Estimating Extensive Margin Responses on Kinked Budget Sets: Evidence from the Earnings Test¹

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March 2016

Abstract

We develop a novel methodology for estimating the impact of incentives to remain employed on the employment rate, and we use it to estimate the impact of the Social Security Old Age and Survivors’ Insurance (OASI) Annual Earnings Test (AET) on the elderly employment rate. The AET reduces OASI claimants’ current OASI benefits as a proportion of earnings, once a claimant earns in excess of an exempt amount, implying that the losses in current benefits due to the AET grow as earnings increase above the exempt amount. Using a Regression Kink Design and administrative data from the Social Security Administration, we document clear visual and statistical evidence that the probability of working decreases discontinuously faster above the exempt amount than below it, paralleling the reduction in benefits. Under this novel estimation strategy, our preferred estimate of the elasticity of the employment rate with respect to the average net-of-tax rate is 0.59, suggesting that the existence of the AET leads to a substantially lower employment rate among affected older workers.

¹ This research was supported by the U.S. Social Security Administration through grant #RRC08098400-06-00 to the National Bureau of Economic Research as part of the SSA Retirement Research Consortium and by grant #G-2015-14005 from the Alfred P. Sloan Foundation. The findings and conclusions expressed are solely those of the author(s) and do not represent the views of SSA, any agency of the Federal Government, or the NBER. We are extremely grateful to David Pattison for running our code on the data. We thank Raj Chetty, Jim Cole, Rebecca Diamond, Joel Slemrod, and seminar participants at the NBER Summer Institute, SIEPR, the University of Chicago, Duke, Notre Dame, UIC and Dartmouth for helpful comments. All errors are our own.
1. Introduction

Taxes and government transfers often create “kink points,” i.e. discontinuous changes the marginal incentives to work or earn more. The progressive individual income tax in many countries, including the U.S., generates a piecewise linear budget set with kinks at each point where the marginal tax rate jumps. Means-tested government transfer programs also introduce piecewise-linear constraints because transfer benefits are “taxed” away as income rises. A swell of recent papers has examined intensive margin responses—i.e. the choice of earnings or hours worked, conditional on earning or working a positive amount—to kink points or notches in the effective tax schedule (e.g. Saez (2010); Chetty et al. (2011); Chetty et al. (2013); Kleven & Waseem (2013)). This follows a long line of previous literature that examined intensive margin earnings or labor supply responses to non-linear budget sets (e.g. Hausman (1981); Burtless & Hausman (1978); Friedberg (2000); Blundell & Hoynes (2004)). This literature has developed methods for estimating the effect of the net-of-tax rate or net-of-tax wage on labor supply or earnings at the intensive margin.

Public programs could also have important effects on the extensive margin—the choice to earn zero or a positive amount (e.g. Cogan (1981); McDonald & Moffitt (1980)). The employment rate is a key outcome across several fields in economics, but there is less work on how to estimate the effect of the net-of-tax rate or net-of-tax wage on labor supply or earnings at the extensive margin, in the presence of a non-linear budget set. In this paper, we develop a novel method for estimating such effects.

Substantively, we show that extensive margin responses to the Social Security Retirement Annual Earnings Test (AET) are important. The AET reduces OASI claimants’ current OASI benefits as a proportion of earnings, once a claimant earns in excess of an exempt amount. For example, for OASI claimants aged 62 to 65 in 2013, current OASI benefits are reduced by 50 cents for every extra dollar earned above $15,120. This could reduce OASI claimants’ incentives for additional work. Reductions in current benefits due to the AET sometimes lead to increases in later benefits; nonetheless, as we discuss in detail, several factors may explain why individuals' earnings still respond to the AET.
The AET is an important policy affecting the elderly in the U.S. The AET’s large benefit reduction rate (BRR), often 50 percent, may lead to a very large kink in the budget set of a magnitude that is rarely observed in public finance contexts. This sizeable variation enables us to document clearly the extensive margin responses to the AET, both in a graphical and statistical sense. The large BRR also raises the possibility that it has substantial effects on employment. The elderly employment rate trended downward throughout most of the second half of the twentieth century, from 1950 to the mid-1980s (Figure 1). This trend reversed in the mid-1980s: the elderly employment rate has generally risen from mid-1980s to the present. This turnaround it coincided with several important changes in the Social Security system—specifically to Old Age and Survivors’ Insurance (OASI), the portion of Social Security outside of Disability Insurance—that could have played an important role. One important set of changes involved the AET. As we describe, the AET became substantially less stringent over the past 30 years, coincident with the rise in the elderly employment rate over this period. The time series evidence could be confounded by other factors that changed over time coincident with changes in the AET (Munnell, Cahill & Jivan (2003); Schirle (2008); Blau & Goodstein (2010); Heiland & Li (2012)). This leads us to turn to the microdata: Social Security Administration (SSA) administrative data on a 25 percent random sample of the U.S. population. Our results using the microdata raise the possibility that the greatly reduced stringency of the AET could have played an important role in increasing the elderly employment rate over the past 30 years.

Our method for identifying the extensive margin effects of a non-linear budget set relies on clear, intuitive patterns in the data. Specifically, using a standard model of earnings determination, we show that at a kink in the budget set where the effective tax rate discontinuously rises, there should robustly be a discontinuous increase at the kink point in the slope of the probability of employment (expressed as a function of desired earnings on a linear budget set), among individuals who are constrained from making small, intensive margin adjustments.\(^2\) Intuitively, the losses in income due to the

\(^2\) Our method also applies at non-convex kinks: the slope of the non-employment rate should discontinuously fall at a non-convex kink. For consistency with the previous literature on kink points that has focused on the effect of taxation, we sometimes use "tax" as shorthand for "tax-and-transfer," while recognizing that the AET reduces Social Security benefits and is not administered through the tax system.
increase in the marginal tax rate above the kink point grow as earnings increase above the kink point; we examine whether nonemployment also grows faster above the kink point than below it, commensurate with the increasing “bite” of the tax. Our methodology points out when such discontinuous changes in slope should occur around budget set kinks. Thus, observing such changes in slope represents a novel type of evidence that there is an extensive margin effect of the kink.

We further show that in this environment, we are able to express the elasticity of participation with respect to the incentive to participation as a function of the discontinuity in slope at the kink, as well as other empirically estimable parameters. All else equal, the larger the discontinuous change in slope at the kink point, the larger will be the estimated elasticity of nonemployment with respect to the effective average net-of-tax rate (ANTR).\(^3\)

In the context of the AET specifically, the BRR discontinuously changes from zero to positive at the exempt amount. Using a Regression Kink Design (RKD), we uncover a novel fact that is clear both graphically and statistically: the slope of the non-employment rate discontinuously increases around the kink created by the AET. This pattern does not occur in a placebo sample not subject to the AET. At a basic level, this constitutes evidence that the AET causes an extensive margin response. We use our RKD design to estimate the elasticity of non-employment with respect to the ANTR. Our baseline estimates imply that the elasticity is 0.59. This large elasticity implies that elderly work decisions are sensitive to the AET, and that policy changes to the AET could impact the elderly employment rate substantially.

Our methodological contribution is most similar to McDonald & Moffitt (1980) and Alpert & Powell (2014), who develop a related but distinct method to simultaneously estimate both intensive and extensive margin responses on non-linear budget sets when capital income is also observed.\(^4\) A key feature of our method is that its central prediction—namely the discontinuity at the kink in the slope of the employment rate—can be readily and transparently confirmed in a simple graph of the

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\(^3\) The average net-of-tax rate (ANTR) refers to the fraction of an individual’s earnings plus Social Security benefits that the individual keeps net of taxes and benefit reduction, when earning a positive amount rather than earning zero.

\(^4\) See also Eissa et al. (2008).
data, in both a visual and a statistical sense. No other existing method applies the newly developed RKD design in the novel context of estimating extensive margin responses to non-linear budget sets. Two recent papers have investigated the effects of a notch—rather than the kinked budget set that is our focus—on an extensive margin decision in very different contexts: Kopczuk & Munroe (Forthcoming) examined the choice to buy a mansion in the context of responses to a tax on mansions that creates a notch, and Kleven et al. (2014) explored the effect of a notched budget set on migration. Our method complements this work and develops methods applicable in a different context, namely a kink rather than a notch.

Our substantive contribution relates to long lines of previous research on the effect of Social Security and other types of pensions on retirement Gruber & Wise (2004); Asch et al. (2005); Coile & Gruber (2001); Liebman et al. (2009); Mastrobuoni (2009); Brown (2013)). With respect to the AET in particular, much previous literature on the AET in particular has examined how the AET affects the choice of how much to earn, given that an individual earns a positive amount (e.g. Burtless & Moffitt (1985); Friedberg (1998); Friedberg (2000); Song & Manchester (2007); Gelber et al. (2014)), and has tended to find moderate substitution elasticities at the intensive margin.

Given the clear but moderate responses at the intensive margin, it is arguably surprising that there is no parallel previous evidence of clear responses at the extensive margin, perhaps particularly since the elderly are often on the margin of whether to retire and therefore may be especially sensitive to incentives. Indeed, much of the earlier empirical literature on the earnings test concludes that the AET has little meaningful effect on the labor supply of older men (e.g. Viscusi 1979; Burtless & Moffit 1985; Gustman & Steinmeier 1985, 1991; Vroman 1985; Honig & Reimers 1989; Leonesio 1990). More recent work has examined the effect of the AET on employment decisions using a differences-in-differences framework, including Gruber & Orzag (2003); Song (2004); Song & Manchester (2007); Haider & Loughran (2008), Song (2004). Song (2004) and Song & Manchester (2007) find evidence that the elimination of the AET is associated with a modest increase in employment, though they note that this increase in employment continues a trend toward greater employment and therefore it is difficult to assess how much of the increase in employment is due to a continuation of the pre-trend
as opposed to the elimination of the AET. None of these previous papers have found clear evidence that the AET affects the employment rate, and as such our results are the first to show clear evidence that the AET affects the employment rate. While this may appear surprising, our results pertain to the 62-64 year-old group, which is a younger group than the group studied in recent literature examining the elimination of the AET for those NRA and above. This younger group is policy-relevant now, as policy-makers consider changes to the AET for those below NRA. The combination of a large administrative dataset (also used in various forms in Song (2004), Song & Manchester (2007), and Haider & Loughran (2008)) and our identification strategy based on individual-level microdata leads to precise estimates of significant effects that correspond to sizeable elasticities. Our results echo those in other countries’ settings, where literature has found that similar Earnings Tests do reduce employment (see Baker and Benjamin 1999 on Canada, and Disney and Smith 2002 on the U.K.).

The paper proceeds as follows. Section 2 describes our model and derives the elasticity as a function of the discontinuity in slope and other parameters. Section 3 describes the policy environment. Section 4 describes our empirical strategy. Section 5 describes the data. Section 6 presents empirical evidence on the response to the AET and uses this evidence to estimate the elasticity. Section 7 concludes with discussion and avenues for future work.

2. Model

In this section, we explore how a kink in the budget set should impact an individual’s decision of whether or not to be employed. In particular, we explore under what conditions the introduction of a kink in the effective tax schedule should cause a corresponding kink in the employment rate.

2.1. Effect of a Kink w/ No Extensive Margin Response

We begin by briefly reviewing the effect of a kink when only intensive margin responses are possible. As shown in Saez (2010), a kinked budget set will result in a discontinuity in the earnings density due to intensive margin responses to the nonlinear budget set. Consider a static model where individuals have preferences over consumption and
earnings, $u(c,z;n)$, where the direct effect of $z$ on utility is negative, as it requires effort to increase earnings, and $n$ is an index of “ability.” That is, let the marginal rate of substitution between $c$ and $z$ be decreasing in $n$ at all levels of $c$ and $z$. Agents maximize utility subject to a per-period budget constraint $c = (1-\tau)z + R$, where $R$ is virtual income. The first-order condition, $(1-\tau)u_c + u_z \equiv 0$ implicitly defines an earnings supply function $z(1-\tau,R;n)$.

Let the heterogeneity in ability, $n$, be smoothly distributed according to the CDF $F(n)$ with pdf $f(n)$. Under a linear tax of $\tau_0$, this translates into a smooth, baseline distribution of earnings $z_0$ with CDF $H_0(z_0)$ and pdf $h_0(z)$. We demonstrate this in Panel A of Figure 3. In the top of the panel, we show the indifference curves of three individuals, “L,” “M,” and “H.” The x-axis measures before-tax income, and the y-axis measures after-tax income. The bottom panel shows a continuous earnings density.

Now, introduce a kink in the budget set at earnings level $z^*$. In particular, let the marginal tax rate jump from $\tau_0$ to $\tau_1$ at $z^*$. In a model without extensive margin responses, the ex-post earnings distribution will feature three key regions. This is demonstrated in Panel B of Figure 3. First, there are individuals for whom $z_0 = z_1 < z^*$, i.e. those initially earning less than individual “L.” These individuals do not respond to the introduction of a kink. The earning density among these agents to the left of the kink remains $h_0(z)$.

The second group locates at the kink in the budget constraint, $z^*$. There will be a

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This implies a standard single-crossing property assumed in these models, which generates rank preservation in earnings, conditional on participating in the labor force.

Consider a general, nonlinear tax schedule $c = z - T(z)$. We follow the standard public finance literature, and rewrite the budget set in a linearized form:

$$c = z - T(z) = z - T'(z)z + T'(z)z - T(z) = (1-\tau)z + R.$$

The virtual income, $R$, is the intercept of a linear tax budget set with slope $(1-\tau)$ that passes through the point $(z,T(z))$. 

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probability mass of “bunchers” at the kink. In particular, these individuals satisfy the following: \( z_i \leq z^* \leq z_0 \), where \( z_i \) are earnings under the higher marginal tax rate, \( \tau_1 \), if it were a simple linear tax. This includes everyone initially earning more than individual “L” and less than individual “H.”

Finally, there are a group of individuals who continue to earn above the threshold \( z^* \)—i.e. those for whom \( z^* < z_i < z_0 \). This includes everyone earning more than individual “H.” These individuals also have a smooth density of earnings, \( h_1(z) \), where \( h_1(z) \) is the density of earnings under a linear tax of \( \tau_1 \) (approximately so if the tax change is big or if there are significant income effects).

As demonstrated in the bottom of Panel B, Figure 3, there is now an excess mass of individuals with earnings at the kink.\(^7\) The excess mass, or “bunching” at the kink, can be defined as:

\[
B = \int_{z^* - \Delta z^*}^{z^*} h_0(\xi) d\xi,
\]

where \( \Delta z^* \) is the earnings adjustment of the buncher with the highest ability, \( n^* + \Delta n \), and \( n^* \) is the ability of the individual initially earning \( z^* \). Note, \( \Delta z^* = \varepsilon \cdot z^* \cdot d\tau / (1 - \tau_0) \), where \( d\tau = \tau_1 - \tau_0 \) and \( \varepsilon = -\left( \partial z / \partial \tau \right) / \left( \partial \tau / (1 - \tau) \right) \) is the elasticity of taxable income with respect to the marginal net of tax rate. The earnings response to the kink can therefore be used to estimate an intensive margin elasticity.

### 2.2. Extensive Margin Responses

We now consider the extensive margin response to a kink. The previous model in Section 2.1 does not allow for general extensive margin responses, as preferences and budget sets are convex. This only allows for labor force entry and exit among those with infinitesimal earnings supply (e.g. Kleven & Kreiner (2005)). In order to capture the realistic pattern of labor force entry and exit among those with non-trivial levels of earnings, we introduce a fixed cost of labor market participation (e.g. Eissa et al.

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\(^7\) We depict this excess mass diffusely distributed about the kink. In practice, individuals typically cannot set earnings precisely at the kink.
We extend the basic static model, relying on the following assumptions:

1. There are two periods.\(^8\) In Period 0, individuals face a linear tax \(\tau_0\), i.e. \(T_0 = \tau_0 z\), and in Period 1, the tax schedule exhibits a kink at \(z^*\), i.e.:

\[
T_1(z) = \begin{cases} 
\tau_0 z & \text{if } z \leq z^* \\
\tau_1 (z - z^*) + \tau_0 z^* & \text{if } z > z^* 
\end{cases}
\]

2. Flow utility in each period, conditional on working, is given by:

\[
u(c_{n,t}, z_{n,t}; n) = v(c_{n,t}, z_{n,t}; n) - q_{n,t} \cdot 1\{z_{n,t} > 0\}
\]

where the time-variant, fixed cost of entry, \(q_{n,t}\), is drawn from a distribution with CDF \(G(q|n, t)\) and pdf \(g(q|n, t)\).

3. If an agent does not work, she receives a reservation level of flow utility of \(u(c^0, 0; n) = v^0.\)

4. Individuals maximize utility subject to a per-period budget constraint:

\[
c_{n,t} = z_{n,t} - T_1(z_{n,t}).\]

We restrict attention to individuals with positive earnings in Period 0, as the empirical outcome of interest will be the probability of exiting the labor force in Period 1, conditional on being in the labor force in Period 0. Period 0 earnings, conditional on having positive earnings, are chosen according the optimization problem in Section 2.1. Consider the extensive margin decision in Period 1. Let \(\bar{z}_{n,t}\) denote the optimal level of earnings in Period 1, conditional on working. The individual will enter the labor force in Period 1 if:

\(^8\) In Appendix A.2, we extend the model to an arbitrary number of periods, and show that our result still holds.

\(^9\) Without loss of generality, the outside option, \(v^0\), does not vary with \(n\) or \(t\). This is because cross-sectional and intertemporal variation in the outside option is not separately identified from the fixed cost of entry \(q_{n,t}\). We therefore collapse all such variation into the fixed cost of entry.

\(^10\) Per-period maximization would arise in a model with no income effects or borrowing constraints. For illustrative purposes, we abstract from saving, discounting and interest rates. However, in Appendix A.2, we present a fully dynamic model with saving and show our results still hold.
The probability of entering the labor force in Period 1 will therefore be:
\[
\Pr(z_{n,1} > 0 \mid z_{n,0}) = \Pr(q_{n,1} \leq v(z_{n,1} - T(z_{n,1})) - v^0 | z_{n,0})
\]
\[
= G(q_{n,1} \mid n, t = 1),
\]
where:
\[
q_{n,1} \equiv v(z_{n,1} - T(z_{n,1})),\quad z_{n,1}, n)
\]

Note that we are allowing the \(G(\cdot)\) function to vary across individuals so that we have two sources of heterogeneity; (1) preferences captured by the \(v(\cdot)\) function pin down intensive margin heterogeneity, but also affect the extensive margin through \(q_{n,1}\) and (2) the general heterogeneity in the \(G(\cdot)\) function allows for arbitrary differences in extensive margin responses, independent of the \(v\) function. The reason to do this is that if extensive margin responses were only driven by the value function \(v(\cdot)\), we might generate unexpected predictions or restrictions on the labor force participation function. For example, in our model, if \(G(\cdot)\) were homogenous, we would necessarily require that labor force participation be downward sloping as a function of earnings in Period 0. Note, we additionally allow the \(G(\cdot)\) to arbitrarily depend on time, so as not to impose any restrictions on the evolution of labor force participation over time (or age).

To demonstrate the impact of a kink on the decision to work, we illustrate the extensive margin incentives created by a kink in Figure 4. Here we plot the average net-of-tax rate (ANTR), i.e. \(1 - a \equiv (z - T(z))/z\). This measures the share of income that is kept, after taxes, when entering the labor force and earning \(z\). With a linear tax schedule, the ANTR is constant at \(1 - \tau_0\). After a kink is introduced, the ANTR decreases to the right of \(z^*\). In particular, the slope of the ANTR changes discontinuously at \(z^*\), i.e. the fraction of one’s income that one keeps begins to decrease discontinuously faster around the exempt amount.

In Figure 5 we illustrate the extensive margin decision after a kink is introduced. We assume that agents can choose a level of earnings along the prevailing tax schedule,
or exit the labor force and receive a level of consumption of $c_0$. In Panel A, the agent’s optimal level of earnings, conditional on staying in the labor force, is $z^*$. In this case, she prefers this to her option outside of the labor force. Her response to the kink is simply a reduction in earnings. In Panel B, the agent similarly has optimal earnings of $z^*$, conditional on having positive earnings. In this case, the individual’s preferences are such that she chooses to exit the labor force, rather than earning at the kink. In the absence of the kink, she would have maintained positive earnings. We formally explore these behavioral responses below.

2.3. The Probability of Positive Earnings with Intensive Margin Responses

Let the employment function, conditional on earnings in period 0, be $\Pr\left(z_{n,1} > 0 \mid z_{n,0}\right)$. That is, we plot the probability of having zero earnings in Period 1 against the level of earnings in Period 0. We have shown that $\Pr\left(z_{n,1} > 0 \mid z_{n,0}\right) = G\left(q_{n,1} \mid n,1\right)$. We now explore the change in this function as $z_{n,0}$ changes. In general, the slope of this function will be:

$$\frac{d \Pr\left(z_{n,1} > 0 \mid z_{n,0}\right)}{dz_{n,0}} = g\left(q_{n,1} \mid n,1\right) \frac{dq_{n,1}}{dz_{n,0}} + \frac{\partial G\left(q_{n,1} \mid n,1\right)}{\partial n} \frac{dn}{dz_{n,0}}$$

We make the following assumptions regarding smoothness in heterogeneity:

**Assumption 1: Smoothness in Heterogeneity**

**Assumption 1.1** $G\left(q_{n,t} \mid n,t\right)$ is continuous.

**Assumption 1.2** The partial derivative of $G\left(q_{n,t} \mid n,t\right)$ w.r.t. $q_{n,t}$, $g\left(q_{n,t} \mid n,t\right)$, is continuous in $q_{n,t}$ and $n$.

**Assumption 1.3** The partial derivative of $G\left(q_{n,t} \mid n,t\right)$ w.r.t. $n$, $\partial G\left(q_{n,t} \mid n,t\right) / \partial n$, is continuous in $q_{n,t}$ and $n$.

**Assumption 1.4** The CDF of $n$, $F(\cdot)$, is continuously differentiable with pdf $f(\cdot)$

We first consider the standard setting where individuals are free to adjust their earnings on the intensive margin, conditional on working. Focusing on the first term in the expression for the slope $d \Pr\left(z_{n,1} > 0 \mid z_{n,0}\right) / dz_{n,0}$ in (1), we have:
\[
\frac{d\bar{q}_{n,1}}{dz_{n,0}} = \frac{\partial v}{\partial z_{n,1}} (\tilde{z}_{n,1} - T_1(z_{n,1}), \tilde{z}_{n,1}; n) \frac{dz_{n,1}}{dn} + \frac{\partial v}{\partial n} (\tilde{z}_{n,1} - T_1(z_{n,1}), \tilde{z}_{n,1}; n) \frac{dn}{dz_{n,0}} = 0
\]

When there are intensive margin responses, we can set the first term on the right side of (2) to zero. For those with \(z_{n,0} < z^*\) or \(z_{n,0} > z^* + \Delta z^*\), \(\partial v(\tilde{z}_{n,1} - T_1(z_{n,1}), \tilde{z}_{n,1}; n) / \partial \tilde{z}_{n,1} = 0\), due to the envelope theorem.\(^{11}\) For those with \(z^* \leq z_{n,0} \leq z^* + \Delta z^*\), \(d\tilde{z}_{n,1} / dz_{n,0} = 0\), since \(\tilde{z}_{n,1} = z^*\) for everyone in this latter set. Substituting for \(d\bar{q}_{n,1} / dz_{n,0}\) in (1) using (2), we have:

\[
\frac{d\text{Pr}(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = g(\bar{q}_{n,1}, n, t) \frac{\partial v}{\partial n} (\tilde{z}_{n,1} - T_1(z_{n,1}), \tilde{z}_{n,1}; n) \frac{dn}{dz_{n,0}} + \frac{\partial G(\bar{q}_{n,1}, n, t)}{\partial n} \frac{dn}{dz_{n,0}},
\]

when individuals are able to adjust on both the intensive and extensive margins.

Our smoothness assumptions imply that this slope is continuous, and in particular, it is continuous at \(z^*\). That is, \(n, \bar{q}_{n,1}, \tilde{z}_{n,1}, T_1(\cdot)\) and \(\partial G(\bar{q}_{n,1}, n, t) / \partial n\) are all continuous in \(z_{n,0}\) at \(z^*\). Furthermore, \(g(\cdot)\) and \(\partial v / \partial n\) are likewise continuous in their arguments. With intensive margin responses in Period 1, we therefore have:

\[
\lim_{z_{n,0} \to z^*} \frac{d\text{Pr}(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} - \lim_{z_{n,0} \to z^*} = 0
\]

That is, the employment probability will not exhibit any first-order change in slope at \(z^*\), despite the fact that the ANTR does feature such a discontinuity.

Figure 6 illustrates this result. The x-axis measures before tax earnings in Period 0. The y-axis plots a hypothetical probability of have positive earnings in Period 1. The dashed line represents a smooth relationship between baseline earnings and labor participation, under a linear tax schedule. The dotted line plots this same relationship,

\(^{11}\) In this and other similar expressions below, we evaluate the partial derivative of \(v\) with respect to \(z\) allowing both earnings and consumption to change via the budget constraint, but holding \(n\) constant.
assuming that a kinked tax schedule is introduced in Period 1, and individuals are able to adjust both on the intensive and extensive margins. First, we see that the pattern is unchanged to the left of $z^*$, as the tax schedule remains the same. To the right of $z^*$, we see a gradual decrease in the probability of positive earnings, owing to the reduction in the share of earnings that are kept that is depicted in Figure 4. However, the kink in the ATNR does not translate into a kink in the employment rate. Intuitively, the ability to adjust on the intensive margin smooths out the first-order changes in rate of change of the average tax rate at $z^*$.

2.4. The Probability of Positive Earnings without Intensive Margin Responses

Now, we consider a special case of the model where there are no intensive margin responses to the kink in Period 1. That is, we make the restriction that $z_{n,1} = z_{n,0}$, conditional on choosing positive earnings. Individuals are still allowed to make extensive margin adjustments in this case. The general expression for $d \Pr(z_{n,1} > 0 | z_{n,0}) / dz_{n,0}$ from (1) still holds. However, we now have a slightly different expression for the critical level of entry costs, which is now evaluated at $z_{n,0}$ instead of $z_{n,1}$: $q_{n,1} = v(z_{n,0} - T_1(z_{n,0}), z_{n,0}; n) - v^0$. Accordingly, we have a different expression for $dq_{n,1} / dz_{n,0}$, as compared to (2). Since $z_{n,1} = z_{n,0}$ for everyone with positive earnings, it is now the case that:

$$
\frac{dq_{n,1}}{dz_{n,0}} = \frac{\partial v(z_{n,0} - T_1(z_{n,0}), z_{n,0}; n)}{\partial z_{n,0}} + \frac{\partial v(z_{n,0} - T_1(z_{n,0}), z_{n,0}; n)}{\partial n} \frac{dn}{dz_{n,0}},
$$

where the key difference is that $\partial v / \partial z$ and $\partial v / \partial n$ are evaluated at $z_{n,0}$ instead of $z_{n,1}$.

For those with $z_{n,0} < z^*$, since $T_1(\cdot) = T_0(\cdot)$ and $z_{n,1} = z_{n,0}$, it is still the case $\partial v(z_{n,0} - T_1(z_{n,0}), z_{n,0}; n) / \partial z_{n,0} = 0$, due to the envelope theorem. The first term in (5) for those with $z_{n,0} > z^*$ is now:
\[
\frac{\partial v(n_{0,0} - T_1(n_{0,0}), n_{0,0}; n)}{\partial z_{n,0}} = (1 - \tau_1) v_c + v_z = \lambda_n \left( (1 - \tau) + \frac{v_z}{v_c} \right)
\]

where \( \lambda_n = v_c \), \( v_c \) and \( v_z \) are the partial derivatives of \( v(\cdot) \) with respect to \( c \) and \( z \), respectively, evaluated at \( (z_{n,0} - T_1(n_{0,0}), z_{n,0}; n) \).

Thus, we have:

\[
d \Pr(z_{n,1} > 0 | z_{n,0}) = \begin{cases} 
  g(q_{n,1} | n,t) \frac{\partial v(z_{n,0} - T_1(n_{0,0}), n_{0,0}; n)}{\partial n} \frac{dn}{dz_{n,0}} + \frac{\partial G(q_{n,1} | n,t)}{\partial n} \frac{dn}{dz_{n,0}} & \text{if } z_{n,0} < z^* \\
  g(q_{n,1} | n,t) \left( (1 - \tau) + \frac{v_z}{v_c} \right) + \frac{\partial G(q_{n,1} | n,t)}{\partial n} \frac{dn}{dz_{n,0}} & \text{if } z^* \leq z_{n,0}
\end{cases}
\]

Note the following limit:

\[
\lim_{z_{n,0} \to z^*} \frac{v_z(n_{0,0} - T_1(n_{0,0}), n_{0,0}; n)}{v_c(n_{0,0} - T_1(n_{0,0}), n_{0,0}; n)} = -(1 - \tau_0)
\]

where we have used the first order condition for \( z_{n,0} \) and the fact that

\[
\lim_{z_{n,0} \to z^*} T_1(n_{0,0}) = T_0(z^*). \]

We now have the following expression for the difference in slopes at \( z^* \):

\[
\lim_{z_{n,0} \to z^*} \frac{d \Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} - \lim_{z_{n,0} \to z^*} \frac{d \Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = \lim_{z_{n,0} \to z^*} g(q_{n,1}' | n,t) \cdot \lambda_n \left( (1 - \tau) - (1 - \tau_0) \right)
\]

\[
= -d \tau \cdot \lambda_n \cdot g(q_{n,1}' | n', t)
\]
where \( q_n^*, n^* \) and \( \lambda_n^* \) all correspond to the individual for whom \( z_n^0 = z^* \).

Returning to Figure 6, the solid line depicts the relationship between earnings in Period 0 and the probability of positive earnings when a kink is introduced in Period 1 and intensive margin adjustment is not possible. Relative to the case where intensive adjustment is possible—i.e. the dotted line—we see there is a discontinuous change in the slope of employment rate. Intuitively, we have shut down one of the channels through which the increase in tax liability is offset, and thus the rate of entry into the labor force decreases more sharply at \( z^* \). Looking at equation (9), the kink in the employment rate is proportional to \( d\tau \), the size of the kink in the tax schedule, \( \lambda_n^* \), the marginal utility of after-tax income and \( g(\bar{q}_{n,t}|n^*,t) \), the density of workers initially who are at the margin entering the labor force. Note that these parameters are local to the individual earning \( z^* \) in Period 0.

### 2.5. Estimating an Extensive Margin Elasticity

When there is a kink created in the employment rate, we may be able to use this behavioral response to estimate an extensive margin elasticity. For the moment, we maintain the assumption of no intensive margin adjustment in Period 1, as in Section 2.4. Define the extensive margin elasticity with respect to the ANTR as:

\[
\eta = \frac{d \Pr(z > 0)}{d(1 - a) \Pr(z > 0)} \left(1 - a\right)
\]

\[
\eta = g(\bar{q}) \frac{\partial \bar{q}}{\partial (1 - a) G(\bar{q})} \left(1 - a\right)
\]

where \( a = T(z)/z \) is the average tax rate. The second line follows from the fact that \( \Pr(z > 0) = G(\bar{q}|n,t) \). If we empirically estimate the kink at \( z^* \)—i.e. a discontinuity—in the slope of the employment probability, \( \beta \), then we have from (9):

\[
g(\bar{q}_{n,t}|n^*,t) = -\frac{\beta}{\lambda_n^* d\tau}
\]

In general, we also have the following, due to the envelope theorem:
\[
\frac{\partial \overline{q}_{n',1}}{\partial (1-a)} = \frac{\partial \left\{ v\left(\overline{z}_{n,1} - T_{1}\left(\overline{z}_{n',1}\right), \overline{z}_{n',1}^*; n^*\right) - v^0 \right\}}{\partial (1-a)} = \frac{\partial \left\{ v\left((1-a)\overline{z}_{n',1}^*, \overline{z}_{n',1}^*; n^*\right) - v^0 \right\}}{\partial (1-a)} = v_c \cdot z^* = \lambda_n^* \cdot z^*,
\] (12)

where \(\lambda_n^*\) is the marginal utility of consumption for the individual earning \(z^*\) in Period 0.

Finally, note that \(G(\overline{q}_{n',1} | n^*, t) = \Pr(z_{n,1} > 0 | z_{n,0} = z^*)\). It follows that we can estimate \(\eta^*\), i.e. the extensive margin elasticity for the individual earning \(z^*\) in Period 0:

\[
\eta^* = g(\overline{q}_{n',1}, n^*, t) \cdot \frac{\partial \overline{q}_{n',1}}{\partial (1-a)} \frac{1-a}{G(\overline{q}_{n',1} | n^*, t)}
\]

\[
= \left( -\frac{\beta}{\lambda_n^* d\tau} \right) \cdot \left( \lambda_n^* z^* \right) \cdot \left( \frac{z^* - T(z^*)}{z^*} \right) \frac{1}{\Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
\]

\[
= -\frac{\beta \cdot (z^* - T(z^*))}{d\tau \cdot \Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
\] (13)

Note that we may rewrite (13) as:

\[
\eta^* = -\frac{\beta \cdot (z^* - T(z^*))}{d\tau \cdot \Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
\]

\[
= -\frac{\beta \cdot z^* \cdot (z^* - T(z^*))}{\frac{d\tau}{1-a} \cdot \left( z^* \right) \Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
\]

\[
= -\frac{\beta \cdot 1-a}{\alpha \cdot \Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
\] (14)

where \(\alpha = -d\tau / z^*\) is the value of the kink at \(z^*\) in the slope of the ANTR rate, \(1-a\).\(^{12}\)

In other words, our model naturally suggests a “fuzzy RKD” approach to estimating the
elasticity of extensive margin participation with respect to the ANTR.

2.6. Interpreting the Observed Elasticity $\hat{\eta}$

In Section 2.5, we are able to estimate the extensive margin elasticity, under the extreme assumption that individuals cannot adjust on the intensive margin in Period 1. In general, individuals may be able to adjust on the intensive margin, at least to some extent. Appendix Figure verifies that, consistent with theory and with past findings in the literature, earnings at the ages subject to the AET (starting at age 62) in fact do show bunching at the exempt amount in our data. This immediately rules out the model where there are no intensive margin responses from Section 2.4. On the other hand, we also observe a kink in the employment function, which also rules out the model with complete intensive margin adjustment from Section 2.3. We must impose further assumptions in order to interpret the observed elasticity:

$$\hat{\eta} = -\frac{\hat{\beta} \cdot (z^* - T(z^*))}{d\tau \cdot \Pr(z_{n,1} > 0 | z_{n,0} = z^*)}$$

(15)

where $\hat{\beta}$ is the estimated kink in the employment function. We provide two cases below that allow for both bunching among some individuals and a kink in the employment function. We show that the observed elasticity, $\hat{\eta}$, can be interpreted as a (weak) lower bound on the general elasticity $\eta^*$, i.e. $\hat{\eta} \leq \eta^*$.

2.6.1. Model with mixture of types

One approach to capturing both bunching and a kink in the employment probability is to posit a model of two types of individuals: Type $A$ that can adjust on the intensive margin and Type $B$ cannot (see e.g. Kleven & Waseem (2013)). We have shown that among type $A$ agents, the employment function has a continuous slope. Among type $B$ agents, there is a discontinuity in the slope at $z^*$. Let $\pi_B^* = \Pr(B | z_{n,0} = z^*)$ be the probability of being type $B$ conditional on having earnings at $z^*$ in Period 0. It follows that:

$$\lim_{z_{n,0} \to z^*} \frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = \lim_{z_{n,0} \to z^*} \frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = -\pi_B^* \cdot d\tau \cdot g(\tilde{q}_{n,1} | n = n^*, t).$$

(16)
Thus, our estimate of the extensive margin elasticity will be attenuated by a factor $\pi_B^*$ and can therefore be considered a lower bound on the elasticity among Type $B$ agents with initial earnings at $z^*$. Notice that this does not mean that there is not an extensive margin response among all agents in response to the introduction of the kink. It is just that we are only able to identify an attenuated measure of the response among a subgroup. Nonetheless, the observed elasticity is a lower bound on the general elasticity among those earning $z^*$ in Period 1:

$$
\hat{\eta} = -\frac{\hat{\beta}}{d\tau} \cdot \frac{\left(z^* - T(z^*)\right)}{\Pr(z_{n,1} > 0 | z_{n,0} = z^*)}
= \pi_B^* \cdot \eta_B^*
\leq \pi_A^* \cdot \eta_A^* + \pi_B^* \cdot \eta_B^*
= \eta^*
$$

(17)

2.6.2. Model with fixed cost of intensive margin adjustment

An alternative model of intensive margin frictions is one where individuals face a fixed cost of adjusting earnings on the intensive margin in Period 1 (see Gelber et al. (2014) for a detailed exposition of this model). In this case, individuals will only adjust if the utility gain of intensive margin adjustment exceeds some fixed cost. Recall that for individuals earning $z_{n,0} < z^*$, there is no change in the tax schedule, and therefore, $z_{n,1} = z_{n,0}$, conditional on working. Also, due to the fixed cost of intensive margin adjustment, individuals earning sufficiently close to $z^*$ from above will also not adjust earnings in Period 1, conditional on working. The reason is that the utility gain from adjusting on the intensive margin converges to zero as $z_{n,0}$ approaches $z^*$—the optimal level of earnings is $z^*$ in Period 1 for this group, once a kink is introduced. However, the fixed cost of intensive margin adjustment remains strictly positive, generating a set of

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13 Under a more restrictive set of assumptions it is possible to separately identify $\pi_B^*$ and $\eta_B^*$, using moments from the data before and after a kink is introduced. Results are available from the authors upon request.
individuals earning just above $z^*$ for whom $z_{n,t} = z_{n,0}$, conditional on working. In this case, individuals behave as in Section 2.4, in a close enough neighborhood around $z^*$, and our results from Section 2.4 follow. In other words, with a fixed cost of intensive margin adjustment, we have $\hat{\eta} = \eta^*$.

2.7. Discussion

Previous literature has reviewed the theoretical effects of the Earnings Test on labor supply, including Blinder et al. (1980), Burkhauser & Turner (1978), Vroman (1985), Burtless & Moffitt (1985), Gustman & Steinmeier (1985), Honig & Reimers (1989), Gustman & Steinmeier (1991) and Friedberg (1998). We provide a number of contributions to this existing work. First, we place a specific focus on the extensive margin effects of the AET; in some cases, more attention has been paid to the intensive margin incentives created by the AET. Second, we introduce a new methodology that exploits the sharp changes right at the AET exempt amount that affects incentives for remaining in the workforce. Our approach therefore relies on clear patterns in the data, verifiable in graphs of the labor force exit rate and thus differs form methods that rely on long term changes in labor supply patterns, or comparisons of behavioral responses between groups of significantly different levels of earnings. Finally, the employment response we model is only detectable in the presence of frictions in intensive margin adjustment—our method therefore also provides an incidental test of intensive margin frictions.

Recall that Figure 4 demonstrates a discontinuous change in extensive margin incentives at the exempt amount $z^*$. Figure 5 accordingly documents how this kink in the budget set may be reflected in the labor force exit rate under alternative models. In the presence of intensive margin frictions, we expect that the slope of the probability of positive earnings decreases discontinuously at the exempt amount, paralleling the decrease in the slope of ANTR at the exempt amount. Phrased differently, the losses in income due to the imposition of the tax grow as earnings increase above $z^*$; we examine whether employment also decreases discontinuously faster above the exempt amount, commensurate with the increasing “bite” of the tax. Thus, the model natural suggests a
“fuzzy RKD” estimation approach, which we discuss in further detail below in Section 4. It is important to note that in our main setting, where the introduction of the AET at age 62 is anticipated, we interpret the elasticity that we estimate as a Frisch elasticity.

Our translation of the behavioral response we observe into an extensive margin elasticity warrants further discussion. Previous papers have examined the extensive margin response to tax policy. For example, in the context of the Earned Income Tax Credit in the U.S., which creates a convex kink that causes intensive margin responses (as well as a non-convex kink), several papers have investigated extensive margin responses (e.g. Eissa & Liebman (1996); Meyer & Rosenbaum (2001); Eissa & Honyes (2004); Gelber & Mitchell (2012)). Our paper estimates an observed elasticity driven by individuals who face intensive margin frictions. We note with our formal model that such an elasticity may differ from the “structural” population extensive margin elasticity, but show that we can interpret our estimate as a lower bound on the population parameter.

3. Policy Environment

The key features of the AET rules from 1978 to 2009 are shown in Figure 2. The dashed line and right vertical axis show the benefit reduction rate (BRR)—the rate at which current benefits are reduced for every dollar in gross earnings above an exempt amount. For those under Normal Retirement Age (NRA) but above the Early Retirement Age of 62, the BRR was 50 percent throughout the sample period (and remains so today).\(^{14}\) Until 1990, the BRR for individuals over NRA subject to the AET was also 50 percent; this fell to 33.33 percent in 1990. Prior to 1983, the AET applied to workers 71 and younger. Starting in 1983, the AET was eliminated for those 70 and older, and in 2000, the AET was eliminated for those NRA and above. The solid line and left vertical axis show the real exempt amount, which has risen modestly over time. Beginning in 1978, the exempt amount became higher for those NRA and above. Thus, the AET generates a reduction in current OASI benefits—we therefore have modeled the AET as creating a

\(^{14}\) The NRA, the age at which workers can claim their full OASI benefits, is 65 for those born 1937 and before, rises by two months a year for cohorts between 1938 and 1943, is constant at age 66 for cohorts between 1943 and 1954, and rises by two months a year until reaching age 67 for those born in 1960 and later.
positive implicit marginal tax rate for some individuals, consistent with the empirical finding in previous literature that some individuals bunch at the AET exempt amount (Friedberg (1998); Friedberg (2000); Song & Manchester (2007); Engelhardt & Kumar (2014)).

When current OASI benefits are lost to the AET, future scheduled benefits may be increased in some circumstances. This is sometimes referred to as “benefit enhancement.” Benefit enhancement can reduce the effective tax rate associated with the AET, in particular for those individuals considering earning enough to trigger the enhancement in the post-1972 period. The potential increase in future benefits depends on one’s age. For beneficiaries under NRA, the actuarial adjustment raises future benefits whenever a claimant earns any amount over the AET exempt amount.15 Future benefits are raised by 0.55 percent per month of benefits withheld for the first three years of AET assessment. In the budget set, this creates a notch in future benefits as well as a kink in current benefits at the AET threshold. This notch has the feature that future benefits increase discontinuously when moving from just under to just over the exempt amount.

For beneficiaries subject to the AET aged NRA and older, a one percent Delayed Retirement Credit (DRC) was introduced in 1972, meaning that each year of benefits foregone leads to a one percent increase in future yearly benefits. The DRC was raised to three percent in 1982 and gradually rose to eight percent for cohorts reaching NRA from 1990 to 2008 (though the AET was eliminated in 2000 for those older than the NRA). An increase in future benefits between seven and eight percent is approximately actuarially fair on average, meaning that a claimant with no liquidity constraints and average life expectancy should be indifferent between either claiming benefits now or delaying claiming and receiving higher benefits once she begins to collect OASI (as Diamond & Gruber (1999) show with respect to the actuarial adjustment for early claiming). OASI claimants’ future benefits are only raised by the DRC when annual earnings are sufficiently high—high enough that the individual loses an entire month’s worth of OASI benefits due to the reductions associated with the AET (Friedberg (1998); Social Security Administration (2012)). Formally, the number of months of

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15 Social Security Administration (2012); Gruber & Orzag (2003)
benefit enhancement received by an OASI recipient with earnings $z$, monthly benefits $MB$, and facing a benefit reduction rate $\tau$ above the exempt amount $z^*$ is

$$\text{floor}\left(\tau \times \left(z - z^*\right)/MB\right)$$

for those NRA and above, and

$$\text{ceiling}\left(\tau \times \left(z - z^*\right)/MB\right)$$

for those below NRA.

Taking into account all of these factors, the AET could affect the earnings decisions of those NRA and older for a number of reasons. Those whose expected lifespan is shorter than average should expect to collect OASI benefits for less long than average, implying that the AET is more financially punitive.\textsuperscript{16} Liquidity-constrained individuals or those who discount faster than average may also reduce work in response to the AET. Delaying claiming for those over NRA was on average roughly actuarially fair only beginning in the late 1990s, and moreover, the DRC only takes effect when an individual earns significantly higher than the exempt amount. Finally, many individuals also may not understand the AET benefit enhancement or other aspects of OASI (Honig & Reimers (1989); Liebman et al. (2009); Brown et al. (2013)). The earnings test is widely viewed as a pure tax. Most popular guides do not note the subsequent adjustment in benefits under the earnings test (Gruber & Orzag (2003)). During the period that we study, the popular guide Your Income Tax (J. K. Lasser Institute (1998)), for example, warned readers that if “you are under age 70, Social Security benefits are reduced by earned income,” but did not note the subsequent benefit adjustment.

We follow most previous work and do not distinguish among the potential reasons for a response to the AET. Following previous literature, our framework presumes that certain individuals treat the AET as creating some effective marginal tax rate above the exempt amount, consistent with the empirical evidence in previous literature cited above, which documents clear responses to the incentives created by the AET.

Other programs, such as Medicaid, Supplemental Security Income, Disability Insurance, or taxes such as unemployment insurance payroll taxes distort earnings incentives near the bottom of the earnings distribution. We eliminate DI recipients from our sample. The kinks created by other such programs would only cause a problem for

\textsuperscript{16} Gelber et al. (2014) fail to find evidence of such a response.
our identification strategy if they caused a discontinuous change in the implicit ANTR near the AET exempt amount; in fact, they typically are safely several thousand dollars away from the AET convex kink.

The AET applies to an individual’s earnings; spouses’ earnings do not count in the earnings total to which the AET is applied. For a retired worker (i.e. primary) beneficiary whose spouse collects spousal benefits, the AET reduces the family’s OASI benefit by the amounts we have described. The family benefit is also reduced when the spouse (separately) earns more than the AET threshold. For a retired worker beneficiary whose spouse is collecting benefits on his or her own earnings record, the AET reduces the retired worker beneficiary’s benefits by the amounts described while not affecting the spouse’s benefits. Thus, following previous literature (e.g. Friedberg (1998); Friedberg (2000)), we model the AET as creating additional marginal tax rates (MTR) associated with the BRRs described above, because the AET reduces family benefits by these amounts (all else equal). Unfortunately, our data do not contain the information necessary to link spouses.

A non-convex kink in the budget set occurs where the AET has completely phased out the OASI benefit, and therefore the BRR jumps from positive to zero. We are not able to investigate this non-convex kink with much precision. First, we do not observe an individual’s OASI benefits in our data extract. Moreover, it is not possible in our data to observe whether an individual has a spouse claiming on their record. If a spouse is claiming on the same record, then the non-convex kink occurs where the family’s entire benefit is completely phased out; thus, we would need information on whether a spouse is claiming on the same record in order to observe the non-convex kink accurately. Given that the non-convex kink occurs at a different earnings level for each individual, the effect of the non-convex kink should be spread out through the earnings distribution. Thus, we would not expect to detect any sharp change in behavior any particular earnings level relative to this non-convex kink.
4. Empirical Strategy

4.1. Regression Kink Design

Recent work has shown that under certain conditions, a change in the slope of treatment intensity can be used to identify local treatment effects by comparing the relative magnitudes of the kink in the assignment variable and the induced kink in the outcome variable. This is known as a “Regression Kink Design” (RKD) (Nielsen et al. (2010)). Estimates can be interpreted as a treatment-on-the-treated parameter (Card et al. (2012)).

In our context, the treatment intensity is the effective ANTR, the assignment variable is the earnings level relative to the exempt amount just prior becoming subject to the AET (which is highly correlated with subsequent earnings, and which we call “initial earnings”), and our primary outcome variable is the probability of zero earnings after becoming subject to the AET. As explained in Section 2.2, the imposition of the AET creates a discontinuity at the exempt amount in the slope of the ANTR as a function of initial earnings (see Figure 4). Meanwhile, the slope of other determinants of the employment rate (such as human capital, work experience, etc.), as a function of initial earnings, should not change discontinuously around the exempt amount. In this case, we can estimate the causal effect of the AET on the probability of non-employment by estimating the change at the exempt amount in the slope of employment as a function of initial earnings, in comparison to the change at the exempt amount in slope of the ANTR. Because we use prior earnings as a proxy for the current desired earnings in the absence of the AET, we technically estimate a “fuzzy” RKD.

Mathematically, we want to estimate the marginal effect of the ANTR, on the probability of employment – i.e. $E_{it} = 1$ – where $i$ indexes individuals and $t$ refers to the time period. The ANTR is a function of earnings conditional on working, which we take as desired earnings on a linear budget set $z_{i0}$ (corresponding to the model in Section 2 above). We will proxy for desired earnings conditional on working using earnings at age 60, i.e. $z_{i60}$, since individuals do not face the AET kink at age 60, and therefore at age 60 their budget set is approximately linear on average in the region of the exempt
amount. Using the RKD, we can estimate the marginal effect of ANTR on the employment probability using the following equation:

$$\frac{\partial \Pr(E_t = 1)}{\partial \text{ANTR}_t} \bigg|_{z_{i0} = z^*} = \lim_{z_{i0} \to z^*} \frac{\partial \Pr(E_t = 1|z_{i60})}{\partial z_{i60}} - \lim_{z_{i0} \to z^*} \frac{\partial \text{ANTR}(z_{i60})}{\partial z_{i60}}.$$  \hfill (18)

That is, the marginal effect we estimate is the change at the bend point in the slope of the employment probability as a function of earnings at age 60, divided by the change at the bend point in the slope of the ANTR as a function of earnings at age 60. Here the ANTR is defined as follows:

$$\text{ANTR}_i(z) = 1 - \left[ \tau_0 + d \tau \frac{z - z^*}{z} 1 \{ z \geq z^* \} \right],$$  \hfill (19)

where $1 \{ \cdot \}$ represents the indicator function. In other words, the ANTR is defined as the fraction of one’s income (inclusive of earnings and OASI benefits) that one keeps if one participates in the labor force at earnings level $z$, as opposed to not participating and earning zero instead.

Identification of the fuzzy RKD estimate of the effect of the AET ANTR on earnings relies on a number of assumptions. Formally, let $E_t(\text{ANTR}_t, z_{i0}, u_t)$ be an indicator for exit the labor force under the AET, which is a function of the ANTR, the desired level of earnings conditional on working under a linear tax schedule, $z_{i0}$, and an unobservable, random variable $u_t$. We have access to a proxy for desired earnings, which may suffer from measurement error:

$$z_{i60} = z_{i0} + p_{it} \cdot v_{it},$$  \hfill (20)

where $p_{it}$ is an indicator variable that equals zero with probability $\pi(z_{i0}, u_t, v_{it})$. In other words, this means that with some positive probability, the level of initial earnings equals

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17 By contrast, we cannot use earnings at ages 62 to 70 as the running variable, because these earnings are potentially endogenous to the AET: we know from the previous literature cited above that claimants bunch at the AET exempt amount. Furthermore, we do not use earnings at age 61, in order to avoid any anticipatory manipulation of earnings.
the desired level of earnings conditional on working in the current period under a linear tax schedule. Following Card et al. (2012), we make the following assumptions:

**Assumption 2: RKD Assumptions**

**Assumption 2.1** $E(\cdot,\cdot,\cdot)$ is a continuous function with $\partial E(ANTR, z_{i0}, u) / \partial ANTR$ continuous in $ANTR$.

**Assumption 2.2** $\partial E(ANTR, z_{i0}, u) / \partial z_{i0}$ is continuous in $z_{i0}$.

**Assumption 2.3** $ANTR(z)$ is $C^1$ with respect to $z$ everywhere in a neighborhood surrounding $z^*$, except for at $z^*$ where:

$$\lim_{z \to z^+} \partial ANTR/\partial z \neq \lim_{z \to z^-} \partial ANTR/\partial z .$$

**Assumption 2.4** The joint density of $(z_0, u, v)$ is continuous and continuously differentiable in $z_0$.

**Assumption 2.5** $\lim_{z \to z^+} \partial ANTR/\partial z \geq \lim_{z \to z^-} \partial ANTR/\partial z$.

**Assumption 2.6** The probability that initial earnings equal current, desired earnings, $\pi(z_{i0}, u, v)$, is smooth.

Assumption 2.1 requires that the marginal effect of the ANTR on the probability of positive earnings be continuous in observed and unobserved variables. Assumption 2.2 requires that the direct marginal effect of baseline earnings on the probability of positive earnings be smooth. Assumption 2.3 can be thought of as a first stage—we need a kink to exist in the tax schedule. Assumption 2.4 requires that the derivative of the conditional probability density function be continuous for all unobservables at the kink, so that the density of the unobserved heterogeneity evolves smoothly with initial earnings at the kink. Assumption 2.5 is a monotonicity assumption, which holds given the tax schedule. Finally, Assumption 2.6 require a smooth evolution of the set of individuals for whom initial earnings equal the current, desired level of earnings.

These assumptions may not hold if we observe sorting in relation to the exempt amount, with respect to initial earnings. We would be worried about sorting if were to observe a kink or discontinuity in the density of the assignment variable at the exempt amount, or observe a change in the distribution of predetermined covariates at the exempt amount. We verify that we do not observe such patterns in the region of the exempt amount, consistent with previous research that has found no bunching at the exempt amount prior to the imposition of the AET (Gelber et al. (2014)). Given these
assumptions, the ratio of kinks in equation (18) provides an average treatment effect among individuals with initial earnings equal to \( z^* \) and whose initial earnings are equal to the current, desired earnings conditional on working under a linear tax schedule.

We use a “fuzzy” RKD, separately estimating the numerator and denominator of (18) using spline regressions (Card et al. (2012)). The numerator is estimated by relating \( \Pr(E_n = 1) \) to initial earnings at age 60, \( z_{i60} \), using the following specification:

\[
\Pr(E_n = 1) = \delta_0 + \sum_{j=1}^J \delta_j (z_{i60} - z^*_i)^j + D \cdot \sum_{j=1}^J \beta_j (z_{i60} - z^*_i)^j + \mu + \epsilon_i, \tag{21}
\]

where \( D = 1\{z_{i60} \geq z^*_i\} \) is an indicator for being above the exempt amount, \( \mu \) represent time fixed effects, \( \epsilon \) is an error, and the change in the slope of \( \Pr(E_n = 1) \) at the exempt amount is given by \( \beta_1 \). Specifically, \( \Pr(E_n = 1) \) is calculated at the individual level by averaging an indicator for employment over a range of years (e.g. 63-64 or 63-70).\(^{18}\)

Similarly, the denominator of (18), i.e. the “first stage,” can be estimated by running models of the form:

\[
ANTR_n = \gamma_0 + \sum_{j=1}^J \gamma_j (z_{i60} - z^*_i)^j + D \cdot \sum_{j=1}^J \alpha_j (z_{i60} - z^*_i)^j + \theta + \nu_i, \tag{22}
\]

Here the change in slope of \( ANTR_n \) at the exempt amount is given by \( \alpha_1 \), \( \theta \) represents a time fixed effect and \( \nu \) is an error. In practice, we use a range of first- through third-order polynomials, which are allowed to differ on either side of the exempt amount.

The first-stage regression in (22) uses earnings at age 60 and the parameters of the AET to estimate the kink in ANTR. As discussed in Section 2.5, if the AET increases the marginal tax rate by \( d\tau \) at the exempt amount, then the kink in the ANTR will be \( \alpha_1 = -d\tau / z^* \). Unfortunately, this value of ANTR is not exactly the theoretically correct measure we require. We would like to evaluate the ANTR in period \( t \) (i.e. ages 63-70) using the rules of the AET and the earnings level \( z_{i60} \). This is the counterfactual level of earnings available to the agent in period \( t \) if there were a linear tax schedule

\(^{18}\) The results we obtain are equivalent to treating each year as a separate observation and clustering standard errors at the individual level.
present—recall the relevant incentives are those faced by agents who cannot adjust earnings on the intensive margin in response to the AET. Note, the observed ANTR of all people during ages 63 to 70 is not an adequate proxy, because the realized ANTR of people at these ages is influenced by intensive margin adjustment and therefore incorporates the responses of both the constrained types and those that are unconstrained at the intensive margin. Thus, in our baseline specification, we proxy for the desired level of earnings under a linear tax, i.e. $z_{i0}$, using initial earnings at age 60, $z_{i60}$.

4.2. Additional Specification Considerations

Interest in the RKD is relatively recent, and many of the details surrounding the econometric theory and empirical implementation of the approach are still actively under discussion. One is the choice of bandwidth. Smaller bandwidths will allow for estimates that are more local to the variation in the ANTR around the exempt amount, but may also limit statistical precision because they limit the sample size. For our main results, we implement a data-driven method for bandwidth selection derived by Calonico et al. (2014). We examine the robustness of our analysis by estimating regressions using a number of alternative bandwidths.

Another issue is the order of the polynomial chosen to estimate the relationship between the assignment and outcome variable. Card et al. (2012) and Dong (2011) argue that when the derivative modeled linearly, the specification is likely to suffer from boundary bias. Card et al. (2012) show that local quadratic regression should have smaller bias than local linear regression using the same bandwidth, although with much higher variance. They use both linear and quadratic specifications in their analysis. Calonico et al. (2014) propose an RKD estimator where the quadratic specification can be used to correct for the bias in the linear estimator, while Ganong & Jäger (2014) advocate for a cubic, spline specification. Our approach is to implement linear, quadratic and cubic versions of equations (21) and (22), to investigate the robustness of our results across all of these different choices. We report confidence intervals, corrected for bias, following Calonico et al. (2014) and use a triangular kernel to weight the data near the exempt amount.
We must also decide whether or not to allow for a discontinuity at the bend point in the level of the outcome variable. Some policy rules result in both a discontinuous change in treatment intensity and a change in treatment intensity at a given value of the assignment variable (e.g., Turner (2014)). That is not the case here, however, as the effect of the AET on benefits does not jump at the bend point. Moreover, when treatment effects are heterogeneous, the imposition of continuity is necessary for a change in slope at the bend point to be considered a causal parameter (Card et al. (2012)). However, there are concerns that imposing continuity increases the likelihood of spurious results (Ando (2013)). Again, given that it remains an open question, we will implement specifications that impose continuity and others that allow for a discontinuity at the bend point.

In relation to other existing approaches for estimating extensive margin elasticities, our method involves a different set of assumptions and therefore has both benefits and costs. We focus here on a comparison to the differences-in-differences approach that has often been used (e.g. Eissa & Liebman (1996); Meyer & Rosenbaum (2001); Eissa & Honyes (2004); Gelber & Mitchell (2012)), though one could also compare to many other approaches (e.g. using a selection correction). Our method allows transparent identification through the RKD approach, allowing clear graphical depiction of the results. Using the RKD approach, one avoids the assumptions of differences-in-differences such as the “parallel trends” assumption. Our formal model also allows us to be explicit about the population that our elasticity refers to, as in the distinction above between the “observed” and “structural” elasticities discussed in Section 2.6.

Our method also has potential disadvantages relative to a differences-in-differences design. Our estimate is local to those in the region of the kink and therefore may not apply more broadly. It may be possible to address this to some extent—for example, in our context, we may investigate whether our estimates are heterogeneous across various samples. Moreover, the differences-in-differences approach also identifies local effects—those local to the region of policy variation—that may not apply more broadly. Our approach also makes the assumptions necessary for an RKD, including the continuity of the first derivative. In our context we are also able to address
these assumptions to some extent, by assessing how our results compare to those in a placebo sample that is not subject to the AET—of course, such placebos may also be available in a differences-in-differences context. Finally, our estimates apply to the marginal labor market participants—those whose decision of whether to work or not is affected at the margin by the variation in the ANTR at the kink.

Ultimately, our approach offers a new and different method, with both potential advantages and disadvantages relative to other methods. Our new approach allows an alternative method that enlarges the set of strategies for estimating extensive margin responses.

4.3. Mapping the RKD Estimates to the Model

As discussed in Section 2.5, our theoretical model naturally suggests a fuzzy RKD estimation approach. First, note that the smoothness conditions imposed on preferences and heterogeneity in Section 2 are analogs to the assumptions presented here in Section 4.1 that allows us to interpret the fuzzy regression kink estimate as a treatment-on-the-treated. Indeed, the restrictions we place on the primitives in Section 2 can be shown to generate an earnings distribution and employment function that satisfy the conditions assumed within our potential frameworks representation of the empirical problem.

Second, our theoretical model predicts a kink in the employment function for those agents who face frictions in adjustment on the intensive margin. Likewise, the fuzzy RKD identifies a treatment effect among the agents for whom the term $p_{it} = 0$ in equation (20). In both cases, a kink in the pattern of outcomes only occurs for those who do not exhibit manipulation of the “running variable.” Furthermore, the probability of falling in this set within the RKD framework, $\pi(z_{it0}, u_{it}, v_{it})$, corresponds to our discussion in Section 2.6 of the interpretation of the “observed elasticity.” In the case where there are two types of agents, $E[\pi(z^*, u, v)] = \pi^*_B$, whereas in the model featuring a fixed cost of adjustment on the intensive margin, $E[\pi(z^*, u, v)] = 1$.

Turning to our regression kink regressions, the coefficients we estimate map directly to the parameters of our model that help identify an extensive margin elasticity.
The parameters $\beta$ and $\alpha$ in equation (14) correspond to the $\beta_1$ and $\alpha_1$ from equations (21) and (22), respectively. We can then calculate the remaining inputs to equation (14) using the data on individuals who have initial earnings equal to $z'$. Finally, we note that just as the fuzzy RKD returns results that are local to agents initially located at the kink, our model likewise identifies parameters that apply to agents initially located at the kink.

5. Data

Our analysis relies on the restricted-access Social Security Administration Master Earnings File (MEF) and Master Beneficiary Record (MBR). Since 1978, the MEF measures uncapped W-2 earnings for all Social Security Numbers (SSNs) in the U.S. for each calendar year. The data contain information on date of birth, date of death, and sex. Our data extract includes individuals born between 1918 and 1923 who have ever claimed social security. Since earnings at age 60 are the key running variable in our analysis, we restrict attention to individuals with positive earnings at age 60 within $40,000 of the AET exempt amount in that year. We then take a 25 percent random subsample of this group. We additional drop individuals with missing values for gender (868 observations) and/or negative earnings between ages 50 and 70 (79 observations). The MBR also contains data on the day, month, and year that people began to claim Social Security.

Several features of the data are worth discussion. First, these administrative data are subject to little measurement error. Second, earnings as measured in the dataset are not subject to manipulation through tax deductions, credits, or exemptions. Third, like most other administrative datasets, the data do not contain information on hours worked, hourly wage rates, or amenities at individuals' jobs.

Our main outcome variable is the fraction of calendar years with positive earnings when individuals are ages 63 and 64. For our purposes, age refers to the highest age a person attains during a given calendar year. The AET begins to apply to claimants at age 62, but it does not make sense to examine the effect of the AET on whether an individual has positive earnings in the calendar year s/he turns age 62. The reason is that we observe calendar year earnings. If an individual claims OASI at age 62, the AET
only applies to months after the individual claims; for example, the individual could only be claiming for a small portion of the year. But if the claimant earns even one dollar during this calendar year—even during months prior to claiming OASI and being potentially subject to the AET—then she will appear as having positive earnings in this calendar year. Thus, a person who is in fact induced by the AET to exit the labor force during the later period of this calendar year would appear in the data with positive earnings, and therefore as having no measured response to the AET. Thus, it is likely that we should not see a measurable response to the AET at age 62—only those born on Jan. 1, who claim on the exact date of their 62nd birthday, should show a response on the extensive margin at an annual level, and this represents a negligible fraction of the sample.

Instead, we expect effects to only begin to appear at age 63. We choose 64 as the oldest age at which to examine decisions because age 60 earnings are a better proxy for desired earnings at ages 63-64 than for older ages. Moreover, at age 65 during the years we investigate, there is partial exposure to the AET for those near the under-NRA exempt amount, as individuals face a much higher exempt amount in the year of turning NRA. These considerations apply all the more to ages older than 65.

We make additional restrictions to the sample based on the year of observation and type of earnings. First, we focus on a sample that is subject to the AET in 1978 and after. The reason is that observe only calendar year earnings, and the AET is effectively applied to calendar year earnings beginning in 1978 (see Gelber et al. (2014), Appendix A). Before 1978, the AET was applied to quarterly earnings, which we do not observe in the data. This limits our focus to individuals born in 1918 or later. Second, we drop those with self-employment earnings in the base year of 1960, since previous research has suggested that these can often reflect reporting responses rather than earnings supply (e.g. Chetty et al. (2013)).

Table 1 shows summary statistics for our sample. 68 percent of the sample is female; this is a consequence of focusing on individuals near the exempt amount, which is a relatively low level of earnings among 60 year-olds. The employment rate at 63-64 is 56.5 percent. Mean earnings at ages 63-64 are around $5,800 ($2012), including zeroes.
6. Empirical Results

6.1. Preliminary Analysis

We begin our empirical analysis with validity checks that demonstrate that our empirical method is valid. In particular, we test for manipulation of the running variable, \textit{i.e.} sorting in earnings at age 60 relative to the exempt amount. \textbf{Error! Reference source not found.} plots the density of earnings, \textit{i.e.} the fraction of observations in $100$ bins. The slope of the density appears continuous around the exempt amount. We run regressions to confirm that the density of observations is smooth in the region of the exempt amount. The results are reported in Table 2. We estimate specifications similar to equation (21), using the fraction of observations in a given bin as the outcome. Using a linear, quadratic and cubic specification, we do not find a statistically significant change in slope at the exempt amount. This suggests that individuals do not appear to sort around the exempt amount when choosing their earnings at age 60.

To test further for sorting and to provide greater context for the validity of our method, we test for bunching, or excess mass, in the earnings distribution as well. In Figure 8 we plot both the density of earnings at age 60 and at age 62. We also report the amount of bunching, using a method similar to that of Chetty et al. (2011). We fit a seventh-order polynomial to the average share of observations in each bin, excluding the data within $2,000$ if the exempt amount. The actual density is then compared to this counterfactual density, to determine whether there is excess mass near the kink. Our measure of excess mass at age 60 is statistically and economically insignificant. For comparison, we perform the same exercise using earnings at age 62, when the AET begins to matter for earnings and benefits. Here, we see a markedly different pattern, with a noticeable spike in the density near the exempt amount. The bunching in this case is confirmed to be statistically significant as well. Thus, the visual, statistical and comparative evidence suggests that there is not sorting in the earnings density at age 60.

6.2. Main Results

We show a graphical version of our primary empirical results in Figure 9. In the figures, the $x$-axis measures earnings at 60, which reflects earnings in the absence of the AET.
On the $y$-axis is the fraction of years with positive earnings, over ages 63 and 64. The data are averaged over $\$500$ bins. The vertical line in each figure represents the exempt amount. The data are pooled across all years of the sample. We see a change in slope occurs at the exempt amount. In the region below the exempt amount, the probability of employment slopes up at a faster rate than it does above the exempt amount.

Even without estimating an elasticity, these results (as well as the regression results below estimating a significant kink in the “reduced form” regressions) constitute a novel source of evidence that the AET causes an extensive margin effect. More generally, observing such a change in slope allows us to verify whether a non-linear budget set causes an extensive margin effect. Recalling our model in Section 2, evidence of a kink in the employment rate is also consistent with a set of agents who face frictions in adjusting earnings on the intensive margin. Our model also allows us to go beyond documenting these two phenomena and also estimate an extensive margin elasticity.

We now confirm that the change in the slope of the employment rate is statistically significant in Table 3 Column 1, where we report the estimated kink from regressions of the form in equation (21). In particular, we report the coefficient $\beta$, the estimated kink in the employment rate at the exempt amount. We use polynomials of first, second and third order, which are allowed to differ on either side of the exempt amount. We report point estimates of the kink and bias-corrected standard errors. In the first row, we estimate a statistically significant kink in the employment rate when using a linear specification. We continue to find significant evidence and slightly larger estimates if we instead use a quadratic or cubic specification. In this table, we report estimates using a data-driven method to select our bandwidth, following Calonico et al. (2014). This results in bandwidths between $\$2,300$ and $\$2,600$ for the linear specification and between $\$4,000$ and $\$4,600$ for the higher-order specifications. We show below that our results are robust to variation in our choice of bandwidth.

### 6.3. Estimating an Extensive Margin Elasticity

Next, we turn to our formula for the extensive margin elasticity, i.e. equation (14). In Table 3 column 2, our extensive margin elasticities range from 0.56 in the baseline linear specification, to 0.82 in the cubic specification.
The elasticities we estimate are relatively large, and fall outside the range of some previous surveys of extensive margin elasticities. Chetty et al. (2012) report Frisch elasticities on the extensive margin between 0.18 and 0.43, in a meta-analysis of microdata estimates (although macro estimates have a mean elasticity of 2.77). Saez (2002) discusses estimates from Negative Income Tax (NIT) experiments, which feature small extensive margin elasticities for males of about 0.2, but participation responses for those less attached to the labor force above 0.5 and sometimes near 1. On this last point, we believe the most relevant range of estimate for our context are those of people with low labor force attachment, as opposed to estimates gathered from prime aged workers. In that case, our estimates are on generally within the range of micro-estimates considered by Saez (2002). Furthermore, it should be noted that our estimates are local to our age groups and our narrow earnings range, and therefore, may be drawing from a more local response than the above-mentioned studies.

6.4. Robustness Checks and Heterogeneity

In Table 3 Column 3 we present the regression estimates from a placebo reduced form regression: employment between ages 56 and 57, as a function of age 60 earnings. During these earlier ages, earnings are not subject to the earnings test. We maintain the same running variable as earlier, since we would like to test for a spurious relationship between employment and earnings at age 60. We once again use a data-driven method to choose bandwidths. Across all three specifications, we fail to reject the null of no kink in lagged employment. The top row of Figure 10 shows graphically that there is no visible kink in employment at ages 56-57, and that there is also no visible kink in mean earnings at these ages. Other graphs within Figure 10 shows that there is no clear visual discontinuity in the slope of predetermined covariates (sex, birth year, and probability of being white). Table 2 shows that in the baseline linear specification, there is no significant discontinuity in the fraction white or female (or in the density), but there is a very small significant discontinuity in the slope of year of birth. However, there is no significant discontinuity in year of birth in the cubic specification, where we do find a significant discontinuity in our main outcome, the age 63-64 employment rate.
Moreover, it is also important to note that controlling for these covariates will have no material affect on the results.

Indeed, in Table 4 we show that our results are robust to controls for demographics (dummies for sex, year of birth, and race), as well as to allowing for a discontinuity in the level of employment at the exempt amount.

In Figure 11 and Table 5 we show that the kink only arises precisely at ages 63 and 64. Figure 10 shows the employment probability by single year of age from 61 to 64. There is no clear kink at ages 61 and 62, consistent with no pre-existing pattern in the data. The visible kink only arises at ages 63 and 64. Table 5 confirms that the kink estimates are insignificant at ages 61 and 62, but are substantial and highly significant at ages 63 and 64.

As another assessment of the validity of our approach, we conduct a permutation test, similar in spirit to that of (Ganong & Jäger, 2014). In particular, we estimate a series of placebo kinks, using data exclusively on either side of the exempt amount, but otherwise similar in specification to our main estimates. We then compare our point estimates to the empirical distribution of these placebo estimates. In Figure 12, we show that our point estimates are located solidly below the distribution of placebo estimates, reinforcing the notion that we are detecting a true kink. In Figure 13, we show following Landais (2015) that the R-squared of the regression in the baseline linear specification is maximized at the actual location of the kink, consistent with the view that we have found a true kink in the data. In Figure 14 we demonstrate how our estimates vary as a function of the bandwidth chosen. The relative size of the kinks remain stable across the majority of bandwidths.

In Table 6 we show the elasticity estimates by demographic group. Consistent with the typical stylized fact that women have larger participation elasticities than men, the elasticity for women is 0.61, and the elasticity for men is 0.36. Whites and non-whites show similar participation elasticities.

7. Conclusion

We develop a novel method for estimating the extensive margin effect of non-linear taxes and transfers, relying on empirically estimable quantities and a transparent RKD
empirical design, and we use this method to estimate the effect of the AET. We find that the AET has a large impact on extensive margin earnings decisions. Under our preferred specifications, we estimate that the elasticity of the employment rate with respect to the ANTR is 0.59. Thus, our results suggest that the AET is an important factor in determining the earnings decisions of the elderly. More generally, our results are consistent with the notion that the retirement decisions of the elderly may be quite sensitive to incentives.

Our results may appear surprising in light of the findings in previous literature, which have not found clear evidence that the AET affects labor force participation. Our paper is not directly comparable to previous findings because our estimates are local to the region of the exempt amount, whereas previous findings have looked at the effects of eliminating the AET in a broader earnings range. Moreover, we investigate a younger group, those 63-64, which previous differences-in-differences work has not examined. Even under the assumption that our findings do generalize to other earnings levels and ages, a closer look at the literature reveals that our results do not contradict past findings. Song & Manchester (2007) find that after the AET was eliminated for those 66-69 in 2000, the employment rate rose for those 66-69 relative to those 62-65 by approximately two percentage points by 2002. However, they note that this relative increase in the employment rate was preceded by pre-trend, i.e. a relative increase in the 66-69 year-old employment rate prior to 2000, and they write that part of the relative increase may therefore be explained by an extension of the pre-trend (though part may be unrelated to the pre-trend). Our results suggest that a substantial fraction of the post-2000 increase may be explained by the causal effect of the elimination of the AET for this group, though their and our confidence intervals do not rule out that part of the increase is also explained by an extension of the pre-trend as Song and Manchester argue. Note that the pre-trend could have been in part caused by, among other things, declining stringency of the AET through the 1990s, as the Delayed Retirement Credit increased.

Using a differences-in-differences design and Current Population Survey (CPS) data from 1974-1999, Gruber & Orzag (2003) find no evidence that the AET threshold, or the imposition of the AET, affects the probability that an individual worked last year.
The standard errors again are consistent with the possibility that the elimination of the AET causes an increase in the participation rate of the magnitude we estimate. Finally, Haider & Loughran (2008) examine the 1983 and 2000 elimination of the AET for those 70-71 and 66-69, respectively, and find no evidence of an increase in participation. Again, our elasticity would imply an effect that is not significantly different from their estimates. In sum, the increase in the participation rate on the order of a couple percentage points, which our results suggest, is within the confidence intervals estimated in previous studies.

Our method is applicable not only in the context of the AET, but also in many other contexts in which individuals make extensive margin decisions. Generally, in the context of the earnings decision, the estimation method requires: (1) a non-linear budget set; (2) information on earnings or labor supply; and (3) a proxy for desired earnings on a linear budget set, such as our measure of earnings prior to the imposition of the AET. There are many contexts in which such information is available, where our method could be fruitfully applied to estimating the effect of non-linear income taxes and transfers.

While we find large substitution effects in our context, our results do not imply that substitution effects are very large in all contexts. For example, elderly individuals are often on the margin of retirement, which could make them more responsive to exogenous changes in substitution incentives than other groups. Indeed, much previous literature has found large labor supply responses among the elderly in particular. It would be interesting to use our method to examine how extensive margin Frisch labor supply elasticities compare when estimated using our method, relative to those that have been estimated previously, in groups of other ages.
Works Cited


Figure 1: Elderly (age 65 and over) employment-to-population ratio

Notes: the figure shows the employment-to-population ratio by year from 1950 to 2000. The data on the employment-to-population ratio come from the Current Population Survey.

Figure 2: Key Earnings Test Rules, 1978 to 2009

Note: The right vertical axis measures the benefit reduction rate in OASI payments for every dollar earned beyond the exempt amount. The left vertical axis measures the real value of the exempt amount over time.
Figure 3: Intensive Margin Response to a Kink

Panel A: Linear Tax Schedule

Panel B: Kinked Tax Schedule

Note: The figure above demonstrates the intensive margin response to the introduction of a kink. In Panel A, agents locate along a linear tax schedule, according to differences in ability. In Panel B, after the introduction of a kink at $z^*$, agents reduce their earnings, with a mass of individuals locating exactly at the kink.
Figure 4: Extensive Margin Incentives

Note: the figure shows the ANTR (y-axis) as a function of before-tax income (x-axis). The figure shows that the slope of this graph discontinuously decreases at the kink point \( z^* \), due to the imposition of the tax on earnings above \( z^* \).
Figure 5: Extensive Margin Response to a Kink

Panel A:

After Tax Income $z - T(z)$

Panel B:

After Tax Income $z - T(z)$

Note: The figure depicts the potential responses to the introduction of a kinked budget set. In Panel A, the agent reduces earnings to the kink at $z^*$, preferring this to the outside level of consumption. In Panel B, the agent prefers the outside option to the optimal level of earnings, conditional on being in the labor force, i.e. earning at the kink.
Figure 6: Extensive Margin Response by Period 0 Earnings

Note: the figure relies on the model described in Section 2. The $x$-axis shows desired income on a linear budget set, observed in period zero. The $y$-axis shows a hypothetical probability of nonemployment in a future period 1, under three scenarios: a linear schedule in period 1 (dashed line); a kinked tax schedule in period 1 when individuals can make intensive margin adjustments (dotted line); and a kinked tax schedule in period 1 when individuals cannot make intensive margin adjustments (solid line).

Figure 7: Reduction in Current Benefits due to AET

Note: the figure shows the reduction in net income ($y$-axis) due to the imposition of the tax, as a function of pre-tax earnings $z$ ($x$-axis). The figure shows that the tax does not reduce net income below the kink $z^*$ but that above the kink, the reduction in net income is proportional to distance above the kink ($z - z^*$). The slope above the kink is $d\tau$, the change in the effective marginal tax rate at the kink.
Figure 8: Intensive Margin Response to the AET

Notes: Figure shows actual earnings density (dotted plot) and smooth fit (solid or dashed line), excluding the region within a $2,000 bandwidth of the exempt amount. Data are averaged over bins of $100 in width. Smooth density is estimated using a seventh-order polynomial.
Figure 9: Employment Rate Near Exempt Amount – Ages 63-64

Notes: Each figure plots the employment rate, i.e. probability of positive earnings, at ages 63 to 64 averaged, as a function of age 60 distance to exempt amount. Sample is people with positive age 60 earnings and no age 60 self-employment income, born 1918-1923.
Figure 10: Predetermined covariates

Note: the figure shows the means of predetermined covariates as a function of the distance to the age 60 exempt amount.
Figure 11: Probability of Positive Earnings by Single Year of Age, Ages 61 to 64

Notes: Each figure plots the employment rate, i.e. probability of positive earnings, for each single year of age from 61 to 64, as a function of age 60 distance to exempt amount. Sample is people with positive age 60 earnings and no age 60 self-employment income, born 1918-1923.
Notes: Figures plot the distribution of placebo kink estimates using data away from the exempt amount. The red vertical line denotes our actual point estimate.
Figure 13: R-squared by Placebo Kink Location

Notes: Figures plot the R-squared of our baseline specification against the “placebo” kink location, relative to the exempt amount. The vertical line denotes the actual location of the exempt amount.
Note: the figure shows the estimate of the change in slope of the probability of employment at the exempt amount, as a function of the bandwidth chosen. The vertical line shows the bandwidth chosen by the procedure of Calonico et al. (2014).
Table 1: Summary Statistics — Mean (Standard Deviation)

<table>
<thead>
<tr>
<th></th>
<th>Mean (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent female</td>
<td>68.0 (46.6)</td>
</tr>
<tr>
<td>Percent white</td>
<td>83.9 (36.7)</td>
</tr>
<tr>
<td>Year of birth</td>
<td>1920.6 (1.7)</td>
</tr>
<tr>
<td>Claim age if claimed</td>
<td>63.1 (2.6)</td>
</tr>
<tr>
<td>Annual earnings, ages 63-64, $1,000s</td>
<td>5.8 (8.2)</td>
</tr>
<tr>
<td>Percent of years with positive earnings, ages 63-64</td>
<td>56.5 (45.9)</td>
</tr>
</tbody>
</table>

Source: Social Security Administration Master Earnings File and Master Beneficiary Record, ten percent random sample. N=103,120. The sample is taken from all people born 1916-1930 with positive earnings at age 60 and no self-employment income in that year. Earnings are in thousands of $2012.

Table 2: Preliminary Tests for Smoothness

<table>
<thead>
<tr>
<th></th>
<th>Estimated Kink</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
</tr>
<tr>
<td>Density</td>
<td>-0.057</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
</tr>
<tr>
<td>White dummy</td>
<td>-0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
</tr>
<tr>
<td>Female dummy</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
</tr>
<tr>
<td>Year of birth</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td>(0.018)***</td>
</tr>
</tbody>
</table>

Notes: Tables present tests for smoothness of density as a function of age 60 distance to the exempt amount. Sample consist of people born 1918-1923 with positive earnings at age 60 and no self-employment income at age 60.
Table 3: Main Results

<table>
<thead>
<tr>
<th></th>
<th>Age 63-64, Pr(PE)</th>
<th>Elasticity, Ages 63-64</th>
<th>Age 56-57, Pr(PE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>-1.85</td>
<td>0.59</td>
<td>-0.59</td>
</tr>
<tr>
<td></td>
<td>(0.72)***</td>
<td>(0.15)***</td>
<td>(0.37)</td>
</tr>
<tr>
<td>Quadratic</td>
<td>-2.47</td>
<td>0.68</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(1.02)***</td>
<td>(0.22)***</td>
<td>(0.53)</td>
</tr>
<tr>
<td>Cubic</td>
<td>-3.11</td>
<td>0.82</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(1.07)***</td>
<td>(0.24)***</td>
<td>(0.60)</td>
</tr>
</tbody>
</table>

Notes: Table presents regression kink estimates of the probability of zero earnings as a function of age 60 distance to the exempt amount. Pr(PE) refers to the fraction of years with positive earnings. Specifications include first-, second- and third-degree polynomials, estimated separately, on either side of the exempt amount. The sample includes people with positive earnings at age 60 and no self-employment income at age 60, born 1918-1923. Robust standard errors, using the procedure of Calonico et al. (2014), are reported in parentheses. Bandwidth is chosen using the method of Calonico et al. (2014).

Table 4: Robustness to Controls

<table>
<thead>
<tr>
<th></th>
<th>Demographic controls</th>
<th>Level discontinuity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>-1.73</td>
<td>-1.85</td>
</tr>
<tr>
<td></td>
<td>(0.46)***</td>
<td>(0.46)***</td>
</tr>
<tr>
<td>Quadratic</td>
<td>-2.28</td>
<td>-2.46</td>
</tr>
<tr>
<td></td>
<td>(0.80)***</td>
<td>(0.81)***</td>
</tr>
<tr>
<td>Cubic</td>
<td>-2.82</td>
<td>-3.07</td>
</tr>
<tr>
<td></td>
<td>(0.91)***</td>
<td>(0.91)***</td>
</tr>
</tbody>
</table>

Notes: Table presents robustness checks on the results in Table 3. The “demographic controls" column shows the kink in the employment probability when we control for dummies for year of birth, sex, and race. The “level discontinuity" column shows the kink in the employment probability when control for a dummy for being above the exempt amount.
Table 5: RKD Estimates by Single Year of Age

<table>
<thead>
<tr>
<th></th>
<th>Age 61</th>
<th>Age 62</th>
<th>Age 63</th>
<th>Age 64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate of slope discontinuity</td>
<td>0.33</td>
<td>0.90</td>
<td>1.74***</td>
<td>1.98***</td>
</tr>
<tr>
<td></td>
<td>(0.81)</td>
<td>(0.91)</td>
<td>(0.65)***</td>
<td>(0.74)***</td>
</tr>
</tbody>
</table>

Notes: Table presents regression kink estimates of the probability of zero earnings as a function of age 60 distance to the exempt amount. The estimates are performed separately for each age, shown in the column heading.

Table 6: RKD Estimates by Demographic group

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
<th>White</th>
<th>Non-white</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate of elasticity</td>
<td>0.36***</td>
<td>0.61***</td>
<td>0.59***</td>
<td>0.64***</td>
</tr>
<tr>
<td></td>
<td>(0.11)***</td>
<td>(0.16)***</td>
<td>(0.14)***</td>
<td>(0.10)***</td>
</tr>
</tbody>
</table>

Notes: Table presents regression kink estimates of the probability of zero earnings as a function of age 60 distance to the exempt amount. The estimates are performed separately for each demographic group, shown in the column heading.
A. Model Discussion

A.1. Extension to Two or More Discrete Job Choices

We now extend the model of no intensive margin adjustments to one where there are multiple, discrete options for earnings. We begin with two options for positive earnings - one at the interior optimum in Period 0, \( z_{n,0} \), and another “part-time” job with some lower level of earnings, \( z_{n,pt} < z_{n,0} \). Let \( v_{n}^{pt} \) be the utility level associated with the part-time level of earnings:

\[
v_{n}^{pt} \equiv v(z_{n,pt} - T(z_{n,pt}), n).
\]

(23)

Note that the probability of remaining in the labor force in Period 1 is 1 minus the probability that exiting dominates both the full-time and part-time jobs:

\[
Pr(z_{n,1} > 0 | z_{n,0}) = 1 - Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \right. \left. \& v^0 \geq v(z_{n,0} - T(z_{n,0}), n), q_{n,1} \right)
\]

\[
= 1 - Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \left| \overline{q}_{n,1} < q_{n,1} \right. \right) \ Pr\left(\overline{q}_{n,1} < q_{n,1} \right)
\]

\[
= 1 - Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \left| \overline{q}_{n,1} < q_{n,1} \right. \right) \left[1 - G(q_{n,1} | n, t)\right].
\]

(24)

The slope of the employment function is now a more complicated expression:

\[
\frac{d Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = \frac{-d Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \left| \overline{q}_{n,1} < q_{n,1} \right. \right) \left[1 - G(q_{n,1} | n, t)\right]}{dz_{n,0}}
\]

\[+ Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \left| \overline{q}_{n,1} < q_{n,1} \right. \right) \left(\frac{dG(\overline{q}_{n,1} | n, t)}{dz_{n,0}}\right)\].

(25)

We will now explore under what conditions this slope reduces that of our earlier model where there is no part-time job option. We will show in general that this is true for those with initial earnings below \( z^* \). Next, we show that for those with earnings initially at or above \( z^* \), the slope is likewise unaffected, so long as the part-time job offers a level of earnings that is discretely lower than the full-time job.

Consider individuals initially to the left of \( z^* \). We first focus on the term

\[
Pr\left(v^0 \geq v_n^{pt} - q_{n,1} \left| \overline{q}_{n,1} < q_{n,1} \right. \right) .
\]

We can show the following for the agents in this set for whom \( \overline{q}_{n,1} > q_{n,1} \):

58
\begin{align*}
v^0 & > v(z_{n,1} - T(z_{n,0}), z_{n,1}; n) - q_{n,1} \\
& = v(z_{n,0} - T(z_{n,0}), z_{n,0}; n) - q_{n,1} \\
& > v_{n}^{pt} - q_{n,1},
\end{align*}

where, in the first line, we used the fact that $q_{n,1} > q_{n,1}$ and the definition of $\bar{q}_{n,1}$. In the second line, we used the fact that $z_{n,1} = z_{n,0}$ and $T_{1}(z_{n,0}) = T_{0}(z_{n,0})$ for individuals with $z_{n,0} < z^{*}$. In the third line, we used the fact that $v(z_{n,0} - T_{0}(z_{n,0}), z_{n,0}; n) \geq v_{n}^{pt}$, due to revealed preference in Period 0. It follows that:

\begin{align*}
\Pr \left( v^0 \geq v_{n}^{pt} - q_{n,1} \mid \bar{q}_{n,1} < q_{n,1}, z_{n,0} < z^{*} \right) &= 1 \\
\frac{d \Pr \left( v^0 \geq v_{n}^{pt} - q_{n,1} \mid \bar{q}_{n,1} < q_{n,1}, z_{n,0} < z^{*} \right)}{dz_{n,0}} &= 0
\end{align*}

In other words, if an individual with initial earnings below $z^{*}$ prefers the outside option in the absence of the part-time job, they continue to prefer it in the presence of the part-time job. Using the results in (27), we can simplify the expression in (26), for those with $z_{n,0} < z^{*}$:

\begin{align*}
\frac{d \Pr \left( z_{n,1} > 0 \mid z_{n,0} < z^{*} \right)}{dz_{n,0}} &= \frac{dG \left( \bar{q}_{n,1} \mid n, t \right)}{dz_{n,0}} \\
& = g \left( \bar{q}_{n,1} \mid n, t \right) \frac{\partial v(z_{n,0} - T_{1}(z_{n,0}), z_{n,0}; n)}{\partial n} \frac{dn}{dz_{n,0}} + \frac{\partial G \left( \bar{q}_{n,1} \mid n, t \right)}{\partial n} \frac{dn}{dz_{n,0}},
\end{align*}

where the second line follows from equation (7). Thus, the presence of a part-time job does not affect the results for this with $z_{n,0} < z^{*}$. Intuitively, the introduction of a kink has not affected the relevant portion of the budget set for these agents.

Now consider individuals with $z_{n,0} \geq z^{*}$. The change in the slope of the employment function at $z^{*}$ depends on the following limit:
First, note the following:

\[
\lim_{z_{n,0} \to z^*} \frac{d \Pr\left(v^0 \geq v^{pt}_n - q_{n,1} \mid \bar{q}_{n,1} < q_{n,1}\right)}{dz_{n,0}} = \lim_{z_{n,0} \to z^*} \frac{d \Pr\left(v^0 \geq v^{pt}_n - q_{n,1} \mid v^0 \geq v\left(z_{n,t} - T_1\left(z_{n,1}, z_{n,1} ; n\right) - q_{n,1}\right)\right)}{dz_{n,0}} \]

(29)

\[
= \Pr\left( v^0 \geq v^{pt}_n - q_{n,1} \mid v^0 \geq v\left(z^* - T_0\left(z^*\right) , z^* ; n\right) - q_{n,1}\right) = 1
\]

(30)

In the second line, we used the fact that \( \lim_{z_{n,0} \to z^*} z_{n,1} = z^* \) and \( T_1\left(z^*\right) = T_0\left(z^*\right) \), and the final line follows from revealed preference—\( z^* \) was initially chosen over the part time job.

As mentioned above, we require that the earnings level offered at the part-time job is discretely lower than that of the full-time job, in the neighborhood of \( z^* \). In particular, we assume the following:

\[
\lim_{z_{n,0} \to z^*} z_{n,pt} < z^*
\]

(31)

This assumption rules out part-time jobs that can be made arbitrarily close to the level of initial earnings. Intuitively, if this were not so, individuals initially earning just above \( z^* \) would essentially be able to replicate intensive margin adjustment, which we have shown smooths out a kink in the employment function.

The assumption in (31) implies that limit in (30) is reached at some level of initial earnings strictly above \( z^* \). That is, as we approach \( z^* \) from above, the likelihood that preferring the outside option without a part-time job implies preferring it in the presence of a part-time job plateaus at 1 at some point before reaching \( z^* \). Thus, we have:

\[
\lim_{z_{n,0} \to z^*} \frac{d \Pr\left(v^0 \geq v^{pt}_n - q_{n,1} \mid \bar{q}_{n,1} < q_{n,1}\right)}{dz_{n,0}} = 0
\]

(32)

As before, (30) and (32) can be used to simplify (29) as follows:
\[
\lim_{z_{n,0} \to z^2} \frac{d \Pr\left( z_{n,1} > 0 \, | \, z_{n,0} \right)}{dz_{n,0}} = \frac{dG\left( \bar{q}_{n,1} \, | \, n^*, t \right)}{dz_{n,0}}
\]

\[
= g\left( \bar{q}_{n,1} \, | \, n^*, t \right) \left[-d\tau \cdot \lambda_n + \frac{\partial v\left( z_{0,1} - T\left( z_{0,1}, z_{0,0} ; n^* \right) \right)}{\partial n} \frac{dn}{dz_{n,0}} \right] \]

\[
+ \frac{\partial G\left( \bar{q}_{n,1} \, | \, n^*, t \right)}{\partial n} \frac{dn}{dz_{n,0}},
\]

where the second line follows from equations (7)-(9). Combining (33) and (28), we have the same result as equation (9) of Section 2.4, i.e. our earlier model with no intensive margin adjustment. Furthermore, this result extends to a model with multiple, discrete earnings options, so long as the options all strictly differ from the initial level of earnings.

A.2. Fully Dynamic Extension of the Model

In this section, we briefly demonstrate how our results hold once our model is extended to a multi-period model with forward-looking agents. We assume that the individual faces \( T \) periods and that preferences and the economic environment yield a dynamic programming problem as follows. Every period, individuals maximize:

\[
u_t\left( c_{n,t}, z_{n,t} ; n \right) = v\left( c_{n,t}, z_{n,t} ; n \right) - q_{n,t} \cdot 1\left\{ z_{n,t} > 0 \right\} + V_t\left( A_{n,t}, z_{n,t} ; n \right),
\]

subject to a dynamic budget constraint:

\[
c_{n,t} = \left( 1 + r_{t-1} \right) A_{n,t-1} + z_{n,t} - T_t\left( z_{n,t} \right) - A_{n,t},
\]

where \( A_{n,t} \) is the level of assets at the end of period \( t \).

The value function for the next period, \( V_t\left( \cdot \right) \), may depend on the level of assets passed forward and potentially the level of earnings in the current period. For example, working today may have some effect on the choice set in the next period.

We once again restrict attention to individuals with positive earnings in period 0, and focus on the probability of having positive earnings in period 1, conditional on the earnings level in period 0: \( \Pr\left( z_{n,t} > 0 \, | \, z_{n,0} \right) \). In addition to the assumptions we have made above in Section 2.3, we assume that the value function is \( C^1 \) in \( A, z \), and \( n \). Agents
choose \( c, z, \) and \( A \) to maximize utility, and the outside value of not working, \( v^0 \), includes the continuation value of future periods.

Note that the first-order conditions when earnings are positive are now:

\[
\begin{align*}
v_z + V_z &= -\lambda \left(1 - T'(z)\right) \\
v_c &= V_A = \lambda,
\end{align*}
\]

where \( \lambda \) is the marginal utility of wealth. Using these conditions, we can show that there will still be bunching among those who can adjust on the intensive margin. As before, individuals will work if the utility conditional on working exceeds that of not working:

\[
v(\tilde{c}_{n,t}, \tilde{z}_{n,t}; n) + V_t(\tilde{A}_{n,t}, \tilde{z}_{n,t}; n) - v^0 - q_{n,t} > 0,
\]

where the “~” denotes optimal levels conditional on working. The probability of working in period 1 is still:

\[
\Pr(z_{n,1} > 0 | z_{n,0}) = G(q_{n,1} | n, t)
\]

where now:

\[
q_{n,t} \equiv v(\tilde{c}_{n,t}, \tilde{z}_{n,t}; n) + V_t(\tilde{A}_{n,t}, \tilde{z}_{n,t}; n) - v^0.
\]

We now show that our main results still hold under this dynamic setting. First, the slope of the labor participation rate will still be:

\[
\frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = g(q_{n,1} | n, t) \frac{d\tilde{q}_{n,1}}{dz_{n,0}} + \frac{dG(q_{n,1} | n, t)}{dn} \frac{dn}{dz_{n,0}}.
\]

We will have a new expression for the first term on the right of equation (40). After substituting for \( c_{n,1} \) in (39) using the dynamic budget constraint in (35) we have:
\[
\frac{d\bar{q}_{n,1}}{dz_{n,0}} = \left[ \frac{\partial v((1 + r_0)A_{n,0} + z_{n,1} - T_1(\tilde{z}_{n,1} - \tilde{A}_{n,1}, \tilde{z}_{n,1}; n))}{\partial \tilde{z}_{n,1}} + \frac{\partial v(A_{n,1}, \tilde{z}_{n,1}; n)}{\partial \tilde{A}_{n,1}} \right] \frac{d\tilde{z}_{n,1}}{dz_{n,0}} \\
+ \left[ \frac{\partial v((1 + r_0)A_{n,0} + \tilde{z}_{n,1} - T_1(\tilde{z}_{n,1} - \tilde{A}_{n,1}, \tilde{z}_{n,1}; n))}{\partial \tilde{A}_{n,1}} + \frac{\partial v(A_{n,1}, \tilde{z}_{n,1}; n)}{\partial \tilde{z}_{n,1}} \right] \frac{d\tilde{A}_{n,1}}{dz_{n,0}} \\
+ \left[ \frac{\partial v((1 + r_0)A_{n,0} + z_{n,1} - T_1(z_{n,1} - A_{n,1}, z_{n,1}; n))}{\partial \tilde{z}_{n,1}} + \frac{\partial v(A_{n,1}, z_{n,1}; n)}{\partial \tilde{A}_{n,1}} \right] \frac{d\tilde{z}_{n,1}}{dz_{n,0}} \\
+ \left[ -v + v_A \right] \frac{dA_{n,1}}{dz_{n,0}} + \left[ v + V_n \right] \frac{dn}{dz_{n,0}}.
\]

\begin{align}
(41)
\end{align}

For those with \( z_{n,0} < z^* \) or \( z_{n,0} > z^* + \Delta z^* \), we can use the first-order conditions in (36) to show that the first term in (41) equals zero when agents are able to adjust on the intensive margin. For those with \( z^* \leq z_{n,0} \leq z^* + \Delta z^* \), the first term in (41) equals zero—

\( \frac{dz_{n,1}}{dz_{n,0}} = 0 \) since \( \tilde{z}_{n,1} = z^* \) for everyone in this latter set due to bunching. Additionally, the second term in (41) equals zero for everyone, due to the first-order condition in (36) for saving.

Thus, when agents are able to adjust on both the intensive and extensive margin, we have:

\begin{align}
\frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} &= g(q_{n,1}, n, t) \left[ \frac{\partial v(c_{n,t}, z_{n,t}; n)}{\partial n} + \frac{\partial v(A_{n,t}, z_{n,t}; n)}{\partial \tilde{A}_{n,t}} \right] \frac{dn}{dz_{n,0}} \\
+ \frac{dG(q_{n,1}, n, t)}{dn} \frac{dn}{dz_{n,0}}.
\end{align}

\begin{align}
(42)
\end{align}

Our smoothness assumptions imply that this slope is continuous, and in particular, it is continuous at \( z^* \). That is, \( n, q_{n,1}, \tilde{z}_{n,1}, \tilde{A}_{n,1}, T_1(\cdot) \) and \( \partial G(q_{n,1}, n, t) / \partial n \) are all continuous in \( z_{n,0} \) at \( z^* \). Furthermore, \( g(\cdot) \), \( \partial v / \partial n \) and \( \partial V / \partial n \) are likewise continuous in their arguments. Our original result follows when there are intensive margin adjustments:

\begin{align}
\lim_{z_{n,0} \to z^*} \frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = \lim_{z_{n,0} \to z^*} \frac{d\Pr(z_{n,1} > 0 | z_{n,0})}{dz_{n,0}} = 0
\end{align}

(43)
Now, turning to the case where individuals are not able to adjust on the intensive margin, we have:

\[
\begin{align*}
\frac{d\tilde{q}_{n,1}}{dz_{n,0}} &= \left[ \frac{\partial \mathcal{V}\left((1+r_0)A_{n,0} + z_{n,0} - T_1(z_{n,0}) - \tilde{A}_{n,1}, z_{n,0}; n\right)}{\partial z_{n,0}} + \frac{\partial \mathcal{V}\left(\tilde{A}_{n,1}, z_{n,0}; n\right)}{\partial z_{n,0}} \right] \\
&+ \left[ \frac{\partial \mathcal{V}\left((1+r_0)A_{n,0} + z_{n,0} - T_1(z_{n,0}) - \tilde{A}_{n,1}, z_{n,0}; n\right)}{\partial \tilde{A}_{n,1}} + \frac{\partial \mathcal{V}\left(\tilde{A}_{n,1}, z_{n,0}; n\right)}{\partial \tilde{A}_{n,1}} \right] \frac{d\tilde{A}_{n,1}}{dz_{n,0}} \\
&+ \frac{\partial \mathcal{V}\left((1+r_0)A_{n,0} + z_{n,0} - T_1(z_{n,0}) - \tilde{A}_{n,1}, z_{n,0}; n\right)}{\partial n} \frac{dn}{dz_{n,0}} \\
&= \left[ (1-\tau_1)v_c + v_z + V_z \right] + \left[ -v_c + V_n \right] \frac{d\tilde{A}_{n,1}}{dz_{n,0}} + \left[ v_n + V_n \right] \frac{dn}{dz_{n,0}}.
\end{align*}
\]

where the primary difference from (41) is that earnings are fixed at \(z_{n,0}\). We still have the second term dropping out of the expression in (44), as assets, \(\tilde{A}_{n,1}\), are still optimally chosen, even when earnings are fixed. Furthermore, for those with \(z_{n,0} \leq z^*\), we have \(T_1 = T_0\), and thus the first term also drops out, due to the envelope theorem. Note that we can rewrite:

\[
(1-\tau_1)v_c + v_z + V_z = \lambda \left( 1-\tau_1 \right) + \frac{v_c + V_c}{v_c}.
\]

Similar to our less dynamic model above in Section 2.4, we therefore have:

\[
\begin{align*}
\frac{d\Pr(z_{n,1} > 0 \mid z_{n,0})}{dz_{n,0}} &= \left\{ \begin{array}{ll}
g\left(\tilde{q}_{n,1} \mid n, t\right) \left[ v_n + V_n \right] \frac{dn}{dz_{n,0}} + \frac{\partial G\left(\tilde{q}_{n,1} \mid n, t\right)}{\partial n} \frac{dn}{dz_{n,0}} & \text{if } z_{n,0} < z^* \\
g\left(\tilde{q}_{n,1} \mid n, t\right) \lambda \left( 1-\tau_1 \right) + \frac{v_n + V_n}{v_n} \frac{dn}{dz_{n,0}} & \text{if } z^* \leq z_{n,0}
\end{array} \right.
\end{align*}
\]

Again, we note the following limit:
\[
\lim_{z_{n,0} \to z^{*}} \frac{v_z \left( (1 + r_0) A_{n,0} + z_{n,0} - T_1(z_{n,0}) - \tilde{A}_{n,1} z_{n,0}; n \right) + V_z \left( \tilde{A}_{n,1} z_{n,0}; n \right)}{v_c \left( (1 + r_0) A_{n,0} + z_{n,0} - T_1(z_{n,0}) - \tilde{A}_{n,1} z_{n,0}; n \right)} = -(1 - \tau_0)
\] (47)

Thus, we have our original result:

\[
\lim_{z_{n,0} \to z^{*}} \frac{d \Pr \left( z_{n,1} > 0 | z_{n,0} \right)}{dz_{n,0}} - \lim_{z_{n,0} \to z^{*}} \frac{d \Pr \left( z_{n,1} > 0 | z_{n,0} \right)}{dz_{n,0}} = \lim_{z_{n,0} \to z^{*}} \frac{g(q_{n,1} | n, t) \cdot \lambda_n \left[ (1 - \tau_1) + \frac{v_z + V_z}{v_c} \right]}{\lambda_{n,1} \left[ (1 - \tau_1) - (1 - \tau_0) \right]} = g \left( \frac{q_{n,1}}{n}, t \right) \cdot \lambda_n \left[ (1 - \tau_1) - (1 - \tau_0) \right] = -d \tau \cdot \lambda_{n,1} \cdot g \left( \frac{q_{n,1}}{n}, t \right)
\] (48)