The Life-Cycle Distribution of Earnings and the Decline in Labor's Share

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Abstract

We estimate the effect of the life-cycle distribution of earnings on labor's share of income. We relax the assumption of perfectly competitive wages and show that the aggregate labor share is no longer a simple function of production parameters, but is instead an earnings-share-weighted harmonic mean of labor shares across demographic groups. We document that the share of earnings accruing to elder workers has risen sharply in recent years, coincidental with the majority of the decline in labor's share. We then use an IV approach to estimate that a one percentage point shift in earnings towards elder workers leads to a 0.29 percentage point decline in labor's share. We rationalize our empirical findings by extending two standard theories of frictional labor markets to include a life-cycle of productivity which endogenously grows faster than earnings.

Preliminary and Incomplete

1 Introduction

In this paper, we relax the assumption that a worker's wage is set under perfect competition and estimate the effect of the age-distribution of earnings on aggregate labor's share. We derive a novel accounting identity which links labor's share and the relative *earnings wedges* [1] of workers across demographic groups. We document that a sharp increase in the earnings shares of elder workers (aged over fifty years) has occurred along side a large decline in labor's share since the late nineties. We estimate the relative earnings wedges between young and elder workers and find that elder workers receive about 3/4 of their marginal product as earnings relative to younger workers. With our estimate of the relative earnings wedge, we predict that labor's share would have barely changed since the late nineties if not for the rise in the earnings of elder workers.

In order to understand how we identify this estimate, consider two increases in labor supply with differential effects on output and earnings. First, one day we find an exogenous one hour increase in the supply of youth labor, followed by a $1 increase in output and the same increase in total labor income. The next day we observe an exogenous one-hour increase in elder labor supply along with a $1 increase in output but only an $0.71 increase in earnings. From this we would infer that elder workers, while equally productive as the young, are able to capture only 75% of their marginal product relative to the young. [2]

Our estimation procedure is quite general and is valid for many theories which may generate life-cycle heterogeneity in the earnings wedge. Specifically, we use population shares as an instrument for exogenous shifts in earnings share. The two series are extremely strongly correlated and we argue that population shares are likely exogenous: all that we require is for twenty-year lagged birth rates and contemporaneous mortality rates to be uncorrelated with current shocks to labor's share.

We also provide evidence from regional and sectoral movements in labor share and earnings shares. The results are largely in line with the aggregate data, although we must use alternative instruments for labor supply since population shares are no longer valid (for sectors they are not even defined). We use a common Bartik style instrument and find that almost all regions have similar relative earnings wedges to the United

[1] As defined formally below, the earnings wedge is the inverse of the fraction of a worker's marginal product paid as salary. A higher wedge means that a worker receives a smaller share of his marginal product as earnings.

[2] For this example we assume that the young earn exactly their marginal product, but this need not be true. We also assumed that both youth and elder labor had the same marginal product, but the logic would be unchanged if they differed.

States, as do two-thirds of sectors for which we have a valid instrument.

The factors determining labor's share and its relation to the aggregate production function were once closed questions, but are now very much open. From Kaldor ([2]) to very recently, macroeconomists typically assumed a constant labor's share of GDP. This assumption, along with the assumption that wages are set in competitive spot markets, restricts the aggregate production function to the Cobb-Douglas form and exactly identifies the elasticity of output with respect to labor. This elasticity is labor's share of GDP. Recent research [3] indicates that the assumption has been unrealistic since the early 1980s and that it has become even less appropriate for the United States since the early 2000's. This decline has coincided with a renewed concern over the distribution of income between factors (Piketty ([8] for example) and the question of how policy should influence the distribution of income.

The existing literature typically maintains a neoclassical aggregate production function along with perfrectly competitive labor markets and relies upon shocks to explain variation in labor's share. Karabarbounis and Neiman ([3]), estimate a CES production function on international data. Their estimates imply that capital and labor are more substitutable than one (the Cobb-Douglas case), so that a decline in the price of investment goods generates a rise in the capital labor ratio and a fall in labor's share. In a recent working paper, Lawrence ([5]) estimates a similar production function but finds just the opposite of Karabarbounis and Neiman - he estimates that the elasticity of substitution is less than one, but that the productivity weighted capital labor ratio has actually fallen over the time period in which labor's share has declined.

Rather than vary the aggregate production technology and attempt to measure the appropriate capital-labor ratios, we relax the assumption that a worker's wage is set under perfect competition. We are not the first to consider non-competitive factors as a cause of labor share's decline. The paper closest to ours is Elsby, et al ([1]), who use industrial data on import competition and labor share. They find that industries which experienced larger increases in import competition also experienced larger declines in labor share. [4] We find a similar result using the same industrial data - industries for which elder workers experienced the most earnings growth tend to have larger declines in labor's share.

We focus on a common class of models with labor market search frictions. In these

[3]The literature is growing. Karabarbounis and Neiman have provided an overview and their research agenda ([4]).

[4]They also argue against capital deepening due to lower investment prices, since there is no relationship between changes in industry level investment goods prices and labor share.

models, the worker-firm relationship generates a match surplus which must be split according to some protocol. We show that the work-horse Diamond-Mortensen-Pissarides ([6]) can generate a rising earnings wedge over the life cycle due to horizon effects. That is, an elder worker has a shorter expected duration, and therefore a smaller match surplus (ceteris paribus). His earnings therefore amount to a smaller fraction of his marginal product. Crucially, this can hold even if he is more productive and therefore receives a higher *level* of earnings.

We also consider an extension to the model by Postel-Vinay and Robin ([9]) in which workers search on the job and employers compete ala Bertrand when two are in contact with the same worker. In this model, workers receive none of the surplus until they receive an outside offer, at which point they receive the minimum of the surplus from the two competing firms (manifest as an increase in earnings). To this model we add growth in match specific productivity. Outside offers are then met with a wage increase, but only to the point that exhausts the surplus of a new (lower productivity) match. This creates an expanding gap between earnings and marginal product as a worker continues on a job, which translates to an increasing life-cycle profile of the earnings wedge.

2 Accounting For Aggregate Labor's Share

2.1 Decomposing Earnings

A tautological explanation for the decline in labor's share is that earnings have grown more slowly than total income. We first decompose the changes in earnings growth to isolate the effect of age-specific changes in earnings growth relative to the effect of shifts in the age-distribution of earnings shares.

The decomposition starts with the identity:

$$g_{t+1} \equiv \frac{E_{t+1} - E_t}{E_t} = \sum_{j=1}^{J} \frac{E_{j,t+1} - E_{j,t}}{E_{j,t}} \frac{E_{j,t}}{E_t} \equiv \sum_{j=1}^{J} g_{j,t+1}\sigma_{j,t} \tag{1}$$

We then rewrite the right-hand side as:

$$\sum_{j=1}^{J} g_{j,t+1}\sigma_{j,t} = \sum_{j=1}^{J} \left[(g_{j,t+1} - \bar{g}_j)(\sigma_{j,t} - \bar{\sigma}_j) + \bar{\sigma}_j g_{j,t+1} + \bar{g}_j \sigma_{j,t-1} - \bar{g}_j \bar{\sigma}_j \right] \tag{2}$$

Where $\bar{x}$ is the average of variable x over some time span. Defining $\bar{g} \equiv \sum_{j=1}^{J} \bar{g}_j \bar{\sigma}_j$ and assuming that the product term $\sum_{j=1}^{J}(g_{j,t+1}-\bar{g}_j)(\sigma_{j,t}-\bar{\sigma}_j) \approx 0$ yields the decomposition:

$$g_{t+1} - \bar{g} \approx \sum_{j=1}^{J} \bar{\sigma}_j (g_{j,t+1} - \bar{g}) + \sum_{j=1}^{J} \sigma_{j,t-1} (\bar{g}_j - \bar{g}) \tag{3}$$

The first term isolates how much aggregate earnings growth differs from average due to changes in age-specific growth rates, keeping earnings shares at the average. The second term isolates how much earnings growth differs from the average due to shifting earnings towards workers who are on the flatter part of their life-cycle earnings profile.

We now provide assumptions on the relationship between earnings and marginal products along with the aggregate production function. We have tried to make these assumptions robust to the specifics of any given labor market theory, though we will later describe a structural model consistent with these assumptions and will use our estimates to discipline the parameters of that model.

2.2 Assumptions

Our first assumption relaxes the assumption that workers are paid 100% of labor's marginal product, although this assumption is nested in our model.

Assumption 1 *At any date t, for each worker $i = 1, ...I_t$, the following equation links a worker's earnings and marginal product:*

$$e_{i,t} = \frac{1}{\alpha_i + \epsilon_t} \frac{\partial Y_t}{\partial n_{i,t}} \tag{4}$$

The parameter α_i reflects an *earnings wedge* between the worker's pay and his marginal product. Specifically, a larger wedge (higher value of α) means that the worker receives a smaller fraction of his marginal product. Note that this parameter is assumed to be constant for a given worker over time, while the error term ϵ_t may vary (but is identical across workers).

Next, we will assume that it is possible to group workers in such a way that they share the same α_i parameter.

Assumption 2 *There exist sets $(\mathbb{D}_j)_{j=1}^{J}$ such that $\forall t \geq 0, \forall i = 1, 2, ...I_t$, there is exactly one $\mathbb{D}_j$ such that $i \in \mathbb{D}_j$ and for every $i, i' \in \mathbb{D}_j, \alpha_i = \alpha_{i'} = \alpha_j$.*

There will be three considerations at play in choosing how to group workers. First, we want an underlying theory for why workers within a given group should have similar earnings wedges. Second, we will want groups that are immune to selection by workers or firms. While a worker may not be in a given group for her entire life, she should have no control over moving across groups. Finally, and related to this point, we are restricted to study groupings for which an instrument for earnings shares can be found. This is necessary to consistently estimate earnings wedges, since unobservable shocks may change both the marginal product of a given group and labor share.

Our next assumption is that the gross domestic product can be represented by a constant returns to scale production function in an aggregate of the capital stock and total labor input.

Assumption 3

$$Y_t = F_t\big(K_t, n_{1,t}, n_{2,t}, ..., n_{I_t,t}\big) \tag{5}$$

with

$$F\big(\lambda K_t, \lambda n_{1,t}, ..., \lambda n_{I_t,t}\big) = \lambda F_t\big(K_t, n_{1,t}, n_{2,t}, ..., n_{I_t,t}\big)$$

We will make one of two further assumptions on the aggregate production function. The first possibility is that the elasticity of output with respect to each worker's labor is constant and equal to α. This could be because the aggregate production function is literally Cobb-Douglas at the aggregate level or because the relevant capital-labor ratios are constant over the period of estimation.[5]

Assumption 4

$$\sum_{i=1}^{I_t} \frac{\partial Y_t}{\partial n_i} = \alpha Y_t \tag{6}$$

The second possibility is that the marginal product of capital is related to a market real interest rate and depreciation rate through a simple polynomial:

Assumption 5 *There is a set of coefficients* $(\psi_\ell)_{\ell=0}^{L}$ *such that:*

$$\frac{\partial F_t}{\partial K_t} = \sum_{\ell=0}^{L} \psi_\ell (r_t + \delta_t)^{\ell-1} \tag{7}$$

Where r_t *is an estimate of the average real interest rate and* δ_t *is an estimate of the average depreciation rate.*

[5] Note that the raw capital-labor ratio is not the relevant variable with heterogeneous workers, but instead we would require a constant value of effective capital utilized per unit of effective labor of each worker type.

2.3 Derivation of Accounting Equation

With these assumptions, we derive our main accounting identity. For each j and each $i \in \mathbb{D}_j$, we can rearrange $\frac{\partial Y_t}{\partial n_{i,t}} = \alpha_j^{-1} e_{i,t}$. Notice that the marginal product on the left-hand side is unobservable, as is the wedge term on the right hand side. However, if we define $E_{j,t} \equiv \sum_{i \in \mathbb{D}_j} e_{i,t} n_{i,t}$ and sum over all of the groups, then we have:

$$Y_t = \frac{\partial F_t}{\partial K_t} K_t + \sum_{j=1}^{J} \alpha_j E_{j,t} + \epsilon_t \sum_{j=1}^{J} E_{j,t} \tag{8}$$

Dividing by $E_t \equiv \sum_{j,t} E_{j,t}$ and defining the earning share of group j as $\sigma_{j,t} = \frac{E_{j,t}}{E_t}$, we arrive at the accounting identity:

$$LS_t^{-1} = \frac{\partial F_t}{\partial K_t} \frac{K_t}{E_t} + \sum_{j=1}^{J} \alpha_j \sigma_{j,t} + \epsilon_t \tag{9}$$

The final step to getting an equation that can be estimated is dealing with the marginal product of capital in Equation (9). Under Assumption (4) we can rewrite the equation as:

$$LS_t^{-1} = \alpha^{-1} \sum_{j=1}^{J} \alpha_j \sigma_{j,t} + \varepsilon_t \tag{10}$$

Under the weaker Assumption (5) we can rewrite Equation (9) as:

$$LS_t^{-1} = \frac{K_t}{E_t} \left(\sum_{\ell=0}^{L} \psi_\ell (r_t + \delta_t)^{\ell-1} \right) + \sum_{j=1}^{J} \alpha_j \sigma_{j,t} + \varepsilon_t \tag{11}$$

Equations (10) and (11) can be taken to data. However, there are two items for which we must rely on theory. The first is the grouping of households by earnings wedge. Any deviation from perfectly competitive wages requires a theory of how workers and employers split the surplus from their relationship and a grouping of workers by wedges relies upon a systematic variation in this split across groups. We develop a theory for which age is the natural dimension along which wedges differ. The second item is the residual ϵ_t, which is of paramount importance for consistently estimating relative wedges for a given grouping of workers. We will specifically worry about the correlation of ϵ_t with different groups' earnings shares since shocks to technology may simultaneously drive down labor's share and shift earnings towards elder workers.

Finally, we note that this equation holds for any level of aggregation for which the production function has a constant elasticity α. We will provide evidence from regional and industrial level below, but this caveat (along with others) will limit our ability to interpret the results structurally.

3 Data

Our measure of aggregate labor share comes from the Bureau of Labor Statistics at the state level, which we then aggregate to compute the national labor share. This has the same dynamics as the index provided by the Federal Reserve Bank of St. Louis, which is readily available. We use annual data from the March Current Population Survey to get earnings shares by group for both the aggregate economy and regions of the united states. We also get population shares by group, which will be used as an instrument in the estimation to follow. This introduces some inconsistency between our earnings and those used for labor's share, since we cannot account for benefits, whereas the BLS includes them in the labor's share computations. As long as there is no consistent difference in the fraction of compensation due to benefits across groups, this will not affect the results. Our industrial level labor share data is provided by Elsby, et al ([1]) and is publicly available through the Brookings website.

Figure 1 shows the aggregate labor share from 1962 to 2013. The period from $1962-2000$ exhibits a slight downward trend, but also extended periods of increase. As recently as 2000 labor's share was at 0.64, which is essentially the average value through 1980. The stark change begins right after 2000, when labor's share begins to decline and hasn't experienced robust increases for any amount of time since. By the end of the sample, in 2013, labor's share has fallen from 0.64 to just below 0.58.

We have 51 observations for labor's share, which limits how finely we can group the population. Our baseline grouping is purely based on age. We split the population into five age groups, the youngest group being 17 to 29 years and the oldest group consisting of 60 years and older. For households with business or farm income, we increase earnings by a proportion of business or farm income consistent with the aggregate labor share. [6]

Figure 2 plots the time series for earnings shares relative to their means. The over-50 group has experienced a dramatic increase in their earnings share. Not shown in this

[6] We perform robustness with respect to this assumption in the appendix, since Elsby, et al ([1]) have shown that the decline in aggregate labor's share is somewhat sensitive to the handling of business income.

graph is that the extent of the increase is greater for older groups: the group aged over sixty years has had earnings share increase by more than 100% and the group aged $50-60$ has experienced a smaller rise.

4 Accounting Results

We present estimates from different specifications and levels of aggregation. A consistent finding is that workers aged 50+ have a larger earnings wedge than younger workers.

4.1 Aggregate Estimation, Cobb-Douglas Production

Equation (10) can be estimated using linear methods once we specify properties of the residual ϵ_t. That is, we will estimate the following regression:

$$LS_t^{-1} = \beta_1 + \sum_{j=2}^{J} \beta_j \sigma_j + \epsilon_t \tag{12}$$

Where $\beta_1 = \frac{1}{\alpha_1}$ and $\beta_j = \frac{1}{\alpha_j} - \frac{1}{\alpha_y}$. For different assumptions on ϵ_t. Since the earnings shares sum to one, we have dropped the first group into the constant, we will then use the estimates to find the relative earnings wedges for groups $j \geq 2$ via:

$$\frac{\hat{\alpha}_j}{\hat{\alpha}_1} = \frac{\hat{\beta}_j + \hat{\beta}_1}{\hat{\beta}_1}$$

4.1.1 Groupings and the Residual

We must first choose the demographic groupings which determine J. We choose to make limit our degrees of freedom to two age groups, with $j = 1$ corresponding to workers younger than 50 and $j = J = 2$ corresponding to those 50 and up. We also estimate the model with a finer age grouping but find that the split between young and elder is most important.

We must also address the residual ϵ_t in Equation (10). First, we assume that $\epsilon_t = \gamma t + \varepsilon_t$, so that there is a linear time trend which affects both age groups symmetrically. Our regression equation is therefore:

$$LS_t^{-1} = \beta_0 + \beta_1 \sigma_{elder,t} + \gamma t + \epsilon_t \tag{13}$$

Where t is defined as the current year minus 1963, which is the first year in our sample.

If the error term ϵ_t is correlated with the mature earnings share,[7] then the above models will not give a consistent estimate of the relative earnings wedges. Our preferred specification allows for this correlation and instruments the elder-earnings share with the elder-population share, similarly to the instrumenting approach used by Shimer (CITE THAT PAPER). If 20-year lagged birth rates, contemporary mortality rates, and immigration/emigration rates are uncorrelated with the residual in Equation (13) then population share satisfies the exclusion restriction. It is clearly strongly correlated with earnings-shares, as can be seen in Figure (7) and the F-statistic is large.

The estimates on relative wedges can be found in Column (1) of Table (1).[8] The coefficient on mature earnings share is 0.57 and the constant is 1.39, which together imply a relative earnings wedge of 0.710. That is, mature workers capture 29% less of their marginal product relative to young workers. The second column in Table (1) reports the IV estimation in which all variables are in first differences. While we prefer to estimate the model in levels so that relative wedges can be constructed, we report this specification as a check of robustness. While the standard errors have increased, the point estimate is essentially unchanged.

The overall fit of this regression can be seen in Figure (8); the predicted labor share tracks the data quite closely, though with such a parsimonious model we should cannot match ever short-run movement. Using the estimates from our preferred specification, we plot the actual labor share time series, the predicted, and a counter factual series in which we set the earnings share of elder workers equal to the initial value. These plots can be seen in Figure (9), which shows that the dynamics of labor's share would have differed dramatically if earnings shares had remained constant. Specifically, we predict that the decline in labor-share since 2000 would have been substantially muted if not for the shift in earnings towards elder workers.

4.2 Aggregate Estimation, General CRS Production

We now relax Assumption 4 and use Assumption 5 to estimate the earnings wedges for a general constant returns production function. In order to do so we must take a stance on which real rate and depreciation rate to use, as well as the order of the polynomial used to approximate the marginal product of capital. For our baseline estimates we define

[7]For example, if it represents a shock that both reduces labor share and increases labor demand for experienced workers.

[8]We have reported Newey-West standard errors for each estimated parameter.

the real rate as the nominal rate on three-month treasury bills less CPI growth and the depreciation rate we take from the Penn World Tables Version 8.1 estimates, which are the same used to calculate the capital stock. Our baseline polynomial is linear.

We estimate two specifications under Assumption 5. The first is Column (3) of Table (1), which excludes the time trend. This gives variation of the marginal product of capital the largest role in changing labor share, but actually implies a larger point estimate of the mature worker's relative earnings wedge (33% instead of 29%, though statistically indistinguishable). The second specification includes a linear time trend, which lowers the coefficients from the marginal product of capital term but has a minor effect on the relative earnings wedge estimate, which rises to 25%.

We perform robustness on both the interest rate measure and order of this polynomial in Tables (2) and (3). The estimates of relative wedges are essentially identical across these specifications.

4.3 Decomposing Earnings Shares

We now look at the underlying forces driving mature earnings share. Specifically, we decompose earnings for a group into that group's population, the labor-force participation of the group, the employment rate, and earnings per employee. These can all be considered relative to average and the change in the log of earnings share decomposed in to the sum of the log-difference of each term.

These decompositions are found in Figures (4) and (5). This highlights that the rise in the earnings share of over-50 households is not purely due to the increase in the raw population. Population has driven some of the rise in earnings share, but earnings-per-worker in this group has risen by even more. Labor-force-participation has in fact dampened the rise in mature earnings shares.

4.4 Sectoral Estimation

It is well known that, even with competitive wages, sectoral reallocation can change the aggregate labor's share if output elasticities differ across sectors. Therefore, labor's share would vary with mature earnings shares if older workers tended to work in low labor elasticity sectors. We therefore estimate the relative earnings wedges using sectoral data, under two common specifications.

We follow Elsby, et al ([1]) to construct sectoral payroll shares from 1987 to 2011, which includes the post-2000 period during which the aggregate labor share fell most sharply. We construct the earnings shares at the industry level using CPS industry codes and match earnings and payroll shares for 11 sectors (4). The average number of observations in the CPS for a sector-year pair is $7,016$. Thus, for each sector we have the payroll share, the earnings shares of individuals under and over 50 years of age, and the national earnings share of individuals over 50 years of age. We first assume that the *slope* coefficient on mature earnings shares is homogenous across sectors, so that we can pool sectoral data and allow for arbitrary time fixed effects. We then allow for fully heterogeneous coefficient across sectors and estimate relative earnings wedges under the Cobb-Douglas assumption in each sector.

4.4.1 Homogeneous Coefficients

We assume that β_1 is constant across sectors and estimate the model in first-differences. We do not estimate the model in levels because that requires that the constant term is identical and we prefer to make as few restrictions as possible. The estimating equation is:

$$\Delta PS_{s,t}^{-1} = \beta_1 \Delta \big[\frac{E_{s,elder,t}}{E_{s,t}}\big] + \psi_t + \eta_{s,t} \tag{14}$$

Since workers can change sectors population shares of mature workers are no longer valid instruments for the earnings share. Therefore, following Nakamura and Steinsson [7], we use the national earnings share of individuals over 50 to instrument for the earnings share of mature workers within a sector, $\frac{E_{s,elder,t}}{E_{s,t}}$. Including the year fixed effects implies that identification of β_1 comes from the cross-sectoral differences in the response of payroll shares (relative to the average) to changes in the sectoral earnings share of mature workers. The first column in Table 5 shows the results from the panel regression, the coefficient on the earnings share of mature workers is positive and significant. In terms of the relative earnings wedges for each sector, the above specification imposes the following restriction:

$$\beta_1 = \alpha_s^{-1}(\alpha_{s,elder} - \alpha_{s,young})$$

for all sectors. Therefore, the positive coefficient in table 5 suggests that mature workers have a larger earnings wedge and receive a smaller portion of their marginal product, consistent with the findings at the national level.

4.4.2 Heterogeneous Coefficients

In our second specification, we assume that equation (13) holds at the sectoral level,

$$PS_{s,t}^{-1} = \beta_{s,0} + \beta_{s,1}\sigma_{s,elder,t} + \gamma_s t + \epsilon_{s,t} \tag{15}$$

where the national earnings share of elder workers is used to instrument for the sector earnings share. We report the average of these sectoral estimates of relative earnings in Column (2) of Table (5).[9] The average is in fact lower than the aggregate estimate (0.44 rather than 0.71), but there is variation across sectors, as can be seen by the minimum and maximum reported in Columns (2) and (3). The point estimates and standard errors for each sector are seen in Figure 11. At the sectoral level the relative wedge estimates are much less precise and the instruments tend to be weaker.[10] However, most point estimates are below one, indicating a larger earnings wedge for elder workers relative to the young.

5 A Theory of Life-Cycle Earnings Wedges

We now describe a parsimonious model in which the earnings wedge rises with age. Life-cycle heterogeneity is driven by the accumulation of general and specific human capital and variation in the arrival rate of outside job offers. Earnings are determined as in Postel-Vinay and Robin (CITE). A shift in the earnings share towards mature workers is driven by declining population growth.

We assume that the economy has three age groups and constant population growth. Workers can be either young, mature, or retired. The probability of transitioning from one state to the next is given by μ.

In addition to age, a worker's employment status and earnings are state variables. An unemployed worker receives flow utility of z from benefits and home production and matches with a firm with probability $f_i, i \in \{y, m\}$ and an employed worker receives an outside offer with probability $\omega_i, i \in \{y, m\}$. Workers do not discount.

Firms produce with labor, the marginal product of which depends on a worker's age and employment history. Young workers have a productivity normalized to one, whereas mature workers' productivity reflects the accumulation of both general and

[9] We weight each point estimate by the inverse of its standard error.

[10] Since the national earnings share was a very weak instrument for Natural Resources and Mining and Leisure and Hospitality, estimates from these sectors are not reported.

match-specific human capital. A mature worker who is currently employed in a job *other than the one held when she was young* has productivity $q > 1$, while a mature worker who is currently employed in the same job she held while young has productivity $p \geq q$. A retired worker has productivity of zero.

The distribution of workers across each age group arises according the the aging rates above and the growth rate of young workers, given by b. Since retired workers do not affect labor share, we are not interested in their measure. The young and mature evolve as:

$$\begin{aligned} N_{m,t+1} &= \mu N_{y,t} + (1-\mu)N_{m,t} && (16)\\ N_{y,t+1} &= (1-\mu)N_{y,t} + bN_{y,t} && (17)\end{aligned}$$

5.1 Worker Value Functions

We list the value functions for unemployed workers of both age groups. For the mature worker we have:

$$U_m = z + \mu\frac{z}{\mu} + (1-\mu)\Big(f_m W_{m,q}\big(w^{\emptyset}_{m,q}\big) + (1-f_m)U_m\Big) \tag{18}$$

Where the term $\frac{z}{\mu}$ represents the discounted expected value from retirement until death, $W_{m,q}$ is the value function for a mature worker with productivity q (since this worker will hold a job other than the one held while young), and $w^{\emptyset}_{m,q}$ is the equilibrium wage of a newly employed mature worker with productivity q.

For the young worker, unemployment has value:

$$\begin{aligned} U_y &= z + \mu\Big(f_m W_{m,q}\big(w^{\emptyset}_{m,q}\big) + (1-f_m)U_m\Big) && (19)\\ &+ (1-\mu)\Big(f_y W_y\big(w^{\emptyset}_{y}\big) + (1-f_y)U_y\Big) \end{aligned}$$

Where the term W_y is the value of a young worker while employed and $w^{\emptyset}_{y}$ is the equilibrium earnings of a newly employed young worker.

An employed mature worker can have productivity $x \in \{p, q\}$ and has value:

$$W_{m,x}(w) = w + \mu\frac{z}{\mu} + (1-\mu)\Big(\omega_m W_{m,x}\big(w^{m}_{m,x}\big) + (1-\omega_m)W_{m,x}\big(w\big)\Big) \tag{20}$$

Where $w^{m}_{m,x}$ is the equilibrium earnings of a mature worker of productivity x who receives an outside offer while mature.

An employed young worker has productivity equal to one and has value:

$$\begin{aligned} W_y(w) &= e + \mu\Big(\omega_m W_{m,x}\big(w^m_{m,x}\big) + (1-\omega_m) W_{m,x}\big(e\big)\Big) \\ &+ (1-\mu)\Big(\omega_y W_y(w^y_y) + (1-\omega_y) W_y(w)\Big) \end{aligned} \tag{21}$$

Where the term w^y_y represents the earnings of a young worker who has received an outside offer while young. Note that we assume that a worker who matures without receiving an outside offer maintains his earnings. For some workers this may mean that earnings while mature are below the mature worker's reservation value (specifically, a newly matured worker who never received an outside offer while young). The fraction of these workers will be small in practice, so we maintain the assumption so that the model remains as clean as possible.

5.2 Employer Value Functions

We now describe the value functions for an employer. The present value depends on the worker's age, productivity, and earnings. For a mature worker with productivity $x \in \{p, q\}$:

$$J_{m,x}(e) = x - w + (1-\mu)\Big(\omega_m J_{m,x}\big(w^m_{m,x}\big) + (1-\omega_m) J_{m,x}(w)\Big) \tag{22}$$

The value for a firm who employs a young worker the value is:

$$\begin{aligned} J_y(w) &= 1 - w + \mu\Big(\omega_m J_{m,p}\big(w^m_{m,p}\big) + (1-\omega_m) J_{m,p}(w)\Big) \\ &+ (1-\mu)\Big(\omega_y J_y\big(w^y_y\big) + (1-\omega_y) J_y(w)\Big) \end{aligned} \tag{23}$$

Note that there are two effects of aging that differ depending on whether a worker is young or mature. A young worker may mature, in which case the firm still has a productive employee for some time, whereas a mature worker retires when she matures. This causes a duration effect in the value of employing a young versus a mature worker. The second is that a mature worker's productivity is fixed for the duration of the job, so the maximal flow of profit for the employer is the current productivity minus the current earnings. For a young worker, maturation leads to an increase in productivity since $p > 1$, which means that profits grow in expectation.

5.3 Earnings Determination

The surplus is split through earnings depending on a worker's history of outside offers. Specifically, when a new match occurs between an employer and an unemployed worker, the firm has full monopsony power and extracts the entire surplus. Once an outside offer arrives, the two employers compete over the worker up until the point that the worker extracts the smaller of the two surpluses. Earnings are therefore determined via the following equations:

$$W_y(w_y^{\emptyset}) - U_y = 0 \tag{24}$$

$$J_y(w_y^y) = 0 \tag{25}$$

$$W_{m,x}(w_{m,x}^{\emptyset}) - U_m = 0, \forall x \in \{p, q\} \tag{26}$$

$$J_{m,q}(w_{m,x}^m) = 0, \forall x \in \{p, q\} \tag{27}$$

Note that Equation (27) determines the mature worker's earnings, independent of his productivity. That is because $q \leq p$, which means that the outside offer for a mature worker will always have the lower surplus.

An interesting feature of this bargaining protocol is that the unemployment value for a worker is independent of the job-finding rate. This can be seen by imposing these conditions on the right-hand side of Equations (18) and (19):

$$U_m = z + \mu \frac{z}{\mu} + (1 - \mu) U_m \tag{28}$$

$$U_y = z + \mu U_m + (1 - \mu) U_y \tag{29}$$

5.4 Characterization of Earnings

The above equations can be solved analytically for earnings of each worker group, though intuition is more easily developed from the expressions before simplification. The first earnings variable, $w_y^\emptyset$, solves:

$$\begin{aligned} w_y^\emptyset &= z \\ &- \mu\Big(\omega_m\big(W_{m,p}(w_{m,p}^m) - U_m\big) + (1-\omega_m)\big(W_{m,p}(w_y^\emptyset) - U_m\big)\Big) \\ &- (1-\mu)\Big(\omega_y\big(W_y(w_y^y) - U_y\big) + (1-\omega_y)\big(W_y(w_y^\emptyset) - U_y\big)\Big) \end{aligned} \tag{30}$$

This shows that the young worker's reservation wage differs from the flow flow value of unemployment through two margins: the expected surplus gains from higher mature productivity and the probability of receiving an outside offer, thereby extracting the entire surplus. Since $w_y^\emptyset < z < 1$, the newly employed young worker has a large earnings wedge. The same is true for a mature worker who has never received an outside offer:

$$\begin{aligned} w_{m,x}^\emptyset &= z \\ &- (1-\mu)\Big(\omega_m\big(W_{m,x}(w_{m,x}^m) - U_m\big) + (1-\omega_m)\big(W_{m,x}(w_{m,x}^\emptyset) - U_m\big)\Big) \end{aligned} \tag{31}$$

For young workers who have received an outside offer, earnings reflect the entire surplus:

$$w_y^y = 1 + \mu\Big(\omega_m J_{m,p}(w_{m,p}^m) + (1-\omega_m) J_{m,p}(w_y^y)\Big) \tag{32}$$

This expression can be used to prove that $w_y^y > 1$. Figure (12) plots the left-hand and right-hand sides of this equation. Since $J_{m,p}(w_{m,p}^m) > J_{m,q}(w_{m,p}^m) = 0$ and $J_{m,p}(1) > 0$, we know that the right-hand side is greater than one when evaluated at $w_y^y = 1$. Furthermore, the right-hand side is decreasing in w_y^y, which means that the equilibrium value of $w_y^y > 1$, which means that young workers who have received an outside offer actually earn more than their static marginal product. This analysis highlights that young workers can capture a larger share of their marginal product than mature workers even if $p = q$, so that there is no match-specific human capital.

5.5 Stationary Equilibrium

The model is not stationary in levels because of exogenous population growth, but population shares are constant in the long run. We will perform comparative statics with respect to the population growth rate b, which affects these long-run population shares.

First, note that the stationary ratio of mature to young workers can be found by dividing Equation (16) by Equation (17): $\frac{N_m}{N_y} = \frac{\mu}{b}$. Under the assumption that all new entrants are young and unemployed, we can write the law of motion for unemployed young workers:

$$U_{y,t+1} = (1-\mu)(1-f_y)U_{y,t} + bN_{y,t} \tag{33}$$

Since all workers are in the labor force, the implied stationary unemployment rate for the young is:

$$u_y = \frac{b}{1-\mu+b-(1-\mu)(1-f_y)} \tag{34}$$

Employed young workers fall into two groups, those with earnings $w_y^{\emptyset}$ and those with w_y^y. The flow equations in levels are:

$$E_{y,t+1}^{\emptyset} = (1-\mu)\Big((1-\omega_y)E_{y,t}^{\emptyset} + f_y U_y\Big) \tag{35}$$

$$E_{y,t+1}^{y} = (1-\mu)\Big(E_{y,t}^{y} + \omega_y E_{y,t}^{\emptyset}\Big) \tag{36}$$

Which in turn give the stationary shares of each employment state:

$$e_y^{\emptyset} = \frac{(1-\mu)f_y}{1-\mu+b-(1-\mu)(1-\omega_y)}u_y \tag{37}$$

$$e_y^{y} = \frac{(1-\mu)\omega_y}{1-\mu+b-(1-\mu)}e_y^{\emptyset} \tag{38}$$

The law of motion for unemployed mature workers follows:

$$U_{m,t+1} = (1-\mu)(1-f_m)U_{m,t} + \mu(1-f_m)U_{y,t} \tag{39}$$

The stationary mature unemployment rate is therefore:

$$u_m = \frac{(1-f_m)b}{1-\mu+b-(1-f_m)(1-\mu)}u_y \tag{40}$$

There are three types of employed mature workers. Those who have never received an outside offer, those who received an outside offer while young but not yet while mature, and those who have received an outside offer while mature. The laws of motion are:

$$E^{\emptyset}_{m,t+1} = (1-\mu)\Big((1-\omega_m)E^{\emptyset}_{m,t} + f_m U_{m,t}\Big) + \mu\Big((1-\omega_m)E^{\emptyset}_{y,t} + f_m U_{y,t}\Big) \tag{41}$$

$$E^{y}_{m,t+1} = (1-\mu)(1-\omega_m)E^{y}_{m,t} + \mu(1-\omega_m)E^{y}_{y,t} \tag{42}$$

$$E^{m}_{m,t+1} = (1-\mu)\Big(E^{m}_{m,t} + \omega_m\big(E^{y}_{m,t} + E^{\emptyset}_{m,t}\big)\Big) + \mu\chi_m\big(E^{y}_{y,t} + E^{\emptyset}_{y,t}\big) \tag{43}$$

The employment rates for mature workers across different earnings therefore solve the following system:

$$e^{\emptyset}_{m} = \frac{1-\mu}{1-\mu+b}\Big((1-\omega_m)e^{\emptyset}_{m} + f_m u_m\Big) + \frac{b}{1-\mu+b}\Big((1-\omega_m)e^{\phi}_{y} + f_m u_y\Big) \tag{44}$$

$$e^{y}_{m} = \frac{(1-\mu)(1-\omega_m)}{1-\mu+b}e^{y}_{m} + \frac{b(1-\omega_m)}{1-\mu+b}e^{y}_{y} \tag{45}$$

$$e^{m}_{m} = \frac{1-\mu}{1-\mu+b}\Big(e^{m}_{m} + \omega_m(e^{y}_{m} + e^{\emptyset}_{m})\Big) + \frac{b\omega_m}{1-\mu+b}\Big(e^{y}_{y} + e^{\emptyset}_{y}\Big) \tag{46}$$

5.6 Comparative Statics

We have built this model in order to understand how life-cycle heterogeneity can generate earnings shares in line with the data. For these reasons, some parameters are much less important than others. The most important are the arrival rates of outside offers (ω_y, ω_m) and the parameters governing productivity growth (p, q). We now show how these parameters affect labor share. The comparative statics are complicated by the fact that some parameters change multiple equilibrium earnings expressions jointly with the stationary share of workers

5.6.1 Arrival Rates

While we cannot directly measure arrival rates of outside offers, there is evidence of a sharp decline in employer-to-employer transitions as workers age. The profile can be seen in Figure (**??**), taken from Bjelland, et al (CITE THE HALTIWANGER). The quarterly flow rate from employer to employer begins at 10% for workers in their early twenties and then falls with age to just above 1% by sixty. The averages for our classification of young and mature workers are roughly 4% and 2%.

This data is not directly comparable to our model since what matters for earnings is the arrival rate of offers, even if they do not lead to an employer switch. We therefore perform comparative statics on both of these arrival rates. We first fix the mature arrival rate to 1.5% per quarter and vary the young arrival rate from 1.5% to 4%. The lower bound of 1.5% assumes that the higher transition rate for younger workers is because they accept outside offers, while increasing toward 4% assumes that the employer to employer flows reflect outside offers perfectly. We then do the same exercise, but fix $\omega_y = 4\%$ and vary ω_m from 1.5% to 4%.

The effect of arrival rates can be seen in Figure **??**. Increasing either of these parameters tends to increase labor's share. First, there is a direct increase in the share of workers of each age group with an outside offer. These workers have large earnings relative to their marginal products, so labor's share increases. There is also a secondary effect in the case of increasing ω_m, since the earnings of a young worker with an outside offer falls with ω_m. This offsetting effect is dominated by the direct effect, however, since labor's share rises.

5.6.2 Match Specific vs General Productivity

The other important set of parameters is q relative to p. Specifically, for mature workers who have received an outside offer while mature, the earnings wedge is exactly $\alpha_m = \frac{q}{p} \leq 1$. This ratio is difficult to measure. There is debate in the empirical literature about the effect of tenure on *earnings*, controlling for experience and match quality. [11]

The model demonstrates that earnings may be uninformative about the growth in specific *productivity*, even if one can perfectly correct for selection effects on tenure. First, the most a mature worker is ever paid is $q < p$. Second, some tenured mature workers will be paid $w_y^y < q$. Finally, newly hired mature workers would have lower earnings than those with long tenure only because $w_m^{\emptyset}$ is below q, but $w_m^{\emptyset}$ is still independent of p. All of this means that an econometrician would estimate a small impact of tenure on wages, even if $\frac{q}{p} \to 0$.

To demonstrate this point, Figure (**??**) plots tenured versus newly hired earnings as a function of $p \geq q$.

6 Conclusion

We have given an account of demographic changes in the United States over the last fifty years and how these changes have manifested in relative earnings. Using a very

[11] See Altoniji & Williams (CITE THAT PAPER) for a discussion and an argument that the returns to tenure are roughly 9% over ten years.

simple extension of production theory (which nests the neoclassical growth model), we have shown that the earnings shares of appropriately chosen groups should account for the aggregate labor's share. The exact relationship between aggregate labor's share and group-specific earnings shares takes a non-linear form and the theory dictates the appropriate weights for averaging.

Empirically, we document that the share of earnings going to older workers (older than 50) has risen dramatically in recent years, precisely when the decline in labor's share has accelerated. Some of the rise in elderly earnings shares has been due to increased labor force attachment of baby boomers, but an increase in average earnings in this group has also been important. In fact, for the 60+ group, most of the rise in earnings share has been due to an increase in their relative earnings-per-worker.

Finally, we have proposed a theoretical connection between the elderly's share of earnings and aggregate labor's share that is deeper than enriching the production function. A search and matching model with life-cycle heterogeneity naturally generates a lower labor's share for elderly workers. An increase in the share of elderly workers in the labor force leads to a unequivocal fall in labor's share and a rise in earnings-per-worker for the elderly may also reduce the aggregate labor's share.

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7 Appendices

This appendix includes robustness and technical derivations where appropriate.

7.1 A2: Alternative Groupings

To be written

7.2 Alternative Methods of Handing Business Income

Have tried all to labor.

8 Figures and Tables

8.1 Figures

8.2 Empirical Tables

Table 1: U.S. regression results

	Cobb-Douglas		General CRS			
	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$
Relative Wedges	0.710***		0.674***		0.747***	
	(0.0493)		(0.0763)		(0.0531)	
Coeff. Mature Share	0.567***	0.554	0.584**	0.983	0.452***	0.789
	(0.123)	(0.546)	(0.187)	(0.558)	(0.116)	(0.563)
Constant	1.390***		1.205***		1.332***	
	(0.0317)		(0.0709)		(0.0580)	
t	0.00190***	0.00303			0.00145***	0.00281
	(0.000346)	(0.00184)			(0.000310)	(0.00191)
$\frac{K_t}{E_t}$			0.0508**	-0.00964	0.0224	-0.0118
			(0.0168)	(0.0259)	(0.0132)	(0.0258)
$(r_t + \delta_t)(\frac{K_t}{E_t})$			0.0421	0.0824*	0.0202	0.0841*
			(0.0666)	(0.0390)	(0.0529)	(0.0389)
Observations	52	51	50	49	50	49
Cragg-Donald F	705.1	35.37	681.5	46.16	750.4	39.42

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: U.S. Regression Results (Different Real Interest Rate Measures)

	T-Bill		Muni. Bonds		Prime Loan	
	(2)	(3)	(5)	(6)	(8)	(9)
	invLS	dinvLS	invLS	dinvLS	invLS	dinvLS
$\frac{\alpha_m^{-1}}{\alpha_y^{-1}}$	0.674***		0.681***		0.679***	
	(0.0763)		(0.0571)		(0.0765)	
σ_{mature}	0.584**	0.983	0.575***	0.838	0.576**	0.810
	(0.187)	(0.558)	(0.142)	(0.549)	(0.190)	(0.564)
Constant	1.205***		1.230***		1.216***	
	(0.0709)		(0.0595)		(0.0674)	
$\frac{K_t}{E_t}$	0.0508**	-0.00964	0.0423**	-0.0123	0.0483*	-0.00558
	(0.0168)	(0.0259)	(0.0153)	(0.0269)	(0.0191)	(0.0273)
$(r_t + \delta_t)(\frac{K_t}{E_t})$	0.0421	0.0824*	0.0915	0.0625	0.0318	0.0326
	(0.0666)	(0.0390)	(0.0474)	(0.0381)	(0.0635)	(0.0412)
Observations	50	49	50	49	50	49
Cragg-Donald F	681.5	46.16	943.2	49.14	658.1	45.97

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: U.S. Regression Results (Different MPK Aproximations)

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$	$\frac{1}{LS}$	$\Delta\frac{1}{LS}$
$\frac{\alpha_m^{-1}}{\alpha_y^{-1}}$	0.674***		0.691***		0.689***	
	(0.0763)		(0.0765)		(0.0754)	
σ_{mature}	0.584**	0.983	0.531**	0.977	0.534**	0.861
	(0.187)	(0.558)	(0.178)	(0.555)	(0.176)	(0.547)
Constant	1.205***		1.187***		1.183***	
	(0.0709)		(0.0682)		(0.0683)	
$\frac{K_t}{E_t}$	0.0508**	-0.00964	0.0548***	-0.00885	0.0566***	-0.00280
	(0.0168)	(0.0259)	(0.0159)	(0.0259)	(0.0164)	(0.0258)
$(r_t + \delta_t)(\frac{K_t}{E_t})$	0.0421	0.0824*	0.255	0.0978	0.122	0.384
	(0.0666)	(0.0390)	(0.203)	(0.117)	(0.466)	(0.267)
$(r_t + \delta_t)^2(\frac{K_t}{E_t})$			-2.359	-0.199	1.094	-7.858
			(2.215)	(1.453)	(11.56)	(6.588)
$(r_t + \delta_t)^3(\frac{K_t}{E_t})$					-24.47	54.92
					(81.43)	(46.07)
Observations	50	49	50	49	50	49
Cragg-Donald F	681.5	46.16	645.1	46.14	629.0	43.66

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Sectors

1	Natural resources and mining
2	Construction
3	Durable goods manufacturing
4	Non-durable goods manufacturing
5	Trade/Transportation and utilities
6	Information
7	Financial activities
8	Professional and business services
9	Education and health services
10	Leisure and hospitality
11	Other services

Table 5: US Sector Results

	Pooled	Sector-by-Sector		
	$\Delta \frac{1}{PS}$	Average	Minimum	Maximum
Coeff. Mature Share	4.487***			
	(1.011)			
Relative Wedges		0.440	0.194	4.639
Observations	264			

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

8.3 Model Tables

Table 6: Parameter Values

Parameter	Value
δ	0.99
η	0.0028
ρ	0.0042
s	0.026
z	0.955
β	0.052
p_y	1.00

Table 7: Counterfactuals

Moment	LF	Earnings	Both	Data
ΔLS	-0.81%	-0.24%	-1.22%	-3.62%
$\Delta \frac{E_o}{E}$	10.9%	1.34%	12.5%	12.1%
$\Delta \frac{\text{EMP}_o}{\text{EMP}}$	10.1%	-0.02%	10.1%	9.8%

Data ΔLS refers to the *predicted fall due to demographics*.

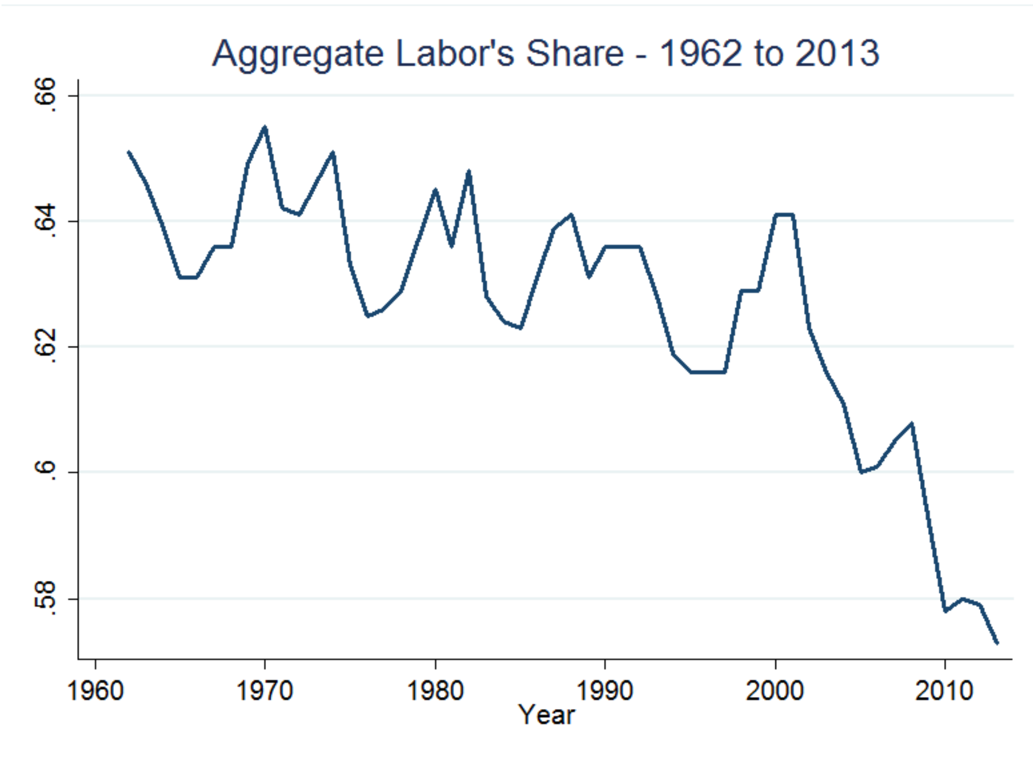

Figure 1: Aggregate Labor Share

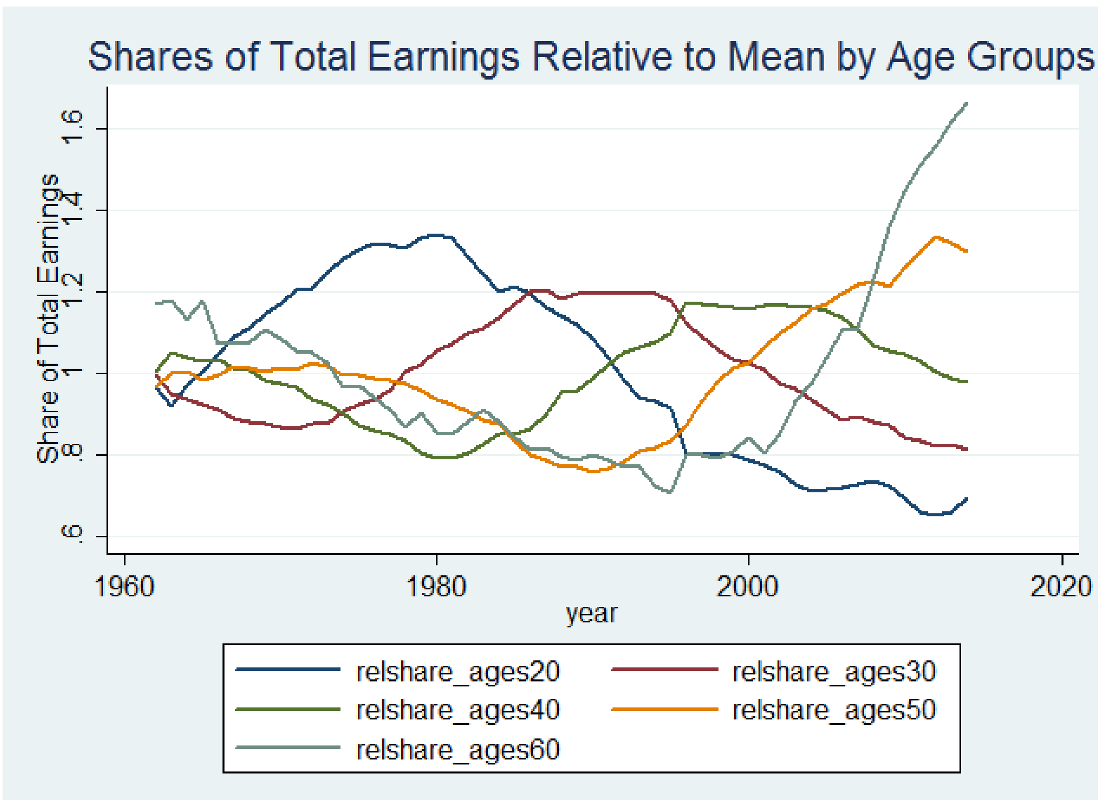

Figure 2: Age Distribution of Earnings Shares

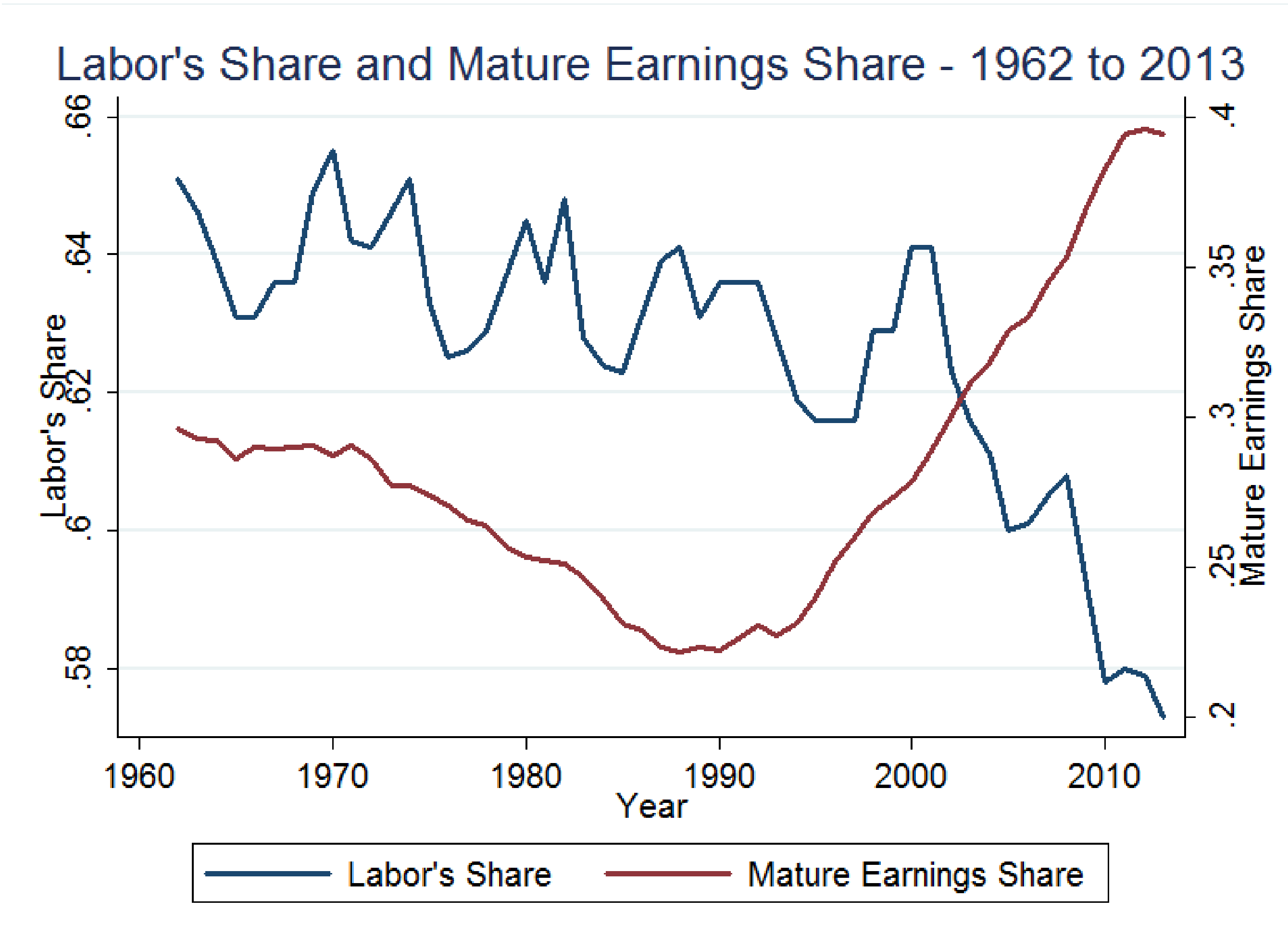

Figure 3: Labor's Share and Mature Earnings Share

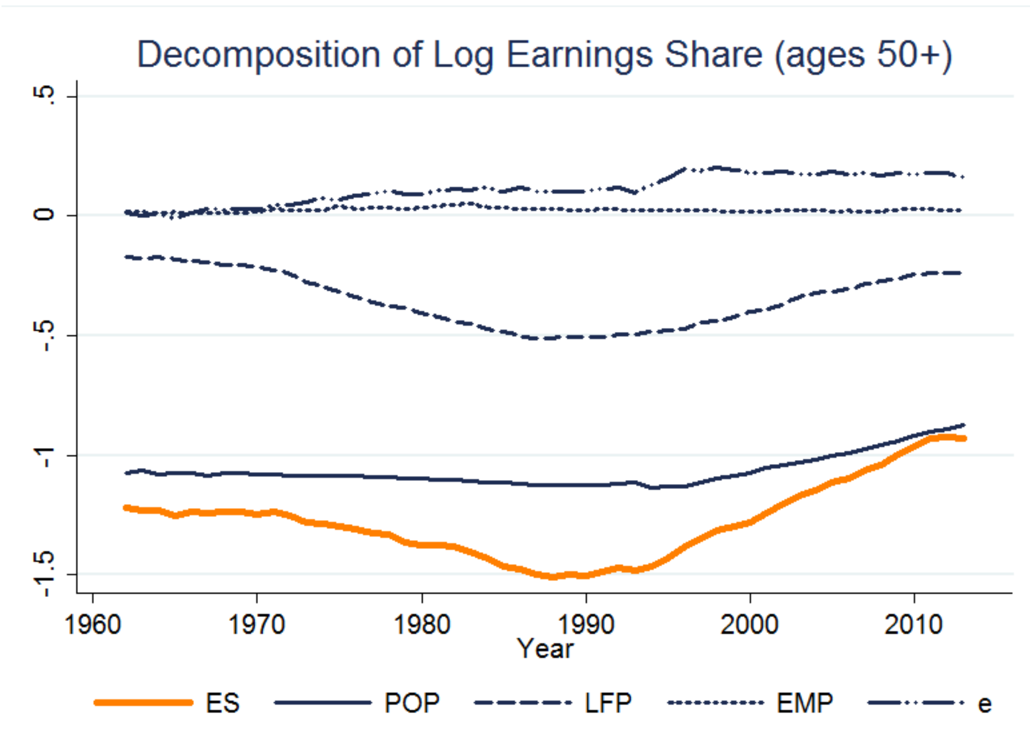

Figure 4: Decomposition of Earnings Share - ages 50+

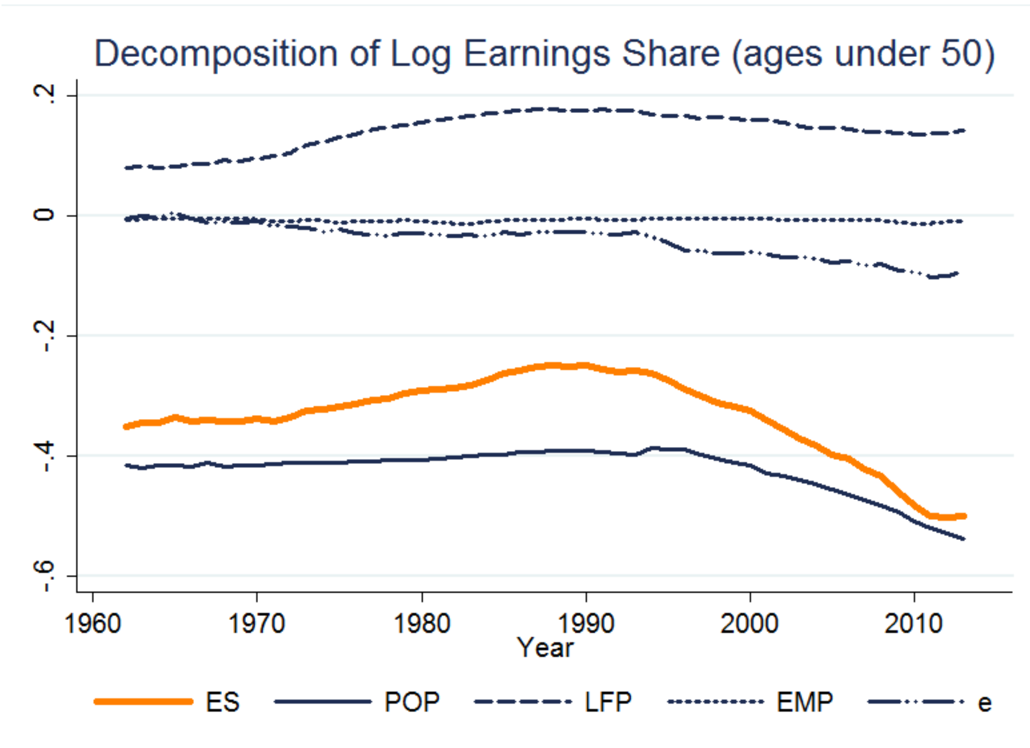

Figure 5: Decomposition of Earnings Share - ages under 50

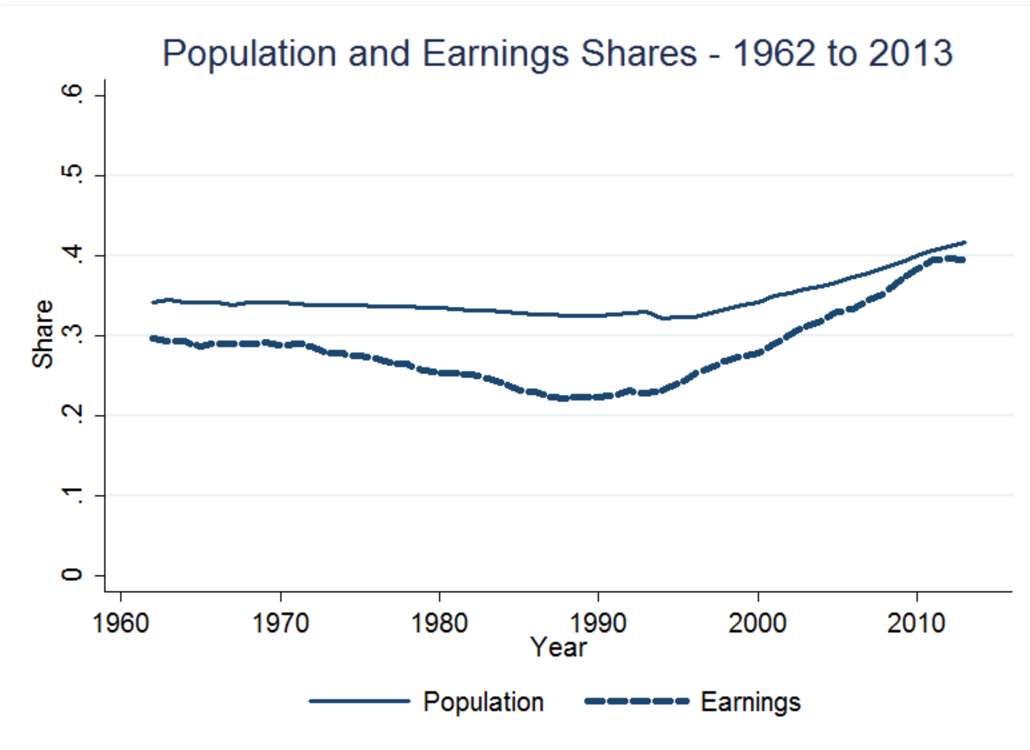

Figure 6: Elder Population Share and Earnings Share

Figure 7: Elder Population Share and Earnings Share

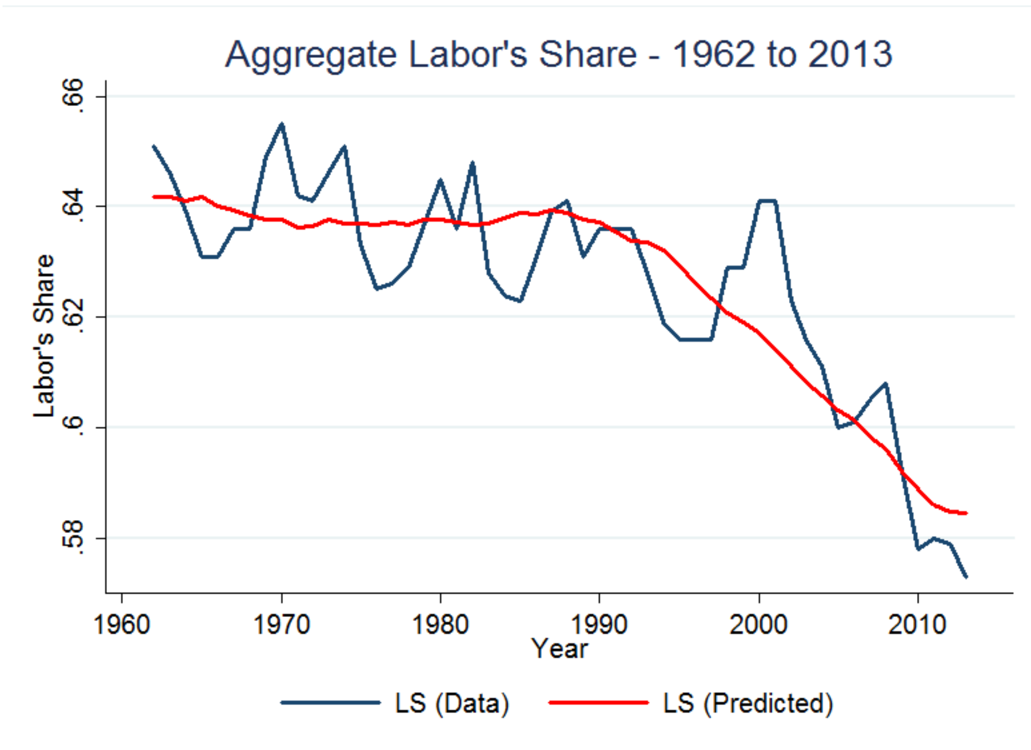

Figure 8: Actual vs Predicted Labor Share

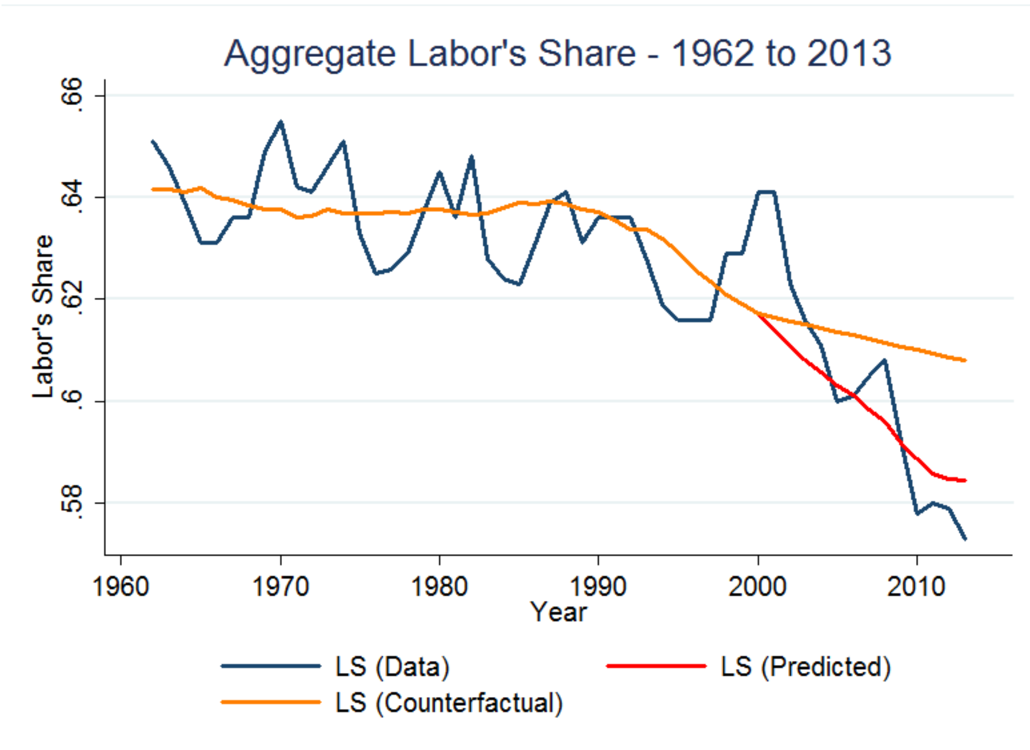

Figure 9: Actual vs Predicted Labor Share

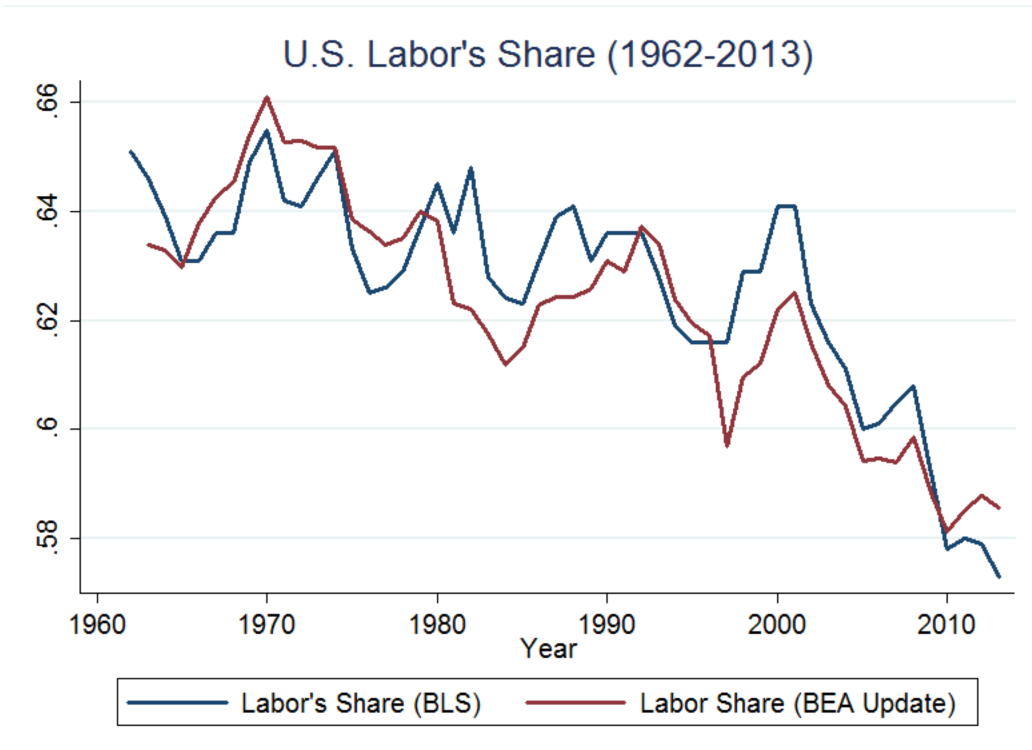

Figure 10: BLS vs BEA Updated Labor Share

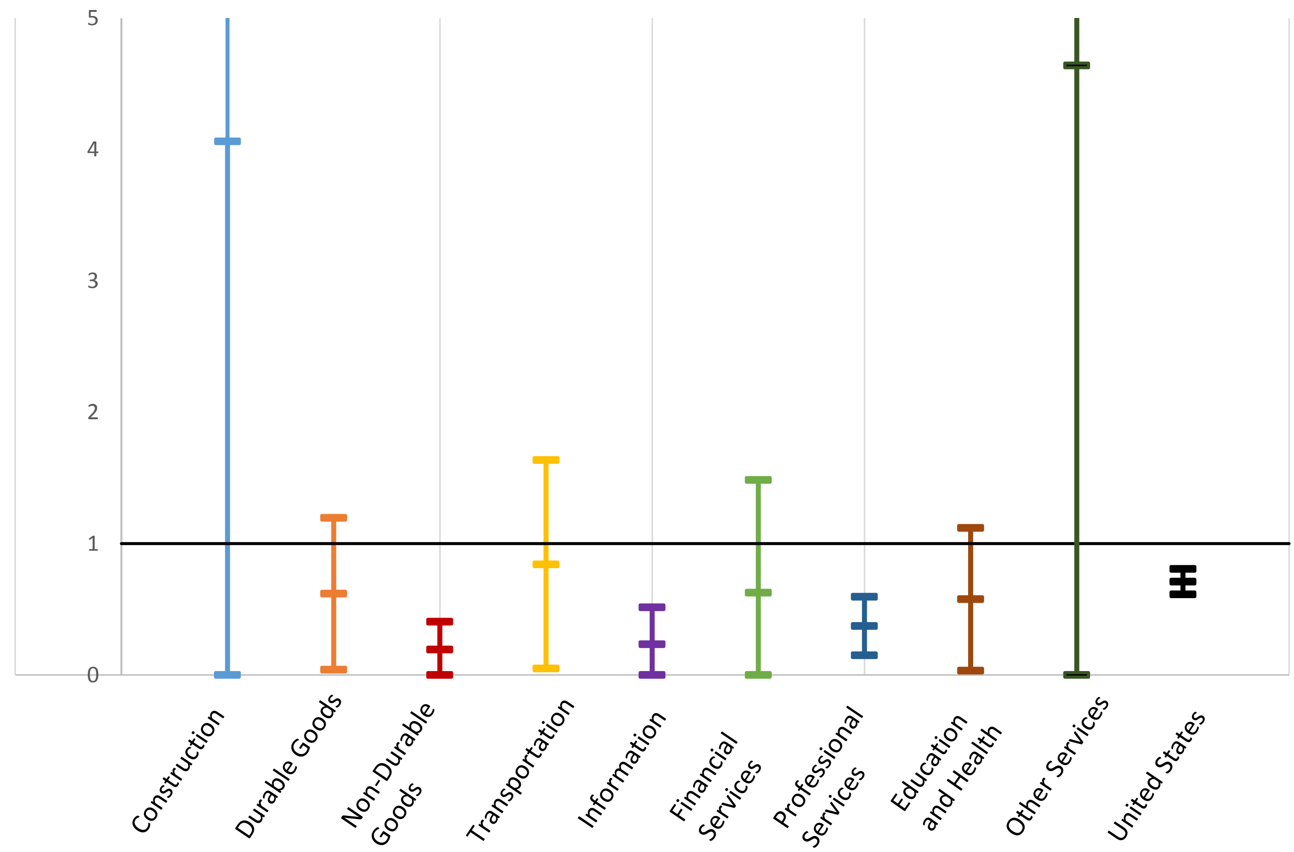

Figure 11: Ratio of Earnings Wedges ($\frac{\hat{\alpha}_j^{-1}}{\hat{\alpha}_1^{-1}}$) Estimates and 95% CI

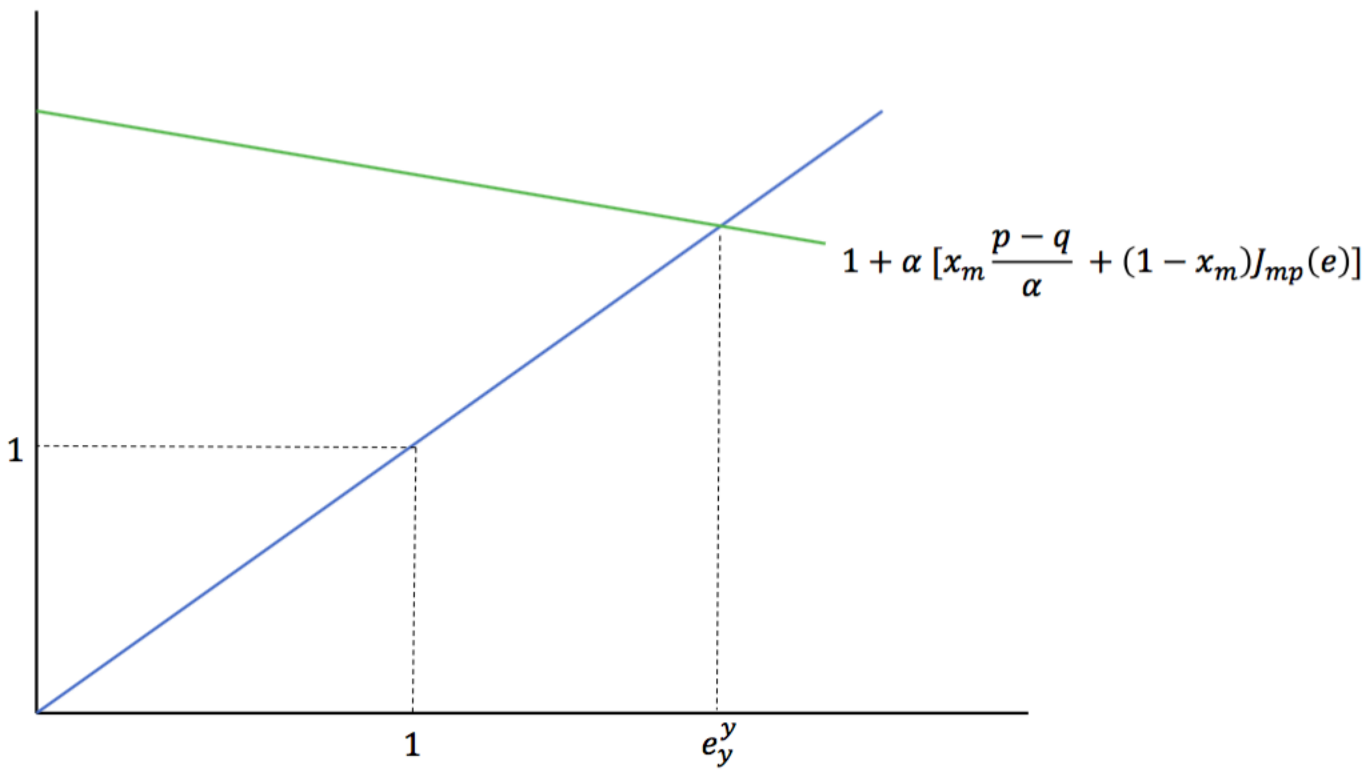

Figure 12: Determining Equilibrium e_y^y