The TIPS Liquidity Premium

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Abstract

We introduce an arbitrage-free dynamic term structure model of nominal and real yields with a liquidity risk factor to account for the liquidity disadvantage of Treasury inflation-protected securities (TIPS) relative to Treasury securities. The identification of the liquidity factor comes from its unique loading, which mimics the idea that, over time, increasing amounts of outstanding securities get locked up in buy-and-hold investors' portfolios. The model is estimated using prices for individual TIPS combined with a standard sample of nominal Treasury yields and delivers liquidity premium estimates for each TIPS. We find that TIPS liquidity premiums have averaged 38 basis points with notable time variation. Furthermore, accounting for liquidity risk improves the model's ability to forecast inflation.

JEL Classification: E43, E47, G12, G13

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1 Introduction

In 1997, the U.S. Treasury started issuing inflation-indexed bonds, which are now commonly known as Treasury inflation-protected securities (TIPS). Since then the U.S. Treasury has steadily expanded the market for TIPS, which by the end of 2013 had a total outstanding amount of $973 billion or 8.2 percent of all marketable Treasury securities.[1]

Despite the apparent large size of the market for TIPS, there is an overwhelming amount of research suggesting that TIPS are less liquid than regular Treasury securities. Fleming and Krishnan (2012) report market characteristics of TIPS that indicate smaller trading volume, longer turnaround time, and wider bid-ask spreads than are normally observed in the nominal Treasury bond market (see also Campbell et al. 2009, Dudley et al. 2009, Gürkaynak et al. 2010, and Sack and Elsasser 2004). Moreover, there is evidence that TIPS yields are elevated for these reasons as investors require a premium for assuming the associated illiquidity risk (see Fleckenstein et al. 2014 for a detailed discussion). However, the degree to which these frictions bias TIPS yields remains a topic of debate as no consensus has emerged on estimates of the TIPS liquidity premium.[2]

In this paper, we introduce an arbitrage-free dynamic term structure model of nominal and real yields with a latent liquidity risk factor to account for the potential liquidity disadvantage of TIPS relative to Treasury securities. The model is an extension of the model of nominal and real yields introduced in Christensen et al. (2010, henceforth CLR), referred to throughout as the CLR model. The identification of the liquidity factor comes from its unique loading for each TIPS that is supposed to mimic the notion that, over time, an increasing amount of its outstanding notional gets locked up in buy-and-hold investors' portfolios. Thanks to investors' forward-looking behavior, this affects its sensitivity over time to variation in the market-wide liquidity as captured by the liquidity factor. By observing a cross section of TIPS prices over time, the liquidity factor can be separately identified.

We estimate the CLR model and its extension using price information for individual TIPS trading from mid-1997 through the end of 2013 combined with a standard sample of nominal Treasury yields from Gürkaynak et al. (2007). To get a clean read of the TIPS liquidity factor, we account explicitly in the model estimation for the time-varying value of the deflation protection option embedded in the TIPS contract using pricing formulas provided in Christensen et al. (2012). In addition to delivering unique liquidity premium estimates for each TIPS, we find that the model that accounts for the TIPS liquidity premium outperforms the model that does not when it comes to forecasting CPI inflation. To have more accurate and less biased estimates of bond investor's inflation expectations embedded in nominal and real yields is crucial for portfolio risk management and monetary policy analysis.

[1]The data is available at: http://www.treasurydirect.gov/govt/reports/pd/mspd/2013/opds122013.pdf

[2]Pflueger and Viceira (2013), D'Amico et al. (2014), and Abrahams et al. (2015) are among the studies that estimate TIPS liquidity premiums.

In terms of the existing literature, Christensen et al. (2016) and Grishchenko et al. (2016) study the pricing of the TIPS deflation protection option, while Pflueger and Viceira (2013), D'Amico et al. (2014), and Abrahams et al. (2015) are among the studies that attempt to account for the TIPS liquidity premium. We do both. Thus, to the best of our knowledge, this is the first paper to account simultaneously for the liquidity premiums and the embedded deflation protection option values in TIPS prices within an arbitrage-free model of nominal and real yields. We make a thorough comparison to this existing literature.

Since we obtain improvement in one-year CPI inflation forecasts from accounting for TIPS liquidity premiums and achieve additional, but smaller improvements from taking the deflation protection option values into account, we are also the first to document the relative importance of accounting for both of these aspects in the pricing of TIPS.

The main finding of the empirical analysis, though, is that TIPS liquidity premiums over the entire sample average 38 basis points, which is lower than the results reported in existing studies of TIPS liquidity premiums. Furthermore, we document on-the-run TIPS liquidity premiums that are slightly lower, averaging 33 basis points and 30 basis points at the five- and ten-year maturity, respectively, and we show that it is an order of magnitude larger than the off-the-run liquidity premium in the Treasury bond market.[3]

We note that our results could have implications for the management of the U.S. Treasury debt. However, to evaluate the benefit to the U.S. Treasury of continuing its TIPS issuance, requires a comprehensive assessment of the sign and magnitude of the inflation risk premium that represents the gain from issuing TIPS relative to the liquidity disadvantage we document (see Christensen and Gillan 2012 for a discussion and analysis). Thus, we caution against drawing policy conclusions from our findings without further analysis.

Finally, we stress that our model approach is amenable to numerous extensions and modifications. First, we estimate the model using yield data only, but the model estimation could include survey data as in Kim and Orphanides (2012). Second, the nominal part of the model can be cast as a shadow-rate model to respect the zero lower bound for nominal yields as in, for example, Priebsch (2013).[4] Third, the model can be modified to allow for stochastic yield volatility to improve its ability to price deflation risk following Christensen et al. (2016). Lastly, the model's objective dynamics can be adjusted for finite-sample bias as described and discussed in Bauer et al. (2012). However, we leave the exploration of these avenues for future research.

The remainder of the paper is structured as follows. Section 2 introduces the general theoretical framework for inferring inflation dynamics from nominal and real Treasury yields. Section 3 describes the CLR model and its extension with a liquidity risk factor. It also

[3]Fontaine and Garcia (2012) document systematic and pervasive positive differences between the prices of recently issued Treasury bonds and those of more seasoned, but otherwise comparable Treasury bonds.

[4]In Appendix E, we do provide a brief comparison to a shadow-rate version of our model using formulas from Christensen and Rudebusch (2015).

details our methodology for deriving model-implied values of the deflation protection options embedded in TIPS. Section 4 contains the data description, while Section 5 presents the empirical results. Section 6 contains an analysis of the estimated TIPS liquidity premium, while Section 7 is dedicated to an analysis of the risk of deflation. Section 8 analyzes the model-implied inflation expectations. Finally, Section 9 concludes and provides directions for future research. Appendices contain additional technical details and results.

2 Decomposing Breakeven Inflation

In this section, we demonstrate how an arbitrage-free term structure model can be used to decompose the difference between nominal and real Treasury yields, also known as the breakeven inflation (BEI) rate, into the sum of the expected inflation and the associated inflation risk premium.

To begin, we follow Merton (1974) and assume a continuum of nominal and real zero-coupon bonds exists with no frictions to their continuous trading. The economic implication of this assumption is that the markets for inflation risk are complete in the limit and spanned by the continuum of nominal and real bond prices. Given nominal and real stochastic discount factors, denoted M_t^N and M_t^R, the no-arbitrage condition enforces a consistency of pricing for any security over time. Specifically, the price of a nominal bond that pays one dollar in τ years and the price of a real bond that pays one unit of the defined consumption basket in τ years must satisfy the conditions that

$$P_t^N(\tau) = E_t^P\left[\frac{M_{t+\tau}^N}{M_t^N}\right] \quad \text{and} \quad P_t^R(\tau) = E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right],$$

where $P_t^N(\tau)$ and $P_t^R(\tau)$ are the observed prices of the zero-coupon, nominal and real bonds for maturity τ on day t and $E_t^P[.]$ is the conditional expectations operator under the real-world (or P-) probability measure. The no-arbitrage condition also requires a consistency between the prices of real and nominal bonds such that the price of the consumption basket, denoted as the overall price level Π_t, is the ratio of the nominal and real stochastic discount factors:

$$\Pi_t = \frac{M_t^R}{M_t^N}.$$

We assume that the nominal and real stochastic discount factors have the standard dynamics given by

$$\begin{aligned} dM_t^N/M_t^N &= -r_t^N dt - \Gamma_t' dW_t^P, \\ dM_t^R/M_t^R &= -r_t^R dt - \Gamma_t' dW_t^P, \end{aligned}$$

where r_t^N and r_t^R are the instantaneous, risk-free nominal and real rates of return, respectively, and Γ_t is a vector of premiums on the risks represented by the Wiener process W_t^P. By Ito's lemma, the dynamic evolution of Π_t is given by

$$d\Pi_t = (r_t^N - r_t^R)\Pi_t dt.$$

Thus, in the absence of arbitrage, the instantaneous growth rate of the price level is equal to the difference between the instantaneous nominal and real risk-free rates.[5] Correspondingly, we can express the stochastic price level at time t+τ as

$$\Pi_{t+\tau} = \Pi_t e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

The relationship between the yields and inflation expectations can be obtained by decomposing the price of the nominal bond as follows

$$\begin{aligned} P_t^N(\tau) &= E_t^P\left[\frac{M_{t+\tau}^N}{M_t^N}\right] = E_t^P\left[\frac{M_{t+\tau}^R/\Pi_{t+\tau}}{M_t^R/\Pi_t}\right] = E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] + cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= P_t^R(\tau) \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right). \end{aligned}$$

Converting this price into a yield-to-maturity using

$$y_t^N(\tau) = -\frac{1}{\tau}\ln P_t^N(\tau) \quad \text{and} \quad y_t^R(\tau) = -\frac{1}{\tau}\ln P_t^R(\tau),$$

we obtain

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^P\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right]$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^P\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation highlights that the inflation risk premium can be positive or negative. It

[5]We emphasize that the price level Π_t is a stochastic process as long as r_t^N and r_t^R are stochastic processes.

is positive if and only if

$$cov_t^P\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences.

Finally, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau), \tag{1}$$

that is, the difference between nominal and real yields of the same maturity and can be decomposed into the sum of the expected inflation and the associated inflation risk premium.

Equation (1) highlights that the decomposition of BEI can be distorted if nominal and real yields are biased by liquidity effects, and the magnitude of the distortion equals the size of the bias. However, the equation also makes clear that it is only the relative liquidity between nominal and real yields that we need to correct BEI rates for any liquidity bias. In the following section, we introduce a dynamic term structure model that accounts for the liquidity differential of TIPS relative to Treasuries and hence provides estimates of the frictionless nominal and real yields that feature in the expectations above.

3 An Arbitrage-Free Model of Nominal and Real Yields with Liquidity Risk

In this section, we first describe how we extend the general framework introduced in the previous section to account for the liquidity risk of a set of real-valued securities relative to a benchmark set of nominal securities. Second, we detail the CLR model that we subject to this extension and apply in the subsequent empirical analysis.

3.1 The General Model with a Liquidity Risk Factor

Due to the lower liquidity of real-valued securities relative to the benchmark nominal securities, the yields of the former are sensitive to liquidity pressures on a relative basis. As a consequence, the discounting of future cash flows from the real-valued securities is not performed with the frictionless real discount function described in Section 2, but rather with a discount function that also accounts for liquidity risk. Recent research by Hu et al. (2013, henceforth HPW) and others suggest that liquidity is indeed a priced risk factor. Thus, we choose to represent this by a single liquidity risk factor denoted X_t^{liq}.[6] Furthermore, since liquidity risk is security-specific in nature, the discount function used to discount the cash

[6] D'Amico et al. (2014) and Abrahams et al. (2015) also only allow for a single TIPS liquidity factor.

flow of a given real-valued security indexed i is assumed to be unique. The single innovation of this paper is to let the individual TIPS discount function take the following form:

$$\overline{r}_t^{R,i} = r_t^R + \beta^i (1 - e^{-\lambda^{L,i}(t - t_0^i)}) X_t^{liq}, \tag{2}$$

where r_t^R is the frictionless real instantaneous rate as before, t_0^i denotes the date of issuance of the security, β^i is its sensitivity to the variation in the liquidity risk factor, and $\lambda^{L,i}$ is a decay parameter. While we could expect the sensitivities to be identical across securities, the results from our subsequent empirical application shows that it is important to allow for the possibility that the sensitivities differ across securities. Furthermore, we allow the decay parameter $\lambda^{L,i}$ to vary across securities as well. Since β^i and $\lambda^{L,i}$ have a nonlinear relationship in the bond pricing formula, it is possible to identify both empirically. Finally, we stress that equation (2) can be included in any dynamic term structure model to account for security-specific liquidity risks.

The inclusion of the issuance date t_0^i in the pricing formula is a proxy for the phenomenon that, as time passes, it is typically the case that an increasing fraction of a given security is held by buy-and-hold investors. This limits the amount of the security available for trading and affects its sensitivity to the liquidity factor. Rational, forward-looking investors will take this dynamic pattern into consideration when they determine what they are willing to pay for the security at any given point in time between the date of issuance and the maturity of the bond. This dynamic pattern is built into the model structure.

3.2 The CLR Model

Building on the insights from the general theoretical discussion in Section 2, we need an accurate model of the instantaneous nominal and real rate, r_t^N and r_t^R, in order to measure the market-implied inflation expectations precisely. With that goal in mind we choose to focus on the tractable affine dynamic term structure model of nominal and real yields introduced in CLR and briefly summarized below. We emphasize that even though the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the canonical Gaussian model.[7]

The CLR model of nominal and real yields is a direct extension of the three-factor, arbitrage-free Nelson-Siegel (AFNS) model developed by Christensen et al. (2011, henceforth CDR) for nominal yields. In the CLR model, the state vector is denoted by $X_t = (L_t^N, S_t, C_t, L_t^R)$, where L_t^N is the level factor for nominal yields, S_t and C_t represent slope and curvature factors common to both nominal and real yields, and L_t^R is the level factor for

[7] These restrictions can be derived explicitly, and the calculations are available upon request.

real yields.[8] The instantaneous nominal and real risk-free rates are defined as

$$r_t^N = L_t^N + S_t, \tag{3}$$
$$r_t^R = L_t^R + \alpha^R S_t. \tag{4}$$

Note that the differential scaling of the real rates to the common slope factor is captured by the parameter α^R. To preserve the Nelson and Siegel (1987) factor loading structure in the yield functions, the risk-neutral (or Q-) dynamics of the state variables are given by the stochastic differential equations:[9]

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\lambda & \lambda & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L^N,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \\ dW_t^{L^R,Q} \end{pmatrix}, \tag{5}$$

where Σ is the constant covariance (or volatility) matrix.[10] Based on this specification of the Q-dynamics, nominal zero-coupon bond yields preserve the Nelson-Siegel factor loading structure as

$$y_t^N(\tau) = L_t^N + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^N(\tau)}{\tau}, \tag{6}$$

where the nominal yield-adjustment term is given by

$$\begin{aligned} \frac{A^N(\tau)}{\tau} &= \frac{\sigma_{11}^2}{6}\tau^2 + \sigma_{22}^2\Big[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau}\Big] \\ &\quad + \sigma_{33}^2\Big[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda\tau} - \frac{1}{4\lambda}\tau e^{-2\lambda\tau} - \frac{3}{4\lambda^2}e^{-2\lambda\tau} + \frac{5}{8\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau}\Big]. \end{aligned}$$

Similarly, real zero-coupon bond yields have a Nelson-Siegel factor loading structure expressed as

$$y_t^R(\tau) = L_t^R + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \alpha^R\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A^R(\tau)}{\tau}, \tag{7}$$

where the real yield-adjustment term is given by

$$\begin{aligned} \frac{A^R(\tau)}{\tau} &= \frac{\sigma_{44}^2}{6}\tau^2 + \sigma_{22}^2(\alpha_S^R)^2\Big[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau}\Big] \\ &\quad + \sigma_{33}^2(\alpha_S^R)^2\Big[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}e^{-\lambda\tau} - \frac{1}{4\lambda}\tau e^{-2\lambda\tau} - \frac{3}{4\lambda^2}e^{-2\lambda\tau} + \frac{5}{8\lambda^3}\frac{1-e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3}\frac{1-e^{-\lambda\tau}}{\tau}\Big]. \end{aligned}$$

[8] Chernov and Mueller (2012) provide evidence of a hidden factor in the nominal yield curve that is observable from real yields and inflation expectations. The CLR model accommodates this stylized fact via the L_t^R factor.

[9] As discussed in CDR, with unit roots in the two level factors, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

[10] As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

3.3 The CLR-L Model

In this section, we augment the CLR model with a liquidity risk factor to account for the liquidity risk in the pricing of TIPS relative to Treasuries, throughout referred to as the CLR-L model.

To begin, let $X_t = (L_t^N, S_t, C_t, L_t^R, X_t^{liq})$ denote the state vector of this five-factor model. As before, L_t^N and L_t^R denote the level factor unique to the nominal and real yield curve, respectively, while S_t and C_t represent slope and curvature factors common to both yield curves. Finally, X_t^{liq} represents the added liquidity factor.

As in the CLR model, we let the frictionless instantaneous nominal and real risk-free rates be defined by equations (3) and (4), respectively, while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa_{liq}^Q \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \theta_{liq}^Q \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L^N,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \\ dW_t^{L^R,Q} \\ dW_t^{liq,Q} \end{pmatrix},$$

where Σ continues to be a diagonal matrix.

Based on the Q-dynamics above, nominal Treasury zero-coupon bond yields preserve the Nelson-Siegel factor loading structure in equation (6). On the other hand, due to the lower liquidity in the TIPS market relative to the market for nominal Treasuries, TIPS yields are sensitive to liquidity pressures. As detailed in Section 3.1, pricing of TIPS is not performed with the frictionless real discount function, but rather with a discount function that accounts for the liquidity risk:

$$\bar{r}_t^{R,i} = r_t^R + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^{liq} = L_t^R + \alpha^R S_t + \beta^i(1 - e^{-\lambda^{L,i}(t-t_0^i)})X_t^{liq}, \tag{8}$$

where t_0^i denotes the date of issuance of the specific TIPS and β^i is its sensitivity to the variation in the liquidity factor. Furthermore, the decay parameter $\lambda^{L,i}$ is assumed to vary across securities as well.

In Appendix A, we show that the net present value of one unit of the consumption basket paid by TIPS i at time $t + \tau$ has the following exponential-affine form

$$\begin{aligned} P_t(t_0^i, \tau) &= E^Q\left[e^{-\int_t^{t+\tau} \bar{r}^{R,i}(s,t_0^i)ds}\right] \\ &= \exp\Big(B_1(\tau)L_t^N + B_2(\tau)S_t + B_3(\tau)C_t + B_4(\tau)L_t^R + B_5(t, t_0^i, \tau)X_t^{Liq} + A(t, t_0^i, \tau)\Big). \end{aligned}$$

This result implies that the model belongs to the class of Gaussian affine term structure models, but unlike standard Gaussian models, $P_t(t_0^i, \tau^i)$ is time-inhomogeneous. Note also that, by fixing $\beta^i = 0$ for all i, we recover the CLR model.

3.4 Valuing the TIPS Deflation Protection Option

TIPS provide inflation protection since their coupons and principal payments are indexed to the headline Consumer Price Index (CPI) produced by the Bureau of Labor Statistics.[11] Importantly, TIPS also provide some protection against price deflation since their principal payments are not permitted to decrease below their original par value. This implies that there is an inflation floor option for the principal embedded in the TIPS contract. In this section, we describe how to value this deflation protection option assuming no frictions to trading.

To begin, consider a TIPS issued at time t_0 with maturity at time $t + \tau$. By time t its accrued inflation compensation, also known as its index ratio, is given by the change in the price level since issuance, i.e., Π_t/Π_0.

To value the embedded deflation protection option, we need to explicitly control for the accrued inflation compensation; that is, the option will only be in the money at maturity provided the change in the price level between t and $t + \tau$ satisfies the following inequality

$$\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}.$$

Thus, for the option to be in the money, the deflation experienced over the remaining life of the bond, $\Pi_{t+\tau}/\Pi_t$, has to negate the accumulated inflation experienced since the bond's issuance.

Now, the present value of the principal payment of the TIPS is given by

$$E_t^Q\left[\frac{\Pi_{t+\tau}}{\Pi_t} \cdot e^{-\int_t^{t+\tau} r_s^N ds} \mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}\}}\right] + E_t^Q\left[1 \cdot e^{-\int_t^{t+\tau} r_s^N ds} \mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \leq \frac{1}{\Pi_t/\Pi_0}\}}\right].$$

The first term represents the present value of the principal payment conditional on the net change in the price index over the bond's remaining time to maturity is not offset by the accrued inflation compensation as of time t; that is, $\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}$. Under this condition, full inflation indexation applies, and the price change adjustment of the principal $\frac{\Pi_{t+\tau}}{\Pi_t}$ is placed within the expectations operator. The second term represents the present value of the *floored* TIPS principal conditional on the net change in the price level until the bond's maturity eroding the accrued inflation compensation as of time t; that is, the price level change $\frac{\Pi_{t+\tau}}{\Pi_t}$ is replaced by a value of one to provide the promised deflation protection.

Next, we exploit the fact that absence of arbitrage implies that the price level change is

[11] The actual indexation has a lag structure since the Bureau of Labor Statistics publishes price index values with a one-month lag; that is, the index for a given month is released in the middle of the subsequent month. The reference CPI is thus set to be a weighted average of the CPI for the second and third months prior to the month of maturity. See Gürkaynak et al. (2010) for a detailed discussion.

given by

$$\frac{\Pi_{t+\tau}}{\Pi_t} = e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

This allows us to rewrite the present value of the principal payment as

$$\begin{aligned} & E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} > \frac{1}{\Pi_t/\Pi_0}\}}\Big] + E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \le \frac{1}{\Pi_t/\Pi_0}\}}\Big] \\ = \; & E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\Big] + E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \le \frac{1}{\Pi_t/\Pi_0}\}}\Big] - E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \le \frac{1}{\Pi_t/\Pi_0}\}}\Big]. \end{aligned}$$

Here, the first term is the net present value of the TIPS principal payment without any deflation protection, while the two remaining terms equal the net present value of the deflation protection option, which is denoted DOV_t and given by

$$DOV_t\Big(\tau, \frac{\Pi_t}{\Pi_0}\Big) \equiv E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^N ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \le \frac{1}{\Pi_t/\Pi_0}\}}\Big] - E_t^Q\Big[e^{-\int_t^{t+\tau} r_s^R ds}\mathbf{1}_{\{\frac{\Pi_{t+\tau}}{\Pi_t} \le \frac{1}{\Pi_t/\Pi_0}\}}\Big]. \qquad (9)$$

This option value needs to be added to the model-implied TIPS price to match the observed TIPS price.

Now, consider the whole value of TIPS i issued at time t_0^i with maturity at $t+\tau^i$ that pays an annual coupon C semi-annually and has accrued inflation compensation equal to Π_t/Π_0. Its price is given by

$$\begin{aligned} \overline{P}_t(t_0^i, \tau^i, C, \frac{\Pi_t}{\Pi_0}) \;\; = \;\; & \frac{C}{2}\frac{(t_1 - t)}{1/2}E^Q\Big[e^{-\int_t^{t_1}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + \sum_{j=2}^{N}\frac{C}{2}E^Q\Big[e^{-\int_t^{t_j}\overline{r}^{R,i}(s,t_0^i)ds}\Big] \\ & + E^Q\Big[e^{-\int_t^{t+\tau^i}\overline{r}^{R,i}(s,t_0^i)ds}\Big] + DOV_t\Big(\tau^i, \frac{\Pi_t}{\Pi_0}\Big), \end{aligned}$$

where the last term is the value of the deflation protection option provided in equation (9) and calculated using the dynamics for the frictionless nominal and real instantaneous short rates, r_t^N and r_t^R, in combination with formulas provided in Christensen et al. (2012).

The only minor omission in the bond price formula above is that we do not account for the lag in the inflation indexation of the TIPS payoff, but the potential error should be modest in most cases, see Grishchenko and Huang (2013) for evidence.

3.5 Market Prices of Risk

So far, the description of the CLR-L model has relied solely on the dynamics of the state variables under the Q-measure used for pricing. However, to complete the description of the model and to implement it empirically, we will need to specify the risk premiums that connect the factor dynamics under the Q-measure to the dynamics under the real-world (or historical) P-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical P-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premiums Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^5$ and $\gamma^1 \in \mathbf{R}^{5\times 5}$ contain unrestricted parameters. Thus, the resulting unrestricted five-factor CLR-L model has P-dynamics given by

$$\begin{pmatrix} dL_t^N \\ dS_t \\ dC_t \\ dL_t^R \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P & \kappa_{14}^P & \kappa_{15}^P \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P & \kappa_{34}^P & \kappa_{35}^P \\ \kappa_{41}^P & \kappa_{42}^P & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P \\ \kappa_{51}^P & \kappa_{52}^P & \kappa_{53}^P & \kappa_{54}^P & \kappa_{55}^P \end{pmatrix} \left(\begin{pmatrix} \theta_1^P \\ \theta_2^P \\ \theta_3^P \\ \theta_4^P \\ \theta_5^P \end{pmatrix} - \begin{pmatrix} L_t^N \\ S_t \\ C_t \\ L_t^R \\ X_t^{liq} \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L^N,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \\ dW_t^{L^R,P} \\ dW_t^{liq,P} \end{pmatrix}.$$

This is the transition equation in the extended Kalman filter estimation.

3.6 Model Estimation and Econometric Identification

Due to the nonlinearity of the TIPS pricing formula, the model cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012), see Appendix B for details. To make the fitted errors comparable across TIPS of various maturities, we scale each TIPS price by its duration.[12] Thus, the measurement equation for the TIPS prices take the following form:

$$\frac{\overline{P}_t(t_0^i, \tau^i)}{D_t(\tau^i)} = \frac{\widehat{P}_t(t_0^i, \tau^i)}{D_t(\tau^i)} + \varepsilon_t^i,$$

where $\widehat{P}_t(t_0^i, \tau^i)$ is the model-implied price of TIPS i and $D_t(\tau^i)$ is its duration, which is fixed and calculated before estimation. Furthermore, to facilitate model estimation when we adjust for the deflation option values, we first estimate the CLR-L model without the option values. Then we use the estimated parameters and filtered state variables to calculate the time series of option values for each TIPS. These are used as a fixed input into a new model estimation at the end of which a new set of option values are calculated, and the process is repeated. This algorithm is continued until convergence is achieved.

From the five-factor model structure above it follows that we will be fitting TIPS yields with two separate factors, the real level factor, L_t^R, and the TIPS liquidity factor, X_t^{liq}, in addition to the common slope and curvature factor that can be identified from the nominal yields. Thus, for reasons of identification, we need to have at least two TIPS securities trading

[12] For robustness, we repeated the estimations using the mid-market yield-to-maturities for each TIPS downloaded from Bloomberg instead and got very similar results. However, we note that those estimations are extremely time consuming since yield-to-maturity is only defined implicitly as a fix point that needs to be calculated for each observation. Hence, we advise against that approach.

at each observation date. This requirement implies that the earliest starting point for the model estimation coincides with the issuance date of the second TIPS in mid-July 1997.

Since the liquidity factor is a latent factor that we do not observe, its level is not identified without additional restrictions. As a consequence, we let the first TIPS issued, that is, the ten-year TIPS with 3.375% coupon issued in January 1997 with maturity on January 15, 2007, have a unit loading on the liquidity factor, that is, $\beta^i = 1$ for this security. This choice implies that the β^i sensitivity parameters measure liquidity sensitivity relative to that of the ten-year 2007 TIPS.

Furthermore, we note that the $\lambda^{L,i}$ parameters can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.01 to 10, which is without practical consequences. Also, for numerical stability during model optimization, we impose the restriction that the β^i parameters fall within the range from 0 to 80, which turns out not to be a binding constraint at the optimum.

Finally, we assume that all fitted nominal yields in equation (6) have *i.i.d.* measurement errors with standard deviation σ_ε^N. Similarly, all TIPS measurement errors are assumed to be *i.i.d.* with standard deviation σ_ε^R.

4 Data

This section briefly describes the data we use in the model estimation.

4.1 Nominal Treasury Yields

The specific nominal Treasury yields we use are zero-coupon yields taken from the Gürkaynak et al. (2007) database with the following maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 4-year, 5-year, 6-year, 7-year, 8-year, 9-year, and 10-year. We use weekly data and limit our sample to the period from July 11, 1997, to December 27, 2013. The summary statistics are provided in Table 1.

Researchers have typically found that three factors are sufficient to model the time-variation in the cross section of nominal Treasury bond yields (e.g., Litterman and Scheinkman, 1991). Indeed, for our weekly nominal Treasury bond yield data, 99.98% of the total variation is accounted for by three factors. Table 2 reports the eigenvectors that correspond to the first three principal components of our data. The first principal component accounts for 95.3% of the variation in the nominal Treasury bond yields, and its loading across maturities is uniformly negative. Thus, like a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 4.5% of the variation in these data and has sizable negative loadings for the shorter maturities and sizable positive loadings for the long maturities. Thus, like a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts

Maturity in months	Mean in %	St. dev. in %	Skewness	Kurtosis
3	2.59	2.14	0.27	1.48
6	2.61	2.15	0.27	1.49
12	2.68	2.13	0.25	1.53
24	2.89	2.02	0.15	1.60
36	3.10	1.89	0.06	1.70
48	3.33	1.76	-0.03	1.82
60	3.54	1.64	-0.11	1.96
72	3.74	1.54	-0.19	2.11
84	3.93	1.45	-0.27	2.27
96	4.10	1.37	-0.34	2.41
108	4.25	1.30	-0.40	2.53
120	4.38	1.25	-0.45	2.63

Table 1: **Summary Statistics for the Nominal Treasury Yields.**
Summary statistics for the sample of weekly nominal Treasury zero-coupon bond yields covering the period from July 11, 1997, to December 27, 2013, a total of 860 observations.

Maturity	Loading on		
in months	First P.C.	Second P.C.	Third P.C.
3	0.35	0.42	-0.50
6	0.35	0.40	-0.25
12	0.35	0.31	0.12
24	0.34	0.12	0.42
36	0.32	-0.01	0.41
48	0.30	-0.12	0.30
60	0.27	-0.19	0.15
72	0.25	-0.25	0.02
84	0.24	-0.29	-0.10
96	0.22	-0.32	-0.19
108	0.21	-0.35	-0.26
120	0.19	-0.36	-0.32
% explained	95.26	4.51	0.21

Table 2: **Eigenvectors of the First Three Principal Components in Nominal Treasury Yields.**
The loadings of yields of various maturities on the first three principal components are shown. The final row shows the proportion of all bond yield variability accounted for by each principal component. The data consist of weekly nominal zero-coupon U.S. Treasury bond yields from July 11, 1997, to December 27, 2013.

for only 0.2% of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. This motivates our use of the AFNS model with its level, slope, and curvature structure for the nominal yields even though we emphasize that the estimated state variables are *not* identical to the principal component factors discussed here.[13]

[13] A number of recent papers use principal components as state variables. Joslin et al. (2011) is an example.

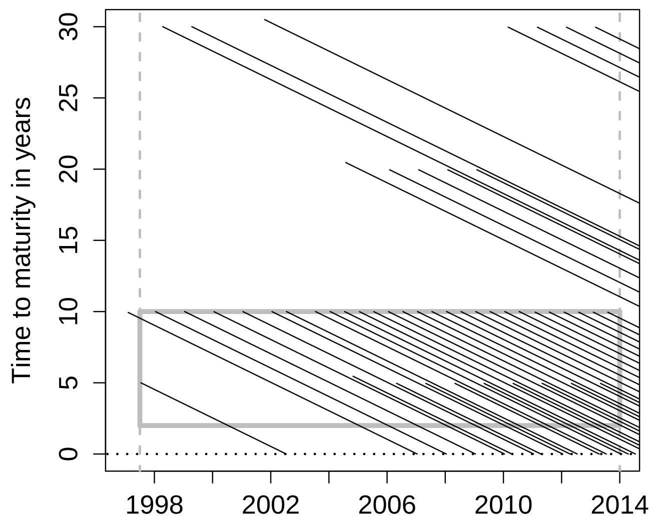


Figure 1: **Maturity Distribution of TIPS.**
Illustration of the maturity distribution of all TIPS issued since the inception of the TIPS program. The solid grey rectangle indicates the subsample used in the main analysis in the paper and characterized by three sample choices: (1) for reasons of identification the sample starts on July 11, 1997; (2) the sample is limited to bonds with less than 10 years to maturity at issuance; (3) the price of each TIPS is censored when it has less than two years to maturity to avoid erratic prices close to expiry.

4.2 TIPS Data

The U.S. Treasury started issuing TIPS in 1997. The first TIPS was issued on February 6, 1997, with maturity on January 15, 2007, and a coupon rate of 3.375%.[14] Since then the U.S. Treasury has issued five-, ten-, twenty-, and thirty-year TIPS. However, only ten-year TIPS have been regularly issued since the inception of the TIPS program. As of the end of 2013, a total of 50 TIPS had been issued and the distribution of their remaining time to maturity across time is shown in Figure 1. The total number of TIPS outstanding at any point in time since the start of the TIPS program is shown with a solid red line in Figure 2. At the end of our sample period there was a total of 37 TIPS outstanding.

To facilitate the empirical implementation and improve model fit, we limit our focus to five- and ten-year TIPS.[15] This reduces the total number of TIPS to 38, while the number of those TIPS outstanding at any point in time is shown with a solid grey line in Figure 2. At the end of 2013, this subset included 25 TIPS. Furthermore, as TIPS prices near maturity

[14]TIPS are issued with a minimum coupon of 0.125%. Since April 2011 this has been a binding constraint for five-year TIPS and occasionally for ten-year TIPS.

[15]As a robustness check, we estimated the model with all available TIPS in combination with nominal yields with maturities up to thirty years. This produced qualitatively similar, but less accurate results. See Appendix F for details.

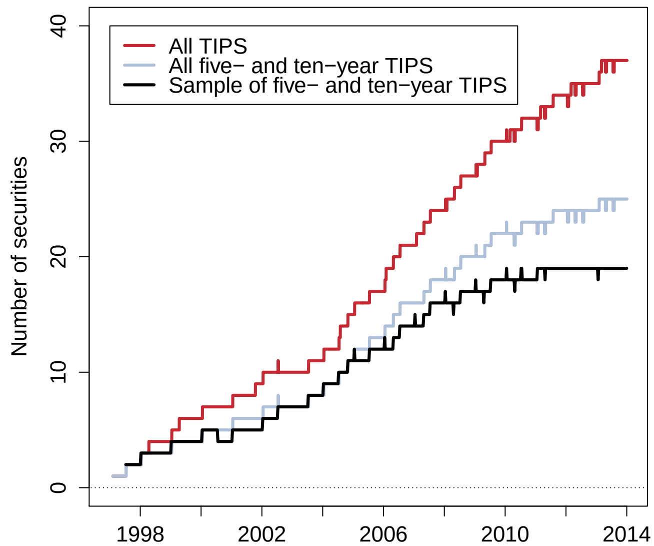


Figure 2: **Number of TIPS Outstanding.**
Illustration of the number of TIPS outstanding. The sample covers the period from February 6, 1997, to December 31, 2013.

tend to exhibit erratic behavior due to seasonal variation in CPI, we drop TIPS from our sample when they have less than two years to maturity, see Gürkaynak et al. (2010). Thus, our analysis is centered around the two- to ten-year maturity range that is the most widely used for both bond risk management and monetary policy analysis. Using this cutoff, the number of TIPS in the sample is further reduced and shown with a solid black line in Figure 2. As of the end of 2013, it included 19 securities. Our sample of TIPS is also indicated with a solid grey rectangle in Figure 1, while summary statistics for our sample of 38 TIPS are reported in Table 3.

To estimate the CLR-L model, we use mid-market clean TIPS prices downloaded from Bloomberg. Since the model has two TIPS specific factors, we start the model estimation on Friday July 11, 1997, when prices become available for the second ever TIPS, the five-year TIPS with maturity on July 15, 2002. We end the sample on December 27, 2013, with 19 TIPS trading. The number of weekly observations for each of our 38 TIPS is also reported in Table 3.

5 Estimation Results

In this section, we first describe the results from the CLR-L model estimated with and without adjustment for the the value of the deflation protection option embedded in the TIPS

TIPS security	No. obs.	Issuance		First reopen		Second reopen	
		Date	Amount	Date	Amount	Date	Amount
(1) 3.375% 1/15/2007 TIPS	393	2/6/97	7,353	4/15/97	8,403	n.a.	n.a.
(2) 3.625% 7/15/2002 TIPS*	158	7/15/97	8,401	10/15/97	8,412	n.a.	n.a.
(3) 3.625% 1/15/2008 TIPS	419	1/15/98	8,409	10/15/98	8,401	n.a.	n.a.
(4) 3.875% 1/15/2009 TIPS	419	1/15/99	8,531	7/15/99	7,368	n.a.	n.a.
(5) 4.25% 1/15/2010 TIPS	418	1/18/00	6,317	7/17/00	5,002	n.a.	n.a.
(6) 3.5% 1/15/2011 TIPS	418	1/16/01	6,000	7/16/01	5,000	n.a.	n.a.
(7) 3.375% 1/15/2012 TIPS	419	1/15/02	6,000	n.a.	n.a.	n.a.	n.a.
(8) 3% 7/15/2012 TIPS	419	7/15/02	10,010	10/15/02	7,000	1/15/03	6,000
(9) 1.875% 7/15/2013 TIPS	419	7/15/03	11,000	10/15/03	9,000	n.a.	n.a.
(10) 2% 1/15/2014 TIPS	419	1/15/04	12,000	4/15/04	9,000	n.a.	n.a.
(11) 2% 7/15/2014 TIPS	419	7/15/04	10,000	10/15/04	9,000	n.a.	n.a.
(12) 0.875% 4/15/2010 TIPS*	181	10/29/04	12,000	4/29/05	9,000	10/28/05	7,000
(13) 1.625% 1/15/2015 TIPS	418	1/18/05	10,000	4/15/05	9,000	n.a.	n.a.
(14) 1.875% 7/15/2015 TIPS	418	7/15/05	9,000	10/17/05	8,000	n.a.	n.a.
(15) 2% 1/15/2016 TIPS	416	1/17/06	9,000	4/17/06	8,000	n.a.	n.a.
(16) 2.375% 4/15/2011 TIPS*	155	4/28/06	11,000	10/31/06	9,181	n.a.	n.a.
(17) 2.5% 7/15/2016 TIPS	390	7/17/06	10,588	10/16/06	9,412	n.a.	n.a.
(18) 2.375% 1/15/2017 TIPS	364	1/16/07	11,250	4/16/07	6,000	n.a.	n.a.
(19) 2% 4/15/2012 TIPS*	156	4/30/07	10,123	10/31/07	7,158	n.a.	n.a.
(20) 2.625% 7/15/2017 TIPS	338	7/16/07	8,000	10/15/07	6,000	n.a.	n.a.
(21) 1.625% 1/15/2018 TIPS	312	1/15/08	10,412	4/15/08	6,000	n.a.	n.a.
(22) 0.625% 4/15/2013 TIPS*	156	4/30/08	8,734	10/31/08	6,266	n.a.	n.a.
(23) 1.375% 7/15/2018 TIPS	286	7/15/08	8,000	10/15/08	6,974	n.a.	n.a.
(24) 2.125% 1/15/2019 TIPS	260	1/15/09	8,662	4/15/09	6,096	n.a.	n.a.
(25) 1.25% 4/15/2014 TIPS*	156	4/30/09	8,277	10/30/09	7,000	n.a.	n.a.
(26) 1.875% 7/15/2019 TIPS	234	7/15/09	8,135	10/15/09	7,055	n.a.	n.a.
(27) 1.375% 1/15/2020 TIPS	207	1/15/10	10,388	4/15/10	8,586	n.a.	n.a.
(28) 0.5% 4/15/2015 TIPS*	155	4/30/10	11,235	10/29/10	10,000	n.a.	n.a.
(29) 1.25% 7/15/2020 TIPS	182	7/15/10	12,003	9/15/10	10,108	11/15/10	10,268
(30) 1.125% 1/15/2021 TIPS	154	1/31/11	13,259	3/31/11	11,493	5/31/11	11,926
(31) 0.125% 4/15/2016 TIPS*	140	4/29/11	14,000	8/31/11	12,367	12/30/11	12,000
(32) 0.625% 7/15/2021 TIPS	128	7/29/11	13,000	9/30/11	11,342	11/30/11	11,498
(33) 0.125% 1/15/2022 TIPS	102	1/31/12	15,282	3/30/12	13,000	5/31/12	13,000
(34) 0.125% 4/15/2017 TIPS*	89	4/30/12	16,430	8/31/12	14,000	12/31/12	14,000
(35) 0.125% 7/15/2022 TIPS	76	7/31/12	15,000	9/28/12	13,000	11/30/12	13,000
(36) 0.125% 1/15/2023 TIPS	49	1/31/13	15,000	3/28/13	13,000	5/31/13	13,000
(37) 0.125% 4/15/2018 TIPS*	37	4/30/13	18,000	8/30/13	16,000	12/31/13	16,000
(38) 0.375% 7/15/2023 TIPS	24	7/31/13	15,000	9/30/13	13,000	11/29/13	13,000

Table 3: **Sample of TIPS.**
The table reports the characteristics, issuance dates, and issuance amounts in millions of dollars for the 38 TIPS used in the analysis. Also reported are the number of weekly observation dates for each TIPS during the sample period from July 11, 1997, to December 27, 2013. Asterisk * indicates five-year TIPS.

contract. Second, we compare the estimated state variables and model fit to those obtained from standard AFNS and CLR models. Upfront we note that, for each model class, we limit the focus to the most parsimonious independent-factor specification to make the results as comparable as possible.[16]

Table 4 contains the estimated dynamic parameters for the CLR-L model estimated with and without accounting for the deflation option values, while Table 5 reports their respective

[16]Since the model fit and the estimated factors are insensitive to the specification of the mean-reversion matrix K^P, this limitation comes at practically no loss of generality for the results presented in this section.

CLR-L model, no adjustment								
K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2408	0	0	0	0	0.0610	σ_{11}	0.0059
	(0.1904)					(0.0082)		(0.0001)
$K^P_{2,\cdot}$	0	0.0939	0	0	0	-0.0301	σ_{22}	0.0099
		(0.1458)				(0.0276)		(0.0002)
$K^P_{3,\cdot}$	0	0	0.4602	0	0	-0.0316	σ_{33}	0.0250
			(0.2934)			(0.0145)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.3139	0	0.0334	σ_{44}	0.0075
				(0.3169)		(0.0099)		(0.0002)
$K^P_{5,\cdot}$	0	0	0	0	0.6987	0.0087	σ_{55}	0.0124
					(0.3953)	(0.0076)		(0.0007)

CLR-L model, option adjusted								
K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2348	0	0	0	0	0.0612	σ_{11}	0.0060
	(0.1907)					(0.0083)		(0.0001)
$K^P_{2,\cdot}$	0	0.0873	0	0	0	-0.0294	σ_{22}	0.0099
		(0.1461)				(0.0297)		(0.0002)
$K^P_{3,\cdot}$	0	0	0.4069	0	0	-0.0323	σ_{33}	0.0249
			(0.2941)			(0.0158)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.2585	0	0.0342	σ_{44}	0.0072
				(0.2782)		(0.0102)		(0.0002)
$K^P_{5,\cdot}$	0	0	0	0	0.7244	0.0074	σ_{55}	0.0124
					(0.3936)	(0.0075)		(0.0007)

Table 4: **Estimated Dynamic Parameters.**
The top panel shows the estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix for the CLR-L model. The estimated value of λ is 0.4473 (0.0019), while $\alpha^R = 0.7620$ (0.0077), $\kappa^Q_{liq} = 0.8257$ (0.0436), and $\theta^Q_{liq} = 0.0016$ (0.0001). The bottom panel shows the corresponding estimates for the CLR-L model with deflation option adjustment. In this case, the estimated value of λ is 0.4442 (0.0019), while $\alpha^R = 0.7584$ (0.0072), $\kappa^Q_{liq} = 0.9004$ (0.0598), and $\theta^Q_{liq} = 0.0014$ (0.0001). The numbers in parentheses are the estimated parameter standard deviations.

estimated β^i and $\lambda^{L,i}$ parameters. Overall, the model parameters, the dynamic parameters in particular, are relatively insensitive to adjusting for the deflation option values.

As for the state variables, Figure 3 shows the estimated paths for (L^N_t, S_t, C_t), which are primarily determined from nominal yields. We note that the estimated paths of these three factors are practically indistinguishable from the estimated paths obtained with a stand-alone AFNS model of nominal yields only. This is also reflected in the summary statistics for the fitted errors of nominal yields reported in Table 6. The results show that the CLR and CLR-L models, with and without deflation option adjustment, provide a very close fit to the entire cross section of nominal yields. Importantly, the fit is as good as that obtained with a stand-alone AFNS model for the nominal yields. Thus, allowing for a joint modeling of nominal and real yields based on the CLR model framework comes at effectively no cost in terms of fit to the nominal yields as also emphasized by CLR.

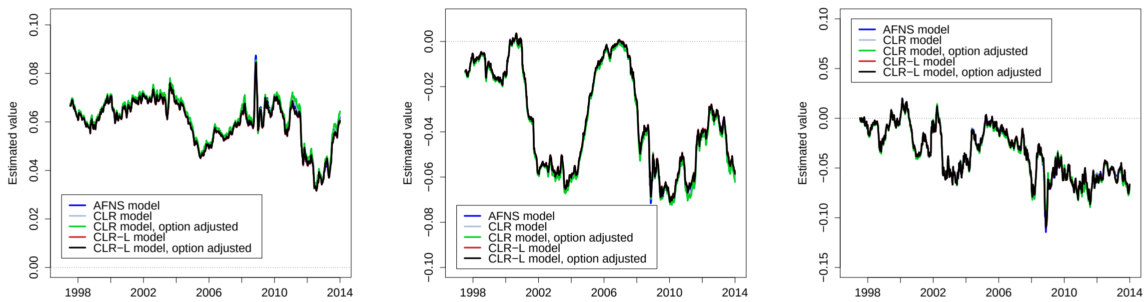
As for the estimated level factor for real yields, Figure 4(a) shows that it is sensitive to

TIPS security	CLR-L model, no adjustment				CLR-L model, option adjusted			
	β^i	Std	$\lambda^{L,i}$	Std	β^i	Std	$\lambda^{L,i}$	Std
(1) 3.375% 1/15/2007 TIPS	1	n.a.	0.7047	0.3563	1	n.a.	0.7892	0.5808
(2) 3.625% 7/15/2002 TIPS*	0.8260	0.1333	8.9414	2.1275	0.8397	0.1402	7.9001	1.9886
(3) 3.625% 1/15/2008 TIPS	2.1317	0.4677	0.1320	0.0493	2.7179	0.5367	0.0954	0.0481
(4) 3.875% 1/15/2009 TIPS	3.0805	0.8843	0.0988	0.0402	3.9884	1.0288	0.0735	0.0404
(5) 4.25% 1/15/2010 TIPS	2.0739	0.1818	0.2360	0.0415	2.2469	0.1968	0.2206	0.0399
(6) 3.5% 1/15/2011 TIPS	2.3928	0.1943	0.2143	0.0320	2.5637	0.2187	0.2098	0.0313
(7) 3.375% 1/15/2012 TIPS	2.4185	0.1833	0.2384	0.0364	2.5856	0.2126	0.2395	0.0341
(8) 3% 7/15/2012 TIPS	2.3956	0.1686	0.2604	0.0412	2.5697	0.1895	0.2597	0.0403
(9) 1.875% 7/15/2013 TIPS	3.0073	0.3761	0.1781	0.0434	3.6307	0.5301	0.1439	0.0397
(10) 2% 1/15/2014 TIPS	5.3622	1.2267	0.0838	0.0264	7.3189	1.6844	0.0620	0.0187
(11) 2% 7/15/2014 TIPS	2.5352	0.1962	0.3410	0.0657	2.8097	0.2341	0.3095	0.0561
(12) 0.875% 4/15/2010 TIPS*	1.9994	0.0802	9.9988	2.2979	2.1262	0.0875	9.9994	2.1352
(13) 1.625% 1/15/2015 TIPS	3.4597	0.3973	0.1871	0.0371	3.8698	0.5574	0.1763	0.0369
(14) 1.875% 7/15/2015 TIPS	2.1376	0.1231	0.9495	0.3939	2.3197	0.1356	0.8955	0.2641
(15) 2% 1/15/2016 TIPS	2.5252	0.1953	0.3848	0.0677	2.7908	0.2285	0.3771	0.0687
(16) 2.375% 4/15/2011 TIPS*	1.9328	0.0798	5.2441	2.0697	2.0565	0.0867	4.6394	2.1190
(17) 2.5% 7/15/2016 TIPS	1.8922	0.1077	6.2509	2.1911	2.0867	0.1202	9.8130	2.1433
(18) 2.375% 1/15/2017 TIPS	1.9108	0.1052	9.9794	2.0559	2.1207	0.1193	9.9189	2.0233
(19) 2% 4/15/2012 TIPS*	1.8565	0.0826	9.9975	1.9748	1.9795	0.0892	10.0000	2.1203
(20) 2.625% 7/15/2017 TIPS	1.5773	0.0929	9.9943	2.3952	1.7740	0.1074	9.9991	2.2029
(21) 1.625% 1/15/2018 TIPS	1.9053	0.1815	0.4481	0.1237	2.1567	0.1953	0.4756	0.1342
(22) 0.625% 4/15/2013 TIPS*	5.3129	1.1659	0.1526	0.0450	4.3600	0.5288	0.2246	0.0536
(23) 1.375% 7/15/2018 TIPS	1.2974	0.1566	0.8968	0.3108	1.5303	0.1734	0.8955	0.3362
(24) 2.125% 1/15/2019 TIPS	28.0660	4.1001	0.0100	0.0018	31.9838	4.1442	0.0105	0.0018
(25) 1.25% 4/15/2014 TIPS*	38.6112	4.1291	0.0230	0.0030	15.1962	3.7701	0.0701	0.0449
(26) 1.875% 7/15/2019 TIPS	1.4761	0.3116	0.4666	0.3420	1.7695	0.3098	0.4706	0.3960
(27) 1.375% 1/15/2020 TIPS	29.9390	4.7859	0.0100	0.0018	35.6542	4.6957	0.0104	0.0017
(28) 0.5% 4/15/2015 TIPS*	23.8180	4.8797	0.0372	0.0084	9.5635	3.6681	0.1165	0.0765
(29) 1.25% 7/15/2020 TIPS	1.8997	1.0819	0.3470	0.5347	2.2639	0.7916	0.4111	0.6262
(30) 1.125% 1/15/2021 TIPS	3.0499	1.6842	0.2461	0.3036	3.6223	1.5942	0.2821	0.3255
(31) 0.125% 4/15/2016 TIPS*	9.8655	6.2960	0.0990	0.0757	6.8538	4.0340	0.1820	0.1326
(32) 0.625% 7/15/2021 TIPS	2.1138	0.7052	0.6286	0.9480	2.8131	0.6675	0.5685	1.1072
(33) 0.125% 1/15/2022 TIPS	3.1227	1.6588	0.4219	0.6766	4.3189	1.8193	0.3685	0.6817
(34) 0.125% 4/15/2017 TIPS*	17.4890	8.2002	0.0530	0.0272	15.7646	8.2915	0.0712	0.0445
(35) 0.125% 7/15/2022 TIPS	2.2077	0.2444	5.4838	8.7416	2.8941	0.3308	4.7134	7.8943
(36) 0.125% 1/15/2023 TIPS	2.7209	0.2336	9.9980	9.7693	3.6017	0.3226	9.9793	8.9224
(37) 0.125% 4/15/2018 TIPS*	24.6981	10.9722	0.0416	0.0199	3.4604	0.9269	0.9891	1.2716
(38) 0.375% 7/15/2023 TIPS	1.8409	0.3227	9.9575	16.7198	2.5613	0.4752	9.9949	13.9168

Table 5: **Estimated Liquidity Sensitivity Parameters.**
The estimated β^i sensitivity and $\lambda^{L,i}$ decay parameters for each TIPS from the CLR-L model with and without deflation option adjustment are shown. Also reported are the estimated parameter standard deviations. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013.

whether a liquidity risk factor is included or not. However, it is only periodically that the differences are sizable. Figure 4(b) shows the estimated liquidity risk factor that is unique to the CLR-L model. We will study this factor and its implications for the TIPS liquidity premiums further below, but for now we note that it has little sensitivity to adjustment for the deflation option values.

To evaluate the fit of the models to the TIPS price data, we calculate the model-implied time series of the yield-to-maturity for each TIPS and compare that to the mid-market TIPS



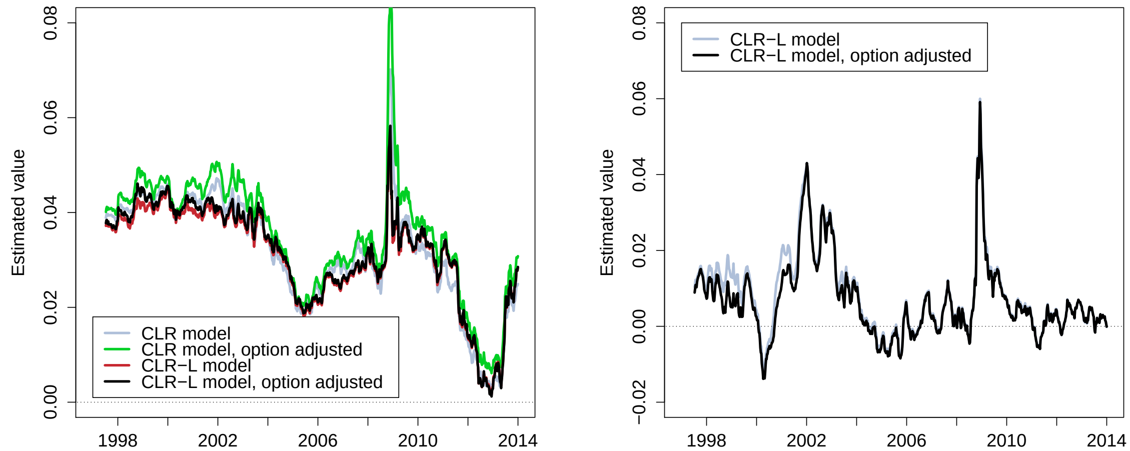
(a) L_t^N. (b) S_t. (c) C_t.

Figure 3: **Estimated State Variables.**
Illustration of the estimated state variables that affect nominal yields from the AFNS model, the CLR model, the CLR model with deflation option adjustment, the CLR-L model, and the CLR-L model with deflation option adjustment. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

Maturity in months	AFNS model		CLR model				CLR-L model			
			No adjustment		Option adjusted		No adjustment		Option adjusted	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
3	-0.99	7.27	-0.99	7.36	-0.97	7.52	-1.15	7.40	-1.14	7.44
6	-0.88	2.64	-0.90	2.66	-0.94	2.67	-0.85	2.70	-0.87	2.70
12	0.95	6.98	0.92	7.02	0.82	7.13	1.19	7.03	1.14	7.04
24	2.54	5.74	2.54	5.98	2.48	6.30	2.78	6.16	2.75	6.17
36	1.32	3.04	1.35	3.43	1.36	3.73	1.35	3.62	1.38	3.63
48	-0.48	2.78	-0.42	2.95	-0.37	2.98	-0.62	2.98	-0.56	2.96
60	-1.72	3.71	-1.65	3.69	-1.61	3.64	-1.95	3.68	-1.88	3.65
72	-2.09	3.83	-2.03	3.80	-2.02	3.83	-2.32	3.82	-2.25	3.80
84	-1.64	3.04	-1.61	3.12	-1.63	3.28	-1.79	3.17	-1.76	3.16
96	-0.56	1.97	-0.58	2.27	-0.61	2.56	-0.59	2.37	-0.60	2.36
108	0.92	2.70	0.85	2.92	0.82	3.10	1.03	3.07	0.98	3.03
120	2.60	5.15	2.47	5.13	2.50	5.16	2.86	5.31	2.76	5.26
All yields	0.00	4.41	0.00	4.52	-0.01	4.64	0.00	4.59	0.00	4.59

Table 6: **Summary Statistics of Fitted Errors of Nominal Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of nominal U.S. Treasury yields from five model estimations are shown. The full sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

yield-to-maturity available from Bloomberg. The summary statistics of the fitted yield errors calculated this way are reported in Table 7. First, we note that the CLR model tends to provide less than ideal fit as measured by RMSEs and with notable bias for some TIPS. Second, accounting for the value of the deflation protection option improves the fit of the model, but this modification is not sufficient for the model to deliver satisfactory fit (as measured by RMSEs) or to eliminate the bias for select TIPS. For all TIPS yields combined, the RMSEs remain close to 15 basis points. Third, incorporating the liquidity factor into the CLR model leads to a significant improvement in model fit for practically all TIPS in



(a) L_t^R. (b) X_t^{liq}.

Figure 4: **Estimated State Variables.**
Panel (a) shows the estimated real yield level factor from the CLR model and the CLR-L model with and without deflation option adjustment. Panel (b) shows the estimated liquidity factor from the CLR-L model with and without deflation option adjustment. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

the sample. Also, and equally important, there is no material bias for any of the TIPS. With the liquidity extension, the fit to the TIPS data is about as good as the fit to the nominal Treasury yields. Finally, accounting for the deflation option values in addition to incorporating the liquidity factor provides a further modest improvement in model fit.

6 The TIPS Liquidity Premium

In this section, we first analyze the estimated TIPS liquidity premiums in detail and compare them to alternative estimates from the existing literature. We then follow Christensen and Gillan (2015, henceforth CG) and study the effects on TIPS liquidity premiums from the Fed's TIPS purchases during its second large-scale asset purchases program—frequently referred to as QE2—that operated from November 2010 through June 2011.

Figure 5 shows the time series of the average yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. This represents the average TIPS liquidity premium for each observation date in our sample. According to the CLR-L model, the sample average of the weekly average liquidity premium is 42.3 basis points. Once we account for the value of the deflation option in the model estimation, the sample average drops to 37.8 basis points. Thus, the assessment of the average TIPS liquidity premium is sensitive to the inclusion of the deflation protection option. Furthermore, we note that over the main sample period analyzed

TIPS security	CLR model				CLR-L model			
	No adjustment		Option adjusted		No adjustment		Option adjusted	
	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
(1) 3.375% 1/15/2007 TIPS	-4.98	13.81	-3.33	10.17	2.53	4.93	2.43	4.80
(2) 3.625% 7/15/2002 TIPS*	5.92	12.56	1.04	10.58	3.25	4.01	3.26	4.04
(3) 3.625% 1/15/2008 TIPS	-2.92	12.10	-1.33	10.44	2.14	4.48	2.15	4.38
(4) 3.875% 1/15/2009 TIPS	0.38	9.77	1.12	9.40	1.29	2.67	1.36	2.69
(5) 4.25% 1/15/2010 TIPS	1.85	8.87	2.41	9.38	0.77	2.94	0.86	3.04
(6) 3.5% 1/15/2011 TIPS	5.52	36.37	3.50	21.00	-0.09	4.33	-0.22	4.27
(7) 3.375% 1/15/2012 TIPS	7.33	23.93	3.79	11.33	0.12	5.16	-0.10	5.19
(8) 3% 7/15/2012 TIPS	5.00	18.30	1.47	10.12	-0.18	4.98	-0.36	4.99
(9) 1.875% 7/15/2013 TIPS	1.54	17.14	-0.48	14.66	-0.76	6.63	-0.98	6.52
(10) 2% 1/15/2014 TIPS	6.61	14.03	5.54	12.25	0.53	3.79	0.39	3.72
(11) 2% 7/15/2014 TIPS	2.65	12.73	3.41	13.80	-0.14	4.43	-0.09	4.49
(12) 0.875% 4/15/2010 TIPS*	1.39	11.43	1.46	9.79	1.86	4.33	2.36	4.46
(13) 1.625% 1/15/2015 TIPS	6.52	12.97	8.33	14.99	0.77	4.36	0.90	4.34
(14) 1.875% 7/15/2015 TIPS	-1.11	10.84	1.66	11.20	0.07	4.38	0.22	4.50
(15) 2% 1/15/2016 TIPS	0.75	11.54	4.27	9.40	1.06	4.67	1.11	4.77
(16) 2.375% 4/15/2011 TIPS*	20.44	48.66	15.05	33.31	4.67	12.20	4.76	12.08
(17) 2.5% 7/15/2016 TIPS	-6.95	15.64	-3.51	10.13	-0.50	5.44	-0.47	5.55
(18) 2.375% 1/15/2017 TIPS	-3.08	17.17	-0.72	8.21	1.92	4.39	1.95	4.46
(19) 2% 4/15/2012 TIPS*	8.94	18.46	19.46	37.86	5.62	11.25	5.59	11.19
(20) 2.625% 7/15/2017 TIPS	-10.77	24.31	-8.43	14.60	0.54	3.70	0.55	3.76
(21) 1.625% 1/15/2018 TIPS	-9.45	29.99	-7.33	18.05	0.45	3.73	0.48	3.73
(22) 0.625% 4/15/2013 TIPS*	-19.22	34.49	-0.14	16.03	0.32	11.55	0.32	11.35
(23) 1.375% 7/15/2018 TIPS	-18.34	38.75	-15.52	26.57	0.27	4.49	0.29	4.58
(24) 2.125% 1/15/2019 TIPS	-6.89	23.27	-7.80	20.35	-0.08	3.14	-0.09	3.23
(25) 1.25% 4/15/2014 TIPS*	-5.91	14.09	0.73	10.98	0.10	4.19	0.25	4.27
(26) 1.875% 7/15/2019 TIPS	-5.35	13.90	-7.96	14.01	0.00	2.32	0.00	2.29
(27) 1.375% 1/15/2020 TIPS	3.68	10.21	0.93	8.14	-0.62	3.71	-0.65	3.62
(28) 0.5% 4/15/2015 TIPS*	3.73	14.41	8.73	15.06	0.38	3.31	0.54	3.23
(29) 1.25% 7/15/2020 TIPS	2.72	10.71	0.13	8.27	-0.32	2.67	-0.28	2.67
(30) 1.125% 1/15/2021 TIPS	12.20	15.49	9.23	11.61	-0.56	3.85	-0.50	3.79
(31) 0.125% 4/15/2016 TIPS*	1.31	7.86	5.53	8.70	-0.22	3.67	-0.14	3.53
(32) 0.625% 7/15/2021 TIPS	6.87	10.12	5.77	8.33	-0.17	2.72	0.11	2.58
(33) 0.125% 1/15/2022 TIPS	15.79	17.85	14.26	15.58	0.11	2.47	0.12	2.32
(34) 0.125% 4/15/2017 TIPS*	-2.66	5.39	1.51	5.20	-0.01	2.58	-0.01	2.53
(35) 0.125% 7/15/2022 TIPS	12.94	14.41	10.88	11.87	0.15	3.84	0.34	3.40
(36) 0.125% 1/15/2023 TIPS	23.53	24.30	19.59	20.46	-0.03	5.93	0.12	5.30
(37) 0.125% 4/15/2018 TIPS*	0.82	3.00	5.47	6.28	-0.68	3.47	-0.21	3.08
(38) 0.375% 7/15/2023 TIPS	15.23	15.60	9.80	10.19	0.44	2.77	0.54	2.62
All TIPS yields	0.42	19.57	1.20	14.58	0.65	4.89	0.66	4.87
Max log L	107,249.3		109,593.5		118,915.3		118,945.5	

Table 7: **Summary Statistics of Fitted Errors of TIPS Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of the yield-to-maturity for individual TIPS according to the CLR-L model with and without deflation option adjustment. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

in CLR (January 3, 2003 to March 28, 2008) and again from early 2010 through the end of 2013, this measure of the TIPS liquidity premium has averaged lower, 27.2 basis points and 34.8 basis points, respectively. Thus, omitting to account for the TIPS liquidity premium as in CLR, may provide a reasonable approximation during normal times. However, from Figure 5 it is also clear that, in parts of the early years of TIPS trading as well as during the

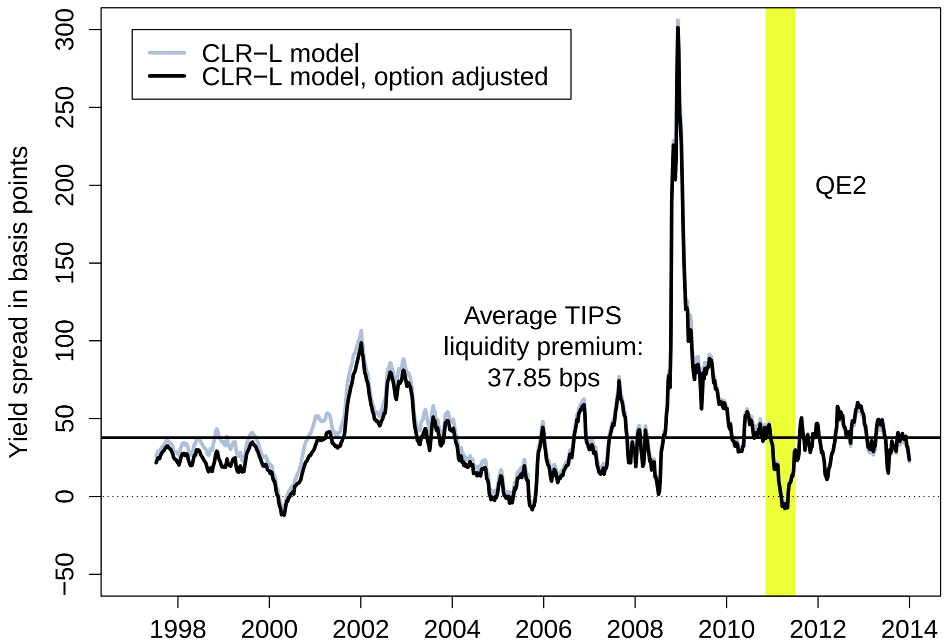


Figure 5: **Average Estimated TIPS Liquidity Premium.**
Illustration of the average estimated TIPS liquidity premium for each observation date implied by the CLR-L model with and without deflation option adjustment. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average TIPS liquidity premium according to the option-adjusted CLR-L model is shown with a solid black horizontal line. The data cover the period from July 11, 1997, to December 27, 2013.

financial crisis, TIPS liquidity premiums were of nontrivial magnitudes with significant time variation.

As an alternative, we analyze the estimated liquidity premiums for the most recently issued (on-the-run) five- and ten-year TIPS, which are shown in Figures 6 and 7. According to the option-adjusted CLR-L model, the five- and ten-year on-the-run TIPS liquidity premium has averaged 33.2 basis points and 29.8 basis points, respectively. For the CLR-L model without option adjustment, the corresponding averages are slightly higher at 37.5 basis points and 31.8 basis points, respectively. Thus, ten-year on-the-run TIPS are more liquid and have lower liquidity premiums than five-year on-the-run TIPS, and this difference is most notable during the financial crisis. For comparison, we also plot the corresponding five- and ten-year off-the-run Treasury yield spreads. First, we note a mild positive correlation with the TIPS liquidity premium measures. Second, we note the level difference, which is evidence that liquidity premiums in the regular Treasury market are an order of magnitude smaller than

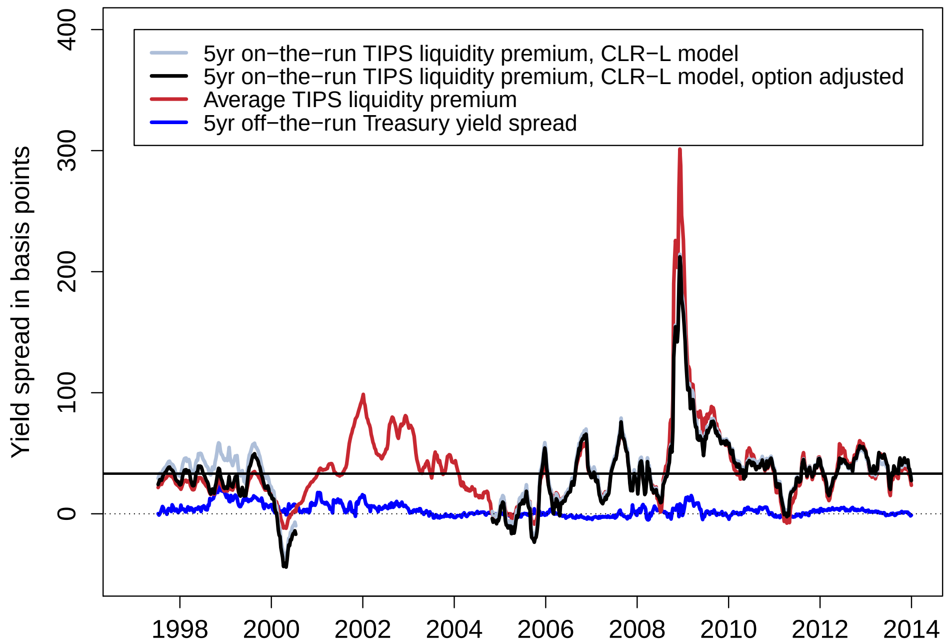


Figure 6: **Estimated Five-Year On-The-Run TIPS Liquidity Premiums.** Illustration of the estimated liquidity premiums for the most recently issued (on-the-run) five-year TIPS on each observation date implied by the CLR-L model with and without deflation option adjustment. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of the individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. Note that between July 14, 2000, and October 29, 2004, there are no five-year TIPS outstanding with more than two years to maturity. The average five-year on-the-run TIPS liquidity premium according to the option-adjusted CLR-L model is shown with a solid black horizontal line. Also shown is the five-year off-the-run Treasury yield spread. The data cover the period from July 11, 1997, to December 27, 2013.

what we find in the TIPS market. This also lends support to our model approach that involves no liquidity correction of the Treasury yields used in model estimation.

In Figure 8, we compare the estimated ten-year on-the-run TIPS liquidity premium from the option-adjusted CLR-L model to alternative ten-year TIPS liquidity premium estimates derived from nominal and real yields. Pflueger and Viceira (2013) use regressions of ten-year BEI on a set of proxies for TIPS liquidity specifically or financial market liquidity more broadly to generate their TIPS liquidity premium estimates.[17] We note that their estimates are almost uniformly above those from the option-adjusted CLR-L model and average 65 basis points, or more than 35 basis points higher. D'Amico et al. (2014) estimate a joint model of nominal and real yields, which is similar to the CLR model in its fit to nominal yields, but distinct

[17]We thank Carolin Pflueger for sharing these data.

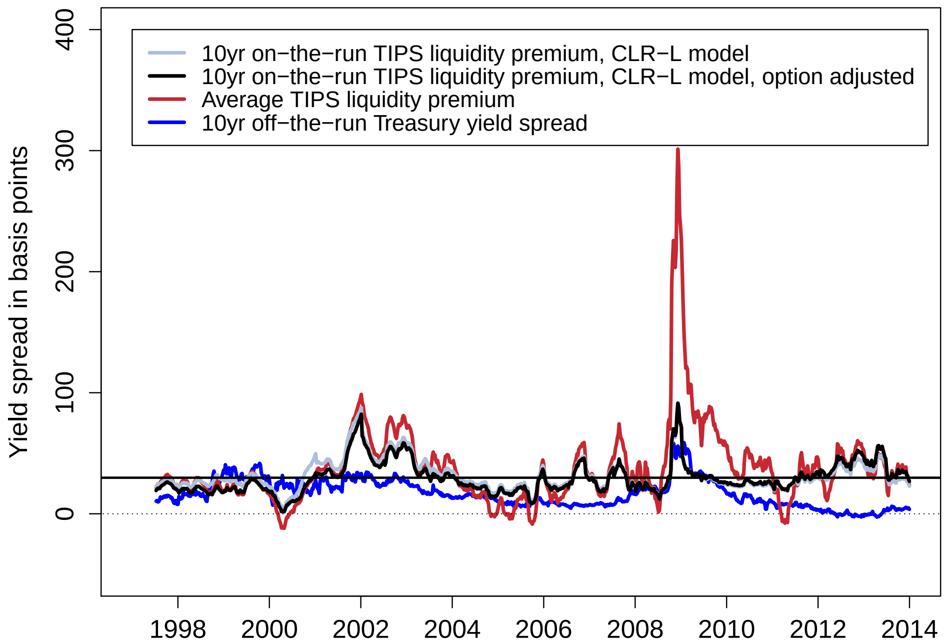


Figure 7: **Estimated Ten-Year On-The-Run TIPS Liquidity Premiums.** Illustration of the estimated liquidity premiums for the most recently issued (on-the-run) ten-year TIPS on each observation date implied by the CLR-L model with and without deflation option adjustment. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of the individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average ten-year on-the-run TIPS liquidity premium according to the option-adjusted CLR-L model is shown with a solid black horizontal line. Also shown is the ten-year off-the-run Treasury yield spread. The data cover the period from July 11, 1997, to December 27, 2013.

in its fit to real yields in that practically all idiosyncratic TIPS yield variation is captured by a separate factor that is labeled liquidity and not allowed to affect the model's generated inflation expectations and risk premiums.[18] In the CLR model, on the other hand, the real yield level factor L_t^R is unique to TIPS yields and matters for the model's expectations and risk premium components. The estimated ten-year TIPS liquidity premium from the former model is mostly above the estimate from the option-adjusted CLR-L model, in particular in the early years of the TIPS market and around the peak of the financial crisis. Over the shown period it averages 61 basis points, or twice the size of the CLR-L model-implied series. Furthermore, it is notably more volatile than the other estimates shown in the figure. Finally, we compare our estimate to the ten-year TIPS liquidity premium estimated from the joint

[18]We thank Michiel de Pooter for sharing the output from this model.

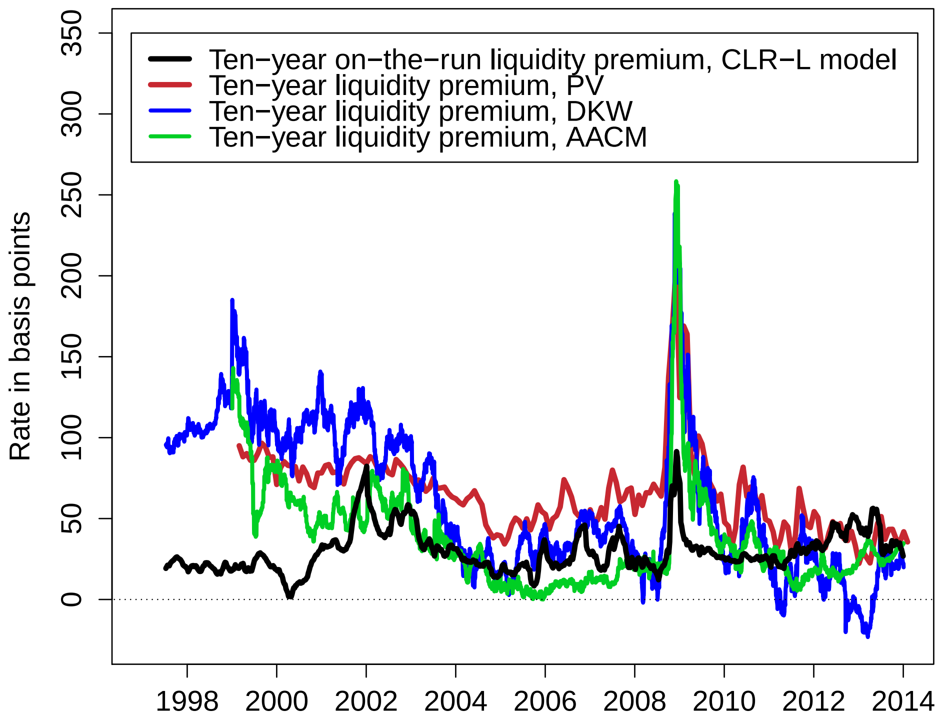


Figure 8: **Comparison of Ten-Year TIPS Liquidity Premium Estimates.**
Illustration of the estimated liquidity premium for the most recently issued (on-the-run) ten-year TIPS on each observation date implied by the CLR-L model with deflation option adjustment. Also shown are the ten-year TIPS liquidity premiums obtained from updates of the analysis in Pflueger and Viceira (PV) (2013), D'Amico et al. (DKW) (2014), and Abrahams et al. (AACM) (2015). The data cover the period from July 11, 1997, to December 27, 2013.

model of nominal and real yields presented in Abrahams et al. (2015).[19] Despite differences in the construction of the estimated liquidity factor in their model relative to the CLR-L model, the two estimated ten-year TIPS liquidity premium series are fairly close to each other for most of the shown period. Still, it is the case that there is a level difference for the early years of the TIPS market, and this causes the measure from Abrahams et al. (2015) to average slightly higher at 37.5 basis points versus the option-adjusted CLR-L model average of 29.8 basis points.

6.1 The Fed's TIPS Purchases during QE2

In Figure 5 above, there is a clear, temporary drop in the average TIPS liquidity premium during the Fed's QE2 program, which included a significant amount of TIPS purchases. CG

[19]We thank Tobias Adrian and Richard Crump for sharing their data.

argue that this decline is evidence of a liquidity channel as a transmission mechanism for central bank large-scale asset purchases—commonly known as quantitative easing (QE)—to affect long-term interest rates. To assess the robustness of their findings, we now repeat parts of their analysis.

To begin, CG argue that QE can reduce priced frictions to trading through a liquidity channel that operates by changing the *shape* of the price distribution of the targeted securities. For evidence of this liquidity channel, they analyze how the Fed's QE2 TIPS purchases affected a measure of priced frictions in TIPS yields and inflation swap rates. They find that, for the duration of the program, their measure of priced frictions averaged 10 to 13 basis points lower than expected, and they conclude that this suggests that QE can improve market liquidity for the targeted securities.

As explained in CG, the Fed purchased about $26 billion of TIPS during the operation of QE2 from November 3, 2010, until June 29, 2011. However, CG did not estimate TIPS liquidity premiums directly unlike what we do in this paper. Thus, equipped with our estimated average TIPS liquidity premium series shown in Figure 5, we can study how this measure of priced frictions in the TIPS market was impacted by the Fed's TIPS purchases. Following CG we use three variables to control explicitly for sources that reflect bond market liquidity more broadly.[20]

The first variable we consider is the VIX options-implied volatility index. It represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The motivation for including this variable is that elevated economic uncertainty would imply increased uncertainty about the future resale price of any security and therefore could cause liquidity premiums that represent investors' guard against such uncertainty to go up.

The second variable included is a market illiquidity measure introduced in the recent paper by HPW.[21] They demonstrate that deviations in bond prices in the Treasury market from a fitted yield curve represent a measure of noise and illiquidity caused by limited availability of arbitrage capital. Their analysis suggests that this measure is a priced risk factor across several financial markets, which they interpret to imply that it represents an economy-wide illiquidity measure that should affect all financial markets. If so, this should include the market for TIPS.

The final variable is the yield difference between seasoned (off-the-run) Treasury securities and the most recently issued (on-the-run) Treasury security, both with ten years to maturity.[22] For each maturity segment in the Treasury yield curve, the on-the-run security is typically

[20]Since we have estimates of TIPS liquidity premiums unlike CG, we do not include bid-ask spreads for inflation swaps in our analysis. Also, since the model estimation explicitly accounts for the value of the TIPS deflation protection options, we are not including deflation option series in the analysis either.

[21]The data are publicly available at Jun Pan's website: https://sites.google.com/site/junpan2/publications.

[22]We do not construct off-the-run spreads for the TIPS market since Christensen et al. (2012) show that such spreads have been significantly biased in the years following the peak of the financial crisis due to the value of the deflation protection option embedded in the TIPS contract.

Explanatory variables	Dependent variable: Avg. estimated TIPS liquidity premium							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.92	-4.87**	0.35	-0.55	-6.06**	-8.47**	-0.53	-6.07**
	(1.33)	(-4.73)	(0.48)	(-0.66)	(-5.56)	(-7.54)	(-0.61)	(-5.45)
AR(1) coefficient	0.98**	0.89**	0.94**	0.93**	0.93**	0.88**	0.93**	0.93**
	(91.68)	(56.22)	(45.12)	(48.05)	(48.13)	(58.62)	(44.02)	(47.25)
VIX		0.45**			0.61**	0.67**		0.61**
		(7.20)			(7.47)	(9.79)		(7.18)
HPW measure			0.59*		-0.94**		0.05	-0.94*
			(2.34)		(-3.01)		(0.12)	(-2.45)
Off-the-run spread				0.22**		-0.97**	0.21	0.00
				(2.99)		(-6.31)	(1.85)	(0.03)
Adjusted R^2	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97

Table 8: **Regression Results for Pre-QE2 Period with AR(1) Specification.**
The table reports the results of regressions with the average estimated TIPS liquidity premium as the dependent variable and an AR(1) term and three measures of market functioning as explanatory variables. T-statistics are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively. The data are weekly covering the period from January 14, 2005, to October 29, 2010, a total of 303 observations.

the most traded security and therefore penalized the least in terms of liquidity premiums. For our analysis, the important thing to note is that if there is a wide yield spread between liquid on-the-run and comparable seasoned Treasuries, we would expect to also see large liquidity premiums in TIPS yields relative to those in the Treasury bond market, that is, a widening of our liquidity premium measure.

Similar to CG, we perform a counterfactual analysis. However, as noted by CG, residuals from regressions in levels are serially correlated. In our case, a simple Durbin-Watson test for the regression with all three explanatory variables included gives a value of 0.14, which indicates highly significant positive serial correlation.

To overcome that problem, we follow CG and include the lagged value of our TIPS liquidity premium measure in the regressions, that is, we use an AR(1) specification. Thus, we run regressions of the following type:

$$LP_t(\tau) = \beta_0 + \rho LP_{t-1}(\tau) + \beta^T X_t + \varepsilon_t, \tag{10}$$

where $LP_t(\tau)$ is the estimated average TIPS liquidity premium from the CLR-L model with adjustment for the deflation option values shown in Figure 5, while X_t represents the exogenous explanatory variables. To replicate the analysis in CG, we estimate the regressions on the sample from January 14, 2005, to November 2, 2010, which delivers the estimated coefficients $\widehat{\beta}_0$, $\widehat{\rho}$, and $\widehat{\beta}$ reported in Table 8 that describe the historical statistical relationship before the introduction of the QE2 program.

Based on the historical dynamic relationship implied by the estimated coefficients in equation (10) and reported in Table 8, we analyze whether the shocks to the liquidity premium measure during QE2 were statistically significantly more negative than in the pre-QE2 period. If so, it would suggest that the QE2 TIPS purchases exerted downward pressure on the liquidity premium measure.

Focusing on regression (8) in Table 8, we calculate realized residuals relative to the counterfactual prediction for the period from November 5, 2010, to July 1, 2011, using

$$\varepsilon_t^R = LP_t(\tau) - \widehat{\beta}_0 - \widehat{\rho} LP_{t-1}(\tau) - \widehat{\beta}^T X_t. \tag{11}$$

Since the residuals from the regressions in Table 8 have fatter tails than the normal distribution (mainly due to the financial crisis), we use a Wilcoxon test of the hypothesis that the mean of the realized residuals in equation (11) is identical to the mean of the residuals in the pre-QE2 regression with the alternative being a lower mean of the realized residuals in light of our previous results. The Wilcoxon test is -1.69 with a p-value of 0.0160. Thus, similar to the results reported by CG, the test suggests that the shocks to the estimated average TIPS liquidity premium experienced during the QE2 program were significantly more negative than what would have been predicted based on the historical dynamic relationships. Therefore, we conclude that the TIPS purchases included in the QE2 program exerted a persistent downward pressure on the priced frictions to trading in the TIPS market.[23] These results indeed suggest that a byproduct of quantitative easing may be improvement in the market liquidity of the assets purchased as argued in CG.

7 Analysis of the Risk of Deflation

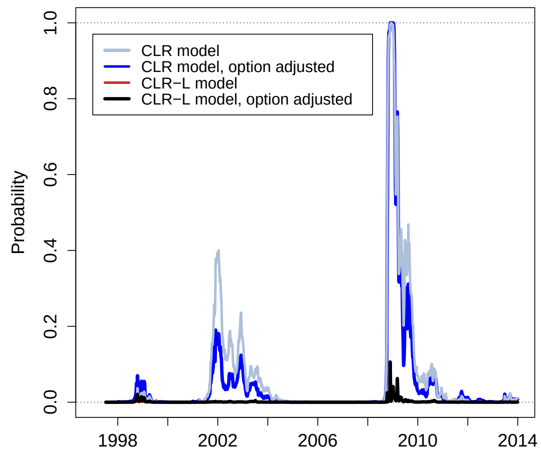
In this section, we focus on the lower tail of the model-implied inflation distributions. Specifically, we analyze the effect of accounting for TIPS liquidity risk on the models' assessment of the risk of deflation.[24]

To begin, Figure 9 shows the estimated probabilities of net deflation over the next year according to the four CLR models discussed in the previous section under both the objective P probability measure and the risk-neutral Q probability measure. These are calculated based on formulas provided in Christensen et al (2012) using the models' frictionless dynamics.

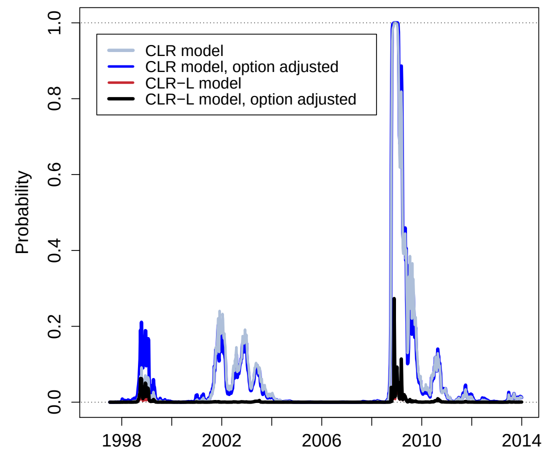
We note that such estimates are significantly affected by whether or not TIPS liquidity premiums are accounted for. When the liquidity factor is omitted, the CLR model suggests that the U.S. economy has experienced three spells of deflation fear, a brief small one in late 1998 (around the time of the Russian sovereign debt crisis), a larger and longer one in 2002-2003, and another, more severe around the peak of the financial crisis in 2008-2009—the two latter spells were also highlighted by Christensen et al. (2012). However, once

[23] For robustness, we repeated the autoregressive counterfactual analysis using samples starting on January 10, 2003, and January 12, 2007, respectively, and obtained qualitatively similar results, see Appendix D.

[24] Fleckenstein et al. (2013) analyze the risk of deflation using inflation cap and floor options, but they do not account for the effects of liquidity risk.



(a) Objective deflation probabilities.



(b) Risk-neutral deflation probabilities.

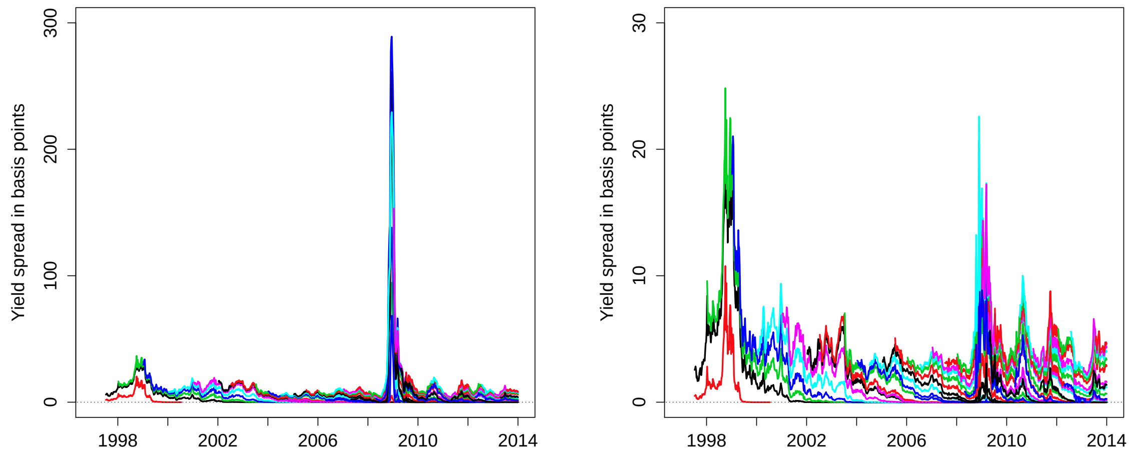
Figure 9: **Estimated One-Year Deflation Probabilities.**
Panel (a) shows the estimated one-year deflation probabilities under the objective P probability measure from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. Panel (b) shows the corresponding estimated one-year deflation probabilities under the risk-neutral Q probability measure. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

the TIPS liquidity premium is accounted for, only the first and the last spell show a small uptick in the risk of deflation. If we focus on the risk-neutral distribution that adjusts the objective probabilities for investors' risk premiums, we see a qualitatively similar picture and the differences from accounting for the TIPS liquidity premium are still clearly notable.

Obviously, these differences matter for the model-implied values of the deflation protection options embedded in the TIPS contract as also noted by Christensen et al. (2016). Figures 10(a) and 10(b) illustrate the deflation protection option values implied by the CLR model and the CLR-L model, respectively, both adjusted for the deflation option values. With the exception of the early years of the TIPS market, the CLR model delivers estimates of the deflation protection option value that are between 5 and 10 times larger than the estimates implied by the CLR-L model. Thus, to assess the severity of tail events such as the risk of deflation, it is crucial to account for the time-varying liquidity premiums in TIPS pricing. Omitting it can lead to severely exaggerated estimates as demonstrated in Figures 9 and 10.

8 Model-Implied Inflation Expectations

In this section, we analyze the properties of the model-implied inflation distributions from the CLR models. First, we evaluate the models' one-year inflation expectations by comparing them to inflation swap rates and survey forecasts. Second, we assess the models' longer-term



(a) CLR model, option adjusted.

(b) CLR-L model, option adjusted.

Figure 10: **Estimated Values of TIPS Deflation Protection Options.**
Panel (a) shows the estimated value of the deflation protection option embedded in each TIPS according to the CLR model estimated with adjustment for the deflation option values. Panel (b) shows the corresponding option values according to the CLR-L model estimated with adjustment for the deflation option values. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

inflation expectations by comparing them to survey forecasts. Overall, the purpose is to demonstrate that better market-based measures of inflation expectations can be obtained by incorporating the TIPS liquidity risk factor into the CLR model.

8.1 Model Selection

For inflation forecasting, the specification of the mean-reversion matrix K^P is critical. To select the best fitting specification of the model's real-world dynamics, we use a general-to-specific modeling strategy in which the least significant off-diagonal parameter of K^P is restricted to zero and the model is re-estimated. This strategy of eliminating the least significant coefficient is carried out down to the most parsimonious specification, which has a diagonal K^P matrix. The final specification choice is based on the values of the Akaike and Bayesian information criteria as in Christensen et al. (2010, 2014).[25]

Note that, to save on computing time in the execution of the model selection, we limit our focus to the CLR-L model without accounting for the deflation option values. Based on our results presented earlier the gain from taking on the added computational burden would likely be very limited in terms of its effect on estimates of the mean-reversion matrix K^P.

The summary statistics of the model selection process are reported in Table 9. The Akaike

[25] The Akaike information criterion is defined as $\text{AIC} = -2\log L + 2k$, where k is the number of model parameters, while the Bayesian information criterion is defined as $\text{BIC} = -2\log L + k\log T$, where T is the number of data observations.

Alternative specifications	Goodness of fit statistics				
	$\log L$	k	p-value	AIC	BIC
(1) Unrestricted K^P	118,970.6	116	n.a.	-237,709.2	-237,157.4
(2) $\kappa_{13}^P = 0$	118,970.6	115	1.00	-237,711.2	-237,164.2
(3) $\kappa_{13}^P = \kappa_{54}^P = 0$	118,970.5	114	0.65	-237,713.0	-237,170.7
(4) $\kappa_{13}^P = \kappa_{54}^P = \kappa_{43}^P = 0$	118,970.5	113	1.00	-237,715.0	-237,177.5
(5) $\kappa_{13}^P = \ldots = \kappa_{35}^P = 0$	118,970.4	112	0.65	-237,716.8	-237,184.0
(6) $\kappa_{13}^P = \ldots = \kappa_{32}^P = 0$	118,970.3	111	0.65	-237,718.6	-237,190.6
(7) $\kappa_{13}^P = \ldots = \kappa_{45}^P = 0$	118,969.8	110	0.32	-237,719.6	-237,196.3
(8) $\kappa_{13}^P = \ldots = \kappa_{15}^P = 0$	118,968.6	109	0.12	-237,719.2	-237,200.7
(9) $\kappa_{13}^P = \ldots = \kappa_{34}^P = 0$	118,968.6	108	1.00	-237,721.2	-237,207.5
(10) $\kappa_{13}^P = \ldots = \kappa_{31}^P = 0$	118,968.5	107	0.65	**-237,723.0**	-237,214.0
(11) $\kappa_{13}^P = \ldots = \kappa_{12}^P = 0$	118,967.3	106	0.12	-237,722.6	-237,218.4
(12) $\kappa_{13}^P = \ldots = \kappa_{24}^P = 0$	118,965.4	105	0.05	-237,720.8	-237,221.3
(13) $\kappa_{13}^P = \ldots = \kappa_{21}^P = 0$	118,961.9	104	< 0.01	-237,715.8	-237,221.1
(14) $\kappa_{13}^P = \ldots = \kappa_{51}^P = 0$	118,956.6	103	< 0.01	-237,707.2	-237,217.2
(15) $\kappa_{13}^P = \ldots = \kappa_{53}^P = 0$	118,955.1	102	0.08	-237,706.2	-237,221.0
(16) $\kappa_{13}^P = \ldots = \kappa_{52}^P = 0$	118,954.2	101	0.18	-237,706.4	**-237,225.9**
(17) $\kappa_{13}^P = \ldots = \kappa_{42}^P = 0$	118,945.4	100	< 0.01	-237,690.8	-237,215.1
(18) $\kappa_{13}^P = \ldots = \kappa_{41}^P = 0$	118,940.8	99	< 0.01	-237,683.6	-237,212.7
(19) $\kappa_{13}^P = \ldots = \kappa_{14}^P = 0$	118,934.7	98	< 0.01	-237,673.4	-237,207.2
(20) $\kappa_{13}^P = \ldots = \kappa_{23}^P = 0$	118,922.3	97	< 0.01	-237,650.6	-237,189.2
(21) $\kappa_{13}^P = \ldots = \kappa_{25}^P = 0$	118,915.3	96	< 0.01	-237,638.6	-237,181.9

Table 9: **Evaluation of Alternative Specifications of the CLR-L Model.**
There are twenty-one alternative estimated specifications of the CLR-L model. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the information criteria (AIC and BIC). The period analyzed covers weekly data from July 11, 1997, to December 27, 2013.

information criterion (AIC) is minimized by specification (10), which has a K^P matrix given by

$$K_{AIC}^P = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & 0 & \kappa_{14}^P & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P & \kappa_{24}^P & \kappa_{25}^P \\ 0 & 0 & \kappa_{33}^P & 0 & 0 \\ \kappa_{41}^P & \kappa_{42}^P & 0 & \kappa_{44}^P & 0 \\ \kappa_{51}^P & \kappa_{52}^P & \kappa_{52}^P & 0 & \kappa_{55}^P \end{pmatrix},$$

while the Bayesian information criterion (BIC) favors the more parsimonious specification

(16) with a K^P matrix given by

$$K^P_{BIC} = \begin{pmatrix} \kappa^P_{11} & 0 & 0 & \kappa^P_{14} & 0 \\ 0 & \kappa^P_{22} & \kappa^P_{23} & 0 & \kappa^P_{25} \\ 0 & 0 & \kappa^P_{33} & 0 & 0 \\ \kappa^P_{41} & \kappa^P_{42} & 0 & \kappa^P_{44} & 0 \\ 0 & 0 & 0 & 0 & \kappa^P_{55} \end{pmatrix}.$$

These preferred specifications are consistent with the results reported in Abrahams et al. (2015). First, the liquidity factor matters for the expected excess return of nominal Treasuries in addition to its effect on TIPS pricing. Second, the slope factor is important for the expected return of both Treasuries and TIPS. This comes about both through its own direct effect on their pricing and through its dynamic interactions with the other state variables.

In addition to these two specifications, we analyze the most flexible model with unrestricted mean-reversion matrix K^P and the most parsimonious model with diagonal K^P. Finally, we also study the specification closest to the specification preferred by CLR with a mean-reversion matrix given by

$$K^P_{CLR} = \begin{pmatrix} \kappa^P_{11} & 0 & 0 & \kappa^P_{14} & 0 \\ \kappa^P_{21} & \kappa^P_{22} & \kappa^P_{23} & 0 & 0 \\ 0 & 0 & \kappa^P_{33} & 0 & 0 \\ \kappa^P_{41} & \kappa^P_{42} & 0 & \kappa^P_{44} & 0 \\ 0 & 0 & 0 & 0 & \kappa^P_{55} \end{pmatrix}.$$

8.2 Inflation Forecast Evaluation

For a start, Figure 11 shows the estimated one-year expected inflation from the CLR and CLR-L models with and without the deflation option adjustment—all specified with a diagonal K^P matrix as in the previous sections—with a comparison to the subsequent realizations of year-over-year headline CPI inflation. We note that the CLR model produces one-year expected inflation estimates that are typically well below the subsequent realizations of year-over-year CPI inflation. On the other hand, the CLR-L model generates one-year inflation forecasts that are notably closer to the actual inflation outcomes.

To evaluate the forecast accuracy more formally, Table 10 reports the statistics of the forecast errors from the four models. It contains the mean errors, the mean absolute errors advocated by Diebold and Shin (2014), and the classic root mean squared errors, and it compares them to the performance of the random walk and the median of forecasts in the Blue Chip Economic Forecast survey. Also, for the most recent period from January 2005 through the end of 2013, the one-year inflation swap rate is included in the forecast evaluation. The table also reports the results for the CLR-L models preferred according to the AIC and

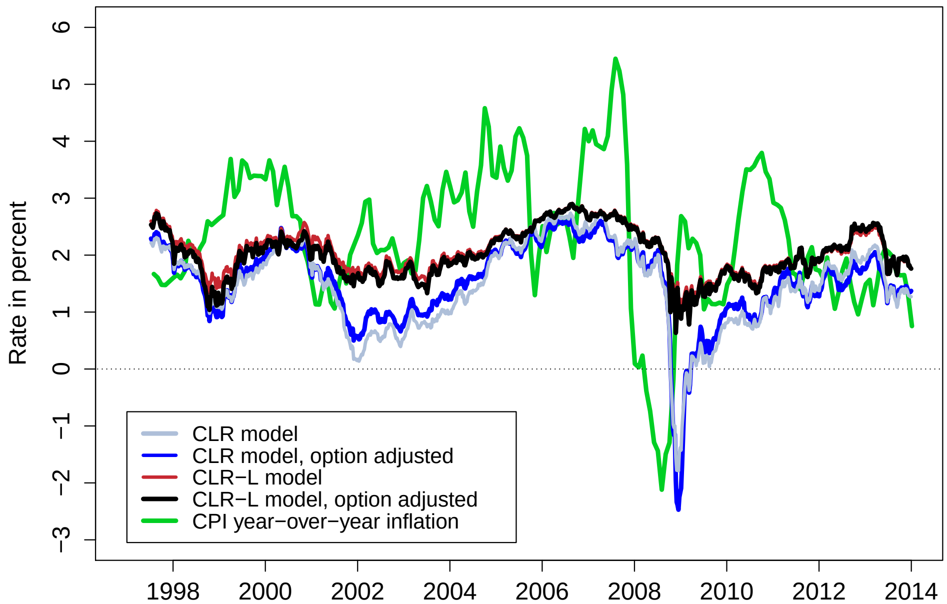


Figure 11: **Estimated One-Year Expected Inflation.**
Illustration of the estimated one-year expected inflation from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. These data cover the period from July 11, 1997, to December 27, 2013. Also shown are the subsequent monthly realizations of year-over-year headline CPI inflation.

BIC and the unrestricted K^P and CLR 2010 benchmark specifications.

Several things are worth highlighting in Table 10. First, the models are able to outperform the random walk at forecasting CPI inflation one year ahead over the full sample; this should be the minimum requirement for any model. Second and more importantly, the CLR-L model systematically outperforms the CLR model, and this conclusion is valid whether or not the model estimation accounts for the values of the deflation options embedded in TIPS. Third, since the fitted frictionless one-year BEI is about as accurate as the CLR-L models' one-year expected inflation forecasts, it follows that the improvement in inflation forecasts comes from accounting for the TIPS liquidity premium and not from the models' estimated inflation risk premiums, which are small at the one-year horizon considered here. Fourth, for the most recent period since 2005, the CLR-L model even outperforms the Blue Chip Economic Forecasts survey at forecasting CPI inflation one year ahead, while that is not the case in the early 1997-2004 period. Finally, the inflation expectations from all models considered are competitive relative to inflation swap rates, which suggests that TIPS prices, if appropriately treated, could be reliable instruments for extracting financial market participants' inflation expectations.

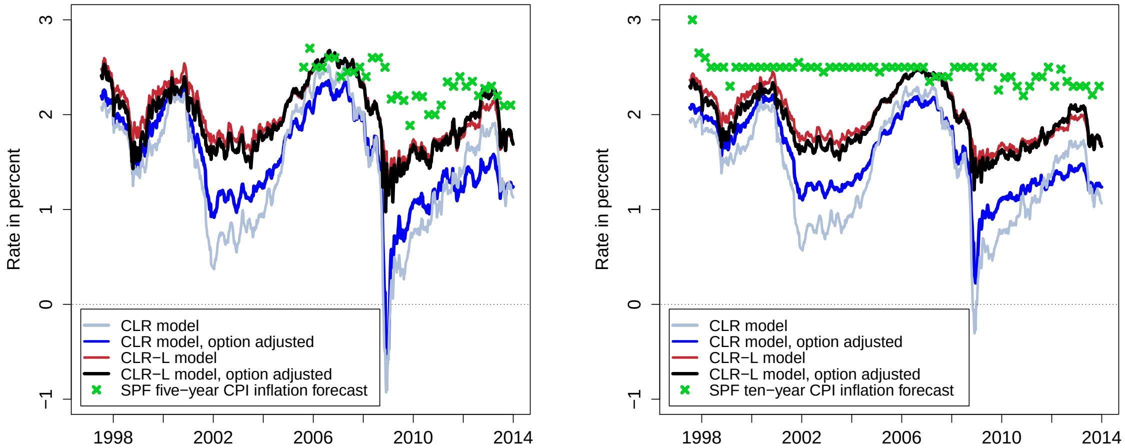
Model	1997-2004			2005-2013			1997-2013		
	Mean	MAE	RMSE	Mean	MAE	RMSE	Mean	MAE	RMSE
Random Walk	20.85	91.50	108.04	-19.16	176.17	230.51	-0.97	137.68	185.17
Blue Chip	23.37	73.43	86.44	-0.54	113.28	150.93	10.33	95.17	125.79
Inflation swap rate	n.a.	n.a.	n.a.	44.32	142.24	198.07	n.a.	n.a.	n.a.
CLR model,	113.54	129.12	147.77	58.09	128.88	162.65	83.30	128.99	156.06
CLR model, option adjusted	101.69	118.06	134.67	59.74	127.15	164.91	78.81	123.02	151.91
CLR-L model	49.95	85.88	99.22	1.72	109.46	141.69	23.64	98.75	124.20
— fitted frictionless BEI	46.42	81.82	95.92	-2.10	111.28	143.31	19.95	97.88	124.04
CLR-L model, option adjusted	61.38	91.12	105.30	3.58	111.17	143.01	29.85	102.05	127.26
— fitted frictionless BEI	57.22	86.57	101.98	1.14	112.45	144.49	26.63	100.69	126.95
CLR-L model, unrestricted K^P	49.44	76.09	90.06	-0.36	109.94	140.86	22.28	94.55	120.46
CLR-L model, CLR 2010	47.60	78.12	92.16	0.66	109.52	140.33	22.00	95.25	120.84
CLR-L model, preferred AIC	48.12	76.71	90.71	-0.21	109.80	140.61	21.76	94.76	120.52
CLR-L model, preferred BIC	46.11	77.36	90.69	1.88	107.68	139.57	21.98	93.90	119.85

Table 10: **Comparison of CPI Inflation Forecasts.**
The table reports summary statistics for one-year forecast errors of headline CPI inflation. The Blue Chip forecasts are mapped to the end of each month from July 1997 to December 2013, a total of 198 monthly forecasts. The comparable model forecasts are generated on the nearest available business day prior to the end of each month. The subsequent CPI realizations are year-over-year changes starting at the end of the month before the survey month. As a consequence, the random walk forecasts equal the past year-over-year change in the CPI series as of the beginning of the survey month. For the subperiod from January 2005 to December 2013, which represents 108 monthly forecasts, one-year inflation swap rates are also available and their forecast performance is reported, while this is not the case for the period from July 1997 to December 2004 that contains 90 monthly forecasts.

Overall, these results are very encouraging as they show that incorporating the liquidity factor does improve model performance along this important dimension. Also, we note that our results are consistent with the findings of Abrahams et al. (2015). They report improvements in inflation forecasts up to three years ahead from accounting for TIPS liquidity premiums, and they show that their results hold for real time forecasts as well. Given the similarity in the estimated TIPS liquidity premiums between their study and ours, their findings may suggest that a similar result would hold in our case. Still, we emphasize that our evidence is not conclusive since the model forecasts are full-sample look-back estimates of expected inflation, while both the BC survey and the inflation swap rates reflect real-time assessments of the inflation outlook. To draw firmer conclusions, a comprehensive real-time forecast analysis is required.

Figure 12 shows the estimated longer-term inflation expectations from the independent-factors CLR and CLR-L models with a comparison to the median of the long-term inflation forecasts from the Survey of Professional Forecasters (SPF). Here, we observe results that are qualitatively similar to the ones shown in Figure 11. The standard CLR models generate estimates of longer-term inflation expectations that appear to be too low and very volatile, while the estimates from the CLR-L models are more stable and of a more reasonable magnitude, although not as high and stable as the SPF forecasts.

In summary, we find that the incorporation of a liquidity factor into the CLR model



(a) Five-year expected inflation.

(b) Ten-year expected inflation.

Figure 12: **Estimated Five- and Ten-Year Expected Inflation.**
Panel (a) shows the estimated five-year expected inflation from the CLR model with and without deflation option adjustment and from the CLR-L model with and without deflation option adjustment. Panel (b) shows the corresponding estimates of the ten-year expected inflation. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

framework leads to better behaved estimates of financial market participants' inflation expectations in addition to the improvement in model fit documented earlier. Regarding model selection, we note from Table 10 that the bottom four preferred or more flexible models do provide further forecast improvements relative to the independent-factor model results discussed above. However, again, a cautionary warning is appropriate. To draw firm conclusions, a comprehensive real-time forecast is required. That said, the evidence presented so far does suggest that the specification of the CLR-L model preferred by the BIC appears to strike a sensible balance between flexibility and performance. For that reason we end the section by using that model to decompose ten-year BEI into a liquidity premium, an expected inflation component, and the associated inflation risk premium.

Figure 13 shows the result of the decomposition. For a start, to have a constant-maturity measure of BEI, we use nominal and real yields from the Gürkaynak et al. (2007, 2010) databases to construct the observed ten-year BEI shown with a solid black line.[26] Next, we calculate the fitted frictionless nominal and real yields implied by the CLR-L model preferred according to the BIC. The difference between the two represents our measure of frictionless BEI shown with a solid blue line. The spread between the frictionless and observed BEI represents an estimate of the TIPS liquidity premium embedded in the observed ten-year BEI and highlighted with yellow shading. Finally, the estimated model P-dynamics allow us to decompose the frictionless ten-year BEI into an inflation expectations component and the

[26]Note that this series is only available starting in January 1999.

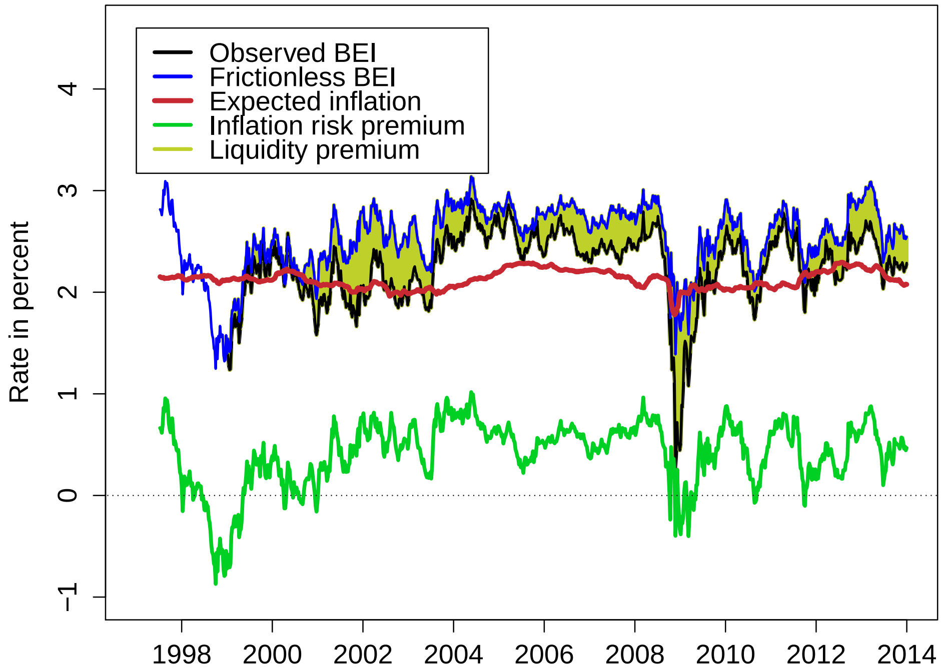


Figure 13: **Ten-Year BEI Decomposition.**
Illustration of the decomposition of the ten-year BEI based on the specification of the CLR-L model without deflation option adjustment preferred according to the BIC. The data cover the period from July 11, 1997, to December 27, 2013. See the main text for details of the decomposition.

associated inflation risk premium as described in Section 2. These are shown with red and green solid lines, respectively. We note that the specification of the CLR-L model favored according to the BIC produces even more stable and slightly higher inflation expectations than the independent-factor specification studied so far and shown in Figure 12(b). As a consequence, most of the variation in the frictionless ten-year BEI is assigned to the inflation risk premium, which is consistent with the results reported in both D'Amico et al. (2014) and Abrahams et al. (2015). However, more research is needed to determine the robustness of this result as already emphasized earlier.

9 Conclusion

In this paper, we extend a model of nominal Treasury and real TIPS yields to include a liquidity risk factor to account for the relative illiquidity of TIPS.

As for the estimated TIPS liquidity premiums, we find that they exhibit large variation. At times, they are low and even negative, which seems to coincide with high inflation when

TIPS are desirable. On the other hand, the TIPS liquidity premiums were large in the early 2000s when the U.S. Treasury committed fully to the TIPS program and increased both the issuance pace and the issuance sizes. Also, the liquidity premiums spiked around the peak of the financial crisis as also emphasized by Campbell et al. (2009) and Fleckenstein et al. (2014). Overall, these results suggest that our estimated TIPS liquidity premiums exhibit a time series pattern consistent with the stylized facts documented elsewhere in the literature.

In addition to estimates of the liquidity premium for each individual TIPS, the model delivers estimates of the frictionless nominal and real yields that can be decomposed into investors' inflation expectations and associated inflation risk premiums. We provide evidence that such decompositions may lead to superior inflation forecasts than simply using BEI without any adjustments. Furthermore, we find that accounting for TIPS liquidity premiums is crucial for the assessment of tail risks to inflation such as the risk of deflation. Without this adjustment, the severity of such risks is likely to be seriously overstated.

In terms of applications, we note that our model framework can be extended in several ways. First, the model can be modified to allow for stochastic yield volatility as in Christensen et al. (2016) to improve the pricing accuracy of the deflation protection option embedded in TIPS. Second, if respecting the zero lower bound for nominal yields is required, the Gaussian nominal short-rate dynamics can be cast as a shadow-rate AFNS model using the formulas provided in Christensen and Rudebusch (2015). However, this modification would make both model estimation and the calculation of model output more time consuming. Thus, we leave it for future research to explore such extensions even though Appendix E provides a brief summary of preliminary results from using this approach that do look promising.

Finally, we emphasize that our model generalization with a liquidity factor can be combined with any term structure model, in particular we envision this approach applied to sovereign bond markets in Europe where market liquidity is thought to be an issue, Denmark and Switzerland are countries that come to mind. Again, we leave that for future research.

Appendix A: Analytical Bond Pricing Formula

In this appendix, we provide the analytical formula for the price of individual TIPS within the CLR model with liquidity factor described in Section 3.3. The net present value of one unit of the consumption basket paid at time $t+\tau$ by TIPS i with liquidity sensitivity parameter, β^i, and liquidity decay parameter, $\lambda^{L,i}$, can be calculated by the formula provided in the following proposition.[27]

Proposition 1:

The net present value of one unit of the consumption basket paid at time $t+\tau$ by TIPS i with liquidity sensitivity parameter, β^i, and liquidity decay parameter, $\lambda^{L,i}$, is given by

$$\begin{aligned}
P^i(t_0^i,t,T) &= E_t^Q\Big[e^{-\int_t^T (r_s^R+\beta^i(1-e^{-\lambda^{L,i}(s-t_0^i)})X_s^{Liq})ds}e^{(\overline{B}^i)'X_T+\overline{A}^i}\Big]\\
&= \exp\Big(B_1^i(t,T)L_t^N + B_2^i(t,T)S_t + B_3^i(t,T)C_t + B_4^i(t,T)L_t^R + B_5^i(t_0^i,t,T)X_t^{Liq} + A^i(t_0^i,t,T)\Big),
\end{aligned}$$

where

$$\begin{aligned}
B_1^i(t,T) &= \overline{B}_1^i,\\
B_2^i(t,T) &= e^{-\lambda(T-t)}\overline{B}_2^i - \alpha^R\frac{1-e^{-\lambda(T-t)}}{\lambda},\\
B_3^i(t,T) &= \lambda(T-t)e^{-\lambda(T-t)}\overline{B}_2^i + \overline{B}_3^i e^{-\lambda(T-t)} + \alpha^R\Big[(T-t)e^{-\lambda(T-t)} - \frac{1-e^{-\lambda(T-t)}}{\lambda}\Big],\\
B_4^i(t,T) &= \overline{B}_4^i - (T-t),\\
B_5^i(t_0^i,t,T) &= e^{-\kappa_{Liq}^Q(T-t)}\overline{B}_5^i - \beta^i\frac{1-e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q} + \beta^i e^{-\lambda^{L,i}(t-t_0^i)}\frac{1-e^{-(\kappa_{Liq}^Q+\lambda^{L,i})(T-t)}}{\kappa_{Liq}^Q+\lambda^{L,i}},\\
A^i(t_0^i,t,T) &= \overline{A}^i - \beta^i\theta_{liq}^Q(T-t) + \theta_{liq}^Q\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big](1-e^{-\kappa_{Liq}^Q(T-t)})\\
&\quad +\beta^i\frac{\kappa_{liq}^Q\theta_{liq}^Q}{\kappa_{Liq}^Q+\lambda^{L,i}}\frac{e^{-\lambda^{L,i}(t-t_0^i)}-e^{-\lambda^{L,i}(T-t_0^i)}}{\lambda^{L,i}} + \frac{\sigma_{11}^2}{2}(\overline{B}_1^i)^2(T-t)\\
&\quad +\sigma_{22}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) - \alpha^R\frac{(\alpha^R+\lambda\overline{B}_2^i)}{\lambda^3}[1-e^{-\lambda(T-t)}] + \frac{(\alpha^R+\lambda\overline{B}_2^i)^2}{4\lambda^3}[1-e^{-2\lambda(T-t)}]\Big]\\
&\quad +\sigma_{33}^2\Big[\frac{(\alpha^R)^2}{2\lambda^2}(T-t) + \alpha^R\frac{\alpha^R+\lambda\overline{B}_2^i}{\lambda^2}(T-t)e^{-\lambda(T-t)} - \frac{(\alpha^R+\lambda\overline{B}_2^i)^2}{4\lambda}(T-t)^2e^{-2\lambda(T-t)}\Big]\\
&\quad -\sigma_{33}^2\frac{(\alpha^R+\lambda\overline{B}_2^i)(3\alpha^R+\lambda\overline{B}_2^i+2\lambda\overline{B}_3^i)}{4\lambda^2}(T-t)e^{-2\lambda(T-t)}\\
&\quad +\sigma_{33}^2\frac{(2\alpha^R+\lambda\overline{B}_2^i+\lambda\overline{B}_3^i)^2+(\alpha^R+\lambda\overline{B}_3^i)^2}{8\lambda^3}[1-e^{-2\lambda(T-t)}] - \sigma_{33}^2\alpha^R\frac{2\alpha^R+\lambda\overline{B}_2^i+\lambda\overline{B}_3^i}{\lambda^3}[1-e^{-\lambda(T-t)}]\\
&\quad +\frac{\sigma_{44}^2}{6}[(\overline{B}_4^i)^3 - (\overline{B}_4^i-(T-t))^3]\\
&\quad +\frac{\sigma_{55}^2}{2}\Bigg[\frac{(\beta^i)^2}{(\kappa_{Liq}^Q)^2}(T-t) + \Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]^2\frac{1-e^{-2\kappa_{Liq}^Q(T-t)}}{2\kappa_{Liq}^Q}\\
&\qquad +\frac{(\beta^i)^2}{(\kappa_{Liq}^Q+\lambda^{L,i})^2}\frac{e^{-2\lambda^{L,i}(t-t_0^i)}-e^{-2\lambda^{L,i}(T-t_0^i)}}{2\lambda^{L,i}} - 2\frac{\beta^i}{\kappa_{Liq}^Q}\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]\frac{1-e^{-\kappa_{Liq}^Q(T-t)}}{\kappa_{Liq}^Q}\\
&\qquad -2(\beta^i)^2\frac{1}{\kappa_{Liq}^Q}\frac{1}{\kappa_{Liq}^Q+\lambda^{L,i}}\frac{e^{-\lambda^{L,i}(t-t_0^i)}-e^{-\lambda^{L,i}(T-t_0^i)}}{\lambda^{L,i}}\\
&\qquad +2\beta^i\Big[\overline{B}_5^i + \frac{\beta^i}{\kappa_{Liq}^Q} - \beta^i\frac{e^{-\lambda^{L,i}(T-t_0^i)}}{\kappa_{Liq}^Q+\lambda^{L,i}}\Big]e^{-\lambda^{L,i}(t-t_0^i)}\frac{e^{-\lambda^{L,i}(T-t)}-e^{-\kappa_{Liq}^Q(T-t)}}{(\kappa_{Liq}^Q)^2-(\lambda^{L,i})^2}\Bigg].
\end{aligned}$$

[27] The calculations leading to this result are available upon request.

Appendix B: The Extended Kalman Filter Estimation

In this appendix we describe the estimation of the CLR-L model based on the standard extended Kalman filter.

For affine Gaussian models, in general, the conditional mean vector and the conditional covariance matrix are[28]

$$
\begin{aligned}
E^P[X_T|\mathcal{F}_t] &= (I-\exp(-K^P\Delta t))\theta^P + \exp(-K^P\Delta t)X_t, \\
V^P[X_T|\mathcal{F}_t] &= \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds,
\end{aligned}
$$

where $\Delta t = T - t$. Conditional moments of discrete observations are computed and the state transition equation is obtained as

$$X_t = (I-\exp(-K^P\Delta t))\theta^P + \exp(-K^P\Delta t)X_{t-1} + \xi_t,$$

where Δt is the time between observations.

In the standard Kalman filter, the measurement equation is linear

$$y_t = A + BX_t + \varepsilon_t$$

and the assumed error structure is

$$\begin{pmatrix} \xi_t \\ \varepsilon_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right],$$

where the matrix H is assumed to be diagonal, while the matrix Q has the following structure

$$Q = \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds.$$

In addition, the transition and measurement errors are assumed to be orthogonal to the initial state.

Now consider Kalman filtering, which is used to evaluate the likelihood function.

Due to the assumed stationarity, the filter is initialized at the unconditional mean and variance of the state variables under the P-measure: $X_0 = \theta^P$ and $\Sigma_0 = \int_0^{\infty} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds$.

Denote the information available at time t by $Y_t = (y_1, y_2, \ldots, y_t)$, and denote model parameters by ψ. Consider period $t-1$ and suppose that the state update X_{t-1} and its mean square error matrix Σ_{t-1} have been obtained. The prediction step is

$$X_{t|t-1} = E^P[X_t|Y_{t-1}] = \Phi_t^{X,0}(\psi) + \Phi_t^{X,1}(\psi)X_{t-1},$$

$$\Sigma_{t|t-1} = \Phi_t^{X,1}(\psi)\Sigma_{t-1}\Phi_t^{X,1}(\psi)' + Q_t(\psi),$$

where $\Phi_t^{X,0} = (I-\exp(-K^P\Delta t))\theta^P$, $\Phi_t^{X,1} = \exp(-K^P\Delta t)$, and $Q_t = \int_0^{\Delta t} e^{-K^P s}\Sigma\Sigma' e^{-(K^P)' s} ds$, while Δt is the time between observations.

In the time-t update step, $X_{t|t-1}$ is improved by using the additional information contained in Y_t:

$$X_t = E[X_t|Y_t] = X_{t|t-1} + \Sigma_{t|t-1}B(\psi)'F_t^{-1}v_t,$$

$$\Sigma_t = \Sigma_{t|t-1} - \Sigma_{t|t-1}B(\psi)'F_t^{-1}B(\psi)\Sigma_{t|t-1},$$

[28] Throughout, conditional and unconditional covariance matrices are calculated using the analytical solutions provided in Fisher and Gilles (1996).

where

$$v_t = y_t - E[y_t|Y_{t-1}] = y_t - A(\psi) - B(\psi)X_{t|t-1},$$

$$F_t = cov(v_t) = B(\psi)\Sigma_{t|t-1}B(\psi)' + H(\psi),$$

$$H(\psi) = diag(\sigma^2_\varepsilon(\tau_1), \dots, \sigma^2_\varepsilon(\tau_N)).$$

At this point, the Kalman filter has delivered all ingredients needed to evaluate the Gaussian log likelihood, the prediction-error decomposition of which is

$$\log l(y_1, \dots, y_T; \psi) = \sum_{t=1}^{T} \Big(-\frac{N}{2}\log(2\pi) - \frac{1}{2}\log|F_t| - \frac{1}{2} v_t' F_t^{-1} v_t \Big),$$

where N is the number of observed yields. Now, the likelihood is numerically maximized with respect to ψ using the Nelder-Mead simplex algorithm. Upon convergence, the standard errors are obtained from the estimated covariance matrix,

$$\widehat{\Omega}(\widehat{\psi}) = \frac{1}{T}\Big[\frac{1}{T}\sum_{t=1}^{T} \frac{\partial \log l_t(\widehat{\psi})}{\partial \psi} \frac{\partial \log l_t(\widehat{\psi})}{\partial \psi}'\Big]^{-1},$$

where $\widehat{\psi}$ denotes the estimated model parameters.

In the CLR-L model, the extended Kalman filter is needed because the measurement equations are no longer affine functions of the state variables. Instead, the measurement equation takes the general form

$$\frac{\overline{P}^i_t(t^i_0, \tau^i)}{D^i_t(t^i_0, \tau^i)} = z(X_t; t^i_0, \tau^i, \psi) + \varepsilon^i_t.$$

In the extended Kalman filter, this equation is linearized using a first-order Taylor expansion around the best guess of X_t in the prediction step of the Kalman filter algorithm. Thus, in the notation introduced above, this best guess is denoted $X_{t|t-1}$ and the approximation is given by

$$z(X_t; t^i_0, \tau^i, \psi) \approx z(X_{t|t-1}; t^i_0, \tau^i, \psi) + \frac{\partial z(X_t; t^i_0, \tau^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}} (X_t - X_{t|t-1}).$$

Thus, by defining

$$A_t(\psi) \equiv z(X_{t|t-1}; t^i_0, \tau^i, \psi) - \frac{\partial z(X_t; t^i_0, \tau^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}} X_{t|t-1} \quad \text{and} \quad B_t(\psi) \equiv \frac{\partial z(X_t; t^i_0, \tau^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}},$$

the measurement equation can be given on an affine form as

$$\frac{\overline{P}^i_t(t^i_0, \tau^i)}{D^i_t(t^i_0, \tau^i)} = A_t(\psi) + B_t(\psi)X_t + \varepsilon^i_t$$

and the steps in the algorithm proceed as previously described.

Appendix C: Factor Structure of TIPS Yield Data

In this appendix, we analyze the factor structure of TIPS yields.

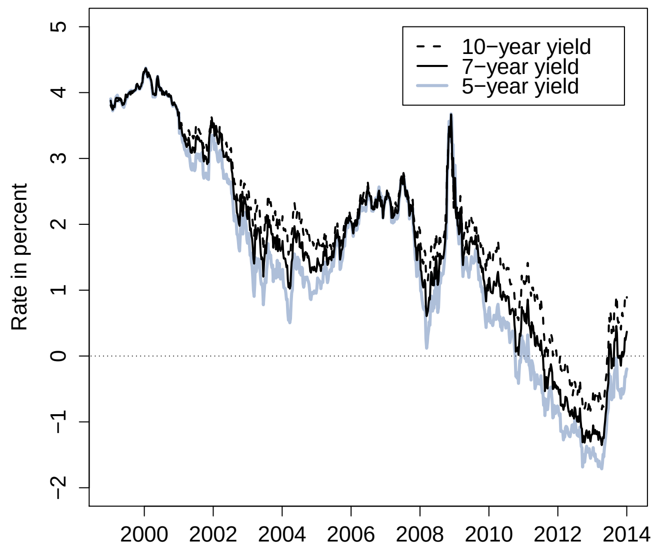


Figure 14: **Real TIPS Yields.**
Illustration of the weekly real TIPS zero-coupon bond yields covering the period from January 8, 1999, to December 27, 2013. The yields shown have maturities in five years, seven years, and ten years, respectively.

To do so, we use a sample of smoothed zero-coupon TIPS yields constructed by the method described in Gürkaynak et al.(2010)[29] and briefly detailed here. For each business day a zero-coupon yield curve of the Svensson (1995)-type

$$y_t(\tau) = \beta_0 + \frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\beta_1 + \Big[\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\Big]\beta_2 + \Big[\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\Big]\beta_3$$

is fitted to price a large pool of underlying TIPS. Thus, for each business day, we have the fitted values of the four factors $(\beta_0(t), \beta_1(t), \beta_2(t), \beta_3(t))$ and the two parameters $(\lambda_1(t), \lambda_2(t))$. From this data set zero-coupon yields for any relevant maturity can be calculated as long as the maturity is within the range of maturities used in the fitting process. As demonstrated by Gürkaynak, Sack, and Wright (2010), this model fits the underlying pool of bonds extremely well. By implication, the zero-coupon yields derived from this approach constitute a very good approximation to the true underlying TIPS zero-coupon yield curve (our results confirms this). We construct real TIPS zero-coupon bond yields with the following six maturities: 5-year, 6-year, 7-year, 8-year, 9-year, and 10-year. We use weekly data and limit our sample to the period from January 8, 1999, to December 27, 2013.[30] The summary statistics are provided in Table 11, while Figure 14 illustrates the constructed time series of the 5-year, 7-year, and 10-year TIPS zero-coupon yields.

Researchers have typically found that three factors are sufficient to model the time-variation in the cross section of nominal Treasury bond yields (e.g., Litterman and Scheinkman, 1991). Indeed, for our weekly

[29]The Board of Governors in Washington DC frequently updates the factors and parameters of this method, see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html

[30]Note that the GSW TIPS yield database only is available starting in early January 1999. The low number of TIPS before then prevents the construction of the yield curve for that early period of the TIPS market.

Maturity in months	Mean in %	St. dev. in %	Skewness	Kurtosis
60	1.49	1.59	-0.11	2.22
72	1.62	1.52	-0.17	2.32
84	1.74	1.46	-0.22	2.40
96	1.84	1.40	-0.25	2.47
108	1.92	1.34	-0.27	2.54
120	2.00	1.29	-0.28	2.59

Table 11: **Summary Statistics for the Real TIPS Yields.**

Summary statistics for the sample of weekly real TIPS zero-coupon bond yields covering the period from January 8, 1999, to December 27, 2013, a total of 782 observations.

Maturity	Loading on		
in months	First P.C.	Second P.C.	Third P.C.
60	0.45	0.68	-0.50
72	0.43	0.28	0.34
84	0.42	-0.01	0.49
96	0.40	-0.22	0.29
108	0.38	-0.39	-0.10
120	0.37	-0.51	-0.55
% explained	99.66	0.34	0.00

Table 12: **Eigenvectors of the First Three Principal Components in Real TIPS Yields.**

The loadings of yields of various maturities on the first three principal components are shown. The final row shows the proportion of all bond yield variability accounted for by each principal component. The data consist of weekly real TIPS zero-coupon bond yields from January 8, 1999, to December 27, 2013.

real TIPS yield data, 99.99% of the total variation is accounted for by three factors. Table 12 reports the eigenvectors that correspond to the first three principal components of our data. The first principal component accounts for 99.7% of the variation in the real TIPS yields, and its loading across maturities is uniformly negative. Thus, like a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 0.3% of the variation in these data and has sizable negative loadings for the shorter maturities and sizable positive loadings for the long maturities. Hence, like a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for only 0.0% of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. This motivates the unique factor structure in the CLR model of nominal and real yields that preserves the AFNS model structure with its level, slope, and curvature structure for the nominal Treasury yields, while accommodating the level, slope, and curvature structure in the TIPS yield data documented above in a very parsimonious way.

Appendix D: Robustness of CG Regressions

Explanatory variables	Dependent var.: Avg. est. TIPS liquidity premium 2003-2011							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.66	-4.51**	0.15	-0.80	-5.07**	-8.12**	-0.64	-4.89**
	(1.22)	(-5.40)	(0.26)	(-1.13)	(-5.79)	(-8.68)	(-0.87)	(-5.42)
AR(1) coefficient	0.98**	0.90**	0.94**	0.94**	0.92**	0.89**	0.93**	0.92**
	(108.28)	(66.02)	(56.58)	(58.36)	(57.20)	(68.13)	(53.72)	(55.81)
VIX		0.41**			0.49**	0.63**		0.51**
		(7.73)			(7.46)	(10.73)		(7.29)
HPW measure			0.57**		-0.49*		0.24	-0.37
			(2.77)		(-2.04)		(0.84)	(-1.31)
Off-the-run spread				0.19**		-0.94**	0.14	-0.07
				(3.13)		(-7.17)	(1.68)	(-0.85)
Adjusted R^2	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97

Explanatory variables	Dependent var.: Avg. est. TIPS liquidity premium 2007-2011							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	1.33	-7.85**	0.42	-0.80	-8.95**	-11.91**	-0.72	-9.00**
	(1.18)	(-4.66)	(0.35)	(-0.57)	(-5.17)	(-6.96)	(-0.50)	(-5.05)
AR(1) coefficient	0.98**	0.87**	0.93**	0.93**	0.91**	0.86**	0.92**	0.91**
	(67.55)	(43.21)	(33.11)	(35.92)	(35.81)	(45.86)	(32.12)	(35.02)
VIX		0.58**			0.72**	0.82**		0.72**
		(6.84)			(6.94)	(9.22)		(6.73)
HPW measure			0.69**		-0.89*		0.17	-0.93
			(2.01)		(-2.32)		(0.35)	(-1.97)
Off-the-run spread				0.25*		-1.09**	0.21	0.02
				(2.49)		(-5.75)	(1.49)	(0.14)
Adjusted R^2	0.96	0.97	0.96	0.96	0.97	0.97	0.96	0.97

Table 13: **Regression Results for Pre-QE2 Period with AR(1) Specification.** The top panel reports the results of regressions with the average estimated TIPS liquidity premium as the dependent variable and an AR(1) term and three measures of market functioning as explanatory variables using a weekly sample from January 10, 2003, to October 29, 2010, a total of 408 observations. The bottom panel reports the results of similar regressions using a weekly sample from January 12, 2007, to October 29, 2010, a total of 199 observations. T-statistics are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

The top panel is based on a sample from January 10, 2003, to October 29, 2010, while the bottom panel is based on a shorter sample from January 12, 2007, to October 29, 2010, a total of 408 and 199 observations, respectively.

For the 2003-2011 sample the Wilcoxon test described in the main text is -1.77 with a p-value of 0.0062, while the Wilcoxon test for the 2007-2011 sample is -0.60 with a p-value of 0.2605. Thus, the counterfactual analysis produces a statistically significant difference in the residuals during the QE2 period when a long sample of data is used.

Appendix E: Results for the Shadow-Rate B-CLR-L Model

In this appendix, we estimate a version of the CLR-L model where the nominal short rate is interpreted as a shadow short rate, s_t, that may be negative, while the short rate used for discounting nominal cash flows is its truncated version, $r_t^N = \max\{0, s_t\}$. Following the notation of Kim and Singleton (2012), we refer to this model as the B-CLR-L model. Up front we note that since computation of deflation option values is complex and time consuming within the B-CLR-L model, we only compare the CLR-L and B-CLR-L models without adjustment for the deflation option values.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.2010	0	0	0	0	0.0579	σ_{11}	0.0060
	(0.1143)					(0.0085)		(0.0001)
$K^P_{2,\cdot}$	0	0.0597	0	0	0	-0.0269	σ_{22}	0.0110
		(0.0972)				(0.0354)		(0.0003)
$K^P_{3,\cdot}$	0	0	0.3197	0	0	-0.0351	σ_{33}	0.0259
			(0.1334)			(0.0198)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.3088	0	0.0324	σ_{44}	0.0066
				(0.1616)		(0.0079)		(0.0001)
$K^P_{5,\cdot}$	0	0	0	0	0.5913	0.0082	σ_{55}	0.0114
					(0.1710)	(0.0077)		(0.0005)

Table 14: **Estimated Dynamic Parameters for the B-CLR-L Model.**
The top panel shows the estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix for the B-CLR-L model. The estimated value of λ is 0.4966 (0.0020), while $\alpha^R = 0.7213$ (0.0047), $\kappa^Q_{liq} = 0.9146$ (0.0318), and $\theta^Q_{liq} = 0.0012$ (0.0000). The numbers in parentheses are the estimated parameter standard deviations.

The estimated dynamic parameters reported in Table 14 are very similar to those reported in the main text for the CLR-L model.

Regarding the liquidity sensitivity parameters reported in Table 15, we note that the differences between the CLR-L model and the B-CLR-L model are, in general, smaller before the financial crisis when the zero lower bound did not matter much.

As shown in Figure 15, the estimated paths for (L_t^N, S_t, C_t, L_t^R) are mostly indistinguishable from each other across the two models. The main difference is that the curvature factor is more negative in the B-CLR-L model after 2011 when short- and medium-term Treasury yields started to be severely constrained by the zero lower bound.

For similar reasons the estimated paths for the TIPS liquidity factor across the two models shown in Figure 16 are very similar throughout the entire sample period. Thus, the estimated TIPS liquidity risk factor is insensitive to accounting for the asymmetric behavior of nominal yields near the zero lower bound.

Tables 16 and 17 compare the model fit of the CLR-L model and the B-CLR-L model. We note that for all yields combined the B-CLR-L model delivers a modest improvement in the fit to both Treasuries and TIPS. Importantly, even security by security, the fit is very similar across the two models.

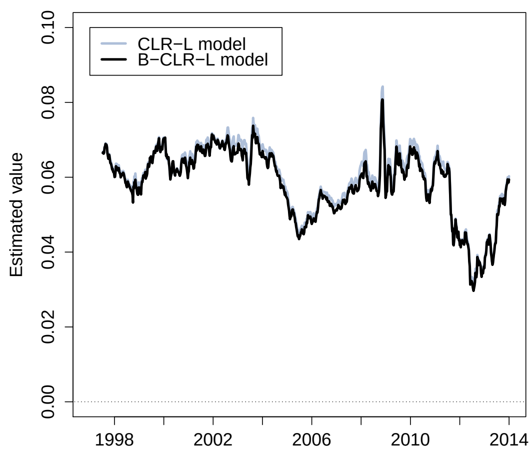
As shown in Figure 17, the B-CLR-L model also provides improvements in the estimated TIPS liquidity premium in addition to the improvement in the overall model fit already discussed above. The average estimated TIPS liquidity premium is reduced from 42.32 basis points to 38.16 basis points, or about 10 percent.

Overall, we conclude that there are benefits from accounting for the zero lower bound of nominal yields by using a shadow-rate modeling approach to those yields, which is consistent with the findings of Christensen and Rudebusch (2015, 2016).

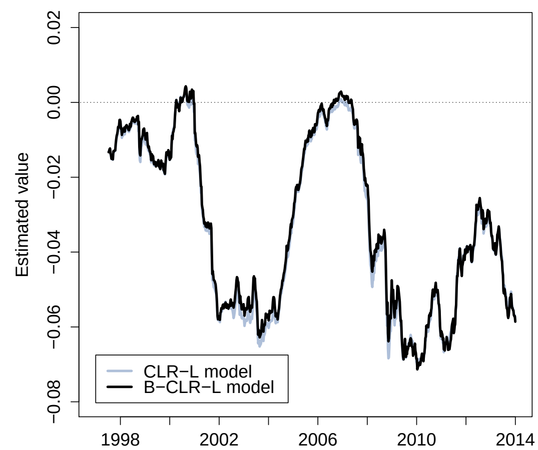
TIPS security	CLR-L model				B-CLR-L model			
	β^i	Std	$\lambda^{L,i}$	Std	β^i	Std	$\lambda^{L,i}$	Std
(1) 3.375% 1/15/2007 TIPS	1	n.a.	0.7047	0.3563	1	n.a.	9.9935	0.1958
(2) 3.625% 7/15/2002 TIPS*	0.8260	0.1333	8.9414	2.1275	0.9989	0.0719	9.7524	0.1825
(3) 3.625% 1/15/2008 TIPS	2.1317	0.4677	0.1320	0.0493	3.2397	0.1883	0.0787	0.0063
(4) 3.875% 1/15/2009 TIPS	3.0805	0.8843	0.0988	0.0402	9.4255	0.1986	0.0286	0.0009
(5) 4.25% 1/15/2010 TIPS	2.0739	0.1818	0.2360	0.0415	3.0870	0.1603	0.1409	0.0114
(6) 3.5% 1/15/2011 TIPS	2.3928	0.1943	0.2143	0.0320	3.1958	0.1271	0.1626	0.0135
(7) 3.375% 1/15/2012 TIPS	2.4185	0.1833	0.2384	0.0364	3.1158	0.1152	0.2015	0.0196
(8) 3% 7/15/2012 TIPS	2.3956	0.1686	0.2604	0.0412	2.9742	0.1059	0.2429	0.0265
(9) 1.875% 7/15/2013 TIPS	3.0073	0.3761	0.1781	0.0434	3.3008	0.1494	0.2212	0.0256
(10) 2% 1/15/2014 TIPS	5.3622	1.2267	0.0838	0.0264	4.3057	0.1780	0.1570	0.0132
(11) 2% 7/15/2014 TIPS	2.5352	0.1962	0.3410	0.0657	2.9066	0.0795	0.4170	0.0504
(12) 0.875% 4/15/2010 TIPS*	1.9994	0.0802	9.9988	2.2979	2.4162	0.0494	2.2158	0.1647
(13) 1.625% 1/15/2015 TIPS	3.4597	0.3973	0.1871	0.0371	3.7528	0.1562	0.2340	0.0235
(14) 1.875% 7/15/2015 TIPS	2.1376	0.1231	0.9495	0.3939	2.5655	0.0512	0.9942	0.1721
(15) 2% 1/15/2016 TIPS	2.5252	0.1953	0.3848	0.0677	3.1403	0.1191	0.3519	0.0394
(16) 2.375% 4/15/2011 TIPS*	1.9328	0.0798	5.2441	2.0697	2.3056	0.0290	4.9222	0.1951
(17) 2.5% 7/15/2016 TIPS	1.8922	0.1077	6.2509	2.1911	2.2655	0.0482	9.9860	0.1945
(18) 2.375% 1/15/2017 TIPS	1.9108	0.1052	9.9794	2.0559	2.2797	0.0432	9.9852	0.2134
(19) 2% 4/15/2012 TIPS*	1.8565	0.0826	9.9975	1.9748	2.2332	0.0316	9.9991	0.1741
(20) 2.625% 7/15/2017 TIPS	1.5773	0.0929	9.9943	2.3952	1.8320	0.0535	9.9934	0.1983
(21) 1.625% 1/15/2018 TIPS	1.9053	0.1815	0.4481	0.1237	2.0743	0.1069	0.5315	0.1023
(22) 0.625% 4/15/2013 TIPS*	5.3129	1.1659	0.1526	0.0450	4.6472	0.1877	0.2334	0.0155
(23) 1.375% 7/15/2018 TIPS	1.2974	0.1566	0.8968	0.3108	1.2968	0.0732	1.4399	0.1888
(24) 2.125% 1/15/2019 TIPS	28.0660	4.1001	0.0100	0.0018	4.9431	0.3497	0.0711	0.0083
(25) 1.25% 4/15/2014 TIPS*	38.6112	4.1291	0.0230	0.0030	9.3828	0.3413	0.1332	0.0077
(26) 1.875% 7/15/2019 TIPS	1.4761	0.3116	0.4666	0.3420	1.1615	0.0869	1.6863	0.3587
(27) 1.375% 1/15/2020 TIPS	29.9390	4.7859	0.0100	0.0018	26.6138	0.3686	0.0100	0.0009
(28) 0.5% 4/15/2015 TIPS*	23.8180	4.8797	0.0372	0.0084	6.7401	0.4092	0.2123	0.0197
(29) 1.25% 7/15/2020 TIPS	1.8997	1.0819	0.3470	0.5347	1.2734	0.2344	0.5453	0.3698
(30) 1.125% 1/15/2021 TIPS	3.0499	1.6842	0.2461	0.3036	22.6288	0.4126	0.0143	0.0017
(31) 0.125% 4/15/2016 TIPS*	9.8655	6.2960	0.0990	0.0757	8.7943	0.4904	0.1566	0.0132
(32) 0.625% 7/15/2021 TIPS	2.1138	0.7052	0.6286	0.9480	1.4487	0.3621	0.5356	0.4548
(33) 0.125% 1/15/2022 TIPS	3.1227	1.6588	0.4219	0.6766	3.5129	0.5912	0.1586	0.0577
(34) 0.125% 4/15/2017 TIPS*	17.4890	8.2002	0.0530	0.0272	13.7716	0.6384	0.0905	0.0073
(35) 0.125% 7/15/2022 TIPS	2.2077	0.2444	5.4838	8.7416	1.3636	0.2099	3.9490	0.6545
(36) 0.125% 1/15/2023 TIPS	2.7209	0.2336	9.9980	9.7693	1.8626	0.1908	9.9991	0.7408
(37) 0.125% 4/15/2018 TIPS*	24.6981	10.9722	0.0416	0.0199	2.8218	0.2575	1.8788	0.7384
(38) 0.375% 7/15/2023 TIPS	1.8409	0.3227	9.9575	16.7198	0.9925	0.4189	9.9877	1.2006

Table 15: **Estimated Liquidity Sensitivity Parameters.**

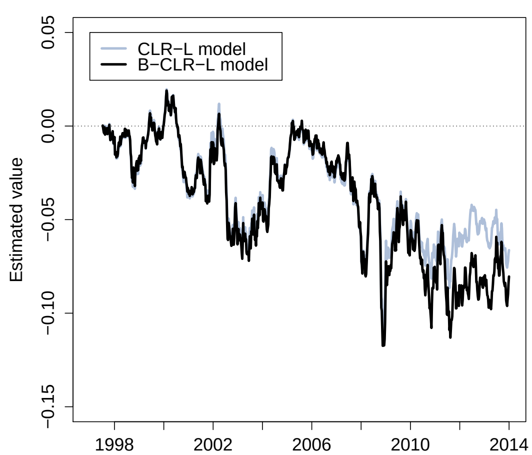
The estimated β^i sensitivity and $\lambda^{L,i}$ decay parameters for each TIPS from the CLR-L model and the B-CLR-L model are shown. Also reported are the estimated parameter standard deviations. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013.



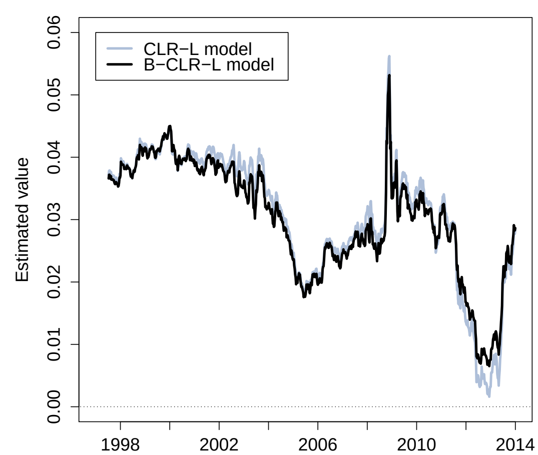
(a) L_t^N.



(b) S_t.



(c) C_t.



(d) L_t^R.

Figure 15: **Estimated State Variables.**

Illustration of the estimated state variables that affect the frictionless nominal and real yields from the CLR-L model and the B-CLR-L model. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

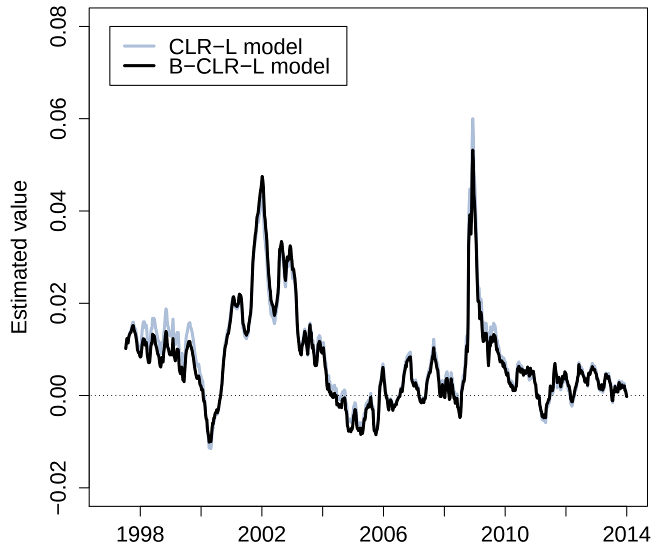


Figure 16: **Estimated Liquidity Factor.**
Illustration of the estimated liquidity factor, X_t^{liq}, from the CLR-L model and the B-CLR-L model. The data are weekly covering the period from July 11, 1997, to December 27, 2013.

Maturity in months	CLR-L model		B-CLR-L model	
	Mean	RMSE	Mean	RMSE
3	-1.15	7.40	-0.83	6.85
6	-0.85	2.70	-0.76	2.82
12	1.19	7.03	0.86	6.64
24	2.78	6.15	2.47	5.27
36	1.35	3.62	1.33	3.16
48	-0.62	2.98	-0.59	3.05
60	-1.95	3.68	-2.02	3.68
72	-2.32	3.82	-2.50	3.67
84	-1.79	3.17	-2.01	2.92
96	-0.59	2.37	-0.75	2.09
108	1.03	3.07	1.02	2.95
120	2.86	5.31	3.06	5.27
All yields	0.00	4.59	-0.06	4.30

Table 16: **Summary Statistics of Fitted Errors of Nominal Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of nominal U.S. Treasury yields from the CLR-L model and the B-CLR-L model estimations are shown. The full sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

TIPS security	CLR-L model		B-CLR-L model	
	Mean	RMSE	Mean	RMSE
(1) 3.375% 1/15/2007 TIPS	2.53	4.93	2.48	5.19
(2) 3.625% 7/15/2002 TIPS*	3.25	4.01	3.21	3.88
(3) 3.625% 1/15/2008 TIPS	2.14	4.48	2.24	4.70
(4) 3.875% 1/15/2009 TIPS	1.29	2.67	1.28	2.60
(5) 4.25% 1/15/2010 TIPS	0.77	2.94	0.71	2.83
(6) 3.5% 1/15/2011 TIPS	-0.09	4.33	0.07	4.04
(7) 3.375% 1/15/2012 TIPS	0.12	5.16	0.08	5.16
(8) 3% 7/15/2012 TIPS	-0.18	4.98	-0.30	4.93
(9) 1.875% 7/15/2013 TIPS	-0.76	6.63	-0.89	6.62
(10) 2% 1/15/2014 TIPS	0.53	3.79	0.56	3.71
(11) 2% 7/15/2014 TIPS	-0.14	4.43	-0.15	4.68
(12) 0.875% 4/15/2010 TIPS*	1.86	4.33	1.89	4.61
(13) 1.625% 1/15/2015 TIPS	0.77	4.36	0.82	4.19
(14) 1.875% 7/15/2015 TIPS	0.07	4.38	0.12	4.40
(15) 2% 1/15/2016 TIPS	1.06	4.67	0.99	4.70
(16) 2.375% 4/15/2011 TIPS*	4.67	12.20	4.79	12.37
(17) 2.5% 7/15/2016 TIPS	-0.50	5.44	-0.42	5.06
(18) 2.375% 1/15/2017 TIPS	1.92	4.39	1.82	4.28
(19) 2% 4/15/2012 TIPS*	5.62	11.25	5.52	10.95
(20) 2.625% 7/15/2017 TIPS	0.54	3.70	0.36	3.47
(21) 1.625% 1/15/2018 TIPS	0.45	3.73	0.33	3.57
(22) 0.625% 4/15/2013 TIPS*	0.32	11.55	0.20	10.97
(23) 1.375% 7/15/2018 TIPS	0.27	4.49	0.15	4.47
(24) 2.125% 1/15/2019 TIPS	-0.08	3.14	-0.01	3.12
(25) 1.25% 4/15/2014 TIPS*	0.10	4.19	0.00	3.62
(26) 1.875% 7/15/2019 TIPS	0.00	2.32	0.04	2.21
(27) 1.375% 1/15/2020 TIPS	-0.62	3.71	-0.24	2.57
(28) 0.5% 4/15/2015 TIPS*	0.38	3.31	0.53	3.57
(29) 1.25% 7/15/2020 TIPS	-0.32	2.67	0.01	1.78
(30) 1.125% 1/15/2021 TIPS	-0.56	3.85	-0.32	2.53
(31) 0.125% 4/15/2016 TIPS*	-0.22	3.67	0.12	3.64
(32) 0.625% 7/15/2021 TIPS	-0.17	2.72	-0.18	2.16
(33) 0.125% 1/15/2022 TIPS	0.11	2.47	0.09	1.61
(34) 0.125% 4/15/2017 TIPS*	-0.01	2.58	-0.02	2.83
(35) 0.125% 7/15/2022 TIPS	0.15	3.84	0.29	2.50
(36) 0.125% 1/15/2023 TIPS	-0.03	5.93	0.13	3.56
(37) 0.125% 4/15/2018 TIPS*	-0.68	3.47	-0.26	3.23
(38) 0.375% 7/15/2023 TIPS	0.44	2.77	0.52	2.16
All TIPS yields	0.65	4.89	0.66	4.77
Max log L	118,915.3		119,838.5	

Table 17: **Summary Statistics of Fitted Errors of TIPS Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of the yield-to-maturity for individual TIPS according to the CLR-L model and the B-CLR-L model. Asterisk * indicates five-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

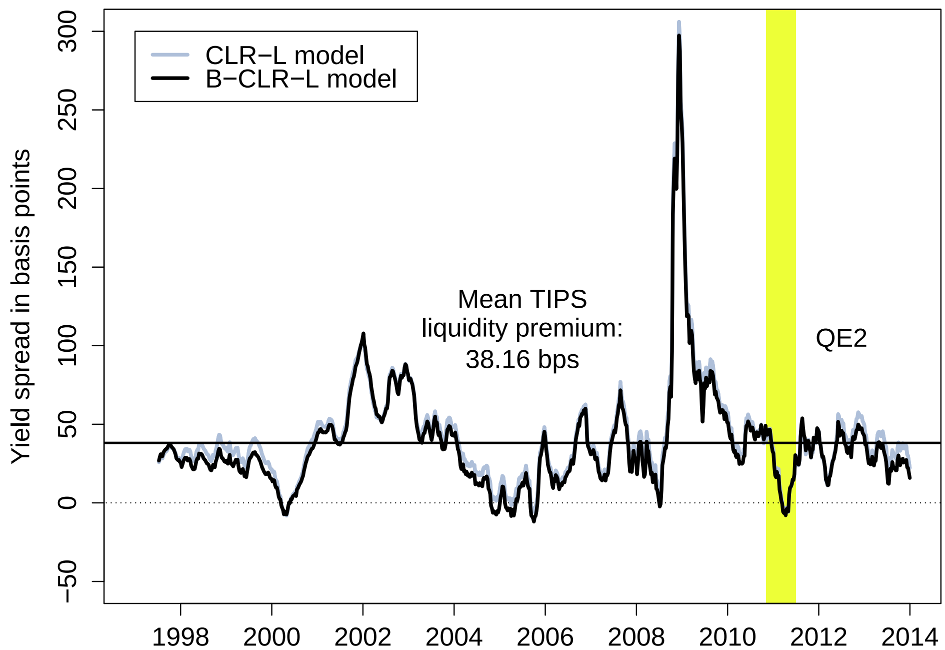


Figure 17: **Average Estimated TIPS Liquidity Premium.**
Illustration of the average estimated TIPS liquidity premium for each observation date implied by the CLR-L model and the B-CLR-L model. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The average TIPS liquidity premium according to the B-CLR-L model is shown with a solid black horizontal line. The data cover the period from July 11, 1997, to December 27, 2013.

Appendix F: Results for the CLR-L Model with All TIPS

In this appendix, we estimate the CLR-L model using the entire universe of all 50 available TIPS with maturities up to thirty years. For that reason the sample of nominal yields is also extended to include maturities up to thirty years.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P		Σ
$K^P_{1,\cdot}$	0.4132	0	0	0	0	0.0671	σ_{11}	0.0074
	(0.1123)					(0.0089)		(0.0001)
$K^P_{2,\cdot}$	0	0.1218	0	0	0	-0.0364	σ_{22}	0.0095
		(0.0824)				(0.0402)		(0.0003)
$K^P_{3,\cdot}$	0	0	0.9680	0	0	-0.0342	σ_{33}	0.0245
			(0.1381)			(0.0198)		(0.0005)
$K^P_{4,\cdot}$	0	0	0	0.3838	0	0.0365	σ_{44}	0.0065
				(0.1580)		(0.0082)		(0.0001)
$K^P_{5,\cdot}$	0	0	0	0	0.2781	0.0350	σ_{55}	0.0223
					(0.1699)	(0.0090)		(0.0006)

Table 18: **Estimated Dynamic Parameters for the CLR-L Model.**
The top panel shows the estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix for the CLR-L model using all available TIPS. The estimated value of λ is 0.3449 (0.0019), while $\alpha^R = 0.8115$ (0.0046), $\kappa^Q_{liq} = 0.8920$ (0.0439), and $\theta^Q_{liq} = 0.0033$ (0.0000). The numbers in parentheses are the estimated parameter standard deviations.

The estimated dynamic parameters reported in Table 18 are qualitatively similar to those reported in the main text for the CLR-L model.

In the model estimation, we change the benchmark TIPS with a unit β^i to be the first thirty-year TIPS. As a consequence, the estimated $(\beta^i, \lambda^{L,i})$ parameters reported in Table 19 are not comparable to those reported for the CLR-L model in the main text.

Tables 20 and 21 contain the summary statistics for the fitted errors to the Treasury and TIPS data, respectively. We note that there is a slight deterioration in the fit to the Treasury yields relative to the results reported in the main text. This was to be expected as the model is now fitting 32 yields with maturities up to thirty years. Therefore, it is also surprising that for the TIPS yields the model fit with all TIPS included is about as accurate as when we only include TIPS with up to ten years to maturity. These results suggest that, absent the added computational burden, there is no material loss in model performance from using all available TIPS information.

Figure 18 shows the estimated average TIPS liquidity premium when all available TIPS are used with a comparison to the result from the main text where twenty- and thirty-year TIPS are dropped from the sample. We note that the main difference is early in the sample when thirty-year TIPS represented more than one-third of the TIPS universe. This pushes up the average TIPS liquidity premium during that period quite notably and increase the overall average TIPS liquidity premium from 42.32 basis points to 56.59 basis points. However, starting in 2006, it makes little difference whether long-term TIPS are censored or not. This supports our choice to focus on the more liquid short- and medium-term TIPS in the main text.

TIPS security	CLR-L model			
	β^i	Std	$\lambda^{L,i}$	Std
(1) 3.375% 1/15/2007 TIPS	0.6156	0.0472	1.6088	0.6044
(2) 3.625% 7/15/2002 TIPS*	0.4930	0.0485	9.9602	2.1677
(3) 3.625% 1/15/2008 TIPS	0.7385	0.0559	0.7739	0.1033
(4) 3.625% 4/15/2028 TIPS+	1	n.a.	0.1984	0.0512
(5) 3.875% 1/15/2009 TIPS	0.9177	0.0754	0.4169	0.0458
(6) 3.875% 4/15/2029 TIPS+	1.0394	0.1223	0.1834	0.0977
(7) 4.25% 1/15/2010 TIPS	0.9962	0.0801	0.4808	0.0449
(8) 3.5% 1/15/2011 TIPS	1.1586	0.0899	0.4114	0.0300
(9) 3.375% 4/15/2032 TIPS+	0.7621	0.0338	9.7114	2.1318
(10) 3.375% 1/15/2012 TIPS	1.1154	0.0825	0.6343	0.0497
(11) 3% 7/15/2012 TIPS	1.0995	0.0776	0.9235	0.0780
(12) 1.875% 7/15/2013 TIPS	1.1185	0.0788	1.5461	0.3363
(13) 2% 1/15/2014 TIPS	1.2125	0.0899	1.5737	0.2828
(14) 2% 7/15/2014 TIPS	1.1827	0.0814	1.2365	0.3514
(15) 2.375% 1/15/2025 TIPS†	0.9217	0.0473	3.1034	2.0001
(16) 0.875% 4/15/2010 TIPS*	1.1426	0.0969	1.0874	0.3536
(17) 1.625% 1/15/2015 TIPS	1.4217	0.1304	0.3816	0.0637
(18) 1.875% 7/15/2015 TIPS	1.1757	0.0792	1.3193	0.6869
(19) 2% 1/15/2016 TIPS	1.3424	0.1044	0.5021	0.0860
(20) 2% 1/15/2026 TIPS†	0.8784	0.0568	9.9998	2.3096
(21) 2.375% 4/15/2011 TIPS*	1.0691	0.0785	9.9983	2.1896
(22) 2.5% 7/15/2016 TIPS	1.1068	0.0749	9.7787	2.1755
(23) 2.375% 1/15/2017 TIPS	1.2281	0.1117	0.6948	0.1877
(24) 2.375% 1/15/2027 TIPS†	0.8508	0.0588	9.3390	2.1504
(25) 2% 4/15/2012 TIPS*	0.9873	0.0703	9.9934	2.1888
(26) 2.625% 7/15/2017 TIPS	0.9846	0.0623	9.8470	2.3489
(27) 1.625% 1/15/2018 TIPS	1.1204	0.0978	0.5947	0.1598
(28) 1.75% 1/15/2028 TIPS†	0.7627	0.0666	3.9167	2.0232
(29) 0.625% 4/15/2013 TIPS*	1.8477	0.3981	0.2434	0.0640
(30) 1.375% 7/15/2018 TIPS	0.8320	0.0818	1.6068	0.5832
(31) 2.125% 1/15/2019 TIPS	1.1683	0.1924	0.3959	0.1469
(32) 2.5% 1/15/2029 TIPS†	0.7754	0.0605	9.9833	4.0255
(33) 1.25% 4/15/2014 TIPS*	3.9061	2.1098	0.1339	0.0869
(34) 1.875% 7/15/2019 TIPS	0.8462	0.0658	1.7844	1.0352
(35) 1.375% 1/15/2020 TIPS	0.9551	0.0829	4.2903	4.2870
(36) 2.125% 2/15/2040 TIPS+	1.2101	0.1069	0.7043	0.8112
(37) 0.5% 4/15/2015 TIPS*	2.6528	0.6092	0.2506	0.0843
(38) 1.25% 7/15/2020 TIPS	0.8429	0.0803	9.7223	5.0713
(39) 1.125% 1/15/2021 TIPS	0.9905	0.1076	9.1372	5.3841
(40) 2.125% 2/15/2041 TIPS+	1.3467	0.0851	9.9917	5.5035
(41) 0.125% 4/15/2016 TIPS*	4.2644	2.6032	0.1432	0.1084
(42) 0.625% 7/15/2021 TIPS	0.8147	0.0920	9.3584	6.4556
(43) 0.125% 1/15/2022 TIPS	7.4918	6.9771	0.0294	0.0305
(44) 0.75% 2/15/2042 TIPS+	1.5184	0.1801	9.9579	7.6768
(45) 0.125% 4/15/2017 TIPS*	1.7643	0.5628	0.6514	0.4668
(46) 0.125% 7/15/2022 TIPS	1.3656	3.1439	0.2365	1.0739
(47) 0.125% 1/15/2023 TIPS	1.8351	5.0833	0.2146	1.1379
(48) 0.625% 2/15/2043 TIPS+	1.5226	0.1953	9.9960	8.6099
(49) 0.125% 4/15/2018 TIPS*	1.3025	0.1227	9.9893	8.8887
(50) 0.375% 7/15/2023 TIPS	2.9068	12.2300	0.0791	0.4317

Table 19: **Estimated Liquidity Sensitivity Parameters.**
The estimated β^i sensitivity and $\lambda^{L,i}$ decay parameters for each TIPS from the CLR-L model are shown. Also reported are the estimated parameter standard deviations. Asterisk * indicates five-year TIPS, dagger † indicates twenty-year TIPS, and plus + indicates thirty-year TIPS. The sample used in the model estimation is weekly covering the period from July 11, 1997, to December 27, 2013.

Maturity in months	AFNS model		CLR-L model	
	Mean	RMSE	Mean	RMSE
3	0.67	15.96	0.49	17.03
6	-0.72	6.96	-0.69	7.72
12	-0.61	8.50	-0.33	8.37
24	0.56	15.04	0.97	15.52
36	0.53	14.90	0.81	15.74
48	0.02	11.88	0.11	12.78
60	-0.42	8.10	-0.53	8.87
72	-0.64	4.91	-0.88	5.34
84	-0.62	3.91	-0.95	3.76
96	-0.44	5.25	-0.80	4.90
108	-0.16	6.98	-0.52	6.71
120	0.13	8.34	-0.18	8.19
132	0.40	9.21	0.15	9.15
144	0.60	9.60	0.44	9.61
156	0.72	9.57	0.64	9.64
168	0.76	9.22	0.76	9.30
180	0.71	8.59	0.79	8.68
192	0.58	7.76	0.73	7.83
204	0.39	6.80	0.60	6.84
216	0.17	5.77	0.42	5.75
228	-0.08	4.73	0.20	4.64
240	-0.33	3.79	-0.04	3.61
252	-0.56	3.10	-0.28	2.87
264	-0.75	2.90	-0.48	2.71
276	-0.87	3.28	-0.64	3.24
288	-0.90	4.10	-0.73	4.21
300	-0.84	5.14	-0.74	5.39
312	-0.65	6.30	-0.64	6.67
324	-0.33	7.54	-0.41	8.00
336	0.14	8.85	-0.06	9.37
348	0.77	10.22	0.45	10.80
360	1.59	11.68	1.12	12.27
All yields	-0.01	8.49	-0.01	8.79

Table 20: **Summary Statistics of Fitted Errors of Nominal Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of nominal U.S. Treasury yields from the CLR-L model estimated with all available TIPS are shown. The full sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

TIPS security	CLR-L model	
	Mean	RMSE
(1) 3.375% 1/15/2007 TIPS	2.53	6.36
(2) 3.625% 7/15/2002 TIPS*	3.43	4.86
(3) 3.625% 1/15/2008 TIPS	1.82	4.71
(4) 3.625% 4/15/2028 TIPS+	1.91	4.13
(5) 3.875% 1/15/2009 TIPS	1.38	3.59
(6) 3.875% 4/15/2029 TIPS+	1.76	4.07
(7) 4.25% 1/15/2010 TIPS	0.36	3.38
(8) 3.5% 1/15/2011 TIPS	-0.28	5.06
(9) 3.375% 4/15/2032 TIPS+	1.03	4.58
(10) 3.375% 1/15/2012 TIPS	0.50	5.83
(11) 3% 7/15/2012 TIPS	0.22	5.40
(12) 1.875% 7/15/2013 TIPS	-0.40	7.15
(13) 2% 1/15/2014 TIPS	0.78	4.69
(14) 2% 7/15/2014 TIPS	-0.28	4.66
(15) 2.375% 1/15/2025 TIPS†	0.73	3.70
(16) 0.875% 4/15/2010 TIPS*	1.68	3.23
(17) 1.625% 1/15/2015 TIPS	0.73	4.91
(18) 1.875% 7/15/2015 TIPS	-0.29	4.70
(19) 2% 1/15/2016 TIPS	1.08	6.06
(20) 2% 1/15/2026 TIPS†	0.84	4.11
(21) 2.375% 4/15/2011 TIPS*	4.69	12.21
(22) 2.5% 7/15/2016 TIPS	-0.51	6.20
(23) 2.375% 1/15/2017 TIPS	0.62	5.27
(24) 2.375% 1/15/2027 TIPS†	0.89	3.32
(25) 2% 4/15/2012 TIPS*	5.56	12.23
(26) 2.625% 7/15/2017 TIPS	0.36	4.93
(27) 1.625% 1/15/2018 TIPS	0.08	5.10
(28) 1.75% 1/15/2028 TIPS†	0.55	3.18
(29) 0.625% 4/15/2013 TIPS*	1.13	12.41
(30) 1.375% 7/15/2018 TIPS	-0.03	5.53
(31) 2.125% 1/15/2019 TIPS	0.05	5.02
(32) 2.5% 1/15/2029 TIPS†	0.33	2.34
(33) 1.25% 4/15/2014 TIPS*	0.09	4.49
(34) 1.875% 7/15/2019 TIPS	0.03	4.45
(35) 1.375% 1/15/2020 TIPS	0.39	4.11
(36) 2.125% 2/15/2040 TIPS+	-1.13	5.33
(37) 0.5% 4/15/2015 TIPS*	0.72	4.72
(38) 1.25% 7/15/2020 TIPS	0.31	3.57
(39) 1.125% 1/15/2021 TIPS	0.63	4.69
(40) 2.125% 2/15/2041 TIPS+	-0.89	4.03
(41) 0.125% 4/15/2016 TIPS*	-0.06	4.11
(42) 0.625% 7/15/2021 TIPS	0.49	3.91
(43) 0.125% 1/15/2022 TIPS	0.14	3.18
(44) 0.75% 2/15/2042 TIPS+	-0.84	4.06
(45) 0.125% 4/15/2017 TIPS*	0.02	2.88
(46) 0.125% 7/15/2022 TIPS	0.19	3.02
(47) 0.125% 1/15/2023 TIPS	0.15	2.93
(48) 0.625% 2/15/2043 TIPS+	-0.89	4.61
(49) 0.125% 4/15/2018 TIPS*	0.01	2.90
(50) 0.375% 7/15/2023 TIPS	0.07	1.78
All TIPS yields	0.76	5.13
Max log L	236,226.7	

Table 21: **Summary Statistics of Fitted Errors of TIPS Yields.**
The mean fitted errors and the root mean squared fitted errors (RMSE) of the yield-to-maturity for individual TIPS according to the CLR-L model estimated with all available TIPS. Asterisk * indicates five-year TIPS, dagger † indicates twenty-year TIPS, and plus + indicates thirty-year TIPS. The sample used in each model estimation is weekly covering the period from July 11, 1997, to December 27, 2013. All numbers are measured in basis points.

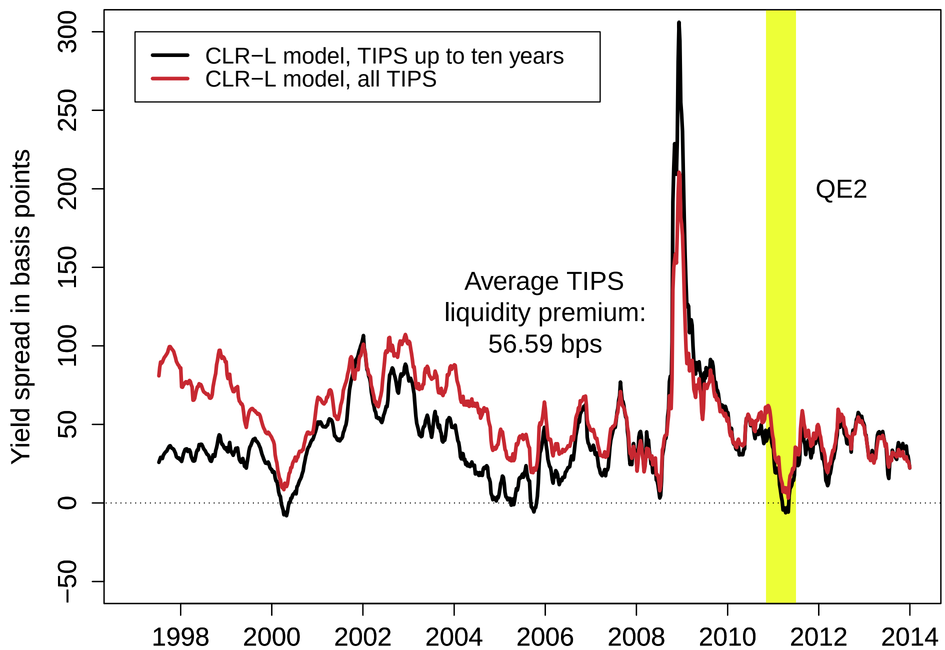


Figure 18: **Average Estimated TIPS Liquidity Premium.**
Illustration of the average estimated TIPS liquidity premium for each observation date implied by the CLR-L model estimated using all available TIPS. TIPS liquidity premiums are measured as the estimated yield difference between the fitted yield-to-maturity of individual TIPS and the corresponding frictionless yield-to-maturity with the liquidity risk factor turned off. The data cover the period from July 11, 1997, to December 27, 2013.

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