

Agency Business Cycles*

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Abstract

We propose a new theory of business cycles. Firms need to randomize over firing or keeping workers who have performed poorly in the past, in order to give them ex-ante an incentive to exert effort on the job. Firms want to coordinate on the outcome of their randomization, as coordination allows them to load the firing probability on states of the world when it is costlier for workers to become unemployed and, hence, it allows them to reduce the overall firing probability. In the unique robust equilibrium, firms use a sunspot to coordinate on the randomization outcomes and the economy experiences aggregate fluctuations that are endogenous—in the sense that they are not driven by exogenous shocks to fundamentals or by exogenous shocks to the selection of equilibrium—and stochastic—in the sense that they follow a non-deterministic path. Our theory of business cycles implies a novel view of recessions which is opposite to view of recessions as “rainy days” proposed by Real Business Cycle theory.

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1 Introduction

We present a new business cycle theory, where aggregate fluctuations are caused by the fact that the agents in the economy randomize over some individual decision in a coordinated fashion. We consider a model where firms need to randomize over firing or keeping workers who have performed poorly in the past, in order to give them ex-ante an incentive to exert effort on the job. Moreover, firms want to coordinate on the outcome of their randomization, as coordination allows them to load the firing probability on states of the world when it is costlier for workers to become unemployed and, hence, it allows them to reduce the overall firing probability. In the unique robust equilibrium of the model, firms coordinate on the randomization outcomes and the economy experiences aggregate fluctuations that are endogenous—in the sense that they are not driven by exogenous shocks to fundamentals or by exogenous shocks to the selection of equilibrium—and stochastic—in the sense that they follow a non-deterministic path. Our theory of business cycles implies a novel view of recessions which is opposite to view of recessions as “rainy days” proposed by the Real Business Cycle theory of Kydland and Prescott (1982).

The theory is cast in the context of a search-theoretic model of the labor market in the spirit of Pissarides (1985) and Mortensen and Pissarides (1994). In particular, we consider a labor market populated by identical risk-averse workers—who look for vacancies when unemployed and produce output when employed—and by identical risk-neutral firms—that attract new workers by posting vacancies and produce according to a technology that has constant returns to scale in labor. Unemployed workers and vacant firms come together through a frictional process that is summarized by a matching function. Matched worker-firm pairs produce under moral hazard: the firm does not observe the worker’s effort but only his output, which is a noisy signal of effort. Matched firm-worker pairs Nash bargain over the terms of an employment contract that specifies the level of effort recommended to the worker, the wage paid by the firm to the worker, and the probability with which the worker is fired by the firm conditional on the output of the worker and on the realization of a sunspot, an inherently meaningless signal that is observed by all market participants.

The theory builds on two assumptions. First, the theory needs some decreasing returns to matching in the labor market. Decreasing returns to matching may either come directly from decreasing returns to scale in the matching function, or it may come from a vacancy cost that increases with the total number of vacancies in the market. Second, the theory needs employment contracts to be incomplete enough that firing takes place along the equilibrium path. In this paper, we simply assume that current wages are paid before observing output

and employment contracts are renegotiated period by period, so that firing is the only tool that firms can use to give workers an incentive to exert effort.

In the first part of the paper, we characterize the properties of the optimal employment contract. We show that the optimal contract is such that the worker is fired with positive probability only when the output of the worker is low and the realization of the sunspot is such that the gains from continued trade accruing to the worker are sufficiently high relative to the gains from continued trade accruing to the firm. The result is intuitive. Firing is costly—as it destroyed a valuable firm-worker relationship—but it is necessary—as it is the only way for the firm to give the worker an incentive to exert effort. However, when firing takes place, it is only the value of the destroyed relationship that would have accrued to the worker that gives incentives. The value of the destroyed relationship that would have accrued to the firm is just “collateral damage.” The optimal contract minimizes the collateral damage by loading the firing probability on the realizations of the sunspot for which the worker’s continuation gains from trade are highest relative to the firm’s. In other words, the optimal contract loads the firing probability on the states of the world where the cost to the worker from losing the job is highest relative to the cost to the firm from losing the worker.

In the second part of the paper, we characterize the relationship between the realization of the sunspot and firing in general equilibrium. We find that there is an equilibrium in which firms fire all of their non-performing workers for some realizations of the sunspot, and firms do not fire none of their non-performing workers for the other realizations. There is a simple logic behind this finding. Suppose that firms load up the firing probability on some realizations of the sunspot. In those states of the world, unemployment is higher and, because of decreasing returns to matching, the job-finding probability of unemployed workers is lower. In turn, if the job-finding probability is lower, the workers have a worse outside option when bargaining with the firms and their wage is lower. If the wage is lower, the workers’ marginal utility of consumption relative to the firms’ is lower and, according to Nash bargaining, the workers’ gains from trade relative to the firms’ are higher. Hence, if the other firms in the market load the firing probability on some realizations of the sunspot, an individual firm has the incentive to load the firing probability on the very same states of the world. In other words, firms want to coordinate the outcome of the randomization between firing and keeping their non-performing workers and the sunspot allows them to do so.

Naturally, alongside the equilibrium in which firms use the sunspot to coordinate on firing or keeping non-performing workers, there are also equilibria in which firms (fully or

partially) ignore the sunspot and randomize on firing or keeping non-performing workers independently. However, these equilibria are an artifact of the simplifying assumption that all firms randomize simultaneously. Indeed, we find that, in a version of the model where firms randomize on firing sequentially, only the equilibrium with perfect coordination survives. In this version of the model, the perfect coordination equilibrium takes the form of a firing cascade where the firing decision of the first few firms uniquely determines the firing decision of all subsequent firms.

In the equilibrium where firms coordinate on firing or keeping non-performing workers, the economy experiences aggregate fluctuations. The aggregate fluctuations in this equilibrium are endogenous. Indeed, they are not caused by exogenous shocks to fundamentals. They are not caused by exogenous shocks to the selection of the equilibrium played by market participants. Instead, aggregate fluctuations are caused by coordinated randomization—i.e. every firm needs to randomize on firing or keeping its non-performing workers, and different firms want to coordinate their randomization outcomes. The aggregate fluctuations in a perfect coordination equilibrium are stochastic. Indeed, the economy does not follow a deterministic cycle as in previous theories of endogenous fluctuations. Instead, the economy follows a stochastic process, in which the probability of a firing burst and, hence, of a recession is an equilibrium outcomes.

In the last part of the paper, we calibrate the model to measure the magnitude and properties of Agency Business Cycles (ABC), i.e. the aggregate fluctuations experienced by the economy in the equilibrium where firms coordinate on firing or keeping non-performing workers. We find that ABC feature fluctuations in unemployment, in the rate at which unemployed workers become employed (UE rate), and in the rate at which employed workers become unemployed (EU rate) that are approximately half as large as those observed in the US labor market and that—as it has been the case in the US labor market since 1984—are uncorrelated with labor productivity.

We then test some of the distinctive features of ABC. First, in ABC, a recession starts with an increase in the EU rate which leads to an increase in the unemployment rate. In turn, the rise in the unemployment rate leads, because of decreasing returns to scale in matching, to a fall in the UE rate. Hence, the EU rate leads both the unemployment rate and the UE rate. We find that the US labor market features the same pattern of leads and lags. Second, in ABC, the probability of a recession is endogenous and depends on the aggregate state of the economy. Specifically, the lower is the unemployment rate, the higher is the probability with which firms need to fire their non-performing workers in order to give

them an incentive to exert effort and, hence, the higher is the probability of a recession. We find that the US labor market also features a negative relationship between unemployment and the probability of a recession. Third, in ABC, a recession is a period when the value of time in the market relative to the value of time at home is abnormally high. Using an admittedly crude approach, we construct time-series for the net value of a job to a worker and we find this series to be strongly countercyclical. The finding is consistent with Davis and Von Wachter (2011) who document that the cost to a worker from losing a job is strongly countercyclical.

The first contribution of the paper is to advance a novel theory of business cycles, where aggregate fluctuations are endogenous and stochastic and emerge because different market participants have to randomize over some decision and find it optimal to coordinate the randomization outcomes. In the business cycle literature, there are theories where aggregate fluctuations are driven by exogenous shocks to the current value of fundamentals (e.g., Kydland and Prescott 1982 or Mortensen and Pissarides 1994), to the future value of fundamentals (e.g., Beaudry and Portier 2004 or Jaimovich and Rebelo 2009), or to the stochastic process of fundamentals (e.g., Bloom 2009). In our theory, all fundamentals are fixed. There are theories where aggregate fluctuations are driven by exogenous shocks to the selection of the equilibrium played by market participants (e.g., Heller 1986, Cooper and John 1988 or Benhabib and Farmer 1994). In our theory, market participants always play the same, unique equilibrium. There are theories where aggregate fluctuations emerge endogenously as limit cycles (e.g., Diamond 1982, Diamond and Fudenberg 1989, Mortensen 1999 or Beaudry, Galizia and Portier 2015) or as chaotic dynamics (e.g., Boldrin and Montrucchio 1986 or Boldrin and Woodford 1990). In our theory, the economy follow a stochastic process. There are theories where aggregate fluctuations are driven by common shocks to higher-order beliefs (e.g., Angeletos and La'O 2013). In our theory there are no such shocks.

The second contribution of the paper is to advance a new view of recessions. In theories where business cycles are caused by fluctuations in productivity—such as in the Real Business Cycle theory of Kydland and Prescott (1982) or in Mortensen and Pissarides (1994)—a recession is a period when the value of time in the market relative to the value of time at home is abnormally low. Indeed, in these theories, a recession is a period when the output of a worker in the market is unusually low. In our theory, a recession is a period when the value of time in the market relative to the value of time at home is abnormally high. Indeed, in our theory, a recession is a period when the output of a worker in the market is not relatively low, but the value of staying at home looking for a job is unusually low. At first glance, the data says that the net value of employment for a worker is countercyclical. But if the net value

of employment is countercyclical, why is there more unemployment in recessions? And why is the rate at which workers lose their job higher and the rate at which unemployed workers find a job lower? That is, if recessions are times when the gains from trade are high, why is there less trade? Our theory provides an answer to this puzzle: when the gains from trade in the labor market are high, firms find it optimal to get rid of their non-performing workers and this creates congestions in the labor market that lowers the speed at which unemployed workers find jobs.

2 Environment and Equilibrium

2.1 Environment

Time is discrete and continues forever. The economy is populated by a measure 1 of identical workers. Every worker has preferences described by $\sum \beta^t [v(c_t) - \psi e_t]$, where $\beta \in (0, 1)$ is the discount factor, $v(c_t)$ is the utility of consuming c_t units of output in period t , and ψe_t is the disutility of exerting e_t units of effort in period t . The utility function $v(\cdot)$ is strictly increasing and strictly concave, with a first derivative $v'(\cdot)$ such that $v'(\cdot) \in [\underline{v}', \bar{v}']$, and a second derivative $v''(\cdot)$ such that $-v''(\cdot) \in [\underline{v}'', \bar{v}'']$, with $\bar{v}' > \underline{v}' > 0$ and $\bar{v}'' > \underline{v}'' > 0$. The consumption c_t is equal to the wage w_t if the worker is employed in period t , and to the value of home production b if the worker is unemployed in period t .¹ The coefficient ψ is strictly positive, and the effort e_t is equal to either 0 or 1. Every worker is endowed with one indivisible unit of labor.

The economy is also populated by a positive measure of identical firms. Every firm has preferences described by $\sum \beta^t c_t$, where $\beta \in (0, 1)$ is the discount factor and c_t is the firm's profit in period t . Every firm operates a constant returns to scale production technology that transforms one unit of labor (i.e. one employee) into y_t units of output, where y_t is a random variable that depends on the employee's effort e_t . In particular, y_t takes the value y_h with probability $p_h(e)$ and the value y_ℓ with probability $p_\ell(e) = 1 - p_h(e)$, with $y_h > y_\ell \geq 0$ and $0 < p_h(0) < p_h(1) < 1$.² Production suffers from a moral hazard problem, in the sense that the firm does not directly observe the effort of its employee, but only the output.

Every period t is divided into five stages: sunspot, separation, matching, bargaining and production. At the first stage, a random variable, z_t , is drawn from a uniform distribution

¹ As the reader can infer from the notation, we assume that workers are banned from the credit market and, hence, they consume their income in every period. The assumption is made only for the sake of simplicity.

² We assume that the worker's effort and output can only take two values. We believe that it would be straightforward to generalize the theory to the case where effort and output are continuous variables.

with support $[0, 1]$.³ The random variable is aggregate, in the sense that it is publicly observed by all market participants. The random variable is a sunspot, in the sense that it does not directly affect technology, preferences or any other fundamentals, although it may serve to coordinate the behavior of market participants.

At the separation stage, some employed workers become unemployed. An employed worker becomes unemployed for exogenous reasons with probability $\delta \in (0, 1)$. In addition, an employed worker becomes unemployed because he is fired with probability $s(y_{t-1}, z_t)$, where $s(y_{t-1}, z_t)$ is determined by the worker's employment contract and it is allowed to depend on the output of the worker in the previous period, y_{t-1} , and on realization of the sunspot in the current period, z_t . For the sake of simplicity, we assume that a worker who becomes unemployed in period t can search for a new job only starting in period $t + 1$.

At the matching stage, some unemployed workers become employed. Firms decide how many job vacancies v_t to create at the unit cost $k > 0$. Then, the u_{t-1} workers who were unemployed at the beginning of the period and the v_t vacant jobs that were created by the firms search for each other. The outcome of the search process is described by a decreasing return to scale matching function, $M(u_{t-1}, v_t)$, which gives the measure of bilateral matches formed between unemployed workers and vacant firms. We denote v_t/u_{t-1} as θ_t , and we refer to θ_t as the tightness of the labor market. We denote as $\lambda(\theta_t, u_{t-1})$ the probability that an unemployed worker meets a vacancy, i.e. $\lambda(\theta_t, u_{t-1}) = M(u_{t-1}, \theta_t u_{t-1})/u_{t-1}$. Similarly, we denote as $\eta(\theta_t, u_{t-1})$ the probability that a vacancy meets an unemployed worker, i.e. $\eta(\theta_t, u_{t-1}) = M(u_{t-1}, \theta_t u_{t-1})/\theta_t u_{t-1}$. We assume that the job-finding probability $\lambda(\theta_t, u_{t-1})$ is strictly increasing in θ_t and strictly decreasing in u_{t-1} and that the job-filling probability $\eta(\theta_t, u_{t-1})$ is strictly decreasing in both θ_t and u_{t-1} . That is, the higher is the labor market tightness, the higher is the job-finding probability and the lower is the job-filling probability. However, for a given labor market tightness, both the job-finding and the job-filling probabilities are strictly decreasing in unemployment.⁴

At the bargaining stage, each firm-worker pair negotiates the terms of a one-period employment contract x_t . The contract x_t specifies the effort e_t recommended to the worker in the current period, the wage w_t paid by the firm to the worker in the current period, and the probability $s(y_t, z_{t+1})$ with which the firm fires the worker at the next separation

³ Assuming that the sunspot is drawn from a uniform distribution with support $[0, 1]$ is without loss in generality.

⁴ Given any constant returns to scale matching function, the job-finding and the job-filling probabilities are only functions of the market tightness. Given any decreasing returns to scale matching function, the job-finding and the job-filling probabilities are also (decreasing) functions of unemployment.

stage, conditional on the output of the worker in the current period and on the realization of the sunspot at the beginning of next period. We assume that the outcome of the bargain between the firm and the worker is the Axiomatic Nash Bargaining Solution.

At the production stage, an unemployed worker home-produces and consumes b units of output. An employed worker chooses an effort level, e_t , and consumes w_t units of output. Then, the output of the worker, y_t , is realized and observed by both the firm and the worker.

A few comments about the environment are in order. We assume that the employment contract cannot specify a wage that depends on the current realization of the worker's output. Hence, the firm cannot use the current wage to give the worker an incentive to exert effort. We also assume that the employment contract is re-bargained every period. Hence, the firm cannot use future wages to give the worker an incentive to exert effort. Overall, firing is the only tool that the firm can use to incentivize the worker. These restrictions on the contract space are much stronger than what we need. Indeed, our theory of business cycles only requires that principals sometimes fire their non-performing agents along the equilibrium path. As we know from Clementi and Hopenhayn (2006), equilibrium firing may obtain under complete contracts as long as the agent is protected by some form of limited liability.

We assume that the matching function $M(u, v)$ has decreasing returns to scale. From the theoretical point of view, one can justify the assumption by noting that the classic urn-ball matching function with finite urns and finite balls has decreasing returns to scale (see, e.g., Burdett, Shi and Wright 2001). From the empirical point of view, it is easy to justify the assumption, since estimating a Cobb-Douglas matching function for the US economy reveals that the exponents on unemployment and vacancies sum up to less than 1.⁵ Moreover, the assumption is not critical. Indeed, the equilibrium conditions of our model are identical to those of a model in which the matching function has constant returns to scale but the cost of a vacancy is strictly increasing in the aggregate number of vacancies.⁶

⁵Petrongolo and Pissarides (2000) show that some empirical studies on the matching function have found increasing returns to scale, some have found constant returns to scale and others have found decreasing returns to scale depending on the data and on the estimation method. Menzio and Shi (2011) show that the estimates of the matching functions are biased if—as most of the studies reviewed by Petrongolo and Pissarides (2000) do—one abstracts from the fact that both employed and unemployed workers search for and match with vacancies.

⁶As the assumption of an increasing marginal cost of a vacancy is more common in than the assumption of a decreasing returns to scale matching function, the reader may be more comfortable with this alternative interpretation of the equilibrium conditions.

2.2 Equilibrium

We now derive the conditions for an equilibrium in our model economy. Let u denote the measure of unemployed workers at the beginning of the bargaining stage. Let $W_0(u)$ denote the lifetime utility of a worker who is unemployed at the beginning of the production stage. Let $W_1(x, u)$ denote the lifetime utility of a worker who is employed under the contract x at the beginning of the production stage. Let $W(x, u)$ denote the difference between $W_1(x, u)$ and $W_0(u)$. Let $F(x, u)$ denote the present value of profits for a firm that, at the beginning of the production stage, employs a worker under the contract x . Let $x^*(u)$ denote the equilibrium contract between a firm and a worker when unemployment is u . Finally, let $\theta(u, \hat{z})$ denote the labor market tightness at the matching stage of next period, when the current unemployment is u and next period's sunspot is \hat{z} . Similarly, let $h(u, \hat{z})$ denote the unemployment at the bargaining stage of next period, when the current unemployment is u and next period's sunspot is \hat{z} .

The lifetime utility $W_0(u)$ of an unemployed worker is such that

$$W_0(u) = v(b) + \beta E_{\hat{z}} [W_0(h(u, \hat{z})) + \lambda(\theta(u, \hat{z}), u)W(x^*(h(u, \hat{z})), h(u, \hat{z}))]. \quad (1)$$

In the current period, the worker home-produces and consumes b units of output. At the matching stage of next period, the worker finds a job with probability $\lambda(\theta(u, \hat{z}), u)$ in which case his continuation lifetime utility is $W_0(h(u, \hat{z})) + W(x^*(h(u, \hat{z})), h(u, \hat{z}))$. With probability $1 - \lambda(\theta(u, \hat{z}), u)$, the worker does not find a job and his continuation lifetime utility is $W_0(h(u, \hat{z}))$.

The lifetime utility $W_1(x, u)$ of a worker employed under the contract $x = (e, w, s)$ is such that

$$\begin{aligned} W_1(x, u) &= v(w) - \psi e + \\ &+ \beta E_{y, \hat{z}} [W_0(h(u, \hat{z})) + (1 - \delta)(1 - s(y, \hat{z}))W(x^*(h(u, \hat{z})), h(u, \hat{z}))|e]. \end{aligned} \quad (2)$$

In the current period, the worker consumes w units of output and exerts effort e . At the separation stage of next period, the worker keeps his job with probability $(1 - \delta)(1 - s(y, \hat{z}))$, in which case his continuation lifetime utility is $W_0(h(u, \hat{z})) + W(x^*(h(u, \hat{z})), h(u, \hat{z}))$. With probability $1 - (1 - \delta)(1 - s(y, \hat{z}))$, the worker loses his job and his continuation lifetime utility is $W_0(h(u, \hat{z}))$.

The difference $W(x, u)$ between $W_1(x, u)$ and $W_0(u)$ represents the gains from trade to a worker employed under the contract x . From (1) and (2), it follows that $W(x, u)$ is such

that

$$W(x, u) = v(w) - v(b) - ce + \beta E_{y, \hat{z}} \{ [(1 - \delta)(1 - s(y, \hat{z})) - \lambda(\theta(\hat{z}, u), u)] W(x^*(h(u, \hat{z})), h(u, \hat{z})) | e \}. \quad (3)$$

We find it useful to denote as $V(u)$ the gains from trade for a worker employed under the equilibrium contract $x^*(u)$, i.e. $V(u) = W(x(u), u)$. We refer to $V(u)$ as the equilibrium gains from trade accruing to the worker.

The present value of profits $F(x, u)$ for a firm that employs a worker under the contract $x = (e, w, s)$ is such that

$$F(x, u) = E_y[y|e] + \beta E_{y, \hat{z}} [(1 - \delta)(1 - s(y, \hat{z})) F(x^*(h(u, \hat{z})), h(u, \hat{z})) | e] \quad (4)$$

In the current period, the firm enjoys a profit equal to the expected output of the worker net of the wage. At the separation stage of next period, the firm retains the worker with probability $(1 - \delta)(1 - s(y, \hat{z}))$, in which case the firm's continuation present value of profits is $F(x^*(h(u, \hat{z})), h(u, \hat{z}))$. With probability $1 - (1 - \delta)(1 - s(y, \hat{z}))$, the firm loses the worker, in which case the firm's continuation present value of profits is zero. We find it useful to denote as $J(u)$ the present value of profits for a firm that employs a worker at the equilibrium contract $x^*(u)$, i.e. $J(u) = F(x^*(u), u)$. We refer to $J(u)$ as the equilibrium gains from trade accruing to the firm.

The equilibrium contract $x^*(u)$ is the Axiomatic Nash Solution to the bargaining problem between the firm and the worker. That is, $x^*(u)$ is such that

$$\max_{x=(e,w,s)} W(x, u) F(x, u), \quad (5)$$

subject to the logical constraints

$$e \in \{0, 1\} \text{ and } s(y, \hat{z}) \in [0, 1],$$

and the worker's incentive compatibility constraints

$$\begin{aligned} \psi &\leq \beta(p_h(1) - p_h(0)) E_{\hat{z}} [(1 - \delta)(s(y_\ell, \hat{z}) - s(y_h, \hat{z})) V(h(u, \hat{z}))], \quad \text{if } e = 1, \\ \psi &\geq \beta(p_h(1) - p_h(0)) E_{\hat{z}} [(1 - \delta)(s(y_\ell, \hat{z}) - s(y_h, \hat{z})) V(h(u, \hat{z}))], \quad \text{if } e = 0. \end{aligned}$$

In words, the equilibrium contract $x^*(u)$ maximizes the product between the gains from trade accruing to the worker, $W(x, u)$, and the gains from trade accruing to the firm, $F(x, u)$, among all contracts x that satisfy the worker's incentive compatibility constraints. The first incentive compatibility constraint states that, if the contract specifies $e = 1$, the cost to the

worker from exerting effort must be smaller than the benefit. The second constraint states that, if the contract specifies $e = 0$, the cost to the worker from exerting effort must be greater than the benefit. The cost of effort is ψ . The benefit of effort is given by the effect of effort on the probability that the realization of output is high, $p_h(1) - p_h(0)$, times the effect of a high realization of output on the probability of keeping the job, $(1 - \delta)(s(y_\ell, \hat{z}) - s(y_h, \hat{z}))$, times the value of the job to the worker, $\beta V(h(u, \hat{z}))$.

The equilibrium market tightness $\theta(u, \hat{z})$ must be consistent with the firm's incentives to create vacancies. The cost to the firm from creating an additional vacancy is k . The benefit to the firm from creating an additional vacancy is given by the job-filling probability, $\eta(\theta(u, \hat{z}), u)$, times the value to the firm of filling a vacancy, $J(h(u, \hat{z}))$. The market tightness is consistent with the firm's incentives to create vacancies if $k = \eta(\theta(u, \hat{z}), u)J(h(u, \hat{z}))$ when $\theta(u, \hat{z}) > 0$, and if $k \geq \eta(\theta(u, \hat{z}), u)J(h(u, \hat{z}))$ when $\theta(u, \hat{z}) = 0$. Overall, the market tightness is consistent with the firm's incentives to create vacancies iff

$$k \geq \eta(\theta(u, \hat{z}), u)J(h(u, \hat{z})) \text{ and } \theta(u, \hat{z}) \geq 0, \quad (6)$$

where the two inequalities hold with complementary slackness.

The equilibrium law of motion for unemployment, $h(u, \hat{z})$, must be consistent with the equilibrium firing probability $s^*(y, \hat{z}, u)$ and with the job-finding probability $\lambda(\theta(u, \hat{z}), u)$. Specifically, $h(u, \hat{z})$ must be such that

$$g(u, \hat{z}) = u - u\mu(J(h(u, \hat{z}), u) + (1 - u)E_y [\delta + (1 - \delta)s^*(y, \hat{z}, u)] , \quad (7)$$

where

$$\mu(J, u) = \lambda(\eta^{-1}(\min\{k/J, 1\}, u), u) ,$$

and $\eta^{-1}(\min\{k/J, 1\}, u)$ is the labor market tightness that solves (6). The first term on the right-hand side of (7) is unemployment at the beginning of the bargaining stage in the current period. The second term in (7) is the measure of unemployed workers who become employed during the matching stage of next period, which is given by unemployment u times the probability that an unemployed worker becomes employed $\mu(J(h(u, \hat{z}), u)$. The last term in (7) is the measure of employed workers who become unemployed during the separation stage of next period. The sum of the three terms on the right-hand side of (7) is the unemployment at the beginning of the bargaining stage in the next period.

We are now in the position to define a recursive equilibrium for our model economy.

Definition 1: A Recursive Equilibrium is a tuple (W, F, V, J, x^*, h) such that: (i) The gains from trade accruing to the worker, $W(x, u)$, and to the firm, $F(x, u)$, satisfy (3) and (4) and

$V(u) = W(x^*(u), u)$, $J(u) = F(x^*(u), u)$; (ii) The employment contract $x^*(u)$ satisfies (5); (iii) The law of motion $h(u, \hat{z})$ satisfies (7).

Over the next three sections, we will characterize the properties of the recursive equilibrium. We are going to carry out the analysis under the maintained assumptions that the equilibrium gains from trade are strictly positive, i.e. $J(u) > 0$ and $V(u) > 0$, and that the equilibrium contract requires the worker to exert effort, i.e. $e^*(u) = 1$. The first assumption guarantees that firms and workers trade in the labor market, and the second assumption guarantees that firms and workers find it optimal to solve the moral hazard problem.⁷

3 Contracts

In this section, we characterize the properties of the Axiomatic Nash Solution to the bargaining problem between the firm and the worker. That is, we characterize the properties of the employment contract that maximizes the product of the gains from trade accruing to the worker and the gains from trade accruing to the firm subject to the worker's incentive compatibility constraint.⁸ We refer to such contract as the optimal employment contract. Our key finding is that the worker is fired if and only if the realization of output is low and the realization of the state of the world is such that the cost to the worker from losing the job relative to the cost to the firm from losing the worker is sufficiently high.

We carry out the characterization of the optimal employment contract in four lemmas. In order to lighten up the notation, and without risk of confusion, we will drop the dependence of the gains from trade to the worker, W , and to the firm, F , as well as the dependence of the optimal contract, x^* , on unemployment in the current period, u . We will also drop the dependence of the continuation gains from trade to the worker and to the firm on unemployment in the current period and write $V(h(u, \hat{z}))$ as $V(\hat{z})$ and $J(h(u, \hat{z}))$ as $J(\hat{z})$.

Lemma 1: Any optimal contract x^* is such that the worker's incentive compatibility holds with equality. That is,

$$\psi = \beta(p_h(1) - p_h(0))E_{\hat{z}} [(1 - \delta)(s^*(y_\ell, \hat{z}) - s^*(y_h, \hat{z}))V(\hat{z})]. \quad (8)$$

⁷It is straightforward to verify that the first assumption is satisfied as long as b is sufficiently low relative to $p_h(1)y_h + p_\ell(1)y_\ell$, and that the second assumption is satisfied as long as y_ℓ is sufficiently low relative to y_h .

⁸As mentioned at the end of Section 2, we will carry out the analysis under the maintained assumptions that the gains from trade are strictly positive and that it is optimal to recommend the worker to exert effort.

Proof: In Appendix A. ■

To understand Lemma 1 consider a contract x such that the worker's incentive compatibility constraint is lax. Clearly, this contract prescribes that the worker is fired with some positive probability after a low realization of output, i.e. $s(y_\ell, \hat{z})$. If we lower $s(y_\ell, \hat{z})$ by some small amount, the worker's incentive compatibility constraint is satisfied. Moreover, if the lower $s(y_\ell, \hat{z})$, the survival probability of the match increases. Since the continuation value of the match is strictly positive for both the worker and the firm, an increase in the survival probability of the match raises the gains from trade accruing to the worker, W , the gains from trade accruing to the firm, F , and the Nash product WF . Therefore, the contract x cannot be optimal.

Lemma 2: Any optimal contract x^* is such that, if the realization of output is high, the worker is fired with probability 0. That is, for all $\hat{z} \in [0, 1]$,

$$s^*(y_h, \hat{z}) = 0. \quad (9)$$

Proof: In Appendix A. ■

To understand Lemma 2 consider a contract x such that the worker is fired with positive probability when the realization of output is high, i.e. $s(y_h, \hat{z}) > 0$. If we lower the firing probability $s(y_h, \hat{z})$, the incentive compatibility constraint of the worker is relaxed. Moreover, if we lower the firing probability $s(y_h, \hat{z})$, the survival probability of the match increases. In turn, the increase in the survival probability of the match raises the gains from trade accruing to the worker, W , the gains from trade accruing to the firm, F , and the Nash product WF . Thus, the contract x cannot be optimal.

Lemma 3: Let $\phi(\hat{z}) = V(\hat{z})/J(\hat{z})$. Any optimal contract x^* is such that, if the realization of output is low, the worker is fired with probability 1 if $\phi(\hat{z}) > \phi^*$, and the worker is fired with probability 0 if $\phi(\hat{z}) < \phi^*$. That is, for all $\hat{z} \in [0, 1]$,

$$s^*(y_\ell, \hat{z}) = \begin{cases} 1, & \text{if } \phi(\hat{z}) > \phi^*, \\ 0, & \text{if } \phi(\hat{z}) < \phi^*. \end{cases} \quad (10)$$

Proof: Let $\rho \geq 0$ denote the multiplier on the worker's incentive compatibility constraint, let $\bar{\nu}(y, \hat{z}) \geq 0$ denote the multiplier on the constraint $1 - s(y, \hat{z}) \geq 0$, and let $\underline{\nu}(y, \hat{z})$ denote the multiplier on the constraint $s(y, \hat{z}) \geq 0$. The first order condition with respect to the firing probability $s(y_\ell, \hat{z})$ is given by

$$(1 - \delta) [F(x)V(\hat{z}) + W(x)J(\hat{z})] = \rho\beta(1 - \delta)(p_h(1) - p_h(0))V(\hat{z}) + \underline{\nu}(y_\ell, \hat{z}) - \bar{\nu}(y_\ell, \hat{z}),$$

together with the complementary slackness conditions $\bar{\nu}(y_\ell, \hat{z}) \cdot (1 - s(y_\ell, \hat{z})) = 0$ and $\underline{\nu}(y_\ell, \hat{z}) \cdot s(y_\ell, \hat{z}) = 0$. The left-hand side is the marginal cost of increasing $s(y_\ell, \hat{z})$. This cost is given by the decline in the product of the worker's and firm's gains from trade caused by a marginal increase in the firing probability $s(y_\ell, \hat{z})$. The right-hand side is the marginal benefit of increasing $s(y_\ell, \hat{z})$. This benefit is given by the value of relaxing the worker's incentive compatibility and the $s(y_\ell, \hat{z}) \geq 0$ constraints net of the cost of tightening the $s(y_\ell, \hat{z}) \leq 1$ constraint.

Using the definition of $\phi(\hat{z})$, we can rewrite the first order condition with respect to the firing probability $s(y_\ell, \hat{z})$ as

$$(1 - \delta)V(\hat{z}) [F(x) + W(x)/\phi(\hat{z}) - \rho\beta(p_h(1) - p_h(0))] = \underline{\nu}(y_\ell, \hat{z}) - \bar{\nu}(y_\ell, \hat{z}), \quad (11)$$

together with $\bar{\nu}(y_\ell, \hat{z}) \cdot (1 - s(y_\ell, \hat{z})) = 0$ and $\underline{\nu}(y_\ell, \hat{z}) \cdot s(y_\ell, \hat{z}) = 0$. The left-hand side of (11) is strictly decreasing in $\phi(\hat{z})$. The right-hand side of (11) is strictly positive if $\underline{\nu}(y_\ell, \hat{z})$ is strictly positive and it is strictly negative if $\bar{\nu}(y_\ell, \hat{z})$ is strictly positive. Therefore, there exists a ϕ^* such that if $\phi(\hat{z}) > \phi^*$, the left-hand side is strictly negative and the solution to (11) requires $\bar{\nu}(y_\ell, \hat{z}) > 0$. In this case, the solution to the first order condition for $s(y_\ell, \hat{z})$ is 1. If $\phi(\hat{z}) < \phi^*$, the left-hand side is strictly positive and the solution to (11) requires $\underline{\nu}(y_\ell, \hat{z}) > 0$. In this case, the solution to the first order condition for $s(y_\ell, \hat{z})$ is 0. ■

Lemma 3 is one of the main results of the paper. It states that any optimal contract x^* is such that, if the realization of output is low, the worker is fired with probability 1 in states of the world \hat{z} in which the continuation gains from trade to the worker, $V(\hat{z})$, relative to the continuation gains from trade to the firm, $J(\hat{z})$, are above some cutoff, and the worker is fired with probability 0 in states of the world in which the ratio $V(\hat{z})/J(\hat{z})$ is below the cutoff. There is a simple intuition behind this result. Firing is costly—as it destroys a valuable relationship—but also necessary—as it is the only tool to provide the worker with an incentive to exert effort. However, only the value of the destroyed relationship that would have accrued to the worker serves the purpose of providing incentives. The value of the destroyed relationship that would have accrued to the firm is “collateral damage.” The optimal contract minimizes the collateral damage by concentrating firing in states of the world in which the value of the relationship to the worker would have been highest relative to the value of the relationship to the firm. In other words, the optimal contract minimizes the collateral damage by concentrating firing in states of the world in which the cost to the worker from losing the job, $V(\hat{z})$, is highest relative to the cost to the firm from losing the worker, $J(\hat{z})$. Notice that this property of the optimal contract follows immediately from the linearity of the problem with respect to the firing probability, and it does not depend on the

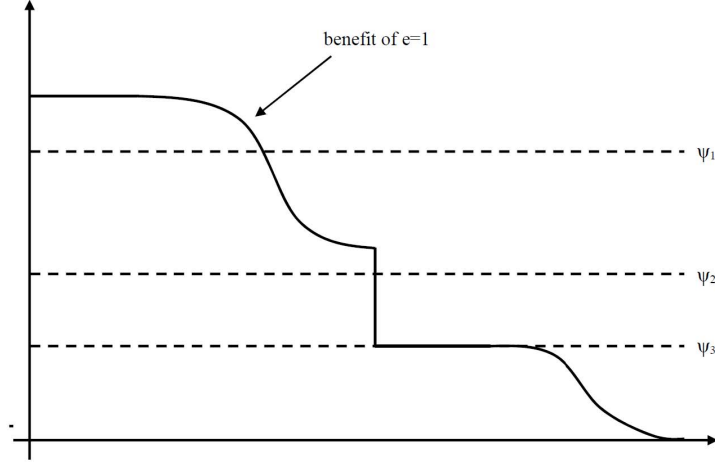


Figure 1: Equilibrium Cutoff ϕ^*

fact that the optimal contract maximizes the product of the gains from trade rather than the gains from trade to the firm taking subject to delivering a given level of gains from trade to the worker. In this sense, the property is rather general to contractual environments in which firing takes place along the equilibrium path.

In any optimal contract x^* , the firing cutoff ϕ^* is such that

$$\psi = \beta(p_h(1) - p_h(0))(1 - \delta) \left[\int_{\phi(\hat{z}) > \phi^*} V(\hat{z}) d\hat{z} + \int_{\phi(\hat{z}) = \phi^*} s^*(y_\ell, \hat{z}) V(\hat{z}) d\hat{z} \right], \quad (12)$$

The above equation is the worker's incentive compatibility constraint (8) written in light of the fact that $s^*(y_h, \hat{z})$ is given by (9) and $s^*(y_\ell, \hat{z})$ is given by (10). Figure 1 plots the right-hand side of (12), which is the worker's benefit from exerting effort, as a function of the firing cutoff ϕ^* . On any interval $[\phi_0, \phi_1]$ where the distribution of the random variable $\phi(\hat{z})$ has positive density, the right-hand side of (12) is strictly decreasing in ϕ^* . On any interval $[\phi_0, \phi_1]$ where the distribution of $\phi(\hat{z})$ has no density, the right-hand side of (12) is constant. At any value ϕ where the distribution of $\phi(\hat{z})$ has a mass point, the right-hand side of (12) can take on an interval of values, as the firing probability $s^*(y_\ell, \hat{z})$ for \hat{z} such that $\phi(\hat{z}) = \phi$ varies between 0 and 1. Overall, the right-hand side of (12) is a weakly decreasing function of the firing cutoff ϕ^* .

In any optimal contract x^* , the firing cutoff ϕ^* is such that the right-hand side of (12) is equal to the worker's cost ψ from exerting effort. There are three cases to consider. First, consider the case in which the right-hand side of (12) equals ψ at a point where the right-hand side of (12) is strictly decreasing in ϕ^* . In this case, the equilibrium firing cutoff is uniquely pinned down. Moreover, since the right-hand side of (12) is strictly decreasing in ϕ^* , the random variable $\phi(\hat{z})$ has no mass point at the equilibrium firing cutoff. Hence, in this case, the firm either fires the worker with probability 0 or with probability 1. This is the case of ψ_1 in Figure 1. Second, consider the case in which the right-hand side of (12) equals ψ at a point where the right-hand side of (12) can take on a range of values. In this case, the equilibrium firing cutoff is uniquely pinned down. However, the random variable $\phi(\hat{z})$ has a mass point at the equilibrium firing cutoff. Hence, in this case, for any realization of $\phi(\hat{z})$ equal to the equilibrium firing cutoff, the firm fires the worker with the probability $s^*(y_\ell, \hat{z})$ that satisfies (12). This is the case of ψ_2 in Figure 1. Finally, consider the case in which there is an interval of values of $\phi(\hat{z})$ such that the right-hand side of (12) equals ψ . In this case, the equilibrium firing cutoff can take on any value in the interval. However, the choice of the cutoff is immaterial, as the probability that the random variable $\phi(\hat{z})$ falls in the interval is zero. This is the case of ψ_3 in Figure 1. In any of the three cases, the equilibrium firing cutoff is effectively unique and so is the firing probability for any realization of the random variable $\phi(\hat{z})$.

Lemma 4: Any optimal contract x^* is such that the wage w^* satisfies

$$\frac{W(x^*)}{F(x^*)} = \frac{v'(w^*)}{1}. \quad (13)$$

Proof: In Appendix A. ■

Lemma 4 states that any optimal contract x^* prescribes a wage such that the ratio of the marginal utility of consumption to the worker to the marginal utility of consumption to the firm, $v'(w^*)/1$, is equal to the ratio of the equilibrium gains from trade accruing to the worker to the equilibrium gains from trade accruing to the firm, $W(x^*)/F(x^*) = V/J$. This is the standard optimality condition for the wage in the Axiomatic Nash Solution. Lemma 4 is important, as it tells us that the relative gains from trade accruing to the worker are higher in states of the world in which the worker's wage is lower. Hence, it follows from Lemma 3 that the optimal contract is such that the firm fires the worker if and only if the realization of output is low and the realization of the state of the world is such that the worker's wage next period would have been sufficiently low.

We are now in the position to summarize the characterization of the optimal contract.

Theorem 1: (*Contracts*) Any optimal contract x^* is such that: (i) the worker is paid the wage w^* given by (13); (ii) If the realization of output is high, the worker is fired with probability $s^*(y_h, \hat{z})$ given by (9); (iii) If the realization of output is low, the worker is fired with probability $s^*(y_\ell, \hat{z})$ given by (10), where the ϕ^* is uniquely pinned down by (12).

4 Sunspots

In the previous section, we showed that, if the realization of the sunspot is associated with some variation in the worker's relative gains from trade, the optimal employment contract will exploit this variation by loading up the firing probability on the states of the world where the worker's relative gains from trade are highest. In the first part of this section, we show that there exist equilibria in which the sunspot is associated with variation in the worker's relative gains from trade and, hence, in firing. In the second part of this section, we show that the equilibria in which the sunspot is not associated with any variation in the worker's relative gains from trade and, hence, in firing are not robust.

There is a simple intuition behind our findings. It is easy to see that the states of the world where the worker's relative gains from trade are highest are the states of the world where unemployment is highest. In fact, in states of the world where unemployment is higher, unemployed workers have a lower job-finding probability because of the assumption of decreasing returns to matching. In turn, if the job-finding probability is lower, workers have a weaker outside option when bargaining with firms and they capture a lower wage. As we know from Lemma 4, if the wage is lower, the worker's relative gains from trade are higher. Hence, the states of the world where the worker's relative gains from trade are highest are the states of the world where unemployment is highest and, consequently, the states of the world where firms find it optimal to fire. Conversely, the states of the world where firms find it optimal to fire are those where unemployment is highest. The self-confirming loop reveals that firms have an incentive to coordinate their firing decisions and supports the existence of equilibria in which this coordination is achieved by means of the sunspot. There are also equilibria in which firms ignore the sunspot and achieve no coordination. However, the no coordination equilibria are an artifact of the assumption that firms make their firing decisions simultaneously.

4.1 Equilibrium Effect of Sunspots

In a recursive equilibrium, the effect of the realization of the sunspot \hat{z} on the probability $s^*(y_\ell, \hat{z})$ with which firms fire non-performing workers, and on the ratio $\phi(\hat{z})$ of the gains from trade accruing to the worker relative to the firm must satisfy two conditions. For any \hat{z} , the firing probability $s^*(y_\ell, \hat{z})$ must be part of the optimal employment contract given the worker's relative gains from trade $\phi(\hat{z})$ and given the probability distribution of the worker's relative gains from trade across realizations of the sunspot. In turn, for any \hat{z} , the worker's relative gains from trade $\phi(\hat{z})$ must be those implied by the evolution of unemployment, given the initial unemployment u and the firing probability $s^*(y_\ell, \hat{z})$. Formally, the equilibrium effect of the sunspot is given by two functions $\phi(\hat{z})$ and $s^*(y_\ell, \hat{z})$ that are a fixed-point of the mapping we have just described. We refer to such a fixed-point as the 1-Period Equilibrium, as it describes the key outcomes of the economy within one period going from the bargaining stage of one period to the bargaining stage of next period.

We first characterize the effect of the firm's firing probability $s^*(y_\ell, \hat{z})$ on the worker's relative gains from trade $\phi(\hat{z})$. Given that unemployment at the beginning of the period is u and that the firing probability at the separation stage is $s(\hat{z}) = s^*(y_\ell, \hat{z})$, the law of motion (7) implies that unemployment at the bargaining stage is $\hat{u}(s(\hat{z}))$ such that

$$\hat{u}(s(\hat{z})) = u - \mu(J(\hat{u}(s(\hat{z}))), u) + (1 - u)(\delta + (1 - \delta)p_\ell(1)s(\hat{z})). \quad (14)$$

We conjecture that the gains from trade accruing to the firm are a strictly increasing function of unemployment, i.e. $J(\hat{u}(s(\hat{z})))$ is strictly increasing in $\hat{u}(s(\hat{z}))$. Under this conjecture, there is a unique $\hat{u}(s(\hat{z}))$ that satisfies (14) and $\hat{u}(s(\hat{z}))$ is strictly increasing in the firing probability $s(\hat{z})$. Intuitively, the higher is the firing probability at the separation stage, the larger is the flow of workers into unemployment and the higher is unemployment at the bargaining stage.

Given that unemployment at the bargaining stage is $\hat{u}(s(\hat{z}))$, the worker's wage is $w^*(\hat{u}(s(\hat{z})))$. Then, it follows from the optimality condition (13) that the worker's relative gains from trade are such that

$$\phi(\hat{z}) = \frac{V(\hat{u}(s(\hat{z})))}{J(\hat{u}(s(\hat{z})))} = \frac{v'(w^*(\hat{u}(s(\hat{z}))))}{1}. \quad (15)$$

We conjecture that the wage is a strictly decreasing function of unemployment, i.e. $w^*(\hat{u}(s(\hat{z})))$ is strictly decreasing in $\hat{u}(s(\hat{z}))$. Under this conjecture, the worker's relative gains from trade $\phi(\hat{z})$ are strictly increasing in the unemployment $\hat{u}(s(\hat{z}))$ and, since $\hat{u}(s(\hat{z}))$ is strictly increasing in $s(\hat{z})$, they are also strictly increasing in the firing probability $s(\hat{z})$. Intuitively, the higher is the firing probability at the separation stage, the higher is unemployment and the

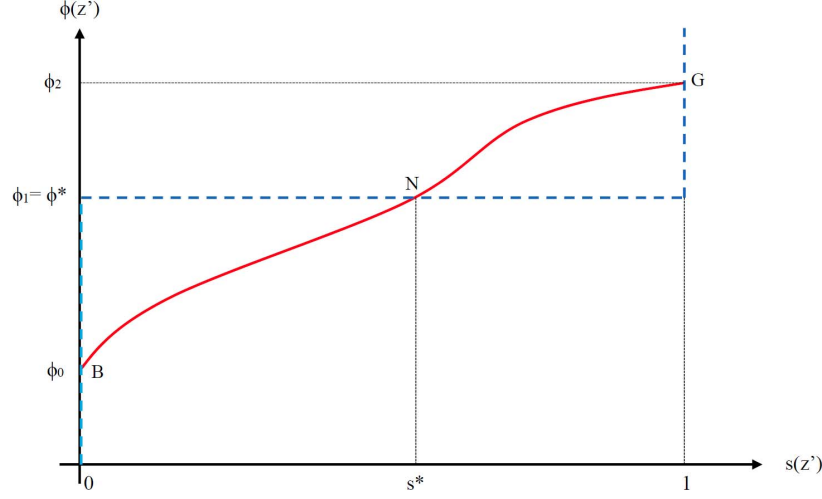


Figure 2: 1-Period Equilibrium

lower is the wage. In turn, the lower is the wage, the higher is the worker's relative marginal utility of consumption and, hence, the worker's relative gains from trade. The solid line in Figure 2 illustrates the effect of the firing probability on the worker's relative gains from trade.

The conjectures that the worker's wage is decreasing in unemployment and the firm's gains from trade are increasing in unemployment will be verified in the next section. Intuitively, these conjectures are true because the matching function has decreasing returns to scale. Indeed, when the matching function has decreasing returns to scale, an increase in unemployment tends to lower the job-finding probability of unemployed workers. In turn, a decline in the job-finding probability lowers the value of unemployment. Since the value of unemployment is the worker's outside option in bargaining, the equilibrium wage falls and the firm's gains from trade increase.

Next, we characterize the effect of the worker's relative gains from trade $\phi(\hat{z})$ on the probability $s(\hat{z}) = s^*(y_\ell, \hat{z})$ with which firms fire their non-performing workers. From the optimality condition (10), it follows that the firing probability $s(\hat{z})$ is such that

$$s^*(y_\ell, \hat{z}) = \begin{cases} 0, & \text{if } \phi(\hat{z}) < \phi^*, \\ \in [0, 1], & \text{if } \phi(\hat{z}) = \phi^*, \\ 1, & \text{if } \phi(\hat{z}) > \phi^*, \end{cases} \quad (16)$$

where ϕ^* is implicitly defined by the worker's incentive compatibility constraint (12). As explained in the previous section, the higher are the worker's relative gains from trade in \hat{z} , the stronger is the firm's incentive to fire its non-performing workers in that state of the world. The dashed line in Figure 2 illustrates the effect of the worker's relative gains from trade $\phi(\hat{z})$ on the firing probability $s(\hat{z})$.

It is now clear from Figure 2 that, for any realization \hat{z} of the sunspot, there are only three possible equilibrium outcomes. In the first equilibrium outcome, firms fire their non-performing workers with probability 0 and the worker's relative gains from trade are smaller than ϕ^* . It is easy to understand why this outcome is an equilibrium. If firms do not fire their non-performing workers, the unemployment rate is relatively low, the wage is relatively high and the worker's relative gains from trade are smaller than ϕ^* , which rationalizes the firms' firing decision. In the second equilibrium outcome, firms fire their non-performing workers with some intermediate probability s^* and the worker's relative gains from trade are equal to ϕ^* . It is easy to understand why this outcome is an equilibrium. If firms fire their non-performing workers with probability s^* , the worker's relative gains from trade are equal to ϕ^* , a value for which firms are indifferent between firing and not firing. In the third equilibrium outcome, firms fire their non-performing workers with probability 1 and the worker's relative gains from trade are greater than ϕ^* . This is an equilibrium because, if firms fire all of the non-performing workers, the unemployment rate is relatively high, the wage is relatively low and, hence, the worker's relative gains from trade are greater than ϕ^* , which rationalizes the firms' firing decision.

Since for any realization \hat{z} of the sunspot there are only three possible outcomes, it follows that any 1-Period Equilibrium will have an identical structure. Let Z_0 denote the realizations of the sunspot for which firms fire non-performing workers with probability 0, and let π_0 denote the measure of Z_0 . Let Z_1 denote the realizations of the sunspot for which firms fire non-performing workers with a probability $s^*(y_\ell, \hat{z}) \in (0, 1)$, and let π_1 denote the measure of Z_1 . Similarly, let Z_2 denote the realizations of the sunspot for which firms fire non-performing workers with probability 1, and let π_2 denote the measure of Z_2 . Depending on the value of π_1 , we can identify three different types of equilibria.

If $\pi_1 = 1$, we have a *No Coordination Equilibrium*. In this type of equilibrium, firms fire the non-performing workers with the same probability independently of the realization of the sunspot, i.e. $s^*(y_\ell, \hat{z}) = s^* \in (0, 1)$ for all $\hat{z} \in [0, 1]$. In this type of equilibrium, the worker's incentive compatibility constraint (12) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1 - \delta)s^*V(\hat{u}(s^*)). \quad (17)$$

When we solve the constraint with respect to s^* , we find that the constant probability with which firms fire their non-performing workers is

$$s^* = \frac{\psi}{\beta(p_h(1) - p_h(0))(1 - \delta)V(\hat{u}(s^*))}. \quad (18)$$

If $\pi_1 \in (0, 1)$, we have a *Partial Coordination Equilibrium*. In this type of equilibrium as in a No Coordination Equilibrium, firms fire their non-performing workers with the same probability for any realization of the sunspot in Z_1 , i.e. $s^*(y_\ell, \hat{z}) = s^* \in (0, 1)$ for all $\hat{z} \in Z_1$. However, if $\hat{z} \notin Z_1$, firms use the realization of the sunspot to coordinate on firing their non-performing workers with either probability zero or probability one. In this type of equilibrium, the worker's incentive compatibility constraint (12) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1 - \delta) [\pi_1 s^* V(\hat{u}(s^*)) + \pi_2 V(\hat{u}(1))]. \quad (19)$$

When we solve the constraint with respect to s^* , we find that the constant probability with which firms fire their non-performing workers for $\hat{z} \in Z_1$ is

$$s_1^* = \frac{\psi - \beta(p_h(1) - p_h(0))(1 - \delta)\pi_2 V(\hat{u}(1))}{\beta(p_h(1) - p_h(0))(1 - \delta)\pi_1 V(\hat{u}(s^*))}. \quad (20)$$

If $\pi_1 = 0$, we have a *Perfect Coordination Equilibrium*. In this type of equilibrium, firms always use the realization of the sunspot to coordinate on either firing their non-performing workers with probability zero or with probability one. In this type of equilibrium, the worker's incentive compatibility constraint (12) becomes

$$\psi = \beta(p_h(1) - p_h(0))(1 - \delta)\pi_2 V(\hat{u}(1)). \quad (21)$$

When we solve the constraint with respect to π_2 , we find that the probability with which firms coordinate on firing all of their non-performing workers is given by

$$\pi_2 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1 - \delta)V(\hat{u}(1))}. \quad (22)$$

The above results are summarized in Theorem 2.

Theorem 2: (*Sunspots*). Three 1-Period Equilibria exist: (i) No Coordination Equilibrium where $s^*(y_\ell, \hat{z}) = s^*$ for all $\hat{z} \in Z_1$, where Z_1 has probability measure $\pi_1 = 1$ and s^* is given by (18); (ii) Partial Coordination Equilibrium where $s^*(y_\ell, \hat{z}) = 0$ for all $\hat{z} \in Z_0$, $s^*(y_\ell, \hat{z}) = s^*$ for all $\hat{z} \in Z_1$, and $s^*(y_\ell, \hat{z}) = 1$ for all $\hat{z} \in Z_2$, where Z_1 has probability measure $\pi_0 \in (0, 1)$ and s^* is given by (20); (iii) Perfect Coordination Equilibrium where $s^*(y_\ell, \hat{z}) = 0$ for all

$\hat{z} \in Z_0$ and $s^*(y_\ell, \hat{z}) = 1$ for all $\hat{z} \in Z_2$, where Z_0 has probability measure $1 - \pi_2$ and Z_2 has probability measure π_2 and $\pi_2 \in (0, 1)$ is given by (22).

In the next subsection, we argue that the Perfect Coordination Equilibrium is the only one that is robust to a small perturbation of the environment. In light of this result, let us discuss some of the key features of this type of equilibrium. First, notice that, in a Perfect Coordination Equilibrium, there are aggregate fluctuations. For realizations of the sunspot in Z_0 , the firms will not fire any of their non-performing workers. In these states of the world, the rate at which workers move from employment to unemployment is low, the unemployment rate is low, the rate at which workers move from unemployment to employment is high and wages are high. For realizations of the sunspot in Z_2 , the firms coordinate on firing all of their non-performing workers. In these states of the world, the rate at which workers move from employment to unemployment is high, the unemployment rate is high, the rate at which workers move from unemployment to employment is low and wages are low. Moreover, the probability with which firms coordinate on firing their non-performing workers is endogenously determined in (22) and it is such that the worker's incentive compatibility constraint holds with equality. As we shall see that the worker's gains from trade are increasing in unemployment, it follows that the probability of an episode of coordinated firing is lower the higher is aggregate unemployment.

Second, in a Perfect Coordination Equilibrium, aggregate fluctuations are not caused by exogenous shocks to fundamentals such as technology (e.g., labor productivity), matching (e.g., vacancy cost) or policy (e.g., unemployment benefits). Indeed, all fundamentals remain constant over time. Aggregate fluctuations are also not caused by exogenous shocks to the equilibrium being played by market participants. Indeed, the equilibrium being played is the same over time. Instead, aggregate fluctuations are endogenous. They are caused by the fact that each firm needs to randomize over firing or keeping its non-performing workers and different firms have an incentive to coordinate the outcome of the randomization.

4.2 Equilibrium Selection

In the model described in Section 2, we assumed that there is a continuum of infinitesimal firm-worker pairs and that all firm-worker pairs decide on whether to break-up or not simultaneously. These assumptions are clearly unrealistic. Certainly, in the actual economy, there is a finite number of firm-worker pairs that, to some degree, make staggered separation decisions. We made these assumptions to make the analysis more tractable. Unfortunately, the assumptions also generate some “artificial” equilibria, i.e. equilibria that only exist because

there is a continuum of firm-worker pairs who make separation decisions simultaneously. In what follows we show that, when these assumptions are relaxed, the No Coordination and the Partial Coordination Equilibria disappear, and only the Perfect Coordination Equilibrium survives. There is a simple intuition behind this result. Firms have an incentive to coordinate firing. However, when firms make firing decisions simultaneously, there are equilibria with less than perfect coordination because—if other firms ignore the sunspot—a firm will also ignore the sunspot. In other words, when firms make their firing decisions simultaneously, the sunspot can be used to coordinate firings, but it can also be ignored. In contrast, when firms make firing decisions sequentially, the firms moving later can always coordinate their firing with the firing of the firms earlier.

Here is a formal description of the modification of the environment of Section 2. Let $1 - u$ denote the measure of employed workers at the bargaining stage of the current period. The measure of employed workers is equally divided into a large number NK firms, each employing one worker of “measure” $(1 - u)/NK$. Firms are clustered into a large number K of groups, each comprising a large number N of firms. Firms and workers bargain over the terms of the one-period employment contract knowing the group to which they belong. At the separation stage of next period, firm-worker pairs in different groups break up sequentially. First, the firm-worker pairs in group 1 simultaneously decide to separate or not. Second, after observing the outcomes of group 1, the firm-worker pairs in group 2 simultaneously decide to separate or not, after observing the outcomes of group 1. Third, after observing the outcomes of groups 1 and 2, the firm-worker pairs in group 3 simultaneously decide to separate or not. The process continues until the firm-worker pairs in group K simultaneously decide to separate or not, after having observed the outcomes of groups 1 through $K - 1$. Naturally, in this version of the model, we do not need the sunspot.

Let T_i denote the measure of workers separating from firms in groups 1 through i . We assume that each firm in group i takes as given the probability distribution of T_i conditional on T_{i-1} , which we denote as $P_i(T_i|T_{i-1})$. The assumption means that each firm views itself as small compared to its group. The assumption is reasonable when N is large. We also assume that $P_i(T_i|T_{i-1})$ is increasing, in the sense of first-order stochastic dominance, in T_{i-1} . The assumption means that firms in a group view themselves as small compared to the whole economy. The assumption is reasonable when K is large. We also approximate the worker’s gains from trade, $V(\hat{u})$, and the firm’s gains from trade, $J(\hat{u})$, with linear functions. The approximation implies that the expectation of the worker’s relative gains from trade over next period’s unemployment, $E[V(\hat{u})]/E[J(\hat{u})]$, is equal to the worker’s relative gains from trade evaluated at the expectation of next period’s unemployment, $V(E[\hat{u}])/J(E[\hat{u}]) = \phi(E[\hat{u}])$.

We can now characterize the optimal contract between a worker and a firm in group $i = 2, 3, \dots, K$. The contract can condition the firing probability $s_i(y, T_{i-1})$ on the realization of the worker's output y and on the measure T_{i-1} of workers separating from firms in groups 1 through $i - 1$. As in Section 3, it is easy to show that the optimal contract is such that: (i) the worker's incentive compatibility constraint holds with equality; (ii) if the realization of output is high, the worker is fired with probability 0, i.e. $s_i(y_h, T_{i-1}) = 0$ for all T_{i-1} ; (iii) if the realization of output is low, the worker is fired with probability 0 if the worker's relative gains from trade are below a cutoff ϕ_i^* , and with probability 1 if they are above the cutoff, i.e. $s_i(y_\ell, T_{i-1}) = 0$ if $\phi(E[\hat{u}|T_{i-1}]) < \phi_i^*$, and $s_i(y_\ell, T_{i-1}) = 1$ if $\phi(E[\hat{u}|T_{i-1}]) > \phi_i^*$. The optimal contract between a worker and a firm in group 1 can only condition the firing probability on the realization of the worker's output y . In this case, the optimal contract is such that $s_1(y_h) = 0$ and $s_1(y_\ell) = s_1$, where s_1 is such that the worker's incentive compatibility constraint holds with equality.

Lemma 5: For $i = 2, \dots, K$, the firing probability $s_i(y_\ell, T_{i-1})$ equals 0 for all $T_{i-1} < T_{i-1}^*$, and it equals 1 for all $T_{i-1} > T_{i-1}^*$.

Proof: We first consider a firm-worker pair in group K . Since $P_K(T_K|T_{K-1})$ is strictly increasing in T_{K-1} and \hat{u} is strictly increasing in T_K , $E[\hat{u}|T_{K-1}]$ is strictly increasing in T_{K-1} . In turn, since $E[\hat{u}|T_{K-1}]$ is strictly increasing in T_{K-1} and $\phi(\hat{u})$ is strictly increasing in \hat{u} , $\phi(E[\hat{u}|T_{K-1}])$ is strictly increasing in T_{K-1} . It then follows from property (iii) of the optimal contract that there exists a T_{K-1}^* such that $s_K(y_\ell, T_{K-1}) = 0$ for all $T_{K-1} < T_{K-1}^*$, and $s_K(y_\ell, T_{K-1}) = 1$ for all $T_{K-1} > T_{K-1}^*$. This establishes that the firing probability $s_K(y_\ell, T_{K-1})$ has the desired threshold property. The threshold property implies that the measure t_K of workers separating from firms in group K is increasing in the measure T_{K-1} of workers separating from firms in groups 1 through $K - 1$.

Next, consider a firm-worker pair in group $K - 1$. Since $P_{K-1}(T_{K-1}|T_{K-2})$ is strictly increasing in T_{K-2} and t_K is increasing in T_{K-1} , it follows that $E[\hat{u}|T_{K-2}]$ and, in turn, $\phi(E[\hat{u}|T_{K-2}])$ are strictly increasing in T_{K-2} . Then property (iii) of the optimal contract implies that there exists a T_{K-2}^* such that $s_{K-1}(y_\ell, T_{K-2}) = 0$ for all $T_{K-2} < T_{K-2}^*$, and $s_{K-1}(y_\ell, T_{K-2}) = 1$ for all $T_{K-2} > T_{K-2}^*$. This establishes that the firing probability $s_{K-1}(y_\ell, T_{K-2})$ has the desired threshold property. The threshold property implies that the measure t_{K-1} of workers separating from firms in group $K - 1$ is increasing in the measure T_{K-2} of workers separating from firms in groups 1 through $K - 2$. By repeating the above argument for firm-worker pairs in groups $K - i$, with $i = 2, 3, \dots, K - 1$, we can establish that the firing probability $s_{K-i}(y_\ell, T_{K-i-1})$ has the threshold property and that the measure t_{K-i} is increasing in T_{K-i-1} . ■

Given the characterization of the optimal firing policy in Lemma 5, we compute the equilibrium probability distribution of t_i conditional on T_{i-1} . A worker separates from a firm in group 1 with probability $\delta + (1 - \delta)p_\ell(1)s_1$. Since N is large, we can use the Central Limit Theorem to approximate the measure t_1 of workers separating from firms in group 1 with a Normal distribution with mean μ_1 and standard deviation σ_1 , where

$$\begin{aligned}\mu_1 &= [\delta + (1 - \delta)p_\ell(1)s_1] (1 - u)/K, \\ \sigma_1 &= \sqrt{[\mu_1(1 - \mu_1)]/N}\end{aligned}$$

Conditional on T_{i-1} , a worker separates from a firm in group $i = 2, 3, \dots, K$ with probability $\delta + (1 - \delta)p_\ell(1)s_i(y_\ell, T_{i-1})$. Since N is large, we can approximate the measure t_i of workers separating from firms in group i with a Normal distribution with mean $\mu_i(T_{i-1})$ and standard deviation $\sigma_i(T_{i-1})$, where

$$\begin{aligned}\mu_i(T_{i-1}) &= [\delta + (1 - \delta)p_\ell(1)s_i(y_\ell, T_{i-1})] (1 - u)/K, \\ \sigma_i(T_{i-1}) &= \sqrt{[\mu_i(T_{i-1})(1 - \mu_i(T_{i-1}))]/N}\end{aligned}$$

We find it convenient to define μ_- as δ , and μ_+ as $\delta + (1 - \delta)p_\ell(1)$.

Lemma 6: For $N \rightarrow \infty$, there is an equilibrium in which firms in groups 2 through K fire their non-performing workers with probability 0 if $T_1 < T_1^*$, and with probability 1 if $T_1 > T_1^*$, where T_1^* is such that

$$\psi = \beta(p_h(1) - p_h(0))(1 - P_1(T_1^*))V(E[\hat{u}|T_K = \mu_1 + (K - 1)\mu_+]) \quad (23)$$

Proof: Let T_1^* be given as in (23) and let $T_i^* = \mu_1 + (i - 1)(\mu_+ - \mu_-)/2$ for $i = 2, 3, \dots, K - 1$. Given the thresholds $\{T_1^*, T_2^*, \dots, T_{K-1}^*\}$, we can compute the unconditional probability distribution of the random variable T_i . For $N \rightarrow \infty$, T_2 is approximately equal to $\mu_1 + \mu_-$ if $T_1 < T_1^*$, and it is approximately equal to $\mu_1 + \mu_+$ if $T_1 > T_1^*$. Since $\mu_1 + \mu_- < T_2^*$ and $\mu_1 + \mu_+ > T_2^*$, T_3 is approximately equal to $\mu_1 + 2\mu_-$ if $T_1 < T_1^*$, and T_3 is approximately equal to $\mu_1 + 2\mu_+$ if $T_1 > T_1^*$. Similarly, for $i = 4, 5, \dots, K$, T_i is approximately equal to $\mu_1 + (i - 1)\mu_-$ if $T_1 < T_1^*$, and it is approximately equal to $\mu_1 + (i - 1)\mu_+$ if $T_1 > T_1^*$. Hence, if $T_1 < T_1^*$, firms in groups 2 through K fire their non-performing workers with probability 0 and the total measure of workers separating from firms is $T_K = \mu_1 + (K - 1)\mu_-$. If $T_1 > T_1^*$, firms in groups 2 through K fire their non-performing workers with probability 1 and the total measure of workers separating from firms is $T_k = \mu_1 + (K - 1)\mu_+$.

The above observations imply that benefit from exerting effort for a worker employed at a firm in group $i = 2, 3, \dots, K$ is given by

$$\beta(p_h(1) - p_h(0))(1 - P_1(T_1^*))V(E[\hat{u}|T_K = \mu_1 + (K - 1)\mu_+]). \quad (24)$$

The benefit in (24) is equal to the cost ψ of exerting effort given the choice of T_1^* . Thus, the incentive compatibility for workers employed by firms in groups $i = 2, 3, \dots, K$ is satisfied. The incentive compatibility constraint for workers employed by firms in group 1 is satisfied given the choice of s_1 . ■

Notice that the equilibrium described in Lemma 6 converges to the Perfect Coordination Equilibrium as the number of groups K goes to infinity. In this sense, the Perfect Coordination Equilibrium is robust to the modification of the environment. Next, we want to show that there are no other equilibria in the modified environment. To this aim, notice that, in any equilibrium, firms in group 2 fire their non-performing workers with probability 0 if $T_1 < T_1^*$, and they fire then with probability 1 if $T_1 > T_1^*$, where T_1^* is such that the worker's incentive compatibility constraint is satisfied, i.e.

$$\psi = \beta(p_h(1) - p_h(0))(1 - P_1(T_1^*))V(E[\hat{u}|T_1 > T_1^*]) \quad (25)$$

For $N \rightarrow \infty$, $T(2)$ is approximately equal to $\mu(1) + \mu_{\text{lo}}$ with probability $P(T^*(1))$ and, it is approximately equal to $\mu(1) + \mu_{\text{hi}}$ with probability $1 - P(T^*(1))$.

Now, suppose that the threshold T_2^* is such that $P_2(T_2^*) > P_1(T_1^*)$. Then, conditional on any T_2 approximately equal to $\mu_1 + \mu_-$, firms in group 3 fire their non-performing workers with probability 0. Conditional on T_2 being approximately equal to $\mu_1 + \mu_+$, firms in group 3 do not fire their non-performing workers with probability $(P_2(T_2^*) - P_1(T_1^*)) / (1 - P_1(T_1^*))$, and they fire their non-performing workers with probability $(1 - P_2(T_2^*)) / (1 - P_1(T_1^*))$. The incentive compatibility constraint for workers employed by firms in group 3 is given by

$$\psi = \beta(p_h(1) - p_h(0))(1 - P_2(T_2^*))V(E[\hat{u}|T_2 > T_2^*]) \quad (26)$$

However, the incentive compatibility constraints (25) and (26) cannot hold simultaneously and, hence, there cannot be an equilibrium in which $P_2(T_2^*) > P_1(T_1^*)$. To see why this is the case, notice that $E[\hat{u}|T_1 > T_1^*]$ is equal to $E[\hat{u}|T_2 > T_2^*] + E[\hat{u}|T_2 < T_2^*, T_1 > T_1^*]$ and $1 - P_1(T_1^*) > 1 - P_2(T_2^*)$. Therefore, the right-hand side of (25) is strictly greater than the right-hand side of (26). Following a similar argument, we can rule also out equilibria in which $P_2(T_2^*) < P_1(T_1^*)$. Hence, in any equilibrium, $P_2(T_2^*) = P_1(T_1^*)$, which implies that firms in group 3 fire their non-performing workers if and only if firms in group 2 do. Repeating the above argument for $i = 4, 5, \dots, K$, we can show that, in any equilibrium, firms in group i fire their non-performing workers if and only if firms in group $i - 1$ do. Therefore, the only equilibrium of the modified environment is the one described in Lemma 6.

We have thus completed the proof of the following theorem.

Theorem 3: (*Refinement*) For $K \rightarrow \infty$ and $N \rightarrow \infty$, the unique equilibrium of the environment with K groups of N firms firing sequentially is the Perfect Coordination Equilibrium.

The analysis of the environment where firms fire sequentially sheds more light on the nature of aggregate fluctuations in our model. The firms in the first group find it optimal to randomize on whether to fire or keep their non-performing workers. If the first group of firms fire enough workers, then all the other firms in the economy find it optimal to fire their non-performing workers. Otherwise, all the other firms in the economy find it optimal to keep their non-performing workers. That is, the equilibrium is such that the firing decision of the first group of firms leads to a “firing cascade”. However, in contrast to the models of herding of Banerjee (1992) and Bikhchandani et alii (1992), the cascades in our model do not take place because the actions of the first group of firms contain information about the realization of an exogenous aggregate shock, but because the actions of the first group of firms affect the incentive of subsequent firms from taking the same action. That is, cascades in our model start from the realization of idiosyncratic shocks, absent any aggregate uncertainty. In this sense, aggregate fluctuations in our model have a granular origin, as in Jovanovic (1987) and Gabaix (2011). However, unlike Jovanovic (1987) and Gabaix (2011), idiosyncratic shocks in our model propagate because of strategic interactions rather than because of the input-output structure.

5 Existence

In this section, we prove the existence of a perfect coordination equilibrium. The gist of the proof is showing that a perfect coordination equilibrium is the fixed point of a mapping that satisfies the conditions of Schauder’s theorem. In the process of proving the existence of the equilibrium, we will characterize several features of the equilibrium objects, i.e. the properties of the law of motion for unemployment, the wage, and the worker’s and firm’s gains from trade.

5.1 Preliminaries

Consider the function $\omega(u, i) = (i - 1)V^+(u) + iJ^+(u)$, with $u \in [0, 1]$ and $i \in \{0, 1\}$. The function $V^+(u)$ denotes the worker’s expected gains from employment at the end of the production stage given that the unemployment rate is u . We refer to $V^+(u)$ as the worker’s continuation gains from trade. Similarly, the function $J^+(u)$ denotes the firm’s expected

gains from employment at the end of the production stage given that the unemployment rate is u . We refer to $J^+(u)$ as the firm's continuation gains from trade. With a slight abuse of notation we will sometime describe the function ω as (V^+, J^+) .

Let Ω denote the set of bounded and continuous functions $\omega : [0, 1] \times \{0, 1\} \rightarrow \mathbb{R}$ such that: (i) for all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the difference $V^+(u_1) - V^+(u_0)$ is greater than $\underline{D}_{V^+}(u_1 - u_0)$ and smaller than $\overline{D}_{V^+}(u_1 - u_0)$; (ii) for all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the difference $J^+(u_1) - J^+(u_0)$ is greater than $\underline{D}_{J^+}(u_1 - u_0)$ and smaller than $\overline{D}_{J^+}(u_1 - u_0)$. In other words, Ω is the set of bounded continuous functions $\omega = (V^+, J^+)$ such that V^+ and J^+ have bounded "derivatives." The bounds on the derivatives are given numbers such that $\overline{D}_{V^+} > \underline{D}_{V^+} > 0$, $\overline{D}_{J^+} > 0 \geq \underline{D}_{J^+}$, and $\underline{D}_{V^+} > \overline{v}'(\overline{D}_{J^+} - \underline{D}_{J^+})$. Following Lemma A.1 in Menzio and Shi (2010), it is straightforward to verify that Ω is a non-empty, bounded, closed and convex subset of the space of bounded continuous functions on $[0, 1] \times \{0, 1\}$ with the sup norm.

In the remainder of this section, we show that the conditions for a perfect coordination equilibrium implicitly define an operator T that takes an arbitrary pair of continuation gains from trade $\omega = (V^+, J^+)$ in Ω and returns an updated pair of continuation value functions $T\omega = (V^{++}, J^{++})$ in Ω . Then we show that the operator T satisfies the conditions for Schauder's fixed point theorem and, hence, there exists an $\omega^* = (V^{+*}, J^{+*})$ such that $T\omega^* = \omega^*$. Finally, we show that the equilibrium objects associated with the fixed point ω^* constitute a perfect coordination equilibrium. Throughout the analysis, we use a few additional pieces of notation. In particular, we use \underline{V} , \overline{V} , \underline{J} and \overline{J} to denote lower and upper bounds on the worker's and firm's gains from trade constructed as in (31). Also, we use $\underline{\mu}_u$ and $\overline{\mu}_u$ to denote the minimum and the maximum of the (absolute value) of the partial derivative of the job-finding probability $\mu(J, u)$ with respect to $i = u$. That is, $\underline{\mu}_u$ denotes $\min |\partial \mu(J, u) / \partial u|$ for $(J, u) \in [\underline{J}, \overline{J}] \times [0, 1]$, and $\overline{\mu}_u$ denotes $\max |\partial \mu(J, u) / \partial u|$ for $(J, u) \in [\underline{J}, \overline{J}] \times [0, 1]$. Similarly, we use $\underline{\mu}_J$ and $\overline{\mu}_J$ to denote the minimum and the maximum on the (absolute value) of the partial derivative of $\mu(J, u)$ with respect to J .

5.2 Wage

Take an arbitrary pair of value functions $\omega = (V^+, J^+)$ in Ω . Then, for any unemployment rate u , the equilibrium wage function $w(u)$ takes a value w such that

$$v(w) - v(b) - \psi + V^+(u) = v'(w) [E[y|e = 1] - w + J^+(u)]. \quad (27)$$

For any unemployment rate u , there is a unique wage that satisfies (27). In fact, the right-hand side of (27) is strictly increasing in w , as the worker's gains from trade $v(w) - v(b) - \psi + V^+(u)$ are strictly increasing in w . The left-hand side of (27) strictly decreasing in w , as the worker's marginal utility of consumption $v'(w)$ and the firm's gains from trade $E[y|e = 1] - w + J^+(u)$ are both strictly decreasing in w . Therefore, there exists a unique wage w that solves (27) for any unemployment rate u .

The next lemma proves that the equilibrium wage function $w(u)$ is strictly decreasing in the unemployment rate u . There is a simple explanation behind this result. Given the bounds on the derivatives of the value functions V_+ and J_+ , an increase in the unemployment rate leads to a larger increase in the worker's gains from trade (the left-hand side of (27)) than in the product of the firm's gains from trade and the worker's marginal utility (the right-hand side of (27)) for any fixed wage w . Therefore, an increase in the unemployment rate requires the equilibrium wage to fall in order to satisfy equation (27). The next lemma also establishes that the "derivative" of the wage function is bounded.

Lemma 7: For all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the wage function $w(u)$ is such that

$$\underline{D}_w(u_1 - u_0) \leq w(u_0) - w(u_1) \leq \overline{D}_w(u_1 - u_0), \quad (28)$$

where the bounds \underline{D}_w and \overline{D}_w are defined as

$$\underline{D}_w = \frac{\underline{D}_{V_+}/\underline{v}' - \overline{D}_{J_+}}{2 + \overline{J}\underline{v}''/\underline{v}'}, \quad \overline{D}_w = \overline{D}_{V_+}/\underline{v}' - \underline{D}_{J_+}. \quad (29)$$

Proof: In Appendix B.

The next lemma proves that the equilibrium wage function w is continuous with respect to the value functions V^+ and J^+ . Specifically, consider two pairs of value functions $\omega_0 = (V_0^+, J_0^+)$ and $\omega_1 = (V_1^+, J_1^+)$ with ω_0, ω_1 in Ω . Denote as w_0 the wage function computed using ω_0 in (27). Similarly, denote as w_1 the wage function computed using ω_1 in (27). Then, if the distance between ω_0 and ω_1 goes to zero, so does the distance between w_0 and w_1 .

Lemma 8: For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $\|\omega_0 - \omega_1\| < \kappa$, we have

$$\|w_0 - w_1\| < \alpha_w \kappa, \quad \alpha_w = 1 + 1/\underline{v}'. \quad (30)$$

Proof: In Appendix B.

5.3 Gains from trade to worker and firm

Given the value functions V^+ and J^+ and the equilibrium wage function w computed using (27), the equilibrium gains from trade accruing to the firm and to the worker, J and V , are respectively given by

$$\begin{aligned} J(u) &= E[y|e = 1] - w(u) + J^+(u), \\ V(u) &= v(w(u)) - v(b) - \psi + V^+(u) \end{aligned} \tag{31}$$

The next lemma proves that the equilibrium gains from trade accruing to the firm are strictly increasing in the unemployment rate. This finding is intuitive. An increase in the unemployment rate leads to a larger decline in the equilibrium wage $w(u)$ than in the firm's continuation gains from trade $J^+(u)$. Similarly, the lemma proves that the equilibrium gains from trade accruing to the worker are strictly increasing in the unemployment rate. This finding is also intuitive. An increase in the unemployment rate increases the equilibrium gains from trade accruing to the firm and lowers the equilibrium wage. Since the equilibrium gains from trade accruing to the worker are proportional to the product between the worker's marginal utility and the firm's gains from trade, the result follows. Finally, the lemma establishes that the equilibrium gains from trade accruing to the firm and the worker have bounded derivatives.

Lemma 9: For all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the equilibrium gains from trade accruing to the firm, $J(u)$, and to the worker, $V(u)$, are such that

$$\begin{aligned} \underline{D}_J(u_1 - u_0) &\leq J(u_1) - J(u_0) \leq \overline{D}_J(u_1 - u_0), \\ \underline{D}_V(u_1 - u_0) &\leq V(u_1) - V(u_0) \leq \overline{D}_V(u_1 - u_0), \end{aligned} \tag{32}$$

where the bounds are defined as

$$\begin{aligned} \underline{D}_J &= \underline{D}_w + \underline{D}_{J^+} > 0, \quad \overline{D}_J = \overline{D}_w + \overline{D}_{J^+}, \\ \underline{D}_V &= \underline{v}'(\underline{D}_w + \underline{D}_{J^+}) > 0, \quad \overline{D}_V = \overline{v}'(\overline{D}_w + \overline{D}_{J^+}) + \overline{v}''\overline{D}_w\overline{J}. \end{aligned} \tag{33}$$

Proof: In Appendix C.

The next lemma proves that the firm's and worker's gains from trade are continuous with respect to the choice of the functions V^+ and J^+ . Specifically, consider two pairs of value functions $\omega_0 = (V_0^+, J_0^+)$ and $\omega_1 = (V_1^+, J_1^+)$ with ω_0, ω_1 in Ω . Denote as w_0 the equilibrium wage computed using ω_0 , and as J_0 and V_0 the equilibrium gains from trade computed using ω_0 and w_0 in (31). Similarly, denote as w_1 the equilibrium wage computed using ω_1 , and as J_1 and V_1 the equilibrium gains from trade computed using ω_1 and w_1 in (31). Then, if the distance between ω_0 and ω_1 goes to zero, so does the distance between J_0 and J_1 and between V_0 and V_1 .

Lemma 10: For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $\|\omega_0 - \omega_1\| < \kappa$, we have

$$\begin{aligned} \|J_0 - J_1\| &< \alpha_J \kappa, \quad \alpha_J = 1 + \alpha_w, \\ \|V_0 - V_1\| &< \alpha_V \kappa, \quad \alpha_V = 1 + \bar{v}' \alpha_w. \end{aligned} \quad (34)$$

Proof: In Appendix C.

5.4 Law of motion for unemployment

Given the value functions V^+ and J^+ , the equilibrium wage function, w , is given by (27) and the equilibrium gains from trade accruing to the firm, J , are given by (31). Given J , the equilibrium law of motion for unemployment, $h(u, \hat{z})$, is such that—for any current unemployment u and any realization of the sunspot \hat{z} —next period's unemployment takes on a value \hat{u} such that

$$\hat{u} = u - u\mu(J(\hat{u}), u) + (1 - u)[\delta + (1 - \delta)p_\ell(1)s(y_\ell, \hat{z})], \quad (35)$$

Since we are looking for a perfect coordination equilibrium, the firing probability $s(y_\ell, \hat{z})$ equals zero if $\hat{z} = Z_0$ and one if $\hat{z} = Z_2$.

Next period's unemployment is uniquely determined by (35). In fact, the left-hand side of (35) equals zero for $\hat{u} = 0$, it is strictly increasing in \hat{u} and it equals one for $\hat{u} = 1$. The right-hand side of (35) is strictly positive for $\hat{u} = 0$ and it is strictly decreasing in \hat{u} , as the worker's job finding probability μ is strictly increasing in the firm's gains from trade J , and J is strictly increasing in \hat{u} . Therefore, there exists one and only one \hat{u} that satisfies (35).

Next period's unemployment is strictly increasing in the current unemployment u . In fact, the left-hand side of (35) is independent of u . The right-hand side of (35) is strictly increasing in u , as its derivative with respect to u is greater than $(1 - \delta)p_h(1) - \mu$, which we assume to be strictly positive. Therefore, the \hat{u} that solves (35) is strictly increasing in u . Moreover, next period's unemployment is strictly higher if the realization of the sunspot \hat{z}' is B rather than G and, more generally, strictly increasing in the probability with which firms fire non-performing workers. To see this, it is sufficient to notice that the left-hand side of (35) is independent of the firing probability, while the right hand side is strictly increasing in it.

The next lemma proves that the equilibrium law of motion for unemployment $h(u, \hat{z}')$ is continuous in u and that its derivative is bounded.

Lemma 11: For all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the equilibrium law of motion for unemployment $h(u, \hat{z}')$ is such that

$$\underline{D}_h(u_1 - u_0) < h(u_1, \hat{z}) - h(u_0, \hat{z}) \leq \overline{D}_h(u_1 - u_0), \quad (36)$$

where the bounds \underline{D}_h and \overline{D}_h are defined as

$$\underline{D}_h = 0, \quad \overline{D}_h = 1 - \delta + \bar{\mu}_u. \quad (37)$$

Proof: In Appendix D.

Next, we prove that the equilibrium law of motion for unemployment, $h(u, \hat{z})$, is continuous with respect to V^+ and J^+ . Formally, we consider two pairs of value functions $\omega_0 = (V_0^+, J_0^+)$ and $\omega_1 = (V_1^+, J_1^+)$ with ω_0, ω_1 in Ω . We denote as J_0 the firm's equilibrium gains from trade computed using ω_0 in (31), and as h_0 the equilibrium law of motion for unemployment computed using J_0 in (35). Similarly, we denote as J_1 the firm's equilibrium gains from trade computed using ω_1 in (31), and as h_1 the equilibrium law of motion for unemployment computed using J_1 in (35). Then, we show that, if the distance between ω_0 and ω_1 goes to zero, so does the distance between h_0 and h_1 .

Lemma 12: For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $\|\omega_0 - \omega_1\| < \kappa$, we have

$$\|h_0 - h_1\| < \alpha_h \kappa, \quad \alpha_h = \bar{\mu}_J \alpha_J. \quad (38)$$

Proof: In Appendix D.

5.5 Updated value functions

Given the value functions V^+ and J^+ , the equilibrium wage function, w , is given by (27). The equilibrium gains from trade accruing to the firm and to the worker, J and V , are given by (31). The equilibrium law of motion for unemployment, h , is given by (35). In turn, given J , V and h , we can construct an update, $V^{+'}$, for the continuation gains from trade accruing to the worker as

$$V^{+'}(u) = \beta E_{\hat{z}'} \{[(1 - \delta)(1 - p_\ell(1)s(y_\ell, \hat{z})) - \mu(J(h(u, \hat{z})), u)]V(h(u, \hat{z}))\} \quad (39)$$

We can also construct an update, $J^{+'}$, for the ex-wage gains from trade accruing to the firm as

$$J^{+'}(u) = \beta E_{\hat{z}'} [(1 - \delta)(1 - p_\ell(1)s(y_\ell, \hat{z}))J(h(u, \hat{z}))] \quad (40)$$

As we are looking for a perfect coordination equilibrium, the firing probabilities $s(y_\ell, Z_2)$ and $s(y_\ell, Z_1)$ in (39) and (40) are set respectively equal to 1 and 0. Similarly, as we are looking for a perfect coordination equilibrium, the probability π_2 that the realization of the sunspot \hat{z}' is Z_2 is set equal to

$$\pi_2 = \frac{\psi}{\beta(p_h(1) - p_h(0))(1 - \delta)V(h(u, Z_2))}. \quad (41)$$

Notice that the probability that the realization of the sunspot is Z_2 is endogenous as it needs to satisfy the worker's incentive compatibility constraint. In particular, the higher is the current unemployment rate u , the higher is next period's unemployment rate $h(u, Z_2)$, the higher are next period's gains from trade accruing to the worker $V(h(u, Z_2))$ and, ultimately, the lower is the probability that the realization of the sunspot is Z_2 .

In the next lemma, we prove that the update of worker's continuation value, $V^{+'}$, is strictly increasing in the unemployment rate. This finding is intuitive. An increase in the unemployment rate causes congestion in the labor market and, for this reason, leads to a decline in the worker's job finding rate and in the worker's value of unemployment. In turn, the decline in the worker's value of unemployment tends to increase the worker's gains from trade (which is only partly mitigated by a decline in the equilibrium wage).

Lemma 13: For all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the updated continuation gains from trade to the worker, $V^{+'}$, and the firm, $J^{+'}$, are such that

$$\begin{aligned} \underline{D}'_{V+}(u_1 - u_0) &< V^{+'}(u_1) - V^{+'}(u_0) \leq \overline{D}'_{V+}(u_1 - u_0), \\ \underline{D}'_{J+}(u_1 - u_0) &< J^{+'}(u_1) - J^{+'}(u_0) \leq \overline{D}'_{J+}(u_1 - u_0). \end{aligned} \quad (42)$$

where the bounds \underline{D}'_{V+} and \overline{D}'_{V+} are defined as

$$\begin{aligned} \underline{D}'_{V+} &= \beta \underline{V}(\underline{\mu}_u - \bar{\mu}_J \bar{D}_J \bar{D}_h) - \frac{2\psi \bar{D}_V \bar{D}_h \bar{V}}{(p_h(1) - p_h(0))(1 - \delta)\underline{V}^2}, \\ \overline{D}'_{V+} &= \beta [\bar{V} \bar{\mu}_u + (1 - \delta) \bar{D}_V \bar{D}_h], \end{aligned} \quad (43)$$

and the bounds \underline{D}'_{J+} and \overline{D}'_{J+} are defined as

$$\begin{aligned} \underline{D}'_{J+} &= -\frac{2c \bar{D}_V \bar{D}_h \bar{J}}{(p_h(1) - p_h(0))(1 - \delta)\underline{V}^2}, \\ \overline{D}'_{J+} &= \beta(1 - \delta) \bar{D}_J \bar{D}_h \end{aligned} \quad (44)$$

Proof: In Appendix E.

The next lemma shows that, under some parametric conditions, there exists a fixed point for the derivatives of the continuation gains from trade, i.e. \underline{D}_{V+} , \overline{D}_{V+} , \underline{D}_{J+} , \overline{D}_{J+} such that $\underline{D}'_{V+} = \underline{D}_{V+}$, $\overline{D}'_{V+} = \overline{D}_{V+}$, $\underline{D}'_{J+} = \underline{D}_{J+}$, and $\overline{D}'_{J+} = \overline{D}_{J+}$, such that the conditions $\overline{D}_{V+} > \underline{D}_{V+} > 0$, $\overline{D}_{J+} > 0 \geq \underline{D}_{J+}$ and $\underline{D}_{V+} > \overline{v}'(\overline{D}_{J+} - \underline{D}_{J+})$ are satisfied. The key parametric condition requires the negative effect of unemployment on the job-finding probability to be sufficiently strong compared to the positive effect of the firm's gains from trade on the job-finding probability. Intuitively, the condition guarantees that the next period's job-finding rate is decreasing in the current period's unemployment rate. In turn, when the discount factor β and the disutility of effort c are small enough, this guarantees that the derivative of the worker's continuation gains from trade is positive and sufficiently larger than the derivative of the firm's continuation gains from trade.

Lemma 14: Assume $\underline{\mu}_u - \overline{\mu}_J(1 - \delta + \overline{\mu}_u) > 0$. Then there exist $\beta^* > 0$ and $c^* > 0$ such that, if $\beta \in (0, \beta^*)$ and $c \in (0, c^*)$, there are bounds \underline{D}_{V+} , \overline{D}_{V+} , \underline{D}_{J+} , \overline{D}_{J+} such that: (i) $\underline{D}'_{V+} = \underline{D}_{V+}$, $\overline{D}'_{V+} = \overline{D}_{V+}$, $\underline{D}'_{J+} = \underline{D}_{J+}$, and $\overline{D}'_{J+} = \overline{D}_{J+}$; (ii) $\overline{D}_{V+} > \underline{D}_{V+} > 0$, $\overline{D}_{J+} > 0 \geq \underline{D}_{J+}$ and $\underline{D}_{V+} > \overline{v}'(\overline{D}_{J+} - \underline{D}_{J+})$.

Proof: In Appendix E.

In the last lemma, we prove that the updated continuation value functions, $V^{+'}$ and $J^{+'}$, are continuous with respect to the original value functions V^+ and J^+ . Specifically, consider two arbitrary functions $\omega_0 = (V_0^+, J_0^+)$ and $\omega_1 = (V_1^+, J_1^+)$ with ω_0, ω_1 in Ω . Denote as $V_0^{+'}$ and $J_0^{+'}$ the continuation value functions computed using V_0^+ , J_0^+ and g_0 in (39) and (40). Similarly, denote as $V_1^{+'}$ and $J_1^{+'}$ the continuation value functions computed using V_1^+ , J_1^+ and g_1 in (39) and (40). Then, if the distance between ω_0 and ω_1 goes to zero, so does the distance between V_0^+ and V_1^+ and between J_0^+ and J_1^+ .

Lemma 15: For any $\kappa > 0$ and any ω_0, ω_1 in Ω such that $\|\omega_0 - \omega_1\| < \kappa$, we have $\|V_0^{+'} - V_1^{+'}\| < \alpha_{V_+'} \kappa$ and $\|J_0^{+'} - J_1^{+'}\| < \alpha_{J_+'} \kappa$, where

$$\begin{aligned} \alpha_{V_+'} &= \left[\frac{2c\overline{V}}{(p_h(1) - p_h(0))(1 - \delta)\underline{V}^2} + 1 \right] (\alpha_V + \overline{D}_V \alpha_h) + \overline{V} \overline{\mu}_J (\alpha_J + \overline{D}_J \alpha_h), \\ \alpha_{J_+'} &= \frac{2c\overline{J} (\alpha_V + \overline{D}_V \alpha_h)}{(p_h(1) - p_h(0))(1 - \delta)\underline{V}^2} + (\alpha_J + \overline{D}_J \alpha_h). \end{aligned} \quad (45)$$

Proof: In Appendix E.

5.6 Existence

In the previous subsections, we have taken a pair of continuation gains from trade V^+ and J^+ and, using the conditions for a perfect coordination equilibrium, we have constructed an

updated pair of continuation gains from trade $V^{+'}$ and $J^{+'}$. We denote as T the operator that takes $\omega(u, i) = (1 - i)V^+(u) + iJ^+(u)$ and returns $\omega'(u, i) = (1 - i)V^{+'}(u) + iJ^{+'}(u)$.

The operator T has three key properties. First, the operator T maps functions that belong to the set Ω into functions that also belong to the set Ω . In fact, for any $\omega = (V^+, J^+) \in \Omega$, $\omega' = (V^{+'}, J^{+'})$ is bounded and continuous and, as established in Lemma 12, it is such that: (i) for all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the difference $V^{+'}(u_1) - V^{+'}(u_0)$ is greater than $\underline{D}_{V_+}(u_1 - u_0)$ and smaller than $\overline{D}_{V_+}(u_1 - u_0)$; (ii) for all $u_0, u_1 \in [0, 1]$ with $u_0 < u_1$, the difference $J^{+'}(u_1) - J^{+'}(u_0)$ is greater than $\underline{D}_{J_+}(u_1 - u_0)$ and smaller than $\overline{D}_{J_+}(u_1 - u_0)$. Second, the operator T is continuous, as established in Lemma 13. Third, the family of functions $T(\Omega)$ is equicontinuous. To see that this is the case, let $\|\cdot\|_E$ denote the standard norm on the Euclidean space $[0, 1] \times \{0, 1\}$. For any $\epsilon > 0$, let $\kappa_\epsilon = \min\{(\max\{\overline{D}_{V_+}, \overline{D}_{J_+}, |\underline{D}_{J_+}|\})^{-1}, 1\}$. Then, for all $(u_0, i_0), (u_1, i_1) \in [0, 1] \times \{0, 1\}$ such that $\|(u_0, i_0) - (u_1, i_1)\|_E < \kappa_\epsilon$, we have

$$|(T\omega)(u_0, i_0) - (T\omega)(u_1, i_1)| < \epsilon, \text{ for all } \omega \in \Omega. \quad (46)$$

Since $T : \Omega \longrightarrow \Omega$, T is continuous and $T(\Omega)$ is equicontinuous, the operator T satisfies the conditions of Schauder's fixed point theorem (see Theorem 17.4 in Stokey, Lucas and Prescott 1989). Therefore there exists a $\omega^* = (V^{+*}, J^{+*}) \in \Omega$ such that $T\omega^* = \omega^*$.

Now, we compute the equilibrium wage function w^* using V^{+*} and J^{+*} in (27). We compute the equilibrium gains from trade V^* and J^* using w^* , V^{+*} and J^{+*} in (31). We compute the equilibrium law of motion for unemployment g^* using J^* in (35). Finally, we construct the equilibrium employment contract $x^*(u)$ as the effort $e^* = 1$, the wage function w^* and the firing probabilities $s^*(y_h, \hat{z}) = 0$, $s^*(y_\ell, Z_0) = 0$ and $s^*(y_h, Z_2) = 1$.

The tuple (V^*, J^*, x^*, h^*) constitute an equilibrium. Indeed, if we substitute (39) and (40) into (31), we find that the gains from trade accruing to the worker, V^* , and to the firm, J^* , satisfy condition (i) in the definition of equilibrium. The law of motion for unemployment, g^* , satisfies condition (iii) in the definition of equilibrium. The employment contract, x^* , prescribes a wage w^* that satisfies (??). It prescribes a firing probability $s^*(y_h, \hat{z})$ that satisfies (12). It prescribes a firing probability $s^*(y_\ell, \hat{z})$ that satisfies (13). To see why this is the case, notice that the gains from trade accruing to the worker relative to those accruing to the firm are such that $v'(w^*(h^*(u, Z_2))) > v'(w^*(h^*(u, Z_0)))$ since the wage function w^* is strictly decreasing in unemployment and $h^*(u, Z_2)$ is strictly greater than $h^*(u, Z_0)$. Moreover, the probability that the realization of the sunspot is Z_2 is such that

the worker's incentive compatibility constraint holds with equality. Therefore, by Theorem 1, the employment contract x^* is optimal and satisfies condition (ii) in the definition of equilibrium. Moreover, the tuple (V^*, J^*, x^*, h^*) is a perfect coordination equilibrium by construction.

We have established the following result.

Theorem 4: (*Existence*) Assume $\underline{\mu}_u - \bar{\mu}_J(1 - \delta + \bar{\mu}_u) > 0$. A perfect coordination equilibrium exists for all (β, ψ) such that $\beta \in (0, \beta^*)$ and $\psi \in (0, \psi^*)$.

Two comments about Theorem 4 are in order. First, notice that, although the parametric conditions in Theorem 4 are sufficient for the existence of a perfect coordination equilibrium, they are not necessary. Indeed, the version of the model that we calibrate in the next section does not always meet these conditions and, yet, it does admit a perfect coordination equilibrium. Second, notice that Theorem 4 does not rule out the existence of equilibria that do not feature perfect coordination in the firing decision of different firms. Indeed, it is easy to show that under the same conditions as in Theorem 4, there exists a no coordination equilibrium. However, as we argued in Section 4, this type of equilibrium is not robust to the introduction of small shocks to fundamentals.

6 Quantifying Agency Business Cycles

In this section, we measure the aggregate labor market fluctuations implied by our theory, and compare them with the aggregate labor market fluctuations observed in the US economy. We refer to the aggregate fluctuations generated by the theory as Agency Business Cycles, or ABC. Using a calibrated version of the model, we construct time-series for the unemployment rate, the rate at which unemployed workers move into employment (the UE rate) and the rate at which employed workers move into employment (the EU rate). First, we find that the model generates fluctuations in unemployment, the UE and the EU rates that are approximately half as large as those observed in the US labor market. Moreover, the fluctuations generated by the model are uncorrelated with labor productivity, just like they have been in the US labor market since 1984. Second, we test the prediction of our model with respect to the morphology of labor market fluctuations. In our model, a recession starts with an increase in the EU rate, which leads to an increase in the unemployment rate which, through decreasing returns to scale in matching, lowers the UE rate. This causal chain manifest itself in the EU rate leading the unemployment rate and the UE rate. We find the same pattern of leads and lags in the US data. Third, we test the prediction of our

model with respect to the stochastic process for recessions. In our model, a recession is less likely the higher is the unemployment rate. Consistently with the model, we find that, in the US labor market, the unemployment rate in a given quarter has a negative effect on the probability of observing the start of a recession in the following quarter.

Lastly, we test the prediction of our model with respect to the cyclicalities of the value of labor in the market relative to its value at home. In our model, a recession is a period when the value of time in the market relative to its value at home is higher than normal. Indeed, in our model, an unemployment rate higher than normal is uncorrelated with the value of the worker's output in the market and it lowers the worker's value of staying at home looking for jobs. In contrast, in models where business cycles are driven by shocks to aggregate productivity, as in the Real Business Cycle model of Kydland and Prescott (1982) or in Mortensen and Pissarides (1994), a recession is a period when the value of time in the market relative to its value at home is lower than normal. Indeed, in one of these models, an unemployment rate higher than normal is caused by a low value of the worker's output in the market. We find that, in the data, the net value of employment to a worker is clearly countercyclical, which is consistent with our model and clearly inconsistent with productivity-driven theories of business cycles. We also find that the value of a worker to a firm is countercyclical, which is also consistent with our model and inconsistent with productivity-driven theories of business cycles where wages are sticky.

6.1 Calibration

We start by calibrating the primitives of the model. Preferences are described by the worker's periodical utility function, $v(c) - \psi e$, and by the discount factor, β . Market production is described by the realizations of output, y_h and y_ℓ , by the probability that output is high given the worker's effort, $p_h(1)$ and $p_h(0)$, and by the exogenous job destruction probability δ . Home production is described by the output of an unemployed worker, b . The search and matching process is described by the vacancy cost, k , and the matching function, $M(u, v)$. We specialize the utility function for consumption to be of the form $v(c) = c^{1-\sigma}/(1-\sigma)$, where σ is the coefficient of relative risk aversion. We specialize the matching function to be of the form $M(u, v) = A(u)m(u, v)$, where $m(u, v) = uv(u^\xi + v^\xi)^{-1/\xi}$ is a constant returns to scale matching function with an elasticity of substitution ξ , and $A(u) = \exp(-\rho u)$ is a matching efficiency function with a semi-elasticity with respect to unemployment of $-\rho$.

We calibrate the parameters of the model to match some key statistics of the US labor market between 1951 and 2014, such as the average unemployment rate, the average UE rate

(i.e. the rate at which unemployed workers move into employment), and the average EU rate (i.e. the rate at which employed workers move into unemployment). We measure the US unemployment rate as the CPS civilian unemployment rate. We measure the UE and the EU rates using the civilian unemployment and short-term unemployment rates from the CPS, following the same methodology as in Shimer (2005). We measure labor productivity as output per worker in the non-farm sector.

We calibrate the basic parameters of the model as is now standard in the literature. We choose the model period to be one month. We set the discount factor, β , so that the annual real interest rate, $(1/\beta)^{1/12} - 1$, is 5 percent. We choose the vacancy cost, k , and the exogenous job destruction probability, δ , so that the model matches the average UE and EU rates in the US economy (respectively, 44% and 2.6%). We normalize the average value of market production, $p_h(1)y_h + (1 - p_h(1))y_\ell$, to 1. We choose the value of home production, b , to be 70% of the average value of market production, which Hall and Milgrom (2010) argue is a reasonable estimate for the US economy.

We calibrate the parameters of the model that determine the extent of the agency problem as follows. The probability that the realization of output is y_h given that the worker exerts effort, $p_h(1)$, affects the number of non-performing workers and, hence, the magnitude of firing bursts. The disutility of effort, ψ , affects the frequency at which firms need to fire non-performing workers, hence, the frequency of firing bursts. Therefore, we choose y_h so that the model generates the same standard deviation in the cyclical component of the EU rate as in the US economy (9.85%). We choose ψ so that, on average, firms coordinate on firing their non-performing workers once every 50 months. The parameters y_h and y_ℓ and $p_h(0)$ cannot be uniquely pinned down. Given that average output is 1, the realizations of output y_h and y_ℓ do not affect the equilibrium, as long as it is optimal for firms to require effort from their workers. Similarly, the probability $p_h(0)$ only affects the equilibrium through the ratio $\psi/(p_h(1) - p_h(0))$. Therefore, we choose some arbitrary values for y_h , y_ℓ and $p_h(0)$ such that firms find it optimal to require effort.

Finally, we need to choose values for the parameters in the utility and the matching functions. We set the coefficient σ of relative risk aversion in the utility function v to 1. This is a standard value from micro-estimates of risk aversion. We set the elasticity ξ of substitution between unemployment and vacancy in the matching function m to 1.24. This is the value estimated by Menzio and Shi (2011).⁹ We set the parameter ρ in the matching

⁹Correctly estimating a matching function requires taking into account the fact that unemployed and employed workers all search for vacancies to some degree. Using a model of search off and on the job,

efficiency function A to 6. This value implies a relationship between the unemployment rate and the UE rate that closely matches the one observed in the US economy.

TABLE 1: CALIBRATED PARAMETERS

| | Description | Value |
|----------|------------------------------------|-------|
| β | discount factor | .9967 |
| σ | worker's relative risk aversion | 1.00 |
| y_h | high output | 1.03 |
| y_ℓ | low output | .000 |
| $p_h(1)$ | probability of y_h given $e = 1$ | .967 |
| $p_h(0)$ | probability of y_h given $e = 0$ | .467 |
| b | UI benefit/value of leisure | .700 |
| ψ | disutility of effort | .007 |
| δ | exogenous job destruction | .025 |
| k | vacancy cost | .257 |
| ξ | elasticity of sub. btw u and v | 1.24 |
| ρ | semi-elasticity of A wrt u | 6.00 |

6.2 Magnitude and Properties of ABC

Table 1 reports the calibrated value of the parameters of the model. Given these values, we simulate the model and create monthly time-series for the unemployment rate, the UE rate, the EU rate and other labor market variables. For each variable, we construct quarterly time-series by taking 3-month averages. We then compute the cyclical component of each variable as the percentage deviation of its quarterly value from a Hodrick-Prescott trend constructed using a smoothing parameter of 10^5 . We use the same procedure to construct the cyclical component of labor market variables in the US data.

In Panel (a) of Figure 3, we present a sample of the time-series generated by the model for the cyclical component of the unemployment rate, the UE rate and the EU rate. The figure clearly illustrates the basic features of ABC. When firms coordinate on firing their non-performing workers, the EU rate spikes up. As a result of the increased flow of workers from employment into unemployment, the unemployment rate increases. In turn, the increase in the unemployment rate leads to a decrease in the efficiency of matching and, hence, to a

Menzio and Shi (2011) estimate the elasticity of substitution between searching workers and vacant jobs in the matching function to be 1.24.

decline in the UE rate. Even though the increase in the EU rate dissipates after just one quarter, the increase in the unemployment rate dies off slowly because of the decline in the UE rate. The decline in the unemployment rate continues until a new episode of coordinated firing starts a new cycle.

Panel (b) in Figure 3 presents the cyclical component of the unemployment rate, the UE rate and the EU rate in the US economy over the period 1990-2014. Qualitatively, the fluctuations in the US labor market resemble those implied by our theory. For instance, the Great Recession of 2008 features a sharp increase in the EU rate, followed by an increase in the unemployment rate and a decline in the UE rate. Moreover, while the increase in the EU rate dies off quickly, the increase in the unemployment rate and the decline in the UE rate dissipate much more slowly.

We now carry out a more formal comparison between ABC and the cyclical fluctuations in the US labor market. Table 2 reports some statistics about the unemployment rate, the UE rate, the EU rate and the labor productivity generated by the model and about the same variables in the data. First, Table 2 shows that ABC can account for a significant fraction of the volatility of the US labor market. Specifically, the standard deviation of unemployment in the model is 55% of what we observe in the data. Similarly, the standard deviation of the UE rate in the model is 33% of its empirical counterpart, and the standard deviation of the EU rate in the model is the same as in the data. The reader should keep in mind that the model is calibrated under the identifying assumption that the volatility in the EU rate observed in the data is entirely explained by our theory. Hence, the reader should interpret the numbers above as an upper bound on the magnitude of ABC.¹⁰

¹⁰Nonetheless, the finding that ABC can create large fluctuations in the unemployment, the UE and the EU rates is important. Indeed, as shown by Shimer (2005), the textbook search-theoretic model of the labor market with productivity shocks explains less than 10% of the empirical volatility of unemployment. Hall (2005), Menzio (2005), Hagedorn and Manovskii (2008), Kennan (2010), Menzio and Shi (2011) develop search-theoretic models in which productivity shocks generate larger fluctuations in unemployment. However, these models counterfactually predict a perfect negative correlation between unemployment and labor productivity.

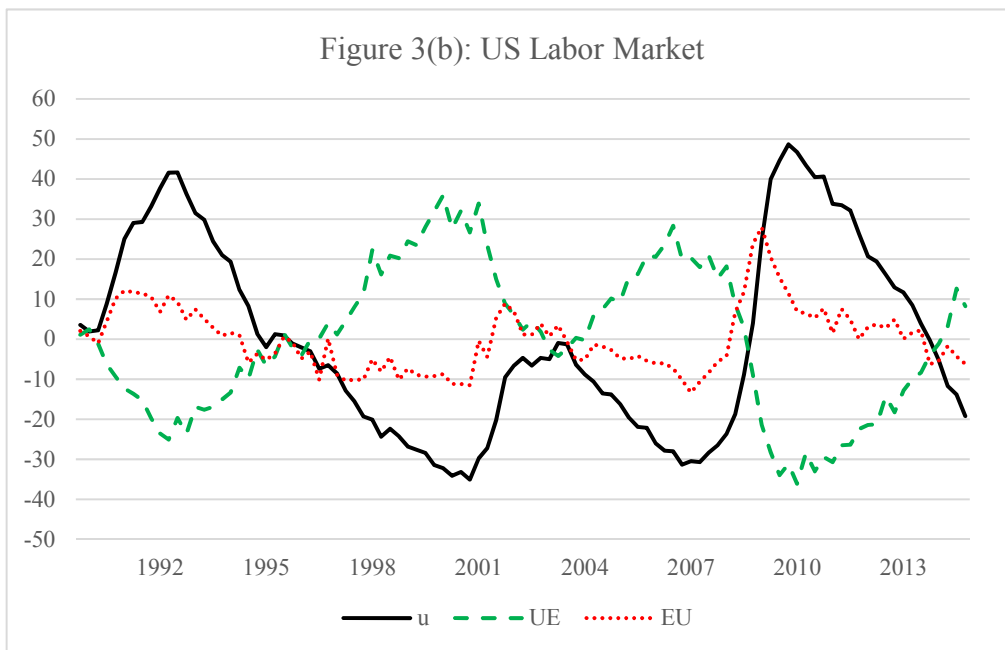
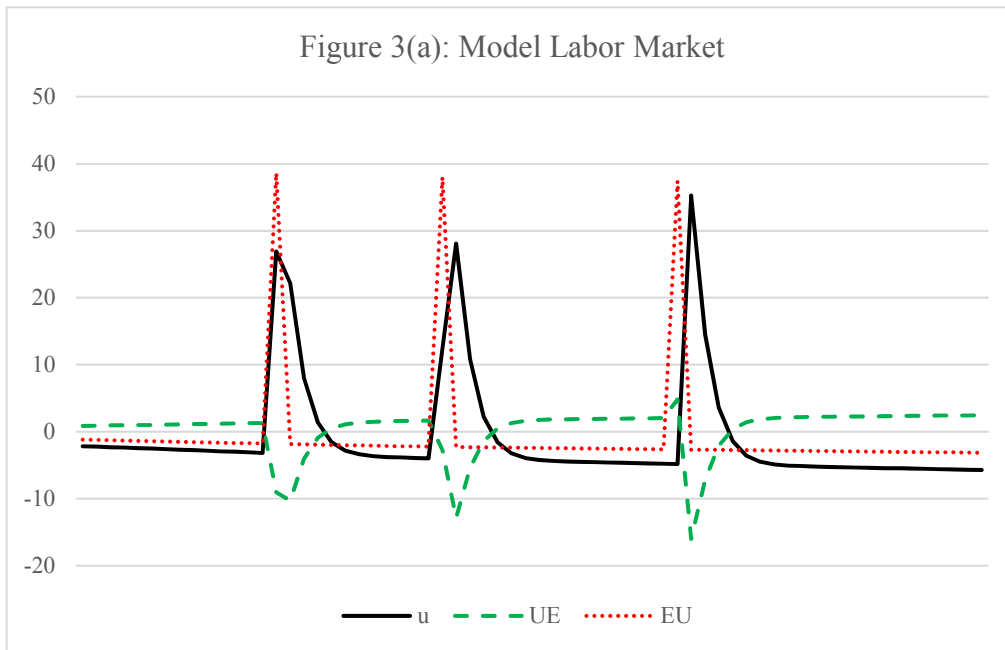


TABLE 2: AGENCY BUSINESS CYCLES

| | | u rate | UE rate | EU rate | APL |
|-----------------|--------------|--------|---------|---------|------|
| Model | std | 9.34 | 4.09 | 9.11 | 0 |
| | cor. wrt u | 1 | -.98 | .32 | - |
| Data: 1951-2014 | std | 16.9 | 12.9 | 9.7 | 1.98 |
| | cor wrt u | 1 | -.94 | .80 | -.37 |
| Data: 1984-2014 | std | 17.3 | 13.8 | 6.91 | 1.38 |
| | cor wrt u | 1 | -.96 | .70 | .09 |

Second, Table 2 shows that ABC feature the same pattern of comovement between the unemployment rate, the UE rate and the EU rate as in the data. In particular, in the model as in the data, the unemployment rate and the UE rate are negatively correlated, while the unemployment rate and the EU rate are positively correlated. However, in the model unemployment and vacancies are mildly positively correlated, while in the data these two variables are almost perfectly negatively correlated. This discrepancy between model and data is an artifact of the simplifying and counterfactual assumption that workers search the labor market only when they are unemployed. Indeed, under the assumption of off-the-job search, an increase in unemployment causes an increase in the number of workers searching the labor market which, in turn, gives firms an incentive to create more vacancies. Under the more realistic assumption of on and off-the-job search, an increase in unemployment does not cause an increase in the number of workers searching the labor market and, hence, does not give firms a clear incentive to create more vacancies.

Finally, Table 2 shows that the model generates relatively large fluctuations in unemployment that are uncorrelated with fluctuations in labor productivity. This is an important feature of the model. The empirical correlation between unemployment and labor productivity—which was significantly negative for the period 1951-1984—has become basically zero for the period 1984-2014. Therefore, over the period 1951-1984, fluctuations in labor productivity may have driven the cyclical movements of the US labor market. In contrast, over the period 1984-2014, fluctuations in labor productivity seem an unlikely driver of cycles in the US labor market. Our theory provides an explanation for the recent lack of comovement between labor productivity and unemployment by identifying a novel, non-technological source of aggregate fluctuations.¹¹

¹¹Gali and van Rens (2014) document in detail the decline in the negative correlation between unemployment and labor productivity. Gali and van Rens (2014), Kaplan and Menzio (2015) and Beaudry, Galizia and Portier (2015) advance theories, alternative to ours, in which labor productivity does not correlate negatively with unemployment.

In Figure 4, we display the correlation between unemployment in quarter t and other labor market variables in quarter $t + x$, with x going from -5 to $+5$. Panel (a) displays the correlations in the model-generated data. Panel (b) displays the same correlations in the US data. The figure clearly shows that ABC feature the same pattern of leads and lags as in the data. In the model as in the data, the UE rate is contemporaneous with unemployment—in the sense that the correlation between unemployment in quarter t and the UE rate in quarter $t + x$ is highest for $x = 0$ —and the EU rate leads unemployment by a quarter—in the sense that the absolute value of the correlation between unemployment in quarter t and the EU rate in quarter $t + x$ is highest for $x = -1$. Moreover, in the model as in the data, the correlation between current unemployment and the future EU rate dies off much more rapidly than the correlation between current unemployment and the future UE rate. The main difference between the correlation functions in the model and in the data is that, in the model, all correlations die off more quickly than in the data. This shortcoming of the model is due to the fact that the driving force behind ABC (i.e. the coordinated firing of non-performing workers) has no persistence.¹²

Fujita and Ramey (2010) were among the first to point out that the EU rate leads the UE and the unemployment rates. As shown in Figure 4, our theory provides a simple explanation for this pattern. Moreover, Fujita and Ramey (2010) used a VAR model to show that, once one takes into account the negative correlation between the current EU rate and the future UE rate, fluctuations in the EU rate account for approximately 60% of the overall volatility of unemployment. Our theory can explain this finding very well. Indeed, in our theory, shocks to the EU rate lead to subsequent declines in matching efficiency and, in turn, in the UE rate. Hence, according to our theory, the negative correlation between the current EU rate and the future UE rate reflects a causal link.¹³ Furthermore, as shown in Table 2, the shocks to the EU rate in our theory account for close to 60% of the empirical volatility of unemployment.

In Figure 5, we plot the average UE rate and the average EU rate conditional on the value of the contemporaneous unemployment rate. We plot these averages separately for

¹²We believe we could create persistence in firings by assuming that firm and workers observe output in a staggered fashion—let’s say half in odd periods and half in even periods.

¹³Ahn and Hamilton (2015) estimate a statistical model of flows in and out of unemployment in which workers differ by their job-finding probability. They find that the increase in the EU rate at the onset of the Great Recession contained a disproportionate fraction of low job-finding probability workers and, hence, caused the subsequent decline in the UE rate. They also find that the low job-finding probability workers are typically those fired from their job.

Figure 4(a): Leads and Lags in the Model

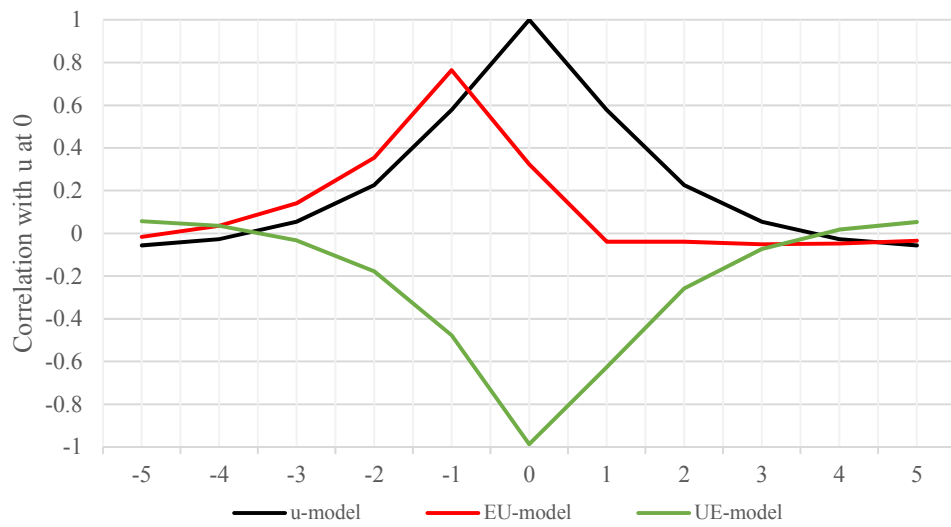
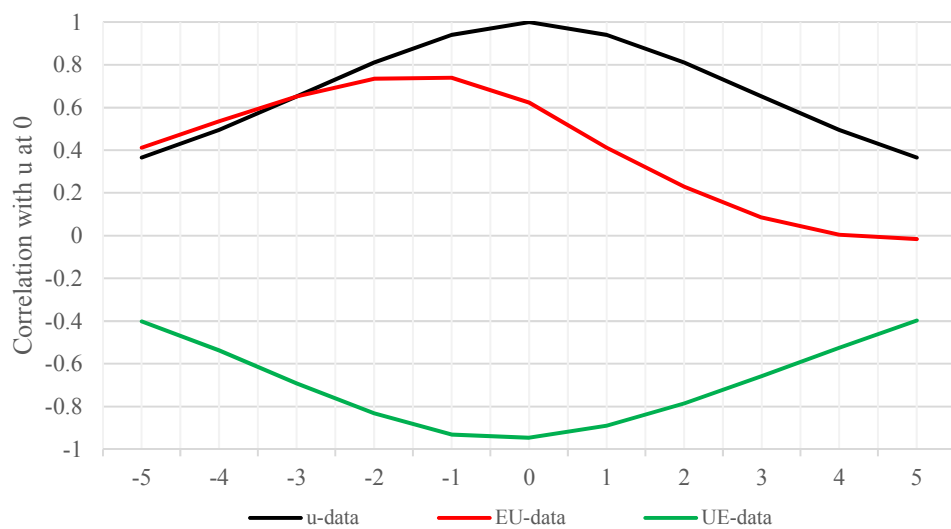


Figure 4(b): Leads and Lags in the Data



recessions—defined as periods when the unemployment rate is increasing—and recoveries—defined as periods when the unemployment rate is falling. Several findings emerge from this Figure. First, conditional on unemployment, the UE rate in the model is the same in recessions and in recoveries. This is because, in our model, the UE rate only moves around because movements in unemployment affect the efficiency of matching. Second, conditional on unemployment, the EU rate in the model is higher in recessions than in recoveries. This is because, in our model, the EU rate is the driving force of recessions and recoveries. Interestingly, the UE rate and the EU rate in the US data have the same features. These observations provide further support to the view that it is the EU rate that drives the fluctuations in unemployment, while the UE rate simply tracks the fluctuations in unemployment and, in doing so, it amplifies them and makes them more persistent.¹⁴

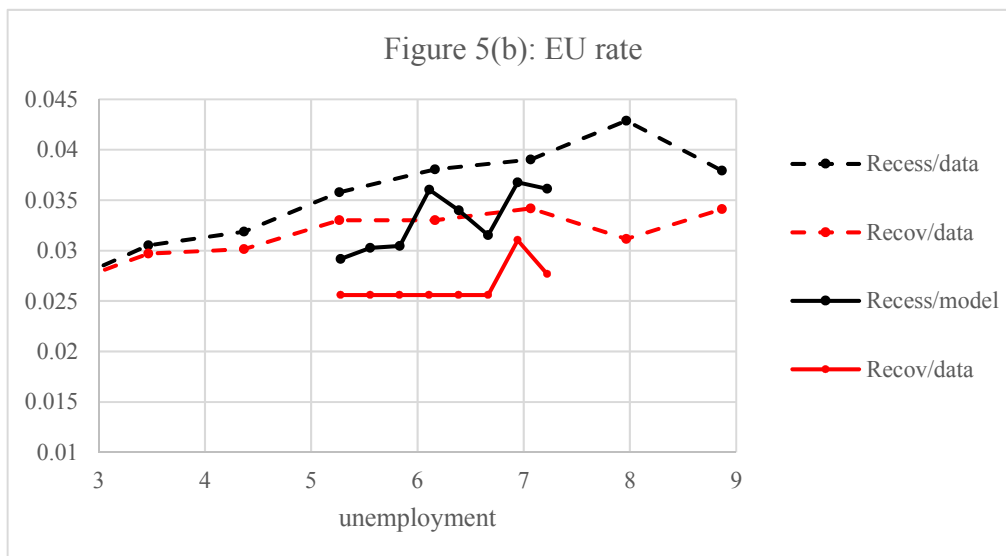
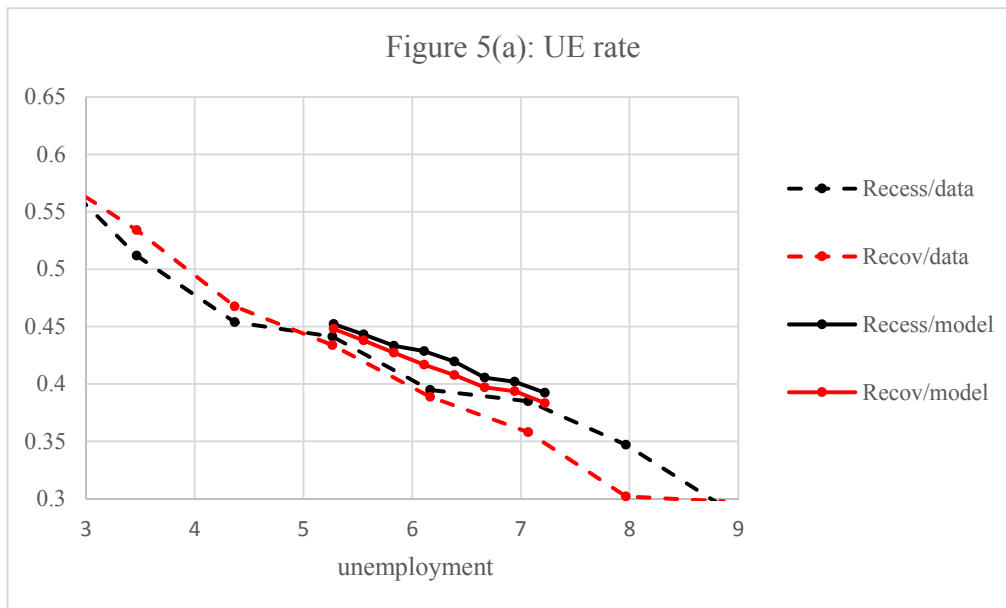
In most business cycle theories, the driving force of the cycle—be it a fundamental as in Real Business Cycle theory, or a sunspot as in non-fundamental theories—follows a stochastic process that is exogenous. In our theory, the driving force of the cycle—i.e., the coordinated firing of non-performing workers—follows a stochastic process that is endogenous and depends on the state of the economy. In particular, in our theory, the lower is unemployment, the lower is the cost to a worker from losing a job relative to the cost to the firm from losing a worker and, for this reason, the higher is the firing probability that is needed to satisfy the worker’s incentive compatibility constraint. As a result, the lower is unemployment, the higher is the probability of a recession. In order to find out whether this feature of the theory is borne out in the data, we take the time-series for the unemployment rate and define the start of a recession as a quarter in which the unemployment rate turns around and grows for at least two consecutive quarters.¹⁵ We then estimate a probit model for the probability of the start of a recession as a function of the unemployment rate. The estimated coefficient on the unemployment rate is $-.21$, with a standard error of 11% .¹⁶ Clearly, it is not possible to precisely estimate the effect of the unemployment rate on the probability that a recession starts because recessions are relatively rare events. In order to gather more observations, we take the time-series of unemployment for Australia, Canada, Italy, Japan, France and the UK.¹⁷ After taking out the average unemployment from the

¹⁴The behavior of the UE rate and EU rate displayed in Figure 5 implies that, in the model and in the data, the unemployment rate increases more quickly in recessions than it falls in recoveries. The reader can immediately verify that this is indeed the case by looking at the time-series for unemployment in Figure 3.

¹⁵The estimates are robust to alternative ways to define the start of a recession.

¹⁶The estimated coefficient implies that the probability of a recession increases from 8% to 11% as the unemployment falls from 5.5% to 4.5% .

¹⁷These are countries for which the OECD provides sufficiently long time-series for unemployment. We drop Germany from the sample because of the large movements in unemployment related to the unification.



time-series of each country, we merge these data to those for the US and re-estimate the probit model. The estimated coefficient on the unemployment rate is $-.20$ with a standard error of 4%.

6.3 ABC versus RBC

We established that ABC generate large fluctuations in labor market variables, they feature the same pattern of comovement and the same pattern of leads and lags between labor market variables as in the data, and they imply a stochastic process for recessions that depends on unemployment just like in the data. In this sense, ABC look like successful theory of cyclical fluctuations in the US labor market, especially in the period after 1984 during which there is no correlation between unemployment and labor productivity.

We now turn to testing what is perhaps the most distinctive feature of our business cycle theory. In our theory, a recession is a period when the value of time in the market relative to its value at home is abnormally high. In contrast, in the Real Business Cycle theory of Kydland and Prescott (1982), in Mortensen and Pissarides (1994), in Kaplan and Menzio (2015) or in any other theory where business cycles are driven by either exogenous or endogenous shocks to the value of production, a recession is a period when the value of time in the market relative to its value at home is abnormally low. To paint a picture, in RBC, a recession is a day when it is raining in the marketplace and, for that reason, workers find it optimal to stay at home. In ABC, a recession is a day when there is no cable at home and, for that reason, firms find it optimal to get rid of their non-performing workers. It is then natural to wonder whether, empirically, the relative value of labor in the market is pro or countercyclical. In order to address this question, we construct some rudimentary empirical measures of the net value of employment to a worker and of the value of a worker to a firm.

We measure the value of employment to a worker, $W_{1,t}$, and the value of unemployment to a worker, $W_{0,t}$, as

$$\begin{aligned} W_{1,t} &= w_t + \beta [h_{t+1}^{EU} W_{0,t+1} + (1 - h_{t+1}^{EU}) W_{1,t+1}] , \\ W_{0,t} &= b_t + \beta [h_{t+1}^{UE} W_{1,t+1} + (1 - h_{t+1}^{UE}) W_{0,t+1}] , \end{aligned} \tag{47}$$

where w_t is a measure of the real wage in month t , b_t is a measure of unemployment benefit/value of leisure in month t , h_{t+1}^{EU} is a measure of the EU rate in month $t + 1$, h_{t+1}^{UE} is a

When estimated for each country separately, the coefficient on unemployment in the probit model is always negative.

measure of the UE rate in month $t + 1$, and $W_{1,t+1}$ and $W_{0,t+1}$ are respectively the value of employment and unemployment in month $t + 1$. We measure the net value of employment to a worker, V_t , in month t as the difference between $W_{1,t}$ and $W_{0,t}$.

We measure the value of a worker to a firm, J_t , as

$$J_t = y_t - w_t + \beta(1 - h_{t+1}^{EU})J_{t+1}, \quad (48)$$

where y_t is a measure of labor productivity in month t , w_t is a measure of the real wage in month t , h_{t+1}^{EU} is a measure of the EU rate in month $t + 1$, and J_{t+1} is a measure the value of a worker to a firm in month $t + 1$.

We measure w_t using the time-series for the hourly wage that have been constructed by Haefke, Sonntag and van Rens (2013). We consider two alternative time-series: the average hourly wage in the cross-section of all employed workers, and the average hourly wage in the cross-section of newly hired workers after controlling for the composition of new hires. The first time-series may be more appropriate when we want to interpret V_t as the cost of losing a job to a worker, the second-time series may be more appropriate when we want to interpret V_t as the benefit of finding a job to a worker. We measure y_t using the time-series of value added per employed worker from the CPS. As in Shimer (2005), we measure h_t^{UE} and h_t^{EU} using, respectively, the values for the EU rate and UE rates implied by the time-series for unemployment and short-term unemployment. In order to make the time-series for w_t , y_t , h_t^{UE} and h_t^{EU} stationary, we construct their Hodrick-Prescott trend using a smoothing parameter of 10^5 . We then take the difference between the value of each variable and its trend and add this difference to the time-series average for that variable. Since w_t and y_t are not measured in the same units (i.e., dollars per hour and value added per worker), we rescale y_t so that its average is 110% of the average of w_t . Since b_t is not directly observable, we tentatively set it to be equal to 70% of the average of w_t .

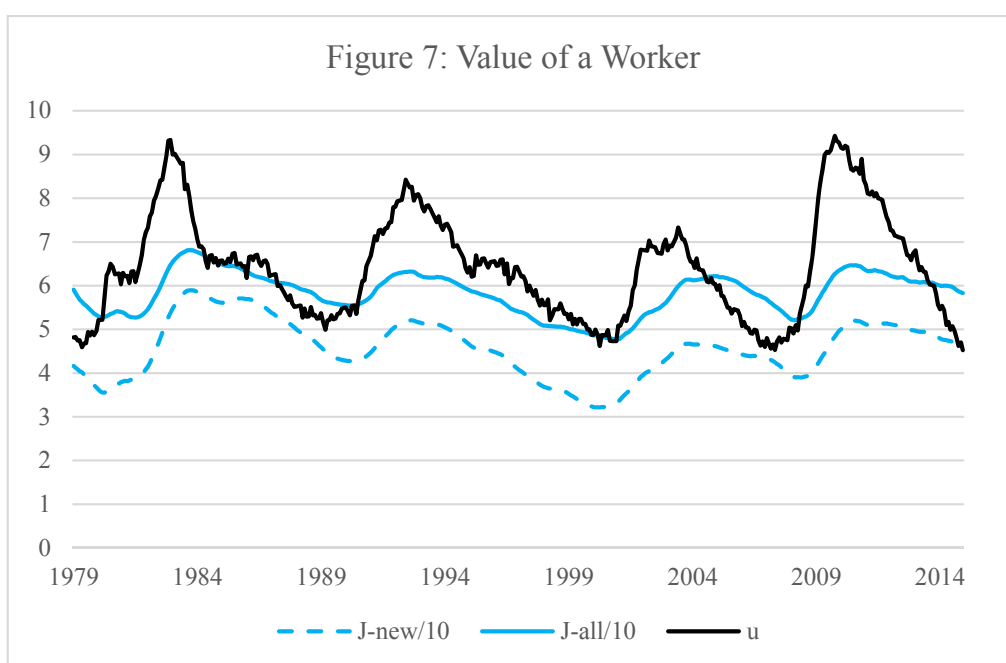
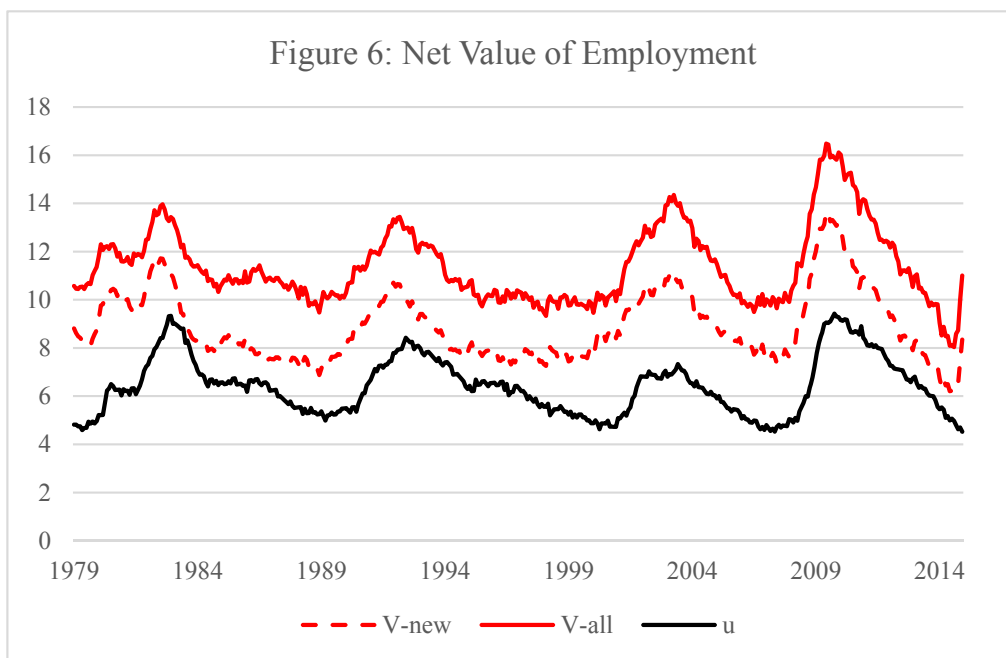
We are now in the position to construct time-series for the value of employment to a worker, $W_{1,t}$, the value of unemployment to a worker, $W_{0,t}$, and the value of a worker to a firm, J_t , over the period going from January 1979 to December 2014. We compute the values for W_1 , W_0 and J in December 2014 by assuming that, from January 2015 onwards, w , y , h^{UE} , and h^{EU} are equal to their historical averages. Given the values for W_1 , W_0 and J in December 2014, we compute the values for W_1 , W_0 and J from November 2014 back to January 1979 by using equations (47) and (48) and the time-series for w , b , y , h^{UE} , and h^{EU} . The reader should notice that the values of W_1 , W_0 and J thus computed differ from

their theoretical counterpart because they are constructed using the realizations of future w , b , y , h^{UE} , and h^{EU} , rather than the expectation of these variables.

Figures 6 and 7 present the result of our calculations. Figure 6 displays the time-series for the net value of employment to a worker, V_t , computed using the average wage of all employed workers (solid black line) and the average wage of newly hired workers (dashed black line). Figure 6 also displays the time-series for the detrended unemployment rate (solid grey line). The figure clearly shows that V_t is countercyclical, in the sense that V_t moves together with the unemployment rate. This is the case whether we measure V_t using the average wage of all employed workers—in which the correlation between V_t and unemployment is 80%—or whether we measure V_t using the average wage of newly hired workers—in which case the correlation between V_t and unemployment is 71%. Mechanically, V_t is countercyclical because, when unemployment increases, the decline in the value of being unemployed caused by the large decline in the UE rate is larger than the decline in the value of being employed caused by the small decline in the wage and by the short-lived increase in the EU rate. The countercyclicity of V_t is a very robust finding. The measure of V_t becomes even more countercyclical if b_t is assumed to be constant fraction of w_t rather than a constant. The correlation between V_t and u_t remains practically unchanged if we do not filter the data.

Figure 7 displays the time-series for the value of a worker to a firm, J_t , computed using the average wage of all employed workers (solid black line) and the average wage of newly hired workers (dashed black line). The figure clearly shows that J_t is also countercyclical. This is the case whether we measure J_t using the average wage of all employed workers—in which the correlation between J_t and unemployment is 56%—or whether we measure J_t using the average wage of newly hired workers—in which case the correlation between J_t and unemployment is 60%. Mechanically, J_t is countercyclical because, while neither the labor productivity nor the wage move much over the cycle, their difference is countercyclical.

The finding that V_t is countercyclical means that recessions are times when unemployed workers find it especially valuable to find a job, and when employed workers find it especially costly to lose a job. The finding is in stark contrast with the view of recessions as “days of rain in the marketplace” advanced by Kydland and Prescott (1982) or by Mortensen and Pissarides (1994). In contrast, the finding is supportive of the view of recessions as “days of no cable at home” advanced by our theory. Notice that our finding that V_t should not be entirely surprising. In fact, using a different, more sophisticated approach and richer data,



Davis and von Wachter (2011) show that the lifetime earning cost of losing a job is much higher in recessions than in expansions.

Our theory offers an explanation for why the net value of employment to a worker is higher in recessions than in expansions. An alternative explanation is that wages are rigid.¹⁸ Indeed, if wages are sufficiently rigid, a decline in productivity may leave the value of employment to a worker nearly unaffected, while lowering the value of unemployment. However, if wage rigidity is the reason for the countercyclicality of the net value of employment V_t , the value of a worker to a firm J_t should be strongly procyclical, as the decline in the worker's productivity is not matched by a decline in the worker's wage. But, as shown in Figure 7, J_t not only is not strongly procyclical, it is countercyclical. In contrast, our theory is consistent with the fact that the net value of employment to a worker is countercyclical and with the fact that the value of a worker to a firm is countercyclical.

7 Conclusions

In this paper, we advanced a novel theory of cyclical fluctuations in the labor market. The theory is based on two key assumptions. First, firms need to fire workers with positive probability in order to give them an ex-ante incentive to exert effort. Under this assumption, we showed that, in order to provide this incentive at the lowest cost, firms load up the firing probability on the states of the world where the worker's cost of losing a job relative to the firm's cost of losing a worker is highest. Second, there are decreasing returns to scale in the matching process. Under this assumption, we showed that the states of the world where the worker's cost of losing a job is highest are the states with highest unemployment and, hence, an individual firm finds it optimal to fire its non-performing workers exactly at the time when other firms fire their non-performing workers. The strategic complementarity between the optimal solution of the agency problem faced by different firms leads to an equilibrium in which firms use the realization of a sunspot to coordinate on firing their non-performing workers at the same time, leading to aggregate fluctuations in the EU, UE and unemployment rates.

The fluctuations generated by our model are genuinely endogenous, in the sense that they do not require any exogenous shocks to preferences, technology or policy, nor they require

¹⁸Hall (2005), Menzio (2005), Gertler and Trigari (2009), Menzio and Moen (2010) and Kennan (2010) develop versions of Mortensen and Pissarides (1994) in which the wage of new hires is sticky, in the sense that it is less responsive to changes in labor productivity than it would under the Axiomatic Nash Solution to bargaining. In these models, the value of a job to a worker may be countercyclical, but the value of a worker to a firm is always procyclical.

any exogenous switch in the selection of the equilibrium, and stochastic, in the sense that the equilibrium features aggregate uncertainty about the firing probability. Quantitatively, we showed that the fluctuations generated by our model can potentially account for a large fraction of the volatility in the unemployment rate and in the UE and EU rates that is observed in the US labor market. Moreover, our model generates fluctuations in labor market variables that are uncorrelated with labor productivity, just as they have been in the US for the past 30 years. We also showed that the fluctuations generated by our model have the same morphology as the labor market fluctuations in the US: the EU rate leads the unemployment rate, while the UE rate and the unemployment rate are contemporaneous, the unemployment rate moves faster in recessions than in recoveries, etc... Finally, we carried out some preliminary calculations that seem to support our view that recessions are times when it is particularly costly to lose a job (or stay unemployed), rather than the standard view that recessions are times when it is particularly cheap to lose a job (or stay unemployed).

Clearly, the theory of labor market fluctuations advanced in this paper is very stark and a lot of work remains to be done. On the modelling side, we want to allow for long-term contracts so that wages and not only firing can be used to provide workers with the incentive to exert effort. On the empirical side, it would be important to find some data to directly calibrate the parameters of the agency problem faced by firms and workers, thus dropping the identifying assumption that all of the volatility in the EU rate observed in the data is due to the theory of cyclical fluctuations proposed in this paper. Finally, we want to explore the welfare properties of the equilibrium and, in particular, address the question of whether the coordination of firms' firings is welfare enhancing or not.

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