Is Bigger Better? Multi-product Firms, Labor Market Imperfections, and International Trade*

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Abstract

International trade is primarily conducted by large, multi-product firms (MPFs) that pay above average wages and exhibit high productivity. In this paper, we introduce labor market frictions and worker heterogeneity into the standard Krugman framework and show that the augmented model simultaneously generates both small, non-exporting single-product firms (SPFs) and large, exporting MPFs. Further, the MPFs that emerge pay higher wages and exhibit higher measured labor productivity than SPFs. Despite their exceptional performance, MPFs are inefficient because they use their ability to extract rents from their workers to expand into high unit input activities. Trade liberalization always raises the real income of high wage workers and lowers real income for low wage workers but has an ambiguous effect on aggregate welfare. Although trade liberalization forces MPFs to become more efficient, it also reallocates resources from small, efficient firms to the large, inefficient firms. The model highlights the need to know why firms "excel" before drawing welfare conclusions regarding cross firm reallocations of resources.

Keywords: International Trade, Labor Market Imperfections, Multi-product Firms, Productivity, Wage Inequality, Welfare.

JEL Classification: F12, ...

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1 Introduction

It is well known that exporters display characteristics that make them appear systematically "better" than non-exporters. As demonstrated by Bernard and Jensen (1999), exporters tend to be much larger, appear to be more productive, and to pay higher wages than do non-exporters. More recently, it has also been demonstrated that they tend to be highly diversified conglomerates that produce many products. For instance, Bernard, Jensen, Redding, and Schott (2007) show that the set of U.S. based firms that export more than 5 products to more than 5 locations account for only 1 percent of exporters but over 90 percent of export value. These patterns appear to be ubiquitous across countries at many levels of development (e.g. Freund and Pierola, 2012).

The dominance of multiproduct firms in international trade raises many interesting questions. First, why do some, but not all, firms expand across often unrelated product categories? Going back to Coase (1937), firm expansion occurs when internal transactions are more efficient than market transactions. If this is the case, then it is a mystery that some firms produce in many industries for many markets while others remain small and highly focused. Second, what is behind the covariance between the apparently better performance attributed in the trade literature to large, conglomerate exporters? This better performance is particularly curious given that a large finance literature has grown to explain why it is that conglomerates appear to be inefficient relative to less diversified firms (e.g. Scharfstein and Stein, 2000). Third, given the importance of multiproduct firms in international trade, how do trade shocks induce factors to reallocate across and within firms, and what are the associated welfare implications?

To address these questions, we introduce labor market frictions into the workhorse model of Krugman (1980). In the model, workers are heterogeneous in terms of their productivity and that productivity is initially known only privately to the worker. Ex ante identical firms have a choice of whether to invest in a human resources (screening) technology that makes it possible to observe the ability of a worker, however. All firms have access to "flexible manufacturing technologies" that allow them to introduce a range of products that enter demand symmetrically but that have increasingly higher marginal cost as these products become further from the firm's "core" variety as in Eckel and Neary (2010).

In the equilibrium of our model, more productive workers sort into the firms that have paid the fixed human resource costs that allows them to identify worker productivities. These workers are then paid a wage that reflects their productivity and is determined by competition across those firms that have invested in the screening technology. Less productive workers are employed by firms that cannot observe their individual productivities and so are paid a wage reflects the average productivity of workers in this pool.

The higher entry costs asssociated with human resource investments has the implication that demand for higher skilled workers is kept relatively low so that their average effective wage, although higher than paid at single product firms, does not fully reflect the high average productivity of their workers. As a result, firms that have invested in human resource activities have lower marginal costs than do firms that have not, and this induces them to enter higher marginal cost activities, such as non-core products and export markets. Hence, the model reproduces the main stylized facts: exporters are larger, display higher labor productivity, pay higher wages and produce more products than non-exporters.

Because multi-product firms exhibit superior performance measures in our model, one might come to the conclusion that they are also socially desirable. This conclusion is incorrect. Because these firms have expanded into non-core products, they are inherently less efficient than single product firms and so their existence lowers aggregate output. Nevertheless, higher productivity workers benefit in real terms from the existence of multi-product firms as they are not forced into a common pool with lower productivity workers.

We then analyze the impact of a generalized reduction in trade costs between countries. As exporters are multiproduct firms that hire better workers, a reduction in trade cost raises the relative demand for better workers and increases the wage premium. Moreover, one can show that the real wage of single product firm workers falls and the real wage of multiproduct workers rises. What one cannot show without knowing exactly the initial conditions is whether aggregate welfare increases or decreases. The higher real wage among high productivity workers induced by the trade liberalization induces firms to trim their least efficient product offerings, which raises aggregate welfare, but it also induces workers to reallocate from single product firms to the inefficient conglomerates, which reduces aggregate welfare.

We bring to the general equilibrium, international trade literature a new perspective on the implications of labor market frictions for resource allocation between and across firms. We demonstrate that multiproduct firms naturally arise in response to the existence of asymmetric information on worker productivities and that the observed characteristics of multiproduct firms relative to single product firms can be traced to their human resource investments that give rise to their internal labor markets. In so doing, our model helps to understand why not all firms are observed to undertake apparently productivity increasing activities associated with management structure (see Bloom et al 2012).

Our paper draws on and contributes to a diverse set of literatures. We are related to the literature on firm organization (e.g. Lazear and coauthors, Waldman 2007, etc) in our focus on the implications of information asymmetries between workers and firms. This literature, however, has been primarily focused on the complex incentive problems inside firms and abstract from general equilibrium conditions surrounding the interaction of firms. We simplify the manner in which information problems are (partially) overcome to isolate its role in generating competitive advantages across firms and to allow us to analyze the general equilibrium implications of information asymmetries.

We also contribute to the literature on multiproduct firms. We share with Eckel and Neary (2010), Bernard, Redding and Schott (2011), and Nocke and Yeaple (2014) a focus on the role of international trade in affecting the product range of multi-product firms and the assumption that firms may produce a range of products that vary in their marginal costs. We go further in analyzing the forces that give rise to the coexistence of single product firms and multiproduct firm and the wage inequality implications of worker sorting in the presence of incomplete information.

Our paper also contributes to the literature on non-traditional mechanisms in explaining growing wage inequality. As in Yeaple (2005) and Sampson (2013) firm heterogeneity arises endogenously from worker heterogeneity and firm choice of technology. In those papers, firm productivity is described by a function that is log-supermodular in technology and worker skill. In our setting, firm choices are over whether to invest in a screening technology that allows these firms to identify, and so better reward, higher quality workers rather than to alter firm productivity directly. Our focus on a screening technology in the presence of worker heterogeneity, we are also related to recent work by Helpman, Itshsoki, and Redding (2010). Firm heterogeneity in their setting does not arise endogeneously, within firm reallocations are not analyzed, and the differences in the nature of labor market imperfections leads to entirely different welfare implications.

The remainder of this paper is divided into three sections. In section one, we specify the model and characterize its equilibrium in both a closed and an open economy setting. We show that multiproduct and single product firms generically exist in equilibrium and that exporters will pay higher wages, have higher labor productivity, and be multiproduct firms. In section two, we analyze the effect of a multilateral trade liberalization. We show that the real income of high wage workers rises while the real income of low wage workers falls as resources are allocated away from non-exporters to exporters. We also show that the sign of the effect on aggregate welfare hinges on whether multiproduct firms were common (welfare tends to rise) or scarce (welfare tends to fall) in the initial equilibrium. The final section concludes.

2 Model

This section lays out the model and solves for its equilibrium. The model assumptions are laid out in section 2.1. In section 2.2, we consider the closed economy equilibrium. We show that single product and multiple product firms coexist in equilibrium and that multiple product firms tend to be larger, to appear more productive, and pay higher wages than single product firms. In section 2.3 we consider a trading environment in which two symmetric countries are separated by fixed and variable costs. We show that under standard assumptions regarding the size of these costs, only multiple product firms export.

2.1 Key Assumptions

2.1.1 Demand

On the demand side, we are not making any new or specific assumption but follow Krugman (1980). Consumers derive utility from the consumption of horizontally differentiated varieties. The utility function of a consumer is CES:

$$U = \left(\int_{i \in \tilde{\Omega}} q(i)^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $q(i)$ is the quantity consumed, σ is the elasticity of substition between any two varieties, and $\tilde{\Omega}$ is the set of potentially consumable varieties.

Direct demand for variety $i \in \Omega$ (the set of actually produced varieties) is then given by

$$x(i) = EP^{\sigma-1} p(i)^{-\sigma}, \tag{2}$$

where $x(i)$ is economy-wide output of variety i and E is aggregate income in the economy. P stands for the price index, defined by

$$P \equiv \left(\int_{i \in \Omega} p(i)^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

2.1.2 Production

There are two types of factors of production: Management H and labor L. Management is a homogeneous factor that is used as our numéraire. As in Yeaple (2005), labor consists of a continuum of heterogeneous workers with skills (or productivity) z. A worker of type z has z units of effective labor. The distribution of skills in the economy is described by the probability density function $g(z)$ with positive support over $[\underline{z}, \infty)$ ($\underline{z} > 0$) and its cumulation distribution function $G(\tilde{z}) = \int_{\underline{z}}^{\tilde{z}} g(z)\, dz$.

Production of a variety $x(i)$ requires a fixed costs f in units of management plus marginal costs in units of (effective) labor. These marginal costs are constant for a specific variety but may vary across varieties. They consist of a unit labor requirement α (in units of effective labor) plus a factor cost component c_j. The factor cost component is firm specific, as denoted by the subscript j.

Unit labor requirements α are given by technology, and all firms have access to the same technology. We follow Eckel and Neary (2010) and assume that all firms posses a certain core competency for a specific variety where their unit labor costs is lowest for all products in their product range. All other products in their product range can then be identified by their (unidimensional) distance to the firm's core competency, denoted by $\omega > 0$. Production of multiple products is subject to flexible manufacturing, which implies that firms can add and drop products to and from their product range freely, but as they add products to their product range and move away from their core competency, unit labor requirements of these products increases. Thus, unit labor requirements α depend on the position ω of a product in a firm's product range, and are increasing in ω:

$$\alpha = \alpha(\omega) \qquad \text{and} \qquad \alpha'(\omega) \equiv \partial\alpha/\partial\omega > 0. \tag{4}$$

To simplify notation we normalize unit labor requirements at the core to one: $\alpha(0) = 1$.

2.1.3 Market Structure and Timing

The market for the homogeneous factor management H is perfectly competitive, and the wage of a unit of management is normalized to one. Workers are fully informed about their own productivity but only know the distribution of productivity in the population, G, which is common knowledge.

In the absence of a screening technology, firms do not observe the productivity of any given worker. It is this information asymmetry that gives rise to the market imperfection. A screening technology exists but is only available to a firm if it incurs a fixed cost f_m (in units of management). One can think of this screening technology as an investment in a human resource staff that can accurately assess productivity. Firms that have incurred f_m can immediately evaluate the productivity of a worker while firms that have not incurred f_m can never observe the productivity of any worker.

This is a one shot game that occurs in five stages. All agents have rational expectations and perfect foresight.

In stage 1, firms enter and decide whether they want to pay f_m and acquire the screening technology or not. This determines their type: Type-m firms pay f_m, type-s firms do not.[1] There is a continuum of firms of both types and their masses will be denoted by n_j ($j \in \{m, s\}$).

Once firms have made their entry and screening technology investments, two labor markets open. Firms that have made the screening investment, $j = m$, operate in one labor market while firms that have not made the screening investment, $j = s$, operate in the other. Let the set of workers that ultimately choose to be in labor market j as Z_j.

We refer to the labor market associated with firms $j = m$ as the "frictionless" labor market because all information regarding workers in that labor market is known by all firms. Perfect competition implies that the wage of worker 1 relative to worker 2 with productivities z_1, $z_2 \in Z_m$ satisfy the no arbitrage condition $w_1/w_2 = z_1/z_2$.

We refer to the labor market associated with firms $j = s$ as the "frictional" labor market because individual worker productivies, $z \in Z_s$, are known only to the workers. The inability of firms $j = s$ to verify workers' productivities requires that there must be a single wage $w = w_S$ for all $z \in Z_s$.

In Stage 2, workers choose whether to enter the frictionless or the frictional labor market. They make this choice with perfect foresight regarding the wage they would receive in each labor market.

In stage 3, the set of products produced (ω) is chosen, and the fixed costs f per product is paid. Firms that produce only their core competency product are called single-product firms (SPF), firms that produce multiple products are called multi-product firms (MPF).

In stage 4, both frictionless and frictional labor markets clear.

In stage 5, production occurs and product markets are cleared. Firms compete via monopolistic competition. Individual products are atomistic and there is no strategic interaction.

2.2 Closed Economy

We begin with the characterization of the closed economy and solve the model backwards.

2.2.1 Product market clearing

Given demand (2) and a market structure of monopolistic competition, the profit-maximizing price charged by division ω of firm j is a constant mark-up over its marginal costs:

$$p(\omega, c_j) = \frac{\sigma}{\sigma - 1} \alpha(\omega) c_j, \tag{5}$$

[1] We have chosen this notation because type m-firms will turn out to be multi-product firms and type s-firms will be single-product firms. This will be proven below in proposition 2.

where j denotes firm type $j \in \{m, s\}$. Since all firms have access to the same technology, and demands are symmetric across all products, all firms within one type will be symmetric. Since firms of different types are drawing their workers from different labor markets, their factor costs c_j may be different, hence the subscipt j.

In order to simplify notation we define

$$A \equiv (\sigma - 1)^{\sigma-1} \sigma^{-\sigma} E P^{(\sigma-1)}. \tag{6}$$

This parameter A depends only on aggregate income E, the price index P, and the elasticity of substitution σ. Since firms are atomistic, A is exogenous to the firm.

Given (2), (5) and (6), output of variety ω can be written as

$$x(\omega, c_j) = (\sigma - 1) A c_j^{-\sigma} \alpha(\omega)^{-\sigma}, \tag{7}$$

and revenues are

$$p(\omega, c_j)\, x(\omega, c_j) = \sigma A c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} \tag{8}$$

Finally, profits per product are variable profits $p(\omega, c_j)\, x(\omega, c_j) / \sigma$ minus fixed costs f:

$$\pi(\omega, c_j) = A c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} - f. \tag{9}$$

2.2.2 Labor market clearing

Worker sorting in stage two leads to segmentation of labor markets by firm type. The labor market equilibrium for type $j \in \{m, s\}$ is

$$n_j \int_0^{\omega_j^d} x(\omega, c_j)\, \alpha(\omega)\, d\omega = \tilde{L}_j, \tag{10}$$

where ω_j^d is the mass of varieties produced by firms of type j,[2] and $\tilde{L}_j$ is the effective supply of labor available to firms of type j. Since workers sort in stage two, and firms decide on their product range in stage 3, both of these variables are given at this stage, and the labor market equilibrium determines the effective wages c_j. In both labor markets $j \in \{m, s\}$ firms are atomistic and take wages as given.

Market clearing of the numéraire factor (management H) is implied in general equilibrium.

[2] The superscript d stands for domestic. It is superfluous in the case of the closed economy, but will be useful in the open economy to distinguish domestic sales from exports.

2.2.3 Product Range

The product range is determined at the firm (or conglomerate) level. Firm level profits Π_j consist of all the profits of its divisions minus the fixed costs for the screening technology (for type-m firms):

$$\Pi_j = \int_0^{\omega_j^d} \pi(\omega, c_j)\, d\omega - \mathbb{I}_j f_m, \tag{11}$$

where $\mathbb{I}_j$ is an indicator variable that takes on the value 1 if $j = m$ and 0 otherwise.

The first order condition with respect to the product range requires that $d\Pi_j / d\omega_j^d = 0$. Since firms are atomistic, they are price takers in both labor markets. Using the Leibniz integral rule, the first order condition can be expressed as

$$\frac{d\Pi_j}{d\omega_j^d} = \pi\left(\omega_j^d, c_j\right) = 0. \tag{12}$$

Thus, using (9), the optimal scope ω_j^d is determined by

$$A c_j^{1-\sigma} \alpha \left(\omega_j^d\right)^{1-\sigma} = f. \tag{13}$$

2.2.4 Worker sorting

Workers can observe whether a firm has invested in the screening technology or not. Thus, they can decide whether they want to apply for a job in a type-m firm or in a type-s firm by choosing the respective labor pool. There are no differences in non-pecuniary job returns, so this decision is entirely based on differences in wages.

The labor market of type-m firms is perfectly competitive. After screening, the true productivity of workers is known by all firms in this labor market segment, and they can pay a wage to individual workers based on this worker's true productivity. Anticipating correctly the effective wage c_m determined in stage 4, firms of type-m pay

$$w_m(z) = c_m z. \tag{14}$$

The labor market of type-s firms is only imperfectly competitive. Firms in this labor market segment have not acquired the screening technology and hence never know the true productivity of their workers. But they do know the distribution of productivities in their labor market pool. Consequently, the wage rate cannot be conditioned on the true productivity of any particular worker, but rather depends on the expected productivity of a representative bundle of workers in this labor market segment:

$$w_s = c_s \mathbb{E}_s(z). \tag{15}$$

Given that wages differ between these two types of firms, each worker can decide whether he or she wants to apply for a job in the frictionless labor market of type-m firms or in the frictional labor market of type-s firm. The wage of a worker with productivity z is thus

$$w = \max\{c_s \mathbb{E}_s(z); c_m z\}. \tag{16}$$

The following proposition describes the sorting outcome:

Proposition 1 (Sorting) *There exists at least one stable equilibrium that is characterized by a $\tilde{z}$ so that workers with $z > \tilde{z}$ will choose to work for type-m firms, and workers with $z < \tilde{z}$ will choose to work for type-s firms. The critical $\tilde{z}$ is determined by*

$$c_s \bar{z}_s(\tilde{z}) = c_m \tilde{z}, \tag{17}$$

where $\bar{z}_s(\tilde{z}) \equiv \int_{\underline{z}}^{\tilde{z}} z dG(z)/G(\tilde{z})$. This equilibrium is stable if $\bar{z}_s(\tilde{z})/\tilde{z}$ is decreasing in $\tilde{z}$.

Proof. Assume a $\tilde{z}$ exists, so that $\mathbb{E}_s(z) = \int_{\underline{z}}^{\tilde{z}} z dG(z)/G(\tilde{z}) = \bar{z}_s(\tilde{z})$. Then rewrite condition (17) as $\bar{z}_s(\tilde{z})/\tilde{z} = c_m/c_s$. Using L'Hôpital's rule, we can determine the limits of $\bar{z}_s(\tilde{z})/\tilde{z}$ as $\tilde{z}$ approaches the boundaries of the support: $\lim_{\tilde{z}\to\underline{z}}[\bar{z}_s(\tilde{z})/\tilde{z}] = 1$ and $\lim_{\tilde{z}\to\infty}[\bar{z}_s(\tilde{z})/\tilde{z}] = 0$. Since $\bar{z}_s(\tilde{z})/\tilde{z}$ is differentiable, this proves existence of (at least) one equilibrium with $\underline{z} < \tilde{z} < \infty$ for $c_m < c_s$. Furthermore, this equilibrium implies sorting where the most productive workers work for type-m firms and the least productive work for type-s firms: $c_m z > c_s \bar{z}_s(\tilde{z})$ for $z > \tilde{z}$ and $c_m z < c_s \bar{z}_s(\tilde{z})$ for $z < \tilde{z}$. This equilibrium is stable if for $\zeta < \tilde{z}$, $c_s \bar{z}_s(\zeta) > c_m \zeta$, and for $\zeta > \tilde{z}$, $c_s \bar{z}_s(\zeta) < c_m \zeta$. Thus, stability implies that $\bar{z}_s(\tilde{z})/\tilde{z}$ is decreasing in $\tilde{z}$ and requires that

$$\frac{\tilde{z} g(\tilde{z})}{G(\tilde{z})} \frac{[\tilde{z} - \bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})} < 1. \tag{18}$$

Since $\bar{z}_s(\tilde{z})/\tilde{z}$ is decreasing globally (from 1 to 0), at least one stable equilibrium must exist. This equilibrium is unique if $\bar{z}_s(\tilde{z})/\tilde{z}$ is monotonically decreasing. ■

[FIGURE 1 here]

In Figure 1 we illustrate the equilibrium and its stability graphically. For illustrative purposes, the function $\bar{z}_s(\zeta)/\zeta$ is not monotonic. Clearly, if $c_s \bar{z}_s(\zeta) > c_m \zeta$, a worker with skill ζ earns higher wages in type-s firms then in type-m firms. Thus, if ζ was a sorting cutoff, this would not be an equilibrium because the marginal worker would want to work for type-s firms, leading to an increase in this cutoff. Therefore, a stable equilibrium requires that the $\bar{z}_s(\zeta)/\zeta$-function intersects c_m/c_s from above. In our Figure 1, equilibria $E1$ and $E3$ are stable, $E2$ is unstable. In what follows we only consider stable equilibria, so we assume that (18) holds.

One important implication of the sorting equilibrium is that

$$c_m = \frac{\bar{z}_s(\tilde{z})}{\tilde{z}} c_s < c_s. \tag{19}$$

Thus, type-m firms that have invested in the screening technology pay a lower effective wage rate (in efficiency units) than type-s firms with no access to the screening technology. This has to hold in equilibrium because the productivity of the marginal worker is discretely higher than the average productivity of all workers with a lower productivity: $\tilde{z} > \bar{z}_s(\tilde{z})$. Therefore, type-$s$ firms have to pay a premium on the effective wage rate of type-m firms in order to compensate their above-average workers for pooling them with below-average workers.

In a sorting equilibrium, we can now also determine the effective supplies of labor $\tilde{L}_j$ for the two types of firms from (10):

$$\tilde{L}_s = LG(\tilde{z})\,\bar{z}_s(\tilde{z}) \qquad \text{and} \qquad \tilde{L}_m = L[1 - G(\tilde{z})]\,\bar{z}_m(\tilde{z}), \tag{20}$$

where $\bar{z}_m(\tilde{z}) \equiv \int_{\tilde{z}}^{\infty} z dG(z) / [1 - G(\tilde{z})]$.

2.2.5 Firm entry

All types of firms can enter and exit freely. Within types, firms are symmetric. This implies that their respective profits are driven down to zero. Given (11), this zero profit condition requires for type-s firms

$$\Pi_s = \int_0^{\omega_s^d} \pi(\omega, c_s)\, d\omega = 0 \tag{21}$$

and for type-m firms

$$\Pi_m = \int_0^{\omega_m^d} \pi(\omega, c_m)\, d\omega - f_m = 0 \tag{22}$$

In this stage, upon entry, type-m firms invest in the screening technology and pay $f_m > 0$.

We can now establish the following proposition regarding firm types:

Proposition 2 (Firm types) *Type-m firms are multi-product firms and type-s firms are single-product firms.*

Proof. For type-s firms, the first order condition for scope (13) and the free entry condition (21) require that $\int_0^{\omega_s^d} \alpha(\omega)^{1-\sigma} d\omega/\omega_s^d = \alpha\left(\omega_s^d\right)^{1-\sigma}$. Since $\alpha'(\omega) > 0$ and $\alpha(0) = 1$ [from flexible manufacturing (4)], this can only hold for $\omega_s^d = 0$, where (by L'Hôpital's rule) $\lim_{\omega_s^d \to 0}\left[\int_0^{\omega_s^d} \alpha(\omega)^{1-\sigma} d\omega/\omega_s^d\right] = 1$. In addition, we have $d\Pi_j/d\omega_s^d\big|_{\omega_s^d=0} = \pi(0, c_s) = 0$. Therefore, type-$s$ firms produce only their core competency product and have no incentives to add any additional products to their product range (the marginal profits of doing so are zero). They become single-product firms. For type-m firms, (13) and (22) imply that $\int_0^{\omega_m^d} \alpha(\omega)^{1-\sigma} d\omega/\omega_m^d = \alpha\left(\omega_m^d\right)^{1-\sigma}(1 + f_m/f)$. Since $f_m > 0$, this condition requires that $\omega_m^d > 0$. Finally, combining (13) for $j \in \{m, s\}$ and $\omega_s^d = 0$ leads to $c_s/c_m = \alpha\left(\omega_m^d\right) > 1$, confirming that our equation (19) holds, and implying that $d\Pi_j/d\omega_m^d\big|_{\omega_m^d=0} = \pi(0, c_m) > \pi(0, c_s) = 0$. Hence, the marginal profits of adding additional varieties evaluated at the core competency variety is positive for type-m firms. Thus, they will become multi-product firms.[3] ■

This proof shows that the sorting equilibrium with is essential for multi-product firms to arise. As stated in our proposition 1, sorting implies that multi-product firms pay a lower effective wage rate than single-product firms ($c_m < c_s$). This allows them to expand into less efficient activities and produce varieties further away from their core competency with higher unit labor requirements. They have an incentive to do so because the screening technology is applicable in all divisions within the firm, so that by adding products to their product range they can lower the fixed costs per product.

Using our normalization of $\alpha(0) = 1$, the free-entry/zero-profit conditions can now be rewritten as

$$Ac_s^{1-\sigma} = f, \tag{23}$$

for single-product firms and

$$Ac_m^{1-\sigma}\bar{\alpha}_m\left(\omega_m^d\right)^{1-\sigma} = \omega_m^d f + f_m \tag{24}$$

for multi-product firms, where $\bar{\alpha}_m\left(\omega_m^d\right) = \left[\int_0^{\omega_m^d} \alpha(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ is the mean of unit labor requirements in multi-product firms.

[3] Note that the set of products produced by single-product firms and the set of products produced by multi-product firms both have positive Lebesgue measure because the cardinality of both sets is $\mathfrak{c} \equiv |\mathbb{R}|$, the cardinality of the continuum (see Briggs and Schaffter, 1979). Individual products have a finite cardinality and, thus, measure zero. A detailed proof is available on request.

Proposition 3 (Co-existence) *In a free entry equilibrium, both types of firms (single-product firms and multi-product firms) will exist.*

Proof. First note from (23) and (24) that $f_m/\omega_m^d > 0$ and $\bar{\alpha}_m\left(\omega_m^d\right) > 1$, so multi-product firms have higher fixed costs per variety and on average higher unit labor requirements. Therefore, a necessary condition for co-existence is that $c_m < c_s$, which is met [see (19)]. Second, we can show that an equilibrium with only one type of firm is inconsistent with free entry: If $\tilde{z} \to \infty$ (no multi-product firms), $\lim_{\tilde{z}\to\infty} c_m = c_s \lim_{\tilde{z}\to\infty} \frac{\bar{z}_s(\tilde{z})}{\tilde{z}} = 0$ and $\lim_{\tilde{z}\to\infty} \Pi_m = +\infty$. Hence, multi-product firms must exist. If $\tilde{z} \to \underline{z}$ (no single-product firms), $\lim_{\tilde{z}\to\underline{z}} c_m = c_s \lim_{\tilde{z}\to\underline{z}} \frac{\bar{z}_s(\tilde{z})}{\tilde{z}} = c_s$ and $\Pi_s > \Pi_m$. Hence, single-product firms must exist. ■

Co-existence of single-product firms and multi-product firms is only possible because the screening technology leads to sorting and allows firms to segment labor markets. A free entry equilibrium with access to identical technologies requires that marginal production costs are equalized across firm types, at least at the margin. This is true here, too [see equations (13) for $j \in \{m, s\}$]:

$$c_m\alpha\left(\omega_m^d\right) = c_s\alpha\left(\omega_s^d\right). \tag{25}$$

But marginal production costs consist of two components: A factor cost component c_j and a unit labor requirement component $\alpha\left(\omega_j\right)$. Differences in one component require differences in the other: $\alpha\left(\omega_m^d\right) > \alpha\left(\omega_s^d\right) \implies c_m < c_s$. Thus, multi-product firms expand into less efficient varieties because they pay a lower effective wage rate.

Propositions 2 and 3 are at the core of our theory. They show how multi-product firms and single-product firms can arise endogenously from ex ante identical firms due to labor market imperfections and different strategies to deal with them. In the frictional labor market, firms pay a wage based on the average productivity of workers in this labor market segment. Such a wage scheme implies an implicit transfer of rents from the more productive workers in this segment to the less productive workers. In the frictionless labor market, firms pay a wage based on the true productivity of workers so that no transfer between workers takes place. Such a wage scheme is particularly beneficial for the more productive workers in the economy who prefer not to be pooled with less productive workers. As a consequence, firms in the frictionless labor market can pay a lower effective wage rate and expand into less efficient products while firms in the frictional labor market have to pay a higher wage rate for pooling and so focus on their core competency to stay competitive.

The differences in the wage schemes between the two labor markets have important implications for the allocative efficiency of resources. In the frictional labor market, rents go from relatively high productive workers to less productive workers. But since all workers are paid the same wage, this does not affect the allocation of resources. In the frictionless labor market, the rents are transfered from (high productivity) workers to firms because firms are paying a lower effective wage. This induces them to expand their product range and implies a misallocation of resources. This will be important in the welfare analysis.

Our framework has a number of interesting implications that are important for empirical work or for welfare analysis. We present them here as corollaries of propositions 1 to 3:

Corollary 1 (Size) *Multi-product firms have higher sales and sell more of their core competency variety than single-product firms.*

Proof. It follows directly from the free entry conditions that multi-product firms have higher fixed costs and thus must have higher revenues in a free entry equilibrium. The output of the core product is determined by (7) when evaluated at $\omega = 0$. Then, it follows directly from $c_m < c_s$ that $x(0, c_m) > x(0, c_s)$. ■

Multi-product firms need higher sales to cover their larger fixed costs, and have higher output of their core variety than a single-product firm because they have lower marginal production costs.

Corollary 2 (Productivity) *Multi-product firms are more productive than single-product firms as measured by revenue per worker.*

Proof. Using (5), (10), and (20), revenues per worker in multi-product firms φ_m can be expressed as

$$\varphi_m \equiv \frac{\int_0^{\omega_m^d} p(\omega)\, x(\omega)\, d\omega}{[1 - G(\tilde{z})]\, L/n_m} = \frac{\sigma}{\sigma - 1} c_m \bar{z}_m \tag{26}$$

Similarily, revenues per worker in single-product firms φ_s can be expressed as

$$\varphi_s \equiv \frac{p(0)\, x(0)}{G(\tilde{z})\, L/n_s} = \frac{\sigma}{\sigma - 1} c_s \bar{z}_s \tag{27}$$

Then, using the sorting condition (17), the ratio of the two productivity measures can be expressed as

$$\frac{\varphi_m}{\varphi_s} = \frac{\bar{z}_m}{\tilde{z}} > 1 \tag{28}$$

■

Multi-product firms generate higher revenues per worker because they employ more productive workers. This is a direct consequence of the sorting result.

Corollary 3 (Wages) *Multi-product firms pay higher wages per worker*

Proof. Single-product firms pay a flat wage of $w_s = c_s \bar{z}_s$. Multi-product firms pay wages based on individual productivities. The average wage in multi-product firms is $\bar{w}_m \equiv \int_{\tilde{z}}^{\infty} w(z)\, dG(z) / [1 - G(\tilde{z})] = c_m \bar{z}_m(\tilde{z})$. Again using (17), the relative average wage in multi-product firms is

$$\frac{\bar{w}_m}{w_s} = \frac{\bar{z}_m}{\tilde{z}} \left(= \frac{\varphi_m}{\varphi_s} \right) > 1 \tag{29}$$

■

Multi-product firms appear more productive despite paying higher wages because they have a more productive labor pool and pass on the gains from the higher labor productivity only incompletely. In a world where labor productivity could be perfectly observed, multi-product firms as modelled here would not exist.

The following figure shows the profile of wages as a function of worker productivity.

[FIGURE 2 here]

In Figure 2, the thick green line depicts the hockey stick profile of wages as a function of workers' productivities. Workers in the range $z \in [\underline{z}, \tilde{z})$ self-select into the frictional labor market and work for single-product firms. They receive a flat wage given by $w_s = c_s \bar{z}_s$. Above $\tilde{z}$, workers decide to go on the frictionless labor market, work for multi-product firms and receive a wage $w_m(z) = c_m z$. This figure also illustrates nicely why a sorting equilibrium implies that the effective wage c_s in the frictional labor market has to be larger than the effective wage c_m in the frictionless labor market. If single-product firms paid the same effective wage as multi-product firms, $w_s = c_m \bar{z}_s$, then the wage for workers with above-average productivity $z > \bar{z}_s$ would be discretely lower in single-product firms than in multi-product firms [$c_m \bar{z}_s < c_m z$ for all $z \in (\bar{z}_s, \tilde{z})$]. Consequently, this could not be a sorting equilibrium. Instead, single-product firms have to pay a premium on the effective wage rate, $c_s > c_m$, in order to compensate their above-average workers for pooling them with below-average worker, so that $c_s \bar{z}_s = c_m \tilde{z}$. Put differently, multi-product firms are able to obtain a rent from their workers in the form of a lower effective wage rate. This rent comes from allowing more productive workers to avoid being pooled with less productive workers.

2.2.6 General equilibrium

For completeness we derive aggregate statics that will be important in the welfare calculations below. With profits driven down to zero, aggregate income consists of labor income and compensation for managers. Since management is used as our numéraire, their compensation is normalized to one:

$$E \equiv L \int_{\underline{z}}^{\infty} w(z)\, dG(z) + H = L \left[c_s \int_{\underline{z}}^{\tilde{z}} z dG(z) + c_m \int_{\tilde{z}}^{\infty} z dG(z) \right] + H. \tag{30}$$

With CES demand, a constant fraction of revenues goes to fixed costs, and variable factors receive the remaining (constant) fraction. In our framework, this implies that $E = \sigma H$, and thus

$$E = \frac{\sigma}{\sigma - 1} L \left\{ c_s G(\tilde{z})\, \bar{z}_s(\tilde{z}) + c_m [1 - G(\tilde{z})]\, \bar{z}_m(\tilde{z}) \right\} = \sigma H. \tag{31}$$

With E determined, and A pinned down by (23), the price index P can be derived easily from (6).

2.3 Open Economy

Let us now consider international trade in an open economy setting with two identical countries. International trade is costly in two dimensions: Entering a foreign market creates fixed costs of exporting f^x, and shipping goods to foreign locations is subject to variable (iceberg) trade costs $\tau > 1$. We follow Melitz (2003) and assume that these trade costs are sufficiently high so that the following condition is met: $f^x \tau^{\sigma-1} > f$.

Profits per product in the domestic market continue to be given by (9). Profits per product in an export market are given by

$$\pi^x(\omega, c_j) = A \tau^{1-\sigma} c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} - f^x. \tag{32}$$

Combining this with the free entry condition in the domestic market leads to our first proposition in the open economy case:

Proposition 4 (Export selection) *Single-product firms do not export.*

Proof. Since $f^x \tau^{\sigma-1} > f$ (by assumption), it follows from (23) that $A \tau^{1-\sigma} c_s^{1-\sigma} < f^x$: The revenues generated by single-product firms in foreign markets are smaller than the fixed costs of entering these markets. Thus, single-product firms do not enter foreign markets and do not export. ■

The intuition behind proposition 4 is analogous to the intuition behind proposition 2. Since $c_s > c_m$, single-product firms pay a higher effective wage rate and have higher marginal production costs. Thus, they can only survive in the market if they focus on the lowest cost activities, like producing only their core competency varieties (proposition 2) and servicing only the domestic market (proposition 4). Multi-product firms, in contrast, have lower marginal production costs, so they can expand into less efficient activities, such as exporting.

Equation (13) continues to determine the optimal product range at home (ω_m^d). The first order condition for the optimal product range of products exported, ω_m^x, is:

$$Ac_m^{1-\sigma}\tau^{1-\sigma}\alpha\left(\omega_m^x\right)^{1-\sigma} = f^x \tag{33}$$

Proposition 5 (Export range) *The range of products exported by multi-product firms is smaller than the range of products sold domestically. It is positive for sufficiently low values of f_m/f.*

Proof. Equations (13) and (33) yield $\alpha\left(\omega_m^x\right)/\alpha\left(\omega_m^d\right) = \left(f/f^x\right)^{\frac{1}{\sigma-1}}/\tau$. Since $f^x\tau^{\sigma-1} > f$, $\alpha\left(\omega_m^x\right)/\alpha\left(\omega_m^d\right) < 1$ and, thus, $\omega_m^x < \omega_m^d$. By the same logic, $\omega_m^x > 0$ implies that $\alpha\left(\omega_m^d\right)^{\sigma-1} > \tau^{\sigma-1}f^x/f > 1$, which in turn requires that f_m/f is sufficiently large (see proof of proposition 2). ■

Since trade is costly, multi-product firms export fewer products than they sell at home. This result is analogous to the selection result in Melitz (2003) and has been pointed out in the context of multi-product firms with CES demand by Bernard, Redding and Schott (2011).

Since single-product firms continue to be active on the domestic market only, their free entry condition has not changed [see equations (21) and (23)]. The free entry condition for multi-product firms changes to $\Pi_m = \int_0^{\omega_m^d}\pi\left(\omega, c_m\right)d\omega + \int_0^{\omega_m^x}\pi^x\left(\omega,\tau,c_m\right)d\omega - f_m = 0$, or

$$Ac_m^{1-\sigma}\bar{\alpha}_m\left(\omega_m^d,\omega_m^x,\tau\right)^{1-\sigma} = \omega_m^d f + \omega_m^x f^x + f_m, \tag{34}$$

where the mean of unit labor requirements in multi-product firms is now $\bar{\alpha}_m\left(\omega_m^d,\omega_m^x,\tau\right) = \left[\int_0^{\omega_m^d}\alpha\left(\omega\right)^{1-\sigma}d\omega + \tau^{1-\sigma}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}$.

By substituting (13) into (33) and both into (34) we obtain two equations that simultaneously determine ω_m^d and ω_m^x:

$$\alpha\left(\omega_m^x\right)^{\sigma-1} = \frac{f}{f^x}\tau^{1-\sigma}\alpha\left(\omega_m^d\right)^{\sigma-1}, \tag{35}$$

$$f\int_0^{\omega_m^d}\left[\frac{\alpha\left(\omega\right)}{\alpha\left(\omega_m^d\right)}\right]^{1-\sigma}d\omega + f^x\int_0^{\omega_m^x}\left[\frac{\alpha\left(\omega\right)}{\alpha\left(\omega_m^x\right)}\right]^{1-\sigma}d\omega = \omega_m^d f + \omega_m^x f^x + f_m. \tag{36}$$

Figure 3 illustrates the equilibrium graphically. Equation (35) describes a positive relation between ω_m^d and ω_m^x based solely on the conditions for optimal scope at home and abroad ("Optimal Scope"). Since $f^x\tau^{\sigma-1} > f$, this line is entirely below the 45° line. Equation (36) describes a negative relation between ω_m^d and ω_m^x based on the free entry condition ("Zero Profit Scope"). The zero profit condition implies that if optimal profits in one market increase, optimal profits in the other market have to fall, thus implying a negative relation between product ranges at home and abroad. The intersection of these two curves determines ω_m^d and ω_m^x.[4]

[FIGURE 3 here]

Knowing ω_m^d, the critical skill level $\tilde{z}$ can be determined by combining the optimal product range at home (13) and the zero profit condition for single-product firms (23) with the sorting condition (17):

$$\frac{\bar{z}_s(\tilde{z})}{\tilde{z}} = \frac{1}{\alpha(\omega_m^d)}. \tag{37}$$

Note that the right hand side of this equation is between 0 and 1 and decreasing in ω_m^d. Since $\bar{z}_s(\tilde{z})/\tilde{z} \in [0,1]$ and decreasing in $\tilde{z}$ (see proposition 1), this has a unique solution in $\tilde{z}$. It determines $\tilde{z}$ as a (positive) function of ω_m^d.

The equations for income (31) and the sorting condition (17) determine simultaneously the two effective wages c_m and c_s for a given $\tilde{z}$:

$$c_m = (\sigma - 1)\left\{\tilde{z}G(\tilde{z}) + [1 - G(\tilde{z})]\,\bar{z}_m(\tilde{z})\right\}^{-1}\frac{H}{L}, \tag{38}$$

$$c_s = (\sigma - 1)\frac{\tilde{z}}{\bar{z}_s(\tilde{z})}\left\{\tilde{z}G(\tilde{z}) + [1 - G(\tilde{z})]\,\bar{z}_m(\tilde{z})\right\}^{-1}\frac{H}{L}. \tag{39}$$

Wages are then given by $w(z) = c_m z$ in multi-product firms and $w_s = c_s \bar{z}_s(\tilde{z})$ in single-product firms.

Since only multi-product firms expand into foreign markets, the labor market clearing conditions for the two labor markets in the open economy case are:

$$n_s(\sigma - 1)\,Ac_s^{-\sigma} = LG(\tilde{z})\,\bar{z}_s(\tilde{z}), \tag{40}$$

$$n_m(\sigma - 1)\,Ac_m^{-\sigma}\bar{\alpha}_m\left(\omega_m^d, \omega_m^x, \tau\right)^{1-\sigma} = L\,[1 - G(\tilde{z})]\,\bar{z}_m(\tilde{z}), \tag{41}$$

Since we have already determined all other endogenous variables, the two labor market clearing conditions pin down the measures of single- and multi-product firms, n_s and n_m.

This concludes our description of the equilibrium in the open economy case. We can now study how this equilibrium changes in response to a trade shock.

[4] The linear and concave curvature of the two curves only hold if the α-function is linear. This has been chosen for the diagram for illustrative purposes only. See appendix A for the mathematical background.

3 Comparative Statics

For our comparative statics we focus on changes in variable trade costs τ.[5] However, our results are qualitatively identical to changes in fixed costs, as we prove in appendix A. Hence our comparative statics really cover a wider range of adjustments typically associated with trade liberalization. Now consider a fall in variable trade costs τ:

Proposition 6 (Firm organization) *Trade liberalization leads to an expansion of the range of products exported by multi-product firms, and a reduction in the range of products sold domestically:* $d\ln\omega_m^x/d\ln\tau < 0$ *and* $d\ln\omega_m^d/d\ln\tau > 0$.

Proof. As τ falls, the Optimal Scope locus in Figure 3 rotates upwards (see Figure 4). Since (36) is independent of τ, the equilibrium moves along the Zero Profit Scope locus, leading to a higher ω_m^x and a lower ω_m^d. See appendix A for a mathematical proof. ■

[FIGURE 4 here]

The intuition for this result is straightforward: The range of products exported ω_m^x rises because exporting is cheaper. As ω_m^x rises, demand for labor increases and the real costs of labor for multi-product firms rises, so that they reduce their product range at home ω_m^d. Again, this mechanism is identical to the selection result in Melitz (2003) for single-product firms and Bernard, Redding and Schott (2011) for multi-product firms.

In a model without labor market imperfections, the expansion of export sales increases demand for labor and leads to a rise in real wages for all workers. Ultimately, this raises welfare, too. Here, however, the labor market (and welfare) consequences are very different.

First, the sorting equilibrium is affected: Knowing how ω_m^d changes, we can calculate the change in $\tilde{z}$ from (37):

Proposition 7 (Sorting threshold) *Trade liberalization leads to a fall in the threshold value for sorting* $\tilde{z}$.

Proof. From (37) in combination with (18) and proposition 6 we obtain

$$\frac{d\ln\tilde{z}}{d\ln\tau} = \frac{\bar{z}_s(\tilde{z})}{\tilde{z}}\left\{1 - \frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z} - \bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})}\right\}^{-1}\varepsilon_\alpha\left(\omega_m^d\right)\frac{d\ln\omega_m^d}{d\ln\tau} > 0, \tag{42}$$

where $\varepsilon_\alpha\left(\omega_m^d\right) \equiv \alpha'\left(\omega_m^d\right)\omega_m^d/\alpha\left(\omega_m^d\right) > 0$. ■

Note that fixed and variable trade costs affect only ω_m^d and ω_m^x directly. All other changes below are driven by changes in $\tilde{z}$ through equation (37). One immediate consequence of the change in $\tilde{z}$ is:

[5] We consider only stable equilibria (see discussion following proposition 1).

Corollary 4 (Employment) *As $\tilde{z}$ falls, employment is pulled out of single-product firms $LG(\tilde{z})$ and into multi-product firms $L[1-G(\tilde{z})]$.*

Since only multi-product firms export, only they benefit from the reduction in trade costs. This leads to an expansion of economic activity of multi-product firms at the expense of single-product firms.

Because $\tilde{z}$ falls in response to trade liberalization, the sorting condition (17) and (18) imply that the relative effective wage rate of multi-product firms must rise:

$$\frac{d\ln c_m}{d\ln\tau} - \frac{d\ln c_s}{d\ln\tau} = \left\{\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z}-\bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})} - 1\right\}\frac{d\ln\tilde{z}}{d\ln\tau} < 0 \tag{43}$$

When the cutoff productivity $\tilde{z}$ falls, the wage that the marginal worker can earn in the frictionless labor market also falls. Thus, single-product firms can lower the premium that they pay per efficiency unit, and c_s/c_m falls.

Furthermore, using (38), we can show that wages for individual workers in multi-product firms $[w(z) = c_m z]$ rise and wages in single-product firms $[w_s = w(\tilde{z}) = c_m\tilde{z}]$ fall:

$$\frac{d\ln w(z)}{d\ln\tau} = -\frac{\tilde{z}G(\tilde{z})}{\tilde{z}G(\tilde{z}) + [1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau} < 0, \tag{44}$$

$$\frac{d\ln w_s}{d\ln\tau} = \frac{[1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}{\tilde{z}G(\tilde{z}) + [1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau} > 0. \tag{45}$$

These results also imply that relative wages of incumbent workers in multi-product firms, $w(z)/w_s$, rise.

[FIGURE 5 here]

The changes in wages are illustrated in Figure 5. The worker who is indifferent between the two labor market segments has a lower productivity after trade liberalization, all incumbent workers in single-product firms earn a lower wage, all incumbent workers in multi-product firms earn a higher wage, and the slope of the wage profile in multi-product firms, c_m, has risen.

From a welfare prospective it is important to calculate changes in real wages (relative to the price index P). They, too, differ across firm types:

Proposition 8 (Real wages) *Trade liberalization raises real wages in multi-product firms and lowers real wages in single-product firms.*

Proof. Using (6), (23) and (31) we can prove that c_s/P is fixed by exogenous parameters:

$$\frac{c_s}{P} = \frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}. \tag{46}$$

Given (46) the changes in real wages follow directly from changes in $\tilde{z}$: The real wage in single-product firms $w_s/P = \bar{z}_s(\tilde{z})(c_s/P)$ clearly falls because $\bar{z}_s(\tilde{z})$ falls, and $w_m(z)/P = (c_m/P)z = (c_m/c_s)(c_s/P)z$ clearly rises because $c_m/c_s = \bar{z}_s(\tilde{z})/\tilde{z}$ rises:

$$\frac{d\ln w_s - d\ln P}{d\ln\tau} = \frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z}-\bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau} > 0 \tag{47}$$

$$\frac{d\ln w_m(z) - d\ln P}{d\ln\tau} = \left\{\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\frac{[\tilde{z}-\bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})} - 1\right\}\frac{d\ln\tilde{z}}{d\ln\tau} < 0 \tag{48}$$

■

Real wages of workers in single-product firms fall because the most productive workers in their labor pool are pulled away into the frictionless labor markets of multi-product firms. Thus, the remaining workforce is on average less productive, and their real wages fall. Real wages in multi-product firms rise because labor demand for exports increases.

Given our CES utility function in (1), a valid measure for welfare is real income $W \equiv E/P$. Given (31), welfare can also be expressed as

$$W = \frac{\sigma}{\sigma-1}\frac{\bar{w}}{P}L, \tag{49}$$

where $\bar{w} \equiv \int_{\underline{z}}^{\infty} w(z)\,dG(z) = c_s G(\tilde{z})\,\bar{z}_s(\tilde{z}) + c_m[1-G(\tilde{z})]\,\bar{z}_m(\tilde{z}) = (\sigma-1)H/L$ is the average wage of all workers. To better understand the forces influencing welfare, we express welfare as

$$W = \frac{\sigma L}{\sigma-1}\left[\frac{w_s}{P}G(\tilde{z}) + \left\{\frac{1}{1-G(\tilde{z})}\int_{\tilde{z}}^{\infty}\frac{w(z)}{P}dG(z)\right\}[1-G(\tilde{z})]\right], \tag{50}$$

where the first term inside the brackets is the average real wage earned by workers in single-product firms multiplied by their share in employment, and the second term is the average real wage in multi-product firms multiplied by their share in employment. Welfare can thus be expressed as a constant multiple $\sigma L/(\sigma-1)$ of a weighed average of average real wages, with the distribution of workers' productivities as weights.

Relative changes in welfare can then be expressed as a weighted sum of the relative changes in these two real wages:[6]

$$\frac{d\ln W}{d\ln\tau}=\underbrace{\frac{w_s}{\bar{w}}G\left(\tilde{z}\right)\frac{d\ln\left(w_s/P\right)}{d\ln\tau}}_{(+)}+\underbrace{\int_{\tilde{z}}^{\infty}\frac{w\left(z\right)}{\bar{w}}dG\left(z\right)\frac{d\ln\left[w\left(z\right)/P\right]}{d\ln\tau}}_{(-)}\gtreqless 0. \tag{51}$$

Equation (51) shows that the the welfare effects of trade liberalization are subject to two counteracting effects: On the one hand, real wages of workers in single-product firms fall. This tends to lower real income. On the other hand, real wages of workers in multi-product firms rise. This tends to raise real income. The net effect depends on the shares of these two groups of workers in the economy. We can establish the following proposition:

Proposition 9 (Welfare) *Trade liberalization has an ambiguous effect on welfare. This effect can be negative if the employment share of single-product firms is large, and positive if the employment share of multi-product firms is large.*

Proof. See appendix. ■

Our proposition 9 shows that trade is not unambiguously welfare increasing. It raises real wages for one group of workers but lowers real wages for the other group. Lower trade costs increase labor demand among exporters, and these exporters are multi-product firms. Consequently, only workers working for multi-product firms benefit from the trade liberalization. Workers in single-product firms experience a reduction in their average productivity, and this lowers their real wage. Whether these changes are good or bad for aggregate real income depends on the initial employment shares of these two types of firms.

The economics of proposition 9 can be best explained by how trade affects the efficiency of the allocation of resources in the economy. Multi-product firms are inherently less efficient than single-product firms because single-product firms produce only their core competency product whereas multi-product firms expand into less efficient products. They can only survive because they are able to overcome labor market imperfections which leads to worker sorting and allows them to recruit more productive workers. Liberalizing trade has two effects: On the one hand, lower trade costs induce multi-product firms to pare the least efficient of their products, thereby increasing their efficiency, and this tends to increase welfare. On the other hand, labor is pulled away from single-product firms and into multi-product firms. This reallocation of labor across firm types reduces the efficiency of production be-

[6] Note that changes in the limits of integration (i.e., changes in the weights) cancel out because of the sorting condition: $\left[w_s/P-w\left(\tilde{z}\right)/P\right]g\left(\tilde{z}\right)=0$. Note also that $d\ln\left[w\left(z\right)/P\right]/d\ln\tau$ is independent of z [see equ. (48)] and can thus be factored out of the integral.

cause multi-product firms are less efficient than single-product firms. The net effect depends on the relative sizes of these opposing effects.

To illustrate the playoff between these two forces, we parameterize the distribution of worker efficiency in the population by assuming that G is Pareto with scale parameter of unity and shape parameter of $\kappa > 1$. This parameterization is natural as the Pareto distribution satisfies (18), and it is consistent with the observed distribution of income. The following Figure plots (50) for the case in which $\kappa = 2$.[7]

[FIGURE 6 here]

Figure 6 shows the parameterized real wage as a function of the cutoff $\widetilde{z}$. When the cutoff is low, most efficiency units of labor are already employed at MPF so that a trade liberalization that lowers $\widetilde{z}$ improves welfare. This is because the reduction in product range at MPF dominates the effect of reallocating labor from SPF to MPF because MPF already have a relatively small product range and so are not that much more inefficient than SPF. When the cutoff is high, little effective labor is allocated to MPF, and those MPF that exist have a high product range that sprawls across very inefficient product lines. Hence, for these cutoffs a trade liberalization induces a reallocation from efficient SPF to very inefficient MPF and this dominates the rationalization in product lines.

To get a sense of which effect would most likely dominate in a country like the United States, we solve for the implied cutoff in our model that is consistent with the 39 percent employment of U.S. workers in exporting firms reported by Bernard, Jensen, and Schott (2008).[8] The implied cutoff is shown as the solid dot on the curve in Figure 6. According to this back-of-the-envelope calculation, the allocation of resources (measured in efficiency units) in the United States, which is highly skewed toward MPF, is consistent with further trade liberalization having a beneficial impact on aggregate real income.

4 Conclusion

In this paper, we developed an information-based model in which multi-product firms and single-product firms arise endogenously from ex ante identical firms. The multi-product firms that emerge display differences from single-product firms that are consistent with key stylized facts: they are larger, appear to be more productive, pay higher wages, and export.

[7] We use $\kappa = 2$ because it is consistent with the highest paid 5 percent of households receiving 22 percent of income. The general shape of the curve plotted in Figure 5 is robust to the choice of κ.

[8] Given a $\kappa = 2$, this implies that roughly 70 percent of labor in terms of efficiency units is employed by MPF in the United States.

The apparent exceptional performance of multi-product firms relative to single-product firms is deceptive, however, because they are endogenously less efficient than single-product firms.

The existence of asymmetric information in our model has the implication that relatively high productivity workers earn relatively less than they would in a frictionless environment. Single product firms cannot overcome the asymmetric information and so pay wages based on the average productivity of their workers. This means that their less capable workers extract a rent from their more capable workers with which they are pooled, but this does not involve any misallocation of resources. Multi-product firms invest in screening workers, and this gives them an informational advantage over single-product firms. This informational advantage allows them to extract a rent from their workers that these firms obtain by allowing high productivity workers the opportunity not to pool with low quality workers. In this case, there is a resource misallocation as the low cost of effective labor induces the firm to expand into less efficient products.

Trade liberalization raises the real wage paid by exporting, multi-product firms and lowers the real wage paid by non-exporting, single-product firms and so induces a reallocation of resources from single-product firms to multi-product firms. The welfare implications of this reallocation are ambiguous. Liberalizing trade has two effects: On the one hand, lower trade costs induce multi-product firms to pare the least efficient of their products, thereby increasing their efficiency, and this tends to increase welfare. On the other hand, labor is pulled away from single-product firms and into multi-product firms. This reallocation of labor across firm types reduces the efficiency of production because multi-product firms are less efficient than single-product firms. The net effect depends on the relative sizes of these opposing effects. The model highlights the need to know why firms "excel" before drawing welfare conclusions regarding cross-firm reallocations of resources.

In this paper, we abstracted from intrinsic firm heterogeneity in order to isolate the role that asymmetric information in the labor market, and firms' options for coping with it, has in generating multi-product firms. It should be clear, however, that the simple structure of our model makes it highly amenable to extension. Such extensions could include: creating a menu of alternative screening technologies, adding ex ante firm productivity heterogeneity, introducing complementarities between firm and worker characteristics, to name just a few. These possibilities present interesting opportunities for future research.

5 Appendix

5.1 Appendix A

Take equations (13), (33) and (34), substitute $A = fc_s^{\sigma-1}$ from (23), and define $\phi^x \equiv f^x/f$ and $\phi_m \equiv f_m/f$ to obtain the following system of equations:

$$\left(\frac{c_s}{c_m}\right)^{\sigma-1} = \alpha\left(\omega_m^d\right)^{\sigma-1} \tag{52}$$

$$\left(\frac{c_s}{c_m}\right)^{\sigma-1} = \phi^x \tau^{\sigma-1}\alpha\left(\omega_m^x\right)^{\sigma-1} \tag{53}$$

$$\left(\frac{c_s}{c_m}\right)^{\sigma-1} = \left(\omega_m^d + \omega_m^x\phi^x + \phi_m\right)\bar{\alpha}_m\left(\omega_m^d, \omega_m^x, \tau\right)^{\sigma-1} \tag{54}$$

These three equations determine c_s/c_m, ω_m^d, and ω_m^x, for given variable and fixed costs τ, f, f^x, and f_m. By substituting out c_s/c_m, these three equations educe to (35) and (36).

By taking derivatives we obtain

$$\begin{pmatrix} 1 & -\varepsilon_\alpha\left(\omega_m^d\right) & 0 \\ 1 & 0 & -\varepsilon_\alpha\left(\omega_m^x\right) \\ \omega_m^d + \omega_m^x\phi^x + \phi_m & \omega_m^d & \omega_m^x\phi^x \end{pmatrix}\begin{pmatrix} d\ln\left(c_s/c_m\right) \\ d\ln\omega_m^d \\ d\ln\omega_m^x \end{pmatrix} = \begin{pmatrix} 0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \tag{55}$$

where $\varepsilon_\alpha\left(\omega_m^d\right) \equiv \alpha'\left(\omega_m^d\right)\omega_m^d/\alpha\left(\omega_m^d\right)$ and $\varepsilon_\alpha\left(\omega_m^x\right) \equiv \alpha'\left(\omega_m^x\right)\omega_m^x/\alpha\left(\omega_m^x\right)$. Furthermore, $\beta_1 \equiv d\ln\tau + \frac{d\ln\phi^x}{(\sigma-1)}$ and $\beta_2 \equiv \gamma^{\sigma-1}\tau^{1-\sigma}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega d\ln\tau + \omega_m^x\phi^x\frac{d\ln\phi^x}{(\sigma-1)} + \phi_m\frac{d\ln\phi_m}{(\sigma-1)}$.

This system of equations yields the following solutions:

$$\begin{aligned} \Delta\varepsilon_\alpha\left(\omega_m^d\right)^{-1}d\ln\left(\frac{c_s}{c_m}\right) &= \left(\omega_m^x\phi^x + \varepsilon_\alpha\left(\omega_m^x\right)\gamma^{\sigma-1}\tau^{1-\sigma}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega\right)d\ln\tau \\ &\quad + \left[1+\varepsilon_\alpha\left(\omega_m^x\right)\right]\omega_m^x\phi^x\frac{d\ln\phi^x}{(\sigma-1)} \\ &\quad + \varepsilon_\alpha\left(\omega_m^x\right)\phi_m\frac{d\ln\phi_m}{(\sigma-1)} \end{aligned} \tag{56}$$

$$\begin{aligned} \Delta d\ln\omega_m^d &= \left(\omega_m^x\phi^x + \varepsilon_\alpha\left(\omega_m^x\right)\gamma^{\sigma-1}\tau^{1-\sigma}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega\right)d\ln\tau \\ &\quad + \left[1+\varepsilon_\alpha\left(\omega_m^x\right)\right]\omega_m^x\phi^x\frac{d\ln\phi^x}{(\sigma-1)} \\ &\quad + \varepsilon_\alpha\left(\omega_m^x\right)\phi_m\frac{d\ln\phi_m}{(\sigma-1)} \end{aligned} \tag{57}$$

$$\begin{aligned}\Delta d\ln\omega_m^x &= -\left(\omega_m^d+\varepsilon_\alpha\left(\omega_m^d\right)\gamma^{\sigma-1}\int_0^{\omega_m^d}\alpha\left(\omega\right)^{1-\sigma}d\omega\right)d\ln\tau \\ &\quad -\left[\varepsilon_\alpha\left(\omega_m^d\right)\left(\omega_m^d+\phi_m\right)+\omega_m^d\right]\frac{d\ln\phi^x}{(\sigma-1)} \\ &\quad +\varepsilon_\alpha\left(\omega_m^d\right)\phi_m\frac{d\ln\phi_m}{(\sigma-1)}\end{aligned} \tag{58}$$

where $\Delta=\varepsilon_\alpha\left(\omega_m^d\right)\varepsilon_\alpha\left(\omega_m^x\right)\left(\omega_m^d+\omega_m^x\phi^x+\phi_m\right)+\varepsilon_\alpha\left(\omega_m^d\right)\omega_m^x\phi^x+\varepsilon_\alpha\left(\omega_m^x\right)\omega_m^d>0$.

The solutions show that

$$\frac{d\ln\omega_m^d}{d\ln\tau}>0,\ \frac{d\ln\omega_m^x}{d\ln\tau}<0,\ \frac{d\ln\left(\frac{c_s}{c_m}\right)}{d\ln\tau}>0 \tag{59}$$

$$\frac{d\ln\omega_m^d}{d\ln\phi^x}>0,\ \frac{d\ln\omega_m^x}{d\ln\phi^x}<0,\ \frac{d\ln\left(\frac{c_s}{c_m}\right)}{d\ln\phi^x}>0 \tag{60}$$

This proves proposition 6 and that changes in variable trade costs and changes in fixed costs have qualitatively identical impacts.

5.2 Appendix B

Given the sorting condition (17) and (46), the average real wage can be expressed as

$$\frac{\bar{w}}{P}\left(\tilde{z}\right)=\left[\bar{z}_s\left(\tilde{z}\right)G\left(\tilde{z}\right)+\frac{\bar{z}_s\left(\tilde{z}\right)}{\tilde{z}}\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]\right]\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}. \tag{61}$$

The derivative with respect to $\tilde{z}$ is

$$\frac{d\left(\bar{w}/P\right)}{d\tilde{z}}=\left\{\frac{\partial\bar{z}_s\left(\tilde{z}\right)}{\partial\tilde{z}}G\left(\tilde{z}\right)+\frac{\partial\left[\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]}{\partial\tilde{z}}\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]\right\}\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}. \tag{62}$$

Define $Z\equiv\int_{\underline{z}}^{\infty}zdG\left(z\right)\in\mathbb{R}^+$. We can establish the following lemmata:

Lemma 1 (Lower limit)

$$\lim_{\tilde{z}\to\underline{z}}\frac{\bar{w}}{P}\left(\tilde{z}\right)=Z\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}} \tag{63}$$

Proof. Given the general properties of a cumulative distribution function and our definition of Z (above), it follows immediately that $\lim_{\tilde{z}\to\underline{z}}G\left(\tilde{z}\right)=0$ and $\lim_{\tilde{z}\to\underline{z}}\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]=Z$. Using L'Hôpital's rule, we can evaluate the following limits: $\lim_{\tilde{z}\to\underline{z}}\bar{z}_s\left(\tilde{z}\right)=\underline{z}$ and

$\lim_{\tilde{z}\to\underline{z}}\left[\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]=1.$ ■

Lemma 2 (Upper limit)

$$\lim_{\tilde{z}\to\infty}\frac{\bar{w}}{P}\left(\tilde{z}\right)=Z\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}} \tag{64}$$

Proof. $\lim_{\tilde{z}\to\infty}\bar{z}_s\left(\tilde{z}\right)G\left(\tilde{z}\right)=Z$ and $\lim_{\tilde{z}\to\infty}\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]=\lim_{\tilde{z}\to\infty}\left[\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]=0.$ ■

Lemma 3 (Global maxima) *The global maxima of the $\bar{w}/P\left(\tilde{z}\right)$-function are at the boundaries of the domain:*

$$\frac{\bar{w}}{P}\left(\tilde{z}\right)\le Z\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}\quad\forall\quad\tilde{z}\in\left[\underline{z},\infty\right] \tag{65}$$

Proof. This lemma holds if we can prove that $\bar{z}_s\left(\tilde{z}\right)G\left(\tilde{z}\right)+\left[\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]\le Z$ $\forall\ \tilde{z}\in\left[\underline{z},\infty\right]$. Substitute $Z=\bar{z}_s\left(\tilde{z}\right)G\left(\tilde{z}\right)+\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]$ and rearrange to obtain $0\le\left[1-\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]$. Since $\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\le1$, this must hold. ■

Lemma 4 (Continuity) *The function $\bar{w}/P\left(\tilde{z}\right)$ and its derivative are continuous in the interval $\left[\underline{z},\infty\right]$.*

Proof. This follows directly from the fact that $g\left(z\right)$ and $G\left(z\right)$ are pdf and cdf of a continuous distribution. ■

Lemma 5 (Lower limit derivative)

$$\lim_{\tilde{z}\to\underline{z}}\frac{d\left(\bar{w}/P\right)}{d\tilde{z}}=-\frac{Z}{2\underline{z}}\frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}<0 \tag{66}$$

Proof. We already know that $\lim_{\tilde{z}\to\underline{z}}G\left(\tilde{z}\right)=0$ and $\lim_{\tilde{z}\to\underline{z}}\bar{z}_m\left(\tilde{z}\right)\left[1-G\left(\tilde{z}\right)\right]=Z$. Using L'Hôpital's rule, we can evaluate the following limits: $\lim_{\tilde{z}\to\underline{z}}\partial\bar{z}_s\left(\tilde{z}\right)/\partial\tilde{z}=\lim_{\tilde{z}\to\underline{z}}g\left(\tilde{z}\right)\left[\tilde{z}-\bar{z}_s\left(\tilde{z}\right)\right]/G\left(\tilde{z}\right)=1/2$ and $\lim_{\tilde{z}\to\underline{z}}\partial\left[\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right]/\partial\tilde{z}=\lim_{\tilde{z}\to\underline{z}}\left(\partial\bar{z}_s\left(\tilde{z}\right)/\partial\tilde{z}-\bar{z}_s\left(\tilde{z}\right)/\tilde{z}\right)/\tilde{z}=-1/\left(2\underline{z}\right)$. ■

These lemmata show that the $\bar{w}/P\left(\tilde{z}\right)$-function has global maxima at $\tilde{z}=\underline{z}$ and $\tilde{z}\to\infty$. At $\tilde{z}=\underline{z}$, $\bar{w}/P\left(\tilde{z}\right)$ falls when $\tilde{z}$ rises, and returns to its $\underline{z}$-value when $\tilde{z}\to\infty$. This proves that $\bar{w}/P\left(\tilde{z}\right)$ has to have (at least) one minimum in $\left(\underline{z},\infty\right)$. This implies that the derivative of $\bar{w}/P\left(\tilde{z}\right)$ with respect to $\tilde{z}$ must be negative for low values of $\tilde{z}$ and positive for high values of $\tilde{z}$.

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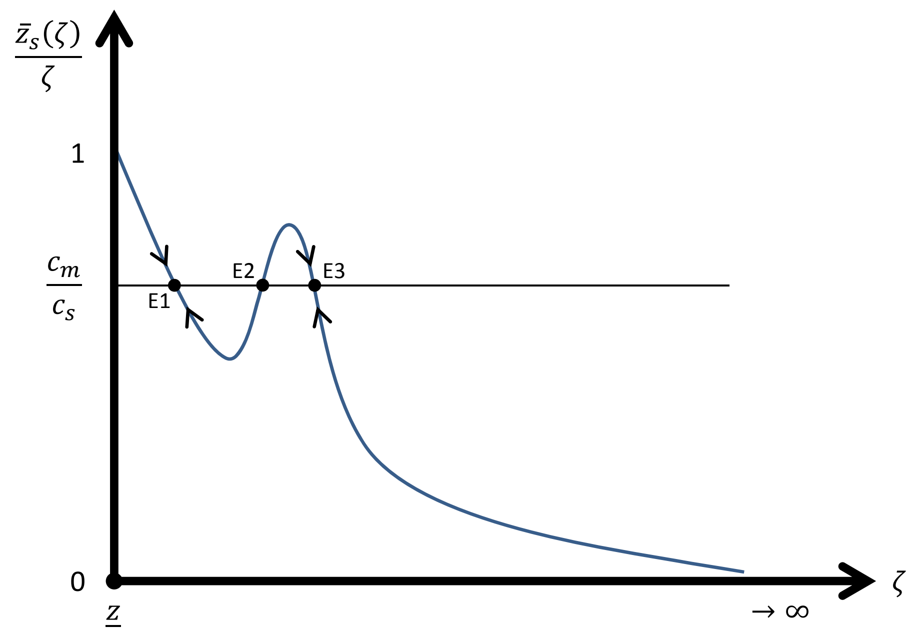

Figure 1: Stability of Sorting Equilibrium

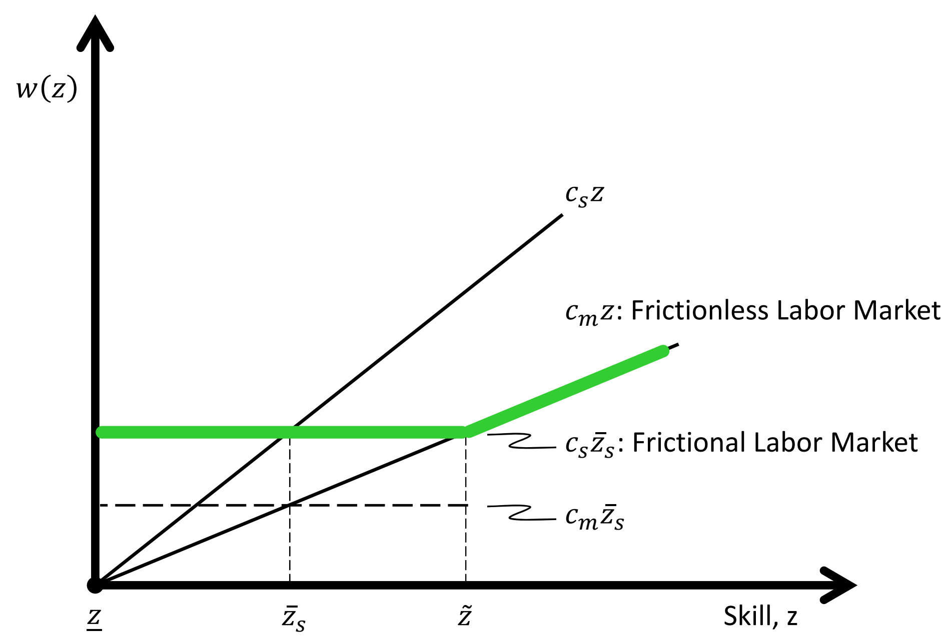

Figure 2: Hockey Stick Wage Profile

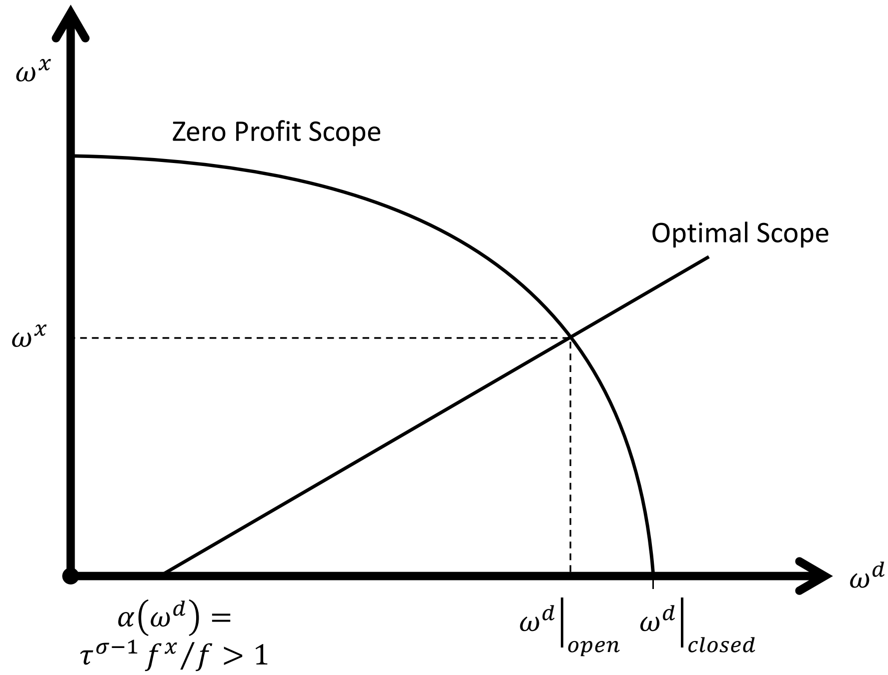

Figure 3: Product Ranges

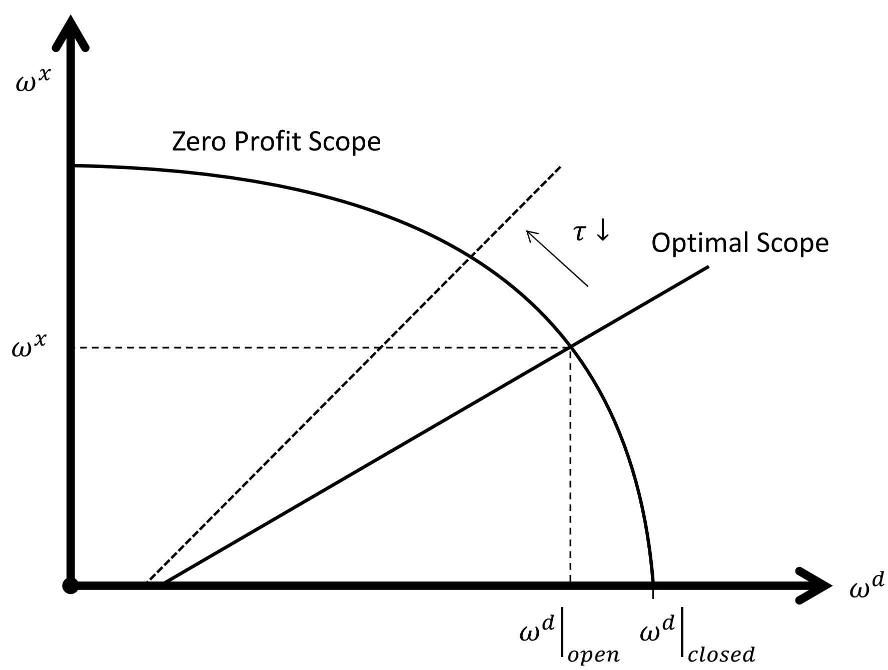

Figure 4: Changes in Product Ranges

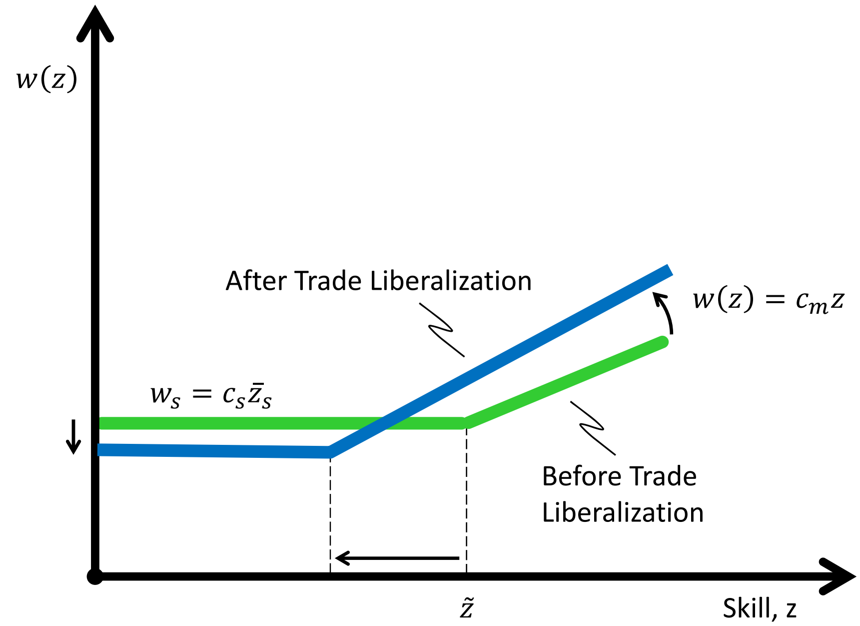

Figure 5: Changes in Wages

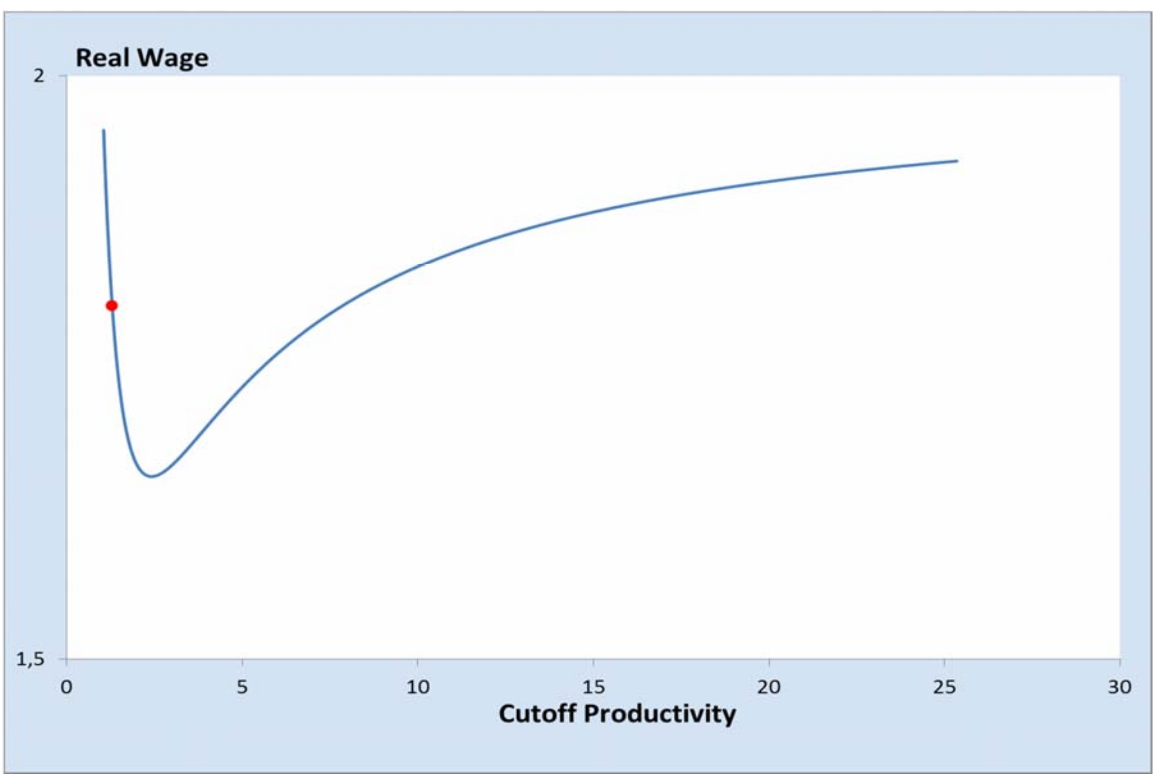

Figure 6: Real Wage Pareto