Draft
Comments welcome

Computation of Standard errors for Purchasing Power Parity (PPP) Exchange Rates from the International Comparison Program (ICP)

Gholamreza Hajargasht
University of Melbourne

and

D.S. Prasada Rao(*)
The University of Queensland

19 May, 2016

In the spirit in which this Workshop/Conference is organized, the paper represents work-in-progress and the authors seek to benefit from the discussion and comments offered at the Workshop.

(*) Author for correspondence: d.rao@uq.edu.au

1. Introduction

The International Comparison Program (ICP) is fast approaching fifty years since its inception in 1968 as a research project at the University of Pennsylvania which was conducted under the auspices of the United Nations Statistical Office and Professors Kravis, Heston and Summers. Over the last five decades, the program has grown in its coverage starting from 10 countries in Phase I in 1970 to 177 countries[1] in the recently completed round of ICP in 2011. There have been numerous developments in the organization of the ICP which has been regionalized with comparisons undertaken at the regional level and subsequently linked in providing global comparisons. The final linking and dissemination of the results were undertaken by the Global Office located at the World Bank. There have been several refinements in the survey framework, a main feature of it being the use of structural product descriptions developed at the Global Office of the ICP. The use of region-specific product lists for regional comparisons and then linking of these comparisons through the use of prices collected for items in the global core list have helped achieve a compromise between comparability and representativity – two competing requirements in the preparation of product lists used in price surveys in the participating countries. The aggregation methodology used in the compilation of purchasing power parities (PPPs) has also been refined. The country-product-dummy (CPD) method is the recommended method for aggregating item level prices leading to PPPs at the basic heading level. For aggregation above the basic heading level, where expenditure weights are available, the Gini-Elteto-Koves-Szulc method is the recommended method. Methods such as the Geary-Khamis and Ikle methods are recommended when additively consistent international comparisons are desired.

The PPPs and the real expenditure aggregates compiled and disseminated through ICP are widely used by national and international organizations, economists and policy makers for a variety of purposes. The demand for data on PPPs and real expenditures has been growing steadily over time and results from ICP are highly sought after. The use of PPPs for household consumption by the World Bank for the purpose of estimating incidence of poverty at the regional and global level has given the PPPs from the ICP a special status. PPPs are used in the compilation of the Human Development Index and in measuring regional and global inequality. ICP data are used in studying catch-up and convergence and also in assessing productivity performance of countries.

The PPPs and real expenditures, published and made available through the reports of the ICP, are compiled using price data collected in the participating countries through specially conducted surveys on a range of products considered relevant for international comparisons. These price data are then aggregated using expenditure share weights in the final computation of PPPs for various national income aggregates. The published PPPs and real expenditures do not provide any indication of the uncertainty or reliability associated with the numbers. The uncertainty may arise due to the fact that price data collected are based on survey data and also on the variability in relative prices in the participating countries. For example, a comparison of prices between USA and Canada may be considered more reliable than a comparison between USA and Uganda which may be considered less reliable or weak. This intuitive notion relies heavily on the dissimilarity in price structures between USA and Uganda.

The need to measure uncertainty associated with PPPs and real expenditures from the ICP has been recognized by Deaton (2012) where he provides a framework to measure uncertainty associated with a range of bilateral and multilateral indexes used in international comparisons. The notion of associating standard errors with price index numbers is closely related to the stochastic approach to index numbers discussed in Selvanathan and Rao (1994). One of the early but not a very satisfactory approach to the computation of standard errors associated with price comparisons using the Geary-Khamis method can be

[1] The 2011 ICP covered 199 countries but only 177 countries had comparisons at the full GDP level. The remaining countries participated only in the comparisons of household consumption.

found in Rao and Selvanathan (1992). However, their approach used a partial framework and therefore not sufficiently general.

The present paper is a step in the direction of compiling standard errors associated with PPPs within the ICP. We report progress made in this important area of research concerning international comparisons of prices. The approach used here is based on the recently developed stochastic approach to the compilation of PPPs within the ICP in Rao and Hajargasht (2015) and the paper reports results from the implementation of this approach for aggregation above the basic heading level and offers a comparative assessment of reliability of PPPs from ICP for the benchmark years 1980, 1985, 1996, 2005 and 2011.

The paper is organized as follows. Section 2 provides an overview of the steps involved in the compilation of PPPs within the ICP. The steps show the sources of uncertainty at various stages of the ICP which can be used in developing a framework to compile measures of uncertainty associated with PPPs. Section 3 describes the econometric methodology that underpins the computation of standard errors for PPPs. Section 4 presents empirical results and the paper is concluded with Section 5 where directions for future work are canvassed.

2. Steps involved in the compilation of PPPs within the ICP

The computation of standard errors for PPPs from the ICP is quite complex. PPPs compiled firstly at the regional level. The regional comparisons are then linked to form global comparisons which are published for different levels of aggregation within the national accounts. The headline aggregate is obviously the gross domestic product.

The following flow chart shows the steps involved in the process followed in ICP 2005 and is drawn from Chapter 1 by Rao (2013) in the *ICP Book* published by the World Bank (2013). We recognize that there have been significant changes to the linking process in the latest round of ICP in 2011. We use the flowchart as a template to discuss the sources of uncertainty in PPPs by identifying the steps involved in the process. The first stage of the process is the compilation of PPPs at the basic heading level[2] using prices collected through surveys conducted in the participating countries. Each participating country provides *national average price* data to the regional coordinating agency for further processing. These data are aggregated to derive PPPs at the basic heading level. The BH PPPs are then aggregated using expenditure weights to derive PPPs for higher level aggregates such as household consumption, government expenditure, gross fixed capital formation and, finally, for the gross domestic product.[3] In 2005, the regional comparisons were linked using linking factors derived from price data provided by a set of selected ring countries from different regions. The 2005 approach to linking was found deficient and was replaced by a more data-intensive approach where prices for goods and services in the *global core list* of products from all the participating countries from all the regions were used in deriving the linking factors. Once the linking factors are obtained, a matrix of PPPs for all the 155 basic headings for all the participating countries is compiled. This information together with the expenditure data in national currency units is used in compiling the final set of PPPs for global comparisons. The global comparisons are derived after imposing the *fixity* condition which ensures that the relativities between countries within a region observed in the regional comparisons are maintained within the global comparisons.

[2] Basic heading level is the lowest level of aggregation at which expenditure data are available. Typically each basic heading represents a collection of items that are similar in nature and are likely to exhibit similar relative prices across countries.
[3] The ICP publication version of the results provides PPPs for 23 major aggregates under the expenditure side of the national accounts.

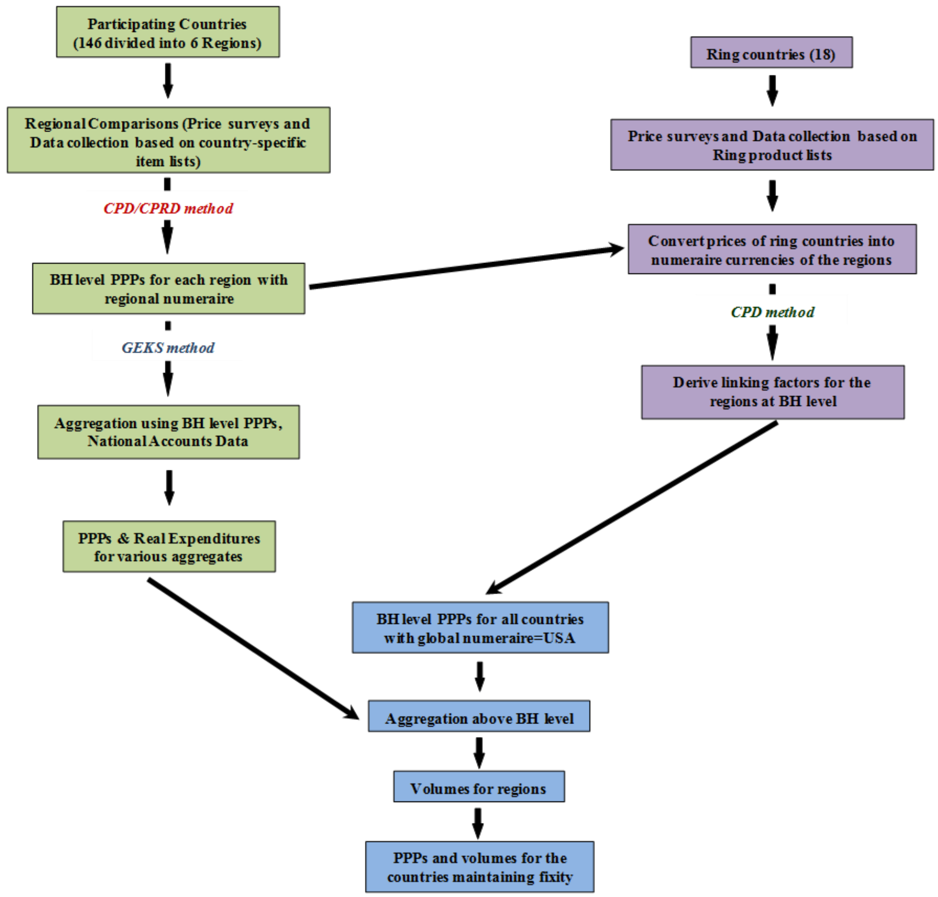


Figure 1: A Schematic diagram of the steps involved in PPP compilation, ICP 2005
Source: Rao (2013), *The Framework of the International Comparison* Program, Chapter 1 in Measuring the Real Size of the World Economy, World Bank 2013, p. 39

Sources of uncertainty

It is difficult to list all the sources of uncertainty associated with the final set of PPPs published at the conclusion of a round of the ICP. The following is a list of sources that we are able to account for in deriving measures of uncertainty associated with PPPs.

1. The first source of uncertainty concerns the use of national average prices. The participating countries usually submit additional information along with national average prices which include the number of quotations used in the averaging process; and the standard deviation of the price quotations used. These together provide an indication of the uncertainty associated with the use of national average prices in the computation of basic heading PPPs.
2. Second, the national average prices are aggregated using the *country-product-dummy* (CPD) method. The CPD method, described further in the next section, is a regression based method that relies on the notion of *law of one price*. Failure of support for this law manifests itself in the residuals of the CPD regression method and contributes to the standard errors associated with estimates of PPPs from the CPD method. The failure of law of one price indicates differences in relative price structures across countries.
3. The next step is to use PPPs at the basic heading level as inputs into the next level of aggregation. In this step, data on expenditures from the national accounts are used as weights. We consider four

aggregation procedures at this stage: Gini-Elteto-Koves-Szulc (GEKS) method; Geary-Khamis (GK) method; Ikle Method and the weighted country-product-dummy (WCPD) method. Diewert (2013) provides a description and discussion of relative merits of the GEKS, GK and Ikle methods. The WCPD method is discussed in Rao (2005) and also in Rao and Hajargasht (2015). The GEKS is the aggregation method used in ICP whereas the remaining are popular alternatives. Deaton (2012) uses residuals from the CPD regression to compute standard errors associated with the PPPs derived using GEKS as well as from binay Fisher and Tornqvist indices. We use a similar but a more direct approach to the computation of PPPs based on the stochastic approach discussed in Rao and Hajargasht (2015). In the approach used by Deaton (2012), uncertainty associated the BH parities derived in step (2) above is not taken into account. The approach developed by Rao and Hajargasht (2015) makes it possible to incorporate uncertainty associated with the BH parities which are used as inputs into computations at this stage.

4. The next level at which uncertainty arises is the computation of the linking factors using either price data on ring products from ring countries as was the case in 2005 or prices of global core list of products from all the countries used in the 2011 ICP. In eithr case, the CPD model is used and therefore it is feasible to obtain measures of uncertainty associated with the linking factors.
5. The linking factors are applied to regional level comparisons resulting in a large matrix of parities for all the 155 basic headings in all the participating countries. This matrix of BH PPPs are used along with national expenditure data to compile PPPs at the global level. It is possible to measure uncertainty associated with PPPs at the global level using the approach described in step (3) above. Deaton (2012) applies the method of measuring uncertainty to global comparisons. As a first attempt at obtaining measures of uncertainty, we follow the same approach and compute standard errors associated with PPPs obtained at the global level. We note here that these PPPs do not satisfy fixity which is described in the next section.

Basically our focus in this paper is on the computation of standard errors for PPPs obtained at the global level using parities for the 155 basic headings obtained after linking the regional comparisons. In this paper we do not incorporate the uncertainty associated with PPPs at the basic heading level supplied by the regions nor do we incorporate the uncertainty associated with national average prices even though the necessary framework has been developed. This work is planned for the next stage of our research which requires collection of additional data from the regional coordinating agencies and World Bank sources.

3. Methodology for computation of the standard errors

In this section, we describe the basic elements necessary to explain how the standard errors for PPPs are computed using the stochastic approach presented in Rao and Hajargasht (2015). As we currently have no access to price data used in the computation or PPPs at the basic heading level from each of the regions, we basically focus on the aggregation of basic headings at the global level. For purpose of this paper, we will have PPPs for the basic headings and expenditure data from different countries. The expenditure data is then used in obtaining expenditure shares for each basic heading in all the participating countries.

We consider the general case involving M countries and N commodities. Let p_{ij} represent the price of i-th commodity in j-th country expressed in local currency units (LCUs). In practice, the price p_{ij} used in ICP is national average of several price quotations. In this case, we will have information on the number of quotations used and on the sampling variance of the price. We consider this general case in section 6. For the purpose of exposition in this section, we treat price p_{ij} as a single quotation. Let PPP_j represent purchasing power parity of currency of country *j* expressed relative to a reference country currency.[4] Then PPP between currencies of any two countries *j* and *k*, PPP_{jk} can be obtained as the ratio: PPP_k / PPP_j . The index number methods used in the ICP ensure that the resulting PPPs are *transitive* and *base invariant*.[5]

This method was originally proposed as a tool to fill gaps in price data but it has been in use as a method for computing *PPP*s at the basic heading level (Kravis, Heston and Summers, 1982; Rao, 2013b).The CPD model in its regression formulation (equation 1 below) is commonly described as "a very simple type of hedonic regression model where the only characteristic of a commodity is the commodity itself" (Diewert, 2005, p. 561). The model in its multiplicative form is referred to as the *law of one price* which postulates that the observed price, p_{ij}, of a commodity is the product of its international price, P_i, and the purchasing power parity of currency of country *j*, PPP_j. The CPD model in its multiplicative form is given by:

$$p_{ij} = P_i \cdot PPP_j \cdot u_{ij}^* \tag{1}$$

where u_{ij}^* s are random disturbance terms which are independently and identically distributed.[6] The additive form of the CPD model is obtained by taking logs on both sides:

$$\ln p_{ij} = \ln P_i + \ln PPP_j + \ln u_{ij}^* = \eta_i + \pi_j + u_{ij} = \sum_{i=1}^{N} \eta_i D_i + \sum_{j=1}^{M} \pi_j D_j^* + u_{ij} \tag{2}$$

where D_i and D_j^* are product and country dummy variables which take values 1 for commodity *i* and country *j* respectively and 0 otherwise. Equation (1) is the reason why this model is known as the country-product-dummy model. Deaton (2012, p4) considers the law of one price interpretation of the CPD model and states that "if there were no trade costs and all goods were tradable and freely traded between countries, so that the law of one price held, the residual terms in (2) would be zero".

The model can be expressed as a standard regression model. Following Rao (2005), we have

$$y_{ij} = \ln p_{ij} = x_{ij}\,\beta + v_{ij} \tag{3}$$

where $\mathbf{x}_{ij} = \left[D_1\, D_2\ ....\ D_N\ \ D_1^*\, D_2^*\ ....\ D_M^* \right]$ and $\boldsymbol{\beta} = \left(\eta_1\, \eta_2 \cdots \eta_N\ \ \pi_1\, \pi_2 \cdots \pi_M \right)'$. Stacking all the *MN* observations, the model can be written in matrix notation as: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ .The model has *MN* observations in (*M+N*) unknowns. We observe that the matrix **X** is of rank (*M+N-1*) and therefore parameters can be estimated only after imposing a linear restriction. Setting $\pi_M = 0$ implies that the currency of country M is the numeraire or reference currency with $PPP_M = 1$, the remaining parameters can be estimated. Dropping the last column of **X**, we have the modified equation:

$$\mathbf{y} = \mathbf{X}^* \boldsymbol{\beta}^* + \mathbf{u} \tag{4}$$

[4] We deliberately omit the subscript for basic heading to keep the notation simple. As aggregation below and above basic heading levels are discussed separately this should not cause any confusion.

[5] It is easy to see from the definition that $PPP_{jk} = PPP_{j\ell} \cdot PPP_{\ell k}$ for all j, k and ℓ, which is the *transitivity* requirement. Base invariance requires symmetric treatment of all the countries in the comparison. See Rao (2013a, 2013b) for further discussion on these two requirements.

[6] As demonstrated in the ensuing sections, this assumption can be easily relaxed.

where $\mathbf{X}^*$ is the same as matrix X but without the last column and $\boldsymbol{\beta}^*$ is the same as vector $\boldsymbol{\beta}$ without the last element. The least squares estimator of $\boldsymbol{\beta}^*$ and its covariance matrix are given by $\hat{\boldsymbol{\beta}}^* = \left(\mathbf{X}^{*\prime}\mathbf{X}^*\right)^{-1}\mathbf{X}^{*\prime}\mathbf{y}$ and $Var\left(\hat{\boldsymbol{\beta}}^*\right) = \sigma^2\left(\mathbf{X}^{*\prime}\mathbf{X}^*\right)^{-1}$.

In our previous work, Hajargasht and Rao (2010) we used distributional assumptions on the disturbance term to generate different types of index number formulae. In particular, assumption of lognormal, Gamma and Inverse Gamma were shown to result in, respectively, the weighted CPD[7], Ikle and an arithmetic system. Hajargasht and Rao (2010) also established that the Geary-Khamis method can be derived using a method of moment estimator with a specific set of moment conditions.

3.1 Three equivalent forms of the CPD model

In this paper we pursue the method of moments estimation of parameters of the CPD model. In order to derive different index number formulae we re-write the CPD model in an equivalent form. For example we can re write the CPD model

$$p_{ij} = P_i \cdot PPP_j \cdot u_{ij}^*$$

in an equivalent form

$$\frac{p_{ij}}{P_i PPP_j} - 1 = u_{ij} \quad \text{where } u_{ij} = u_{ij}^* - 1 \qquad \text{with } E(u_{ij}) = 0 \text{ and } Var(u_{ij}) = \sigma^2$$

which can then be written in a more general form:

$$r_{ij}(p_{ij}, P_i, PPP_j) = u_{ij} \tag{5}$$

The function form for $r_{ij}(p_{ij}, P_i, PPP_j)$ will depend on how the CPD equation in (1) is written. We use the following three forms for the CPD model:

$$Geometric: \quad r_{ij} = \ln p_{ij} - \ln P_i - \ln PPP_j \tag{6}$$

$$Arithmetic: \quad r_{ij} = \frac{p_{ij}}{P_i PPP_j} - 1 \tag{7}$$

$$Harmonic: \quad r_{ij} = \frac{P_i PPP_j}{p_{ij}} - 1 \tag{8}$$

The geometric form lends itself to the use of least-squares estimation. However the arithmetic and harmonic forms are in the form of a non-additive non-linear regression model. These can be estimated using the method of moment estimator. Details of the procedure are provided in Rao and Hajargasht (2015). There are N parameters representing international prices and M parameters representing PPPs. We denote these (N+M) parameters by vector $\boldsymbol{\beta} = \left[\mathbf{P}\ \mathbf{PPP}\right]$.

[7] In Rao (2005), the weighted CPD is shown to be equal to the Rao (1990) system. Diewert (2005) has shown a way to derive the Geary-Khamis method for the special case of two countries.

3.2 Method of moments estimation

The parameters in vector $\boldsymbol{\beta} = \left[\mathbf{P}\ \mathbf{PPP}\right]$ can be estimated using any one of the three specifications, equations (6) to (8) along with a set of moment conditions. The method of moments requires the specification of a set of (N+M) moment conditions defined by matrix $\mathbf{R}$ which is of order (N+M) by (N+M) and of the form:

$$\frac{1}{NM}\mathbf{R}'\mathbf{r} = \mathbf{0} \tag{9}$$

Once the moment condition matrix is specified, it is just a matter of solving (9) for the unknown parameters $\boldsymbol{\beta} = \left[\mathbf{P}\ \mathbf{PPP}\right]$. It is a well-known result that the optimum choice of $\mathbf{R}$ is given by:

$$\mathbf{R} = E\left[\partial \mathbf{r} / \partial \boldsymbol{\beta}'\right] \tag{10}$$

Once the moment conditions are specified solutions to (9) will provide different index numbers methods. Rao and Hajargasht (2015) show that by choosing different set of moment conditions, it is possible to derive the unweighted geometric *Jevons*, arithmetic and harmonic averages of price relatives, and also the *Dutot* index.[8]

3.3 Weighted versus unweighted method of moments estimation

Use of the method of moments estimator in (9) along with a choice of $\mathbf{R}$, similar to that shown in (10), leads to unweighted index numbers. Rao and Hajargasht (2015) show that by choosing different set of moment conditions, it is possible to derive the unweighted geometric *Jevons*, arithmetic and harmonic averages of price relatives, and also the *Dutot* index.[9]

As we are interested in index numbers that make use of weights that reflect the relative importance of different commodities in different countries, we use weighted method of moments estimator. Within the ICP, BH parity is interpreted as a price for the *composite commodity* associated with the basic heading. We recall that at the BH level, expenditure data are available. Let e_{ij} represent expenditure on i-th basic heading in country j expressed in national currency units[10]. Using this information we can define "*quantity*" as $q_{ij} = e_{ij} / p_{ij}$ and expenditure shares as: $w_{ij} = e_{ij} \Big/ \sum_{i=1}^{N} e_{ij}$. It is standard in index number methodology to use expenditure share weights to derive price index numbers. The same approach is also used in the ICP. However the Geary-Khamis method uses quantity share weights of the form $q_{ij}* = q_{ij} \Big/ \sum_{j=1}^{M} q_{ij}$

Let $\mathbf{W} = \left\{w_{ij}\right\}$ be a diagonal matrix with expenditure shares in its diagonal. We can incorporate the expenditure share weighting matrix $\mathbf{W}$ in the MOM estimation of non-linear additive models in a straightforward manner using the moment conditions using :

[8] See Table 1 in Rao and Hajargasht (2015) for a summary of possible specifications.
[9] See Table 1 in Rao and Hajargasht (2015) for a summary of possible specifications.
[10] We note here that e_{ij} may be zero for some basic headings in some countries. The ICP practice has been one of imputing PPPs to BHs even when there are zero expenditures.

$$\frac{1}{NM}\mathbf{R}'\mathbf{W}\mathbf{r} = \mathbf{0}. \tag{11}$$

In this equation, we may also use quantity share weights $q_{ij} * = q_{ij} \Big/ \sum_{j=1}^{M} q_{ij}$. If we apply the weighted method of moments estimator by solving (11), we can derive the following systesms.

1. ***Weighted CPD – Rao system*** can be obtained by using geometric specification of the CPD model in equation (6) with expenditure share weights. Since the model is linear in log-prices, the method of moments estimator is identical to the least squares estimator. The weight least squares estimator of the geometric version in (6) leads to the following normal equations.

$$\ln PPP_j = \sum_{i=1}^{N} w_{ij} \ln \frac{p_{ij}}{\hat{P}_i} \qquad \ln \hat{P}_i = \sum_{j=1}^{M} w_{ij}^{*} \ln \frac{p_{ij}}{PPP_j} \tag{12}$$

2. ***Arithmetic system –*** The weighted arithmetic system has not been used in international comparisons before but it complements the geometric and harmonic versions which have been in use. The arithmetic choice for r_{ij} in equation(7) with optimal choice of moment conditions following equation (10) leads to the following system of simultaneous equations.

$$PPP_j = \sum_{i=1}^{N} w_{ij} \frac{p_{ij}}{\hat{P}_i} \qquad \hat{P}_i = \sum_{j=1}^{M} w_{ij}^{*} \frac{p_{ij}}{PPP_j} \tag{13}$$

3. ***Ikle system*** – The harmonic specification for r_{ij} along with expenditure share weights matrix **W** leads to the following system of equations which are identical to the system of equations that define the Ikle system.

$$\frac{1}{PPP_j} = \sum_{i=1}^{N} w_{ij} \frac{\hat{P}_i}{P_{ij}} \qquad \frac{1}{P_i} = \sum_{i=1}^{M} w_{ij}^{*} \frac{PPP_j}{P_{ij}} \tag{14}$$

We can see the similarity between the Ikle and the arithmetic systems in that they, respectively, use harmonic and arithmetic means to define the systems.

4. ***Geary-Khamis system*** – The GK system can be obtained by using the arithmetic choice for r_{ij} in equation (7) with optimal choice of moment conditions but with quantity share weights instead of expenditure share weights used in the arithmetic system defined in (13). The resulting system of equations are:

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} \hat{P}_i q_{ij}} \qquad \hat{P}_i = \sum_{j=1}^{M} \left(\frac{p_{ij} q_{ij}}{PPP_j} \right) \Big/ \sum_{j=1}^{M} q_{ij} \tag{15}$$

5. ***Gini-Elteto-Koves-Szulc (GEKS)*** **based on Tornqvist binary index numbers using CPD model -** From the CPD model in logarithmic form we have,

$$\ln p_{ij} = \eta_i + \pi_j + u_{ij} \quad \text{and } \ln p_{ik} = \eta_i + \pi_j + u_{ik}$$
$$\Rightarrow \ln p_{ik} - \ln p_{ij} = \ln\left(\frac{p_{ik}}{p_{ij}}\right) = \pi_k - \pi_j + v_{ij}$$

The last equation here is the same as the model in Selvanathan and Rao (1994). Using arithmetic average of expenditure shares on *i*-th commodity in countries *j* and *k* as weights, Selvanthan and Rao (1994) show that the weighted least squares of the parameters of the model in log of price changes leads to the Tornqvist-based GEKS indexes and their standard errors. This means that the stochastic approach based on the CPD model can be used in deriving Tornqvist-based GEKS indexes. Standard errors for Tornqvist-based GEKS are shown in Selvanathan and Rao (1994).

Having demonstrated that the feasibility of obtaining commonly used aggregation procedures from the CPD model, we now turn to the formulae for computing standard errors for the PPPs derived from weighted method of moments estimation.

Standard errors for PPPs from different aggregation procedures

In order to obtain standard errors for PPPs, and also Ps, we need to detail stochastic specifications for the disturbances in the CPD model in equation (1)

$$p_{ij} = P_i \cdot PPP_j \cdot u_{ij}^*$$

Let $\mathbf{\Omega}$ represent the covariance matrix of the disturbance terms u_{ij}^* for i = 1,2,…,N and j = 1,2,….M. We consider three specifications. The first specification is one of homoscedasticity where variances of the disturbance term are the same for all the commodities and all the countries. The second specification is where disturbances are heteroskedastic for commodities and countries. The last assumption is where variances of disturbances are different for different countries but the same for all commodities. As the disturbances are not observed we estimate the covariance matrix using the residuals from different specifications given in equations (6), (7) and (8) where the unknown vectors of **P** and **PPP** vectors are replaced by their estimates $\hat{\mathbf{P}}$ and $\mathbf{P\hat{P}P}$. The estimated covariance matrices are given for three specifications as:

Homoskedastic disturbances: The estimated covariance matrix is given by

$$\hat{\mathbf{\Omega}} = \hat{\sigma}^2 \mathbf{I}_{MN \times MN} \tag{16}$$

where where $\hat{\sigma}^2 = \dfrac{\hat{\mathbf{r}}'\hat{\mathbf{r}}}{MN}$ and $\hat{\mathbf{r}}$ is a vector of residuals computed using the models specified in equations (6), (7) or (8).

Unrestricted White's Heteroscedastic model: The estimated covariance matrix is given by

$$\hat{\mathbf{\Omega}} = Diag(\hat{r}_{ij}^2) \tag{17}$$

where $\hat{r}_{ij}$ is the residual computed from equations (6), (7) or (8) which ever is appropriate for the method and computed for a specific commodity in a given country.

Heteroscedasticity – different variances in different countries: The estimated covariance matrix is given by

$$\hat{\mathbf{\Omega}} = Diag\left(\sum_{i=1}^{N} \hat{u}_{ij}^2 / N\right) \otimes \mathbf{I}_N \tag{18}$$

where $\hat{r}_{ij}$ is the residual pecific commodity in a given country computed from equations (6), (7) or (8) which ever is appropriate for the method.

The covariance matrices for the estimated international average prices and purchasing power parities, $\hat{\mathbf{P}}$ and $\mathbf{P\hat{P}P}$, for different methods are given by:

Weighted CPD (geometric) Method: The covariance matrix is given by:

$$Var(\ln \mathbf{P}, \ln \mathbf{PPP}) = \left[\mathbf{X}^{*}{}'\mathbf{W}\mathbf{X}^{*}\right]^{-1} \mathbf{X}^{*}{}'\mathbf{W}\hat{\mathbf{\Omega}}\mathbf{W}\mathbf{X}^{*}{}'\left[\mathbf{X}^{*}{}'\mathbf{W}\mathbf{X}^{*}{}'\right]^{-1} \qquad (19)$$

where $\mathbf{X^*}$ is given in equation (4); $\mathbf{W}$ is expenditure share matrix; and $\hat{\mathbf{\Omega}}$ is the covariance matrix following one of the three alternatives given in (16) , (17) and (18).

Ikle and Geary-Khamis systems: The form of the covariance matrix is the same for both the systems but the actual matrices R and W differ for the two systems. The form of the covariance matrix is given by:

$$Var(\hat{\boldsymbol{\beta}}_{MM}) = \left[\hat{\mathbf{R}}'\mathbf{W}\hat{\mathbf{D}}\right]^{-1} \hat{\mathbf{R}}'\mathbf{W}\hat{\mathbf{\Omega}}\mathbf{W}\hat{\mathbf{R}}\left[\hat{\mathbf{D}}'\mathbf{W}\hat{\mathbf{R}}\right]^{-1} \qquad (20)$$

where $\hat{\mathbf{R}}$ is the matrix that defines the moment conditions where unknown international prices and purchasing parities are replaced by their estimates, $\hat{\mathbf{P}}$ and $\mathbf{P\hat{P}P}$.

Standard errors for the binary Tornqvist indices: The binary Tornqvist indices are obtained by running ordinary least squares on the equation

$$\ln\left[p_{ij}/p_{iM}\right] = \ln PPP_j + u_{ij}$$

The estimated variance of $\ln PPP_j$ is given by:

$$Var(\ln PPP_j) = \sum_{i=1}^{N} w_{ij}^{*2}\hat{\sigma}_j^2 \qquad (21)$$

where $\hat{\sigma}_j^2 = \frac{1}{N}\sum_{i=1}^{N}\left\{\ln\left[p_{ij}/p_{iM}\right] - \ln PPP_j\right\}^2$.

4. Standard Errors for PPPs from selected benchmark ICP Comparisons

In this section we present empirical results based on the analysis of benchmark data for the years 1980, 1985, 1996, 2005 and 2011. We selected these benchmarks as they cover sufficient large countries and also basic headings. We have downloaded our data from the PWT 8.0 website. For the purpose of illustrating the stochastic approach used in the computation of standard errors, we have abstracted from all the steps and focus just on making global comparisons without fixity requirement. As international comparisons until 2005 were undertaken only at the global level, our approach allows comparisons in the standard errors derived across different benchmarks. We make use of PPPs for all the basic headings that cover gross domestic product (GDP) and the expenditure data at the basic heading levels as inputs into the computation of PPPs at the GDP level. In this process we ignore the standard errors and covariances associated with

PPPs for different countries at the basic heading level.[11] The coverage of countries and details are shown in the table below.

Bench mark year	Number of Basic Headings	Number of countries
1980	151	61
1985	139	64
1996	31	115
2005	129	146
2011	155	180

We have computed PPPs and their standard errors for all the countries participating in each of the benchmarks using all the aggregation methods: weighted CPD; Geary-Khamis and Ikle Methods. All the standard errors are based on hetroskedasticity specification used in equation (20) which states that the disturbances in the CPD model vary with countries and not with commodities.[12] As the standard errors are associated with the method of moments and therefore essentially asymptotic in nature, we computed standard errors using the simple jack-knife method used on basic headings. The jack-knife method involves computation of PPPs for all the countries dropping one basic heading at a time and then compute the standard deviation of the complete set of PPPs computed for each country. For example, if there are 155 basic headings, the jack-knife method provides 155 different estimates of PPP for the currency of any given country. The standard errors measure the variability in these PPPs.[13] All the PPPs in this section refer to PPPs of currencies of countries with the US dollar as the reference currency.

Following Deaton (2012), we plot the standard errors expressed as a percentage of PPP against the Laspeyres-Paasche spread.The Laspeyres-Paashe spread is a commonly used measure of dissimilarity in price and quantity structures between countries. The measure we use is:

$$LPS_j = \ln\left[\frac{LASPEYRES_{US,j}}{PAASCHE_{US,j}}\right]$$

This measure is generally expected to be positive as Laspeyres index is usually greater than the Paasche index. As the United States is used as the reference countriy, the Laspeyres-Paasche spread is small for countries in the OECD and expected to be large for low income countries in Africa and Asia.

Standard errors for weighted CPD, ICP 2005

In the following charts we present the standard errors for PPPs computed using the weighted CPD which is closely related to the binary Tornqvist index number. The following chart shows the standard errors for

[11] The stochastic approach proposed in Rao and Hajargasht (2015) provides a method of incorporating uncertainty associated with basic heading level PPPs.
[12] For the purpose of checking robustness of the standard errors agains different assumptions, we have also used White's robust standard errors. We find the standard errors for PPP under specifications (19) and (20) yield fairly similar magnitudes for the standard errors.
[13] We are in the process of devising a more formal bootstrap method which can be used in computing standard errors of PPPs or used in crosschecking standard errors obtained from our formulae in section 3.4.

PPPs for the 2005 benchmark year. For this year, we show the standard errors based on equation (19) along with the jack-knife standard errors and standard errors for the binary Tornqvist index shown in (21).

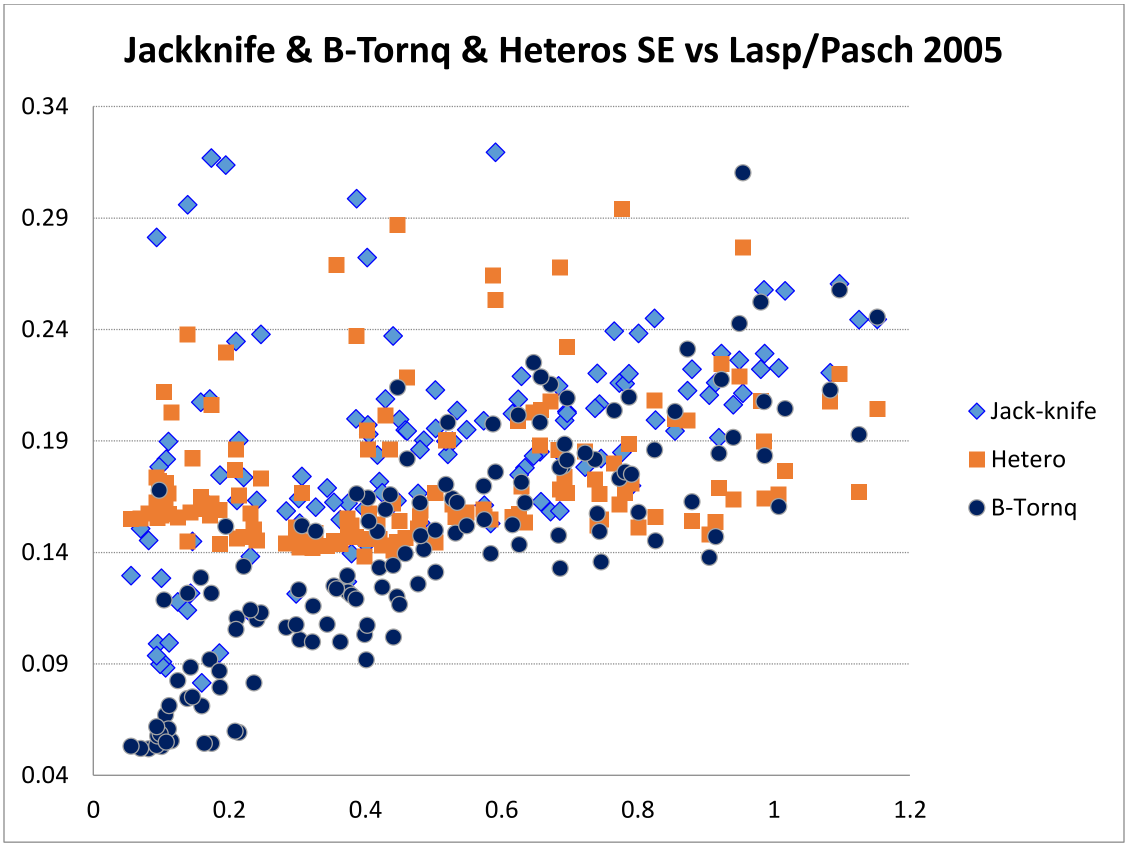
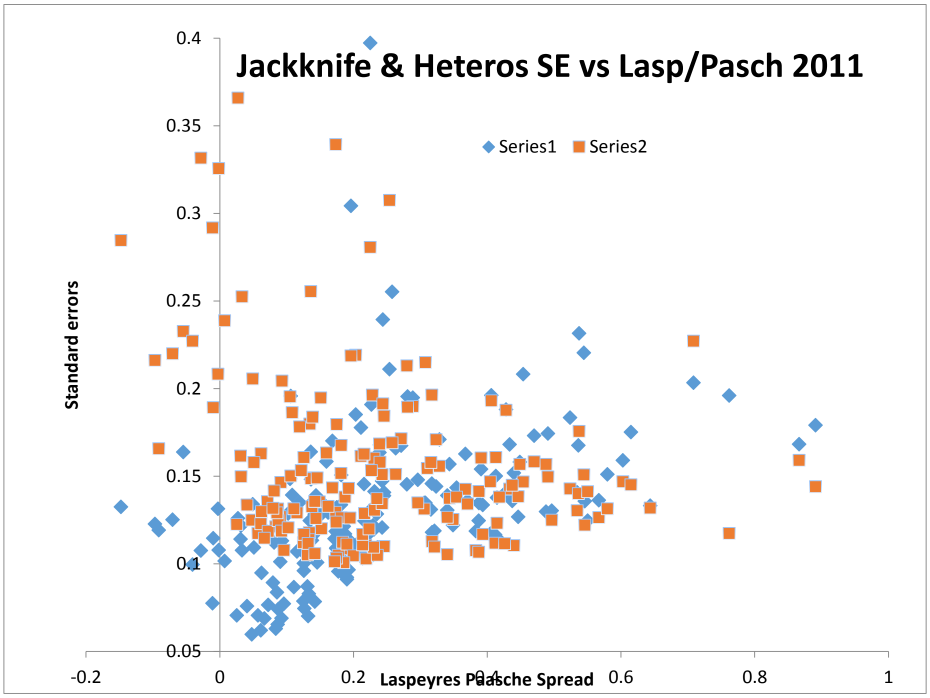


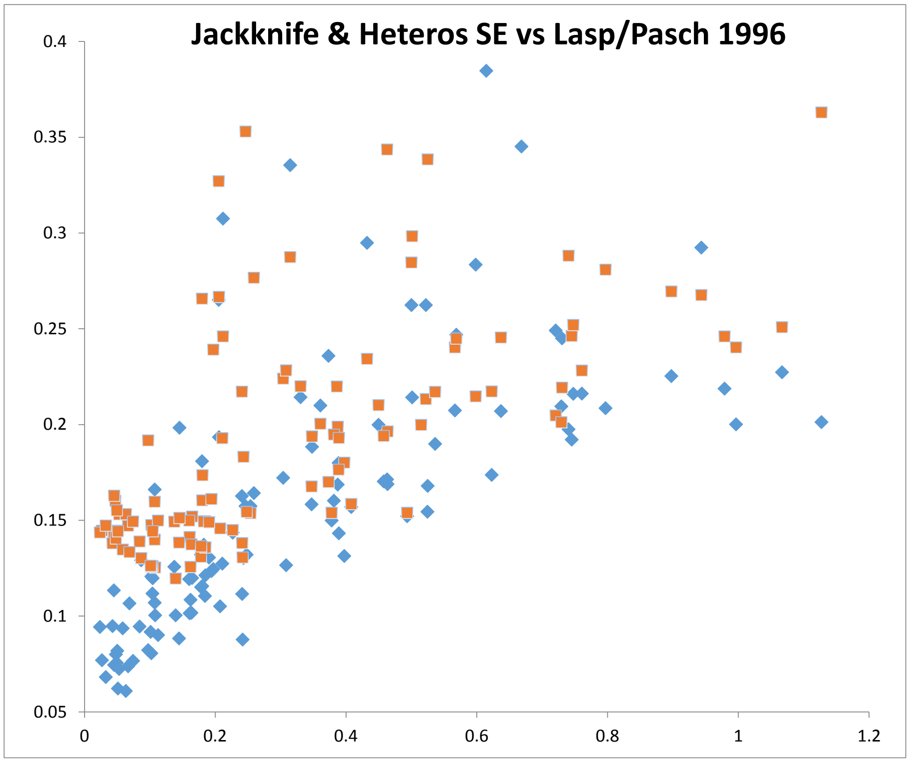
Chart: Standard errors for PPPs based on weighted CPD and Binary Tornqvist indices
2005 ICP Benchmark

The orange dots show heteroskedastic standard errors for weighted CPD and these are of the order 12% to 30%. The standard errors based on jackknife method in contrast are generally lower and in the range 7% to 32%. Standard errors for binary Tornqvist index are generally lower than the weighted CPD standard errors. These results suggest that the standard errors tend to increase when PPPs are compiled on a multilateral basis satisfying transitivity. These results are consistent with the results reported by Deaton (2012).

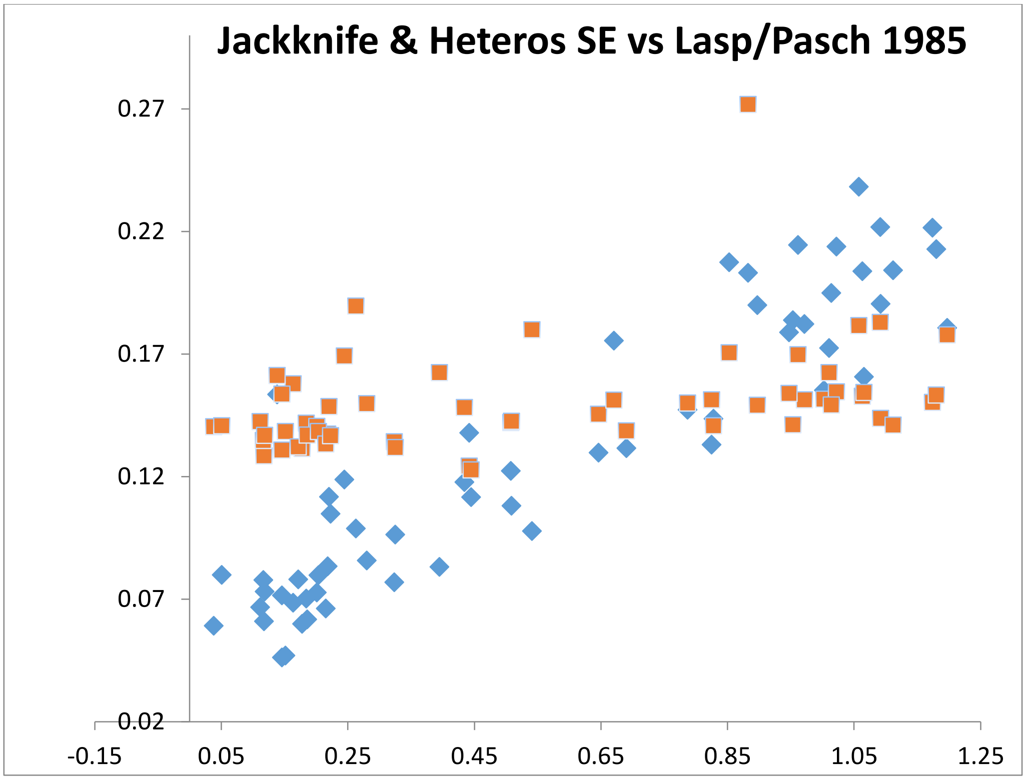
Standard errors for PPPs from different benchmarks

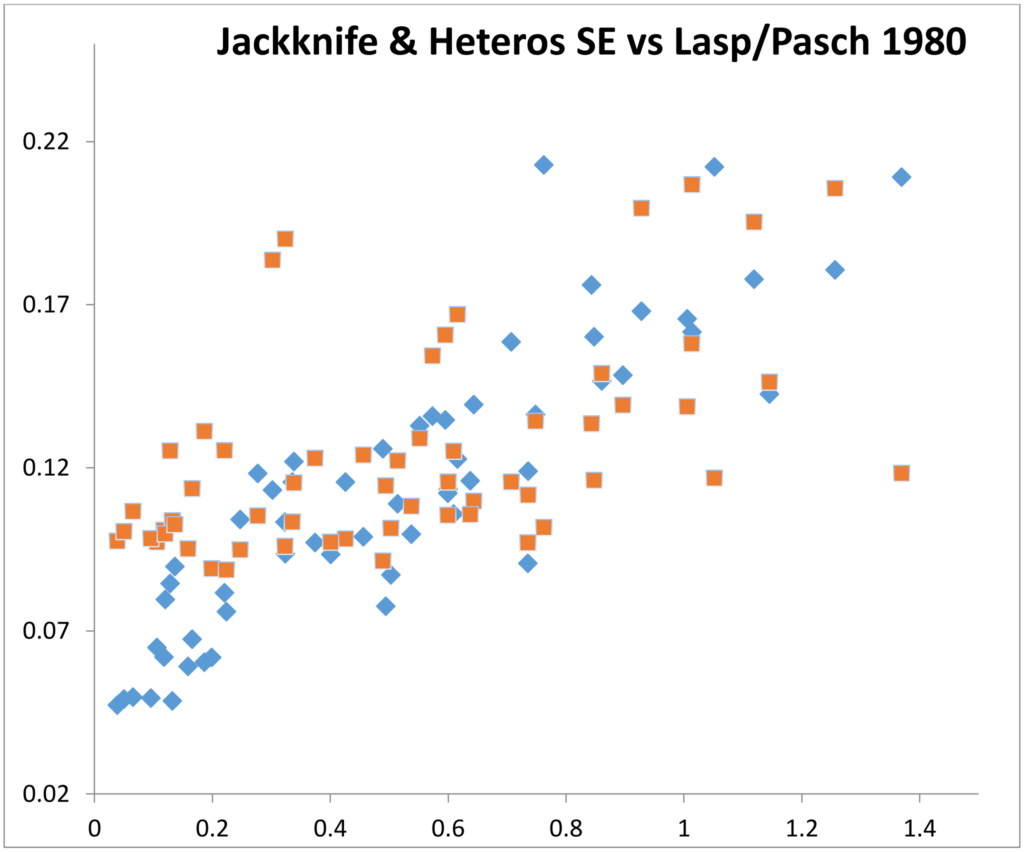
We present standard errors for all the five benchmarks considered here. These provide a temporal comparison of the patterns of standard errors.





Charts: Heteroskedastic and Jacknife Standard Errors, ICP 2011 and 1996





Charts: Heteroskedastic and Jacknife Standard Errors, ICP 1985 and 1980

These charts reveal important structure in standard errors from different methods.

- Result for 2011 show that there a number of countries with negative LPS. This is the only benchmark year for which we observe negative LPS. This may be due to the fact that 2011 has the most extensive coverage of the countries and a lot of small countries from Africa and Carabbean have been included. Further examination of these results would be useful.
- Standard errors for the 1980 and 1985 benchmarks appear to be in smaller bands than those observed for the remaining benchmark years. It is not clear whether these results are due to the fact that comparisons in those benchmark years were based on a global approach to price surveys and PPP compilation. The use of regionalized approach coupled with the use of price survey data for linking purposes may have contributed to larger standard errors. We hope to investigate this further when we attempt to compute standard errors incorporating all the sources of uncertainty.
- The jackknife standard errors are lower than the heteroskedastic standard errors in all the cases. As jackknife is not as rigorous as the bootstrap method, we propose to use bootstrap method and then investigate if significant differences persist.
- Multilateral indices based on weighted CPD have higher standard errors than the bilateral Tornqvist indices.

Finally we present standard errors for the three competing methods, the weighted CPD, Geary-Khamis and Ikle methods. The following chart shows standard errors based on data for the 2005 benchmark comparisons.

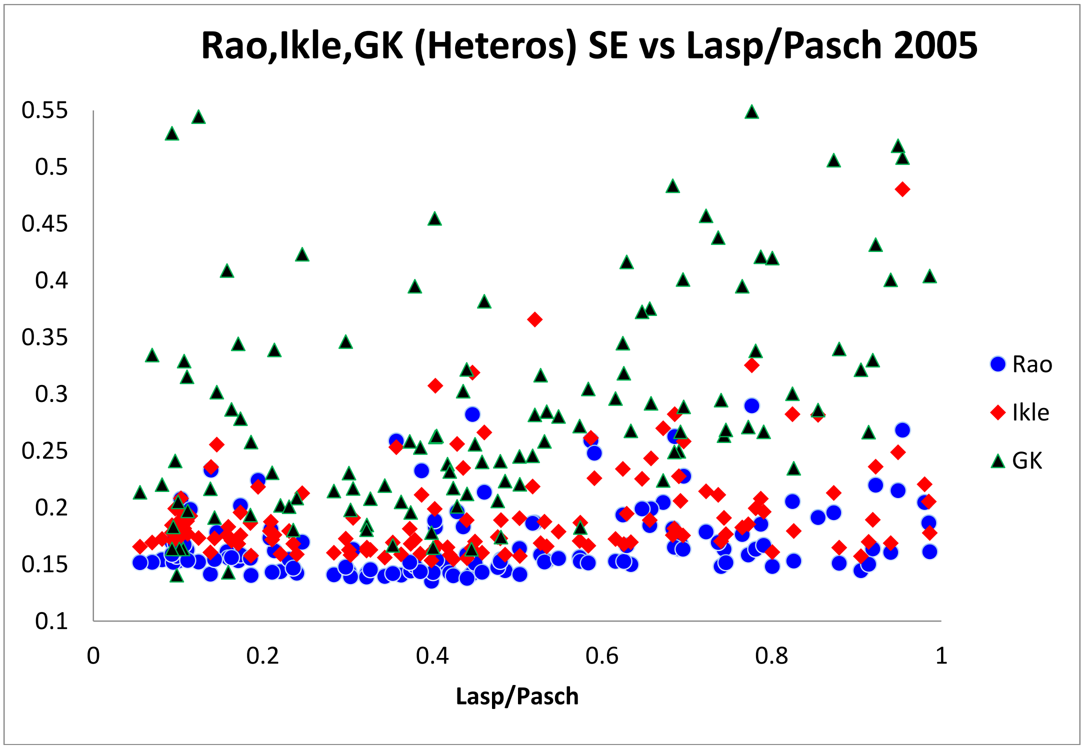


Chart: SE's for CPD, Ikle and GK methods, ICP 2005

This chart exhibits interesting pattrns. Standard errors for CPD appear to be within a band from 12 to 20% whereas the Ikle standard errors show larger spread. The GK exhibits low standard errors for countries with low LPS, most likely countries within the OECD, but very large standard errors up to 45% for countries with large LPS.

5. Conclusions

In this paper we have empirically implemented the econometric tools developed to measure standard errors associated with PPPs from the ICP. Given the current data availability, the research has focused only on PPPs compiled at the very last stage of the ICP. These PPPs are computed using PPPs for all the basic headings included in the comparison along with expenditure data from national accounts without imposing fixity. These are a sort of global comparisons which form the basis for the *country aggregation and redistribution* (CAR) method used in the 2011 round of ICP. This approach is somewhat similar to non-regionalised global comparisons in the 1980, 1985 and 1996 ICP comparisons. Thus our results provide a comparative assessment of standard errors associated with PPPs in different benchmark years. The results suggest that standard errors were lower in the 1980 and 1985 benchmarks compared to results from more recent rounds. The results also provide an indication of increase in uncertainty resulting from multilateral comparisons instead of simple binary comparisons. Binary comparisons have much lower standard errors compared to their multilateral countrparts. Of the aggregation methods, the Geary-Khamis method appears to result in a large range for the standard errors – this may be due to the use of quantity share weights which tend to exhibit larger variation than the expenditure share weights used in weighted CPD and Ikle methods. One of the reasons for choosing to compile standard errors for PPPs across different benchmarks is to see if there is any way we can identify benchmark comparisons that are superior to other benchmarks. It is not immediately clear how this can be answered even though we find the 1980 and 1985 comparisons to have smaller levels of uncertainty associated with PPPs. While our recent work has made progress in the development of a stochastic framework for the compilation of PPPs within ICP, more work is needed in refining this approach and also provide reliable measures of uncertainty associated with PPPs.

References

Deaton, A. (2012), "Calibrating Measurement Uncertainty in Purchasing Power Parity Exchange Rates", Research Program in Development Studies, mimeographed, Princeton.

Diewert, W.E. (1995), "On the Stochastic Approach to Index Numbers" Discussion Paper No. 95-31, Department of Economics, University of British Columbia.

Diewert, W.E. (2005), "Weighted Country Product Dummy Variable Regressions and Index Number Formulae", *Review of Income and Wealth*, 51, 561-70.

Diewert, W.E. (2013), "Methods of Aggregation above the Basic Heading Level within Regions", Chapter 5 in World Bank (ed) *Measuring the Real Size of the World Economy*, World Bank, Washington DC. pp. 121-167.

Hajargasht, G and D. S. Prasada Rao (2010), "Stochastic Approach to Index Numbers for Multilateral Price Comparisons and their Standard Errors", *Review of Income and Wealth*, Series 56, *Special Issue 1*, June, 2010, pp. S32-S58.

Hajargasht, G. and D.S. Prasada Rao (2013), "On Conditions for the Existence and Uniqueness of Index Numbers for International Price Comparisons", *mimeographed.*

Hill, R.J. and I. A. Syed (2012) "Accounting for Unrepresentative Products and Urban-Rural Price Differences in International Comparisons of Real Income: An Application to the Asia-Pacific Region," *Graz Economics Papers 2012-07*, University of Graz, Department of Economics.

ILO, IMF, OECD, UN ECE, Eurostat, and World Bank. 2004. *Consumer Price Index Manual: Theory and Practice*, (ed. Peter Hill). Geneva: International Labour Office.

Kravis, I.B., A. Heston and R. Summers (1982), *World Product and Income: International Comparisons of Real Gross Product*, The Johns Hopkins University Press, Baltimore, USA.

Rao, D.S. Prasada (1990), 'A System of Log-Change Index Numbers for Multilateral Comparisons', in *Comparisons of Prices and real Products in Latin America* (eds. Salazar-Carrillo and Rao),

Contributions to Economic Analysis Series, Amsterdam: North-Holland Publishing Company.
Rao, D.S. Prasada (2005). "On the Equivalence of Weighted Country–Product–Dummy (CPD) Method and the Rao-System for Multilateral Price Comparisons." *Review of Income and Wealth* (51): 571–80.
Rao, D.S. Prasada, (2013a), "The Framework for the International Comparison Program (ICP)", Chapter 1 in World Bank (ed) *Measuring the Real Size of the World Economy*, World Bank, Washington DC. pp. 13-45.
Rao, D.S. Prasada, (2013b), "Computation of Basic Heading Purchasing Power Parities (PPPs) for Comparisons within and between Regions", Chapter 4 in World Bank (ed) *Measuring the Real Size of the World Economy*, World Bank, Washington DC. pp. 93-119.
Selvanathan, E.A. and D.S. Prasada Rao (1994), *Index Numbers: A Stochastic Approach*, Ann Arbor: The University of Michigan Press.
Summers, R. (1973). "International Price Comparisons Using Incomplete Data." *Review of Income and Wealth* (19): 1–6.
Wooldridge, J.M. (2010), *Econometric Analysis of Cross Section and Panel Data*, Second Edition, MIT Press.
World Bank (2013), *Measuring the Real Size of the World Economy* (Edited book), World Bank, Washington, DC.