of debt as a tool of redistributive politics.

The paper is also related to work on legislative bargaining with endogenous status quo. Kalandrakis (2004) studies a classic divide-the-dollar problem where the division agreed to in one period is the status quo for the next period. Bowen, Chen and Eraslan (2014) study a model in which two parties decide unanimously how to allocate a given budget to spending on a public good and private transfers. The focus is on the comparison between two political institutions: discretionary vs. mandatory public good spending (private transfers are discretionary in both cases).[9] When the public good is discretionary (mandatory), the status quo level of the public good is zero (the one from the previous period). By contrast, we focus on the interplay between debt and entitlements.

A very different approach to understanding public debt is explored by Azzimonti et al. (2014). They propose a multi-country model with incomplete markets, and they show that governments may choose higher public debt when financial systems are more integrated. They thus offer an explanation of the rise in debt as driven by an increase in financial integration. Related to our discussion of the comovement of entitlements and debt, they also show that debt increases with the level of idiosyncratic risk.

3 Model

In this section we present the model. Section 9 provides a discussion of some of the assumptions.

3.1 Demography and economy

There are two groups, A and B, who each live for two periods. In each period t there is an endowment of 1 that can be allocated to private goods for the two groups x_A^t and x_B^t, or to a public good g^t. Preferences in each period are given by:

$$u_i\left(x_i^t, x_j^t, g^t\right) = h\left(x_i^t\right) + v\left(g^t\right),$$

where $h(\cdot)$ and $v(\cdot)$ are concave, and twice continuously differentiable. We also assume that the public good is sufficiently valuable that $v'(0) = \infty$, implying that it is not optimal for one group to spend all the resources on its own private good. Utility is additive across periods and there is no discounting. There are no capital markets to either privately save or borrow.[10]

[9] In a modified version of this model–with two periods and no private transfers–Bowen et al. (2015) allow the party making the take-it-or-leave-it offer to choose whether the public good is discretionary or mandatory. The focus is on the efficiency of the public good provision under various budgetary institutions.

[10] It can be shown that the results are unchanged if we allow for private savings and there is a positive tax on capital, which is clearly empirically plausible.

The resource constraints in periods 1 and 2 are given by:

$$\begin{aligned} x_A^1 + x_B^1 + g^1 &\leq 1 + d \\ x_A^2 + x_B^2 + g^2 &\leq 1 - d, \end{aligned}$$

where d represents debt or surplus, and the interest rate is assumed to be zero. We assume $x_A^t, x_B^t, g^t \geq 0$, and thus $d \in [-1, 1]$.

We assume no default on debt, but we revisit this assumption in Section 10.

3.2 Political structure and entitlements

Save for Section 7, throughout the paper the political structure is such that group A decides the allocation in period 1; group B decides the allocation in period 2 subject to debt and entitlements, as specified below.

In period 1, group A chooses the quintuple:

$$\left(x_A^1, x_B^1, g^1, d, E\right),$$

subject to the resource constraint. E is a nonnegative number that represents group A's entitlements in the future. In period 2, group B chooses the triple:

$$\left(x_A^2, x_B^2, g^2\right),$$

subject to the resource constraint and to the additional constraint that entitlements need to be honored:

$$x_A^2 \geq E.$$

Note that like debt, entitlements (E) are placed beyond group B's political ability to renegotiate.

4 Equilibrium Analysis

In this section, we start by studying the properties of a benchmark case in which entitlements are set to zero. We then compare these properties to those of the equilibrium with both debt and entitlements.

Definition 1 ***(second-period policies)*** *Define second-period policy choices conditional on a budget commitment of c as the set $X(c), G(c)$ that solves:*

$$\max_{(x,g)} h(x) + v(g) \;\; s.t. \;\; x + g \leq 1 - c.$$

$X(c)$ represents the amount of private good that a group (either A or B, whichever has the power to choose the allocation) would allocate itself in period 2, subject to the constraint that a fraction c of period 2's endowment has been reserved for other purposes. $G(c)$ represents the corresponding amount of public good.

Lemma 1 ***(well-behaved second-period policies)*** *Period-2 policy choices $X(c)$ and $G(c)$ are single-valued differentiable functions that are decreasing in c. Thus, increasing the fraction of the second period budget which is committed lowers private and public consumption in the second period.*

Proof. See Appendix A. ■

In period 2, group B is in power. We can use Definition 1 to describe group B's allocation choice.

Corollary 1 ***(equilibrium allocation in period 2)*** *Assume period 2 starts with predefined commitments d of debt and E of entitlements. Then in period 2 group B allocates exactly E to group A's private good, allocates $X(d+E)$ in private good to itself, and allocates $G(d+E)$ to the public good.*

Definition 2 ***(first-period policies)*** *Define first-period policy choices as the set (x^*, g^*, d^*, E^*) that solves:*

$$\max_{(x,g,d,E)} h(x) + v(g) + h(E) + v(G(d+E)) \;\; s.t. \;\; x + g \leq 1 + d. \tag{1}$$

The set (x^*, g^*, d^*, E^*) maximizes group A's lifetime payoff. This payoff is partly accrued in period 1 (the first two addends in equation (1)) and partly in period 2 (the last two addends in equation (1)). However, in period 2 group A does not control the allocation; therefore, it only receives private consumption in the amount it chose in entitlements in the first period, and it receives public consumption in whatever amount group B chooses to provide given the (uncommitted) resources available in the second period.

A useful feature of our model is that, absent entitlements, it behaves similarly to prior literature on debt, notably the literature following Alesina and Tabellini (1990). In this literature, debt arises whenever the currently powerful generation fears the loss of political power in the future.

Proposition 1 ***(No entitlement benchmark)*** *Fix $E \equiv 0$. Then in equilibrium:*

1. *Group A runs up debt in period 1, $d^*_{E=0} > 0$;*
2. *Group A's private consumption decreases between periods 1 and 2;*

3. Public good provision must decreases between periods 1 and 2.

Proof. See Appendix A. ■

To gain some intuition about these results, it is useful to consider the first order condition determining debt. Equation (2) is obtained by differentiating the objective function (1) with respect to d (and fixing $E = 0$):

$$h'(x) = -v'(G(d))\,G'(d). \tag{2}$$

If group A were in charge in both periods, then the term $G'(d)$ would not appear in the optimality condition of the agent. This term captures the fact that an extra dollar left to period 2 only increases public consumption by $G'(d) < 1$, the marginal amount chosen by group B, with the remainder going to its private consumption. We call the presence of this term the *crowdout effect*. The crowdout effect gives an incentive to increase debt. Furthermore, there is a *smoothing effect* that works as follows: because of concavity in the utility function, group A wants to smooth consumption over time. If public consumption is smaller in period 2 (which is the case when debt is positive), then the smoothing effect gives an incentive to decrease debt. The balance of these two effects determines the equilibrium level of debt.

Part 2 follows directly from the fact that group B has no incentives to allocate resources to group A's private consumption since it does not benefit from doing so.

Part 3 follows directly from part 1: since debt is positive, there are fewer resources effectively available for consumption, hence less public good, in period 2 than in period 1.

We now describe a result that offers an initial characterization of the equilibrium of the model with entitlements.[11]

Proposition 2 ***(Equilibrium characterization)*** *In equilibrium:*

1. *Total government obligations are always positive and, if $v(G(\cdot))$ is concave (refer to Lemma 4 in Appendix G for sufficient conditions), they are larger than in the case without entitlements: $d^* + E^* > d^*_{E=0}$;*

2. *Group A does not fully commit period 2's budget: $d^* + E^* < 1$;*

3. *Public consumption decreases between periods 1 and 2;*

4. *Group A's private consumption is perfectly smoothed;*

[11] It is easy to see that Proposition 2 holds when private saving is allowed and there is a positive tax τ on capital. In that case, private saving is 0 in equilibrium. Indeed, suppose that there is a positive equilibrium saving rate s by group A. This increases private consumption by $s(1-\tau)$ in the second period. Group A can increase its welfare by instead setting savings to zero and making a budget neutral change in entitlements and debt (that is, increase entitlements by $s(1-\tau)$ and decrease debt by the same amount). This leaves an extra amount of resources $s\tau$ available to group A, thereby raising its welfare. The same logic explains why other Propositions are also robust to allowing for private savings.

5. *Group A may choose to run a surplus; for example, if* $h(x) = v(x)$ *have a CRRA form* $(x)^{1-\rho}/(1-\rho)$, *then group A runs a surplus if and only if* $\rho > 1$.

Proof. See Appendix A. ■

Part 1 of Proposition 2 shows that allowing for entitlements increases government obligations. In our model both types of government obligations arise because in period 2 group A lacks political control and understands that a fraction of any uncommitted dollar will be diverted from public consumption to group B's private consumption. Absent entitlements, the only way to pre-commit period 2 dollars is to consume them today (by issuing debt). But, due to the concavity of the utility function, group A would prefer to allocate (at least some of) these period 2 dollars to its private consumption in period 2. This is exactly what entitlements allow group A to do. This additional commitment channel raises the value of committing period 2 dollars and therefore leads to larger government obligations.

Part 2 highlights the moderating role that the public good plays in our model. Group A does not wish to commit the entirety of tomorrow's resources, because this would lead to zero public consumption. We believe that this feature of the model is realistic in that one reason that current generations refrain from full fiscal depredation is that they realize that they will need some public goods when they are retired. These public goods can only be provided if whoever is in power then is left with some uncommitted fiscal capacity.

Part 3 indicates that the pre-commitment of period 2's budget results in a reduction in public good provision. This feature has an empirical counterpart in the crowding out of discretionary expenditures that is associated with the increase in entitlements (refer back to Figure 1).

Parts 4 and 5 of Proposition 2 highlight the differences with the no-entitlements benchmark. Absent entitlements, private consumption cannot be perfectly smoothed across periods. This is because group A lacks any political control over how money is spent in period 2, and group B has absolutely no incentive to provide private consumption for group A. By contrast, the availability of entitlements in conjunction with debt allows group A full control over its own private consumption in period 2. This is why the presence of entitlements allows for better intertemporal resource allocation (part 4) and lessens the proclivity to run up the debt (part 5).

As we did for the case without entitlements, it is useful to consider the two key first-order conditions that determine debt and entitlements. Equations (3) and (4) are obtained by differentiating the objective function (1) with respect to E and d respectively:

$$h'(E) = -v'(G(d+E))\,G'(d+E), \tag{3}$$

$$h'(x) = -v'(G(d+E))\,G'(d+E). \tag{4}$$

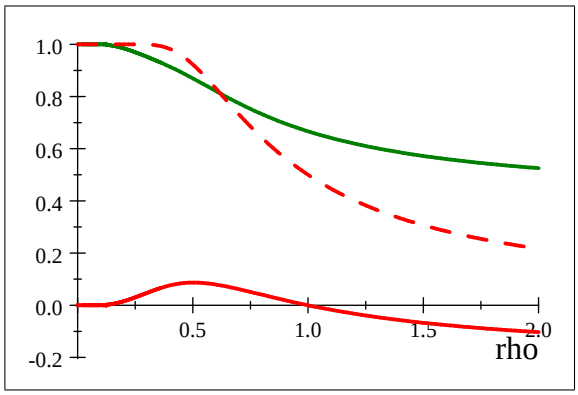
These equations illustrate the different roles of debt and entitlements. Group A uses debt to smooth consumption over time and entitlements to smooth consumption over types

of goods in period 2. As in the case of the model without entitlements, it is useful to discuss how crowdout and smoothing effects determine debt and entitlements. As in the model with a single "commitment instrument," the crowdout effect gives an incentive to group A to increase government obligations; this effect pushes toward both higher debt and higher entitlements. The smoothing effect, however, is quantitatively different in terms of how it affects debt, and qualitatively different in terms of how it affects entitlements.

Regarding debt, the smoothing effect is amplified by the presence of entitlements and in fact still remains strong even when debt is zero: part of period 2 resources are pre-committed to entitlements, and thus for any given level of debt, the imbalance in public consumption between periods 1 and 2 is even more pronounced. To smooth public consumption across periods, group A may therefore choose to run a surplus. Except for this important different, in their effect on debt, the role of the crowdout and smoothing effects are qualitatively similar to the case without entitlements discussed after Proposition 1.

In the case of entitlements, the smoothing effect can be either positive or negative because entitlements may be either lower or higher than public consumption depending on the importance of public consumption and the degree of risk aversion. In contrast, in the case of debt, the sign of the smoothing effect is always negative because public consumption is always lower in the second period.

Let us consider CRRA preferences (that is, $h(x) = (x)^{1-\rho}/(1-\rho)$ and $v(x) = \alpha h(x)$) and a low value of $\alpha = .5$, which represents an environment with relatively high distributive conflict since the value of public consumption is relatively low.[12] Figure 4 represents that case (with and without entitlements).



Green: E, Red: b with entitlements, Red Dash: b w/o entitlements

Consider first the case without entitlements. The dashed red line depicts equilibrium debt when entitlements are not allowed. In this case, debt is always positive. When $\rho \to 0$, the conflict among the two groups becomes extreme because group B would spend the entire budget on its private consumption, leaving nothing for the public good. The crowdout effect is thus absent, and group A choses maximal debt. As ρ increases, the

[12] A case with lower distributive conflict, where $\alpha = 1.5$, is discussed in Appendix B.

smoothing effect starts to matter more and more, and debt falls.

When entitlements are allowed (represented by plain lines), just as in the case with no entitlements, when $\rho \to 0$ there is an extreme conflict of views in period 2. However, the consequence is very different: in the limit there is no debt and full entitlements. Entitlements are indeed a superior way to capture the second period resources. Just as before, when ρ increases it becomes desirable to devote part of the budget to the public good, so entitlements drop. Debt responds non-monotonically to an increase in ρ. While resources become available for the public good over both periods, the amount invested in the public good is different in the two periods; for low values of ρ, the crowdout effect dominates, and for high values of ρ the smoothing effect dominates.

5 Strategic Substitutes Property

We now look more generally at the relation between debt and entitlements. The next proposition establishes that the effects of tightening a debt ceiling are partly (but not fully) offset by a strategic adjustment in entitlements–and viceversa, that the effects of constraining entitlements are partly (but not fully) offset by a strategic increase in debt.

Proposition 3 *Assume $v(G(\cdot))$ is concave (refer to Lemma 4 in Appendix G for sufficient conditions).*

1. ***(strategic substitutes property)** Fix debt at a level $\overline{d}$. The entitlement level $E\left(\overline{d}\right)$ that maximizes group A's lifetime utility conditional on $\overline{d}$ is a decreasing function of $\overline{d}$;*

2. ***(strategic substitutes property)** Fix entitlements at a level $\overline{E}$. The debt level $d\left(\overline{E}\right)$ that maximizes group A's lifetime utility conditional on $\overline{E}$ is a decreasing function of $\overline{E}$;*

3. ***(debt ceilings are partially effective)** Tightening a binding debt ceiling $\overline{d}$ reduces $\overline{d} + E\left(\overline{d}\right)$, the sum of debt and entitlements in equilibrium;*

4. ***(entitlement caps are partially effective)** Tightening a binding cap on entitlements $\overline{E}$ reduces $d\left(\overline{E}\right) + \overline{E}$, the sum of debt and entitlements in equilibrium.*

Proof. See Appendix B. ■

To understand the mechanism at work in part 1 (which also applies to part 2), imagine that group A is constrained and can only run debt up to $\overline{d}$. This may be because of a fiscal rule or for other reasons. Imagine now that the fiscal rule is relaxed. The relaxation causes a reduction in group B's fiscal capacity in the second period, and therefore a reduction in public-good spending in that period. This reduction raises the marginal cost of entitlements for group A.

Parts 3 and 4 show that, despite the partial crowding out, constraining either debts or entitlements still reduces the total obligations that group A bequeathes to group B.

Though it is simple, Proposition 3 has a number of important implications. First, consider the important literature that has highlighted the role of debt as an instrument that perpetuates temporary power (e.g., Alesina and Tabellini 1990, Tabellini and Alesina 1990, Persson and Svensson 1989).[13] If, consistent with this literature, entitlements were left out of our model (i.e., implicitly set to zero), Proposition 3 part 2 indicates that the equilibrium level of debt would be larger than if entitlements were accounted for by the model. That is, by abstracting from the presence of entitlements, a model overestimates how much debt is created in an effort to take advantage of temporary power. However, we also know from Proposition 2 part 1 that such a model would underestimate the level of governement obligations (i.e., the sum of debt and entitlements).

Second, Proposition 3 has consequences for policy evaluation. The implementation of fiscal rules or debt ceilings may have the unintended (and difficult-to-measure) consequence of increasing entitlements, thus partially offsetting the reduction in debt. By the same token, implementing welfare reform will make it harder to ensure fiscal stability. This latter trade-off seems confirmed by the current structure of the EU's *Stability and Growth Pact*, a fiscal rule that binds EU states. In 2005, it was agreed that the spending ceiling enacted by the Pact would be relaxed for countries who were implementing structural reforms. As in our Proposition 3, structural reform and deficit reduction are treated as strategic substitutes.

Third, the result yields a testable implication: entitlements should be larger where balanced-budget rules are more stringent.[14] This speaks to the literature exploring the effects of fiscal rules (Poterba 1996, Alesina and Perotti 1999, Badinger and Reuter 2015).

6 Welfare Effects of Constraining Debt or Entitlements

Absent entitlements, debt is harmful from the perspective of utilitarian social welfare: there is no insurance motive (no shock, recession, war, or natural disaster) in the model that could make debt desirable from the perspective of an omnipotent social planner. In this section we study the welfare effects of constraining debt or entitlements in our setting, where policies are not set by the social planner but by groups A and B.

We start with debt. The following proposition shows that, due to the general equilibrium effect of capping debt, the welfare effect of introducing such a cap can be rather nuanced, even at the margin.

[13] This determinant of debt is a major component of recent developments in the political economy theory of public debt (see, e.g., Battaglini and Coate 2008, Battaglini 2010, Barseghyan et al. 2014, and Azzimonti et al. 2015).

[14] A suggestive piece of prima facie evidence comes from the interaction between entitlements (proxied by the percentage of pensions unfunded)and the stringency of balanced-budget rules in US states (as measured by Hou and Smith 2006). The correlation between the two is indeed positive (0.173).

Proposition 4 ***(welfare effects of constraining debt).***

1. *Suppose debt is unconstrained and* $h(x) = v(x)$ *have a CRRA form* $(x)^{1-\rho}/(1-\rho)$. *Introducing a barely-binding debt cap leaves group A indifferent at the margin; group B is made strictly better off if* $\rho > (\log 2/\log 6)$ *and strictly worse off if* $\rho < (\log 2/\log 6)$*;*

2. *For some parameter configurations, relaxing a binding constraint on debt is a strict Pareto improvement: for example, if* $h(x) = v(x)$ *have a CRRA form* $(x)^{1-\rho}/(1-\rho)$, *then relaxing a binding constraint on debt makes both groups A and B better off at the margin if* $\rho < (\log 2/\log 6) \simeq 0.39$.

Proof. See Appendix C. ■

The intuition of part 1 of Proposition 4 is as follows. A marginal constraint on debt transfers resources from period 1 to period 2. This increases group B's utility in period 2 (total obligations go down, thus available resources to group B go up), but decreases it in period 1 (debt goes down, hence public good provision). To understand the net welfare effect, note that (i) group B's marginal utility of consumption is higher in period 2 than in period 1 (because public good provision is higher in the second period), and (ii) due to the endogeneous reaction of entitlements to a change in debt, only a portion of the resources transferred to period 2 benefit group B. Thus, the net welfare effect for group B is positive if the difference in marginal utility across periods is sufficient to compensate for the "loss" in resources.

Proposition 4 part 2 is rather intriguing. Despite the fact that debt is used by group A as a tool to expropriate group B, in equilibrium, allowing more debt can be socially beneficial. A central driver for Proposition 4 part 2 is the fact that, in our model, resources are used less efficiently in period 2 than in period 1. Indeed, group A chooses the level of entitlements, taking into account that only a fraction of the remaining budget in period 2 will be devoted to the public good by group B. As a consequence, group A entitles itself excessively (from a social perspective). By decreasing the budget in period 2, debt helps reduce that inefficiency. However, increasing debt is not without cost: it leads to a decrease in other types of consumption in period 2, which has higher marginal utility than period-1 consumption. Thus we find that it is not always true that relaxing a debt limit is a good thing.

The effect of a limit on entitlements is more straightforward. In our model, entitlements have positive and negative features: they allow group A to expropriate group B, but they also are a tool for group A to guarantee itself some consumption in period 2, allowing for some consumption smoothing across periods for group A. Accordingly, the following proposition shows that it is good to constrain entitlements a bit, but not too much. If one believes that the real-world status quo is one in which entitlements have been relatively

unconstrained thus far, then part 1 of Proposition 4 reassures us that a bit of reform might indeed be a good thing.

Proposition 5 ***(welfare effects of constraining entitlements)****.*

1. *Assume $v(G(\cdot))$ is concave (refer to Lemma 4 in Appendix G for sufficient conditions). There exists a constraint on entitlements that increases utilitarian social welfare;*

2. *Eliminating entitlements altogether decreases utilitarian welfare relative to any allocation with positive entitlements.*

Proof. See Appendix C. ■

7 Persistence in Power

An important question in prior literature (e.g., Alesina and Tabellini 1990) is whether debt is larger when the government is more unstable. The idea is that instability causes the conflicts between groups in charge in different periods to be particularly strong, so debt may respond to this. In fact, Alesina and Tabellini (1990) show that, under certain mild conditions on preferences, debt does increase as the political system becomes more unstable, as measured by the probability that the government remains in charge. The question that we wish to address is whether this result still holds once we allow for entitlements and, more generally, what is the effect of political instability on total government obligations.

We follow Alesina and Tabellini and assume that, conditional on being in charge in the first period, group A stays in charge with probability π, whereas power changes hands (group B takes over) with probability $1-\pi$. Thus, π is a measure of persistence of the political system, while $1-\pi$ is a measure of the instability of the political system.

In the event that group A persists in power, one needs to specify how entitlements constrain group A's period-2 decision. At issue is whether group A, if in power in period 2, might be forced to allocate at least E to its own private good, even if it prefers to reduce some of its own entitlements in favor of financing more public goods. We assume that this is not the case; that is, the entitlements set by group A in period 1 do not bind group A itself in the event that group A persists in power in period 2. This is because we feel that any generation should be able to give up some pre-existing entitlements easily if they wish.[15]

[15] For some assumptions on preferences, this assumption is not binding. Furthermore, for more general preferences, it is possible to relax this assumption without affecting the results qualitatively.

Proposition 6 ***(effects of increasing the probability π that group A stays in power in period 2).*** *Assume $v(G(\cdot))$ is concave (refer to Lemma 4 in Appendix G for sufficient conditions). In equilibrium:*

1. *absent entitlements, that is, if $E \equiv 0$, debt is decreasing in π.*
2. *debt and entitlements move in opposite directions when π varies;*
3. *total government obligations move in the same direction as debt when π varies;*
4. *debt and entitlements are monotonic in π; debt (entitlements) is monotonically decreasing (increasing) if equilibrium debt is positive at $\pi = 0$, and debt (entitlements) is monotonically increasing (decreasing) if equilibrium debt is negative at $\pi = 0$.*

Proof. See Appendix D. ■

The intuition behind part 1 is the following. When there are no entitlements–i.e., $E = 0$–the direct effect is that when group A remains in charge tomorrow, debt is harmful for that group because it reduces both private and public consumption (while it only reduces public consumption if B is in charge tomorrow). Thus, more political persistence (higher π) leads to a reduction in debt.

The intuition for Proposition 6 parts 2 and 3 is as follows. Given that entitlements are relevant only if group B is in power, their level does not depend directly on π. Rather, the effect of a change of π on entitlements is indirect–through the effect of π on debt. Given that debt and entitlements are substitutes, they move in opposite directions when π changes (part 2). Given that the elasticity of entitlements to debt is larger than -1, total government obligations move in the same direction as debt when π changes (part 3).

In order to gain an intuition for part 4, note that when group A stays in power, debt (or surplus) introduces an intertemporal distortion. Group A would prefer debt to be zero in this event. By increasing π, the probability of that distortion increases; hence debt must get closer to zero. The monotonicity for entitlements follows from the monotonicity proved in part 2.

Proposition 6 speaks to the testable implications sought by the literature following Alesina and Tabellini (1990). That literature seeks to explain debt accumulation using power persistence as the explanatory variable.[16] Our analysis suggests that entitlements are an important moderating variable in this relationship. For example, the evolution of debt should be different in a jurisdiction that has the ability to incur debt but not to alter entitlements (e.g., many European cities), compared to a jurisdiction that has the ability to alter both debt and entitlements (national governments, for example). By the same token, the evolution of entitlements should be different in jurisdictions in which debt is severely constrained (e.g., US states) compared to national governments.

[16] Ozler and Tabellini (1991), Crain and Tollison (1993), Lambertini (2003), and Franzese (2002).

8 Comovement of Debt and Entitlement

Debt and entitlements have risen together since the late 1970s. This is a puzzle since, as we saw above, they are (imperfect) substitute instruments of intertemporal redistribution. This puzzle is particularly acute in the case of a mechanical and exogenous growth of entitlement programs, such as the one that happened after the first oil shock. Indeed, politicians being forced, to some extent, to redistribute resources intertemporally through entitlements should be less willing to use other instruments of intertemporal redistribution (i.e., debt). This raises the following question: what are the countervailing factors that lead to the observed comovement of debt and entitlements?

The strategic substitute property highlighted in Proposition 3 indicates that the comonotonicity of the two variables cannot be explained by a relaxing of fiscal rules–i.e., procedures that constrain debt or entitlements (or both)–because in that case the two variables should have evolved in opposite directions. For the same reason, increases in political turnover cannot explain this co-evolution of debt and entitlements (see our analysis in Section 7). Therefore, the observed co-movement must be ascribed to a change in a different model parameter.

In this section we explore a possible mechanism that can explain the rise of both debt and entitlements: an increase in political polarization or disagreement. It is well established that since the late 1970s in the US, the polarization of political parties (McCarty, Poole and Rosenthal 2006) as well as the level of political disagreement (Azzimonti 2015) has increased dramatically. We operationalize an increase in polarization as a decrease in the value of the public good for both groups. The logic is that disagreement between groups increases when everyone's value for the public good decreases: all agents become less public-spirited and desire to push the allocation of resources away from public consumption toward more private consumption for themselves. Of course, this is a very specific way to capture polarization or political disagreement. However, it is related to the way that Tabellini and Alesina (1990) model polarization.

Proposition 7 ***(polarization in the CRRA case)*** *Suppose* $h(x) = (x)^{1-\rho}/(1-\rho)$ *and* $v(x) = \alpha h(x)$, *where* $\alpha > 0$ *is a parameter that captures polarization. Then, as polarization increases (*α *decreases): for* $\rho < 1$ *(respectively:* $\rho > 1$*), debt and entitlements increase jointly if and only if* α *is smaller (respectively: larger) than* $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.

Proof. See Appendix E. ■

This proposition can be contrasted with Alesina and Tabellini (1990). In their model, an increase in polarization always leads to an increase in debt. In our model, this result holds for entitlements–but not for debt: debt and entitlements co-move with polarization only under some configurations of parameters. The following figure illustrates those

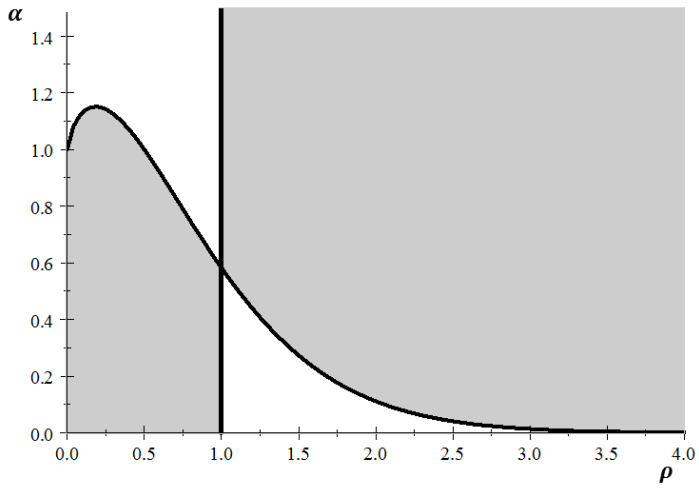


Figure 3: Shaded areas give the combinations of α and ρ such that debt and entitlements co-move when polarization increases.

conditions.

9 Discussion of Modeling Assumptions

This section marshals evidence in support of our key modeling assumptions.

9.1 Number of Periods

Our model only has two periods. The two-period (finite-horizon) model facilitates comparison with some of the prior work done in the literature on debt. We have worked out the model for a longer finite horizon, and the main results remain unchanged. An infinite-horizon version of our model would require some substantial modifications in the setup, but we believe that the major forces identified in this paper should still be present in the richer model.

9.2 Interpreting Groups A and B as Workers and Retirees

In our model, there are only two groups and they alternate in power. However, the strategic analysis remains unchanged if we reinterpret the model as follows: group A represents a single generation that is young in period 1 and old in period 2; and group B represents two different generations: the old in period 1, and the young in period 2. The analysis remains unchanged because in our model there are no wealth effects or other connections to link group B's period 2 objective function to the allocation that group B received in period 1, so group B's period-2 self is unaffected by what happened to its period-1 self.

Thus, our equilibrium can be reinterpreted as the equilibrium of a two-period overlapping generation model. In our preferred interpretation, group A represents workers in period 1 and retirees in period 2; group B represents successively period-1 retirees and period-2 workers. In each period, workers hold political power.

The next two subsections provide arguments in favor of this interpretation.

9.2.1 Conflict of Interest: Age or Wealth?

Interpreting groups A and B as workers and retirees means that the conflict of interest in our model is generational. A different perspective could be that the key conflict for public policy is not workers vs. pensioners, but rather rich vs. poor. For some public policies this alternative view is likely correct (in the case of taxes, for example). However, welfare policy appears to be relatively orthogonal to the rich-poor cleavage. Indeed, Bonoli (2000), p. 5 argues:

> "the main political cleavage in social policy-making seems to be shifting from the left-right axis to an opposition between governments, to a large extent regardless of their political orientation, and a pro-welfare coalition of interest groups, which is often led by the labour movement. This has long been the case in France where the Socialist governments of the 1980s clashed with the unions on a number of occasions. As new left-of-centre governments have been voted into power in Europe, this shift in the dominant cleavage in the politics of social policy has become more evident. In Germany, Italy and, to a lesser extent, Britain, the left-of-centre governments of the late 1990s are committed to continue reforming their welfare states, and the main confrontation is between themselves and the labour movement. Figure 4 supports this view because it shows that in most OECD countries a relatively small fraction of public spending is means-tested."

Figure 4 supports this view because it shows that in most OECD countries a relatively small fraction of public spending is means-tested. In this sense, the cleavage in the allocation of welfare benefits is not rich vs. poor.

9.2.2 Workers, Not Retirees, Hold the Political Power

Throughout most of this paper workers, not retirees, hold political power. (The assumption is relaxed in Section 7, where we assume that group A stays in power with probability π.) Is this a natural assumption?

In a sense, the assumption that workers have all the political power is natural: there are more workers than pensioners, and thus the median voter must be a worker, not a pensioner. But what of the idea that–because there are a lot of older voters and they are

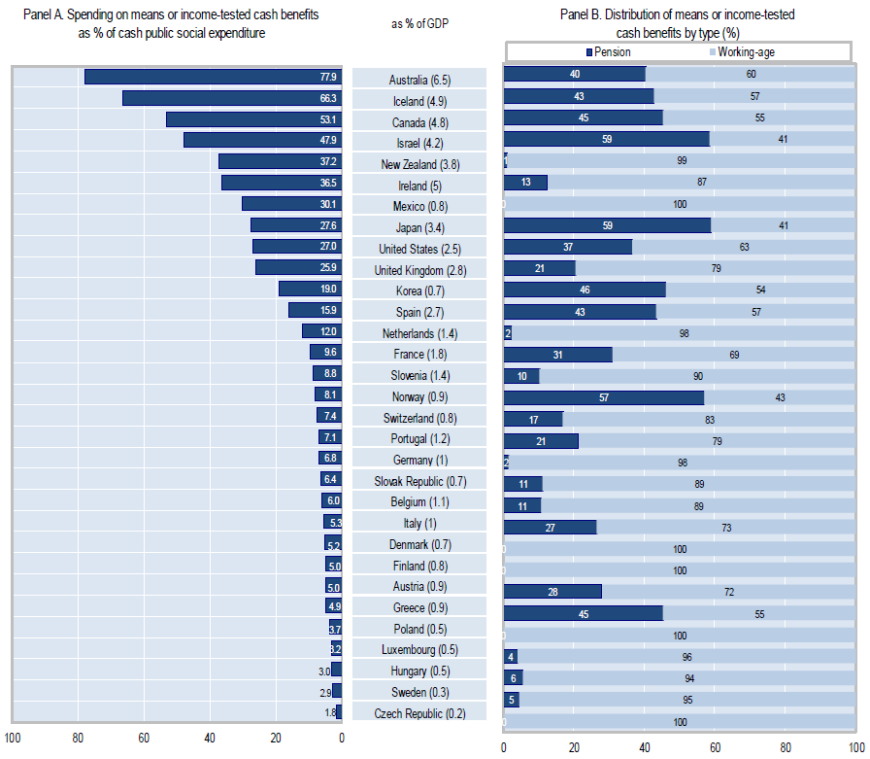


Figure 4: Public spending on income and means-test benefits as a percent of public social spending on cash benefits (and GDP in brackets), 2012 or lastest year available. Source: OECD (2014).

more likely to vote–older voters should have a disproportionate influence on policy? The idea has some merit, but the fact is that while older voters are more likely to vote, the median voter's age is far lower than pensionable age (see, e.g., Galasso and Profeta 2004 and Galasso 2008). In the US, for example, the median voter's age in the 2008 election was about 45. The fact that the median voter is squarely of working age holds true across all democracies, including demographically older democracies.

If the median voter is a worker, what are the consequences for welfare policy? According to the median-voter model, all the political power should rest with the workers. Assuming that workers' interests regarding welfare policy are relatively homogenous, it should not matter whether the retiree population is getting older or larger (provided retirees do not exceed 50%): in a median-voter model, welfare policy should only reflect the preference of workers. Consistent with this hypothesis, Vanhuysse (2012) finds that, while there is a lot of cross-national variation in the pro-elderly bias of welfare spending in the OECD, "population aging actually cannot explain very much of this pro-elderly bias variance. For instance, countries such as Denmark, Finland and Sweden are demographically old societies, yet they boast among the lowest pro-elderly spending biases in the OECD world." For the US, Mulligan and Sala-i-Martin (1999) share the view that this pro-elderly bias cannot be solely explained by demographics. These findings are consistent with our modeling assumption that welfare policy (i.e., entitlements) is chosen by workers and protected by pensioners.

9.3 Modeling Entitlements

We model entitlements as spending floors on private consumption only. This choice was made for tractability. We also explored a different, more flexible specification in which group A can create entitlements on public goods E_g as well as on their private consumption. In this specification, there is a "budget constraint" for commitment: $E + E_g \leq \overline{E}$. The logic is that there is a limit $\overline{E}$ to the intergenerational commitments that a court system will enforce or a political system will protect. Under plausible parameter values, we found that group A would choose $E_g = 0$: that is, group A would use whatever commitment power is available on private, not public, goods.[17] This finding supports our modeling choice of simply assuming no ability to commit on public good provision.

It might appear that the model gives generation A a lot of "intergenerational power" by allowing them to set their own entitlements in period 2. But if our model is interpreted as a building block of an overlapping generation model, then it does not take much power to set entitlements. This is because in period 1 there are only two groups who are alive: the worker generation who will benefits from entitlements in the next period; and the retiree generation who will not be around in the next period. Therefore, there is no conflict of

[17] The intuition is as follows: given that the level of public good is positive in period 2, it is too costly to set E_g such that it affects the level of public good provided in that period.

interest regarding entitlements between the generations who are alive in period 1. This observation suggests that *setting* entitlements may not in reality require a lot of power. In contrast, *defending* entitlements in period 2 would require power.

We assume that retirees can fully defend whatever entitlements they previously arranged for themselves. Of course, such an extreme form of commitment is not necessary to get the flavor of our results. All that is required is that new generations cannot easily renegotiate the older generations' entitlements. Surely generations that "fight for their right" to entitlements must believe that these entitlements cannot easily be renegotiated, otherwise the political fights would hardly be worthwhile. But is this belief factually correct? Have pensioners been able to defend their entitlements? We take up this issue in the following section.

9.4 What Are the Sources of the Commitment to Entitlements?

The promises of the welfare state have been resilient beyond expectations. In the 1980s, some leading political scientists expected the welfare state to retrench.[18] In the US and the UK, Reagan and Thatcher had brought conservative, anti-welfare agendas to power; union membership was declining across OECD countries and the "working class" was decreasing; and government debt was shooting up, creating fiscal pressure. And yet, despite this unfavorable environment, there was no retrenchment. Instead, the welfare state continued to expand (see OECD 2012, p.5). The welfare state also survived cataclysmic events, such as the fall of communist regimes (see Vanhuysse 2006, p. 77-8).

In this section we explore the sources of the commitment to these promises. In our view, the forces discussed in this subsection can, when taken together, explain the remarkable durability of welfare commitments. An important qualification is in order: while the mechanisms mentioned below suffice to *defend* the status quo (entitlements), they do not create sufficient power to *improve upon* the status quo. In other words, these mechanisms would not allow the pensioners to increase the generosity of existing policies.[19] This is an important observation because, in our model, we do not contemplate such renegotiation of entitlements.

9.4.1 Entitlements Defended Through Political Institutions

In the seminal book *Dismantling the Welfare State*, Pierson (1994) called attention to the remarkable durability of the welfare state, founding the literature on the "new politics of the welfare state."[20] Pierson (1994, pp. 1-2) argues that

[18] For an intellectual history, see Myles and Guadagno (2002) and Gingrich (2015).

[19] The economic literature on pensions recognizes the importance of the status quo, and studies how this importance differs across political institutions (see, e.g., Hansson and Stuart 1989, and Azariadis and Galasso 2002). It also explores how lobbying helps explain how the elderly defend their entitlements (see, e.g., Grossman and Helpman 1998 and Mulligan and Sala-i-Martin 1999).

[20] For a review of this literature see Gingrich (2015).

> "retrenchment is a distinctive and difficult political enterprise. It is in no sense a simple mirror image of welfare state expansion.[...] Retrenchment advocates must operate on a terrain that the welfare state itself has fundamentally transformed. Welfare states have created their own constituencies. If citizens dislike paying taxes, they nonetheless remain fiercely attached to public social provision. That social programs provide concentrated and direct benefits while imposing diffuse and often indirect costs is an important source of their continuing political viability. Voters' tendency to react more strongly to losses than to equivalent gains also gives these programs strength."

In other words, Pierson argues that entitlements could be defended because welfare policies created constituencies that became entrenched, and because the benefits of entitlements are relatively concentrated (according to our model, on the old), but their costs are diffuse.[21] In a similar vein, Pierson (1994, p. 42) points out that undoing pay-as-you-go pension entitlements would impose concentrated costs on the "switch generation," which would need to fund two pensions. Bonoli (2000, p. 5) identifies some institutional features that make it easier to defend entitlements:

> "The degree of influence that pro-welfare interest groups have on policy depends to a large extent on the opportunities provided by the political institutions. Absence of veto points means that governments will be able to go much further in the restructuring of their welfare state. In contrast, political systems that offer veto points will find it more difficult to adapt their welfare states and pension systems to a changing economic and demographic environment."

Thus, Bonoli argues that universalism contributes to status-quo bias: countries where change requires the consensus of broad coalitions find it difficult to move away from a high-welfare status quo.[22]

If commitments were defended exclusively through political power, we would expect groups with more political power to be betterable to defend their entitlements. Instead, when entitlements have been retrenched (typically, for pension benefits), the adjustments have been made gradually and with grandfather clauses designed to *protect senior citizens in proportion to their seniority*, and thus *in inverse proportion to their electoral/political*

[21] In this vein, Vanhuysse (2006) argues that the cohesion of threatened workers and pensioners was instrumental in preserving the commitment to pensions in certain Eastern European countries during the transition from Communism.

[22] Bonoli's leading example is Switzerland, but the same argument could apply to France, Germany, and Italy. In the same vein, Orenstein (2000, p. 2) argues that countries with more "veto actors"–social and institutional actors with an effective veto over reform–engaged in less radical reform. According to Orenstein, Poland and Hungary have generated less radical change than Kazakhstan, partly because they had more representative political systems, to which more associations, interest groups, and "proposal actors" have access.

power.[23,24] This observation suggests that commitments to entitlements are not defended through political power alone.

9.4.2 Judicial Protection of Entitlements

Judicial recourse has been a powerful protection against the renegotiation of welfare policies. There are many examples in which the courts have prohibited legislatures from impairing entitlements. In Illinois, for instance, the *New York Times* reports that:

> "All seven members of the state's highest court found that a pension overhaul lawmakers had agreed to almost a year and a half ago violated the Illinois Constitution. The changes would have curtailed future cost-of-living adjustments for workers, raised the age of retirement for some and put a cap on pensions for those with the highest salaries. But under the state Constitution, benefits promised as part of a pension system for public workers "shall not be diminished or impaired." (Davey 2015)

Illinois is the norm, not the exception, among US states. In their report on legal constraints on changing public pensions, Munnell and Quinby (2012) indicate that "[f]or the vast majority of states, changing future benefits for current employees is extremely difficult."[25] Outside the US, courts have invalidated welfare reforms in, e.g., Turkey (2006),[26]

[23] Pension retrenchment has taken two main forms: so-called *parametric reforms* (e.g., increases in retirement age or decreases in benefits) and so-called *systemic reforms* (that is, moving from public, defined-benefit systems to private, defined-contribution systems). Parametric reforms have been more common in Europe (OECD 2013), whereas systemic reforms have been more common in Latin America (Mesa-Lago and Márquez 2007).

[24] Regardless of the form retrenchments have taken, a broadly shared principle has held true: current pensioners have been grandfathered in. Indeed, current pensioners have been automatically protected against increases to the pensionable age (pensioners cannot be recalled to work), and they have also typically been protected against decreases in benefit levels (with the occasional exception of cost-of-living adjustments). Some academic authors actually promote grandfathering of current pensioners as welfare-improving (see, e.g., Conesa and Garriga 2008, Aubuchon et al. 2011). As concerns current workers, decrease in pension benefits have typically been phased in gradually; for example, increases in the retirement age have typically been larger for workers who were younger at the time of the reform. Quoting from Arpaia et al. (2009), emphasis ours: "Almost all countries increased the statutory retirement age, *the majority opting for a smooth transition towards higher retirement ages.* [...] The age of eligibility to a state pension was *progressively* increased from 65 to 67 in Denmark, Sweden and Germany, in the latter *with a very long phasing-in period.* [...] The retirement age was also *progressively* increased in the Czech Republic (2003) [...], in Hungary (1997) up to 62, Slovenia (1999) and Romania (2000)." If, as argued in Section 9.2.2, the median voter is a 45 year-old worker, then the effect of gradual phase-ins has been to provide a higher level of protection to citizens who are farther away from the median voter.

[25] Similarly, Monahan (2013, p. 5) states that "Changes to a participant's benefit once she has retired will be extremely difficult to make in any state." At the US federal level, interestingly, the entitlement to Social Security is not legally enforceable. "Congress has the power legislatively to promise to pay individuals a certain level of Social Security benefits, and to provide legal evidence of Congress's 'guarantee' of the obligation of the federal government to provide for the payment of such benefits in the future. While Congress may decide to take whatever measures necessary to fulfill such an obligation, courts would be unlikely to find that Congress's unilateral promise constitutes a contract which could not be modified in the future." Page ii, Lanza and Nicola (2014).

[26] Stewart (2008).

Latvia (2009),[27] Portugal (2014),[28] and Italy (2015).[29]

9.4.3 Commitment to Entitlements Arising from Threat of Breakdown in the Social Contract

The economic literature identifies conditions under which the younger generation does not renege on entitlements that benefit the older generation because they know that this would lead to tomorrow's younger generation following suit and reneging on the entitlements that benefit them. By reneging on pensions today, the young generation would deprive itself of pensions tomorrow. When the economy is dynamically inefficient, they are better off not reneging. A pay-as-you-go pension system (or any similar form of entitlements) may thus be part of an equilibrium, hence sustained without any institutional or legal defense of the pension system (see, e.g., Samuelson 1958 and Aaron 1966, and the discussion in Weil 2008). Browning (1975) shows that, because workers see past contributions as a sunk cost, a pay-as-you-go pension system can be political sustainable even when the economy is dynamically inefficient.

10 Extension: Default

We now discuss how the possibility of default may impact debt and entitlements differentially. This is a rich question with many possible angles. For instance, an important difference between debt and entitlements arises because debt is partly owed to outsiders (sovereign debt), while entitlements are only "owed" to a specific group of voters. This difference potentially generates different political incentives to default. We believe that this is an important difference, and we plan to purse it in detail in follow-up work. Yet, it requires a major departure from the model that we have worked with so far. Tabellini (1991) also points out that there is an additional potential difference among the coalitions supporting default, even if he focuses on domestic debt and does not focus on the comparative statics of default.

Here we discuss some preliminary analysis of the consequences of default for the size of debt and entitlements. To fix ideas, let us begin by assuming that there are exogenous probabilities δ and η with which debt repayments and entitlement payments are reduced by a fixed amount λ (the default size). In equilibrium, for investors to be willing to lend, the interest rate on debt has to be adjusted to reflect this probability of default. This, of course, affects the willingness of group A to take on debt. This market discipline effect is absent in the case of entitlements. In fact, we can construct scenarios in which debt decreases with the probability and size of default, while entitlements are increasing in

[27] See Social Protection – Human Rights (2009).
[28] See Fox (2014).
[29] See Merler (2015).

the same quantities. Note that these are statements about the effect of default on one given obligation–say entitlements–on the endogenous size of that obligation. The effect of an increase in the default probability for entitlements on the equilibrium size of debt is complex and may well be positive: if lenders expect pensions to be reduced, they may be more willing to lend. Thus, it is difficult to use changes in default environments to understand the historical evolution of debt and entitlements. Such a study would also require historical data on default probabilities going back to the 1970s.

A richer model of default incorporates an endogenous default response by group B to the size of debt and entitlements. A particularly simple way to do this is to assume that there is a default technology for debt $F\left(d, d^r, I^r\right)$ and one for entitlements $H\left(E, E^r, I^E\right)$. These describe the cost for group B to change the amount (entitlements) from d to d^r (E to E^r). In order to introduce uncertainty about these possibilities, we add some shocks to the size of the endowment available in period 2. In turn group A may, at some cost, build institutions I^E and I^d in period 1 that raise the cost of defaulting on these promises in the subsequent period. We conjecture that in equilibrium, group A would over-entitle itself relative to the target level of desired entitlements and build institutions to protect debt and entitlements in anticipation of partial default on both.

11 Conclusions

Entitlements are well understood to be a key determinant of the fiscal sustainability of government obligations. However, entitlements are more difficult to measure than debt, and they have tended to take a backseat in the policy debate: what is not measured can't be managed. The political economy literature has not yet focused on the interplay between debt and entitlements. And yet there is a lot we can learn from taking a closer look at the interplay between these two quantities. For instance, there are some surprises in recent measures of total government obligations compiled by the EU; such as the example that once entitlements are factored in, the healthiest country in the EU is Italy.

In this paper we have presented a very simple politico-economic model where entitlements and debt are jointly determined. The main findings are the following. Debt and entitlement are strategic substitutes: constraining debt increases entitlements (and vice versa). Does this generate unintended consequences of fiscal rules? From a welfare perspective, it is good to constrain entitlements, but not too strictly; constraining debt may be detrimental (even in the absence of shocks to be smoothed). Debt and entitlements move in opposite directions in response to political instability. Surprisingly, debt, and even the expected total fiscal burden can go down in response to political instability when entitlements are endogenous. We have proposed that the joint growth of debt and entitlements could be attributed to increases in political polarization.

Appendix A: Proofs and Additional Example for Section 4

Proof of Lemma 1. Uniqueness follows directly from the concavity of the problem, and differentiability from the implicit function theorem. Using the constraint to substitute for x and taking the first order conditions with respect to c we have:

$$-h'(1-c-g)+v'(g)=0. \tag{5}$$

Replacing g with $G(c)$ and differentiating yields:

$$v''(G(c))G'(c)=-h''(1-c-G(c))(1+G'(c)),$$

hence

$$G'(c)=-\frac{h''(1-c-G(c))}{v''(G(c))+h''(1-c-G(c))}\in(-1,0).$$

Because at the optimum the constrain holds with equality, we have:

$$X(c)+G(c)=1-c$$

Differentiating, we have $X'(c)=-1-G'(c)\in(-1,0)$. ■

Proof of Proposition 1.

1. With the constraint $E^*\equiv 0$, group A's problem (1) specializes to:

$$\max_{(x,g,d)} h(x)+v(g)+v(G(d)) \text{ s.t. } x+g\leq 1+d.$$

Form the Lagrangean and take first-order conditions to get:

$$v'(g^*_{E=0})=-v'(G(d^*_{E=0}))G'(d^*_{E=0})=h'(x^*_{E=0}). \tag{6}$$

The proof of Lemma 1 shows that $G'(c)\in(-1,0)$, hence the first equality in (6) implies $v'(G(d^*_{E=0}))>v'(g^*_{E=0})$, which in turn implies:

$$G(d^*_{E=0})<g^*_{E=0}. \tag{7}$$

Now take the second equality in (6), substitute from equation (5) in the proof of Lemma 1, and again use $G'(c)\in(-1,0)$ to write:

$$h'(1-d^*_{E=0}-G(d^*_{E=0}))>h'(x^*_{E=0}),$$

whence:

$$1-d^*_{E=0}-G(d^*_{E=0})<x^*_{E=0}. \tag{8}$$

Adding up (7) and (8) yields:

$$1-d^*_{E=0}<x^*_{E=0}+g^*_{E=0}.$$

The last line can be rewritten as $X(d^*_{E=0})+G(d^*_{E=0})<x^*_{E=0}+g^*_{E=0}$, which implies that

more resources are allocated to private and public good consumption in in period 1 than in period 2 (the implication follows because group 2 in period 1, as well as group 1 in period 2, are allocated zero private consumption). So it must be $d^*_{E=0} > 0$.

2. In equilibrium group 1 is not allocated any private good in period 2.

3. See (7).

■

Proof of Proposition 2.

1. Let us first prove that $d^* + E^* > 0$. Suppose, by contradiction, that $d^* + E^* \leq 0$. Given that $E^* > 0$, we have from Lemma 1 and the proof of part 3 of this Proposition, that public good provision must be higher in period 2 than in period 1 (i.e. $G(d^* + E^*) \geq g^*$). This implies that $v'(g^*) > -v'(G(d^* + E^*))\, G'(d^* + E^*)$. Therefore, group A could increase its lifetime payoff by increasing public good provision in period 1. To do so, group A just has to increase debt (or reduce surplus). So, $d^* + E^* \leq 0$ cannot be true. We now prove that $d^* + E^* > d^*_{E=0}$. From Proposition 3 part 4, we know that, for $\overline{E} < E^*$, increasing $\overline{E}$ increases $d\left(\overline{E}\right) + \overline{E}$. Thus, to get the result, it suffices to note that, in $\overline{E} = 0$, $d\left(\overline{E}\right) + \overline{E} = d^*_{E=0}$, and that, in $\overline{E} = E^*$, $d\left(\overline{E}\right) + \overline{E} = d^* + E^*$.

2. If, by contradiction, group A were to choose $d^* + E^* = 1$ then period 2's budget constraint implies the no public good could be provided. Because of Inada conditions for group A's utility function, this could never be an optimal choice for group A.

3. Fix any d (for example, $d = d^*$) and consider the vector (x, g, E) that solves problem (1) conditional on the debt level being set at d. The conditional problem is separable in the sense that the x and g that solve the conditional problem (1) are the solutions to the following simpler problem which does not involve E:

$$\max_{(x,g)} h(x) + v(g) \text{ s.t. } x + g \leq 1 + d.$$

This problem was introduced in Definition 1, and so the g that solves the conditional problem (1) must be exactly $G(-d)$. The solution to the unconditional problem (1) is then $g^* = G(-d^*)$. Now from Lemma 1 we know that $G(\cdot)$ is a decreasing function, so if d^* is positive then $g^* = G(-d^*) > G(d^* + E^*)$. The inequality is the desired conclusion.

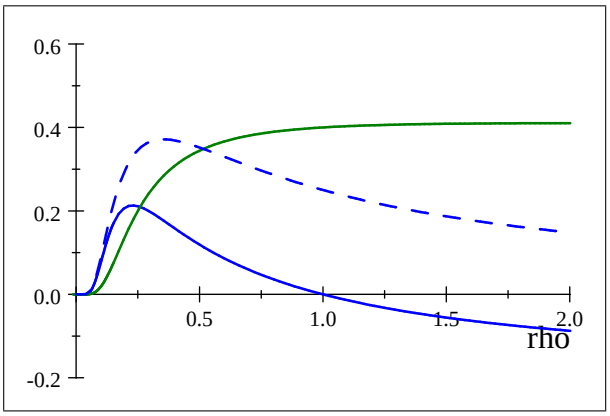
4. Suppose, by contradiction, that the marginal value of group A's private consumption was larger in period 2. Then group A could reduce debt (or increase surplus) by one unit, and simultaneously increase entitlements by one unit. This operation would not change group B's optimization problem in period 2, hence public good provision in period 2 would be unchanged; and it would increase group A's lifetime utility.

5. See Proposition 8 part 3 in the Appendix F.

■

Additional Example

Consider: $\alpha = 1.5$. When $\rho \to 0$, there is essentially no conflict between groups because group B spends the entire budget on the public good. Thus, in this case, the optimal debt (entitlements) level goes to zero as ρ goes to zero. When ρ increases, conflict starts mattering, but the effect differs for debt and entitlements. For debt, and the crowdout effect first dominates so debt rises until it is overtaken by the smoothing effect and debt drops. For entitlements, both effects pull in the same direction. Because group B starts allocating resources to its private consumption, the crowdout effect pulls entitlements up. Because of the change in the concavity of the utility function, group A wants to balance private and public consumption more in period 2. Given that we start from zero private consumption (and full public consumption), this requires an increase in entitlements.



Green: E, Blue: b with entitlements, Green Dash: E w/o debt Blue Dash: d w/o entitlements, Black: E+d

Appendix B: Proofs of Section 5

Proof of Proposition 3.

1. Fix any d and consider the vector (x, g, E) that solves problem (1) conditional on the debt level being set at d. The entitlement level that solves the conditional problem is the solution to the following simpler problem which does not involve x or g:

$$\max_{E} h(E) + v(G(d+E)). \tag{9}$$

The first order conditions read:

$$h'(E) = -v'(G(d+E))\,G'(d+E). \tag{10}$$

The LHS is an decreasing function of E. Because $v(G(d+E))$ is concave in E, its first derivative with respect to E, $v'(G(\cdot))\,G'(\cdot)$, is a decreasing function of E. The RHS is its opposite, and therefore an increasing function of E. Now increase d. The LHS function stays

unchanged. The RHS function shifts up. Therefore the two functions now cross at a lower level of E.

2. Fix any E and consider the vector (x, g, d) that solves problem (1) conditional on the entitlement level being set at E. The debt level that solves the conditional problem is the solution to the following simpler problem:

$$\max_{(x,g,d)} h(x) + v(g) + v(G(d+E)) \text{ s.t. } x + g \leq 1 + d.$$

Define the following value function:

$$U(d) = \max_{(x,g)} h(x) + v(g) \text{ s.t. } x + g - 1 \leq d. \tag{11}$$

Because $h(\cdot)$ and $v(\cdot)$ are concave, $U(d)$ is concave in d. Our problem can then be rewritten as follows:

$$\max_{d} U(d) + v(G(d+E)). \tag{12}$$

This problem is isomorphic to (9), and the rest of the proof follows the argument in part 1.

3. Consider any $\overleftarrow{d} < \overline{d}$. Suppose by contradiction that $\overleftarrow{d} + E\left(\overleftarrow{d}\right) > \overline{d} + E\left(\overline{d}\right)$. Then, because the RHS of (10) is an increasing function of $d + E$, we have:

$$-v'\left(G\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right)\right) G'\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right) > -v'\left(G\left(\overline{d} + E\left(\overline{d}\right)\right)\right) G'\left(\overline{d} + E\left(\overline{d}\right)\right). \tag{13}$$

Now, we know from part part 1 that $E\left(\overleftarrow{d}\right) > E\left(\overline{d}\right)$. Because $h'(\cdot)$ is a decreasing function it follows that:

$$h'\left(E\left(\overleftarrow{d}\right)\right) < h'\left(E\left(\overline{d}\right)\right). \tag{14}$$

By definition of $E\left(\overline{d}\right)$, the RHS of (13) must equal the RHS of (14). But then it follows that:

$$h'\left(E\left(\overleftarrow{d}\right)\right) < -v'\left(G\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right)\right) G'\left(\overleftarrow{d} + E\left(\overleftarrow{d}\right)\right),$$

which contradicts the definition of $E\left(\overleftarrow{d}\right)$.

4. Problem (12) is isomorphic to (9), and the rest of the proof follows the argument in part 3.

■

Appendix C: Proofs of Section 6

Proof of Proposition 4. The proof starts with a foundational result. Let $g^*(\overline{d})$ and $E\left(\overline{d}\right)$ denote the optimal choice of period-1 public good and entitlements, respectively, conditional on a debt level $\overline{d}$. Suppose a debt ceiling $\overline{d}$ is binding for group A. We now show that relaxing the debt ceiling increases group B's lifetime utility if

$$-G'\left(\overline{d} + E(\overline{d})\right) \frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} > 1 + E'\left(\overline{d}\right).$$

Let $x^*(\overline{d})$ and $g^*\left(\overline{d}\right)$ denote the optimal choice of private and public goods, respectively, in the first period conditional on a debt level $\overline{d}$. Similarly, $X(\overline{d}+E)$ and $G\left(\overline{d}+E\right)$ denote the optimal choice of private and public goods, respectively, by group B in the second period conditional on a debt level $\overline{d}$ and an entitlements level E.

(i) Period-1 social return to relaxing debt limit.

Suppose a binding debt cap is increased by one dollar (so the budget available to group A increases). The variation in period-1 utility for group A is:

$$\frac{\partial W_A^1}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right).$$

The variation in period-1 utility for group B is:

$$\frac{\partial W_B^1}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}}$$

(ii) Period-2 social return to relaxing debt limit.

Let $E\left(\overline{d}\right)$ denote the solution to problem (9) for $d=\overline{d}$. The period-2 total variation in utility for group A is given by

$$\frac{\partial W_A^2}{\partial \overline{d}} = E'(\overline{d})h'\left(E(\overline{d})\right) + \left(1+E'\left(\overline{d}\right)\right)G'\left(\overline{d}+E(\overline{d})\right)v'\left(G\left(\overline{d}+E(\overline{d})\right)\right).$$

Use the first-order conditions (10) to simplify this expression:

$$\frac{\partial W_A^2}{\partial d} = G'\left(\overline{d}+E(\overline{d})\right)v'\left(G\left(\overline{d}+E(\overline{d})\right)\right).$$

The total variation in utility for group B is given by

$$\frac{\partial W_B^2}{\partial \overline{d}} = \left(1+E'\left(\overline{d}\right)\right)\left[X'(\overline{d}+E(\overline{d}))h'\left(X(\overline{d}+E(\overline{d}))\right) + G'\left(\overline{d}+E(\overline{d})\right)v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)\right].$$

The first order conditions for group B require that $h'\left(X(\overline{d})\right) = v'\left(G\left(d+E(\overline{d})\right)\right)$, whence

$$\frac{\partial W_B^2}{\partial \overline{d}} = \left(1+E'\left(\overline{d}\right)\right)\cdot\left[X'(\overline{d}+E(\overline{d})) + G'\left(\overline{d}+E(\overline{d})\right)\right]\cdot v'\left(G\left(\overline{d}+E(\overline{d})\right)\right).$$

From the proof of Lemma 1 we have that $X'(\cdot)+G'(\cdot) = -1$. Substitute into group B's total variation to get:

$$\frac{\partial W_B^2}{\partial \overline{d}} = -\left(1+E'\left(\overline{d}\right)\right)\cdot v'\left(G\left(\overline{d}+E(\overline{d})\right)\right).$$

(iii) Conditions for improvement in group B's lifetime utility:

By assumption the debt limit is binding for group A, which implies:

$$\frac{\partial W_A^1}{\partial \overline{d}} + \frac{\partial W_A^2}{\partial \overline{d}} = v'\left(g^*\left(\overline{d}\right)\right) + v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)G'\left(\overline{d}+E(\overline{d})\right) > 0. \tag{15}$$

Group B's lifetime utility is given by:

$$\begin{aligned}\frac{\partial W_B^1}{\partial \overline{d}} + \frac{\partial W_B^2}{\partial \overline{d}} &= v'\left(g^*\left(\overline{d}\right)\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} - v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)\left(1+E'\left(\overline{d}\right)\right) \\ &> v'\left(G\left(\overline{d}+E(\overline{d})\right)\right)\left[-G'\left(\overline{d}+E(\overline{d})\right)\frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} - \left(1+E'\left(\overline{d}\right)\right)\right], \qquad (16)\end{aligned}$$

where the inequality follows from (15). This expression has the same sign as the expression in brackets. The desired statement follows (remember that we now work under the assumption that $h(x) = v(x)$ have a CRRA form $(x)^{1-\rho}/(1-\rho)$).

1. If debt is unconstrained, that is, $\overline{d} = d^*$, then the inequality in (15) is replaced by an equality. Consequently, the inequality in (16) is replaced by an equality. Thus the variation in group B's lifetime utility from forcing group A to incur slightly more debt than they would choose to is exactly equal to the RHS of (16). If $\rho > (\log 2/\log 6)$, Proposition 11 part 3 ensures that the bracketed term in expression (16) is negative for any d. Hence group B suffers from relaxing the binding constraint. Group A is indifferent at the margin because we assumed that the constraint was barely binding. Conversely, therefore, tightening any barely-binding constraint on debt makes group A no worse off and group B strictly better off at the margin. The case where $\rho < (\log 2/\log 6)$ is exactly symmetric.

2. If debt is constrained at $\overline{d} < d^*$, then the variation in group B's lifetime utility from allowing group A to incur slightly more debt is bounded below by the RHS of (16). Since $\rho < (\log 2/\log 6)$, Proposition 11 part 3 ensures that the bracketed term in expression (16) is positive for any d. Hence group B benefits from relaxing the binding constraint. Group A benefits too because we assumed that the constraint was binding. Conversely, therefore, tightening any binding constraint on debt strictly hurts both groups.

■

Proof of Proposition 5.

1. Group A's allocation problem can be described as a constrained optimization problem. If a constraint on entitlements barely binds, its Lagrange multiplier will be close to zero. In the limit, it will be exactly zero. Thus the marginal impact on group A's lifetime payoff of introducing a constraint on entitlements is zero. Let's now consider the effects on group B's lifetime payoff. Proposition 3 part 4 ensures that tightening a cap on entitlements reduces the total obligations that group A bequeathes to group B. This means that group B is strictly better off in period 2. Also, group B is strictly better off in period 1 because constraining entitlements leads group A to increase debt (Proposition 3 part 2), and part of the additional period-1 resources will be allocated to the public good in period 1, from which group B benefits.

2. Follows from the Inada condition on group A's utility for the private good.

■

Appendix D: Proofs of Section 7

Proof of Proposition 6.

1. Substitute $E \equiv 0$ into equation (18). Repeat the argument that follows equation (18). Proposition 1 part 1 ensures that at $\pi = 0$ we have $d^* > 0$. The desired conclusion then follows from part 4.

2. If group A persists in power it faces the following period-2 allocation problem:

$$\max_{x,g} h(x) + v(g) \text{ s.t. } x + g \leq 1 - d.$$

The solution to this problem is given by $X(d)$, $G(d)$ (refer to Lemma 1). Using these expressions we can write group A's allocation problem as follows:

$$\max_{d,E} U(d) + \pi \left[h(1 - d - G(d)) + v(G(d)) \right] + (1 - \pi) \left[h(E) + v(G(d + E)) \right], \tag{17}$$

where $U(d)$ is defined in (11). Fix d; due to multiplicative separability of E and π, the optimal level of entitlements $E(d)$ is independent of π. This means that any effect of varying π on E^*, the equilibrium level of entitlements, must be channeled through the choice of debt, that is:

$$\frac{\partial E^*}{\partial \pi} = \frac{\partial E(d)}{\partial d} \frac{\partial d^*}{\partial \pi}.$$

Because $E(d)$ is independent of π, Proposition 3 part 1 applies, ensuring that $\partial E(d)/\partial d < 0$. Thus we have proved that equilibrium debt and entitlement move in opposite directions when π varies.

3. It follows directly from the previous step that

$$\frac{\partial d^*}{\partial \pi} + \frac{\partial E^*}{\partial \pi} = \frac{\partial d^*}{\partial \pi} \left(1 + \frac{\partial E(d)}{\partial d} \right).$$

It thus remains to prove that $1 + \frac{\partial E(d)}{\partial d} > 0$. Differentiating equation (3) implicitly, we obtain:

$$\frac{\partial E}{\partial d} = - \frac{v''(G_B(d + E)) (G_B'(d + E))^2 + v'(G_B(d + E)) G_B''(d + E)}{h''(E) + v''(G_B(d + E)) (G_B'(d + E))^2 + v'(G_B(d + E)) G_B''(d + E)}$$

From the concavity of $v(G_B(\cdot))$ and the concavity of $h(\cdot)$, we have that this is larger than -1.

4. The optimal d is identified as the zero of the derivative of problem (17) wrt d, when E is replaced by $E(d)$. That derivative reads:

$$U'(d) + \frac{\partial}{\partial d} \left[h(E(d)) + v(G(d + E(d))) \right] + \pi \frac{\partial}{\partial d} \left[h(1 - d - G(d)) + v(G(d)) - h(E(d)) - v(G(d + E(d))) \right]. \tag{18}$$

The equilibrium debt d^* is a zero of the function (18). This expression is not guaranteed to be decreasing in d without further assumptions, and thus it can have many zeros. However, this function is linear with respect to π, and so its derivative with respect to π is independent of the level of π. This implies that, fixing all parameters except π, the function shifts either

up or down with π, independent of the level of π. If the function shifts up, the equilibrium d^* shifts to the right, i.e., increases monotonically with π; if the function shifts down, the equilibrium d^* decreases monotonically with π. Either way, debt reacts monotonically to an increase in π. To conclude the proof, note that we have $d^* = 0$ at $\pi = 1$ (when A is the social planner, smoothing requires no debt or surplus); thus if debt started out positive at $\pi = 0$ then it must monotonically decrease with π; and if debt started out negative at $\pi = 0$ then it must monotonically increase in π. Together with part 2 of this Proposition, this shows that entitlements are monotonic in π (in the opposite direction as debt).

■

Appendix E: Proofs of Section 8

Proof of Proposition 7. From Proposition 10 in Appendix F. we have that: E^* is decreasing in α for any $\rho > 0$; for $\rho < 1$, d^* is decreasing in α if and only if α is smaller than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$; for $\rho > 1$, d^* is decreasing in α if and only if α is greater than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$. ■

Appendix F: CRRA Preferences

In this Appendix we provide closed-form solutions for the equilibrium under the assumption that groups have CRRA preferences, that is, when:

$$u_i\left(x_i^t, x_j^t, g^t\right) = \frac{\left(x_i^t\right)^{1-\rho_i}}{1-\rho_i} + \alpha_i \frac{\left(g^t\right)^{1-\rho_i}}{1-\rho_i},$$

with $\rho_i > 0$.

Note that we allow the possibility that $\rho_A \neq \rho_B$ and $\alpha_A \neq \alpha_B$. This type of heterogeneity is not contemplated in the main model, but this extension poses no difficulties. To simplify notation, $\rho_A = \rho$ and $\alpha_A = \alpha$.

Period 2

Group B's problem is

$$\max_g \frac{(1-d-E-g)^{1-\rho_B}}{1-\rho_B} + \alpha_B \frac{g^{1-\rho_B}}{1-\rho_B}.$$

The first order wrt g read:

$$(1-d-E-g)^{-\rho_B} = \alpha_B g^{-\rho_B},$$

and solving for g yields the public good allocation in period 2:

$$G\left(d+E\right)=\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-d-E\right). \tag{19}$$

Group B's private consumption in period 2, $X\left(d+E\right)$, can be recovered from the budget constraint $G\left(d+E\right)+X\left(d+E\right)=1-d-E$.

Note that this implies $v\left(G\left(d+E\right)\right)=\frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-d-E)\right)^{1-\rho_B}}{1-\rho_B}$.

Period 1

Group A's problem is:

$$\max_{g,E,d}\frac{(1+d-g)^{1-\rho}}{1-\rho}+\alpha\frac{g^{1-\rho}}{1-\rho}+\frac{E^{1-\rho}}{1-\rho}+\alpha\frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\left(1-d-E\right)\right)^{1-\rho}}{1-\rho}.$$

Let $g^*(\overline{d})$ and $E\left(\overline{d}\right)$ denote the optimal choice of period-1 public good and entitlements, respectively, conditional on a debt level $\overline{d}$.

Lemma 2 ***(period-1 equilibrium allocations conditional on a given debt level in the CRRA case)***

1. *The equilibrium allocation of period-1 public good conditional on a debt level $\overline{d}$, is* $g^*(\overline{d})=\left(1+\overline{d}\right)\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}$;
2. *The equilibrium allocation of entitlements conditional on a debt level $\overline{d}$, is* $E(\overline{d})=\left(1-\overline{d}\right)/\left[1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}\right]$.

Proof.

1. The function $g^*(\overline{d})$ solves the following problem:

$$g^*(\overline{d})=\arg\max_g\frac{\left(1+\overline{d}-g\right)^{1-\rho}}{1-\rho}+\alpha\frac{g^{1-\rho}}{1-\rho}.$$

The first order wrt g reads:

$$\left(1+\overline{d}-g\right)^{-\rho_B}=\alpha g^{-\rho},$$

and solving for g yields:

$$g^*(\overline{d}) = \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\left(1+\overline{d}\right)$$

2. The function $E(\overline{d})$ solves the following problem:

$$E(\overline{d}) = \arg\max_{E} \frac{E^{1-\rho}}{1-\rho} + \alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{1-\rho} \frac{\left(1-\overline{d}-E\right)^{1-\rho}}{1-\rho}.$$

The first order wrt E read:

$$\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{1-\rho}\left(1-\overline{d}-E\right)^{-\rho} = E^{-\rho},$$

and solving for E yields, after some algebra:

$$E(\overline{d}) = \frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}\left(1-\overline{d}\right). \tag{20}$$

■

Lemma 3 *The equilibrium level of debt in the CRRA case is determined by the following condition:*

$$\frac{(1-d)}{(1+d)} = \left(1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{\left(1+\alpha^{\frac{1}{\rho}}\right)}. \tag{21}$$

Proof. Substitute the expressions for $g^*(\overline{d})$ and $E(\overline{d})$ into group A's period-1 problem:

$$\begin{aligned}
&\max_d \frac{(1+d-K(1+d))^{1-\rho}}{1-\rho} + \alpha\frac{(K(1+d))^{1-\rho}}{1-\rho} + \frac{(Z(1-d))^{1-\rho}}{1-\rho} + \alpha\frac{\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-d-Z(1-d))\right)^{1-\rho}}{1-\rho} \\
= &\max_d \frac{(1+d)^{1-\rho}}{1-\rho}\left[(1-K)^{1-\rho}+\alpha K^{1-\rho}\right] + \frac{(1-d)^{1-\rho}}{1-\rho}\left[Z^{1-\rho}+\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-Z)\right)^{1-\rho}\right],
\end{aligned} \tag{22}$$

where we have denoted $K = \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}$ and $Z = 1/\left[\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}+1\right]$. The problem is concave in d. The first order conditions wrt d are:

$$(1+d)^{-\rho}\left[(1-K)^{1-\rho}+\alpha K^{1-\rho}\right] = (1-d)^{-\rho}\left[Z^{1-\rho}+\alpha\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}(1-Z)\right)^{1-\rho}\right],$$

or equivalently:

$$\frac{(1-d)}{(1+d)} = \left\{ \left[Z^{1-\rho} + \alpha \left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}} (1-Z) \right)^{1-\rho} \right] \left[\frac{1}{(1-K)^{1-\rho} + \alpha K^{1-\rho}} \right] \right\}^{1/\rho}.$$

After much algebra, the first term in square brackets simplifies to:

$$\left(\alpha^{\frac{1}{\rho}} \left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}} \right)^{\frac{1-\rho}{\rho}} + 1 \right)^{\rho},$$

and the second term in square brackets simplifies to:

$$\left(1+\alpha^{\frac{1}{\rho}}\right)^{-\rho}.$$

Substituting back into the first order conditions we get equation (21). ■

Proposition 8 ***(properties of the equilibrium in the CRRA case).*** *In the CRRA case (with the same α):*

1. *the equilibrium level of debt is strictly interior: $d^* \in (-1, 1)$;*
2. *$d^* = 0$ when $\rho = 1$;*
3. *$d^* > 0$ if and only if $\rho < 1$;*
4. *the equilibrium level of entitlements is strictly positive, $E^* > 0$;*
5. *in equilibrium, total obligations, that is, debt and entitlements, are related by the following proportion: $(1+d^*)/E^* = \left(1+\alpha^{\frac{1}{\rho}}\right)$.*

Proof.

1. The RHS of (21) is nonnegative. The LHS of (21) goes from $+\infty$ at $d = -1$ to 0 at $d = 1$; and it is a decreasing function of d on $[-1, 1]$. Therefore, the equilibrium level of debt $d^* \in (-1, 1)$.
2. The RHS of (21) equals 1 when $\rho = 1$, hence $d^* = 0$ when $\rho = 1$.
3. $d^* > 0$ if and only if the RHS of (21) is smaller than 1, that is, if and only if:

$$\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}} \right)^{\frac{1-\rho}{\rho}} < 1,$$

   which is the case if and only if $1-\rho > 0$ or $\rho < 1$.
4. $E^* = E(d^*)$. The conclusion follows from expression (20) and the fact that $d^* < 1$.

5. Combine expressions (20) and (21) to get:

$$\begin{aligned} E(d^*) &= \frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho B}}}{1+\alpha_B^{\frac{1}{\rho B}}}\right)^{\frac{1-\rho}{\rho}}}(1-d^*) \\ &= \frac{1+d^*}{\left(1+\alpha^{\frac{1}{\rho}}\right)}. \end{aligned}$$

■

Proposition 9 *In the CRRA case (with the same α) we have: $d^* = \frac{1-h(z)}{1+h(z)}$ and $E^* = \frac{2}{[1+h(z)](1+z)}$ where $z = \alpha^{\frac{1}{\rho}}$ and $h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}$.*

Proof.

$$\frac{(1-d)}{(1+d)} = \left(1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{\left(1+\alpha^{\frac{1}{\rho}}\right)}.$$

Make the change of variable: $\alpha^{\frac{1}{\rho}} = z > 0$. Because $\rho > 0$ by assumption, z is a monotonic function of α. Then

$$\begin{aligned} \frac{(1-d)}{(1+d)} &= \left(1+z\left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}\right)\frac{1}{(1+z)} \\ &= \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}. \end{aligned}$$

Denote:

$$h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}.$$

Then we have:

$$\begin{aligned} \frac{(1-d)}{(1+d)} &= h(z), \\ d^* &= \frac{1-h(z)}{1+h(z)}. \end{aligned} \tag{23}$$

Take the expression for $E(d)$ from Lemma 2, and substitute z to get:

$$\begin{aligned} E(d) &= \frac{(1-d)}{1+z\left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}} \\ &= \frac{(1-d)}{1+(1+z)\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}} \\ &= \frac{(1-d)}{(1+z)h(z)}. \end{aligned}$$

Substitute from (23) to get:

$$\begin{aligned} E^* &= \frac{(1-d^*)}{(1+z)\,h(z)} \\ &= \left(1-\frac{1-h(z)}{1+h(z)}\right)\frac{1}{(1+z)\,h(z)} \\ &= \frac{2}{[1+h(z)](1+z)} \end{aligned}$$

■

Proposition 10 ***(comparative statics with respect to*** α*)*** *In the CRRA case (with the same* α*) :*

1. E^* *is decreasing in* α *for any* $\rho > 0$.
2. d^* *is decreasing in* α*: for* $0 < \rho < 1$ *if and only if* α *is larger than* $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$; *for* $\rho > 1$ *if and only if* α *is smaller than* $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.

Proof.

1. E^* is increasing in z if and only if $[1+h(z)](1+z)$ is decreasing in z. First, compute:

$$h(z) = \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}}.$$

$$h'(z) = -\frac{1}{(1+z)^2} + \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)^2}.$$

Then

$$\begin{aligned} & \frac{\partial}{\partial z}[1+h(z)](1+z) \\ = \;& h'(z)(1+z) + 1 + h(z) \\ = \;& \left[-\frac{1}{(1+z)^2} + \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)^2}\right](1+z) + 1 + \frac{1}{(1+z)} + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}} \\ = \;& \frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\frac{1}{(1+z)} + 1 + \left(\frac{z}{1+z}\right)^{\frac{1}{\rho}} > 0. \end{aligned}$$

Hence E^* is decreasing in z, and hence in α, for any $\rho > 0$.

2. We have:

$$\begin{aligned} d^* &= \frac{1-h(z)}{1+h(z)} \\ \frac{\partial}{\partial z}d^* &= \frac{-h'(z)(1+h(z))-(1-h(z))h'(z)}{(1+h(z))^2} \end{aligned}$$

This has the same sign as its numerator:

$$\begin{aligned} & -h'(z)(1+h(z))-(1-h(z))h'(z) \\ = \; & -2h'(z) \\ = \; & 2\frac{1}{(1+z)^2}\left[1-\frac{1}{\rho}\left(\frac{z}{1+z}\right)^{\frac{1}{\rho}-1}\right]. \end{aligned}$$

This expression is positive iff

$$\rho > \left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}.$$

So debt is increasing in z if and only if $\rho > \left(\frac{z}{1+z}\right)^{\frac{1-\rho}{\rho}}$.

In the case $0 < \rho < 1$, this condition rewrites as:

$$\begin{aligned} 0 &< \rho < \left(\frac{\alpha^{1/\rho}}{1+\alpha^{1/\rho}}\right)^{\frac{1-\rho}{\rho}} \\ 0 &< \rho^{\frac{\rho}{1-\rho}}\left(1+\alpha^{1/\rho}\right) < \alpha^{1/\rho} \\ -\rho^{\frac{\rho}{1-\rho}}\alpha^{1/\rho} &< \rho^{\frac{\rho}{1-\rho}} < \alpha^{1/\rho}-\rho^{\frac{\rho}{1-\rho}}\alpha^{1/\rho} \\ -\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\alpha^{1/\rho} &< \frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)} < \alpha^{1/\rho} \\ -\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}\alpha &< \left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho} < \alpha \end{aligned}$$

So if $0 < \rho < 1$ then debt is decreasing in α for α larger than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.

In the case $1 < \rho$, the condition rewrites as:

$$\begin{aligned} 0 &< \rho < \left(\frac{\alpha^{1/\rho}}{1+\alpha^{1/\rho}}\right)^{\frac{1-\rho}{\rho}} \\ \infty &> \rho^{\frac{\rho}{1-\rho}}\left(1+\alpha^{1/\rho}\right) > \alpha^{1/\rho} \\ \infty &> \rho^{\frac{\rho}{1-\rho}} > \alpha^{1/\rho} - \rho^{\frac{\rho}{1-\rho}}\alpha^{1/\rho} \\ \infty &> \frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)} > \alpha^{1/\rho} \\ \infty &> \left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho} > \alpha \end{aligned}$$

So if $1 < \rho$ then debt is decreasing in α for α smaller than $\left[\frac{\rho^{\frac{\rho}{1-\rho}}}{\left(1-\rho^{\frac{\rho}{1-\rho}}\right)}\right]^{\rho}$.

■

Appendix G: Deferred Results and Proofs

We often assume that $v(G(\cdot))$ is concave. Because $G(\cdot)$ is endogenous, it is helpful to provide sufficient conditions on the primitives that ensure the desired property. This is the purpose of the next result.

Lemma 4 ***(sufficient conditions for concavity of*** $v(G(\cdot))$***)*** *$v(G(\cdot))$ is concave if $G(x)$ is concave.*

1. *$G(x)$ is concave if and only if $\frac{v''\left([v']^{-1}(x)\right)}{h''\left([h']^{-1}(x)\right)}$ is nonincreasing in x.*
2. *(symmetric case) $G(x)$ is concave if $h(x) = v(x)$.*
3. *(proportional CRRA functions) $G(x)$ is concave if $v(x)$ is CRRA and $h(x) = \alpha v(x)$ for $\alpha > 0$.*
4. *(CRRA functions with different curvatures) Suppose $v(x) = x^p/p$ and $h(x) = x^q/q$, with $p, q < 1$. Then $G(x)$ is strictly concave if and only if $p < q$.*

Proof.

1. $X(c), G(c)$ solve:
$$\max_{(x,g)} h(x) + v(g) \text{ s.t. } x + g \leq 1 - c.$$
$G(c)$ is concave in c iff $G(1-c)$ is concave in c. So, let's make the change of variables $k = 1 - c$ and write the following auxiliary problem:
$$\max_{(x,g)} h(x) + v(g) \text{ s.t. } x + g \leq k.$$

Denote the solutions to the auxiliary problem by $\widetilde{X}(k), \widetilde{G}(k)$. Let us derive necessary and sufficient conditions for $\widetilde{G}(k)$ to be globally concave.

Form the auxiliary problem's Lagrangean to get the first order conditions:

$$h'\left(\widetilde{X}(k)\right) = \lambda(k) = v'\left(\widetilde{G}(k)\right). \tag{24}$$

Differentiate with respect to k :

$$h''\left(\widetilde{X}(k)\right)\widetilde{X}'(k) = \lambda'(k) = v''\left(\widetilde{G}(k)\right)\widetilde{G}'(k).$$

Note that $\lambda'(k) < 0$. Use (24) to substitute for $\widetilde{X}(k)$ and $\widetilde{G}(k)$:

$$h''\left(\left[h'\right]^{-1}(\lambda(k))\right)\widetilde{X}'(k) = \lambda'(k) = v''\left(\left[v'\right]^{-1}(\lambda(k))\right)\widetilde{G}'(k).$$

Eliminate $\lambda'(k)$ to get:

$$\frac{\widetilde{X}'(k)}{\widetilde{G}'(k)} = \frac{v''\left(\left[v'\right]^{-1}(\lambda(k))\right)}{h''\left(\left[h'\right]^{-1}(\lambda(k))\right)}.$$

Since the constraint $x + g \leq k$ must hold with equality, we must have $\widetilde{X}'(k) + \widetilde{G}'(k) = 1$, whence our equation can be rewritten as follows:

$$\frac{1}{\widetilde{G}'(k)} - 1 = \frac{v''\left(\left[v'\right]^{-1}(\lambda(k))\right)}{h''\left(\left[h'\right]^{-1}(\lambda(k))\right)}.$$

Therefore $\widetilde{G}'(k)$ is decreasing in k if and only if:

$$\frac{v''\left(\left[v'\right]^{-1}(\lambda(k))\right)}{h''\left(\left[h'\right]^{-1}(\lambda(k))\right)} \text{ is nondecreasing in } k.$$

Since $\lambda(k)$ is decreasing in k (recall that $\lambda'(k) < 0$), the above condition is equivalent to:

$$\frac{v''\left(\left[v'\right]^{-1}(x)\right)}{h''\left(\left[h'\right]^{-1}(x)\right)} \text{ nonincreasing in } x.$$

2. In this case we can see directly that $\widetilde{G}(c)$ is linear. Indeed, symmetry and concavity guarantee that $\widetilde{X}(c) = \widetilde{G}(c) = c/2$. Thus $\widetilde{G}(c)$ is (weakly) concave.

3. Consider now $\widetilde{v}(x) = \alpha v(x)$. Then:

$$\begin{aligned} \widetilde{v}'(x) &= \alpha v'(x) \\ \widetilde{v}''(x) &= \alpha v''(x) \\ \left[\widetilde{v}'\right]^{-1}(x) &= \left[v'\right]^{-1}\left(\frac{x}{\alpha}\right). \end{aligned}$$

When $v(x) = x^p/p$ we get:

$$\begin{aligned} v'(x) &= x^{p-1} \\ v''(x) &= (p-1)\,x^{p-2} \\ [v']^{-1}(x) &= (x)^{1/(p-1)}. \end{aligned}$$

Thus:

$$\begin{aligned} \widetilde{v}''(x) &= \alpha(p-1)\,x^{p-2} \\ [\widetilde{v}']^{-1}(x) &= [v']^{-1}\left(\frac{x}{\alpha}\right) = \left(\frac{x}{\alpha}\right)^{1/(p-1)} = \left(\frac{1}{\alpha}\right)^{1/(p-1)} [v']^{-1}(x). \end{aligned}$$

So

$$\begin{aligned} & \widetilde{v}''\left([\widetilde{v}']^{-1}(x)\right) \\ = \;& \alpha(p-1)\left([\widetilde{v}']^{-1}(x)\right)^{p-2} \\ = \;& \alpha(p-1)\left(\left(\frac{1}{\alpha}\right)^{1/(p-1)} [v']^{-1}(x)\right)^{p-2} \\ = \;& \left(\frac{1}{\alpha}\right)^{(p-2)/(p-1)} \alpha(p-1)\left([v']^{-1}(x)\right)^{p-2} \\ = \;& \left(\frac{1}{\alpha}\right)^{(p-2)/(p-1)} \alpha v''\left([v']^{-1}(x)\right) \\ = \;& \alpha^{\frac{1}{p-1}} v''\left([v']^{-1}(x)\right). \end{aligned}$$

Thus

$$\frac{\widetilde{v}''\left([\widetilde{v}']^{-1}(x)\right)}{v''\left([v']^{-1}(x)\right)} = \alpha^{\frac{1}{p-1}} \text{ independent of } x.$$

Thus the condition in part 1 of the lemma is verified trivially.

4. Given the functional forms of $v(x)$ and $h(x)$ we get:

$$\begin{aligned} v''\left([v']^{-1}(x)\right) &= (p-1)\,(x)^{(p-2)/(p-1)}, \\ h''\left([h']^{-1}(x)\right) &= (q-1)\,(x)^{(q-2)/(q-1)}, \end{aligned}$$

so that

$$\frac{v''\left([v']^{-1}(x)\right)}{h''\left([h']^{-1}(x)\right)} = \left(\frac{p-1}{q-1}\right) x^{\frac{(p-2)}{(p-1)} - \frac{(q-2)}{(q-1)}}.$$

Because $p, q < 1$ the term in parentheses is positive. Therefore, the RHS is decreasing in x

if and only if

$$\begin{aligned}\frac{(p-2)}{(p-1)} &< \frac{(q-2)}{(q-1)}\\ \frac{1}{(p-1)} &> \frac{1}{(q-1)}.\end{aligned}$$

Because $p, q < 1$ this equation is equivalent to $q > p$.

■

Proposition 11 ***(welfare properties of a constrained equilibrium in the CRRA case).*** *In the CRRA case with $\alpha_B = \alpha$ and $\rho_B = \rho$:*

1. *a zero-debt cap is binding if and only if $\rho < 1$;*
2. *relaxing a binding zero-debt cap is Pareto-optimal if:*

$$\left(\alpha^{\frac{1}{\rho}}\right)^{\frac{2\rho-1}{\rho}}\left(1+\alpha^{\frac{1}{\rho}}\right)^{\frac{1-\rho}{\rho}} + \left(\alpha^{\frac{1}{\rho}}\right)^{\frac{\rho+1}{\rho}} > \left(1+\alpha^{\frac{1}{\rho}}\right)^{2};$$

3. *if $\alpha = 1$ then relaxing a zero-debt cap is Pareto-optimal if $\rho < \log 2/\log 6 \simeq 0.39$.*
4. *if $\rho = 1 - \varepsilon$ then relaxing a binding zero-debt cap is never Pareto-optimal for any α provided that ε is sufficiently small.*

Proof.

1. Expression (22) describes group A's period-1 allocation problem as a function of d alone. In the case $\rho < 1$ the unconstrained optimum d^* exceeds 0 (see Proposition 8 part 3). A zero-debt cap rule out d^*. Because problem (22) is concave in d, the constrained optimum is for group A to choose debt right up to the ceiling. Thus a zero-debt cap is binding. If $\rho > 1$ Proposition 8 part 3 indicates that a zero-debt cap is not binding.
2. In the CRRA case, we can leverage (19) and Lemma (2) to write:

$$\begin{aligned}-G'\left(\overline{d}+E(\overline{d})\right) &= \frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}};\\ \frac{\partial g^*\left(\overline{d}\right)}{\partial \overline{d}} &= \frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}};\\ E'\left(\overline{d}\right) &= -\frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_B^{\frac{1}{\rho_B}}}{1+\alpha_B^{\frac{1}{\rho_B}}}\right)^{\frac{1-\rho}{\rho}}}.\end{aligned}$$

Recall the condition Proposition 4 states that if a debt ceiling $\overline{d}$ is binding for group A, then relaxing the debt ceiling increases group B's lifetime utility if

$$-G'\left(\overline{d}+E(\overline{d})\right)\frac{\partial g^{*}\left(\overline{d}\right)}{\partial\overline{d}}>1+E'\left(\overline{d}\right).$$

Substitute to get:

$$\frac{\alpha_{B}^{\frac{1}{\rho_{B}}}}{1+\alpha_{B}^{\frac{1}{\rho_{B}}}}\frac{\alpha^{\frac{1}{\rho}}}{1+\alpha^{\frac{1}{\rho}}}>1-\frac{1}{1+\alpha^{\frac{1}{\rho}}\left(\frac{\alpha_{B}^{\frac{1}{\rho_{B}}}}{1+\alpha_{B}^{\frac{1}{\rho_{B}}}}\right)^{\frac{1-\rho}{\rho}}}$$

Set $\alpha_B=\alpha$ and $\rho_B=\rho$, then manipulate the previous equation to get:

$$\left(\alpha^{\frac{1}{\rho}}\right)^{\frac{2\rho-1}{\rho}}\left(1+\alpha^{\frac{1}{\rho}}\right)^{\frac{1-\rho}{\rho}}+\left(\alpha^{\frac{1}{\rho}}\right)^{2}>\left(1+\alpha^{\frac{1}{\rho}}\right)^{2}. \tag{25}$$

3. When $\alpha=1$ condition (25) specializes to:

$$2^{\frac{1-\rho}{\rho}}+1>4.$$

After some manipulation, this condition is seen to be equivalent to:

$$\rho<\frac{\log 2}{\log 6}.$$

4. When $\rho=1$ condition (25) specializes to:

$$\alpha+\alpha^{2}>(1+\alpha)^{2}.$$

This condition never holds for any $\alpha>0$. Therefore, if $\rho=1-\varepsilon$ condition (25) must also fail for ε small enough.

■

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