A Contagious Malady? Open Economy Dimensions of Secular Stagnation*

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Abstract

We propose an open economy model of secular stagnation and show how it can be transmitted from one country to another via current account imbalances. While current account surpluses normally lower interest rates in the recipient country, in a secular stagnation, surpluses transmit recessions due to the zero lower bound on nominal interest rates. In general monetary policies and those directed at competitiveness have negative externalities on trading partners in these circumstances, while fiscal policies and those directed at stimulating domestic demand have positive externalities. This, in a positive sense, explains why the world has relied so much on monetary policies relative to fiscal policies in the wake of the financial crisis and in a normative sense points towards the desirability of fiscal policies. Fiscal policies in response to a secular stagnation are self-financing as in De Long and Summers (2012) in our numerical experiments and a one shot increase in debt will raise demand and is fiscally sustainable. While expansionary monetary policy only provides for a possibility of a better outcome without excluding the possibility of continuing secular stagnation appropriate fiscal policy eliminates secular stagnation by directly raising the natural rate of interest as in Eggertsson and Mehrotra (2014).

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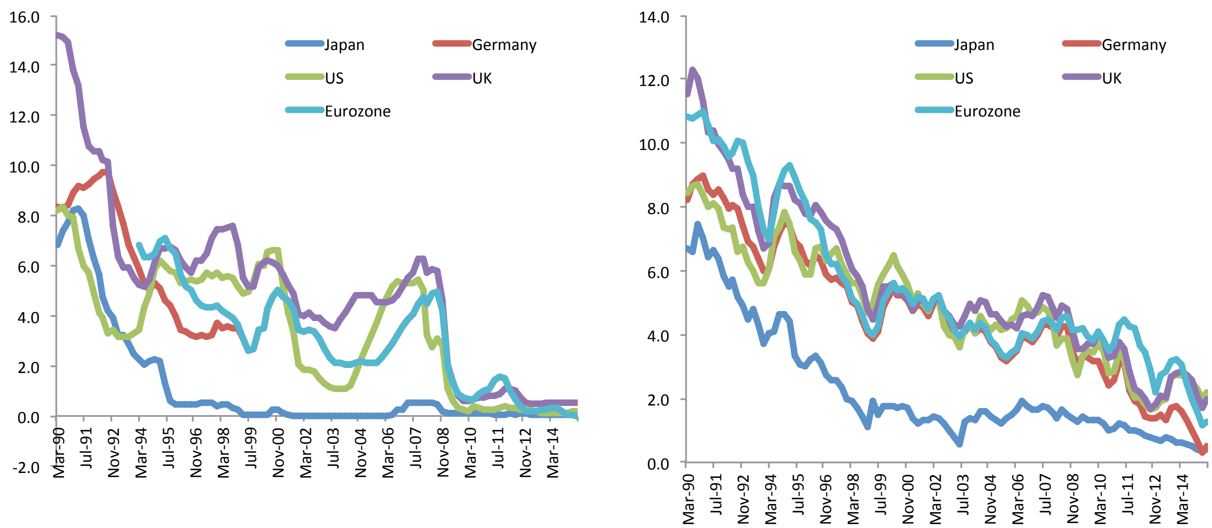
Figure 1: Short and Long-Term Interest Rates



1 Introduction

In this paper we analyze the open economy dimensions of secular stagnation. The concept of secular stagnation, dating back to Hansen (1939), was recently resurrected by Summers (2013), (see also Summers (2014), Summers (2015*a*) and Summers (2015*b*)). The key idea, in Summers formulation, is that the natural rate of interest - the real interest rate the Federal Reserve needs to track to achieve full employment - is permanently negative. This poses a major challenge for policy due to the fact the nominal interest rate cannot be cut below the zero lower bound (ZLB). Eggertsson and Mehrotra (2014) offer the first attempt to formally model secular stagnation using an overlapping generation model (OLG) in the spirit of Samuelson (1958) in a closed economy. However, low interest rates and lackluster growth are a global phenomenon. To understand secular stagnation in a open economy, we here consider a two country open economy OLG framework with varying degrees of financial market imperfection across countries.

Broadly speaking the paper makes three central points. First, secular stagnation – which can be thought of as economies permanently facing the possibility of the ZLB without any natural force to normalcy – may be important in modeling modern economies. It can be an important phenomenon in the global economy either because the world economy as a whole is in secular stagnation or if only part of it is in secular stagnation. In the latter case we show how it can be transmitted from one part of the global economy to another via capital flows and the associated trade dislocations.

Second, in the open economy, policies that are stimulative for the home economy can have very different impacts on other economies and on the choices open to other countries in a secular stagnation. In general monetary policies and those directed at competitiveness have negative externalities and fiscal policies and those directed at stimulating domestic demand have positive externalities. This, in a positive sense, explains why the world has relied so much on monetary policies relative to fiscal policies in the wake of the financial crisis. In a normative sense it points towards the desirability of fiscal policies.

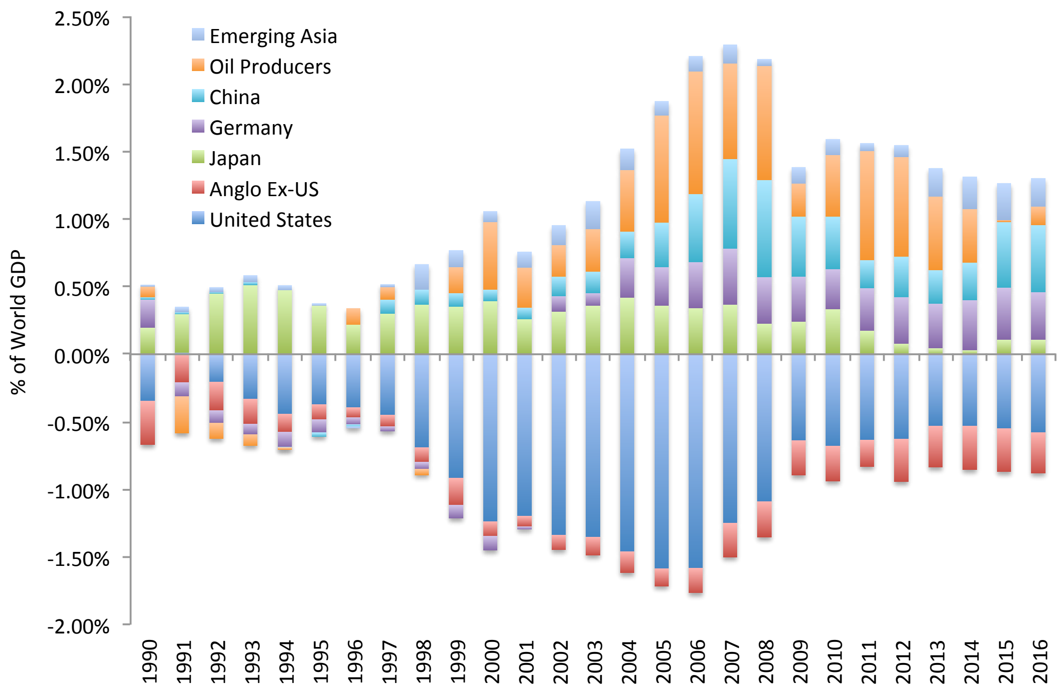
Third, fiscal policies in response to secular stagnation with the associated positive externalities are consistent with the government's long run budget constraint with three consideration being central. First, they may pay for themselves as in De Long and Summers (2012) as we verify in our model in a numerical experiment. Second, balanced budget policies like tax financed spending or the expansion of pay as you go social security have positive fiscal impacts. Third, a one shot increase in debt will raise demand and is clearly sustainable in a secular stagnation.

At the time of writing, the US has just raised the Federal Funds rate for the first time since 2008 based upon the hope that recovery is well underway and inflation will rise back to target. Meanwhile, the rest of the world remains stuck at the ZLB with some central banks seeking further stimulus. Our analysis suggest extent that the rise in US rates increase capital flows to the US, these flows put downward pressure on the natural rate of interest in the US. If these forces are strong enough, the Federal Reserve will be forced to cut rates back to zero. Capital flight from emerging markets, such as China, to the US will have the same effect.

The paper is organized as follows with some key results highlighted. Section 2 proposes a two country overlapping generations endowment economy with imperfect capital integration. Here, we provide a simple framework that rationalizes the secular decline in short and long term interest rates, seen in Figure 1, throughout the world in the past quarter of a century. We incorporate imperfect capital integration across countries to explain persistent differences in real interest rates across countries, thus rationalizing how one part of the world can be in a secular stagnation, while the other is not. Indeed, Japan hit the zero bound in the mid 1990's - well before the Great Recession brought the rest of the developed world to the ZLB. While the closed economy literature on secular stagnation emphasized secular forces like demographic trends, inequality, the fall in relative price of investment, and debt deleveraging, our framework shows how these forces can be transmitted across regions via "global imbalances" - countries with excess savings and low real interest rates will export savings to those countries where returns are higher.

In Section 3, we extend the model to include government debt, taxes, and reserve accumulation. The goal is to show how the global saving glut hypothesis of Bernanke (2005) fits naturally into our framework. According to this hypothesis, current account deficits in the US prior to the Great Recession were a consequence of heightened demand for US debt, including the accumulation of US Treasuries by various foreign government. Figure 2 displays global imbalances leading up to the financial crisis. We show in this section how the global savings glut hypothesis fits naturally with the secular stagnation hypothesis - the forces articulated in each theory can generated

Figure 2: Global Imbalances



a persistent decline in the natural rate of interest in the US.

Section 4 extends the simple endowment economy to include production and price determination and formally defines equilibrium in the full model. Section 5 shows how the model can be collapsed into a simple set of steady state relationships that can by analyzed via basic aggregate demand and supply diagrams. This simplifies our expressions considerably, moreover it corresponds to a limiting case in which case the secular stagnation is everlasting without any pullback to full employment, a natural benchmark. We consider secular stagnation under both imperfect and perfect capital integration. In the former case, one country is in secular stagnation while the other is not; we think of this version of the model as capturing the salient features of the global imbalances period pre 2008 and, particularly, the US and Japan prior to 2008. In the latter case, we think of the model as capturing features of the US and Eurozone interaction from 2008 to today. While in the simple endowment economy in section 2 then current account imbalances transmitted low real rate, in the general model then capital markets may propagate output shortfalls and a binding zero lower bound rather than lower real interest rates. This insight allows us to formalizes the idea of neomercantilism - a policy regime that encourages exports and discourages imports, with the aim of increasing a country's net foreign asset position. If a country targets a positive net foreign asset position vs its trading partner (e.g. by running large trade surpluses), this policy will exert a negative externality on the trading partner. A policy of this type can, in principle, generate a recovery at home, depending on the details of how the increase in net asset holdings is financed.

Neomercantilism is therefore example of a beggar-thy-neighbor policy. Another example we illustrate is what happens if a country increases wage flexibility. While this policy raises output in the country undertaking the reform, it comes at the expense of its trading partner. Moreover, aggregate world output declines as a consequence. This finding is suggestive that "structural reforms" of this form in Southern Europe may not be the magic bullet for restoring growth in Europe.

Section 6 studies monetary and exchange rate policy. We find that exchange rate depreciation, in general, is at the expense of the trading partner if both countries are in secular stagnation. It is thus yet another example of beggar thy neighbor policy. Increasing the inflation target in one country can be effective, but will also have similarly strong negative externalities on the trading partner if the trading partner does not inflate as well. We also identify a key difficulty with monetary policy in a secular stagnation which extends the previous result in Eggertsson and Mehrotra (2014) to an open economy. While a higher inflation target, if credible, allows for a better equilibrium, the secular stagnation equilibrium cannot not be excluded.

Section 7 introduces fiscal policy. In contrast to monetary policy, trade policy, or structural reforms, expansionary fiscal policy generates positive externalities for the trading partners in a secular stagnation as we have already stressed. Moreover, fiscal policy also does not suffer from the multiple equilibria problem that we illustrate for monetary policy. A sufficiently robust fiscal expansion eliminates the secular stagnation equilibrium altogether. Fiscal policy "jump starts" the economy.

One reason fiscal policy is so powerful in our model is that Ricardian equivalence does not hold due to the assumption of finite lifetimes. This means that government debt is far from neutral. Higher levels of public debt raise the natural rate of interest directly and thus eliminate the need for a negative real interest rate, thereby pulling the economy out of secular stagnation. While this policy is quite powerful, we also discuss some possible drawbacks. We also consider the effect of increasing government spending with direct taxation on the working population. This balanced budget policy, as well, directly increases the natural rate of interest and pulls the economy out of secular stagnation. As with debt expansion, this policy has strong positive externalities for the trading partners. Given that fiscal policy carries positive externalities, countries will tend to undersupply fiscal expansion. We show formally how fiscal expansion absent coordination is undersupplied relative to cooperation, and show what factors influence the severity of the coordination problem.

In Section 8, we calibrate our model to quantify the two particular episodes already alluded to: the asymmetric stagnation of Japan and the US pre 2008 and the symmetric stagnation of US and Eurozone from 2008-2015. Our calibration suggests that Japan greatly benefited from capital flows to the US, as this allowed it to export its excess savings during this period. Our calibration also suggest that the US in fact may have benefited from closing capital markets in the latter period, as it would have avoided the zero bound. Finally, we establish that a fiscal expansion in a secular stagnation, at least in our numerical example, is self-financing, much as in De Long as Summers (2012). Interestingly, we obtain this result even without assuming hysteresis. These numerical examples are meant to highlight that the framework presented here can easily be parameterized to draw concrete quantitative conclusions on the sources of secular stagnation and possible policy responses. We expect it can be enhanced considerably to yield more detailed quantitative predictions.

1.1 Related Literature

We have already pointed out that our paper draws heavily on Summers original reformulation of the secular stagnation hypothesis, and the model proposed in Eggertsson and Mehrotra (2014). Our paper also relates to the emerging literature on models of economic stagnation, including including Kocherlakota (2013), Caballero and Farhi (2014), Schmitt-Grohé and Uribe (2013), Benigno and Fornaro (2015), Bianchi and Kung (2014), Guerron-Quintana and Jinnai (2014). With the exception of Kocherlakota (2013) and Caballero and Farhi (2014), these models feature a steady state real interest rate that is always positive and determined solely by the discount factor of the representative household.

Our results are closest in spirit to recent work by Caballero, Farhi and Gourinchas (2015) which was developed independently and concurrently with our work. One important difference is that the recession in their model is driven by shortage of safe assets. In contrast, our framework has a whole category of forces that drive the natural rate of interest, which have been more closely tied to the secular stagnation hypothesis, such as demographics, debt deleveraging, fall in relative price of investment and income inequality, along with global imbalances. While, we do not focus on the shortage of safe asset story, we do not view it as incompatible with our framework. Interestingly, despite a very different theoretical setup, some of our main policy conclusion are at a broad level complementary.

At a conceptual level, one key difference in our model from Caballero, Farhi and Gourinchas (2015) is our focus imperfect financial integration across countries. Our model allows for the possibility that only part of the world is in a secular stagnation while the rest of the world is not. This feature is necessary to capture the long lasting slump in Japan, and the fact that the US is appears to exiting the ZLB while Europe and Japan are not. This difference is due to the fact that we explicitly model incomplete financial integration which allows us to model violations of interest rate parity.

One key advantage of our framework, relative to others is that our model delivers a locally unique equilibria which allows for comparative statics.[1] It is thus easier to quantify and delivers

[1] Schmitt-Grohé and Uribe (2013), Benigno and Fornaro (2015) are examples of models in which the equilibria is locally indeterminate.

sharper implications for policy relative to the literature that focuses on the deflation steady state in a standard representative agent model.

Our model is closest in structure to the model of Coeurdacier, Guibaud and Jin (2015) which examines how financial integration accounts to declining real interest rates and capital flows from emerging markets to advanced economies. Our model also shares features of models that examine the global demand for safe assets and the persistent US current account deficit: e.g. Caballero, Farhi and Gourinchas (2008) and Gourinchas and Jeanne (2013). Interestingly, when the natural rate turns negative, our model can generate a current account deficit for debtor countries in steady state. Finally, our results on the gains from monetary and fiscal coordination build on earlier work by Clarida, Galı and Gertler (2002), Dixit and Lambertini (2003) and Benigno and Benigno (2006).

2 Capital Integration and the Natural Rate of Interest

We start by showing how the real interest rate is determined in an endowment economy, allowing for varying degrees of financial integration. In the more general model we introduce later, the real interest rate we derive here maps directly into the natural rate of interest in each country. To consider intermediate cases in between autarky and full financial integration, we introduce a constraint on international capital flows. Our focus is to show how the domestic real interest rate is affected by the degree of financial market integration.

There are two countries, domestic and foreign. Each country trades a one period risk free bond with returns r_t and r_t^* respectively. Without loss of generality, we focus here on the case in which $r_t \geq r_t^*$, - a situation in which the returns on the asset in the domestic economy dominates that in the foreign country so long as capital markets are imperfectly integrated.

Consider a simple overlapping generation economy. Households live for three periods: they are born in period 1 (young), earn income in period 2 (middle aged), and retire in period 3 (old). We assume there are no aggregate savings, but that the generations can borrow and lend to one another. We assume that only the middle age receive income, Y_t and Y_t^* respectively. This will imply that the middle aged generation in each country lend to the young in order to save for retirement. A key constraint we impose is on the borrowing of the young. The young are constrained by a borrowing limit $(1 + r_t)B_t^y \leq D_t$ and $(1 + r_t^*)B_t^{*y} \leq D_t$ as in Eggertsson and Krugman (2012). Implicitly, we think of this limit as emerging from some type of incentive constraint, however, for our purposes, we take this limit to be exogenous.

If the real interest rate is higher in one country than the other, savings will flow to the country with the highest yield. If there are no constraints on capital flows, then the real interest rate in equilibrium is equalized across the two countries. We impose a simple quantity constraint on international capital flows which we denote K_t. In particular, we assume that the domestic debt held by the foreign middle generation has to be lower than some K_t. Again, implicitly, we are assuming this constraint reflects some sort of incentive constraint, perhaps due to incomplete enforcement of contracts across national borders, home bias for investors, or other limits to arbitrage. For the purpose of our analysis, we will simply treat this constraint as exogenous. One could similarly interpret this as representing some form of capital controls since it places a direct quantity limit on how much capital can move across countries. When the constraint is not binding, then real interest rates must be equalized across the two countries.[2]

Formally, consider the following overlapping generation model. A domestic household born at time t has the following utility function:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\}$$

subject to the following (real) budget constraints:

$$C_t^y = B_t^y \tag{1}$$

$$C_{t+1}^m = Y_{t+1} - (1+r_t)B_t^y - A_{t+1}^D - A_{t+1}^I \tag{2}$$

$$C_{t+2}^o = (1+r_{t+1})A_{t+1}^D + (1+r_{t+1}^*)A_{t+1}^I \tag{3}$$

$$(1+r_t)B_t^i \leq D_t \tag{4}$$

$$0 \leq A_{t+1}^I \leq K_{t+1} \tag{5}$$

Here C_t^i denotes consumption for each generation i, B_t^yborrowing in a one period risk-free bond that carries an interest rate r_t. A_t^D is the asset holding of the middle aged household of the domestic bond that carries interest rate r_t while A_{t+1}^I is the middle generation holdings of the foreign asset. The foreign economy has the same set of preferences and faces the same set of constraints. We assume that there is no short-selling of the foreign asset. While the middle generation can accumulate a positive position in A_{t+1}^I, which earns interest r_t^*, it cannot issue its own debt in the foreign currency at rate r_t^*.

We consider an equilibrium in which the borrowing constraint for the young is binding:

$$C_t^y = B_t^y = \frac{D_t}{1+r_t} \tag{6}$$

In equilibrium, the middle generation lend to the young to save for their retirement. Their savings decision satisfies a consumption Euler equation:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t (1+r_t) \frac{1}{C_{t+1}^o} \tag{7}$$

while the old consume all their income - principal and interest on domestic and foreign savings.

$$C_t^o = (1+r_{t-1})A_{t-1}^D + (1+r_{t-1}^*)A_{t-1}^I \tag{8}$$

[2]We derive similar results when there is a credit spread function that depends on the level of the capital flow between the two countries. We adopt the quantity restriction here given that the resulting equilibrium conditions are a generalization of the closed economy case considered in Eggertsson and Mehrotra (2014).

The residents of the foreign economy satisfy the same conditions where we denote each variable with a star. The model is closed by bond market clearing in each country. For the domestic market it is given by:

$$N_t B_t^y = N_{t-1} A_t^D + N_{t-1}^* A_t^{I*} \tag{9}$$

while the foreign bond market clearing condition is given below:

$$N_t^* B_t^{y*} = N_{t-1}^* A_t^{D*} + N_{t-1} A_t^I \tag{10}$$

which closes the model.[3]

Without loss of generality, we consider the case in which $r_t > r_t^*$. In this case, the international lending constraint is binding (5). Define $1 + g_t = \frac{N_t}{N_{t-1}}$ and $\omega_t = \frac{N_t}{N_t + N_t^*}$. Then we can express the domestic asset market clearing constraint as:

$$(1 + g_t) B_t^y = A_t^D + \frac{1 - \omega_{t-1}}{\omega_{t-1}} K_t^*$$

The left-hand side is the domestic demand for loans, L_t^d, and the right-hand side the supply of loans, L_t^s from domestic and foreign sources. The domestic demand for loans can be expressed in terms of the collateral constraint (6) so that:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t$$

Assuming perfect foresight, we obtain the domestic supply of loans by substituting the budget constraint of the old (8) to solve for the consumption of the middle aged using the Euler equation, (7). We then substitute the resulting expression for C_t^m into the middle aged budget constraint (2), use (6) and solve for A_t^D to obtain:

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t^m - D_{t-1}) + \frac{1 - \omega_{t-1}}{\omega_{t-1}} K_t^*$$

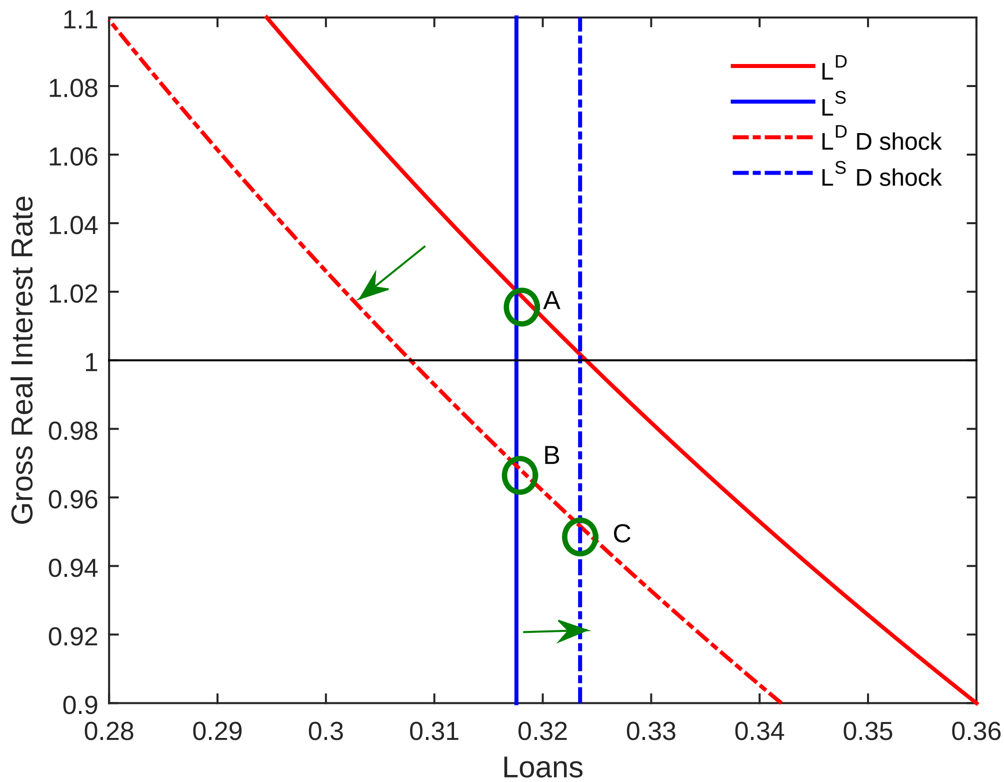
Figure 3 depicts the demand and supply for loans in the domestic economy. The demand for loans increases as the real interest rate fall. A lower interest rate increases the borrowing capacity of the young, allowing them to take on more debt. As emphasized by Eggertsson and Mehrotra (2014), both the debt deleveraging shock D_t as well as slowdown in population growth can reduce the real interest rate. Either mechanism will shift down the demand for loans, as shown at point B in Figure 3, leading to a drop in the real interest rate.

By contrast, the supply for loans remains unchanged with deleveraging and population growth shocks.[4] This assumption implies that the middle aged are simply saving a fixed fraction of their

[3]For a given set of exogenous processes $\{D_t, N_t, Y_t\}$ and $\{D_t^*, N_t^*, Y_t^*\}$, an equilibrium in the global economy is now characterized by a collection of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, B_t^y, A_t^I\}$ and $\{C_t^{y*}, C_t^{o*}, C_t^{m*}, r_t^*, B_t^{y*}, A_t^{I*}\}$ that solve (1), (2), (5), (6), (7), and (8) for the domestic and the foreign households respectively along with asset market clearing conditions (9) and (10).

[4]This is not a general feature of the model, but is due to the assumption of log preferences and the fact that all income is accrued in middle aged. Eggertsson and Mehrotra (2014) treat the more general cases, that we omit here for simplicity.

Figure 3: Equilibrium in the asset market



disposable income. As a result, the supply of savings is a vertical line in Figure 3. One interesting mechanism that shifts the supply for loans is a permanent debt deleveraging shock. This shock leads to a reduction in D_{t-1} triggering a further reduction in the real interest rate by shifting out the supply for loans in the next period as shown by point C in Figure 3 in line with Eggertsson and Mehrotra (2014) but in contrast to the earlier literature on deleveraging such as Eggertsson and Krugman (2012). The fact that the young can take on less debt after a persistent decrease in D_t means that these households have greater disposable income in middle age and thus a higher supply of savings. Thus a permanent tightening of the collateral constraint leads to a permanent reduction in the real interest rate.

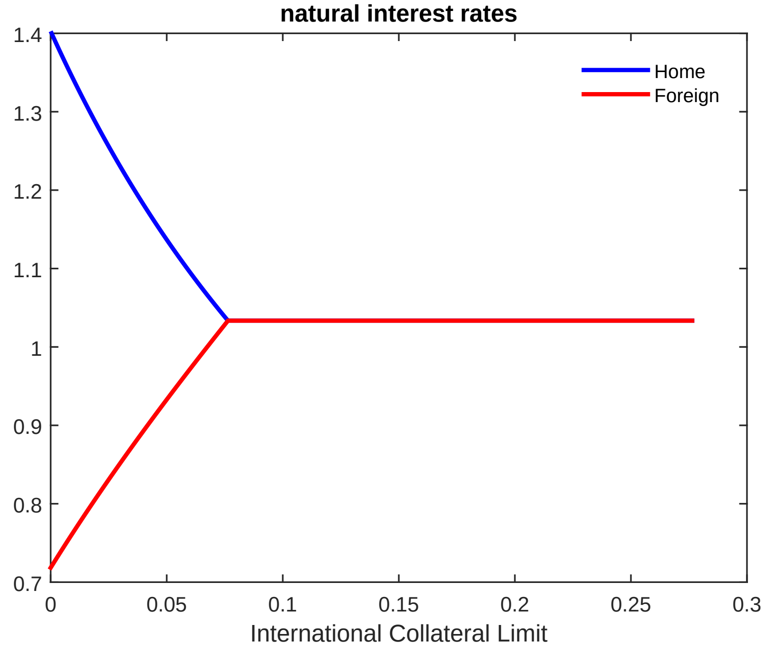
Relative to Eggertsson and Mehrotra (2014),the new element in this model that impacts the interest rate determination is the presence of international lending K_t^* in the supply of loans. The inflow of foreign capital will directly shift out the supply of loans, thereby reducing the real interest rate. This provides for an additional force that can lead to secular stagnation.

Equating loan supply and loan demand, we obtain an expression for the domestic interest rate:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t - D_{t-1} + \frac{1-\omega_{t-1}}{\omega_{t-1}} \frac{1+\beta}{\beta} K_t^*} \tag{11}$$

The most important implication of our supply and demand framework for loanable funds is that

Figure 4: Effect of an increase in international lending on natural rate



there is no inherent reason to expect the equilibrium real interest rate to be either positive. Whether rates are positive or negative depends on the relative size of demand and supply for loanable funds. While we show above how population dynamics and debt deleveraging may affect this relationship, the earlier literature has also emphasized other forces which could easily be incorporated such as an increase in income inequality (which increases the supply of savings), a fall in the relative price of investment, or an increase in uncertainty. Importantly, the liberalization of capital markets - to the extent it leads to a capital inflow - also puts downward pressure on the domestic interest rates via increases in K_t^*.

Analogously, using the foreign asset market clearing condition (10) and combining foreign budget constraints, we can obtain an expression for the foreign real interest rate:

$$1 + r_t^* = \frac{1+\beta}{\beta} \frac{(1+g_t^*)D_t^* + \frac{1+r_t}{1+\beta}K_t^*}{Y_t^* - D_{t-1}^* - K_t^*} \tag{12}$$

where the foreign real interest rate will be influenced by the domestic interest rate, since the foreign old collect higher interest income from the domestic borrowers. Figure 4 shows the domestic and foreign interest rate determination graphically. The x-axis reflects ranges of K_t^*, and the y-axis shows the gross real interest rate. In autarky then $K^* = 0$ so the two interest rates are determined independently of each other. In the current example the foreign interest rate is negative while the domestic rate is positive. We see that as K_t^* increases, the two interest rates converge.

Finally when K_t^* is high enough, the international lending constraint (5) is no longer binding and interest rates are equalized. The point of convergence may happen at either positive or negative world interest rates, depending on parameter values. Beyond this point, there is a single world interest rate given by:

$$1+r_t^W = \frac{1+\beta}{\beta}\frac{\omega_{t-1}\left(1+g_t\right)D_t + \left(1-\omega_{t-1}\right)\left(1+g_t^*\right)D_t^*}{\omega_{t-1}\left(Y_t - D_{t-1}\right) + \left(1-\omega_{t-1}\right)\left(Y_t^* - D_{t-1}^*\right)} \tag{13}$$

Under full integration, the world interest rate will lie in between the two autarky rates.

Proposition 1. *If* $r_t^{aut} > r_t^{aut*}$*, then* $r_t^{aut} > r_t^W > r_t^{aut*}$*.*

Proof. Follows directly from the expression for the world interest rate under integration and the domestic/foreign interest rates under autarky. □

Observe that in this equilibrium, as long as $r_t > r_t^*$ in autarky then capital will flow into the domestic economy. The domestic economy's net foreign asset position under full integration is given by:

$$NFA_t^{full} = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1}\right) - \left(1+g_t\right)\frac{D_t}{1+r_t} < 0$$

The trade balance is simply the change in the net foreign asset position adjusted for interest payments and population growth. In the case of the domestic economy, the trade balance is given by the following expression:

$$TB_t = NFA_t - \frac{1+r_{t-1}}{1+g_{t-1}}NFA_{t-1} \tag{14}$$

In steady state, if the real interest rate exceeds the growth rate of population, the trade balance takes the opposite sign of the net foreign asset position. Debtor countries with negative NFA positions must run a trade surplus in steady state. However, if $r < g$, the trade balance and NFA share the same sign. Debtor countries can run perpetual trade deficits.

As we show in coming sections, the assumption of incomplete integration will be helpful to make sense of the fact that Japan has been experiencing conditions consistent with a secular stagnation for a far longer period than the rest of the world. Incomplete integration will also help us analyze the spillover from reserve accumulation and rising global imbalances in the pre 2008 period.

3 Government Debt and the Global Savings Glut

The global saving glut hypothesis argues that the reduction in real interest rates in the US and developed countries in recent years has been triggered by reserve accumulation by East Asian and oil producing countries. We have shown how these forces could work via private capital flows, where interest rates fall in higher interest rate countries as the lending constraint slackens. The emphasis in the global savings glut literature, however, has usually been on government policies that put downward pressures on US interest rates via purchases of US Treasuries. We now extend our model to focus more squarely on reserve accumulation and fiscal policies, which will, in general, impact the determination of interest rates in an OLG economy. One interesting feature of our environment is that the effects of reserve accumulation depend both on how reserve accumulation is financed and the extent of capital market integration.

We denote a the lump sum tax levied on each generation by T_t^y, T_t^m, T_t^o. The domestic government issues public debt and levies taxes on each generation to make interest payments on past government debt and finances some level of government expenditure G_t. The government's budget constraint is given as:

$$B_t^g + \frac{1}{1+g_{t-1}} T_t^o + T_t^m + (1+g_t) T_t^y = G_t + \frac{1+r_{t-1}}{1+g_{t-1}} B_{t-1}^g \tag{15}$$

Our aim here will not be to analyze fiscal policy in general (we defer that discussion until we have incorporated endogenous production), but instead clarify how foreign reserve accumulation can lead to a drop in the natural rate of interest. For now, assume that $T_t^y = 0$ and $G_t = 0$. Also, as in Eggertsson and Mehrotra (2014), we assume that both governments adopt a particular fiscal rule that eliminates any loan supply effects of taxation:

$$T_{t+1}^o = \beta \left(1+r_t\right) T_t^m \tag{16}$$

The overall level of taxes will adjust to ensure the government budget constraint is satisfied. The foreign government also issues public debt and levies taxes on each generation to make interest payments on past government debt. However, the foreign government also chooses to accumulate some of the debt issued by the other country, IR_t:

$$B_t^{g*} + \frac{1}{1+g_{t-1}^*} (T_t^{o*} + (1+r_t)\, IR_{t-1}) + T_t^{m*} + (1+g_t^*) T_t^{y*} = G_t^* + \left(1+r_{t-1}^*\right) B_{t-1}^{g*} + IR_t \tag{17}$$

Here the left-hand side of the equation tallies the revenues of the government while the right-hand side gives government expenditures. We express the variables in term in terms of spending/reserves per middle age person. In particular, a positive level of IR_t denotes foreign reserve assets accumulated by the foreign government which are in the form of the bond issued by the domestic government. Observe that we assume that the government is not constrained by K_t which only applies to private capital flows.[5]

Fiscal policy impacts interest rates through its effects on the asset market clearing conditions:

$$N_t B_t^y + N_{t-1} B_t^g - N_{t-1}^* IR_t = N_{t-1} A_t^D + N_{t-1}^* K_t \tag{18}$$

$$N_t^* B_t^{y*} + N_{t-1}^* B_t^g = N_{t-1}^* A_t^{D*} \tag{19}$$

[5]The rationale for assuming that the reserve accumulation decision is not subject to the international lending constraint is that emerging market economies are, typically, quite closed to private portfolio flows despite considerable official capital flows. Further, some emerging market economies accumulate low interest US Treasuries for non-pecuniary reasons (i.e. insurance against sudden stops, exchange rate manipulation to favor traded sector, etc.).

To avoid unnecessary notation, we assume symmetric country size for now so that $\omega_t = 1/2$ and no population growth, $g_t = g_t^* = 0$ If the capital constraint is binding, then the equilibrium real interest rate in debtor and creditor countries is given by:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{D_t}{(Y_t - D_{t-1}) + \frac{1+\beta}{\beta}(K_t^* - B_t^g + IR_t)} \tag{20}$$

$$1 + r_t^* = \frac{1+\beta}{\beta} \frac{D_t^* + \frac{1+r_t}{1+\beta} K_t^*}{Y_t^* - D_{t-1}^* - K_t^* - \frac{1+\beta}{\beta} B_t^{g*}} \tag{21}$$

while if capital markets are perfectly integrated, the single world interest rate given by:

$$1 + r_t^W = \frac{1+\beta}{\beta} \frac{D_t + D_t^*}{Y_t + Y_t^* - D_{t-1} - D_{t-1}^* - B_t^g - B_t^{g*} - IR_t} \tag{22}$$

The equations above offer several insights. First of all, notice that an increase in government debt will always raise the real interest rate in that country. However, fully integrated capital markets are necessary for a rise in the foreign country debt to have an effect on the interest rate in the domestic economy. Therefore, the manner in which foreign reserve accumulation is financed has different effects under complete versus incomplete integration.

Consider first incomplete integration. We see in equation (20) that an increase in reservers will directly reduce the real interest rate in the domestic economy. However, the foreign economy only has an influence on the domestic real interest rate through IR_t and K_t^*. Hence from the perspective of the domestic economy, it does not matter whether the increase in reserves by are financed by debt or taxes (for example, some commentators have suggested that the accumulation of foreign reserves in commodity rich countries was mostly driven by increased revenues from high commodity prices, while in cases such as China, it was driven by a proportional increase in Chinese debt).This is likely the empirically relevant case given that the reserve accumulation is concentrated in countries with low degrees of integration with world financial markets.

However, under perfectly integrated financial markets, however, we see that the financing of foreign reserves, IR_t matters a great deal. In particular imagine that the increase in IR_t is met by a proportional increase in the debt of the foreign country B_t^{g*}. In that case, foreign reserve accumulation has no effect on the world real interest rate as the increased supply of bonds offset the decline in debt held by the public.

The effect of reserve accumulation on global rates we have just outlined is fully consistent with the argument advanced in Bernanke (2015). Hence, we have seen that the global saving glut is a natural complement to other forces that may trigger secular stagnation, like a fall in population growth or deleveraging shocks. A final point to emphasize is that IR_t reflects the a policy choice of the government. While we would not expect private capital to flow from one country in our model to another unless there is a positive interest rate differential, no such interest rate differential is needed for reserve accumulation. This matters, since a large driver of current account deficits we documented in the introduction stems from countries such as China or oil producing countries. It is not obvious that rates of returns in the US dominate the returns in these countries. The fact that those countries still choose to invest in US Treasuries still acts as a negative force on the US natural rate of interest, which (as we will show) can have negative consequences when we take nominal frictions and the zero lower bound into account. Foreign reserve accumulation, in this way, exerts a negative externality on the US.

4 Prices, Production and Exchange Rates

That the natural rate of interest is negative need not to be problem in itself. It only becomes a problem once we incorporate the zero lower bound and nominal frictions. We now introduce nominal price determination, the zero lower bound, production and nominal frictions. Critically, we will be assuming that each country may run its own monetary policy. Accordingly, each country has a currency which determines the price level in terms of that nominal unit. On the production side, will assume frictions in the adjustment of nominal wages defined in the price level of each country.

4.1 Prices

We follow the literature by introducing nominal price determination via the Woodford "cashless" economy. Each country now trades, in addition to the real bond, a nominal bond denominated in each country's price level. We assume that households in either country can hold these nominal bonds implying arbitrage equations between the real and the nominal bonds within a country, but also arbitrage equations across nominal assets denominated in different currencies.[6] Let us denote the domestic price level by P_t and the foreign price level with P_t^*. The nominal exchange rate is $S_t = \frac{P_t}{P_t^*}$

The presence of the two nominal bonds implies two new Euler equations for the middle generation in each country:

$$\frac{1}{C_t^m} = (1 + i_t)\beta E_t \frac{1}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \tag{23}$$

and an equivalent Euler equation for the foreign middle generation. Each middle generation household also must be indifferent between real and nominal debt implying the Fisher relation:

$$(1 + r_t)E_t \frac{1}{C_{t+1}^o} = (1 + i_t)E_t \frac{1}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \tag{24}$$

[6] In equilibrium, we assume that the nominal bonds may be in zero net supply. Hence these equations are only important for pricing, i.e. the resulting pricing equations for these nominal bonds is what pins down the nominal price level in each country - see equations (23)- (24). This is convenient because it implies that, in equilibrium, the budget constraint will be identical to in the endowment economy so that the previous derivations continue to hold.

4.2 Monetary Policy

We assume that each country follows a strict inflation targeting regime, so that:

$$\Pi_t = \bar{\Pi} \text{ if } i_t \geq 0 \text{ otherwise} i_t = 0 \text{ and } \Pi_t < 1 \tag{25}$$

$$\Pi_t^* = \bar{\Pi}^* \text{ if } i_t^* \geq 0 \text{ otherwise} i_t^* = 0 \text{ and } \Pi_t^* < 1 \tag{26}$$

Each country will set its nominal interest rate so as to achieve its inflation target. If the inflation target cannot be achieved, then the central bank sets its nominal interest rate equal to zero. The zero interest rate then closes the model instead of the inflation target.[7] This assumption conveniently abstracts altogether from a particular feedback rule while focusing on the possible problems a country may face if it cannot achieve its inflation target due to the zero bound. Fiscal policy follows the same fiscal rule as outlined in Section 3.

4.3 Production

We assume that firms are price takers on product and labor markets. However, we assume that wages are downwardly rigid. This assumption is sufficient to generate a long-run trade-off between inflation and output, which is what is needed to generate a secular stagnation.[8]

Households supply labor inelastically at $\bar{L}$. We assume that only the middle aged supply labor. There is one firm per middle aged household. Firms hire labor to produce output using a decreasing returns to scale technology. Firms maximize profits, taking wages and prices as given:

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t \tag{27}$$

$$\text{s.t. } Y_t = L_t^\alpha \tag{28}$$

The optimality condition for firm labor demand is standard:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \tag{29}$$

If prices and wages are flexible, the model is closed by setting aggregate labor supply equal to labor demand:

$$L_t = \bar{L} \tag{30}$$

Under this assumption, the economy is identical to the endowment economy we have already studied, except for the determination of nominal prices and exchange rates.

[7]The monetary policy here is a simple Taylor rule as the response coefficient is taken to infinity.

[8]In Eggertsson and Mehrotra (2014), we examine alternative nominal frictions that incorporate forward looking behavior, like Calvo pricing, but find that it added much complexity with little additional insight. In that environment, the long run trade-off between inflation and output stems from inefficient price dispersion and misallocation across identical producers.

What will separate our model from the endowment economy is that we replace the market clearing relationship (30) with the assumption that wages do not fully adjust. In particular, we assume that workers will never be willing to supply labor to firms if the firm offers a wage that falls below some wage norm $\tilde{W}_t$ (the classic example of this is the Keynesian idea that workers will never accept wages lower than last years nominal wages). This constraint is asymmetric, that is, workers would happily accept higher nominal wages. Accordingly, if the wage rate implied by competitive markets is above $\tilde{W}_t$, then wages get bid up and the market clears. What this assumption implies is that if the wage norm is binding, then real wages can be higher than they would need to be for the market to clear. In this case, we assume that employment is rationed.

To be more specific, we assume that wages are downwardly rigid and given by:

$$W_t = \max\{\tilde{W}_t, W_t^{flex}\}$$

where $\tilde{W}_t$ is a wage norm determined by:

$$\tilde{W}_t = \gamma W_{t-1}\bar{\Pi} + (1-\gamma)P_t\alpha\bar{L}^{\alpha-1}$$

When $\gamma = 1$ and $\bar{\Pi} = 1$ wages are perfectly downwardly rigid and when $\gamma = 0$, wages are flexible and real wages always attain their market clearing level. We allow for possibility that the wage norm is rising at the inflation target of the central bank, $\bar{\Pi}$ which implies costs of inflation below the central bank's target. With a positive inflation target, outright deflation is not needed to generate a secular stagnation. When inflation is less than target, $W_t > W_t^{flex}$, and, therefore, $L_t < \bar{L}$ because firms' labor demand does not exhaust the labor endowment and employment is rationed. Let us denote output when labor is fully employed as $Y_f \equiv \bar{L}^{\alpha}$.

Combining labor demand, the production function, and the wage norm, we can obtain an aggregate supply curve of the form:

$$Y_t = \begin{cases} Y^f & \text{if } \Pi_t \geq (\frac{Y^f}{Y_{t-1}})^{\frac{1-\alpha}{\alpha}} \\ \left[\gamma \frac{Y_{t-1}^{\frac{\alpha-1}{\alpha}}}{\Pi_t} + (1-\gamma) Y_f^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} & \text{otherwise} \end{cases} \tag{31}$$

Analogously, for the foreign economy, we have:

$$Y_t^* = \begin{cases} Y^{*f} & \text{if } \Pi_t^* \geq (\frac{Y^{*f}}{Y_{t-1}^*})^{\frac{1-\alpha}{\alpha}} \\ \left[\gamma^* \frac{{Y_{t-1}^*}^{\frac{\alpha-1}{\alpha}}}{\Pi_t^*} + (1-\gamma^*) {Y_f^*}^{\frac{\alpha-1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}} & \text{otherwise} \end{cases} \tag{32}$$

With production, we now adjust the middle generation household budget constraint to take account of labor income and profits, replacing Y_t in (2) with $\frac{W_t}{P_t}L_t + \frac{Z_t}{P_t}$. Noting that $Y_t = \frac{W_t}{P_t}L_t + \frac{Z_t}{P_t}$, the budget constraints takes on exactly the same form as before, and hence the first order conditions for the each generation maximization problem we derived in the endowment economy still apply. Hence, following the same steps as before, we can express the interest rate in each country as in equations (11) and (12) while under full integration, we use (22).

We now have all the pieces together to explicitly define the equilibrium in the model and, without loss of generality, we restrict our attention to the case in which $r_t \geq r_t^*$.

Definition 1. *An equilibrium under incomplete capital integration is a set of quantities* $\{Y_t, C_t^y, C_t^m, C_t^o, B_t^y, A_t^m, T_t^m, T_t^o\}$ *and a set or prices* $\{r_t, i_t, \Pi_t\}$ *for the domestic economy, an analogous set of quantities and prices for the foreign economy, a set of exogenous processes for* $\{D_t, D_t^*, N_t, N_t^*, G_t, G_t^*, B_t^g, B_t^{g*}, IR_t\}$ *that satisfies (1), (2), (3), (4), (7), (16), (23), (25), (31), for the domestic and foreign economies, along with government budget constraints (15), (17) and and asset market clearing conditions (18), (19) with (5) binding. If* $r_t = r_t^*$*, then (22) replaces the domestic and foreign asset market clearing conditions and (5) not binding.*

5 Open Economy Secular Stagnation

While the equilibrium defined above may appear somewhat unwieldy, the model can be reduced to only a few equations. By focusing on steady states, we can collapse the model to two equations relating output and inflation in steady state that can be graphically illustrated. Our focus on steady states follows naturally from our interest in analyzing the protracted slumps across developed countries. For simplicity, we assume below that the countries are of the same size, there is no population growth, and $r \geq r^*$.[9]

Monetary policy in (25) and (26) is useful to organize our thinking about global secular stagnation. It helps us limiting the equilibrium conditions to the essentials. In particular, we consider the four possible scenarios that represent possible combinations of monetary policy.

Definition 2. *An inflation target equilibrium represents 4 scenarios at time t:*

1. ***Scenario 1: Full-Employment:*** *Both countries set* $\Pi_t = \bar{\Pi}$ *and* $\Pi_t^* = \bar{\Pi}^*$ *while* $i_t \geq 0$ *and* $i_t^* \geq 0$.

2. ***Scenario 2: Global Secular Stagnation:*** *Both countries miss their inflation targets* $\Pi_t < \bar{\Pi}$ *and* $\Pi_t^* < \bar{\Pi}^*$ *and set* $i_t = i_t^* = 0$.

3. ***Scenario 3: Foreign Secular Stagnation:*** *Home sets* $\Pi_t = \bar{\Pi}$ *while* $i_t \geq 0$*. Foreign misses its inflation target* $\Pi_t^* < \bar{\Pi}^*$ *and sets* $i_t^* = 0$.

4. ***Scenario 4: Domestic Secular Stagnation:*** *Home misses its inflation target* $\Pi_t < \bar{\Pi}$ *and sets* $i_t = 0$*. Foreign country sets* $\Pi_t^* = \bar{\Pi}^*$ *while* $i_t^* \geq 0$

[9] None of these assumptions are critical and are in fact relaxed in our numerical examples.

Notable in our definition of the inflation target equilibrium is what it excludes. We do not consider the possibility that inflation is above $\bar{\Pi}$ in each country. The idea is that the central bank could always eliminate this equilibrium by raising interest rates. In other words, the only reason inflation is not on target according to this definition is because of the zero bound.[10] We will explore later the effect of the central bank deliberately increasing its inflation target.

The definition below establishes the equilibrium conditions satisfied by a steady state in our model:

Definition 3. *The inflation targeting steady state of the model steady state consists of a vector* $(Y, Y^*, \Pi, \Pi^*, i, i^*, r, r^*)$ *that satisfies the following eight conditions: if* r¿$r^* Y = \left(1 + \frac{1+\beta}{\beta}\frac{1+g}{1+r}\right) D - \frac{1+\beta}{\beta}\left(\frac{1-\omega}{\omega}K^* - B^g + IR\right)$ $Y^* = \left(1 + \frac{1+\beta}{\beta}\frac{1+g}{1+r^*}\right) D^* + \left(1 + \frac{1}{\beta}\frac{1+r}{1+r^*}\right) K^* + \frac{1+\beta}{\beta} B^{g*}$

$$\textit{if } r = r^* = r^w \begin{cases} \omega Y + (1-\omega) Y^* = & \left(\frac{1+\beta}{\beta}\frac{1+g}{1+r^w} + 1\right)(\omega D + (1-\omega) D^*) \\ & + (\omega (B^g - IR) + (1-\omega) B^{g*}) \end{cases} \tag{33}$$

$$Y = \begin{cases} Y_f & \textit{if } \Pi \geq 1 \\ Y_f \left(\frac{1 - \frac{\gamma \bar{\Pi}}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} & \textit{otherwise} \end{cases} \tag{34}$$

$$Y^* = \begin{cases} Y_f^* & \textit{if } \Pi^* \geq 1 \\ Y_f^* \left(\frac{1 - \frac{\gamma^* \bar{\Pi}^*}{\Pi^*}}{1-\gamma^*}\right)^{\frac{\alpha}{1-\alpha}} & \textit{otherwise} \end{cases} \tag{35}$$

$$\Pi = \bar{\Pi} \textit{ or } i = 0 \tag{36}$$

$$\Pi^* = \bar{\Pi}^* \textit{ or } i^* = 0 \tag{37}$$

$$1 + r = \frac{1+i}{\Pi} \tag{38}$$

$$1 + r^* = \frac{1+i^*}{\Pi^*} \tag{39}$$

The first two equations apply under incomplete capital market integration. They are equivalent to a basic IS equations in most macroeconomic models. The lower the real interest rate, output demanded rises. If the value of K^* is high enough, then interest rates are equated across the two countries, the third equation (35) is operative. World demand depends on a world real interest rate, r^w. Equations (36) and (37) describe aggregate supply under both imperfect and perfect integration. Under this specification, if inflation is above target, output is at its full-employment level and wages are equal to their market clearing wage. If inflation falls below the inflation target then real wages rise above their market clearing level (due to the binding wage norm) so labor demand

[10] Observe that, in general, there may be equilibria consistent with inflation above target at zero interest rates.

falls below the labor endowment. Equations (38) and (39) describe the policy rules, while the last two equations are Fisher relation.

For future reference it will be useful to define the natural rate of interest. It is the real interest rate and output that emerges if the central bank hits its inflation target that corresponds to the interest rate we derived in the endowment economy. It is easy to confirm that in our general model the natural rate of interest is given by equations (20)-(22) where output in each equation is replaced by full employment output $Y_f \equiv \bar{L}^{\alpha}$.

Definition 4. *The natural rate of interest r_t^n, r_t^{n*} is the real interest rate in (33) and (34) with output at Y_f and Y_f^* respectively.*

5.1 Stagnation under Imperfect Financial Integration

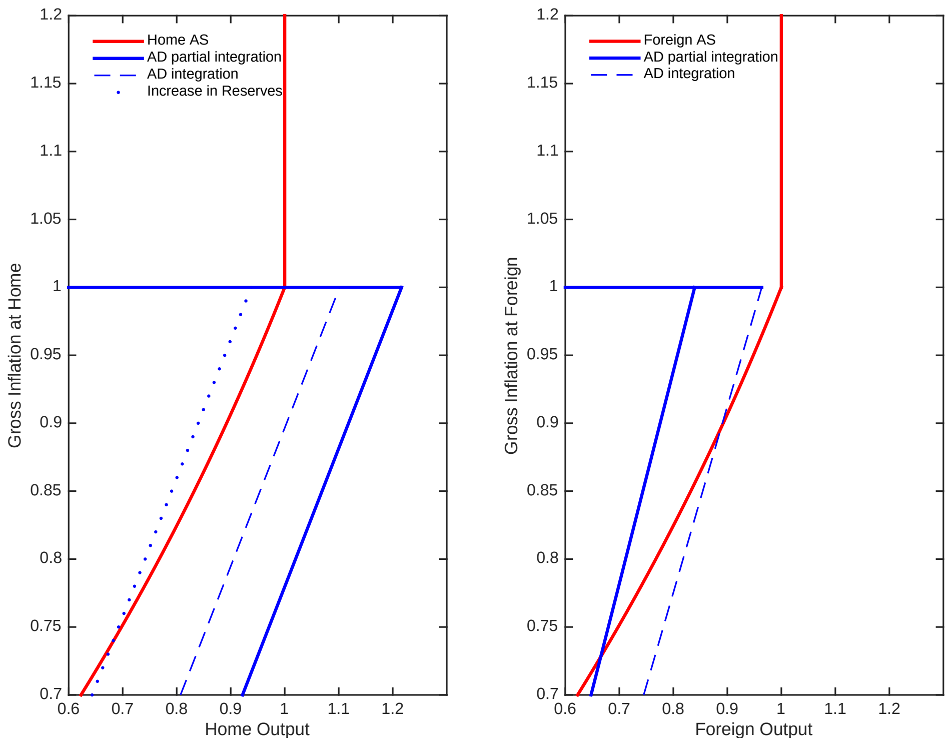
We start by considering the case when one country is in a secular stagnation while the other is not. This case shows how secular stagnation can be transmitted through greater capital market integration. In particular, this case can show how current account surpluses in Japan during the late 1990's and early 2000 reduced interest rates in US while easing the effects of stagnation in Japan. In Section 8, we analyze quantitatively the spillovers from Japan to the US in the pre 2008 global imbalances period. This section also answers a broader question: how can we have a world capital flows in which one country suffers from secular stagnation while the other does not? Imperfect arbitrage on capital flows allows from this outcome. The prospect of asymmetric stagnation once again becomes relevant as the US seeks to normalize interest rates in 2016 while other developed economies remain stuck at the zero lower bound.

We can plot graphically the equilibria in Definition 3 via simple diagrams. The panels of Figure 5 plots steady state output and inflation for the home and foreign country. Aggregate demand is determined by combining the IS equation (33) with the monetary policy rule (38) and the Fisher equation (40). The demand curve is horizontal at the inflation target of the domestic economy, which, for simplicity, is set at $\Pi = 1$. The central bank will set interest rates at whatever is needed to achieve this target. We can then back the required nominal interest rate to achieve the inflation target out of the IS equation (33). However, at some point, the inflation target may require a negative nominal interest rate. In that case there is a kink in the aggregate demand curve as shown in the figure. Once interest rates hit zero, the aggregate demand curve is increasing in inflation and, since inflation reduces real interest rates, the AD curve slopes upward. Below the kink, the AD curve (home country) is given by combining the IS equation (33) with (40) and imposing the zero bound:

$$Y = \left(1 + \frac{1+\beta}{\beta}\Pi\left(1+g\right)\right) D - \frac{1+\beta}{\beta}\left(\frac{1-\omega}{\omega}K^* - B^g + IR\right)$$

The aggregate supply given by equation (36) is also plotted up in Figure 5. At positive inflation this relationship is vertical as the wage rate is equal to its flex price level and all the labor

Figure 5: Effect of an increase in international lending



endowment is employed. If inflation is below zero (or more generally below the inflation target of the central bank), then the wage norm becomes binding in equation (36). Accordingly, the AS curve is upwards sloping in inflation and output.

The left panel in Figure 5 depicts the equilibrium in the domestic economy when the natural rate of interest is positive at home. The aggregate demand and aggregate supply curve intersect at full employment. The right panel in Figure 5 shows the equilibrium in the foreign economy under the assumption that the foreign natural rate of interest is negative. The solid line shows the case when $K^* = 0$ and the dashed line the case in which $K^* > 0$ and capital moves from the foreign country to the domestic. Indeed, we can prove under general conditions that a unique asymmetric stagnation equilibrium exists. In this equilibrium, the foreign country in stagnation accumulates assets in the domestic economy.

Proposition 2. *If the international lending constraint, K^*, is binding so that $r > r^*$ and $r^n > 0, r^{n*} < 0$, $\bar{\Pi} = \bar{\Pi}^* = 1$ and $\gamma^* > 0$, there exists a unique, locally determinate secular stagnation equilibrium in the creditor country with $i^* = 0, \Pi^* < 1$, and $Y^* < Y^*_f$.*

Proof. See Appendix A. □

As shown in Figure 5, an increase in international lending leads to capital flows from foreign country to the domestic economy. This has no effect on output in the domestic economy, but reduces the domestic real interest rate. For the foreign economy, greater international lending allows the foreign economy to export its excessive savings and thereby reduce the downward pressures on the real interest rate. In a secular stagnation, this increase in demand raises output by raising the inflation rate.

There is no reason to assume that this process of exporting excess savings does not push the domestic economy to the zero lower bound. The condition needed for this is simply that K^* is large enough so that the natural rate of interest is negative in the home country as well. Interestingly, it may be beneficial for the home country to close its capital markets to prevent secular stagnation from spreading, and we will investigate this possibility with some numerical examples in Section 8.

The effect of increasing private foreign capital holdings on the domestic economy is exactly the same as if the foreign government directly invests in the foreign economy through reserve accumulation (as we see in equation 33). In either case, the foreign government is exporting excess savings and putting downward pressure on the real interest rate in the domestic economy. This capital inflow, in principle may be large enough so as to drag the domestic economy into a secular stagnation.[11]. In this case, foreign capital inflows no longer transmits lower interest rates, but instead transmits a recession. We see this case in the second dotted line in Figure 5, whereby foreign official capital flows (given by IR) pushes the domestic economy into a secular stagnation. It is worth noting that this increase in IR need not have any effect on the foreign economy (see equation 34)) if fiscal policy has no loan supply effects.

Proposition 3. *If the international lending constraint at* $K^* = 0$, $r > r^*$ *and* $r^n > 0$, $r^{n*} < 0$, $\bar{\Pi} = \bar{\Pi}^* = 1$ *and* $\gamma, \gamma^* > 0$, *then* $\frac{\partial r}{\partial IR} < 0$ *and* $\frac{\partial Y}{\partial IR} = 0$ *when* $i > 0$ *while* $\frac{\partial r}{\partial IR} > 0$ *and* $\frac{\partial Y}{\partial IR} < 0$ *when i=0 and the domestic economy enters secular stagnation.*

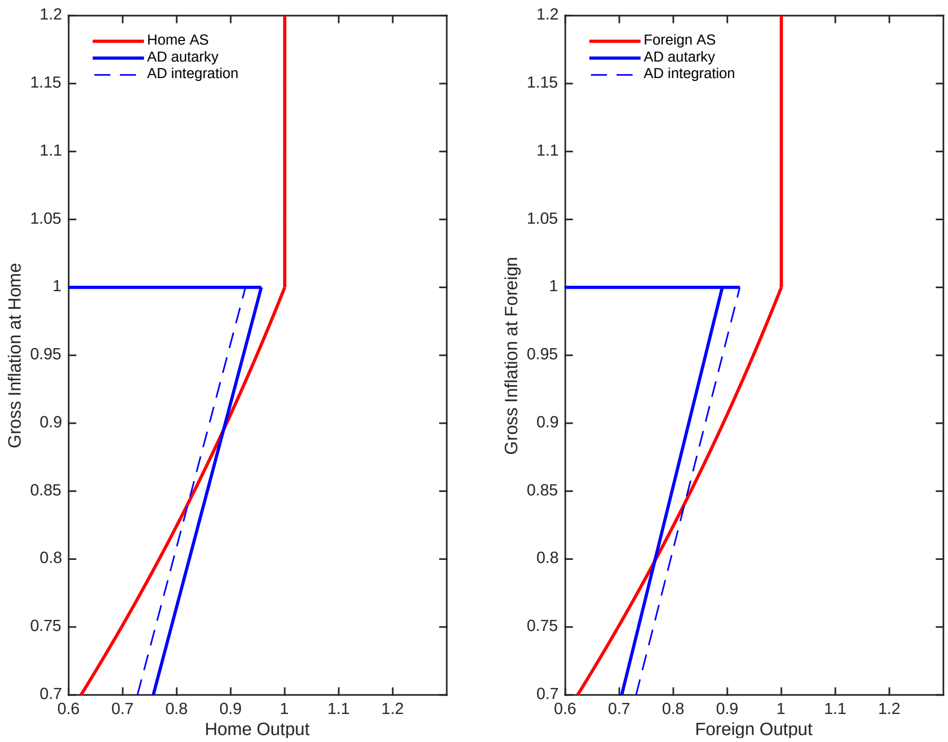
Proof. See Appendix A. □

Proposition 3 shows that while capital inflows typically lowers real interest rate, once the zero bound becomes binding, capital inflows instead transmit a recession and *higher real interest rate*. We have framed Proposition 3 in terms of the government policy variable IR which measures the amount of domestic assets (reserves) acquired by the foreign government. We do this for simplicity. We have seen that an increase in capital flows as measured by an increase in K^*, has exactly the same effect.[12] The effect of the increase in capital inflows in the domestic economy is showed by a dotted line in Figure 5.

[11]Substantial reserve accumulation in the pre 2008 era may have made the US more vulnerable to negative demand shocks by increasing the likelihood of hitting the zero lower bound

[12]Framing the proposition in term of K^* instead of IR, however, involves some complications. We then need to keep track of that there still remains positive interest rate differentials $r \geq r^*$ at all times, otherwise the capital flow can reverse (leading to different special cases depending on parameters), a complication that we avoid by characterizing our insight in terms of IR in the proposition.

Figure 6: Effect of international reserve accumulation



Our model offers a framework for thinking about neomercantilism, which are policies that attempt to boost a country's net foreign asset position by target persistent trade surpluses for some time. In our model these policies may be expansionary for the country that implements them but even if so, it will always be at the expense of the trading partner. Thus if successful, it corresponds to a classic beggar they neighbor policy.

Under imperfect integration, private capital flows from the country with lower rates to the country with higher rates. Given our assumptions, however, reserve accumulation need not go in the same direction as private capital flows, moreover, we can imagine that a country can curtail private capital inflows with capital controls. Reserve accumulation will always worsen the stagnation in the debtor country. An increase in international reserves, say by China purchasing US Treasuries, reduces the US Treasuries held by the US residents. This lowers the natural rate of interest in the US. If US is at the ZLB and in a secular stagnation, this has the effect of pushing inflation further below target and worsening the output shortfall. Figure 6 displays the effect of reserve accumulation with the solid line representing the effect of increased reserve accumulation by the foreign country.

For the country building up reserves, e.g. China, it will in general matter how the reserve accumulation is financed even if this is irrelevant from the perspective of the US under incomplete financial integration. If this reserve accumulation is financed by taxation of the savers or by issuance of public debt, this policy has the effect of raising the natural rate of interest and boosting inflation/output. Alternatively, if both the middle-aged and old are taxed according to fiscal rule (16), then reserve accumulation has no impact on equilibrium inflation and output in the creditor country. In this particular case, reserve accumulation worsens secular stagnation in the debtor country while providing no benefit for the creditor country.

Reserve accumulation is equivalent to a net foreign asset target for the country acquiring reserves. Along the transition path, the country accumulating reserves will need to run a trade surplus. If equilibrium interest rates are negative, then these current account surpluses can even be permanent as we saw in equation (14). Significant trade surpluses in Japan and Germany along with continued reserve accumulation are policies that may have alleviated or eliminated output gaps in those countries while exerting a significant drag on the US economy.

5.2 Perfect Capital Integration and Multiple Equilibria

Consider now a world in a secular stagnation with perfect financial integration. In this case, the world interest rate is now determined by a single equation (35). The zero lower bound and the inflation target in each country now place a lower bound on the *equilibrium* world real interest rate. The world real interest rate cannot fall below the inverse of the inflation target: $1 + r^W \geq \frac{1}{\bar{\Pi}^W}$ where $\bar{\Pi}^W = \max\{\bar{\Pi}, \bar{\Pi}^*\}$. As we show, if the world natural rate falls below this bound there exists a symmetric stagnation equilibrium with output shortfalls in both countries. This symmetric secular stagnation equilibrium is graphically depicted in Figure 7 where we now plot up global output and global inflation with inflation rates equalized across the two countries.

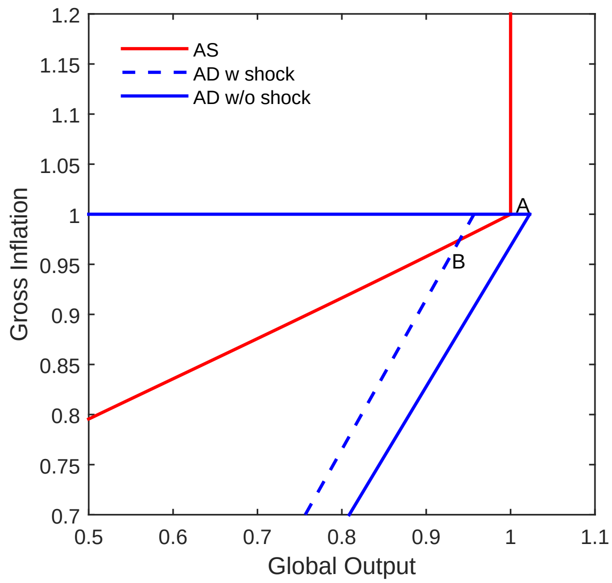
If the world natural rate of interest is negative, then a symmetric stagnation equilibrium exists. This is essentially an application of Proposition 1 from Eggertsson and Mehrotra (2014) where the AD curve is now the global AD curve in equation (35) and the global AS curve is obtained by the population weighted average of the domestic and foreign AS equations. Moreover, the symmetric stagnation equilibrium is locally determinate in contrast to the deflation steady state considered in Benhabib, Schmitt-Grohé and Uribe (2001). If we assume a symmetric inflation target for both countries, then we can formally establish these results in Proposition 4:

Proposition 4. *If $r^{W,n} < \bar{\Pi}^{-1}$, there exists a locally determinate secular stagnation equilibrium with $Y < Y_f$, $Y^* < Y_f^*$, $i = i^* = 0$ and $\Pi < \bar{\Pi}$.*

Proof. See Appendix A. □

Even if both countries are in a secular stagnation, this need not imply that both countries experience the same output gap. The output gap in each country is determined by the deviation of inflation below the inflation target and the degree of wage rigidity. Assuming a symmetric

Figure 7: Global stagnation



inflation target of $\bar{\Pi} = 1$, the output gap in each country is given by the following equations:

$$Y = \left(\frac{1 - \frac{\gamma}{\Pi}}{1 - \gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f \tag{40}$$

$$Y^* = \left(\frac{1 - \frac{\gamma^*}{\Pi}}{1 - \gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{41}$$

$$1 + r = \frac{1}{\Pi} \tag{42}$$

where equation (42) and equation (43) are the domestic and foreign AS curves, and (44) is the Fisher equation defining the real interest rate when the zero lower bound is binding in both the home and foreign country. Equations (35), (42) - (44) jointly determine the endogenous variables r, Π, Y, and Y^* in a symmetric stagnation equilibrium.

Note that the country with the higher degree of wage rigidity, as given by the higher γ or γ^* will suffer a higher output gap. The wage norm reacts more sluggishly in the less flexible labor market, moving the real wage rate further from its market-clearing level. Holding constant the world interest rate, it is unambiguously beneficial for each country to increase its wage flexibility in a secular stagnation. This, policy however worsens the demand shortfall and increases global deflation. Overall, a structural reform policy leads to a paradox of flexibility, lowering global output. An asymmetric labor market reform that increases wage flexibility is a beggar-thy-neighbor policy.

Perfect capital integration also raises the possibility of multiple equilibria. Under the same conditions as in Proposition 4 some of the other scenarios in Definition 2 may also be feasible. It is possible for the domestic (foreign) country to be in stagnation with inflation below target while the foreign (domestic) economy has positive nominal rates and output at its full employment level.[13] Real rates are still equalized because the rate of deflation in the domestic economy equals the nominal interest rate in the foreign economy (without loss of generality, assume that the inflation target $\bar{\Pi} = 1$ for both the domestic and foreign economies).

The left panel of Figure 8 displays the asymmetric equilibria. There exist two distinct asymmetric equilibria. In one case, the home country is in secular stagnation (HS) while the foreign country is not; the mirror case is shown at (FS) when the foreign country is in stagnation. To see this possibility, again refer to Definition 2 and now look for an equilibrium in which the home country is a secular stagnation and the foreign country is not. Then, $Y^* = Y_f^*$ and $\Pi^* = 1$. Then world output satisfies the following aggregate supply equation:

$$Y^w = \omega \left(\frac{1 - \frac{\gamma}{\Pi}}{1 - \gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f + (1 - \omega) Y_f^* + \tag{43}$$

The global AD curve is unchanged from equation (35) A analogous equation for world output obtains if the foreign economy is in secular stagnation and domestic is not with γ replace γ^* and Π with Π^*.

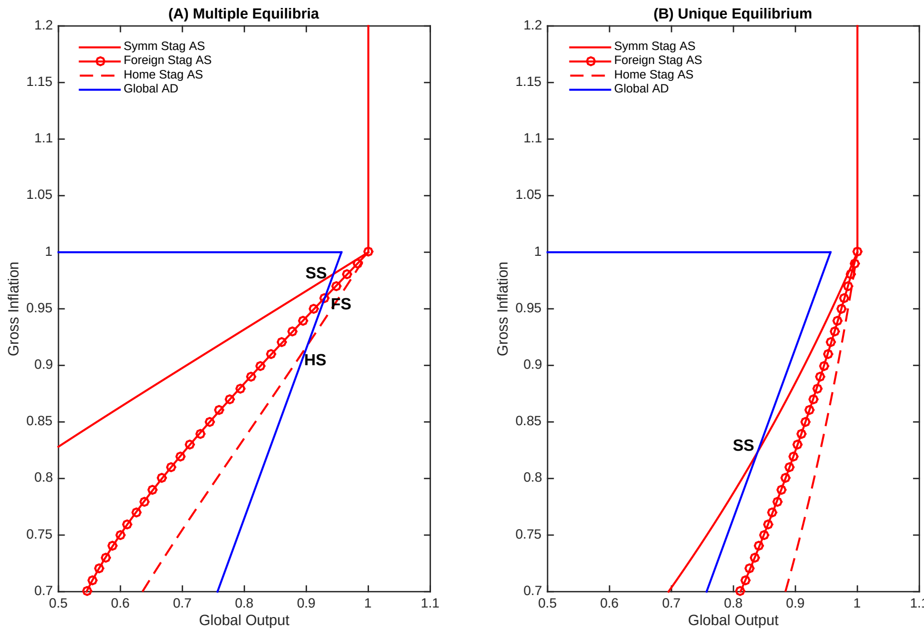
Figure 8 shows the intersection of the aggregate world demand and supply. The dashed line corresponds to world aggregate supply curve when the domestic economy is in the secular stagnation and foreign is not. The circled line corresponds to aggregate world supply when the foreign country is in secular stagnation - the AS curve in equation (45). In drawing this figure, we have assumed that $\gamma < \gamma^*$, - wages are more flexible at home than abroad. This implies that world output is lower if only the home country is in secular stagnation relative to the case of foreign-only stagnation. The large output shortfall with greater wage flexibility is an instance of the paradox of flexibility - greater flexibility increases deflation and leads to higher real interest rates depressing demand.

In an asymmetric secular stagnation, the one country in secular stagnation must absorb the entire shortfall in world demand. Therefore, holding parameters constant, if one country escapes a secular stagnation while the other does not, it will always do so at the expense of its trading partner. Moreover, even if one country escapes and benefits in terms of output/welfare, world output as a whole will fall in an asymmetric stagnation. Since one country must absorb the entire shortfall in demand, deflation (or shortfall in inflation from target) must be higher in an asymmetric stagnation than in a symmetric stagnation. Graphically, as shown Figure 8, the world AS curve under asymmetric stagnation must always lie below the world AS curve under symmetric stagnation. Since the AD curve is upward sloping, world output is lower under a symmetric stagnation.

The existence of these asymmetric stagnation equilibria with perfect financial integration is not assured. If the global collateral constraint exceeds the level of output of the country that is not in

[13] While this can also occur under imperfect capital market integration, it is a bit more transparent in this setting.

Figure 8: Asymmetric stagnation



stagnation, then we can guarantee existence of an asymmetric stagnation.[14] Likewise, relatively high degrees of wage rigidity make it more likely for multiple equilibria to emerge. The right-hand panel of Figure 8, illustrates the case where wage rigidity is sufficiently low that only a symmetric stagnation exists.[15]

Proposition 5 establishes conditions and properties of an asymmetric stagnation equilibrium, in the case when home is in stagnation and foreign is not. The analogous conditions establish when the mirror case occurs: home country at full employment, foreign country in stagnation. Depending on parameter values, both, one or neither of these asymmetric stagnation equilibria may emerge.

Proposition 5. *If $r^{W,Nat} < \bar{\Pi}^{-1}$, $D^W > (1-\omega) Y_f^*$, $\gamma > 0$, there exists a unique, locally determinate asymmetric secular stagnation with $r = r^*$, $Y < Y_f$, $Y^* = Y_f^*$, $i = 0$, and $\Pi < \bar{\Pi}$.*

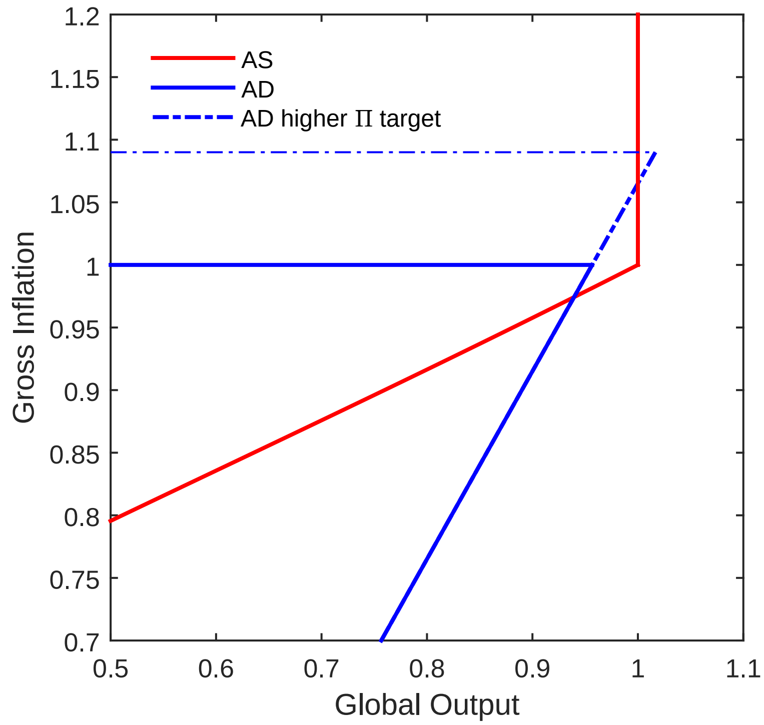
Proof. See Appendix A. □

As we have shown in this section, the possibility of a natural rate of interest that falls below a central bank's inflation target leads to secular stagnation equilibria in which one or both countries

[14]This is a sufficient, not a necessary condition.

[15]In an asymmetric stagnation, only one country must absorb the entire shortfall in world output. Intuitively, supply exceeds demand and if higher interest rates drive down global demand faster than global supply no equilibria exists. The failure of the AD and AS curves to cross is due to the fact that global supply in an asymmetric stagnation is bounded below by the full-employment level of output in the country not in stagnation. In a symmetric stagnation, there always exists a sufficiently high rate of deflation that drives global output to zero while demand remains bounded away from zero.

Figure 9: Effect of raising the inflation target



may fall. While capital controls may prevent secular stagnation from spreading to countries with a sufficiently autarky natural rate, other policy options may be preferable. We next turn to monetary and fiscal policy to investigate whether these policies can eliminate the possibility of secular stagnation.

6 Monetary Policy

6.1 Increasing the Inflation Target

Imagine that both countries have an inflation target of $\Pi = \Pi^* = 1$ and that the world natural rate of interest is negative in a world with perfect capital integration. Assume that the domestic economy announces a higher inflation target while the foreign economy does not. If the domestic economy reaches this new inflation target, the only possible equilibrium is an asymmetric secular stagnation where the foreign economy remains trapped and the domestic economy is not. This outcome will be important when we analyze the behavior of exchange rates and monetary policy formulated in terms of exchange rate. There is no guarantee that the domestic economy reaches its new target - one cannot exclude the possibility of the domestic economy finding itself in a secular stagnation as well.

Consider now a symmetric secular stagnation where both countries increase their inflation target. Figure 9 displays the effect of an increase in the global inflation target. Our model does not have much new to day about this policy change. In essence, it is equivalent to the experiment examined in Eggertsson and Mehrotra (2014) in a closed economy. An increase in both inflation targets shifts up the kink point in the AD curve. This allows for the possibility of a new equilibrium at the intersection of the two curves at the new inflation target. While the higher inflation target allows for a full employment equilibria, it does not exclude the secular stagnation equilibria. This multiplicity provides one motivation for fiscal policy. Before getting there, however, it is worth highlighting that the model does have something new to say about about how the relative monetary policies of the two countries affects equilibrium.

6.2 Currency Wars

In formulating monetary policy, our key assumption is that the central bank sets policy using the short term nominal interest rate. The logic for this assumption is that the government controls the money supply, and the short-term risk-free nominal interest rate is the opportunity cost of holding money. Thus the nominal rate can be set from a money demand equation - for a particular value of the nominal stock of money one can determine the nominal interest rate. The only complication is that the return on money can never go below zero (as then people would hold money as an asset) and hence the zero bound. At the zero bound, any supply of money above the satiation level is consistent with a zero nominal interest rate (see Eggertsson and Woodford (2003)).

Strictly speaking, the government does not control the nominal exchange rate in the model. The exchange rate is the relative price of the currencies (one unit of dollar in terms of euro) of the two countries which is given by the relative price level $\frac{P_t}{P_t^*}$. Once the zero bound become binding, the central bank no longer has direct control of the price level due to the zero bound. Accordingly, it does not have control of the exchange rate either.

Many authors have suggested that the exchange rate can act as an additional instrument of policy and that the central bank can affect the exchange rate by printing money and buying foreign currencies (see, for example, Bernanke (2000)). In the model, however, the exchange rate is the price of interest rate differentials between the two countries. This equation does not directly depend on the relative asset position of either central bank beyond the effect the money supply has on the nominal interest.

As has been argued by Eggertsson (2006), one way in which foreign asset market purchases could have an effect is by signaling something about future monetary policy, for example making future interest rate policy credible. Without trying to model that mechanism here, it is interesting to ask the following question: Suppose the government could commit to a particular path for the nominal exchange rate (without modeling exactly how and through what mechanism it can achieve that nominal peg). What are the implication for secular stagnation?

Consider first the case of an asymmetric secular stagnation. Under asymmetric stagnation, one country is producing at full capacity, while the other is not. As a result, the nominal exchange rate is continuously appreciating for the country in stagnation. If, for example, the domestic economy is in stagnation, so that $i = 0$ but the foreign economy achieves price stability (inflation target of $\bar{\Pi}^* = 1$), then:

$$\Pi^D = \left(\frac{S_{t+1}}{S_t}\right)^{-1}$$

The nominal exchange rate domestic currency is continuously appreciating at the rate of deflation of the domestic economy.

Consider a policy in which the domestic economy pegs its exchange rate to the foreign currency so that $S_{t+1} = S_t$. In this case, straightforward proposition follows:

Proposition 6. *Suppose $r^{W,n} < 0$, capital markets are perfectly integrated, and the domestic and foreign inflation targets are given by $\bar{\Pi} = \bar{\Pi}^* = 1$, and the nominal exchange rate is pegged at $\bar{S}_{t+1} = S_t = \bar{S}$. Then the global symmetric secular stagnation equilibria is the unique solution of the model.*

The proof of this proposition follows directly from the fact that if nominal peg is constant, the inflation rates of the two countries must be the same. Since their inflation target of $\bar{\Pi} = \bar{\Pi}^* = 1$ cannot be achieved due to the negative world natural interest rate, the only equilibria is the symmetric secular stagnation equilibria.

An interesting implication of this proposition is that if a given country finds itself in a secular stagnation (and the other does not) and reacts by pegging its exchange rate, it does not escape stagnation. Instead, it exports deflation to its trading partner. An exchange rate policy of this form is a beggar-thy-neighbor policy.

Can a country escape a secular stagnation all together via exchange rate policy? Denote the rate deflation of the foreign country if it finds itself in an asymmetric secular stagnation by $\Pi^{FS*} < 1$. Suppose now that the nominal exchange rate of the domestic economy is such that $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$. We then obtain the following proposition:

Proposition 7. *Suppose $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$ and the world natural rate of interest is negative. Then there exists no equilibrium in which the domestic economy is in a secular stagnation, but, if $\bar{\Pi}^* = 1$, the foreign economy must always be secular stagnation.*

The proof of this proposition follows directly from the fact that $\frac{S_{t+1}}{S_t} < \Pi^{FS*}$ implies that the inflation rate in domestic economy has to be higher than in the foreign economy. Since the inflation target of the foreign economy is $\Pi^* = 1$, we can exclude the possibility that neither one is in a secular stagnation, and since the inflation rate of the domestic economy is higher, the only feasible equilibria is one in which the domestic economy is not in stagnation so it is achieving its inflation target $\bar{\Pi}$ (which can be any number greater than unity), while the foreign economy is trapped.

An interesting element of the last two propositions, is that a policy commitment that is framed in terms of the nominal exchange rate when the world natural rate of interest is negative, is always going to come at the expense of the trading partner. This is in contrast to the higher inflation target of both countries we studied first in which case both countries win. A problem there, however, was that we could not exclude the secular stagnation equilibria. Can this be done? The answer is yes, through the use of fiscal policy.

7 Fiscal Policy

7.1 Government Spending

One natural policy, emphasized in the zero bound literature, is an increase in government spending. However, in contrast to some of the existing literature, the effects of an increase in government spending depend crucially on how that spending is financed. For example, an increase in government spending financed via a tax on the credit-constrained young, has no effect, since every dollar of increased government spending is offset by a corresponding cut in the spending by the young.[16] For now, let us consider a balanced budget and perfectly integrated capital markets. The most natural balanced budget assumption is that spending is financed via tax on the working age population (the middle generation).

Assuming $G_t = T_t^m$ and setting taxes on the old to zero in the government budget constraint (equation (15)), we can write the IS equation under perfect capital integration in steady state as:

$$\omega Y + (1-\omega)\, Y^* = \left(1 + \frac{1+\beta}{\beta}\frac{1+g}{1+r^w}\right)(\omega D + (1-\omega)\, D^*) + \omega G + (1-\omega)\, G^* \tag{44}$$

As this expression shows, the weighted average of government purchases in both countries matters for the real rate. Higher government spending in one country has benefits for both countries.

In normal times, government purchases would leave output unaffected given that labor is supplied inelastically. This implies that an increase in either G or G^* is met with an increase in the r^w leaving aggregate world output unchanged. In a symmetric secular stagnation, higher government purchases instead lower the real interest rate. They do this by raising inflation and crowd in consumption resulting in multipliers great than one. We can illustrate this by way of a numerical example. In Table 1, we use numbers from a quantitative experiment we describe in detail in Section 8, where we calibrate our model to fit the US and Eurozone secular stagnation episodes from 2008-2015.

The table shows that small changes in government spending carry sizable multipliers. Moreover, it does not matter which country does increases government purchase - the symmetric and asymmetric fiscal expansions deliver the same increase in output in both the US and Eurozone.

[16] See Eggertsson and Mehrotra (2014) for further discussion on fiscal multipliers under different financing regimes.

Table 1: Government Spending Multipliers

Regime	US	Eurozone
Baseline output gap	10%	15%
Symmetric expansion output gap	7.1%	10.7%
Asymmetric expansion output gap	7.1%	10.7%
Welfare (rel. to symmetric)	-0.2%	+0.2%

The welfare implications are somewhat different, however. If only US undertakes a fiscal expansion, then US working age population bears the greater burden of taxation reducing domestic consumption. The Eurozone meanwhile enjoys the benefits of higher global inflation and higher consumption. Quantitatively, these welfare are not particularly large compared to the overall benefit of higher output in both countries. The inflation rate rises from 1% in the baseline calibration to 1.2% - a twenty basis point reduction in the global real interest rate.

As equation (46) reveals and our numerical experiment demonstrates, output in a symmetric secular stagnation depends on overall global government spending. Spending in one country carries positive demand externalities - the foreign country benefits from a purely domestic increase in government spending. If government spending stimulus is costly, then each of the government may provide too little fiscal expansion if it only cares about the welfare of its own citizens and does not coordinate policy with its trading partner.

To formalize this insight, let us assume that each government has a loss function given by:

$$L_t = (\Pi_t - 1)^2 + (Y_t - Y^f)^2 + (G_t - G^{\text{target}})^2$$

and the loss function for the foreign government is the same but in terms of foreign variables. Let us call a solution that selects G and G^* jointly to maximize both countries objective simultaneously the cooperative solution. Let us call the solution if each country maximizes its own objective, taking the other countries spending as given the non-cooperative solution. In the Appendix, we prove the following proposition:

Proposition 8. *Government spending in the noncooperative solution is less than in the cooperative solution, with coordination losses maximized when* $\omega = \frac{1}{2}$.

Proof. See Appendix B □

The logic of this proposition is straightforward. Because fiscal stimulus is costly (government is larger than its optimal size in absence of frictions) each country supplies too little stimulus to stabilize world output since the benefits are shared by foreigners. As we establish in a corollary in Appendix B, the coordination problem - the gap between government spending in the cooperative and noncooperative solutions - is increasing in the number of countries and, in the case of two countries, is worst when these countries are the same size. This result has implications for the Eurozone where fiscal expansion by smaller periphery economies will have little impact on overall Eurozone demand. Likewise, our results suggest a substantial coordination problem between the US and Eurozone given the similar size of their economies to the extent that capital markets are perfectly integrated.[17]

7.2 Debt Policy

As we have already shown in Section 3, our model can be generalized to include government debt. Foreign reserve accumulation in the absence of increase in public debt issuance puts downward pressure on the natural rate of interest, as can be seen in equation (22).

Equation (22) has other important implications. In particular, an increase in government debt will directly increase the natural rate of interest. Thus, a straightforward solution to secular stagnation is to issue government debt. That foreign reserve accumulation puts downward pressure on the natural rate of interest is essentially the inverse of this - an increase in reserve accumulation reduced the total amount of government bonds held by the private sector.[18]

But will increasing government debt always work? Our model is silent on what type of limitations may constrain a government's ability to issue debt. However, it is not very difficult to list some reasonable limitations. One possibility is, given a probability that the forces that give rise to a secular stagnation ultimately reverse themselves, that interest rates eventually may rise. If the government has accumulated large amounts of debt, the real cost of servicing this debt may be quite high. Higher distortionary taxation may result in welfare losses, and hence put limits on the amount of debt the government is willing to issue. Another possibility is that if debt rises above a certain level, this triggers uncertainty about if debt will get repaid again. To the extent that government debt serves as collateral in various private transactions, this may even have a negative effect on a broad class of what had been considered safe assets. This is another mechanism we have not modeled, but one could imagine may, in principle, be important.

To the extent such constraints exist on the government's willingness to issue debt, a reasonable approximation to the government's objective function might take a similar form as we saw in the last section. The government would choose the optimal level of debt to minimize a loss function of the following form (in this case holding G_t constant for simplicity):

$$L_t = (\Pi_t - 1)^2 + \lambda_Y(Y_t - Y_t^f) + \lambda_b(B_t - B^{safe})^2$$

where we denote B^{safe} as the level above which agents start putting some probability on a gov-

[17]With imperfect integration, the benefits of government spending are fully realized by the country undertaking the fiscal expansion *conditional* on the lending constraint remaining binding.

[18]A permanent increase in government debt has the same effects as a helicopter drop at the zero lower bound. Issuance of money to the private sector can eliminate a secular stagnation.

ernment default. If the loss function of the government of each country takes this form, then policy will be subject to exactly the same problem as we considered in previous section: each government has the incentive to ”free ride” on the fiscal stimulus of the other country. Austerity will be oversupplied, and the coordination problem worsens with the number of countries.

8 Quantitative Examples

8.1 Asymmetric Stagnation: US-Japan, 2000-2008

We now illustrate how this framework can be used to rationalize recent developments in the global economic environment. The first question, is whether our model is consistent with the fact that Japan appears to have been in a secular stagnation with zero interest rates since the mid/late 1990's while, in the US, the nominal interest rate only fell to zero following the economic crisis of 2008. Here we consider, for simplicity, a world in which Japan and US are the only two countries. The key insight of the experiment is to confirm quantitatively one of the main conclusion we had reached previously reached qualitatively, that a country in a secular stagnation benefits greatly from the opening of financial markets so that it can export its excess savings. The host of these capital inflows, then, will experience a drop in the real interest rate and a credit boom in the private sector.

We calibrate the model to get a sense of the magnitudes of various parameters and the implications of capital flows for interest rates, output, and the external balance. In Table 2, we fit the model to match several targets for the US and Japan between 2002 and 2008. Typical parameters such as the rate of time preference and labor share are set to conventional values: $\beta = 0.96$, $\alpha = 0.7$. We must choose population growth rates, inflation targets, and collateral constraints for each country. Additionally, we must choose the wage rigidity parameter γ^* for Japan. We must also set the international lending constraint K^*.

For the US, the population growth rate is set at 1%, the US inflation target is set at 2% - the unofficial target of the Federal Reserve, and the nominal interest is set at 3% to match the average real interest rate from 2002-2007. The real interest rate determines the level of the collateral constraint D. For Japan, the population growth rate is set at 0, the inflation target is set at 0% given that the Bank of Japan has only recently announced a 2% target. The rate of inflation is set at -0.5% to match the average real interest rate in Japan from 2002-2007. Given the OLG structure, periods last 20 years and all rates are converted accordingly.

Measuring the output gap is somewhat more difficult. We set the output gap at 10% based on the discussion in Hausman and Wieland (2014). US potential output is normalized to one, while potential output in Japan is set at 0.35 based on Japanese GDP (as a percentage of US GDP in 2007) at market exchange rates. The output gap and inflation rate in Japan pin down the foreign

Table 2: Parameterization: US and Japan, 2002-2008

Panel A: Common parameters	*Symbol*	*Value*	
Labor share	α	0.7	
Discount rate	β	0.96^{20}	
Int'l lending constraint	K	0.14	
Panel B: Country-specific parameters	*Symbol*	*US*	*Japan*
Inflation target	$\bar{\Pi}, \bar{\Pi}^*$	2%	0%
Population growth	g, g^*	1%	0%
Potential output	Y_f, Y_f^*	1	0.34
Wage adjustment	γ, γ^*	N/A	0.296
Collateral constraint	D, D^*	0.237	0.071

collateral constraint D^* and γ^*. The international lending constraint K is set to match the bilateral net foreign asset position. Based on TIC data from the Department of the Treasury, net Japanese holdings of debt securities in the US were approximately $2 trillion in June 2008. The K value in Table 2 is the net foreign asset position as a percentage of 20-year GDP ($2 trillion/$14.5 trillion x 1/20).

These targets imply a modest degree of wage rigidity with $\gamma^* = 0.3$ - when $\gamma^* = 1$, wages are fully rigid. The collateral constraint is looser for the US but comparable across both countries. Given this calibration, we can consider the implications of autarky for secular stagnation in Japan. If $K = 0$, Japan's inflation rate would fall to -1.38% per year and the output gap would rise drastically from 10% to 28.6%. Based on this numerical example, the $2 trillion net asset position in the US significantly ameliorated Japan's output gap. Conversely, full capital market integration between Japan and US would pull Japan out of a secular stagnation with the world natural rate of interest. However, equilibrium world real interest rates would be quite low in this scenario at 0.87%.

By contrast, the effects of this large negative NFA position were fairly modest for the US. Setting $K = 0$, the US nominal (and real) interest rate would be 7 basis points higher and the household debt to GDP ratio would be 1.5% percent lower relative to the baseline. However, these calculation ignore substantial capital inflows into the US during this time for other emerging market and oil-producing countries. These patterns qualitatively fit the rise in household debt and easing of collateral constraints experienced in the US during the credit boom between 2001 and 2008.[19]

[19] This mechanism is also at work in the quantitative lifecycle model of Favilukis, Ludvigson and Van Nieuwerburgh (2015) who consider the effect of the global savings glut on US house prices and asset prices.

In short, in a world with incomplete financial integration, we can construct numerical examples that match the broad patterns in the data seen in the US and Japan. Japan gained significantly from capital market integration, as that allowed it to export some of its excess savings to the US. In the US, this capital flow reduced interest rates, easing lending constraints, and boosted household debt - albeit by a modest amount. One interesting implication of this is that both countries would have benefited from full financial integration, as this would have pulled Japan out of a secular stagnation, a conclusion that, however, can be overturned in different setting as we now shall see.

8.2 Symmetric Stagnation: US-Europe 2008-2015

Under perfectly integrated financial markets, one country may benefit from closing its capital account. To illustrate this possibility, we calibrate the model to model the interaction of the US and Eurozone in the post 2008 Great Recession period. In this numerical experiment, we find that US interest rates would be positive in the absence of capital inflows from Europe. We also show that our calibration implies that the Eurozone suffers a greater shortfall in output due to a higher degree of wage rigidity

To calibrate the model, we choose the wage rigidity parameters in the US and the Eurozone to match output gaps in each region and chose the collateral constraints to match global interest rates and the net foreign asset position of the Eurozone in the US. Standard parameters - the rate of time preference β and the labor share α are set as before. The growth rate g is set at 1% annually in accordance with the average population growth rate across the regions.

Both the US and Eurozone nominal rates are set at zero given the zero lower bound has remained binding in each region over this period. The inflation target is set at 1.75% to reflect the Eurozone's somewhat lower desired inflation target. The inflation rate is set at 1% in both regions to equate the world real interest rate at -1% - approximately consistent with US and Eurozone short-term real rates between 2008-2015. The full-employment level of output is normalized to unity in the US and 0.96 in the Eurozone based on GDP relative to the US evaluated at market exchange rates in 2008.

We target an output gap in the US and Eurozone at 10% and 15% respectively reflecting the deviation of real GDP per capita in the US and Eurozone relative to pre-recession trend. The average output gap and global real interest rate determines the average collateral constraint. In June of 2013, Eurozone holdings of US debt securities net of US holding of Euro debt equaled $2 trillion. The net foreign asset target is computed as a percentage of 20-year GDP ($2 trillion/$16.5 trillion x 1/20). The foreign asset position determines the difference between the US and Eurozone collateral constraints: D and D^*.

In Table 3, we show the implied parameter values that match the targets described above. Wage rigidity in the US and Eurozone are comparable and, for the Eurozone, imply a somewhat

Table 3: Parameterization: US and Eurozone, 2008-2015

Panel A: Common parameters	*Symbol*	*Value*	
Labor share	α	0.7	
Discount rate	β	0.96	
Inflation target	$\bar{\Pi}$	1.75%	
Population growth	g	1%	
Panel B: Country-specific parameters	*Symbol*	*US*	*Eurozone*
Potential output	Y_f, Y_f^*	1	0.96
Wage adjustment	γ, γ^*	0.217	0.297
Collateral constraint	D, D^*	0.157	0.136
Panel C: Counterfactual under autarky	*Symbol*	*US*	*Eurozone*
Output gap	Y, Y^*	0%	21.3%
Nominal rate	i, i^*	0.25%	0%
Welfare (rel. to integration)	U, U^*	+7.5%	−4.2%

greater degree of wage adjustment than found in Schmitt-Grohé and Uribe (2011). In particular, the US display more flexible wages than the Eurozone implying that, for a given level of inflation below target, the shortfall in output is less in the US than the Eurozone. These wage rigidity values are also consistent with the more prominent role of unions in wage-setting in the Eurozone.

The collateral constraints are also comparable across regions and somewhat tighter in the Eurozone to reflect the fact that net capital flows towards the US. Interestingly, in steady state, the US runs a trade deficit with the Eurozone despite the fact that the US has a negative net foreign asset position. Since the equilibrium world interest rate is negative, the US is, in effect, paid to borrow from the Eurozone. This permanent trade deficit is however quite small - only 0.22% of GDP in steady state.

Table 3, Panel C displays the counterfactual case of financial autarky. In the absence of capital integration, the natural rate of interest would be -1.5% in the US and -2.1% in the Eurozone. At the assumed inflation target, the US would be able to remain at full-employment. In other words, net capital flows from the Eurozone pushed the US into a secular stagnation. These values also suggest that only a modest increase in inflation expectations is needed to attain the world natural rate of interest. Under autarky, the output gap in the Eurozone would worsen by 6 percentage points and the inflation rate would fall to 0.7%. As Table 3 shows, US welfare increases under autarky while Eurozone welfare worsens. Any gains for further consumption smoothing under integration are offset by the gains from a smaller output gap in the US.

9 Conclusion

In this paper, we extend secular stagnation to an open economy setting, showing how capital markets act as a mechanism to transmit low natural rates. In the presence of the zero lower bound and nominal frictions, negative natural rates of interest can result in a secular stagnation in one or both countries characterized by a binding zero lower bound, low inflation or deflation, and a persistent output gap. Our two country setting illuminates possible monetary and fiscal policy spillovers. Uncoordinated changes in monetary policy has beggar-thy-neighbor effects improving conditions in one country at the expense of the other. Fiscal policy, by contrast, has positive externalities across countries. These positive externalities may give rise to a coordination problem whereby fiscal expansion is undersupplied.

While our model emphasizes the analysis of a steady state of an economy in which natural rate of interest is permanently negative, this does not imply that the zero bound must always be binding in our framework. The steady state is locally unique and determinate. It is therefore straightforward to consider local perturbations of the model to shocks. We can therefore have ”secular stagnation” business cycles, in which interest rates rise from time to time above steady state. A key prediction of the theory is that, if the steady state natural rate is negative, then the zero bound will be hit much more frequently, and contractions will be more violent, as they cannot be offset by interest rate cuts. The failure of long-term real interest rate to rise recently, even in the wake of Fed's decision to increase the Federal Funds rate, raises the distressing possibility that the US may find itself in this scenario in coming years.

At the end of 2015, as the US embarks on the first interest rate increase in nearly a decade, the monetary policy stance is increasingly moving in different direction in US relative to the Eurozone and Japan. Our model suggests that, at the very least, continued low interest rates in other developed economies will act as an anchor on US interest rates, limiting the degree to which nominal interest rates in the US will rise. Our model also suggests a reemergence of global imbalances as higher rates in the US increase capital inflows and widen the US current account deficit. Indeed, in recent quarters, US export growth has substantially slowed and the trade deficit has increased, lowering the growth rate of US GDP.

Finally, it is worth considering the implications of the slowdown in economic growth in major emerging market economies, and its implications for the global economy. Chinese economic growth is decelerating while previously robust economies like Russia and Brazil are exhibiting substantial weakness. The drop in global oil prices may have further negative implications for other emerging market economies. On one hand, slow growth in these countries may reduce demand for US Treasuries and safe assets, thereby providing a force to raise US and global interest rates. China is already reducing its stock of US Treasuries to stabilize its exchange rate. On the other hand, slower growth will likely reduce FDI flows from developed to emerging market economies. To the extent that the fall in portfolio flows to the US is offset by a fall in FDI flows to emerging market economies, net capital flows to the US may be unchanged or even increasing. Such an outcome would place further downward pressure on US rates in the coming years.

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A Existence, Uniqueness and Local Determinacy

Here we provide formal proofs for various propositions presented in the body of the text.

Proposition 2 *If the international lending constraint is binding,* $r^n > 0$*,* $r^{n*} < 0$*,* $\bar{\Pi} = \bar{\Pi}^* = 1$ *and* $\gamma^* > 0$*, there exists a unique, locally determinate secular stagnation equilibrium in the creditor country with* $i^* = 0$*,* $\Pi^* < 1$*, and* $Y^* < Y_f^*$*.*

Proof. Under the assumptions of the proposition and the monetary policy rule, the zero lower bound is binding for the creditor country and the equilibrium real interest rate in steady state is given by $r^* = \frac{1}{\Pi^*}$. Equilibrium inflation and output in steady state in the creditor country solve the following equations:

$$Y^* = D^* + K^* + \psi^* \Pi^* \tag{A.1}$$

$$Y^* = \left(\frac{1 - \frac{\gamma^*}{\Pi^*}}{1 - \gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{A.2}$$

where $\psi^* = \frac{1+\beta}{\beta}(1+g)\left(D^* + \frac{1+r^n}{1+\beta}K^*\right) > 0$. We may define the difference equation $\Delta(\Pi^*)$ by taking the difference between ($A.1$) and ($A.2$) . This function is continuous in Π^* with $\Delta(\gamma^*) > 0$ and $\Delta(1) < 0$. Therefore, there exists a $\gamma^* < \Pi^* < 1$ such that $\Delta(\Pi^*) = 0$. Since $\Pi^* < 1$, it follows that $Y^* < Y_f^*$.

To establish uniqueness, we first assume that their exist multiple distinct values of Π^* at which $\Delta(\Pi^*) = 0$. Graphically, in inflation-output space (output on the x-axis), the AS curve (equation ($A.2$)) lies above the AD curve (equation ($A.2$)) when inflation equals γ^* and the AS curve lies below the AD curve for inflation at unity. Thus, if multiple steady states exist, given that AS is a continuous function, there must exist at least three distinct points at which the AS and AD curve intersect.

At the first intersection point, the slope of AS curve crosses the AD line from above and, therefore, at the second intersection the AS curve crosses the AD curve from below. Since the AD curve is a line, the AS curve is locally convex in output in this region. Similarly, between the second and third intersection, the AS curve is locally concave in output. Thus, as output Y^* increases, the AS curve must first have a positive second derivative followed by a negative second derivative.

We compute the second derivative of inflation with respect output of the AS curve and derive the following expression (we drop the $*$ for simplicity):

$$\frac{d^2\Pi}{dY^2} = G(Y)\left((1+\phi)(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi} + (\phi-1)\right) \tag{A.3}$$

$$G(Y) = \frac{\phi\gamma(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}}{Y^2\left(1-(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}\right)} \tag{A.4}$$

$$\phi = \frac{1-\alpha}{\alpha} \tag{A.5}$$

As can be seen, over the region considered, the function $G(Y)$ is positive and, therefore, the convexity of the AS curve is determined by the second term. This term may be negative if $\phi < 1$, but this expression is increasing in Y between 0 and Y_f. Therefore, the second derivative cannot switch signs from positive to negative. Thus, we have derived a contradiction by assuming multiple steady states. Therefore, there must exist a unique intersection point.

As established before, it must be the case that the AS curve has a lower slope than the AD curve at the point of intersection. The slope of the AS curve is:

$$\frac{d\Pi^*}{dY^*} = \frac{1-\alpha}{\alpha}\frac{1}{\gamma^*}\frac{\Pi^*}{Y^*}(\Pi^* - \gamma^*) \tag{A.6}$$

If the slope of the AS curve is less than the slope of the AD curve at the intersection point, then it must be the case that:

$$\begin{aligned}
\frac{1-\alpha}{\alpha}\frac{\Pi^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*}-1\right) &< (\psi^*)^{-1} \\
\frac{1-\alpha}{\alpha}\frac{\psi^*\Pi^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*}-1\right) &< 1 \\
\frac{1-\alpha}{\alpha}\frac{Y^*-D^*-K^*}{Y^*}\left(\frac{\Pi^*}{\gamma^*}-1\right) &< 1 \\
s_y\frac{\alpha}{1-\alpha}+1 &> \frac{\Pi^*}{\gamma^*} \\
\frac{\gamma^*}{\Pi^*}\left(s_y\frac{\alpha}{1-\alpha}+1\right) &> 1
\end{aligned}$$

Linearizing the equilibrium conditions around the secular stagnation steady state, we obtain the following linearized AD and AS equations:

$$0 = E_t\pi_{t+1} - s_y y_t^* + d_t^* + s_d d_{t-1}^* \tag{A.7}$$

$$y_t^* = \gamma_w^* y_{t-1} + \gamma_w^*\frac{\alpha}{1-\alpha}\pi_t^* \tag{A.8}$$

where d_t^* is the collateral shocks and various coefficients are given in terms of their steady state values.

$$\gamma_w^* = \frac{\gamma^*}{\bar{\Pi}^*}$$
$$s_y = \frac{\bar{Y}^*}{\bar{Y}^* - \bar{D}^* - \bar{K}^*}$$
$$s_d = \frac{\bar{D}}{\bar{Y}^* - \bar{D}^* - \bar{K}^*}$$

Substituting ($A.8$) into ($A.7$), we obtain a forward looking difference equation in y_t^*. The local determinacy condition requires the coefficient on $E_t y_{t+1}^*$ to be less than one. This condition is the same as the slope condition. Therefore, the unique secular stagnation steady state is always locally determinate as required. □

Proposition 3 *If $r^{W,Nat} < \bar{\Pi}^{-1}$, there exists a locally determinate secular stagnation equilibrium with $Y < Y_f$, $Y^* < Y_f^*$, $i = i^* = 0$ and $\Pi < \bar{\Pi}$.*

Proof. Under the assumptions of the proposition, monetary policy in both countries cannot track the world natural rate of interest and $i = i^* = 0$. Perfect capital market integration requires equalization of the domestic and foreign real interest rate, hence $\Pi = \Pi^*$. Steady state inflation, domestic output, and foreign output jointly satisfy the following equilibrium conditions:

$$\omega Y + (1+\omega) Y^* = D^W + \psi^W \Pi \tag{A.9}$$

$$Y = \left(\frac{1 - \frac{\gamma \bar{\Pi}}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f \tag{A.10}$$

$$Y^* = \left(\frac{1 - \frac{\gamma^* \bar{\Pi}}{\Pi}}{1-\gamma^*} \right)^{\frac{\alpha}{1-\alpha}} Y_f^* \tag{A.11}$$

where $D^W = \omega D + (1-\omega) D^*$ and $\psi^W = \frac{1+\beta}{\beta}(1+g) D^W > 0$. We may define the difference equation $\Delta(\Pi)$ by taking the difference between ($A.9$) and the weighted sum of ($A.10$) and ($A.11$). Without loss of generality, assume that $\gamma < \gamma^*$. We assume that output is bounded below by zero - that is, if $\Pi < \gamma^*$, then $Y^* = 0$. Given this assumption, the function $\Delta(\Pi)$ is continuous (but not necessarily differentiable), with $\Delta(\gamma) > 0$ and $\Delta(\bar{\Pi}) < 0$. Therefore, there exists a global inflation rate Π_{ss} with $\Pi_{ss} < \bar{\Pi}$ implying that, in steady state, $Y_{ss} < Y_f$ and $Y_{ss}^* < Y_f^*$.

To establish that this steady state is locally determinate, we observe that, graphically, at $\Pi = \gamma$ the global AS curve (weighted sum of equations ($A.10$) and ($A.11$)). At $\Pi = \bar{\Pi}$, the global AD curve lies above the AS curve. Thus, there exists at least one equilibrium in which the AD curve is locally steeper than the AS curve. We first derive the condition for local determinacy. The log-linearized equilibrium conditions for a symmetric stagnation equilibrium are given below:

$$E_t\pi_{t+1} = \bar{\omega} s_y y_t + (1-\bar{\omega})\, y_t^* + shocks \tag{A.12}$$

$$y_t = \gamma_w y_{t-1} + \gamma_w \phi \pi_t \tag{A.13}$$

$$y_t^* = \gamma_w^* y_{t-1}^* + \gamma_w^* \phi \pi_t \tag{A.14}$$

where $\phi = \frac{\alpha}{1-\alpha}$ and the other coefficients are defined below:

$$\begin{aligned}
\gamma_w &= \frac{\gamma\bar{\Pi}}{\Pi_{ss}} \\
\gamma_w^* &= \frac{\gamma\bar{\Pi}}{\Pi_{ss}} \\
s_y &= \frac{\omega Y_{ss} + (1-\omega)\, Y_{ss}^*}{\omega(Y_{ss} - D) + (1-\omega)\,(Y_{ss}^* - D^*)} \\
\bar{\omega} &= \frac{\omega Y_{ss}}{\omega Y_{ss} + (1-\omega)\, Y_{ss}^*}
\end{aligned}$$

This linearized system can be expressed as:

$$\begin{aligned}
AE_t x_{t+1} &= Bx_t + shocks \\
E_t x_{t+1} &= A^{-1}Bx_t + A^{-1}shocks
\end{aligned}$$

where $x_t = [\pi_t, y_{t-1}, y_{t-1}^*]'$ and the A, B are square matrices with suitably defined coefficients. Local determinacy requires that the matrix $A^{-1}B$ has exactly one eigenvalue outside the unit circle.

Since the matrix B has a row of zeros, one eigenvalue of the system is zero. The characteristic polynomial that determines the remaining eigenvalues is:

$$\lambda^2 - \left(\phi s_y \left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma_w^*\right) + \gamma_w + \gamma_w^*\right)\lambda + \gamma_w\gamma_w^*\,(1 + s_y\phi) = 0$$

Since the characteristic polynomial is positive at $\lambda = 0$, the condition that ensures local determinacy is that the characteristic polynomial is negative at $\lambda = 1$. This condition requires:

$$1 + \gamma_w\gamma_w^*\,(1 + s_y\phi) < \phi s_y \left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma_w^*\right) + \gamma_w + \gamma_w^* \tag{A.15}$$

It remains to show that this local determinacy condition is identical to the slope condition that must be satisfied in equilibrium. The slope of the global AS curve and global AD curve is given below:

$$\begin{aligned}
\frac{dY_{AS}^W}{d\Pi} &= \phi\left(\omega\gamma_w \frac{Y_{ss}}{\Pi_{ss} - \gamma\bar{\Pi}} + (1-\omega)\,\gamma^* \frac{Y_{ss}^*}{\Pi_{ss} - \gamma^*\bar{\Pi}}\right) \\
\frac{dY_{AD}^W}{d\Pi} &= \psi^W
\end{aligned}$$

A steeper slope for the AD curve relative to the AS curve implies:

$$\frac{dY^W_{AS}}{d\Pi} > \frac{dY^W_{AD}}{d\Pi}$$
$$\phi\left(\omega\gamma_w \frac{Y_{ss}}{\Pi_{ss} - \gamma\bar{\Pi}} + (1-\omega)\,\gamma^*_w \frac{Y^*_{ss}}{\Pi_{ss} - \gamma^*\bar{\Pi}}\right) > \psi^W$$
$$\phi\left(\bar{\omega}\frac{\gamma_w}{1-\gamma_w} + (1-\bar{\omega})\,\frac{\gamma^*_w}{1-\gamma^*_w}\right) > \psi^W \frac{\Pi_{ss}}{Y^W}$$
$$\phi s_y\left(\bar{\omega}\gamma_w\left(1-\gamma^*_w\right) + (1-\bar{\omega})\,\gamma^*_w\left(1-\gamma_w\right)\right) > (1-\gamma_w)\left(1-\gamma^*_w\right)$$
$$\phi s_y\left(\bar{\omega}\gamma_w + (1-\bar{\omega})\,\gamma^*_w - \gamma_w\gamma^*_w\right) > 1 - \gamma_w - \gamma^*_w + \gamma_w\gamma^*_w$$
$$\phi s_y\left(\bar{\omega}\gamma_w - (1-\bar{\omega})\,\gamma^*_w\right) + \gamma_w + \gamma^*_w > 1 + \gamma_w\gamma^*_w\left(1 + s_y\phi\right)$$

where the last inequality is identical to the determinacy condition derived in equation ($A.15$).

□

Proposition 4 *If $r^{W,Nat} < \bar{\Pi}^{-1}$, $D^W > (1-\omega)\,Y^*_f$, $\gamma > 0$, there exists a unique, locally determinate asymmetric secular stagnation with $r = r^*$, $Y < Y_f$, $Y^* = Y^*_f$, $i = 0$, and $\Pi < \bar{\Pi}$*

Proof. Under the assumptions of the proposition and the monetary policy rule, the zero lower bound is binding for the home country and not binding for the foreign country. Nevertheless, real interest rates are equalized across both countries: $\frac{1}{\Pi} = r = r^* = \frac{i^*}{\Pi^*}$ where $i^* > 0$. Equilibrium inflation and output in the home country solve the following equations:

$$Y = \frac{1}{\omega}\left(D^W - (1-\omega)\,Y^*_f + \psi^W\Pi\right) \tag{A.16}$$
$$Y = \left(\frac{1 - \frac{\gamma\bar{\Pi}}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y_f \tag{A.17}$$

where $D^W = \omega D + (1-\omega)\,D^*$ and $\psi^W = \frac{1+\beta}{\beta}(1+g)\,D^W > 0$. We may define the difference equation $\Delta(\Pi)$ by taking the difference between ($A.16$) and ($A.17$) . This function is continuous in Π with $\Delta(\gamma) > 0$ (since $D^W > (1-\omega)\,Y_f$) and $\Delta(\bar{\Pi}) < 0$ since $r^{W,Nat} < \bar{\Pi}^{-1}$. Therefore, there exists a $\gamma < \Pi < \bar{\Pi}$ such that $\Delta(\Pi) = 0$. Since $\Pi < \bar{\Pi}$, it follows that $Y < Y_f$.

Uniqueness of an asymmetric stagnation equilibrium under perfect integration is established identically as in Proposition 2. Graphically, the global AD curve (equation ($A.16$) form a line in domestic inflation-output space. The domestic AS curve (equation ($A.17$)) is identical to equation ($A.2$) and cannot cross the AD curve more than once given that the second derivative cannot switch signs from positive to negative.

It must be the case that the AS curve has a lower slope than the AD curve at the point of intersection. The slope of the AS curve is identical to equation ($A.6$) If the slope of the AS curve is less than the slope of the AD curve at the intersection point, then it must be the case that:

$$\frac{1-\alpha}{\alpha}\frac{\Pi}{Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right)<\left(\frac{\psi^W}{\omega}\right)^{-1}$$
$$\frac{1-\alpha}{\alpha}\frac{\psi^W\Pi}{\omega Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right)<1$$
$$\frac{1-\alpha}{\alpha}\frac{\omega Y-D^W-(1-\omega)Y_f^*}{\omega Y}\left(\frac{\Pi}{\gamma\bar{\Pi}}-1\right)<1$$
$$s_y\frac{\alpha}{1-\alpha}+1>\frac{\Pi}{\gamma\bar{\Pi}}$$
$$\frac{\gamma\bar{\Pi}}{\Pi}\left(s_y\frac{\alpha}{1-\alpha}+1\right)>1$$

The linearization of the global AD curve (equation ($A.16$)) and the domestic AS curve (equation ($A.17$)) around the asymmetric stagnation steady state imply identical expressions to the linearized equilibrium conditions in Proposition 2 where the coefficients are given by:

$$\gamma_w=\frac{\gamma\bar{\Pi}}{\Pi_{ss}}$$
$$s_y=\frac{\omega Y_{ss}}{\omega Y_{ss}-D^W-(1-\omega)Y_f^*}$$

where Π_{ss} and Y_{ss} are the solution to steady state equilibrium conditions ($A.16$) and ($A.17$). Substituting the linearized AS curve into the linearized AD curve as in Proposition 2 provides a forward-looking difference equation in y_t. Local determinacy requires the coefficient on $E_t y_{t+1}$ to be less than unity. This condition is identical to slope condition derived above implying that the asymmetric stagnation equilibrium is always locally determinate, as required. □

B Fiscal Policy Coordination

Non-Cooperative Game

The government in each country *unilaterally* chooses its own level of government spending G to minimize the deviations of output, inflation and level of government spending from their own respective target levels. We assume both countries have identical aggregate supply curves - same full-employment level of output, labor share, and degree of wage rigidity. The policy objective and constraints is given below:

$$\min_{G,Y,Y^*,\Pi} (Y - Y_f)^2 + (\Pi - 1)^2 + (G - G_{target})^2$$

$$\text{s.t. } \omega Y + (1-\omega) Y^* = D^W + \omega G + (1-\omega) G^* + \psi^W \Pi$$

$$Y = \left(\frac{1 - \frac{\gamma}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f$$

$$Y^* = \left(\frac{1 - \frac{\gamma}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f$$

where $D^W = \omega D + (1-\omega) D^*$ and $\psi^W = \frac{1+\beta}{\beta}(1+g) D^W$.

By substituting the domestic and foreign aggregate supply curves into the objective function and global aggregate demand curve, we obtain the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}(Y_{AS}(\Pi) - Y_f)^2 + \frac{1}{2}(\Pi - 1)^2 + \frac{1}{2}(G - G_{target})^2 \\ + \lambda \left(\omega Y_{AS}(\Pi) + (1-\omega) Y^*_{AS}(\Pi) - D^W - \omega G - (1-\omega) G^* - \psi^W \Pi \right)$$

The first-order conditions are given below:

$$0 = \frac{dY_{AS}}{d\Pi}(Y_{AS}(\Pi) - Y_f) + \Pi - 1 + \lambda \left(\frac{dY_{AS}}{d\Pi} - \psi^W \right)$$

$$0 = G - G_{target} - \lambda \omega$$

where λ is the Lagrange multiplier on the global aggregate demand curve. Given that the domestic economy is in secular stagnation: $Y < Y_f$ and $\Pi < 1$ and $\frac{dY_{AS}}{d\Pi} > \psi^W > 0$ - and the slope of the AS curve exceeds the slope of the AD curve, the multiplier $\lambda > 0$ and the fiscal authority in secular stagnation always chooses a level of government spending that exceeds the target. The level of government spending above target is increasing in ω.

Cooperative Game

We now consider the optimal level of government spending when both countries jointly maximize their welfare. The loss function of the global planner is the weighted sum of each country's loss function. Given that the aggregate supply curve are identical and if we assume that the target level of government expenditures is the same, we obtain the following loss function subject to a global aggregate demand and global aggregate supply constraints:

$$\min_{G,Y,\Pi} (Y - Y_f)^2 + (\Pi - 1)^2 + (G - G_{target})^2$$

$$\text{s.t. } Y = D^W + G + \psi^W \Pi$$

$$Y = \left(\frac{1 - \frac{\gamma}{\Pi}}{1-\gamma} \right)^{\frac{\alpha}{1-\alpha}} Y_f$$

where $D^W = \omega D + (1-\omega) D^*$ and $\psi^W = \frac{1+\beta}{\beta}(1+g) D^W$. In the cooperative setup, Y is global output (instead of output of the domestic country only) and G is global government spending. Relative to the non-cooperative setup, the only difference is that the planner chooses G and G^* simultaneously.

The first-order conditions for the optimal level of global government spending is given below:

$$0 = \frac{dY_{AS}}{d\Pi}\left(Y_{AS}(\Pi) - Y_f\right) + \Pi - 1 + \lambda\left(\frac{dY_{AS}}{d\Pi} - \psi^W\right)$$
$$0 = G - G_{target} - \lambda$$

where the multiplier is the same as in the case of the non-cooperative game. The only difference is that ω no longer appears in the second optimality condition.

Proposition 7 *Consider two countries in symmetric secular stagnation with identical aggregate supply parameters, loss functions, and target levels of government spending. Then coordinated optimal government spending exceeds uncoordinated government spending. Coordination losses are maximized when $\omega = \frac{1}{2}$.*

Proof. Global government spending under coordination and absent coordination are given below along with the Lagrange multiplier, λ:

$$G_{coop} = G_{target} + \lambda$$
$$G_{non-coop} = G_{target} + \lambda\left(\omega^2 + (1-\omega)^2\right)$$
$$\lambda = \frac{\frac{dY_{AS}}{d\Pi}\left(Y_f - Y_{AS}(\Pi)\right) + 1 - \Pi}{\frac{dY_{AS}}{d\Pi} - \psi^W}$$

where G_{coop} is global government spending under cooperation and $G_{non-coop}$ is global government spending absent coordination. Since $\omega \leq 1$, $G_{non-coop} \leq G_{coop}$. The term $\omega^2 + (1-\omega)^2$ is minimized at $\omega = \frac{1}{2}$ implying that losses from coordination are maximized when the two countries have the same size. □

Corollary *Consider N countries in a symmetric secular stagnation with identical aggregate supply parameter, loss function, and target levels of government spending. Global government spending absent coordination goes to zero as $N \to \infty$.*

Proof. Since countries are identical, $\omega = \frac{1}{N}$. The optimality condition for government spending for each country is given by the first-order condition of the non-cooperative game. Therefore, global government spending absent coordination is given by the expression below:

$$G_{non-coop} = G_{target} + \lambda N \omega^2$$
$$= G_{target} + \lambda \frac{1}{N}$$

where the second term goes to zero as $N \to \infty$. □