The Treatment of Owner Occupied Housing in the CPI and its Implications for Monetary Policy

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Abstract:

The impact of movements in house prices on the CPI depends on how owner-occupied housing (OOH) is included. Most national statistical institutes (NSIs) measure the cost of OOH using either the acquisitions or rental equivalence approaches. We argue here that the user cost approach is potentially better than either acquisitions or rental equivalence. However, the performance of the user cost method depends critically on how capital gains (actual or expected) are treated. From an axiomatic perspective we argue that a case can be made for excluding capital gains. This also makes the CPI more responsive to the housing market, which may be desirable from a monetary policy perspective. Using detailed micro data for Sydney, Australia we then compare empirically the impact of these approaches on the CPI. Our results indicate that the CPI is very sensitive to the way OOH costs are measured. In the case of Sydney with its booming housing market, however, the user cost method with capital gains excluded pushes up the CPI so much as to undermine the feasibility of this approach. A user cost approach that extrapolates expected real capital gains over a long time horizon of about 30 years therefore may be preferable. These findings have important implications for the debate over how monetary policy should respond to booms and busts in the housing market.

PRELIMINARY DRAFT

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1 Introduction

Inflation targeting has spread rapidly round the world since its introduction in New Zealand in 1990. Since the global financial crisis (GFC), however, there has been much debate over whether central banks should also respond to movements in house prices (or other asset prices) when setting monetary policy. The perspective that central banks should raise interest rates in response to a booming housing market (or stock market) is known as "leaning against the wind" (see Cecchetti 2006 and Mishkin 2011).

A prerequisite to such a debate though (in the case of the housing market) is to first consider the extent to which movements in house prices are already incorporated into the target measure of inflation. The typical inflation target – the consumer price index (CPI) – includes an imputation for the cost of owner-occupied housing (OOH). There is widespread disagreement however on how OOH should be included in the CPI. The three main approaches are user cost, rental equivalence and acquisitions. The impact of house prices on the CPI will depend on which approach is used. Furthermore, it is not just a matter of which approach is used, but also of exactly how that approach is implemented.

This paper has two main objectives. The first is to consider the conceptual arguments for each approach for including OOH in the CPI. We argue in favor of the user cost approach. Implementation of the user cost approach however encounters a number of problems. Probably the most important of these is the treatment of capital gains. From an axiomatic perspective we argue that capital gains should be excluded from user cost when it is being used as an input into the CPI. The inclusion of capital gains will impart a downward bias to the CPI, and will make it relatively unresponsive to a housing boom. By contrast, when capital gains are excluded from user cost, the CPI becomes more responsive to movements in house prices and hence an inflation targeting central bank will naturally engage in some leaning against the wind.

The second objective is to investigate the empirical sensitivity of the CPI to the treatment of OOH. We do this using detailed micro-level data for Sydney, Australia. Our source data consist of over 1 million price and rent observations from Sydney over the period 2004-2014. Applying hedonic and quantile regression methods to this data set we impute a price and rent for each dwelling in each of these 11 years. Also, we track the status of each dwelling over time, in terms of whether it is owner-occupied or rented. We then compute price and rent indexes, and impute the average rental equivalence and user cost OOH expenditure shares in the CPI. We estimate alternative CPIs based on the rental equivalence and user cost approaches and contrast them with the official CPI for Sydney computed by the Australian Bureau of Statistics (ABS) using the acquisitions approach.

We show that the way OOH is treated can have a huge impact on the responsiveness of the CPI to movements in house prices. Of particular importance is the treatment of capital gains in the user cost approach. Our results therefore demonstrate that before one can discuss how monetary policy should respond to house prices, it is first necessary to establish how OOH is included in the CPI, and hence how responsive the CPI is to movements in house prices.

2 Ways of Including OOH in the CPI

There are three main ways of including OOH in the CPI. These are the acquisitions, rental equivalence, and user cost approaches. The payments approach have also been proposed in the literature, but in our opinion it lacks sufficient theoretical foundations to warrant consideration here. The weaknesses of the payments approach are discussed in Diewert (2002, 2009). In the next three subsections we briefly discuss each of the three main approaches. In addition, we then also discuss the opportunity cost approach of Diewert, Nakamura and Nakamura (2009), which combines aspects of rental equivalence and user cost.

2.1 The acquisitions approach

The acquisitions approach is used by Australia, New Zealand, and on an experimental basis by the member states of the European Union. A decision will be taken in 2018 on whether to include OOH in the harmonized index of consumer prices (HICP), which is the official inflation target of the European Central Bank.

The acquisitions approach seeks to treat housing in the same way as consumer durables. The purchase by a consumer of a new car for example is included in its entirety in the CPI, while the purchase of a second-hand car is excluded (unless it is new to the consumer sector). Likewise, the acquisitions approach includes the purchase of a new residential dwelling in the CPI, but completely excludes the purchase of existing dwellings unless they are new to the residential sector.

An important difference however exists between cars and housing. A dwelling consists of a structure and land. Under the acquisitions approach it is typically argued that since the land has not been produced, it should be excluded from the CPI. In other words, the objective is to construct a price index and expenditure shares for new residential structures, excluding land. The expenditure shares are usually obtained from gross fixed capital formation (GFCF) in the national accounts. Australia and New

Zealand use cost indexes for residential construction building materials. By contrast, national statistical institutes (NSIs) in the European Union use price indexes for new residential housing (i.e., based on actual transaction prices and which hence include the land component).

Household expenditure on OOH under the acquisitions approach is calculated as follows:

Y_t = New dwelling purchase by owner-occupiers + Maintenance and repair of dwellings + Property rates and charges.

The average expenditure per household is obtained by dividing Y_t through by the total number of households H_t (i.e., both owner-occupiers and renters).

$$y_t = \frac{Y_t}{H_t}.$$

2.2 The rental equivalence approach

Both the rental equivalence and user cost approaches attempt to measure the expenditure on OOH services. Given that OOH services are derived from both the structure and land it follows that there is no need to try and separate land from structure in the house price index. Rental equivalence as the name suggests imputes a rental expenditure for owner-occupied dwellings. This is usually done with surveys of owner-occupiers who are asked the hypothetical question: How much do you think your dwelling would cost to rent?

Average household expenditure on OOH under rental equivalence (y_t) is the average imputed rent on OOH dwellings ($\hat{R}_t$).

$$y_t = \hat{R}_t$$

The rental equivalence price index can be obtained from the ratio of average imputed rent: $\hat{R}_{t+1}/\hat{R}_t$.

Rental equivalence is used for example by the USA, Canada, and the UK.

2.3 The user cost approach

The user cost approach tries to measure the cost of OOH services directly. For each dollar invested in OOH the user cost is usually assumed to consist of the following components (or something similar):

$$u_t = r_t + \delta_t + \omega_t + \gamma_t - \pi_t - g_t, \tag{1}$$

where:
u is per dollar user cost
r is the interest rate
δ is depreciation
ω is running and average transaction costs
γ is the risk premium
π is the expected rate of inflation
g is the expected real capital gain on housing

The formula becomes more complicated if owner-occupiers can tax deduct mortgage interest payments. This however is not the case in Australia.

Average household expenditure on OOH (y_t) under the user cost approach is calculated as follows:

$$y_t = P_t u_t,$$

where P_t is the average price of an OOH dwelling in period t.

The user cost price index is a house price index calculated over OOH dwellings. It is given by P_{t+1}/P_t.

To implement the user cost approach it is therefore necessary to compute per dollar user cost u_t and the average price of OOH dwellings P_t.

The only country that claims to use a version of user cost in its CPI is Iceland which uses a highly simplified version of it (see Gudnason and Jónsdóttir 2009, and Diewert 2009).

2.4 The opportunity cost approach

Diewert, Nakamura and Nakamura (2009) suggest setting the expenditure on OOH each period equal to the maximum of rental equivalence and user cost. The argument is that an owner has the choice between owner-occupying and renting out a dwelling. The cost incurred by the owner is therefore equal to the maximum of user cost and the amount the dwelling could be rented for. This approach presumably uses the same price index as the user cost approach.

Empirically, how the opportunity cost approach behaves depends crucially on how the user cost approach is implemented. We return this issue in section 6.

3 Disadvantages of the Acquisitions Approach

The main rationale for the acquisitions approach is that it treats OOH in the same way as consumer durables such as cars, and refrigerators. However, a house is quite different from a car or refrigerator. It consists of land and a structure. While the structure is produced the land is not. The acquisitions approach focuses exclusively on the structure, and hence completely ignores the important role played by land. The only sensible way of dealing with land in this context is to focus on the stream of services it provides. Once this approach is taken for land, it makes sense to do likewise for the structure as well.

This leads us naturally towards either the user cost or rental equivalence methods. Focusing on the stream of services provided by OOH also ensures consistency with the treatment of rental dwellings (where the focus is also on the stream of services provided).

A second weakness of the acquisitions approach is that new residential construction is a very volatile component of GDP, that rises strongly during housing booms only then to collapse when house prices start falling. Hence the expenditure weights on OOH under the acquisitions approach will tend to fluctuate very significantly over the housing cycle, thus imparting undesirable instability to the CPI. In the European context this is potentially particularly problematic given that the housing cycles of many Eurozone countries seem to be out of sync. By implication, in any given period Eurozone countries with rising house prices will have large acquisitions expenditure weights while countries with stagnant housing markets will have small acquisitions expenditure weights, thus undermining the comparability of the HICP across countries.

A third weakness is that the acquisitions approach may also be subject to systematic bias. A similar issue arises in the context of user cost. We will only briefly sketch the issue here and then consider it in more detail when we return to user cost. Our conjecture is that when house prices are rising, the level of new builds is high. Hence under the acquisitions approach, when prices are rising OOH receives a large weight in the CPI. Conversely, when prices fall, new builds fall dramatically, and hence OOH receives a much smaller weight in the CPI. It follows that if house prices rise and then return back to their original level (while everything else in the economy is constant), the CPI will end up higher than it started, thus implying an upward bias. This argument does not really apply in the case of Australia since the weights are only updated every 5-6 years. It is more applicable in the European context where weights are updated on an annual basis. In the Australian context, the problem is that the weights may be highly sensitive to the choice of base year, and whether residential construction is booming or weak at this time.

4 Disadvantages of the Rental Equivalence Approach

The services a household obtains from renting a dwelling are not the same as the services obtained by owner-occupying. As an owner-occupier one knows one can live there indefinitely. A tenant by contrast knows that he or she could be evicted as soon as the lease expires. Hence any maintenance and improvements are likely to be valued more by owner occupiers. Also, the possibility of having to move say in half a year can be a source of stress. Moving, even from one rental dwelling to another, incurs substantial transaction costs (in both time and money). Conceptually the cost of OOH that should be included in the CPI is the cost of actually owning, and not the imputed cost of renting exactly the same dwelling. Hence it is clear that the theoretically correct concept is the user cost.

Nevertheless, it could be argued that rental equivalence is easier to use, and that it provides a good approximation to the cost of owner occupying.

We take issue with both these claims. In some countries a significant part of the rental market is subject to rent control. Also, the rental market is sometimes so small that rental equivalence is not even feasible. This is an important issue in the European Union. The HICP requires that all member states use the same method to measure the costs of OOH. The share of the rental market of most eastern European countries, and some western countries such as Spain and Italy is less than 20 percent, and in addition to being small it is not representative of the overall housing stock. The rental sector tends to be focused on apartments in urban areas. More generally, even when the rental market is larger, it is not clear how accurately rents can be imputed for OOH. Hill and Syed (2016) find that owner-occupied dwellings are of systematically higher quality than rental dwellings, even when one controls for observed characteristics (such as location, number of bedrooms and land area). This implies that the expenditure weights will tend to be too low when rental equivalence is used. Also, Hill and Syed find that the magnitude of the quality difference between owner occupied and rental dwellings changes significantly over time. Hence the downward bias in the rental equivalence expenditure weights would not be constant over time. Another complication is that the price-rent ratio tends to vary depending on what part of the housing market is being considered. Bracke (2015) and Hill and Syed (2016) find that dwellings at the higher end of the market in London and Sydney respectively tend to have higher price-rent ratios, while Heston and Nakamura (2009) find that some cities (those with higher incomes) tend to have higher price-rent ratios.[1]

[1]Some reasons for this systematic effect are considered by Bracke (2013, 2015), and Hill and Syed (2016).

If instead imputed rents are obtained from a survey of owner-occupiers anecdotal evidence suggests that they may be too high either because owners are overly optimistic or because they value the particular features of their property more than the average renter (see Heston 2009).

Turning now to the OOH price index, there is ample evidence that rent indexes and price indexes can follow very different paths over the short to medium term. Rents tend to change much more slowly than prices. In general during a housing boom, house prices rise first, and then rents gradually adjust upwards (see for example Hill and Syed 2016). The reverse happens during a housing bust.

Rental equivalence has indeed been implicated in contributing to the global financial crisis (GFC), in that rental prices hardly rose in the US during the housing boom that ended in 2006. As a result, the US CPI was largely unaffected by the housing boom. By contrast, if OOH entered into the CPI in a way that made the CPI more responsive to a housing boom, this could have pushed the Federal Reserve to start raising interest rates sooner and more aggressively. This in turn could have prevented the housing market from reaching as high a level by 2006. A smaller boom would presumably have led to a smaller subsequent bust.

5 The Case for Excluding Real Capital Gains from the User Cost of OOH

Given the problems with acquisitions and rental equivalence discussed above, why then is the user cost approach not currently used by any NSIs? The reason is that attempts to measure the user cost of OOH encounter a number of difficulties, some of which have not yet been satisfactorily resolved. Each of the components of the per dollar user cost u_t is problematic in its own way. Most problematic of all are real capital gains. Expected real capital gains cannot be observed directly, and a few studies (e.g., Verbrugge 2008, Garner and Verbrugge 2009, and Hill and Syed 2016) have shown that the estimated user cost can be highly sensitive to the choice of time horizon for expectation formation when computing the expected real capital gain. We return to this issue shortly.

The user cost equilibrium condition states that in equilibrium a household should be indifferent between owner-occupying and renting. Hence the cost of owner-occupying (the user cost) should equal the cost of renting. This yields the following equation:

$$u_t P_t = R_t,$$

which can be rearranged as follows:

$$\frac{P_t}{R_t} = \frac{1}{u_t}.$$

This approach therefore provides an estimate of the equilibrium price-rent ratio. Departures from equilibrium can therefore be detected by comparing the actual and equilibrium price-rent ratios (see for example Himmelberg, Mayer, and Sinai 2005, and Hill and Syed 2016).

In this context it makes sense to include expected real capital gains in the per dollar user cost u_t, since households will account for expectations of future house price movements when deciding whether to buy or rent.

However, when the objective is to measure the cost of OOH in the CPI the case for including expected real capital gains is less clear. To see why, consider the following example. Suppose over the interval $t = 1$ to $t = T$, real house prices rise (or fall) and then return to their original level. Suppose further that over this same interval that prices and expenditure of all other components in the CPI remain constant. A standard price index axiom is that when all prices in period T are the same as in period 1, then the price index in period T should be the same as in period 1.

This axiom is not satisfied when real capital gains are included in the user cost. More specifically, when real capital gains (actual or expected) are included in OOH costs, the CPI will tend to have a downward bias. A numerical example is provided in the Appendix in Table A1 which considers a situation where real house prices rise by four percent a year for four years, after which they fall by four per cent a year for four years. The initial price of a house in period 1 is \$200 000. Normalizing the price index to 1 in period 1, house prices peak in period 5 at 1.16986, before falling back to 1 by period 9. It is assumed that the prices of all other components of the CPI (except OOH) remain constant, and that the total non-OOH expenditure remains fixed at \$90 000. Finally, it is also assumed that all components of per dollar user cost except capital gains remain constant and that these components sum to 0.05. This numerical example is constructed to make sure that the user cost never goes negative irrespective of how capital gains are treated.

In what follows we consider three ways of dealing with real capital gains.

(i) Include ex post real capital gains. The user cost of OOH can then be written as follows:

$$P_t u_t = P_t x_t - P_t \left(\frac{I_{t+1} - I_t}{I_t} \right),$$

where P_t denotes the price of the average dwelling in period t, u_t is the per dollar user cost, x_t is all components of per dollar user cost except for real capital gains, and I_t

and I_{t+1} are the level of the house price index in periods t and $t+1$, respectively, in constant dollars. The term $(I_{t+1} - I_t)/I_t$ therefore represents the per dollar real capital gain.

(ii) Exclude real capital gains. The user cost of OOH can then be written as follows:

$$P_t u_t = P_t x_t.$$

(iii) Include expected real capital gains. The user cost of OOH can then be written as follows:

$$P_t u_t = P_t x_t - P_t \left(\frac{E_t I_{t+1} - I_t}{I_t} \right),$$

where $E_t I_{t+1}$ is the level of the house price index in period $t+1$ expected at the beginning of period t (again in constant dollars). The question now is how do households compute $E_t I_{t+1}$? We assume expectations are computed as follows:

$$\left(\frac{E_t I_{t+1} - I_t}{I_t} \right) = \left(\frac{I_t - I_{t-k}}{I_{t-k}} \right)^{1/k},$$

which can be rearranged as:

$$E_t I_{t+1} = I_t \left(\frac{I_t - I_{t-k}}{I_{t-k}} \right)^{1/k} + I_t.$$

It is assumed therefore that households compute the compounded rate of return over the last k periods, and then expect this rate of return in period t. In Table A1, we try setting k equal to $1, 2, \ldots, 8$.

We then compute chained Törnqvist, Paasche, Laspeyres, and Fisher price indexes over the first five periods and the full nine periods. These results are summarized for the case of Fisher in Table 1.

From an axiomatic perspective, the chained Fisher price index calculated over the full nine periods should equal 1. This is the result obtained when either real capital gains are excluded or when expectations are extrapolated over the previous eight years. When expected real capital gains are extrapolated over a shorter time horizon or if they are included ex post, then the chained Fisher price index ends up below 1, implying a downward bias in the CPI. The size of this bias gets bigger as the time horizon over which expectations are extrapolated gets smaller. The biggest bias occurs when real capital gains are included ex post.

These findings lead us to the conclusion that the inclusion of expected real capital gains or ex post real capital gains in the user cost of OOH imparts a downward bias to the CPI. The magnitude of this bias decreases as the time horizon over which expectations are extrapolated rises. The intuition for this result is that when real house prices

Table 1: Hypothetical Example of the Impact of the Treatment of Real Capital Gains on the CPI

Treatment of Real Capital Gains	Level of the CPI in Given Period CPI_1	CPI_5	CPI_9
Excluded	1.0000	1.0170	1.0000
Ex post	1.0000	1.0068	0.9808
Expected - 1 year	1.0000	1.0053	0.9810
Expected - 2 year	1.0000	1.0069	0.9854
Expected - 3 year	1.0000	1.0087	0.9898
Expected - 4 year	1.0000	1.0104	0.9942
Expected - 5 year	1.0000	1.0118	0.9974
Expected - 6 year	1.0000	1.0127	0.9991
Expected - 7 year	1.0000	1.0133	0.9998
Expected - 8 year	1.0000	1.0138	1.0000

Note: In this example, house prices rise from period 1 to period 5 and then fall back to their original value by period 9.

are rising, the inclusion of expected real capital gains acts to lower the expenditure share of OOH. Conversely, when real house prices are falling, the inclusion of expected real capital gains acts to raise the expenditure share of OOH. These effects combine to generate a downward bias. The extent of this bias depends on the length of the time horizon over which expectations are formed relative to the length of the cycle in house prices. Holding the length of the price cycle constant, as the expectation formation horizon gets longer the magnitude of the bias decreases. It disappears completely once the expectation formation time horizon is longer than the price cycle. However, if we turn this argument around, for a given expectation formation time horizon the inclusion of expected real capital gains in user cost will generate a downward bias in the presence of a long enough price cycle. It should be emphasized that similar results are obtained if house prices fall and then recover again to their original levels.

Before drawing conclusions regarding the efficacy of excluding capital gains it is important to check how this version of user cost performs on real data. This is the objective of the next section. Empirically we find that excluding capital gains from the user cost is also problematic.

6 Empirical Strategy

6.1 The data set

The data set used in this paper covers the period 2004 to 2014 for Australia's largest city, Sydney. The data set was purchased from Australian Property Monitors (APM).[2] It consists of 358 738 actual transaction prices (measured in Australian dollars) for houses sold over this period. We also have corresponding data for apartments, but have not used these data thus far in our calculations. We intend to include apartments in the next draft of the paper. The data set also includes 310 314 asking rents (the rents are quoted on a weekly basis) for houses (again data for apartments are available and will be included in the next draft).

For each price and rent observation we have information on the following characteristics: exact date of sale (or posting of the asking rent), land area, number of bedrooms, number of bathrooms, exact address, postcode identifier, and exact longitude and latitude. Houses with land areas greater than 5 000 square meters, or more than 6 bedrooms or bathrooms were deleted (since a significant number of these outliers contain data entry errors). The longitudes lie within [150.60,151.35] and the latitudes within [-34.20,-33,40]. Summary statistics are provided in Table 2.

In the sales data set 76.4% and in the rental data set 98.6% of all observations are completely observed, i.e., all characteristics are available. Some properties appear more than once in the data sets as they are sold and rented or sold / rented multiple times and it is possible to use these repeated observations to reconstruct some incomplete observations.[3] The reconstruction algorithm consists of three steps: Missing observations are, if possible, refilled separately first within the sales and second within the rental data set. In the third step, the sales and rental observations are pooled and the reconstruction algorithm is applied on the combined sample. A missing characteristic is refilled using information of a completely observed observation of the same property, subject to certain constraints. First, if there are several completely observed values, the algorithm checks whether the observed values differ and refills only if the same value is observed all the times. For instance, if a dwelling appears three times in the data set and a characteristic is completely observed two out of three times, refilling is permitted only when the same value is observed for both complete observations. Second, if a dwelling appears twice within a period of six month, this may be a signal for renova-

[2]APM provides real estate related research service and data for the Australian market. See http://apm.com.au in order obtain access to their data sets.

[3]The algorithm applied in this paper is similar as in Waltl (2015, 2016) but extended to cross-refilling between sales and rental observations.

Table 2: Summary statistics.

	Rent	Price		Land area in m^2	
Minimum	20	56,500	Minimum	100	100
1st quartile	360	440,000	1st quartile	404	465
Median	470	640,000	Median	572	589
Mean	565	820,065	Mean	626	629
3rd quartile	650	935,000	3rd quartile	713	720
Maximum	10,000	32,000,000	Maximum	4,999	4,998
	No of bedrooms in %			**No of bathrooms** in %	
1	2.38	0.30	1	60.44	44.06
2	16.13	8.65	2	31.20	39.75
3	51.62	45.70	3	7.21	13.59
4	24.52	34.50	4	0.95	2.09
5	4.68	9.32	5	0.16	0.41
6	0.67	1.53	6	0.04	0.10
	No observations				
All	312,239	421,284			
Complete	310,314	358,738			
in % of all	99.4%	85.2%			

Note: The table reports summary statistics for rental (left columns) and sales data (right columns). In the previous section *all* refers to all observations and *complete* to all fully observed or fully reconstructed observations.

tions. Repeat-sales indexes usually discard such observations from their calculations for this very reason. The reconstruction is very successful: The share of complete observations in the sales data set is increased from 76.4% to 85.2% and in the rental data set from 98.6% to 99.4%. The empirical analysis is then performed on all completely observed or successfully refilled observations.

6.2 Imputing prices and rents for individual houses in Sydney

Step 1: Estimating quantile regression models. Separately for each year within 2004 and 2014, two types of quantile regression models are estimated: one based on rental observations and one based on sold dwellings. All models are of the structure

$$\log p \sim \beta_0 + \beta_1 \log(area) + \sum_{j=2}^{4} \beta_j^{bed} \mathbb{1}_{\{j\}}(bed) + \sum_{j=2}^{4} \beta_j^{bath} \mathbb{1}_{\{j\}}(bath) + f(long, lat), \quad (2)$$

where p denotes either the transaction price or the observed rent. Due to few observations with five or six bed- or bathrooms, the four, five and six rooms are merged to a single category. $f(long, lat)$ denotes a smoothly estimated geographical spline measuring locational effects on a grid spanned by longitudes and latitudes. Models are estimated for nine different quantile levels $\vartheta \in \{0.1, 0.2, \ldots, 0.9\}$. All in all, there are hence $11 \times 2 \times 9 = 198$ models. In the following, we refer to a model for rental (sales) observations in year t and quantile level ϑ by $mod(R, t, \vartheta)$ ($mod(S, t, \vartheta)$).

Step 2: Allocating dwellings to segments. Each observation is allocated to a unique price segment indicating the position in the price or rent distribution. For instance, let h_{it}^R be a dwelling that was rented in year t. The observed rent is denoted by p_{it}^R and its set of characteristics by x_{it}^R. To assign an appropriate segment, we impute rents based on the characteristics x_{it}^R using models for period t and *all* quantile levels yielding nine different prices

$$\begin{aligned} mod(R, t, 0.1) &\longrightarrow \hat{p}_{it}^R(\vartheta = 0.1), \\ mod(R, t, 0.2) &\longrightarrow \hat{p}_{it}^R(\vartheta = 0.2), \\ &\vdots \\ mod(R, t, 0.9) &\longrightarrow \hat{p}_{it}^R(\vartheta = 0.9). \end{aligned}$$

Imputed rents are compared to the observed rent. The model yielding an imputed rent closest to the observed rent is the most appropriate for a particular observation. Hence, observation h_{it}^r is assigned to price segment ϑ^* given by

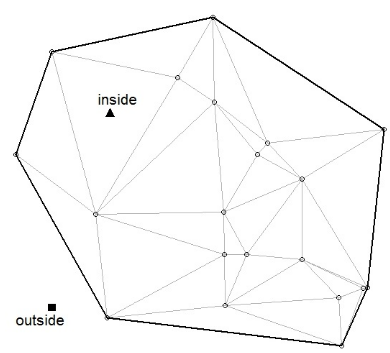
$$\vartheta^* = \arg\min_{\vartheta} \left| \hat{p}_{it}^R(\vartheta) - p_{it}^R \right| .$$

The segment is treated like an additional characteristic of each observation indicated by $h_{it}^R(\vartheta^*)$. See Davino and Vistocco (2008).

Step 3: Imputing prices and rents. For each observation appearing at least once in the data set, a rent and a price is imputed for each year.[4] For instance, for observation $h_{it}^R(\vartheta^*)$ a rent and a price for period s is obtained by evaluating model $mod(R, s, \vartheta^*)$ and $mod(S, s, \vartheta^*)$ for the set of characteristics x_{it}^R.
Implicitly we assume that segments are comparable between rented and sold houses in the sense that a house that belongs to a top segment in the sales distribution would also belong to a top segment in the rents distribution and vice versa.

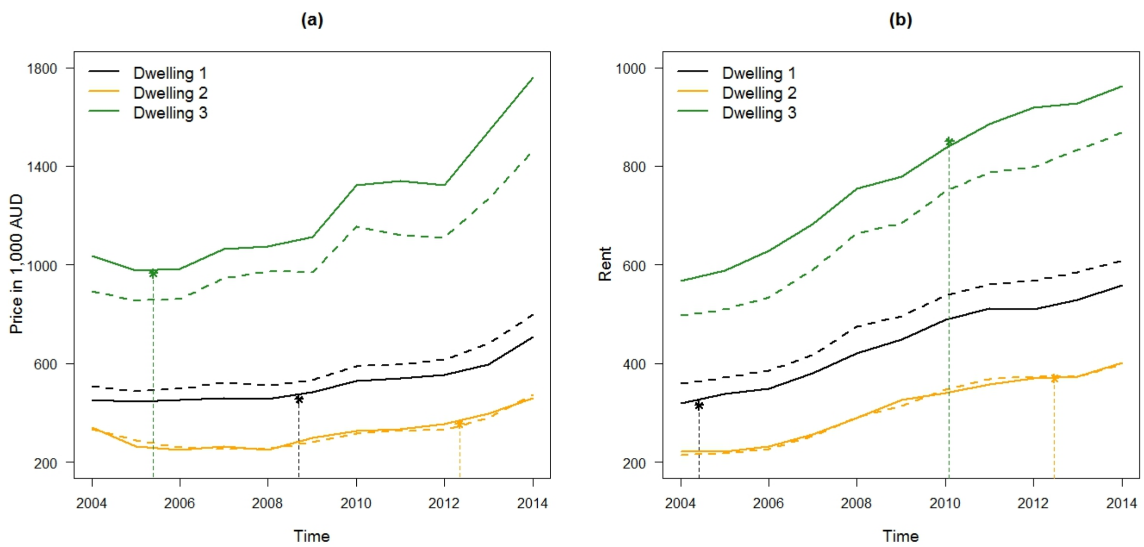
The main advantage of using quantile regression models to impute prices and rents is that observed prices are much better reproduced. If one would use a linear model (or a generalized linear model), evaluating the model for a specific set of characteristics x would yield an estimate of the *conditional mean* price, $\widehat{\mathbb{E}[\log p|x]}$. Imputed prices and rents are hence much stronger clustered around the mean than they would be in reality. Quantile regression by estimating conditional quantile prices $\widehat{Q_\vartheta[\log p|x]}$ reconstructs observed price and rent distributions much more realistically. Figure 1 shows imputed prices and rents together with their observed counterparts for three selected dwellings. Observed prices match very well with imputations from conditional quantile models whereas imputations from conditional mean models[5] perform worse. Dwelling 1 in Figure 1 was assigned to segment 2, i.e., a low price segment. The conditional mean model as expected overestimates prices and rents. Dwelling 2 is assigned to segment 5, the median segment, and in this case the conditional mean model predicts prices and rents well. Dwelling 3 was assigned to a very high price segment, segment 8. The conditional mean model hence underestimates prices and rents.

[4] The support of a particular spline $f(long, lat)$ is the convex hull of all locational coordinates of dwellings used to estimate the model. Locational effects are obtained for each triangle created from the coordinates using a Delaunay triangulation (see Hansen et al., 1998, and Koenker and Mizera, 2004). It is therefore not possible to directly impute a locational effect for coordinates falling outside the convex hull (see illustration on the right). One could include additional dummy vertices into the Delauny triangulation to increase the support, however this would lead to extrapolation of locational effects. We refrain here from extrapolation and therefore exclude observations that fall outside the convex hull in a least one model which reduces the sample size by 0.2%.



[5] I use penalized least squares to estimate the specification (2) separately for each year. The locational spline is based on *thin plate regression splines* (see Hill and Scholz 2014). The predicted prices and rents from these models are denoted by $\tilde{p}_i^S$ and $\tilde{p}_i^R$, respectively.

Figure 1: Examples of imputed prices and rents.



Note: The figure plots the temporal development of imputed prices, panel (a), and imputed weekly rents, panel (b), for three dwellings that were sold and rented some time within the period of observation. The solid lines depict imputed values from conditional quantile models, the dashed lines imputed values from conditional mean models and the stars indicate observed prices and rents. Dwelling 1 is located in the suburban region *Penrith-Windsor*, has four bedrooms and two bathrooms, a land area of $550m^2$ and was assigned to segment 2. Dwelling 2 is located in the metropolitan region *Fairfield-Liverpool*, has three bedrooms and one bathroom, a land area of $612m^2$ and was assigned to segment 5. Dwelling 3 is located in the inner-city region *Inner West*, has three bedrooms and two bathrooms, a land area of $491m^2$ and was assigned to segment 8. Sales prices are in 1,000 AUD.

Average absolute deviations over all observations are very small:

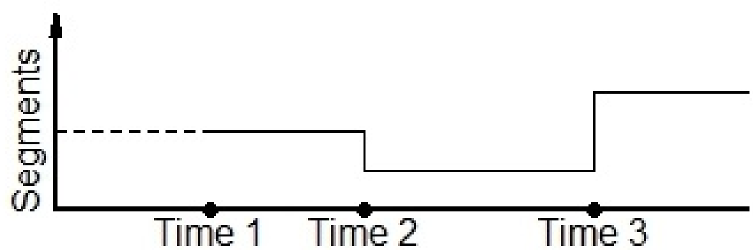
$$\frac{1}{n_R}\sum_{i=1}^{n_R}\left|\frac{\hat{p}_i^R - p_i^R}{p_i^R}\right| = 3.0\% \quad \text{and} \quad \frac{1}{n_S}\sum_{i=1}^{n_S}\left|\frac{\hat{p}_i^S - p_i^S}{p_i^S}\right| = 3.3\%,$$

where n_R and n_S denotes the number of rental and sales observations. The success of reconstructing observed prices is remarkable. When using a conditional mean model instead of conditional quantile models, average absolute prediction errors are much higher:

$$\frac{1}{n_R}\sum_{i=1}^{n_R}\left|\frac{\tilde{p}_i^R - p_i^R}{p_i^R}\right| = 14.1\% \quad \text{and} \quad \frac{1}{n_S}\sum_{i=1}^{n_S}\left|\frac{\tilde{p}_i^S - p_i^S}{p_i^S}\right| = 13.9\%.$$

Step 4: Adjusting imputations for dwellings appearing multiple times in the data set. There are many dwellings that appear more than once in the data set either as rental or sales observations (52.1% are unique observations, 26.0% appear twice and 21.9% at least three times). It happens regularly that a dwelling is not assigned to the same price segment all the times it appears in the data set. Reasons for changes include renovation, depreciation of the structure or changes in locational amenities.[6] Figure 2 illustrates a possible path: The dwelling appears first in the data set at time 1 and is at that point in time assigned to a medium segment. The structure depreciates over time such that it is assigned to a low price segment when it re-appears at time 2. The dwelling undergoes renovation and appears on the market again at time 3 and is then assigned to a very high segment. To obtain unique imputed prices and rents per year, we allow changes in the allocation to segments and use the respective imputations. For the illustrated path in Figure 2 this implies that the dwelling is assigned to the medium segment in the time interval [2004, time 2), to the low segment in [time 2, time 3) and to the high segment in [time 3, 2014].

Figure 2: Illustration of temporal changes in the segment allocation.



Note:

Step 5: Identification of owner occupied and rented houses. Generally, we assume that houses sold are owner occupied and houses rented are not. The allocation of a specific dwelling may – similar as in step 4 – change over time. If a house was sold at time 1, rented at time 2 and again sold at time 3, we allocate the dwelling to the OOH sample in [2004, time 2) and [time 3, 2014]. In the interval [time 2, time 3) it is assigned to the rental sample.

[6]Of course, measurement errors as well as errors resulting from differences between segments according to the price and rent distribution may also lead to changes in the segment allocation.

6.3 Average rents and prices per quarter

Table 3 reports median and mean sales and rental prices for the OOH and rent sample obtained from imputations based on conditional quantile models. Mean prices and rents are consistently higher than median prices and rents as the price distributions are right-skewed.

Table 3: Average prices and rents obtained from imputations.

	OOH			Rents		
Year	Median	Mean	Obs.	Median	Mean	Obs.
2004	572,563	698,394	243,653	331.4	401.3	117,316
2005	549,602	684,980	242,672	340.9	413.5	118,297
2006	548,346	695,894	241,671	356.1	431.6	119,298
2007	568,280	741,572	241,002	386.8	469.1	119,967
2008	573,520	739,513	239,376	440.0	522.8	121,593
2009	600,923	762,735	241,823	457.1	533.9	119,146
2010	675,589	841,853	238,988	492.6	577.3	121,981
2011	684,436	832,252	235,791	519.2	606.9	125,178
2012	697,481	832,452	232,825	529.3	616.1	128,144
2013	771,807	925,139	230,199	542.3	629.9	130,770
2014	912,388	1,074,373	226,228	557.2	649.4	134,741

Note: The table reports the median and mean sales price for OOH and the median and mean rent per year. Results are obtained from imputations based on conditional quantile models. Additionally, the number of observations per year is reported. Numbers vary slightly as the type of a particular dwelling – OOH or rent – may change over time.

6.4 Construction of hedonic price indexes and rent indexes

We construct rental and sales prices indexes using a Törnqvist hedonic imputation approach. The formula is illustrated below for the case of the price index. The price index for multiple periods is obtained by chaining the indexes between adjacent periods.

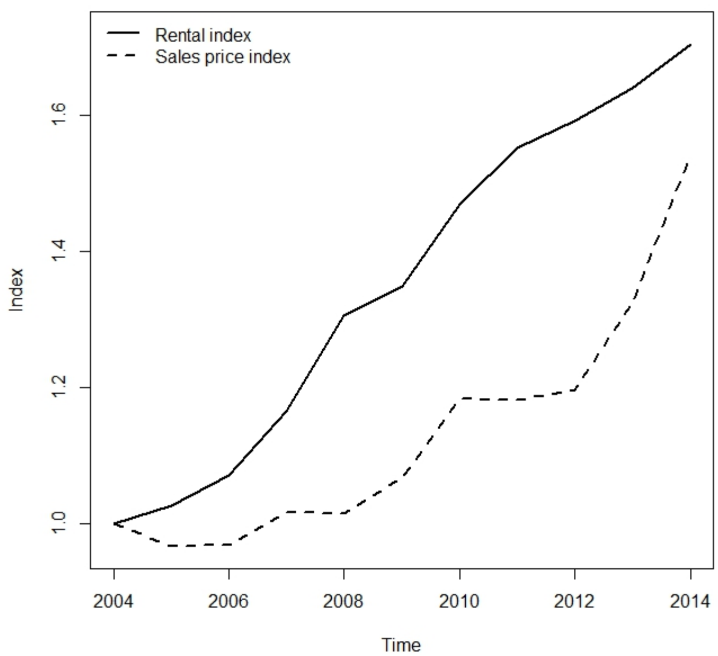
$$\text{Paasche} - \text{Type Imputation}: \; P^{PI}_{t,t+1} = \prod_{h=1}^{H_{t+1}} \left[\left(\frac{\hat{p}_{t+1,h}}{\hat{p}_{t,h}(z_{t+1,h})} \right)^{1/H_{t+1}} \right] \tag{3}$$

$$\text{Laspeyres} - \text{Type Imputation}: \; P^{LI}_{t,t+1} = \prod_{h=1}^{H_t} \left[\left(\frac{\hat{p}_{t+1,h}(z_{t,h})}{\hat{p}_{t,h}} \right)^{1/H_t} \right] \tag{4}$$

$$\text{Törnqvist Imputation}: \; P^{TI}_{t,t+1} = \sqrt{P^{PI}_{t,t+1} \times P^{LI}_{t,t+1}} \tag{5}$$

We do not distinguish between OOH and rental houses here. All houses sold enter the sales price index and all houses rented the rental index. Double imputation is used to address a potential omitted variables bias. Imputations are obtained from quantile regression models as described in the previous section.

Figure 3: Rental and sales prices indexes.



Year	Rent	Price
2004	1.000	1.000
2005	1.027	0.967
2006	1.071	0.969
2007	1.165	1.016
2008	1.305	1.015
2009	1.349	1.068
2010	1.470	1.184
2011	1.553	1.183
2012	1.591	1.197
2013	1.640	1.324
2014	1.705	1.542

Note: Results are based on the geometric Törnqvist index formula. We use imputed prices and rents from conditional quantile models.

6.5 Estimating the components of the user cost of OOH

Here we draw on Hill and Syed (20106) when computing the components of user cost for Sydney. We set r_t as the 10-year interest rate on Australian government bonds (Source:

Reserve Bank of Australia). The bond rate ranged between a minimum value of 2.89 percent in 2012 and a maximum value of 6.59 percent in 2008.[7]

Structures depreciate while land does not. Hence the appropriate depreciate rate should depend both on the age of the structure and on the share of the structure in the total value of the dwelling. This implies that every dwelling will have its own unique depreciation rate. In the context of the CPI, the important thing though is to get the average about right. We set depreciation $\delta = 1.1$ percent. This is the depreciation rate estimated by Stapledon (2007) for Sydney and used by Fox and Tulip (2014).

We set the running and average transaction costs $\omega_t = 1.9$ percent. We again follow Fox and Tulip (2014) who estimate running costs in the Australian context of 1.2 percent (see their Table A1, p. 29).[8] The main components of transaction costs are stamp duty and real estate agent commissions. Average transaction costs are obtained by amortizing the total amount over a ten year period. Again these estimates are obtained from Table A1 in Fox and Tulip (2014). Fox and Tulip estimate average transaction costs to equal 0.7 percent. Combining these components yields a total of 1.9 percent.

We set the risk premium γ_t to zero. In the housing literature, Flavin and Yamashita (2002) estimate the risk premium to be 2 percent. This is also the estimate used by Himmelberg et al. (2005). Flavin and Yamashita's estimate, however, focuses on the risk associated with owning a house without considering the risk of renting. Sinai and Souleles (2005) find that the volatility of real rents is about half that of real house prices in the US. Given that γ_t is supposed to measure the differential between the risks of owning and renting, we think Flavin and Yamashita's estimate is too high. Hill and Syed (2016) set the risk premium to 1 percent. We exclude the risk premium here both because there is so much uncertainty over its value and because we think the user cost of OOH in the CPI should focus on directly incurred costs of ownership. It should be noted that Fox and Tulip (2014) also exclude the risk premium. Their reasoning is similar to ours.

[7]Here we follow Himmelberg, et al. (2005) who, in a US context, use the 10-year Treasuries interest rate. Alternatively, we could have used the mortgage interest rate. Whether this is appropriate depends on the loan-to-value ratio of purchasers. The relevant interest rate for a purchaser with a 100 percent loan-to-value ratio is the mortgage interest rate r^M, while for a purchaser with a 0 percent loan-to-value ratio it is the risk-free 1-year rate r^{rf}. According to Green and Wachter (2005, Table 2), the average loan-to-value ratio in Australia is 63 percent. Assuming this figure remains constant, we could calculate r as follows: $r^* = 0.37 \times (1-t) \times r^{rf} + 0.63 \times r^M$, where t is the marginal tax rate. Setting $t = 0.33$, the choice between using the 10-year government bond rate or r^* should not have that much impact on our results.

[8]Fox and Tulip include repair costs as part of running costs. In our setup, we exclude repair costs from running costs since they are included in gross depreciation.

> In contrast to other studies, we do not explicitly include a 'risk premium'. It is not clear that the financial risks of home ownership should outweigh renters' insecurity of tenure and uncertainty regarding future rents, or how this might be quantified. Moreover, aversion to risk is just one of many unobserved subjective factors that may influence the decision to buy a house. We prefer to calculate expected returns and allow households to compare these with their own weighting of subjective factors. (Fox and Tulip, 2014, pages 9-10)

The expected rate of inflation π_t is assumed to be 2.5 percent. This is very close to the average rate of inflation for Sydney over the 2004-2014 period which equalled 2.6 percent. It is also the middle of the Reserve Bank of Australia's inflation target (which is 2-3 percent).

g is the expected real capital gain. The expected real capital gain in year t is assumed to equal the geometric average of real capital gain over the preceding x years. We consider three different values of x (i.e., 10, 20 and 30 years). More precisely, the expected real capital gain in year t is calculated as follows:

$$\text{Expected real capital gain}_t = \left(\frac{EHPI_t / CPI_t}{EHPI_{t-x} / CPI_{t-x}} \right)^{1/x}.$$

Here $EHPI_t$ is the level of the Established House Price Index and CPI_t is the level of the consumer price index for Sydney in year t. Both the EHPI and CPI are computed by the Australian Bureau of Statistics (ABS).[9]

Annualized expected real capital gains based on extrapolating over 10, 20 and 30 year horizons are shown in Table 4. Diewert, citing evidence on the length of housing booms and busts from Girouard et al. (2006), argues that a longer time horizon (e.g., 20 years) is more plausible in terms of how market participants form their expectations (see also Bracke 2013).

Also, shown in Table 4 are the implied values of the per dollar user cost u_t. It can be seen from Table 4 that the assumed time horizon of past performance over which expected real capital gains are calculated plays a pivotal role in determining the value of u_t. The volatility of per dollar user cost when expected capital gains are extrapolated

[9]The Established House Price Index (EHPI) is computed using the stratified-median approach, which may fail to fully adjust for quality changes over time. Given the EHPI is probably the most widely followed house price index for Sydney, it nevertheless is a useful benchmark for describing expectations of capital gains. The EHPI only goes back to 1986. To obtain prices back to 1984 or 1974 (for the cases where x=20 or 30), the EHPI was spliced together with an index calculated by Abelson and Chung (2005).

Table 4: Expected Real Capital Gains and Per Dollar User Costs: Sydney 2004-2014

	g(0)	g(10)	g(20)	g(30)	r	u(0)	u(10)	u(20)	u(30)
2004	0.0000	0.0660	0.0501	0.0331	0.0585	0.0635	0.0000	0.0133	0.0303
2005	0.0000	0.0591	0.0476	0.0335	0.0514	0.0564	0.0000	0.0088	0.0229
2006	0.0000	0.0555	0.0436	0.0328	0.0574	0.0624	0.0069	0.0188	0.0295
2007	0.0000	0.0533	0.0449	0.0345	0.0620	0.0670	0.0138	0.0221	0.0326
2008	0.0000	0.0481	0.0415	0.0354	0.0659	0.0709	0.0228	0.0293	0.0355
2009	0.0000	0.0338	0.0184	0.0301	0.0556	0.0606	0.0268	0.0422	0.0305
2010	0.0000	0.0393	0.0312	0.0293	0.0533	0.0583	0.0190	0.0271	0.0290
2011	0.0000	0.0400	0.0327	0.0262	0.0516	0.0566	0.0166	0.0239	0.0304
2012	0.0000	0.0217	0.0300	0.0274	0.0300	0.0350	0.0132	0.0050	0.0075
2013	0.0000	0.0071	0.0305	0.0312	0.0354	0.0404	0.0333	0.0099	0.0092
2014	0.0000	0.0067	0.0359	0.0354	0.0370	0.0420	0.0353	0.0061	0.0066
Average	0.0000	0.0391	0.0369	0.0317	0.0507	0.0557	0.0171	0.0188	0.0240

Note: In the per dollar user cost formula we hold depreciation fixed at $\delta = 0.011$, running and average transaction costs fixed at $\omega = 0.019$, and expected inflation fixed at $\pi = 0.025$. r is the yield on 10-year government bonds. $g(x)$ is the expected real capital gain and $u(x)$ the per dollar user cost obtained by extrapolating expectations of capital gains over an x year time horizon. The per dollar user cost is calculated using the formula in (1).

from past performance over short time horizons has been noted previously by Verbrugge (2008), Garner and Verbrugge (2009), and Diewert (2009). We restrict the per dollar user cost to be nonnegative. This constraint is binding for u(10) in 2004 and 2005.

It is because of these complications, especially regarding the treatment of capital gains, that we think the user cost approach has not received more attention from NSIs. Our recommendation is that in a CPI context both expected capital gain and the risk premium should be excluded. This is also a position that NSIs are more likely to be comfortable with given their preference for focusing on actual tangible costs incurred by owner occupiers, as opposed to capital gains and risk adjustments. It should also be noted that given the long-run upward trend in real house prices, when expectations are formed over a long horizon the expected real capital gain should be positive and reasonably stable. In this case it follows that the risk premium (assuming it is positive) and expected real capital gains will partially offset each other in the user cost formula. Hence excluding both should not have that big an effect on measured user cost as compared with when both are included.

The other terms in the user cost formula while still problematic do not present insurmountable hurdles. An NSI should be able to decide on an appropriate interest rate, depreciation rate, transaction and running cost rate, and expected rate of inflation. Once this is done, the only other thing that is needed to implement the user cost approach is an estimate of the average value of an OOH dwelling. Such estimates are provided in Table 3.

6.6 Computing average OOH expenditures for the official Sydney CPI

The 16th series of the Australian CPI uses expenditure weights derived from the 2009-2010 household expenditure survey (see Australian Bureau of Statistics 2011). Average expenditures in Australian dollars for each component of the CPI are provided for Sydney for the June quarter 2011 (here denoted by t).

Corresponding average expenditures (y) for other quarters s can be obtained as follows:

$$y_{s,n} = y_{t,n} \times \left(\frac{p_{s,n}}{p_{t,n}} \right).$$

In this way we are able to construct average OOH-acquisitions expenditures for Sydney for each quarter.

6.7 Average OOH expenditures compared

To obtain the average OOH expenditure under rental equivalence it is necessary to adjust the average rents in Table 3 for the proportion of owner-occupiers versus renters. In Sydney about two-thirds of households are owner-occupiers and 0ne-third are renters (Australian Bureau of Statistics, Census of Population and Housing). It follows that the average rent should be multiplied by 2/3 to make it representative of the whole population of households.

Combining the results in Table 4 and Table 3 we obtain the user cost expenditure of an owner-occupying household:

Average OOH user cost of owner-occupier in period t = average value of an OOH dwelling (P_t) $\times$ Per dollar user cost (u_t).

Again to obtain the average OOH expenditure it is necessary to multiply by 2/3.

Average OOH expenditures are compared in Table 5 for the following methods:
User cost excluding real capital gains

User cost with expected real capital gains extrapolated from the previous 10 years
User cost with expected real capital gains extrapolated from the previous 20 years
User cost with expected real capital gains extrapolated from the previous 30 years
Rental equivalence
Acquisitions

Table 5: Average Monthly OOH Expenditures in Dollars: Sydney 2004-2014

	u(0)	u(10)	u(20)	u(30)	Rental Equiv.	Acquis.
2004	2462.0	0.0	517.8	1176.0	1162.5	606.2
2005	2146.5	0.0	335.5	873.2	1197.8	629.4
2006	2411.6	265.2	726.2	1141.9	1250.3	637.8
2007	2762.3	567.0	910.8	1341.7	1358.9	646.2
2008	2911.5	935.5	1205.3	1456.5	1514.5	672.6
2009	2566.5	1133.7	1787.1	1292.2	1546.6	689.9
2010	2725.5	888.1	1265.8	1355.3	1672.3	711.2
2011	2617.0	767.9	1106.3	1404.9	1758.1	732.7
2012	1616.3	610.9	229.3	348.8	1784.7	744.9
2013	2076.4	1712.1	510.0	473.6	1824.7	770.8
2014	2508.4	2107.1	362.8	392.9	1881.2	802.3
Average	2436.7	817.0	814.3	1023.4	1541.1	694.9

It is noticeable in Table 5 that the user cost approach with expected real capital gains excluded, u(0), has by far the largest OOH expenditures. The zero value for the OOH expenditure of user cost with 10-year extrapolation of expected real capital gains, u(10), in 2004 and 2005 is due to the housing boom that started in about 1992 and ended in 2004. The implication is that in 2004, under u(10) households expected very high real capital gains, and this acted to push down the user cost at the beginning of our sample period.

Rental equivalence generates higher OOH expenditure levels than u(10), u(20), u(30). The reason u(0) is so much higher than u(10), u(20), u(30) is that the Sydney housing market has performed strongly since the 1970s. It is also noticeable that acquisitions OOH expenditure level are lower than their user cost and rental equivalence counterparts. This can be attributed to acquisitions focus on only new residential construction, and its exclusion of land. Also, it should be noted that our estimates of user cost and rental equivalence expenditures are based exclusively on houses (i.e., apartments are excluded).

The OOH expenditure shares derived from Table 5 are shown in Table 6.

Table 6: Average Monthly OOH Expenditure Shares: Sydney 2004-2014

	u(0)	u(10)	u(20)	u(30)	Rental Equiv.	Acquis.
2004	0.3560	0.0000	0.1042	0.2089	0.2070	0.1198
2005	0.3170	0.0000	0.0676	0.1588	0.2057	0.1198
2006	0.3398	0.0536	0.1342	0.1959	0.2106	0.1198
2007	0.3678	0.1067	0.1609	0.2203	0.2225	0.1198
2008	0.3707	0.1592	0.1961	0.2276	0.2346	0.1198
2009	0.3361	0.1828	0.2607	0.2031	0.2338	0.1198
2010	0.3428	0.1453	0.1950	0.2060	0.2424	0.1198
2011	0.3271	0.1248	0.1705	0.2070	0.2462	0.1198
2012	0.2280	0.1004	0.0402	0.0599	0.2459	0.1198
2013	0.2683	0.2321	0.0826	0.0772	0.2437	0.1198
2014	0.2985	0.2633	0.0580	0.0625	0.2419	0.1198
Average	0.3229	0.1244	0.1336	0.1661	0.2304	0.1198

When we bring apartments into the computations, this should bring down the user cost and rental equivalence expenditures quite significantly. As a rough approximation, houses in Sydney are about 50 percent more expensive than apartments, and houses consist of about 4/7 of total transactions. Combining these statistics it follows that including apartments would reduce the average imputed rent and house price by about 15 percent. The approximate impact on the user cost and rental equivalence OOH expenditure shares is shown in Table 7.

The opportunity cost method of Diewert, Nakamura and Makamura (2009) discussed above sets OOH expenditure equal to the maximum of user cost and imputed rent. It can be seen from Table 7 that when u(0) is used as the user cost component of opportunity cost, then user cost always exceeds imputed rent. Hence in this case, opportunity cost is identical with u(0) over the time horizon considered here. Conversely, when any of u(10), u(20) or u(30) is the user cost component, then in every year except one imputed rent exceeds user cost, and hence opportunity cost is almost the same as rental equivalence. These findings do, however, depend also on the treatment of the other components of user cost besides capital gains. For example, if a risk premium is included, this would push up the user cost expenditure shares thus potentially making the opportunity cost a more interesting mix of u(10), u(20) or u(30)-type user cost and imputed rent. This is an issue that warrants further investigation.

Table 7: Approximate Average Monthly OOH Expenditure Shares Adjusting for the House-Apartment Mix: Sydney 2004-2014

	u(0)	u(10)	u(20)	u(30)	Rental Equiv.	Acquis.
2004	0.3197	0.0000	0.0899	0.1833	0.1816	0.1198
2005	0.2829	0.0000	0.0581	0.1383	0.1804	0.1198
2006	0.3043	0.0459	0.1164	0.1716	0.1849	0.1198
2007	0.3309	0.0922	0.1402	0.1937	0.1957	0.1198
2008	0.3337	0.1386	0.1717	0.2003	0.2067	0.1198
2009	0.3009	0.1597	0.2306	0.1781	0.2059	0.1198
2010	0.3072	0.1262	0.1707	0.1806	0.2139	0.1198
2011	0.2924	0.1081	0.1487	0.1815	0.2173	0.1198
2012	0.2007	0.0867	0.0344	0.0514	0.2170	0.1198
2013	0.2376	0.2044	0.0711	0.0664	0.2150	0.1198
2014	0.2656	0.2330	0.0497	0.0536	0.2134	0.1198
Average	0.2887	0.1086	0.1165	0.1454	0.2029	0.1198

6.8 The impact of OOH on the CPI

The 16th series of the Australian CPI is computed using a Laspeyres-type price index formula as follows:[10]

$$\frac{CPI_{t+1}}{CPI_t} = \sum_{n=1}^{N} \left[s_{b,n} \left(\frac{p_{t+1,n}}{p_{t,n}} \right) \right],$$

1 where CPI_{t+1}/CPI_t is the change in the CPI from period t to $t+1$, $s_{b,n}$ denotes the expenditure weight for heading n in the base period. which here is June 2011.

Under the acquisitions approach, OOH consists of three headings:
New dwelling purchase of owner occupiers,
Maintenance and repair of the dwelling,
Property rates and charges.
Here we will classify these headings as headings $N-2$, $N-1$, and N. To determine the impact on the CPI of switching from acquisitions to user cost or rental equivalence, it is necessary to separate the OOH components of the CPI from the rest of it, as follows:

$$\left. \frac{CPI_{t+1}}{CPI_t} \right| OOH = \sum_{n=1}^{N-3} \left[s_{b,n} \left(\frac{p_{t+1,n}}{p_{t,n}} \right) \right]$$

[10]More precisely, when the weights are fixed, this price index formula is referred to as a Young index (see chapter 1 of the Consumer Price Index Manual (2004).

$$= \frac{CPI_{t+1}}{CPI_t} - s_{b,N-2}\left(\frac{p_{t+1,N-2}}{p_{t,N-2}}\right) - s_{b,N-1}\left(\frac{p_{t+1,N-1}}{p_{t,N-1}}\right) - s_{b,N}\left(\frac{p_{t+1,N}}{p_{t,N}}\right).$$

Our variants on the official CPI are then calculated as follows:

$$\frac{CPI^*_{t+1}}{CPI^*_t} = \left(\frac{1}{\sum_{n=1}^{N-3} s_{b,n} + s^*_{t,N+1}}\right)\sum_{n=1}^{N-3}\left[s_{b,n}\left(\frac{p_{t+1,n}}{p_{t,n}}\right)\right] + s^*_{t,N+1}\left(\frac{p^*_{t+1,N+1}}{p^*_{t,N+1}}\right),$$

where $s^*_{t,N+1}$ and $p^*_{t+1,N+1}/p^*_{t,N+1}$ are expenditure shares and price relatives for OOH obtained using either rental equivalence or user cost. It should be noted that in the case of rental equivalence and user cost, OOH is represented by a single heading here denoted by $N+1$, while under acquisitions it is represented by the three headings $N-1$, $N-1$, and N.

The impact on the Sydney CPI of each approach to including OOH is shown in Table 8. As it stands the user cost approach excluding expected capital gains u(0) seems to generate unrealistically high CPI inflation. the (geometric) average inflation rate is 8.9% as compared with 2.7% for the official CPI (computed using the acquisitions approach). Indeed all our alternative estimates based on either user cost or rental equivalence are gigher than the official CPI.

Table 8: CPI for Sydney (with 2004 as the Base)

	u(0)	u(10)	u(20)	u(30)	Rental Equiv.	Acquis.
2004	1.000	1.000	1.000	1.000	1.000	1.000
2005	1.072	1.022	1.015	1.028	1.040	1.025
2006	1.161	1.065	1.050	1.071	1.101	1.064
2007	1.266	1.081	1.075	1.111	1.156	1.083
2008	1.397	1.120	1.120	1.171	1.251	1.129
2009	1.538	1.146	1.157	1.221	1.301	1.144
2010	1.717	1.208	1.252	1.295	1.389	1.177
2011	1.871	1.252	1.308	1.356	1.486	1.222
2012	2.001	1.268	1.335	1.395	1.550	1.238
2013	2.132	1.307	1.367	1.430	1.634	1.270
2014	2.342	1.414	1.414	1.477	1.727	1.305
Average Inflation	8.9%	3.5%	3.5%	4.0%	5.6%	2.7%

As noted above though our user-cost and rentail equivalence CPI estimates need to be revised downwards to account for the house-apartment mix. Using our rough adjustment from above, we obtain the user cost and rental equivalence CPIs as shown in Table 9.

Table 9: CPI for Sydney (Approximately Adjusting for Apartments)

	u(0)	u(10)	u(20)	u(30)	Rental Equiv.	Acquis.
2004	1.000	1.000	1.000	1.000	1.000	1.000
2005	1.059	1.022	1.014	1.023	1.034	1.025
2006	1.135	1.065	1.050	1.063	1.088	1.064
2007	1.220	1.081	1.072	1.096	1.134	1.083
2008	1.328	1.119	1.114	1.149	1.216	1.129
2009	1.439	1.141	1.143	1.188	1.254	1.144
2010	1.583	1.195	1.224	1.250	1.327	1.177
2011	1.704	1.235	1.273	1.303	1.408	1.222
2012	1.799	1.249	1.294	1.333	1.457	1.238
2013	1.900	1.284	1.325	1.365	1.523	1.270
2014	2.060	1.375	1.367	1.408	1.596	1.305
Average Inflation	7.5%	3.2%	3.2%	3.5%	4.8%	2.7%

The average CPI inflation rate of 7.5% for u(0) is still too high in Table 9. We are led to the conclusion therefore that in spite of its desirable axiomatic properties, it would be impractical to use u(0) to construct a CPI for monetary policy or wage and contract indexation purposes. However, u(30) should be viewed as a viable alternative to rental equivalence or acquisitions.

The results in Table 9 indicate that rental equivalence pushes up the CPI more than any of u(10), u(20) or u(30). This finding may be an artifact of the particular time period considered in this study. House prices peaked in 2004 after rising strongly for about 12 years. Prices then fell slightly before gradually resuming an upward trend. Given that rents tend to lag trends in house prices, the main impact of the 1992-2004 boom on rents was only felt after 2004. Hence during our sample period 2004-2104, rents rose more strongly than house prices as can be seen in Figure 3. This situation is probably unusual. Most of the time house prices rise and fall more than rents over the housing cycles.

7 Conclusion

The CPI is sensitive to the way OOH expenditures and prices are measured. While an axiomatic case can be made in favor of the user cost approach with capital gains excluded, u(0), empirically this method seems to generate a CPI that is too sensitive to movements in house prices. Nevertheless, the user cost approach has advantages over rental equivalence and the acquisitions approach. Our preferred variant of user cost for Sydney assumed expected real capital gains are extrapolated over a 30 year time horizon.

Whichever approach to including OOH in the CPI is actually used, it is important that this is clarified prior to any discussion of how monetary policy should respond to house prices. For example, when u(0) is used, the CPI is already highly sensitive to movements in house prices. Conversely, the acquisitions approach currently used by the Australian Bureau of Statistics is relatively insensitive to the housing market. Rental equivalence and other varieties of user cost, such as u(10), u(20) and u(30) lie somewhere in between these extremes.

References

Abelson, P. and D. Chung (2005), "The Real Story of Housing Prices in Australia from 1970 to 2003," *Australian Economic Review* 38(3), 265-81

Australian Bureau of Statistics (2011), Australian Consumer Price Index: Concepts, Sources and Methods (6461.0). Australian Bureau of Statistics: Canberra.

Bracke, P. (2013), "How Long Do Housing Cycles Last? A Duration Analysis for 19 OECD Countries," *Journal of Housing Economics* 22, 213-230.

Bracke, P. (2015), "House Prices and Rents: Micro Evidence from a Matched Dataset in Central London," *Real Estate Economics* 43, 403-431.

Cecchetti, S. (2006), "The Brave New World of Central Banking: Policy Challenges Posed by Asset Price Booms and Busts," *National Institute Economic Review* 196, April, 1-13.

Consumer Price Index Manual: Theory and practice (2004), International Labour Office (ILO), Geneva.

Davino, C. and D. Vistocco (2008). Quantile regression for the evaluation of student satisfaction. *Italian Journal of Applied Statistics*, 20(3-4):179–196.

Diewert W. E. (2003), "The Treatment of Owner Occupied Housing and Other Durables in a Consumer Price Index," Discussion Paper No. 08-03. Department of Economics, University of British Columbia: Vancouver.

Diewert, W. E. (2009), "Durable and Owner-Occupied Housing in a Consumer Price Index," in Diewert W. E., J. S. Greenless and C. R. Hulten (eds.), *Price Index Concepts and Measurements*. NBER: University of Chicago Press, Chapter 12, 445-500.

Diewert, W. E., A. O. Nakamura, and L. I. Nakamura (2009), "The Housing Bubble and a New Approach to Accounting for Housing in a CPI," *Journal of Housing Economics* 18(3), September, 156-171.

Flavin, M. and T. Yamashita (2002), "Owner-Occupied Housing and the Composition of the Household Portfolio," *American Economic Review* 92(1), 345-362.

Fox, R. and P. Tulip (2014), "Is Housing Overvalued?," Research Discussion Paper RDP 2014-06, Reserve Bank of Australia, Sydney.

Garner, T. I. and R. Verbrugge (2009), "Reconciling User Costs and Rental Equivalence: Evidence from the US Consumer Expenditure Survey," *Journal of Housing Economics* 18(3), September, 172-192

Girouard, N., M. Kennedy, P. van den Noord, and C. André (2006), "Recent House Price Developments: The Role of Fundamentals," OECD Economics Department Working Papers 475. Paris: OECD.

Green, R. K. and S. M. Wachter (2005), "The American Mortgage in Historical and International Context," *Journal of Economic Perspectives* 19(4), 93-114.

Gudnason, R., and G. R. Jónsdóttir (2009), "Owner occupied housing in the Icelandic CPI," in *Price and Productivity Measurement, vol. 1, Housing*, ed. W. E. Diewert, B. M. Balk, D. Fixler, K. J. Fox, and A. O. Nakamura, 147– 50. Victoria, B.C.: Trafford Press. Available at: www.vancouvervolumes.com.

Hansen, M., C. Kooperberg, and S. Sardy (1998). Triogram models. *Journal of the American Statistical Association*, 93(441):101–119.

Heston, A. (2009), "Comment on Durable and Owner-Occupied Housing in a Consumer Price Index," in Diewert W. E., J. S. Greenless and C. R. Hulten (eds.), *Price Index*

Concepts and Measurements. NBER: University of Chicago Press, Chapter 12, 500-505.

Heston, A. and A. O. Nakamura (2009), "Questions About the Equivalence of Market Rents and User Costs for Owner Occupied Housing," *Journal of Housing Economics* 18(3), September, 273-279.

Hill, R. J. and M. Scholz (2014). Incorporating geospatial data in house price indexes: A hedonic imputation approach with splines. *Graz Economics Papers*, 2014-05.

Hill, R. J. and I. Syed (2016), "Hedonic Price-to-Rent Ratios, User Cost, and the Detection of Departures from Equilibrium in the Housing Market," *Regional Science and Urban Economics* 56, January, 60-72.

Himmelberg, C., C. Mayer and T. Sinai (2005), "Assessing High House Prices: Bubbles, Fundamentals and Misperceptions," *Journal of Economic Perspectives* 19(4), 67-92.

Koenker, R. and I. Mizera (2004). Penalized triograms: total variation regularization for bivariate smoothing. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 66(1):145–163.

Mishkin, F. S, (2011), "Monetary Policy Strategy: Lessons from the Crisis," NBER Working Paper No. 16755, February.

Sinai, T. and N. Souleles (2005), "Owner-Occupied Housing as a Hedge Against Rent Risk," *Quarterly Journal of Economics* 120(2), 763-789.

Stapledon, N. D. (2007), "Long Term Housing Prices in Australia and Some Economic Perspectives," PhD Dissertation, University of New South Wales, Australian School of Business School of Economics.

Verbrugge, R. (2008), "The Puzzling Divergence of Rents and User Costs," *Review of Income and Wealth* 54(4), 671-699.

Waltl, S. R. (2015). Variation across price segments and locations: A comprehensive quantile regression analysis of the Sydney housing market. *Graz Economics Papers*, 2015-09.

Waltl, S. R. (2016). A hedonic house price index in continuous time. *International Journal of Housing Markets and Analysis*, forthcoming.

APPENDIX: Table A1

The Impact of Alternative Treatments of Capital Gains in the User Cost of OOH

Excluding capital gains

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	NA	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	NA	0.05	10400	90000	0.1036	1.004	1.004	1.004	1.004	1.004	1.004
3	1.0816	NA	0.05	10816	90000	0.1073	1.004	1.004	1.004	1.004	1.004	1.004
4	1.124864	NA	0.05	11248.64	90000	0.1111	1.004	1.004	1.004	1.004	1.004	1.004
5	1.169859	NA	0.05	11698.59	90000	0.1150	1.005	1.004	1.004	1.004	1.004	1.004
6	1.124864	NA	0.05	11248.64	90000	0.1111	0.996	0.995	0.996	0.996	0.996	0.996
7	1.0816	NA	0.05	10816	90000	0.1073	0.996	0.996	0.996	0.996	0.996	0.996
8	1.04	NA	0.05	10400	90000	0.1036	0.996	0.996	0.996	0.996	0.996	0.996
9	1	NA	0.05	10000	90000	0.1000	0.996	0.996	0.996	0.996	0.996	0.996
Chained price index comparing years 1 and 9									1.000	1.000	1.000	1.000
Chained price index comparing years 1 and 5									1.017	1.017	1.017	1.017

Including ex post capital gains

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0.04	0.01	2000	90000	0.0217	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.04	0.01	2080	90000	0.0226	1.001	1.001	1.001	1.001	1.001	1.001
3	1.0816	0.04	0.01	2163.2	90000	0.0235	1.001	1.001	1.001	1.001	1.001	1.001
4	1.124864	0.04	0.01	2249.728	90000	0.0244	1.001	1.001	1.001	1.001	1.001	1.001
5	1.169859	-0.03846	0.088462	20697.5	90000	0.1870	1.007	1.001	1.004	1.001	1.007	1.004
6	1.124864	-0.03846	0.088462	19901.44	90000	0.1811	0.993	0.993	0.993	0.993	0.993	0.993
7	1.0816	-0.03846	0.088462	19136	90000	0.1753	0.993	0.993	0.993	0.993	0.993	0.993
8	1.04	-0.03846	0.088462	18400	90000	0.1697	0.993	0.993	0.993	0.993	0.993	0.993
9	1	0	0.05	10000	90000	0.1000	0.996	0.993	0.995	0.993	0.996	0.995
Chained price index comparing years 1 and 9									0.981	0.976	0.985	0.981
Chained price index comparing years 1 and 5									1.007	1.004	1.010	1.007

Including ex ante capital gains (with expectations extrapolated over a 1 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.04	0.01	2080	90000	0.0226	1.001	1.004	1.002	1.004	1.001	1.002
3	1.0816	0.04	0.01	2163.2	90000	0.0235	1.001	1.001	1.001	1.001	1.001	1.001
4	1.124864	0.04	0.01	2249.728	90000	0.0244	1.001	1.001	1.001	1.001	1.001	1.001
5	1.169859	0.04	0.01	2339.717	90000	0.0253	1.001	1.001	1.001	1.001	1.001	1.001
6	1.124864	-0.03846	0.088462	19901.44	90000	0.1811	0.993	0.999	0.996	0.999	0.993	0.996
7	1.0816	-0.03846	0.088462	19136	90000	0.1753	0.993	0.993	0.993	0.993	0.993	0.993
8	1.04	-0.03846	0.088462	18400	90000	0.1697	0.993	0.993	0.993	0.993	0.993	0.993
9	1	-0.03846	0.088462	17692.31	90000	0.1643	0.994	0.993	0.993	0.993	0.993	0.993
Chained price index comparing years 1 and 9									0.981	0.986	0.976	0.981
Chained price index comparing years 1 and 5									1.005	1.007	1.004	1.005

Including ex ante capital gains (with expectations extrapolated over a 2 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.019804	0.030196	6280.788	90000	0.0652	1.003	1.004	1.003	1.004	1.003	1.003
3	1.0816	0.04	0.01	2163.2	90000	0.0235	1.001	1.003	1.002	1.003	1.001	1.002
4	1.124864	0.04	0.01	2249.728	90000	0.0244	1.001	1.001	1.001	1.001	1.001	1.001
5	1.169859	0.04	0.01	2339.717	90000	0.0253	1.001	1.001	1.001	1.001	1.001	1.001
6	1.124864	0	0.05	11248.64	90000	0.1111	0.996	0.999	0.997	0.999	0.996	0.997
7	1.0816	-0.03846	0.088462	19136	90000	0.1753	0.993	0.996	0.994	0.996	0.993	0.994
8	1.04	-0.03846	0.088462	18400	90000	0.1697	0.993	0.993	0.993	0.993	0.993	0.993
9	1	-0.03846	0.088462	17692.31	90000	0.1643	0.994	0.993	0.993	0.993	0.993	0.993
Chained price index comparing years 1 and 9									0.985	0.990	0.981	0.985
Chained price index comparing years 1 and 5									1.007	1.009	1.005	1.007

Including ex ante capital gains (with expectations extrapolated over a 3 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.013159	0.036841	7662.844	90000	0.0785	1.003	1.004	1.004	1.004	1.003	1.004
3	1.0816	0.026492	0.023508	5085.255	90000	0.0535	1.002	1.003	1.003	1.003	1.002	1.003
4	1.124864	0.04	0.01	2249.728	90000	0.0244	1.001	1.002	1.002	1.002	1.001	1.002
5	1.169859	0.04	0.01	2339.717	90000	0.0253	1.001	1.001	1.001	1.001	1.001	1.001
6	1.124864	0.013159	0.036841	8288.132	90000	0.0843	0.997	0.999	0.998	0.999	0.997	0.998
7	1.0816	-0.01299	0.062988	13625.67	90000	0.1315	0.995	0.997	0.996	0.997	0.995	0.996
8	1.04	-0.03846	0.088462	18400	90000	0.1697	0.993	0.995	0.994	0.995	0.993	0.994
9	1	-0.03846	0.088462	17692.31	90000	0.1643	0.994	0.993	0.993	0.993	0.993	0.993
Chained price index comparing years 1 and 9									0.990	0.994	0.985	0.990
Chained price index comparing years 1 and 5									1.009	1.010	1.007	1.009

Including ex ante capital gains (with expectations extrapolated over a 4 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.009853	0.040147	8350.491	90000	0.0849	1.003	1.004	1.004	1.004	1.003	1.004
3	1.0816	0.019804	0.030196	6532.02	90000	0.0677	1.003	1.003	1.003	1.003	1.003	1.003
4	1.124864	0.029852	0.020148	4532.652	90000	0.0479	1.002	1.003	1.002	1.003	1.002	1.002
5	1.169859	0.04	0.01	2339.717	90000	0.0253	1.001	1.002	1.001	1.002	1.001	1.001
6	1.124864	0.019804	0.030196	6793.301	90000	0.0702	0.997	0.999	0.998	0.999	0.997	0.998
7	1.0816	0	0.05	10816	90000	0.1073	0.996	0.997	0.997	0.997	0.996	0.997
8	1.04	-0.01942	0.069419	14439.22	90000	0.1383	0.995	0.996	0.995	0.996	0.995	0.995
9	1	-0.03846	0.088462	17692.31	90000	0.1643	0.994	0.995	0.994	0.995	0.993	0.994
Chained price index comparing years 1 and 9									0.994	0.999	0.990	0.994
Chained price index comparing years 1 and 5									1.010	1.012	1.009	1.010

Including ex ante capital gains (with expectations extrapolated over a 5 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.007875	0.042125	8762.002	90000	0.0887	1.003	1.004	1.004	1.004	1.003	1.004
3	1.0816	0.015812	0.034188	7395.55	90000	0.0759	1.003	1.003	1.003	1.004	1.003	1.003
4	1.124864	0.023812	0.026188	5891.7	90000	0.0614	1.002	1.003	1.003	1.003	1.002	1.003
5	1.169859	0.031874	0.018126	4240.97	90000	0.0450	1.002	1.002	1.002	1.002	1.002	1.002
6	1.124864	0.023812	0.026188	5891.7	90000	0.0614	0.998	0.998	0.998	0.998	0.998	0.998
7	1.0816	0.007875	0.042125	9112.482	90000	0.0919	0.996	0.998	0.997	0.998	0.996	0.997
8	1.04	-0.00781	0.057813	12025.2	90000	0.1179	0.995	0.996	0.996	0.996	0.995	0.996
9	1	-0.02326	0.073258	14651.54	90000	0.1400	0.995	0.995	0.995	0.995	0.994	0.995
Chained price index comparing years 1 and 9									0.997	1.001	0.994	0.997
Chained price index comparing years 1 and 5									1.012	1.013	1.010	1.012

Including ex ante capital gains (with expectations extrapolated over a 6 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.006558	0.043442	9035.895	90000	0.0912	1.004	1.004	1.004	1.004	1.004	1.004
3	1.0816	0.013159	0.036841	7969.358	90000	0.0813	1.003	1.004	1.003	1.004	1.003	1.003
4	1.124864	0.019804	0.030196	6793.301	90000	0.0702	1.003	1.003	1.003	1.003	1.003	1.003
5	1.169859	0.026492	0.023508	5500.212	90000	0.0576	1.002	1.003	1.003	1.003	1.002	1.003
6	1.124864	0.019804	0.030196	6793.301	90000	0.0702	0.997	0.998	0.997	0.998	0.997	0.997
7	1.0816	0.013159	0.036841	7969.358	90000	0.0813	0.997	0.997	0.997	0.997	0.997	0.997
8	1.04	0	0.05	10400	90000	0.1036	0.996	0.997	0.996	0.997	0.996	0.996
9	1	-0.01299	0.062988	12597.7	90000	0.1228	0.995	0.996	0.996	0.996	0.995	0.996
Chained price index comparing years 1 and 9									0.999	1.002	0.996	0.999
Chained price index comparing years 1 and 5									1.013	1.014	1.012	1.013

Including ex ante capital gains (with expectations extrapolated over a 7 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.005619	0.044381	9231.314	90000	0.0930	1.004	1.004	1.004	1.004	1.004	1.004
3	1.0816	0.011269	0.038731	8378.303	90000	0.0852	1.003	1.004	1.004	1.004	1.003	1.004
4	1.124864	0.016951	0.033049	7435.139	90000	0.0763	1.003	1.003	1.003	1.003	1.003	1.003
5	1.169859	0.022665	0.027335	6395.648	90000	0.0663	1.003	1.003	1.003	1.003	1.003	1.003
6	1.124864	0.016951	0.033049	7435.139	90000	0.0763	0.997	0.997	0.997	0.997	0.997	0.997
7	1.0816	0.011269	0.038731	8378.303	90000	0.0852	0.997	0.997	0.997	0.997	0.997	0.997
8	1.04	0.005619	0.044381	9231.314	90000	0.0930	0.996	0.997	0.997	0.997	0.996	0.997
9	1	-0.00559	0.055587	11117.46	90000	0.1099	0.996	0.996	0.996	0.996	0.996	0.996
Chained price index comparing years 1 and 9									1.000	1.002	0.998	1.000
Chained price index comparing years 1 and 5									1.013	1.014	1.012	1.013

Including ex ante capital gains (with expectations extrapolated over a 8 year horizon)

Year	HPI	g	u	Pu	Exp	Weight	GP	GL	Tornqvist	Laspeyres	Paasche	Fisher
1	1	0	0.05	10000	90000	0.1000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.04	0.004915	0.045085	9377.758	90000	0.0944	1.004	1.004	1.004	1.004	1.004	1.004
3	1.0816	0.009853	0.040147	8684.511	90000	0.0880	1.003	1.004	1.004	1.004	1.003	1.004
4	1.124864	0.014816	0.035184	7915.34	90000	0.0808	1.003	1.003	1.003	1.004	1.003	1.003
5	1.169859	0.019804	0.030196	7065.033	90000	0.0728	1.003	1.003	1.003	1.003	1.003	1.003
6	1.124864	0.014816	0.035184	7915.34	90000	0.0808	0.997	0.997	0.997	0.997	0.997	0.997
7	1.0816	0.009853	0.040147	8684.511	90000	0.0880	0.997	0.997	0.997	0.997	0.996	0.997
8	1.04	0.004915	0.045085	9377.758	90000	0.0944	0.996	0.997	0.996	0.997	0.996	0.996
9	1	0	0.05	10000	90000	0.1000	0.996	0.996	0.996	0.996	0.996	0.996
Chained price index comparing years 1 and 9									1.000	1.002	0.998	1.000
Chained price index comparing years 1 and 5									1.014	1.015	1.013	1.014