Optimal Macroprudential and Monetary Policy in a Currency Union*

Dmitriy Sergeyev†

March 22, 2016

Abstract

I solve for optimal macroprudential and monetary policies for members of a currency union in an open economy model with nominal price rigidities, demand for safe assets, and collateral constraints. Monetary policy is conducted by a single central bank, which sets a common interest rate. Macroprudential policy is set at a country level through the choice of reserve requirements. I emphasize two main results. First, with asymmetric countries and sticky prices, the optimal macroprudential policy has a country-specific stabilization role beyond optimal regulation of financial sectors. This result holds even if optimal fiscal transfers are allowed among the union members. Second, there is a role for global coordination of country-specific macroprudential policies. This is true even when countries have no monopoly power over prices of internationally traded goods or assets. These results build the case for coordinated macroprudential policies that go beyond achieving financial stability objectives.

*I would like to thank the ECB for financial support through the Lamfalussy fellowship. For useful comments and conversations I thank Luigi Iovino, Tommaso Monacelli, Nicola Pavoni, Ricardo Reis, Margarita Rubio, Michael Woodford, and workshop participants at the Bank of Finland.

†Bocconi University, Department of Economics, e-mail: dmytro.sergeyev@unibocconi.it

1 Introduction

Macroprudential regulation—policies that target financial stability by emphasizing the importance of general equilibrium effects—has become an important tool of financial regulation in recent years (Hanson *et al.*, 2011). For example, the 2010 Basel III accord, an international regulatory framework for banks, introduced a set of tools that require financial firms to hold larger liquidity and capital buffers, which could depend on the credit cycle (BSBC, 2010).

Macroprudential regulation may be in conflict with traditional monetary policy that stabilizes inflation and output (Stein, 2013, 2014). On the one hand, variation in the monetary policy rate shapes private incentives to take on risks, use leverage, and short-term debt financing. On the other hand, changes in macroprudential regulation constrain financial sector borrowing, which affects aggregate output.

In contrast, regional macroprudential policies may help achieve traditional monetary policy objectives in a currency union. Monetary policy cannot fully stabilize asymmetric shocks in a currency union, because fixed nominal exchange rate and a single monetary policy rate are constraints that prevent full stabilization. Macroprudential regulation at a regional level can help mitigate asymmetric shocks, because tighter financial regulation can affect local business cycles.

The goal of this paper is to solve for optimal union-wide monetary and regional macroprudential policies in an environment where these policies interact. I address the optimal policy problem by solving a model that combines a standard New Keynesian model with a recent literature on macroprudential regulation of the financial sector, which I then extend to a currency union setting.

The first step is to define a fundamental market failure that justifies policy interventions. I consider a model environment, which is a variant of the model proposed in Stein (2012), with the following key features. Households value safe securities above and beyond their pecuniary returns because these securities are useful for transactions. This is formally introduced via a safe-assets-in-advance constraint. Financial firms can manufacture a certain amount of these securities by posting durable goods as collateral. The resulting endogenous collateral constraint on safe debt issuance, which features durable goods price, leads to a negative pecuniary externality (a fire-sale externality). Financial firms issue too many safe securities, which leads to social welfare losses. This provides a role for macroprudential policy to limit issuance of safe debt by financial firms.

Financial regulation can address this externality using a number of tools.[1] In this paper, I study reserve requirements (with interests paid on reserves) applied universally

[1] See Claessens (2014) for a recent review of various macroprudential tools.

to all riskless liabilities of all financial firms. I follow Kashyap and Stein (2012) and Woodford (2011), who argue that this tool can address financial stability concerns in a closed economy. Universal reserve requirements resemble traditional reserve requirements and the liquidity coverage ratio, introduced in the Basel III accord. Traditional reserve requirements policy orders banks to keep a minimum amount of central bank reserves relative to their deposits. The liquidity coverage ratio broadens the scope of traditional reserve requirements by obliging various types of financial firms (and not just traditional banks) to hold a minimum amount of liquid assets relative to various liabilities, and not just deposits. The macroprudential policy tool in this paper differs from the liquidity coverage ratio, in that financial firms are required to hold central bank reserves only.

In a closed economy version of the model, optimal monetary and macroprudential policies are not in conflict. Optimal monetary policy achieves the flexible price allocation, and optimal macroprudential policy only corrects the fire-sale externality. However, if any of the two policies is suboptimal, there is a scope for the other policy to address both inefficiencies.

The proposed model is extended to a currency union setting along the lines of Obstfeld and Rogoff (1995) and Farhi and Werning (2013). Households have preferences over traded and non-traded goods. The safe-assets-in-advance constraint is applied to both traded and non-traded goods. Durable goods are produced by local financial firms out of non-traded goods. The last assumption allows local macroprudential policy to affect output of non-traded goods. Only safe securities are traded internationally.

The international dimension of the model adds three additional externalities that affect welfare. First, the price of durable goods, which enters the collateral constraint, depends on the traded and non-traded goods composition of aggregate consumption. Private agents do not internalize their effect on the durable goods price through their effect on the composition of aggregate consumption. This introduces an additional negative pecuniary externality. This type of pecuniary externality is emphasized in the literature on prudential capital controls (Bianchi, 2011). Second, the relative price of traded to non-traded goods is present in the safe-assets-in-advance constraint. This creates a positive pecuniary externality. Third, in the presence of sticky prices, fixed exchange rate, and non-traded goods, an increase in safe debt issuance by financial firms affects the households pattern of spending. This effect generates another macroeconomic externality. Similar macroeconomic externality underlies benefits of fiscal transfers (Farhi and Werning, 2012). All of these international externalities will affect the trade-offs faced by local financial regulators.

I emphasize two main results in this paper. First, optimal macroprudential policy is used to stabilize business cycles. When monetary and macroprudential policies are set optimally in a coordinated way across monetary union members, optimal macroprudential policy is country-specific, and it depends on the amount of slack in a country. Optimal monetary policy sets average across countries labor wedge to zero. However, the central bank cannot replicate flexible price allocation in each country. This provides a stabilization role for regional financial regulation. Optimal macroprudential policy trades off its financial stability objective, mitigation of pecuniary externalities, and stabilization of inefficient business cycle fluctuations due to presence of sticky prices.

Optimal macroprudential policy is used to stabilize business cycles even when fiscal transfers are allowed among the union members, and these transfers are set optimally. Optimal fiscal transfers equalize the social marginal value of traded goods across countries. However, in general, the fiscal transfers cannot achieve a flexible price allocation in every country. As a result, macroprudential policy are partly used to stabilize inefficient business cycle fluctuations. This result emphasizes that optimal regional macroprudential policy must be directed toward business cycle stabilization even when some regional stabilization tools are available.

The second main results underscores the benefits of global coordination of regional macroprudential policies. There are three sources of gains from coordination stemming from the three externalities that arise in the international context. Intuitively, a tighter financial regulation in a particular country reduces the supply of safe assets in this country. This affects consumption of traded and non-traded goods in this country and in all other counties of the union. Variation in consumption of traded and non-traded goods changes the collateral constraints, the amount of goods that can be bought with safe debt, and the size of labor wedge in all other countries. As a result, local macroprudential policy has international spillovers that are not internalized by the local regulator. This results in the benefits of coordination.[2] The logic behind this result does not require countries to have any monopoly power over price of internationally traded goods or assets.

Related Literature. The elements of the model are related to several strands of literature. The model builds on the recent paper by Stein (2012) who argues that the fire-sale externality creates a role for macroprudential interventions.[3] The idea that it is useful to use safe and liquid securities for transactions, and the financial sector can create such securities, is rationalized in Gorton and Pennacchi (1990); Dang *et al.* (2012). Woodford

[2]If fiscal transfers are not available, there is an additional source of international spillovers. Because social marginal benefits from traded goods are not equalized, there are first order gains from redistribution of traded consumption across countries.

[3]Stein (2012) relies on the earlier literature which emphasizes fire-sales. See, for example, Shleifer and Vishny (1992); Gromb and Vayanos (2002); Lorenzoni (2008). A number of recent papers suggested that a system of Pigouvian taxes can be used to bring financial sector incentives closer to social interests (Bianchi, 2011; Jeanne and Korinek, 2010).

(2011) introduces a model similar to Stein (2012) into a standard closed-economy New Keynesian model and shows that optimal monetary policy must partly address the fire-sale externality when macroprudential policy is suboptimal. In a related model, Caballero and Farhi (2015) show that unconventional monetary policy can be more effective than traditional monetary policy in fighting the shortage of safe assets. In this paper, I extend the model with a special role for safe assets to an international setting following the New Open Economy Macro literature.[4] I build on the models of Obstfeld and Rogoff (1995) and Farhi and Werning (2013).

The results in this paper are connected to three strands of literature. First, the optimal currency area literature deals with the inability of traditional monetary policy to fully stabilize asymmetric shocks in a currency union. This literature proposes that factors mobility (Mundell, 1961), higher level of openness (McKinnon, 1963), and fiscal integration (Kenen, 1969) are necessary for stabilization of asymmetric shocks. More recent contributions emphasize the importance of regional fiscal purchases (Beetsma and Jensen, 2005; Gali and Monacelli, 2008), distortionary fiscal taxes (Ferrero, 2009), fiscal transfers (Farhi and Werning, 2013), and capital controls (Schmitt-Grohe and Uribe, 2012). Adao *et al.* (2009); Farhi *et al.* (2014) show that with a sufficient number of fiscal tools the flexible price allocation can be achieved in a monetary union. However, it is possible that a sufficient number of policy tools is not available to policy makers. The current paper complements this literature by analyzing regional macroprudential policy as a potential macroeconomic stabilization tool.

Second, there is a growing literature that studies macroprudential and monetary policy in a small open economy. Benigno *et al.* (2013), Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010) study macroprudential capital controls in models where foreign borrowing by a country is limited by a collateral constraint. Fornaro (2012) compares different exchange rate policies, and Ottonello (2013) solves for optimal exchange rate policy in a model with wage rigidity and occasionally binding borrowing constraints. Otrok *et al.* (2012) compare different monetary and macroprudential policies in an environment with sticky prices and collateral constraints. In my environment, there is an explicit financial sector that can be a source of the fire-sale externality even without international capital flows. This allows me to separate capital controls and financial sector regulation policies. In addition, I am interested in deriving optimal policy in a currency union instead of a small open economy.

Finally, there are papers that address joint conduct of monetary and macroprudential policies in a currency union. Beau *et al.* (2013) and Brzoza-Brzezina *et al.* (2015) compare

[4]See, for example, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Benigno and Benigno (2003) for early contributions and Corsetti *et al.* (2010) for a recent overview.

effects of several specifications of monetary and macroprudential policies on macroeconomic variables, and Rubio (2014) does it for welfare. Quint and Rabanal (2014) solve for the best monetary and macroprudential policies in the class of simple policy rules that are predetermined functions of macroeconomic variables. In this paper, I solve for optimal monetary and macroprudential policies and derive implications for coordination of these policies.

The rest of the paper is organized as follows. Section 2 presents a closed economy model with sticky prices and nonpecuniary demand for safe assets. Section 3 extends the model to a currency union setting. Section 4 concludes.

2 A 2-period Closed Economy Model

I first present a closed-economy model that introduces specific modeling assumptions in the most transparent way. Section 3 extends the model to a multi-country setting.

The economy goes on for two dates, $t = 0, 1$. Uncertainty affects only preferences over durable goods in period 1, the state of the world is denoted by s_1 (all endogenous variables in period 1 can depend on s_1). There are three types of goods in the economy: durable goods, (final) consumption goods and a continuum of differentiated intermediate goods. The economy is populated by a continuum of identical multi-member households with a unit mass, a continuum of final-good producing firms with a unit mass, and the government. Any state-contingent security is traded between periods 0 and 1.

2.1 Households

Each household consists of four types of agents: a firm, a banker, a consumer and a worker.[5] Household preferences are

$$\mathbb{E}\left\{u\left(c_0\right) - v\left(n_0\right) + \beta\left[\mathbb{U}\left(c_1, \underline{c}_1\right) + X_1(s_1)g\left(h_1\right) - v\left(n_1\right)\right]\right\} \tag{1}$$

where n_t is labor supply in $t = 0, 1$; c_t is consumption which can be bought on credit in $t = 0, 1$; $\underline{c}_1$ is consumption in period 1 that can be bought with safe assets only, h_1 is consumption of durable goods.[6] $u(\cdot)$ is strictly increasing and concave, $v(\cdot)$ is strictly

[5]The multi-member household construct allows to study situations in which different agents have different trading opportunities but keeps the simplicity of the representative household. See, for example Lucas (1990).

[6]The fact that preferences are not symmetric over the two periods is without loss of generality. Assuming that household enters period 0 with an endowment of safe assets and endowment of durable goods allows to make preferences symmetric without changing the results.

increasing and convex, $g(\cdot)$ is strictly increasing, concave, and $-g''(h_1)\, h_1 / g'(h_1) < 1$. Random variable $X_1(s_1)$ takes on two values $X_1 \in \{1, \theta\}$ with corresponding probabilities μ and $1-\mu$. Utility from consumption of perishable goods in period 1 is given by

$$\mathbb{U}(c_1, \underline{c}_1) = u(c_1 + \underline{c}_1) + \nu u(\underline{c}_1),$$

where ν is the parameter that controls demand for goods bought with safe securities.[7]

A worker competitively supplies n_t units of labor and receives income $W_t n_t$, where W_t is the nominal wage in period t. A firm is a monopolist and it uses a linear technology to produce differentiated good j

$$y_t^j = A_t n_t^j,$$

The firm hires labor on a competitive market at nominal wage W_t, but pays $W_t \cdot (1+\tau_t^L)$, where τ_t^L is the labor tax (or subsidy if negative). Price P_0^j of differentiated good period 0 is sticky, however, price P_1^j is flexible. I do not model the reason for price P_0^j stickiness. One can assume that the price was set before period 0 conditional on expectations about future economic conditions, and the economic conditions turn out to be different from expected. The final goods producer's demand for each variety is $y_t(P_t^j/P_t)^{-\epsilon}$, where $P_t = (\int (P_t^j)^{1-\epsilon} dj)^{1/(1-\epsilon)}$ is the price of final goods. The profits of the firm producing variety j is

$$\Pi_0^j = \left(P_0^j - \frac{(1+\tau_0^L)\, W_0}{A_0}\right) y_0 \left(\frac{P_0^j}{P_0}\right)^{-\epsilon},$$

$$\Pi_1^j(s_1) = \left(P_1^j(s_1) - \frac{(1+\tau_1^L)\, W_1(s_1)}{A_1}\right) y_1(s_1) \left(\frac{P_1^j(s_1)}{P_1(s_1)}\right)^{-\epsilon}.$$

A banker buys k_0 units of final goods in period 0 and immediately produces $h_1 = G(k_0)$ units of durable goods that he sells to consumers in the next period at flexible nominal price $\Gamma_1(s_1)$. To finance the purchase of final goods the banker issues safe bonds with face value D_1^b, and he receives $D_1^b / (1+i_0)$, where i_0 is the nominal interest rate on safe bonds. In addition to safe debt, the banker can issue any state contingent security, including equity. The banker is required that at least fraction z_0 of his safe liabilities is covered by central bank reserves. Formally, the banker buys R_1^b reserves by paying $R_1^b/(1+i_0^r)$, where i_0^r is the interest rate on reserves to satisfy

$$z_0 \leq \frac{R_1^b}{D_1^b}. \tag{2}$$

[7] Under this assumption on preferences, the overall production of consumption goods can be determined without reference to the supply of liquid assets (see Woodford, 2003 for details).

For banker's safe debt to be safe, this debt must be guaranteed to be repaid in the worst state of the economy in period 1. Formally, this implies

$$D_1^b \leq \min_{s_1}\{\Gamma_1(s_1)\}G\left(k_0\right) + R_1^b,$$

where $\min_{s_1} \Gamma_1(s_1)$ is the smallest possible price of durable goods in period 1.

A consumer decides on the assets allocation of the household: he buys any state-contingent security that bank issues, and he also buys D_1^c safe bonds, and pays $D_1^c/(1 + i_0)$. A consumer also buys final goods c_t on credit in both periods, and final goods $\underline{c}_1$ in period 1 with safe assets. Formally,

$$P_1\underline{c}_1 \leq D_1^c, \tag{3}$$

where P_1 is the nominal price in $t = 1$. This inequality states that consumption $\underline{c}_1$ must be purchased using risk-free assets D_1^c.[8] There is a long tradition in macroeconomic literature to assume that part of consumption goods must be bought with nominal liabilities of a central bank (Svensson, 1985; Lucas and Stokey, 1987) because of transaction frictions. I assume that not only central bank liabilities, but also other safe assets can be used to purchase these goods. These securities include government and private safe bonds.[9] I will call this constraint the " safe-assets-in-advance (SAIA) constraint."

Consolidated household budget constraints in periods 0 and 1 are

$$\begin{aligned}
&T_0 + P_0c_0 + \frac{D_1^c}{1+i_0} + \frac{R_1^b}{1+i_0^r} + P_0k_0 \leq \frac{D_1^b}{1+i_0} + W_0n_0 + \Pi_0^j, \\
&P_1\left(c_1 + \underline{c}_1\right) + T_1 + \Gamma_1h_1 + D_1^b \leq D_1^c + R_1^b + W_1n_1 + \Gamma_1G(k_0) + \Pi_1^j,
\end{aligned}$$

where P_0 is the price of perishable goods in period 0; T_0,T_1 are lump-sum taxes.[10]

If the interest rate on reserves is strictly smaller than the interest rate on other safe securities ($i_0^r < i_0$), the bankers optimally choose not to hold more reserves than required ($R_1^b = z_0D_1^b$). As a result, the collateral constraint and the budget constraints in both

[8] In this simple model, there is not going to be inflation risk. Thus it is not necessary to specify if the securities must be safe in real or nominal terms.

[9] See Krishnamurthy and Vissing-Jorgensen (2012a,b) for recent evidence that the U.S. treasuries and some financial sector liabilities command both safety and liquidity premia.

[10] Note that this representation of the budget constraint does not feature state-contingent securities issued by banks and state-contingent securities bought by the consumers. This is without loss of generality because bankers and consumers are members of multi-member households. It can be thought that bankers issue state-contingent securities within its household.

periods can be written as

$$\widetilde{D}_1^b \le \min_{s_1}\{\Gamma_1(s_1)\}G\left(k_0\right), \tag{4}$$

$$T_0 + P_0 c_0 + \frac{D_1^c}{1+i_0} + P_0 k_0 \le \frac{\widetilde{D}_1^b}{1+i_0}(1-\tau_0^b) + W_0 n_0 + \Pi_0^j, \tag{5}$$

$$P_1\left(c_1 + \underline{c}_1\right) + T_1 + \Gamma_1 h_1 + \widetilde{D}_1^b \le D_1^c + W_1 n_1 + \Gamma_1 G(k_0) + \Pi_1^j, \tag{6}$$

where $\widetilde{D}_1^b \equiv D_1^b - R_1^b$ is bankers safe debt liabilities net of reserves deposited at the central bank, and $\tau_0^b \equiv z_0/(1-z_0)\cdot(i_0 - i_0^r)/(1+i_0^r)$. Constraints (4)-(6) do not separately depend on i_0^r and z_0 but only through their combination expressed by τ_0^b, which can be interpreted as the Pigouvian tax on safe debt issuance. I will call τ_0^b a "macroprudential tax." If the interest rate on reserves are equal to the interest rate on other safe securities ($i_0^r = i_0$), a banker may choose to hold excess reserves in which case the reserve requirements constraint does not bind, but the constraints faced by the household are still identical to (4)-(6) with $\tau_0^b = 0$.

A household maximizes (1) subject to (3)-(6) by choosing consumption $c_0, c_1, \underline{c}_1$, h_1, safe debt position D_1^c, $\widetilde{D}_1^b$, labor supply n_0, n_1, investment in production of durable goods k_0 and price P_1^j (price P_0^j is exogenously fixed).

Household optimality conditions with respect to consumption, asset allocation, and labor supply can be summarized as follows

$$u'(c_0) =(1+i_0)\beta\mathbb{E}_0\left\{\frac{P_0}{P_1}u'(c_1+\underline{c}_1)\left[1+\frac{\nu u'(\underline{c}_1)}{u'(c_1+\underline{c}_1)}\right]\right\}, \tag{7}$$

$$\frac{\Gamma_1(s_1)}{P_1} =\frac{X_1(s_1)g'(h_1)}{u'\left(c_1+\underline{c}_1\right)}, \tag{8}$$

$$\frac{W_0}{P_0} =\frac{v'(n_0)}{u'(c_0)}, \tag{9}$$

$$\frac{W_1}{P_1} =\frac{v'(n_1)}{u'(c_1+\underline{c}_1)}. \tag{10}$$

Equation (7) is the Euler equation for safe bonds. It features a term in square brackets that represents safety yield. Equation (8) represents the demand for durable goods. Optimality conditions (9) and (10) are labor supply schedules in both periods.

The Lagrange multiplier on the SAIA constraint expressed in utility units is $\eta_1 = \nu u'(\underline{c}_1)/u'(c_1+\underline{c}_1)$. It is positive (the constraint binds) if the marginal utility $u'(\cdot)$ is positive. The bankers' optimal choice of investment in durable goods production and issuance of safe assets implies

$$u'(c_0) = \beta G'(k_0)\,\mathbb{E}_0 u'(c_1 + \underline{c}_1)\left\{\frac{\Gamma_1}{P_1} + \zeta_0 \frac{\min_{s_1}\{\Gamma_1\}}{P_1}\right\}, \tag{11}$$

where ζ_0 is the Lagrange multiplier on the collateral constraint (4) expressed in units of utility, and it equals

$$\zeta_0 = \frac{1-\tau_0^B}{1+i_0} \cdot \frac{u'(c_0)/P_0}{\mathbb{E}_0\left[u'(c_1+\underline{c}_1)/P_1\right]} - 1 \geq 0. \tag{12}$$

Optimal choice of prices for intermediate goods in a symmetric equilibrium in which all firms set the same price P_1 leads to

$$P_1 = \frac{\epsilon}{\epsilon - 1} \cdot \frac{W_1(1+\tau_1^L)}{A_1}. \tag{13}$$

The expression states that the firm sets its period-1 price equal to a markup over its marginal costs.

2.2 Final Goods Firms

Final goods are produced by competitive firms that combine a continuum of varieties $j \in [0,1]$ using the CES technology

$$y_t = \left(\int y_t^{j\,\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}},$$

with elasticity $\epsilon > 1$. Each firm solves in $t = 0, 1$

$$\max_{y_t^j} P_t y_t - \int P_t^j y_t^j dj,$$

Optimal choice of inputs leads to differentiated goods demand $y_t^j = y_t (P_t^j / P_t)^{-\epsilon}$ and the aggregate price index is defined as $P_t = (\int (P_t^j)^{1-\epsilon} dj)^{1/(1-\epsilon)}$.[11]

[11] It must be noted that the formulation of final goods firm's problem implicitly assumes that they sell goods at a single price to those who buy goods on credit and to those who buy goods with safe assets. This assumption rules out an equilibrium in which final goods producers sell their output at different prices to those who buy with credit and to those who buy with safe assets.

2.3 Government

The government consists of financial regulation, monetary and fiscal authorities.

Financial regulation policy. The financial regulation authority chooses the level of reserve requirements z_0 and the interest on reserves i_0^r, which is equivalent to choosing τ_0^b. It rebates the proceeds to the fiscal authority.

Monetary policy. The monetary authority sets the nominal interest rate i_0 on safe assets and targets inflation rate $\Pi^* = P_1/P_0$ which is assumed not to depend on state s_1. To motivate the monetary authority control over the nominal price level in period 1 and the nominal interest rate between periods 0 and 1, one can assume that fraction $\kappa \in [0,1]$ of purchases has to be made with monetary authority nominal liabilities M_0, M_1 that do not pay nominal interest (cash).[12] Formally, $\kappa P_0 c_0 = M_0, \kappa P_1(c_1 + \underline{c}_1) = M_1$. Because consumers want to economize on cash holdings when the safe nominal interest rate is strictly positive, there is demand for cash which depends on nominal interest rate. By setting nominal interest rate i_0, the monetary authority is ready to satisfy any demand for cash in period 0. The price level in period 1 is $P_1 = M_1/[\kappa(c_1 + \underline{c}_1)]$. When announcing the price level for period 1, the monetary authority adjusts M_1 to keep the price level fixed at the announced level. Allowing κ and M_0, M_1 to go to zero so that ratios $M_0/\kappa, M_1/\kappa$ stay positive and finite, the government determines price P_1, but there is no need to explicitly consider equilibrium in the cash market. This limit is sometimes called "a cashless economy."

Fiscal Policy. The fiscal authority sets lump sum taxes T_0, T_1, and proportional labor taxes τ_0^L, τ_1^L. The fiscal authority corrects monopolistic competition friction in period 1 by setting $\tau_1^L = -1/\epsilon$. The labor tax in period 0 will not affect the equilibrium conditions because the period-0 price is assumed to be exogenously fixed.

The government issues D_1^g of safe securities. This amount consists of safe government bonds and the reserves purchased by the banks. Note that under the assumption that the financial regulator sets the interest rate on reserves i_0^r and the reserve requirement z_o, the quantity of reserves must adjust to satisfy banks reserves demand. The identity of the authority that issues public safe securities does not matter for equilibrium as long as the consolidated government budget constraint is satisfied. However, it matters whether the overall amount of public safe securities D_1^g reacts to changes in the economy. For example, if the reserve requirement constraint binds ($R_1^b = z_0 D_1^b$), the total amount of

[12] See Mankiw and Weinzierl (2011) for similar treatment of monetary policy in a 2-period model.

reserves demanded by the banks is an endogenous variable, which may affect the overall public safe securities supply. I assume that the government targets the overall public safe securities supply D_1^g, which implies that any equilibrium variation in the amount of outstanding reserves is offset by the mirror change in the supply of safe public bonds.

The consolidated government budget constraints in both periods are

$$\begin{aligned} 0 &= T_0 + \tau_0^b \frac{\widetilde{D}_1^b}{1+i_0} + \frac{D_1^g}{1+i_0} + \tau_0^L W_0 n_0, && (14) \\ D_1^g &= T_1 + \tau_1^L W_1 n_1. && (15) \end{aligned}$$

2.4 Auxiliary Variables

It will prove useful to introduce a number of new variables which will simplify notation. $d_1^c \equiv D_1^c/P_1$ is the real household demand for safe assets, $\widetilde{d}_1^b \equiv \widetilde{D}_1^b/P_1$ is the real private supply of safe assets by bankers, $d_1^g \equiv D_1^g/P_1$ is the real public supply of safe assets, $\gamma_1 \equiv \Gamma_1/P_1$ is the price of durable goods expressed in units of period-1 consumption goods, $w_t \equiv W_t/P_t$ is the real wage, and $r_0 \equiv (1+i_0)P_0/P_1$ is the safe real interest rate. Let us define the durable goods price elasticity as

$$\epsilon_\Gamma \equiv -\frac{\partial \log \gamma_1}{\partial \log h_1} = -\frac{g''(h_1)h_1}{g'(h_1)}.$$

The elasticity is positive and, as was assumed earlier, less than one. It can depend on the durable goods consumption. The labor wedge is defined as

$$\tau_0 \equiv 1 - \frac{v'(n_0)}{A_0 u'(c_0)}.$$

The labor wedge is zero when a marginal benefit of consumption equals a marginal cost of working. The labor wedge equals zero if prices are flexible (economy is stabilized). It is positive when equilibrium labor and consumption are too small (a recession). It is negative when labor and consumption are too high (a boom).

Finally, I introduce the safety wedge as the ratio of the marginal utility of consumption bought with safe debt to the marginal utility of overall consumption in period 1

$$\tau_A \equiv \frac{\nu u'(\underline{c}_1)}{u'(c_1 + \underline{c}_1)}.$$

The wedge distorts the safe debt Euler equation (7). The safety wedge is zero in the absence of preferences over goods bought with safe assets.

2.5 Equilibrium

An equilibrium specifies consumption $c_0, c_1, \underline{c}_1$, labor n_0, n_1, investment in durable goods k_0, durable goods consumption h_1, real safe debt supply by bankers and the government $\widetilde{d}_1^b, d_1^g$, safe debt demand d_1^c by consumers, real wage w_t, nominal interest rate i_0, government lump-sum taxes T_0, T_1, labor taxes τ_0^L, τ_1^L, and macroprudential tax τ_0^b such that households and firms maximize, the government budget constraints are satisfied as equalities in every period, final goods markets clear

$$k_0 + c_0 = A_0 n_0, \tag{16}$$
$$c_1 + \underline{c}_1 = A_1 n_1, \tag{17}$$

durable goods market clears

$$h_1 = G(k_0), \tag{18}$$

and safe assets market clears

$$d_1^c = \widetilde{d}_1^b + d_1^g. \tag{19}$$

The complete set of equilibrium conditions (4)-(19) can be simplified as follows. First, because prices are flexible in period 1, household labor supply (10), firms choice of prices (13), and goods clearing condition (17) imply $A_1 u'(A_1 n_1) = v'(n_1)$: a marginal benefit of working is equal to a marginal cost of working. This expression determines equilibrium amount of labor only as a function of productivity A_1 in period 1. I denote it by n_1^*, and the corresponding level of output by $y_1^* = A_1 n_1^*$. Second, because the SAIA constraint binds in equilibrium, consumption bought with safe assets $\underline{c}_1$ is determined by the amount of safe assets acquired in the previous period. This implies that consumption c_1 and $\underline{c}_1$ do not depend on the realization of preferences over durable goods $X_1(s_1)$. Third, the only period-1 endogenous variable that depends on realization of s_1 is the price of durable goods. Equation (8) implies that there are only two possible realizations of the price: $\gamma(X_1 = 1) = g'(h_1)/u'(y_1^*)$ and $\gamma(X_1 = \theta) = \theta g'(h_1)/u'(y_1^*) = \theta\gamma(X_1 = 1)$.

It is intuitive that allocations in period 1 do not depend on the realization of state s_1. Realization of s_1 directly affects durable goods price γ_1 by changing the marginal utility of durable goods. However, the realization of this price only redistributes resources between bankers and consumers. Because bankers and consumers belong to a large household, they effectively pool their resources together in the end of period 1. Thus, the allocation in period 1 is not affected.

The SAIA constraint binds in equilibrium ($\underline{c}_1 = \widetilde{d}_1^b + d_1^g$) because marginal utility of consumption bought with safe assets is always positive. Taking this into account, Euler equation (7) links three endogenous variables: consumption c_0, real interest rate on safe debt $(1+i_0)/\Pi^*$ and safe debt supply $\widetilde{d}_1^b$:

$$u'(c_0) = \beta \frac{1+i_0}{\Pi^*} u'(y_1^*) \left[1 + \frac{\nu u'(\widetilde{d}_1^b + d_1^g)}{u'(y_1^*)}\right], \tag{20}$$

Household demand for durable goods (8), bankers' choice of investment in durable goods (11), and durable goods market clearing condition (18) lead to

$$u'(c_0) = \beta g'[G(k_0)] G'(k_0) [\mu + (1-\mu)\theta + \zeta_0 \theta], \tag{21}$$

where multiplier ζ_0, given by (12), can be rewritten taking into account safe assets Euler equation (20) as follows

$$\zeta_0 = \left(1 - \tau_0^B\right) \frac{u'(c_0)}{u'(y_1^*)} \cdot \frac{\Pi^*}{\beta(1+i_0)} - 1 \geq 0.$$

Next, collateral constraint (4) can be expressed in real terms, taking into account equilibrium durable goods price (8), as follows

$$\widetilde{d}_1^b \leq \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0). \tag{22}$$

Note that the minimal real durable good price $\gamma_1 = \theta g'[G(k_0)]/u'(y_1^*)$ depends on the level of investment in durable goods production. This price in the collateral constraint is a source of pecuniary externality that affects welfare. I will call it a fire-sale externality.

Equations (20)-(22), together with complementarity slackness conditions on the last inequality, describe equilibrium. This system determines the remaining unknown endogenous variables $c_0, k_0, \widetilde{d}_1^b$.

2.6 Ramsey Planning Problem

The financial regulation and monetary authorities face all of the equilibrium conditions (4)-(19) as constraints when choosing their optimal policies. The full system of equilibrium conditions was reduced to system (20)-(22).[13] Note that the full set of equilibrium conditions can be unambiguously recovered from (20)-(22).

Following the public finance literature (Lucas and Stokey, 1983), I further drop certain variables and constraints from the optimal policy problem. First, given quantities

[13] In addition, the collateral and liquid-assets-in-advance constraints are accompanied by the complementarity slackness conditions.

$c_0, k_0, \widetilde{d}_1^b$, optimality with respect to choice of investment in durable goods (21) can be dropped because it can be used to express optimal macroprudential tax τ_0^b when the collateral constraint binds. We will see that whenever the planner's optimal choice of investment in durable goods does not lead to binding collateral constraints, the planner's optimum will coincide with the private one. Finally, (20) can be dropped because it can be used to back out the nominal interest rate.

Proposition 1. *An allocation $c_0, k_0, \widetilde{d}_1^b$ form part of an equilibrium if and only if condition (22) holds.*

I now solve the Ramsey problem by choosing the competitive equilibrium that maximizes the social welfare. Formally, the planner solves

$$\max_{\{c_0,k_0,d_1^b\}} u(c_0) - v\left(\frac{c_0 + k_0}{A_0}\right) + \beta\left[u(A_1 n_1^*) - v(n_1^*) + (\mu + (1-\mu)\theta)\, g\,(G(k_0)) + \nu u(\widetilde{d}_1^b + d_1^g)\right]$$

$$s.t.: \quad \widetilde{d}_1^b \leq \theta \frac{g'\,(G(k_0))}{u'(A_1 n_1^*)} G(k_0).$$

The representation of the planner's objective takes into account that only preferences over durable goods depend on realization of s_1, which in expectation is $\mathbb{E}X_1(s_1) = \mu + (1-\mu)\theta$. The planner's optimal behavior leads to

$$\begin{aligned}
\tau_0 &= 0, \\
\widetilde{\zeta}_0 &= \tau_A, \\
u'(c_0) &= \beta g'[G(k_0)]G'(k_0)\left[\mu + (1-\mu)\theta + \widetilde{\zeta}_0\theta\,(1-\epsilon_\Gamma)\right],
\end{aligned}$$

where $\widetilde{\zeta}_0 \geq 0$ are the Lagrange multiplier on the collateral constraint expressed in units of period-1 consumption goods. The first equation states that the labor wedge equals zero (the economy is stabilized). The second line states that in planner's optimum the collateral constraints binds. Moreover, the multiplier on the collateral constraint equals the liquidity wedge τ_A. The third equation is the choice of investment in durable goods production. Compared to private optimal choice of investment in durable goods (21), the planner's optimal condition reveals that she internalizes the impact of durable goods investment on future durable goods price, which is formally represented by term $1 - \epsilon_\Gamma$ on the right-hand side of the equation. As a result, the planner invests less compared to the private choice of bankers.

I next characterize implementation of the constrained efficient allocation. Comparison of planner's optimality with private optimality condition (21) leads to following result.

Proposition 2. *Constrained efficient allocation can be implemented by setting the macroprudential tax and the nominal interest rate so that*

$$\tau_0^b = \frac{\epsilon_\Gamma \tau_A}{1+\tau_A} \text{ and } \tau_0 = 0. \tag{23}$$

When monetary and macroprudential policies are chosen optimally, the economy is stabilized (the labor wedge equals zero) and the financial regulation reduces welfare losses due to pecuniary externality. Macroprudential tax τ_0^b is proportional to durable goods price elasticity and the safety wedge. When durable goods price elasticity ϵ_Γ is zero, the macroprudential tax is also zero because private investment decisions do not affect future price of durable goods. In addition, in the absence of safety wedge, the planner makes the collateral constraint slack resulting in zero prudential tax.

The result above does not depend on the fact that the two policies are set cooperatively. This is because both policies are chosen to maximize the same objective conditional on the same constraints.[14] If the macroprudential policy is chosen optimally, it is optimal for the monetary policy to set flexible price output (the labor wedge is zero).

It is sometimes proposed that monetary policy should be directed towards financial stability objectives because macroprudential policy may not be chosen optimally. For example, Stein (2013, 2014) argues that some market participants may evade macroprudential regulation leading to inability of the financial regulators to set optimal policy. However, monetary policy has a universal effect on all market participants. Symmetrically, one can argue that sometimes monetary policy may not be set optimally, for example, due to the zero lower bound or because a country belongs to a monetary union, which precludes control over the nominal interest rate. In this case, the macroprudential policy should be directed toward the stabilization of inefficient business cycle fluctuations due to sticky prices. The model of this section can be used to analyze these two situations. The following proposition describes the optimal monetary policy when macroprudential policy is not set optimally and the optimal macroprudential policy when monetary policy is not set optimally.

Proposition 3. *(i) Optimal monetary policy when the macroprudential tax is set at $\tau_0^b \neq \epsilon_\Gamma \tau_A / (1+\tau_A)$ is such that*

$$\tau_0 = -\frac{1}{Z_1}\left(\tau_0^b - \frac{\epsilon_\Gamma \tau_A}{1+\tau_A}\right), \tag{24}$$

where $Z_1 > 0$ is a variable that depends on the optimal allocation;

[14] Paoli and Paustian (2013) show that there is a scope for coordination between the two policy choices when the objectives of monetary and macroprudential authorities differ.

(ii) optimal macroprudential tax when monetary policy is set such that $\tau_0 \neq 0$ is

$$\tau_0^b = \frac{1}{1-\tau_0}\left(\frac{\epsilon_\Gamma \tau_A}{1+\tau_A} - \tau_0 Z_2\right), \tag{25}$$

where $Z_2 > 1$ is a variable that depends on the optimal allocation.

Proof and the formal expressions for Z_1, Z_2 are in Appendix A.1.3. The first part of Proposition 3 states that the optimal monetary policy generates a recession ($\tau_0 > 0$) if the prudential authority sets the macroprudential tax below the optimal level, $\tau_0^b < \epsilon_\Gamma \tau_A / (1+\tau_A)$. If the macroprudential tax is above its optimum, the optimal monetary policy is associated with negative labor wedge (the monetary authority generates an inefficient recession). Formally, the planner solves a problem in which she has an additional constraint: banker's optimality condition with respect to investment in durable goods.

The second part of proposition 3 shows that the optimal macroprudential tax not only corrects the fire-sale externality (the first term in the brackets of equation (25)), but also stabilizes inefficient business cycle fluctuations if monetary policy is not set optimally. The last term in the brackets represents the effect due to sticky prices: an effect from an increase in the demand for goods due to an increase in durable goods production, when $\tau_0 > 0$. Finally, there is a feedback effect. Because the planner wants to stabilize the economy it makes bankers invest and issue more safe asset. This increases losses due to pecuniary externality which the regulator wants to undo. This effect is formally expressed by the presence of multiplier $1/(1-\tau_0)$.

3 A 2-period Model of Currency Union

This section extends the model presented in the previous section to a multi-country setting, and presents the main results of the paper.

The model features traded and non-traded goods as in Obstfeld and Rogoff (1995) and Farhi and Werning (2013). Non-traded goods are produced with labor, while there is inelastic supply of traded goods. Durable goods are produced with non-traded goods and are consumed only locally. Labor is immobile across countries. Agents can trade safe bonds across borders. Only non-traded goods prices in period 0 are sticky, all of the other prices are flexible. There is a continuum of countries of measure one.

The following household preferences extend closed-economy preferences (1) by adding traded and non-traded goods

$$\mathbb{E}\Bigg\{U\left(c^i_{NT,0},c^i_{T,0}\right)-v\left(n^i_0\right)+\beta U\left(c^i_{NT,1}+\underline{c}^i_{NT,1},c^i_{T,1}+\underline{c}^i_{T,1}\right) \\ -\beta v\left(n^i_1\right)+\beta X_1(s_1)g\left(h^i_1\right)+\beta\nu^i U\left(\underline{c}^i_{NT,1},\underline{c}^i_{T,1}\right)\Bigg\} \tag{26}$$

where superscript i is the country index, $c^i_{NT,t}, c^i_{T,t}$ is country i household consumption of non-traded (NT) and traded goods (T) in period t and $\underline{c}^i_{NT,1}, \underline{c}^i_{T,1}$ is non-traded and traded goods consumption in period 1 that must be purchased with safe assets. $U(\cdot,\cdot)$ is strictly increasing and concave.

Household's consolidated budget constraint in period 0 is

$$T^i_0+P^i_{NT,0}c^i_{NT,0}+P_{T,0}c^i_{T,0}+\frac{D^{c,i}_1}{1+i_0}+P^i_{NT,0}k^i_{NT,0} \\ \le P_{T,0}e^i_{T,0}+\frac{\widetilde{D}^{b,i}_1}{1+i_0}\left(1-\tau^{b,i}_0\right)+W^i_0n^i_0+\Pi^i_0(j), \tag{27}$$

where $P^i_{NT,0}$ is the sticky price index of non-traded goods in country i in period 0, $P_{T,0}$ is the flexible price of traded goods in period 0, $e^i_{T,0}$ is the household endowment of traded goods in period 0, $k^i_{NT,0}$ is the input in production of durable goods, $D^{c,i}_1$ is country i consumer nominal purchases of safe debt, $\widetilde{D}^{b,i}_1(s_0)$ is country i banker nominal issuance of safe debt net of reserves held at the central bank, i_0 is safe debt nominal interest rate, $\Pi^i_0(j)$ are the profits of non-traded goods firm that produces differentiated good j

$$\Pi^i_0(j)=\left(P^i_{NT,0}(j)-\frac{1+\tau^{L,i}_0}{A^i_0}\right)y^i_0\left(\frac{P^i_{NT,0}(j)}{P^i_{NT,0}}\right)^{-\epsilon}.$$

Budget constraint (27) is an international extension of the closed-economy budget constraint (5).

Household budget constraint in period 1 is

$$P^i_{NT,1}\left(c^i_{NT,1}+\underline{c}^i_{NT,1}\right)+P_{T,1}\left(c^i_{T,1}+\underline{c}^i_{T,1}\right)+T^i_1+\Gamma^i_1h^i_1+\widetilde{D}^{b,i}_1 \\ \le P_{T,1}e^i_{T,1}+D^{c,i}_1+W^i_1n^i_1+\Gamma^i_1G\left(k^i_{NT,0}\right)+\Pi^i_1(j). \tag{28}$$

where $P^i_{NT,1}, P_{T,1}, \Gamma^i_1$ are non-traded, traded and durable goods nominal prices in period 1, W^i_1 is the nominal wage, $\Pi^i_1(j)$ are the profits of the firm that produces non-traded goods

$$\Pi_1^i(j) = \left(P_{NT,1}^i(j) - \frac{1+\tau_1^{L,i}}{A_1^i}\right) y_1^i \left(\frac{P_{NT,1}^i(j)}{P_{NT,1}^i}\right)^{-\epsilon}.$$

Traded goods nominal prices $P_{T,0}, P_{T,1}$ and nominal interest rate i_0 have no country superscripts reflecting the fact that countries belong to a monetary union.

Country i banker constraint on the issuance of safe debt is

$$\widetilde{D}_1^{b,i} \leq \min_{s_1}\{\Gamma_1^i\} G\left(k_{NT,0}^i\right), \tag{29}$$

Part of traded and non-traded consumption in period 1 must be purchased with safe assets. The following constraint extends the closed-economy safe-assets-in-advance constraint (3) to incorporate traded and non-traded goods

$$P_{NT,1}^i \underline{c}_{NT,1}^i + P_{T,1} \underline{c}_{N,1}^i \leq D_1^{c,i}. \tag{30}$$

A typical household in country i maximizes (26) subject to (27)-(30) by choosing consumption of traded and non-traded goods $c_{NT,0}^i$, $c_{T,0}^i$, $c_{NT,1}^i$, $c_{T,1}^i$, $\underline{c}_{NT,1}^i$, $\underline{c}_{T,1}^i$, consumption of durable goods h_1^i, safe assets portfolio $D_1^{c,i}$, $\widetilde{D}_1^{b,i}$, labor supply n_0^i, n_1^i, investment in production of durable goods $k_{NT,0}^i$, and period-1 non-traded goods price $P_{NT,1}^i$.

The household's optimality conditions with respect to consumption are[15]

$$\frac{U_{NT,0}^i}{P_{NT,0}^i} = \frac{U_{T,0}^i}{P_{T,0}}, \tag{31}$$

$$\frac{U_{NT,1}^i}{P_{NT,1}^i} = \frac{U_{T,1}^i}{P_{T,1}}, \tag{32}$$

$$\frac{\underline{U}_{NT,1}^i}{U_{NT,1}^i} = \frac{\underline{U}_{T,1}^i}{U_{T,1}^i}. \tag{33}$$

The first two equations characterize optimal intraperiod consumption choices in both periods. The last equation describes optimal choice between traded and non-traded goods

[15] $U_{NT,t}^i, U_{T,t}^i, \underline{U}_{NT,1}^i, \underline{U}_{T,1}^i$ are partial derivatives of household preferences with respect to $c_{NT,t}^i, c_{T,t}^i, \underline{c}_{NT,1}^i, \underline{c}_{T,1}^i$.

bought with safe debt. Household optimal labor supply satisfies

$$\frac{v'\left(n_0^i\right)}{U^i_{NT,0}} = \frac{W_0^i(s_0)}{P^i_{NT,0}}, \tag{34}$$

$$\frac{v'\left(n_1^i\right)}{U^i_{NT,1}} = \frac{W_1^i}{P^i_{NT,1}}. \tag{35}$$

Durable goods demand is described by

$$\frac{X_1(s_1)g'\left(h_1^i\right)}{U^i_{T,1}} = \frac{\Gamma_1^i}{P_{T,1}}. \tag{36}$$

Household optimal choice of safe bonds is summarized by the following Euler equation

$$U^i_{T,0} = \beta\mathbb{E}_0 \frac{1+i_0}{P_{T,1}/P_{T,0}} U_{T,1}\left(1+\frac{\nu^i \underline{U}^i{}_{T,1}}{U^i_{T,1}}\right), \tag{37}$$

Optimal choice of investment in durable goods leads to

$$U^i_{NT,0} = \beta\mathbb{E}_0 U^i_{T,1} G^i_{NT,0}(k^i_{NT,0})\left(\frac{\Gamma_1^i}{P_{T,1}} + \zeta_0^i \min_{s_1}\frac{\Gamma_1^i}{P_{T,1}}\right), \tag{38}$$

where the Lagrange multiplier on the collateral constraint is

$$\zeta_0^i = \frac{1-\tau_0^{b,i}}{1+i_0}\cdot\frac{U^i_{T,0}/P_{T,0}}{\beta\mathbb{E}_0 U_{T,1}/P_{T,1}} - 1 \geq 0. \tag{39}$$

Optimal choice of prices for intermediate goods in period 1 leads to

$$P^i_{NT,1} \quad = \quad \left(1+\tau_1^{L,i}\right)\frac{\epsilon}{\epsilon-1}\cdot\frac{W_1^i}{A_1^i}. \tag{40}$$

3.1 Government

The government consists of a union-wide monetary authority, national fiscal and financial regulation authorities. The monetary authority sets the nominal interest rate on safe bonds i_0 and period-1 price of traded goods $P_{T,1}$, so that the price level does not depend on state s_1.[16] A financial regulator in country i sets the level of reserve requirements z_0^i

[16]Similarly to the closed-economy case, I motivate the monetary authority control over the nominal price level of tradable goods in period 1 and the nominal interest rate between periods 0 and 1 by assuming that fraction $\kappa \in [0,1]$ of traded goods purchases has to be bought with monetary authority nominal liabilities

and the interest rate on reserves $i_0^{r,i}$, which is equivalent to setting macroprudential tax $\tau_0^{b,i}$ on local issuance of safe debt. It rebates the proceeds to the local fiscal authority. Local fiscal authority sets lump sum taxes T_0^i, T_1^i, labor taxes $\tau_0^{L,i}$, $\tau_1^{L,i}$ and issues safe bonds $D_1^{g,i}$. The consolidated government budget constraints in both periods are

$$T_0^i + \tau_0^{L,i} W_0^i n_0^i + \tau_0^{b,i} \frac{\widetilde{D}_1^{b,i}}{1+i_0} + \frac{D_1^{g,i}}{1+i_0} = 0, \tag{41}$$

$$T_1^i + \tau_1^{L,i} W_1^i n_1^i = D_1^{g,i}. \tag{42}$$

Government budget constraint in period 0 states that the revenue from lump-sum taxes T_0^i (transfers if negative), revenue from labor taxes $\tau_0^{L,i} W_0^i n_0^i$, revenue from reserve requirement policy $\tau_0^{b,i} D_1^{b,i} / (1+i_0)$, and revenue from issuing government safe debt $D_1^{g,i} / (1+i_0)$ must add up to zero. Period 1 budget constraints requires the fiscal authority to repay safe debt $D_1^{g,i}$ by collecting lump-sum taxes T_1^i and proportional labor taxes $\tau_1^{L,i} W_1^i n_1^i$.

Fiscal authority in country i corrects monopolistic competition friction in period 1 by setting $\tau_1^{L,i} = -1/\epsilon$. The choice of labor tax $\tau_0^{L,i}$ does not affect equilibrium.

3.2 Auxiliary Variables

Similarly to the closed-economy model, I introduce real variables and several wedges. First, I express period-1 nominal non-traded goods and durable goods prices in units of traded goods as follows: $p_1^i \equiv P_{NT,1}^i / P_{T,1}, \gamma_t^i \equiv \Gamma_1^i / P_{T,1}$, workers nominal wages in units of traded goods $w_t^i = W_t^i / P_{T,1}$, and the interest rate on safe debt deflated by traded goods inflation $r_0 \equiv (1+i_0) P_{T,0} / P_{T,1} - 1$. Second, I express nominal quantities in units of traded goods: $\widetilde{d}_1^{b,i} \equiv \widetilde{D}_1^{b,i} / P_{T,1}$, $d_1^{g,i} \equiv D_1^{g,i} / P_{T,1}$, $d_1^{c,i} \equiv D_1^{c,i} / P_{T,1}$. Finally, the labor and safety wedges are defined as

$$\tau_0^i \equiv 1 - \frac{v'(n_0^i)}{A_0^i U_{NT,0}^i}, \quad \tau_A^i \equiv \frac{\nu^i \underline{U}_{T,1}^i}{U_{T,1}^i}.$$

3.3 Equilibrium

An equilibrium specifies consumption $c_{NT,t}^i, c_{T,t}^i, \underline{c}_{NT,1}^i, \underline{c}_{T,1}^i$, labor n_t^i, investment in durable goods $k_{NT,0}^i$, durable goods production h_1^i, real (in terms of tradable goods) safe debt supply by bankers and the government $\widetilde{d}_1^{b,i}, d_1^{g,i}$, real safe debt demand $d_1^{c,i}$, wages w_t^i,

---

M_0, M_1 that do not pay interest (cash): $\kappa P_{T,0} \int c_{T,0}^i di = M_0, \kappa P_{T,1} \int (c_{T,1}^i + \underline{c}_{T,1}^i) di = M_1$. Making κ, M_0, M_1 tend to zero in a way that keeps $M_0/\kappa, M_1/\kappa$ finite and bounded from zero allows the monetary authority to have a control over nominal variables $P_{T,1}, i_0$ but does not require explicit treatment of cash.

traded and non-traded goods prices $P^i_{NT,0}, P_{T,0}, P^i_{NT,1}, P_{T,1}$, real interest rate r_0, government lump-sum taxes T^i_0, T^i_1, and labor taxes $\tau^{L,i}_0, \tau^{L,i}_1$ in every country $i \in [0,1]$ such that households and firms maximize, the government budget constraints are satisfied, final non-traded goods markets in both periods clear in every country

$$k^i_{NT,0} + c^i_{NT,0} = A^i_0 n^i_0, \tag{43}$$
$$c^i_{NT,1} + \underline{c}^i_{NT,1} = A^i_1 n^i_1, \tag{44}$$

tradable goods market clears in both periods

$$\int c^i_{T,0} di = \int e^i_0 di, \tag{45}$$
$$\int \left(c^i_{T,1} + \underline{c}^i_{T,1}\right) di = \int e^i_1 di, \tag{46}$$

durable goods markets clear in every country

$$h^i_1 = G(k^i_{NT,0}), \tag{47}$$

and international safe assets market clears

$$\int d^{c,i}_1 di \;\; = \;\; \int d^{g,i}_1 di + \int \tilde{d}^{b,i}_1 di. \tag{48}$$

3.4 Equilibrium Characterization

This section simplifies the complete set of equilibrium conditions (27)-(48) before turning to the optimal policy characterization. First, let me introduce the following assumption

Assumption 1. *Utility function $U(c_{NT}, c_T)$ has the following form*

$$U(c_{NT}, c_T) = \log\left(c^a_{NT} c^{1-a}_T\right).$$

This simplifying assumption states that the intratemporal elasticity of substitution between traded and non-traded goods and the coefficient of relative risk aversion equal one. Intratemporal optimality conditions (31)-(33) and assumption 1 can be used to express consumption of traded goods as follows: $c^i_{T,0} = (1-a)/a \cdot c^i_{NT,0} P^i_{NT,0}/P_{T,0}$, $c^i_{T,1} = (1-a)/a \cdot p^i_1 c^i_{NT,1}$, $\underline{c}^i_{T,1} = (1-a)/a \cdot p^i_1 \underline{c}^i_{NT,1}$.

The flexibility of prices in period 1, household labor supply (35), firms choice of prices (40), goods clearing condition (44) in period 1, and assumption 1 imply $v'(n^i_1) = A^i_1 U^i_{NT,1}$. This expression determines equilibrium amount of labor only as a function of productivity. I denote it as $n_1^{i,*}$, the corresponding level of output is denoted $y_{NT,1}^{i,*} \equiv A_1^i n_1^{i,*}$. Note that equilibrium labor and output in period 1 do not depend on the realization of state s_1.

Household budget constraint (27), government budget constraint (41), and non-traded goods market clearing condition can be combined to express country i consolidated budget constraint in period 0

$$\frac{1-a}{a} c_{NT,0}^i \frac{P_{NT,0}^i}{P_{T,0}} - e_{T,0}^i = \frac{\widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i}}{1+r_0}. \tag{49}$$

It states that country i excess consumption of traded goods (the left-hand side) must be financed by issuing safe bonds on the international market. Similarly, (28), (42), and (44) can be combined to express country-wide budget constraint in period 1

$$y_{NT,1}^{i,*} p_1^i \frac{1-a}{a} - e_{T,1}^i = d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}, \tag{50}$$

It shows that excess consumption of traded goods in period 1 results from the accumulation of safe claims on other countries. The household Euler equation and the market clearing conditions imply

$$\frac{1}{c_{NT,0}^i} = (1+r_0)\beta \frac{P_{NT,0}^i}{P_{T,0} p_1^i} \left(\frac{1}{y_{NT,1}^{i,*}} + \frac{\nu^i}{\underline{c}_{NT,1}^i} \right), \tag{51}$$

Durable goods demand (36) and supply (38) lead to

$$\frac{a}{c_{NT,0}^i} = \beta g' \left[G\left(k_{NT,0}^i \right) \right] G'\left(k_0^i \right) \left(\mu + (1-\mu)\theta^i + \zeta_0^i \theta^i \right), \tag{52}$$

where $\zeta_0^i = \left(1 - \tau_0^{b,i}\right) \left\{ \nu^i y_{NT,1}^{i,*} / \underline{c}_{NT,1}^i + 1 \right\} - 1 \geq 0$ is the Lagrange multiplier on the collateral constraint in country i. The real interest rate on safe debt expressed in units of traded goods is related to price level of traded goods as follows

$$P_{T,0} = \frac{1+r_0}{1+i_0} P_{T,1}. \tag{53}$$

Recall that the central bank has a control over i_0 and $P_{T,1}$. The last expression states that price $P_{T,0}$ is related to real interest rate r_0 and monetary policy choices $i_0, P_{T,1}$.

Finally, collateral constraint (29) and safe-assets-in-advance constraint (30) can be simplified as follows

$$d_1^{b,i} \leq \theta^i \frac{g'[G(k_0^i)]}{a/y_{NT,1}^{i,*}} G(k_0^i) p_1^i, \tag{54}$$

$$\frac{\underline{c}_{NT,1}^i p_1^i}{a} \leq d_1^{c,i}. \tag{55}$$

Equations (45), (46), (49)-(55), and the complementarity slackness conditions on the last two inequalities, describe equilibrium. This system determines the remaining unknown endogenous variables $c_{NT,0}^i, \underline{c}_{NT,1}^i, k_{NT,0}^i, \widetilde{d}_1^{b,i}, d_1^{c,i}, p_1^i, r_0, P_{T,0}$. There are two cross-country equations to determine interest rate r_0 and price level $P_{T,0}$, which are common across countries. There are six conditions for every country i to determine six country-level endogenous variables $c_{NT,0}^i, \underline{c}_{NT,1}^i, k_{NT,0}^i, \widetilde{d}_1^{b,i}, d_1^{c,i}, p_1^i$.

3.5 Ramsey Planning Problem

The financial and monetary authorities face all equilibrium conditions (27)-(48) as constraints when choosing their optimal policies. The full system of equilibrium conditions was reduced to system (45), (46), (49)-(55). Note that the full set of equilibrium conditions can be unambiguously recovered from (45), (46), (49)-(55).

I further drop certain variables and constraints from the optimal policy problem. First, given quantities $c_{NT,0}^i, \underline{c}_{NT,1}^i, k_{NT,0}^i, \widetilde{d}_1^{b,i}, d_1^{c,i}$ and prices $r_0, P_{T,0}, P_{NT,0}^i, p_1^i$, the optimal condition with respect to choice of investment in durable goods (52) can be dropped because it can be used to express optimal macroprudential tax $\tau_0^{b,i}$ when the collateral constraint binds. Finally, (53) can be dropped because it can be used to express the ratio of the nominal interest rate and price of traded goods in period 1.

Proposition 4. *An allocation $c_{NT,0}^i, k_{NT,0}^i, d_1^{b,i}, d_1^{c,i}$ and prices $r_0, P_{T,0}, P_{NT,0}^i$ form part of an equilibrium if and only if conditions (45), (46), (49), (50), (54) and (55) hold.*

After taking into account intratemporal consumption choice by the household, the household objective in country i can be simplified as in the following lemma.

Lemma 1. *Country i indirect household utility is*

$$\begin{aligned} &V^i\left(c_{NT,0}^i, k_{NT,0}^i, P_{NT,0}^i/P_{T,0}, r_0, p_1^i\right) \\ &\quad = \log c_{NT,0}^i - v\left(n_0^i\right) + \beta\left\{\nu^i \log \underline{c}_{NT,1}^i + X_1(s_1) g\left[G\left(k_{NT,0}^i\right)\right]\right\} \\ &\qquad + (1-a)\log\left(\frac{P_{NT,0}^i}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p_1^i + O, \end{aligned} \tag{56}$$

where O is the term which depends only on exogenous variables and model parameters, and $n_0^i = (c_{NT,0}^i + k_{NT,0}^i)/A_0^i$.

3.5.1 Local Planner

I start by solving a local planner problem. In this case, the planner maximizes local welfare taking international prices as given. I will later compare this solution to a union wide planner's solution. The two solutions will turn out to be different.

Formally, the local planner maximizes (56) subject to country budget constraints (49) and (50), banker's collateral constraint (54), safe-assets-in-advance constraint (55), and Euler equation (51) by choosing allocation $c_{NT,0}^i, \underline{c}_{NT,1}^i, k_{NT,0}^i, \widetilde{d}_1^{b,i}, d_1^{c,i}$ and price p_1^i conditional on prices r_0, $P_{T,0}$. The solution to the planner's problem is derived in Appendix A.2.4. The following Lemma presents the implementation of the planner's solution.

Proposition 5. *Constrained Pareto efficient allocation in country i, given international prices, can be implemented by setting the macroprudential tax to*

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i}\left(\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i + Z_3^i \widetilde{d}_1^{b,i} - Z_3^i a d_1^{c,i} - \frac{a}{1-a}\tau_0^i Z_4^i\right) \tag{57}$$

where $Z_2^i > 1, Z_3^i > 0, Z_4^i > 0$ *are variables that depend on the optimal allocation.*

Proof and expressions for Z_2^i, Z_3^i, Z_4^i are in Appendix A.2.4. The interpretation of this formula highlights the externalities that the planner takes into when choosing the optimal macroprudential tax. There are five terms in the parentheses. They correspond to five externalities. With only the first two terms, the optimal macroprudential tax (57) would look like the optimal tax (25) that the planner sets in a closed economy when monetary policy is not set optimally. In this case, the planner mitigates fire-sale externality, like in Stein (2012), and tries to close the labor wedge that creates the aggregate demand externality (Farhi and Werning, 2013; Korinek and Simsek, 2014).

The last three terms represent externalities that arise in an international context. The third term in the parentheses reflects a negative pecuniary externality due to the presence of non-traded goods relative to traded goods prices in the bankers collateral constraint (54). This externality is often used to justify prudential capital controls (Bianchi, 2011). The affect of this externality on the macroprudential tax is proportional to the amount of safe debt issued by bankers in country i. The fourth term reflects the positive pecuniary externality due to the presence of relative price of non-traded and traded goods in the SAIA constraint (55). The effect of this externality on prudential tax is proportional to the amount of safe securities held by consumers. The last term in the parenthesis reflects the externality due to distorted private choice of traded and non-traded goods under sticky non-traded goods prices. Farhi and Werning (2012) use this externality to justify the role of fiscal transfers policy in a monetary union. The contribution of this externality to the optimal macroprudential tax is proportional to the labor wedge τ_0^i and to the relative expenditure share of non-traded to traded goods $a/(1-a)$.

3.5.2 Global Planner

In this section, I solve the global planner's problem and show that global planner chooses a different allocation compared to the independent local planners. Global planner optimally chooses union-wide monetary and regional macroprudential policies. Formally, the global planner maximizes a weighted average, with Pareto weights $\{\omega^i\}$, of country-specific welfare functions (56) subject to (45)-(55) by choosing allocation $\{c^i_{NT,0}, k^i_{NT,0}, \underline{c}^i_{NT,1}, \widetilde{d}_1^{b,i}, d_1^{c,i}\}$ and prices $r_0, P_{T,0}, \{p^i_{NT,0}\}$.

The full characterization of the global planner problem is in Appendix A.2.5. The following proposition summarizes the optimal monetary and macroprudential policy implementation.

Proposition 6. *At a constrained Pareto efficient equilibrium*
(i) average (across countries) labor wedge is zero

$$\int \omega_i \tau_0^i di = 0,$$

(ii) optimal choice of $\{c^i_{NT,0}, k^i_{NT,0}, \underline{c}^i_{NT,1}, \widetilde{d}_1^{b,i}, d_1^{c,i}, p^i_{NT,0}\}$ is implemented by setting macroprudential tax

$$\tau_0^{b,i} = \tau_0^{b,i}\Big|_{local} + \frac{\widetilde{\psi}_0}{1-\tau_0^i} Z_5^i, \tag{58}$$

where $Z_5^i > 0$, $\tau_0^{b,i}\Big|_{local}$ is the expression identical to local prudential tax (57), and $\widetilde{\psi}_0$ can be both positive or negative.

Proof and expressions for Z_5^i and $\widetilde{\psi}_0$ are in Appendix A.2.5. The first part of proposition 6 show that the monetary authority sets the average labor wedge across countries to zero. This result is similar to the one derived in Farhi and Werning (2012). The linearized version of this condition would equalize the average output gap to zero (Benigno, 2004; Gali and Monacelli, 2008). If all of the countries are symmetric, the monetary authority stabilizes all economies with just one policy tool.

The second part of the proposition characterizes the implementation of macroprudential policy. This characterization highlights that the global planner's optimal macroprudential tax deviates from local planner's tax. Specifically, the deviation is proportional to $\widetilde{\psi}_0$ that captures the net effect of all of the spillovers of local prudential policy on the other countries.[17] There are two distinct types of spillovers. The global planner internalizes the redistributional effects of local macroprudential policy absent other tools that equalize the marginal utilities of consumption of traded goods across countries.

In addition to redistributional effects, the local macroprudential policy interacts with the two pecuniary externalities, which arise in the international context, in the other countries. First, consider the effect of macroprudential policy in country i on private safe debt issuance in all other countries. Higher prudential tax in country i reduces private supply of safe debt in country i. This leads to a smaller consumption of traded goods in country i in period 0 and a higher consumption of traded goods in period 1. Formally, this can be seen in country-wide budget constraints (49) and (50). The effects are opposite in the rest of the union: traded consumption increases in period 0 and declines in period 1. With lower consumption of traded goods in the rest of the union in period 1, the relative price of traded to non-traded goods falls.[18] As a result, collateral constraints (54) are tighter in the other countries of the union. This increases bankers costs of financing in those countries. The global planner internalizes this spillover of local prudential policy unlike the local planner. Second, higher prudential tax $\tau_0^{b,i}$ in country i, which, as discussed, leads to a decline in price of non-traded relative to traded goods, relaxes the safe-assets-in-advance constraints (55) in the other countries. This is a positive international spillover of a tighter local macroprudential policy that the global regulator takes into account.

The difference between the global and local planner solutions underscores the importance of coordination of macroprudential policies across countries. Because the local regulator does not internalize her effects on the other countries of the union, she chooses an outcome which is sub-optimal from the global regulator perspective.

3.5.3 Additional Policy Tools

The results presented so far were derived under the assumption that there are no other policy tools. In this section, I study how the optimal choice of additional tools affects the optimal macroprudential policy. Specifically, I allow the authorities in different countries to use fiscal transfers and portfolio taxes in a coordinated manner.

Fiscal transfers. I assume the local fiscal authority lump-sum taxes are represented as the sum of two terms: $T_t^i + \widehat{T}_t^i$, where T_t^i is a local lump-sum tax and $\widehat{T}_t^i$ is a cross-border

[17] $\widetilde{\psi}_0$ is expressed in equation (A.5) in Appendix A.2.5.

[18] Formally, $p_1^j = a/(1-a) \cdot c_{T,1}^j / c_{NT,1}^j = a/(1-a) \cdot c_{T,1}^j / y_{NT,1}^{j,*}$.

transfer. The cross-border transfers sum to zero

$$\int \widehat{T}_t^i di = 0. \tag{59}$$

The addition of fiscal transfers changes the country-wide budget constraints and adds (59) in both periods as new constraints to the global planner problem. The new country-wide budget constraints are

$$\begin{aligned} c_{NT,0}^i \frac{P_{NT,0}^i}{P_{T,0}} \cdot \frac{1-a}{a} - e_{T,0}^i + \frac{d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}}{1+r_0} + \widehat{Tr}_0^i &= 0, \\ y_{NT,1}^{i,*} p_1^i \frac{1-a}{a} - e_{T,1}^i + \widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i} + \widehat{Tr}_1^i &= 0, \end{aligned} \tag{60}$$

where $\widehat{Tr}_t^i \equiv \widehat{T}_t^i / P_{T,t}$ are transfers expressed in units of traded goods. The next proposition summarizes the implementation of the global planner's solution.

Proposition 7. *At a constrained Pareto efficient equilibrium with optimally chosen fiscal transfers, monetary and macroprudential policies,*
(i) average across countries labor wedge is zero

$$\int \omega_i \tau_0^i di = 0,$$

(ii) marginal social value of traded goods are equalized across countries in both periods;
(iii) the allocation can be supported by the following macroprudential tax

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i} \left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i + Z_6^i \right],$$

where $Z_2^i > 0, Z_6^i$ are the variables that depend on the optimal allocation.

Proof and expressions for Z_2^i, Z_6^i are in Appendix A.2.6. The first part of the proposition states that optimal monetary policy equalizes labor wedges across countries in period 0. The second part states that optimal fiscal transfers policy equalizes social marginal value of traded goods in every country. Note that without optimal fiscal transfers this is not necessarily the case. The last part of the proposition represents the implementation of optimal allocation through appropriate choice of macroprudential tax. The first two terms in the square brackets are exactly like in formulas (57) and (58): optimal financial regulation corrects fire-sale externality and tries to close the labor wedge. The third term Z_6^i, which can be either positive or negative, represents the combined effect of the international externalities in country i and all other countries in the union. The last part of the proposition highlights that optimal macroprudential policy takes macroeconomic stabilization into account even if the fiscal transfers are allowed and they are chosen optimally. The reason for this is that the addition of fiscal transfers is not enough to close the labor wedge in each individual economy.

Fiscal transfers and portfolio taxes. Assume now that in addition to fiscal transfers, the global planner can tax consumers holdings of safe assets differently across countries. Specifically, assume that when a consumer in country i buys $D_1^{c,i}$ units of safe debt, he must pay $(1+\tau_0^{c,i})D_1^{c,i}/(1+i_0)$, where $\tau_0^{c,i}$ is a portfolio tax. The fiscal revenue of this policy is rebated to the local fiscal authority.

The introduction of this policy effectively brings back the local monetary policy, because both bankers and consumers effectively face country-specific safe interest rates. Formally, the global planner will not face the Euler equation as one of its constraints. The following proposition summarizes the optimal financial regulation policy when transfers and portfolio taxes are chosen optimally.

Proposition 8. *At a constrained Pareto efficient equilibrium with optimal fiscal transfers and portfolio taxes, the optimal macroprudential policy depends on the labor wedge as follows*

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i}\left(\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i\right),$$

where $Z_2^i > 0$ is a variable that depends on the optimal allocation.

Proof is in Appendix A.2.7. The proposition states that macroprudential policy must be directed towards stabilization of the economy even if portfolio taxes are added to planner's tools. Compared to the optimal prudential tax without fiscal transfers and portfolio taxes (57), the above expression does not feature the last two terms present in (57). This is because portfolio taxes and fiscal transfers allow the planner to set her marginal value of safe assets equal to the private marginal value of safe assets. This frees macroprudential tax $\tau_0^{b,i}$ from addressing the pecuniary externalities associated with the relative price of traded and non-traded goods in the collateral and the SAIA constraints. However, in general, the addition of portfolio taxes does not help fully stabilize every country in the union, which leaves some stabilization role for macroprudential policy.

4 Conclusion

When monetary and macroprudential policies are set optimally in a currency union, regional macroprudential policy has a regional macroeconomic stabilization role beyond the correction of the fire-sale externality in the financial sector. There are gains from delegating local macroprudential policy decision making to a central financial regulator.

The proposed model considered only a macroprudential regulation tool. One direction for future research is to consider unconventional monetary policy tools. For example, directed purchases of regional risky assets by the central bank in exchange of newly created reserves can also be used to stabilize local business cycles.

References

ADAO, B., CORREIA, I. and TELES, P. (2009). On the relevance of exchange rate regimes for stabilization policy. *Journal of Economic Theory*, **144** (4), 1468–1488.

BEAU, D., CAHN, C., CLERC, L. and MOJON, B. (2013). *Macro-Prudential Policy and the Conduct of Monetary Policy*. Working Papers Central Bank of Chile 715, Central Bank of Chile.

BEETSMA, R. M. and JENSEN, H. (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics*, **67** (2), 320–352.

BENIGNO, G. and BENIGNO, P. (2003). Price Stability in Open Economies. *Review of Economic Studies*, **70** (4), 743–764.

—, CHEN, H., OTROK, C., REBUCCI, A. and YOUNG, E. (2013). *Capital Controls or Real Exchange Rate Policy? A Pecuniary Externality Perspective*. Research Department Publications IDB-WP-393, Inter-American Development Bank, Research Department.

BENIGNO, P. (2004). Optimal monetary policy in a currency area. *Journal of international economics*, **63** (2), 293–320.

BIANCHI, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*, **101** (7), 3400–3426.

— and MENDOZA, E. G. (2010). *Overborrowing, Financial Crises and 'Macro-prudential' Taxes*. NBER Working Papers 16091, National Bureau of Economic Research, Inc.

BRZOZA-BRZEZINA, M., KOLASA, M. and MAKARSKI, K. (2015). Macroprudential policy and imbalances in the euro area. *Journal of International Money and Finance*, **51** (C), 137–154.

BSBC (2010). The basel committee's response to the financial crisis: Report to the g20. *Basel Committee On Banking Supervision*.

CABALLERO, R. J. and FARHI, E. (2015). *The Safety Trap*. Working Paper 233766, Harvard University OpenScholar.

CLAESSENS, S. (2014). *An overview of macroprudential policy tools*. 14-214, International Monetary Fund.

CORSETTI, G., DEDOLA, L. and LEDUC, S. (2010). Optimal Monetary Policy in Open Economies. In B. M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics*, *Handbook of Monetary Economics*, vol. 3, *16*, Elsevier, pp. 861–933.

— and PESENTI, P. (2001). Welfare And Macroeconomic Interdependence. *The Quarterly Journal of Economics*, **116** (2), 421–445.

DANG, T. V., GORTON, G. and HOLMSTROM, B. (2012). Ignorance, debt and financial crises. *working paper*.

EGGERTSSON, G. B. and KRUGMAN, P. (2012). Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. *The Quarterly Journal of Economics*, **127** (3), 1469–1513.

FARHI, E., GOPINATH, G. and ITSKHOKI, O. (2014). *Fiscal Devaluations*. Scholarly Articles 12336336, Harvard University Department of Economics.

— and WERNING, I. (2012). *Fiscal Unions*. NBER Working Papers 18280, National Bureau of Economic Research, Inc.

— and — (2013). *A theory of macroprudential policies in the presence of nominal rigidities*. Tech. rep., National Bureau of Economic Research.

FERRERO, A. (2009). Fiscal and monetary rules for a currency union. *Journal of International Economics*, **77** (1), 1–10.

FORNARO, L. (2012). *Financial Crises and Exchange Rate Policy*. 2012 Meeting Papers 726, Society for Economic Dynamics.

GALI, J. and MONACELLI, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, **76** (1), 116–132.

GORTON, G. and PENNACCHI, G. (1990). Financial intermediaries and liquidity creation. *Journal of Finance*, **45** (1), 49–71.

GROMB, D. and VAYANOS, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, **66** (2-3), 361–407.

HANSON, S. G., KASHYAP, A. K. and STEIN, J. C. (2011). A Macroprudential Approach to Financial Regulation. *Journal of Economic Perspectives*, **25** (1), 3–28.

JEANNE, O. and KORINEK, A. (2010). *Managing Credit Booms and Busts: A Pigouvian Taxation Approach*. Working Paper Series WP10-12, Peterson Institute for International Economics.

KASHYAP, A. K. and STEIN, J. C. (2012). The optimal conduct of monetary policy with interest on reserves. *American Economic Journal: Macroeconomics*, **4** (1), 266–82.

KENEN, P. (1969). The theory of optimum currency areas: an eclectic view. In R. Mundell and A. Swoboda (eds.), *Monetary Problems of the International Economy*, Chicago University Press.

KORINEK, A. and SIMSEK, A. (2014). *Liquidity Trap and Excessive Leverage*. NBER Working Papers 19970, National Bureau of Economic Research, Inc.

KRISHNAMURTHY, A. and VISSING-JORGENSEN, A. (2012a). The aggregate demand for treasury debt. *Journal of Political Economy*, **120** (2), 233 – 267.

— and — (2012b). Short-term debt and financial crises: What we can learn from u.s. treasury supply. *working paper*.

LORENZONI, G. (2008). Inefficient credit booms. *Review of Economic Studies*, **75** (3), 809–833.

LUCAS, J., ROBERT E and STOKEY, N. L. (1987). Money and Interest in a Cash-in-Advance Economy. *Econometrica*, **55** (3), 491–513.

LUCAS, R. E. J. (1990). Liquidity and interest rates. *Journal of Economic Theory*, **50** (2), 237 – 264.

LUCAS, R. J. and STOKEY, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, **12** (1), 55–93.

MANKIW, N. G. and WEINZIERL, M. (2011). An exploration of optimal stabilization policy. *Brookings Papers on Economic Activity*, **42** (1 (Spring), 209–272.

MCKINNON, R. I. (1963). Optimum currency areas. *The American Economic Review*, **53** (4), pp. 717–725.

MUNDELL, R. A. (1961). A theory of optimum currency areas. *The American Economic Review*, pp. 657–665.

OBSTFELD, M. and ROGOFF, K. (1995). Exchange Rate Dynamics Redux. *Journal of Political Economy*, **103** (3), 624–60.

OTROK, C., BENIGNO, G., CHEN, H., REBUCCI, A. and YOUNG, E. R. (2012). *Monetary and Macro-Prudential Policies: An Integrated Analysis*. Working Papers 1208, Department of Economics, University of Missouri.

OTTONELLO, P. (2013). Optimal exchange rate policy under collateral constraints and wage rigidity.

PAOLI, B. D. and PAUSTIAN, M. (2013). *Coordinating monetary and macroprudential policies*. Staff Reports 653, Federal Reserve Bank of New York.

QUINT, D. and RABANAL, P. (2014). Monetary and Macroprudential Policy in an Estimated DSGE Model of the Euro Area. *International Journal of Central Banking*, **10** (2), 169–236.

RUBIO, M. (2014). *Macroprudential Policy Implementation in a Heterogeneous Monetary Union*. Discussion Papers 2014/03, University of Nottingham, Centre for Finance, Credit and Macroeconomics (CFCM).

SCHMITT-GROHE, S. and URIBE, M. (2012). *Prudential Policy for Peggers*. NBER Working Papers 18031, National Bureau of Economic Research, Inc.

SHLEIFER, A. and VISHNY, R. (1992). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance*, **47** (4), 1343–66.

STEIN, J. C. (2012). Monetary policy as financial stability regulation. *The Quarterly Journal of Economics*, **127** (1), 57–95.

— (2013). *Overheating in Credit Markets: Origins, Measurement, and Policy: as speech at the "Restoring Household Financial Stability after the Great Recession: Why Household Balance Sheets Matter" research symposium sponsored by the Federal Reserve Bank of St. Louis, St. Louis, Missouri, February 7, 2013*. Speech, Board of Governors of the Federal Reserve System (U.S.).

— (2014). *Incorporating Financial Stability Considerations into a Monetary Policy Framework : a speech at the International Research Forum on Monetary Policy, Washington, D.C., March 21, 2014*. Speech 796, Board of Governors of the Federal Reserve System (U.S.).

SVENSSON, L. E. O. (1985). Money and asset prices in a cash-in-advance economy. *Journal of Political Economy*, **93** (5), 919–44.

WOODFORD, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press Princeton, NJ.

— (2011). Monetary policy and financial stability. *presentation at NBER Summer Institute*.

A Appendix

A.1 A 2-period Closed Economy Model

This section presents closed economy derivations and proofs omitted from the main text.

A.1.1 Household Problem Solution

The Lagrangian for the household problem is

$$\begin{aligned}
\mathcal{L}_0 =& \mathbb{E}\Bigg\{ u\left(c_0\right) - v\left(n_0\right) + \beta\left[u\left(c_1 + \underline{c}_1\right) + X_1 g\left(h_1\right) + \nu u\left(\underline{c}_1\right) - v\left(n_1\right)\right] \\
& - \frac{\lambda_0}{P_0}\left[T_0 + P_0 c_0 + \frac{D_1^c}{1+i_0} + P_0 k_0 - \frac{\widetilde{D}_1^b}{1+i_1}\left(1 - \tau_0^B\right) - W_0 n_0 - \Pi_0^j\right] \\
& - \beta\frac{\lambda_1}{P_1}\left[P_1\left(c_1 + \underline{c}_1\right) + T_1 + \Gamma_1 h_1 + \widetilde{D}_1^b - D_1^c - W_1 n_1 - \Gamma_1 G(k_0) - \Pi_1^j\right] \\
& - \beta\frac{\lambda_1}{P_1}\zeta_0\left[\widetilde{D}_1^b - \min_{s_1}\{\Gamma_1\} G\left(k_0\right)\right] \\
& - \beta\frac{\lambda_1}{P_1}\eta_1\left[P_1 \underline{c}_1 - D_1^c\right]\Bigg\},
\end{aligned}$$

where

$$\Pi_0^j = \left(P_0^j - \frac{\left(1+\tau_0^L\right) W_0}{A_0}\right) y_0\left(\frac{P_0^j}{P_0}\right), \quad \Pi_1^j(s_1) = \left(P_1^j - \frac{\left(1+\tau_1^L\right) W_1}{A_1}\right) y_1\left(\frac{P_1^j}{P_1}\right)^{-\epsilon}.$$

The first order conditions are

$$\begin{aligned}
\partial c_0 :& u'\left(c_0\right) = \lambda_0, \\
\partial c_1 :& u'\left(c_1 + \underline{c}_1\right) = \lambda_1, \\
\partial \underline{c}_1 :& u'\left(c_1 + \underline{c}_1\right) + \nu u'\left(\underline{c}_1\right) = \lambda_1\left(1 + \eta_1\right), \\
\partial D_1^c :& \frac{\lambda_0}{P_0(1+i_0)} = \beta\mathbb{E}_0\frac{\lambda_1}{P_1}(1+\eta_1), \\
\partial \widetilde{D}_1^b :& \frac{\lambda_0}{P_0(1+i_0)}(1-\tau_0^B) = \beta\mathbb{E}_0\frac{\lambda_1}{P_1}(1+\zeta_0), \\
\partial n_0 :& v'(n_0) = \lambda_0\frac{W_0}{P_0}, \\
\partial n_1 :& v'(n_1) = \lambda_1\frac{W_1}{P_1}, \\
\partial h_1 :& X_1 g'(h_1) = \lambda_1\frac{\Gamma_1}{P_1}, \\
\partial k_0 :& \lambda_0 = \beta G'\left(k_0\right)\mathbb{E}_0\left\{\lambda_1\frac{\Gamma_1}{P_1} + \lambda_1\zeta_0\frac{\min_{s_1}\{\Gamma_1\}}{P_1}\right\}, \\
\partial P_1^j :& P_1 = (1+\tau_1^L)\frac{\epsilon}{\epsilon-1}\cdot\frac{W_1}{A_1}.
\end{aligned}$$

where the last condition takes into account that in symmetric equilibrium all of the firms set identical prices: $P_1^j = P_1$. The complementarity slackness conditions are

$$CSC_1 : \widetilde{D}_1^b \leq \min_{s_1}\{\Gamma_1\}G(k_0),\ \zeta_0 \geq 0,\ [\widetilde{D}_1^b - \min_{s_1}\{\Gamma_1\}G(k_0)]\zeta_0 = 0,$$
$$CSC_2 : P_1\underline{c}_1 \leq D_1^c,\ \eta_1 \geq 0,\ [P_1\underline{c}_1 - D_1^c]\eta_1 = 0.$$

The first order condition can be simplified as follows

$$\begin{aligned}
\zeta_0 &= (1-\tau_0^B)\frac{1}{1+i_0}\cdot\frac{u'(c_0)/P_0}{\mathbb{E}_0\left[u'(c_1+\underline{c}_1)/P_1\right]} - 1 \geq 0,\\
&\quad \widetilde{D}_1^b \leq \min_{s_1}\{\Gamma_1\}G(k_0),\ [\widetilde{D}_1^b - \min_{s_1}\{\Gamma_1\}G(k_0)]\zeta_0 = 0,\\
\eta_1 &= \frac{\nu u'(\underline{c}_1)}{u'(c_1+\underline{c}_1)} \geq 0,\ P_1\underline{c}_1 \leq D_1^c, [P_1\underline{c}_1 - D_1^c]\eta_1 = 0,\\
u'(c_0) &= (1+i_0)\beta\mathbb{E}_0\left\{\frac{P_0}{P_1}u'(c_1+\underline{c}_1)\left[1+\frac{\nu u'(\underline{c}_1)}{u'(c_1+\underline{c}_1)}\right]\right\},\\
\frac{v'(n_0)}{u'(c_0)} &= \frac{W_0}{P_0},\\
\frac{v'(n_1)}{u'(c_1+\underline{c}_2)} &= \frac{W_1}{P_1},\\
u'(c_0) &= \beta G'(k_0)\mathbb{E}_0\left\{X_1 g'(h_1) + \zeta_0\min_{s_1}\{X_1 g'(h_1)\}\right\}.
\end{aligned}$$

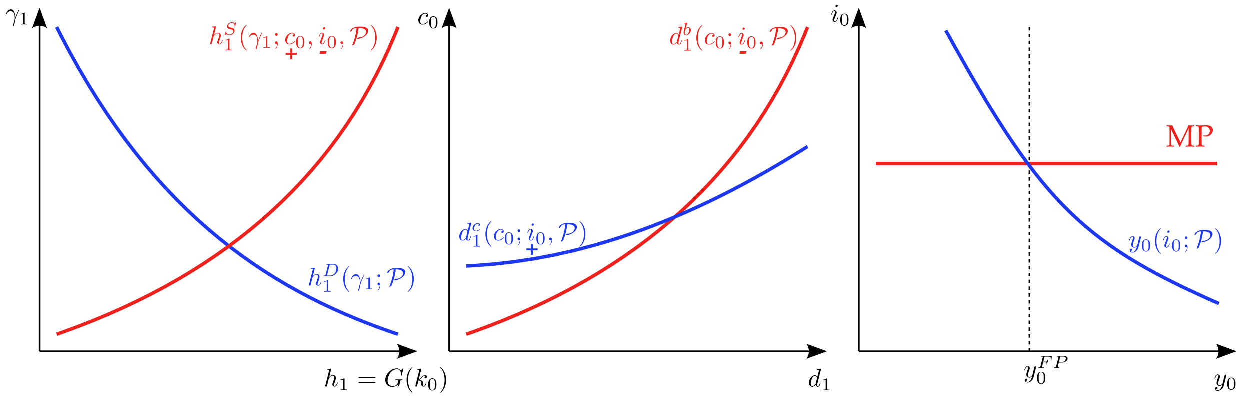
The optimality conditions, the market clearing conditions, and the fact that only durable goods price is affected by s_1 lead to the following full set of equilibrium equations

$$\begin{aligned}
\zeta_0 &= (1-\tau_0^B)\left\{\frac{\nu u'(\underline{c}_1)}{u'(A_1 n_1^*)} + 1\right\} - 1 \geq 0,\\
&\quad \widetilde{d}_1^b \leq \theta\frac{g'[G(k_0)]}{u'(A_1 n_1)}G(k_0),\\
&\quad \left[\widetilde{d}_1^b - \theta\frac{g'[G(k_0)]}{u'(A_1 n_1)}G(k_0)\right]\zeta_0 = 0,\\
\eta_1 &= \frac{\nu u'(\underline{c}_1)}{u'(A_1 n_1^*)} \geq 0,\ \underline{c}_1 \leq d_1^g + \widetilde{d}_1^b,\ \left[\underline{c}_1 - d_1^g - \widetilde{d}_1^b\right]\eta_0 = 0,\\
u'(c_0) &= \frac{(1+i_0)\beta}{\Pi^*}\left[u'\left(A_1 n_1^*\right) + \nu u'\left(\underline{c}_1\right)\right],\\
u'(A_1 n_1) &= \frac{v'(n_1)}{A_1},\\
u'(c_0) &= \beta g'[G(k_0)]G'(k_0)\left(\mu + (1-\mu)\theta + \zeta_0\theta\right),\\
y_0 &= A_0 n_0,\\
y_0 &= c_0 + k_0.
\end{aligned}$$

A.1.2 Equilibrium Determination

Collateral constraint (22) may be slack depending on the severity of financial regulation policy. I will discuss the equilibrium determination assuming that the collateral constraint binds. However, I solve for the optimal policy allowing for the constraint to possibly be slack.

Equilibrium in period 0 can be conveniently described by plotting equations (20), (21), (22), where the last one holds as equality. Figure 1 plots the equations. $\mathcal{P} \equiv \{\theta, \nu, A_0, \tau_0^B\}$ represents all of the variables that agents in the model take as given.



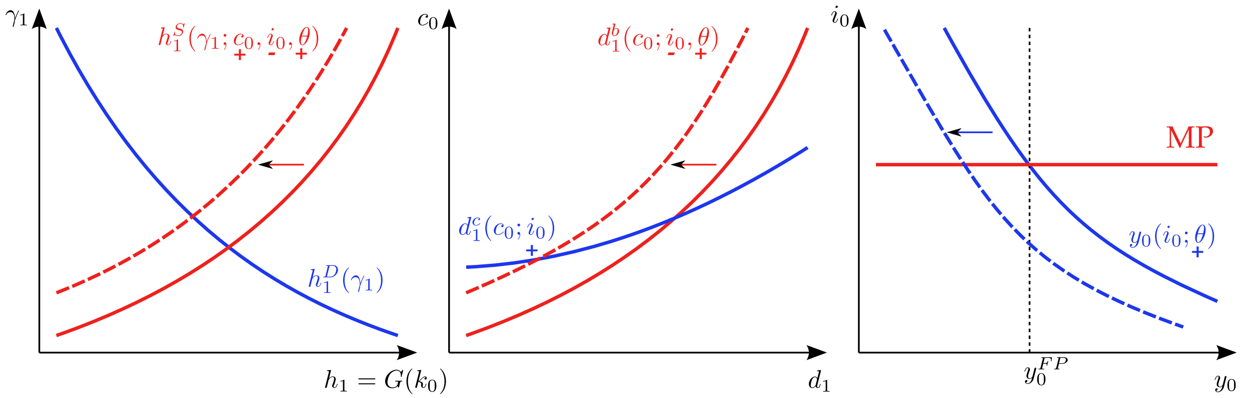
(a) Durable market equilibrium (b) Liquid assets supply and demand (c) Monetary Policy and IS curves

Figure 1: Equilibrium in period 0.

Panel (a) represents equation (21) as an intersection of supply and demand for durable goods. The downward sloping demand curve is the household optimal choice of durable goods described by equation (8), while the upward sloping supply curve is banker's optimality with respect to investment in durable goods (11). These curves are plotted for a given level of consumption c_0, policy rate i_0, as well as various variables $\mathcal{P}$. Banker's optimal durable supply depends positively on consumption c_0 and negatively on interest rate i_0, this can be formally seen from (21). Intuitively, higher consumption in period 0 reduces opportunity cost of investing, and higher interest rate on safe bonds reduces benefits of issuing liquid bonds which acts to reduce investment in durable goods. Thus, when durable goods supply equals durable goods demand, higher consumption increases durable goods output and decreases their price, while an increase in the interest has the opposite effect. (22), assuming it holds as equality, defines bankers safe debt supply as a function of consumption c_0, interest rate i_0, and variables $\mathcal{P}$. When elasticity of durable goods price is smaller than one, the safe debt supply is positively related to investment in durable goods production and the liquid debt supply is positively related to consumption c_0 and negatively to i_0

$$\frac{\partial \widetilde{d}_1^b}{\partial k_0} = \theta \frac{g'[G(k_0)]}{u'\left(A_1 n_1^*\right)} G'(k_0)\left[1 - \epsilon_\Gamma\right] > 0,\ if\ \epsilon_\Gamma < 1.$$

Panel (b) of figure 1 plots the safe assets demand and supply schedules as functions of consumption c_0. The supply is positively related to consumption c_0. The demand for safe assets, given by $\underline{c}_1$, is positively related to both consumption c_0 and interest rate i_0, as can be formally seen from (20). Intuitively, given interest rate i_0 and the total level of consumption in period 1, higher c_0 makes households willing to buy more goods with safe assets in the future. Similarly, given the total level of consumption in period 1 and consumption c_0, higher interest rate i_0 makes it more attractive to invest in safe debt. I will focus on the case in which the liquid debt demand schedule increases faster with consumption c_0 compared to the safe debt supply schedule. If this is not the case the model counterfactually implies that increase in policy rate i_0 stimulates the economy. The sufficient condition for safe debt demand to be more sensitive to consumption than safe debt supply is



(a) Durable market equilibrium (b) Liquid assets supply and demand (c) Monetary Policy and IS curves

Figure 2: Equilibrium in period 0 with different values of θ.

$$\partial d_1^c / \partial c_0 > \partial \tilde{d}_1^b / \partial c_0 \ \text{ if } \ \frac{1-\epsilon_\Gamma}{\frac{G''(k_0)G(k_0)}{[G'(k_0)]^2} - \epsilon_\Gamma} \cdot \frac{\underline{c}_1 u''(\underline{c}_1)}{u'(\underline{c}_1)} < 1. \tag{A.1}$$

Intuitively, this expression states that the elasticity of marginal utility of consumption bought with safe debt is not too high. Because safe debt demand increases and safe debt supply falls with interest rate i_0, the intersection of safe debt supply and demand on panel (b) implies that consumption is negatively related to interest rate i_0.

Panel (c) of figure 1 plots output in period 0 as a function of the policy rate. Recall that output in period 0 is a sum of consumption and investment in production of durable goods: $y_0 = c_0 + k_0$. We deduced from panel (b) that consumption is negatively related to interest rate i_0. We also discussed, see panel (a), that investment in durable goods is negatively related to interest rate for a given level of consumption c_0. There is an additional negative effect of k_0 because consumption falls with interest rate. As a result, output is negatively related to interest rate. This is represented by a downward sloping curve on panel (c). Due to price stickiness, the central bank can affect output by setting interest rate i_0, represented by horizontal line on panel (c). If prices were flexible output would equal y_0^{FP}, see vertical dashed line on panel (c). In the figure the central bank sets the interest rate so that output equals flexible price output.

A low value of parameter θ acts as a negative aggregate demand "shock". Figure 2 compares equilibria with low and high values of θ. The dashed lines correspond to the low value of θ. With smaller θ bankers reduce their supply of durable goods as well as the supply of safe debt. For a given level of interest rate i_0, households must reduce their consumption to equate supply and demand for safe debt. As a result consumption c_0 is smaller. Output is represented by the dashed curve shifted to the left. If monetary authority does not adjust the nominal interest rate the economy experiences recession in period 0.

It is instructive to contrast the effect of a change in θ to an effect of deleveraging shock considered in a recent paper by Eggertsson and Krugman (2012). The authors study an economy populated by heterogeneous consumers who endogenously become borrowers and savers. An exogenous negative shock to borrowing capacity of borrowers makes them delever. In an environment with sticky prices, given a nominal interest rate, borrowers delever by reducing current consumption which results in recession. In the current paper the bankers resemble borrowers from Eggertsson and Krugman (2012). A low value of θ does not allow them to borrow much. As a result, the durable goods investment demand is small which, given the nominal interest rate, has a direct negative effect on output. However, in the current paper there is an additional negative effect on output. Because bankers issue less safe debt, households will buy fewer goods with safe assets resulting in higher future marginal utility $u'(\underline{c}_1)$. Households reduce current consumption in an attempt to buy more safe debt which results in an additional negative effect on output. Smaller consumption c_0 reduces bankers incentives to invest in durable goods which reduces the supply of safe assets even more. The logic repeats leading to an amplification of the direct effect from low realization in θ. This amplification mechanism can potentially lead to existence of multiple equilibria. However, condition (A.1) rules out this possibility.

A.1.3 Proof of Proposition 3

(i) Let's first consider optimal monetary policy conditional on macroprudential policy being set at τ_0^b. The planner solves

$$\max_{c_0,k_0,d_1^b} \quad u(c_0) - v\left(\frac{c_0+k_0}{A_0(s_0)}\right) + \beta\left[u(y_1^*) + (\mu + (1-\mu)\theta)\, g[G(k_0)] + \nu u(\widetilde{d}_1^b + d_1^g) - v(n_1^*)\right]$$

$$s.t. :\widetilde{d}_1^b \leq \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0),$$

$$u'(c_0) = \beta g'\left[G(k_0)\right] G'(k_0)\left(\mu + \left[(1-\tau_0^B)\left\{\frac{\nu u'(\widetilde{d}_1^b + d_1^g)}{u'(y_1^*)} + 1\right\} - \mu\right]\theta\right).$$

This problem differs from the problem of optimally choosing both monetary and macroprudential policy. The banker's choice of optimal investment is now taken as a constraint because τ_0^B is not chosen optimally. The Lagrangian of this problem is

$$\begin{aligned}
\widetilde{\mathcal{L}}_0 =& u(c_0) - v\left(\frac{c_0+k_0}{A_0(s_0)}\right) + \beta\left[u(y_1^*) + (\mu + (1-\mu)\theta)\, g[G(k_0)] + \nu u(\widetilde{d}_1^b + d_1^g) - v(n_1^*)\right] \\
&- \beta u'(A_1 n_1^*)\widetilde{\zeta}_0\left[\widetilde{d}_1^b - \theta\frac{g'[G(k_0)]}{u'(y_1^*)}G(k_0)\right] \\
&- \widetilde{\chi}_0\left\{u'(c_0) - \beta g'\left[G(k_0)\right] G'(k_0)\left(\mu + \left[(1-\tau_0^B)\left\{\frac{\nu u'(\widetilde{d}_1^b + d_1^g)}{u'(y_1^*)} + 1\right\} - \mu\right]\theta\right)\right\}.
\end{aligned}$$

The first order optimality conditions for this problem are

$$\begin{aligned}
&\partial c_0 : u'(c_0) = \frac{v'(n_0)}{A_0} + \widetilde{\chi}_0 u''(c_0), \\
&\partial d_1^b : \nu u'(\widetilde{d}_1^b + d_1^g) + \widetilde{\chi}_0 g'\left[G(k_0)\right] G'(k_0)\theta(1-\tau_0^B)\frac{\nu u''(\widetilde{d}_1^b + d_1^g)}{u'(y_1^*)} = u'(y_1^*)\widetilde{\zeta}_0(s_0), \\
&\partial k_0 : \beta g'[G(k_0)]G'(k_0)\Bigg\{\mu + (1-\mu)\theta + \widetilde{\zeta}_0\theta\left(1-\epsilon_\Gamma\right) \\
&\qquad + \widetilde{\chi}_0\left[(1-\theta)\mu + (1-\tau_0^B)\left(1+\tau_A\right)\theta\right]\left[\frac{G''(k_0)}{G'(k_0)} - \epsilon_\Gamma\frac{G'(k_0)}{G(k_0)}\right]\Bigg\} = \frac{v'(n_0)}{A_0}.
\end{aligned}$$

and the complementarity slackness conditions are

$$CSC_1 :\widetilde{d}_1^b \leq \theta\frac{g'[G(k_0)]}{u'(y_1^*)}G(k_0),\ \widetilde{\zeta}_0 \geq 0,\ \left\{\widetilde{d}_1^b - \theta\frac{g'[G(k_0)]}{u'(y_1^*)}G(k_0)\right\}\widetilde{\zeta}_0 = 0.$$

These conditions can be reduced to

$$\begin{aligned}
\widetilde{\chi}_0 &= \tau_0 \frac{u'(c_0)}{u''(c_0)}, \\
\widetilde{\zeta}_0 &= \tau_A \left[1 + \theta \frac{g'\left[G(k_0)\right] G'(k_0)}{u'(y_1^*)} (1 - \tau_0^B) \frac{u'\left(c_0\right) u''\left(\widetilde{d}_1^b + d_1^g\right)}{u''\left(c_0\right) u'\left(\widetilde{d}_1^b + d_1^g\right)} \tau_0 \right], \\
u'(c_0) &= \beta \frac{g'[G(k_0)] G'(k_0)}{1 - \tau_0} \Bigg\{ \mu + (1-\mu)\theta + \widetilde{\zeta}_0 \theta \left(1 - \epsilon_\Gamma\right) \\
&\quad + \tau_0 \frac{u'(c_0)}{u''(c_0)} \left[\mu + (1-\mu)\theta + \theta \left(\left(1 - \tau_0^B\right)\left(1 + \tau_A\right) - 1\right)\right] \left[\frac{G''(k_0)}{G'(k_0)} - \epsilon_\Gamma \frac{G'(k_0)}{G(k_0)}\right] \Bigg\}.
\end{aligned}$$

Combining the last two equations with private durable investment optimality conditions leads to

$$\tau_0 = -\frac{1}{Z_1} \left[\tau_0^b - \frac{\epsilon_\Gamma \tau_A}{1 + \tau_A}\right],$$

where

$$\begin{aligned}
Z_1 = &\left(\frac{\mu + (1-\mu)\theta + \theta\tau_A}{\theta(1+\tau_A)} - \tau_0^b\right) \left(1 + \frac{u'(c_0)}{u''(c_0)} \left[\frac{G''(k_0)}{G'(k_0)} - \epsilon_\Gamma \frac{G'(k_0)}{G(k_0)}\right]\right) \\
&+ \frac{\tau_A\left(1 - \epsilon_\Gamma\right)}{(1 + \tau^A)} \theta \frac{g'\left[G(k_0)\right] G'(k_0)}{u'(y_1^*)} (1 - \tau_0^b) \frac{u'\left(c_0\right) u''\left(\widetilde{d}_1^b + d_1^g\right)}{u''\left(c_0\right) u'\left(\widetilde{d}_1^b + d_1^g\right)} > 0.
\end{aligned}$$

Part (ii) Let's consider optimal macroprudential policy conditional on monetary policy being set at i_0. The planner solves

$$\begin{aligned}
&\max_{c_0, k_0, d_1^b} \quad u(c_0) - v\left(\frac{c_0 + k_0}{A_0(s_0)}\right) + \beta \left[u(y_1^*) + (\mu + (1-\mu)\theta)\, g[G(k_0)] + \nu u(\widetilde{d}_1^b + d_1^g) - v(n_1^*)\right] \\
&s.t. : \widetilde{d}_1^b \leq \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0), \\
&\quad \beta \frac{1 + i_0}{\Pi^*} u'\left(A_1 n_1^*\right) \left[1 + \frac{\nu u'(\widetilde{d}_1^b + d_1^g)}{u'\left(y_1^*\right)}\right] = u'\left(c_0\right).
\end{aligned}$$

The regulator's problem is characterized by the following Lagrangian

$$\begin{aligned}
\widetilde{\mathcal{L}}_0 = &u(c_0) - v\left(\frac{c_0 + k_0}{A_0(s_0)}\right) + \beta \left[u(y_1^*) + (\mu + (1-\mu)\theta)\, g[G(k_0)] + \nu u(\widetilde{d}_1^b + d_1^g) - v(n_1^*)\right] \\
&- \beta u'(y_1^*) \widetilde{\zeta}_0 \left[\widetilde{d}_1^b - \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0)\right] - \widetilde{\phi}_0 \left\{\beta \frac{1 + i_0}{\Pi^*} u'\left(y_1^*\right) \left[1 + \frac{\nu u'(\widetilde{d}_1^b + d_1^g)}{u'\left(y_1^*\right)}\right] - u'\left(c_0\right)\right\}.
\end{aligned}$$

The first order conditions are

$$
\begin{aligned}
&\partial c_0 : u'(c_0) = \frac{v'(n_0)}{A_0} - \widetilde{\phi}_0 u''(c_0), \\
&\partial \underline{c}_1 : \nu u'(\widetilde{d}_1^b + d_1^g) = u'(y_1^*)\widetilde{\zeta}_0 + \widetilde{\phi}_0 \frac{1+i_0}{\Pi^*} \nu u''(\widetilde{d}_1^b + d_1^g), \\
&\partial k_0 : \beta g'[G(k_0)]G'(k_0)\left[\mu + (1-\mu)\theta + \widetilde{\zeta}_0 \theta\left(1-\epsilon_\Gamma\right)\right] = \frac{v'(n_0)}{A_0}.
\end{aligned}
$$

and the complementarity slackness conditions are

$$
CSC_1 : \widetilde{d}_1^b \leq \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0),\ \widetilde{\zeta}_0 \geq 0,\ \left\{\widetilde{d}_1^b - \theta \frac{g'[G(k_0)]}{u'(y_1^*)} G(k_0)\right\} \widetilde{\zeta}_0 = 0.
$$

The first order conditions can be rewritten as follows

$$
\begin{aligned}
\tau_0 &= -\widetilde{\phi}_0 \frac{u''(c_0)}{u'(c_0)}, \\
\widetilde{\zeta}_0 &= \tau_A \left[1 - \widetilde{\phi}_0 \frac{1+i_0}{\Pi^*} \cdot \frac{u''(\widetilde{d}_1^b + d_1^g)}{u'(\widetilde{d}_1^b + d_1^g)}\right], \\
(1-\tau_0)u'(c_0) &= \beta g'[G(k_0)]G'(k_0)\left[\mu + (1-\mu)\theta + \widetilde{\zeta}_0 \theta\left(1-\epsilon_\Gamma\right)\right].
\end{aligned}
$$

Comparing planner's optimal choice of investment in durable goods to private optimum I get

$$
\tau_0^b(s_0) = \frac{1}{1-\tau_0}\left\{\frac{\epsilon_\Gamma \tau_A}{1+\tau_A} - \tau_0 Z_2\right\},
$$

where

$$
Z_2 = \frac{\mu(1-\theta) + \theta(1+\tau_A)}{\theta(1+\tau_A)} + \frac{\tau_A(1-\epsilon_\Gamma)}{1+\tau_A} \cdot \frac{1+i_0}{\Pi^*} \cdot \frac{u'(c_0)}{u''(c_0)} \cdot \frac{u''\left(\widetilde{d}_1^b + d_1^g\right)}{u'\left(\widetilde{d}_1^b + d_1^g\right)} > 1, \tag{A.2}
$$

The two terms in Z_2 reflects two effects that bankers do not internalize when they decide to issue safe debt. First, higher level of safe debt allows a banker increase its investment in durable goods production. This increases "aggregate demand" in period 0. This has a positive welfare effect if a country is in recession, i.e., $\tau_0 > 0$. Second, higher level of safe debt increases consumers safe debt holdings, which allows them to buy more goods with safe debt in period 1. When the nominal (and real) interest rate does not adjust, higher consumption of goods bought with safe debt in period 1 lead to higher consumption of goods in period 0. As a result, "aggregate demand" increases. When the country is in recession, this has a positive welfare effects.

A.2 A 2-period Model of Currency Union

This section presents monetary union derivations and proofs omitted from the main text.

A.2.1 Household Problem Solution

A typical household in country i solves the following problem

$$
\begin{aligned}
\mathcal{L}_0 = & \mathbb{E}\Bigg\{ U\left(c^i_{NT,0}, c^i_{T,0}\right) - v\left(n^i_0\right) + \beta\Bigg[U\left(c^i_{NT,1} + \underline{c}^i_{NT,1}, c^i_{T,1} + \underline{c}^i_{T,1}\right) - v\left(n^i_1\right) \\
& \qquad + X_1(s_1) g\left(h^i_1\right) + \nu^i U\left(\underline{c}^i_{NT,1}, \underline{c}^i_{T,1}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
& - \Lambda^i_0 \Big[T^i_0 + P^i_{NT,0} c^i_{NT,0} + P_{T,0} c^i_{T,0} + \frac{D^{c,i}_1}{1+i_0} + P^i_{NT,0} k^i_{NT,0} \\
& \qquad - P_{T,0} e^i_{T,0} - \frac{\widetilde{D}^{b,i}_1}{1+i_0}\left(1 - \tau^{b,i}_0\right) - W^i_0 n^i_0 - \Pi^i_0 \Big] \\
& - \beta \Lambda^i_1 \Big[P^i_{NT,1}\left(c^i_{NT,1} + \underline{c}^i_{NT,1}\right) + P_{T,1}\left(c^i_{T,1} + \underline{c}^i_{T,1}\right) + T^i_1 + \Gamma^i_1 h^i_1 + \widetilde{D}^{b,i}_1 \\
& \qquad - P_{T,1} e^i_{T,1} - D^{c,i}_1 - W^i_1 n^i_1 - \Gamma^i_1 G\left(k^i_{NT,0}\right) - \Pi^i_1 . \Big] \\
& - \beta \Lambda^i_1 \zeta^i_0 \left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\} G(k^i_{NT,0}) \right] \\
& - \beta \Lambda^i_1 \eta^i_1 \left[P^i_{NT,1} \underline{c}^i_{NT,1} + P_{T,1} \underline{c}^i_{NT,1} - D^c_1 \right].
\end{aligned}
$$

Let's introduce the following notation

$$
\begin{aligned}
U^i_{NT,0} &\equiv \frac{\partial U\left(c^i_{NT,0}, c^i_{T,0}\right)}{\partial c^i_{NT,0}}, \; U^i_{T,0} \equiv \frac{\partial U\left(c^i_{NT,0}, c^i_{T,0}\right)}{\partial c^i_{T,0}}, \\
U^i_{NT,1} &\equiv \frac{\partial U\left(c^i_{NT,1} + \underline{c}^i_{NT,1}, c^i_{T,1} + \underline{c}^i_{T,1}\right)}{\partial c^i_{NT,1}}, \quad U^i_{T,1} \equiv \frac{\partial U\left(c^i_{NT,1} + \underline{c}^i_{NT,1}, c^i_{T,1} + \underline{c}^i_{T,1}\right)}{\partial c^i_{T,1}}, \\
\underline{U}^i_{NT,1} &\equiv \frac{\partial U\left(\underline{c}^i_{NT,1}, \underline{c}^i_{T,1}\right)}{\partial \underline{c}^i_{NT,1}}, \; \underline{U}^i_{T,1} \equiv \frac{\partial U\left(\underline{c}^i_{NT,1}, \underline{c}^i_{T,1}\right)}{\partial \underline{c}^i_{T,1}}, \\
G^i_{NT,0} &\equiv G'\left(k^i_{NT,0}\right).
\end{aligned}
$$

The first order conditions can be written as follows

$$
\begin{aligned}
\partial c^i_{NT,0} &: U^i_{NT,0} = \Lambda^i_0 P^i_{NT,0}, \\
\partial c^i_{T,0} &: U^i_{T,0} = \Lambda^i_0 P_{T,0}, \\
\partial c^i_{NT,1} &: U^i_{NT,1} = \Lambda^i_1 P^i_{NT,1}, \\
\partial c^i_{T,1} &: U^i_{T,1} = \Lambda^i_1 P_{T,1}, \\
\partial \underline{c}^i_{NT,1} &: U^i_{NT,1} + \nu^i \underline{U}^i_{NT,1} = P^i_{NT,1}\Lambda^i_1 \left(1+\eta^i_1\right), \\
\partial \underline{c}^i_{T,1} &: U^i_{T,1} + \nu^i \underline{U}^i_{T,1} = P_{T,1}\Lambda^i_1 \left(1+\eta^i_1\right), \\
\partial D^{c,i}_1 &: \frac{\Lambda^i_0}{1+i_0} = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\eta^i_1\right), \\
\partial \widetilde{D}^{b,i}_1 &: \frac{\Lambda^i_0}{1+i_0}\left(1-\tau^{b,i}_0\right) = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\zeta^i_0\right), \\
\partial n^i_0 &: v'\left(n^i_0\right) = \Lambda^i_0(s_0) W^i_0, \\
\partial n^i_1 &: v'\left(n^i_1\right) = \Lambda^i_1(s_1) W^i_1, \\
\partial h^i_1 &: X_1(s_1) g'\left(h^i_1\right) = \Lambda^i_1 \Gamma^i_1, \\
\partial k^i_{NT,0} &: \Lambda^i_0 P^i_{NT,0} = \beta \mathbb{E}_0 G^i_{NT,0} \Lambda^i_1 \left(\Gamma^i_1 + \zeta^i_0 \min_{s_1}\{\Gamma^i_1\}\right),
\end{aligned}
$$

$$
\partial P^i_{NT,1} : P^i_{NT,1} = \left(1+\tau^{L,i}_1\right)\frac{\epsilon}{\epsilon-1}\cdot\frac{W^i_1}{A^i_1}.
$$

as well as complementarity slackness conditions

$$
\begin{aligned}
CSC_1 :& \widetilde{D}^{b,i}_1 \le \min_{s_1|s_0}\{\Gamma^i_1(s_1)\} G\left(k^i_{NT,0}\right), \zeta^i_0 \ge 0, \left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\} G\left(k^i_{NT,0}\right)\right]\zeta^i_0 = 0, \\
CSC_2 :& P^i_{NT,1}\underline{c}^i_{NT,1} + P_{T,1}\underline{c}^i_{T,1} \le D^{c,i}_1, \eta^i_1 \ge 0, \left[P^i_{NT,1}\underline{c}^i_{NT,1} + P_{T,1}\underline{c}^i_{N,1} - D^{c,i}_1\right]\eta^i_1 = 0.
\end{aligned}
$$

A.2.2 Equilibrium

The full set of equilibrium conditions is

$$
\begin{aligned}
\zeta_0^i &= \frac{1-\tau_0^{b,i}}{1+i_0}\cdot\frac{U_{T,0}^i/P_{T,0}}{\beta\mathbb{E}_0\left(U_{T,1}^i/P_{T,1}\right)} - 1 \geq 0, \tilde{d}_1^{b,i} \leq G(k_{NT,0}^i)\min_{s_1}\frac{X_1(s_1)g'(h_1^i)}{U_{T,1}^i}, \\
&\left[\tilde{d}_1^{b,i} - G(k_{NT,0}^i)\min_{s_1|s_0}\frac{X_1(s_1)g'(h_1^i)}{U_{T,1}^i}\right]\zeta_0^i = 0, \\
\eta_1^i &= \frac{\nu^i\underline{U}_{T,1}^i}{U_{T,1}^i} \geq 0,\ p_1^i\underline{c}_{NT,1}^i + \underline{c}_{T,1}^i \leq d_1^{c,i}, [p_1^i\underline{c}_{NT,1}^i + \underline{c}_{T,1}^i - d_1^{c,i}]\eta_1 = 0, \\
U_{T,0}^i &= (1+r_0)\beta\mathbb{E}_0\left(U_{T,1}^i + \nu^i\underline{U}_{T,1}^i\right), \\
\frac{v'(n_0^i)}{U_{NT,0}^i} &= \frac{W_0^i}{P_{NT,0}^i}, \\
\frac{v'(n_1^i)}{U_{NT,1}^i} &= A_1^i, \\
U_{NT,0}^i &= \beta G_{NT,0}^i\mathbb{E}_0\left[X_1(s_1)g'(h_1^i) + \Lambda_1^i\zeta_0^i\min_{s_1}\left\{\frac{X_1(s_1)g'(h_1^i)}{\Lambda_1^i}\right\}\right], \\
\frac{U_{NT,0}^i}{U_{T,0}^i} &= \frac{P_{NT,0}^i}{P_{T,0}}, \\
\frac{U_{NT,1}^i}{U_{T,1}^i} &= p_1^i, \\
\frac{\underline{U}_{NT,1}^i}{\underline{U}_{T,1}^i} &= \frac{U_{NT,1}^i}{U_{T,1}^i}, \\
c_{T,0}^i - e_{T,0}^i &+ \frac{d_1^{c,i} - d_1^{b,i} - d_1^{g,i}}{1+r_0} = 0, \\
c_{T,1}^i + \underline{c}_{T,1}^i &- e_{T,1}^i + d_1^{b,i} + d_1^{g,i} - d_1^{c,i} = 0, \\
A_0^i n_0^i &= k_{NT,0}^i + c_{NT,0}^i, \\
A_1^i n_1^i &= c_{NT,1}^i + \underline{c}_{NT,1}^i, \\
h_1^i &= G(k_{NT,0}^i),
\end{aligned}
$$

$$
\int d_1^{c,i}di = \int d_1^{g,i}di + \int d_1^{b,i}di.
$$

Using assumption 1 and the fact that equilibrium allocation does not depend on the realization of state s_1, the full set of equilibrium conditions can be written as follows

$$\zeta_0^i = (1-\tau_0^{b,i})\left\{\nu^i \frac{\widetilde{c}_{NT,1}^i}{\underline{c}_{NT,1}^i}+1\right\}-1 \geq 0, \widetilde{d}_1^{b,i} \leq \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i), \left[\widetilde{d}_1^{b,i} - \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i)\right]\zeta_0^i = 0$$

$$\eta_1^i = \frac{\nu^i \widetilde{c}_{NT,1}^i}{\underline{c}_{NT,1}^i} \geq 0, \; \frac{\underline{c}_{NT,1}^i p_1^i}{a} \leq d_1^{c,i}, \left[\frac{\underline{c}_{NT,1}^i p_1^i}{a} - d_1^{c,i}\right]\eta_1^i = 0,$$

$$\frac{a}{c_{NT,0}^i} = (1+r_0)\frac{P_{NT,0}^i}{P_{T,0} p_1^i}\beta\left[\frac{a}{\widetilde{c}_{NT,1}^i} + \nu^i \frac{a}{\underline{c}_{NT,1}^i}\right],$$

$$A_1^i \frac{a}{\widetilde{c}_{NT,1}^i} = v'\left(\frac{\widetilde{c}_{NT,1}^i}{A_1^i}\right),$$

$$\frac{v'(n_0^i)}{U_{NT,0}^i} = \frac{W_0^i}{P_{NT,0}^i},$$

$$\frac{a}{1-a}\cdot\frac{c_{T,0}^i}{c_{NT,0}^i} = \frac{P_{NT,0}^i}{P_{T,0}},$$

$$\frac{a}{c_{NT,0}^i} = \beta g'\left[G\left(k_{NT,0}^i\right)\right] G'(k_0^i)(\mu + (1-\mu)\theta^i + \zeta_0^i\theta^i),$$

$$c_{NT,0}^i \frac{P_{NT,0}^i}{P_{T,0}}\cdot\frac{1-a}{a} - e_{T,0}^i + \frac{d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}}{1+r_0} = 0,$$

$$\widetilde{c}_{NT,1}^i p_1^i \frac{1-a}{a} - e_{T,1}^i + \widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i} = 0,$$

$$\int d_1^{c,i} di = \int d_1^{g,i} di + \int \widetilde{d}_1^{b,i} di,$$

where $\widetilde{c}_{NT,1}^i(s_0) = c_{NT,1}^i(s_0) + \underline{c}_{NT,1}^i(s_0)$.

A.2.3 Proof of Lemma 1

$$\begin{aligned}
& V^i\left(c_{NT,0}^i, k_{NT,0}^i, \underline{c}_{NT,1}^i, P_{NT,0}^i/P_{T,0}, p_1^i\right) \\
&= \log\left(c_{NT,0}^i\left[\frac{1-a}{a}\cdot\frac{P_{NT,0}^i}{P_{T,0}}\right]^{1-a}\right) - v\left(\frac{c_{NT,0}^i + k_{NT,0}^i}{A_0^i}\right) \\
&\quad + \beta\log\left(y_{NT,1}^{i,*}\left[\frac{1-a}{a}p_1^i\right]^{1-a}\right) + \beta\nu^i\log\left(\underline{c}_{NT,1}^i\left[\frac{1-a}{a}p_1^i\right]^{1-a}\right) \\
&\quad - \beta v\left(\frac{y_{NT,1}^{i,*}}{A_1^i}\right) + \beta X_1(s_1) g\left[G\left(k_{NT,0}^i\right)\right] =
\end{aligned}$$

$$\begin{aligned}
&= \log c_{NT,0}^i - v\left(\frac{c_{NT,0}^i + k_{NT,0}^i}{A_0^i}\right) + \beta\left\{\nu^i \log \underline{c}_{NT,1}^i(s_0) + X_1(s_1) g\left[G\left(k_{NT,0}^i\right)\right]\right\} \\
&\quad + (1-a)\log\left(\frac{P_{NT,0}^i}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p_1^i \\
&\quad + \beta\left[\log y_{NT,1}^{i,*} - v\left(y_{NT,1}^{i,*}/A_1^i\right)\right] + [1+\beta(1+\nu^i)]\log[(1-a)/a].
\end{aligned}$$

Note that the term on the last line does not depend on endogenous variables.

A.2.4 Proof of Proposition 5

The regulator's problem can be summarized as follows

$$\max_{\substack{k^i_{NT,0}, c^i_{NT,0}, \underline{c}^i_{NT,1}, \\ \widetilde{d}^{b,i}_1, d^{c,i}_1, p^i_1}} \mathbb{E}\left\{ \log c^i_{NT,0} - v\left(\frac{c^i_{NT,0} + k^i_{NT,0}}{A^i_0}\right) + \beta\left\{\nu^i \log \underline{c}^i_{NT,1} + X_1(s_1) g\left[G\left(k^i_{NT,0}\right)\right]\right\}\right.$$

$$+ (1-a)\log\left(\frac{P^i_{NT,0}}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p^i_1$$

$$\text{s.t.: } \frac{1}{c^i_{NT,0}} = (1+r_0)\beta \frac{P^i_{NT,0}}{P_{T,0} p^i_1}\left(\frac{1}{y^{i,*}_{NT,1}} + \frac{\nu^i}{\underline{c}^i_{NT,1}}\right),$$

$$\widetilde{d}^{b,i}_1 \leq \theta^i \frac{g'[G(k^i_0)]}{a/y^{i,*}_{NT,1}} G(k^i_0) p^i_1,$$

$$\frac{\underline{c}^i_{NT,1} p^i_1}{a} \leq d^{c,i}_1,$$

$$\frac{1-a}{a} c^i_{NT,0} \frac{P^i_{NT,0}}{P_{T,0}} - e^i_{T,0} = \frac{\widetilde{d}^{b,i}_1 + d^{g,i}_1 - d^{c,i}_1}{1+r_0},$$

$$\widetilde{c}^i_{NT,1} p^i_1 \frac{1-a}{a} - e^i_{T,1} = d^{c,i}_1 - \widetilde{d}^{b,i}_1 - d^{g,i}_1.$$

Denote the Lagrange multipliers on the above constraints as $\widetilde{\phi}^i, \beta\widetilde{\lambda}^i_1\widetilde{\zeta}^i_0, \beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0, \widetilde{\lambda}^i_0, \beta\widetilde{\lambda}^i_1$ respectively. The first order conditions are

$$\begin{aligned}
\partial c^i_{NT,0} :& \widetilde{\lambda}^i_0 = \frac{1-a}{c^i_{T,0}}\left(1 + \frac{a}{1-a}\tau^i_0 + \frac{\widetilde{\phi}_i}{(1-a)c^i_{NT,0}}\right), \\
\partial k_{NT,0} :& \frac{v'(n^i_0)}{A^i_0} = \beta G^{i,\prime}_0 g^{i,\prime}_0\left[\mu + (1-\mu)\theta^i + \theta^i \widetilde{\zeta}^i_0 \frac{\widetilde{\lambda}^i_1 p^i_1}{a/y^{i,*}_{NT,0}}\left(1 - \epsilon^i_\Gamma\right)\right], \qquad (A.3) \\
\partial \underline{c}^i_{NT,1} :& \nu^i = \widetilde{\phi}_i(1+r_0)\frac{P^i_{NT,0}}{P_{T,0} p^i_1} \cdot \frac{\nu^i}{\underline{c}^i_{NT,1}} + \widetilde{\lambda}^i_1 \widetilde{\zeta}^i_0 d^{c,i}_1, \\
\partial \widetilde{d}^{b,i}_1 :& \widetilde{\zeta}^i_0 = -1 + \frac{1}{\beta}\frac{\widetilde{\lambda}^i_0}{\widetilde{\lambda}^i_1} \cdot \frac{1}{1+r_0},
\end{aligned}$$

$$\begin{aligned}
\partial d^c_1 :& \widetilde{\zeta}^i_0 = \widetilde{\eta}^i_0, \\
\partial p^i_1 :& \widetilde{\lambda}^i_1 = \frac{1-a}{\widetilde{c}^i_{T,1}} \cdot \frac{1 + \nu^i - \frac{\widetilde{\phi}_i}{\beta(1-a)c^i_{NT,0}}}{1 + \widetilde{\zeta}^i_0 \frac{d^{c,i}_1 - \widetilde{d}^{b,i}_1}{\widetilde{c}^i_{T,1}}}.
\end{aligned}$$

The first order conditions for p_1^i and $\underline{c}_{NT,1}^i$ can be used to express the unknown Lagrange multipliers $\widetilde{\zeta}_0, \widetilde{\lambda}_1^i$ through $\widetilde{\phi}^i$ as follows

$$\widetilde{\lambda}_1^i = \frac{1-a}{\widetilde{c}_{T,1}^i}\left\{1 + \frac{\nu^i\left(\widetilde{d}_1^{b,i} - ad_1^{c,i}\right)}{(1-a)d_1^{c,i}} - \frac{\widetilde{\phi}^i}{\beta(1-a)}\left[\frac{1}{c_{NT,0}^i}\cdot\frac{\widetilde{d}_1^{b,i}}{d_1^{c,i}} + \left(1 - \frac{\widetilde{d}_1^{b,i}}{d_1^{c,i}}\right)\beta(1+r_0)\frac{P_{NT,0}^i}{P_{T,0}p_1^i y_{NT,1}^{i,*}}\right]\right\},$$

$$\widetilde{\zeta}_0^i = \frac{\frac{\nu^i}{1-a}\left[1 - \widetilde{\phi}^i(1+r_0)\frac{P_{NT,0}^i}{P_{T,0}p_1^i \underline{c}_{NT,1}^i}\right]\frac{\widetilde{c}_{T,1}^i}{d_1^{c,i}}}{1 + \frac{\nu^i\left(\widetilde{d}_1^{b,i} - ad_1^{c,i}\right)}{(1-a)d_1^{c,i}} - \frac{\widetilde{\phi}^i}{\beta(1-a)}\left[\frac{1}{c_{NT,0}^i}\cdot\frac{\widetilde{d}_1^{b,i}}{d_1^{c,i}} + \left(1 - \frac{\widetilde{d}_1^{b,i}}{d_1^{c,i}}\right)\beta(1+r_0)\frac{P_{NT,0}^i}{P_{T,0}p_1^i y_{NT,1}^{i,*}}\right]}.$$

The first order conditions for $\widetilde{d}_1^{b,i}$ can then be used to express $\widetilde{\phi}^i$ as follows

$$\frac{\widetilde{\phi}^i}{\beta c_{NT,0}^i} = \frac{\frac{\nu^i\left(\widetilde{d}_1^{b,i} - ad_1^{c,i}\right)}{\widetilde{c}_{T,1}^i} - (1+\tau_A^i)\frac{ad_1^{c,i}}{\widetilde{c}_{T,1}^i}\tau_0^i}{\frac{\tau_A^i}{1+\tau_A^i} + \frac{d_1^{c,i}}{(1+r_0)c_{T,0}^i} + \frac{d_1^{c,i} + \tau_A^i \widetilde{d}_1^{b,i}}{\widetilde{c}_{T,1}^i(1+\tau^i)}}.$$

Finally, comparing private durable goods investment optimality condition (52) to the regulator's condition (A.3), I can express optimal prudential tax as follows

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i}\left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i \frac{\mu + (1-\mu)\theta^i + \theta\tau_A^i}{\theta^i\left(1+\tau_A^i\right)} + \frac{\tau_A^i\left(1-\epsilon_\Gamma^i\right)}{1+\tau_A^i}\left(1 - \frac{\widetilde{\lambda}_1^i \widetilde{\zeta}_0^i}{U_{T,0}^i \tau_A^i}\right)\right].$$

The last term in the square brackets can be further simplified as follows

$$\frac{\tau_A^i\left(1-\epsilon_\Gamma^i\right)}{1+\tau_A^i}\left(1 - \frac{\widetilde{\lambda}_1^i \widetilde{\zeta}_0^i}{U_{T,0}^i \tau_A^i}\right) = \frac{\tau_A^i\left(1-\epsilon_\Gamma^i\right)}{1+\tau_A^i}\widetilde{\phi}^i(1+r_0)\frac{P_{NT,0}^i}{P_{T,0}p_1^i \underline{c}_{NT,1}^i} = \Omega^i\left(\frac{\tau_A^i}{1+\tau_A^i}\cdot\frac{\left(\widetilde{d}_1^{b,i} - ad_1^{c,i}\right)}{\widetilde{c}_{T,1}^i} - \frac{a}{1-a}\tau_0^i\right),$$

where

$$\Omega^i \equiv \frac{\tau_A^i\left(1-\epsilon_\Gamma^i\right)}{1+\tau_A^i}\left(\frac{\tau_A^i}{1+\tau_A^i} + \frac{d_1^{c,i}}{(1+r_0)c_{T,0}^i} + \frac{d_1^{c,i} + \tau_A^i \widetilde{d}_1^{b,i}}{\widetilde{c}_{T,1}^i\left(1+\tau_A^i\right)}\right)^{-1}.$$

This optimal macroprudential tax can now be expressed as follows

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i}\left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i + Z_3^i\left(\widetilde{d}_1^{b,i} - ad_1^{c,i}\right) - \frac{a}{1-a}\tau_0^i Z_4^i\right],$$

where

$$Z_2^i \equiv \frac{\mu + (1-\mu)\theta^i + \theta\tau_A^i}{\theta^i\left(1+\tau_A^i\right)}, \quad Z_3^i \equiv \frac{\tau_A^i}{1+\tau_A^i}\cdot\frac{\Omega^i}{\widetilde{c}_{T,1}^i}, \quad Z_4^i \equiv \Omega^i. \tag{A.4}$$

A.2.5 Proof of Proposition 6

This section solves the global Ramsey planner problem that corresponds to optimal union-wide monetary and regional macroprudential policies.

$$
\begin{aligned}
\max_{\substack{\{k^i_{NT,0}, c^i_{NT,0}, \underline{c}^i_{NT,1}, p^i_1, \\ d^{b,i}_1, d^{c,i}_1\}, P_{T,0}, r_0}} \mathbb{E} \int \omega_i \Bigg\{ & \log c^i_{NT,0} - v\left(\frac{c^i_{NT,0} + k^i_{NT,0}}{A^i_0}\right) + \beta\left\{\nu^i \log \underline{c}^i_{NT,1} + X^i_1(s_1) g\left[G\left(k^i_{NT,0}\right)\right]\right\} \\
& + (1-a)\log\left(\frac{P^i_{NT,0}}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p^i_1 \Bigg\} di, \\
\text{s.t.: } & d^{b,i}_1 \le \theta^i \frac{g'[G(k^i_{NT,0})]}{a/y^{i,*}_{NT,1}} G(k^i_{NT,0}) p^i_1, \\
& \frac{\underline{c}^i_{NT,1} p^i_1}{a} \le d^{c,i}_1, \\
& c^i_{NT,0} \frac{P^i_{NT,0}}{P_{T,0}(s_0)} \frac{1-a}{a} - e^i_{T,0} + \frac{d^{c,i}_1 - \widetilde{d}^{b,i}_1 - d^{g,i}_1}{1+r_0} = 0, \\
& y^{i,*}_{NT,1} p^i_1 \frac{1-a}{a} - e^i_{T,1} + \widetilde{d}^{b,i}_1 + d^{g,i}_1 - d^{c,i}_1 = 0, \\
& \frac{1-a}{a} \int \frac{P^i_{NT,0}}{P_{T,0}} c^i_{NT,0} di = \int e^i_{T,0} di, \\
& \frac{1-a}{a} \int p^i_1 y^{i,*}_{NT,1} di = \int e^i_{T,1} di, \\
& \frac{1}{c^i_{NT,0}} = (1+r_0)\beta \frac{P^i_{NT,0}}{P_{T,0} p^i_1}\left(\frac{1}{y^{i,*}_{NT,1}} + \frac{\nu^i}{\underline{c}^i_{NT,1}}\right).
\end{aligned}
$$

Note that the traded goods market clearing condition in one of the periods is redundant. On of the two conditions can be obtained by summing country-wide budget constraints across countries in both periods, and then using the traded goods market clearing condition in the other period. Thus, I drop global market clearing condition for traded goods in period 1. The first order conditions are

$$
\begin{aligned}
\partial c^i_{NT,0} : & \widetilde{\lambda}^i_0 = \frac{1-a}{c^i_{T,0}}\left(1 + \frac{a}{1-a}\tau^i_0 + \frac{\widetilde{\phi}_i}{(1-a)c^i_{NT,0}} - \frac{\widetilde{\psi}_0 c^i_{T,0}}{\omega^i(1-a)}\right) \\
\partial \underline{c}^i_{NT,1} : & \nu^i = \widetilde{\phi}_i(1+r_0)\frac{P^i_{NT,0}}{P_{T,0}p^i_1} \cdot \frac{\nu^i}{\underline{c}^i_{NT,1}} + \widetilde{\lambda}^i_1 \widetilde{\eta}^i_0 d^{c,i}_1, \\
\partial k_{NT,0} : & \beta G^{i,\prime}_0 g^{i,\prime}_0 \left[\mu + (1-\mu)\theta^i + \theta^i \widetilde{\zeta}^i_0 \frac{\widetilde{\lambda}^i_1 p^i_1}{a/y^{i,*}_{NT,0}}\left(1 - \epsilon^i_\Gamma\right)\right] = \frac{v'(n^i_0)}{A^i_0}, \\
\partial P_{T,0} : & \int \omega_i \tau^i_0 di = 0, \\
\partial \widetilde{d}^{b,i}_1 : & \widetilde{\zeta}^i_0 = -1 + \frac{1}{\beta}\frac{\widetilde{\lambda}^i_0}{\widetilde{\lambda}^i_1} \cdot \frac{1}{1+r_0}, \\
\partial d^{c,i}_1 : & \widetilde{\zeta}^i_0 = \widetilde{\eta}^i_0, \\
\partial r_0 : & \int \omega_i \left[\frac{\widetilde{\phi}_i}{c^i_{NT,0}} - \widetilde{\lambda}^i_0\left(c^i_{T,0} - e^i_{T,0}\right)\right] di = 0,
\end{aligned}
$$

$$\partial p_1^i : \widetilde{\lambda}_1^i = \frac{1-a}{\widetilde{c}_{T,1}^i} \cdot \frac{1+\nu^i - \frac{\widetilde{\phi}_i}{\beta(1-a)c_{NT,0}^i}}{1+\widetilde{\zeta}_0^i \frac{d_1^{c,i}-d_1^{b,i}}{\widetilde{c}_{T,1}^i}}.$$

Optimality condition with respect to investment in durable goods can be rearranged as follows

$$\beta G'(k_{NT,0}^i) g' \left[G(k_{NT,0}^i) \right] \left[1 + \theta^i \widetilde{\zeta}_0^i \frac{\widetilde{\lambda}_0^i}{U_{T,0}^i} \left(1 - \epsilon_\Gamma^i \right) \right] = (1-\tau_0^i) U_{NT,0}^i.$$

This expression is identical to local planner's optimality condition with respect to investment in durable goods. The first order condition with respect to the real interest rate on safe bonds leads to

$$\widetilde{\psi}_0 = \frac{\int \omega_i \left(1 - \frac{\widetilde{\lambda}_0^i e_{T,0}^i}{U_{T,o}^i c_{T,0}^i} \right) di}{\int e_{T,0}^i di}.$$

This equation states that the Lagrange multiplier on the traded goods market clearing condition in period 0 equals the average deviation of planner's marginal value of traded goods from the private agents marginal utility multiplied by the share of endowment in consumption of traded goods. This expression can be alternatively written as

$$\widetilde{\psi}_0 = (1+r_0) \frac{\int \omega^i \frac{1-a}{c_{T,0}^i} \left(c_{T,0}^i - e_{T,0}^i \right) \left(1 + \frac{a}{1-a} \tau_0^i \right) di - \int \omega^i \frac{e_{T,0}^i}{\Theta^i c_{T,0}^i} \beta \left[\frac{\nu^i (d_1^{b,i} - a d_1^{c,i})}{\widetilde{c}_{T,1}^i} - (1+\tau_A^i) \frac{a d_1^{c,i}}{\widetilde{c}_{T,1}^i} \tau_0^i \right] di}{\int \frac{e_{T,0}^i}{c_{T,0}^i} \cdot \frac{d_1^{c,i}}{\Theta^i} di}, \quad \text{(A.5)}$$

where

$$\Theta^i = \frac{\tau_A^i}{1+\tau_A^i} + \frac{d_1^{c,i}}{(1+r_0) c_{T,0}^i} + \frac{d_1^{c,i} + \tau_A^i \widetilde{d}_1^{b,i}}{\widetilde{c}_{T,1}^i \left(1 + \tau^i \right)}.$$

It is easy to see that

$$\frac{\widetilde{\phi}^i}{\beta c_{NT,0}^i} = \frac{\frac{\nu^i \left(d_1^{b,i} - a d_1^{c,i} \right)}{\widetilde{c}_{T,1}^i} - (1+\tau_A^i) \frac{a d_1^{c,i}}{\widetilde{c}_{T,1}^i} \tau_0^i + \widetilde{\psi}_0 \frac{d_1^{c,i}}{\beta(1+r_0)\omega_i}}{\frac{\tau_A^i}{1+\tau_A^i} + \frac{d_1^{c,i}}{(1+r_0) c_{T,0}^i} + \frac{d_1^{c,i} + \tau_A^i d_1^{b,i}}{\widetilde{c}_{T,1}^i \left(1 + \tau_A^i \right)}}.$$

Next, I express optimal tax rate by comparing private and regulator optima with respect to investment in durable goods

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i} \left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i + Z_3^i \cdot \left(d_1^{b,i} - a d_1^{c,i} \right) - \frac{a}{1-a} \tau_0^i Z_4^i + \widetilde{\psi}_0 Z_5^i \right] \quad \text{(A.6)}$$

where Z_2^i, Z_3^i, Z_4^i are similar to (A.4) and

$$Z_5^i \equiv \frac{1}{1+\tau_A^i} \cdot \frac{\widetilde{c}_{T,1}^i}{a} \cdot \frac{Z_4^i}{\beta(1+r_0)\omega_i} > 0.$$

If I denote

$$\tau_0^{b,i}(s_0)\bigg|_{local} \equiv \frac{1}{1-\tau_0^i}\left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i Z_2^i + Z_3^i \cdot \left(d_1^{b,i} - a d_1^{c,i}\right) - \frac{a}{1-a}\tau_0^i Z_4^i\right],$$

the optimal prudential tax can be alternatively written as

$$\tau_0^{b,i} = \tau_0^{b,i}\bigg|_{local} + \frac{\widetilde{\psi}_0}{1-\tau_0^i} Z_5^i.$$

Because global regulator internalizes the effects of its choice of prudential policy in country i on the rest of the union, it sets different prudential tax. Depending on the sign of $\widetilde{\psi}_0$, the tax can be higher or lower than that of local regulator. Variable $\widetilde{\psi}_0$ combines the net effect of several forces. First, absent additional instruments, like fiscal transfers, the global regulator understands that changing the local allocation through changing the local tax effectively redistributes tradable goods across countries. Second, even when the marginal values additional consumption of tradable goods are equalized across countries, so that redistribution is no longer a concern, the global regulator internalizes the positive and negative international spillovers of local macroprudential policy. To see this formally, consider the case in which all countries are identical. In this case, equation A.5 becomes

$$\begin{aligned}
\widetilde{\psi}_0 &= -\frac{(1+r_0)\beta\left[\frac{\nu(d_1^b - a d_1^c)}{\widetilde{c}_{T,1}^i} - (1+\tau_A)\frac{a d_1^c}{\widetilde{c}_{T,1}}\tau_0\right]}{d_1^c} \\
&= -\frac{(1+r_0)\beta}{\widetilde{c}_{T,1}^i}\left[\frac{\nu(d_1^b - a d_1^c)}{d_1^c} - (1+\tau_A)a\tau_0\right] \\
&= -\frac{(1+r_0)\beta}{\widetilde{c}_{T,1}^i}\left\{\frac{\nu\left[(1-a)d_1^b - d_1^g\right]}{d_1^c} - (1+\tau_A)a\tau_0\right\}.
\end{aligned}$$

In case when countries are symmetric, the union-wide monetary policy stabilizes all economies, i.e., $\tau_0 = 0$. This further simplifies the expressions for $\widetilde{\psi}_0$ as follows

$$\widetilde{\psi}_0 = -\frac{(1+r_0)\beta}{\widetilde{c}_{T,1}^i} \cdot \frac{\nu\left[(1-a)d_1^b - d_1^g\right]}{d_1^c}.$$

$\widetilde{\psi}_0$ captures the effect of local macroprudential tax on the two pecuniary externalities in the other countries. The intuition behind this formula is summarized in the main text after the statement of Proposition 6. If the supply of government debt d_1^g is small enough, the above formula implies that $\widetilde{\psi}_0 < 0$, and the global planner sets smaller prudential tax. In other words, uncoordinated macroprudential policy over-regulates the financial sector.

A.2.6 Proof of Proposition 7

$$
\begin{aligned}
\max_{\substack{\{k^i_{NT,0}, c^i_{NT,0}, \underline{c}^i_{NT,1}, p^i_1,\\ \tilde{d}^{b,i}_1, d^{c,i}_1, \widehat{Tr}^i_0, \widehat{Tr}^i_1\}\\ P_{T,0}, r_0}} \mathbb{E} \int \omega_i \Bigg\{ & \log c^i_{NT,0} - v\left(\frac{c^i_{NT,0} + k^i_{NT,0}}{A^i_0}\right) + \beta \left\{ \nu^i \log \underline{c}^i_{NT,1} + X^i_1(s_1) g\left[G\left(k^i_{NT,0}\right)\right]\right\} \\
& + (1-a)\log\left(\frac{P^i_{NT,0}}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p^i_1 \Bigg\} di, \\
\text{s.t.: } & \tilde{d}^{b,i}_1 \le \theta^i \frac{g'[G(k^i_{NT,0})]}{a/y^{i,*}_{NT,1}} G(k^i_{NT,0}) p^i_1, \\
& \frac{\underline{c}^i_{NT,1} p^i_1}{a} \le d^{c,i}_1, \\
& c^i_{NT,0} \frac{P^i_{NT,0}}{P_{T,0}(s_0)} \frac{1-a}{a} - e^i_{T,0} + \frac{d^{c,i}_1 - \tilde{d}^{b,i}_1 - d^{g,i}_1}{1+r_0} + \widehat{Tr}^i_0 = 0, \\
& y^{i,*}_{NT,1} p^i_1 \frac{1-a}{a} - e^i_{T,1} + \tilde{d}^{b,i}_1 + d^{g,i}_1 - d^{c,i}_1 + \widehat{Tr}^i_1 = 0, \\
& \int \widehat{Tr}^i_0 di = 0, && \text{(A.7)} \\
& \int \widehat{Tr}^i_1 di = 0, && \text{(A.8)} \\
& \frac{1-a}{a} \int \frac{P^i_{NT,0}}{P_{T,0}} c^i_{NT,0} di = \int e^i_{T,0} di, \\
& \frac{1-a}{a} \int p^i_1 y^{i,*}_{NT,1} di = \int e^i_{T,1} di, \\
& \frac{1}{c^i_{NT,0}} = (1+r_0)\beta \frac{P^i_{NT,0}}{P_{T,0} p^i_1} \left(\frac{1}{y^{i,*}_{NT,1}} + \frac{\nu^i}{\underline{c}^i_{NT,1}} \right).
\end{aligned}
$$

I can use country-wide budget constraints to eliminate transfers. Constraints (A.7),(A.8), after taking into account traded goods market clearing conditions, can be replaced by one safe bonds debt market clearing condition. The problem takes the following form now

$$
\begin{aligned}
\max_{\substack{\{k^i_{NT,0}, c^i_{NT,0}, \underline{c}^i_{NT,1}, p^i_1,\\ \tilde{d}^{b,i}_1, d^{c,i}_1\}, P_{T,0}, r_0}} \mathbb{E} \int \omega_i \Bigg\{ & \log c^i_{NT,0} - v\left(\frac{c^i_{NT,0} + k^i_{NT,0}}{A^i_0}\right) + \beta \left[\nu^i \log \underline{c}^i_{NT,1} + X^i_1(s_1) g\left(G\left(k^i_{NT,0}\right)\right)\right] \\
& + (1-a)\log\left(\frac{P^i_{NT,0}}{P_{T,0}}\right) + \beta(1-a)(1+\nu^i)\log p^i_1 \Bigg\} di, \\
\text{s.t.: } & \tilde{d}^{b,i}_1 \le \theta^i \frac{g'[G(k^i_{NT,0})]}{a/y^{i,*}_{NT,1}} G(k^i_{NT,0}) p^i_1, \qquad \left[\beta\tilde{\zeta}_0\omega^i\right] \\
& \frac{\underline{c}^i_{NT,1} p^i_1}{a} \le d^{c,i}_1, \qquad \left[\beta\tilde{\eta}_0\omega^i\right]
\end{aligned}
$$

$$\int \left(d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}\right) di = 0, \qquad \left[\beta\widetilde{\xi}\right]$$

$$\frac{1-a}{a}\int \frac{P^i_{NT,0}}{P_{T,0}} c^i_{NT,0} di = \int e^i_{T,0} di, \qquad [\widetilde{\psi}_0]$$

$$\frac{1-a}{a}\int p^i_1 y^{i,*}_{NT,1} di = \int e^i_{T,1} di, \qquad [\beta\widetilde{\psi}_1]$$

$$\frac{1}{c^i_{NT,0}} = (1+r_0)\beta \frac{P^i_{NT,0}}{P_{T,0}p^i_1}\left(\frac{1}{y^{i,*}_{NT,1}} + \frac{\nu^i}{\underline{c}^i_{NT,1}}\right). \qquad \left[\widetilde{\phi}^i \omega^i\right]$$

The first order conditions are

$$\partial c^i_{NT,0} : \widetilde{\psi}_0 = \omega^i \frac{1-a}{c^i_{T,0}}\left(1 + \frac{a}{1-a}\tau^i_0 + \frac{\widetilde{\phi}_i}{(1-a)c^i_{NT,0}}\right)$$

$$\partial \underline{c}^i_{NT,1} : \nu^i = \widetilde{\phi}_i(1+r_0)\frac{P^i_{NT,0}}{P_{T,0}p^i_1}\cdot\frac{\nu^i}{\underline{c}^i_{NT,1}} + \widetilde{\eta}^i_0 d_1^{c,i},$$

$$\partial k_{NT,0} : \beta G_0^{i,\prime} g_0^{i,\prime}\left[\mu + (1-\mu)\theta^i + \theta^i\widetilde{\zeta}^i_0 \frac{p^i_1}{a/y^{i,*}_{NT,0}}\left(1-\epsilon^i_\Gamma\right)\right] = \frac{v'(n^i_0)}{A^i_0},$$

$$\partial P_{T,0} : \int \omega_i \tau^i_0 di = 0$$

$$\partial \widetilde{d}_1^{b,i} : \omega^i \widetilde{\zeta}^i_0 = \widetilde{\xi},$$

$$\partial d_1^{c,i} : \omega^i \widetilde{\eta}^i_0 = \widetilde{\xi},$$

$$\partial r_0 : \int \omega_i \frac{\widetilde{\phi}_i}{c^i_{NT,0}} di = 0,$$

$$\partial p^i_1 : \widetilde{\psi}_1 = \omega^i \frac{1-a}{c^i_{T,1}}\left(1 + \nu^i + \widetilde{\zeta}^i_0 \frac{d_1^{b,i} - d_1^{c,i}}{1-a} - \frac{\widetilde{\phi}_i}{\beta(1-a)c^i_{NT,0}}\right)$$

Comparing the optimality condition for investment in durable goods to private optimality, I can express the optimal macroprudential tax as follows

$$\tau_0^{b,i} = \frac{1}{1-\tau^i_0}\left[\frac{\tau^i_A \epsilon^i_\Gamma}{1+\tau^i_A} - \tau^i_0 Z^i_2 + \frac{(1-\epsilon^i_\Gamma)}{1+\tau^i_A}\tau^i_A\left(1 - \frac{\widetilde{\xi}/\omega^i}{\nu^i \underline{U}^i_{T,1}}\right)\right],$$

where $\widetilde{\xi}$ is the planner's marginal value of safe asset, which is common across all of the countries, $\underline{U}^i_{T,1}$ is private marginal value of safe assets in country i, and $Z^i_2 = \left[\mu + (1-\mu)\theta^i + \theta\tau^i_A\right] / \left[\theta^i\left(1+\tau^i_A\right)\right]$. Using the first order condition for $\underline{c}^i_{NT,1}$ one can show that

$$\widetilde{\xi} = \frac{\int \omega^i \frac{\nu^i}{d^{c,i}} d^{c,i} \frac{1+\tau^i_A}{\tau^i_A} di}{\int d^{c,i} \frac{1+\tau^i_A}{\tau^i_A} di}.$$

This expression is a weighted average across countries of marginal utilities from consumption bought with safe assets, with weights being $\Phi^i \equiv d^{c,i}\frac{1+\tau_A^i}{\tau_A^i} / \int d^{c,i}\frac{1+\tau_A^i}{\tau_A^i}di$. The last expression implies

$$\tau_A^i\left(1-\frac{\widetilde{\xi}/\omega^i}{\nu^i \underline{U}_{T,1}^i}\right)=\frac{1}{\omega^i U_{T,1}^i}\left(\omega^i\frac{\nu^i}{d^{c,i}}-\int\omega^i\frac{\nu^i}{d^{c,i}}\Phi^i di\right).$$

This leads to

$$\begin{aligned}\tau_0^{b,i}&=\frac{1}{1-\tau_0^i}\left[\frac{\tau_A^i\epsilon_\Gamma^i}{1+\tau_A^i}-\tau_0^i Z_2^i+\frac{1-\epsilon_\Gamma^i}{1+\tau_A^i}\cdot\frac{1}{\omega^i U_{T,1}^i}\left(\omega^i\frac{\nu^i}{d^{c,i}}-\frac{\int\omega^i\frac{\nu^i}{d^{c,i}}d^{c,i}\frac{1+\tau_A^i}{\tau_A^i}di}{\int d^{c,i}\frac{1+\tau_A^i}{\tau_A^i}di}\right)\right]\\&=\frac{1}{1-\tau_0^i}\left[\frac{\tau_A^i\epsilon_\Gamma^i}{1+\tau_A^i}-\tau_0^i Z_2^i+Z_6^i\right].\end{aligned}$$

A.2.7 Proof of Proposition 8

$$\begin{aligned}\max_{\substack{\{k_{NT,0}^i,c_{NT,0}^i,\underline{c}_{NT,1}^i,\\ p_1^i,\widetilde{d}_1^{b,i},d_1^{c,i}\},P_{T,0}}}\mathbb{E}\int\omega_i\Big\{&\log c_{NT,0}^i-v\left(\frac{c_{NT,0}^i+k_{NT,0}^i}{A_0^i}\right)+\beta\left\{\nu^i\log\underline{c}_{NT,1}^i+X_1^i(s_1)g\left[G\left(k_{NT,0}^i\right)\right]\right\}\\&+(1-a)\log\left(\frac{P_{NT,0}^i}{P_{T,0}}\right)+\beta(1-a)(1+\nu^i)\log p_1^i\Big\}di,\end{aligned}$$

$$\begin{aligned}\text{s.t.: }&\widetilde{d}_1^{b,i}\le\theta^i\frac{g'[G(k_{NT,0}^i)]}{a/y_{NT,1}^{i,*}}G(k_{NT,0}^i)p_1^i, && \left[\beta\widetilde{\zeta}_0\omega^i\right]\\&\frac{\underline{c}_{NT,1}^i p_1^i}{a}\le d_1^{c,i}, && \left[\beta\widetilde{\eta}_0\omega^i\right]\\&\int\left[d_1^{c,i}-d_1^{b,i}-d_1^{g,i}\right]di=0, && \left[\beta\widetilde{\xi}\right]\\&\frac{1-a}{a}\int\frac{P_{NT,0}^i}{P_{T,0}}c_{NT,0}^i di=\int e_{T,0}^i di, && \left[\widetilde{\psi}_0\right]\\&\frac{1-a}{a}\int p_1^i y_{NT,1}^{i,*}di=\int e_{T,1}^i di. && \left[\beta\widetilde{\psi}_1\right]\end{aligned}$$

The first order conditions are

$$\begin{aligned}\partial c_{NT,0}^i:&\widetilde{\psi}_0=\omega^i\frac{1-a}{c_{T,0}^i}\left(1+\frac{a}{1-a}\tau_0^i\right),\\\partial\underline{c}_{NT,1}^i:&\nu^i=\widetilde{\eta}_0^i d_1^{c,i},\\\partial k_{NT,0}:&\beta G_0^{i,\prime}g_0^{i,\prime}\left[\mu+(1-\mu)\theta^i+\theta^i\widetilde{\zeta}_0^i\frac{p_1^i}{a/y_{NT,0}^{i,*}}\left(1-\epsilon_\Gamma^i\right)\right]=\frac{v'(n_0^i)}{A_0^i},\\\partial P_{T,0}:&\int\omega_i\tau_0^i di=0,\\\partial\widetilde{d}_1^{b,i}:&\widetilde{\zeta}_0^i=\widetilde{\xi},\end{aligned}$$

$$\partial d_1^{c,i} : \widetilde{\eta}_0^i = \widetilde{\xi},$$

$$\partial p_1^i : \widetilde{\psi}_1 = \omega^i \frac{1-a}{c_{T,1}^i} \left(1 + \nu^i + \widetilde{\zeta}_0^i \frac{\widetilde{d}_1^{b,i} - d_1^{c,i}}{1-a}\right).$$

Combine FOCs for $\underline{c}_{NT,1}^i$ and p_1^i

$$\widetilde{\psi}_1 = \omega^i \frac{1-a}{c_{T,1}^i} \left(1 + \widetilde{\eta}_0^i d_1^{c,i} + \widetilde{\zeta}_0^i \frac{\widetilde{d}_1^{b,i} - d_1^{c,i}}{1-a}\right) = \omega^i \frac{1-a}{c_{T,1}^i} \left(1 + \widetilde{\xi} \frac{\widetilde{d}_1^{b,i} - a d_1^{c,i}}{1-a}\right).$$

Summing the first order conditions for $c_{NT,0}^i$ and p_1^i across all of the countries, one can find

$$\widetilde{\psi}_0 = \frac{1-a}{\int e_{T,0}^i di},$$

$$\widetilde{\psi}_1 = \frac{1-a}{\int e_{T,1}^i di} \left(1 + \widetilde{\xi} \frac{\int \left(d_1^{b,i} - a d_1^{c,i}\right) \omega^i di}{1-a}\right).$$

This first order condition imply

$$\tau_0^{b,i} = \frac{1}{1-\tau_0^i} \left[\frac{\tau_A^i \epsilon_\Gamma^i}{1+\tau_A^i} - \tau_0^i \frac{\mu + (1-\mu)\theta^i + \theta \tau_A^i}{\theta^i \left(1+\tau_A^i\right)}\right].$$