Profit-Sharing, Wisdom of the Crowd, and Theory of the Firm*

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Abstract

I show that simple profit-sharing contracts with decentralized control could empower individuals with wisdom of the crowd by coordinating their actions guided by dispersed private information. This result parallels existing theories for financial markets, where the equilibrium market price achieves an information aggregation effect through rational expectations. The wisdom of the crowd effect of a well-designed profit-sharing contract sheds new light on the nature of the firm: on a macro-level, joint-stock companies endogenously emerge to complete the market; while on a micro-level, profit-sharing speaks to optimal corporate governance structures, and guides security design for some new financing practices (e.g. crowdfunding).

Keywords: contract versus market, crowdfunding, information aggregation, incomplete financial market, investment theory, security design, rational expectation

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Many economic activities benefit from a population's collective wisdom. The recent emergence of crowdfunding has given a good example. This paper shows that by coordinating actions guided by dispersed private information, *simple* profit-sharing contracts can empower individuals with wisdom of the crowd. While decentralized possession of information has long been recognized in analyzing the market economy in general (Hayek (1944, 1945)) and the financial market in specific (e.g. Hellwig (1980), Diamond and Verrecchia (1981)), less is known about its implications for non-market institutions. Studying optimal profit-sharing through the lens of wisdom of the crowd achieves multiple goals: first, it provides practical guidance for some new financing practices (e.g. crowdfunding); second, it sheds new light on an old tension between firms and markets (Coase (1937)); third, it identifies a point of departure for future economic analysis that is microfounded on the economic doctrine of private benefit maximization.[1]

I cast the current paper in a setting where multiple individuals provide a homogeneous productive input to a production technology. To illustrate, consider a simple thought experiment. Alice and Bob are two deep-pocketed and identically risk-averse investors. They individually decide on how much money to invest in a risky business opportunity. Alice and Bob have different assessments of the return from the business, as they rely on independent yet unbiased private information sources to update their posterior beliefs. Although neither investor has access to the other's private information, it is common knowledge that Alice's private signal is more accurate than Bob's.[2] Then the question is, does Alice ever

[1]Ideological arguments over profit-sharing abound. For example, the Bible champions equal pay regardless of the amount of contribution (Matthew 20: The Parable of the Workers in the Vineyard); the Jewish classic Babylonian Talmud suggests a quasi-equal "loss" sharing principle in bankruptcy settlement (Kethuboth 93a); the socialism principle "to each according his contribution" (Marx (1875)), applied to an investment setting, suggests splitting profits according to individual investment amount (pro-rata sharing).

[2]To be mathematically precise, assume that the net return and private signals all follow normal distributions, i.e. net return $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ while investor i's signal $s_i = r + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$, $\epsilon_i \perp\!\!\!\perp \tilde{r}$, $i \in \{A, B\}$, and r denotes the realization of $\tilde{r}$. Information sources being independent indicates that $\epsilon_A \perp\!\!\!\perp \epsilon_B$, while Alice's information being more accurate indicates that $\tau_A > \tau_B$. Since both investors are deep-pocketed, their preferences feature no wealth effect, and could be summarized by a constant absolute risk-aversion (CARA) utility function $u(W) = -e^{-\rho \cdot W}$ for some $\rho > 0$. Such CARA-normal setup is

benefit from sharing investment profits with Bob? If yes, what is the preferred sharing rule considering Alice's information advantage?

Section 1 analyzes this example in detail and proves a somewhat surprising result: despite Alice's superior information quality, as a Nash equilibrium outcome they both prefer a fifty-fifty profit-sharing contract. Under a fifty-fifty deal, both Alice and Bob will obtain a payoff that is exactly the same *as if* he or she had access to the other's private information (even though he/she actually does not). In another word, (appropriate) profit-sharing harnesses the wisdom of the crowd. Although the fifty-fifty contract is just a special result due to identical preference, it embodies a more general insight: in a world featuring decentralized possession of information among many heterogeneous players, some *simple* profit-sharing contracts, independent of each individual's private information or informedness, could coordinate individual actions to the effect of empowering them with wisdom of the crowd.

The optimal profit-sharing contract developed in this paper has three nice properties. First, it is *first best*. Section 2 proves that optimal profit-sharing fully employs wisdom of the crowd when preferences feature no confounding wealth effects (as is standard in the investment literature), production technology is constant return to scale (as in most investment cases), and private signals fall into a large class of exponential family distributions (including standard cases of normal idiosyncratic noises).[3] Other than these standard assumptions, perfect coordination holds for *any* prior distributions of the business return. Second, it is *simple*. The optimal sharing contract *only* requires information on risk preferences. In particular, it does not depend on how well-informed each individual is, which is private information and often hard to solicit. Such simplicity makes implementing the optimal profit-sharing contracts particularly easy. For example, on a crowdfunding platform, all information needed to determine the optimal sharing rule can be proxied by standard questionnaire items on

standard among investment models in finance started by Lintner (1965)).

[3] Approximate results under more general setups will be discussed in Section 5.

income, wealth, investment experience, investment objectives, etc. that are readily available at the time of account opening. Third, it is *cost-effective*. Because profit-sharing does not involve direct exchange of private information, no sophisticated communication technology is required, no incentive to encourage participation in communication is asked for, and no worry of individuals telling lies is warranted. A simple profit-sharing contract is all we need.

An optimal profit-sharing contract that harnesses wisdom of the crowd slightly differs from traditional equity contracts. An equity contract is a "revenue-sharing" contract, which splits proportionally the total payoff including each individual's initial investment.[4] Yet a profit-sharing contract splits according to a pre-specified rule the total payoff excluding initial investment. While a traditional equity contract is no worse than a profit-sharing contract when private information is absent, it is strictly dominated by an optimal profit-sharing contract whenever collective wisdom is present. In this sense, equity contracts may be a historical legacy, and modifications are warranted as we enter a knowledge economy.

Even though explicit profit-sharing contracts as described in the current paper are yet to be widely adopted, for hundreds of years implicit profit-sharing contracts have underlied the structures of many joint-stock companies, in which multiple joint-owners share residual earnings (according to a pre-specified rule) from and (individually) provide productive inputs to a particular technology.[5] Section 3 formalizes this idea, presents historical and contemporary evidence, and illustrates the rise of (inefficient) "moonlighting" incentives when a profit-sharing contract is not well-designed.[6]

While the role of joint-stock companies in harnessing wisdom of the crowd has received little attention, similar rationales have long been accepted as a main function of market prices. In the literature on financial innovation (e.g. Grossman (1977)), a new security/market

[4]Indeed revenues are often, though not always, spit in proportion to each individual's initial investment.
[5]This anatomy of joint-stock companies makes precise the nexus of contracts view in theory of the firm.
[6]A joint-owner moonlights when she contributes to her private enterprise in addition to the firm in which she holds partial ownership.

provides a new price (an endogenous variable) for decisions to be contingent upon, thus permitting indirect information aggregation. Analogously, in this paper a well-designed profit-sharing contract makes each participant's action contingent upon others' actions (also endogenous variables). Both mechanisms help individuals make more informed decisions and largely complement each other, yet a joint-stock company is particularly helpful when the market is absent or "noises" prevent the price from fully revealing. A joint-stock company could thus be interpreted as an institutional innovation in response to market incompleteness. Section 4 uses the setup developed in Section 3 to formalize the insight that the creation of a joint-stock company completes the market. The analysis also provides a new perspective to look at the long-discussed relationship between a firm and its surrounding market economy.

I would be remiss not to mention the multiple forces in reality that could counteract the power of a profit-sharing contract. Section 5 investigates these forces in the context of a joint-stock company. In such a context, these forces also help delineate the boundary of a firm. A firm's boundary is determined by a trade-off between the benefits from wisdom of the crowd and the costs due to free-riding or adverse market power (decreasing return to scale technology). In the presence of these frictions, profit-sharing cannot coordinate individuals perfectly, yet it still dominates alternatives including direct communication of private information (even when it is costless). This is because truthful communication is strictly incentive incompatible under those frictions. For example, in the presence of adverse market power, each player has strict incentive to understate her private information when asked, in hope of less competition if her lie is believed. This strict incentives to lie shuts down the direct communication channel, and compensations featuring more or less profit-sharing elements remain dominant. I illustrate this point with a simple numerical example.

The rest of the paper is organized as follows. Section 1 analyzes the solution to the Alice-Bob question. Section 2 contains more discussions on sufficient conditions for perfect information aggregation. Section 3 sets up a workhorse model for joint-stock companies, and derives the optimal profit-sharing contract. Section 4 explores the relationship between a firm and the market within my framework and illustrates how a firm endogenously arise in an incomplete market. Section 5 investigates forces that shape firm boundaries. Section 6 discusses general implications on various corporate governance topics. Section 7 relates existing literature. Section 8 concludes.

1 Analysis of the Alice-Bob Example

This section proves that, despite more accurate private signal, Alice always finds it optimal for herself to *equally* share total investment profit with Bob. So does Bob.

As a recap of the example, both Alice and Bob have preferences summarized by a utility function $u(W) = -e^{-\rho \cdot W}$ for some $\rho > 0$, and they individually decide on how much money to invest in a business opportunity with net return $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$.[7] The two investors have different opinions on $\tilde{r}$, as they rely on independent yet unbiased private information sources to update posterior beliefs. Mathematically, investor i has an unbiased private signal of the project return $s_i = r + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$, ϵ_i is independent of $\tilde{r}$, $i \in \{A, B\}$, and r denotes the realization of $\tilde{r}$. Neither investor has access to the other's private information.

Given an equal division of investment profits, investor i's problem is to choose an investment amount x_i based on s_i such that

$$x_i(s_i) = \operatorname{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x + \tilde{x}_{-i}(s_{-i})]} | s_i],$$

where $i \in \{A, B\}$ and $-i = \{A, B\} \backslash \{i\}$. Because the optimum to the right hand side depends on i's belief of $x_{-i}(s_{-i})$, the solution is given in a Nash equilibrium.

[7] In another word, if the total payoff to the business is $\tilde{v}$, then $\tilde{r} = \tilde{v} - 1$ given no discounting over time. The normality assumption here is just for exposition ease. As Section 2 will show, $\tilde{r}$ could follow any prior distribution and results remain unchanged.

Definition A Nash Equilibrium under equal profit-sharing in the Alice-Bob example consists of two investment functions $x_A(\cdot)$ and $x_B(\cdot)$ such that

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x+\tilde{x}_{-i}(s_{-i})]}|s_i], \tag{1}$$

where $i \in \{A, B\}$ and $-i = \{A, B\} \backslash \{i\}$.

Before solving the Nash equilibrium explicitly, let's first discuss intuitively how could a profit-sharing contract change Alice's and Bob's incentives and thus empower them with their collective wisdom (of a crowd of size two).

The exponent on the right hand side of (1) is the sum of two parts: $-\frac{1}{2}\rho\tilde{r}x$ and $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$. The first part $-\frac{1}{2}\rho\tilde{r}x$, compared to $-\rho\tilde{r}x$ when there is no profit-sharing, divides the sensitivity of i's payoff to her (his) investment decision by two. Hence it appears *as if* profit-sharing makes investor i half less risk-averse.[8] This observation indicates that investor i could invest more *aggressively*, and in particular be more responsive to her (his) own signal, thus enhancing aggregate use of private information.

Such aggressiveness however could potentially lead to (inefficient) overuse of prior information. This negative effect, however, is counteracted by the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$, which involves an interaction between $\tilde{r}$ and investor $-i$'s investment. Because private signals are correlated due to the common component r, Alice (Bob) would worry that when she (he) has a high signal and invests a lot, so would Bob (Alice). Intuitively this concern would make Alice (Bob) act more *conservatively* to the (public) prior, balancing the overuse tendency. An optimal profit-sharing contract is expected to obtain a perfect balance.

To understand why a fifty-fifty contract is optimal when Alice and Bob have the same preference, we again look at the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$. Because investor $-i$'s investment is a function of $-i$'s private information, the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$ *effectively* exposes

[8] In certain sense, risk aversion deters prevents the full use of information. Profit-sharing, thus like risk-sharing, encourages the use of information.

investor i to $-i$'s private information. Given the same preference, only when Alice and Bob agree to follow a fifty-fifty sharing contract would Alice (Bob) act exactly the same as what Bob (Alice) would like to had he (she) got access to her (his) private information. In another word, a fifty-fifty profit-sharing profit-sharing contract perfectly aligns both investors' incentives and makes each investor a perfect "agent" for the other. The aggregate use of private information gets most enhanced in this way.

I explicitly solve the equilibrium and confirm the intuitions above. I will prove the existence and uniqueness (up to a constant) of the Nash equilibrium under a more general setting in Section 2. However, in a special case in which all random variables are normally distributed, a linear Nash equilibrium (which happens to be the unique Nash equilibrium) is easily obtained via guess and verify. Assume

$$x_i(s_i) = \alpha + \beta_i s_i,$$

then equation (1) leads to

$$\alpha + \beta_i s_i \quad = \quad \text{argmax}_x - \mathbb{E}[e^{\left(-\frac{1}{2}\rho\tilde{r}\right)(x+\alpha+\beta_{-i}\tilde{s}_{-i})}|s_i]. \tag{2}$$

The conditional expectation on the right hand side of (2) is similar to the moment-generating function of a non-central χ^2-distributed random variable (because both $-\frac{1}{2}\rho\tilde{r}$ and $x+\alpha+\beta_{-i}\tilde{s}_{-i}$, an affine transformation of the normal variable $\tilde{s}_{-i}$, follow normal distributions), which has a closed-form expression given by the following lemma.

Lemma 1.1. *If* $\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right)$, *where* $(\rho\sigma_1\sigma_2 - 1)^2 > \sigma_1^2\sigma_2^2$ *then*

$$\mathbb{E}\left[e^{\tilde{y}_1\tilde{y}_2}\right] = \frac{\exp\left\{(\theta_2^2\sigma_1^2 - 2\rho\theta_1\theta_2\sigma_1\sigma_2 + \theta_1^2\sigma_2^2 + 2\theta_1\theta_2)/(2[(\rho\sigma_1\sigma_2-1)^2 - \sigma_1^2\sigma_2^2])\right\}}{\sqrt{(\rho\sigma_1\sigma_2-1)^2 - \sigma_1^2\sigma_2^2}}.$$

Proof. Standard integration. $\square$

Plug in $-\frac{1}{2}\rho\tilde{r}$ and $x+\alpha+\beta_{-i}\tilde{s}_{-i}$ into Lemma 1.1, and notice that conditional on s_i,

$$\begin{bmatrix} -\frac{1}{2}\rho\tilde{r} \\ x+\alpha+\beta_{-i}\tilde{s}_{-i} \end{bmatrix}_{|s_i} \sim \mathcal{N}\left(\begin{bmatrix} -\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \\ x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \end{bmatrix}, \begin{bmatrix} \frac{\rho^2}{4(\tau_r+\tau_i)} & -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} \\ -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} & \beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) \end{bmatrix} \right),$$

thus the expectation on the right hand side of (2) is equal to

$$-\frac{\exp\left\{\frac{\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\frac{\rho^2}{4(\tau_r+\tau_i)}-\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\frac{\rho\beta_{-i}}{\tau_r+\tau_i}+\left(\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)-\rho\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)}{2\left[\left(-\frac{1}{2}\frac{\rho\beta_{-i}}{\tau_r+\tau_i}-1\right)^2-\frac{\rho^2}{4(\tau_r+\tau_i)}\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)\right]}\right\}}{\sqrt{\left(-\frac{1}{2}\frac{\rho\beta_{-i}}{\tau_r+\tau_i}-1\right)^2-\frac{\rho^2}{4(\tau_r+\tau_i)}\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)}}.$$

Notice that x, the variable we maximize over, only enters the numerator of the exponent in the above expression in a linear-quadratic function, thus (2) leads to

$$\begin{aligned} \alpha+\beta_i s_i &= \operatorname{argmin}_x \Bigg[\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\frac{\rho^2}{4(\tau_r+\tau_i)}-\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\frac{\rho\beta_{-i}}{\tau_r+\tau_i} \\ &+ \left(\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)-\rho\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\Bigg] \\ &= \frac{2}{\rho}(\tau_r\bar{r}+\tau_i s_i)-\alpha. \end{aligned}$$

Matching coefficients gives $\alpha=\frac{1}{\rho}\tau_r\bar{r}$ and $\beta_i=\frac{2}{\rho}\tau_i$, leading to

$$\begin{cases} x_A = \frac{1}{\rho}(\tau_r\bar{r}+2\tau_A s_A) \\ x_B = \frac{1}{\rho}(\tau_r\bar{r}+2\tau_B s_B) \end{cases}.$$

The 2-s in the second terms of the above expressions confirm our first intuition that under profit-sharing, each investor becomes more responsive to her (his) private information, while the absence of 2-s in the first terms reflects a perfect "canceling out" effect between the simultaneous aggressiveness and conservativeness under an optimal profit-sharing contract.

For any particular joint realization of return and private signals, investor i's payoff under equal profit-sharing is

$$r\frac{x_A(s_A)+x_B(s_B)}{2} = \frac{r}{\rho}(\tau_r\bar{r}+\tau_A s_A+\tau_B s_B). \tag{3}$$

Let's compare this outcome with a full-information benchmark. When Alice could (hypothetically) make independent investment decisions based on both her private signal and Bob's, her investment amount would be given by

$$\begin{aligned} x'_A(s_A,s_B) &= \text{argmax}_x \mathbb{E}[-e^{-\rho r x}|s_A,s_B] \\ &= \text{argmax}_x - e^{-\rho\mathbb{E}(r|s_A,s_B)x+\frac{1}{2}\text{Var}(r|s_A,s_B)\rho^2x^2} \\ \Rightarrow x'_A(s_A,s_B) &= \frac{\mathbb{E}(r|s_A,s_B)}{\rho\text{Var}(r|s_A,s_B)} \\ &= \frac{1}{\rho}(\tau_r\bar{r}+\tau_A s_A+\tau_B s_B). \end{aligned}$$

and thus payoff $rx'_A(s_A,s_B)=\frac{r}{\rho}(\tau_A s_A+\tau_B s_B)$, which is exactly the same as under profit-sharing (expression (3)), confirming our second intuition that the increased total use of information benefits both investors perfectly.

The above observation is summarized below.

Theorem 1.2. *For all realizations of the state of nature $\{r, s_A, s_B\}$, each investor's payoff under equal division of profits is always equal to that under a full-information benchmark. So does the expected utility. A well-designed profit-sharing contract perfectly empowers collective wisdom, leading to best outcomes for both investors.*

It is worth noting that unlike other studies on the efficient use of information (e.g.

Angeletos and Pavan (2007), Amador and Weill (2010)), where strategic complementarity/substitutability in technology plays an important role, in my model there is no strategic interaction between Alice and Bob from the technology itself.[9] It is the profit sharing contract that introduces strategic interdependencies between the two investors. To see the point more concretely, consider a hypothetical case in which Alice is forced to only get half of her investment profit, and does not enjoy the half contributed by Bob. In this no strategic interdependence case Alice would invest $\frac{1}{\rho}(2\tau_r\bar{r} + 2\tau_A s_A)$. In comparison, under profit-sharing there is no 2 in front of the first term in x_A (Alice's investment), due to Alice's "conservativeness" toward Bob's correlated actions when she is exposed to Bob's contribution.

The power with wisdom of the crowd involves averaging conditionally (on r) independent signals (yet unconditional correlated because of the common term r), which is mathematically founded on law of large numbers. It apparently reminds of what has become conventional wisdom since the seminal work of Markowitz (1952) that proper diversification achieves optimal return-risk trade-off.[10] Indeed in the CARA-normal setup, under profit-sharing part of investor i's compensation comes from one half of the expected value of her contribution while bearing only a quarter of its variance. However, there are several differences between Theorem 1.2 and traditional diversification arguments. First, diversification in portfolio theory relies on pooling multiple assets, yet Theorem 1.2 only considers one single business, and "diversification" is achieved via teaming agents. Second, portfolio theory usually does not involve asymmetric information, yet Theorem 1.2 requires dispersed private information. Without private information (i.e., $\tau_e = 0$), profit-sharing would make no difference. Third, as the next section will show, my result extends beyond normal distributions, while traditional mean variance analysis crucially depends on the absence of higher (than second) moments.

[9] The setup is flexible enough to incorporate primitive strategic interdependencies in technology, which will be discussed in Section 5.4 and Appendix D.

[10] It is obvious that signals being conditionally (on r) independent is not necessary. As long as they are conditionally imperfectly correlated, wisdom of the crowd has power.

2 Profit-sharing as an Information Aggregator

This section relaxes the normality assumption in the previous example, and explores sufficient conditions for profit-sharing to be a perfect information aggregator.

Denote $u(W) = -e^{-\rho W}$, and consider general distributions of r and $s_i, i \in \{A, B\}$. Under a full-information benchmark $x_i(s_i, s_{-i})$ maximizes

$$\begin{aligned}
& \mathbb{E}\left[u(x\tilde{r})|s_i, s_{-i}\right] \\
= & \int u(xr) f(r|s_i, s_{-i}) dr \\
= & \int u(xr) f(s_{-i}|r, s_i) f(s_i|r) f(r) \frac{1}{f(s_i, s_{-i})} dr \\
= & \int u(xr) f(s_{-i}|r) f(s_i|r) f(r) \frac{1}{f(s_i, s_{-i})} dr, (\because s_i \perp\!\!\!\perp s_{-i}|r),
\end{aligned}$$

or equivalently $x(s_i, s_{-i})$ maximizes

$$\int u(xr) f(s_{-i}|r) f(s_i|r) f(r) dr \tag{4}$$

Assume the profit-sharing agreement stipulates that investor i gets α_i of the total profit ($\sum_i \alpha_i = 1$), then under profit-sharing $x_i(s_i)$ maximizes (in a Nash equilibrium)

$$\begin{aligned}
& \mathbb{E}\left[u\left(\alpha_i x\tilde{r} + \alpha_i x_{-i}(s_{-i})\tilde{r}\right)|s_i\right] \\
= & \iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(r, s_{-i}|s_i) ds_{-i} dr \\
= & \iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(s_{-i}|r) f(s_i|r) f(r) \frac{1}{f(s_i)} ds_{-i} dr,
\end{aligned}$$

or equivalently $x(s_i)$ maximizes

$$\iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr \tag{5}$$

Taking first-order conditions we have that

$$
\begin{aligned}
(4) \Rightarrow \quad & \int u'(x_i(s_i, s_{-i})r) r f(s_{-i}|r) f(s_i|r) f(r) dr = 0 && (6)\\
(5) \Rightarrow \quad & \iint u'\left(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r\right) r f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr = 0, && (7)
\end{aligned}
$$

where (with some abuse of notation) $x(s_i, s_{-i})$ denotes the optimal investment amount given signal s_i and s_{-i} under the full information benchmark, while $x_i(s_i)$ denotes investor i's investment amount given signal s_i in the profit-sharing Nash equilibrium.

In order to keep tractability, in the spirit of Breon-Drish (2015), I further assume that the likelihood function of r given private signals $s_i, i \in \{A, B\}$ lies in an *exponential family*, an assumption extensively used in Bayesian statistics and decision theories to preserve closed-form expression.[11] Precisely, assume that

$$
f(s_i|r) = h_i(s_i) e^{r k_i s_i} g(r)
$$

for some constant k_i and positive function $h_i(\cdot)$. then

$$
\begin{aligned}
(6) \quad \Rightarrow \quad & \int e^{-\rho x_i(s_i, s_{-i}) r} r h_{-i}(s_{-i}) e^{r k_{-i} s_{-i}} g(r) h_i(s_i) e^{r k_i s_i} g(r) f(r) dr = 0 && (8)\\
(7) \quad \Rightarrow \quad & \iint e^{-\rho(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r)} r h_{-i}(s_{-i}) e^{r k_{-i} s_{-i}} g(r) h_i(s_i) e^{r k_i s_i} g(r) f(r) ds_{-i} dr = 0 && (9)
\end{aligned}
$$

thus (factoring out $h_i(s_i)$)

$$
\begin{aligned}
(8) \quad \Rightarrow \quad & \int e^{-\rho x_i(s_i, s_{-i}) r + r k_{-i} s_{-i} + r k_i s_i} r g^2(r) f(r) dr = 0 && (10)\\
(9) \quad \Rightarrow \quad & \iint e^{-\rho(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r)} r h_{-i}(s_{-i}) e^{r k_{-i} s_{-i} + r k_i s_i} g^2(r) f(r) ds_{-i} dr = 0 \\
\Rightarrow \quad & \int e^{-\rho \alpha_i x_i(s_i) r + r k_i s_i} r g^2(r) f(r) \left(\int e^{-\rho \alpha_i x_{-i}(s_{-i}) r} h_{-i}(s_{-i}) e^{r k_{-i} s_{-i}} ds_{-i} \right) dr = 0 && (11)
\end{aligned}
$$

[11] E.g. exponential family is particularly useful for deriving conjugate priors.

We thus have the following result

Theorem 2.1. *Under the full-information benchmark, equation (10) has a unique solution, which is linear in s_i and s_{-i}. Similarly, under any profit-sharing agreement (i.e. for any given α_i), equation (11) has a unique Nash equilibrium, in which investor i's strategy is linear in s_i, $\forall i$. When the profit-sharing agreement is optimally designed, profit-sharing obtains the same payoff as in the full-information benchmark.*

Proof. Consider the equation of x

$$\int e^{xr} r g^2(r) f(r) dr = 0. \tag{12}$$

Taking derivative with respect to x immediately tells that equation (12) has at most one solution, denoted as X. Compared to equation (10) we get $x_i(s_i, s_{-i}) = \frac{1}{\rho}(k_{-i}s_{-i} + k_i s_i - X)$.

Similarly, consider the equation of x

$$\int e^{rx} r g^2(r) f(r) H_{-i}(r) dr = 0,$$

where $H_{-i}(r) = \int e^{-\rho\alpha_i x_{-i}(s_{-i})r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} > 0$. Taking derivative with respect to x immediately tells that the equation features at most one solution (for a given $x_{-i}(s_{-i})$). Compared to equation (11) we get that $x_i(s_i) = \frac{k_i s_i - C}{\rho\alpha_i}$, where C is a constant such that

$$\int e^{rC} r g^2(r) f(r) H_{-i}(r) dr = 0. \tag{13}$$

By the same logic, $x_{-i}(s_{-i}) = \frac{k_{-i}s_{-i} - C'}{\rho\alpha_{-i}}$, where C' is also a constant such that

$$\int e^{rC'} r g^2(r) f(r) H_i(r) dr = 0, \tag{14}$$

where $H_i(r) = \int e^{-\rho\alpha_{-i}x_i(s_i)r} h_i(s_i) e^{rk_i s_i} ds_i > 0$. Plug in x_i and x_{-i} into (13) , we have

$$\int e^{rC} rg^2(r) f(r) \int e^{\frac{\alpha_i}{\alpha_{-i}}(C' - k_{-i}s_{-i})r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} dr = 0. \tag{15}$$

If $\alpha_i = \alpha_{-i} = \frac{1}{2}$, equation (15) simplifies into (after factoring out $\int h_{-i}(s_{-i}) ds_{-i}$)

$$\int e^{r(C'+C)} rg^2(r) f(r) dr = 0. \tag{16}$$

Since equation (12) has at most one solution, we have $C + C' = X$. Thus under profit-sharing the payoff to investor i for a given realization of r and private signals is

$$\begin{aligned}
& \alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r \\
= \; & \alpha_i \frac{k_i s_i - C}{\rho \alpha_i} r + \alpha_i \frac{k_{-i} s_{-i} - C'}{\rho \alpha_{-i}} r \\
= \; & \frac{r}{\rho}(k_i s_i - C + k_{-i} s_{-i} - C'), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2} \\
= \; & \frac{r}{\rho}(k_i s_i + k_{-i} s_{-i} - X), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2},
\end{aligned}$$

which exactly equals to $rx(s_i, s_{-i})$, or the payoff under a full information benchmark. □

Theorem 2.1 can be easily extended to scenarios with more than two investors of heterogeneous risk preferences. The exact math is tedious but straightforward, and is omitted for the sake of brevity.[12] Section 3, however, provides an illustrative example with normal distributions, which also embeds a workhorse model to describe a joint-stock company.

[12] I conjecture yet has been able to prove that under more general settings nonlinear sharing rules exist to obtain perfect information aggregation.

3 Workhorse Model for a Joint-Stock Company

The structure of modern corporations have evolved into prohibitive complicacy, yet they do share some common features. At a high level, a firm's business could be captured by a risky production technology, to which a set of "owners" provide (usually relatively homogeneous) production inputs (capital, labor, or raw ingredients, etc.) based on their assessment of the business productivity, and from which the same set of owners share residual earnings according to a pre-specified rule. This primitive description of firms with emphases on profit-sharing and risk-taking follows corporate legal literature (Hansmann (2009)), and is linguistically consistent with English tradition – among the synonyms of a firm are "company" (profit-sharing among multiple owners), and "venture" (risky business). Firms in this form has been accompanying human history ever since Queen Elizabeth granted the East India Company its first Royal Charter on December 31st, 1600 AD, and even today still underlies partnerships (e.g. private equity/venture capital firms), producer cooperatives, joint-ventures, and (to a less extent) all other firms except for sole-proprietorship.

Insights learned from the Alice-Bob example suggest a possibility that joint-stock companies, when properly structured, could empower their owners with wisdom of the crowd. This section formalizes this idea.

3.1 The Firm as a Profit-Sharing Coalition

A firm is defined by a charter, which stipulates a compensation scheme among its n owners (players). Upon firm creation in period $t = 0$, the charter entitles owner i, who has a constant absolute risk aversion parameter ρ_i, of a_i of the firm's residual earning to be realized by the end of period $t = 1$, where $\sum_{i=1}^{n} a_i = 1$. The firm has a constant-return-to-scale production technology $Y = vX$, where Y is the total revenue, $v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$ is a stochastic factor productivity, and X is total amount of productive input contributed by all the owners, i.e. $X = \sum_{j=1}^{n} x_i$, where x_i is player i's productive input contribution.[13] The unit cost of productive input is denoted as p, which is a constant and without loss of generality normalized to zero in this section for brevity.[14] The wisdom of the crowd assumption indicates that each player has some private knowledge in assessing the stochastic factor productivity. Assume player i's private knowledge $s_i = v + e_i$, where $v \perp\!\!\!\perp e_i$ and $e_i \sim \mathcal{N}(0, \tau_j^{-1})$. Each firm owner independently decides on how much production input to contribute to the firm.

The input provided by the n owners of the firm is given in a Nash equilibrium. In particular, player i chooses x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i v(x_i + \sum_{k \neq i} x_k)\right]\right)|s_i\right], \tag{17}$$

given her perception of other players' equilibrium productive input x_k, $k \neq i$. The following theorem provides a linear Nash equilibrium solution for a given player.

Theorem 3.1. *A linear Nash equilibrium exists only when* $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n} \frac{1}{\rho_i}}$, *and in equilibrium player i's productive input could be given by*

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right) \tau_i s_i, \tag{18}$$

Proof. To be subsumed in the proof for Theorem 3.3. □

Given no complementarities in productive inputs, one may be tempted to think that players should be indifferent between running a sole proprietorship or taking part in a firm, nor would firm creation affect real allocation. However, Theorem 3.1 shows that a player

[13] By assuming a constant-return-to-scale production technology, I shutdown any complementarity in players' inputs which would mechanically favor firm creation. For different modeling purposes, existing literature usually assumes non-separable production technologies, e.g. Alchian and Demsetz (1972). In these models agents' productive input choices impose (usually positive) externalities on each other. Such externalities can either come from output (e.g. Kandel and Lazear (1992)) or cost (e.g. Edmans, Goldstein, and Zhu (2011)).

[14] Or equivalently one can assume that $v \sim \mathcal{N}(\bar{v} - p, \tau_v^{-1})$. I will endogenize p in Section 4. I also do not consider any private cost to each player's input supply, which I will address in Section 5.

becomes more dedicated to the project when in a firm whenever she has positive assessment of the project prospect (i.e., x_i increases with n when $s_i > 0$). Because a player's assessment are more likely to be positive for a high-value (v) project, creating a firm (rather than keeping multiple sole proprietorships) helps a good business to receive (probablisticly) higher total productive input. Firm creation improves real allocation.

This result stems from the fact that a firm coordinates its owners and empower them with wisdom of the crowd. Notice that if player i has full information, her input supply would be

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \frac{\sum_{i=1}^{n} \tau_i}{\rho_i} i^*, \text{ where } i^* = \frac{\sum_{k=1}^{n} \tau_k s_k}{\sum_{k=1}^{n} \tau_k}.$$

Thus player i's payoff is

$$v \left[\frac{\tau_v \bar{v}}{\rho_i} + \frac{\sum_{i=1}^{n} \tau_i}{\rho_i} i^* \right]$$

under both full information and in a firm. This is summarized in the following theorem.

Theorem 3.2. *Given her private information, a player's expected utilities of participating in a properly structured n-owner firm is identical to as if she could obtain other $n-1$ players' private information without cost while running a sole proprietorship.*

A direct implication of this result is that creating a joint-stock company raises owners' expected utilities, and thus is a voluntary outcome of economic evolution. The reason why a joint-stock company benefits participating players is again because, first, a well-designed profit-sharing contract empowers them with wisdom of the crowd, and second, it provides a means for human-capital diversification. The relation between the information aggregation effect and risk-reduction suggests that profit-sharing could be alternatively interpreted as an institutional innovation, compared to "financial innovation" based on security design à la Allen and Gale (1994).

3.2 "Moonlighting" under a Suboptimal Sharing Contract

If a firm disregard each partner's risk preference and stipulates arbitrary sharing rules, its owners will have incentives to "moonlight", or to contribute to private enterprises in addition to the firm in which they hold partial ownership. To see this, consider a player i who has a_i shares in a firm choosing x_i and X_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_j v(x_i+\sum_{k\neq i}x_k)+vX_i\right]\right)|s_i\right], \tag{19}$$

given her (correct) anticipation of other players' equilibrium input provision to the firm x_k, $k\neq i$. The following theorem provides a linear Nash equilibrium solution.

Theorem 3.3. *Each player's expected utility is maximized when ownership shares in the firm is divided according to players' risk preferences, i.e.* $a_i=\frac{\frac{1}{\rho_i}}{\sum_{i=1}^n\frac{1}{\rho_i}}$. *In the resulting linear Nash equilibrium, players do not moonlight, that is* $X_i=0$.

Proof of Theorem 3.3. A linear symmetric equilibrium is given by $x_k+\frac{X_k}{a_k}=\pi_k+\gamma_k s_k+\frac{\pi_k+\Gamma_k s_k}{a_k}$ for some π_k and γ_k. Because

$$\begin{bmatrix} -a_i\rho_i v \\ x_i+\frac{X_i}{a_i}+\sum_{k\neq i}x_k\end{bmatrix}_{|s_i}\sim$$

$$\mathcal{N}\left(\begin{bmatrix} -\rho_i a_i\mathbb{E}(v|s_i) \\ x_i+\frac{X_i}{a_i}+\sum_{k\neq i}\pi_k+\sum_{k\neq i}\gamma_k\mathbb{E}(v|s_i)\end{bmatrix},\begin{bmatrix}\rho_i^2a_i^2\mathrm{Var}(v|s_i) & -\rho_i a_i\sum_{k\neq i}\gamma_k\mathrm{Var}(v|s_i)\\ -\rho_i a_i\sum_{k\neq i}\gamma_k\mathrm{Var}(v|s_i) & (\sum_{k\neq i}\gamma_k)^2\mathrm{Var}(v|s_i)+\sum_{k\neq i}\gamma_k^2\tau_k^{-1}\end{bmatrix}\right)$$

by Lemma 1.1, player i equivalently minimizes

$$\begin{aligned}
& \theta_2^2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\theta_2\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + \theta_1^2[(\sum_{k\neq i}\gamma_k)^2\mathrm{Var}(v|s_i) + \sum_{k\neq i}\gamma_k^2\tau_k^{-1}] + 2\theta_1\theta_2 \\
\stackrel{\text{FOC}}{\Longrightarrow} \quad & 2\theta_2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + 2\theta_1 = 0, \\
& \text{where } \theta_1 = -\rho_i a_i \mathbb{E}(v|s_i) \text{ and } \theta_2 = x_i + \frac{X_i}{a_i} + \sum_{k\neq i}\pi_k + \sum_{k\neq i}\gamma_k \mathbb{E}(v|s_i).
\end{aligned}$$

Plugging in $x_i + \frac{X_i}{a_i} = \pi_i + \gamma_i s_i + \frac{\pi_k + \Gamma_k s_k}{a_k}$ leads to

$$[\sum_{k\neq i}\pi_k + \pi_i + \gamma_i s_i + \frac{\pi_k + \Gamma_k s_k}{a_k} + \sum_{k\neq i}\gamma_k\mathbb{E}(v|s_i)]\rho_i^2 a_i^2\mathrm{Var}(v|s_i) - \rho_i a_i\mathbb{E}(v|s_i)\rho a_i\sum_{k\neq i}\gamma_k\mathrm{Var}(v|s_i) - \rho_i a_i \mathbb{E}(v|s_i) = 0$$

and matching coefficients renders $\gamma_i + \frac{\Gamma_i}{a_i} = \frac{\tau_i}{\tau_v\bar{v}}(\Pi + \frac{\pi_i}{a_i})$, $\frac{1}{\tau_v\bar{v}}(\Pi + \frac{\pi_i}{a_i})\rho_i a_i = 1$ ($\Pi \doteq \sum_{i=1}^n \pi_i$), $\square$

Thus expected utility is given by

$$-\frac{\sqrt{\tau_v + \tau_i}\exp\{-\frac{(\tau_v+\tau_i)\mathbb{E}^2(v|s_i)}{2}\}}{\sqrt{\tau_v + \tau_i + 2\rho_i a_i\sum_{k\neq i}\gamma_k - \rho_i^2 a_i^2\sum_{k\neq i}\gamma_k^2\tau_k^{-1}}},$$

which is maximized at $\gamma_i = \frac{\tau_i}{\rho_i a_i}$. Plugging in $\gamma_i + \frac{\Gamma_i}{a_i} = \frac{\tau_i}{\tau_v\bar{v}}(\Pi + \frac{\pi_i}{a_i})$ and $\frac{1}{\tau_v\bar{v}}(\Pi + \frac{\pi_i}{a_i})\rho_i a_i = 1$ lead to the at the optimal γ_i, $\Gamma_i = 0$. Thus for any given sharing rule a_k ($k = 1, \cdots, n$), there exists a linear equilibrium in which each player optimally chooses her amount of input supply both within and outside of a firm. In particular, when a_i is chosen to be $\frac{\frac{1}{\rho_i}}{\sum_{i=1}^n \frac{1}{\rho_i}}$, input supply in the firm can be stipulated so that no player has incentive to work outside of the firm, and the resulting equilibrium gives the highest expected utilities to all players.[15]

[15] In particular, when the players have homogeneous risk preference (but possibly heterogeneous knowledge precisions), a $\frac{1}{n}$ equal sharing rule is optimal. This observation provides an alternative interpretation why some knowledge intensive partnership firms stick to equal profit-sharing. For example, as Dick Kramlich, founder of the famous Silicon Valley venture capital firm New Enterprise Associates (NEA) puts: "...some culture of NEA has never changed: always maintained a democracy among the partners, wherein they all

4 Arise of a Joint-Stock Firm in a Market Economy

An important question in the theory of the firm concerns the relationship between a firm and the outside market. To this end, this section generalizes the workhorse model by allowing the product input cost p to be endogenously determined as an equilibrium market outcome. The setup for the productive input market resembles classic noisy rational expectation models à la Hellwig (1980) and Diamond and Verrecchia (1981). I show that while the market also communicates information through equilibrium price, it is nevertheless dominated by profit-sharing due to the presence of market "noise".[16] In this sense, a joint-stock company arises as a response to market incompleteness caused by asymmetric information.

The market for the productive input consists of a continuum of players with player i having a constant absolute risk aversion ρ_i, $i \in [0,1]$. On $t = 0$, a risky business opportunity with factor of productivity $v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$ emerges. Player i decides on x_i, the optimal amount of productive input to provide to the business. When making decisions, player i has a private signal of the business productivity $s_i = v + e_i$, where v and e_i are independent and $e_i \sim \mathcal{N}(0, \tau_i^{-1})$. A quantity noise $z \sim \mathcal{N}(\bar{z}, \sigma_z^2)$ measures the aggregate demand for the productive input for alternative uses other than the new business opportunity, which carries no information about the new business opportunity, i.e., z is independent of v.

Assume that players $1, 2, \cdots, n$ agree to created a joint-stock company and share profits. Then player i's problem is given by choosing x_i and maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i(v-p)(x_i + \sum_{k\neq i} x_k)\right]\right)|s_i, p\right], \tag{20}$$

have the same draw from the firm's fee and the same participation in the carried interest, or investment profits, from their funds...", see Finkel and Greising (2009) P180.

[16]See Black (1986) for a comprehensive assessment of "noise". Existing literature often attributes the non-informative "noise" to quantity shocks (or noise traders), but this is not necessary. The noise could be interpreted as a reduced-form description of investors' incomplete knowledge about market architecture. Alternatively, it could be viewed as a partial equilibrium outcome, in which some un-modeled outside market also influences price (this is indeed a justification for treating the risk-free rate as exogenous in most noisy rational expectation equilibrium models).

where $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n} \frac{1}{\rho_i}}$. The solution will look like a mixture of a Nash equilibrium studied in Section 3.1, and a noisy rational expectation equilibrium à la Hellwig (1980).

For ease of comprehension, I abstract from the noisy rational expectation equilibrium in the productive input market at the moment, and first present a general result of their optimal input provisions and corresponding expected utilities when the equilibrium input price is (exogenously) given by an arbitrary linear price system.

Theorem 4.1. *In an market economy in which the equilibrium input cost follows a linear function $p = \mu + \pi v - \gamma z$, the optimal input provision amount of a player i in a firm of size n is given by*

$$x_i = \frac{1}{\rho_i}\left[\tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_z^2}(\mu - \gamma \bar{z})\right] + \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right) \tau_i s_i - \frac{1}{\rho_i}\left[\sum_{k=1}^{n} \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_z^2} - \frac{\pi}{\gamma^2 \sigma_z^2}\right] p \quad (21)$$

and her expected utility

$$-\frac{\exp\left(-\frac{1}{2}\frac{1}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}\left[\tau_i s_i - \frac{\pi}{\gamma^2\sigma_z^2}(\mu - \gamma\bar{z}) + \tau_v \bar{v} - (\tau_i + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2} - \frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)}{\sqrt{\frac{\sum_{k=1}^{n} \tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}}}$$

Proof. See Appendix. □

In comparison, under a (hypothetically) symmetric information benchmark, each player will base her decision on the weighted average of the private signals of all n players. Denote $\sum_{k=1}^{n} \tau_k s_k / \sum_{k=1}^{n} \tau_k$ as $s^* = v + e^*$, then $v \perp\!\!\!\perp e^*$ and $e^* \sim \mathcal{N}(0, \frac{1}{\sum_{k=1}^{n} \tau_k})$. With a linear cost system in which $p = \mu + \pi v - \gamma z$, the input provision by member i in an signal-sharing alliance is given by maximizing $\mathbb{E}\left[-\exp(-\rho_i(v-p)x_i|s^*,p\right]$, and thus

$$\begin{aligned} x_i' &= \frac{\mathbb{E}(v|s^*,p)-p}{\rho_i \mathrm{Var}(v|s^*,p)} \\ &= \frac{1}{\rho_i}[\tau_v \bar{v} - \frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z}) + \sum_{k=1}^{n}\tau_k s^* - (\sum_{k=1}^{n}\tau_k + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2} - \frac{\pi}{\gamma^2\sigma_z^2})p] \end{aligned} \tag{22}$$

$$\text{(Notice that } \begin{cases} \mathbb{E}(v|s^*,p) &= \frac{\gamma^2\sigma_z^2\sum_{k=1}^{n}\tau_k s^* + \pi(p-\mu+\gamma\bar{z}) + \gamma^2\sigma_z^2\tau_v\bar{v}}{\gamma^2\sigma_z^2\sum_{k=1}^{n}\tau_k + \pi^2 + \gamma^2\sigma_z^2\tau_v} \\ \mathrm{Var}(v|s^*,p) &= \frac{\gamma^2\sigma_z^2}{\gamma^2\sigma_z^2\sum_{k=1}^{n}\tau_k + \pi^2 + \gamma^2\sigma_z^2\tau_v} \end{cases} \text{)}$$

Compared (21) with (22), it is easy to verify that the total (*ex post*) input provision from a firm and that from players under full information benchmark are identical. Hence equilibrium market price is the same across both structures. In another word, there is an isomorphism in terms of price between under profit-sharing and under symmetric information. This result leads to a similar isomorphism in terms of (*interim*) expected utility, as summarized below.

Theorem 4.2. *A player's interim expected utilities conditioning on her own private signal are the same in a joint-stock company and under a symmetric information benchmark.*

Proof. See Appendix. □

Because profit-sharing benefits from the wisdom of the crowd effect, the following result follows immediately.

Corollary 4.3. *When a joint-stock company is adequate small (so that it has negligible influence to the productive input market), each owner's expected utility strictly increases with the firm size n.*

For any particular market structure, the price function $p = \mu + \pi v - \gamma z$ will be given by market clearing. An example is provided in Section 5.4.

Comment (relationship between a joint-stock company and the outside market):

The expected utility equivalence between a profit-sharing equilibrium and a full information benchmark suggests that a competitive market with dispersed private information is intrinsically unstable. Rational agents will always have incentive to partner with others.[17]

As compared to rational expectation inference from the market price, profit-sharing provides a non-price based mechanism to obtain the effect of aggregating information dispersed among market participants. Indeed, the idea of rational expectation in its most general form assumes that when agents make decisions, they rely on not only their own information, but also further inference from any endogenous variables in the economic system. In a financial market, such endogenous variables are usually equilibrium market prices. When the market price is absent or noisy, creating a new market (thus creating new prices) provides a new information source and completes the market.

When we go beyond the financial market, a lot of other endogenous variables arise, including the total profit contributed by multiple parties. When a profit-sharing contract explicitly links each player's compensation to the other's actions, it makes each player's action contingent on what others do (in a Nash equilibrium), thus empowering them wisdom of the crowd. This effect is particularly useful when a centralized market does not exist, e.g. in private equity/venture capital investment or crowdfunding.

The complementarity between a joint-stock company and the market is reminiscent of the complementarity between banks and markets discussed in Boot and Thakor (1997). In a dynamic version of that model, Song and Thakor (2010) highlight that banks and markets exhibit three forms of interaction: competition, complementarity and co-evolution. My paper thus provides an informational perspective on the complementarity between market

[17] In an investment setting, Ross (2005) develops a theory of delegated wealth management based on the unstableness of noisy rational expectation equilibrium. Similar in spirit, Indjejikian, Lu, and Yang (2014) suggest that the strategic Kyle (1985) equilibrium is not stable, because the most informed would have incentive to leak information to an uninformed trader, so that the other informed traders will trade less aggressively.

and more general institutional structures.

5 Forces that Shape Firm Boundaries

Corollary 4.3 suggests that firm size features (locally) "the more the merrier". However, as a firm looms large, several natural forces would kick in to restrict firm size. An optimal firm size could thus be determined. This section discuss their effect.

5.1 The Boundary of the Wise Crowd

Because the benefit of profit-sharing in the above theory comes from wisdom of the crowd, a straightforward force to restrict firm size is the boundary of the (wise) crowd. Such a boundary is plausible when the particular business in question features agree to disagree or overconfidence, etc. Since overconfident players believe that they are the smartest and having nothing to learn from others, they have no incentive to form a firm with others. Indeed as Muth (1961) in his seminal article on rational expectation points out, rational expectation is built on the assumption of people's humbleness. If this assumption fails, then there is no room for rational expectation, nor for a theory of the firm based on rational expectation.[18] Although whether a particular business line features wisdom of the crowd or not is an empirical question, we can conclude that it would be schizophrenic if we study the market with rational expectation, while refusing to look at firms via the perspective of information aggregation.

[18] Even in an economy where the wisdom of the crowd applies, search frictions (i.e. it takes time to find another players interested in the same business) naturally limits firm size. Note that search dynamic is a key element in information percolation models built on direct communication.

5.2 Costly Information Acquisition

Another force shaping the firm boundary arises when players have to incur some acquisition cost to get their private information. When such acquisition cost is private and not contractible, profit-sharing would lead to a free-riding problem. Free-riding costs trade-off benefits from information aggregation, and determines an optimal firm size. We shall note that such free-riding effect is also present in existing studies of the financial market. Grossman and Stiglitz (1980) rely on this free-riding problem to prove the impossibility of strong form market efficiency. Thus free-riding in information acquisition shall not concern us if our focus is on comparing the relative role of a firm versus the market in knowledge aggregation.

5.3 Private Cost in Providing the Productive Input

Another form of free-riding cost arises when the cost of the productive input is private. Such cost resembles the moral hazard in teams problem studied by Holmström (1982). In this case, the decision to form a firm involves a trade-off between the benefit from wisdom of the crowd and a cost due to free-riding. For ease of exposition, I illustrate in a simple case with identical preference, i.e. $\forall i, \rho_i = \rho$, same for Section 5.4 below.

If the cost of productive input has to be incurred by player i in its entirety, she would choose x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho\left[a_i v(x_i + \sum_{k\neq i} x_k) - p x_i\right]\right) |s_i\right], \tag{23}$$

given her anticipation of other players' equilibrium productive input level x_k, $k \neq i$. Using similar solution technique, the player i's equilibrium input amount to the firm is given by

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right) \tau_i s_i - \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right)\left(\tau_v + \sum_{k=1}^{n} \tau_k\right) p \tag{24}$$

The equilibrium input amount consists of two parts. The first part $\frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right) \tau_i s_i$ represents a benefit from wisdom of the crowd, while the second part $-\left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right)(\tau_v + \sum_{k=1}^{n} \tau_k)\, p$ represents a free-riding cost. The relative magnitudes of the two effects could determine optimal firm size.[19]

In the context of information gathering agencies, Millon and Thakor (1985) also analyze how moral hazard related intrafirm costs within a partnership pin down a finite optimal size of the firm. In addition to differences in context, in Millon and Thakor (1985) the benefit of forming a partnership comes from *direct* information communication, while in my paper the benefit comes better coordination of actions led by the profit-sharing contract.[20]

The case where private cost c equals 0 corresponds to costless intrafirm monitoring. This assumption appears in Ramakrishnan and Thakor (1984), who develop a theory of financial intermediation, where information producers also write *ex ante* contracts on *ex post* payoffs. The specific sharing rule in the current paper is similar their independent (not IMJC) contract. The difference is that, in Ramakrishnan and Thakor (1984), it is the information producers who are producing information about players. In contrast in my paper, players directly form a coalition. This rules out the kinds of joint contracts that Ramakrishnan and Thakor (1984) consider.

5.4 Decreasing Return to Scale Technology

When a non-zero measure of players in the productive input market engage in profit-sharing, they become more responsive to their information, making the equilibrium price more effi-

[19] That said, if we combine profit-sharing with a "massacre" or "scapegoat" penalizing mechanism as introduced in Rasmusen (1987), first best result may still be maintained. Some further discussion on moral hazard in teams: Williams and Radner (1988) develop examples showing how partnerships preserve efficiency when the joint output is uncertain. Legros and Matsushima (1991) present a necessary and sufficient condition for achieving efficiency in partnerships. Strausz (1999) studies how sequential partnerships sustain efficiency. These considerations, however, are beyond the scope of the current paper.

[20] For examples of how partnering (partially) incentivizes truthful *direct* communication, see Garicano and Santos (2004) on efficient case referral among lawyers.

cient and thus lowering the value of private information. This decline in information value generates decreasing return to scale in the business opportunity.

To illustrate the decline in value of private information when players agree to share profits, Corollary 5.1 calculates the price efficiency, input provision behavior, and expected utility when *all* players in the continuum join their respective partnerships of finite size n.

In this case, but equilibrium price changes because now a non-zero measure (indeed a measure one) of players respond more aggressively to their private information when making input provision amount decisions. However, since the size of each firm n is finite, it is still justified to assume that each player acts as a price-taker.[21] Thus the equilibrium price is given by the market clearing condition

$$\int_0^1 x_i di + z = 0,$$

where total supply is normalized to zero, and for each investor i, x_i is given by (20). The result is summarized below.

Corollary 5.1. *When all players in the continuum join their respective partnerships of finite size n, the equilibrium price becomes $p = \mu + \pi v - \gamma z$, where the coefficients are given by market clearing*

$$\begin{cases} \mu &= \frac{\sigma_z^2\rho^2}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2}\left(\tau_v\bar{v} - \frac{n\tau_e}{\rho\sigma_z^2}\bar{z}\right) \\ \pi &= -\frac{n\tau_e}{\rho}\gamma = \frac{n\tau_e(n\tau_e+\sigma_z^2\rho^2)}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2} \\ \gamma &= -\frac{n\tau_e\rho+\sigma_z^2\rho^3}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2} \end{cases}, \tag{25}$$

and player i's equilibrium demand is given by

$$\frac{1}{\rho}[\tau_v\bar{v} - n\tau_e(\frac{\tau_v\bar{v}+\rho\bar{z}}{n\tau_e+\sigma_z^2\rho^2}) + n\tau_e s_i - (n\tau_e + \frac{\rho^2\sigma_z^2\tau_v}{\rho^2\sigma_z^2+n\tau_e})p], \tag{26}$$

[21] Appendix D further relaxes the price-taking assumption.

which gives player i's expected utility as

$$-\frac{\exp\left(-\frac{\rho^2\sigma_z^2\left[-\rho^2\sigma_z^2\tau_v\bar{v}+n\rho\tau_e\bar{z}-\tau_e\left(n\tau_e+\rho^2\sigma_z^2\right)s_i+(n\tau_e^2+\rho^2\sigma_z^2\tau_e+\rho^2\sigma_z^2\tau_v)p\right]^2}{2(n\tau_e+\rho^2\sigma_z^2)^2(n^2\tau_e^2+\rho^2\sigma_z^2(\tau_e+\tau_v))}\right)}{\sqrt{\frac{\frac{n^2\tau_e^2}{\rho^2\sigma_z^2}+n\tau_e+\tau_v}{\frac{n^2\tau_e^2}{\rho^2\sigma_z^2}+\tau_e+\tau_v}}}, \tag{27}$$

Proof. Appendix. □

Maximizing equation (27) gives a interior (socially) optimal n^*.

Furthermore, as a joint-stock company looms large, the price impact of each individual's input provision amount is also no longer negligible, forcing them to "shred orders" à la Kyle (1989). Appendix D derives input provisions and equilibrium productive input price.

In general, a decreasing return to scale production technology prevents the size of a joint-stock company to grow to infinity, drawing the "boundaries of the firm". As long as the price impact in the productive input market is adequate small, the optimal compensation still features partial profit-sharing. Appendix D illustrates with a numerical example. Although with decreasing return to scale profit-sharing no longer perfectly coordinates actions to the effect of empowering wisdom of the crowd, it still strictly dominates alternatives, and in particular any form of direct information communication. This is because with externalities in the productive input market all players have incentives to lie to others in direct communication, while profit-sharing is incentive compatible and immune to lies.

In a knowledge economy, it is important to coordinate individuals with dispersed information within an institution. However, many obstacles in reality deter effective direct communication. First, truthful-telling may not be incentive compatible. The fear of one's valuable information being abused, as well as the jeopardy of unintentional divulgence or blatant re-sale by others, often deter truthful communication.[22] Second, communication often

[22]Preventing knowledge stealth is empirically a serious concern, as Bhide (1994) reports that 71% of the firms included in the Inc 500 (a list of young, fast growing firms) were founded by people who replicated or modified an idea encountered in their previous employment. Theoretically the concern over critical knowledge

takes time. If a market opportunity is short-lived and requires immediate reaction, delays might negate benefits from communication.[23] Third, when the number of people involved increases, direct communication would become too costly, if not infeasible.[24] This is a particular pressing issue with the rise of some new financing methods like crowdfunding, where a large number of individuals with low or no affinity are involved in the decision making for the same venture. Profit-sharing is a simple mechanism to overcome all these obstacles.

6 General Implications for Corporate Finance

Aside from elucidating the complementarity between a joint-stock company and its surrounding market, looking at profit-sharing contracts through the lens of wisdom of the crowd also broadens the content of corporate governance studies. Shleifer and Vishny (1997) defines corporate governance as a study that "deals with the ways in which suppliers of finance to corporations assure themselves of getting a return on their investment", with "a straightforward agency perspective, sometimes referred to as separation of ownership and control". Recent advances in corporate governance practices, along with a wisdom of the crowd perspective on profit-sharing design could thus add to this established view.

A long-lasting question in the theory of the firm asks what is the "glue" that keeps a firms together? Traditional property-based theory of the firm identifies physical assets as

stealth is also the departure point in Rajan and Zingales (1998). In the context of financial markets, resale has been brought up in discussions on information production (e.g. Hirshleifer (1971)). Controlling information usage motivates studies on optimal selling of information (e.g. Admati and Pfleiderer (1986) and Admati and Pfleiderer (1990)). Allen (1990) develops a theory of financial intermediation based on indirect selling of information. See Dewatripont and Tirole (2005) for further discussions on potential costs and biases in knowledge transfers.

[23] Bolton and Dewatripont (1994) consider the time involved in information transmission. The "information percolation" literature explicitly models the slow diffusion process of information in the financial market over repeated direct communication, see Duffie, Giroux, and Manso (2010) and Andrei and Cujean (2013), etc.

[24] Furthermore, conversations, meetings, and discussions take time, at the cost of leisure, actual work, and missing opportunities; misinterpretation and oblivion create additional attrition to communication; "soft" information like haphazard know-hows, amorphous business acumen, and tacit knowledge à la Grant (1996) are simply too hard to codify and impossible to convey; cognitive capacity limits cap the amount of knowledge an individual can possess (see the rational inattention literature as in e.g. Veldkamp (2011)).

such glues.[25] However, as we enter an information age, in many business sectors traditional asset intensive firms are now being gradually peripheralized by human capital intensive ones, leading to the call for a "search of new foundations" (of corporate finance / theory of the firm) in Zingales (2000).[26] There has been many attempts in direction, including Acharya, Myers, and Rajan (2011) and Rajan (2012)).[27] Profit-sharing gives an alternative.

The decentralized control in my profit-sharing analysis is reminiscent of the meritocracy spirit of Aghion and Tirole (1997), in which formal authority is distinguished from real authority. An agent with formal authority will exercise her power if and only if she acquires the necessary knowledge to do so, or otherwise she delegates decision-making to her more knowledgeable subordinates.[28] In the profit-sharing relation studied above, players make decisions without others' interference, as their private knowledge (either about the mapping from the information set to the optimal action or the information set itself) grant them real authority. This decentralized governance structure lines up with the flat organization, teamwork focus, and advocated "workforce democracy" found in most human capital intensive firms. It also differentiates my model from the social choice problem of Wilson (1968).

Viewing firms as profit-sharing mechanisms also helps interpret some recent trends in capital structure changes. In a recent discussion on "secular stagnation" among industrialized economies, Summers (2014) pinpoints the "reductions in demand for debt-financed investment", and contends that "probably to a greater extent, it is a reflection of the changing character of productive economic activity."[29] Traditional asset-intensive industries are

[25]For example, Hart (1995) argues that "a firm's non-human assets, then, simply represent the glue that keeps the firm together ... If non-human assets do not exist, then it is not clear what keeps the firm together" (p. 57)."

[26]Gluing human capital and preventing talent attrition is an important consideration in modern corporate governance. The consequence of neglecting it is vividly illustrated in the case of the British advertising agency Saatchi and Saatchi documented in Rajan and Zingales (2000).

[27]Berk, Van Binsbergen, and Liu (2014) and Cheng, Massa, Spiegel, and Zhang (2012), among others, focus on a particular type of human-capital intensive firms – mutual fund families.

[28]Although the focus of Aghion and Tirole (1997) is on how the allocation of formal authority alters agents' *ex ante* knowledge acquisition incentives.

[29]Summers elaborates further: "Ponder that the leading technological companies of this age – I think, for

debt-friendly, as assets serve as collaterals and allow outside investors to take a passive role in the firm's operation (except in default).[30] Firms with intensive knowledge inputs, however, require a more active role of all input providers, e.g. the more active roles played by venture capitalists than commercial banks, the adoption of equity-based employee compensation, and less involvement of passive creditors in knowledge-intensive firms (law firms, strategic management (but not IT) consulting firms, etc.) – even though such industry might be most subject to insider moral hazard or unverifiable cash flow, which traditional theories (e.g. Innes (1990) and Townsend (1979)) would predict in favor of debt-financing.[31] In a broader sense, my theory relates to the partnership model of outside equity investors in Myers (2000).

The profit-sharing view of a firm also provides new perspectives on valuation. The nexus of *explicit* contracts view of the firm à la Alchian and Demsetz (1972) and Jensen and Meckling (1979) assumes that compensations to all stakeholders but shareholders are explicitly contracted. Since equity holders are the only residual rights owner, maximizing shareholder value equates to maximizing social welfare for all stakeholders. However, this powerful argument is no long that clear-cut once employee human capital is taken into consideration, as residual rights owners can no longer be summarized as one representative person, and their internal relations do matter.[32]

example, of Apple and Google – find themselves swimming in cash and facing the challenge of what to do with a very large cash hoard. Ponder the fact that WhatsApp has a greater market value than Sony, with next to no capital investment required to achieve it. Ponder the fact that it used to require tens of millions of dollars to start a significant new venture, and significant new ventures today are seeded with hundreds of thousands of dollars..."

[30] Williamson (1988) sympathizes this perspective.

[31] See also Berk, Stanton, and Zechner (2010) and Berk and Walden (2013) on the implications of human capital on capital structure and asset pricing.

[32] Several recent papers have investigated the valuation implication of firm's non-tangible assets. For example, Eisfeldt and Papanikolaou (2013) document that firms with more organization capital have average returns that are 4.6% higher than firms with less organization capital. Zhang (2014) studies the implications of employee's limited commitment to the firm on cash flow volatility.

7 Other Related Literature

Theoretical results on the coordination effect of profit-sharing are related to studies on the equilibrium and efficient use of information. In a linear-quadratic setup featuring asymmetric information and strategic complementarity/substitutability, Angeletos and Pavan (2007) show that redistribution among individuals can achieve as an equilibrium outcome efficient use of information. The coordination effect of profit-sharing also obtains efficient use of information, although strategic complementarity/substitutability are not present.

Casting my results in the financial market reminds of studies on indirect sale of information (Admati and Pfleiderer (1990), etc.) When informed investors manage delegated portfolios for a fee, they indirectly sell information to those uninformed. Following this logic, when information in the economic system is dispersed, investors would have incentives to delegate their wealth to each other. The result of such mutual delegation appears like profit-sharing studied in this paper. Admati and Pfleiderer uses their insight of indirect sale of information to explain the rise of institutional investors. Their results could thus be viewed as an important example of creating a firm to counter market information frictions.[33]

Cooperative game theory provides useful tools for many profit-sharing problems.[34] However these studies do not consider the wisdom of the crowd effect. Compared to cooperative game theory in which the value for a particular subset of players is exogenously specified, my solution is entirely based on non-cooperative game theory, and the value created by any subset of players is endogenous, dependent on the particular sharing contract among them.

Technically, a profit-sharing cooperative could be viewed as a game-theoretical implementation for a rational expectation equilibrium. So my result is connected to the implementation theory literature in mechanism design, whose focus is on designing mechanisms

[33] Also see García and Vanden (2009) on wealth delegation with endogenous information acquisition.

[34] See e.g. Nash Jr (1950) and Shapley (1952) for original references, Aoki (1984) for a review of applications in the theory of the firm, Brandenburger and Nalebuff (2011) for a popular introduction, and Aumann and Maschler (1985) for a cooperative game theoretical analysis for the Talmud bankruptcy problem.

to achieve one equilibrium outcome via another equilibrium concept. Palfrey (2002) provides a nice introduction to implementation theory, while Blume and Easley (1990) studies implementation of Walrasian expectations equillibrium in a general setting.

This paper also contributes to the literature on the theory of the firm. Over 70 years' academic endeavors on this fundamental topic makes an exhaustive reference almost impossible, so I only attempt to classify some well-known contributions around major lines of thoughts and connect them with the current paper.

The neoclassic theory views firms as production technology sets, and firms *per se* are void of meaningful definitions. The first and foremost question, raised by the seminal work of Coase (1937), asks what essentially defines a firm, and how within-firm organization is distinguished from market contracting. Coase identifies authority, which is useful when contracting is costly, as the defining feature that differs within-firm transactions apart from market contracting. Two questions remain to be answered in this argument, the first being what constitutes contracting costs, and the second being a formal definition of authority.

On contracting costs, Williamson (1975), Klein, Crawford, and Alchian (1978), Williamson (1979), and Williamson (1985) identifies *ex post* haggling as a source of cost to contracting. In my humble opinion, another important friction to contracting lies in the nonexistence of Pareto optimal, incentive compatible, and budget-balancing bilateral bargaining outcomes under two-sided asymmetric information (Myerson and Satterthwaite (1983)), although to my knowledge no resolutions have yet been proposed in this direction.

The formalization of authority spearheads the development of the incomplete contracting approach. Grossman and Hart (1986) and Hart and Moore (1990) argue that asset ownership determines the allocation of residual rights (or authority). In this property rights theory of the firm, physical assets play vital roles for the very existence of firms as they entangle other production inputs around it and give birth to firms.[35] However, as human capital intensive

[35]See Hart (1989) and Holmström and Tirole (1989) for reviews. This view is partially supported by

firms arise in the knowledge economy, Rajan and Zingales (1998) point out the narrowness of property rights theory and develop a new theory based on "access" to critical resources.[36]

The Coase authority-cost paradigm is not the only framework for understanding firms. For example, the "nexus of contracts" theory views a firm as a legal illusion no more than a central contracting party to subsume a complex of multilateral contracts (Alchian and Demsetz (1972)). Jensen and Meckling (1979) specifically focus on the principal-agent contracting problem between a firm owner and the management subject to moral hazard. In the market intermediary theory, Spulber (1999) interprets firms as centralized exchanges to reduce market search costs. A few papers take a knowledge perspective on the essence of the firm. Demsetz (1988) emphasizes information cost reduction as a foundation of firms, and Grant (1996) proposes a knowledge-based theory of the firm. However, both papers take information cost reduction and knowledge aggregation with a firm as given, abstracting from supporting micro-foundations. Bolton and Dewatripont (1994), Winter (2006), and Winter (2010) analyze how organizations minimize information processing and communicating costs, but they do not consider communication incentives, which is emphasized in this paper. While my paper is built on dispersed information, Dicks and Fulghieri (2014) study optimal governance structure based on disagreement between owners and managers that is endogenously generated by ambiguity aversion.

8 Conclusion

This paper shows that a profit-sharing contracts, similar but different from traditional equity contracts, could coordinate individual actions guided by dispersed private information and obtain an *as if* information aggregation effect. The optimal profit-sharing contract is simple and easy to implement, is immune to lying or abuse, and remains dominant even when

Kaplan, Sensoy, and Strömberg (2009).

[36]Also see Rajan and Zingales (2001).

frictions prevent perfect coordination. This result is deeply connected to the discussions on the relationship between firms and markets, has supports from centuries of joint-stock company histories, and more importantly could provide practical guidance for many financing practices in a foreseeable future. As the human race enters the information age, contract design that unleashes the power of collective wisdom has far-reaching implications for business organization, corporate governance, entrepreneurship, productivity, and economic growth. Security design should progress in accordance with the evolution of our business world.

Many further implications are left for future research. For example, how is profit-sharing related to other non-market-price-based information aggregation mechanisms, such as auctions or voting?[37] Are methods developed in this paper applicable to describing the formation of networks (e.g. Stanton, Walden, and Wallace (2015)) and the rise of "too-big-to-fail" banks (e.g. Erel (2011))? Does profit-sharing speak to the governance of PE/VC, the organization of R&D activities, or the rise of mutual fund families? How is my results on financial disintermediation (i.e. crowd-funding) related to existing theories on financial intermediation, e.g. Diamond (1984), Ramakrishnan and Thakor (1984))? Are there profit-sharing arrangements in existence to tunnel Chinese walls (on information communication)? Does profit-sharing camouflage insider trading? Casting in an even wider range, does profit-sharing have implications for optimal taxation? Does the investigation into the relationship between the firm and market suggest a connection between the Welfare Theorem and Coase Theorem? Is there an equivalence between capitalism and socialism in terms of information aggregation? Further developments are down the road.

[37] see e.g. Feddersen and Pesendorfer (1997).

Appendix

A Proof of Theorem 4.1

Proof of Theorem 4.1: When n players agree to partner, partner i's investment in the risky project maximizes

$$\mathbb{E}\left[-\exp(-\rho_i a_i(v-p)(x_i+\sum_{k\neq j}x_k))|s_i,p\right] \tag{28}$$

Focusing on symmetric linear equilibria, assume $x_k = \alpha_0 + \alpha_{1k}s_k + \alpha_{2k}p$. Notice that

$$x_i + \sum_{k\neq i}x_k = x_i + \sum_{k\neq i}\alpha_0 + \sum_{k\neq i}\alpha_{1k}v + \sum_{k\neq i}\alpha_{2k}p + \sum_{k\neq i}\alpha_{1k}e_k, \tag{29}$$

thus

$$\begin{pmatrix} -\rho_i a_i(v-p) \\ x_i+\sum_{k\neq i}x_k \end{pmatrix}_{|s_i,p} \sim \mathcal{N}\left(\begin{array}{c} \begin{bmatrix} -\rho_i a_i(\mathbb{E}(v|s_i,p)-p) \\ x_i+\sum_{k\neq i}\alpha_0+\sum_{k\neq i}\alpha_{1k}\mathbb{E}(v|s_i,p)+\sum_{k\neq i}\alpha_{2k}p \end{bmatrix}, \\ \begin{bmatrix} \rho_i^2a_i^2\mathrm{Var}(v|s_i,p) & -\rho_i a_i\sum_{k\neq i}\alpha_{1k}\mathrm{Var}(v|s_i,p) \\ -\rho_i a_i\sum_{k\neq i}\alpha_{1k}\mathrm{Var}(v|s_i,p) & (\sum_{k\neq i}\alpha_{1k})^2\mathrm{Var}(v|s_i,p)+\sum_{k\neq i}\alpha_{1k}^2\tau_k^{-1} \end{bmatrix} \end{array} \right) \tag{30}$$

By Lemma 1.1, the certainty equivalent of (28) is

$$-\frac{\exp(\frac{A}{2B})}{\sqrt{B}}, \tag{31}$$

where

$$\begin{aligned}
A &= \rho_i^2a_i^2(\mathbb{E}(v|s_i,p)-p)^2[(\sum_{k\neq i}\alpha_{1k})^2\mathrm{Var}(v|s_i,p)+\sum_{k\neq i}\alpha_{1k}^2\tau_k^{-1}] \\
&+ [x_i+\sum_{k\neq i}\alpha_0+\sum_{k\neq i}\alpha_{1k}\mathbb{E}(v|s_i,p)+\sum_{k\neq i}\alpha_{2k}p]^2\rho_i^2a_i^2\mathrm{Var}(v|s_i,p) \\
&- 2\rho_i a_i(\mathbb{E}(v|s_i,p)-p)[x_i+\sum_{k\neq i}\alpha_0+\sum_{k\neq i}\alpha_{1k}\mathbb{E}(v|s_i,p)+\sum_{k\neq i}\alpha_{2k}p][1+\rho_i a_i\sum_{k\neq i}\alpha_{1k}\mathrm{Var}(v|s_i,p)] \\
B &= [1+\rho_i a_i\sum_{k\neq i}\alpha_{1k}\mathrm{Var}(v|s_i,p)]^2 \\
&- \rho_i^2a_i^2\mathrm{Var}(v|s_i,p)[(\sum_{k\neq i}\alpha_{1k})^2\mathrm{Var}(v|s_i,p)+\sum_{k\neq i}\alpha_{1k}^2\tau_k^{-1}]
\end{aligned}$$

Taking FOC w.r.t. x_i we get

$$
\begin{aligned}
& [x_i + \sum_{k \neq i} \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i, p) + \sum_{k \neq i} \alpha_{2k} p] \rho_i a_i \mathrm{Var}(v|s_i, p) && (32) \\
= \;& (\mathbb{E}(v|s_i, p) - p)[1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \mathrm{Var}(v|s_i, p)]. && (33)
\end{aligned}
$$

Given $p = \mu + \pi v - \gamma z$, we have

$$
\begin{cases}
\mathbb{E}(v|s_i, p) = \bar{v} + \frac{\gamma^2 \sigma_z^2 \tau_v^{-1}(s_i - \bar{v}) + \pi \tau_v^{-1} \tau_i^{-1}(p - \mu - \pi \bar{v} + \gamma \bar{z})}{\gamma^2 \sigma_z^2 \tau_v^{-1} + \pi^2 \tau_v^{-1} \tau_i^{-1} + \gamma^2 \sigma_z^2 \tau_i^{-1}} = \frac{\gamma^2 \sigma_z^2 \tau_v^{-1} s_i + \pi \tau_v^{-1} \tau_i^{-1}(p - \mu + \gamma \bar{z}) + \gamma^2 \sigma_z^2 \tau_i^{-1} \bar{v}}{\gamma^2 \sigma_z^2 \tau_v^{-1} + \pi^2 \tau_v^{-1} \tau_i^{-1} + \gamma^2 \sigma_z^2 \tau_i^{-1}} \\
\mathrm{Var}(v|s_i, p) = \tau_v^{-1} - \frac{\gamma^2 \sigma_z^2 \tau_v^{-2} + \pi^2 \tau_v^{-2} \tau_e^{-1}}{\gamma^2 \sigma_z^2 \tau_v^{-1} + \pi^2 \tau_v^{-1} \tau_i^{-1} + \gamma^2 \sigma_z^2 \tau_i^{-1}} = \frac{\gamma^2 \sigma_z^2 \tau_i^{-1} \tau_v^{-1}}{\gamma^2 \sigma_z^2 \tau_v^{-1} + \pi^2 \tau_v^{-1} \tau_i^{-1} + \gamma^2 \sigma_z^2 \tau_i^{-1}}
\end{cases} \quad (34)
$$

In equilibrium $x_i = \alpha_{0j} + \alpha_{1j} s_i + \alpha_{2j} p$, and Equation (33) leads to

$$
\alpha_{1j} s_i + \sum_{k=1}^n \alpha_0 + \sum_{k \neq i} \alpha_{1k} \mathbb{E}(v|s_i, p) + \sum_{k=1}^n \alpha_{2k} p = \frac{(\mathbb{E}(v|s_i, p) - p)[1 + \rho_i a_i \sum_{k \neq i} \alpha_{1k} \mathrm{Var}(v|s_i, p)]}{\rho_i a_i \mathrm{Var}(v|s_i, p)}, \quad (35)
$$

thus

$$
\rho_i a_i \mathrm{Var}(v|s_i, p) \left[\alpha_{1j} s_i + \sum_{k=1}^n \alpha_0 + \sum_{k=1}^n \alpha_{2k} p \right] = \mathbb{E}(v|s_i, p) - p - \rho_i a_i \sum_{k \neq i} \alpha_{1k} \mathrm{Var}(v|s_i, p) p \quad (36)
$$

Plug in (34),

$$
\begin{aligned}
& \rho_i a_i \gamma^2 \sigma_z^2 \tau_i^{-1} \tau_v^{-1} \left[\alpha_{1j} s_i + \sum_{k=1}^n \alpha_0 + \sum_{k=1}^n \alpha_{2k} p \right] = \gamma^2 \sigma_z^2 \tau_v^{-1} s_i + \pi \tau_v^{-1} \tau_i^{-1}(p - \mu + \gamma \bar{z}) + \gamma^2 \sigma_z^2 \tau_i^{-1} \bar{v} \\
- \;& (\gamma^2 \sigma_z^2 \tau_v^{-1} + \pi^2 \tau_v^{-1} \tau_i^{-1} + \gamma^2 \sigma_z^2 \tau_i^{-1}) p - \rho_i a_i \sum_{k \neq i} \alpha_{1k} \gamma^2 \sigma_z^2 \tau_i^{-1} \tau_v^{-1} p
\end{aligned}
$$

Equalizing coefficients:

$$
\begin{cases}
\rho_i a_i \sum_{k=1}^n \alpha_0 = \tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_z^2}(\mu - \gamma \bar{z}) \\
\rho_i a_i \alpha_{1j} = \tau_i \\
\rho_i a_i \sum_{k=1}^n \alpha_{2k} = -(\tau_i + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_z^2}) + \frac{\pi}{\gamma^2 \sigma_z^2} - \rho_i a_i \sum_{k \neq i} \frac{\tau_k}{\rho_k a_k}
\end{cases} \quad (37)
$$

$\Rightarrow$

$$
\begin{cases}
a_i = \frac{\frac{1}{\rho_i}}{\sum_{k=1}^n \frac{1}{\rho_k}} \\
\sum_{k=1}^n \alpha_0 = \left(\sum_{k=1}^n \frac{1}{\rho_k} \right) \left[\tau_v \bar{v} - \frac{\pi}{\gamma^2 \sigma_z^2}(\mu - \gamma \bar{z}) \right] \\
\alpha_{1j} = \left(\sum_{k=1}^n \frac{1}{\rho_k} \right) \tau_i \\
\sum_{k=1}^n \alpha_{2k} = \left(\sum_{k=1}^n \frac{1}{\rho_k} \right) \left[-(\sum_{k=1}^n \tau_k + \tau_v + \frac{\pi^2}{\gamma^2 \sigma_z^2}) + \frac{\pi}{\gamma^2 \sigma_z^2} \right]
\end{cases} \quad (38)
$$

Thus

$$x_i = \frac{1}{\rho_i}\left[\tau_v\bar{v} - \frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})\right] + \left(\sum_{k=1}^{n}\frac{1}{\rho_k}\right)\tau_i s_i - \frac{1}{\rho_i}\left[\sum_{k=1}^{n}\tau_k + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2} - \frac{\pi}{\gamma^2\sigma_z^2}\right]p \tag{39}$$

Plug (38) in (31) shows that

$$\begin{aligned}
\mathbb{E}(v|s_i,p) &= \frac{\tau_i s_i + \frac{\pi}{\gamma^2\sigma_z^2}(p-\mu+\gamma\bar{z}) + \tau_v\bar{v}}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v} \\
\mathrm{Var}(v|s_i,p) &= \frac{1}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}.
\end{aligned}$$

$$\begin{aligned}
A &= -(\mathbb{E}(v|s_i,p)-p)^2\left(\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v\right) \\
B &= \frac{\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v},
\end{aligned}$$

thus the expression for the expected utility of player i is given by

$$\begin{aligned}
& -\frac{\exp\left(-\frac{(\mathbb{E}(v|s_i,p)-p)^2(\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v)}{2}\right)}{\sqrt{\frac{\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}} \\
= & -\frac{\exp\left(-\frac{1}{2}\frac{1}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_i s_i - \frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z}) + \tau_v\bar{v} - (\tau_i+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2} - \frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)}{\sqrt{\frac{\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}}
\end{aligned}$$

Notice that this is identical to the expected utility achieved under complete information (but no partnership creation). □

B Proof of Theorem 4.2

Proof. In a symmetric information benchmark, player i's expected utility is given by

$$\begin{aligned}
-\exp\Big\{ & -\left[\tau_v\bar{v} - \tfrac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z}) + \textstyle\sum_{k=1}^{n}\tau_k s^* - (\sum_{k=1}^{n}\tau_k + \tau_v + \tfrac{\pi^2}{\gamma^2\sigma_z^2} - \tfrac{\pi}{\gamma^2\sigma_z^2})p\right][\mathbb{E}(v|s^*,p)-p] + \\
& \tfrac{1}{2}\left[\tau_v\bar{v} - \tfrac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z}) + \textstyle\sum_{k=1}^{n}\tau_k s^* - (\sum_{k=1}^{n}\tau_k + \tau_v + \tfrac{\pi^2}{\gamma^2\sigma_z^2} - \tfrac{\pi}{\gamma^2\sigma_z^2})p\right]^2 \mathrm{Var}(v|s^*,p)\Big\}
\end{aligned}, \tag{40}$$

and expected utility before entering full information benchmark conditional on s_i is

$$
\begin{aligned}
& -\mathbb{E}\left\{\exp\left(-\left[\tau_v\bar{v}-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\sum_{k=1}^n\tau_k s^*-(\sum_{k=1}^n\tau_k+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right][\mathbb{E}(v|s^*,p)-p]\right.\right. \\
& \left.\left.+\frac{1}{2}\left[\tau_v\bar{v}-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\sum_{k=1}^n\tau_k s^*-(\sum_{k=1}^n\tau_k+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\mathrm{Var}(v|s^*,p)\right)\,\middle|\, s_i,p\right\} \\
= & -\mathbb{E}\left\{\exp\left(-\frac{1}{2}\frac{1}{\sum_{k=1}^n\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_v\bar{v}-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\sum_{k=1}^n\tau_k s^*-(\sum_{k=1}^n\tau_k+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)\,\middle|\, s_i,p\right\} \\
= & -\frac{\exp\left(-\frac{1}{2}\frac{1}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_i s_i-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\tau_v\bar{v}-(\tau_i+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)}{\sqrt{\frac{\sum_{k=1}^n\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}}
\end{aligned}
$$

The first equation is from (4), while the second one uses the following two facts:

1. If $x\sim\mathcal{N}(\mu,\sigma^2),\mathbb{E}[e^{Ax^2}]=\frac{\exp\left(\frac{A\mu^2}{1-2A\sigma^2}\right)}{\sqrt{1-2A\sigma^2}}$;
2. $\tau_v\bar{v}-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\sum_{k=1}^n\tau_k s^*-(\sum_{k=1}^n\tau_k+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\,\big|_{s_i,p}\sim$
$\mathcal{N}\left(\frac{\sum_{k=1}^n\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_i s_i-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\tau_v\bar{v}-(\tau_i+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right],\frac{\sum_{k=1}^n\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\sum_{k=1,k\neq i}^n\tau_k\right)$

□

C Proof of Corollary 5.1

Proof of Corollary 5.1: When all players create firms, in a linear equilibrium, project valuation and player investment still follows

$$
\begin{aligned}
\tilde{p} &= \mu+\pi\tilde{v}-\gamma\tilde{z} &\quad (41)\\
x_k &= n\alpha_0+n\alpha_1 s_k+n\alpha_2 p,\ \text{for player } k, &\quad (42)
\end{aligned}
$$

where μ,π,γ and α_i $(i=0,1,2)$ are all functions of n to be determined.

Integrate each individual players' investment over the continuum and by market clearing,

$$
n\alpha_0+n\alpha_1 v+n\alpha_2(\mu+\pi v-\gamma z)+z = 0,\ \text{thus} \quad (43)
$$

$$
\left\{\begin{array}{lcl}\alpha_0 &=& -\alpha_2\mu\\ \alpha_1 &=& -\alpha_2\pi\\ \alpha_2 &=& \frac{1}{n\gamma}\end{array}\right. \quad (44)
$$

Thus

$$\begin{cases} \mu &=& \frac{\sigma_z^2\rho^2}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2}\left(\tau_v\bar{v}-\frac{n\tau_e}{\rho\sigma_z^2}\bar{z}\right) \\ \pi &=& -\frac{n\tau_e}{\rho}\gamma=\frac{n\tau_e(n\tau_e+\sigma_z^2\rho^2)}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2} \\ \gamma &=& -\frac{n\tau_e\rho+\sigma_z^2\rho^3}{(n\tau_e+\tau_v)\sigma_z^2\rho^2+n^2\tau_e^2} \end{cases} \tag{45}$$

Plug in (39) and (31) and we get the optimal demand and expected payoff given in (26). □

D Market Powers in the Productive Input Market

First consider a heterogeneous agents extension to Kyle (1989). In this case there are N investors who do not form partnerships. Perceiving a residual supply curve $p = p_i + \lambda_i x_i$, investor i chooses x_i to maximize

$$\begin{aligned} & \mathbb{E}\left[-\exp(-\rho_i(v-p(x_i))x_i)|s_i,p\right], \text{ or} \\ & \mathbb{E}\left[-\exp(-\rho_i(v-p_i-\lambda_i x_i)x_i)|s_i,p_i\right] \\ \Leftrightarrow \quad & -\exp\left[-\rho_i(\mathbb{E}(v|s_i,p_i)-p_i)x_i+\rho_i\lambda_i x_i^2+\frac{1}{2}\rho_i^2\mathrm{Var}(v|s_i,p_i)x_i^2\right] \\ \stackrel{\text{FOC}}{\Longrightarrow} \quad & x_i=\frac{(\mathbb{E}(v|s_i,p_i)-p_i)}{2\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i)} \end{aligned}$$

thus

$$p = p_i+\lambda_i x_i \tag{46}$$

$$= p_i+\frac{\lambda_i(\mathbb{E}(v|s_i,p_i)-p_i)}{2\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i)} \tag{47}$$

$$\stackrel{\text{FOC}}{\Longrightarrow} (2\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i))p=(\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i))p_i+\lambda_i\mathbb{E}(v|s_i,p_i) \tag{48}$$

$$\Rightarrow p_i=\frac{(2\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i))p-\lambda_i\mathbb{E}(v|s_i,p_i)}{(\lambda_i+\rho_i\mathrm{Var}(v|s_i,p_i))} \tag{49}$$

thus (given that p_i and p are informationally equivalent)

$$x_i=\frac{\mathbb{E}(v|s_i,p)-p}{(\lambda_i+\rho_i\mathrm{Var}(v|s_i,p))}$$

Conjecture a linear strategy profile $x_i=\mu_i+\beta_i s_i-\gamma_i p$, then by market clearing

$$\sum_{k=1}^N\mu_k+\sum_{k=1}^N\beta_k s_k-\sum_{k=1}^N\gamma_k p+z=0 \tag{50}$$

or

$$x_i + \sum_{k\neq i} \mu_k + \sum_{k\neq i} \beta_k s_k - \sum_{k\neq i} \gamma_k p + z = 0 \tag{51}$$

$$\Rightarrow \quad p = \frac{x_i + \sum_{k\neq i} \mu_k + \sum_{k\neq i} \beta_k s_k + z}{\sum_{k\neq i} \gamma_k} \tag{52}$$

$$\Rightarrow \quad \lambda_i = \frac{1}{\sum_{k\neq i} \gamma_k}, p_i = \frac{\sum_{k\neq i} \mu_k + \sum_{k\neq i} \beta_k s_k + z}{\sum_{k\neq i} \gamma_k} \tag{53}$$

thus

$$\mathbb{E}(v|s_i, p_i) = \bar{v} + \frac{(\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(s_i - \bar{v}) + \sum_{k\neq i} \beta_k \tau_i^{-1} [\sum_{k\neq i} \beta_k (s_k - \bar{v}) + z - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} \tag{54}$$

$$\mathrm{Var}(v|s_i, p_i) = \frac{\tau_i^{-1}[\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} \tag{55}$$

and

$$\begin{aligned}
x_i &= \frac{\bar{v} + \frac{(\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(s_i - \bar{v}) + \sum_{k\neq i} \beta_k \tau_i^{-1} [\sum_{k\neq i} \beta_k (s_k - \bar{v}) + z - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{\sum_{k\neq i} \mu_k + \sum_{k\neq i} \beta_k s_k + z}{\sum_{k\neq i} \gamma_k}}{2\frac{1}{\sum_{k\neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} && (56) \\
&= \mu_i + \beta_i s_i - \gamma_i p && (57) \\
&= \mu_i + \beta_i s_i - \gamma_i \frac{\sum_{k=1}^N \mu_k + \sum_{k=1}^N \beta_k s_k + z}{\sum_{k=1}^N \gamma_k} && (58)
\end{aligned}$$

Equating coefficients we get

$$\frac{\bar{v} + \frac{(\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(-\bar{v}) + \sum_{k\neq i} \beta_k \tau_i^{-1} [\sum_{k\neq i} \beta_k (-\bar{v}) - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{\sum_{k\neq i} \mu_k}{\sum_{k\neq i} \gamma_k}}{2\frac{1}{\sum_{k\neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} = \mu_i - \gamma_i \frac{\sum_{k=1}^N \mu_k}{\sum_{k=1}^N \gamma_k} \tag{59}$$

$$\frac{\frac{\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}}{2\frac{1}{\sum_{k\neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} = \beta_i - \gamma_i \frac{\beta_i}{\sum_{k=1}^N \gamma_k} \tag{60}$$

$$\frac{\frac{\sum_{k\neq i} \beta_k \tau_i^{-1}}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{1}{\sum_{k\neq i} \gamma_k}}{2\frac{1}{\sum_{k\neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i} \beta_k)^2 + \tau_v \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} = -\frac{\gamma_i}{\sum_{k=1}^N \gamma_k} \tag{61}$$

Equation (60) and (61) lead to

$$\frac{[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2](\sum_{k=1}^N\gamma_k-\sum_{k\neq i}\gamma_k\beta_i\rho_i\tau_i^{-1})}{\tau_i^{-1}[(\sum_{k\neq i}\beta_k)^2+\tau_v\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2}=2\beta_i \tag{62}$$

$$\frac{\sum_{k=1}^N\gamma_k\sum_{k\neq i}\beta_k\tau_i^{-1}+\gamma_i\rho_i\tau_i^{-1}[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i}\beta_k)^2+\tau_v\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2}=\frac{-\gamma_i}{\sum_{k\neq i}\gamma_k}+1 \tag{63}$$

(59) leads to

$$\tau_v\tau_i^{-1}[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2]\bar{v}-\sum_{k\neq i}\beta_k\tau_i^{-1}\bar{z}=(\mu_i-\gamma_i\frac{\sum_{k=1}^N\mu_k}{\sum_{k=1}^N\gamma_k})\rho_i\tau_i^{-1}[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2]$$

$$+[(\mu_i-\gamma_i\frac{\sum_{k=1}^N\mu_k}{\sum_{k=1}^N\gamma_k})\frac{1}{\sum_{k\neq i}\gamma_k}+\frac{\sum_{k=1}^N\mu_k}{\sum_{k=1}^N\gamma_k}]\left[\tau_i^{-1}[(\sum_{k\neq i}\beta_k)^2+\tau_v\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2\right] \tag{64}$$

The three equations above defines the equilibrium.

Profit-sharing Now consider $M\leq N$ investors agree to partner. Perceiving a residual supply curve $p=p_i+\lambda_i x_i$, partner i chooses x_i to maximize

$$\mathbb{E}\left[-\exp(-\rho_i a_i(v-p_i-\lambda_i x_i)(x_i+\sum_{k\neq i,k=1}^M x_k))|s_i,p\right],$$

Conjecture a linear strategy profile $x_i=\mu_i+\beta_i s_i-\gamma_i p$, thus

$$x_i+\sum_{k\neq i,k=1}^M x_k=x_i+\sum_{k\neq i,k=1}^M\mu_k+\sum_{k\neq i,k=1}^M\beta_k s_k-\sum_{k\neq i,k=1}^M\gamma_k p \tag{65}$$

and

$$\begin{pmatrix}-\rho_i a_i(v-p_i-\lambda_i x_i)\\ x_i+\sum_{k\neq i,k=1}^M x_k\end{pmatrix}_{|s_i,p}\sim\mathcal{N}\left(\begin{bmatrix}-\rho_i a_i(\mathbb{E}(v|s_i,p)-p_i-\lambda_i x_i)\\ x_i+\sum_{k\neq i,k=1}^M\mu_k+\sum_{k\neq i,k=1}^M\beta_k\mathbb{E}(v|s_i,p)-\sum_{k\neq i,k=1}^M\gamma_k p\end{bmatrix},\begin{bmatrix}\rho_i^2a_i^2\mathrm{Var}(v|s_i,p) & -\rho_i a_i\sum_{k\neq i,k=1}^M\beta_k\mathrm{Var}(v|s_i,p)\\ -\rho_i a_i\sum_{k\neq i,k=1}^M\beta_k\mathrm{Var}(v|s_i,p) & (\sum_{k\neq i,k=1}^M\beta_k)^2\mathrm{Var}(v|s_i,p)+\sum_{k\neq i,k=1}^M\beta_k^2\tau_k^{-1}\end{bmatrix}\right)$$

whose certainty equivalent is (by Lemma 1.1)

$$-\frac{\exp(\frac{A}{2B})}{\sqrt{B}}, \tag{66}$$

where

$$
\begin{aligned}
A &= \rho_i^2 a_i^2(\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i)^2[(\sum_{k\neq i,k=1}^{M}\beta_k)^2\mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M}\beta_k^2\tau_k^{-1}] \\
&+ [x_i + \sum_{k\neq i,k=1}^{M}\mu_k + \sum_{k\neq i,k=1}^{M}\beta_k\mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M}\gamma_k p]^2\rho_i^2 a_i^2\mathrm{Var}(v|s_i,p) \\
&- 2\rho_i a_i(\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i)[x_i + \sum_{k\neq i,k=1}^{M}\mu_k + \sum_{k\neq i,k=1}^{M}\beta_k\mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M}\gamma_k p] \\
&\quad [1 + \rho_i a_i\sum_{k\neq i,k=1}^{M}\beta_k\mathrm{Var}(v|s_i,p)] \\
B &= [1 + \rho_i a_i\sum_{k\neq i,k=1}^{M}\beta_k\mathrm{Var}(v|s_i,p)]^2 \\
&- \rho_i^2 a_i^2\mathrm{Var}(v|s_i,p)[(\sum_{k\neq i,k=1}^{M}\beta_k)^2\mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M}\beta_k^2\tau_k^{-1}]
\end{aligned}
$$

Taking FOC w.r.t. x_i we get

$$
\begin{aligned}
&-\lambda_i\rho_i^2 a_i^2(\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i)[(\sum_{k\neq i,k=1}^{M}\beta_k)^2\mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M}\beta_k^2\tau_k^{-1}] \\
&+ [x_i + \sum_{k\neq i,k=1}^{M}\mu_k + \sum_{k\neq i,k=1}^{M}\beta_k\mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M}\gamma_k p]\rho_i^2 a_i^2\mathrm{Var}(v|s_i,p) \\
&+ \rho_i a_i\lambda_i[x_i + \sum_{k\neq i,k=1}^{M}\mu_k + \sum_{k\neq i,k=1}^{M}\beta_k\mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M}\gamma_k p][1 + \rho_i a_i\sum_{k\neq i,k=1}^{M}\beta_k\mathrm{Var}(v|s_i,p)] \\
&- \rho_i a_i(\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i)[1 + \rho_i a_i\sum_{k\neq i,k=1}^{M}\beta_k\mathrm{Var}(v|s_i,p)] = 0.
\end{aligned}
$$

thus plug in $x_i = \mu_i + \beta_i s_i - \gamma_i p$, and since

$$
\begin{aligned}
p &= \frac{x_i + \sum_{k\neq i,k=1}^{N} \mu_k + \sum_{k\neq i,k=1}^{N} \beta_k s_k + z}{\sum_{k\neq i,k=1}^{N} \gamma_k} = \frac{\sum_{k=1}^{N} \mu_k + \sum_{k=1}^{N} \beta_k s_k + z}{\sum_{k=1}^{N} \gamma_k} \\
\lambda_i &= \frac{1}{\sum_{k\neq i,k=1}^{N} \gamma_k} \\
p_i &= \frac{\sum_{k\neq i,k=1}^{N} \mu_k + \sum_{k\neq i,k=1}^{N} \beta_k s_k + z}{\sum_{k\neq i,k=1}^{N} \gamma_k} \\
\mathbb{E}(v|s_i,p_i) &= \bar{v} + \frac{(\sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(s_i - \bar{v}) + \sum_{k\neq i,k=1}^{N} \beta_k \tau_i^{-1} [\sum_{k\neq i} \beta_k (s_k - \bar{v}) + z - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N} \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2} \\
\mathrm{Var}(v|s_i,p_i) &= \frac{\tau_i^{-1}[\sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N} \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2}
\end{aligned}
$$

Equating coefficients we get

Constants:

$$
\begin{aligned}
& [\sum_{k=1}^{M} \mu_k + (\sum_{k\neq i,k=1}^{M} \beta_k)\bar{v} - (\sum_{k\neq i,k=1}^{M} \beta_k) \frac{(\sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2)\bar{v} + \sum_{k\neq i,k=1}^{N} \beta_k \tau_i^{-1}[\sum_{k\neq i} \beta_k \bar{v} + \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N} \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2} \\
& - \sum_{k=1}^{M} \gamma_k \frac{\sum_{k=1}^{N} \mu_k}{\sum_{k=1}^{N} \gamma_k}] \left[\rho_i a_i \mathrm{Var}(v|s_i,p) + \lambda_i + \lambda_i \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p) \right] \\
= \; & (\bar{v} + \frac{(\sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(-\bar{v}) + \sum_{k\neq i,k=1}^{N} \beta_k \tau_i^{-1}[\sum_{k\neq i} \beta_k (-\bar{v}) - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N} \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k\neq i,k=1}^{N} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{\sum_{k=1}^{N} \mu_k}{\sum_{k=1}^{N} \gamma_k}) \\
& \left[1 + \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p) + \lambda_i \rho_i a_i [(\sum_{k\neq i,k=1}^{M} \beta_k)^2 \mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M} \beta_k^2 \tau_k^{-1}] \right] \qquad (67)
\end{aligned}
$$

$$
\begin{aligned}
& s_i : \\
& [\beta_i + (\sum_{k\neq i,k=1}^{M} \beta_k)\frac{(\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2)}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N}\beta_k)^2+\tau_v\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2} - \sum_{k=1}^{M}\gamma_k\frac{\beta_i}{\sum_{k=1}^{N}\gamma_k}] \\
& \left[\rho_i a_i \mathrm{Var}(v|s_i,p) + \lambda_i + \lambda_i\rho_i a_i \sum_{k\neq i,k=1}^{M}\beta_k \mathrm{Var}(v|s_i,p)\right] \\
= \; & (\frac{(\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2)}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N}\beta_k)^2+\tau_v\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2} - \frac{\beta_i}{\sum_{k=1}^{N}\gamma_k}) \\
& \left[1+\rho_i a_i \sum_{k\neq i,k=1}^{M}\beta_k \mathrm{Var}(v|s_i,p) + \lambda_i\rho_i a_i[(\sum_{k\neq i,k=1}^{M}\beta_k)^2\mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M}\beta_k^2\tau_k^{-1}]\right] \qquad (68)
\end{aligned}
$$

$$
\begin{aligned}
& z : \\
& [(\sum_{k\neq i,k=1}^{M} \beta_k)\frac{\sum_{k\neq i,k=1}^{N}\beta_k\tau_i^{-1}}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N}\beta_k)^2+\tau_v\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2} - \frac{\sum_{k=1}^{M}\gamma_k}{\sum_{k=1}^{N}\gamma_k}] \\
& \left[\rho_i a_i \mathrm{Var}(v|s_i,p) + \lambda_i + \lambda_i\rho_i a_i \sum_{k\neq i,k=1}^{M}\beta_k \mathrm{Var}(v|s_i,p)\right] \\
= \; & (\frac{\sum_{k\neq i,k=1}^{N}\beta_k\tau_i^{-1}}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^{N}\beta_k)^2+\tau_v\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i,k=1}^{N}\beta_k^2\tau_k^{-1}+\sigma_z^2} - \frac{1}{\sum_{k=1}^{N}\gamma_k}) \\
& \left[1+\rho_i a_i \sum_{k\neq i,k=1}^{M}\beta_k \mathrm{Var}(v|s_i,p) + \lambda_i\rho_i a_i[(\sum_{k\neq i,k=1}^{M}\beta_k)^2\mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M}\beta_k^2\tau_k^{-1}]\right] \qquad (69)
\end{aligned}
$$

Special case: *ex ante* identical investors Simplify (68) and (69) for $1 \leq j \leq M$ as well as (62) and (63) for $M+1 \leq j \leq N$ we have that under profit-sharing:

$$
\begin{aligned}
& \Bigg[\frac{1}{\gamma_1(M-1)+\gamma_2(N-M)} + \frac{\rho(\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e)}{M\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}} \\
& + \frac{\beta_1(M-1)\rho[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{M[\gamma_1(M-1)+\gamma_2(N-M)]\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}}\Bigg] \\
& \left[\beta_1 - \frac{\beta_1\gamma_1 M}{\gamma_1 M+\gamma_2(N-M)} + \frac{\beta_1(M-1)\tau_e(\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e)}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right] \\
= \; & \left[-\frac{\beta_1}{\gamma_1 M+\gamma_2(N-M)} + \frac{\tau_e[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right] \\
& \Bigg[1 + \frac{\beta_1(M-1)\rho[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{M\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}} \\
& + \frac{\beta_1^2(M-1)\rho\left[1+\frac{(M-1)\tau_e[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right]}{M[\gamma_1(M-1)+\gamma_2(N-M)]\tau_e}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\left[-\frac{\gamma_1 M}{\gamma_1 M+\gamma_2(N-M)}+\frac{\beta_1(M-1)(\beta_1(M-1)+\beta_2(N-M))\tau_e}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right]\\
&\left[\frac{1}{\gamma_1(M-1)+\gamma_2(N-M)}+\frac{\rho(\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e)}{M\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}}\right.\\
&\left.+\frac{\beta_1(M-1)\rho[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{M[\gamma_1(M-1)+\gamma_2(N-M)]\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}}\right]\\
=\;&\left[\frac{1}{-\gamma_1 M+\gamma_2(M-N)}+\frac{[\beta_1(M-1)+\beta_2(N-M)]\tau_e}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right]\\
&\left[1+\frac{\beta_1(M-1)\rho[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{M\{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)\}}\right.\\
&\left.+\frac{\beta_1^2(M-1)\rho\left[1+\frac{(M-1)\tau_e[\beta_1^2(M-1)+\beta_2^2(N-M)+\sigma_z^2\tau_e]}{-2\beta_1\beta_2(M-1)(M-N)\tau_e+\beta_2^2(M-N)[(-1+M-N)\tau_e-\tau_v]+\sigma_z^2\tau_e(\tau_e+\tau_v)+\beta_1^2(M-1)(M\tau_e+\tau_v)}\right]}{M[\gamma_1(M-1)+\gamma_2(N-M)]\tau_e}\right]
\end{aligned}
$$

$$
\frac{[M\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2](M\gamma_1+(N-M)\gamma_2-(M\gamma_1+(N-M-1)\gamma_2)\beta_2\rho\tau_e^{-1})}{(M\beta_1+(N-M-1)\beta_2)^2+\tau_v(M\beta_1^2+(N-M-1)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2+M\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2}=2\beta_2
$$

$$
\frac{(M\gamma_1+(N-M)\gamma_2)[M\beta_1+(N-M-1)\beta_2]+\gamma_2\rho\tau_e^{-1}[M\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2]}{(M\beta_1+(N-M-1)\beta_2)^2+\tau_v(M\beta_1^2+(N-M-1)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2+M\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2}=\frac{-\gamma_2}{M\gamma_1+(N-M-1)\gamma_2}+1
$$

Under information sharing, however equation (60) and (61) lead to

$$
\begin{aligned}
&\frac{[(M-1)/M\beta_1^2+(N-M)\beta_2^2+\tau_e\sigma_z^2]((M\gamma_1+(N-M)\gamma_2)M-((M-1)\gamma_1+(N-M)\gamma_2)\beta_1\rho\tau_e^{-1})}{[((M-1)\beta_1+(N-M)\beta_2)^2+\tau_v((M-1)\beta_1^2/M+(N-M)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2]+((M-1)\beta_1^2/M+(N-M)\beta_2^2)+\tau_e\sigma_z^2}=2\beta_1\\
&\frac{(M\gamma_1+(N-M)\gamma_2)((M-1)\beta_1+(N-M)\beta_2)+\gamma_1\rho\tau_e^{-1}[(M-1)\beta_1^2/M+(N-M)\beta_2^2+\tau_e\sigma_z^2]}{((M-1)\beta_1+(N-M)\beta_2)^2+\tau_v((M-1)\beta_1^2/M+(N-M)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2+M((M-1)\beta_1^2/M+(N-M)\beta_2^2)+M\tau_e\sigma_z^2}=\frac{-\gamma_1}{(M-1)\gamma_1+(N-M)\gamma_2}+1\\
&\frac{[\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2](M\gamma_1+(N-M)\gamma_2-(M\gamma_1+(N-M-1)\gamma_2)\beta_2\rho\tau_e^{-1})}{(M\beta_1+(N-M-1)\beta_2)^2+\tau_v(\beta_1^2+(N-M-1)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2+\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2}=2\beta_2\\
&\frac{(M\gamma_1+(N-M)\gamma_2)(M\beta_1+(N-M-1)\beta_2)+\gamma_2\rho\tau_e^{-1}[\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2]}{(M\beta_1+(N-M-1)\beta_2)^2+\tau_v(\beta_1^2+(N-M-1)\beta_2^2)\tau_e^{-1}+\tau_v\sigma_z^2+\beta_1^2+(N-M-1)\beta_2^2+\tau_e\sigma_z^2}=\frac{-\gamma_2}{M\gamma_1+(N-M-1)\gamma_2}+1
\end{aligned}
$$

If we further assume $\sigma_z^2\to\infty$ then under profit-sharing we have:

$$
\begin{aligned}
&\left[\frac{1}{\gamma_1(M-1)+\gamma_2(N-M)}+\frac{\rho}{M(\tau_e+\tau_v)}+\frac{\beta_1(M-1)\rho}{M[\gamma_1(M-1)+\gamma_2(N-M)](\tau_e+\tau_v)}\right]\left[\beta_1-\frac{\beta_1\gamma_1 M}{\gamma_1 M+\gamma_2(N-M)}+\frac{\beta_1(M-1)\tau_e}{(\tau_e+\tau_v)}\right]\\
=\;&\left[-\frac{\beta_1}{\gamma_1 M+\gamma_2(N-M)}+\frac{\tau_e}{(\tau_e+\tau_v)}\right]\left[1+\frac{\beta_1(M-1)\rho}{M(\tau_e+\tau_v)}+\frac{\beta_1^2(M-1)\rho\frac{M\tau_e+\tau_v}{\tau_e+\tau_v}}{M[\gamma_1(M-1)+\gamma_2(N-M)]\tau_e}\right]
\end{aligned}
\tag{70}
$$

$$
\begin{aligned}
&\left[\frac{1}{\gamma_1(M-1)+\gamma_2(N-M)}+\frac{\rho}{M(\tau_e+\tau_v)}+\frac{\beta_1(M-1)\rho}{M[\gamma_1(M-1)+\gamma_2(N-M)](\tau_e+\tau_v)}\right]\gamma_1 M\\
=\;&\left[1+\frac{\beta_1(M-1)\rho}{M(\tau_e+\tau_v)}+\frac{\beta_1^2(M-1)\rho\frac{M\tau_e+\tau_v}{\tau_e+\tau_v}}{M[\gamma_1(M-1)+\gamma_2(N-M)]\tau_e}\right]
\end{aligned}
\tag{71}
$$

$$\frac{[M\gamma_1 + (N-M)\gamma_2]\tau_e - (M\gamma_1 + (N-M-1)\gamma_2)\beta_2\rho}{\tau_v + \tau_e} = 2\beta_2 \tag{72}$$

$$\frac{\gamma_2\rho}{\tau_v + \tau_e} = \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1 \tag{73}$$

Plug (71) into (70) we get

$$\left[\beta_1 - \frac{\beta_1\gamma_1 M}{\gamma_1 M + \gamma_2(N-M)} + \frac{\beta_1(M-1)\tau_e}{(\tau_e+\tau_v)}\right] = \left[-\frac{\beta_1}{\gamma_1 M + \gamma_2(N-M)} + \frac{\tau_e}{(\tau_e+\tau_v)}\right]\gamma_1 M$$

$$\Rightarrow \quad \beta_1 = \frac{\tau_e}{M\tau_e + \tau_v}\gamma_1 M \tag{74}$$

Plug (74) into (71) we get

$$\left[\frac{1}{\gamma_1(M-1)+\gamma_2(N-M)} + \frac{\rho}{M(\tau_e+\tau_v)} + \frac{\gamma_1(M-1)\rho\tau_e}{[\gamma_1(M-1)+\gamma_2(N-M)](\tau_e+\tau_v)(M\tau_e+\tau_v)}\right]\gamma_1 M$$

$$= \left[1 + \frac{\gamma_1(M-1)\rho\tau_e}{(\tau_e+\tau_v)(M\tau_e+\tau_v)} + \frac{\gamma_1^2 M(M-1)\rho\tau_e}{[\gamma_1(M-1)+\gamma_2(N-M)](\tau_e+\tau_v)(M\tau_e+\tau_v)}\right] \tag{75}$$

$$\Rightarrow \quad \frac{\gamma_1 M}{\gamma_1(M-1)+\gamma_2(N-M)} + \frac{\rho\gamma_1}{M\tau_e+\tau_v} = 1 \tag{76}$$

The results are summarized in the following theorem

Theorem D.1. *Under profit sharing, we have*

$$\begin{aligned} \beta_1 &= \frac{M\tau_e\gamma_1}{M\tau_e+\tau_v} \\ \beta_2 &= \frac{\tau_e\gamma_2}{\tau_v+\tau_e}, \end{aligned}$$

while γ_1 and γ_2 are determined by

$$\begin{aligned} \frac{\gamma_1\rho}{M\tau_e+\tau_v} &= \frac{-\gamma_1 M}{(M-1)\gamma_1+(N-M)\gamma_2} + 1 \\ \frac{\gamma_2\rho}{\tau_v+\tau_e} &= \frac{-\gamma_2}{M\gamma_1+(N-M-1)\gamma_2} + 1. \end{aligned}$$

Under information sharing, equation (62) and (63) lead to

$$\frac{(M\gamma_1 + (N-M)\gamma_2)M\tau_e - ((M-1)\gamma_1 + (N-M)\gamma_2)\beta_1\rho}{\tau_v + \tau_e} = 2\beta_1 \tag{77}$$

$$\frac{\gamma_1\rho}{\tau_v + M\tau_e} = \frac{-\gamma_1}{(M-1)\gamma_1 + (N-M)\gamma_2} + 1 \tag{78}$$

$$\frac{[M\gamma_1 + (N-M)\gamma_2]\tau_e - (M\gamma_1 + (N-M-1)\gamma_2)\beta_2\rho}{\tau_v + \tau_e} = 2\beta_2 \tag{79}$$

$$\frac{\gamma_2\rho}{\tau_v + \tau_e} = \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1 \tag{80}$$

(77) $\Rightarrow$

$$\beta_1 = \frac{(M\gamma_1 + (N-M)\gamma_2)M\tau_e}{\rho[(M-1)\gamma_1 + (N-M)\gamma_2] + 2(\tau_v + \tau_e)} \tag{81}$$

(78) $\Rightarrow \gamma_1\rho[(M-1)\gamma_1 + (N-M)\gamma_2] = -(\tau_v + M\tau_e)\gamma_1 + (\tau_v + M\tau_e)[(M-1)\gamma_1 + (N-M)\gamma_2]$

$$\begin{aligned}
\therefore \quad & \{\rho[(M-1)\gamma_1 + (N-M)\gamma_2] + (\tau_v + M\tau_e)\}\gamma_1 = (\tau_v + M\tau_e)[(M-1)\gamma_1 + (N-M)\gamma_2] \\
\Rightarrow \quad & \{\rho[(M-1)\gamma_1 + (N-M)\gamma_2] + 2(\tau_v + M\tau_e)\}\gamma_1 = (\tau_v + M\tau_e)[M\gamma_1 + (N-M)\gamma_2] \\
\Rightarrow \quad & \frac{M\gamma_1 + (N-M)\gamma_2}{\rho[(M-1)\gamma_1 + (N-M)\gamma_2] + 2(\tau_v + M\tau_e)} = \frac{\gamma_1}{\tau_v + M\tau_e}
\end{aligned}$$

Plug in (81) we have

$$\beta_1 = \frac{M\tau_e\gamma_1}{\tau_v + M\tau_e},$$

and similarly we have

$$\beta_2 = \frac{\tau_e\gamma_2}{\tau_v + \tau_e},$$

while γ_1 and γ_2 are jointly determined by solving (78) and (80). The results are summarized below

Theorem D.2. *Under information sharing, we have*

$$\begin{aligned}
\beta_1 &= \frac{M\tau_e\gamma_1}{M\tau_e + \tau_v} \\
\beta_2 &= \frac{\tau_e\gamma_2}{\tau_v + \tau_e},
\end{aligned}$$

while γ_1 and γ_2 are determined by

$$\begin{aligned}
\frac{\gamma_1\rho}{M\tau_e + \tau_v} &= \frac{-\gamma_1}{(M-1)\gamma_1 + (N-M)\gamma_2} + 1 \\
\frac{\gamma_2\rho}{\tau_v + \tau_e} &= \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1.
\end{aligned}$$

Compared to the the profit-sharing case, both β_i and γ_i $(i \in \{1, 2\})$ are smaller under profit-sharing.

Under full information, investors' payoffs are $\mathbb{E}\left[-\exp(-\rho(v-p(x_i))x_i)|s_i,p\right]$ when $\sigma_z^2 \to \infty$.[38]

$$
\begin{aligned}
& \mathbb{E}\left[-\exp(-\rho(v-p(x_i))x_i)|s_i,p_i\right] \\
= \ & \mathbb{E}\left[-\exp(-\rho v \frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}})|s_i,p_i\right] \\
& \exp\left[\rho(p_i+\frac{1}{(M-1)\gamma_1+(N-M)\gamma_2}\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}})\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}\right] \\
= \ & -\exp\left[-\rho\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)\frac{\tau_i s_i}{\tau_v+\tau_i}}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}+\frac{1}{2}\rho^2\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)^2\frac{1}{\tau_v+\tau_i}}{(2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i})^2}\right. \\
+ \ & \left.\rho\left(p_i+\frac{1}{(M-1)\gamma_1+(N-M)\gamma_2}\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}\right)\frac{(\frac{\tau_i s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}\right]
\end{aligned}
$$

thus the expected utility for those who participate in information sharing is

$$
\begin{aligned}
& -\exp\left[-\rho\frac{(\frac{M\tau_e s_i}{\tau_v+\tau_i}-p_i)\frac{M\tau_e s_i}{\tau_v+\tau_i}}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}+\frac{1}{2}\rho^2\frac{(\frac{M\tau_e s_i}{\tau_v+\tau_i}-p_i)^2\frac{1}{\tau_v+\tau_i}}{(2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i})^2}\right. \\
+ \ & \left.\rho\left(p_i+\frac{1}{(M-1)\gamma_1+(N-M)\gamma_2}\frac{(\frac{M\tau_e s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}\right)\frac{(\frac{M\tau_e s_i}{\tau_v+\tau_i}-p_i)}{2[(M-1)\gamma_1+(N-M)\gamma_2]+\rho\frac{1}{\tau_v+\tau_i}}\right].
\end{aligned}
$$

thus expected utility is

$$
-\exp\left[-\frac{\rho(M\tau_e s_i-p_i(M\tau_e+\tau_v))^2(\gamma_2(N-M)\rho-2(M\tau_e+\tau_v)+4\gamma_1^2(M-1)^2(M\tau_e+\tau_v)+4\gamma_2^2(M-N)^2(M\tau_e+\tau_v)+\gamma_1(M-1)(\rho-8\gamma_2(M-N)(M\tau_e+\tau_v)))}{2(\gamma_1(M-1)+\gamma_2(N-M))(M\tau_e+\tau_v)(\rho+2(\gamma_1(M-1)+\gamma_2(N-M))(M\tau_e+\tau_v))^2}\right]
$$

While under profit sharing, partner i's expected utility is given by

$$
-\frac{\exp(\frac{A}{2B})}{\sqrt{B}}, \tag{82}
$$

[38]Notice that the conditioning on p_i is important even when $\sigma_z^2 \to \infty$, i.e. price itself does not aggregate information.

where

$$\begin{aligned}
A &= \frac{\rho^2}{M^2}(\frac{\tau_e s_i}{\tau_v+\tau_e} - p_i - \lambda_i x_i)^2[((M-1)\beta_1)^2\frac{1}{\tau_v+\tau_e} + (M-1)\beta_1^2\tau_e^{-1}] \\
&+ [x_i + (M-1)\beta_1\frac{\tau_e s_i}{\tau_v+\tau_e} - (M-1)\gamma_1(p_i+\lambda_i x_i)]^2\frac{\rho^2}{M^2}\frac{1}{\tau_v+\tau_e} \\
&- 2\frac{\rho}{M}(\frac{\tau_e s_i}{\tau_v+\tau_e} - p_i - \lambda_i x_i)[x_i + (M-1)\beta_1\frac{\tau_e s_i}{\tau_v+\tau_e} - (M-1)\gamma_1(p_i+\lambda_i x_i)] \\
&\quad [1+\frac{\rho}{M}(M-1)\beta_1\frac{1}{\tau_v+\tau_e}] \\
B &= [1+\frac{\rho}{M}(M-1)\beta_1\frac{1}{\tau_v+\tau_e}]^2 - \frac{\rho^2}{M^2}\frac{1}{\tau_v+\tau_e}[((M-1)\beta_1)^2\frac{1}{\tau_v+\tau_e} + (M-1)\beta_1^2\tau_e^{-1}]
\end{aligned}$$

D.1 Numerical Illustration on Decreasing Return to Scale

Numerical illustration: I solve a numerical example to illustrate effect of decreasing return to scale of the business opportunity on the structure of optimal compensation contracts. Consider choosing a_1 and a_2 to maximize

$$\gamma_1\mathbb{E}[-e^{-\rho_1[r-\lambda(x_1+x_2)](a_1x_1+a_2x_2)-\rho_1 b}|s_1] + \gamma_2\mathbb{E}[-e^{-\rho_2[r-\lambda(x_1+x_2)][(1-a_1)x_1+(1-a_2)x_2]+\rho_2 b}|s_2],$$

where γ_1 and γ_2 are Pareto weights, and b measures (if any) side payments. λ measures the degree to decreasing return to scale. Notice that $\lambda = 0$ corresponds to the Alice-Bob example. I calibrate the parameters to be $\rho_1 = 10, \rho_2 = 15, \tau_1 = 7, \tau_2 = 5, \tau_r = 3, \gamma_1 = \gamma_2 = .5, b = 0$, and solve for optimal a_1 and a_2 when λ varies from 0 to 1%. Results are summarized below.

λ	a_1	a_2
0	60%	61%
0.1%	56%	61%
0.2%	53%	62%
0.3%	50%	62%
0.4%	48%	62%
0.5%	47%	62%
0.6%	45%	62%
0.7%	44%	62%
0.8%	42%	62%
0.9%	41%	62%
1.0%	40%	62%

With decreasing return to scale, optimal compensation also contains a bonus part in addition to profit-sharing. That said, the profit-sharing component still constitutes a large proportion of each player's compensation. The less degree of decreasing return to scale, the more profit-sharing components in compensation. Having some profit-sharing elements in compensations could make signing parties better-off because of the benefit from wisdom of the crowd.

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