Abstract

We build a heterogeneous-agent life-cycle incomplete-markets model of the US economy with multiple aggregate shocks (income, financial deregulation, and beliefs) leading to fluctuations in equilibrium house prices. Through a series of counterfactual numerical experiments, we address three questions. First, what was the main source of the boom and bust in house prices? We find that the belief shock plays a chief role in the behavior of prices and rents. Financial deregulation alone was unimportant, but its interaction with the other shocks explains the dynamics of home-ownership, leverage, and cash-out refinancing. The model’s cross-sectional implications are in line with the ‘new narrative’ of the crisis that emphasizes the role of middle- and high-income households. Second, how much of the dynamics of US nondurable consumption around the Great Recession was caused by the boom-bust of house prices? Our model suggests that changes in house prices alone can explain at least 1/2 of the corresponding boom-bust in expenditures, mostly occurring through a wealth effect rather than tightening access to credit. Third, would a massive debt forgiveness policy have cushioned the macroeconomic bust and accelerated the recovery? Our simulations imply that such government intervention would have dramatically reduced foreclosure rates, but would have had a trivial impact on the collapse of house prices and consumption expenditures. Finally, our model illustrates how the size of the elasticity of consumption to house prices depends crucially on the underlying shock that moves house prices. This finding has consequences for the use of sufficient-statistic approaches in this context.

Keywords: Beliefs, Consumption, Financial Deregulation, Great Recession, House Prices.

JEL Classification: E21, E30, E40, E51.
1 Introduction

During the Great Recession, the US economy experienced its sharpest drop in household consumption expenditures in the postwar period (De Nardi, French, and Benson, 2011; Petev, Pistaferr, and Eksten, 2011). Consumption declined precipitously across all categories, not just durables. Figure 1 (right panel) shows that real detrended expenditures in nondurable consumption dropped by over 10 percent from their peak in early 2007 to their trough roughly five years later. This rapid fall occurred at the end of a decade of markedly above-trend growth.

The leading interpretation of these atypical aggregate consumption dynamics emphasizes the extraordinary swings in U.S. housing net worth that occurred since the end of the 1990s (Mian and Sufi, 2014). The left panel of Figure 1 shows that house prices grew 3 percent per year above trend in the 1997-2006 decade, and then collapsed with a cumulative fall exceeding 40 percent in the next five years.

The aim of this paper is to quantitatively investigate the link between house prices and consumption around the Great Recession in order to understand the sources of house price fluctuations, their transmission into household consumption, and whether the policy interventions that were discussed at the time of the crisis, but implemented only timidly and with delay, could have dampened the bust.

For this purpose, we build a stochastic equilibrium model of the US economy. Our model economy is populated by overlapping generations of finitely-lived households who are subject to uninsurable earnings shocks. Households can save into a financial asset, a bond, whose price is set on the world market. They spend on non-durable consumption —the final good numeraire— and housing services. Housing services can be obtained by either renting or buying houses. Buyers have access to long-term mortgages priced competitively by financial intermediaries, and to one-period home-equity lines of credit (HELOCs). Homeowners, every period, can either choose to make their mortgage payment, refinance their mortgage, sell the house subject to a transaction fee, or default on their mortgage debt. A construction sector supplies new housing units and a rental sector intermediates housing services between owners and renters.

The key feature of our economy is that house prices (and rents) are not exogenous, as assumed by much of the literature, but they are determined in equilibrium and fluctuate endogenously in response to aggregate shocks. We model three sources of shocks, which have been identified as the main potential culprits of the boom-bust in the recent academic and policy debate: labor income shocks, shocks to the degree of financial deregulation in the mortgage market —captured by fluctuations in a subset of model parameters that determine households
Figure 1: Left panel: FHFA national house price index deflated by the price index of expenditures in nondurable and services. Right panel: Real consumption expenditures in nondurables and services (BEA, NIPA Table 1.1.5). Both series are deviations from linear time trends estimated separately over the period 1975-1996. The Boom is defined as the decade 1997-2007 and the bust as the post-2007 period.

borrowing limits and borrowing costs—and changing beliefs about future house price growth. This latter shock is modeled as a shift between two regimes that differ in the likelihood of a transition into a third regime where all households have stronger preference for housing, relative to nondurable consumption: this modeling expedient has a ‘bubble-like’ flavor, but it allows us to maintain a rational-expectation solution of the dynamic equilibrium. To discipline the calibration of this last stochastic process, we use survey data on expectations of future house appreciation during the early 2000s.

The model is parameterized to match life-cycle and cross-sectional patterns of income, consumption, assets and liabilities of US households in the period preceding the boom and bust (end of the 1990s). The boom-bust episode is modeled as a particular realization of our aggregate shocks where: (i) labor income rises and then falls back; (ii) financial constraints in mortgage markets loosen (in the boom) and then tighten again (in the bust); and (iii) a switch takes place from a regime where the transition towards a high taste for housing state is unlikely into a regime where such transition is likely (boom) and back (bust). We study IRF’s of the aggregate economy to these shocks and run a series of counterfactual experiments to answer three questions that have been at the core of the academic and public policy debate on the housing crisis.

First, which one of these shocks was responsible for the boom-bust in house prices around the Great Recession? Our model suggests unequivocally that the change in beliefs was the
driving force of house price dynamics. Financial deregulation alone plays only a minor part in accounting for the observed evolution of house prices, but its interaction with beliefs is important in explaining homeownership, leverage, and refinancing behavior. As we explain in the paper, the limited role of loosening and tightening of loan-to-value ratio limits finds an explanation in the existence of rental markets and the fact that mortgages are long-term contracts, two realities that are omitted in most of the literature.

We also show that, because expectations of house appreciation are shared by all borrowers —rich and poor, low-risk and high-risk—the model predicts that mortgage credit growth occurs uniformly across the household distribution. This implication is consistent with a new narrative of the crisis that has recently emerged thanks to more detailed micro data (Adelino, Schoar, and Severino, 2016; Albanesi, De Giorgi, Nosal, and Ploenzke, 2016; Foote, Loewenstein, and Willen, 2016).

Second, how much, and through which mechanism, did household consumption fall because of the collapse in house prices? Our model suggests that at a minimum half of the boom-bust in consumption over the 1997-2011 period is attributable to house price dynamics —more if one takes the view that a portion of the decline in labor income was caused by the collapse in housing net worth. Our model attributes only a small role to forced deleveraging and collateral effects and implies that aggregate consumption dropped because of a wealth effect.

Third, during the housing crisis, a number of policymakers and commentators advocated the implementation of a massive government-sponsored debt relief programs as a way to cushion the collapse of house prices and expenditures. The policies of the Obama administration consisted of interventions (e.g., HAMP and HARP) that, because of their complex rules and narrow scope, had very limited success (Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski, and Seru, 2012). We use the model to run a counterfactual where, at the onset of the bust, the government unexpectedly announces a principal reduction program whereby all mortgage debt in excess of 95 percent of home values is cancelled, with the government repaying the lender. In the model, this policy helps roughly 1/4 of homeowners. In spite of its large scale, we show that the policy would only have a trivial effect on house prices and consumption, despite significantly lessening aggregate leverage and, as a consequence, foreclosure rates.

Finally, our structural equilibrium model sheds new light on the ‘common factor’ problem that plagues the empirical literature that attempts to estimate, at the micro and macro level, the elasticity of consumption expenditures to house prices.¹ We argue that the search for ‘the’ right value of this elasticity is vacuous because the elasticity itself varies dramatically depending

on which underlying shock generates the movement in house prices: changes in house prices induced by aggregate income shocks have the highest elasticity, whereas those caused by relaxations or tightening of collateral constraints have the lowest impact on consumption (possibly negative). Expenditure elasticities to house prices movements brought about by belief and interest rate shocks are in between. We emphasize that, rather than looking for ‘exogenous’ sources of variation in house prices to estimate an elasticity that has no clear and interesting empirical counterpart, it is more productive to grasp the reality that different, historically relevant, macroeconomic shocks transmit to household consumption, through house prices, with different intensity.

This result has implications for the use of the sufficient statistic approach (as advocated, for example, in Chetty (2008)) to this question. In a recent paper, Berger, Guerrieri, Lorenzoni, and Vavra (2015) propose a clever and easy to compute ‘sufficient statistic’ to estimate the individual (micro) elasticity of consumption to house prices. We show that their sufficient statistic provides a good approximation to the true value (through the eyes of our model) in the context of the Great Recession. The reason is that the belief shock, which drives equilibrium price dynamics in this boom-bust episode, is largely orthogonal to other determinants of consumption decisions, and thus it is akin to house price shocks in housing demand models where such prices are exogenous, as in Berger et al. (2015). At the same time we caution against the use of this approach by showing that when house prices dynamics are caused by other shocks the sufficient statistic fails, sometimes quite dramatically, in mimicking the true elasticity.

1.1 Related Literature

[To Be Completed]

The rest of the paper is organized as follows. Section 2 outlines the model, the equilibrium concept, and our approach to numerical computation. Section 3 describes the model’s parameterization and its empirical fit. Section 4 presents the results from all our numerical experiments on the boom-bust. Section 5 discusses the shock- and state-dependence of the elasticity of aggregate consumption to house prices, and the implications for the sufficient statistic approach to this question. Section 6 concludes the paper. The Appendix [TBC] includes more details about the computational algorithm, including some accuracy tests.
2 Model

It is useful to succinctly delineate the main features of the model, before providing a formal description. The economy is populated by overlapping generations of households whose life-cycle is divided between work and retirement. During the working stage, they are subject to uninsurable idiosyncratic shocks to their efficiency units of labor, supplied inelastically. Household can save into a financial asset, a bond, whose price is set on the world market. They consume non-durable consumption—the final good numeraire—and housing services. Housing services can be obtained by either renting or buying houses that come in a finite number of sizes. The rental stock is owned by a competitive rental sector. Buyers have access to long-term mortgages priced competitively by financial intermediaries. Homeowners who do not sell can either choose to make their mortgage payment, refinance or default on their mortgage. Defaulting results in foreclosure by the intermediary which entails a utility loss, and an exacerbated depreciation for the house. Owning a house allows the homeowner to open HELOCs, modeled as one-period non-defaultable debt-contracts. A competitive construction sector supplies new housing units every period.

Three types of exogenous aggregate shocks can impact the economy every period: interest rates, the degree of financial regulation in the mortgage market, and beliefs about future taste for housing. It is convenient to postpone the exact definition of these shocks to Section 2.5, after we have outlined the rest of the model in detail.

In illustrating the model, we begin with all the model primitives that are needed to describe household decisions, and we lay out the household problems. Next, we present the financial intermediation sector, the rental side, and the production side of the economy. Finally, we define the equilibrium. Throughout, we adopt a recursive formulation of the economic environment in discrete time.

2.1 Households

Demographics: The economy is populated by a measure-one continuum of finitely-lived households. Age is indexed by \( j = 1, 2, \ldots, J \). Households work in the first phase of their life cycle and, at age \( J_{\text{ret}} \), they retire. They all die with certainty at age \( J \).

Preferences: Expected lifetime utility of the household is given by:

\[
E_0 \left[ \sum_{j=1}^{J} \beta^{j-1} u_j(c_j, s_j) + \beta^J v(y) \right]
\]  

(1)
where $\beta > 0$ is the discount factor, $c_j > 0$ is consumption of non-durables at age $j$, $s_j > 0$ is the consumption of housing services. Nondurable consumption is the numeraire good of the economy. The expectation is taken over sequences of aggregate and idiosyncratic shocks that we specify below. The function $v$ measures the felicity from leaving bequests $b > 0$.\footnote{This bequest motive prevents households from selling their house and dis-saving too much during retirement, which would be counterfactual.} Specifically, for $u$ we assume:

$$u_j(c_j,s_j) = e_j \left[ (1-\phi) c_j^{1-\gamma} + \phi s_j^{1-\gamma} \right]^{\frac{1}{1-\theta}} \frac{1}{1-\theta} - 1,$$

where $\phi$ measures the relative taste for housing, $1/\gamma$ measures the elasticity of substitution between housing services and non-durables, and $1/\theta$ measures the IES. The expenditures equivalence scale $\{e_j\}$ captures deterministic changes in household size and composition over the life cycle and explains why the intra-period utility function $u$ is indexed by $j$.

The warm-glow bequest motive at age $j$ takes the functional form:

$$v(b) = \psi (b + \flat)^{1-\theta} - 1,$$

as proposed by De Nardi et al. (2011): the term $\psi$ measures the strength of the bequest motive, while $\flat$ reflects the extent to which bequests are luxury goods.

**Endowments:** Working households receive an idiosyncratic labor income endowment $y_{jw}$ given by:

$$\log y_{jw} = Z + \chi_j + \epsilon_j$$

where $Z$ is an index of aggregate labor productivity. Individual labor productivity has two orthogonal components: $\chi_j$ is a deterministic age profile, and $\epsilon_j$ follows an idiosyncratic first-order Markov process. Households are born with initial wealth endowment $b_1$ drawn from an exogenous distribution that integrates up to the overall amount of wealth bequeathed in the economy by the deceased households. The draw is correlated with the initial draw of $\epsilon_1$. We also denote by $Y_j$ the age-dependent transition matrix for earnings and by $Y_j^*$ the earnings distribution at age $j$.

**Housing:** In order to consume housing services, households have the option of renting or owning a home. Houses are characterized by their size, whose number is finite. For owner-
occupied housing, house size belongs to the set $\mathcal{H} = \{h_0, \ldots, h_N\}$, where $h_0 < h_1, \ldots, h_{N-1} < h_N$. For rental units, size belongs to the set $\tilde{\mathcal{H}} = \{\tilde{h}_0, \ldots, \tilde{h}_N\}$.

Renting generates housing services one-for-one with the size of the house, i.e. $s_j = h_j$. To capture the fact there may be additional utility from home ownership, we assume that an owner-occupied house generates $s_j = \omega h_j$ units of housing services, with $\omega > 1$. The rental rate of a unit of housing is denoted by $\rho$. The per-unit price of housing is denoted by $p_h$. Owner-occupied houses carry a per-period maintenance and tax cost of $(\delta h + \tau h)p_hh$, expressed in units of the numeraire good. Maintenance fully offsets physical depreciation of the dwelling $\delta h$. When a household sells its home, it incurs a transaction cost $\kappa h(p_hh)$, linear in the house value.

Financial Instruments: Households can save in one-period bonds, $b$, at the price $q_b$ exogenously determined by the net supply of financial assets from the rest of the world. It is convenient to also define the interest rate on bonds $r_b := 1/q_b - 1$. We allow homeowners to access HELOCs: they can borrow up to a fraction $\lambda b$ of the value of their house at an interest equal to $r_b = r_b(1 + \iota_b)$, where $\iota_b > 0$ is an intermediation wedge.\footnote{In what follows to lighten the exposition, with a slight abuse of notation, we keep denoting the interest rate on liquid assets $r_b$ but it is implicit that it equals $r_b$ when $b < 0$. We use a similar convention for $q_b$.}

Housing purchases can be financed by taking on mortgages. All mortgages are long-term and amortized over the remaining life of the buyer at the real interest rate $r_m$ equal to $r_b$ times the wedge $(1 + \iota_m)$, with $\iota_m > 1$. Newly originated mortgages are subject to a fixed origination cost $\kappa_m$. They must also respect a maximum loan-to-value (LTV) ratio limit: the initial principal balance $m$ must be less than a fraction $\lambda m$ of the value of the home. Note that, once a mortgage is originated, there is no further requirement that $m < \lambda m p_hh$. This realistic assumption, crucial to understanding deleveraging behavior and, thus, the consumption response to house price shocks, sets our model apart from several notable contributions in this literature (Favilukis, Ludvigson, and Van Nieuwerburgh, 2010; Iacoviello and Neri, 2010)).

A household of age $j$ that takes out a mortgage with principal balance $m$ receives $q_m m$ units of the numeraire good in the current period, with $q_m \leq 1$. Thus, the down payment required by the household at origination is $p_hh - q_m m$. Note that the principal due on a mortgage of size $m$ is not equal to the funds received from the bank at the time of purchase ($q_m m$) because the pricing of the mortgage accounts for the possibility of default, a choice that depends on all individual and aggregate states.\footnote{One can interpret this gap as so-called “points” or other up-front interest rate charges that households face when taking out their loans.} Section 2.2 below provides the exact expression for the equilibrium price $q_m$.\footnote{Section 2.2 below provides the exact expression for the equilibrium price $q_m$.}
Going forward, the household makes $J - j$ equal mortgage payments $\pi_m$ that must exceed the minimum mortgage payment:

$$\pi^* (m) = \frac{r_m(1 + r_m)^{J-j}}{(1 + r_m)^{J-j} - 1} m,$$

and the remaining principal evolves as $m' = m(1 + r_m) - \pi$. Even though all households pay the same interest rate $r_m$ on the principal due, the heterogeneity in principals $m$ and prices $q_m$ maps into heterogeneous effective interest rates. In simulations, when a household originates a mortgage of size $m$, we can use $q_m$ and $\pi^* (m)$ to solve for the effective interest rate $r^*_m$ on the mortgage through the relationship:

$$\frac{\pi^* (m)}{q_m} = \frac{r^*_m(1 + r^*_m)^{J-j}}{(1 + r^*_m)^{J-j} - 1}.$$  (6)

This formula solves for the interest rate $r^*_m$ that would yield constant mortgage payment schedule $\pi^* (m)$ on an outstanding balance of $q_m m$ (the funds received at origination).5

Mortgage holders have the option to refinance, by repaying the residual principal balance and originating a new mortgage at cost $\kappa_m$. If a household chooses to sell its home, it is also required to pay off its remaining mortgage balance. Households also have the option to default on their mortgage debt. Upon default, mortgages are designated as the primary lien on the house, implying that the proceeds from the foreclosure are disbursed to the mortgagee. We assume no recourse in case of foreclosure. Foreclosing reduces the value of the house to the lender because it is the lender who must pay property taxes $\tau_h$ and maintenance, and the foreclosed house depreciates at a higher rate than regular houses, i.e. depreciation in case of foreclosure is $\delta^d_h > \delta_h$. Thus the lender recovers $\min \{ (1 - \delta^d_h - \tau_h) p_h h, (1 + r_m) m \}$. A household who defaults incurs a utility penalty $\xi$ in the period of default.

**Government:** The government spends an amount $G$ on services that are not valued by households. It also runs a PAYG social security system. Retirees receive social security benefits $y^\text{ret}(c_{1w})$, where the argument of the benefit function proxies for average gross lifetime earnings. In what follows, we adopt the notation $y_j$ for income at age $j$, with the convention that if $j < J^\text{ret}$ then $y_j = y^w_j$ and $y_j = y^\text{ret}$ otherwise. We also denote by $Y^*_\text{ret}$ the income distribution for retirees.

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5A richer model would allow the households to simultaneously choose the amortization interest rate $r_m$ and the principal $m$ so that effectively they could choose $q_m m$, as is common in the data. However, this formulation would add a state variable (the amortization rate). For tractability we impose the fixed amortization interest rates.
To finance these expenditures, the government levies a property tax $\tau_h$ on the value of the house, a flat payroll tax $\tau^{ss}$ and a progressive labor income tax $\tau^y(y_j)$. Households can deduct the interest paid on mortgages against their taxable income. We denote the combined income tax liability function $T(y_j, m_j)$. A final source of revenues for the government comes from the proceedings of the sale of new land permits for construction, as described in more detail in Section 2.3 below.

### 2.1.1 Household Decision Problems

To simplify the notation, we let $\Omega \in \mathcal{O}$ denote the vector of aggregate state variables, defined below. We begin by stating the problem of non-homeowners (renters and buyers). Next we state the problem of home-owners (sellers, keepers who repay, keepers who refinance, and households who default). Finally, we describe the problem of the retiree in its last period of life, when the warm-glow bequest motive is active.

**Renters and Buyers:** Let $V^n$ denote the value function of households who start the period without owning any housing. These households choose between being a renter and buying a house to become an owner by solving:

$$V^n(b_j, y_j; \Omega) = \max \{ V^r(b_j, y_j; \Omega), V^o(b_j, y_j; \Omega) \}, \quad (7)$$

where we let $g^o(b_j, y_j; \Omega) \in \{0, 1\}$ denote the decision to own a house.\footnote{It is implicit that, when this decision takes the value of zero, the household chooses to be a renter.}

Those who choose to rent solve:

$$V^r(b_j, y_j; \Omega) = \max_{c_j, h_j, b_{j+1}} u_j(c_j, s_j) + \beta \mathbb{E}_{y_j, \Omega} \left[ V^n(b_{j+1}, y_{j+1}; \Omega') \right] \quad (8)$$

s.t.

$$c_j + \rho(\Omega) h_j + q_b b_{j+1} \leq b_j + y_j - T(y_j, 0)$$

$$b_{j+1} \geq 0$$

$$s_j = h_j \in \mathcal{H}$$

$$y_{j+1} = Y_j(y_j), \quad \Omega' = \Gamma(\Omega)$$

Let $x_j \equiv (b_j, h_j, m_j)$ denote the household portfolio of assets and liabilities. Those who...
choose to buy and become owners solve:

$$V^o(b_j, y_j; \Omega) = \max_{c_j, b_{j+1}, h_{j+1}, m_{j+1}} \left[ u_j(c_j, s_j) + \beta \mathbb{E}_{y_j, \Omega} \left[ V^h(x_{j+1}, y_{j+1}; \Omega') \right] \right]$$

subject to:

$$c_j + q_b b_{j+1} + p_h(\Omega) h_{j+1} + \kappa_m \leq b_j + y_j - T(y_j, 0) + q_m(x_j, y_j; \Omega) m_{j+1}$$

$$m_{j+1} \leq \lambda^m p_h(\Omega) h_{j+1}$$

$$b_{j+1} \geq 0$$

$$h_{j+1} \in \mathcal{H}, \quad s_j = \omega h_{j+1}$$

$$y_{j+1} = Y_j(y_j), \quad \Omega' = \Gamma(\Omega)$$

where $V^h(\cdot)$ is the value function of a household that starts off the next period as a homeowner that we describe below.

**Homeowners:** A homeowner has the option to keep the house and make its mortgage payment, refinance the house, sell the house, or default (obviously, this latter option can be optimal only if the household has some residual mortgage debt).

$$V^h(x_j, y_j; \Omega) = \max \begin{cases} 
\text{Pay:} & V^p(x_j, y_j; \Omega) \\
\text{Refinance:} & V^f(x_j, y_j; \Omega) \\
\text{Sell:} & V^n(b^n_j, y_j; \Omega) \\
\text{Default:} & V^d(b_j, y_j; \Omega) 
\end{cases}$$

It is convenient to denote the refinance decision by $g^f(x_j, y_j; \Omega)$, the selling decision by $g^n(x_j, y_j; \Omega)$, and the mortgage default decision by $g^d(x_j, y_j; \Omega)$. All these decisions are dummy variables in $\{0, 1\}$ and it is implicit that, when they are all zeros, the homeowner chooses to make a payment on its mortgage during that period. We now describe all these four options one by one.
A household that chooses to make a mortgage payment solves:

\[ V(x_j, y_j; \Omega) = \max_{c_j, b_{j+1}, \pi} u(c_j, s_j) + \beta \mathbb{E}_{y_j, \Omega} \left[ V(x_{j+1}, y_{j+1}; \Omega') \right] \tag{10} \]

s.t.
\[ c_j + q_b b_{j+1} + (\delta_h + \tau_h) p_h (\Omega) h_j + \pi \leq b_j + y_j - \mathcal{T}(y_j, m_j) \]
\[ \pi \geq \pi^* (m_j) \]
\[ m_{j+1} = (1 + r_m) m_j - \pi \]
\[ b_{j+1} \geq -\lambda^b p_h (\Omega) h_{j+1} \]
\[ s_j = \omega h_j, \quad h_{j+1} = h_j \]
\[ y_{j+1} = Y_j (y_j), \quad \Omega' = \Gamma (\Omega) \]

Note that because the mortgage is long-term, there is no requirement that the principal outstanding on the mortgage be less than \( \lambda^m \) times the current value of the home. If the aggregate house price had declined, the household could be underwater on its mortgage, but so long as it continues to make its mortgage payment it is not forced to deleverage. HELOCs—because they are refinanced each period—are instead subject to a period-by-period constraint on the balance relative to the current home value.

An homeowner who chooses to refinance its mortgage solves the following problem:

\[ V^f(x_j, y_j; \Omega) = \max_{c_j, b_{j+1}, m_{j+1}} u(c_j, s_j) + \beta \mathbb{E}_{y_j, \Omega} \left[ V^h(x_{j+1}, y_{j+1}; \Omega') \right] \tag{11} \]

s.t.
\[ c_j + q_b b_{j+1} + (\delta_h + \tau_h) p_h (\Omega) h_j + (1 + r_m) m_j \]
\[ \leq b_j + y_j - \mathcal{T}(y_j, m_j) - \kappa_m + q_m (x_j, y_j; \Omega) m_{j+1} \]
\[ m_{j+1} \leq \lambda^m p_h (\Omega) h_{j+1} \]
\[ b_{j+1} \geq -\lambda^b p_h (\Omega) h_{j+1} \]
\[ s_j = \omega h_j, \quad h_{j+1} = h_j \]
\[ y_{j+1} = Y_j (y_j), \quad \Omega' = \Gamma (\Omega) \]

A homeowner that chooses to sell its home solves the problem as if it started the period without any housing, i.e., with value function \( V^w \) given by (7) with financial assets equal to its...
previous holdings plus the net-of-costs proceeds from the sale of the home, i.e.

\[ b_j^H (x_j; \Omega) = b_j + (1 - \delta_h - \tau_h - \kappa_h) \ p_h (\Omega) \ h_j - (1 + r_m) \ m_j \]  

(12)

The timing ensures that a household can sell and buy a new home within the period.

Finally, a household that has defaulted on its mortgage incurs a utility penalty \( \xi \) and must rent for a period, thus solving (7) with financial assets equal to \( b_j \). Only in the following period the household can buy another house.

**Bequest:** In the last period of life, \( j = J \), the warm-glow inheritance motive, apparent from preferences in (1), induces households to leave a bequest. For example, a retired homeowner of age \( J \) (who does not sell its house in this last period) would solve:

\[ V^p (x_J, y_J; \Omega) = \max_{c_J, s_J} u (c_J, s_J) + \beta v (b) \]  

(13)

\[ s.t.
\]

\[ c_J + q_h b_{J+1} + (1 + r_m) m_J \leq b_J + y_J - T (y_J, m_J) \]

\[ b_{J+1} \geq 0 \]

\[ s_J = \omega h_J, \ h_{J+1} = h_J \]

\[ \Omega' = \Gamma (\Omega) \]

In other words, in the last period of life households pay off their residual mortgage and HELOC and take into account that their residual housing wealth contributes to bequests only as the expected net-of-costs proceedings from the sale, next period.

### 2.2 Financial Intermediaries

The financial intermediation sector is perfectly competitive with free entry. Loans are therefore priced through a zero-profit condition that holds loan by loan. The pricing of the mortgage can be defined recursively as in Chatterjee and Eyigungor (2013) long-term sovereign debt default model, adapted here to collateralized debt and finite lifetimes.

Mortgage prices depend on the age \( j \) of the homeowner, all its choices of assets and liabilities for next period \( x_{j+1} := (b_{j+1}, h_{j+1}, m_{j+1}) \), on its current income state \( (y_J) \), and on the current
aggregate state vector $\Omega$. Thus, we can write:

$$q_m(x_j+1, y_j; \Omega) = \frac{1}{(1 + r_m)m_{j+1}} \cdot \mathbb{E}_{y_j, \Omega} \left\{ g^n(x_{j+1}, y_{j+1}; \Omega') + g^f(x_{j+1}, y_{j+1}; \Omega') \right\} (1 + r_m)m_{j+1}$$

(14)

$$+ g^d(x_{j+1}, y_{j+1}; \Omega') \min\left( 1 - \frac{\delta^d}{\delta^h - \tau_h}, \frac{p_h(\Omega') h_{j+1, (1 + r_m)m_{j+1}}} \right)$$

$$+ \left[ 1 - g^n(\cdot) - g^f(\cdot) - g^d(\cdot) \right] \left\{ \pi_m(x_{j+1}, y_{j+1}; \Omega') + q_m(x_{j+2}, y_{j+1}; \Omega')m_{j+2} \right\}$$

Intuitively, if the households sells ($g^n = 1$) or refinances ($g^f = 1$) its home, it has to payoff the mortgage, so the financial intermediary receives the full principal plus interest. If the household defaults on the mortgage ($g^d = 1$), the intermediary forecloses and recovers the minimum between the depreciated value of the home and the value of the residual mortgage debt. If the household makes a payment on the home ($g^n = g^f = g^d = 0$), the value to the intermediary is the contemporaneous value of the mortgage payment, plus the continuation value of the remaining balance of the mortgage going forward —which is compactly represented by the pricing function.\footnote{Note that a lender who observes $x_{j+1}$ can compute next-period decisions $x_{j+2}$ for each possible future realization $y_{j+1}, \Omega'$.}

Finally, one should note that, these zero-profit conditions hold in expectation only. Thus, strictly speaking, because of the aggregate risk, along the equilibrium path the financial intermediaries would be making profits and losses. We assume that the financial intermediaries (only) have access to a full set of Arrow securities that span the aggregate risk with the rest of the world and therefore make zero profits period by period.

### 2.3 Production

Production in the economy is divided between two sectors: the final good sector which produces non-durable consumption (the numeraire good of the economy), and a construction sector which produces new houses. Labor is perfectly mobile across sectors.

**Final Good Sector:** The final good sector operates a constant returns to scale technology

$$Y = ZN_c$$

(15)

where $Z$ is the aggregate productivity level, and $N_c$ are units of labor services. From the competitive firm problem the wage is simply $w = Z$.\footnote{Note that a lender who observes $x_{j+1}$ can compute next-period decisions $x_{j+2}$ for each possible future realization $y_{j+1}, \Omega'$.}
**Construction Sector:** The competitive construction sector operates with production technology $I_h = (ZN_h)^\alpha (\bar{L})^{1-\alpha}$, with $\alpha \in (0, 1)$, where $N_h$ are units of labor services in this sector, and $\bar{L}$ is the amount of new buildable land available for construction: each period the government issues new permits equivalent to $\bar{L}$ units of buildable land. We follow Favilukis et al. (2010) in assuming that permits are sold at market price to developers, and thus all rents accrue to the government. The developer therefore solves the static problem:

$$
\begin{align*}
\max_{N_h} p_h(\Omega) I_h - wN_h \\
\text{s.t.} \\
I_h = (ZN_h)^\alpha (\bar{L})^{1-\alpha}
\end{align*}
$$

which, after substituting the equilibrium condition $w = Z$, implies labor demand and housing investment functions:

$$
\begin{align*}
N_h(\Omega) &= [\alpha p_h(\Omega)]^{\frac{1}{1-\alpha}} \bar{L}/Z, \\
I_h(\Omega) &= [\alpha p_h(\Omega)]^{\frac{\alpha}{1-\alpha}} \bar{L}.
\end{align*}
$$

Note that the aggregate housing supply price-elasticity is $\alpha / (1 - \alpha)$.

### 2.4 Rental Sector

A competitive rental sector owns housing units and rents them out to households. Rental companies, owned by risk-neutral agents, can buy and sell units frictionlessly on the housing market and incur an operating cost $\psi$ for each unit of housing they rent. The problem of the representative rental company is therefore:

$$
J(\bar{H}; \Omega) = \max_{\bar{H}'} -p_h(\Omega) [\bar{H}' - (1 - \delta_h - \tau_h) \bar{H}] + (\rho(\Omega) - \psi) \bar{H}' + \left( \frac{1}{1 + r^b} \right) E_\Omega [J(\bar{H}'; \Omega')]
$$

Optimization implies that the equilibrium rental rate equals the user cost of housing, or:

$$
\rho(\Omega) = \psi + p_h(\Omega) - \left( \frac{1 - \delta_h - \tau_h}{1 + r^b} \right) E_\Omega [p_h(\Omega')],
$$

which establishes a standard ‘Jorgensonian’ user cost relationship between equilibrium rent and current and future equilibrium house prices.\(^8\)

---

\(^8\)The implicit assumption we make here is that when the rental company buys owner occupied houses of vari-
2.5 Aggregate Risk

There are three sources of aggregate shocks in our economy, all of which are assumed to follow stationary Markov chains. First, aggregate labor productivity $Z$. Second, a set of time-varying parameters that characterizes the degree of financial regulation in mortgage markets: we follow Favilukis et al. (2010) and choose the maximum loan-to-value ratio at mortgage origination $\lambda^m$, the HELOC borrowing limit $\lambda^b$, and the mortgage origination cost $\kappa_m$ which we combine into an index of financial deregulation $F = (\lambda^m, \lambda^b, \kappa_m)$. Finally, we introduce aggregate uncertainty about future taste for housing: the parameter $\phi$ follows a discrete Markov chain with three states $(\phi_L, \phi^*_L, \phi_H)$ with $\phi_H > \phi_L = \phi^*_L$. The difference between the two low states is that when the economy hits the $\phi^*_L$ state it is more likely to transit into the high state $\phi_H$. Therefore, a transition between $\phi_L$ and $\phi^*_L$ is a news/belief shock about future demand for housing, whereas a shift between $\phi_L$ (or $\phi^*_L$) and $\phi_H$ is an actual preference shock.

In what follows, we compactly denote the vector of exogenous shocks $(Z, F, \phi)$ as $\mathcal{Z}$. Because of aggregate risk and incomplete markets, the equilibrium distribution of households $\mu$ is a state variable needed to forecast next period house prices and rents. Thus the vector of aggregate states used in the recursive description of the household problem is $\Omega = (\mathcal{Z}, \mu)$.

2.6 Equilibrium

To ease notation, in the definition of equilibrium we denote the vector of individual states for age-$j$ homeowners and non-homeowners as $x^h_j := (b_j, h_j, m_j, y_j) \in X^h_j$ and $x^n_j := (b_j, y_j) \in X^n_j$. Let $\mu_j := (\mu^h_j, \mu^n_j)$ be the measure of these different types of households at age $j$, with

$$\sum_{j=1}^I (\mu^h_j + \mu^n_j) = 1.$$

A recursive competitive equilibrium consists of value functions $\{V^m(x^m_j; \Omega), V^r(x^n_j; \Omega), V^o(x^h_j; \Omega), V^p(x^h_j; \Omega), V^f(x^h_j; \Omega), V^d(x^n_j; \Omega)\}$, decision rules

$$\{s^o(x^h_j; \Omega), s^n(x^h_j; \Omega), s^f(x^h_j; \Omega), s^d(x^n_j; \Omega), c^h_j(x^h_j; \Omega), c^n_j(x^n_j; \Omega), b^h_{j+1}(x^n_j; \Omega), b^n_{j+1}(x^n_j; \Omega), h_{j+1}(x^n_j; \Omega), m_{j+1}^n(x^n_j; \Omega), m_{j+1}^h(x^h_j; \Omega)\},$$

a rental function $\rho(\Omega)$, house price function $p_h(\Omega)$, mortgage price function $q_m(x^h_j; \Omega)$, aggregate functions for construction labor, rental units stock, property housing stock, housing investment, and government expenditures $\{N_h(\Omega), \tilde{H}(\Omega), H(\Omega), I_h(\Omega), G(\Omega)\}$, and a law of motion for the aggregate states $\Gamma$ such that:

ous sizes in $\mathcal{H}$, it can freely recombine these units into housing sizes in $\tilde{\mathcal{H}}$. 

15
1. Household optimize, by solving problems (7)-(13), with associated value functions \( \{ V_n, V_r, V^o, V^p, V^f, V^d \} \) and decision rules \( \{ g^o, g^n, g^f, g^d, c^h_j, c^n_j, b^h_{j+1}, b^n_{j+1}, h_{j+1}, h^m_j, m^m_{j+1} \} \).

2. Firms in the construction sector maximize profits, by solving (16), with associated labor demand and housing investment functions \( \{ N_h(\Omega), I_h(\Omega) \} \).

3. The labor market clears at the wage rate \( w = Z \), and labor demand in the final good sector is determined residually as \( N_c = 1 - N_h(\Omega) \).

4. The financial intermediation market clears loan-by-loan with pricing function \( q_m(x^h_{j+1}; \Omega) \) determined by condition (14).

5. The rental market clears at price \( \rho(\Omega) \) given by (19), and the equilibrium quantity of rental units satisfies:

\[
\hat{H}'(\Omega) = \sum_{j=1}^I \left[ \int_{\chi^h_j} \tilde{h}_j \left( b^p_n \left( x^h_j; \Omega \right), y_j; \Omega \right) \left[ 1 - g^o \left( x^n_j; \Omega \right) \right] g^n \left( x^n_j; \Omega \right) d\mu^h_j 
+ \int_{\chi^h_j} \tilde{h}_j \left( x^n_j; \Omega \right) g^d \left( x^h_j; \Omega \right) d\mu^h_j 
+ \int_{\chi^h_j} \tilde{h}_j \left( x^n_j; \Omega \right) [1 - g^o \left( x^n_j; \Omega \right)] d\mu^n_j \right]
\]

where the LHS is the total supply of rental units and the RHS is the demand of rental units by households who sell and become renters, households who default on their mortgage, plus renters who stay renters. The function \( b^p_n \left( x^h_j; \Omega \right) \) represents the financial wealth of the seller, after the transaction, see equation (12).

6. The housing market clears at price \( p_h(\Omega) \) and the equilibrium quantity of housing, measured at the end of the period after all decisions are made, satisfies:

\[
I_h(\Omega) - \delta_h H(\Omega) = \left[ \hat{H}'(\Omega) - (1 - \delta_h) \hat{H}(\Omega) \right] + \sum_{j=1}^I \left[ \int_{\chi^h_j} h_{j+1} \left( x^n_j; \Omega \right) g^o \left( x^n_j; \Omega \right) d\mu^n_j
- \int_{\chi^h_j} h_j \left[ g^n \left( x^h_j; \Omega \right) + \left( 1 - (\delta^d - \delta_h) \right) g^d \left( x^h_j; \Omega \right) \right] d\mu^h_j
- \int_{\chi^h_j} h_{j+1} \left( x^h_j; \Omega \right) d\mu^h_j \right]
\]

The left hand side represents the net addition to the capital stock of owner occupied houses, or the new houses on the market. The right hand side combines the houses pur-
chased by the rental company and by new owners (first line) minus the sale of houses and the foreclosed properties that are back on the market after depreciation (second line), minus the houses sold on the market when the wills of the deceased are executed (third line).

7. The final good market clears:

\[ Y = \sum_{j=1}^{J} \left\{ \int_{\chi_{j}^{h}} c_{j}^{h} \left( x_{j}^{h}; \Omega \right) d\mu_{j}^{h} + \int_{\chi_{j}^{n}} c_{j}^{n} \left( x_{j}^{n}; \Omega \right) d\mu_{j}^{n} + \int_{\chi_{j}^{h}} \kappa_{h} \cdot p_{h} \left( \Omega \right) h_{j} g_{n} \left( x_{j}^{h}; \Omega \right) d\mu_{j}^{h} \right\} + \kappa_{m} \left[ \int_{\chi_{j}^{n}} m_{j+1}^{n} \left( x_{j}^{n}; \Omega \right) g_{n} \left( x_{j}^{n}; \Omega \right) d\mu_{j}^{n} + \int_{\chi_{j}^{h}} m_{j+1}^{h} \left( x_{j}^{h}; \Omega \right) g_{f} \left( x_{j}^{h}; \Omega \right) d\mu_{j}^{h} \right] + \tau_{h} p_{h} H \left( \Omega \right) + \left[ p_{h} \left( \Omega \right) I_{h} \left( \Omega \right) - w N_{h} \left( \Omega \right) \right] \] (20)

where the first two terms on the RHS are expenditures in nondurable consumption, the third term is transaction fees on sales, the terms on the second line are mortgage origination and refinancing costs, the third line represents intermediation costs on mortgage and HELOC credit, and the last line includes operating costs of the rental company, government expenditures on the numeraire good, and net exports \( NX \).

8. The government budget constraint holds, with expenditures \( G \left( \Omega \right) \) adjusting residually to absorb shocks:

\[ G \left( \Omega \right) + \left( \frac{\bar{J} - J_{ret} + 1}{J} \right) \int_{\chi_{ret}} y_{ret} dY_{ret} = \sum_{j=1}^{J} \left[ \int_{\chi_{j}^{h}} T \left( y_{j}, m_{j} \right) d\mu_{j}^{h} + \int_{\chi_{j}^{n}} T \left( y_{j}, 0 \right) d\mu_{j}^{n} \right] + \tau_{h} p_{h} H \left( \Omega \right) + \left[ p_{h} \left( \Omega \right) I_{h} \left( \Omega \right) - w N_{h} \left( \Omega \right) \right] \] (21)

where expenditures on goods and pension payments (the LHS) are financed by income (net of mortgage interest deduction) and property taxes, and the revenues from selling new licences to developers.

9. The aggregate law of motion \( \Gamma \) is consistent with individual behavior.

2.6.1 Numerical computation of equilibrium

Our computation strategy follows the insight developed in Krusell and Smith (1998): since it is computationally infeasible to keep track of the entire equilibrium distribution \( \left\{ \mu_{j} \right\}_{j=1}^{J} \), we
substitute it with a lower dimensional vector that, ideally, provides sufficient information to agents to make accurate forecasts.

In our model, in every period, there is one “deep” price that the households need to know and need to forecast when making decisions: \( p_h \), the price of owner-occupied housing. Knowing its law of motion is sufficient to pin down both the full mortgage pricing schedule (see eq. 14) and the rental rate (see eq. 19). A key difference with the original Krusell and Smith framework, is that the total stock of owner occupied houses, \( H \), is not predetermined (as is capital in Krusell and Smith), but it is determined in equilibrium to clear the housing market. Thus our problem is akin to the Krusell and Smith economy with a risk-free bond, or with endogenous labor supply.

To approximate the exact equilibrium, we propose to simply forecast next-period price of housing \( p_h \), as a function of the current price, the current exogenous states and and next period exogenous states. This strategy has promise, because as reflected in equation (18), housing investment is entirely pinned down by the price of housing. In sum, we conjecture a law of motion for \( p_h \) of the form:

\[
\log p_h'(p_h, Z, Z') = a_0(Z, Z') + a_1(Z, Z') \log p_h
\]  

(22)

and iterate, using actual market-clearing prices at each step, until we achieve convergence on the vector of coefficients \( \{a_0(Z, Z'), a_1(Z, Z')\} \).

Appendix [TBC] provides more details on the computation strategy.

3 Parameterization

There are two groups of parameters in the model. Values for the first group are assigned externally, without the need to solve for the model’s equilibrium. The values for the second group are, instead, chosen internally: they are determined by a minimum-distance algorithm that aims at setting a number of equilibrium moments from the model’s stochastic steady state as close as possible to their data counterpart.

The model’s parameterization is meant to capture certain key cross-sectional features of the US economy before the start of boom-bust in the housing market, i.e., in the late 1990s. In particular, to benchmark our economy to the data, we use information from the 1998 wave of the SCF. The parameter values are summarized in Tables 1 and the targeted moments in Table 2.

The stochastic processes for the aggregate shocks are described in Section 3.1.
**Demographics:** The model period is equivalent to 2 years of life. We think of households entering the model at age 21. Thus, set the maximum lifetime $J$ to 30 periods (age 81) and the retirement age $J^{ret}$ to 22 (age 65).

**Preferences:** We set the elasticity of substitution in (2) to 1.25 based on the estimates of Piazzesi, Schneider, and Tuzel (2007). We set the IES to 0.5, hence $\sigma = 2$. The consumption expenditures equivalence scale $\{e_j\}$ reproduces the McClements scale, a commonly used consumption equivalence measure. The additional utility from owner-occupied housing $\omega (h_j)$ is assumed to be linear in $h_j$ and the coefficient $\omega$ is set to match the average homeownership rate in the US economy before the boom-bust episode, i.e., 66 pct.

The warm-glow bequest motive function (3) is indexed by two parameters: $\nu$ measures the strength of the bequest motive, while $\flat$ reflects the extent to which bequests are luxury goods. Parameters $\flat$ and $\nu$ are chosen to match the fraction of households leaving a positive inheritance in the bottom half of the distribution and the home-ownership rate for the elderly (70 and older).

The disutility from mortgage default, $\xi$, is chosen to generate an equilibrium foreclosure rate of 0.5 pct, the empirical counterpart for the late 1990s.

Finally the discount factor $\beta$ is chosen to replicate a ratio of aggregate net worth to aggregate annual income of 3.5.9

**Endowments:** The deterministic earnings component of income $\{\chi_j\}$ is chosen, as in Kaplan and Violante (2014) to replicate the fact that average earnings grow roughly by a factor of 3 to their age 50 peak, and the decline slowly over the rest of the working life. The stochastic component of earnings $y_j$ is modeled as an AR(1) process in logs with annual persistence of 0.97, annual standard deviation of innovations of 0.20, and initial standard deviation of 0.42. This parameterization implies a rise in the variance of log earnings of 2.5 between the ages of 21 and 64 (in line with Heathcote, Perri, and Violante, 2010)). We normalize earnings so that median annual household earnings ($52,000 in the 1998 SCF) equal 1 in the model.

---

9The model also generates a median net worth to income ratio of 0.6, virtually equal to its empirical counterpart from the SCF.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Internal</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
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</tr>
<tr>
<td>$J^w$</td>
<td>Age of retirement</td>
<td>N</td>
<td>22 (period = 2 yrs)</td>
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<tr>
<td>$J$</td>
<td>Length of life</td>
<td>N</td>
<td>30</td>
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<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/\gamma$</td>
<td>Elast. subst $(c, s)$</td>
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<td>${e_j}$</td>
<td>Equivalence scale</td>
<td>N</td>
<td>McClements scale</td>
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<tr>
<td>$\omega$</td>
<td>Additional utility from owning</td>
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<td>$\psi$</td>
<td>Strength of bequest motive</td>
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<td>$\beta$</td>
<td>Extent of bequest as luxury</td>
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<tr>
<td>$\xi$</td>
<td>Utility cost of foreclosure</td>
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<td>$\beta$</td>
<td>Discount factor</td>
<td>Y</td>
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<td>${\chi_j}$</td>
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<td>$\mathcal{H}$</td>
<td>Owner-occupied house sizes</td>
<td>Y</td>
<td>{1.5, 2.0, 2.5, 3, 25, 4, 0, 5.5}</td>
</tr>
<tr>
<td>$\mathcal{H}$</td>
<td>Rental house sizes</td>
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<td>{1.0, 1.5, 2.0}</td>
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<td>$\delta_h$</td>
<td>Housing maintenance/depr.</td>
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<td>Loss from foreclosure</td>
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<tr>
<td>$\kappa_h$</td>
<td>Transaction cost</td>
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<td>$\psi$</td>
<td>Operating cost rental comp.</td>
<td>Y</td>
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<td>$\alpha/(1-\alpha)$</td>
<td>Housing supply elasticity</td>
<td>N</td>
<td>1.5</td>
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<td>$\bar{L}$</td>
<td>New permits</td>
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<td>Mortgage rate wedge</td>
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<td>0.33</td>
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<td>$\iota_h$</td>
<td>HELOC rate wedge</td>
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<td>Government</td>
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<tr>
<td>$\tau_y$, $\tau_y$</td>
<td>Income tax function</td>
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<td>$\tau_h$</td>
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Table 1: Parameter values
<table>
<thead>
<tr>
<th>Moment</th>
<th>Empirical value</th>
<th>Model Value</th>
</tr>
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<tbody>
<tr>
<td>Aggr. home-ownership rate</td>
<td>0.66</td>
<td>0.65</td>
</tr>
<tr>
<td>Fraction of bequests in bottom half of wealth dist.</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Home-ownership rate at age &gt; 70</td>
<td>0.78</td>
<td>0.79</td>
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<tr>
<td>Foreclosure rate</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>Aggr. NW / Aggr. labor income (median ratio)</td>
<td>7 (1.2)</td>
<td>5.8 (1.04)</td>
</tr>
<tr>
<td>P10 Housing NW / total NW for owners</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td>P50 Housing NW / total NW for owners</td>
<td>0.50</td>
<td>0.42</td>
</tr>
<tr>
<td>P90 Housing NW / total NW for owners</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>Avg.-size owned house / rented house</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Avg. earnings owners / renters</td>
<td>2.05</td>
<td>2.02</td>
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<tr>
<td>Home-ownership rate of &lt; 30 y.o.</td>
<td>0.27</td>
<td>0.26</td>
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<tr>
<td>Relative size of construction sector</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
<td>Fraction of homeowners with HELOC</td>
<td>0.06</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2: Targeted moments in the calibration

The mean and variance of the initial distribution of bequests are chosen to mimic the empirical distribution of financial assets and its correlation with earnings at age 21 (as computed by Kaplan and Violante, 2014).

Housing: To discipline the set $H$, we choose 3 parameters: the minimum size of owner-occupied units, the number of house sizes in that set, and the gap between house sizes. We target three moments of the distribution of the ratio of housing net worth to total net worth, the P10, P50, and P90, respectively 0.11, 0.50, and 0.95. Similarly, for $H'$ we choose 2 parameters: the minimum minimum size of rental units, and the the number of house sizes in that set (the gap between rental unit sizes is the same as for owner-occupied houses). We target the average house size and the average earnings of of owners vs renters, respectively 1.5 and 2, from the SCF 1998.

The maintenance cost that fully offsets depreciation $\delta_h$ is set to 0.03 (of the value of the house) to replicate an annual depreciation rate of the housing stock of 1.5 pct (BEA Table 7.4.5, consumption of fixed capital divided by the stock of residential housing). In the event of a mortgage default, the depreciation rate rises to $\delta^d_h = 0.25$, consistently with a loss of value of 22 pct for foreclosed properties (Pennington-Cross, 2009). The transaction cost upon selling the house $\kappa_h$, linear in the value of the house, equals 0.07 (Federal Reserve Board). The operating cost of the rental company $\psi$ affects the relative cost of renting vs buying, a decision especially relevant for young households, so to set its value we target the home ownership rate of households younger than 30, 26 pct.

The construction technology parameter $\alpha$ is set to 0.6 so that the elasticity of the housing supply function $\alpha / (1 - \alpha)$ equals 1.5, the median housing supply price-elasticity estimated by
Saiz (2010). The value of new permits \( \hat{L} \) is set to 0.31 to match the relative size of the construction sector.

**Financial Instruments:** The proportional intermediation wedge on mortgages \( \iota_m \) is set to 0.33 consistent with the gap between the average rate on 30-year fixed-term mortgages and the 10-year T-Bill rate in the late 1990s (FRED series MORTGAGE30US and GS10). The proportional wedge on HELOCs \( \iota_b \) is set to 0.33 to match a take-up rate of HELOCs of 7 pct among homeowners (SCF 1998).

**Government:** For the income tax function \( T(\cdot) \), we adopt the simple functional form in Heathcote, Storesletten, and Violante (2015), i.e.,
\[
T(y_j; m_j) = \tau^0_y (y_j - r_m \min \{m_j, \bar{m}\})^{1-\tau^1_y}.
\]
The parameter \( \tau^0_y \) measures the average level of taxation and is set so that aggregate tax revenues are 20\% of output in the stochastic steady state of the model. The parameter \( \tau^1_y \), which measures the degree of progressivity of the US tax/transfer system, is set to 0.15, based on the estimates of Heathcote, Storesletten, and Violante (2014). The argument of the function is taxable income, defined as income net of the deductible portion of mortgage interest payments. This specification takes into account that interests on mortgages are only deductible up to a limit, with \( \bar{m} \) corresponding to $1,000,000. The property tax \( \tau_h \) is set to 0.02, or 1\% annually, the median value across US states.

### 3.1 Aggregate Shocks and Boom-Bust Episode

As discussed in Section 2.5, the macroeconomy is subject to three aggregate shocks: labor income \( Z \), financial deregulation \( F = (\lambda^m, \lambda^b, \kappa_m) \), and preference for housing \( \phi \).

**Stochastic Processes:** All these stochastic processes are modeled as discrete Markov chains, independent of each other. The aggregate labor income process follows a two-point Markov process estimated based on the NIPA series ‘Wages and Salaries’ divided by the Labor Force.

Also all the elements in the vector \( F \) follow a two-state process. In normal times, \( \lambda^m = 0.85 \) to replicate the FHFA conforming loan limit for the late 1990s. The maximum HELOC limit, as a fraction of the home value, \( \lambda^b \), is set to 0.2, which corresponds to the 75th percentile of its distribution (SCF 1998). The origination cost for mortgages \( \kappa_m \) is set to $2,000 in the model (corresponding to application, attorney, appraisal and inspection fees (FRB Cost of Refinancing)).\(^{10}\) In times of financial deregulation, we increase \( \lambda^m \) to 1.0 and decrease \( \kappa_m \) to $1,200, based

\(^{10}\)http://www.federalreserve.gov/pubs/refinancings/default.htm

22
on the evidence presented by Favilukis et al. (2010). Moreover, we increase $\lambda^b$ to 0.3, the 75th percentile of the distribution of HELOC limits as a fraction of home values at the peak of the boom (SCF 2007). We assume that both states are extremely persistent, meaning that all agents in the economy think that the current state will not change during their lifetime.

As explained, taste for housing follows a three state Markov chain:

$$
\begin{bmatrix}
\phi_L & \phi^*_L & \phi_H \\
\phi_L & q_{LL} & q_{LL} & q_{LH} \\
\phi^*_L & q_{LH} & q_{LH} & q_{LH} \\
\phi_H & q_{HL} & q_{HL} & q_{HH}
\end{bmatrix},
$$

where the rows all sum to 1. The value $\phi_L$ is set to 0.11 so that the average share of housing on total expenditures is 0.16 (NIPA). We choose the remaining parameters to match the expected house prices growth reported by Case, Quigley, and Shiller (2011) during the early 2000s. In addition, we target statistics related the average duration and frequency of house price booms and busts reported by Burnside, Eichenbaum, and Rebelo (2011). We set $q_{LL}$ equal to 0.95 so that a belief or demand shock occurs on average once every forty years, and impose that from $q_{LL} = q_{LL}$ to match the two large post-war booms in real house prices (in the 1980s and 2000s). In order to generate large expected movements in house prices, we first need to generate large realized movements in house prices when $\phi_H$ realizes. To generate the large increase in house prices, the level $\phi_H$ and the persistence, $q_{HH}$, complement each other. As such, we calibrate $\phi_H$ to 0.18 and $q_{HH}$ to 0.85 to target a 40% movement in prices. Next, to generate large expected movements in state $\phi^*_L$ requires that the likelihood of transitioning to $H$ is much higher than reverting back to $L$. As such, we set $\phi_{L-H} / \phi_{L-L} = 10$. \footnote{Note, that despite the degrees of freedom we possess, it is very difficult to match the data on expected house price appreciated because of rational expectations. Increasing $\phi_{L-H}$ increases the contemporaneous price when in state $L^*$, thus having an offsetting effect.}

**Boom-Bust Episode:** The boom-bust episode is a particular joint realization of these stochastic processes that corresponds to the decade 1997-2007 (boom) and post-2007 (bust). In the pre-boom period, the economy is in a regime with low income, normal times for financial conditions, and taste for housing equal to $\phi_L$. The boom corresponds to a switch to a high income, financial deregulation, and taste for housing equal to $\phi^*_L$, meaning that all agents in the economy (borrowers and lenders) believe that a future increase in the demand for housing is more likely. The bust, occurring in 2007, is a sudden reversion to all variables to their pre-boom val-
ues. Note therefore that our modelling of the boom and bust does not entail any actual change in preferences for housing, but only a change in the belief that this might happen.

Figure 2 plots the realized paths for the various components of the shocks over the boom-bust episode. The third panel reports what an agent in the economy rationally expects about average house price growth. We note that these expectations are in line with the survey evidence discussed in Case et al. (2011) for the boom period.

3.2 Life-cycle and cross-sectional implications

Lifecycle: The top panels of Figure 3 plots the average labor income (pension after retirement), nondurable and housing consumption profile for households in our economy, and the corresponding variances of logs. The strong precautionary saving motive, together with the changing scale of the household, produce a hump shape in average expenditures in nondurables and housing. The age profile of the variances of log income and nondurable consumption are in line with their empirical counterparts (Heathcote et al., 2010).

The bottom-left panel plots the lifecycle profile of home ownership: consistently with the data (SCF 1998), home ownership rises steadily from 10 pct at age 25 to 80 pct at age 55, and then stabilizes. The bottom-right panel plots the fraction of homeowners with mortgage debt and, conditional on borrowing, leverage (the debt-housing net worth ratio). The model tends to overshoot the fraction of homeowners borrowing at young ages (in the data, some inherits houses whereas in the model inheritances consist only of financial wealth only) and undershoot it at older ages. The model tracks leverage well until retirement, then leverage drops excessively. The reason is that households in the model are more sensitive, relative to the data, to the mortgage interest deduction. This tax break becomes much smaller as retirees slide down the income brackets.
Figure 3: Top-left panel: Average earnings, nondurable and housing expenditures by age in the model. Top-right panel: Age profile of the variance of the logs for these same variables in the model. Bottom-left panel: homeownership in the model and in the data (source: SCF 1998). Bottom-right panel: fraction of homeowners with debt and leverage ratio in the model and in the data (source: SCF 1998).

**Cross-Section:** Table 3 reports some additional cross-sectional moments of interest on the distribution of leverage in the model and in the data (SCF 1998). We have also estimated the consumption insurance coefficients with respect to income shocks, following the strategy proposed by Blundell, Pistaferri, and Preston (2008). The model is aligned with the data also in this dimension, an important one since one of the aims of this paper is quantifying the transmission of housing wealth shocks into consumption.

### 4 Results

We organize our quantitative finding around three questions: (i) What were the sources of the boom-bust dynamics of house prices? (ii) What was the main transmission mechanism from house prices to consumption?; and (ii) How effective would a large-scale mortgage modification program have been at limiting the collapse in house prices? Our results are based on an analysis of the simulated IRF of the aggregate economy to the realized paths for the three shocks describe above.
4.1 What caused the boom and bust in house prices?

House price and consumption dynamics The benchmark model (with incorporates all three shocks) generates an increase in house prices of 30 pct and a fall of a similar size (Figure 4, left panel). The decomposition into the three separate shocks in isolation illustrates that the key source of the observed house price dynamics is the shift in beliefs about future house appreciation. Changes in credit conditions have a trivial impact on house prices. Productivity contributes by a very small amount only, to the extent that housing is a normal good and demand for housing responds to income fluctuations.

Aggregate nondurable expenditures (right panel) rise by 7 pct and fall by a similar amount. The belief shock explains around 4 pct points on both the up and the down, so nearly half of the data. The dynamics of labor income explain another 3 pct points. Overall, we conclude that around half of the boom and bust in consumption can be attributed to the boom and bust in house prices.

It is useful to compare our shock to expectations about future demand for housing to an actual realized change in preferences of the same size. Figure 5 draws this comparison. While both shocks induce a similar boom-bust in prices, the implications of the preference shock for consumption are entirely counterfactual: as households want more housing, they substitute away from (nondurable) consumption, causing it to drop sharply (and counterfactually) in the boom and to rise in the bust. We conclude that the joint dynamics of consumption expenditures and house prices speak loudly against an actual housing demand shift and are, instead, consistent with an expected one.

As such, the model generates plausible movements in the current account of 2-3% of output over the boom-bust episode.

In a richer model with nominal or real rigidities where the collapse in house prices causes a decline in aggregate labor demand, part of the drop in labor income would be attributed to house prices. In this sense, our estimate is a lower bound.

We note that with strong complementarity between housing and nondurable consumption in preferences, the model would generate a rise in consumption expenditures under a preference shock. However, such degree of complementarity would imply, counterfactually, a very unstable housing share of consumption over time.

Table 3: Other implied cross-sectional moments

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<thead>
<tr>
<th>Moment</th>
<th>Empirical value</th>
<th>Model Value</th>
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<tbody>
<tr>
<td>Fraction homeowners w/ mortgage</td>
<td>0.66</td>
<td>0.57</td>
</tr>
<tr>
<td>Aggr. mortgage debt / housing value</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>P10 LTV ratio for mortgagors</td>
<td>0.15</td>
<td>0.04</td>
</tr>
<tr>
<td>P50 LTV ratio for mortgagors</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td>P90 LTV ratio for mortgagors</td>
<td>0.92</td>
<td>0.81</td>
</tr>
<tr>
<td>BPP consumption insurance coeff.</td>
<td>0.36</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Figure 4: House prices and aggregate consumption. Benchmark is the model’s simulation with all shocks hitting the economy. The other lines correspond to counterfactuals where all shocks are turned off except one.

**Home ownership dynamics and the rent-price ratio** Understanding the dynamics of home ownership requires first an understanding of the dynamics of rents. As illustrated in Figure 6, the model can replicate 2/3 of the fall in the rent-price ratio. Belief shocks are instrumental for the model to be consistent with the data along this dimension: the equilibrium condition dictates that when prices increase, rents increase too. Thus, without any belief shift, the rent-price ratio would remain stable. The change in expected appreciation pushes down rents and aligns the model to its empirical counterpart. The fact that in the boom, rents rise a lot less than prices, means that renting is more appealing relative to owning, and we should thus expect that the belief shock, by itself, counterfactually reduces home ownership.

The model’s implications for home ownership are illustrated in the top left panel of Figure 7. As expected, the belief shock alone reduces home ownership. With only the belief shock, there are two forces working against the increase in home ownership. First, rents are cheaper relative to prices which, at the margin, moves people towards renting. Second, the large increase in prices induced by the shift in beliefs makes the downpayment constraint binding for more households. Financial deregulation alone allows some households that were previously
Figure 5: House prices and aggregate consumption. Benchmark is the model’s simulation with the belief shock hitting the economy. Demand Only is the model’s simulation with the shock to taste for housing hitting the economy.

constrained to buy, but quantitatively this only explains about half of the increase in homeownership. It is the interaction of the belief about future housing demand and the relaxation of credit limits that yields the rise in home ownership in the model, allowing the model match the data in this dimension.

Figure 8 compares the change in home ownership across the age distribution in the data and in the model. As in the data, in the model it is the young that go in and out of the housing market during the boom-bust and account for the rise-fall in home ownership. However, they buy houses of similar size as those they rented and, as a consequence, do not contribute much to push up housing demand and prices. Prices go up because existing homeowners, in the anticipation of price appreciation, upgrade to bigger houses: they move up the ladder of house sizes and raise demand. We thus reiterate that home ownership and the ramp-up in prices are largely determined by different forces, financial deregulation and shifts in expectations, respectively.

The important point here is that one can get strong effects of financial deregulation on house prices only if households are initially constrained in their housing choice so that, when credit limits are relaxed, they demand more housing units. This is part of what happens in Favilukis et al. (2010), for example, and in similar models where households must buy to enjoy housing services. In our model, households are not much constrained in their housing choices: those who cannot afford the minimum down payment can always rent, and when they buy, they buy a house of similar size as the one they were renting, so aggregate housing demand is
Figure 6: Ratio of equilibrium rental rate to house price. Benchmark is the model’s simulation with all three shocks hitting the economy. The other lines correspond to counterfactuals where all shocks are turned off except one.

largely unaffected. Furthermore, most existing homeowners are not financially constrained in their housing choices: median leverage of mortgagors is only about 50% and more than 1/3 of homeowners have no mortgage at all — implying that those households have sufficient equity to make a 20% downpayment on houses significantly larger than they currently occupy. Therefore, the presence of a rental market and matching a realistic lifetime profile of leverage play a key role in determining the effect of relaxed borrowing constraints on house prices.

Leverage, refinancing and foreclosures The remaining panels of Figure 7 show the model’s implications for the dynamics of leverage, refinancing, and foreclosures over the boom and bust. Mechanically, the belief shift induces a fall in leverage (housing debt/housing value) during the boom, and a rise during the bust, simply because the denominator of leverage is changing. Changing credit conditions cause an expansion of leverage during the boom and deleveraging during the bust. It’s therefore, once again, the combination of the two shocks that allows leverage to be flat in the boom, consistent with the US data. It’s also important to note that the aggregate debt-to-income ratio increases dramatically in the model, as in the data, since leverage is constant and house prices grow four times more than income.

The change in credit conditions entirely accounts for the rise and fall in cash-out refi’s. It is the decline in the fixed origination cost that, by shrinking the inaction region, induces significant growth in the number of households who choose to refinance their mortgage.
Finally, the spike in foreclosure occurs when the collapse in prices pushes many households underwater. Tightening of credit conditions alone cannot explain the foreclosure crisis because (i) they do not move prices and (ii) because the fact that maximum loan-to-value limits tighten is relevant only at origination in a model with long-term debt.\footnote{See Corbae and Quintin (2011) for a discussion of “teaser” rate mortgages and their contribution to the foreclosure boom} However, even for this case there is a strong interaction between credit conditions and beliefs. Financial deregulation amplifies the effect of belief shock on foreclosures because buyers during the boom are able to obtain bigger mortgages and are more easily underwater when beliefs, and prices, revert.

**Narrative of the housing crisis revisited** Since almost as soon as the housing crisis unfolded, conventional wisdom has held that the its explanation lies in a popular narrative: easy credit pouring to low-income and high-risk borrowers caused house prices to accelerate and subsequently to crash. This narrative builds on the finding by Mian and Sufi (2009) that growth

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\footnote{See Corbae and Quintin (2011) for a discussion of “teaser” rate mortgages and their contribution to the foreclosure boom}
in purchase-mortgage originations at the ZIP code level turned from positively to negatively correlated with per-capita income in the run-up to the financial crisis, particularly in regions with strong house appreciation. As a consequence of these empirical findings, much prominence has been given to the role of financial deregulation in channeling credit to low-income and subprime borrowers and in causing the boom-bust in house prices and consumption.

Our model speaks against this interpretation. Instead, our findings are consistent with a new narrative of the crisis that has recently emerged, thanks to the availability of new and more refined micro data. According to several authors (Adelino et al., 2016; Albanesi et al., 2016; Foote et al., 2016), credit growth during the boom was not concentrated among the low-income and high-risk households but was a lot more widespread across the household distribution, and even reached high-income and prime borrowers. The top-left panel of Figure 9, from Foote et al. (2016), shows that growth in the stock of mortgage debt over the boom occurred at similar rates across the whole household income distribution. The top-right panel replicates this figure in our model. The bottom-left panel reports mortgage originations by FICO score over the boom from Adelino et al. (2016): growth in mortgage debt is uniform across credit risk category. Our model has no immediate counterpart of a credit score, but for each borrower we can compute a probability of default at origination (embedded in the effective interest rate paid on debt, see equation 6) and group households by risk level. The bottom-right panel of Figure 9 shows that our model predicts fairly even credit growth between low-risk and high-risk borrowers.

In light of our analysis of the economic forces at work in the model, it is not surprising that our simulated cross-section of debt growth lines up well with its empirical counterpart. In the model, aggregate housing demand rises because all households – rich and poor, prime and subprime – expect large capital gains from holding this asset. High-income, low-risk households,

who are already in a position to access low-cost credit even without looser loan-to-value constraints, are among those who choose to increase the housing share in their portfolio to take advantage of expected appreciation.\footnote{Interestingly, this is exactly the interpretation given by the authors uncovering these new facts. Adelino et al. (2016) (page 32) write that this suggests that demand-side effects and possibly also expectations of future house prices increases could have been important drivers in the mortgage expansion as borrowers and lenders bought into expected increases in asset values. Foote et al. (2016) (page 35) conclude that their findings are more consistent with an alternative story, in which exogenous borrowing constraints play no role and the causality runs from house prices, or house-price expectations, to the widespread accumulation of mortgage debt. During the boom, optimistic views of house-price growth were widely shared by potential home buyers (in all income classes) as well as mortgage lenders.}

Whose beliefs matter? Our rational expectations approach to modeling a belief shock imposes that all agents in the economy share the same beliefs about future house prices, as well as the same preferences over housing services and beliefs about future preferences. Thus, when the belief shock hits there are three channels through which the shock affects behavior. First,
individual households believe that they themselves are likely to desire more housing services relative to non-durable consumption in the future. Since there are adjustment costs on housing, this may lead them to change their housing demand immediately, even in the absence of any change in house prices. Second, individual households believe that all other households are likely to desire more housing services relative to non-durable consumption in the future, and rationally foresee that this may lead to increase in the price of housing in the future. Thus a household may be tempted to change their demand for housing due to a speculative motive, even if their own preferences are not affected by the change. Third, the financial sector understands the implied dynamics for house prices and hence lenders change the optimal credit contracts that they offer to home owners. These changes in lending contracts may also affect households’ demand for housing.

First, we investigate the role of the lenders’ beliefs. In Figure 10, the endogenous borrowing rate (the inverse of the price of a unit of mortgage debt, a function of the individual characteristics that predict default) in different aggregate states is plotted for a 30 year old household that owns a $150K house and earns median income. The blue line is before any financial deregulation and in the absence of a belief shock. The maximum LTV that a household can take out is 0.85. Note that for low levels of leverage the interest rate is flat, but starts increasing steadily above leverage of about 0.7. The green line shows that the belief shock flattens the q function in the boom. The lenders have the same expectations as households: they also expect prices to rise. Thus, they expect the probability of default to fall, and as a result they offer more favorable credit conditions to all borrowers, but especially to the risky ones, who now become a lot safer in the eyes of the lender (the gap between the blue and the green line widens more for high LTVs). Thus, viewed through the lens of our model, the increased credit flow to riskier borrowers was an endogenous response to an increased belief in expected house price appreciation.

In order to quantify the importance of lenders’ beliefs in explaining the aggregate dynamics surrounding the boom-bust episode, we consider a counterfactual economy where lenders do not share the same beliefs about house price appreciation as the households. Specifically, we assume that the banks place zero probability on transitioning to $\phi_L$ or $\phi_H$, so that the forecasted default probabilities do not take into account expected future house price growth. We solve for the new equilibrium of this economy with “pessimistic” lenders.

When simulating the same boom-bust episode in the pessimistic lender economy, we find that the lender’s beliefs only have a modest effect on the dynamics of house prices and non-durable consumption: house prices and consumption rise about 5 and 0.3 percentage points less, respectively. The lenders pessimistic beliefs, however, have a large impact on homeownership, leverage and foreclosure. Contrary to the benchmark economy, when the belief shock hits
the home-ownership rate and leverage fall to below their pre-boom levels. The biggest impact, however, can be seen on the foreclosure rate, which falls from a peak of 2.5% of mortgages to only 0.5%. Without the expected house price gains, otherwise high risk borrowers don’t experience the endogenous relaxation of credit, causing them to either become renters or take lower leveraged mortgages, which are less prone to default in the bust. Thus, while lenders’ beliefs only had a small effect on house prices, it is critical for the lenders to also have believed house prices would increase to match the joint dynamics of homeowner, leverage and foreclosure.

Second, we investigate the importance of the common knowledge of preference dynamics for house price movements. In order to separate the speculative motive for demanding additional housing from the direct effect that comes from the actual future change in preferences, we consider the housing demand of a single household who faces the equilibrium price dynamics from our benchmark economy, but whose preferences for housing remain fixed. [TO BE COMPLETED]

Beliefs about demand or supply? In our benchmark economy, we generate expected future increases in house prices by shocking households’ relative preferences for housing. However, the qualitative and quantitative results of the model still obtain in a model where the expected future increases in prices are due to future expected changes on the supply side of housing.

The results from this counterfactual are consistent with the findings of Gerardi, Lehnert, Sherlund, and Willen (2008), who argue that to rationalize the increase in sub-prime borrowing, lenders needed to have assigned sufficiently low probability to a large decline in house prices.
We consider the same three state belief structure, but instead of changing the preference parameter $\phi$, we assume that what changes is the future size of land permits made available to the construction company from the government. Thus, the boom is generated by households believing that less land will be made available for construction in the future.\footnote{It is important that we maintain the structure of a shock to future availability of land. While decreasing the land available contemporaneously would lead to an increase in prices, it could lead to a counterfactual drop in housing investment (similar to the counterfactual drop in consumption for the demand shock in the benchmark).} Thus, we remain somewhat agnostic as to the underlying fundamental shock which causes the increase in house prices. However, what is important is that it operates through future expected house price growth in order to jointly match the dynamics of housing investment\footnote{This point has similarly been made in Berger et al. (2015).} and consumption in the data.

**Movements in the risk-free rate** One other candidate explanation for the boom in house prices was the dramatic decline in the risk-free rate experienced in the US in the 2000s. We did not consider this shock in our benchmark because, while it has the potentially to successfully explain the housing and consumption boom, real interest rates have not risen again to pre-2000 levels, making it difficult for them to be instrumental in the bust. In order to evaluate this additional possible channel, we resolve our economy with an exogenous, but stochastic real rate. For computational tractability, we then make the financial deregulation and productivity shocks perfectly correlated. We assume that the real rate falls 100 basis points in 2001 and stays low through 2020. The impulse responses from this experiment are provided in the appendix. The drop in the interest rate does yield larger movements in house prices in the boom, but qualitatively, the joint dynamics are very similar to the benchmark.

**4.2 How does the fall in house prices transmit into consumption?**

To understand the transmission mechanism from house prices to consumption expenditures in the model, we analyze how the drop in household consumption during the bust (from 2007 to 2011) correlates with initial leverage and the initial housing share of net wealth in 2007. Leverage is computed as total debt divided by net wealth (including human wealth, computed as the expected discounted present value of future after-tax earnings and social security benefits), and the housing share of net wealth is computed as housing equity divided by net wealth. If forced deleveraging (the collateral effect) is a strong force in the model, one should expect leverage (the debt share of net wealth) to be negatively correlated with the change in consumption. Whereas, if the transmission mechanism if mainly through a wealth effect, one should expect
the change in consumption to be negatively correlated with the housing share of net wealth.

Figure 11 shows that leverage and the change in consumption are uncorrelated, whereas households with the largest housing share are those with the largest drop in expenditures, an indication that they key channel of transmission in the model is a wealth effect. Note that renters have a small, but positive, change in consumption as a result of the equilibrium drop in rents. When we split the sample between workers and retirees, the contrast is even starker for workers – the plot for leverage remains flat, but the negative wealth effect is much more pronounced. The reason is that, in the model, retirees have an active luxury-bequest motive and by adjusting their bequest can dis-save to smooth consumption.

4.3 Policy experiment: a debt forgiveness program

We now use the model to run a counterfactual policy experiment. We implement a massive debt relief program in the model — a policy intervention that a number of economists and policy-makers thought could have cushioned the housing crash and accelerated the recovery in aggregate expenditures. All homeowners with more than 95 percent LTV ratio see their excess debt forgiven, with the government stepping in and reimbursing the banks. The residual mortgage is paid off by households under the baseline amortization formula. This principal reduction program is implemented in 2009, it is unexpected and households do not believe that such a program will ever be implemented again in the future. Overall, the counterfactual program affects around 1/4 of all mortgagors, i.e. it displays a much larger scale (and a better timing) than the programs, such as HAMP (Home Affordable Modification Program) and HARP (Home Affordable Refinance Program), initiated by the Obama administration.

Figure 12 shows that the program is very effective in cutting the number of underwater households and in reducing foreclosures. Thus, to the extent that foreclosures imply a utility loss and accelerated depreciation of the property, there is a gain for the households and banks.
who can avoid it. Leverage, mechanically rises less. However, the effect on house prices and consumption is trivial. First, by limiting foreclosure and the associated large depreciation, the supply of housing goes up. This, in turn, pushes the price per unit of housing down (an unintended consequence of the policy). Second, foreclosure is an insurance vehicle for consumption smoothing. By limiting foreclosures, those households who now do not foreclose, but would have in absence of the policy, consume less. It is only the households with high LTV ratios that would not have foreclosed who now have lower mortgage payments and consume more. These beneficiaries of the policy, though, account for a relatively small share of aggregate consumption. These results are consistent with the empirical findings of Agarwal et al. (2012) who use a diff-diff approach to evaluate the effects of HAMP and document that the regions where the policy was used most intensively experienced no change in non-durable consumption.\textsuperscript{20}

The results of this policy counterfactual also highlight how in the model, as in the data, leverage increased throughout the income distribution. Even households who were not underwater after the bust still had a desire to deleverage (mostly by simply paying down the mortgage as prescribed by amortization). This slow deleveraging by the bulk of homeowners is behind the tepid consumption recovery following the bust and explains why the proposed interventions have had a limited impact in speeding the recovery in consumption and house prices.

5 Implications for the elasticity of consumption to house prices

A vast empirical literature in economics tries to estimate, with household-level longitudinal data (Campbell and Cocco, 2007; Browning et al., 2013), geographical-level panel data (Mian et al., 2013), or time-series data (Carroll et al., 2011) the elasticity of consumption expenditures to changes in house prices. This literature is, however, plagued by the ‘comovement problem’. First, the endogeneity of house prices makes it extremely challenging to find exogenous sources of variation in the data that can identify correctly the parameter of interest. Second, even under the best identification scenario, we argue that measuring the response of consumption to an ‘exogenous’ change in house prices is not especially relevant, since it is an event that does never occur in practice. Since prices are an equilibrium object, house prices changes are never exogenous. Rather, house price changes are the result of the realization of some combination of underlying shocks that influence either the demand or supply of housing, or both, and thus cause equilibrium house prices to move.

\textsuperscript{20}Studying the HARP program, Mitman (2016) finds small effects on foreclosure and modest effects on consumption for households who were able to refinance under the program
This distinction is important only if the elasticity of aggregate consumption to house prices varies a lot depending on the many possible macroeconomic shocks that lead to house price movements. With our structural equilibrium multi-shock model in hand, we are in the best position to answer this question. Table 4 reports the time series elasticity of aggregate nondurable consumption expenditures to house prices computed by simulating the model under four possible sources of price dynamics: productivity, financial deregulation, beliefs, and preferences. Elasticities are computed for both the boom and the bust.

Our main result is that these elasticities are remarkably different across shocks. Income shocks generate the largest elasticities of consumption to house prices, well above one, as income shocks are quite persistent and affect consumption directly, not just through their effect through house prices. The elasticities induced by shocks to the taste for housing are, instead, negative: households with an increased preference for housing spend more on housing and less on nondurable consumption. The relaxation of collateral constraints is associated to a negative elasticity of consumption to house prices, because a number of households can now afford to buy a house and cut expenditures to finance the required down payment. Finally, the belief shock induces a moderate response of consumption to house prices: many homeowners stay...
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<tr>
<th></th>
<th>Boom</th>
<th>Bust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Productivity</td>
<td>1.60</td>
<td>1.28</td>
</tr>
<tr>
<td>Credit conditions</td>
<td>−0.61</td>
<td>0.32</td>
</tr>
<tr>
<td>Belief shift</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>Taste for housing</td>
<td>−0.28</td>
<td>−0.23</td>
</tr>
</tbody>
</table>

Table 4: Elasticities of Nondurable Consumption to House Prices during the Boom and the Bust. Results of counterfactuals when a single shock affects house prices.

in their house and extract more equity to increase consumption. At the same time, this force is counteracted by the fact that other households upgrade their housing and, in the process, limit their consumption expenditures in order to meet the larger down-payment requirement and the transaction costs.

These results have implications for the use of the sufficient statistic approach (as advocated, for example, in Chetty, 2008) to this question. In a recent paper, Berger et al. (2015) propose a clever and easy way to compute a ‘sufficient statistic’ to estimate the individual (micro) elasticity of consumption to a permanent change in house prices. Their sufficient statistic amounts to the product between the individual marginal propensity to consume out of a transitory income shock and her beginning-of-period quantity of housing.

The left panel of Figure 13 shows that their sufficient statistic, once aggregated across the population, provides an excellent approximation to the true elasticity computed by simulating data from the model when the underlying shock is the belief shock, i.e., in the context of the recent boom-bust episode for which we argued that fluctuations in beliefs about future housing demand account for much of house price dynamics. The reason is that the belief shock is largely orthogonal to other determinants of consumption decisions, and thus it is akin to house price shocks in housing-demand models where such prices are exogenous, as in the framework that Berger et al. (2015) use to derive their sufficient statistic.

However, we also emphasize that one should exercise caution when using this approach: the sufficient statistic could fail quite dramatically in replicating the true elasticity, depending on the shock underlying house price movements. The right panel illustrates one such stark example when the source of fluctuations is shocks to the taste for housing. By design, the sufficient statistic is positive, whereas as shown in Table 4 the true elasticity is negative.
6 Conclusions

Equilibrium models with heterogeneous households, incomplete markets, and aggregate shocks are becoming a standard tool to analyze the macroeconomy and study the welfare and distributional consequences of government policy. In this paper we have built a rich version of this framework to examine the co-movement between consumption and house prices around the Great Recession and in the ensuing slump.

Through the lens of this model, it is virtually impossible to generate the large house price growth observed in the data during the decade leading to the crash in absence of a shock that induces all agents in the economy, households and lenders, to (rationally) believe that future demand for housing will increase. This expected future appreciation also helps keeping rents stable, as in the data, and interacts with financial deregulation in mortgage markets to produce a rise in home ownership and stable aggregate leverage during the boom. A reversion of beliefs induces the bust in house prices, the spike in foreclosures, and the sudden mechanical surge in leverage.

According to the model, at least one half of the boom and bust in aggregate consumption can be accounted for by the dynamics of house prices, and the transmission mechanism from housing wealth to consumption is a wealth effect. Deleveraging, through tighter collateral constraints play a minor role for the transmission of the shock, consistent with the result that they cannot explain the drop in equilibrium house prices.

Thus, in the model the macro elasticity of consumption to house prices is around 0.25. However, through counterfactual simulations we demonstrate that when the sources of house
prices movements are other than belief shifts—for example, interest rates, income, or relaxation/tightening of borrowing limits—the elasticity can be dramatically different. This result sheds new light on the ‘common factor’ problem that affects the empirical literature trying to estimate the elasticity of consumption to house prices with macro or micro data, and cautions against using the sufficient statistic approach in this context.

Finally, we conclude that a large-scale debt forgiveness policy implemented at the onset of the housing crisis would have significantly reduced foreclosures, but it would have been entirely ineffective in cushioning the fall in house prices or speeding up the recovery in consumption.
References


Figure 14: House prices and aggregate consumption with exogenous drop in interest rate

7 Appendix

7.1 Alternative Shocks

7.1.1 Interest Rate

[To Be Completed]
Figure 15: Homeownership, leverage, refinancing, and foreclosures.