# Measuring uncertainty and its impact on the economy* 

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#### Abstract

We propose a new framework for measuring uncertainty and its effects on the economy, based on a large VAR model with errors whose stochastic volatility is driven by two common unobservable factors, representing aggregate macroeconomic and financial uncertainty. The uncertainty measures can also influence the levels of the variables so that, contrary to most existing measures, ours reflect changes in both the conditional mean and volatility of the variables, and their impact on the economy can be assessed within the same framework. Moreover, identification of the uncertainty shocks is simplified with respect to standard VAR-based analysis, in line with the FAVAR approach and with heteroskedasticity-based identification. Finally, the model, which is also applicable in other contexts, is estimated with a new Bayesian algorithm, which is computationally efficient and allows for jointly modeling many variables, while previous VAR models with stochastic volatility could only handle a handful of variables. Empirically, we apply the method to estimate uncertainty and its effects using US data, finding that there is indeed substantial commonality in uncertainty, sizable effects of uncertainty on key macroeconomic and financial variables with responses in line with economic theory, and some uncertainty about uncertainty and its effects.


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J.E.L. Classification: E44, C11, C13, C33, C55.

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## 1 Introduction

In the aftermath of the 2008 financial crisis and the Great Recession the interest of economists and policymakers is markedly focused on the analysis of macroeconomic and financial uncertainty and their effects on the economy. Reflecting such an interest, the literature on the topic has mushroomed in the last few years. Econometric studies on measuring uncertainty and its effects on the economy started with the seminal paper by Bloom (2009), and other relevant contributions include, among others, Bachmann, Elstner, and Sims (2013), Baker, Bloom and Davis (2015), Basu and Bundick (2015), Caggiano, Castelnuovo and Groshenny (2014), Gilchrist, Sim and Zakrajek (2014), Jo and Sekkel (2015), Jurado, Ludvigson, and Ng (2015), and Ludvigson, Ma, and Ng (2015); Bloom (2014) surveys related work.

A common denominator of most of the contributions in the literature is the fact that some measures of uncertainty (either financial, or macroeconomic, or both) are estimated in a preliminary step and then used as if they were observable data series in the subsequent econometric analysis of its impact on macroeconomic variables. For example, Bloom (2009) and Caggiano, Castelnuovo and Groshenny (2014) use the VIX ${ }^{1}$, Basu and Bundick (2015) the $\mathrm{VXO}^{2}$, Bachmann, Elstner, and Sims (2013) the disagreement in business expectations, Jurado, Ludvigson, and Ng (2015) an average of the volatilities of the residuals of a set of factor augmented regressions, Jo and Sikkel (2015) the common factor in the forecast errors resulting from the use of SPF forecasts for a few variables, Baker, Bloom and Davis (2015) an index based on newspapers coverage, and Gilchrist, et al. (2014) a sequence of estimated time fixed effects capturing common shocks to (constructed) firm-specific idiosyncratic volatilities. They all then include their preferred uncertainty measure, together with a small set of macroeconomic variables, in a homoskedastic VAR model and compute the responses of the macro variables to the uncertainty shock.

While the approach outlined above has the merit of bringing to the fore the effects that uncertainty can have on the macroeconomy, the fact that the uncertainty measure is not fully embedded in the econometric model at the estimation stage inevitably can complicate the task of making statistical inference on its effects, for several reasons.

First, the two-step approach treats uncertainty - which is estimated in the first step -

[^1]as an observable variable in the second step. Therefore the inference heavily relies on the first-step estimates being consistent, which might or might not be the case, depending on whether the size of the cross section is large enough, or whether the model used in the first step is correctly specified. ${ }^{3}$ If consistency is not achieved, the second step can potentially suffer from measurement errors in the regressors, which might lead to an endogeneity bias. A related problem is that the uncertainty around the uncertainty estimates can not be accounted for in such a setup, since the proxy for uncertainty is treated as data.

Second, even if in the first step a large enough cross section of variables is considered in estimating uncertainty, the second step invariably relies on rather small systems, typically including a handful of macroeconomic variables. The use of small VAR models to assess the effects of uncertainty can make the results subject to the common omitted variable bias and non-fundamentalness of the errors, besides the obvious shortcoming of providing results on the impact to just a few economic indicators.

Third, the models used in the first and second step are somewhat contradictory. While the estimation of the uncertainty measure(s) in the first step is predicated on the assumptions that macroeconomic data feature time-varying volatilities, the vector autoregression (VAR) used in the second step features homoskedastic errors. Moreover, in the first step volatilities are assumed not to affect the conditional means of the variables (even though the final goal is to actually assess the conditional mean effects of uncertainty on economic variables), while in the second step the uncertainty measure only affects the conditional means, but not the conditional variances (which as mentioned above are assumed to be constant over time).

Fourth, most of the structural analyses carried out in the existing work rely on a Cholesky scheme for identification. ${ }^{4}$ While such a scheme has some merits, it requires taking a stand on the appropriate ordering of the variables, a choice which is not obvious, since it is unclear whether uncertainty is an impulse or propagation mechanism.

Motivated by these considerations, in this paper we develop an econometric model and method for jointly and coherently (1) constructing measures of uncertainty (macroeconomic

[^2]and financial) and (2) conducting inference on its impact on the macroeconomy in a way that avoids all of the issues highlighted above. Specifically, we build a large, heteroskedastic VAR model in which the error volatilities evolve over time according to a factor structure. The volatility of each variable in the system is driven by a common component, and an idiosyncratic component. Changes in the common component of the volatilities of the VAR's variables provide contemporaneous, identifying information on uncertainty.

In our setup, uncertainty and its effects are estimated in a single step within the same model, which avoids both the estimated regressors problem and the use of two contradictory models typical of the two-step approach. The model uses a large cross section of data and allows for time variation in the volatilities, which avoids problems of misspecification, omitted variable bias, and non-fundamentalness. Finally, the fact that uncertainty is defined as the common component of the time-varying volatilities allows us to uniquely identify uncertainty shocks without having to resort to a Cholesky (or other) identification scheme.

In the discussion so far we have generically referred to uncertainty. More specifically, we consider both macroeconomic and financial uncertainty. Each of these measures of uncertainty is modeled as the common component of the volatilities of macroeconomic and financial variables, respectively. The vector containing the two measures of uncertainty is assumed to depend on its own past values as well as past values of macroeconomic variables. Hence, macroeconomic uncertainty can affect financial uncertainty and vice versa, and both can be affected by the business cycle and financial fluctuations. Moreover, the vector of macro and financial uncertainty enters the conditional means of the large VAR equations. As a consequence, macro and financial uncertainty are allowed to contemporaneously affect the macroeconomy and financial conditions.

The model is estimated via a new MCMC algorithm, which is computationally efficient and makes tractable estimation of large models with stochastic volatility (SV). Since uncertainty is explicitly treated as an unobservable random variable, the estimation procedure returns its entire posterior distribution, which is readily available for inference and allows us to measure uncertainty around uncertainty. The model can be also interpreted as a factor model, or a factor augmented VAR (FAVAR), in which the factor affects not only the levels but also the conditional volatility of the variables. As such, it relates to the vast literature on factor models; see, e.g., Stock and Watson (2015) for an overview.

Our proposed modeling framework extends the seminal work of Jurado, Ludvigson, and

Ng (2015) [hereafter, JLN] in several ways. In the JLN approach, individual measures of uncertainty are built for each variable, using an augmented factor model for each variable assuming that uncertainty does not affect the conditional mean, and an aggregate uncertainty measure is formed as an average of these individual estimates. Then, in a second step, the uncertainty estimate is treated as data and inserted into a separate small VAR to compute its effects on the macroeconomy. This approach relies on an identification assumption about structural shocks, and it rules out an assessment of the uncertainty around uncertainty in conducting inference on uncertainty's macroeconomic effects. In our VAR-based framework, the estimate of uncertainty is obtained from a joint model in which uncertainty affects the conditional mean and variances of each variable in the VAR, there is no need to resort to standard VAR based procedures to identify the uncertainty shock, and the estimates of the effects of uncertainty reflect the uncertainty around the measure of uncertainty.

In light of research and practical interest in the interaction of macroeconomic and financial uncertainty, and their effects on the economy, Ludvigson, Ma, and Ng (2015) [hereafter, LMN] develop a model featuring both financial and macroeconomic uncertainty. LMN estimate financial uncertainty using the methodology of JLN, applied to a large set of financial indicators. They then model financial uncertainty, GDP growth and JLN-type macroeconomic uncertainty first in a 3 -variable VAR and then in a slightly larger VAR, and study the transmission of financial and macroeconomic uncertainty shocks, using a novel identification procedure that avoids a Cholesky ordering of the variables. Their findings indicate that "higher uncertainty about real economic activity in recessions is fully an endogenous response to other shocks that cause business cycle fluctuations, while uncertainty about financial markets is a likely source of the fluctuations. Financial market uncertainty has quantitatively large negative consequences for several measures of real activity including employment, production, and orders." However, in their approach the uncertainty measures are still both estimated in a preliminary step (using a model that assumes volatilities do not affect the conditional means of variables) and then plugged into a small scale homoskedastic model.

Creal and Wu (2016) develop a model of bond yields and a small set of macroeconomic variables that extends a typical term structure model to allow uncertainty about monetary policy to affect economic activity and bond yields. Their model, like ours, jointly treats uncertainty as a factor in volatility and in conditional means of macroeconomic variables
and interest rates. To borrow their wording, the model internalizes the uncertainty. Their estimates show two volatility factors to be important, capturing uncertainty about monetary policy and the term premium. Our model can be seen as generalizing theirs in some respects, by moving to a VAR structure, permitting a relatively large data set, and allowing both uncertainty factors in our specification to affect all variables, not just macro variables.

Our approach also significantly extends some other, previous econometric work on modeling uncertainty, all using small models. Alessandri and Mumtaz (2014) develop a small nonlinear VAR in four variables that allows volatility to enter the conditional means. However, in order to estimate the common factor they adopt the common volatility specification of Carriero, Clark and Marcellino (2015a), which is more restrictive than the formulation we present here, because there cannot be idiosyncratic volatility and the loadings on the common volatility factor must be all equal across variables. Moreover, their model cannot handle a large dataset, which is instead key for a proper estimation of aggregate uncertainty, a result which was emphasized by JLN and that is also confirmed by the empirical evidence we will provide.

Other contributions in the literature have also proposed the inclusion of volatility in the conditional mean of a VAR, without resorting to a common factor specification for the volatilities. Jo (2014) studies the effects of oil price uncertainty on global real economic activity using a VAR model with stochastic volatility in mean and finds that the effects are sizable. While these results on oil price interesting are useful, the VAR is for a small set of variables and the volatilities for each variable are treated as independent processes. Shin and Zhong (2014) introduce a new small VAR model with stochastic volatility, also allowing for volatility-in-mean, in order to study the real effects of uncertainty shocks, which are identified by imposing restrictions on the first and second moment responses of the variables to the uncertainty shock. They provide theoretical methods for estimation and inference for the new model, with the more general structural identification procedure; empirically, they find evidence that an increase in uncertainty leads to a decline in industrial production only if associated with a deterioration in financial conditions. With respect to their specification, we can model a much larger number of variables and allow for a factor structure in the volatilities, permitting us to define uncertainty as the common factor in volatility.

We apply our proposed model to monthly US data for the period 1959-2014, finding
substantial evidence of commonality in volatilities, as well as not-negligible idiosyncratic movements in the volatilities. Uncertainty around estimated uncertainty is sizable. Yet, a clear and significant pattern of time variation emerges, with increases in macro uncertainty associated with economic recessions. However, we find less little evidence of the "Great Moderation." This is mainly due to the use of a large information set, as already pointed out by Giannone, Lenza and Reichlin (2008), and to the monthly frequency of the variables we analyze, as indeed we find somewhat stronger evidence of a moderation in volatility after the mid 1980s when repeating the analysis with quarterly data.

As noted above, we separately identify a macroeconomic uncertainty measure and a financial uncertainty measure. In impulse response analysis, we document sizable effects of uncertainty shocks on many macroeconomic and financial variables. Shocks (surprise increases) to macroeconomic and financial uncertainty both lead to significant and persistent declines in economic activity. But a shock to financial uncertainty does not affect some measures of economic activity (notably, the response of the housing market and consumption expenditures to financial uncertainty is insignificant) as much as a shock to macro uncertainty does. Both types of shocks also cause the credit spread in the model to rise (modestly but significantly). However, for other financial variables, results are more mixed: we find that surprise increases to financial uncertainty reduce measures of aggregate stock prices and returns, whereas the effects of increases in macro uncertainty are not significant.

Therefore, the overall picture emerging from our empirical application is that macroeconomic uncertainty has large, significant effects on real activity, but has a limited impact on financial variables, whereas financial uncertainty shocks directly impact financial variables and subsequently transmit to the macroeconomy, a finding in line with, e.g., LMN.

The paper is structured as follows. Section 2 discusses model specification and estimation. Section 3 presents the data. Section 4 presents our estimates of aggregate uncertainty. Section 5 studies its effects on the economy. Section 6 summarizes our main findings and concludes.

## 2 A joint model of uncertainty and business cycle fluctuations

In this section we present the model that we use to estimate aggregate uncertainty and its effects on the economy. We start by summarizing the main features of the model,
highlighting the relation and differences with other approaches. Then we discuss, in turn, model specification and estimation. We also detail the univariate autoregressive (AR) model with stochastic volatility we use in some comparisons.

### 2.1 The model

The model for the macroeconomic and financial variables of interest - collected in the vector $y_{t}$ - is a heteroskedastic VAR, similar to those widely used in macroeconomic analysis since the contributions of Cogley and Sargent (2005) and Primiceri (2005). However, rather than using a small cross section and assuming that volatilities for each variable evolve independently, we use a large cross section of variables, and we assume that volatilities follow a factor structure, i.e. have a common and an idiosyncratic component. ${ }^{5}$ Our measures of macroeconomic and financial uncertainty are defined as the common components in the volatility of either macroeconomic or financial variables. These common components are state variables of the model, and they are assumed to follow a bivariate VAR augmented with lags of the macroeconomic and financial variables of interest. Hence, the economic and financial variables of $y_{t}$ are allowed to have a feedback effect on uncertainty. The measures of uncertainty enter the conditional mean of the VAR in $y_{t}$. Therefore, our modeling approach allows for uncertainty to contemporaneously affect the macroeconomy, through both first (means) and second order (variances) effects. Actually, the latter is the key idea in this literature, but often the relationship is only imposed in a separate auxiliary model and not used at the uncertainty estimation level, so that the estimated measure of uncertainty only reflects the conditional second moments of the variables. In our specification, instead, the measure of uncertainty reflects information in the levels of the variables. ${ }^{6}$ Finally, it is worth emphasizing that our model features time variation in the volatilities, and this time variation is driven in part by the uncertainty measures, which means that shocks to the measures of uncertainty are uniquely identified, without resorting to a Cholesky or other identification scheme.

[^3]
### 2.2 Model specification

Let $y_{t}$ denote the $n \times 1$ vector of variables of interest, split into $n_{m}$ macroeconomic and $n_{f}=n-n_{m}$ financial variables. Let $v_{t}$ be the corresponding $n \times 1$ vector of reduced form shocks to these variables, also split into two groups of $n_{m}$ and $n_{f}$ components. The reduced form shocks are modelled as:

$$
\begin{equation*}
v_{t}=A^{-1} \Lambda_{t}^{0.5} \epsilon_{t}, \epsilon_{t} \sim \operatorname{iid} N(0, I), \tag{1}
\end{equation*}
$$

where $A$ is an $n \times n$ lower triangular matrix with ones on the main diagonal, and $\Lambda_{t}$ is a diagonal matrix of volatilities with generic $j$-th element

$$
\lambda_{j t}=\left\{\begin{array}{c}
m_{t}^{\beta_{m, j}} \cdot h_{j, t}, j=1, \ldots, n_{m}  \tag{2}\\
f_{t}^{\beta_{f, j}} \cdot h_{j, t}, j=n_{m}+1, \ldots, n
\end{array},\right.
$$

which implies that the log-volatilities follow a linear factor model:

$$
\ln \lambda_{j t}=\left\{\begin{array}{c}
\beta_{m, j} \ln m_{t}+\ln h_{j, t}, j=1, \ldots, n_{m}  \tag{3}\\
\beta_{f, j} \ln f_{t}+\ln h_{j, t}, j=n_{m}+1, \ldots, n
\end{array} .\right.
$$

We discuss below the rationale for the block specification of (3), in which only the factor $m$ enters the $\lambda$ process of macro variables, and only the factor $f$ enters the $\lambda$ process of financial variables. The variables $h_{j, t}$ capture idiosyncratic volatility components associated with the $j$-th variable in the VAR, and are assumed to follow (in logs) an autoregressive process:

$$
\begin{equation*}
\ln h_{j, t}=\gamma_{j, 0}+\gamma_{j, 1} \ln h_{j, t-1}+e_{j, t}, j=1, \ldots, n, \tag{4}
\end{equation*}
$$

with $\nu_{t}=\left(e_{1, t}, \ldots, e_{n, t}\right)^{\prime}$ jointly distributed as i.i.d. $N\left(0, \Phi_{\nu}\right)$ and independent among themselves, so that $\Phi_{\nu}=\operatorname{diag}\left(\phi_{1}, \ldots, \phi_{n}\right)$. These shocks are also independent from the conditional errors $\epsilon_{t}$.

The variable $m_{t}$ is our measure of (unobservable) aggregate macroeconomic uncertainty, and the variable $f_{t}$ is our measure of (unobservable) aggregate financial uncertainty. Although our specification does not rule out the inclusion of additional uncertainty factors, we believe two factors to be appropriate. One reason is that we are interested in aggregate uncertainty, which suggests the use of a single macro factor and a single financial factor, in keeping with the concepts of studies such as JLN. A second reason is that a general factor analysis in Carriero, Clark, and Marcellino (2016) suggests two dynamic factors. ${ }^{7}$

[^4]Together, the two measures of uncertainty (in logs) follow an augmented VAR process:

$$
\left[\begin{array}{c}
\ln m_{t}  \tag{5}\\
\ln f_{t}
\end{array}\right]=D(L)\left[\begin{array}{c}
\ln m_{t-1} \\
\ln f_{t-1}
\end{array}\right]+\left[\begin{array}{c}
\delta_{m}^{\prime} \\
\delta_{f}^{\prime}
\end{array}\right] y_{t-1}+\left[\begin{array}{c}
u_{m, t} \\
u_{f, t}
\end{array}\right],
$$

where $D(L)$ is a lag-matrix polynomial of order $d$. The shocks to the uncertainty factors $u_{m, t}$ and $u_{f, t}$ are independent from the shocks to the idiosyncratic volatilities $e_{j, t}$ and the conditional errors $\epsilon_{t}$, and they are jointly normal with mean 0 and variance $\operatorname{var}\left(u_{t}\right)=$ $\operatorname{var}\left(\left(u_{m, t}, u_{f, t}\right)^{\prime}\right)=\Phi_{u}=\left[\begin{array}{ll}\phi_{n+1} & \phi_{n+3} \\ \phi_{n+3} & \phi_{n+2}\end{array}\right]$. The specification in (5) implies that the uncertainty factors depend on their own past values as well as the previous values of the variables in the model, and therefore they respond to business cycle fluctuations. ${ }^{8}$ Importantly, financial uncertainty affects macro uncertainty and vice-versa, and the error terms $u_{m, t}$ and $u_{f, t}$ are allowed to be correlated, with correlation $\phi_{n+3}$, reflecting the idea that a common shock can affect both uncertainties.

For identification, we set $\beta_{m, 1}=1$ and $\beta_{f, n_{m}+1}=1$ and assume $\ln m_{t}$ and $\ln f_{t}$ to have zero unconditional mean. In addition, for identification, we deliberately include the block restrictions of factor loadings in the volatilities specification of (2) in order to allow the comovement between uncertainties captured in the VAR structure and correlated innovations of (5). Conceptually, we believe these block restrictions to be consistent with broad definitions of uncertainty: macro uncertainty is the common factor in the error variances of macro variables, and finance uncertainty is the common factor in the error variances of finance variables. However, these uncertainties are not necessarily independent; they can move together due to correlated innovations to the uncertainties, the VAR dynamics of uncertainty captured in $D(L)$, and responses to past fluctuations in macro and finance variables $\left(y_{t-1}\right)$.

The uncertainty variables $m_{t}$ and $f_{t}$ can also affect the levels of the macro and finance variables of interest $y_{t}$, contemporaneously and with lags. In particular, $y_{t}$ is assumed to follow:

$$
\begin{equation*}
y_{t}=\Pi(L) y_{t-1}+\Pi_{m}(L) \ln m_{t}+\Pi_{f}(L) \ln f_{t}+v_{t}, \tag{6}
\end{equation*}
$$

where $p$ denotes the number of $y_{t}$ lags in the VAR, $\Pi(L)=\Pi_{1}-\Pi_{2} L-\cdots-\Pi_{p} L^{p-1}$, with each $\Pi_{i}$ an $n \times n$ matrix, $i=1, \ldots, p$, and $\Pi_{m}(L)$ and $\Pi_{f}(L)$ are $n \times 1$ lag-matrix polynomials of order $p_{m}$ and $p_{f}$. The specification above ensures that business cycle fluctuations respond to movements in uncertainty (macro and financial), both through the conditional

[^5]variances (contemporaneously, via movements in $v_{t}$ ) and through the conditional means (contemporaneously and with lag, via the coefficients collected in $\Pi_{m}(L)$ and $\Pi_{f}(L)$ ).

The model in (1)-(6) is related to Carriero, Clark and Marcellino (2015a), who impose $\Pi_{m}(L)=\Pi_{f}(L)=0$ and consider a small model for computational reasons. However, as discussed in the introduction, when measuring uncertainty it appears to be important to allow $n$ to be large and to permit direct effects of uncertainty on the endogenous macroeconomic and financial variables $\left(\Pi_{m}(L) \neq 0, \Pi_{f}(L) \neq 0\right)$. The model is also related to Cogley and Sargent (2005) and Primiceri (2005), who also impose $\Pi_{m}(L)=\Pi_{f}(L)=0$ and, in addition, assume that there is no factor structure in the volatilities, which amounts to setting $\beta_{j}=0 .{ }^{9}$ Finally, the model is related to Alessandri and Mumtaz (2014), who assume that $\beta_{j}=1$ for all $j$, and $\ln h_{j, t}=0$. Augmented by allowing the common volatility factor to affect the conditional mean of $y_{t}$, this corresponds to the CSV specification of Carriero, Clark and Marcellino (2015a), which, however, is not suited in this context, as with $n$ large both restrictions are not likely to hold in the data (and indeed Alessandri and Mumtaz (2014) analyze four variables only).

The model in (1)-(6) is also related to parametric factor models, such as Stock and Watson (1989), where $\Pi(L)=0$ and $v_{t} \sim \operatorname{iid} N(0, \Sigma)$, or Marcellino, Porqueddu and Venditti (2015), who allow for stochastic volatility both in $v_{t}$ and in the error driving the common factor, $u_{t}$.

It is worth mentioning that our model could be applied in a variety of other contexts where volatilities are likely to follow a factor structure and affect the levels of the variables, for example models for stock returns or the term structure of interest rates.

Working with a model as general as (1)-(6) substantially complicates estimation, as we discuss in the next subsection. The reader not interested in technicalities can skip to Section 3. In implementation, we set the VAR lag order at $p=6$ in monthly data (we use $p=4$ in some supplemental results with quarterly data). We set the lag order for the uncertainty factors in the VAR's conditional mean ( $p_{m}$ and $p_{f}$ ) at 2 , so that, for both uncertainty measures, the model includes the current value and two lags. We also conduct a robustness check with a model in which the current value of uncertainty is zeroed out, so that there are no contemporaneous effects of uncertainty on the conditional means of the

[^6]VAR. Finally, we set the lag order of the bivariate VAR in the uncertainty factors (d) to 2 .

### 2.3 Triangularization for estimation

In a Bayesian setting, estimation and inference on the model parameters and unobservable states are based on their posterior distributions. The latter can be obtained by combining the likelihood of the model with prior distributions for the parameters and states. Often, analytical posteriors are not available but draws from them can be obtained by MCMC samplers. However, this is in general so computationally intensive for models with stochastic volatilities that practical implementation of these models has been limited to a handful of variables, with $n$ typically in the range of 3 to 5 . To make estimation feasible in a model with a large number of variables and stochastic volatility, we exploit the VAR triangularization of Carriero, Clark and Marcellino (2016), who consider the case $\Pi_{m}(L)=\Pi_{f}(L)=0$ and stochastic volatilities without a common factor structure. With the triangularization, our estimation algorithm will block the conditional posterior distribution of the system of VAR coefficients in $n$ different blocks. In the step of the typical Gibbs sampler that involves drawing the set of VAR coefficients $\Pi$, all of the remaining model coefficients are given. Consider again the reduced form residuals $v_{t}=A^{-1} \Lambda_{t}^{0.5} \epsilon_{t}$ :

$$
\left[\begin{array}{c}
v_{1, t}  \tag{7}\\
v_{2, t} \\
\ldots \\
v_{n, t}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & \ldots & 0 \\
a_{2,1}^{*} & 1 & & \ldots \\
\ldots & & 1 & 0 \\
a_{n, 1}^{*} & \ldots & a_{n, n-1}^{*} & 1
\end{array}\right]\left[\begin{array}{cccc}
\lambda_{1, t}^{0.5} & 0 & \ldots & 0 \\
0 & \lambda_{2, t}^{0.5} & & \ldots \\
\ldots & & \ldots & 0 \\
0 & \ldots & 0 & \lambda_{n, t}^{0.5}
\end{array}\right]\left[\begin{array}{c}
\epsilon_{1, t} \\
\epsilon_{2, t} \\
\ldots \\
\epsilon_{n, t}
\end{array}\right]
$$

where $a_{j, i}^{*}$ denotes the generic element of the matrix $A^{-1}$ which is available under knowledge of $A$. The VAR can be written as:

$$
\begin{aligned}
y_{1, t}= & \sum_{i=1}^{n} \sum_{l=1}^{p} \pi_{1, l}^{(i)} y_{i, t-l}+\sum_{l=0}^{p_{m}} \pi_{l, 1}^{(m)} \ln m_{t-l}+\sum_{l=0}^{p_{f}} \pi_{l, 1}^{(f)} \ln f_{t-l}+\lambda_{1, t}^{0.5} \epsilon_{1, t} \\
y_{2, t}= & \sum_{i=1}^{n} \sum_{l=1}^{p} \pi_{2, l}^{(i)} y_{i, t-l}+\sum_{l=0}^{p_{m}} \pi_{l, 2}^{(m)} \ln m_{t-l}+\sum_{l=0}^{p_{f}} \pi_{l, 2}^{(f)} \ln f_{t-l}+a_{2,1}^{*} \lambda_{1, t}^{0.5} \epsilon_{1, t}+\lambda_{2, t}^{0.5} \epsilon_{2, t} \\
& \cdots \\
y_{n, t}= & \sum_{i=1}^{n} \sum_{l=1}^{p} \pi_{n, l}^{(i)} y_{i, t-l}+\sum_{l=0}^{p_{m}} \pi_{l, N}^{(m)} \ln m_{t-l}+\sum_{l=0}^{p_{f}} \pi_{l, N}^{(f)} \ln f_{t-l}+a_{n, 1}^{*} \lambda_{1, t}^{0.5} \epsilon_{1, t}+\cdots \\
& \cdots+a_{n, n-1}^{*} \lambda_{n-1, t}^{0.5} \epsilon_{n-1, t}+\lambda_{n, t}^{0.5} \epsilon_{n, t},
\end{aligned}
$$

with the generic equation for variable $j$ :

$$
\begin{align*}
& y_{j, t}-\left(a_{j, 1}^{*} \lambda_{1, t}^{0.5} \epsilon_{1, t}+\cdots+a_{j,, j-1}^{*} \lambda_{j-1, t}^{0.5} \epsilon_{j-1, t}\right) \\
= & \sum_{i=1}^{n} \sum_{l=1}^{p} \pi_{j, l}^{(i)} y_{i, t-l}+\sum_{l=0}^{p_{m}} \pi_{l, j}^{(m)} \ln m_{t-l}+\sum_{l=0}^{p_{f}} \pi_{l, j}^{(f)} \ln f_{t-l}+\lambda_{j, t} \epsilon_{j, t} . \tag{8}
\end{align*}
$$

Consider estimating these equations in order from $j=1$ to $j=n$. When estimating the generic equation $j$ the term of the left hand side in (8) is known, since it is given by the difference between the dependent variable of that equation and the estimated residuals of all the previous $j-1$ equations. Therefore we can define:

$$
\begin{equation*}
y_{j, t}^{*}=y_{j, t}-\left(a_{j, 1}^{*} \lambda_{1, t}^{0.5} \epsilon_{1, t}+\cdots+a_{j, j-1}^{*} \lambda_{j-1, t}^{0.5} \epsilon_{j-1, t}\right), \tag{9}
\end{equation*}
$$

and equation (8) becomes a standard generalized linear regression model for the variable in equation (9) with Gaussian disturbances with mean 0 and variance $\lambda_{j, t}$.

Accordingly, drawing on results detailed in Carriero, Clark and Marcellino (2016), the posterior distribution of the VAR coefficients can be factorized as:

$$
\begin{aligned}
p\left(\Pi \mid A, \beta, f_{T}, m_{T}, h_{T}, y_{T}\right)= & p\left(\pi^{(n)} \mid \pi^{(n-1)}, \pi^{(n-2)}, \ldots, \pi^{(1)}, A, \beta, f_{T}, m_{T}, h_{T}, y_{T}\right) \\
& \times p\left(\pi^{(n-1)} \mid \pi^{(n-2)}, \ldots, \pi^{(1)}, A, \beta, f_{T}, m_{T}, h_{T}, y_{T}\right) \\
& \times p\left(\pi^{(1)} \mid A, \beta, f_{T}, m_{T}, h_{T}, y_{T}\right),
\end{aligned}
$$

where the vector $\beta$ collects the loadings of the uncertainty factors and $f_{T}, m_{T}, h_{T}=$ $\left(h_{1, T}, \ldots, h_{n, T}\right)$, and $y_{T}$ denote the history of the states and data up to time $T$. As a result, we are able to estimate the coefficients of the VAR on an equation-by-equation basis. For reasons discussed in Carriero, Clark and Marcellino (2016), this greatly speeds estimation and permits us to consider much larger systems than we would otherwise be able to consider. Below we provide further details on the estimation algorithm, allowing for the presence of the unobservable uncertainty factors $f_{T}$ and $m_{T}$ in the conditional means and variances.

Importantly, although the expression (7) and the following triangular system are based on a Cholesky-type decomposition of the variance $\Sigma_{t}$, the decomposition is simply used as an estimation device, not as a way to identify structural shocks. The ordering of the variables in the system does not change the joint (conditional) posterior of the reduced form coefficients, so changing the order of the variables is inconsequential to the results. ${ }^{10}$

[^7]We now discuss in turn the general organization of the MCMC algorithm, estimation of the model coefficients, unobservable states, and the details of the MCMC algorithm used to draw from the joint posterior of coefficients and states.

### 2.4 General steps of MCMC algorithm

Our exposition of priors, posteriors, and estimation makes use of the following additional notation. The vector $a_{j}, j=2, \ldots, n$, contains the $j^{\text {th }}$ row of the matrix $A$ (for columns 1 through $j-1$ ). We define the vector $\gamma=\left\{\gamma_{1}, \ldots, \gamma_{n}\right\}$, which collects the coefficients appearing in the conditional means of the transition equations for the states $h_{T}$, and $\delta=$ $\left\{D(L), \delta_{m}^{\prime}, \delta_{f}^{\prime}\right\}$, which collects the coefficients appearing in the conditional means of the transition equations for the states $m_{T}$ and $f_{T}$. The coefficient matrices $\Phi_{v}$ and $\Phi_{u}$ defined above collect the variances of the shocks to the transition equations for the idiosyncratic states $h_{T}$ and the common uncertainty factors $m_{T}$ and $f_{T}$, respectively. In addition, we group the parameters of the model in (1)-(6), except the vector of factor loadings $\beta$, into

$$
\begin{equation*}
\Theta=\left\{\Pi, A, \gamma, \delta, \Phi_{v}, \Phi_{u}\right\} . \tag{10}
\end{equation*}
$$

Let $s_{T}$ denote the time series of the mixture states used in the Kim, Shephard, and Chib (1998) algorithm (explained below) to draw $h_{T}$.

We use an MCMC algorithm to obtain draws from the joint posterior distribution of model parameters $\Theta$, loadings $\beta$, and latent states $h_{T}, m_{T}, f_{T}, s_{T}$. Specifically, we sample in turn from the following two conditional posteriors (for simplicity, we suppress notation for the dependence of each conditional posterior on the data sample $y_{T}$ ):

1. $h_{T}, \beta \mid \Theta, s_{T}, m_{T}, f_{T}$
2. $\Theta, s_{T}, m_{T}, f_{T} \mid h_{T}, \beta$.

The first step relies on a state space system. Defining the rescaled residuals $\tilde{v}_{t}=A v_{t}$, taking the log squares of (1), subtracting out the known (in the conditional posterior)
system might be affected by the ordering of the variables in the system due to an entirely different reason: the diagonalization typically used for the error variance $\Sigma_{t}$ in stochastic volatility models. Since priors are elicited separately for $A$ and $\Lambda_{t}$, the implied prior of $\Sigma_{t}$ will change if one changes the equation ordering, and therefore different orderings would result in different prior specifications and then potentially different joint posteriors. This problem is not a feature of our triangular algorithm, but rather it is inherent to all models using the diagonalization of $\Sigma_{t}$. As noted by Sims and Zha (1998) and Primiceri (2005), this problem will be mitigated in the case (as the one considered in this paper) in which the covariances $A$ do not vary with time, because the likelihood information will soon dominate the prior.
contributions of the common factors, and using (3) yields the observation equations ( $\bar{c}$ denotes an offset constant used to avoid problems with near-zero values):

$$
\left\{\begin{array}{c}
\ln \left(\tilde{v}_{j, t}^{2}+\bar{c}\right)-\beta_{m, j} \ln m_{t}=\ln h_{j, t}+\ln \epsilon_{j, t}^{2}, j=1, \ldots, n_{m}  \tag{11}\\
\ln \left(\tilde{v}_{j, t}^{2}+\bar{c}\right)-\beta_{f, j} \ln f_{t}=\ln h_{j, t}+\ln \epsilon_{j, t}^{2}, j=n_{m}+1, \ldots, n .
\end{array}\right.
$$

For the idiosyncratic volatility components, the transition and measurement equations of the state-space system are given by (4) and (11), respectively. The system is linear but not Gaussian, due to the error terms $\ln \epsilon_{j, t}^{2}$. However, $\epsilon_{j, t}$ is a Gaussian process with unit variance; therefore, we can use the mixture of normals approximation of Kim, Shepard and Chib (1998) [hereafter, KSC] to obtain an approximate Gaussian system, conditional on the mixture of states $s_{T}$. To produce a draw from $h_{T}, \beta \mid \Theta, s_{T}, m_{T}, f_{T}$ we then proceed as usual by (a) drawing the time series of the states given the loadings using ( $h_{T} \mid \beta, \Theta$, $s_{T}, m_{T}, f_{T}$ ), following Primiceri's (2005) implementation of the KSC algorithm, and by then (b) drawing the loadings given the states using $\left(\beta \mid h_{T}, \Theta, s_{T}, m_{T}, f_{T}\right)$, using the conditional posterior detailed below in (22). ${ }^{11}$

The second step conditions on the idiosyncratic volatilities and factor loadings to produce draws of the model coefficients $\Theta$, common uncertainty factors $m_{T}$ and $f_{T}$, and the mixture states $s_{T}$. Draws from the posterior $\Theta, s_{T}, f_{T} \mid h_{T}, \beta$ are obtained in three substeps from, respectively: (a) $\Theta \mid m_{T}, f_{T}, h_{T}, \beta$; (b) $m_{T}, f_{T} \mid \Theta, h_{T}, \beta$; and (c) $s_{T} \mid \Theta, m_{T}$, $f_{T}, h_{T}, \beta$. More specifically, for $\Theta \mid m_{T}, f_{T}, h_{T}, \beta$ we use the posteriors detailed below, equations (20), (21), (23), (24), (25), and (26). For $m_{T}, f_{T} \mid \Theta, h_{T}, \beta$, we use the particle Gibbs step proposed by Andrieu, Doucet, and Holenstein (2010). For $s_{T} \mid \Theta, m_{T}, f_{T}, h_{T}$, $\beta$, we use the 10 -state mixture approximation of Omori, et al. (2007) that improves on Kim, Shephard, and Chib's (1998) 7-state approximation.

### 2.4.1 Coefficient priors and posteriors

This subsection details the priors and posteriors we use in the algorithm characterized above. We specify the following (independent) priors for the parameter blocks of the model

[^8](parameterization details are given in the appendix):
\[

$$
\begin{align*}
\operatorname{vec}(\Pi) & \sim N\left(\operatorname{vec}\left(\underline{\mu}_{\Pi}\right), \underline{\Omega}_{\Pi}\right),  \tag{13}\\
a_{j} & \sim N\left(\underline{\mu}_{a, j}, \underline{\Omega}_{a, j}\right), j=2, \ldots, n,  \tag{14}\\
\beta_{j} & \sim N\left(\underline{\mu}_{\beta}, \underline{\Omega}_{\beta}\right), j=2, \ldots, n_{m}, n_{m+2}, \ldots, n,  \tag{15}\\
\gamma_{j} & \sim N\left(\underline{\mu}_{\gamma}, \underline{\Omega}_{\gamma}\right), j=1, \ldots, n,  \tag{16}\\
\delta & \sim N\left(\underline{\mu}_{\delta}, \underline{\Omega}_{\delta}\right),  \tag{17}\\
\phi_{j} & \sim I G\left(d_{\phi} \cdot \underline{\phi}, d_{\phi}\right), j=1, \ldots, n,  \tag{18}\\
\Phi_{u} & \sim I W\left(d_{\Phi_{u}} \cdot \underline{\Phi_{u}}, d_{\Phi_{u}}\right) . \tag{19}
\end{align*}
$$
\]

Under these priors, the parameters $\Pi, A, \beta, \gamma, \delta, \Phi_{v}$, and $\Phi_{u}$ have the following closed form conditional posterior distributions:

$$
\begin{align*}
\operatorname{vec}(\Pi) \mid A, \beta, m_{T}, f_{T}, h_{T}, y_{T} & \sim N\left(\operatorname{vec}\left(\bar{\mu}_{\Pi}\right), \bar{\Omega}_{\Pi}\right),  \tag{20}\\
a_{j} \mid \Pi, \beta, m_{T}, f_{T}, h_{T}, y_{T} & \sim N\left(\bar{\mu}_{a, j}, \bar{\Omega}_{a, j}\right), j=2, \ldots, n,  \tag{21}\\
\beta_{j} \mid \Pi, A, \gamma, \Phi, m_{T}, f_{T}, h_{T}, s_{T}, y_{T} & \sim N\left(\bar{\mu}_{\beta}, \bar{\Omega}_{\beta}\right), j=2, \ldots, n_{m}, n_{m+2}, \ldots, n,  \tag{22}\\
\gamma_{j} \mid \Pi, A, \beta, \Phi, m_{T}, f_{T}, h_{T}, y_{T} & \sim N\left(\bar{\mu}_{\gamma}, \bar{\Omega}_{\gamma}\right), j=1, \ldots, n,  \tag{23}\\
\delta \mid \Pi, A, \gamma, \beta, \Phi, m_{T}, f_{T}, h_{T}, y_{T} & \sim N\left(\bar{\mu}_{\delta}, \bar{\Omega}_{\delta}\right),  \tag{24}\\
\phi_{j} \mid \Pi, A, \beta, \gamma, m_{T}, f_{T}, h_{T}, y_{T} & \sim I G\left(d_{\phi} \cdot \underline{\phi}+\sum_{t=1}^{T} \nu_{j t}^{2}, d_{\phi}+T\right), j=1, \ldots, n,  \tag{25}\\
\Phi_{u} \mid \Pi, A, \beta, \delta, \gamma, m_{T}, f_{T}, h_{T}, y_{T} & \sim I W\left(d_{\Phi_{u}} \cdot \underline{\Phi_{u}}+\sum_{t=1}^{T} u_{t}^{2}, d_{\Phi_{u}}+T\right) . \tag{26}
\end{align*}
$$

Expressions for $\bar{\mu}_{a, j}, \bar{\mu}_{\delta}$, and $\bar{\mu}_{\gamma}$ are straightforward to obtain using standard results from the linear regression model. In the interest of brevity, we omit details for these posteriors; the general solutions for these components are readily available in other sources (e.g., Cogley and Sargent (2005) for the treatment of $\bar{\mu}_{a, j}$ ). In the posterior for the factor loadings $\beta$, the mean and variance take a GLS-based form, with dependence on the mixture states used to draw volatility, as indicated above. In the case of the VAR coefficients $\bar{\mu}_{\Pi}$, with smaller models it is possible to rely on the GLS solution for the posterior mean given in sources such as Carriero, Clark and Marcellino (2015b). However, as discussed above, with larger models, it is far faster to exploit the triangularization discussed above and estimate the VAR coefficients on an equation-by-equation basis. ${ }^{12}$ Specifically, using the factorization in

[^9](10) together with the model in (8) allows us to draw the coefficients of the matrix $\Pi$ in separate blocks. Again, let $\pi^{(j)}$ denote the $j$-th row of the matrix $\Pi$, and let $\pi^{(1: j-1)}$ denote all the previous rows. Then draws of $\pi^{(j)}$ can be obtained from:
\[

$$
\begin{equation*}
\pi^{(j)} \mid \pi^{(1: j-1)}, A, \beta, f_{T}, m_{T}, h_{T}, y_{T} \sim N\left(\bar{\mu}_{\pi^{(j)}}, \bar{\Omega}_{\pi^{(j)}}\right) \tag{27}
\end{equation*}
$$

\]

with

$$
\begin{align*}
\bar{\mu}_{\pi^{(j)}} & =\bar{\Omega}_{\pi^{(j)}}\left\{\sum_{t=1}^{T} X_{j, t} h_{j, t}^{-1} y_{j, t}^{* \prime}+\underline{\Omega}_{\pi^{(j)}}^{-1}\left(\underline{\mu}_{\pi^{(j)}}\right)\right\}  \tag{28}\\
\bar{\Omega}_{\pi^{(j)}}^{-1} & =\underline{\Omega}_{\pi^{(j)}}^{-1}+\sum_{t=1}^{T} X_{j, t} h_{j, t}^{-1} X_{j, t}^{\prime} \tag{29}
\end{align*}
$$

where $y_{j, t}^{*}$ is defined in (9) and where $\underline{\Omega}_{\pi^{(j)}}^{-1}$ and $\underline{\mu}_{\pi^{(j)}}$ denote the prior moments on the $j$-th equation, given by the $j$-th column of $\underline{\mu}_{\Pi}$ and the $j$-th block on the diagonal of $\bar{\Omega}_{\Pi}^{-1}$. Note we have implicitly used the fact that the matrix $\underline{\Omega}_{\Pi}^{-1}$ is block diagonal, which is the case in our application, as our prior on the conditional mean coefficients is independent across equations. ${ }^{13}$

### 2.4.2 Unobservable states

For the unobserved volatility states $f_{t}, m_{t}$, and $h_{j, t}, j=1, \ldots, n$, we need to specify priors for the period 0 values, detailed in the appendix. Given the priors and the law of motion for the unobservable states in (4)-(5), draws from the posteriors can be obtained using the algorithm of Kim, Shepard and Chib (1998, KSC) for the idiosyncratic volatilities and the particle Gibbs step of Andrieu, Doucet, and Holenstein (2010) for the common volatility factors. ${ }^{14}$

### 2.5 AR-SV model

To facilitate some comparisons to the uncertainty estimates of JLN and LMN, we use an AR model with stochastic volatility (AR-SV) to form measures of uncertainty using a

[^10]methodology similar to theirs. The AR-SV model for a scalar series $y_{t}$ takes the following form:
\[

$$
\begin{align*}
y_{t} & =\pi_{0}+\pi(L) y_{t-1}+\lambda_{t}^{0.5} \epsilon_{t}, \epsilon_{t} \sim \operatorname{iid} N(0,1)  \tag{30}\\
\ln \lambda_{t} & =\gamma_{0}+\gamma_{1} \ln \lambda_{t-1}+e_{t}, e_{t} \sim i i d N(0, \phi) \tag{31}
\end{align*}
$$
\]

For each series, we estimate the model using the full sample of data. We follow the approach of JLN in computing, at each moment in time $t$, the forecast error variance using the error variance $\lambda_{t}$ and the estimated AR coefficients, for horizons up to 12 months ahead. We do so for each draw of the posterior distribution (using an MCMC algorithm that is a simplification of that used for our multivariate model) and form the median estimate of uncertainty at each horizon. Finally, we obtain a measure of uncertainty by averaging uncertainty estimates across variables, for cross sections ranging from 8 through 129 series.

## 3 Data

Our macroeconomic data (and some financial indicators) are taken from the FRED-MD monthly dataset detailed in McCracken and Ng (2015) and available from the Federal Reserve Bank of St. Louis. The FRED-MD dataset is similar to that underlying common factor model analyses, such as Stock and Watson (2005, 2006) and Ludvigson and Ng (2011). Accordingly, the dataset is also similar to the one used by JLN. After dropping out a few series with significant numbers of missing observations and dropping the series non-borrowed reserves because it became extremely volatile with the Great Recession, the total dataset comprises 129 series, over a sample of January 1959 through mid or late 2014, depending on the series. Each series is transformed as in McCracken and Ng (2015) to achieve stationarity. ${ }^{15}$

For financial variables, we use the return on the S\&P 500, the spread between the Baa bond rate and the 10 -year Treasury yield, and a set of additional variables available in datasets constructed by Kenneth French and available on his webpage. ${ }^{16}$ Specifically, in our baseline results, we use the French series on CRPS excess returns, four risk factors for SMB (Small Minus Big), HML (High minus Low), R15_R11 (small stock value spread), and momentum - and sector-level returns for a breakdown of five industries (consumer, manufacturing, high technology, health, other). We obtained similar results when, instead of

[^11]these 10 variables from Kenneth French, we used a 10 -sector breakdown of returns available from his datasets.

This specification reflects some choice as to what constitutes a macroeconomic variable rather than a financial variable. Reflecting the typical factor model analysis, the McCrackenNg dataset includes a number of indicators - of stock prices, interest rates, and exchange rates - that may be considered financial indicators. In our model specification, the variables in question are the federal funds rate, the credit spread, and the S\&P 500 index. As the instrument of monetary policy, it seems most appropriate to treat the funds rate as a macro variable. For the other two variables, the distinction between macro and finance is admittedly less clear. Whereas JLN and JMN treat these indicators as macro variables that bear on macroeconomic uncertainty and not directly on financial uncertainty (in JMN, finance uncertainty is based on the volatilities of various measures of stock returns and risk factors, we instead include the credit spread and the S\&P 500 index in the set of financial variables.

Our primary VAR results are based on a baseline specification given by 30 macroeconomic and financial variables of interest, which are listed in Table 1 below. Following examples such as JLN, after transforming each series for stationarity as needed, we standardize the data (demean and divide by the simple standard deviation) before estimating the model.

Table 1: variables in the baseline model

| Macroeconomic variables | Financial variables |
| :--- | :--- |
| All Employees: Total nonfarm | S\&P 500 |
| IP Index | Spread, Baa-10y Treasury |
| Capacity Utilization: Manufacturing | Excess return |
| Help wanted to unemployed ratio | SMB FF factor |
| Unemployment rate | HML FF factor |
| Real personal income | Momentum factor |
| Weekly hours: goods-producing | R15_R11 |
| Housing starts | Industry 1 return |
| Housing permits | Industry 2 return |
| Real consumer spending | Industry 3 return |
| Real manuf. and trade sales | Industry 4 return |
| ISM: new orders index | Industry 5 return |
| Orders for durable goods |  |
| Avg. hourly earnings, goods-prod. |  |
| PPI, finished goods |  |
| PPI, commodities |  |
| PCE price index |  |
| Federal funds rate |  |

In some additional results, we use all of the 129 macro series to fit AR-SV models and form measures of macro uncertainty with an approach similar to that of JLN. We also consider larger financial data sets that include more disaggregate industry breakdowns (of 43 and 93 sectors) of stock returns, constructed by Kenneth French. In considering how the size of the dataset affects uncertainty estimates obtained with an approach like that of JLN, we also report AR-SV-based macro uncertainty estimates based on 8 or 60 series and AR-SV-based finance uncertainty estimates based on larger sets of series. We also make some comparisons to the measures of uncertainty estimated by JLN and LMN, obtained from the website of Professor Ludvigson.

## 4 Measuring Aggregate Uncertainty

In the following results, we focus on estimates of our baseline model with 30 variables, in monthly data. We discuss some aspects of robustness in the next section.

Table 2 provides posterior estimates of the factor loadings - the $\beta_{m}$ and $\beta_{f}$ coefficients of equations (2) and (3). All of the posterior mean estimates are positive and clustered around a value of 1 (as noted above, the loadings on employment and S\&P returns are fixed at 1). Some loadings are somewhat below 1 (e.g., orders for durable goods), and others are modestly above 1 (e.g., industrial production). In all cases, the loadings appear to
be estimated with reasonable precision. However, the loadings on the finance factor have posterior standard deviations that are noticeably lower than those for the loadings on the macro factor (despite a prior that is the same for the macro and finance factors).

Table 2: Posterior estimates of factor loadings

| Variable Posterior median (st. dev.) |  |
| :---: | :---: |
| Loadings of macro | variables on macro factor |
| Employment | 1.000 (NA) |
| Ind. prod. | 1.255 (0.272) |
| Capacity utilization | 0.685 (0.272) |
| Help wanted/unemployment | 0.737 (0.290) |
| Unemployment rate | 0.851 (0.239) |
| Real personal income | 0.787 (0.306) |
| Weekly hours, goods | 0.815 (0.325) |
| Housing starts | 1.138 (0.246) |
| Housing permits | 1.183 (0.306) |
| Real consumer spending | 1.162 (0.266) |
| Real manuf. and trade sales | 0.619 (0.260) |
| ISM index, new orders | 0.704 (0.236) |
| Orders for durable goods | 0.615 (0.280) |
| Avg. hourly earnings, goods | 1.122 (0.301) |
| PPI, finished goods | 1.172 (0.295) |
| PPI, commodities | 0.734 (0.324) |
| PCE price index | 1.139 (0.263) |
| Federal funds rate | 1.303 (0.321) |
| Loadings of finance variables on finance factor |  |
| S\&P 500 | 1.000 (NA) |
| Spread, Baa-10y Treasury | 1.119 (0.165) |
| Excess return | 1.002 (0.128) |
| SMB | 0.931 (0.118) |
| HML | 1.021 (0.115) |
| Momentum | 1.521 (0.152) |
| R15-R11 | 0.771 (0.141) |
| Industry 1 | 0.833 (0.122) |
| Industry 2 | 0.781 (0.155) |
| Industry 3 | 0.825 (0.120) |
| Industry 4 | 0.802 (0.143) |
| Industry 5 | 0.903 (0.161) |

Figure 1 displays the posterior distribution of the measures of macro (top panel) and financial uncertainty (bottom panel). In these charts, we define macro uncertainty as the square root of the common volatility factor $m_{t}$ and financial uncertainty as the square root of the common volatility factor $f_{t}$, such that macroeconomic and financial uncertainty correspond to a standard deviation. In order to facilitate comparability with other studies, the figure also displays the measures (macro in the top panel and financial in the bottom panel) obtained by JLN (macro) and LMN (financial). In the interest of brevity, we do not compare our uncertainty measures with other proposals in the literature, such as the VIX
or the cross-sectional variation in SPF forecasts or in firms' profits; studies such as JLN and Caldara, et al. (2014) provide such comparisons.

The results indicate the correlation of our uncertainty estimates with the JLN and LMN estimates are quite high, about 0.771 for macro uncertainty and 0.765 for financial uncertainty. There are, however, some differences. One difference is that our estimates are more variable. This variability stems in part from the inclusion of $y_{t-1}$ in the VAR process of the factors. The estimates of the coefficients $\delta_{m}$ and $\delta_{f}$ are generally small but not zero, such that movements in $y_{t-1}$ lead to movements in $m_{t}$ and $f_{t}$. Another difference is that the peak in the JLN measure around the early 1980s recession is quite higher than that around the mid-1970s recession, while the two values are similar for our model. Figure 1 also reports the $15 \%-85 \%$ credible set bands around our estimated measures of uncertainty, which, as mentioned, are correctly considered random variables in our approach. The bands are sizable, which suggests that taking uncertainty around uncertainty into account could indeed be important for inference about the significance of changes in uncertainty and effects on the macroeconomy.

The estimated macro and financial uncertainties are also somewhat correlated with each other. Using the time series of the posterior median uncertainties (again, defined in this section as standard deviations), the correlation between macro and financial uncertainty is 0.41. The uncertainty estimates of JLN (macro) and LMN (finance) are similarly correlated, with a simple correlation of 0.56 (using their 1-step ahead uncertainty series).

From a broader macroeconomic point of view, it is interesting that our measures of uncertainty do not present clear evidence of the sharp decline in volatility commonly referred to as the Great Moderation. This finding is in line with Giannone, Lenza and Reichlin (2008), who stress that the Great Moderation appears smaller with models based on larger datasets than with models based on smaller datasets. However, they do not consider large models with SV, as methodology existing before our paper did not make it tractable. Yet their result seems to hold up even once stochastic volatility is allowed in larger models. One possible explanation for weak evidence of a Great Moderation in our uncertainty measure is the use of monthly rather than quarterly data. However, data frequency does not seem to produce a significant difference in low frequency movements in common volatility around the Great Moderation. The broad contours of the uncertainty estimates obtained with quarterly estimates of our model, shown in Figure 2, are similar to those obtained with monthly data.

When we instead consider in Figure 3 the reduced form volatilities of each variable - defined as the diagonal elements of $\Sigma_{t}$, which reflect both the common uncertainty factors and idiosyncratic components - Great Moderation effects become evident for some variables. Arguably, for some variables, typically real quantities such as employment (PAYEMS), the Great Moderation effects appear larger in quarterly data (supplementary appendix available upon request) than monthly data (Figure 3). In either case, even in monthly data, the volatility of the federal funds rate (and related term spreads) exhibits a major decrease after the early 1980s, suggesting that a more predictable monetary policy contributed to the stabilization of the other volatilities.

Finally, about the financial uncertainty factor, it is worth noting that it increases during recessions, as the macro uncertainty factor, but also in other periods of financial turmoil. This different temporal pattern may help in disentangling macroeconomic and financial uncertainty.

## 5 Measuring the impact of uncertainty

### 5.1 Identification

With our uncertainty measure(s) entering each of the equations of the VAR in $y_{t}$, we can easily compute impulse response functions to unexpected aggregate uncertainty shocks. What we do is similar to shock identification in factor augmented VAR models, such as Bernanke, Boivin and Eliasz (2005) or Marcellino and Sivec (2016), but also allowing for (common) stochastic volatility.

Our approach features two important differences with respect to the existing structural analysis exercises on the impact of uncertainty on the macroeconomy.

First, in our specification, a shock to uncertainty affects not only the conditional mean of $y_{t}$ but also the conditional variance. In analysis such as Bloom (2009), JLN, or Caldara, et al. (2014), it is common to conduct inference on the former while ignoring the latter. Moreover, our approach takes into account the uncertainty around uncertainty, while these studies condition on the point estimates of uncertainty, thereby abstracting from the variance of uncertainty estimates. To avoid a similar practice, we are able to use our model and impulse response functions to conduct inference on the effects of uncertainty shocks to $y_{t}$ taking account of their effects on not only the conditional mean but also the conditional variance. Our estimates also account for the variance of the uncertainty measure in the sense that
our estimates of the VAR's coefficients reflect the fact that uncertainty is a latent state and not an observed series.

Second, the specification of our model permits us to identify an uncertainty shock and its effects without having to rely on a recursive identification scheme. This is not possible in some of the other approaches in the literature, such as Bloom (2009) and JLN. In our case, changes in the (common) volatilities of the VAR's variables provide contemporaneous, identifying information on uncertainty.

In our approach, shocks to uncertainty are identified by the very fact that uncertainty appears not only in the conditional means, but also in the conditional variances of an heteroskedastic VAR. To clarify this point consider the following simple example. Consider a homoskedastic VAR with 2 variables only, uncertainty $\left(\ln m_{t}\right)$ and output $\left(y_{t}\right)$ :

$$
\left[\begin{array}{c}
\ln m_{t}  \tag{32}\\
y_{t}
\end{array}\right]=\left[\begin{array}{cc}
\pi_{1} & \pi_{2} \\
\pi_{3} & \pi_{4}
\end{array}\right]\left[\begin{array}{c}
\ln m_{t-1} \\
y_{t-1}
\end{array}\right]+\left[\begin{array}{c}
v_{1, t} \\
v_{2, t}
\end{array}\right]
$$

where the variance of the disturbances is given by $\Sigma$. The VAR above can be written as:

$$
\left[\begin{array}{c}
\ln m_{t}  \tag{33}\\
y_{t}
\end{array}\right]=\left[\begin{array}{ll}
\pi_{1} & \pi_{2} \\
\pi_{3} & \pi_{4}
\end{array}\right]\left[\begin{array}{c}
\ln m_{t-1} \\
y_{t-1}
\end{array}\right]+\underbrace{\left[\begin{array}{cc}
c_{1} & 0 \\
c_{2} & c_{3}
\end{array}\right]}_{C} \underbrace{\left[\begin{array}{ll}
q_{1} & q_{2} \\
q_{3} & q_{4}
\end{array}\right]}_{Q}\left[\begin{array}{c}
\varepsilon_{1, t} \\
\varepsilon_{2, t}
\end{array}\right] ;
$$

where $Q$ is such that $Q Q^{\prime}=I$ and $\varepsilon_{t}=\left(\varepsilon_{1, t}, \varepsilon_{2, t}\right)^{\prime}$ is a vector of mutually independent standard normals. It follows that $\operatorname{VAR}\left(C Q \varepsilon_{t}\right)=C C^{\prime}=\Sigma$, therefore $Q$ does not appear in the likelihood. It follows that the elements in the matrices $Q$ and $C$ are not separately identified; only their product is. Typically identification is achieved by restricting $Q$ in some way, which then permits identification of $C$. Different choices of $Q$ correspond to different identification schemes. For example $Q=I$ gives a Cholesky scheme with $\ln m_{t}$ ordered first. Other possible choices of $Q$ can yield other Cholesky orderings or set identification schemes (i.e. sign restrictions).

Note that in the standard framework described above, uncertainty $\left(\ln m_{t}\right)$ is simply another variable in an otherwise standard homoskedastic $V A R$, and does not impact the error variance $\Sigma$. In our setup instead, uncertainty enters both the conditional mean and the error variance of the VAR. In this simplified example, this would give:

$$
\left[\begin{array}{c}
\ln m_{t}  \tag{34}\\
y_{t}
\end{array}\right]=\left[\begin{array}{ll}
\pi_{1} & \pi_{2} \\
\pi_{3} & \pi_{4}
\end{array}\right]\left[\begin{array}{c}
\ln m_{t-1} \\
y_{t-1}
\end{array}\right]+\underbrace{\left[\begin{array}{cc}
c_{1} & 0 \\
c_{2} & c_{3}=m_{t}^{0.5 \beta}
\end{array}\right]}_{C} \underbrace{\left[\begin{array}{ll}
q_{1} & q_{2} \\
q_{3} & q_{4}
\end{array}\right]}_{Q}\left[\begin{array}{l}
\varepsilon_{1, t} \\
\varepsilon_{2, t}
\end{array}\right] .
$$

Note that the fact that uncertainty appears in the error variance provides identifying information on the matrix $C$. The (time varying) coefficient $c_{3}=m_{t}^{0.5 \beta}$ is identified because
$m_{t}$ appears in both the conditional mean and variance. In equation (34), differently from what happens in equation (33), alternative rotation matrices $Q$ will result in alternative likelihoods, since changing the elements of $Q$ will change the equation for $\ln m_{t}$, and this in turn will change the value of $c_{3}=m_{t}^{0.5 \beta}$ and the likelihood.

More generally, any $Q \neq I$ will yield a model which is different from the one initially specified. This happens because in the standard setup using a different rotation matrix $Q$ only impacts on the error variance part of the model: different $Q$ s imply different structural errors, but identical reduced form errors, which only appear in the likelihood in the form of their error variance $\left(C C^{\prime}\right)$. In our setup instead, the elements of $C$ are related also with the conditional mean of the model, so that $C$ appears also in the conditional mean part of the likelihood, and this provides identification.

Going back to the more general model, note that in our setup the shocks to uncertainty contemporaneously affect $y_{t}$ and are orthogonal to $\epsilon_{t}$. The orthogonality between $\left(u_{m, t}, u_{f, t}\right)^{\prime}$ and $\epsilon_{t}$ stems from our modeling strategy that separates the total variance of the residual $A v_{t}=\Lambda_{t}^{0.5} \epsilon_{t}$ into three orthogonal components: a common component, an idiosyncratic component, and a component due to the conditionally independent shock $\epsilon_{t}$ see equation (11). When a large shock (represented by $\Lambda_{t}^{0.5} \epsilon_{t}$ ) hits the economy, we let the data distinguish whether this is a large shock in the conditional error $\epsilon_{t}$ (so an outlier in a standard normal distribution, with a variance that is not moving) or rather a relatively ordinary shock (in terms of size of $\epsilon_{t}$ ) accompanied by an increase in the variance $\Lambda_{t}^{0.5}$. Hence these two components have to be orthogonal to one another in order to be separately identified. ${ }^{17}$ However, by including $y_{t-1}$ in the process for the factor $m_{t}$ (and the factor $f_{t}$ ), our model allows for previous $\epsilon$ shocks to affect the factor and, in turn, $\Lambda_{t}^{0.5}$. In general, our approach exploits the ability to identify the uncertainty factors from observed volatilities and then identify the effects of uncertainty from the first-moment relationship of $y_{t}$ to the uncertainty factors. In this sense, there is some parallel between our identification and heteroskedasticity-based identification approaches such as Rigobon (2003) and Lanne and Lutkepohl (2008).

As a consequence, we are able to allow uncertainty to contemporaneously affect the macroeconomy and financial markets and contemporaneously respond to macroeconomic and financial developments. The former effect is captured by the inclusion of the uncertainty

[^12]factors in the VAR's conditional mean given in equation (6). The latter effect is captured in the following way: when a large shock to the innovation $v_{t}$ of equation (6) occurs, and it reflects a shift in volatility that is common across variables, the uncertainty factors will move higher. That is, large surprises to $y_{t}$ can yield movements in uncertainty. In contrast, under the more common approach in the literature of obtaining an estimate of uncertainty from elsewhere (e.g., as an average of univariate volatilities as in JLN), and then adding the uncertainty estimate to an otherwise standard VAR, identification requires an additional step in the VAR, which is typically based on the use of a recursive identification scheme, which is not immune to criticism, some of which is represented in studies such as Caldara, et al. (2014) and LMN. ${ }^{18}$

While the vector of uncertainty measures $u_{t}=\left(u_{m, t}, u_{f, t}\right)^{\prime}$ is identified for the reasons outlined above, in order to separately identify the effects of macro and financial uncertainty, an identification assumption is needed for the system in (5). In line with common wisdom that financial variables are "fast" while macroeconomic variables are "slow", we assume a Cholesky identification scheme in which financial uncertainty $f_{t}$ is ordered last, and hence it contemporaneously responds to both $u_{m, t}$ and $u_{f, t}$, while macroeconomic uncertainty responds contemporaneously to $u_{m, t}$ but responds to $u_{f, t}$ with some delay.

For simplicity, we compute the responses using the levels of uncertainty and $\Sigma_{t}$ at just the end of the sample, in period $t=T$. This is not restrictive, in the sense that, with the responses driven by the constant coefficients of the VAR and the log factor processes, considering an earlier point in the sample wouldn't yield really different results. We report impulse response functions computed in the usual way; we have verified that response functions computed with the generalized approach of Koop, Pesaran, and Potter (1996) as implemented in studies such as Benati (2008) yield the same estimates. The equivalence and the validity of the simple approach - stem from the independence among the conditional VAR shocks $\epsilon$, the shocks to the volatility factors, and the shocks to the idiosyncratic volatility components. For each of the $j=1, \ldots, 5000$ retained draws of the VAR's parameters and latent states, we compute impulse response functions. We report the posterior medians and 70 percent credible sets of these functions.

[^13]
### 5.2 Results

### 5.2.1 Impulse responses

Figure 4 provides the impulse response estimates of a one-standard deviation shock to log macro uncertainty $\left(\ln m_{t}\right)$ in our 30 variable (monthly frequency) VAR specification. Note that, although the model is estimated with standardized data, the impulse responses are scaled and transformed back to the units typical in the literature. We do so by using the model estimates to: (1) obtain impulse responses in standardized, sometimes (i.e., for some variables) differenced data; (2) multiply the impulse responses for each variable by the standard deviations used in standardizing the data before model estimation; and (3) accumulate the impulse responses of step (2) as appropriate to get back impulse responses in levels or log levels. (With smaller cross-sections, we have verified that estimating a model in log levels or levels yields results very similar to those obtained with the approach we actually use in the paper.) Accordingly, the units of the reported impulse responses are percentage point changes (based on 100 times log levels for variables in logs or rates for variables not in log terms). As examples, the response of employment is the percentage point response (again, 100 times the $\log$ ); the response of the unemployment rate is the percentage point change in the unemployment rate; and the response of the federal funds rate is the percentage point change in the annualized federal funds rate. However, there is one complication to the reading of results on stock prices and returns, relating to the source data: for the S\&P 500 variable, we display the response in percentage changes of the price level (the response of 100 times the log level of the S\&P index), but for the CRSP excess return, we display the response of the return (computed as a monthly return), rather than a price level.

As shown in the penultimate panel of Figure 4, the shock to log macro uncertainty produces a rise in uncertainty that gradually dies out, over the course of about one year. As indicated in the last panel of Figure 4, financial uncertainty rises in response, also for about a year, although the response of finance uncertainty is estimated less precisely than the response of macro uncertainty.

Now consider the effects of the macro uncertainty shock on industrial production and employment, which are both significantly negative, with a modestly larger response of production than employment. The responses are qualitatively similar to those obtained by JLN, who only focus on these two variables, but in their case the effects are more short-lived,
becoming not significant about one year after the shock. ${ }^{19}$
In the labour market, we also find that hours worked generally decrease (with peak effect after about six months) and unemployment increases (with peak effect after about 20 months), in line with firms trying to avoid hiring adjustment costs, as, e.g., in Nickell (1986) and Bloom (2009). Interestingly, there are no significant effects on hourly earnings (average hourly earnings decline, but the estimate is too imprecise to be meaningful), suggesting that wages are rather sticky in the face of uncertainty shocks.

The overall effects on real personal income, real personal consumption expenditures and real M\&T (manufacturing and trade) sales are significantly negative and persistent. The fall in consumption is likely due to lower current and future expected income but also, likely, to the need to increase precautionary savings (e.g., Bansal and Yaron (2004)) and the preference to postpone buying durable goods until uncertainty declines, e.g. Eberly (1994) and Bertola, Guiso and Pistaferri (2005).

In terms of other indicators of production, we detect a significant, persistent decrease in capacity utilization. Utilization bottoms out after about 15 months (with a peak response of about 30 basis points) and then slowly rises, but remains below baseline for the full four year horizon covered in the impulse responses. Orders of durable goods and the new orders component of the ISM index also fall significantly, signaling a clear decrease in actual and expected investment. This is in line with the presence of sizable investment adjustment costs, e.g. Ramey and Shapiro (2001) and Cooper and Haltiwanger (2006), that firms try to avoid in the presence of higher uncertainty. An even more significant effect emerges in the building sector, where adjustment costs can be expected to be even higher, with prolonged decreases in housing starts and building permits.

One other notable result in the responses of economic activity to the shock in macro uncertainty concerns timing: for some, but not all indicators, the response to the shock is immediate (contemporaneous) and sizable. Relatively quick and large responses occur for housing starts and permits, the ISM index of new orders, and weekly hours worked (which presumably reflects an intensive margin of adjustment, rather than the extensive margin captured by employment). Slower, although eventually large and significant, responses occur for variables such as employment, unemployment, and industrial production.

[^14]Despite the significant decline of economic activity in response to the macro uncertainty shock, there doesn't appear to be evidence of a broad decline in prices. The PPI for finished goods does decline steadily and by as much as 2 percentage points, although the response is estimated relatively imprecisely. Neither the PPI for commodity prices nor overall consumer prices as captured by the PCE price index (in earlier versions of the model, we obtained the same result for core PCE prices) evidence a significant change. Overall, this picture of price responses is in line with New-Keynesian models, such as Leduc and Liu (2015), Basu and Bundick (2015), and Fernández-Villaverde, Guerrón-Quintana, Kuester, and RubioRamirez (2015), which predict a small effect of uncertainty on inflation due to sticky prices (and possibly wages), such that lower consumption does not stimulate investment.

In the face of this sizable deterioration in the real economy and absence of much movement in prices, the federal funds rate gradually falls. The reaction of the federal funds rate is minimal for the first few months. Then, there is a steady, statistically significant decline for about 20-22 months. The response of the funds rate reaches about -20 basis points, not quite as large as the movement in employment but almost double the peak response of the unemployment rate. Such a response appears to be about in line with the parameterization of the Taylor (1999) rule, if one replaces the rule's output gap with an unemployment gap and assumes that Okun's law justifies roughly doubling Taylor's coefficient of 1 on the output gap.

The responses of financial indicators to the shock to macro uncertainty are - collectively speaking - muted and imprecisely estimated. The one exception is the credit spread, between the Baa and 10 year Treasury yields, which displays a modest, but persistent and significant, rise, with a hump-shape pattern. The substantial increase in the credit spread likely increases borrowing costs for firms, further reducing their investment, as in studies looking at the effects of uncertainty in models with financial constraints, such as Arellano, Bai, and Kehoe (2012), Christiano, Motto, and Rostagno (2014), and Gilchrist, Sim, and Zakrasjek (2014). Aggregate stock prices and returns as captured by the S\&P 500 price index and the excess CRPS return decline, in line with common wisdom and findings in the finance literature (e.g., Bansal and Yaron 2004), but the estimated responses are sufficiently imprecise that they should not be judged meaningful. The responses of the other financial indicators, including the risk factors and industry-level returns, are also overall insignificant, signaling that financial variables are less sensitive to macroeconomic uncertainty than they
are to financial uncertainty. This brings us to the next point, to the effects of surprise changes in financial uncertainty.

As we discussed above, in our setup financial uncertainty is estimated in a single step, together with macroeconomic uncertainty, using the identification assumption that it represents the common volatility across a set of financial indicators. The effects of a shock to financial uncertainty are displayed in Figure 5.

As reported in the last panel of Figure 5, the shock to log finance uncertainty produces a rise in uncertainty that only gradually dies out, over the course of almost two years. In response, macro uncertainty changes very little, by an amount that is not significant. Based on this and the corresponding result for a shock to macro uncertainty, our estimates and identification attribute the comovement between macro and finance uncertainty to finance uncertainty (relatively fast moving) moving in response to a change in macro uncertainty (relatively slow moving).

As to broader effects of finance uncertainty, when compared to a macro uncertainty shock, a finance uncertainty shock has similar, but sometimes smaller and more delayed macroeconomic effects and larger financial effects. More specifically, the effect on industrial production and employment follow a pattern similar to that obtained for the case of macroeconomic uncertainty, with a significantly negative response, more persistent in the case of employment. The effect on the labour market show an increase in unemployment and a decrease in hours worked, but the reaction of the latter is smaller on impact and in general slower than what happens in the case of the macroeconomic uncertainty shock. Real personal income and real personal consumption expenditures show the same negative response observed for a macro uncertainty shock, but the former is slower and insignificant on impact with respect to the case of a macro uncertainty shock, while the latter is largely insignificant. In perhaps the most notable difference with respect to results for a macro uncertainty shock, a finance uncertainty shock does not have significant effects on the housing sector (starts and permits). Overall, the responses of prices to the finance uncertainty shock are no more significant than the corresponding responses to the macro uncertainty shock.

Turning our attention to the financial variables, on balance they respond more to the finance uncertainty shock than the macro uncertainty shock, although in some cases the responses are imprecisely estimated. The shock to finance uncertainty produces a persis-
tent and significant rise in the credit spread, with a hump-shape pattern. It also produces a sizable falloff in aggregate stock prices and returns. The response of the S\&P500 price level is negative and significant, with no sign of a rebound after the forecast horizon (4 years). The CRSP excess returns display a negative jump and recover only after 6 months. The industry-level returns included in the model also decline, but the responses are estimated very imprecisely. The responses of the risk factors included in the model are also insignificant.

### 5.2.2 Forecast error variance decompositions

To assess the overall importance of uncertainty shocks, we apply a variance decomposition to our estimated model. Specifically, for each draw of the model's parameters and latent states, we compute forecast error variances - due to the macro uncertainty shock, finance uncertainty shock, and the conditional errors $\epsilon$ of the VAR - for horizons 1 through 48 months ahead, and the shares of total forecast error variance due to the two different uncertainty shocks. ${ }^{20}$ For simplicity, in these results, we compute the variance decompositions using the levels of uncertainty and $\Sigma_{t}$ at just the end of the sample, in period $t=T$. We further simplify the calculations by abstracting from the expectations of the future error variance matrix $\Sigma_{T+h}$ that enter the forecast error variances and simply treat the expected future $\Sigma_{T+h}$ as equaling $\Sigma_{T}$; in doing so, we follow studies such as Cogley and Sargent (2005) in abstracting from movements over the forecast horizon. With this simplification and the independence among the innovations to uncertainty and idiosyncratic volatility, we are able to greatly streamline the calculations by making use of an analytical solution that is comparable to those used with simpler models.

In line with the impulse responses, the shares of variance due to a macro uncertainty shock reported in Figure 6 are often sizable, more so for economic activity than inflation or the model's financial variables. For example, in the case of employment and the unemployment rate, the share of variance due to the macro uncertainty shock rises sharply for about the first year of the forecast horizon and then stabilizes at a little more than 25 percent. The contours of the variance shares are similar for other economic activity variables, although the shares are not quite as high for some of these variables (e.g., industrial production) as for employment or unemployment. Probably reflecting the importance of uncertainty shocks to the variability of economic activity, these shocks also account for a significant portion

[^15]of the variation in the federal funds rate. But, consistent with the impulse responses, the macro uncertainty shock drives relatively little variation in price measures (e.g., the PCE price index), and the responses are estimated with considerable imprecision.

The shares of variance due to a financial uncertainty shock provided in Figure 7 are broadly comparable to those reported for the macro uncertainty shock. But in keeping with the impulse responses, for the macroeconomic variables, the financial uncertainty shock generally drives somewhat less variation than does the macro uncertainty shock. For example, for employment, the share of variance due to the financial uncertainty shock peaks at less than 15 percent, compared to a peak of more than 25 percent for the macro uncertainty shock. For the housing variables and consumer spending, the share of variance due to the finance uncertainty shock is small and imprecisely estimated.

### 5.2.3 Robustness

In this section we consider some robustness checks of our results.
First, we consider a special case of our model in which the contemporaneous effect of the measures of uncertainty on the conditional means of the VAR is shut down (for simplicity, we shut down these coefficients by imposing an extremely tight prior on the contemporaneous coefficients on $\ln m_{t}$ and $\ln f_{t}$ in equation (6)). The point of this exercise is to examine the extent to which our baseline estimates of uncertainty effects are driven by the contemporaneous terms we include in the VAR's conditional mean. As noted above, our baseline approach yields identification, so this is not a check of identification; rather, it is simply a check of how much of our estimated baseline effect is influenced by the contemporaneous reaction of macro and financial variables to uncertainty. The results of this exercise are provided in Figures 8 (macro uncertainty shock) and 9 (financial uncertainty shock).

Zeroing out the contemporaneous terms in the VAR's conditional mean has some material effects on the impulse response estimates, more so for the macro uncertainty shock than the financial uncertainty shock. For one, following its shock, macro uncertainty returns to baseline much faster, and the associated rise in financial uncertainty is greater than in our baseline estimates of Figure 4. For another, the macroeconomic effects of the shock to macro uncertainty are smaller and much less precisely estimated than in the baseline case. With the restricted model, only the responses of housing starts and permits remain significant. In the case of the shock to financial uncertainty, the results with the restricted model are
more similar to those from the baseline specification. Even with the restriction, the shock to financial uncertainty produces significant declines in most measures of economic activity and in stock prices and returns. On balance, we take these results as being consistent with the point of LMN about challenges in separating the effects of macro and financial uncertainty, and with the value of contemporaneous information (again, legitimately used for identification) in our framework, for the purpose of identifying uncertainty and its effects.

Second, we consider the robustness of our results to the use of a quarterly, rather than monthly, data frequency. Qualitatively, these quarterly results, provided in Figures 10 and 11, are very similar to our baseline estimates with monthly data. There are some differences, such as the more significant response of macro uncertainty to finance uncertainty in the quarterly estimates than the monthly estimates, but our key results from the monthly frequency carry over to the quarterly case.

Third, we consider how estimates of uncertainty depend on the size of the cross section. In this exercise, for simplicity we rely on estimates of SV from the AR-SV model described in section 2, and we consider measures of uncertainty defined as the simple average of the time-varying standard deviations obtained from the SV estimates, for cross-sections of different size. We begin with macroeconomic uncertainty. In the upper panel of Figure 12, we report AR-SV-based uncertainty measures resulting from averaging the SVs of $8,18,60$ and 129 macroeconomics variables (all monthly data). The different subsets of variables mainly differ for the level of considered disaggregation, as detailed in Table 3. Although not shown directly, the measure of uncertainty obtained by averaging the SV estimates of 129 variables is highly correlated with the series of JLN. As indicated in the top of Figure 12 , uncertainty estimates based on 129 and 60 variables are extremely similar; they can be hardly distinguished in the figure. Compared to these estimates, the uncertainty estimates based on 18 or 8 variables differ somewhat. For example, the measure based on just 8 variables presents a substantially higher peak around the recession of the early 1980's and lower values from the early 1990s onwards, particularly so from 2010 onwards. That said, the estimates implied by the two smaller variable sets are still highly correlated (correlations in excess of 0.9 ) with the estimates based on the two larger variable sets.

Now consider the measures of financial uncertainty obtained by averaging AR-SV estimates, shown in the lower panel of Figure 12. We consider two estimates obtained by averaging volatilities of two different industry-breakdowns of returns from the Kenneth

French datasets, one (the most disaggregate) with 93 industries and the other (less disaggregate) with 43 industries. Although not shown directly in the chart, the series based on 93 returns is very highly correlated with the uncertainty estimate of LMN. The series based on a smaller set of 43 returns is extremely similar. Using a smaller set of 12 financial variables - either the set of 12 financial variables included in our baseline model (corresponding to the $\mathrm{N}=12$ set covered in chart) or the alternative set of 12 variables that replaces the five risk factors and 5 industry returns in the baseline set with 10 industry returns - yields modestly different estimates. Again, though, these estimates are highly correlated (correlations in excess of 0.9 ) with those obtained by using the large datasets. Overall, we view these results as supporting our use of a model that has 18 macro variables and 12 financial variables in order to reasonably capture uncertainty and its effects.

Finally, a relevant question is whether the recent financial crisis had a substantial impact on the results we have obtained. For example, Alessandri and Mumtaz (2014), using a nonlinear VAR model, find that uncertainty has a much stronger (negative) impact on output in periods of financial stress than otherwise. To assess whether this is the case in our baseline results, we recompute the impulse responses using estimates of the model with data through just December 2007. Figures 13 and 14 provide the results. In the shorter sample, it continues to be the case that shocks to both macro uncertainty and financial uncertainty have significant effects on economic activity and some financial variables, with patterns generally similar to those obtained for the full sample of data. However, in the case of the shock to macro uncertainty, the effects are modestly smaller and less precisely estimated than in the baseline case. In the case of the shock to financial uncertainty, the effects for the 1960-2007 sample are quite similar to those for the 1961-2014 period. Overall, we judge that the crisis period has provided useful information for assessing the effects of changes in macro uncertainty but by no means drives it, whereas the crisis period has less effect on the measurement of financial uncertainty and its effects.

## 6 Conclusions

This paper developed a new framework for measuring uncertainty and its effects on the macroeconomy and financial conditions. Specifically, we developed a VAR model for a possibly large set of variables whose volatility is driven by two common unobservable factors, which can be interpreted as the underlying aggregate macroeconomic and financial uncer-
tainty, respectively. These uncertainty measures reflect common changes in the volatility of the variables under analysis, but can also influence their levels. Hence, contrary to most existing measures, ours reflect changes in both the conditional mean and volatility of the underlying variables, and they are estimated taking explicitly into account the existence of aggregate uncertainty. Creal and Wu (2016) pursue a broadly similar idea of internalizing the treatment of uncertainty, but in a different, much smaller model with a different question.

Moreover, our approach allows simultaneous estimation of the uncertainty measures and their impact on the economy, providing also a coherent measure of the uncertainty around them, while most existing studies rely on a two-step approach with one model used to estimate uncertainty and a second one to assess its effects. Finally, identification of the uncertainty shock is simplified with respect to standard VAR based analysis, in line with the FAVAR approach and heteroskedasticity-based identification.

We introduced a new Bayesian estimation method for the model, which can be also applied in other contexts, is computationally efficient, and allows for estimation even of large models, while previous VAR models with stochastic volatility could only handle a handful of variables.

We applied the method to estimate uncertainty and its effects using US data, finding that there is indeed substantial commonality in uncertainty, sizable effects of uncertainty on key macroeconomic and financial variables with responses in line with economic theory, and some uncertainty about uncertainty and its effects. We provided results separately for macroeconomic and financial uncertainty, showing that macro uncertainty shocks have a major impact on macroeconomic variables but their effects do not transmit substantially to financial variables, while financial uncertainty shocks have significant effects on financial variables but also substantiallt transmit to the macroeconomy.

## A Appendix

In this appendix, we provide the details of the priors we use in the multivariate model estimation.

## A. 1 Priors

For the VAR coefficients contained in $\Pi$, we use a Minnesota-type prior. With the variables of interest transformed for stationarity, we set the prior mean of all the VAR coefficients to 0 . We make the prior variance-covariance matrix $\underline{\Omega}_{\Pi}$ diagonal. The variances are specified to make the prior on the intercept, $\log m_{t}$, and $\log f_{t}$ terms uninformative and the prior on the lags of $y_{t}$ take a Minnesota-type form. Specifically, for the intercept, $\log m_{t}$, and $\log f_{t}$ terms of equation $i$, the prior variance is $\theta_{3}^{2} \sigma_{i}^{2}$. For lag $l$ of variable $j$ in equation $i$, the prior variance is $\frac{\theta_{1}^{2}}{l^{2}}$ for $i=j$ and $\frac{\theta_{\frac{2}{2} \theta_{2}^{2}}^{l^{2}} \frac{\sigma_{i}^{2}}{\sigma_{j}^{2}} \text { otherwise. In line with common settings, we set overall }}{}$ shrinkage $\theta_{1}=0.2$, cross-variable shrinkage $\theta_{2}=0.5$, and intercept/factor shrinkage $\theta_{3}=$ 1000. With these settings, we have deliberately made the prior on the uncertainty terms in the VAR uninformative (making the prior less uninformative by setting $\theta_{3}=10$ yielded results very similar to the baseline estimates). Finally, consistent with common settings, the scale parameters $\sigma_{i}^{2}$ take the values of residual variances from $\operatorname{AR}(p)$ models from the estimation sample.

As to the volatility-related components of the model, for the rows $a_{j}$ of the matrix $A$, we follow Cogley and Sargent (2005) and make the prior fairly uninformative, with prior means of 0 and variances of 10 for all coefficients. For the coefficients ( $\gamma_{i, 0}, \gamma_{i, 1}$ ) (intercept, slope) of the idiosyncratic processes of equation $i, i=1, \ldots, n$, the prior mean is $\left(\log \sigma_{i}^{2}\right.$, 0.0 ), where $\sigma_{i}^{2}$ is the residual variance of an $\operatorname{AR}(p)$ model over the estimation sample. The prior standard deviations (assuming 0 covariance) are $\left(2^{0.5}, 0.4\right)$. For the factor loadings $\beta_{j}$, $j=2, \ldots, n$, we use a prior mean of 1 and a standard deviation of 0.4 . For the coefficients of the VAR process of the factors, the prior means are zero, except that the first-order lag coefficients of each factor has a mean of 0.8 . The prior standard deviations are set to 0.2 for the elements of $D(L)$ and 0.4 for the elements of $\delta_{m}$ and $\delta_{f}$. For the innovations to the idiosyncratic components of volatility $\left(\phi_{1}, \ldots, \phi_{n}\right)$, we use a mean of 0.03 , with 10 degrees of freedom for each. For the variance-covariance matrix of innovations to the factor processes $\left(\Phi_{u}\right)$, we use a mean of 0.01 times an identity matrix, with 10 degrees of freedom. For the period 0 values of $\log m_{t}$ and $\log f_{t}$, and $\log h_{i, t}$, we set the mean at 0 and in each draw use the variance implied by the VAR representation of the factors (treating the $\delta_{m}$ and $\delta_{f}$ coefficients as 0 ) and the draws of the coefficients and error variance matrix. Finally, for the period 0 values of $\log h_{i, t}$, we set the mean and variance at $\log \sigma_{i}^{2}$ and 2.0 , respectively.

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Table 3: Variable Combinations

| variable | mnemonic | included in $N=60 \text { set }$ | included in $N=40 \text { set }$ | included in $N=18 \text { set }$ | included in $N=8$ set |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Real Personal Income | RPI | Y | Y | Y |  |
| RPI ex. Transfers | W875RX1 |  |  |  |  |
| Real PCE | DPCERA3M086SBEA | Y | Y | Y | Y |
| Real M\&T Sales | CMRMTSPLx | Y | Y | Y |  |
| Retail and Food Services Sales | RETAILx | Y |  |  |  |
| IP Index | INDPRO | Y | Y | Y | Y |
| IP: Final Products and Supplies | IPFPNSS |  |  |  |  |
| IP: Final Products | IPFINAL |  |  |  |  |
| IP: Consumer Goods | IPCONGD |  |  |  |  |
| IP: Durable Consumer Goods | IPDCONGD | Y | Y |  |  |
| IP: Nondurable Consumer Goods | IPNCONGD | Y | Y |  |  |
| IP: Business Equipment | IPBUSEQ | Y |  |  |  |
| IP: Materials | IPMAT |  |  |  |  |
| IP: Durable Materials | IPDMAT | Y |  |  |  |
| IP: Nondurable Materials | IPNMAT | Y |  |  |  |
| IP: Manufacturing | IPMANSICS | Y | Y |  |  |
| IP: Residential Utilities | IPB51222S |  |  |  |  |
| IP: Fuels | IPFUELS |  |  |  |  |
| ISM Manufacturing: Production | NAPMPI | Y |  |  |  |
| Capacity Utilization: Manufacturing | CAPUTLB00004S | Y | Y | Y |  |
| Help-Wanted Index for US Help wanted indx | HWI |  |  |  |  |
| Help Wanted to Unemployed ratio | HWIURATIO | Y | Y | Y |  |
| Civilian Labor Force | CLF16OV |  |  |  |  |
| Civilian Employment | CE16OV | Y | Y |  |  |
| Civilian Unemployment Rate | UNRATE | Y | Y | Y | Y |
| Average Duration of Unemployment | UEMPMEAN | Y | Y |  |  |
| Civilians Unemployed < Weeks | UEMPLT5 |  |  |  |  |
| Civilians Unemployed 5-14 Weeks | UEMP5TO14 |  |  |  |  |
| Civilians Unemployed > 15 Weeks | UEMP15OV |  |  |  |  |
| Civilians Unemployed 15-26 Weeks | UEMP15T26 |  |  |  |  |
| Civilians Unemployed > 27 Weeks | UEMP27OV | Y |  |  |  |
| Initial Claims | CLAIMSx | Y | Y |  |  |
| All Employees: Total nonfarm | PAYEMS | Y | Y | Y | Y |
| All Employees: Goods-Producing | USGOOD |  |  |  |  |
| All Employees: Mining and Logging | CES1021000001 |  |  |  |  |
| All Employees: Construction | USCONS |  |  |  |  |
| All Employees: Manufacturing | MANEMP |  |  |  |  |
| All Employees: Durable goods | DMANEMP | Y |  |  |  |
| All Employees: Nondurable goods | NDMANEMP | Y |  |  |  |
| All Employees: Service Industries | SRVPRD | Y |  |  |  |
| All Employees: TT\&U | USTPU |  |  |  |  |
| All Employees: Wholesale Trade | USWTRADE |  |  |  |  |
| All Employees: Retail Trade | USTRADE |  |  |  |  |
| All Employees: Financial Activities | USFIRE |  |  |  |  |
| All Employees: Government | USGOVT |  |  |  |  |
| Hours: Goods-Producing | CES0600000007 | Y | Y | Y |  |
| Overtime Hours: Manufacturing | AWOTMAN | Y | Y |  |  |
| Hours: Manufacturing | AWHMAN |  |  |  |  |
| ISM Manufacturing: Employment | NAPMEI | Y |  |  |  |

Table 1, continued: Variable Combinations

| variable | mnemonic | included in $N=60 \text { set }$ | included in $N=40 \text { set }$ | included in $N=18 \text { set }$ | included in $N=8$ set |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Starts: Total | HOUST | Y | Y | Y |  |
| Starts: Northeast | HOUSTNE |  |  |  |  |
| Starts: Midwest | HOUSTMW |  |  |  |  |
| Starts: South | HOUSTS |  |  |  |  |
| Starts: West | HOUSTW |  |  |  |  |
| New Private Housing Permits (SAAR) | PERMIT | Y | Y | Y |  |
| New Private Housing Permits, Northeast (SAAR) | PERMITNE |  |  |  |  |
| New Private Housing Permits, Midwest (SAAR) | PERMITMW |  |  |  |  |
| New Private Housing Permits, South (SAAR) | PERMITS |  |  |  |  |
| New Private Housing Permits, West (SAAR) | PERMITW |  |  |  |  |
| ISM: PMI Composite Index | NAPM |  |  |  |  |
| ISM: New Orders Index | NAPMNOI | Y | Y | Y |  |
| ISM: Supplier Deliveries Index | NAPMSDI | Y | Y |  |  |
| ISM: Inventories Index | NAPMII |  |  |  |  |
| Orders: Durable Goods | AMDMNOx | Y | Y | Y |  |
| Unfilled Orders: Durable Goods | AMDMUOx | Y |  |  |  |
| Total Business Inventories | BUSINVx |  |  |  |  |
| Inventories to Sales Ratio | ISRATIOx | Y | Y |  |  |
| Money Stock | M1SL |  |  |  |  |
| Money Stock | M2SL | Y | Y |  |  |
| Real M2 Money Stock | M2REAL |  |  |  |  |
| St. Louis Adjusted Monetary Base | AMBSL |  |  |  |  |
| Total Reserves | TOTRESNS |  |  |  |  |
| Commercial and Industrial Loans | BUSLOANS | Y | Y |  |  |
| Real Estate Loans | REALLN |  |  |  |  |
| Total Nonrevolving Credit | NONREVSL |  |  |  |  |
| Credit to PI ratio | CONSPI | Y |  |  |  |
| S\&P: Composite | S\&P 500 | Y | Y |  |  |
| S\&P: Industrials | S\&P: indust |  |  |  |  |
| S\&P: Dividend Yield | S\&P div yield | Y | Y |  |  |
| S\&P: Price-Earnings Ratio | S\&P PE ratio |  |  |  |  |
| Effective Federal Funds Rate | FEDFUNDS | Y | Y | Y | Y |
| Month AA Comm. Paper Rate CPF3M Comm paper | CP3M |  |  |  |  |
| 3-Month T-bill | TB3MS |  |  |  |  |
| 6-Month T-bill | TB6MS |  |  |  |  |
| 1-year T-bond | GS1 | Y | Y |  |  |
| 5-year T-bond | GS5 |  |  |  |  |
| 10-year T-bond | GS10 | Y | Y |  |  |
| Corporate Bond Yield Aaa bond | AAA | Y |  |  |  |
| Corporate Bond Yield Baa bond | BAA | Y | Y |  |  |
| CP - FFR spread CP-FF spread | COMPAPFF |  |  |  |  |
| 3 Mo . - FFR spread 3 mo -FF spread | TB3SMFFM |  |  |  |  |
| 6 Mo. - FFR spread 6 mo-FF spread | TB6SMFFM |  |  |  |  |
| 1 yr . - FFR spread $1 \mathrm{yr}-\mathrm{FF}$ spread | T1YFFM | Y | Y |  |  |
| 5 yr. - FFR spread 5 yr-FF spread | T5YFFM |  |  |  |  |
| 10 yr. - FFR spread 10 yr-FF spread | T10YFFM | Y | Y |  | Y |
| Aaa - FFR spread Aaa-FF spread | AAAFFM | Y |  |  |  |
| Baa - FFR spread Baa-FF spread | BAAFFM | Y | Y |  |  |

Table 1, continued: Variable Combinations

| variable | mnemonic | included in $N=60$ set | included in $N=40 \text { set }$ | included in $N=18 \text { set }$ | included in $N=8$ set |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Switzerland / U.S. FX Rate | EXSZUS | Y |  |  |  |
| Japan / U.S. FX Rate | EXJPUS |  |  |  |  |
| U.S. / U.K. FX Rate | EXUSUK | Y | Y |  |  |
| Canada / U.S. FX Rate | EXCAUS |  |  |  |  |
| PPI: Finished Goods | PPIFGS | Y | Y | Y |  |
| PPI: Finished Consumer Goods | PPIFCG |  |  |  |  |
| PPI: Intermediate Materials | PPIITM | Y |  |  |  |
| PPI: Crude Materials | PPICRM | Y |  |  |  |
| Crude Oil Prices: WTI | oilprice |  |  |  |  |
| PPI: Commodities | PPICMM | Y | Y | Y | Y |
| ISM Manufacturing: Prices | NAPMPRI | Y |  |  |  |
| CPI: All Items | CPIAUCSL | Y |  |  |  |
| CPI: Apparel | CPIAPPSL |  |  |  |  |
| CPI: Transportation | CPITRNSL |  |  |  |  |
| CPI: Medical Care | CPIMEDSL |  |  |  |  |
| CPI: Commodities | CUSR0000SAC |  |  |  |  |
| CPI: Durables | CUUR0000SAD |  |  |  |  |
| CPI: Services | CUSR0000SAS |  |  |  |  |
| CPI: All Items Less Food | CPIULFSL |  |  |  |  |
| CPI: All items less shelter | CUUR0000SA0L2 |  |  |  |  |
| CPI: All items less medical care | CUSR0000SA0L5 |  |  |  |  |
| PCE: Chain-type Price Index | PCEPI | Y | Y | Y | Y |
| PCE: Durable goods | DDURRG3M086SBEA | Y | Y |  |  |
| PCE: Nondurable goods | DNDGRG3M086SBEA |  |  |  |  |
| PCE: Services | DSERRG3M086SBEA | Y | Y |  |  |
| Ave. Hourly Earnings: Goods | CES0600000008 | Y | Y | Y |  |
| Ave. Hourly Earnings: Construction | CES2000000008 |  |  |  |  |
| Ave. Hourly Earnings: Manufacturing | CES3000000008 |  |  |  |  |
| MZM Money Stock | MZMSL |  |  |  |  |
| Consumer Motor Vehicle Loans | DTCOLNVHFNM |  |  |  |  |
| Total Consumer Loans and Leases | DTCTHFNM |  |  |  |  |
| Securities in Bank Credit | INVEST | Y |  |  |  |



Figure 1: Uncertainty estimates, monthly data: posterior median (black line) and 15\%/70\% quantiles (blue lines), with macro uncertainty in the top panel and financial uncertainty in the bottom panel. The green line represents the corresponding estimates from JLN (top) and LMN (bottom).


Figure 2: Uncertainty estimates, quarterly data: posterior median (black line) and 15\%/70\% quantiles (blue lines), with macro uncertainty in the top panel and financial uncertainty in the bottom panel. The green line represents the corresponding estimates from JLN (top) and LMN (bottom).


Figure 3: Reduced-form volatilities, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue lines)


Figure 3: Continued, reduced-form volatilities, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue lines)


Figure 3: Continued, reduced-form volatilities, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue lines)


Figure 4: Impulse responses, one st. dev. shock to macro uncertainty, 30 -variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 4: Continued, impulse responses, one st. dev. shock to macro uncertainty, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 5: Impulse responses, one st. dev. shock to financial uncertainty, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 5: Continued, impulse responses, one st. dev. shock to financial uncertainty, 30variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 6: Variance shares, shock to macro uncertainty, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 6: Continued, variance shares, shock to macro uncertainty, 30-variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 7: Variance shares, shock to financial uncertainty, 30 -variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 7: Continued, variance shares, shock to financial uncertainty, 30 -variable model, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 8: Impulse responses, one st. dev. shock to macro uncertainty, 30-variable model without contemporaneous uncertainty effects, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 8: Continued, impulse responses, one st. dev. shock to macro uncertainty, 30 -variable model without contemporaneous uncertainty effects, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 9: Impulse responses, one st. dev. shock to financial uncertainty, 30-variable model without contemporaneous uncertainty effects, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 9: Continued, impulse responses, one st. dev. shock to financial uncertainty, 30variable model without contemporaneous uncertainty effects, monthly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 10: Impulse responses, one st. dev. shock to macro uncertainty, 30-variable model, quarterly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 10: Continued, impulse responses, one st. dev. shock to macro uncertainty, 30variable model, quarterly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 11: Impulse responses, one st. dev. shock to financial uncertainty, 30 -variable model, quarterly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 11: Continued, impulse responses, one st. dev. shock to financial uncertainty, 30variable model, quarterly data: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 12: Uncertainty obtained from AR-SV estimates, different variable sets. The top panel of macro measures includes averages across different variable sets indicated in Table 3. The variable set $\mathrm{N}=18$ corresponds to the set of macroeconomic variables included in the baseline model. The bottom panel of financial measures includes averages across the $\mathrm{N}=12$ financial variables included in the baseline model, an alternative set of 12 variables that drop the Fama-French factors and use a 10 -industry breakdown, a set of 43 industry-level portfolio returns, and a set of 93 industry-level portfolio returns.


Figure 13: Impulse responses, one st. dev. shock to macro uncertainty, 30 -variable model, monthly data ending in Dec. 2007: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 13: Continued, impulse responses, one st. dev. shock to macro uncertainty, 30variable model, monthly data ending in Dec. 2007: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 14: Impulse responses, one st. dev. shock to financial uncertainty, 30 -variable model, monthly data ending in Dec. 2007: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


Figure 14: Continued, impulse responses, one st. dev. shock to financial uncertainty, 30variable model, monthly data ending in Dec. 2007: posterior median (black line) and $15 \% / 70 \%$ quantiles (blue shading)


[^0]:    *The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Federal Reserve System. We thank Brent Bundick, Josh Chan, Soojin Jo, Gary Koop, Haroon Mumtaz, Rodrigo Sekkel, Minchul Shin, Stephen Terry, Molin Zhong, and seminar participants at the Federal Reserve Bank of Cleveland, SBIES conference in Philadelphia, John Hopkins University, and the Barcelona summer meeting for several useful comments on an earlier draft. We are particularly grateful to Drew Creal for his advice on the particle Gibbs sampler. Carriero gratefully acknowledges support for this work from the Economic and Social Research Council [ES/K010611/1], and Marcellino from the Baffi-Carefin center.

[^1]:    ${ }^{1}$ According to the CBOE, which created the VIX, it "is based on the S\&P 500 Index (SPX), the core index for U.S. equities, and estimates expected volatility by averaging the weighted prices of SPX puts and calls over a wide range of strike prices."
    ${ }^{2}$ The CBOE's VXO is based on trading of S\&P 100 (OEX) options.

[^2]:    ${ }^{3}$ One might worry less about this in a large cross-section approach such as that of Jurado, Ludvigson, and Ng (2015). However, the formal analytics underlying the consistency of factor estimates are based on cases in which the dependent variables are data series rather than (stochastic volatility) estimates from a model. Even if one is not concerned with such complications, it is preferable to have an approach which works well in not-large cross sections.
    ${ }^{4}$ An important exception is Ludvigson, Ma, and Ng (2015), which is, however, based on the two-step approach and a small cross-section.

[^3]:    ${ }^{5}$ The literature on forecasting with large datasets - see, e.g., Banbura, Giannone and Reichlin (2010) and Stock and Watson (2002) - has shown that typically the size of the information set matters and can reduce forecast errors and their volatility, even though there is a debate on how "large" large is, with studies such as Koop (2013) and Carriero, Clark and Marcellino (2015b) suggesting that about 20 carefully selected macroeconomic and financial variables could be sufficient.
    ${ }^{6}$ Conditional heteroskedasticity in-mean was introduced by French, et al. (1987) with the GARCH-inmean model. Macroeconomic applications include Elder (2004) and Elder and Serletis (2010) for, respectively, inflation and oil price uncertainty. Koopman and Uspensky (2002) and Chan (2015) introduce univariate stochastic volatility-in-mean models, and Jo (2014) and Shin and Zhong (2014) consider multivariate VAR extensions with independent volatility processes.

[^4]:    ${ }^{7}$ Carriero, Clark, and Marcellino (2016) estimate a BVAR with stochastic volatility with 125 variables (including macroeconomic indicators, an array of interest rates, some stock return measures, and exchange rates). A factor analysis of innovations to volatility indicates two components to account for the vast majority of innovations to volatilities.

[^5]:    ${ }^{8}$ The inclusion of $y_{t-1}$ in the volatility factor processes can be seen as a version of the leverage effect sometimes included in stochastic volatility models of financial returns.

[^6]:    ${ }^{9}$ However, Primiceri's (2005) model permits the innovations to the volatilities to be correlated across variables, while in our specification they are not, and any correlation among volatilities are forced onto the common factor, a restriction that is standard in factor model analysis.

[^7]:    ${ }^{10}$ This statement refers to drawing from the conditional posterior of the conditional mean parameters, when $\Sigma_{t}$ belongs to the conditioning set. One needs also to keep in mind that the joint distribution of the

[^8]:    ${ }^{11}$ In drawing the loadings, we make use of the information in the observable $\ln \left(\tilde{v}_{j, t}^{2}\right)$, with the following transformation of the observation equations:

    $$
    \ln \left(\tilde{v}_{j, t}^{2}+\bar{c}\right)-\ln h_{j, t}=\left\{\begin{array}{c}
    \beta_{m, j} \ln m_{t}+\ln \epsilon_{j, t}^{2}, j=1, \ldots, n_{m}  \tag{12}\\
    \beta_{f, j} \ln f_{t}+\ln \epsilon_{j, t}^{2}, j=n_{m}+1, \ldots, n .
    \end{array}\right.
    $$

    With the conditioning on $h_{T}$ and $s_{T}$ in the posterior for $\beta$, we use this equation, along with the mixture mean and variance associated with the draw of $s_{T}$, for sampling the factor loadings with a conditionally normal posterior with mean and variance represented in a GLS form.

[^9]:    ${ }^{12}$ Since the triangularization obtains computational gains of order $n^{2}$, the cross-sectional dimension of the

[^10]:    system can be extremely large, and indeed Carriero, Clark and Marcellino (2016) present results for a VAR with 125 variables.
    ${ }^{13}$ Our prior specification for the conditional mean parameters follows the Minnesota prior beliefs, which are uncorrelated across equations. Modifying the algorithm to allow for dependent prior beliefs is discussed in Carriero, Clark, and Marcellino (2016).
    ${ }^{14}$ Chan (2015) provides a sampler designed to jointly sample the log-volatilities when they appear in the conditional means, but his sampler is only viable in the case of independent log-volatilities, which is not the case of this paper.

[^11]:    ${ }^{15}$ The McCracken-Ng transformations are very similar to those of JLN.
    ${ }^{16}$ LMN take their financial variables from the datasets constructed by Professor French.

[^12]:    ${ }^{17}$ In this sense, our specification builds on the standard, simpler stochastic volatility common in finance and introduced into macroeconomics by Cogley and Sargent (2005) and Primiceri (2005).

[^13]:    ${ }^{18}$ In the interest of brevity, we provide in a supplementary appendix available upon request results for a constant parameter/constant volatility BVAR in 31 variables - our 30 variables and the 1-step ahead uncertainty estimate (logged, for consistency with our results) of JLN. These results have some similarity to the baseline model results discussed below.

[^14]:    ${ }^{19}$ Also, and again as in JLN, there is no rebound in employment and industrial production in the long run, contrary, e.g., to Bloom (2009). However, this pattern may be driven by the use of HP-detrended data in Bloom (2009).

[^15]:    ${ }^{20}$ We do so by making use of squares of the cumulated impulse responses functions.

