

# Micro to Macro: Optimal Trade Policy with Firm Heterogeneity\*

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## Abstract

The empirical observation that “large firms tend to export, whereas small firms do not” has transformed the way economists think about the determinants of international trade. Yet, it has had surprisingly little impact about how economists think about trade policy. In this paper, we characterize optimal trade policy in a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). At the micro-level, we find that optimal import taxes discriminate against the most profitable foreign exporters, while optimal export taxes are uniform across domestic exporters. At the macro-level, we demonstrate that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that dampen the incentives for terms-of-trade manipulation, and in turn, the overall level of trade protection.

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# 1 Introduction

There are large firms and small firms. The former tend to export whereas the latter do not. What are the policy implications of that empirical observation?

Models of firm heterogeneity have transformed the way economists think about the determinants of international trade. Yet, the same models have had surprisingly little impact about how economists think about trade policy.<sup>1</sup> The goal of this paper is to fill this large gap on the normative side of the literature and uncover the general principles that should guide the design of optimal trade policy when heterogeneous firms select into exporting.

Our basic environment is a generalization of the model of intra-industry trade with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). On the supply side, we let firms be heterogeneous in terms of both their variable costs and their fixed costs. We impose no restrictions on the joint distribution of these costs across firms and markets. On the demand side, we maintain the assumption that the elasticity of substitution between all varieties from a given country is constant, but we impose no restrictions on the substitutability between domestic and foreign goods.

The first part of our analysis studies the ad-valorem taxes that maximize domestic welfare, which we label unilaterally optimal taxes, when governments are free to impose different taxes on different firms. At the micro-level, we find that optimal trade policy requires firm-level import taxes that discriminate against the most profitable foreign exporters. In contrast, export taxes that discriminate against or in favor of the most profitable domestic exporters can be dispensed with. The fact that optimal import taxes discriminate against the most profitable exporters from abroad is reminiscent of an anti-dumping duty.<sup>2</sup> The rationale, however, is very different. Here, discriminatory taxes do not reflect a desire to deter the entry of the most profitable exporters. They reflect instead a desire to promote the entry of the marginally unprofitable exporters who, if they were to face the same tariff, would prefer not to export at all.

At the macro-level, standard terms-of-trade considerations pin down the overall level of trade taxes. Specifically, the only reason why a welfare-maximizing government would like to implement aggregate imports and exports that differ from those in the decentral-

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<sup>1</sup>The last handbook of international economics, Gopinath, Helpman and Rogoff, eds (2014), is a case in point. In their chapter on heterogeneous firms, Melitz and Redding (2014) have only one trade policy paper to cite. In his chapter on trade policy, Maggi (2014) has no paper with firm heterogeneity to review.

<sup>2</sup>According to U.S. law, dumping occurs when foreigners' export prices, adjusted for transportation costs, are lower than their domestic prices. The latter prices, however, are rarely available in practice, especially for non-market economies like China. Hence, as noted by Ruhl (2014), anti-dumping duties tend to be imposed on the most productive foreign exporters. This is the feature that we emphasize here as well.

ized equilibrium is because it internalizes the impact of both quantities on the price of the infra-marginal units that it buys and sells on the world markets. Like in a Walrasian economy, the higher the elasticity of world prices with respect to exports and imports, in absolute value, the larger the trade restriction that it optimally imposes.

The second part of our analysis focuses on optimal taxation under the polar assumption that governments are constrained to impose the same tax on all firms from the same country selling in a given market. Our main finding in this environment is a new optimal tariff formula that generalizes existing results in the literature. Under monopolistic competition with homogeneous firms, [Gros \(1987\)](#) has shown that optimal tariffs are determined by the elasticity of substitution between domestic and foreign goods and the share of expenditure on local goods abroad. Our new formula establishes that, conditional on these two statistics, firm heterogeneity lowers the overall level of trade protection if and only if it creates aggregate nonconvexities abroad.<sup>3</sup> When strong enough, these aggregate nonconvexities may even turn the optimal import tariff into a subsidy. In such circumstances, a government may *lower* the price of its imports by *raising* their volume, an example of the Lerner paradox.

The final part of our analysis extends our basic environment to incorporate intra- and inter-industry trade. In this case, sector-level increasing returns to scale, the so-called home market effects, can also shape optimal trade policy. The common wisdom in the literature ([Helpman and Krugman, 1989](#)) is that such effects provide a very different rationale for trade protection. Our last set of results suggests a different interpretation, one according to which home-market effects matter to the extent that they shape terms-of-trade elasticities, but not beyond.

While both the positive and normative implications of imperfectly competitive markets for international trade have been studied extensively, the same cannot be said of the heterogeneous firms operating in these markets. On the positive side, the pioneering work of [Melitz \(2003\)](#) has lead numerous researchers to revisit various results of [Helpman and Krugman \(1985\)](#) under the assumption that firms are heterogeneous and select into exporting. On the normative side, however, much less energy has been devoted to revisit the classical results of [Helpman and Krugman \(1989\)](#).

To the best of our knowledge, only three papers—[Demidova and Rodríguez-Clare \(2009\)](#), [Felbermayr, Jung and Larch \(2013\)](#), and [Haaland and Venables \(2014\)](#)—have used the work of [Melitz \(2003\)](#) to explore the implications of firm heterogeneity for optimal

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<sup>3</sup>With homogeneous firms (and the standard assumption in that context that there are no fixed costs of trade), aggregate production possibility frontiers are necessarily linear. With heterogeneous firms and fixed trade costs, nonconvexities are likely to arise, as the mild sufficient conditions of Section 5.4 establish.

trade policy. All three papers are restricted to environments where utility functions are CES; fixed costs of exporting are constant across firms; distributions of firm-level productivity are Pareto; and, importantly, trade taxes are uniform across firms.<sup>4</sup> In this paper, we relax all of these assumptions, we derive new results about optimal trade taxes at the micro-level, and we generalize prior results about optimal trade taxes at the macro-level. Beside greater generality, these results uncover a novel connection between firm heterogeneity, aggregate nonconvexities, and lower levels of trade protection.

In terms of methodology, our analysis builds on the work of [Costinot, Lorenzoni and Werning \(2014\)](#) and [Costinot, Donaldson, Vogel and Werning \(2015\)](#) who characterize the structure of optimal trade taxes in a dynamic endowment economy and a static Ricardian economy, respectively. Like in the two previous papers, we use a primal approach and general Lagrange multiplier methods to characterize optimal wedges rather than explicit policy instruments. The novel aspect of our analysis is to break down the problem of finding optimal wedges into a series of micro subproblems, where we study how to choose micro-level quantities to deliver aggregate quantities at the lowest possible costs, and a macro problem, where we solve for the optimal aggregate quantities. The solutions to the micro and macro problems then determine the structure of optimal micro and macro taxes described above. This decomposition helps to highlight the deep connection between standard terms-of-trade arguments, as in [Baldwin \(1948\)](#) and [Dixit \(1985\)](#), and the design of optimal trade policy in models of monopolistic competition.

In spite of their common rationale, i.e., terms-of-trade manipulation, the specific policy prescriptions derived under perfect and monopolistic competition differ sharply. In [Costinot, Donaldson, Vogel and Werning \(2015\)](#), optimal export taxes should be heterogeneous, whereas optimal import tariffs should be uniform. This is the exact opposite of what we find under monopolistic competition. In a Ricardian economy, goods exported by domestic firms could also be produced by foreign firms. This threat of foreign entry limits the ability of the domestic government to manipulate world prices and leads to lower export taxes on goods for which its firms have a weaker comparative advantage. Since the previous threat is absent under monopolistic competition, optimal export taxes are uniform instead. Conversely, lower import tariffs on the least profitable foreign firms under monopolistic competition derive from the existence of fixed exporting costs, which

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<sup>4</sup>A fourth paper by [Demidova \(2015\)](#) analyzes optimal trade policy under the assumption of quadratic utility functions, similar to those in [Melitz and Ottaviano \(2008\)](#). All other assumptions are the same as in the aforementioned papers. In this environment, markups vary across firms, which leads to domestic distortions even within the same industry and opens up the possibility of terms-of-trade manipulation even at the firm-level. Our baseline analysis abstracts from these issues and instead focuses on the implication of the self-selection of heterogeneous firms into export markets, as in [Melitz \(2003\)](#). We come back to this point in our concluding remarks.

are necessarily absent under perfect competition.

The previous discussion is related to recent results by [Ossa \(2011\)](#) and [Bagwell and Staiger \(2012b,a, 2015\)](#) on whether imperfectly competitive markets create a new rationale for the design of trade agreements. We hope that our analysis can contribute to the application of models with firm heterogeneity to study this question as well as other related trade policy issues. [Bagwell and Lee \(2015\)](#) offer an interesting first step in that direction. They study trade policy in a symmetric version of the [Melitz and Ottaviano \(2008\)](#) model that also features the selection of heterogeneous firms into exporting. They show that this model provides a rationale for the treatment of export subsidies within the World Trade Organization.

The rest of the paper is organized as follows. Section 2 describes our basic environment. Section 3 sets up and solves the micro and macro planning problems of a welfare-maximizing country manipulating its terms-of-trade. Section 4 shows how to decentralize the solution to the planning problems through micro and macro trade taxes when governments are free to discriminate across firms. Section 5 studies the polar case where governments can only impose uniform taxes. Section 6 explores the sensitivity of our results to the introduction of multiple industries. Section 7 offers some concluding remarks.

## 2 Basic Environment

### 2.1 Technology, Preferences, and Market Structure

Consider a world economy with two countries, indexed by  $i = H, F$ ; one factor of production, labor; and a continuum of differentiated goods or varieties. Labor is immobile across countries.  $w_i$  and  $L_i$  denote the wage and the inelastic supply of labor in country  $i$ , respectively.<sup>5</sup>

**Technology.** Producing any variety in country  $i$  requires an overhead fixed entry cost,  $f_i^e > 0$ , in terms of domestic labor. Once the overhead fixed cost has been paid, firms randomly draw a blueprint  $\varphi \in \Phi$ .  $N_i$  denotes the measures of entrants in country  $i$  and  $G_i$  denotes the distribution of blueprints  $\varphi$  across firms in that country. Each blueprint describes how to produce and deliver a firm's differentiated variety to any country.  $l_{ij}(q, \varphi)$  denotes the total amount of labor needed by a firm from country  $i$  with blueprint  $\varphi$  in

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<sup>5</sup>The two-country assumption will help us relate our results to those from the existing literature in Section 5. With more than two countries, all the micro-level predictions described in Section 4 would remain unchanged. We discuss how macro-level predictions would change in Section 6.

order to produce and deliver  $q \geq 0$  units in country  $j$ . We assume that

$$\begin{aligned} l_{ij}(q, \varphi) &= a_{ij}(\varphi)q + f_{ij}(\varphi), \text{ if } q > 0, \\ l_{ij}(q, \varphi) &= 0, \text{ if } q = 0. \end{aligned}$$

Technology in [Krugman \(1980\)](#) corresponds to the special case in which  $G_i$  has all its mass at a single blueprint with zero fixed cost. Technology in [Melitz \(2003\)](#) corresponds to the special case in which firms have heterogeneous productivity, but face homogenous iceberg trade costs,  $a_{ij}(\varphi) \equiv \tau_{ij}/\varphi$ , and homogenous fixed costs of selling in the two markets,  $f_{ij}(\varphi) \equiv f_{ij}$ .

**Preferences.** In each country there is a representative agent with a two-level homothetic utility function,

$$\begin{aligned} U_j &= U_j(Q_{Hj}, Q_{Fj}), \\ Q_{ij} &= \left[ \int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi) \right]^{\mu_i}. \end{aligned}$$

where  $Q_{ij}$  denotes the subutility aggregator from consuming varieties from country  $i$  in country  $j$ ,  $q_{ij}(\varphi)$  denotes country  $j$ 's consumption of a variety with blueprint  $\varphi$  produced in country  $i$ , and  $\mu_i \equiv \sigma_i/(\sigma_i - 1)$ , with  $\sigma_i > 1$  the elasticity of substitution between varieties from country  $i$ . [Krugman \(1980\)](#) and [Melitz \(2003\)](#) correspond to the case where  $\mu_H = \mu_F \equiv \mu$  and  $U_j(Q_{Hj}, Q_{Fj}) \equiv [Q_{Hj}^{1/\mu} + Q_{Fj}^{1/\mu}]^\mu$ .<sup>6</sup> Here, we do not restrict the elasticities of substitution,  $\sigma_H$  and  $\sigma_F$ , to be the same, nor do we restrict the upper-level utility function,  $U_j$ .

**Market Structure.** All goods markets are monopolistically competitive with free entry. All labor markets are perfectly competitive. Foreign labor is our numeraire,  $w_F = 1$ .

## 2.2 Decentralized Equilibrium with Taxes

Our focus is on a scenario where governments have access to a full set of ad-valorem consumption and production taxes. That is, we let taxes vary across markets *and* across firms. Section 5 considers a more restricted case, where taxes can vary across markets but are required to be uniform across firms.

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<sup>6</sup>In Section 5.3, we will exploit this slight generalization to disentangle terms-of-trade motives for protection from second-best arguments related to markup distortions.

We view the availability of a rich set of taxes as a useful benchmark for our analysis. In theory, there is a priori no reason within the model that we consider why different goods should face the same taxes. In an Arrow-Debreu economy, imposing the same taxes on arbitrary subsets of goods would be ad-hoc. Changing the market structure from perfect to monopolistic competition does not make it less so. In practice, perhaps more importantly, different firms do face different trade taxes, even within the same narrowly defined industry. Anti-dumping duties often act as import tariffs imposed on the most productive firms. Loan subsidies provided to small exporters in many countries can also be thought of as export subsidies that vary with firms' productivity.<sup>7</sup>

Formally, we let  $t_{ij}(\varphi)$  denote the tax charged by country  $j$  on the consumption in country  $j$  of a variety with blueprint  $\varphi$  produced in country  $i$ . Let  $s_{ij}(\varphi)$  denote the subsidy paid by country  $i$  on the production by a domestic firm of a variety with blueprint  $\varphi$  sold in country  $j$ . For  $i \neq j$ ,  $t_{ij}(\varphi) > 0$  corresponds to an import tariff while  $t_{ij}(\varphi) < 0$  corresponds to an import subsidy. Similarly,  $s_{ij}(\varphi) > 0$  corresponds to an export subsidy while  $s_{ij}(\varphi) < 0$  corresponds to an export tax. Tax revenues are rebated to domestic consumers through a lump-sum transfer,  $T_i$ .<sup>8</sup>

In a decentralized equilibrium with taxes, consumers choose consumption to maximize their utility subject to their budget constraint; firms choose their output to maximize profits, taking their residual demand curves as given; firms enter up to the point at which expected profits are zero; markets clear; and the government's budget is balanced in each country. Let  $\bar{p}_{ij}(\varphi) \equiv \mu_i w_i a_{ij}(\varphi) / (1 + s_{ij}(\varphi))$  and  $\bar{q}_{ij}(\varphi) \equiv [(1 + t_{ij}(\varphi)) \bar{p}_{ij}(\varphi) / P_{ij}]^{-\sigma_i} Q_{ij}$ . Using the previous notation, we can characterize a decentralized equilibrium with taxes as schedules of output,  $\mathbf{q}_{ij} \equiv \{q_{ij}(\varphi)\}$ , schedules of prices,  $\mathbf{p}_{ij} \equiv \{p_{ij}(\varphi)\}$ , aggregate output levels,  $Q_{ij}$ , aggregate price indices,  $P_{ij}$ , wages,  $w_i$ , and measures of entrants,  $N_i$ , such

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<sup>7</sup>Firms operating in the same industry, say "Cotton, not carded or combed" (HS8 520100), may also face different tariffs because they are producing different varieties. As Kim (2016) notes, the most favoured nation (MFN) tariff rate applied by the United States for "Cotton, not carded or combed, having staple length of 28.575 mm or more but under 34.925 mm (HS8 52010038)" is 14%, as of 2013, whereas "Cotton, not carded or combed, having a staple length under 19.05mm (3/4 inch), harsh or rough (HS8 52010005)" is duty free.

<sup>8</sup>When firm-level taxes are allowed, as in our baseline analysis, our focus on ad-valorem rather than specific taxes is without loss of generality. Ruling out non-linear taxes, like two part-tariffs, is not. We discuss how the introduction of such instruments would affect our results in Section 4.4.



that

$$q_{ij}(\varphi) = \begin{cases} \bar{q}_{ij}(\varphi) & \text{if } \mu_i a_{ij}(\varphi) \bar{q}_{ij}(\varphi) \geq l_{ij}(\bar{q}_{ij}(\varphi), \varphi), \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

$$p_{ij}(\varphi) = \begin{cases} \bar{p}_{ij}(\varphi) & \text{if } \mu_i a_{ij}(\varphi) q_{ij}(\varphi) \geq l_{ij}(q_{ij}(\varphi), \varphi), \\ \infty & \text{otherwise,} \end{cases} \quad (2)$$

$$Q_{Hj}, Q_{Fj} \in \arg \max_{\tilde{Q}_{Hj}, \tilde{Q}_{Fj}} \{U_j(\tilde{Q}_{Hj}, \tilde{Q}_{Fj}) | \sum_{i=H,F} P_{ij} \tilde{Q}_{ij} = w_j L_j + T_j\}, \quad (3)$$

$$P_{ij}^{1-\sigma_j} = \int_{\Phi} N_i [(1 + t_{ij}(\varphi)) p_{ij}(\varphi)]^{1-\sigma_i} dG_i(\varphi), \quad (4)$$

$$f_i^e = \sum_{j=H,F} \int_{\Phi} [\mu_i a_{ij}(\varphi) q_{ij}(\varphi) - l_{ij}(q_{ij}(\varphi), \varphi)] dG_i(\varphi), \quad (5)$$

$$L_i = N_i [\sum_{j=H,F} \int_{\Phi} l_{ij}(q_{ij}(\varphi), \varphi) dG_i(\varphi) + f_i^e], \quad (6)$$

$$T_i = \sum_{j=H,F} \left[ \int_{\Phi} N_j t_{ji}(\varphi) p_{ji}(\varphi) q_{ji}(\varphi) dG_j(\varphi) - \int_{\Phi} N_i s_{ij}(\varphi) p_{ij}(\varphi) q_{ij}(\varphi) dG_i(\varphi) \right]. \quad (7)$$

Conditions (1) and (2) assume that firms that are indifferent between producing and not producing, produce. This is without loss of generality since, to simplify, we assume indifference is measure zero. We do so by adopting the sufficient condition that conditional on a positive value for the fixed cost  $f_{ij} > 0$  the distribution over the variable cost  $a_{ij}$  is continuous.<sup>9</sup> Throughout our analysis, we restrict attention to the interesting nontrivial cases where preferences and trade costs are such that the utility maximization problem (3) admits an interior solution. This rules out equilibria without trade ( $Q_{FH} = Q_{HF} = 0$ ) or without domestic production ( $Q_{HH} = Q_{FF} = 0$ ).

## 2.3 Unilaterally Optimal Taxation

We assume that the government of country  $H$ , which we refer to as the home government, is strategic, whereas the government of country  $F$ , which we refer to as the foreign government, is passive. Namely, the home government sets ad-valorem taxes,  $\mathbf{t}_{HH} \equiv \{t_{HH}(\varphi)\}$ ,  $\mathbf{t}_{FH} \equiv \{t_{FH}(\varphi)\}$ ,  $\mathbf{s}_{HH} \equiv \{s_{HH}(\varphi)\}$ , and  $\mathbf{s}_{HF} \equiv \{s_{HF}(\varphi)\}$ , and a lump-sum transfer  $T_H$  in order to maximize home welfare, whereas foreign taxes are all equal to zero. We conjecture that our qualitative results would remain unchanged in the presence of taxes

<sup>9</sup>Note that this condition rules out mass points but only for strictly positive fixed costs. The distribution conditional on  $f_{ij} = 0$  for  $a_{ij}$  may have mass points. Thus, the model in Krugman (1980), which has no heterogeneity but no fixed costs of exporting, satisfies our requirement.



abroad.<sup>10</sup> This leads to the following definition of the home government's problem.

**Definition 1.** *The home government's problem is*

$$\max_{T_H, \{t_{jH}, s_{Hj}\}_{j=H,F}, w_H, \{q_{ij}, Q_{ij}, p_{ij}, P_{ij}, N_i\}_{i,j=H,F}} U_H(Q_{HH}, Q_{FH})$$

*subject to conditions (1)-(7).*

The goal of the next two sections is to characterize unilaterally optimal taxes, i.e., taxes that prevail at a solution to the domestic government's problem. To do so we follow the public finance literature and use the primal approach. Namely, we will first approach the optimal policy problem of the domestic government in terms of a relaxed planning problem in which domestic consumption, output, and the measure of entrants can be chosen directly (Section 3). We will then establish that the optimal allocation can be implemented through linear taxes and characterize the structure of these taxes (Section 4).

### 3 Micro and Macro Planning Problems

In this section, we focus on a relaxed version of the home government's problem that abstracts from all constraints in which Home's tax instruments,  $T_H, \{t_{jH}, s_{Hj}\}_{j=H,F}$ , and Home's prices,  $w_H, \{p_{Hj}\}_{j=H,F}$ , appear. This relaxed problem can be interpreted as the problem of a fictitious planner who directly controls the quantities demanded by home consumers,  $\mathbf{q}_{HH} \equiv \{q_{HH}(\varphi)\}$  and  $\mathbf{q}_{FH} \equiv \{q_{FH}(\varphi)\}$ , as well as the quantities exported by home firms,  $\mathbf{q}_{HF} \equiv \{q_{HF}(\varphi)\}$ , and the measure of home entrants,  $N_H$ . Specifically, we drop conditions (2), (5), and (7) for  $i = H$ ; we drop condition (3) for  $j = H$ ; and we relax conditions (1) and (4) for  $i = H$  or  $j = H$  by imposing instead

$$\int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi) \geq Q_{ij}^{1/\mu_i}, \text{ for } i = H \text{ or } j = H. \quad (8)$$

We refer to this new problem as Home's relaxed planning problem; see Appendix A.

In order to solve this relaxed problem, we take advantage of the nested structure of preferences in this economy and follow a two-step approach. First, we take macro quantities,  $\{Q_{ij}\}$ , and the measure of entrants,  $\{N_i\}$ , as given and solve for the micro quantities,  $\{q_{ij}\}$ , that deliver macro quantities at the lowest possible costs. Second, we solve for the

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<sup>10</sup>Accordingly, our results should also hold in the Nash equilibrium of a simultaneous game in which both countries behave strategically.

optimal macro quantities and measure of entrants. The solution to these micro and macro problems will determine the optimal micro and macro taxes, respectively, in Section 4.<sup>11</sup>

### 3.1 First Micro Problem: Producing Domestic Varieties

Consider the problem of minimizing the labor cost of producing  $Q_{HH}$  units of aggregate consumption for Home and  $Q_{HF}$  units of aggregate consumption for Foreign taking as given the measure of entrants,  $N_H$ , and constraint (8) for  $i = H$  and  $j = H, F$ . This can be expressed as

$$L_H(Q_{HH}, Q_{HF}, N_H) \equiv \min_{\mathbf{q}_{HH}, \mathbf{q}_{HF}} N_H \left[ \sum_{j=H,F} \int_{\Phi} l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) + f_H^e \right] \quad (9a)$$

$$\int_{\Phi} N_H(q_{Hj}(\varphi))^{1/\mu_H} dG_H(\varphi) \geq Q_{Hj}^{1/\mu_H}, \text{ for } j = H, F. \quad (9b)$$

This minimization problem is infinite dimensional and non-smooth. More precisely, since there are fixed costs, the objective function is neither continuous nor convex around  $q_{Hj}(\varphi) = 0$  for any  $\varphi$  such that  $f_{Hj}(\varphi) > 0$ . To deal with the previous issues and derive necessary properties that any solution to (9) must satisfy, we adopt the following strategy.

First, we consider a planning problem that extends (9) by allowing for randomization: conditional on  $\varphi$ , we let the planner select a distribution of output levels.<sup>12</sup> Since this problem is convex, we can invoke Lagrangian necessity theorems. We then show that randomization is not employed at any solution to the extended problem, so that the planner effectively solves (9). It follows that any solution to (9) must minimize the Lagrangian,

$$\mathcal{L}_H \equiv N_H \left[ \sum_{j=H,F} \int_{\Phi} \left( l_{Hj}(q_{Hj}(\varphi), \varphi) - \lambda_{Hj}(q_{Hj}(\varphi))^{1/\mu_H} \right) dG_H(\varphi) + f_H^e \right],$$

for some Lagrange multipliers,  $\lambda_{Hj} > 0$ . The complete argument can be found in Appendix A.1.

Second, we use the additive separability of the Lagrangian  $\mathcal{L}_H$  in  $\{q_{Hj}(\varphi)\}$  to minimize it variety-by-variety and market-by-market, as in [Everett \(1963\)](#), [Costinot, Lorenzoni and Werning \(2014\)](#), and [Costinot, Donaldson, Vogel and Werning \(2015\)](#). Although the

<sup>11</sup>Together with the foreign equilibrium conditions, the previous variables determine all foreign prices at the solution of Home's relaxed planning problem. For expositional purposes, we omit the description of these variables from the main text and present them in Appendix A.2.

<sup>12</sup>There are two interpretations of this randomization. In the first, a firm with a blueprint  $\varphi$  is randomly assigned a  $q$  according to this conditional distribution; in the second, there is a continuum of firms for a given  $\varphi$  and each firm is assigned a different  $q$  so that the population is distributed according to the conditional distribution.

discontinuity at zero remains, it is just a series of one-dimensional minimization problems that can be solved by hand. Namely, for a given variety  $\varphi$  and a market  $j$ , consider the one-dimensional subproblem

$$\min_q l_{Hj}(q, \varphi) - \lambda_{Hj} q^{1/\mu_H},$$

The solution to this problem follows a simple cut-off rule, which must then apply to any solution,  $q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H)$ , to the original constrained problem (9),

$$q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H) = \begin{cases} (\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma_H}, & \text{if } \varphi \in \Phi_{Hj}, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

with  $\Phi_{Hj} \equiv \{\varphi : (\mu_H - 1)a_{Hj}(\varphi)(\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma_H} \geq f_{Hj}(\varphi)\}$  the set of domestically produced varieties sold in country  $j$ .<sup>13</sup>

By comparing equations (1) and (2) with equation (10), one can already see that for all produced varieties, relative output levels at the solution to the relaxed planning problem coincide with those in the decentralized equilibrium with zero taxes. This derives from the fact that firms charge a constant markup over marginal cost under monopolistic competition with Constant Elasticity of Substitution (CES) utility. As shown in Section 4, this implies that the home government may want to impose a uniform import tariff or an export tax—in order to manipulate the fraction of labor allocated to domestic production rather than exports—but that it never wants to impose taxes that vary across domestic firms selling in a particular market.

### 3.2 Second Micro Problem: Importing Foreign Varieties

Next consider the problem of minimizing the cost of Home's imports,  $C_{FH}(Q_{FH}, Q_{FF}, N_F)$ , conditional on Foreign's aggregate output in the two markets,  $Q_{FH}$  and  $Q_{FF}$ , as well as the measure of foreign entrants,  $N_F$ , subject to condition (8) for  $i = F$  and  $j = H$  as well as Foreign's equilibrium conditions, i.e, conditions (1) and (4) for  $i = F$  and  $j = F$  and (2),

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<sup>13</sup>Given the previous definition, equation (10) implies that when indifferent between producing or not, the planner chooses producing. Since indifference is measure zero, this is without loss of generality.

(5), (6) for  $i = F$ . This second micro problem can be reduced to

$$C_{FH}(Q_{FH}, Q_{FF}, N_F) \equiv \min_{q_{FH}} \int_{\Phi} N_F \mu_F a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi) \quad (11a)$$

$$\int_{\Phi} N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi) \geq Q_{FH}^{1/\mu_F}, \quad (11b)$$

$$N_F f_F^e = \Pi_{FF}(Q_{FF}, N_F) + N_F \int_{\Phi} [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi), \quad (11c)$$

$$L_F = N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_{\Phi} l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi), \quad (11d)$$

$$\mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi), \quad (11e)$$

where  $\Pi_{FF}(Q_{FF}, N_F)$  and  $L_{FF}(Q_{FF}, N_F)$  denote the total profits and total employment associated with the local sales of foreign firms, which themselves depend on the local prices and quantities in Foreign,  $\mathbf{p}_{FF}(Q_{FF}, N_F)$ ,  $P_{FF}(Q_{FF}, N_F)$ , and  $\mathbf{q}_{FF}(Q_{FF}, N_F)$ . The previous values can be computed using conditions (1), (2), and (4), as discussed in Appendix A.2.

In what follows, we let  $\mathcal{Z}_F$  denote the set of output levels and measures of foreign entrants,  $(Q_{FH}, Q_{FF}, N_F)$ , for which there exist  $\mathbf{q}_{FH}$  that satisfy constraints (11b)-(11e). To deal with the non-smoothness and non-convexities of the minimization problem (11) and derive necessary properties that any of its solution must satisfy, we can follow a similar strategy as in Section 3.1. Technical details can be found in Appendix A.2.

Consider the one-dimensional subproblem of finding the amount of foreign imports of variety  $\varphi$  that solves

$$\min_q (1 - \lambda_E) \mu_F a_{FH}(\varphi) q + (\lambda_E + \lambda_L) l_{FH}(q, \varphi) - \lambda_{FH} q^{1/\mu_F} \quad (12a)$$

$$\mu_F a_{FH}(\varphi) q \geq l_{FH}(q, \varphi), \quad (12b)$$

where  $\lambda_{FH} \geq 0$ ,  $\lambda_E$ , and  $\lambda_L$  are the Lagrange multipliers associated with (11b)-(11d).<sup>14</sup> The solution to the unconstrained problem, ignoring inequality (12b), is given by

$$q_{FH}^u(\varphi) = \begin{cases} (\mu_F \chi_{FH} a_{FH}(\varphi) / \lambda_{FH})^{-\sigma_F}, & \text{if } \theta_{FH}(\varphi) \geq (\max\{0, (\lambda_E + \lambda_L) / \chi_{FH}\})^{1/\sigma_F}, \\ 0, & \text{otherwise,} \end{cases}$$

with  $\theta_{FH}(\varphi) \equiv (\lambda_{FH} / \mu_F \chi_{FH}) [(\mu_F - 1)(a_{FH}(\varphi))^{1-\sigma_F} / f_{FH}(\varphi)]^{1/\sigma_F}$  and  $\chi_{FH} \equiv \mu_F(1 -$

<sup>14</sup>Compared to Section 3.1,  $\lambda_{FH}$  may, in general, be zero, whereas  $\lambda_L$  and  $\lambda_E$ , which are associated with equality constraints, may be negative.

$\lambda_E) + (\lambda_L + \lambda_E) > 0$ .<sup>15</sup> In what follows, we refer to  $\theta_{FH}(\varphi)$  as the “profitability” of foreign varieties in Home’s market. If  $q_{FH}^u(\varphi)$  satisfies constraint (12b), then it is also a solution to (12). If it does not, then the solution to (12) is given either by zero or by  $q_{FH}^c(\varphi) > q_{FH}^u(\varphi)$  such that (12b) exactly binds, that is

$$q_{FH}^c(\varphi) = f_{FH}(\varphi) / ((\mu_F - 1)a_{FH}(\varphi)).$$

The former case occurs if  $\chi_{FH}a_{FH}(\varphi)q_{FH}^c(\varphi) + (\lambda_E + \lambda_L)f_{FH}(\varphi) - \lambda_{FH}(q_{FH}^c(\varphi))^{1/\mu_F} > 0$ , while the latter case occurs otherwise. Accordingly, we can express the solution to our second micro problem in a compact way as

$$q_{FH}(\varphi | Q_{FH}, Q_{FF}, N_F) = \begin{cases} (\mu_F \chi_{FH} a_{FH}(\varphi) / \lambda_{FH})^{-\sigma_F} & , \text{ if } \varphi \in \Phi_{FH}^u, \\ f_{FH}(\varphi) / ((\mu_F - 1)a_{FH}(\varphi)) & , \text{ if } \varphi \in \Phi_{FH}^c, \\ 0 & , \text{ otherwise,} \end{cases} \quad (13)$$

with  $\Phi_{FH}^u \equiv \{\varphi : \theta_{FH}(\varphi) \in [(\max\{1, (\lambda_E + \lambda_L)/\chi_{FH}\})^{1/\sigma_F}, \infty)\}$  and  $\Phi_{FH}^c \equiv \{\varphi : \theta_{FH}(\varphi) \in [(\chi_{FH} + (\mu_F - 1)(\lambda_E + \lambda_L))/\mu_F \chi_{FH}, 1)\}$ .

The set  $\Phi_{FH}^c$  will play a key role in our subsequent analysis. For varieties  $\varphi \in \Phi_{FH}^c$ , Home finds it optimal to alter its importing decision to make sure that foreign firms are willing to produce and export strictly positive amounts. Whether the previous set is empty or not depends on the importance of selection, as captured by constraint (11e), relative to general equilibrium considerations, as captured by constraints (11c) and (11d). In the case of a small open economy that cannot affect the free entry and labor market clearing conditions in Foreign ( $\lambda_E = \lambda_L = 0$ ), one can check that both intervals,  $[(\max\{1, (\lambda_E + \lambda_L)/\chi_{FH}\})^{1/\sigma_F}, \infty)$  and  $[(\chi_{FH} + (\lambda_E + \lambda_L)(\mu_F - 1))/\mu_F \chi_{FH}, 1)$ , are non-empty. If so, Home will find it optimal to impose lower taxes on the least profitable firms, as shown in Section 4.

### 3.3 Macro Problem: Manipulating Terms-of-Trade

Finally, consider the choice of macro quantities,  $\{Q_{ij}\}$ , and the measure of entrants,  $\{N_i\}$ , that maximize  $U_H(Q_{HH}, Q_{FH})$  subject to the domestic resource constraint (6) and the remaining foreign equilibrium condition, namely utility maximization in Foreign. Given

<sup>15</sup>  $\chi_{FH} > 0$  is necessary for the solution of the Lagrangian problem to satisfy constraints (11b)-(11d).

the analysis of Section 3.1, constraint (6) can be expressed as

$$L_H(Q_{HH}, Q_{HF}, N_H) = L_H. \quad (14)$$

Given the analysis of Section 3.2, condition (3) for  $j = F$ , in turn, implies

$$U_{HF}/U_{FF} = P_{HF}/P_{FF}(Q_{FF}, N_F), \quad (15)$$

$$P_{FF}(Q_{FF}, N_F)Q_{FF} + P_{HF}Q_{HF} = L_F, \quad (16)$$

where  $U_{iF} \equiv \partial U_F(Q_{HF}, Q_{FF})/\partial Q_{iF}$  denotes the marginal utility abroad of the aggregate good from country  $i = H, F$  and  $P_{FF}(Q_{FF}, N_F)$  is the price index for locally produced goods in Foreign. Equation (15) determines Home's export price,  $P_{HF}(Q_{HF}, Q_{FF}, N_F)$ , as a function of macro quantities and the measure of foreign entrants. Together with the budget constraint (16)—and the free entry and labor market clearing conditions, (11c) and (11d)—this leads to the following trade balance condition,

$$P_{HF}(Q_{HF}, Q_{FF}, N_F)Q_{HF} = C_{FH}(Q_{FH}, Q_{FF}, N_F). \quad (17)$$

Combining equations (14) and (17), we conclude that optimal aggregate quantities must solve the following macro problem,

$$\max_{Q_{HH}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}) \quad (18a)$$

$$P(Q_{FH}, Q_{HF})Q_{HF} = Q_{FH}, \quad (18b)$$

$$L_H(Q_{HH}, Q_{HF}) = L_H, \quad (18c)$$

where the price of Home's exports relative to its imports is given by

$$P(Q_{FH}, Q_{HF}) \equiv \max_{Q_{FF}, N_F: (Q_{FH}, Q_{FF}, N_F) \in \mathcal{Z}_F} \{P_{HF}(Q_{HF}, Q_{FF}, N_F)Q_{FH}/C_{FH}(Q_{FH}, Q_{FF}, N_F)\},$$

and Home's production possibility frontier is determined by

$$L_H(Q_{HH}, Q_{HF}) \equiv \min_{N_H} L_H(Q_{HH}, Q_{HF}, N_H).$$

At this point, it should be clear that we are back to a standard terms-of-trade manipulation problem, with Home's planner internalizing the impact of its aggregate imports and exports,  $Q_{FH}$  and  $Q_{HF}$ , on its terms-of-trade,  $P$ . Compared to the decentralized equilibrium where consumers and firms take  $P$  as given, this introduces curvature into Home's

consumption possibility frontier; see e.g. [Baldwin \(1948\)](#). Like in a perfectly competitive model of international trade, foreign technology, endowments, and preferences only matter through their combined effect on Home's terms-of-trade.

Let  $(Q_{HH}^*, Q_{FH}^*, Q_{HF}^*)$  be an interior solution of (18), which will be the focus of our analysis. The necessary first-order conditions imply

$$U_{HH}^* = \Lambda_H L_{HH}^*, \quad (19)$$

$$U_{FH}^* = \Lambda_T (1 - (P^* Q_{HF}^* / Q_{FH}^*) \eta_{FH}^*), \quad (20)$$

$$\Lambda_T P^* (1 + \eta_{HF}^*) = \Lambda_H L_{HF}^*, \quad (21)$$

where  $U_{iH}^* \equiv \partial U_H(Q_{HH}^*, Q_{FH}^*) / \partial Q_{iH}$  denotes the marginal utility at home of the aggregate good from country  $i = H, F$ ;  $L_{Hj}^* \equiv \partial L_H(Q_{HH}^*, Q_{HF}^*) / \partial Q_{Hj}$  denotes the marginal cost of producing and delivering one unit of the home good in country  $j = H, F$ ;  $\Lambda_T$  and  $\Lambda_H$  are the Lagrange multipliers associated with constraints (18b) and (18c); and  $\eta_{ij}^* \equiv \partial \ln P(Q_{FH}^*, Q_{HF}^*) / \partial \ln Q_{ij}$ , with  $i \neq j$ , denotes the elasticity of Home's terms of trade with respect to exports and imports. For future reference, note that the trade balance condition (18b) and the first-order conditions (19)-(21) imply

$$MRS_H^* = MRT_H^* P^* / (1 + \tau^*), \quad (22)$$

where  $MRS_H^* \equiv U_{HH}^* / U_{FH}^*$  denotes the marginal rate of substitution at home,  $MRT_H^* \equiv L_{HH}^* / L_{HF}^*$  denotes the marginal rate of transformation, and

$$\tau^* = -(\eta_{HF}^* + \eta_{FH}^*) / (1 + \eta_{HF}^*)$$

is the wedge that captures the terms-of-trade motive. Absent this motive, the only difference between  $MRS_H^*$  and  $P^*$  would be coming from the cost of producing Home's aggregate good for the domestic market relative to the foreign market, that is  $MRT_H^*$ . If there are no trade frictions, including no fixed exporting costs, then  $MRT_H^* = 1$  and equation (22) reduces to the familiar condition for an optimal tariff:  $MRS_H^* = P^* / (1 + \tau^*)$ .

## 4 Optimal Taxes

In the previous section, we have derived three necessary conditions—equations (10), (13), and (22)—that micro and macro quantities solving Home's relaxed planning problem must satisfy. We now use these conditions to derive necessary properties that ad-valorem



taxes implementing such a solution must satisfy (Sections 4.1-4.3). We will then use these properties to establish the existence of such taxes (Section 4.4). Since they replicate the solution to Home's relaxed planning problem, they a fortiori solve the home government's problem described in Definition 1.

#### 4.1 Micro-level Taxes on Domestic Varieties

Consider first a schedule of domestic taxes,  $\{s_{HH}^*(\varphi)\}$  and  $\{t_{HH}^*(\varphi)\}$ , that implements the optimal micro quantities,  $\{q_{HH}^*(\varphi) \equiv q_{HH}(\varphi|Q_{HH}^*, Q_{HF}^*, N_H^*)\}$ . Fix a benchmark variety  $\varphi_{HH}$  that is sold domestically,  $q_{HH}^*(\varphi_{HH}) > 0$ . Denote by  $s_{HH}^* \equiv s_{HH}^*(\varphi_{HH})$  and  $t_{HH}^* \equiv t_{HH}^*(\varphi_{HH})$  the domestic taxes imposed on that variety. Now take any other variety  $\varphi \in \Phi_{HH}$  that is sold domestically. By equations (1) and (2), we must have

$$\frac{q_{HH}^*(\varphi_{HH})}{q_{HH}^*(\varphi)} = \left( \frac{(1+t_{HH}^*)a_{HH}(\varphi_{HH})}{(1+s_{HH}^*)} \frac{(1+t_{HH}^*(\varphi))}{(1+s_{HH}^*(\varphi))a_{HH}(\varphi)} \right)^{-\sigma_H}.$$

Combining this expression with equation (10), we obtain our first result.

**Lemma 1.** *In order to implement an allocation solving the relaxed planning problem, domestic taxes should be such that*

$$(1+s_{HH}^*(\varphi))/(1+t_{HH}^*(\varphi)) = (1+s_{HH}^*)/(1+t_{HH}^*) \text{ if } \varphi \in \Phi_{HH}. \quad (23)$$

While we have focused on domestic taxes, there is nothing in the previous proposition that hinges on domestic varieties being sold in the domestic market rather than abroad. Thus, we can use the exact same argument to characterize the structure of export taxes,  $\{s_{HF}^*(\varphi)\}$ , that implements  $\{q_{HF}^*(\varphi) \equiv q_{HF}(\varphi|Q_{HH}^*, Q_{HF}^*, N_H^*)\}$ . In line with the previous analysis, let  $\varphi_{HF}$  denote a benchmark variety that is exported, with  $s_{HF}^* \equiv s_{HF}^*(\varphi_{HF})$ . The following result must hold.

**Lemma 2.** *In order to implement an allocation solving the relaxed planning problem, export taxes should be such that*

$$s_{HF}^*(\varphi) = s_{HF}^* \text{ if } \varphi \in \Phi_{HF}. \quad (24)$$

#### 4.2 Micro-level Taxes on Foreign Varieties

Now consider a schedule of import taxes,  $\{t_{FH}^*(\varphi)\}$ , that implements the desired allocation,  $\{q_{FH}^*(\varphi) \equiv q_{FH}(\varphi|Q_{FH}^*, Q_{FF}^*, N_F^*)\}$ . Fix a benchmark variety  $\varphi_{FH} \in \Phi_{FH}^u$  that is imported. In line with our previous analysis, let  $t_{FH}^* \equiv t_{FH}^*(\varphi_{FH})$  denote the import tax

imposed on that benchmark variety. For any other variety  $\varphi \in \Phi_{FH} \equiv \Phi_{FH}^u \cup \Phi_{FH}^c$  that is imported, equations (1) and (2) now imply

$$\frac{q_{FH}^*(\varphi_{FH})}{q_{FH}^*(\varphi)} = \left( \frac{(1 + t_{FH}^*)a_{FH}(\varphi_{FH})}{(1 + t_{FH}^*(\varphi))a_{FH}(\varphi)} \right)^{-\sigma_F}. \quad (25)$$

There are two possible cases to consider. If  $\varphi \in \Phi_{FH}^u$ , then equations (13) and (25) imply

$$t_{FH}^*(\varphi) = t_{FH}^*.$$

If  $\varphi \in \Phi_{FH}^c$ , then equations (13) and (25) imply

$$t_{FH}^*(\varphi) = (1 + t_{FH}^*)\theta_{FH}(\varphi) - 1.$$

This leads to our third result.

**Lemma 3.** *In order to implement an allocation solving the relaxed planning problem, import taxes should be such that*

$$t_{FH}^*(\varphi) = (1 + t_{FH}^*) \min\{1, \theta_{FH}(\varphi)\} - 1 \text{ if } \varphi \in \Phi_{FH}, \quad (26)$$

with the profitability index  $\theta_{FH}(\varphi) \equiv (\lambda_{FH}/\chi_{FH}\mu_F)[(\mu_F - 1)(a_{FH}(\varphi))^{1-\sigma_F}/f_{FH}(\varphi)]^{1/\sigma_F}$ .

In the context of a canonical model of intra-industry trade where heterogeneous firms select into exporting, optimal import taxes are higher for more profitable exporters. However, such heterogeneous taxes do not reflect the home government's desire to prevent imports from more profitable exporters. Instead, they reflect the desire to import from less profitable exporters as well. This motive leads to import taxes that are constant among the most profitable exporters, but vary among the least profitable ones.

### 4.3 Overall Level of Taxes

Our next goal is to characterize the overall level of taxes that is necessary for a decentralized equilibrium to implement the desired allocation. In Sections 4.1 and 4.2, we have already expressed all other taxes as a function of  $t_{HH}^*$ ,  $t_{FH}^*$ ,  $s_{HH}^*$ , and  $s_{HF}^*$ . So, this boils down to characterizing these four taxes. To do so, we compare the ratio between the marginal rates of substitution at home and abroad—determined by equations (15), (A.5) and (22)—and their ratio in the decentralized equilibrium with taxes—determined by equations (4)-(2). As expected, and as established formally in Appendix B.1, the wedge,

$\tau^* = -(\eta_{HF}^* + \eta_{FH}^*)/(1 + \eta_{HF}^*)$ , that appears in the first-order conditions of Home's macro planning problem anchors the overall level of taxes in the decentralized equilibrium.

**Lemma 4.** *In order to implement an allocation solving the relaxed planning problem, the overall level of optimal taxes,  $t_{HH}^*$ ,  $t_{FH}^*$ ,  $s_{HH}^*$ , and  $s_{HF}^*$ , should be such that*

$$\frac{(1 + t_{FH}^*)/(1 + t_{HH}^*)}{(1 + s_{HF}^*)/(1 + s_{HH}^*)} = \frac{(1 + \tau^*) \int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\})^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{\int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}. \quad (27)$$

Two remarks are in order. First, if  $\Phi_{FH}^c$  is measure zero, then  $\min\{1, \theta_{FH}(\varphi)\} = 1$  for all  $\varphi \in \Phi_{FH}$  so optimal import taxes are uniform and equation (27) reduces to

$$\frac{(1 + t_{FH}^*)/(1 + t_{HH}^*)}{(1 + s_{HF}^*)/(1 + s_{HH}^*)} = (1 + \tau^*).$$

This is what would happen in the absence of fixed exporting costs, as in [Krugman \(1980\)](#). We come back to this situation more generally in Section 5 when we study optimal uniform taxes. Second, if  $\Phi_{FH}^c$  is not measure zero, then  $\mu_F > 1$  implies

$$\frac{(1 + t_{FH}^*)/(1 + t_{HH}^*)}{(1 + s_{HF}^*)/(1 + s_{HH}^*)} > (1 + \tau^*).$$

This merely reflects our choice of benchmark variety for imports.  $t_{FH}^*$  is the tax on varieties  $\varphi \in \Phi_{FH}^u$ , and we know from Lemma 3 that import taxes should be lower on varieties  $\varphi \in \Phi_{FH}^c$ . So in order to implement the same wedge, the domestic government must now impose import taxes on varieties  $\varphi \in \Phi_{FH}^u$  that, relative to other taxes, are strictly greater than  $1 + \tau^*$ .

## 4.4 Implementation

Lemmas 1-4 provide necessary conditions that linear taxes have to satisfy so that the decentralized equilibrium replicates a solution to the relaxed planning problem. In the next lemma, which is proven in Appendix B.2, we show that that if the previous taxes are augmented with high enough taxes on the goods that are not consumed,  $\varphi \notin \Phi_{HH}$ ,  $\varphi \notin \Phi_{HF}$ , and  $\varphi \notin \Phi_{FH}$ , then they are also sufficient to implement any allocation that solves the relaxed planning problem.

**Lemma 5.** *There exists a decentralized equilibrium with taxes that implements any allocation that solves the relaxed planning problem.*

Since Home's relaxed planning problem is, as its name indicates, a relaxed version of Home's government problem introduced in Definition 1, the taxes associated with a decentralized equilibrium that implements a solution to the relaxed planning problem must a fortiori solve Home's government problem. Lemmas 2-5 therefore imply that any taxes that solve Home's government problem must satisfy conditions (23), (24), (26), and (27). To summarize, we can characterize unilaterally optimal taxes as follows.

**Proposition 1.** *At the micro-level, unilaterally optimal taxes should be such that: (i) domestic taxes are uniform across all domestic producers (condition 23); (ii) export taxes are uniform across all exporters (condition 24); (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones (condition 26). At the macro-level, unilaterally optimal taxes should reflect standard terms-of-trade considerations (condition 27).*

Note that condition (27) only pins down the relative levels of optimal taxes. In the proof of Lemma 5, we show how to implement the desired allocation using only import taxes,  $t_{HH}^* = s_{HH}^* = s_{HF}^* = 0$ . There is, however, a continuum of optimal taxes that would achieve the same allocation. For instance, we could have used a uniform export tax,

$$s_{HF}^* = \frac{\int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{(1 + \tau^*) \int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\})^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)},$$

while setting the overall level other taxes such that  $t_{HH}^* = s_{HH}^* = t_{FH}^* = 0$ . This is an expression of Lerner symmetry, which must still hold under monopolistic competition. In this case, all varieties  $\varphi \in \Phi_{FH}^c$  would receive an import subsidy equal to  $\theta_{FH}(\varphi) - 1 < 0$ . As alluded to in Section 3.1, the fact that domestic taxes can be dispensed with derives from the efficiency of the decentralized equilibrium with monopolistic competition and CES utility. Here, as in Bhagwati (1971), trade taxes are the preferred instruments to exploit monopoly and monopsony power in world markets.<sup>16</sup>

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<sup>16</sup>In the present environment, however, the introduction of non-linear taxes would raise Home's welfare. By imposing two-part tariffs, Home could incentivize foreign firms to sell at marginal costs and compensate them (exactly) for the fixed exporting costs that they incur. Qualitatively, optimal taxes would remain uniform across domestic firms and optimal import tariffs would remain biased against Foreign's most profitable exporters. The main difference is that discrimination would take the form of higher fixed fees for entering Home's market rather than higher linear taxes.

## 5 Optimal Uniform Taxes

In the last two sections, we have characterized optimal trade policy under the assumption that the home government is not only free to discriminate between firms from different countries by using trade taxes, but also unlimited in its ability to discriminate between firms from the same country. While this provides a useful benchmark to study the normative implications of firm heterogeneity for trade policy, informational or legal constraints may make this type of taxation infeasible in practice. Here, we turn to the other polar case in which the home government is constrained to set uniform taxes:  $t_{HF}(\varphi) = \bar{t}_{HF}$ ,  $t_{HH}(\varphi) = \bar{t}_{HH}$ ,  $s_{HF}(\varphi) = \bar{s}_{HF}$ , and  $s_{HH}(\varphi) = \bar{s}_{HH}$  for all  $\varphi$ .

### 5.1 Micro to Macro Once Again

To solve for optimal uniform taxes, we can follow the same approach as in Sections 3 and 4. The only difference is that the micro problems of Sections 3.1 and 3.2 should now include an additional constraint:

$$q_{ij}(\varphi')/q_{ij}(\varphi) = (a_{ij}(\varphi')/a_{ij}(\varphi))^{-\sigma_F} \text{ for any } \varphi, \varphi' \text{ such that } q_{ij}(\varphi'), q_{ij}(\varphi) > 0. \quad (28)$$

By construction, whenever the solution to Home's relaxed planning problem satisfies (28), it can be implemented with uniform taxes over the goods that are being produced. Furthermore, since Home always prefers to produce or import the most profitable goods, any solution that satisfies (28) can also be implemented with the same uniform taxes over the goods that are not produced or imported. Like in Section 4.4, strictly higher taxes on those goods can be dispensed with.

For varieties from Home that are sold in any market,  $i = H$  and  $j = H, F$ , constraint (28) is satisfied by the solution to the relaxed problem (9). In this case, optimal taxes were already uniform, as established in Lemmas 1 and 2. So the value of  $L_H(Q_{HH}, Q_{HF})$  remains unchanged. In contrast, for foreign varieties that are imported by Home,  $i = F$  and  $j = H$ , constraint (28) may bind at the solution to (11). This leads to new costs of imports and new terms of trade for Home, which we describe in the next subsection.

The other equations that characterize the solution to Home's relaxed planning problem are unchanged. In particular, one can still reduce Home's macro planning problem to (18). Following the same reasoning as in Section 4, one can therefore show that optimal

uniform taxes must satisfy

$$\frac{(1 + \bar{t}_{FH}^*)/(1 + \bar{t}_{HH}^*)}{(1 + \bar{s}_{HF}^*)/(1 + \bar{s}_{HH}^*)} = 1 + \tau^*. \quad (29)$$

Like in Section 4, the optimal wedge,  $\tau^* = -(\eta_{HF}^* + \eta_{FH}^*)/(1 + \eta_{HF}^*)$ , still depends exclusively on the terms-of-trade elasticities.

In order to help compare our results to those in the existing literature, we set domestic and export taxes to zero in the rest of this section:  $\bar{t}_{HH}^* = \bar{s}_{HH}^* = \bar{s}_{HF}^* = 0$ . For the same reasons as in Section 4.4, this is without loss of generality. Under this normalization, we can talk equivalently about optimal uniform taxes and optimal uniform tariffs,  $\bar{t}_{FH}^* = \tau^*$ .

## 5.2 Terms-of-Trade Elasticities

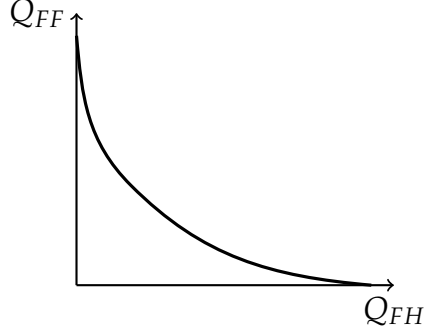
In Section 3, terms-of-trade elasticities are complex objects that depend both on supply and demand conditions in Foreign as well as the optimal micro-level choices of Home's government. With uniform trade taxes, the constraints imposed on the latter makes the determinants of terms-of-trade elasticities simpler. We now take advantage of this simplicity to explore the deeper determinants of terms-of-trade elasticities. In the next subsection, this information will allow us to address whether going from an economy without firm heterogeneity to an economy with firm heterogeneity affects the overall level of trade protection by changing the terms-of-trade elasticities.

In the absence of taxes that vary at the micro-level, it is convenient to summarize technology in Foreign by the function

$$L_F(Q_{FH}, Q_{FF}) \equiv \min_{q_{FH}, q_{FF}, N_F} N_F \left[ \sum_{j=H,F} \int_{\Phi} l_{Fj}(q_{Fj}(\varphi), \varphi) dG_F(\varphi) + f_F^e \right] \quad (30a)$$

$$N_F \int_{\Phi} (q_{Fj}(\varphi))^{1/\mu_F} dG_F(\varphi) \geq Q_{Fj}^{1/\mu_F}, \text{ for } j = H, F. \quad (30b)$$

By construction, Foreign's production possibility frontier corresponds to the set of aggregate output levels  $(Q_{FH}, Q_{FF})$  such that  $L_F(Q_{FH}, Q_{FF}) = L_F$ . This is just the counterpart of equation (18c) for Home in Section 3.3. Building on the efficiency of the decentralized equilibrium under monopolistic competition with CES utility, one can then show that Foreign necessarily operates on its production possibility frontier with the marginal rate of transformation being equal to the price of foreign exports relative to foreign domestic output. On the demand side, we already know that the marginal rate of substitution must be equal to the price of foreign imports relative to foreign domestic consumption. Thus



**Figure 1:** Aggregate Nonconvexities with Firm Heterogeneity

foreign equilibrium conditions can be described compactly as follows; see Appendix C.1 for a formal proof.

**Lemma 6.** *Conditional on  $Q_{HF}$  and  $Q_{FH}$ , the decentralized equilibrium abroad satisfies*

$$MRS_F(Q_{HF}, Q_{FF}(Q_{FH})) = P_{HF}/P_{FF}, \quad (31)$$

$$MRT_F(Q_{FH}, Q_{FF}(Q_{FH})) = \tilde{P}_{FH}/P_{FF}, \quad (32)$$

with  $\tilde{P}_{FH}$  the untaxed price of Home's imports and  $Q_{FF}(Q_{FH})$  given by the implicit solution of

$$L_F(Q_{FH}, Q_{FF}) = L_F. \quad (33)$$

The key insight of Lemma 6 is that the decentralized equilibrium abroad under monopolistic competition with CES utility is isomorphic, in terms of aggregate quantities and prices, to a perfectly competitive equilibrium with three goods, one that is produced and consumed domestically (in quantity  $Q_{FF}$ ), one that is produced but not consumed (exported in quantity  $Q_{FH}$ ), and one that is consumed but not produced (imported in quantity  $Q_{HF}$ ). The only distinction between the two equilibria is that under monopolistic competition, Foreign's production set may not be convex, as depicted in Figure 1. We come back to this point below.

Using Lemma 6, we can relate the elasticity of Home's terms of trade to the elasticities of substitution and transformation in Foreign. Let  $\epsilon \equiv -d \ln(Q_{HF}/Q_{FF})/d \ln(P_{HF}/P_{FF})$  denote the elasticity of substitution between imports and domestic goods and let  $\kappa \equiv d \ln(Q_{FH}/Q_{FF})/d \ln(\tilde{P}_{FH}/P_{FF})$  denote the elasticity of transformation between exports and domestic goods (both in Foreign). Since the marginal rate of substitution and the marginal rate of transformation abroad are both homogeneous of degree zero,<sup>17</sup>

<sup>17</sup>The homogeneity of degree zero of the marginal rate of substitution derives directly from our as-



equations (31) and (32) imply

$$\epsilon = -1/(d \ln MRS_F(Q_{HF}/Q_{FF}, 1)/d \ln(Q_{HF}/Q_{FF})), \quad (34)$$

$$\kappa = 1/(d \ln MRT_F(Q_{FH}/Q_{FF}, 1)/d \ln(Q_{FH}/Q_{FF})). \quad (35)$$

By equations (31) and (32), Home's terms of trade can also be expressed directly as

$$P(Q_{FH}, Q_{HF}) = MRS_F(Q_{HF}, Q_{FF}(Q_{FH}))/MRT_F(Q_{FH}, Q_{FF}(Q_{FH})). \quad (36)$$

Differentiating equation (36) with respect to Home's aggregate exports and imports,  $Q_{HF}$  and  $Q_{FH}$ , and using equations (34) and (35), we obtain Home's terms-of-trade elasticities,

$$\eta_{HF} = -1/\epsilon, \quad (37)$$

$$\eta_{FH} = -(1/x_{FF} - 1)/\epsilon - 1/(x_{FF}\kappa), \quad (38)$$

where  $x_{FF} \equiv P_{FF}Q_{FF}/L_F$  is the share of expenditure on domestic goods in Foreign.<sup>18</sup>

When  $\kappa \geq 0$ , Foreign's production set is convex and, everything else being equal, an increase in Home's imports tends to worsen its terms of trade by raising the opportunity cost of foreign exports in terms of foreign domestic output. This is the mechanism at play in a neoclassical environment. When  $\kappa < 0$  instead, aggregate nonconvexities imply that an increase in Home's imports tends to *lower* the opportunity cost of foreign exports, and in turn, *improve* its terms of trade.

### 5.3 A Generalized Optimal Tariff Formula

Combining equation (29)—under the restriction that  $\bar{t}_{HH}^* = \bar{s}_{HH}^* = \bar{s}_{HF}^* = 0$ —with equations (37) and (38), we obtain the following characterization of optimal uniform tariffs under monopolistic competition with firm heterogeneity.

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sumption that the foreign utility function is homothetic. Establishing the homogeneity of degree zero of the marginal rate of transformation is more subtle since the transformation function,  $L_F(Q_{FH}, Q_{FF})$ , is not homogeneous of degree one. We do so formally in Appendix C.2.

<sup>18</sup>To derive equation (38), we have also used the fact that

$$d \ln Q_{FF}(Q_{FH})/d \ln Q_{FH} = Q'_{FF}(Q_{FH})Q_{FH}/Q_{FF}(Q_{FH}) = -Q_{FH}MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))/Q_{FF}(Q_{FH}).$$

Together with equation (32), this implies

$$d \ln Q_{FF}(Q_{FH})/d \ln Q_{FH} = -(\tilde{P}_{FH}Q_{FH})/(P_{FF}Q_{FF}) = -(1/x_{FF} - 1).$$

**Proposition 2.** *Optimal uniform tariffs are such that*

$$\bar{t}_{FH}^* = \frac{1 + (\epsilon^* / \kappa^*)}{(\epsilon^* - 1)x_{FF}^*}, \quad (39)$$

where  $\epsilon^*$ ,  $\kappa^*$ , and  $x_{FF}^*$  are the values of  $\epsilon$ ,  $\kappa$ , and  $x_{FF}$  evaluated at those taxes.

Equation (39) is a strict generalization of the optimal tariff formula derived under monopolistic competition by [Gros \(1987\)](#), [Demidova and Rodríguez-Clare \(2009\)](#), and [Felbermayr, Jung and Larch \(2013\)](#). It applies to any economy in which: (i) Home's optimal choices of exports and imports correspond to the solution to a planning problem that can be reduced to (18); and (ii) the decentralized equilibrium in the rest of the world can be reduced to equations (31)-(33). Within that class of models, alternative assumptions about technology, preferences, and market structure only matter for the overall level of trade protection if they affect the three sufficient statistics:  $\epsilon^*$ ,  $\kappa^*$ , and  $x_{FF}^*$ .

[Gros \(1987\)](#) focuses on an economy à la [Krugman \(1980\)](#). There is no firm heterogeneity, no market-specific fixed costs, and the elasticity of substitution between domestic and foreign goods is constant,  $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$ . In this case, all firms export to all markets. Thus, equation (32) implies that the marginal rate of transformation abroad is constant and given by

$$MRT_F = \frac{(\int_{\Phi} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}{(\int_{\Phi} (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}.$$

In turn, the elasticity of transformation  $\kappa^*$  goes to infinity and equation (39) becomes

$$\bar{t}_{FH}^* = \frac{1}{(\sigma - 1)x_{FF}^*} > 0.$$

Proposition 2 demonstrates that [Gros's \(1987\)](#) formula remains valid for arbitrary distributions of firm-level productivity and arbitrary upper-level utility functions provided that Foreign's production possibility frontier is linear. A sufficient condition for this to be the case is that foreign firms face no fixed costs of selling in both markets,  $f_{Fj}(\varphi) = 0$  for  $j = H, F$ .

Beside greater generality, a benefit of our analysis is that it helps identify the economic forces that determine optimal trade policy under monopolistic competition. Under the restriction that  $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$ , the optimal tariff formula derived by [Gros \(1987\)](#) can be interpreted in two ways, as discussed by [Helpman and Krugman \(1989\)](#). One can think of Home as manipulating its terms-of-trade, as we have emphasized in this paper, or of Home imposing a tariff equal to the markup charged on domestic goods so that the relative price of foreign to domestic goods equals the country's true opportunity cost of

domestic goods. Indeed, the difference between Home firms' price and marginal cost for their domestic sales is equal to  $\mu_H - 1 = 1/(\sigma_H - 1)$ , which is the optimal tariff that a small open economy would choose when  $\epsilon^* = \sigma_H$ . By allowing the upper-level elasticity of substitution,  $\epsilon^*$ , to differ from the lower-level elasticities of substitution,  $\sigma_H$  and  $\sigma_F$ , our analysis suggests that the first of these two interpretations is the most robust. When  $\epsilon^* \neq \sigma_H$ , Home firms still charge a markup  $\mu_H = \sigma_H/(\sigma_H - 1)$  on the goods that they sell domestically. Yet, the only relevant elasticity in this case is  $\epsilon^*$  because it is the one that shapes Home's terms-of-trade elasticities, as shown in equations (37) and (38). We come back to this issue in Section 6.3.

As noted above, Proposition 2 also generalizes the results of Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013) who focus on an economy à la Melitz (2003). Compared to the present paper, they assume a constant elasticity of substitution between domestic and foreign goods,  $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$ . They also assume that taxes are uniform across firms, that firms only differ in terms of their productivity, and that the distribution of firm-level productivity is Pareto. Under these assumptions, the decentralized equilibrium with taxes can be solved in closed-form. As discussed in Feenstra (2010), models of monopolistic competition with Pareto distributions lead to an aggregate production possibility frontier with constant elasticity of transformation,

$$\kappa^* = -\frac{\sigma\nu - (\sigma - 1)}{\nu - (\sigma - 1)} < 0, \quad (40)$$

where  $\nu > \sigma - 1$  is the shape parameter of the Pareto distribution; see Appendix C.3.<sup>19</sup> Combining equations (39) and (40) and imposing  $\epsilon^* = \sigma$ , we obtain

$$\bar{t}_{FH}^* = \frac{1}{(\nu\mu - 1)x_{FF}^*} > 0,$$

as in Felbermayr, Jung and Larch (2013). In the case of a small open economy, the previous expression simplifies further into  $1/(\nu\mu - 1)$ , as in Demidova and Rodríguez-Clare (2009).

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<sup>19</sup>In his analysis of models of monopolistic competition with Pareto distributions, Feenstra (2010) concludes that firm heterogeneity leads to strictly convex production sets. In contrast, equation (40) implies that Foreign's production set is non-convex:  $\kappa^* < 0$ . Both results are mathematically correct. The apparently opposite conclusions merely reflect the fact that we have defined the aggregate production possibility frontier abroad as a function of the CES quantity aggregates,  $Q_{FH}$  and  $Q_{FF}$ , whereas Feenstra (2010) defines them, using our notation, in terms of  $Q_{FH}^{1/\mu_F}$  and  $Q_{FF}^{1/\mu_F}$ .

## 5.4 Firm Heterogeneity, Aggregate Nonconvexities, and Trade Policy

Since  $\nu > \sigma - 1$ , an intriguing implication of the results in [Demidova and Rodríguez-Clare \(2009\)](#) and [Felbermayr, Jung and Larch \(2013\)](#) is that conditional on  $\epsilon^* = \sigma$  and  $x_{FF}^*$ , the optimal level of trade protection is lower when only a subset of firms select into exports than when they all do,  $1/((\nu\mu - 1)x_{FF}^*) < 1/((\sigma - 1)x_{FF}^*)$ . This specific parametric example, however, is silent about the nature and robustness of the economic forces leading up to this result.

Our general analysis isolates aggregate nonconvexities as the key economic channel through which firm heterogeneity tends to lower the overall level of trade protection. Mathematically, the previous observation is trivial. From equations (21) and (37), we know that  $\epsilon^* - 1 > 0$ . Since  $\kappa^* \rightarrow \infty$  when firms are homogeneous, we arrive at the following corollary of Proposition 2.

**Corollary 1.** *Conditional on  $(\epsilon^*, x_{FF}^*)$ , optimal uniform tariffs are strictly lower with than without firm heterogeneity if and only if firm heterogeneity creates aggregate nonconvexities,  $\kappa^* < 0$ .*

Economically speaking, Home's trade restrictions derive from the negative effects of exports and imports on its terms of trade. By reducing the elasticity of Home's terms of trade with respect to its imports, in absolute value, aggregate nonconvexities dampen this effect, and in turn, reduce the optimal level of trade protection.

The final question that remains to be addressed is how likely it is that the selection of heterogeneous firms into exporting will lead to aggregate nonconvexities. It is instructive to consider first a hypothetical situation in which the measure of foreign firms,  $N_F$ , is exogenously given. In that situation, the selection of heterogeneous firms would necessarily lead to aggregate nonconvexities. To see this, note that equation (32) implies

$$MRT_F = \frac{(\int_{\Phi_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}{(\int_{\Phi_{FF}} (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}},$$

with the set of foreign varieties sold in market  $j = H, F$  such that

$$\Phi_{Fj} = \{\varphi : (\mu_F - 1)a_{Fj}^{1-\sigma_F}(\varphi)(N_F \int_{\Phi_{Fj}} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi))^{-\mu_F} Q_{Fj} \geq f_{Fj}(\varphi)\}.$$

If selection is active in market  $j$ , in the sense that some foreign firms are indifferent between selling and non-selling in market  $j$ , then  $\Phi_{Fj}$  must expand as  $Q_{Fj}$  increases. Since consumers love variety, this must lead to a decrease in  $(\int_{\Phi_{Fj}} (a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}$ .<sup>20</sup>

<sup>20</sup>Formally, this requires that  $G_F$  has strictly positive density around blueprints  $\varphi$  with profitability such

And since labor market clearing requires  $Q_{FF}$  to be decreasing in  $Q_{FH}$ , this implies that  $MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))$  is decreasing in  $Q_{FH}$ , i.e. that there are aggregate nonconvexities.

Intuitively, an increase in foreign exports,  $Q_{FH}$ , has two effects. First, it expands the set of foreign firms that export, which lowers the unit cost of Foreign's exports. Second, it lowers  $Q_{FF}$ , which reduces the set of foreign firms that sell domestically and raises the unit cost of Foreign's domestic consumption. Both effects tend to lower Foreign's opportunity cost of exports in terms of domestic consumption.

Our next result provides sufficient conditions such that the previous selection forces dominate any additional effect that changes in aggregate exports,  $Q_{FH}$ , may have on the number of foreign entrants,  $N_F$ , and in turn, the monotonicity of  $MRT_F$ . Let  $N_F(Q_{FH}, Q_{FF})$  denote the measure of foreign firms associated with the solution to (30).

**Lemma 7.** *If the measure of foreign entrants increases with aggregate output to any market,  $\partial N_F(Q_{FH}, Q_{FF}) / \partial Q_{Fj} \geq 0$  for  $j = H, F$ , then firm heterogeneity creates aggregate nonconvexities,  $\kappa^* \leq 0$ , with strict inequality whenever selection is active in at least one market.*

We view the monotonicity condition in Lemma 7 as very mild. The measure of foreign entrants,  $N_F(Q_{FH}, Q_{FF})$ , is determined by free entry.<sup>21</sup> When a change in aggregate output in any of the two markets changes firms' expected profits, the measure of foreign entrants adjusts to bring them back to the fixed entry costs,  $f_F^e$ . In the absence selection effects, an increase in aggregate output in any market raises profits and, in turn, the measure of foreign entrants. In this case, the monotonicity condition in Lemma 7 would necessarily be satisfied. In the presence of selection effects, an increase in aggregate output in market  $j$  may actually *decrease* expected profits if the decrease in the price index associated with an expansion of  $\Phi_{Fj}$  is large enough to offset the direct positive effect of aggregate output,  $Q_{Fj}$ , on firms' profits. For the monotonicity condition in Lemma 7 to be violated, there must be large selection effects in one, but only one of the two markets so that expected profits shift in opposite directions in response to changes in  $Q_{FH}$  and  $Q_{FF}$ . Under these circumstances,  $N_F^*$  cannot be increasing in both  $Q_{FH}$  and  $Q_{FF}$ . For the interested reader, Appendix C.5 constructs one such example in which, in spite of the selection of heterogeneous firms, Foreign's aggregate production set remains locally convex.

Combining Corollary 1 and Lemma 7, we arrive at the following proposition.

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that foreign firms are indifferent between selling and not selling in market  $j$ . Whenever we say that selection is active in market  $j$ , we assume that this is the case.

<sup>21</sup>Since the decentralized equilibrium is efficient, one can always interpret  $N_F(Q_{FH}, Q_{FF})$  as the measure of foreign entrants in the decentralized equilibrium, conditional on the equilibrium values of  $Q_{FH}$  and  $Q_{FF}$ . This is the interpretation we adopt here. Formally,  $N_F(Q_{FH}, Q_{FF})$  is given by equation (C.3) in the proof of Lemma 6.

**Proposition 3.** *If the measure of foreign entrants increases with aggregate output to any market, then conditional on  $(\epsilon^*, x_{FF}^*)$ , optimal uniform tariffs are lower with than without firm heterogeneity, with strict inequality whenever selection is active in at least one market.*

The active selection of heterogeneous firms may actually lower the overall level of trade protection so much that the optimal uniform tariff may become an *import subsidy*, an instance of the Lerner paradox. To see this, note that as  $\epsilon^*$  goes to infinity, the optimal uniform tariff in equation (39) converges towards

$$\bar{t}_{FH}^* = 1/(\kappa^* x_{FF}^*),$$

which is strictly negative if there are aggregate nonconvexities abroad,  $\kappa^* < 0$ . In this limit case, foreign preferences are linear, which eliminates the last neoclassical force calling for an import tariff: diminishing marginal rates of substitution in Foreign. More generally, equation (39) implies that an import subsidy is optimal,  $\bar{t}_{FH}^* < 0$ , if and only if nonconvexities on the supply-side dominate convexities on the demand-side,  $\kappa^* > -\epsilon^*$ .<sup>22,23</sup>

The “new” trade theory synthesized by Helpman and Krugman (1985) and Helpman and Krugman (1989) is rich in paradoxical results. For instance, a country with higher demand for a particular good may be a net exporter of that good, the so-called home-market effect. Such paradoxes derive from the presence of increasing returns at the sector-level: when employment in a sector expands, more firms enter, and since consumers love varieties, the associated price index goes down. In a one-sector economy, however, these considerations are mute, which explains why the optimal tariff formula derived by Gros (1987) under monopolistic competition à la Krugman (1980) is the same as in a perfectly competitive Armington model, or why the formula for gains from trade derived by Arkolakis, Costinot and Rodríguez-Clare (2012) is the same for the two models.

Interestingly, the possibility of the Lerner paradox presented above derives from a very different type of nonconvexities, one that is unique to monopolistically competitive models with firm heterogeneity and selection, and one that matters for trade policy, even with only one sector. In a neoclassical environment with diminishing marginal returns,

<sup>22</sup>Allowing for non-CES utility functions such that  $\epsilon^* \neq \sigma$  is important for an import subsidy to be optimal. With CES utility functions, the same parameter  $\sigma$  would affect the curvature of both the production possibility frontier and the indifference curve abroad. In the Pareto case, for instance, one can check that the restriction  $\nu > \sigma - 1$  further implies  $\kappa^* < -\sigma$ . So, in the environments considered by Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013), an import subsidy would never be optimal.

<sup>23</sup>A careful reader may wonder whether the inequality  $\kappa^* > -\epsilon^*$  conflicts with our previous decision to restrict our analysis to interior solutions of Home’s relaxed planning problem, as characterized by equations (19)-(21). The answer is no. Provided that the upper-level elasticity of substitution between domestic and foreign goods *at home* is low enough, Home’s planner must prefer an interior allocation, regardless of whether  $\kappa^* > -\epsilon^*$  or  $\kappa^* \leq -\epsilon^*$  abroad.

consumers and firms do not internalize the fact that, at the margin, an increase in imports must raise their opportunity costs and, in turn, the price of all infra-marginal units, which calls for a positive import tax. Here, in contrast, a government may lower the price of its imports by raising their volume and inducing more foreign firms to become exporters, which explains why an import subsidy may be optimal.

## 6 Optimal Taxes with Intra- and Inter-Industry Trade

The monopolistically competitive model of Section 2 is commonly interpreted as a model of intra-industry trade where domestic and foreign firms specialize in differentiated varieties of the same product. We now consider a more general environment with both intra- and inter-industry trade across multiple sectors indexed by  $k = 1, \dots, K$ . Formally, the utility function of the representative agent in each country is given by

$$\begin{aligned} U_i &= U_i(U_i^1, \dots, U_i^K), \\ U_i^k &= U_i^k(Q_{Hi}^k, Q_{Fi}^k), \\ Q_{ji}^k &= \left[ \int_{\Phi} N_j^k(q_{ji}^k(\varphi))^{1/\mu_j^k} dG_j^k(\varphi) \right]^{\mu_j^k}, \end{aligned}$$

with  $U_i^k$  the utility from consuming all varieties from sector  $k$  in country  $i$ ,  $Q_{ji}^k$  the subutility associated with varieties from country  $j$  in that sector, and  $\mu_j^k \equiv \sigma_j^k / (\sigma_j^k - 1)$ , with  $\sigma_j^k > 1$  the elasticity of substitution between varieties from country  $j$  in sector  $k$ . The model of Section 2 corresponds to the special case in which  $K = 1$  and  $U_i = Q_i^1$ . In line with our previous analysis, we assume that  $U_i^k(\cdot, \cdot)$  is homothetic for all  $i$  and  $k$ . Assumptions on technology and market structure are unchanged.

### 6.1 More Micro Problems and a More Complex Macro Problem

The first goal of this section is to show that the micro-to-macro approach that we have followed in previous sections readily extends to an economy with multiple industries. At the micro-level, all our qualitative results about the structure of optimal trade taxes are unchanged. At the macro-level, domestic taxes are now necessary to correct for the differences in markups across sectors, but, other than that, the overall level of trade protection still reflects the manipulation of Home's terms of trade, both within and between sectors.

Let us start with the micro problems of Sections 3.1 and 3.2. Within each sector  $k = 1, \dots, K$ , one can still define the minimum labor cost at home of producing domestic output,



$Q_{HH}^k$ , and exports,  $Q_{HF}^k$ , conditional on the measure of entrants,  $N_H^k$ ,

$$L_H^k(Q_{HH}^k, Q_{HF}^k, N_H^k) \equiv \min_{q_{HH}^k, q_{HF}^k} N_H^k \left[ \sum_{j=H,F} \int_{\Phi} l_{Hj}(q_{Hj}^k(\varphi), \varphi) dG_H^k(\varphi) + f_H^{e,k} \right]$$

$$N_H^k \int_{\Phi} (q_{Hj}^k(\varphi))^{1/\mu_H^k} dG_H^k(\varphi) \geq (Q_{Hj})^{1/\mu_H^k}, \text{ for } j = H, F,$$

as well as the minimum cost of imports, conditional on Foreign's aggregate output in the two markets,  $Q_{FH}^k$  and  $Q_{FF}^k$ , the measure of foreign entrants,  $N_F^k$ , and sectoral employment,  $L_F^k$ ,

$$C_{FH}^k(Q_{FH}^k, Q_{FF}^k, N_F^k, L_F^k) \equiv \min_{q_{FH}^k} \int_{\Phi} N_F^k \mu_F^k a_{FH}(\varphi) q_{FH}^k(\varphi) dG_F^k(\varphi)$$

$$\int_{\Phi} N_F^k (q_{FH}^k(\varphi))^{1/\mu_F^k} dG_F^k(\varphi) \geq Q_{FH}^k,$$

$$N_F^k f_F^{e,k} = \Pi_{FF}^k(Q_{FF}^k, N_F^k)$$

$$+ N_F^k \int_{\Phi} [\mu_F^k a_{FH}^k(\varphi) Q_{FH}^k q_{FH}^k(\varphi) - l_{FH}(q_{FH}^k(\varphi), \varphi)] dG_F^k(\varphi),$$

$$L_F^k = N_F^k f_F^{e,k} + L_{FF}^k(Q_{FF}^k, N_F^k) + N_F^k \int_{\Phi} l_{FH}(q_{FH}^k(\varphi), \varphi) dG_F^k(\varphi),$$

$$\mu_F^k a_{FH}(\varphi) q_{FH}^k(\varphi) \geq l_{FH}(q_{FH}^k(\varphi), \varphi).$$

The exact same arguments as in Sections 4.1 and 4.2 imply that domestic taxes should be uniform across firms within the same sector, but that import taxes should be lower on the least profitable exporters from Foreign. Uniform domestic taxes, in particular, still reflect the fact that firms charge a constant markup over marginal cost under monopolistic competition with CES subutility.

Compared to the one-sector case, the key difference is that Home's planner can choose the level of foreign employment,  $L_F^k$ , in each sector  $k$  subject to the foreign resource constraint,  $\sum_k L_F^k = L_F$ , and the optimality of foreign consumption across sectors,

$$(\partial U_F / \partial U_F^{k_1}) / (\partial U_F / \partial U_F^{k_2}) = P_F^{k_1} / P_F^{k_2}, \text{ for all } k_1, k_2,$$

where  $P_F^k \equiv \min_{Q_{HF}, Q_{FF}} \{P_{HF}^k Q_{HF} + P_{FF}^k Q_{FF} | U_F^k(Q_{HF}, Q_{FF}) \geq 1\}$  denotes the foreign price index in sector  $k$ .<sup>24</sup> These considerations can be summarized in a compact way through

<sup>24</sup> Alternatively, one could have assumed no labor mobility across sectors in Foreign, so that the amount of labor,  $L_F^k$ , is exogenously given in each sector. In this case, the previous first-order conditions would pin down instead the foreign wage in sector  $k$ ,  $w_F^k$ , as a function of Home's imports and exports. When foreign preferences are Cobb-Douglas, this is a simple environment to analyze, though one without home-market

a new feasible set,  $\mathcal{Z}_F$ , that includes all  $\mathbf{Q}_{FH} \equiv (Q_{FH}^1, \dots, Q_{FH}^K)$ ,  $\mathbf{Q}_{FF} \equiv (Q_{FF}^1, \dots, Q_{FF}^K)$ ,  $\mathbf{N}_F \equiv (N_F^1, \dots, N_F^K)$ , and  $\mathbf{L}_F \equiv (L_F^1, \dots, L_F^K)$  such that, for all  $k$ , there exist  $\mathbf{q}_{FH}^k$  that satisfy the sector-level counterparts of constraints (11b)-(11e). Let  $\mathbf{Q}_{HH} \equiv (Q_{HH}^1, \dots, Q_{HH}^K)$  and  $\mathbf{Q}_{HF} \equiv (Q_{HF}^1, \dots, Q_{HF}^K)$  denote the vector of domestic output and exports across sectors, respectively. Like in Section 3.3, Home's macro planning problem is simply to maximize the utility of its representative agent subject to a trade balance condition and a resource constraint,

$$\max_{\mathbf{Q}_{HH}, \mathbf{Q}_{FH}, \mathbf{Q}_{HF}} U_H(U_H^1(Q_{HH}^1, Q_{FH}^1), \dots, U_H^K(Q_{HH}^K, Q_{FH}^K)) \quad (41a)$$

$$\sum_k P_{HF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}) Q_{HF}^k = \sum_k \tilde{P}_{FH}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}) Q_{FH}^k, \quad (41b)$$

$$\sum_k L_H^k(Q_{HH}^k, Q_{HF}^k) = L_H, \quad (41c)$$

where the export and import prices,  $P_{HF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$  and  $\tilde{P}_{FH}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$ , now reflect Home's optimal choices of Foreign's local output,  $\mathbf{Q}_{FF} \equiv (Q_{FF}^1, \dots, Q_{FF}^K)$ , measures of entrants,  $\mathbf{N}_F \equiv (N_F^1, \dots, N_F^K)$ , and employment,  $\mathbf{L}_F \equiv (L_F^1, \dots, L_F^K)$  over  $\mathcal{Z}_F$ ; see Appendix D.1.

Using the associated first-order conditions, one can check that Home's planner would like to set the marginal rate of substitution between sectors equal to the marginal rate of transformation. Whenever markups differ across sectors, however, the marginal rate of transformation will not be equal to the ratio of price indices across sectors in the decentralized equilibrium. Building on this observation, one can then show that Home's government would find it optimal to impose domestic taxes that vary across sectors. This is just an application of the targeting principle: domestic instruments should be used to correct the domestic distortions, here variable markups across sectors.

In terms of macro-level trade taxes, the above analysis implies that terms-of-trade manipulation—i.e. the fact that the home government internalizes the impact of aggregate imports and exports,  $\mathbf{Q}_{FH}$  and  $\mathbf{Q}_{HF}$ , on prices,  $\tilde{P}_{FH}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$  and  $P_{HF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$ , whereas private agents do not—is the sole determinant of the overall level of trade protection. This is an important observation, which we come back to in the next subsection. But it should be clear that, like in a neoclassical environment with arbitrarily many sectors, see e.g. Bond (1990), there is little that can be said about the optimal structure of trade protection. In general, export and import prices in each sector depend on export and import decisions in *all* sectors,  $\mathbf{Q}_{FH}$  and  $\mathbf{Q}_{HF}$ , not just export and import in that sector,  $Q_{FH}^k$

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effects, like in earlier sections.

and  $Q_{HF}^k$ .<sup>25</sup>

## 6.2 A Simple Example with Homogeneous and Differentiated Goods

To provide further insights into the forces that shape terms-of-trade manipulation under monopolistic competition, both within and between sectors, we turn to a simple example that has received particular attention in the previous literature. Namely, we assume that there are two sectors, a homogeneous outside sector ( $k = O$ ) and a differentiated sector ( $k = D$ ), and that Foreign consumers have Cobb-Douglas preferences, as in the model with homogeneous firms of [Venables \(1987\)](#), [Ossa \(2011\)](#), and [Campolmi, Fadinger and Forlati \(2014\)](#), and the model with heterogeneous firms of [Haaland and Venables \(2014\)](#).

To facilitate the connection between previous results in the literature and ours, we also restrict all taxes to be uniform within the same sector, as in Section 5. This is equivalent to adding the sector-level counterpart of constraint (28) to the sector-level micro problems in Section 6.1. We let  $\bar{t}_{HH}^D$ ,  $\bar{t}_{FH}^D$ ,  $\bar{s}_{HH}^D$ , and  $\bar{s}_{HF}^D$  denote the uniform ad-valorem taxes in the differentiated sector and  $\bar{t}_H^O$  denote the ad-valorem trade tax-cum-subsidy in the homogeneous sector.<sup>26</sup>

In the outside sector, we assume that  $\sigma_F^O \rightarrow \infty$ , that there are no fixed costs of production and no trade costs, and that all firms at home and abroad have the same productivity, which we normalize to one. So, one can think of the homogeneous good as being produced by perfectly competitive firms in both countries. In the rest of this section, we use the outside good as our numeraire. As in the previous sections, we impose no restriction on the distributions of firm-level productivity and fixed costs in the differentiated sector,  $G_H^D$  and  $G_F^D$ , nor on the sector-level aggregator,  $U_H^D$  and  $U_F^D$ , which determines the substitutability between domestic and foreign varieties in both countries. Finally, we let  $\beta_F$  denote the share of expenditure on differentiated goods in Foreign. Given our Cobb-Douglas assumption, this share is constant.

Let  $X_H^O \equiv Q_{HF}^O - Q_{FH}^O$  denote Home's exports of the outside good. Under the previous

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<sup>25</sup>Throughout our analysis, we have restricted ourselves to a world economy with only two countries. We can deal with multi-country environments in the same way as we have dealt with multi-sector environments. Indeed, one can always reinterpret varieties from different countries as varieties from different sectors, leading to a macro planning problem similar to (41). At the micro-level, our qualitative results would still hold country-by-country. At the macro-level, the key difference, relative to the multi-sector case, is that optimal taxes would now also reflect the incentives to manipulate relative wages (since labor is immobile across countries).

<sup>26</sup>For notational convenience, we focus throughout this section on the structure of optimal trade taxes under the normalization that domestic taxes are zero in the homogeneous sector. As mentioned above, the difference in markups between the differentiated and homogeneous sectors implies that the optimal level of domestic taxes in the differentiated sector,  $\bar{t}_{HH}^D$  and  $\bar{s}_{HH}^D$ , will no longer be zero.

assumptions, Home's macro planning problem reduces to

$$\begin{aligned} \max_{Q_H^O, X_H^O, Q_{HH}^D, Q_{FH}^D, Q_{HF}^D} \quad & U_H(Q_H^O - X_H^O, U_H^D(Q_{HH}^D, Q_{FH}^D)) \\ & \tilde{P}_{FH}^D(X_H^O, Q_{FH}^D)Q_{FH}^D = P_{HF}^D(X_H^O, Q_{FH}^D, Q_{HF}^D)Q_{HF}^D + X_H^O, \\ & Q_H^O + L_H^D(Q_{HH}^D, Q_{HF}^D) = L_H, \end{aligned}$$

where Home's import and export prices in the differentiated sector,  $\tilde{P}_{FH}^D(X_H^O, Q_{FH}^D)$  and  $P_{HF}^D(X_H^O, Q_{HF}^D, Q_{FH}^D)$ , are such that

$$\begin{aligned} \tilde{P}_{FH}^D(X_H^O, Q_{FH}^D) &= \mu_F^D L_{FH}^D(Q_{FH}^D, Q_{FF}^D(Q_{FH}^D, L_F^D(X_H^O))), \\ P_{HF}^D(X_H^O, Q_{HF}^D, Q_{FH}^D) &= \mu_F^D L_{FF}^D(Q_{FH}^D, Q_{FF}^D(Q_{FH}^D, L_F^D(X_H^O))) MRS_F^D(Q_{HF}^D, Q_{FF}^D(Q_{FH}^D, L_F^D(X_H^O))), \end{aligned}$$

with  $L_{Fi}^D \equiv \partial L_F^D / \partial Q_{Fi}$  the marginal cost in Foreign of aggregate output for market  $i = H, F$  and  $MRS_F^D(Q_{HF}^D, Q_{FF}^D(Q_{FH}^D, L_F^D(X_H^O))) \equiv (\partial U_F^D(Q_{HF}^D, Q_{FF}^D) / \partial Q_{HF}^D) / (\partial U_F^D(Q_{HF}^D, Q_{FF}^D) / \partial Q_{FF}^D)$  the marginal rate of substitution in the differentiated sector.

With two sectors, foreign production of the differentiated good for its local market,  $Q_{FF}^D(Q_{FH}^D, L_F^D(X_H^O))$ , not only depends on foreign exports of the differentiated good,  $Q_{FH}^D$ , but also on the total amount of labor allocated to the differentiated sector,  $L_F^D(X_H^O)$ , which now appears as a second argument. Given Cobb-Douglas preferences, this only depends on Home's net imports of the outside good. Since Foreign always spends  $(1 - \beta_F)L_F$  on the outside good, the amount of labor allocated to that sector must be equal to  $(1 - \beta_F)L_F - X_H^O$  and the amount allocated to the differentiated sector must be equal to  $L_F$  minus this number,  $L_F^D(M_H^O) = \beta_F L_F + X_H^O$ .

In spite of the introduction of an outside sector, the relative price of Home's exports in the differentiated sector,  $P^D \equiv P_{HF}^D / \tilde{P}_{FH}^D$ , still satisfies  $P^D = MRS_F^D / MRT_F^D$ . Since there are now three aggregate goods that are traded internationally—Home's and Foreign's differentiated goods as well as the homogeneous outside good—there are two relative prices,  $P^D$  and  $\tilde{P}_{FH}^D$ , that Home can manipulate to improve its terms-of-trade both within and between sectors. Mathematically, these considerations are captured by the first-order conditions of Home's new macro problem, which imply

$$\begin{aligned} MRS_H^D &= MRT_H^D P^D / (1 + \tau^D), \\ MRS_H^{FO} &= \tilde{P}_{FH}^D (1 + \tau^O), \end{aligned}$$

with  $MRS_H^{FO} \equiv (\partial U_H / \partial Q_{FH}^D) / (\partial U_H / \partial U_H^O)$  the marginal rate of substitution for Home between Foreign's differentiated good and the homogeneous good, and the two optimal

wedges,  $\tau^D$  and  $\tau^O$ , such that

$$\tau^D = -\frac{\eta_{HF}^D + z\eta_{FH}^D + (z-1)\zeta_{FH}}{1 + \eta_{HF}^D}, \quad (42)$$

$$\tau^O = -\frac{z\eta_{FH}^D + yz\eta_X^D + (z-1)(y\zeta_X + \zeta_{FH})}{1 + yz\eta_X^D + (z-1)y\zeta_X}, \quad (43)$$

with  $\eta_{HF}^D \equiv \partial \ln P^D / \partial \ln Q_{HF}^D$ ,  $\eta_{FH}^D \equiv \partial \ln P^D / \partial \ln Q_{FH}^D$ ,  $\eta_X^D \equiv \partial \ln P^D / \partial \ln X_H^O$ ,  $\zeta_{FH} \equiv \partial \ln \tilde{P}_{FH}^D / \partial \ln Q_{FH}^D$ ,  $\zeta_X \equiv \partial \ln \tilde{P}_{FH}^D / \partial \ln X_H^O$ ,  $y \equiv \tilde{P}_{FH}^D Q_{FH}^D / X_H^O$ , and  $z \equiv \tilde{P}_{HF}^D Q_{HF}^D / P_{FH}^D Q_{FH}^D$ . The first group of price elasticities,  $\eta_{HF}^D$ ,  $\eta_{FH}^D$ , and  $\eta_X^D$ , determine Home's incentives to manipulate terms of trade within the differentiated sector, whereas the second group of elasticities,  $\zeta_{FH}$  and  $\zeta_X$ , determine its incentives to manipulate terms of trade between the differentiated sector and the homogeneous sector. When there is no inter-industry trade,  $z = 1$ , only the first group of elasticities affects Home's optimal wedges.<sup>27</sup>

The previous wedges, in turn, pin down the relative level of optimal trade taxes. Using the same argument as in Section 4, one can show that

$$\frac{(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_{HH}^D)}{(1 + \bar{s}_{HF}^D) / (1 + \bar{s}_{HH}^D)} = 1 + \tau^D, \quad (44)$$

$$(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_H^O) = 1 + \tau^O. \quad (45)$$

Finally, one can check, as we do in Appendix D.2, that given the difference in markups between the differentiated and homogeneous sectors, the domestic government would like to use domestic taxes in order to undo the markup distortion,

$$(1 + \bar{t}_{HH}^D) / (1 + \bar{s}_{HH}^D) = 1 / \mu_H^D.$$

When there is no active selection of firms in the differentiated sector, as in the model with homogeneous firms of Venables (1987), Ossa (2011), and Campolmi, Fadinger and Forlati (2014), equations (42)-(45) imply

$$\frac{(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_{HH}^D)}{(1 + \bar{s}_{HF}^D) / (1 + \bar{s}_{HH}^D)} = 1 + \frac{1}{(\epsilon^D - 1)x_{FF}^D}, \quad (46)$$

$$(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_H^O) = 1 - \frac{(1 - r_{FF}^D)(z/r_{FF}^D + (1 - z)\epsilon^D)}{\epsilon^D(\sigma_F^D - 1) + (1 - r_{FF}^D)(\sigma_F^D z/r_{FF}^D + (1 - z)\epsilon^D)}. \quad (47)$$

<sup>27</sup>The definitions of  $\eta_X^D$  and  $\zeta_X$  implicitly assume that Home is an exporter of the homogeneous good,  $X_H^O > 0$ . If Home is an importer of the homogeneous good, one can simply rewrite all our formulas in terms of  $\partial \ln P^D / \partial \ln(-X_H^O)$  and  $\partial \ln P_{FH}^D / \partial \ln(-X_H^O)$ . None of our results depends on this convention.

with  $\epsilon^D$  the elasticity of substitution within the differentiated sector in Foreign and  $x_{FF}^D \equiv P_{FF}^D Q_{FF}^D / (P_{FF}^D Q_{FF}^D + P_{HF}^D Q_{HF}^D)$  and  $r_{FF}^D \equiv P_{FF}^D Q_{FF}^D / (P_{FF}^D Q_{FF}^D + \tilde{P}_{FH}^D Q_{FH}^D)$  the domestic expenditure and revenue shares, respectively. The formal derivation can be found in Appendix D.3. From equation (46), we see that Gros's (1987) formula, which determines the optimal level of trade protection within the differentiated sector remains unchanged. Although the domestic government now wants to manipulate its terms-of-trade both within and between sectors, the latter consideration only affects the choice of  $(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_H^O)$ .

According to equation (47), if Home is an exporter of the homogeneous good,  $z < 1$ , then optimal taxes must be such that  $(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_H^O) < 1$ . This can be achieved, for example, by subsidizing imports of the differentiated good,  $\bar{t}_{FH}^D < 0$  with  $\bar{t}_H^O = 0$ , or by subsidizing exports of the homogeneous good,  $\bar{t}_{FH}^D = 0$  with  $\bar{t}_H^O > 0$ . Intuitively, an increase in Home's exports of the homogeneous good creates a home-market effect: it increases Foreign's employment in the differentiated sector,  $\beta_F L_F + X_H^O$ , which leads to more entry of foreign firms in this sector and, because of love of variety, a lower price of Foreign's differentiated goods relative to the homogeneous good. When Home is an exporter of the homogeneous good, this creates a first improvement in its terms of trade. In addition, an increase in either imports of the differentiated good or exports of the homogeneous good raises foreign production of the differentiated good for its local market. Since  $P^D \propto P_{HF}^D / P_{FF}^D = MRS_F^D$  in the absence of selection, this must be accompanied by a decrease in the relative price of Foreign's differentiated goods relative to Home's differentiated goods, a second improvement in Home's terms of trade.<sup>28</sup> When Home is a small open economy in the sense that  $r_{FF}^D = 1$ , it cannot manipulate entry or output abroad, which leads to zero subsidies:  $(1 + \bar{t}_{FH}^D) / (1 + \bar{t}_H^O) = 1$ . The same is true when  $\sigma_F^D$  goes to infinity. In this case, the relative price of Foreign's differentiated goods relative to the homogeneous good is fixed. Hence, Home can only manipulate  $P^D$ , which it will do optimally by setting an import tariff or an export tax in the differentiated sector according to equation (46).

When there is active selection, equations (42)-(45) offer a strict generalization of the results of Haaland and Venables (2014). In line with the papers cited in Section 5.3, they assume a constant elasticity of substitution between domestic and foreign goods,  $\epsilon^D = \sigma_H^D = \sigma_F^D \equiv \sigma^D$ , that firms only differ in terms of their productivity, and that the distribution of firm-level productivity is Pareto. Crucially, they also assume that Home is

<sup>28</sup>If Home is an importer of the homogeneous good,  $z > 1$ , then Home's terms of trade unambiguously improve if both  $P^D$  and  $P_{HF}^D$  increase. Although a decrease in Home's imports of the homogeneous good imports of differentiated goods necessarily increases  $P^D$  and lowers  $P_{FH}^D$ , it only increases  $P_{HF}^D$  if Foreign's elasticity of substitution between domestic and foreign goods,  $\epsilon^D$ , is low enough. Accordingly, Home only taxes imports of the homogeneous good in this case if  $\epsilon^D < z / (r_{FF}^D(z - 1))$ .

small relative to Foreign in the sense that it cannot affect the number of foreign entrants,  $N_F^D$ , nor local output,  $Q_{FF}^D$ , in the differentiated sector. This implies  $\zeta_X = \eta_X^D = 0$  and  $\zeta_{FH} = 1/\kappa^D$ . Under this restriction, Appendix D.4 establishes that

$$\frac{(1 + \bar{t}_{FH}^D)/(1 + \bar{t}_{HH}^D)}{(1 + \bar{s}_{HF}^D)/(1 + \bar{s}_{HH}^D)} = 1 + \frac{1 + \epsilon^D/\kappa^D}{\epsilon^D - 1}, \quad (48)$$

$$(1 + \bar{t}_{FH}^D)/(1 + \bar{t}_H^O) = 1 + 1/\kappa^D. \quad (49)$$

By equation (48), the structure of optimal trade protection within the differentiated sector is again exactly the same as in the one-sector case, with firm heterogeneity lowering trade protection if and only if there is active selection of foreign firms into exporting.<sup>29</sup> Furthermore, by equation (49), the same aggregate nonconvexities,  $\kappa^D < 0$ , should lead to less trade protection in the differentiated sector relative to the homogeneous sector:  $(1 + \bar{t}_{FH}^D)/(1 + \bar{t}_H^O) < 1$ . This reflects the fact that given aggregate nonconvexities, the import price in the differentiated sector,  $\tilde{P}_{FH}^D$ , is a *decreasing* function of import volumes,  $Q_{FH}^D$ . This can again be achieved by subsidizing imports of the differentiated good,  $\bar{t}_{FH}^D < 0$  with  $\bar{t}_H^O = 0$ , or by subsidizing exports of the homogeneous good,  $\bar{t}_{FH}^D = 0$  with  $\bar{t}_H^O > 0$ , an expression of Lerner symmetry.

### 6.3 Terms-of-Trade Manipulation and Optimal Trade Policy Redux

The existing literature on optimal trade policy under monopolistic competition draws a sharp distinction between models with only intra-industry trade, like the one studied by Gros (1987), and models with both intra- and inter-industry, like the one studied by Venables (1987). In the former class of models, the standard view, as put forward by Helpman and Krugman (1989), is that terms-of-trade manipulation can be thought of as the rationale behind optimal trade policy since a strategic country can affect its relative wage. In the latter class of models, however, the standard view would be that such terms-of-trade motives are absent whenever the existence of an outside good pins down relative wages between countries, and accordingly, that the rationale behind trade policy must lie somewhere else, like the existence of so-called home-market effects.

Our analysis offers a different perspective, one suggesting that the terms-of-trade mo-

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<sup>29</sup>All formulas in this section are implicitly derived under the assumption that Home and Foreign produce in both sectors. A small open economy, however, is likely to be completely specialized in only one of them. When Home is completely specialized in the differentiated sector, one can show that both equations (48) and (49) must still hold. When Home is completely specialized in the outside sector, equation (49) must again hold, but equation (48), while consistent with an optimum, is no longer necessary. Details are available upon request.



tive has greater scope than previously recognized. According to this view, imperfect competition and firm heterogeneity matter for the design of macro-level trade taxes, but only to the extent that they affect terms-of-trade elasticities. In the simple example of Section 6.2, Home's relative wage is fixed, whereas the number of foreign entrants in the differentiated sector is free to vary. Yet, if all elasticities of world prices are zero, that is if Home has no market power, then optimal wedges and optimal trade taxes are zero, as can be seen from equations (42) and (43). Our analysis echoes the results of Bagwell and Staiger (2012b,a, 2015) who argue that terms-of-trade externalities remain the sole motive for international trade agreements under various market structures.

The importance of the terms-of-trade motive in our analysis clearly depends on the availability of a full set of domestic instruments. In the presence of domestic distortions, trade policy can also be used as a second-best instrument, which means that if one were to restrict the set of domestic taxes, such considerations would also affect the level optimal trade taxes, as in Flam and Helpman (1987). This is true regardless of whether markets are perfectly or monopolistically competitive and we have little to add to this observation.

The core of the difference between the standard view and ours has a simpler origin. We define terms-of-trade manipulation at the macro-level as the manipulation of the relative price of sector-level aggregate prices, not the manipulation of relative wages. In the one-sector case studied by Gros (1987), the two definitions coincide, but not otherwise. While one may view the previous distinction as semantic, this does not mean that it is either irrelevant or trivial. Part of the reason why one builds theory is to develop a common language that can be applied under seemingly different circumstances. The perspective pushed forward in this paper is that within the class of models that we consider, international trade remains another transformation activity that turns aggregate exports into aggregate imports, as summarized by the trade balance condition in (41), the shape of which determines the structure of optimal trade policy at the macro-level.

## 7 Concluding Remarks

Few economic mechanisms have received as much empirical support as the selection of heterogeneous firms into exporting; see e.g. Bernard and Jensen (1999), Bernard, Eaton, Jensen and Kortum (2003), Bernard, Jensen, Redding and Schott (2007), and Eaton, Kortum and Kramarz (2011). Policy makers have paid attention. As documented in the World Trade Report 2016, there were only two regional trade agreements (RTA) with provisions related to small- and medium-sized enterprises (SME) prior to 1990. As of March 2016, 133 RTAs, representing 49% of all the notified RTAs, include at least one provision

mentioning explicitly SMEs. The Trans-Pacific Partnership (TPP) is now the first U.S. free trade agreement to include a separate chapter on SMEs.<sup>30</sup>

Ironically, there has been very little work to date about the policy implications of the endogenous selection of firms into exporting. In this paper, we have tackled this issue in the context of a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). We have organized our analysis around two polar assumptions about the set of available policy instruments. In our baseline environment, ad-valorem taxes are unrestricted so that governments are free to impose different taxes on different firms. In our extensions, ad-valorem taxes are uniform so that governments cannot discriminate between firms from the same country.

When ad-valorem taxes are unrestricted, we have shown that optimal trade policy requires micro-level policies. Specifically, a welfare-maximizing government should impose firm-level import taxes that discriminate against the most profitable foreign exporters. In contrast, export taxes that discriminate against or in favor of the most profitable domestic exporters can be dispensed with. When taxes are uniform, we have shown that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that lowers the overall level of trade protection. Under both assumptions, we have highlighted the central role that terms-of-trade manipulation plays in determining the structure of optimal trade taxes at the macro-level, thereby offering a unifying perspective on previous results about trade policy under monopolistic competition.

We conclude by pointing out a number of limitations of the present analysis that could be relaxed in future research. The first one is the assumption that all firms charge a constant markup. In general, a government that manipulates its terms-of-trade may do so by imposing different taxes on different firms in order to incentivize them to charge different markups. In practice, we know that firms of different sizes tend to have different markups and different pass-through rates; see e.g. Berman, Martin and Mayer (2012), Goldberg, Loecker, Khandelwal and Pavcnik (2015), and Amiti, Itskhoki and Konings (2015). While this channel is not directly related to the selection of heterogeneous firms into exporting, this is another potentially important mechanism through which firm heterogeneity may affect the design of optimal trade policy.

The second limitation is that fixed exporting costs are assumed to be paid in the exporting country. This implies that all trade is trade in goods. If fixed costs were paid in the importing country, trade would also include trade in services, and manipulating the prices of such services would also be part of the objective of a welfare-maximizing

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<sup>30</sup>Details of the chapter on SMEs can be found at <https://medium.com/the-trans-pacific-partnership/small-and-medium-sized-businesses-8de15a02d843#.c68wv1rwc>

government. More generally, our analysis abstracts from intermediate goods and global supply chains, which is another exciting area for future research on optimal trade policy; see [Blanchard, Bown and Johnson \(2015\)](#) for a first step in this direction.

The third limitation is that governments have access to a full set of tax instruments. As discussed in the previous section, when domestic instruments are restricted, trade policy would be called for not only to improve a country's terms of trade, but also to help in mitigating domestic distortions. We know little about the implications of trade models with firm heterogeneity for the design of optimal industrial policy. They may be particularly relevant in economies where credit markets are imperfect. In short, many normative questions remain open.

The final limitation is that we have only characterized the optimal policy of a country when the rest of the world consists of a single country that imposes no taxes of any sort. To further understand the implications of firm heterogeneity and selection for trade policy, future research should strive to characterize the Nash equilibrium in which all countries attempt to manipulate their terms of trade, and then study how trade agreements would be structured to avoid the associated negative welfare implications. Much remains to be done on the normative side of the literature to close the gap between theory and practice.

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## A Proofs of Section 3

In Sections 3.1-3.3, we have described the solution to Home's relaxed planning problem:

$$\max_{\{\mathbf{q}_{ij}, Q_{ij}\}_{i,j=H,F}, \mathbf{P}_{FF}, \mathbf{P}_{FH}, P_{FF}, P_{HF}, \{N_i\}_{i=H,F}} U_H(Q_{HH}, Q_{FH}) \quad (\text{A.1a})$$

subject to:

$$Q_{ij}^{1/\mu_i} \leq \int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi), \text{ for } i = H \text{ or } j = H, \quad (\text{A.1b})$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu_F a_{FF}(\varphi) \bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi), \varphi), \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A.1c})$$

$$P_{FF}^{1-\sigma_j} = \int_{\Phi} N_F[p_{FF}(\varphi)]^{1-\sigma_j} dG_F(\varphi), \quad (\text{A.1d})$$

$$p_{Fj}(\varphi) = \begin{cases} \bar{p}_{Fj}(\varphi) & \text{if } \mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) \geq l_{Fj}(q_{Fj}(\varphi), \varphi), \\ \infty & \text{otherwise,} \end{cases} \text{ for } j = H, F, \quad (\text{A.1e})$$

$$f_F^e = \sum_{j=H,F} \int_{\Phi} [\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi), \varphi)] dG_F(\varphi), \quad (\text{A.1f})$$

$$Q_{HF}, Q_{FF} \in \arg \max_{\tilde{Q}_{HF}, \tilde{Q}_{FF}} \{U_F(\tilde{Q}_{HF}, \tilde{Q}_{FF}) \mid \sum_{i=H,F} P_{iF} \tilde{Q}_{iF} = w_F L_F\}, \quad (\text{A.1g})$$

$$L_i = N_i \left[ \sum_{j=H,F} \int_{\Phi} l_{ij}(q_{ij}(\varphi), \varphi) dG_i(\varphi) + f_i^e \right], \text{ for } i = H, F. \quad (\text{A.1h})$$

We now provide the formal arguments used to characterize this solution.

### A.1 Necessity of Lagrangian Minimization (Section 3.1)

This appendix discusses some technical details behind our analysis and characterization of optimal policy. Our approach in Section 3.1 was to derive necessary conditions for optimality, appealing to global Lagrangian necessity theorems. This appendix clarifies how we can invoke such results, despite the apparent non convexity of the problems.

As stated, the problem in 3.1 is not a convex optimization problem. We first convexify this problem by allowing randomization: instead of specifying a single quantity  $q(\varphi)$  for each blueprint  $\varphi$ , we let the planner choose a distribution over  $q$  conditional on  $\varphi$ . Formally, for each  $\varphi$  there is a CDF over  $q_{Hj}$  given by  $M_{Hj}(q; \varphi)$ . Letting  $\mathbf{M}_{Hj} \equiv \{M_{Hj}(q; \varphi)\}$ , the planning problem becomes

$$L_H(Q_{HH}, Q_{HF}, N_H) \equiv \min_{\mathbf{M}_{HH}, \mathbf{M}_{HF} \in \mathcal{M}} N_H \left( \sum_{j=H,F} \int_{\Phi} \int_{[0,\infty)} l_{Hj}(q, \varphi) dM_{Hj}(q; \varphi) dG_H(\varphi) + f_H^e \right) \\ N_H \int_{\Phi} \int_{[0,\infty)} q^{1/\mu_H} dM_{Hj}(q; \varphi) dG_H(\varphi) \geq Q_{Hj}^{1/\mu_H}, \text{ for } j = H, F,$$

where  $\mathcal{M}$  is the set of all families of CDFs.



Note that  $\mathcal{M}$  is a convex subset of a vector space. As stated, the above planning problem is linear and, thus, convex in  $\mathbf{M}_{HH}$  and  $\mathbf{M}_{HF}$ . We have relaxed the equality to an inequality constraint to ensure that there exists an interior point, i.e. an  $\mathbf{M}_{Hj}$  such that the constraint holds with strict inequality. Thus, we can apply a Lagrangian necessity theorem such as Theorem 1, p. 217 from [Luenberger \(1969\)](#). This guarantees that there exists  $\lambda_{Hj} \geq 0$  such that any solution to the above problem must also minimize

$$N_H \left( \sum_{j=H,F} \int_{\Phi} \int_{[0,\infty)} l_{Hj}(q, \varphi) dM_{Hj}(q; \varphi) dG_H(\varphi) + f_H^e \right) \\ + \sum_{j=H,F} \lambda_{Hj} \left( Q_{Hj}^{1/\mu_H} - N_H \int_{\Phi} \int_{[0,\infty)} q^{1/\mu_H} dM_{Hj}(q; \varphi) dG_H(\varphi) \right)$$

over  $\mathbf{M}_{HH}, \mathbf{M}_{HF} \in \mathcal{M}$ .

Next, we argue that this minimization must be attained without randomization. In other words, it can be described by two functions  $q_{Hj}(\varphi)$  for  $j = H, F$ . This follows from the fact that: (i) any  $\mathbf{M}_{Hj} \in \mathcal{M}$  is dominated by  $\hat{\mathbf{M}}_{Hj} \in \mathcal{M}$  that assigns probability one to the set of points where  $l_{Hj}(q; \varphi) - q^{1/\mu_H}$  is minimized; and that: (ii) the set of minimizers of  $l_{Hj}(q; \varphi) - q^{1/\mu_H}$  is almost everywhere unique. To verify (ii), note that from the characterization in Section 3.1,  $l_{Hj}(q; \varphi) - q^{1/\mu_H}$  has multiple minimizers only when

$$(\mu_H - 1)(\mu_H / \lambda_{Hj})^{-\sigma_H} (a_{Hj}(\varphi))^{1-\sigma_H} = f_{Hj}(\varphi).$$

Since we have assumed that for any  $f_{Hj} > 0$  the distribution over  $a_{Hj}$  is smooth, this condition can only hold on a set with probability zero.

At this point, we have established that the solution to the relaxed version of the problem in 3.1, with randomization, needs to minimize the associated Lagrangian and that the solution to the Lagrangian problem does not involve randomization. This implies that any solution to the original problem in 3.1, without randomization, must also minimize

$$N_H \left[ \sum_{j=H,F} \int_{\Phi} \left( l_{Hj}(q_{Hj}(\varphi), \varphi) - \lambda_{Hj}(q_{Hj}(\varphi))^{1/\mu_H} \right) dG_H(\varphi) + f_H^e \right].$$

Finally, note that for the solution of the Lagrangian problem to satisfy (9b),  $\lambda_{Hj}$  must also be non-zero, as stated in the main text.

## A.2 Import Cost Minimization (Section 3.2)

The full problem of minimizing the cost of Home's imports,  $C_{FH}(Q_{FH}, Q_{FF}, N_F)$ , conditional on Foreign's aggregate output in the two markets,  $Q_{FH}$  and  $Q_{FF}$ , as well as the measure of foreign entrants,  $N_F$ , subject to condition (8) for  $i = F$  and  $j = H$  as well as Foreign's equilibrium conditions,

i.e, conditions (1) and (4) for  $i = F$  and  $j = F$  and (2), (5), (6) for  $i = F$ , is given by

$$C_{FH}(Q_{FH}, Q_{FF}, N_F) \equiv \min_{q_{FF}, q_{FH}, p_{FF}, p_{FH}, P_{FF}} \int_{\Phi} N_F p_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi) \quad (\text{A.2a})$$

$$\int_{\Phi} N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi) \geq Q_{FH}^{1/\mu_F}, \quad (\text{A.2b})$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu_F a_{FF}(\varphi) \bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi), \varphi), \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A.2c})$$

$$P_{FF}^{1-\sigma_j} = \int_{\Phi} N_F [p_{FF}(\varphi)]^{1-\sigma_j} dG_F(\varphi), \quad (\text{A.2d})$$

$$p_{Fj}(\varphi) = \begin{cases} \bar{p}_{Fj}(\varphi) & \text{if } \mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) \geq l_{Fj}(q_{Fj}(\varphi), \varphi), \\ \infty & \text{otherwise,} \end{cases} \quad \text{for } j = H, F, \quad (\text{A.2e})$$

$$f_F^e = \sum_{j=H,F} \int_{\Phi} [\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi), \varphi)] dG_F(\varphi), \quad (\text{A.2f})$$

$$L_F = N_F [\sum_{j=H,F} \int_{\Phi} l_{Fj}(q_{Fj}(\varphi), \varphi) dG_F(\varphi) + f_F^e], \quad (\text{A.2g})$$

Constraints (A.2c)-(A.2e) can be used to solve for the local micro quantities and prices in Foreign,

$$q_{FF}(\varphi|Q_{FF}, N_F) = \begin{cases} \bar{q}_{FF}(\varphi|Q_{FF}, N_F) & , \text{if } \mu_F a_{FF}(\varphi) \bar{q}_{FF}(\varphi|Q_{FF}, N_F) \geq l_{FF}(\bar{q}_{FF}(\varphi|Q_{FF}, N_F), \varphi), \\ 0 & , \text{otherwise;} \end{cases} \quad (\text{A.3})$$

$$p_{FF}(\varphi|Q_{FF}, N_F) = \begin{cases} \mu_F a_{FF}(\varphi) & , \text{if } \mu_F a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF}, N_F) \geq l_{FF}(q_{FF}(\varphi|Q_{FF}, N_F), \varphi), \\ \infty & , \text{otherwise;} \end{cases} \quad (\text{A.4})$$

$$P_{FF}(Q_{FF}, N_F) = \left( \int_{\Phi} N_F (p_{FF}(\varphi|Q_{FF}, N_F))^{1-\sigma_F} dG_F(\varphi) \right)^{1/(1-\sigma_F)}, \quad (\text{A.5})$$

with  $\bar{q}_{FF}(\varphi|Q_{FF}, N_F) \equiv [\mu_F a_{FF}(\varphi) / P_{FF}(Q_{FF}, N_F)]^{-\sigma_F} Q_{FF}$ . Total profits and total employment associated with the local sales of foreign firms,  $\Pi_{FF}(Q_{FF}, N_F)$  and  $L_{FF}(Q_{FF}, N_F)$ , are then given by

$$\Pi_{FF}(Q_{FF}, N_F) \equiv N_F \left[ \int_{\Phi} \mu_F a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF}, N_F) dG_F(\varphi) - \int_{\Phi} l_{FF}(q_{FF}(\varphi|Q_{FF}, N_F), \varphi) dG_F(\varphi) \right], \quad (\text{A.6})$$

$$L_{FF}(Q_{FF}, N_F) \equiv N_F \left[ \int_{\Phi} l_{FF}(q_{FF}(\varphi|Q_{FF}, N_F), \varphi) dG_F(\varphi) \right]. \quad (\text{A.7})$$

Substituting the previous expressions into problem A.2, we get that Home's optimal import quantities and prices must solve

$$C_{FH}(Q_{FH}, Q_{FF}, N_F) \equiv \min_{\mathbf{q}_{FH}, \mathbf{p}_{FH}} \int_{\Phi} N_F p_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi) \quad (\text{A.8a})$$

$$\int_{\Phi} N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi) \geq Q_{FH}^{1/\mu_F}, \quad (\text{A.8b})$$

$$p_{FH}(\varphi) = \begin{cases} \bar{p}_{FH}(\varphi) & \text{if } \mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi), \\ \infty & \text{otherwise,} \end{cases} \quad (\text{A.8c})$$

$$\begin{aligned} N_F f_F^e &= \Pi_{FF}(Q_{FF}, N_F) \\ &+ N_F \int_{\Phi} [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi), \end{aligned} \quad (\text{A.8d})$$

$$L_F = N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_{\Phi} l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi). \quad (\text{A.8e})$$

After solving for Home's import prices using (A.8c), we get that Home's import quantities must solve (11). For future reference, note that given quantities,  $q_{FH}(\varphi|Q_{FH}, Q_{FF}, N_F)$ , optimal import prices are given by

$$p_{FH}(\varphi|Q_{FH}, Q_{FF}, N_F) = \begin{cases} \mu_F a_{FH}(\varphi) & , \text{ if } \mu_F a_{FH}(\varphi) q_{FH}(\varphi|Q_{FH}, Q_{FF}, N_F) \geq l_{FH}(q_{FH}(\varphi|Q_{FH}, Q_{FF}, N_F), \varphi), \\ \infty & , \text{ otherwise.} \end{cases} \quad (\text{A.9})$$

Finally, we can derive necessary conditions for optimality, appealing to global Lagrangian necessity theorems, as we did in Section 3.1. Specifically, allowing for randomization we can rewrite the problem as a choice over two CDFs,  $M_{Fj}(q; \varphi)$  for  $j = H, F$ . The problem then becomes

$$\begin{aligned} \min_{\mathbf{M}_{FH} \in \mathcal{M}_{FH}} & \int_{\Phi} \int_{[0, \infty)} N_F \mu_F a_{FH}(\varphi) q dM_{FH}(q; \varphi) dG_F(\varphi) \\ & \int_{\Phi} \int_{[0, \infty)} N_F q^{1/\mu_F} dM_{Fj}(q; \varphi) dG_F(\varphi) \geq Q_{FH}^{1/\mu_F}, \\ N_F f_F^e &= \Pi_{FF}(Q_{FF}, N_F) \\ &+ N_F \int_{\Phi} \int_{[0, \infty)} [\mu_F a_{FH}(\varphi) q - l_{FH}(q, \varphi)] dM_{FH}(q; \varphi) dG_F(\varphi), \\ L_F &= N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_{\Phi} \int_{[0, \infty)} l_{FH}(q, \varphi) dM_{FH}(q; \varphi) dG_F(\varphi), \end{aligned}$$

where

$$\mathcal{M}_{FH} = \{\mathbf{M}_{FH} \in \mathcal{M} : \int_{\Phi} \int_{\{q: \mu_F a_{FH}(\varphi) q - l_{FH}(q, \varphi) < 0\}} dM_{FH}(q; \varphi) dG_F(\varphi) = 0\}$$

is the set of probability distributions that ensures positive profits almost everywhere. As stated, this problem is linear and, thus, convex. It features an inequality and two equality constraints.

Invoking the Lagrangian necessity theorem given by Theorem 1, p. 217 from [Luenberger \(1969\)](#) extended in Exercise 8.8.7 (p. 236), there exist multipliers  $\lambda_{FH} \geq 0$  for the inequality constraint and  $\lambda_E$  and  $\lambda_L$  for the equality constraints so that any solution must also maximize the Lagrangian

$$\int_{\Phi} \int_{[0,\infty)} [(1 - \lambda_E)\mu_F a_{FH}(\varphi)q + (\lambda_E + \lambda_L)l_{FH}(q, \varphi) - \lambda_{FH}q^{1/\mu_F}] dM_{FH}(q; \varphi) dG_F(\varphi)$$

over  $\mathbf{M}_{FH} \in \mathcal{M}_{FH}$ . As before, since the objective is linear in  $\mathbf{M}_{FH}$  it follows that we can focus on a “bang bang” solution that puts full weight on any point  $q$  for each  $\varphi$  that minimizes

$$(1 - \lambda_E)\mu_F a_{FH}(\varphi)q + (\lambda_E + \lambda_L)l_{FH}(q, \varphi) - \lambda_{FH}q^{1/\mu_F}$$

over the set of  $q$  satisfying  $\mu_F a_{FH}(\varphi)q \geq l_{FH}(q, \varphi)$ . By virtue of the analysis carried out in Section 3.2, the solution to this problem is unique almost everywhere. This follows since indifference obtains only if  $f_{FH}(\varphi) > 0$  and for at most two values of  $\theta_{FH}(\varphi)$ , namely  $\theta_{FH}(\varphi) = ((\lambda_E + \lambda_L)/\chi_{FH})^{1/\sigma_F}$ , if  $(\lambda_E + \lambda_L) > \chi_{FH}$ , or  $\theta_{FH}(\varphi) = (\chi_{FH} + (\mu_F - 1)(\lambda_E + \lambda_L))/\mu_F \chi_{FH}$ , otherwise. But under our assumption that for any  $f_{FH} > 0$  the distribution over  $a_{FH}(\varphi)$  is smooth, it follows that indifference happens with probability zero.

## B Proofs of Section 4

Let  $(\{\mathbf{q}_{ij}^*, Q_{ij}^*\}_{i,j=H,F}, \mathbf{p}_{FF}^*, \mathbf{p}_{FH}^*, P_{FF}^*, P_{HF}^*, \{N_i^*\}_{i=H,F})$  denote a solution to Home’s relaxed planning problem. In the main text, we have already described some of these variables. Before establishing Lemmas 4 and 5, we provide a complete characterization of this solution.

The measure of domestic entrants,  $N_H^*$ , is such that

$$N_H^* \in \arg \min_{N_H} L_H(Q_{HH}^*, Q_{HF}^*, N_H). \quad (\text{B.1})$$

Since the Lagrangian  $\mathcal{L}_H$  associated with  $L_H(Q_{HH}, Q_{HF}, N_H)$  is linear in  $N_H$ , a necessary and sufficient for  $N_H^*$  to satisfy (B.1) is

$$f_H^e = \sum_{j=H,F} \int_{\Phi} [\mu_H a_{Hj}(\varphi) q_{Hj}^*(\varphi) - l_{Hj}(q_{Hj}^*(\varphi), \varphi)] dG_H(\varphi). \quad (\text{B.2})$$

The measure of foreign entrants,  $N_F^*$ , and the local output in Foreign,  $Q_{FF}^*$ , are given by

$$Q_{FF}^*, N_F^* \in \arg \max_{Q_{FF}, N_F: (Q_{FH}^*, Q_{FF}, N_F) \in \mathcal{Z}_F} \{P_{HF}(Q_{HF}^*, Q_{FF}, N_F) Q_{FH}^* / C_{FH}(Q_{FH}^*, Q_{FF}, N_F)\}.$$

Other macro quantities,  $(Q_{HH}^*, Q_{HF}^*, Q_{FH}^*)$ , are given by equations (18b)-(21).

Conditional on macro quantities and the measures of entrants, the domestic micro quantities,  $\mathbf{q}_{HH}^* = \{q_{HH}(\varphi | Q_{HH}^*, Q_{HF}^*, N_H^*)\}$  and  $\mathbf{q}_{HF}^* = \{q_{HF}(\varphi | Q_{HH}^*, Q_{HF}^*, N_H^*)\}$  can be computed

using equation (10). The foreign micro quantities,  $\mathbf{q}_{FH}^* = \{q_{FH}(\varphi|Q_{FH}^*, Q_{FF}^*, N_F^*)\}$  and  $\mathbf{q}_{FF}^* = \{q_{FF}(\varphi|Q_{FF}^*, N_F^*)\}$ , can be computed using equations (13) and (A.3). Finally, the prices of foreign varieties,  $\mathbf{p}_{FF}^* = \{p_{FF}(\varphi|Q_{FF}^*, N_F^*)\}$  and  $\mathbf{p}_{FH}^* = \{p_{FH}(\varphi|Q_{FH}^*, Q_{FF}^*, N_F^*)\}$ , are given by equations (A.4) and (A.9), whereas the aggregate price indices,  $P_{HF}^* = P_{HF}(Q_{HF}^*, Q_{FF}^*, N_F^*)$  and  $P_{FF}^* = P_{FF}(Q_{FF}^*, N_F^*)$ , are given by equations (15), and (A.5). We also let  $\tilde{P}_{FH}^* \equiv C_{FH}(Q_{FH}^*, Q_{FF}^*, N_F^*)/Q_{FH}^*$  denote the average cost of imports at home. Note that  $\tilde{P}_{FH}^*$  differs from the import price index faced by Home consumers in the decentralized equilibrium,  $P_{FH}$ , which is inclusive of taxes.

## B.1 Lemma 4

*Proof of Lemma 4.* First, consider the marginal rate of substitution,  $MRS_j^* \equiv U_{Hj}^*/U_{Fj}^*$ , in country  $j = H, F$  at a solution to Home's relaxed planning problem. In Foreign, equations (15) and (A.5) imply

$$MRS_F^* = \frac{P_{HF}^*}{(\int_{\Phi} N_F^*(p_{FF}^*(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \quad (\text{B.3})$$

At home, we already know from equation (22) that

$$MRS_H^* = MRT_H^* P_{HF}^* / ((1 + \tau^*) \tilde{P}_{FH}^*).$$

By the Envelope Theorem, we also know that

$$MRT_H^* = (\lambda_{HH}/\lambda_{HF})(Q_{HH}^*/Q_{HF}^*)^{-1/\sigma_H}.$$

From equations (9b) and (10), we also know that the Lagrange multipliers satisfy

$$\lambda_{Hj} = [N_H^* \int_{\Phi_{Hj}} (\mu_H a_{Hj}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} (Q_{Hj}^*)^{1/\sigma_H}.$$

Combining the two previous expressions, we get

$$MRT_H^* = \frac{(\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}, \quad (\text{B.4})$$

and in turn,

$$MRS_H^* = \frac{(\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)} P_{HF}^*}{(1 + \tau^*) (\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)} \tilde{P}_{FH}^*}. \quad (\text{B.5})$$

Next, consider a decentralized equilibrium with taxes that implements a solution to the relaxed planning problem. The marginal rate of substitution for each of the two countries is determined by conditions (2)-(4). Using the fact that the set of varieties available for consumption in the decentralized equilibrium must be the same as in the solution to the relaxed planning problem,

we obtain

$$MRS_F^* = \frac{(\int_{\Phi_{HF}} N_H^* (\mu_H w_H a_{HF}(\varphi) / (1 + s_{HF}^*(\varphi)))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi} N_F^* (p_{FF}^*(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}, \quad (\text{B.6})$$

$$MRS_H^* = \frac{(\int_{\Phi_{HH}} N_H^* ((1 + t_{HH}^*(\varphi)) \mu_H w_H a_{HH}(\varphi) / (1 + s_{HH}^*(\varphi)))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi_{FH}} N_F^* ((1 + t_{FH}^*(\varphi)) \mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \quad (\text{B.7})$$

Combining equations (B.3), (B.5), (B.6), and (B.7) with the micro-level taxes in Lemmas 1-3, we get

$$\frac{(1 + t_{FH}^*) / (1 + t_{HH}^*)}{(1 + s_{HF}^*) / (1 + s_{HH}^*)} = \frac{(1 + \tau^*) \tilde{P}_{FH}^*}{(\int_{\Phi_{FH}} N_F^* (\min\{1, \theta_{FH}(\varphi)\} \mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \quad (\text{B.8})$$

By definition of  $\tilde{P}_{FH}^*$ , we know that

$$\tilde{P}_{FH}^* Q_{FH}^* = \int_{\Phi} N_F^* \mu_F a_{FH}(\varphi) q_{FH}^*(\varphi) dG_F(\varphi).$$

Together with equation (13), this implies

$$\frac{\tilde{P}_{FH}^* Q_{FH}^*}{(N_F^*)^{1/(1-\sigma_F)} \mu_F} = \frac{\int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi_{FH}^c} ((\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{(\mu_F \chi_{FH} / \lambda_{FH})^{\sigma_F} (N_F^*)^{\sigma_F/(1-\sigma_F)}}.$$

At a solution to Home's relaxed planning problem, constraint (11b) must be binding. Otherwise, Home could raise its imports,  $Q_{FH}$ , and hence the utility of its representative agent, without increasing the cost of its imports,  $C_{FH}(Q_{FH}, Q_{FF}, N_F)$ . Using equations (13) and (11b), one can also check that

$$\frac{(\mu_F \chi_{FH} / \lambda_{FH})^{\sigma_F-1} (Q_{FH}^*)^{1/\mu_F}}{N_F^*} = \int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi_{FH}^c} (\theta_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi).$$

Combining the two previous expressions, we then obtain

$$\frac{\tilde{P}_{FH}^*}{(N_F^*)^{1/(1-\sigma_F)} \mu_F} = \frac{\int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi_{FH}^c} ((\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{(\int_{\Phi_{FH}^u} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi_{FH}^c} (\theta_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{\sigma_F/(\sigma_F-1)}}. \quad (\text{B.9})$$

Substituting into equation (B.8) and using the definition of  $\Phi_{FH}^u$  and  $\Phi_{FH}^c$  we get equation (27).  $\square$

## B.2 Lemma 5

*Proof of Lemma 5.* In order to show the existence of a decentralized equilibrium that implements the desired allocation, we follow a guess and verify strategy. Consider: (i) quantities such that

$$q_{ij}(\varphi) = q_{ij}^*(\varphi), \quad (\text{B.10})$$

$$Q_{ij} = Q_{ij}^*; \quad (\text{B.11})$$

(ii) measures of entrants such that

$$N_i = N_i^* \text{ for all } i; \quad (\text{B.12})$$

(iii) wages such that

$$w_H = P_{HF}^* / \mu_H L_{HF}^*, \quad (\text{B.13})$$

$$w_F = 1; \quad (\text{B.14})$$

(iv) goods prices such that

$$p_{Hj}(\varphi) = \begin{cases} \bar{p}_{Hj}(\varphi) & , \text{ if } \mu_H a_{Hj}(\varphi) q_{Hj}(\varphi) \geq l_{Hj}(q_{Hj}(\varphi), \varphi), \\ \infty & , \text{ otherwise,} \end{cases} \quad (\text{B.15})$$

$$p_{Fj}(\varphi) = p_{Fj}^*(\varphi), \quad (\text{B.16})$$

and

$$P_{HH}^{1-\sigma_H} = \int_{\Phi} N_H [(1 + t_{HH}(\varphi)) p_{HH}(\varphi)]^{1-\sigma_H} dG_H(\varphi), \quad (\text{B.17})$$

$$P_{HF} = P_{HF}^*, \quad (\text{B.18})$$

$$P_{FH} = (1 + \tau^*) \tilde{P}_{FH}^* \quad (\text{B.19})$$

$$P_{FF} = P_{FF}^*, \quad (\text{B.20})$$

with  $\tau^*$  given by equation (22); (v) taxes such that

$$s_{Hj}(\varphi) = s_{Hj}^*, \text{ for all } \varphi \text{ and for } j = H, F, \quad (\text{B.21})$$

$$t_{HH}(\varphi) = t_{HH}^*, \text{ for all } \varphi, \quad (\text{B.22})$$

$$t_{FH}(\varphi) = t_{FH}^*(\varphi), \text{ if } \varphi \in \Phi_{FH}, \quad (\text{B.23})$$

$$t_{FH}(\varphi) \geq t_{FH}^*, \text{ otherwise,} \quad (\text{B.24})$$



with  $s_{Hj}^* = 0$  for  $j = H, F$ ,  $t_{HH}^* = 0$ ,  $t_{FH}^*(\varphi)$  given by equation (26), and  $t_{FH}^*$  given by equation (27); and (vi) a lump-sum transfer such that

$$T_H = \sum_{j=H,F} \left[ \int_{\Phi} N_j t_{jH}(\varphi) p_{jH}(\varphi) q_{jH}(\varphi) dG_j(\varphi) - \int_{\Phi} N_H s_{Hj}(\varphi) p_{Hj}(\varphi) q_{Hj}(\varphi) dG_H(\varphi) \right]. \quad (\text{B.25})$$

We now check that the previous allocation and prices satisfy the equilibrium conditions (1)-(7).

First, consider condition (7). Since it is equivalent to equation (B.25), it is trivially satisfied by construction.

Second, consider condition (2). For goods that are produced by home firms, they are equivalent to equations (B.15). So, it is again trivially satisfied. For goods that are produced by foreign firms, condition (2) derives from equations (A.4), (A.9), (B.10), and (B.16).

Third, consider condition (4). For goods locally sold by home firms, it directly derives from condition (B.17). For goods exported by home firms, one can use the same argument as in the proof of Lemma 4 to show that

$$\begin{aligned} L_{HF}^* &= \lambda_{HF} (Q_{HF}^*)^{-1/\sigma_H} / \mu_H, \\ \lambda_{HF} &= [N_H^* \int_{\Phi_{HF}} (\mu_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} (Q_{HF}^*)^{1/\sigma_H}, \end{aligned} \quad (\text{B.26})$$

which imply

$$L_{HF}^* = [N_H^* \int_{\Phi_{HF}} (\mu_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} / \mu_H. \quad (\text{B.27})$$

Combining the previous expression with equation (B.13), we get

$$P_{HF}^* = [N_H^* \int_{\Phi_{HF}} (\mu_H w_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)}.$$

Condition (4) then derives from the previous equation and equations (B.12), (B.15), and (B.21). Next, consider goods locally sold by foreign firms. For those, condition (4) derives from equations (A.4), (A.5), (B.10), (B.12), (B.16), and (B.20). Finally, for goods exported by foreign firms, we already know from equation (B.8) that

$$\frac{(1 + t_{FH}^*) / (1 + t_{HH}^*)}{(1 + s_{HF}^*) / (1 + s_{HH}^*)} = \frac{(1 + \tau^*) \tilde{P}_{FH}^*}{(\int_{\Phi_{FH}} N_F^* (\min\{1, \theta_{FH}(\varphi)\} \mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}.$$

Combining the previous expression with equations (26), (B.23), (B.24), and using the fact that  $s_{Hj}^* = 0$  for  $j = H, F$  and  $t_{HH}^* = 0$ , we then get

$$\left( \int_{\Phi_{FH}} N_F^* ((1 + t_{FH}(\varphi)) \mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) \right)^{1/(1-\sigma_F)} = (1 + \tau^*) \tilde{P}_{FH}^*.$$

Condition (4) derives from the previous expression and equations (A.9), (B.12), (B.16), and (B.19).

Fourth, consider condition (1). For goods locally sold by foreign firms, condition (1) directly derives from equations (A.3), (B.10), (B.11), and (B.20). For goods exported by home firms, note that by equations (B.10), (B.13), (B.15), (B.11), and (B.21) with  $s_{HF}^* = 0$ , condition (1) holds if

$$(\mu_H a_{HF}(\varphi) / \lambda_{HF})^{-\sigma_H} = [P_{HF}^* a_{HF}(\varphi) / (L_{HF}^* P_{HF})]^{-\sigma_H} Q_{HF}^*.$$

Since the previous equation follows from equations (B.26) and (B.18), condition (1) must hold for goods exported by home firms. We can use a similar logic to analyze micro-level quantities sold at Home. Given equations (B.10), (B.13), (B.15), (B.11), and (B.22) with  $t_{HH}^* = 0$ , condition (1) holds for goods locally sold by home firms if

$$(\mu_H a_{HH}(\varphi) / \lambda_{HH})^{-\sigma_H} = (P_{HF}^* a_{HH}(\varphi) / (L_{HF}^* P_{HH}))^{-\sigma_H} Q_{HH}^*. \quad (\text{B.28})$$

Using the same argument as in the proof of Lemma 4, one can also show that

$$L_{HH}^* = \lambda_{HH} (Q_{HH}^*)^{-1/\sigma_H} / \mu_H.$$

Hence, condition (B.28) is equivalent to

$$P_{HF}^* / P_{HH} = L_{HF}^* / L_{HH}^*, \quad (\text{B.29})$$

which follows from equations (B.4), (B.15), (B.17), (B.18), (B.21), (B.22), as well as the fact that condition (4) holds for goods exported by home firms. Lastly, consider goods exported by foreign firms. Given equations (B.10), (B.11), (B.14), (B.16), (B.23), and (B.24), condition (1) holds if

$$\begin{aligned} (\mu_F \chi_{FH} a_{FH}(\varphi) / \lambda_{FH})^{-\sigma_F} &= [(1 + t_{FH}^*) \mu_F a_{FH}(\varphi) / P_{FH}]^{-\sigma_F} Q_{FH}^*, \text{ if } \varphi \in \Phi_{FH}^u, \\ f_{FH}(\varphi) / ((\mu_F - 1) a_{FH}(\varphi)) &= [(1 + t_{FH}^*) \theta_{FH}(\varphi) \mu_F a_{FH}(\varphi) / P_{FH}]^{-\sigma_F} Q_{FH}^*, \text{ if } \varphi \in \Phi_{FH}^c, \end{aligned}$$

Given the definitions of  $\theta_{FH}(\varphi)$ , both conditions reduce to

$$\lambda_{FH} / \mu_F = P_{FH} / (1 + t_{FH}^*). \quad (\text{B.30})$$

Using equations (13) and (11b), one can again use the same strategy as in the proof of Lemma 4 to show that

$$\lambda_{FH} / \mu_F = \left( \int_{\Phi_{FH}} N_F^* (\mu_F (\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) \right)^{1/(1-\sigma_F)}.$$

Since condition (4) holds for goods exported by foreign firms, we also know from equations (B.12),

(B.23), and (B.24) that

$$P_{FH} = (1 + t_{FH}^*) \left( \int_{\Phi_{FH}} N_F^* (\mu_F(\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) \right)^{1/(1-\sigma_F)}.$$

Equation (B.30) derives from the two previous observations. Hence, condition (1) must also hold for goods exported by foreign firms.

Fifth, consider the free entry condition (5). Abroad, this condition derives from equations (11c), (A.6), (B.10), (B.11), and (B.12). At home, it derives from equations (B.2) and (B.10).

Sixth, consider the labor market condition (6). Abroad, this condition derives from equations (11d), (A.7), (B.10), (B.11), and (B.12). At home, constraint (18c) implies

$$L_H(Q_{HH}^*, Q_{HF}^*) = L_H. \quad (\text{B.31})$$

Condition (6) then derives from the definition of  $L_H(Q_{HH}, Q_{HF})$  and equations (B.10), (B.11), (B.12), and (B.31).

Finally, consider condition (3). Abroad, we know from equations (15) and (16) that at the desired allocation,

$$\begin{aligned} U_{HF}^*/U_{FF}^* &= P_{HF}^*/P_{FF}^*, \\ P_{FF}^* Q_{FF}^* + P_{FH}^* Q_{FH}^* &= L_F. \end{aligned}$$

Thus equations (B.11), (B.14), (B.18), and (B.20) imply that condition (3) holds abroad. At Home, we know from equations (19)-(21) that at the desired allocation

$$U_{FH}^*/U_{HH}^* = (1 + \tau^*) (L_{HF}^* \tilde{P}_{FH}^* / L_{HH}^* P_{HF}^*), \quad (\text{B.32})$$

Equations (B.29) and (B.32) imply

$$U_{FH}^*/U_{HH}^* = (1 + \tau^*) (\tilde{P}_{FH}^* / P_{HH}).$$

By equation (B.19), we then get

$$U_{FH}^*/U_{HH}^* = P_{FH}/P_{HH}. \quad (\text{B.33})$$

At the desired allocation, constraint (18b) also implies

$$\tilde{P}_{FH}^* Q_{FH}^* = P_{HF}^* Q_{HF}^*,$$

and in turn, using equation (B.11),

$$P_{HH} Q_{HH} + \tilde{P}_{FH}^* Q_{FH} = P_{HH} Q_{HH} + P_{HF}^* Q_{HF}. \quad (\text{B.34})$$

Since conditions (1) and (4) hold for goods sold by home firms at home and abroad, we know that

$$P_{Hj}Q_{Hj} = N_H \int_{\Phi} p_{Hj}(\varphi) q_{Hj}(\varphi) dG_H(\varphi).$$

Combining this observation with equations (B.15), (B.18), (B.21), and (B.22) we get

$$\begin{aligned} P_{HH}Q_{HH} + P_{HF}^*Q_{HF} &= N_H w_H \left( \int_{\Phi} \mu_H a_{HH}(\varphi) q_{HH}(\varphi) dG_H(\varphi) \right. \\ &\quad \left. + \int_{\Phi} \mu_H a_{HF}(\varphi) q_{HF}(\varphi) dG_H(\varphi) \right). \end{aligned}$$

Since condition (5) holds at home, this can be rearranged as

$$P_{HH}Q_{HH} + P_{HF}^*Q_{HF} = N_H w_H \left( \sum_{j=H,F} \int_{\Phi} l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) + f_H^e \right).$$

Since condition (6) also holds, we then get

$$P_{HH}Q_{HH} + P_{HF}^*Q_{HF} = w_H L_H.$$

Combining this expression with equation (B.34), we obtain

$$P_{HH}Q_{HH} + \tilde{P}_{FH}^*Q_{FH} = w_H L_H. \quad (\text{B.35})$$

Since conditions (1) and (4) hold for goods sold by foreign firms at home, we must have

$$P_{FH}Q_{FH} = N_F \int_{\Phi} (1 + t_{FH}(\varphi)) p_{FH}(\varphi) q_{FH}(\varphi) dG_H(\varphi),$$

which, using equation (B.16), leads to

$$P_{FH}Q_{FH} = N_F \int_{\Phi} \mu_F (1 + t_{FH}(\varphi)) a_{FH}(\varphi) q_{FH}(\varphi) dG_H(\varphi). \quad (\text{B.36})$$

From the definition of  $\tilde{P}_{FH}^*$  as well as equations (B.11) and (B.12), we also know that

$$\tilde{P}_{FH}^*Q_{FH} = N_F \int_{\Phi} \mu_F a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi). \quad (\text{B.37})$$

Combining equation (B.25) with equations (B.35), (B.36), and (B.37), we finally obtain

$$P_{HH}Q_{HH} + P_{FH}Q_{FH} = w_H L_H + T_H. \quad (\text{B.38})$$

Condition (3) at home derives from equations (B.33) and (B.38).  $\square$

## C Proofs of Section 5

### C.1 Lemma 6

*Proof of Lemma 6.* Equation (31) derives directly from utility maximization abroad. To establish equations (32) and (33), we can follow similar steps as in Section 3.1. Any solution to (30) must be such that the optimal quantity of good  $\varphi$  produced for country  $j = H, F$  satisfies

$$q_{Fj}^*(\varphi) = \begin{cases} (\mu_F a_{Fj}(\varphi) / \lambda_{Fj})^{-\sigma_F}, & \text{if } \varphi \in \Phi_{Fj}, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{C.1})$$

with the set of varieties with non-zero output such that

$$\Phi_{Fj} = \{\varphi : \mu_F a_{Fj}(\varphi) (\mu_F a_{Fj}(\varphi) / \lambda_{Fj})^{-\sigma_F} \geq l_{Fj}((\mu_F a_{Fj}(\varphi) / \lambda_{Fj})^{-\sigma_F}, \varphi)\},$$

and the Lagrange multiplier associated with (30b) such that

$$\lambda_{Fj} = [N_F^* \int_{\Phi_{Fj}} (\mu_F a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}. \quad (\text{C.2})$$

Any solution to (30) must also be such that

$$\sum_{j=H,F} \int_{\Phi_{Fj}} [\mu_F a_{Fj}(\varphi) q_{Fj}^*(\varphi) - l_{Fj}(q_{Fj}^*(\varphi), \varphi)] dG_F(\varphi) = f_F^e. \quad (\text{C.3})$$

The comparison of equations (1), (4), (2), and (5), on the one hand, and equations (C.1), (C.2), and (C.3), on the other hand, imply that the outputs of foreign varieties and the measure of foreign entrants in the decentralized equilibrium must coincide with the solution of (30), conditional on  $Q_{FH}$  and  $Q_{FF}$ . Since the outputs of foreign varieties and the measure of foreign entrants satisfy (6), we must therefore have

$$L_F(Q_{FH}, Q_{FF}) = L_F,$$

which establishes equation (33) and implicitly defines  $Q_{FF}(Q_{FH})$ . To conclude, note that by the Envelope Theorem, we must have

$$\partial L_F(Q_{FH}, Q_{FF}) / \partial Q_{Fj} = \lambda_{Fj} Q_{Fj}^{-1/\sigma_F} / \mu_F. \quad (\text{C.4})$$

Conditional on  $Q_{FH}$  and  $Q_{FF}$ , equations (4) (with  $w_F = 1$ ) and (C.2) further imply that

$$\lambda_{Fj} = P_{Fj} Q_{Fj}^{1/\sigma_F}. \quad (\text{C.5})$$

Equation (32) follows from equations (C.4) and (C.5).  $\square$

## C.2 Marginal Rate of Transformation is Homogeneous of Degree Zero (Section 5.2)

In Section 5.2, we have argued that  $MRT_F(Q_{FH}, Q_{FF})$  is homogeneous of degree zero. We now establish this result formally. In the proof of Lemma 6, we have already shown that the solution of (30) satisfies equations (C.1), (C.2), and (C.3). Combining these three conditions, one can check that the measure of foreign firms is such that

$$N_F(Q_{FH}, Q_{FF}) = (M_F(Q_{FH}, Q_{FF}))^{1/\mu_F},$$

with  $M_F(Q_{FH}, Q_{FF})$  implicitly given by the solution to

$$M_F = \frac{\sum_{j=H,F} Q_{Fj} \mathbb{A}_{Fj}(M_F/Q_{Fj})}{(\sigma_F - 1) \left[ f_F^e + \sum_{j=H,F} \mathbb{F}_{Fj}(M_F/Q_{Fj}) \right]}, \quad (\text{C.6})$$

with

$$\mathbb{A}_{Fj}(M_F/Q_{Fj}) \equiv \left( \int_{\Phi_{Fj}(M_F/Q_{Fj})} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi) \right)^{1/(1-\sigma_F)}, \quad (\text{C.7})$$

$$\mathbb{F}_{Fj}(M_F/Q_{Fj}) \equiv \int_{\Phi_{Fj}(M_F/Q_{Fj})} f_{Fj}(\varphi) dG_F(\varphi), \quad (\text{C.8})$$

and

$$\Phi_{Fj}(M_F/Q_{Fj}) \equiv \{ \varphi : a_{Fj}^{1-\sigma_F}(\varphi) \geq \frac{f_{Fj}(\varphi)}{(\mu_F - 1)} \frac{M_F}{Q_{Fj}} \left( \int_{\Phi_{Fj}(M_F/Q_{Fj})} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi) \right)^{\mu_F} \}. \quad (\text{C.9})$$

From equation (32), we know that

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{[\int_{\Phi_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)}}{[\int_{\Phi_{FF}} (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)}}.$$

Using the notation above, this can be rearranged as

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{\mathbb{A}_{FH}(M_F^*(Q_{FH}, Q_{FF})/Q_{FH})}{\mathbb{A}_{FF}(M_F^*(Q_{FH}, Q_{FF})/Q_{FF})}.$$

By equation (C.6),  $M_F(Q_{FH}, Q_{FF})$  is homogeneous of degree one. Together with the previous expression, this implies that  $MRT_F(Q_{FH}, Q_{FF})$  is homogeneous of degree zero.

## C.3 Marginal Rate of Transformation in the Pareto Case (Section 5.3)

In Section 5.3, we have argued that under the assumptions that (i) firms only differ in terms of their productivity,  $f_{ij}(\varphi) = f_{ij}$ , (ii) the distribution of firm-level productivity is Pareto,  $a_{ij}(\varphi) =$

$\tau_{ij}/\varphi$  with  $G_F(\varphi) = 1 - (b_F/\varphi)^{\nu_F}$  for all  $\varphi \geq b_F$ , and (iii) there is active selection of Foreign firms in both the Foreign and Home markets, then the elasticity of transformation,  $\kappa^*$ , satisfies equation (40). We now establish this result formally.

The same arguments as in the proof of Lemma 6 imply

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{(\lambda_{FH} Q_{FH}^{-1/\sigma_F} / \mu_F)}{(\lambda_{FF} Q_{FF}^{-1/\sigma_F} / \mu_F)} \quad (C.10)$$

with the Lagrange multipliers such that

$$\lambda_{Fj} = [N_F^* \int_{\Phi_{Fj}} (\mu_F a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}$$

and the set of imported varieties such that

$$\Phi_{Fj} = \{\varphi : \mu_F a_{Fj}(\varphi) (\mu_F a_{Fj}(\varphi) / \lambda_{Fj})^{-\sigma_F} \geq I_{Fj} ((\mu_F a_{Fj}(\varphi) / \lambda_{Fj})^{-\sigma_F}, \varphi)\}.$$

Under assumption (i), the set of imported varieties must be such that  $\Phi_{Fj} = \{\varphi \geq \varphi_{Fj}^*\}$ , with the productivity cut-off such that

$$(\mu_F - 1)(\tau_{Fj}/\varphi_{Fj}^*)^{1-\sigma_F} (\mu_F/\lambda_{Fj})^{-\sigma_F} = f_{Fj}, \quad (C.11)$$

while assumptions (ii) and (iii) imply that  $\varphi_{Fj}^* \geq b_F$  and that the Lagrange multiplier must be such that

$$\lambda_{Fj} = [N_F^* \nu_F (b_F)^{\nu_F} \int_{\varphi_{Fj}^*} (\mu_F \tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}. \quad (C.12)$$

Equations (C.10) and (C.12) imply

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{(\int_{\varphi_{FH}^*} (\tau_{FH}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi)^{1/(1-\sigma_F)}}{(\int_{\varphi_{FF}^*} (\tau_{FF}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi)^{1/(1-\sigma_F)}}, \quad (C.13)$$

whereas equations (C.11) and (C.12) imply

$$\varphi_{Fj}^* = \frac{\tau_{Fj} (f_{Fj}/(\mu_F - 1))^{1/(\sigma_F-1)} Q_{Fj}^{1/(1-\sigma_F)}}{[N_F^* \nu_F (b_F)^{\nu_F} \int_{\varphi_{Fj}^*} (\tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi]^{\sigma_F/((\sigma_F-1)(1-\sigma_F))}}.$$

We can use the last expression to solve for  $\varphi_{Fj}^*$ . We obtain

$$\varphi_{Fj}^* = \frac{\tau_{Fj}^{(\sigma_F-1)/((\sigma_F-1)-\nu_F\sigma_F)} (f_{Fj}/(\mu_F - 1))^{(1-\sigma_F)/((\sigma_F-1)-\nu_F\sigma_F)} Q_{Fj}^{(\sigma_F-1)/((\sigma_F-1)-\nu_F\sigma_F)}}{[N_F^* \nu_F (b_F)^{\nu_F} / (\sigma_F - \nu_F - 1)]^{\sigma_F/((\sigma_F-1)-\nu_F\sigma_F)}},$$



and, in turn,

$$\left( \int_{\varphi_{Fj}^*} (\tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi \right)^{1/(1-\sigma_F)} = \frac{\tau_{Fj}^{\frac{\nu_F(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} (f_{Fj}/(\mu_F-1))^{\frac{\nu_F-(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} Q_{Fj}^{-\frac{\nu_F-(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}}}{[N_F^* \nu_F (b_F)^{\nu_F}]^{\frac{\sigma_F(\sigma_F-\nu_F-1)}{(1-\sigma_F)((\sigma_F-1)-\nu_F\sigma_F)}} (\sigma_F - \nu_F - 1)^{-\frac{\sigma_F-1}{\nu_F\sigma_F-(\sigma_F-1)}}}.$$

Substituting into equation (C.13) leads to

$$MRT_F(Q_{FH}, Q_{FF}) = (\tau_{FH}/\tau_{FF})^{\frac{\nu_F(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} (f_{FH}/f_{FF})^{\frac{\nu_F-(\sigma_F-1)}{\sigma_F\nu_F-(\sigma_F-1)}} (Q_{FH}/Q_{FF})^{-\frac{\nu_F-(\sigma_F-1)}{\sigma_F\nu_F-(\sigma_F-1)}}.$$

For  $\nu_F = \nu$  and  $\sigma_F = \sigma$ , the previous expression and equation (35) imply equation (40).

## C.4 Lemma 7

*Proof of Lemma 7.* In Section C.2, we have established that

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{\mathbb{A}_{FH}(M_F(Q_{FH}, Q_{FF})/Q_{FH})}{\mathbb{A}_{FF}(M_F(Q_{FH}, Q_{FF})/Q_{FF})}.$$

with  $M_F$ ,  $\mathbb{A}_{FH}$ , and  $\mathbb{A}_{FF}$  implicitly determined by equations (C.6)-(C.9). Taking log and totally differentiating the previous expression with respect to  $Q_{FH}$ , we get

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = \epsilon_{FH}^{\mathbb{A}}(-(1 - \epsilon_{FH}^M) + \epsilon_F^Q \epsilon_{FF}^M) + \epsilon_{FF}^{\mathbb{A}}(-\epsilon_{FH}^M + \epsilon_F^Q(1 - \epsilon_{FF}^M)),$$

with

$$\begin{aligned} \epsilon_{Fj}^{\mathbb{A}} &= \frac{d \ln \mathbb{A}_{Fj}(M_F/Q_{Fj})}{d \ln(M_F/Q_{Fj})} \geq 0, \\ \epsilon_F^Q &= \frac{d \ln Q_{FF}(Q_{FH})}{d \ln Q_{FH}} < 0, \\ \epsilon_{Fj}^M &= \frac{\partial \ln M_F^*(Q_{FH}, Q_{FF})}{\partial \ln Q_{Fj}}, \end{aligned}$$

where the non-negativity of  $\epsilon_{Fj}^{\mathbb{A}}$  directly follows from equations (C.7) and (C.9). In Section C.2, we have already argued that  $M_F(Q_{FF}, Q_{FH})$  is homogeneous of degree one. Thus, we must have  $\epsilon_{FH}^M + \epsilon_{FF}^M = 1$ , which leads to

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = (\epsilon_{FF}^{\mathbb{A}} \epsilon_{FH}^M + \epsilon_{FH}^{\mathbb{A}} \epsilon_{FF}^M) (\epsilon_F^Q - 1). \quad (\text{C.14})$$

Since  $\epsilon_F^Q - 1 < 0$ ,  $\epsilon_{Fj}^A \geq 0$ , and  $\epsilon_{FH}^M, \epsilon_{FF}^M \geq 0$  with  $\epsilon_{FH}^M + \epsilon_{FF}^M = 1$ , we can conclude that if selection is active in at least one market,  $\epsilon_{FF}^A > 0$  or  $\epsilon_{FH}^A > 0$ , then

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} < 0,$$

which is equivalent to  $\kappa^* < 0$  by equation (35).  $\square$

## C.5 Locally Convex Production Sets with Selection (Section 5.4)

The goal of this subsection is to construct an economy where: (i) the number of entrants in Foreign is strictly decreasing with aggregate output in one market and (ii) Foreign's production set is locally convex.

Suppose that firms in Foreign differ only in terms of their productivity,  $a_{ij}(\varphi) = \tau_{ij}/\varphi$  and  $f_{ij}(\varphi) = f_{ij}$  for all  $\varphi$ , and that fixed exporting costs are equal to zero,  $f_{FH} = 0$ , whereas fixed costs of selling domestically are not,  $f_{FF} > 0$ . Starting from these assumptions and the characterization of the solution to (30)—equations (C.1)–(C.3) in the proof of Lemma 6—we can follow the same strategy as in Section C.2 and write  $M_F(Q_{FH}, Q_{FF}) = (N_F(Q_{FH}, Q_{FF}))^{\mu_F}$  as the implicit solution of

$$M_F = \frac{Q_{FH} \mathbb{A}_{FH} + Q_{FF} \mathbb{A}_{FF}(M_F/Q_{FF})}{(\sigma_F - 1) [f_F^e + \mathbb{F}_{FF}(M_F/Q_{FF})]}, \quad (C.15)$$

with

$$\mathbb{A}_{FH} = \tau_{FH} \left( \int_{\Phi} \varphi^{\sigma_F - 1} dG_F(\varphi) \right)^{1/(1 - \sigma_F)}, \quad (C.16)$$

$$\mathbb{A}_{FF}(M_F/Q_{FF}) = \left( \int_{\varphi_{FF}^*} \varphi^{\sigma_F - 1} dG_F(\varphi) \right)^{1/(1 - \sigma_F)}, \quad (C.17)$$

$$\mathbb{F}_{FF}(M_F/Q_{FF}) = f_{FF}(1 - G_F(\varphi_{FF}^*)), \quad (C.18)$$

and the productivity cut-off for foreign firms in their domestic market such that

$$(\varphi_{FF}^*)^{\sigma_F - 1} = \frac{f_{FF} M_F}{(\mu_F - 1) Q_{FF}} \left( \int_{\varphi_{FF}^*} \varphi^{\sigma_F - 1} dG_F(\varphi) \right)^{\mu_F}. \quad (C.19)$$

By equation (C.15), a sufficient condition for  $M_F$  to be decreasing in  $Q_{FH}$  is that

$$\epsilon_{FF}^A \frac{\mathbb{A}_{FF}(M_F/Q_{FF})}{(Q_{FH}/Q_{FF}) \mathbb{A}_{FH} + \mathbb{A}_{FF}(M_F/Q_{FF})} - \epsilon_{FF}^F \frac{\mathbb{F}_{FF}(M_F/Q_{FF})}{f_F^e + \mathbb{F}_{FF}(M_F/Q_{FF})} > 1,$$

with  $\epsilon_{FF}^A = d \ln \mathbb{A}_{FF}(M_F/Q_{FF}) / d \ln(M_F/Q_{FF}) \geq 0$  and  $\epsilon_{FF}^F \equiv d \ln \mathbb{F}_{FF}(M_F/Q_{FF}) / d \ln(M_F/Q_{FF})$ . In the limit, when  $Q_{FH}/Q_{FF} \rightarrow 0$  and  $f_F^e / \mathbb{F}_{FF}(M_F/Q_{FF}) \rightarrow 0$ , the previous condition reduces to

$$\epsilon_{FF}^A - \epsilon_{FF}^F > 1. \quad (C.20)$$

We will now provide sufficient conditions on  $G_F$  such that the previous inequality holds. By equation (C.19), we know that

$$\epsilon_{FF} = \frac{1}{\sigma_F - 1 + \frac{\mu_F \varphi_{FF}^* g_F(\varphi_{FF}^*)}{\int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi)}},$$

with  $\epsilon_{FF} \equiv d \ln \varphi_{FF}^*(M_F / Q_{FF}) / d \ln(M_F / Q_{FF})$ . Combining this expression with equations (C.17) and (C.18), we get

$$\begin{aligned} \epsilon_{FF}^{\mathbb{A}} &= \frac{1 - (\sigma_F - 1)\epsilon_{FF}}{\sigma_F}, \\ \epsilon_{FF}^{\mathbb{F}} &= -\frac{(1 - (\sigma_F - 1)\epsilon_{FF}) \int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi)}{\mu_F(1 - G_F(\varphi_{FF}^*))}. \end{aligned}$$

and, in turn,

$$\epsilon_{FF}^{\mathbb{A}} - \epsilon_{FF}^{\mathbb{F}} = \left( \frac{1 - (\sigma_F - 1)\epsilon_{FF}}{\sigma_F} \right) \left( 1 + \frac{(\sigma_F - 1) \int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi)}{1 - G_F(\varphi_{FF}^*)} \right).$$

Hence, the sufficient condition (C.20) can be rearranged as

$$\frac{1}{\sigma_F - 1} \frac{\varphi_{FF}^* g_F(\varphi_{FF}^*)}{1 - G_F(\varphi_{FF}^*)} > \frac{(\sigma_F - 1) \int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi) + \mu_F \varphi_{FF}^* g_F(\varphi_{FF}^*)}{(\sigma_F - 1) \int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi) + 1 - G_F(\varphi_{FF}^*)}$$

Now taking  $\sigma_F = 2$  and noting that  $\int_{\varphi_{FF}^*} (\varphi / \varphi_{FF}^*)^{\sigma_F - 1} dG_F(\varphi) > 1$ , a sufficient condition is

$$H(\varphi_{FF}^*) \equiv \frac{1}{1 - G_F(\varphi_{FF}^*)} - \frac{1}{\varphi_{FF}^* g_F(\varphi_{FF}^*)} > 1,$$

which can always be satisfied by picking  $G_F$  with finite support  $[\underline{\varphi}, \bar{\varphi}]$  and setting  $\frac{f_{FF}}{(\mu_F - 1)}$  such that given equation (C.19),  $\varphi_{FF}^*$  converges to  $\bar{\varphi}$  and  $H(\varphi_{FF}^*)$  goes to infinity. At this point, we have established that there exist sufficient conditions under which  $M_F(Q_{FH}, Q_{FF})$  and hence  $N_F(Q_{FH}, Q_{FF})$  is strictly decreasing in  $Q_{FH}$ . To conclude, recall that by equation (C.14), we must have

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = \left( \epsilon_{FF}^{\mathbb{A}} \epsilon_{FH}^M + \epsilon_{FH}^{\mathbb{A}} \epsilon_{FF}^M \right) (\epsilon_F^Q - 1).$$

In the present economy, equations (C.16) and (C.17) imply  $\epsilon_{FF}^{\mathbb{A}} > 0$  and  $\epsilon_{FH}^{\mathbb{A}} = 0$ . We have just provided sufficient conditions under which  $\epsilon_{FH}^M < 0$ . Since  $\epsilon_F^Q - 1 < 0$ , we therefore obtain

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} > 0,$$

which concludes our proof.

## D Proofs of Section 6

### D.1 Import and Export Prices (Section 6.1)

In the main text, we have already described the import cost,  $C_{FH}^k(Q_{FH}^k, Q_{FF}^k, N_F^k, L_F^k)$ . Like in Section 3.3, the export price index in sector  $k$ ,  $P_{HF}^k(Q_{HF}^k, Q_{FF}^k, N_F^k)$ , is implicitly determined by the following first-order condition,

$$U_{HF}^k / U_{FF}^k = P_{HF}^k / P_{FF}^k(Q_{FF}^k, N_F^k),$$

where  $P_{FF}^k(Q_{FF}^k, N_F^k)$  is the price index for locally produced goods in Foreign, which is given by

$$P_{FF}^k = \left( \int_{\Phi_{FF}^k} N_F^k (\mu_F^k a_{FF}(\varphi))^{1-\sigma_F^k} dG_F^k(\varphi) \right)^{1/(1-\sigma_F^k)},$$

with  $\Phi_{FF}^k \equiv \{\varphi : (\mu_F^k - 1)a_{FF}(\varphi)(\mu_F^k a_{FF}(\varphi)/P_{FF}^k)^{-\sigma_F^k} Q_{FF}^k \geq f_{FF}(\varphi)\}$ .

Using the previous notation, Home's macro problem can be expressed as

$$\begin{aligned} \max_{\mathbf{Q}_{HH}, \mathbf{Q}_{FH}, \mathbf{Q}_{HF}} \quad & U_H(U_H^1(Q_{HH}^1, Q_{FH}^1), \dots, U_H^K(Q_{HH}^K, Q_{FH}^K)) \\ \max_{(\mathbf{Q}_{FF}, \mathbf{N}_F, \mathbf{L}_F) : (\mathbf{Q}_{FH}, \mathbf{Q}_{FF}, \mathbf{N}_F, \mathbf{L}_F) \in \mathcal{Z}_F} \quad & \sum_k P_{HF}^k(Q_{HF}^k, Q_{FF}^k, N_F^k) Q_{HF}^k - \sum_k C_{FH}^k(Q_{FH}^k, Q_{FF}^k, N_F^k, L_F^k) = 0, \\ \sum_k \min_{N_H^k} L_H^k(Q_{HH}^k, Q_{HF}^k, N_H^k) = L_H. \end{aligned}$$

Let  $\mathbf{Q}_{FF}(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$ ,  $\mathbf{N}_F(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$ , and  $\mathbf{L}_F(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})$  denote Home's optimal choices of Foreign's local output, measures of entrants, and employment, conditional on Home's aggregate imports and exports,

$$\begin{aligned} \mathbf{Q}_{FF}(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}), \mathbf{N}_F(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}), \mathbf{L}_F(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}) \equiv \\ \arg \max_{(\mathbf{Q}_{FF}, \mathbf{N}_F, \mathbf{L}_F) : (\mathbf{Q}_{FH}, \mathbf{Q}_{FF}, \mathbf{N}_F, \mathbf{L}_F) \in \mathcal{Z}_F} \left\{ \sum_k P_{HF}^k(Q_{HF}^k, Q_{FF}^k, N_F^k) Q_{HF}^k - \sum_k C_{FH}^k(Q_{FH}^k, Q_{FF}^k, N_F^k, L_F^k) \right\}. \end{aligned}$$

Then by setting

$$\begin{aligned} P_{HF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}) &\equiv P_{HF}^k(Q_{HF}^k, Q_{FF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}), N_F^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})), \\ \tilde{P}_{FH}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}) &\equiv C_{FH}^k(Q_{FH}^k, Q_{FF}^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}), N_F^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF}), L_F^k(\mathbf{Q}_{FH}, \mathbf{Q}_{HF})) / Q_{FH}^k, \end{aligned}$$

and substituting into the macro problem above, we obtain (41).

## D.2 Domestic Taxes (Section 6.2)

The goal of this subsection is to show that if the assumptions of Section 6.2 hold then,

$$(1 + \bar{t}_{HH}^D)/(1 + \bar{s}_{HH}^D) = 1/\mu_H^D. \quad (\text{D.1})$$

The first-order conditions associated with Home's relaxed planning problem imply

$$MRS_H^{HO} = L_{HH}^D, \quad (\text{D.2})$$

with  $MRS_H^{HO} \equiv (\partial U_H / \partial Q_{HH}^D) / (\partial U_H / \partial U_H^O)$  the marginal rate of substitution for Home between Home's differentiated good and the homogeneous good and  $L_{HH}^D \equiv \partial L_H^D / \partial Q_{HH}$  the marginal cost of aggregate output for the local market at home. Like in Section 4.3, one can use the Envelope Theorem to show that

$$L_{HH}^D = \left( \int_{\Phi_{HH}^D} N_H^D (a_{HH}(\varphi))^{1-\sigma_H^D} dG_H^D(\varphi) \right)^{1/(1-\sigma_H^D)}. \quad (\text{D.3})$$

In the decentralized equilibrium with taxes, utility maximization at home implies

$$MRS_H^{HO} = P_{HH}^D, \quad (\text{D.4})$$

with the aggregate price index such that

$$P_{HH}^D = \left( \int_{\Phi_{HH}^D} N_H^D ((1 + \bar{t}_{HH}^D) \mu_H^D a_{HH}(\varphi) / (1 + \bar{s}_{HH}^D))^{1-\sigma_H^D} dG_H^D(\varphi) \right)^{1/(1-\sigma_H^D)}. \quad (\text{D.5})$$

Equations (D.2)-(D.5) imply that in order to implement the solution of Home's relaxed planning problem, domestic taxes must be such that equation (D.1) holds.

## D.3 Trade Taxes without Active Selection (Section 6.2)

The goal of this subsection is to establish equations (46) and (47) under the assumption that there is no active selection. We first compute Home's terms-of-trade elasticities within the differentiated sector,  $\eta_{HF}^D \equiv \partial \ln P^D / \partial \ln Q_{HF}^D$  and  $\eta_{FH}^D \equiv \partial \ln P^D / \partial \ln Q_{FH}^D$ . Since  $P^D \equiv P_{HF}^D / P_{FH}^D$ , still satisfies  $P^D = MRS_F^D / MRT_F^D$ ,  $\eta_{HF}^D$  and  $\eta_{FH}^D$  must satisfy the counterparts of equations (37) and (38),

$$\eta_{HF}^D = -1/\epsilon^D, \quad (\text{D.6})$$

$$\eta_{FH}^D = -(1/r_{FF}^D - 1)/\epsilon^D - 1/(r_{FF}^D \kappa^D), \quad (\text{D.7})$$

where  $\epsilon^D$  and  $\kappa^D$  denote the elasticities of substitution and transformation, respectively, within the differentiated sector in Foreign and  $r_{FF}^D \equiv P_{FF}^D Q_{FF}^D / (P_{FF}^D Q_{FF}^D + P_{FH}^D Q_{FH}^D)$  denote Foreign's do-

mestic share of revenue in the differentiated good.<sup>31</sup> In the absence of active selection, Foreign's production possibility frontier for the differentiated sector is linear,  $\kappa^D \rightarrow \infty$ , so equation (D.7) simplifies into

$$\eta_{FH}^D = -(1/r_{FF}^D - 1)/\epsilon^D. \quad (D.8)$$

Now, consider  $\eta_X^D \equiv \partial \ln P^D / \partial \ln X_H^O$ . The same steps used to compute  $\eta_{FH}^D$  implies

$$\eta_X^D = (d \ln Q_{FF}^D / d \ln X_H^O) / \epsilon^D. \quad (D.9)$$

In the decentralized equilibrium abroad, we know that

$$Q_{FF}^D = (\beta_F L_F + X_H^O) / P_{FF}^D - (P_{FH}^D / P_{FF}^D) Q_{FH}^D$$

with price indices such that

$$\begin{aligned} P_{FF}^D &= \left( \int_{\Phi_{FF}} N_F^D (\mu_F^D a_{FF}(\varphi))^{1-\sigma_F^D} dG_F(\varphi) \right)^{1/(1-\sigma_F^D)}, \\ P_{FH}^D &= \left( \int_{\Phi_{FH}} N_F^D (\mu_F^D a_{FH}(\varphi))^{1-\sigma_F^D} dG_F(\varphi) \right)^{1/(1-\sigma_F^D)}, \\ N_F^D &= \frac{\beta_F L_F + X_H^O}{(\sigma_F - 1) [f_F^e + \sum_{j=H,F} \int_{\Phi_{Fj}} f_{Fj}(\varphi) dG_F(\varphi)]}. \end{aligned}$$

In the absence of active selection, we can treat  $\Phi_{FF}$  and  $\Phi_{FH}$  as fixed. Thus, the previous equations imply

$$d \ln Q_{FF}^D / d \ln X_H^O = (\mu_F^D X_H^O) / (P_{FF}^D Q_{FF}^D).$$

Combining this expression with equation (D.9), we obtain

$$\eta_X^D = (\mu_F^D X_H^O) / (\epsilon^D P_{FF}^D Q_{FF}^D). \quad (D.10)$$

Finally, consider  $\zeta_{FH} \equiv \partial \ln P_{FH}^D / \partial \ln Q_{FH}^D$  and  $\zeta_X \equiv \partial \ln P_{FH}^D / \partial \ln X_H^O$ . In the absence of active selection, we must have

$$\zeta_{FH} = 0, \quad (D.11)$$

$$\zeta_X = \frac{1}{1 - \sigma_F^D} \frac{X_H^O}{(P_{FF}^D Q_{FF}^D + P_{FH}^D Q_{FH}^D)}. \quad (D.12)$$

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<sup>31</sup>In Section 5.2, we have expressed  $\eta_{FH}$  as a function of the expenditure share,  $x_{FF} \equiv P_{FF} Q_{FF} / L_F$ . It should be clear that with only one sector, shares of revenues and expenditures are equal by trade balance.

Combining equations (42) and (43) with equations (D.6), (D.8), (D.10), (D.11), and (D.12), we obtain

$$\tau^D = \frac{1}{(\epsilon^D - 1)x_{FF}^D},$$

$$\tau^O = -\frac{(1 - r_{FF}^D)(z/r_{FF}^D + (1 - z)\epsilon^D)}{\epsilon^D(\sigma_F^D - 1) + (1 - r_{FF}^D)(\sigma_F^D z/r_{FF}^D + (1 - z)\epsilon^D)},$$

where the first expression uses the fact foreign expenditure and revenue shares are related through  $(1/x_{FF}^D - 1) = (1/r_{FF}^D - 1)z$ . Equations (46) and (47) derive from equations (44) and (45) and the two previous expressions.

## D.4 Trade Taxes in a Small Open Economy (Section 6.2)

The goal of this subsection is to establish equations (48) and (49) under the assumption that Home is a small open economy. We have already argued in the main text that if Home is a small open economy, then  $\zeta_X = \eta_X^D = 0$  and  $\zeta_{FH} = 1/\kappa^D$ . In addition, setting  $r_{FF}^D = 1$  in equation (D.7), we obtain  $\eta_{FH}^D = -1/\kappa^D$ . The last elasticity,  $\eta_{HF}^D$ , is unaffected by the fact that Home is a small open economy:  $\eta_{HF}^D = -1/\epsilon^D$  by equation (D.6). Combining the previous observations with equations (42) and (43), we get

$$\tau^D = (1 + \epsilon^D/\kappa^D)/(\epsilon^D - 1),$$

$$\tau^O = 1/\kappa^D.$$

Equations (48) and (49) derive from equations (44) and (45) and the two previous expressions.