RESERVE REQUIREMENTS AND OPTIMAL CHINESE STABILIZATION POLICY

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ABSTRACT. We build a two-sector DSGE model of the Chinese economy to study the role of reserve requirement (RR) policy for capital realloation and business cycle stabilizaton. In the model, state-owned enterprises (SOEs) have lower average productivity than private firms, but they have superior access to bank loans because of government guarantees. Private firms rely on "shadow" bank financing. Commercial banks are subject to RR regulations but shadow banks are not. Our framework implies a tradeoff for RR policy: Increasing RR acts as a tax on SOE activity and reallocates resources to private firms, raising aggregate productivity. This reallocation is supported by empirical evidence. However, raising RR also increases the incidence of costly SOE failures. Under our calibration, RR policy can be complementary to interest-rate policy and useful for stabilizing macro fluctuations and improving welfare.

I. Introduction

China's central bank, the Peoples Bank of China (PBOC), frequently uses reserve require-ments (RR) as a policy instrument for macroeconomic stabilization. Since 2006, the PBOC has adjusted RR ratio at least 40 times. Changes have also been substantial. During the tightening cycles from 2006 to 2011, the RRR increased from 8.5 percent to 21.5 percent (see Figure 1). The literature has claimed that these changes in reserve requirements are an important policy tool for the PBOC [e.g. (Ma et al., 2013)].[1]

Under China's existing financial system, state-owned enterprises (SOEs) have better access to bank loans than private firms (Elliott et al., 2015). The Chinese government provides explicit or implicit guarantees for loans to SOEs, creating an incentive for banks to prefer lending to SOEs, despite their lower average productivity than private firms (Song et al., 2011). Financing of private firms, especially small and medium-sized firms, largely relies on informal financial intermediaries, such as shadow banks (Lu et al., 2015).[2]

Since commercial banks are subject to RR regulations but shadow banks are not, raising RR acts as a tax on conventional banking and thus also on SOE activity. When RR increases, capital should flow from the SOE sector to the private sector. Empirical evidence supports this reallocation mechanism. Figure 2 shows the macroeconomic effects of a positive shock to RR that raises the required reserve ratio in a Bayesian vector-autoregression (BVAR) model. The impulse responses estimated from the BVAR model show that the share of SOE investment falls significantly, although the shock has ambiguous effects on real GDP and

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[1]Some studies in the literature suggest that the PBOC uses RR to help address external imbalances. For example, Ma et al. (2013) argues that the PBOC uses RR to mop up foreign exchange revenues under China's tightly controlled capital account. Following the global financial crisis, Chinas limited capital mobility combined with low foreign interest rates raised the fiscal cost of sterilizing capital inflows. Chang et al. (2015b) demonstrate that there is a tradeoff between sterilization costs and domestic price stability, raising the possibility of welfare gains through the use of RR as an alternative policy tool. China is not the only country that employs RR as a stabilization tool. Federico et al. (2014) find that about two-thirds of the emerging market countries in their study use RR for stabilization. Many of these other emerging market economies have open capital accounts, and primarily rely on RR policy as a mechanism for tightening activity without attracting further expansionary capital inflows, as conventional interest rate increases would do (Montoro and Moreno, 2011).

[2]In China, the government directly controls the volume of commercial bank loans (through "window guidance" of lending) and imposes caps on the loan-to-deposit ratio. These limitations on lending, combined with sharp increases in the RR ratio, have contributed to rapid expansion in shadow banking activity (Hachem and Song, 2015; Elliott et al., 2015). Lending by China's shadow banking sector increased by over 30% per year between 2009 and 2013. While shadow banks can help reduce intermediation costs, their unregulated activity raises risks to financial and macroeconomic stability [e.g. (Gorton and Metrick, 2010; Verona et al., 2013; Elliott et al., 2015)].

the nominal interest rate.[3] Since private firms are on average more productive than SOEs (Hsieh and Klenow, 2009; Hsieh and Song, 2015), this capital reallocation should improve aggregate productivity and therefore raise aggregate output.[4] On the other hand, increases in RR reduce aggregate demand. Furthermore, the increased funding costs can increase the share of SOE bankruptcies.

Our DSGE model highlights the tradeoff for the use of RR policy. In our model, a homogeneous intermediate good is produced by firms in two sectors—an SOE sector and a private (POE) sector— using the same production technology. Consistent with empirical evidence, we assume that POEs have higher average productivity than SOEs. The representative household purchases a final good for consumption and capital investment. The household also supplies labor and capital to intermediate good firms. The final good is a composite of retail goods. Each retailer uses the homogenous intermediate good as input to produce a differentiated retail product. Retailers are price takers in the input market but monopolistic competitors in the product markets. Retail price adjustments are costly [e.g. (Rotemberg, 1982)].

To incorporate financial frictions, we build on the framework of Bernanke et al. (1999) (BGG) with costly state verification. We generalize the BGG framework to our two-sector environment. In particular, we assume that firms in each sector needs to finance working capital with both internal net worth and external debt. Production and financing decisions are made after observing an aggregate productivity shock. As in BGG, we assume that loan contracts are signed before the realization of idiosyncratic shocks, implying that the loan rate is identical for all firms. In equilibrium, there is a threshold level of idiosyncratic productivity, above which firms repays the loan at the contractual rate, and earn non-negative profits. Firms with productivity below the threshold level, however, may choose to default. In the event of a default, the lender pays a cost to liquidate the project.

To capture the features of China's financial system, we deviate from the BGG framework in several dimensions: First, we assume that credit markets are segmented, with banks

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[3]The BVAR model includes the required reserve ratio, the 3-month nominal deposit rate, the real GDP (in log units), and the share of business fixed investment in the SOE sector in aggregate business fixed investment. The sample ranges from 1995:Q1 to 2013:Q4. The time-series data are taken from Chang et al. (2015a). The BVAR is estimated with 4-quarterly lags, with the Sims-Zha priors, and with the RR ordered first for Cholesky identification. Under this identification assumption, the RR responds to all shocks in the impact period, while the other 3 variables do not respond to the RR shock in the impact period. The qualitative results do not change if RR is ordered last.

[4]Average productivity of SOEs is lower than that of private firms, but firm-level evidence also shows that there is substantial heterogeneity of productivity among firms within each sector. For example, Brandt (2015) shows that, in SOE-dominant industries, both SOEs and POES have lower productivity than firms in industries with less SOE presence.

lending to SOE firms only. POE firms borrow from informal financial intermediaries, which we call shadow banks. This complete segmentation of credit markets is adopted for analytic simplicity. However, the bulk of commercial bank lending in China is directed towards SOEs, while private firms, especially small and medium-sized firms, are much more dependent on non-bank funding [e.g. (Lardy, 2014), (Elliott et al., 2015)].

Second, the government provides guarantees on bank loans to SOEs and, in the event of an SOE default, the government steps in to cover the bank's loan losses. This leaves bank loans risk-free. The guarantee represents an implicit subsidy to SOEs that reduces their funding costs. In contrast, loans to private firms are not guaranteed, and the financial frictions facing POEs mimic those in the standard BGG environment. In particular, the loan rate offered to POEs includes a default premium (or credit spread) that compensates the lender for expected bankruptcy losses.[5]

Third, commercial banks are subject to RR policy, and need to hold a fraction of their deposits as reserves at the central bank. Since banks do not earn any interests on reserves, RR policy drives a wedge between the deposit interest rate and the lending rate.

We study a calibrated version of our model to illustrate the tradeoff incurred by adjusting RR. We first examine the steady-state effects of RR policy. Consistent with the mechanism described above, we find that an increase in the steady-state RR ratio improves aggregate TFP through reallocation of resources toward the more productive POE sector, but it also raises the social cost of SOE bankruptcies. As a consequence, there is an interior optimal steady-state level of the RR ratio that maximizes social welfare.

We then examine the implications of a simple reserve-requirement rule for macroeconomic stability and social welfare when the economy is buffeted by an aggregate technology shock and an aggregate demand shock (in particular, a government spending shock). We compare the stabilizing performance of the reserve-requirement rule to that of an interest-rate rule. Under each policy rule, the policy instrument (the nominal deposit rate or the RR ratio) reacts to fluctuations in inflation and real GDP growth. We search for the coefficients in the reaction functions that maximize the representative household's welfare.

Compared to our benchmark economy in which the monetary authority follows a Taylor rule and maintains a constant RR ratio, we find that following optimal RR and interest rate rules improve welfare. The optimal interest-rate rule is more effective for stabilizing fluctuations in output and inflation than the optimal RR rule, although the optimal RR

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[5]Chang et al. (2015a) provide evidence that China's credit policy favors capital-intensive (or heavy) industries at the expense of labor-intensive (or light) industries. Although not all heavy industries are state-owned, Chang et al. (2015a) find that the share of SOEs in capital-intensive industries has increased steadily after the large-scale SOE reform in the late 1990s.

rule is more effective for reallocating resources between the SOE sector and the POE sector. Since the government provides guarantees for SOE loans, lenders (conventional banks) face no default risks. The financial accelerator mechanism of the BGG framework is thus muted for the SOE sector, but not for the POE sector, rendering the POE sector more responsive to macroeconomic shocks. By shifting resources between the two sectors, adjustments in the RR ratio can help stabilize aggregate fluctuations.

As a result, when the planner is allowed to optimally choose the coefficients in both policy rules, social welfare can be improved substantially relative to each individual optimal rule. In the case with jointly optimal policy rules, the effectiveness of interest-rate policy for stabilization is substantially enhanced by also pursuing optimal RR policy. This result suggests that RR can be a complementary policy instrument to the conventional interest-rate policy.

Our work is related to the important contribution by Song et al. (2011), who study China's transition dynamics in a two-period overlapping generations model with SOEs and POEs. As in our paper, SOEs have lower productivity, but enjoy superior access to bank credit. Indeed, the SOE sector in their model is fully financially integrated with neoclassical firms. In contrast, high-productivity POEs save and self-finance their investment. Their model's transition dynamics explain some puzzling characteristics about the Chinese economy, such as high growth being accompanied by high saving rates.

Our model differs from theirs in three important dimensions. First, our model is a DSGE model with an infinite horizon, which allows us to study both the steady state equilibrium and business cycle dynamics. Second, we model financial frictions for both the SOE sector and the private sector in the spirit of Bernanke et al. (1999). Although SOE loans are guaranteed by the government, some SOEs do fail in equilibrium if they have sufficiently low productivity relative to loan repayment liabilities. POEs rely on shadow-bank financing, with an agency problem similar to that in the BGG framework. Third, we study the implications of RR policy relative to the conventional interest-rate policy in an environment with nominal rigidities and financial frictions. Our finding suggests that, in this second-best environment, RR policy is useful for not just steady-state reallocation, but also for business cycle stabilization.

II. The model

The economy is populated by a continuum of infinitely-lived households. The representative household consumes a basket of differentiated goods purchased from retailers. Retailers produce differentiated goods using homogeneous intermediate goods as inputs. These intermediate goods are produced by two types of firms: state-owned firms (SOEs) and private

firms (POEs). The two types of firms have identical production technologies ex-ante except that the average productivity of SOEs is lower than POEs.

Firms face working capital constraints. Each firm finances wages and rental payments using both internal net worth and external debt. Following Bernanke et al. (1999), we assume that external financing is subject to a costly state verification problem. In particular, each firm can observe its own idiosyncratic productivity shocks. Firms with sufficiently low productivity choose to default on the debt. In the event of a default, the lender needs to pay a liquidation cost to take over the project and obtain the revenue.

We generalize the BGG framework to a two-sector environment with SOEs and POEs have access to different forms of external financing. SOEs can borrow from conventional banks and POEs can borrow only from nonbank financial intermediaries. Bank loans are subject to reserve requirements, but banks face no default risks because of government guarantees. Non-bank financial intermediaries (which we call "shadow banks") are not subject to reserve-requirement regulations, but they need to internalize default costs for their loans to private firms, as in the standard BGG framework.[6]

II.1. **Households.** There is a continuum of infinitely-lived and identical households with a unit mass. The representative household has preferences represented by the expected utility function

$$U = \sum_{t=0}^{\infty} \beta_t \left[\ln(C_t) - \Psi \frac{H_t^{1+\eta}}{1+\eta} \right], \tag{1}$$

where C_t denotes consumption and H_t denotes labor hours. The parameter $\beta \in (0,1)$ is a subjective discount factor, $\eta > 0$ is the inverse Frisch elasticity of labor supply, and $\Psi > 0$ is a weight on the disutility of working.

We assume that labor is imperfectly mobile across SOEs and POEs. In particular, total hours H_t is a CES composite of the hours worked in SOEs H_{st} and in POEs H_{pt}. Specifically, we have

$$H_t = \left(\mu H_{st}^{1+\sigma_L} + (1-\mu) H_{pt}^{1+\sigma_L} \right)^{\frac{1}{1+\sigma_L}}. \tag{2}$$

where the parameter σ_L measures the elasticity of substitution between labor hours devoted to the two sectors. In the special case with $\sigma_L = 0$, the two types of hours are perfect substitutes and labor becomes perfectly mobile across sectors. In general, labor hours are

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[6]The non-bank financial intermediaries in our model include not just narrowly defined shadow banking activity such as wealth-management products and local government financing vehicles, they also include broadly other forms of non-bank financing activity such as private loans and corporate bonds. In China, large and profitable private firms have no difficulties accessing bank loans, but they rely more on non-bank channels such as equity and corporate bond markets for raising funds to avoid implicit taxes through reserve requirements on bank loans.

imperfect substitutes across sectors. This assumption not just capture China's reality with highly restricted labor mobility, it also helps obtain an interior equilibrium allocation of capital between SOEs and POEs despite that POEs have higher steady-state productivity.

The household faces the sequence of budget constraints

$$C_t + I_t + \frac{D_{st} + D_{pt}}{P_t} = w_{st}H_{st} + w_{pt}H_{pt} + r_t^k K_{t-1} + R_{t-1}\frac{D_{s,t-1} + D_{p,t-1}}{P_t} + T_t. \tag{3}$$

where I_t denotes capital investment, D_{st} and D_{pt} denote deposit in banks (to be lent to SOEs) and in nonbank intermediaries (to be lent to POEs), w_{st} and w_{pt} denote the real wage rates in SOEs and in POEs, r_t^k denotes the real rent rate on capital, K_{t-1} denotes the level of capital stock at the beginning of period t, R_{t-1} is the gross nominal interest rate on household savings determined based on period $t-1$ information, P_t denotes the prive level, and T_t denotes the lump-sum transfers from all types of firms and the government.

The capital stock evolves according to the law of motion

$$K_t = (1-\delta)K_{t-1} + [1 - \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2]I_t, \tag{4}$$

where we have assumed that changes in investment incur an adjustment cost with the parameter Ω_k measuring the size of the adjustment costs. The constant g_I denotes the steady-state growth rate of investment.

The household maximizes (1), subject to the constraints (3) and (4). The optimizing conditions are summarized by the following equations:

$$\Lambda_t = \frac{1}{C_t}, \tag{5}$$

$$w_{st} = \frac{\Psi H_t^{\eta-\sigma_L}\mu H_{st}^{\sigma_L}}{\Lambda_t}, \tag{6}$$

$$w_{pt} = \frac{\Psi H_t^{\eta-\sigma_L}(1-\mu)H_{pt}^{\sigma_L}}{\Lambda_t}, \tag{7}$$

$$1 = \mathrm{E}_t\beta R_t\frac{\Lambda_{t+1}}{\Lambda_t\pi_{t+1}}, \tag{8}$$

$$1 = q_t^k\left[1 - \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \Omega_k\left(\frac{I_t}{I_{t-1}} - g_I\right)\frac{I_t}{I_{t-1}}\right] + \beta\mathrm{E}_t q_{t+1}^k\frac{\Lambda_{t+1}}{\Lambda_t}\Omega_k\left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{9}$$

$$q_t^k = \beta\mathrm{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}[q_{t+1}^k(1-\delta) + r_{t+1}^k],. \tag{10}$$

where Λ_t denotes the Lagrangian multiplier for the budget constraint (3), $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the inflation rate from period $t-1$ to period t, and $q_t^k \equiv \frac{\Lambda_t^k}{\Lambda_t}$ is Tobin's q, with Λ_t^k being the Lagrangian multiplier for the capital accumulation equation (4).

II.2. **Retail sector and price setting.** There is a continuum of retailers, each producing a differentiated retail product indexed by $z \in [0,1]$. The retail goods are produced using a homogeneous intermediate input, with a constant-returns technology. Retailers are price takers in the input market and face monopolistic competition in their product markets. They can adjust their prices subject to a quadratic cost, as in Rotemberg (1982).

Denote by $Y_t(z)$ the quantity of retail product of type z and $P_t(z)$ its price. The final consumption good (denoted by Y_t^f) is a Dixit-Stiglitz composite of retail products given by

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)}, \tag{11}$$

where $\epsilon > 1$ denotes the elasticity of substitution between retail goods. The final good producer's optimizing decision implies a downward-sloping demand schedule for each retail product z:

$$Y_t^d(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\epsilon} Y_t^f. \tag{12}$$

The zero-profit condition for the final good producer implies that the price level P_t is related to retail prices by

$$P_t = \left[\int_0^1 P_t(z)^{(1-\epsilon)} dz \right]^{1/(1-\epsilon)}. \tag{13}$$

Production of one unit of retail goods requires one unit of intermediate goods.

Each retailer takes as given the demand schedule (12) and the price level P_t, and set a price $P_t(z)$ to maximize profits. Price adjustments are costly, with the cost function given by

$$\frac{\Omega_p}{2} \left(\frac{P_t(z)}{\pi P_{t-1}(z)} - 1 \right)^2 C_t,$$

where Ω_p measures the size of the adjustment cost and π is the steady-state inflation rate. Retailer z chooses $P_t(z)$ to maximize the expected discounted profits

$$\sum_{i=0}^\infty \beta^i \mathrm{E}_t \Lambda_{t+i} \left[\frac{P_{t+i}(z) - P_{t+i}^w}{P_{t+i}} Y_{t+i}^d(z) - \frac{\Omega_p}{2} \left(\frac{P_{t+i}(z)}{\pi P_{t+i-1}(z)} - 1 \right)^2 C_{t+i} \right], \tag{14}$$

where P_t^w is the nominal price of the intermediate input and $Y_{t+i}^d(z)$ is given by the demand schedule (12).

We focus on a symmetric equilibrium in which $P_t(z) = P_t$ for all z. The optimal price-setting decision implies that

$$\frac{1}{x_t} = \frac{\epsilon - 1}{\epsilon} + \frac{\Omega_p}{\epsilon} \frac{1}{Y_t} \left[\left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} C_t - \beta \mathrm{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} C_{t+1} \right]. \tag{15}$$

where $x_t = P_t/P_t^w$ is the markup of the retail price over the wholesale price.

II.3. **Intermediate goods sectors.** Intermediate goods are produced by firms in both the SOE sector and the POE sector. We present a representative firm's optimizing problem in sector $j \in \{s, p\}$.

In each sector j, firms produce a homogeneous intermediate good using capital K_{jt} and two types of labor inputs— household labor H_{jt} and entrepreneurial labor H_{jt}^e. The representative firm in sector $j \in \{s, p\}$ has access to the production technology

$$Y_{jt} = A_t \bar{A}_j \omega_{jt} (K_{jt})^{1-\alpha} \left[(H_{jt}^e)^{1-\theta} H_{jt}^\theta \right]^\alpha. \tag{16}$$

where Y_{jt} denotes the quantity of output, A_t denotes an aggregate productivity shock, $\bar{A}_j$ measures the average level of total factor productivity (TFP) of sector c, and the parameters $\alpha \in (0, 1)$ and $\theta \in (0, 1)$ are input elasticities in the production technology. The term ω_{jt} is an idiosyncratic productivity shock that is i.i.d. across firms and across time, and it is drawn from the distribution $F(\cdot)$ with a non-negative support.

Aggregate productivity A_t contains a deterministic trend component g^t and a stationary component A_t^m. In particular, we assume that $A_t = g^t A_t^m$, where the stationary component A_t^m follows the AR(1) stochastic process

$$\ln A_t^m = \rho_a \ln A_{t-1}^m + \epsilon_{at}. \tag{17}$$

where we normalize the steady-state level of A^m to unity, $\rho_a \in (-1, 1)$ is a persistence parameter, and the term ϵ_{at} is an i.i.d. innovation to productivity shock drawn from a log-normal distribution $N(0, \sigma_a)$.

Firms face working capital constraints. In particular, they need to pay wage bills and capital rents before production takes place. Firms finance working capital payments by both its beginning-of-period net worth $N_{j,t-1}$ and external debt B_{jt}. The working capital constraint for a firm in sector $j \in \{s, p\}$ is given by

$$\frac{N_{j,t-1} + B_{jt}}{P_t} = w_{jt} H_{jt} + w_{jt}^e H_{jt}^e + r_t^k K_{jt}. \tag{18}$$

where w_{jt}^e is the real wage rate of managerial labor.

Given the working capital constraints in Eq. (18), cost-minimizing implies the factor demand functions

$$w_{jt} H_{jt} = \alpha \theta \frac{N_{j,t-1} + B_{jt}}{P_t}, \tag{19}$$

$$w_{jt}^e H_{jt}^e = \alpha(1 - \theta) \frac{N_{j,t-1} + B_{jt}}{P_t}. \tag{20}$$

$$r_t^k K_{jt} = (1 - \alpha) \frac{N_{j,t-1} + B_{jt}}{P_t}. \tag{21}$$

Substituting these optimal choices of input factors in the production function (16), we obtain the firm's revenue (in final good units)

$$\frac{Y_{jt}}{x_t} = \tilde{A}_{jt}\omega_{jt}\frac{N_{j,t-1} + B_{jt}}{P_t}, \tag{22}$$

where the term $\tilde{A}_{jt}$ is given by

$$\tilde{A}_{jt} = \frac{1}{x_t}A_t\bar{A}_j \left(\frac{1-\alpha}{r_t^k}\right)^{1-\alpha} \left[\left(\frac{\alpha(1-\theta)}{w_{jt}^e}\right)^{1-\theta}\left(\frac{\alpha\theta}{w_{jt}}\right)^\theta\right]^\alpha. \tag{23}$$

We interpret $\tilde{A}_{jt}$ as the rate of return on the firm's investment financed by external debt and internal funds.

II.4. **Financial intermediaries and debt contracts.** At the beginning of each period t, commercial banks obtain household deposits D_{st} at the interest rate R_t. They put aside a fraction τ_t of the deposit as required reserves, which earn no interest and loan out the remaining deposit $B_{st} = (1 - \tau_t)D_{st}$ to SOEs. Since the government guarantees repayments of SOE loans, there is no default risks on bank loans and the banks charge a risk-free loan rate of R_{st}. The commercial banks earn zero profit in equilibrium. However, the reserve requirements drive a wedge between the loan rate and the deposit rate such that

$$(R_{st} - 1)(1 - \tau_t) = (R_t - 1). \tag{24}$$

The funding cost for banks (i.e., the opportunity cost of bank loans) is given by R_{st}.

Nonbank financial intermediaries also obtain household deposits D_{pt} at the interest rate R_t. They lend to the POEs. Since these nonbank intermediaries are not subject to the reserve requirement regulation, their funding cost is given by $R_{pt} = R_t$ and the amount of loans B_{pt} equals the amount of deposit D_{pt}.

Commercial banks and nonbank intermediaries both earn zero profit. They design their debt contract to ensure that they can recover their funding costs R_{st} and R_{pt}. Since lenders can only observe a borrower's realized returns at a cost, they charge a state-contingent gross interest rate Z_{jt} on loans to cover the monitoring and liquidation costs. Under this financial arrangement, firms with sufficiently low levels of realized productivity are not able to make repayments. There is a cut-off level of productivity $\bar{\omega}_{jt}$ such that firms with $\omega_{jt} < \bar{\omega}_{jt}$ choose to default. The default decision is described by,

$$\omega_{jt} < \bar{\omega}_{jt} \equiv \frac{Z_{jt}B_{jt}}{\tilde{A}_{jt}(N_{j,t-1} + B_{jt})}. \tag{25}$$

If the firm fail to make the repayments, the lender pays a liquidation cost and obtains the revenue. In the process of liquidating, a fraction m_{jt} of output is lost. Furthermore, depending on the type of the firm, the government may take over the firm and cover a

fraction l_j of the loan losses financed by lump-sum taxes collected from the households. We also assume that $l_s = 1$ and $l_p = 0$ such that the government covers the entire loss to lenders for SOE defaults but nothing for POE defaults.

We now describe the optimal contract. Under the bond contract featured by $\bar{\omega}_{jt}$ and B_{jt}, the expected nominal income for the type-c firm is given by,

$$\int_{\bar{\omega}_{jt}}^{\infty} \tilde{A}_{jt}\omega_{jt}(N_{j,t-1} + B_{jt})dF(\omega) - (1 - F(\bar{\omega}_{jt}))Z_{jt}B_{jt}$$

$$= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})[\int_{\bar{\omega}_{jt}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_{jt}))\bar{\omega}_{jt}]$$

$$\equiv \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\bar{\omega}_{jt}).$$

where $f(\bar{\omega}_{jt})$ is the share of production revenue going to the low-risk firm under the bond contract.

The expected nominal income for the lender is given by,

$$(1 - F(\bar{\omega}_{jt}))Z_{jt}B_{jt} + \int_{0}^{\bar{\omega}_{jt}} \{(1 - m_{jt})\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})$$

$$+ l_j[Z_{jt}B_{jt} - (1 - m_{jt})\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})]\}dF(\omega)$$

$$= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})\{[1 - (1 - l_j)F(\bar{\omega}_{jt})]\bar{\omega}_{jt} + (1 - m_{jt})(1 - l_j)\int_{0}^{\bar{\omega}_t} \omega dF(\omega)\}$$

$$\equiv \tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_{jt}(\bar{\omega}_{jt}). \tag{26}$$

where $g_j(\bar{\omega}_{jt})$ is the share of production revenue going to the lender under the bond contract with type-c firm. Note that

$$f(\bar{\omega}_{jt}) + g_{jt}(\bar{\omega}_{jt}) = 1 - m_{jt}\int_{0}^{\bar{\omega}_{jt}} \omega dF(\omega) + l_j\int_{0}^{\bar{\omega}_{jt}} [\bar{\omega}_{jt} - (1 - m_{jt})\omega]dF(\omega).$$

The optimal contract is the pair $(\bar{\omega}_{jt}, B_{jt})$ that the firm choooses at the beginning of period t to maximize the expected income in period t,

$$\max \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\bar{\omega}_{jt}) \tag{27}$$

subject to the participation constraint for the lender,

$$\tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_j(\bar{\omega}_{jt}) \geq R_{jt}B_{jt} \tag{28}$$

The optimal condition for the contract characterizes the relation between the leverage ratio and the cut-off productivity as follows,

$$\frac{N_{j,t-1}}{B_{jt} + N_{j,t-1}} = -\frac{g_j'(\bar{\omega}_{jt})}{f'(\bar{\omega}_{jt})}\frac{\tilde{A}_{jt}f(\bar{\omega}_{jt})}{R_{jt}}, \tag{29}$$

Following Bernanke et al. (1999), we assume that a manager in sector $j \in \{s, p\}$ survives at the end of each period with probability ξ_j. Thus, the average lifespan for the firm is $\frac{1}{1-\xi_j}$.

The $1 - \xi_j$ fraction of exiting managers is replaced by an equal mass of new managers, so that the population size of managers stays constant. New managers have start-up funds equal to their managerial labor income $w_{jt}^e H_{jt}^e$. For simplicity, we follow the literature and assume that each manager supply one unit of labor inelastically and the managerial labor is sector specific (so that $H_{jt}^e = 1$ for $j \in \{s, p\}$).

The end-of-period aggregate net worth of all type-c firms consists of profits earned by surviving firms and also managerial labor income. In particular, we have

$$N_{jt} = \xi_j \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\overline{\omega}_{jt}) + P_t w_{jt}^e H_{jt}^e. \tag{30}$$

II.5. **Government policy.** The government conducts monetary policy by following the Taylor rule

$$R_t = \bar{R} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\psi_{rp}} \left(\frac{GDP_t}{GDP_{t-1}g} \right)^{\psi_{rp}}, \tag{31}$$

where $\bar{R}$ and $\bar{\pi}$ denote the steady-state nominal deposit rate and inflation rate, respectively, and the parameters ψ_{rp} and ψ_{ry} are the response coefficients in the interest-rate rule.[7]

In the benchmark economy, we assume that the government fixes the required reserve ratio at $\tau_t = \bar{\tau}$. We will also consider an alternative reserve requirement policy under which the government varies τ_t in response to fluctuations in inflation and output (Section IV.2).

Government spending is financed by lump-sum taxes collected from the household. We assume the ratio of government spending to GDP ($g_t^c \equiv \frac{G_t}{GDP_t}$) is exogenous and follows the stationary stochastic process

$$\ln(g_t^c/g^c) = \rho_g \ln(g_{t-1}^c/g^c) + \epsilon_{gt}. \tag{32}$$

where the parameter g^c is the steady-state ratio of government consumption to GDP, ρ_g is a persistence parameter, and ϵ_t^g is an i.i.d. innovation drawn from a log-normal distribution $N(0, \sigma_g)$.

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[7]In the standard Taylor rule, the nominal interest rate responds to fluctuations of inflation and output gap, where output gap is measured by deviations of real GDP from the welfare-relevant potential output. In our model, there are multiple sources of distortions so that potential output may not be efficient in the flexible-price equilibrium, even if steady-state subsidies are available to offset monopolistic markups. In particular, reserve requirement policy itself introduces additional distortions since it taxes banking and SOE activity. We consider an interest rate rule that responds to fluctuations in inflation and real GDP growth, which is consistent with the PBOC's mandate. The literature has considered interest rate rules that responds to both output gap and real GDP growth (Smets and Wouters, 2007).

II.6. **Market clearing and equilibrium.** The final good is used for consumption, investment, government spending, paying price adjustment costs, and covering bankruptcy costs. Final-good market clearing implies that

$$Y_t^f = C_t + I_t + G_t + \frac{\Omega_p}{2}(\frac{\pi_t}{\pi} - 1)^2 C_t + \tilde{A}_{st}\frac{N_{s,t-1} + B_{st}}{P_t}m_t \int_0^{\bar{\omega}_{st}} \omega dF(\omega)$$
$$+ \tilde{A}_{pt}\frac{N_{p,t-1} + B_{pt}}{P_t}m_t \int_0^{\bar{\omega}_{pt}} \omega dF(\omega). \tag{33}$$

Intermediate goods market clearing implies that

$$Y_t^f = Y_{st} + Y_{pt}. \tag{34}$$

Capital market clearing implies that

$$K_{t-1} = K_{st} + K_{pt}. \tag{35}$$

Bonds market clearing implies that

$$B_{st} = (1 - \tau_t)D_{st}, \quad B_{pt} = D_{pt}. \tag{36}$$

In addition, in specifying the production technologies, we have implicitly imposed the labor market clearing condition.

For convenience of discussion, we define real GDP as the final output net of the costs of firm bankruptcies and price adjustments. In particular, real GDP is defined as

$$GDP_t = C_t + I_t + G_t. \tag{37}$$

We also define two measures of aggregate TFP, one based on gross output and the other based on value added (i.e., GDP). The output-based TFP is defined as

$$\tilde{A}_{Y,t} = \frac{Y_{st} + Y_{pt}}{(K_{st} + K_{pt})^{1-\alpha}H_t^{\alpha\theta}}. \tag{38}$$

The value-added based TFP is defined as

$$\tilde{A}_{GDP,t} = \frac{GDP_t}{(K_{st} + K_{pt})^{1-\alpha}H_t^{\alpha\theta}}. \tag{39}$$

Note that, using firms' optimal production decisions, we can express the output-based aggregate TFP as an average of the SOE TFP and the POE TFP, with the weights determined by the relative hours used by each sector:

$$\tilde{A}_{Y,t} = \mu^{1-\alpha}(\frac{H_{st}}{H_t})^{(1+\sigma_L)(1-\alpha)+\alpha\theta}A_t A_s + (1 - \mu)^{1-\alpha}(\frac{H_{pt}}{H_t})^{(1+\sigma_L)(1-\alpha)+\alpha\theta}A_t A_p.$$

III. Calibration

A period in the model corresponds to a quarter of a year. Parameters are selected to best capture the steady-state ratios and microeconomic evidence in the Chinese economy. Table 1 summarizes the calibrated parameter values.

We follow the literature to set the subjective discount factor to $\beta = 0.996$. We set the steady-state balanced growth rate to $g = 1.0125$, implying an average annual growth rate of 5%. We set the steady-state inflation target $\bar{\pi}$ to 2% per annual. We calibrate the elasticity of substitution between differentiated retail goods ϵ at 10, implying an average gross markup of 11%. We set $g^c = 13\%$ to match the average ratio of China's government consumption to GDP. We set $\Omega_p = 22$, implying an average duration of price contracts of about four quarters.[8]

For the preference parameters in the utility function, we set $\eta = 2$, implying a Frisch labor elasticity of 0.5, which lies in the range of empirical studies. We set Ψ such that the steady state value of total labor hours is about 1/3 of total time endowment (which itself is normalized to one). We set the elasticity of substitution between labor hours supplied to the two sectors to $\sigma_L = 1$ following the estimate of Horvath (2000). We calibrate the share of labor hours used by the SOE sector to $\mu = 0.5$, consistent with the data in Chinese industrial sector.[9]

Regarding the technological parameters, we set the capital depreciation rate δ to 0.035, implying an annual depreciation rate of 14%. We have less guidance for calibrating the investment adjustment cost parameter Ω_k. We use $\Omega_k = 3$ as a benchmark, which lies in the range of empirical estimates of DSGE models (Christiano et al., 2005; Smets and Wouters, 2007). For the production technology, we calibrate the labor income share to $\alpha = 0.5$, consistent with empirical evidence in Chinese data (Brandt et al., 2008; Zhu, 2012). Out of the total labor income, we calibrate the share of household labor to $\theta = 0.94$; accordingly, the managerial labor share is 0.06.

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[8]Log-linearizing the optimal pricing decision equation (15) around steady state leads to a linear form of Phillips curve relation with the slope of the Phillips curve given by $\kappa = \frac{\epsilon-1}{\Omega_p}\frac{C}{Y}$. Our calibration implies a steady state ratio of consumption to gross output of about 48%. The values of $\epsilon = 5$ and $\Omega_p = 22$ imply that $\kappa = 0.086$. In an economy with Calvo-type price contracts, the slope of the Phillips curve is given by $(1-\beta\alpha_p)(1-\alpha_p)/\alpha_p$ where α_p is the probability that a firm cannot re-optimize prices. To obtain a slope of 0.086 for the Phillips curve in the Calvo model, α_p must be set equal to 0.75, which corresponds to an average duration of price contracts of about four quarters.

[9]Both the employment share and the operating cost share of SOEs in the industrial sector are around 0.5 on average from 2007 to 2014, which implies $\frac{H_s}{H_p} = 1$ and $\frac{N_s+B_s}{N_p+B_p} = 1$ in the model. Putting these numbers into Eq.(6),(7) and (19), we have: $\frac{\mu}{1-\mu} = \frac{w_s}{w_p}\left(\frac{H_s}{H_p}\right) = \frac{w_s}{w_p} = \frac{N_s+B_s}{N_p+B_p} = 1$, which leads to $\mu = 0.5$.

We assume that the idiosyncratic productivity shocks ω are drawn from a Pareto distribution with the cumulative density function $F(\omega) = 1 - (\frac{\omega_m}{\omega})^k$ over the range $[\omega_m, \infty)$. We calibrate the scale parameter ω_m and the shape parameter k to match empirical estimates of cross-firm dispersions of TFP in China's data. In particular, Hsieh and Song (2015) estimated that the standard deviation of logarithm of TFP across firms is around 1.2, implying a standard deviation of the level of TFP across firms of about 3.22 (assuming that the TFP level is log-normally distributed with mean of one). We set $k = 2.14$ and $\omega_m = 0.53$ such that $var(\omega) = 3.22$. We normalize the scale of SOE TFP to $\bar{A}_s = 1$ and calibrate the scale of POE TFP parameter to target the average ratio of SOE output to POE output of 0.3 in the data. This implies that the relative TFP level of the POE sector is $\bar{A}_p = 1.58$. This average TFP gap is also consistent with the empirical evidence obtained in the TFP-accounting literature.[10]

For the parameters associated with financial frictions, we follow Bernanke et al. (1999) and set the liquidation cost parameters to $m_s = m_p = 0.15$. We set the SOE manager's survival rate to $\xi_s = 0.98$, implying an average term for the SOE manager of around 16 years. We set the POE manager's survival rate to $\xi_p = 0.62$, implying an average term of around 8 months. These survival rates are chosen to target the steady state outcome that the annual bankruptcy ratio is around 0.25 for both SOEs and POEs.[11]

For the monetary policy parameters, we set the required reserve ratio to $\tau = 0.15$. We set the Taylor rule parameters to $\psi_{rp} = 1.5$ and $\psi_{ry} = 0.5$.

We consider two shocks: an aggregate TFP shock that follows an AR(1) stochastic process with a persistence parameter of 0.95 and a standard deviation of the innovation of 0.0014, and a government consumption shock that follows an AR(1) stochastic process with a persistence parameter of 0.95 and a standard deviation of the innovation of 0.045. The persistence and the standard deviation of the government spending shock are obtained from an AR(1) regression using Chinese aggregate data from 2000Q1 to 2015Q4. The persistence of the TFP shock is calibrated following the estimate of Liu (2008). The standard deviation of the TFP shock is calibrated to target the real GDP volatility in the data from 2000Q1 to 2015Q4 in the simulated benchmark model.

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[10]Brandt and Zhu (2010) estimatae TFP gap of 2.3 in 2004. Using a different methodology, Hsieh and Klenow (2009) estimate a "revenue-TFP gap" of 1.42.

[11]The NBS industrial survey reports that annual fraction of industrial firms that earns negative profits is around 24% for SOEs and 10% for POEs. However, the number is likely to be underestimated, especially for POEs, because the NBS industrial survey collects information from large industrial firms and it excludes the service sector, in which many more POEs operate than SOEs.

IV. Quantitative results

We now illustrate the implications of adjusting reserve requirements (τ) for aggregate productivity and welfare in the calibrated model. We study how τ affect both the steady-state equilibrium and aggregate dynamics. We find that reserve requirement policy is important in both cases.

IV.1. **Optimal steady-state reserve requirements.** We begin by exploring how steady-state equilibrium allocations and welfare depend on the required reserve ratio. We focus on the deterministic steady-state equilibrium, in which all exogenous shocks are turned off. As we have discussed above, reserve requirements act like a tax on SOE activity since SOEs rely on bank credit for external financing. An increase in the reserve requirements thus diverts resources from SOEs to POEs. Since POEs are on average more productive than SOEs, this resource reallocation raises aggregate TFP. However, an increase in reserve requirements also raises the incidents of SOE bankruptcies; although banks do not suffer from loan losses with government guarantees, SOE bankruptcies are socially costly. Changing reserve requirements thus incurs a tradeoff between allocation efficiency and bankruptcy costs.

This tradeoff is illustrated in Figure 3, which displays the relations between the steady-state required reserve ratio (τ) and the levels of several macroeconomic variables. The figure also shows the welfare gains associated with different values of τ relative to the steady-state level of $\tau = 0.15$. Consistent with the mechanism described above, an increase in τ reduces SOE output relative to POE output. As resources are reallocated from SOEs to POEs, aggregate TFP rises. However, with increased funding costs, the bankruptcy rate of SOEs rises. The increase in costly bankruptcies reduces the resources available for consumption and investment and thus may reduce welfare.

The tradeoff between efficiency gains and bankruptcy losses implies that there should be an interior optimum for the required reserve ratio that maximizes social welfare. Under our calibration, this is indeed the case. As shown in the lower-right panel of Figure 3, the representative household's steady-state welfare has a hump-shaped relation with τ and reaches the maximum at $\tau^* = 0.73$.

IV.2. **Optimal simple policy rules.** We have shown that reserve requirement policy plays an important role in reallocating resources between SOEs and POEs in the steady state. We now examine the effectiveness of reserve requirement policy for macroeconomic stabilization over the business cycles.

We consider two types of shocks —an aggregate TFP shock and a government spending shock. The central bank can use either the nominal deposit rate or the required reserve ratio (or both) as a policy instrument for stabilizing macroeconomic fluctuations. We assume that

the central benk follows simple rules, under which the relevant policy instrument (R or τ) is adjusted to respond to fluctuations in inflation and real GDP growth.

As a benchmark, we assume that the central bank follows the standard Taylor rule for the nominal deposit rate and keeps the required reserve ratio constant. Relative to this benchmark policy regime, we evaluate the performance of three counterfactual policy regimes for macroeconomic stability and social welfare: an optimal interest-rate rule, an optimal reserve-requirement rule, and jointly optimal rules.

Specifically, the interest rate rule is given by in Eq (31), which we rewrite here in logarithm form:

$$\ln\left(\frac{R_t}{R}\right) = \psi_{rp}\ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \psi_{ry}\ln\left(\frac{GDP_t}{GDP_{t-1}g}\right). \tag{40}$$

The reserve-requirement rule takes a similar form:

$$\ln\left(\frac{\tau_t}{\tau}\right) = \psi_{\tau p}\ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \psi_{\tau x}\ln\left(\frac{GDP_t}{GDP_{t-1}g}\right), \tag{41}$$

where the parameters $\psi_{\tau p}$ and $\psi_{\tau y}$ measure the responsiveness of the required reserve ratio to changes in inflation and real GDP growth.

Under the optimal interest-rate rule, the reaction coefficients ψ_{rp} and ψ_{ry} in (40) are set to maximize the representative household's welfare, while the required reserve rate is kept at the benchmark value (i.e., $\tau_t = \tau$). Under the optimal reserve-requirement rule, the reaction coefficients $\psi_{\tau p}$ and $\psi_{\tau y}$ are set to maximize welfare, while the interest rate follows the benchmark Taylor rule in (40), with $\psi_{rp} = 1.5$ and $\psi_{ry} = 0.5$ fixed. Under the jointly optimal rule, all 4 reaction coefficients ψ_{rp}, ψ_{ry}, $\psi_{\tau p}$, and $\psi_{\tau y}$ are optimally set to maximize welfare.

We measure welfare gains under each counterfactual policy relative to the benchmark model as the percentage change in permanent consumption such that the representative household is indifferent between living in an economy under a given optimal policy rule and in the benchmark economy. Denote by C_t^b and H_t^b the allocations of consumption and hours worked under the benchmark policy regime. Denote by V^a the value of the household's welfare obtained from the equilibrium allocations under an alternative policy regime. Then, the welfare gain under the alternative policy relative to the benchmark is measured by the constant Δ, which is implicitly solved from

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t^b(1+\Delta)) - \Psi\frac{(H_t^b)^{1+\eta}}{1+\eta}\right] = V^a. \tag{42}$$

IV.2.1. *Macroeconomic stability and welfare under alternative policy rules.* Table 2 shows the macroeconomic volatilities under the four different policy regimes and also the welfare gains under each optimal simple rule relative to the benchmark regime.

Under the optimal reserve-requirement rule, the required reserve ratio τ_t increases with inflation and decreases with real GDP growth. With government guarantees of SOE debt, bank loans are risk-free and the standard BGG type of financial accelerator mechanism is muted for SOE activity. But there is no such guarantees of POE debt and the loan rate to POEs reflects a credit spread over the funding cost to cover the monitoring costs. To mitigate inefficient fluctuations in the POE sector associated with the financial accelerator mechanism, a lower τ_t is called for to shift resources from POEs to SOEs when the economy experiences an expansion (i.e., when real GDP increases). Thus, optimal τ_t decreases with real GDP growth. Since the central bank keeps the interest-rate rule parameters at their benchmark values, adjustments in reserve requirements also serve to stabilize inflation. An increase in inflation calls for a policy tightening through raising τ_t (beyond the automatic tightening through the Taylor rule). The optimal reserve-requirement rule leads to moderate welfare gains relative to the benchmark policy with a constant required reserve ratio.

The optimal interest-rate rule is more aggressive against inflation fluctuations than the benchmark policy, although it assigns a smaller weight to real GDP growth. As shown in Table 2, the optimal interest-rate rule produces better macroeconomic stability and higher welfare than both the benchmark policy and the optimal reserve-requirement rule.

Under the jointly optimal policy rule, the central bank can adjust the nominal interest rate to stabilize macroeconomic fluctuations. At the same time, it can adjust the required reserve ratio to reallocate resources between the two sectors to mitigate the financial accelerator effects in the POE sector. Thus, the jointly optimal rule achieves superior outcomes for both macroeconomic stability and social welfare than the benchmark policy. It also outperforms each individually optimal rule. This finding suggests that the nominal interest rate and the required reserve ratio are complementary policy instruments for macroeconomic stabilization and allocative efficiency.

IV.2.2. *The economic mechanism.* To help understand the economic mechanism underlying our quantitative results, we examine the impulse responses of several key macroeconomic variables and sector-level variables following each of the two types of shocks.

First, consider the dynamic effects of a positive TFP shock. Figure 4 displays the impulse responses of real GDP, inflation, the nominal deposit rate, and the required reserve ratio following the shock under the benchmark policy and the three alternative policy regimes. Figure 5 shows the impulse responses of output, leverage, the bankruptcy rates, and credit spreads in each of the two sectors.

In the benchmark economy (the black solid lines), a positive TFP shock raises real GDP and lowers inflation. Under the benchmark policy regime, the nominal deposit rate declines to accommodate the fall of inflation while the required reserve ratio stays constant.

The decline in the nominal deposit rate lowers funding costs for both banks and nonbank intermediaries. However, SOE debts are guaranteed by the government, so that bank loans are free from default risks. The bank loan rate to SOEs is simply a constant markup over the deposit rate, with the wedge determined by the constant required reserve ratio τ. Thus, SOE credit spread does not respond to changes in macroeconomic conditions, neither does the SOE leverage ratio under our calibration (with the idiosyncratic productivity shocks drawn from the Pareto distribution). With a constant credit spread and leverage, the financial accelerator mechanism of the BGG framework is muted for the SOE sector. In contrast, the BGG financial accelerator mechanism works for the POE sector since there is no government guarantee of the POE debt. The decline in the deposit rate following the decline in inflation leads to an expansion of POE leverage, which amplifies the responses of POE output to the TFP shock. As shown in Figure 5, the leverage ratio and the bankruptcy ratio in the POE sector both rises following the TFP shock, as does POE output.

Under the optimal reserve-requirement rule (the red dashed lines in the figures), the TFP shock raises real GDP and lowers inflation and the nominal deposit rate, similar to the benchmark model. Now, the central bank can adjust the reserve requirement to alleviate the social cost of POE bankruptcies associated with the expansion in leverage. But since the central bank cannot optimize its interest-rate policy, it relies heavily on changing reserve requirements to mitigate the impact of the TFP shock. In particular, the central bank raises the required reserve ratio τ_t, so that the bank lending rate rises, partially offsetting the decline in real marginal cost following the TFP shock. The increase in τ shifts capital from SOEs to POEs, as shown in Figure 5, and it leads to a greater expansion in POE leverage and more bankruptcies in both sectors than in the benchmark model. The increase in bankruptcy losses implies that less resources are available for consumption and investment, leading to a more muted increase in real GDP than in the benchmark model, as shown in Figure 4. But since real marginal cost declines less, the optimal reserve requirement policy also mitigates the drop in inflation following the TFP shock.

Now consider the impulse responses under the optimal interest-rate rule. As shown in Figure 4, the optimal interest rate rule (the blue dashed lines) is more effective in stabilizing fluctuations in inflation and real GDP than the benchmark policy, which is consistent with the simulation results reported in Table 2. However, since the real interest rate declines more than that in the benchmark model, the policy leads to more expansion in POE leverage and a greater increase in POE bankruptcies (see Figure 5). Overall, however, the gain from macroeconomic stability under the optimal interest rate rule outweighs the loss from increased POE bankruptcies, leading to moderate welfare gains relative to the benchmark (see Table 2).

Under the jointly optimal policy rule (the magenta dashed lines in the figures), the required reserve ratio declines on impact along with the nominal interest rate. The easing of monetary policy through both instruments alleviates the decline in inflation further compared to the benchmark policy and also individually optimal policy rules. More importantly, the reduction in τ shifts capital from POEs to SOEs to mitigate the financial accelerator effects in the POE sector and thereby reducing bankruptcy losses (see Figure 5). This policy leads to a greater expansion of real GDP than in the benchmark model. Since the interest-rate rule parameters are also optimized, the jointly optimal rule leads to smaller declines in inflation than the optimal reserve-requirement rule. As shown in Table 2, the jointly optimal policy achieves better macroeconomic stability and higher welfare than the benchmark policy and each individually optimal policy rule.

Next, consider the dynamic effects of a positive government spending shock. The impulse responses are shown in Figures 6 and 7.

Since the shock leads to an increase in aggregate demand, both real GDP and inflation rises in the benchmark model (the black solid lines). Under the benchmark policy regime, the nominal deposit rate increases. Banks pass through the increase in deposit rate to loan rate that they charge SOEs. Although SOE leverage rate does not respond to the shock because of government guarantees of SOE debt, the increase in funding costs slightly raises the SOE bankruptcy rate.

Under the optimal reserve-requirement rule (the red dashed lines), the central bank raises the required reserve ratio to alleviate the SOE bankruptcy losses. Increases in reserve requirements shift capital from the SOE sector to the POE sector, reducing the SOE output relative to POEs (see Figure 7). Accordingly, the POE leverage and bankruptcy rates both more than in the benchmark. The increase in bankruptcy costs is partially offset by the improvement in aggregate productivity associated with the reallocation effects of the policy.

The optimal interest-rate rule (the blue dashed lines) is much more effective for stabilizing inflation than the benchmark policy (see Figure 6). However, since the the optimal rule is more aggressive against inflation, the real interest rate rises more than under the benchmark policy. The aggressive tightening through interest-rate adjustments depresses POE activity and leverage relative to the benchmark policy regime, as shown in Figure 7. Similar to the benchmark model, the government spending shock has relative muted effects on SOE activity.

Under the jointly optimal policy, the central bank uses the interest rate to stabilize inflation. At the same time, it adjusts the required reserve ratio to reallocate resources between SOEs and POEs to cut down bankruptcy losses. As shown in Figures 6 and 7 (the magenta

dashed and dotted lines), the deposit rate rises in response to a positive government spending shock. The required reserve ratio initially drops and subsequently rises sharply. These policy actions together lead to smaller inflation fluctuations and less bankruptcy losses.

These impulse responses illustrate the tradeoff between allocation efficiencies and bankruptcy costs associated with changes in reserve requirements. Overall, the interest-rate policy is effective for stabilizing fluctuations in real GDP and inflation, while the reserve requirements are more effective for stabilizing sectoral allocations at the business cycle frequencies. In particular, the use of reserve requirements helps partially offset inefficient fluctuations in POE activity relative to SOE activity, given that the financial accelerator mechanism is present only in the POE sector.

V. Conclusion

We have studied the role of adjusting RR as a policy instrument to offset or alleviate other distortions in a two-sector DSGE model with Chinese characteristics. Our model builds on the standard financial accelerator model of Bernanke et al. (1999) and generalizes to include two key forms of frictions. First, the model features segmented credit markets, in which SOE firms are able to obtain bank loans, while POE firms have to rely on shadow bank lending. Second, and more importantly, the government provides guarantees for bank loans to SOE firms, but not to shadow bank lending. We show that government guarantees of SOE loans are an important source of distortions and that adjustments in reserve requirements can be an effective second-best policy. In particular, our analysis here suggests that RR adjustments can not only alleviate steady-state distortions but also help stabilize business cycle fluctuations.

Our finding that the central bank's use of reserve requirements to alleviate distortions caused by government guarantees of SOE loans suggests that a more effective reform is to reduce or eliminate such guarantees. More broadly, it calls for coordination between bank regulatory and monetary policy.

Our model is a closed economy environment, where private firms rely on domestic shadow banking loans to finance their operation. This is a good approximation to China's current financial system because China has maintained tight controls over the capital account, so that it is difficult for domestic firms to obtain foreign funding. The Chinese government has set out plans to loosen capital controls. Similar to the shadow banking sector in our model, having improved access to foreign funds would help make financing POE production more readily available and, to the extent that private firms remain more productive than SOE firms, improve overall allocation efficiency in China. However, opening to foreign asset markets may also crowd out some domestic shadow banking activity, although risk may be

better diversified with foreign lenders present. A full analysis of the consequences of opening the capital account in such an environment requires an open-economy model with these features. Future research along that line should be promising.

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Table 1. Calibrated values.

Variable	Description	Value
	A. Households	
β	Household discount rate	0.996
η	Inverse Frisch elasticity of labor supply	2
Ψ	Weight of disutility of working	23
σ_L	Substitutability of labor between SOE and POE	0.8
μ	Share of labor supplied to SOEs	0.5
Ω_k	Capital adjustment cost	1
δ	Capital depreciation rate	0.035
g	Steady state growth rate	1.0125
π	Steady state inflation rate	1.005
	B. Retailers	
ϵ	Elasticity of substitution between differentiated retail goods	10
Ω_p	Price adjustment cost	22
	C. Firms	
k	Shape parameter for Pareto distribution of idiosyncratic shock	2.14
ω_m	Scale parameter for Pareto distribution of idiosyncratic shock	0.53
α	Capital income share	0.5
θ	Share of household labor	0.94
A_s	SOE TFP scale (normalized)	1
A_p	POE TFP scale	1.58
m_s	SOE monitoring cost	0.15
m_p	POE monitoring cost	0.15
ξ_s	SOE manager's survival rate	0.99
ξ_p	POE manager's survival rate	0.62
	D. Government policy	
l_s	Fraction of losses guaranteed by government for bank loans to SOEs	1
l_p	Fraction of losses guaranteed by government for shadow bank loans to POEs	0
τ	Required reserve ratio	0.15
ψ_{rp}	Response coefficient to inflation in interest rate rule	1.5
ψ_{ry}	Response coefficient to GDP growth in interest rate rule	0.5
	E. Steady state targets	
Y_s/Y_p	Ratio of SOE output to POE output	0.3
W_s/W_p	Ratio of SOE wage to POE wage	1
$F(\overline{\omega_s})$	SOE bankruptcy ratio	0.25/4
$F(\overline{\omega_p})$	POE bankruptcy ratio	0.25/4
	F. Shock process	
ρ_a	Persistence of TFP shock	0.95
ρ_g	Persistence of government spending shock	0.95
σ_a	Standard deviation of TFP shock	0.01
σ_g	Standard deviation of government consumption shock	0.045

TABLE 2. Volatilities and welfare under alternative policy rules

Variables	Benchmark	Optimal τ rule	Optimal R rule	Jointly optimal rule
Policy rule coefficients				
ψ_{rp}	1.50	1.50	1.93	1.51
ψ_{ry}	0.50	0.50	0.32	-0.14
$\psi_{\tau p}$	0.00	374	0.00	232
$\psi_{\tau y}$	0.00	417	0.00	-913
Volatility				
GDP	5.351%	5.375%	5.321%	5.325%
π	0.617%	0.598%	0.381%	0.398%
C	4.956%	4.954%	4.926%	4.925%
H	0.749%	0.723%	0.792%	0.855%
R	0.525%	0.511%	0.475%	0.724%
Y_s	5.374%	5.412%	5.363%	6.887%
Y_p	5.468%	5.534%	5.493%	5.438%
Welfare				
Welfare gains	—	0.019%	0.023%	0.493%

Note: The welfare gain under each optimal policy rule is the consumption equivalent relative to the benchmark economy (see the text in Section IV.2 for details).

China required reserve ratio

Percent

Source:Bloomberg

FIGURE 1. China's required reserve ratio (daily frequencies).

FIGURE 2. Impulse responses to a shock to the required reserve ratio estimated from the BVAR model.
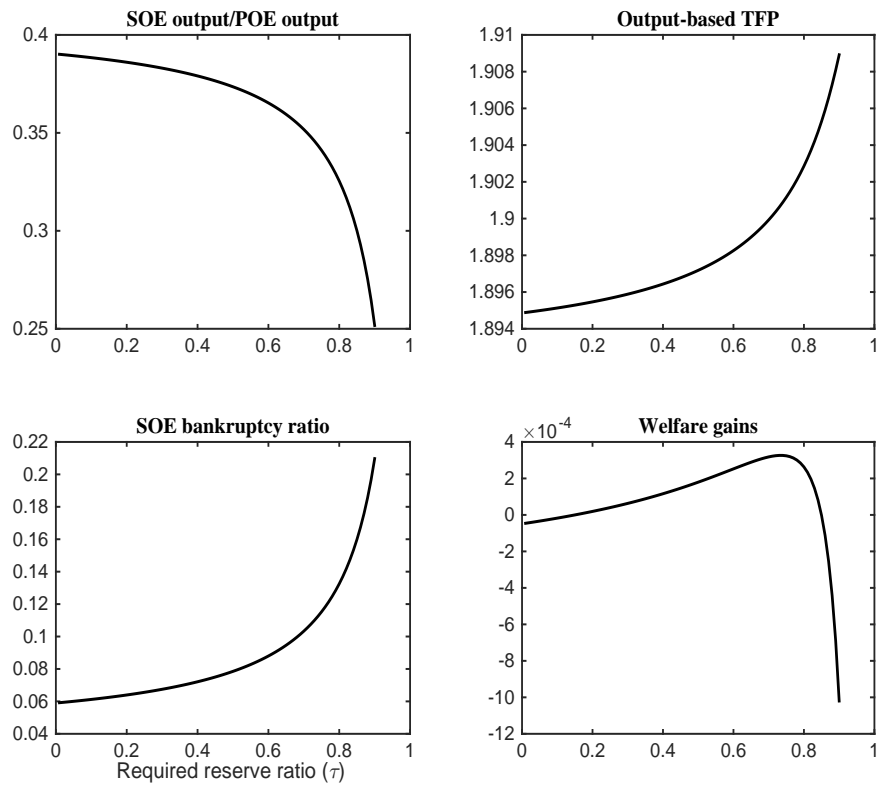
FIGURE 3. Steady-state implications of the required reseve ratio (τ) for macroeconomic variables and welfare. Welfare gains are measured as consumption equivalent relative to the steady state in the benchmark model with $\tau = 0.15$.
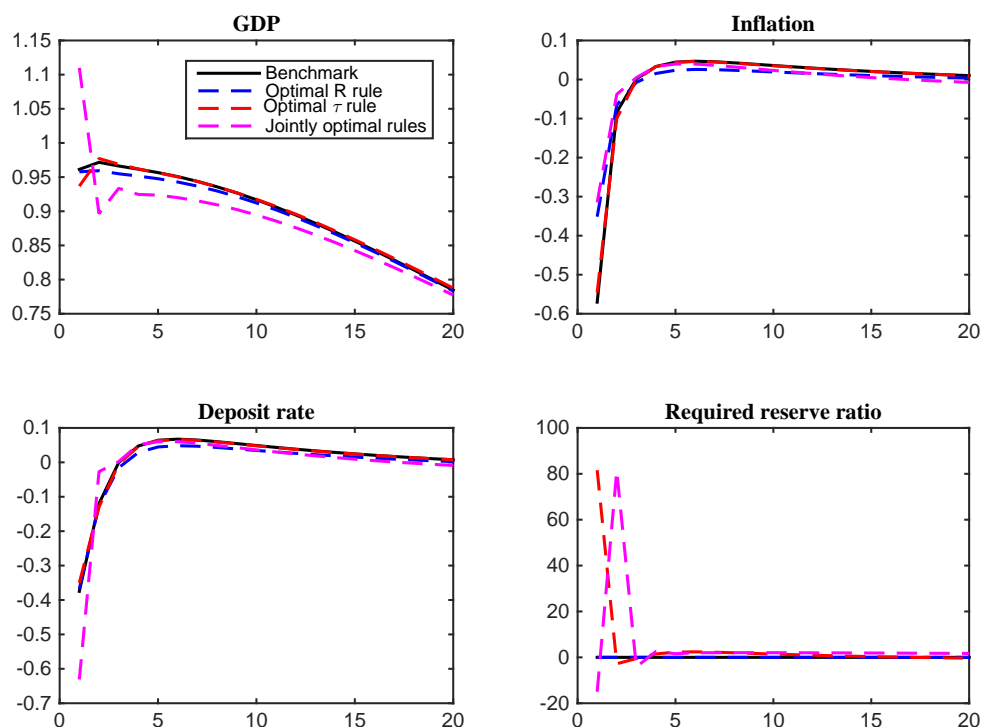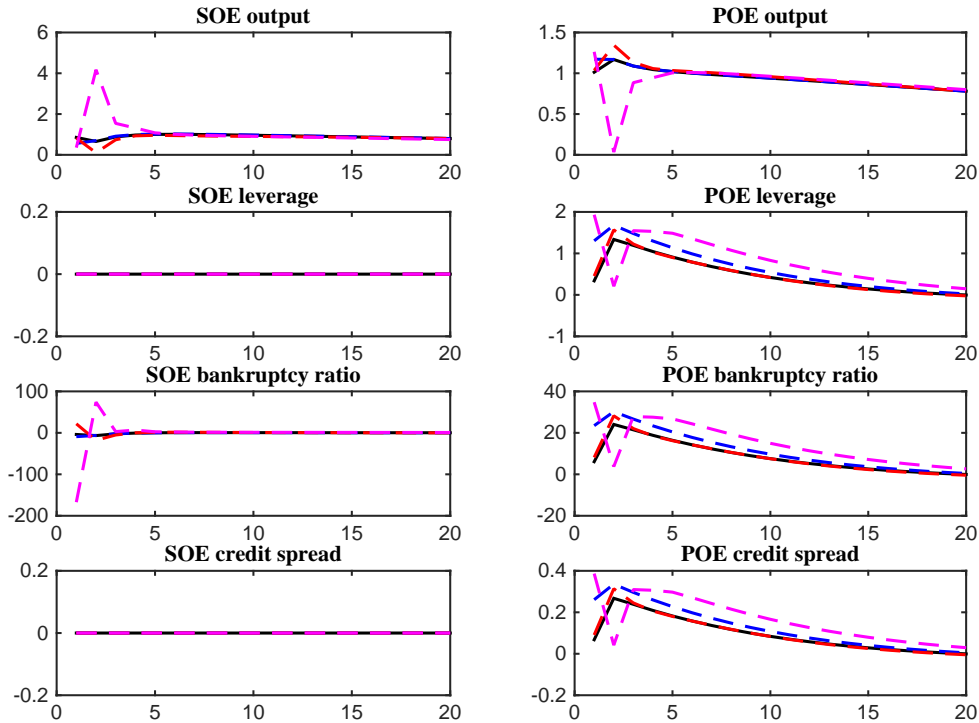
FIGURE 4. Impulse responses of aggregate variables to a positive TFP shock under alternative policy rules. Benchmark rule: black solid lines; optimal interest rate rule: blue dashed lines; optimal reserve requirement rule: red dashed lines; jointly optimal rule: magenta dashed-dotted lines. The vertical-axis unit of the required reserve ratio is the percentage-point deviations from the steady state level. The vertical-axis units for all other variables are percent deviations from the steady state levels. The variables displayed include real GDP (GDP_t), inflation (π_t), the nominal deposit rate (R_t), and the required reserve ratio (τ_t).

Impulse responses to TFP shock

FIGURE 5. Impulse responses of sector-specific variables to a positive TFP shock under alternative policy rules. Benchmark rule: black solid lines; optimal interest rate rule: blue dashed lines; optimal reserve requirement rule: red dashed lines; jointly optimal rule: magenta dashed-dotted lines. The vertical-axis units are percent deviations from the steady state levels. The variables displayed include SOE output (Y_{st}), POE output (Y_{pt}), SOE leverage ratio (B_{st}/N_{st}), POE leverage ratio (B_{pt}/N_{pt}), SOE bankruptcy ratio $(F(\overline{\omega}_{st}))$, POE bankruptcy ratio $(F(\overline{\omega}_{pt}))$, SOE credit spread (Z_{st}/R_t), and POE credit spread (Z_{pt}/R_t).

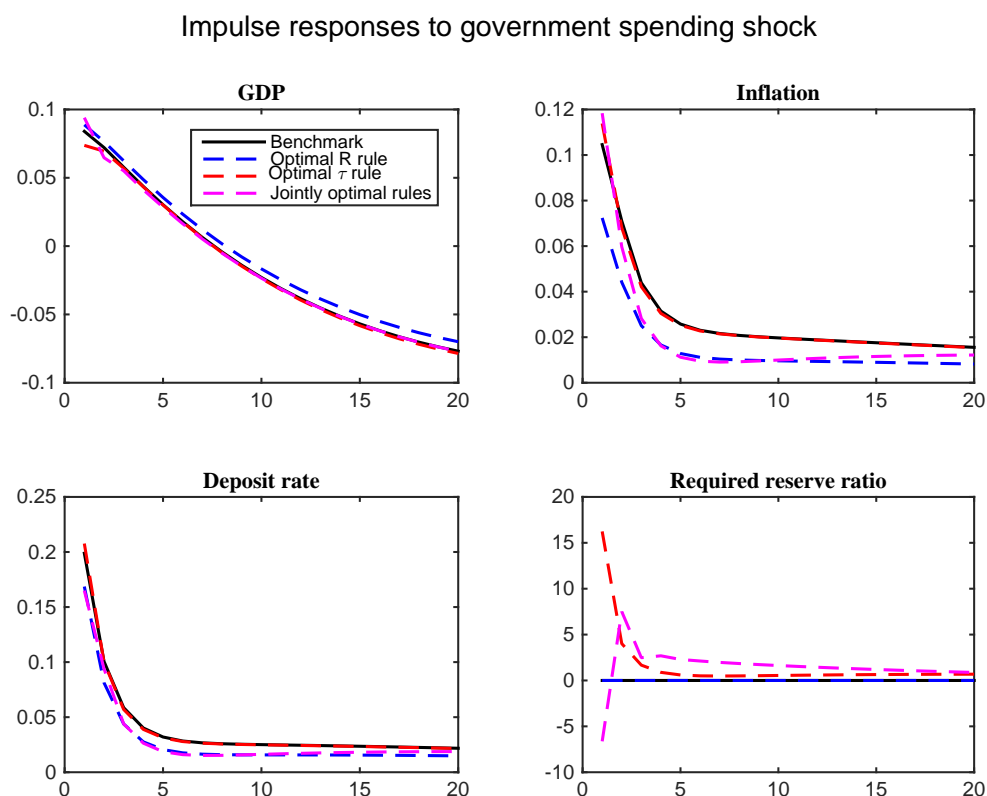Impulse responses to government spending shock

FIGURE 6. Impulse responses of aggregate variables to a positive government spending shock under alternative policy rules. Benchmark rule: black solid lines; optimal interest rate rule: blue dashed lines; optimal reserve requirement rule: red dashed lines; jointly optimal rule: magenta dashed-dotted lines. The vertical-axis unit of the required reserve ratio is the percentage-point deviations from the steady state level. The vertical-axis units for all other variables are percent deviations from the steady state levels. The variables displayed include real GDP (GDP_t), inflation (π_t), the nominal deposit rate (R_t), and the required reserve ratio (τ_t).

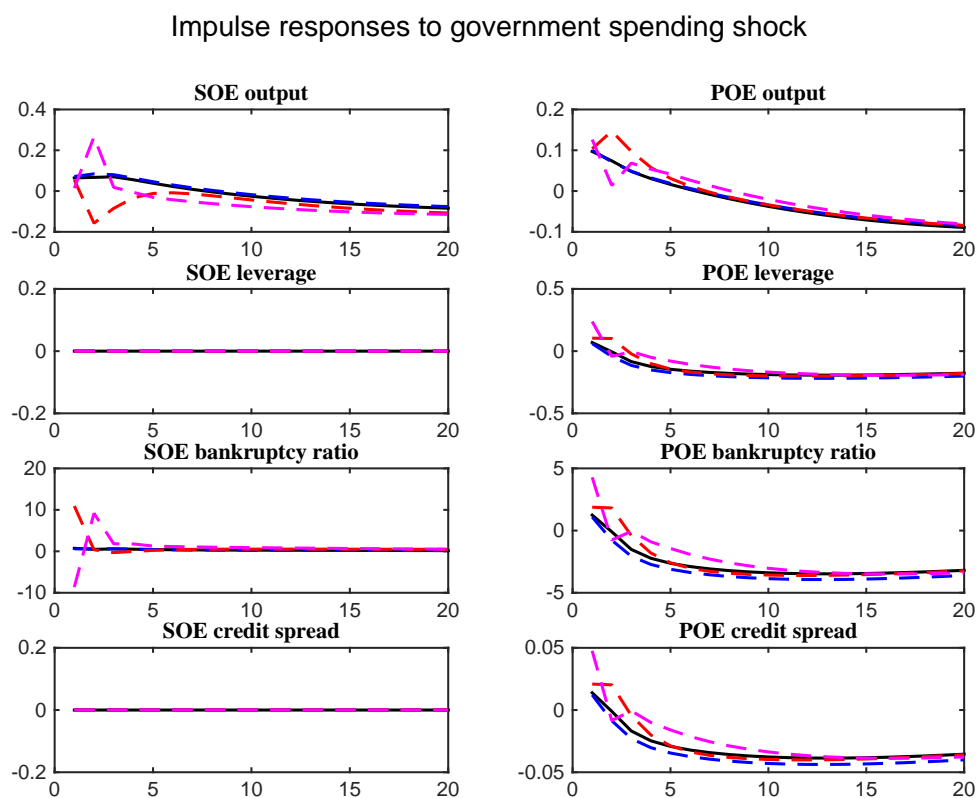Impulse responses to government spending shock

FIGURE 7. Impulse responses of sector-specific variables to a positive government spending shock under alternative policy rules. Benchmark rule: black solid lines; optimal interest rate rule: blue dashed lines; optimal reserve requirement rule: red dashed lines; jointly optimal rule: magenta dashed-dotted lines. The vertical-axis units are percent deviations from the steady state levels. The variables displayed include SOE output (Y_{st}), POE output (Y_{pt}), SOE leverage ratio (B_{st}/N_{st}), POE leverage ratio (B_{pt}/N_{pt}), SOE bankruptcy ratio $(F(\overline{\omega}_{st}))$, POE bankruptcy ratio $(F(\overline{\omega}_{pt}))$, SOE credit spread (Z_{st}/R_t), and POE credit spread (Z_{pt}/R_t).

APPENDIX A. BALANCED-GROWTH PATH EQUILIBRIUM CONDITIONS

On a balanced growth path, output, consumption, investment, real bank loans and real wage rates all grow at a constant rate g. To obtain balanced growth, we make the stationary transformations

$$y_t = \frac{Y_t}{g^t}, c_t = \frac{C_t}{g^t}, i_t = \frac{I_t}{g^t}, k_t = \frac{K_t}{g^t}, \lambda_t = \Lambda_t g^t, \lambda_t^k = \Lambda_t^k g^t,$$

$$y_{st} = \frac{Y_{st}}{g^t}, y_{pt} = \frac{Y_{pt}}{g^t}, k_{st} = \frac{K_{st}}{g^t}, k_{pt} = \frac{K_{pt}}{g^t}, \tilde{w}_{st} = \frac{w_{st}}{g^t}, \tilde{w}_{pt} = \frac{w_{pt}}{g^t}, \tilde{w}_{s,e,t} = \frac{w_{s,e,t}}{g^t}, \tilde{w}_{p,e,t} = \frac{w_{p,e,t}}{g^t},$$

$$n_{st} = \frac{N_{st}}{P_t g^t}, n_{pt} = \frac{N_{pt}}{P_t g^t}, b_{st} = \frac{B_{st}}{P_t g^t}, b_{pt} = \frac{B_{pt}}{P_t g^t}.$$

On the balanced growth path, the transformed variables, the interest rate and the inflation rate are all constants.

The balanced growth equilibrium is summarized by the following equations:

1) Households.

$$k_t = \tfrac{1-\delta}{g} k_{t-1} + i_t [1 - \tfrac{\Omega_k}{2} (\tfrac{i_t g}{i_{t-1}} - g)^2], \tag{A1}$$

$$\lambda_t = \tfrac{1}{c_t}, \tag{A2}$$

$$\lambda_t \tilde{w}_{st} = \Psi H_t^{\eta - \sigma_L} \mu H_{st}^{\sigma_L}, \tag{A3}$$

$$\lambda_t \tilde{w}_{pt} = \Psi H_t^{\eta - \sigma_L} (1 - \mu) H_{pt}^{\sigma_L}, \tag{A4}$$

$$\lambda_t = \beta \lambda_{t+1} \tfrac{R_t}{\pi_{t+1} g}, \tag{A5}$$

$$\lambda_t = \lambda_t^k [1 - \tfrac{\Omega_k}{2} (\tfrac{i_t g}{i_{t-1}} - g)^2 - \Omega_k (\tfrac{g i_t}{i_{t-1}} - g) \tfrac{i_t g}{i_{t-1}}] + \Omega_k \beta \tfrac{\lambda_{t+1}^k}{g} (\tfrac{i_{t+1} g}{i_t} - g)(\tfrac{i_{t+1} g}{i_t})^2, \tag{A6}$$

$$\lambda_t^k = \beta [(1-\delta) \tfrac{\lambda_{t+1}^k}{g} + \tfrac{\lambda_{t+1}}{g} r_{t+1}^k]. \tag{A7}$$

2) Firms and banks.

$$y_{st} = a_t \bar{A}_s k_{st}^{1-\alpha} (H_{st}^\theta)^\alpha, \tag{A8}$$

$$\tilde{w}_{st} H_{st} = \alpha \theta (\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}), \tag{A9}$$

$$\tilde{w}_{s,e,t} = (\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}) \alpha (1 - \theta), \tag{A10}$$

$$k_{st} r_t^k = (1 - \alpha)(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}), \tag{A11}$$

$$\tilde{A}_{st} = \tfrac{y_{st}}{x_t} \tfrac{1}{\frac{n_{s,t-1}}{\pi_t g} + b_{st}}, \tag{A12}$$

$$\tilde{A}_{st}(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st})g_s(\overline{\omega}_{st}) = b_{st}R_{st}, \tag{A13}$$

$$\frac{\frac{n_{s,t-1}}{\pi_t g}}{\frac{n_{s,t-1}}{\pi_t g} + b_{st}} = -\frac{g_s'(\overline{\omega}_{st})f(\overline{\omega}_{jt})}{f'(\overline{\omega}_{st})}\frac{\tilde{A}_{st}}{R_{st}}, \tag{A14}$$

$$n_{st} = \tilde{w}_{s,e,t} + \xi_s \tilde{A}_{st}(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st})f(\overline{\omega}_{st}), \tag{A15}$$

$$y_{pt} = a_t \bar{A}_p k_{pt}^{1-\alpha}(H_{pt}^\theta)^\alpha, \tag{A16}$$

$$\tilde{w}_{pt}H_{pt} = \alpha\theta(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A17}$$

$$\tilde{w}_{p,e,t} = \alpha(1-\theta)(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A18}$$

$$r_t^k k_{pt} = (1-\alpha)(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A19}$$

$$\tilde{A}_{pt} = \frac{y_{pt}}{x_t}\frac{1}{\frac{n_{p,t-1}}{\pi_t g} + b_{pt}}, \tag{A20}$$

$$\tilde{A}_{pt}(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt})g_p(\overline{\omega}_{pt}) = b_{pt}R_{pt}. \tag{A21}$$

$$\frac{\frac{n_{p,t-1}}{\pi_t g}}{\frac{n_{p,t-1}}{\pi_t g} + b_{pt}} = -\frac{g_p'(\overline{\omega}_{pt})f(\overline{\omega}_{pt})}{f'(\overline{\omega}_{pt})}\frac{\tilde{A}_{pt}}{R_{pt}}, \tag{A22}$$

$$n_{pt} = \tilde{w}_{p,e,t} + \xi_p \tilde{A}_{pt}(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt})f(\overline{\omega}_{pt}), \tag{A23}$$

$$(R_{st} - 1)(1 - \tau_t) = R_t - 1, \tag{A24}$$

$$R_{pt} = R_t. \tag{A25}$$

3) Pricing, market clearing and monetary policy.

$$\frac{1}{x_t} = \frac{\epsilon-1}{\epsilon} + \frac{\Omega_p}{\epsilon}\frac{1}{y_t}[(\tfrac{\pi_t}{\pi}-)1\tfrac{\pi_t}{\pi}c_t - \beta\tfrac{\lambda_{t+1}}{\lambda_t}(\tfrac{\pi_{t+1}}{\pi} - 1)\tfrac{\pi_{t+1}}{\pi}c_{t+1}], \tag{A26}$$

$$\ln(\tfrac{R_t}{R}) = \psi_{ry}\ln(\tfrac{gdp_t}{gdp_{t-1}}) + \psi_{rp}\ln(\tfrac{\pi_t}{\pi}), \tag{A27}$$

$$\ln(\tfrac{\tau_t}{\tau}) = \psi_{\tau y}\ln(\tfrac{gdp_t}{gdp_{t-1}}) + \psi_{\tau p}\ln(\tfrac{\pi_t}{\pi}), \tag{A28}$$

$$y_t = i_t + c_t + c_t\frac{\Omega_p}{2}(\tfrac{\pi_t}{\pi} - 1)^2 + g_t + \tilde{A}_{st}(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st})m_t\int_0^{\overline{\omega}_{st}}\omega dF(\omega)$$
$$+\tilde{A}_{pt}(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt})m_t\int_0^{\overline{\omega}_{pt}}\omega dF(\omega), \tag{A29}$$

$$g_t = gdp_t g_t^c, \tag{A30}$$

$$gdp_t = g_t + i_t + c_t, \tag{A31}$$

$$y_t = \frac{y_{st} + y_{pt}}{x_t}, \tag{A32}$$

$$H_t = (\mu H_{st}^{1+\sigma_L} + (1-\mu)H_{pt}^{1+\sigma_L})^{\frac{1}{1+\sigma_L}}, \tag{A33}$$

$$\frac{k_{t-1}}{g} = k_{st} + k_{pt}. \tag{A34}$$

where

$$f(\overline{\omega}_{st}) = \frac{1}{k-1}\omega_m^k \overline{\omega}_{st}^{1-k}, \tag{A35}$$

$$f'(\overline{\omega}_{st}) = -\omega_m^k \overline{\omega}_{st}^{-k}, \tag{A36}$$

$$g_s(\overline{\omega}_{st}) = \omega_m \frac{k}{k-1}(1-l_s)(1-m) + l_s\overline{\omega}_{st} + (1-l_s)[1 - \frac{(1-m)k}{k-1}]\omega_m^k \overline{\omega}_{st}^{1-k} \tag{A37}$$

$$g_s'(\overline{\omega}_{st}) = l_s + (1-l_s)(1-mk)\omega_m^k \overline{\omega}_{st}^{-k}, \tag{A38}$$

$$f(\overline{\omega}_{pt}) = \frac{1}{k-1}\omega_m^k \overline{\omega}_{pt}^{1-k}, \tag{A39}$$

$$f'(\overline{\omega}_{pt}) = -\omega_m^k \overline{\omega}_{pt}^{-k}, \tag{A40}$$

$$g_p(\overline{\omega}_{pt}) = \omega_m \frac{k}{k-1}(1-l_p)(1-m) + l_p\overline{\omega}_{pt} + (1-l_p)[1 - \frac{(1-m)k}{k-1}]\omega_m^k \overline{\omega}_{pt}^{1-k} \tag{A41}$$

$$g_p'(\overline{\omega}_{pt}) = l_p + (1-l_p)(1-mk)\omega_m^k \overline{\omega}_{pt}^{-k}, \tag{A42}$$

$$\int_0^{\overline{\omega}_{st}} \omega dF(\omega) = \frac{k}{k-1}(\omega_m - \omega_m^k \overline{\omega}_{st}^{1-k}), \tag{A43}$$

$$\int_0^{\overline{\omega}_{pt}} \omega dF(\omega) = \frac{k}{k-1}(\omega_m - \omega_m^k \overline{\omega}_{pt}^{1-k}). \tag{A44}$$

The system of 34 equations from (47) to (80) determine the equilibrium solution for the 34 endogenous variables summarized in the vector,

$$[y_t, c_t, i_t, g_t, gdp_t, k_t, \lambda_t, \lambda_t^k, H_t, H_{st}, H_{pt},$$

$$y_{st}, y_{pt}, k_{st}, k_{pt}, n_{st}, n_{pt}, b_{st}, b_{pt}, \tilde{A}_{st}, \tilde{A}_{pt}, \overline{\omega}_{st}, \overline{\omega}_{pt},$$

$$\tilde{w}_{st}, \tilde{w}_{pt}, \tilde{w}_{s,e,t}, \tilde{w}_{p,e,t}, r_t^k, R_t, R_{st}, R_{pt}, \pi_t, x_t, \tau_t]$$