Discrimination and Worker Evaluation*

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Abstract

We develop a model of self-sustaining discrimination in wages coupled with higher unemployment and shorter employment duration among blacks. While white workers are hired and retained indefinitely without monitoring, black workers are monitored and fired if a negative signal is received. The fired workers, who return to the pool of job-seekers, lower the average productivity of black job-seekers, perpetuating the cycle of lower wages and discriminatory monitoring. Under suitable parameter values the model has two steady states, one corresponding to each population group. Discrimination can persist even if the productivity of blacks exceeds that of whites.

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1 Introduction

African-Americans have lower wages, higher unemployment, longer unemployment duration and shorter employment duration than their apparently similar white counterparts. Moreover, these disadvantages are less marked and, perhaps, in some cases nonexistent for the most skilled workers as measured by education or performance on the Armed Forces Qualifying Test (AFQT). While there is a plethora of models intended to explain one or two of these empirical regularities, none generates all of them.[1]

In this paper, we develop a simple model in which differences in wages, unemployment and job duration arise naturally and their relation to skill is plausible. There are multiple equilibria in our model, a property it shares with models of rational stereotyping or self-confirming expectations (Coate and Loury, 1993). However, in our model discrimination is not simply a product of coordination failure; instead, history matters. A group that begins with a low level of skills for which only the bad equilibrium exists will remain in that equilibrium even if its skill level rises to a level consistent with the existence of both the good and bad equilibria. Even if blacks are, on average, more skilled than whites, whites can be in the good steady-state and blacks in the bad steady-state because of a history of lower access to schooling and other human capital investments. Equalizing the human capital that blacks and whites bring to the labor market need not be sufficient to equalize labor market outcomes. In contrast, in the standard self-confirming expectations models, if we could just convince blacks to invest in themselves and employers that blacks have invested, we would immediately jump to the good equilibrium.

There are three key elements to our model. First, ability is match-specific but some people are more likely to be good matches. Therefore, someone revealed to be a poor match at one job is more likely to be a poor match at the next job. Second, since workers who have been fired for cause from a previous job have an incentive to hide that information, a worker's employment history cannot be credibly conveyed, and therefore a firm must statistically infer worker ability. Third, the quality of the match is revealed to the firm and worker only if resources are invested in monitoring to evaluate the worker.

If the probability that a worker is well-matched is sufficiently high or sufficiently low, it will not be worth investing resources to determine match quality. However, if the cost of determining the match quality is not too high, there will be an intermediate range at which this investment is worthwhile. Employers expect black workers to have a relatively low likelihood of being well-matched with this job. Consequently, they are subject to heightened scrutiny and are more likely to

[1] Many models (e.g. Aigner and Cain, 1977; Becker, 1971; Bjerk, 2008; Charles and Guryan, 2011; Coate and Loury, 1993; Fryer, 2007; Lang, 1986; Lang and Manove, 2011; Lundberg and Startz, 1983; Moro and Norman, 2004) assume market clearing and therefore cannot address unemployment patterns. Search models (e.g. Black, 1995; Bowlus and Eckstein, 2002; Lang and Manove, 2003; Lang, Manove and Dickens, 2005; Rosen, 1997) can explain unemployment differentials, but assume otherwise homogeneous workers and thus cannot address wage differentials at different skill levels. Peski and Szentes (2013) treat wages as exogenous. In general, discrimination models have not addressed employment duration. See the review in Lang and Lehmann (2012).

be found to be a poor match and fired. The increased scrutiny ensures that the pool of unemployed black workers has a higher proportion of workers who have been found to be a poor match at one or more prior jobs. And therefore employers' expectations that black workers are more likely to be poor matches is correct in equilibrium. This generates lower initial wages, lower expected lifetime wages, shorter employment duration, and higher unemployment duration for blacks. While our model treats worker type as exogenous, blacks gain more than whites do from being a good type rather than a bad type, which, loosely speaking, is consistent with blacks getting more education than whites with similar cognitive scores (Lang and Manove, 2011).

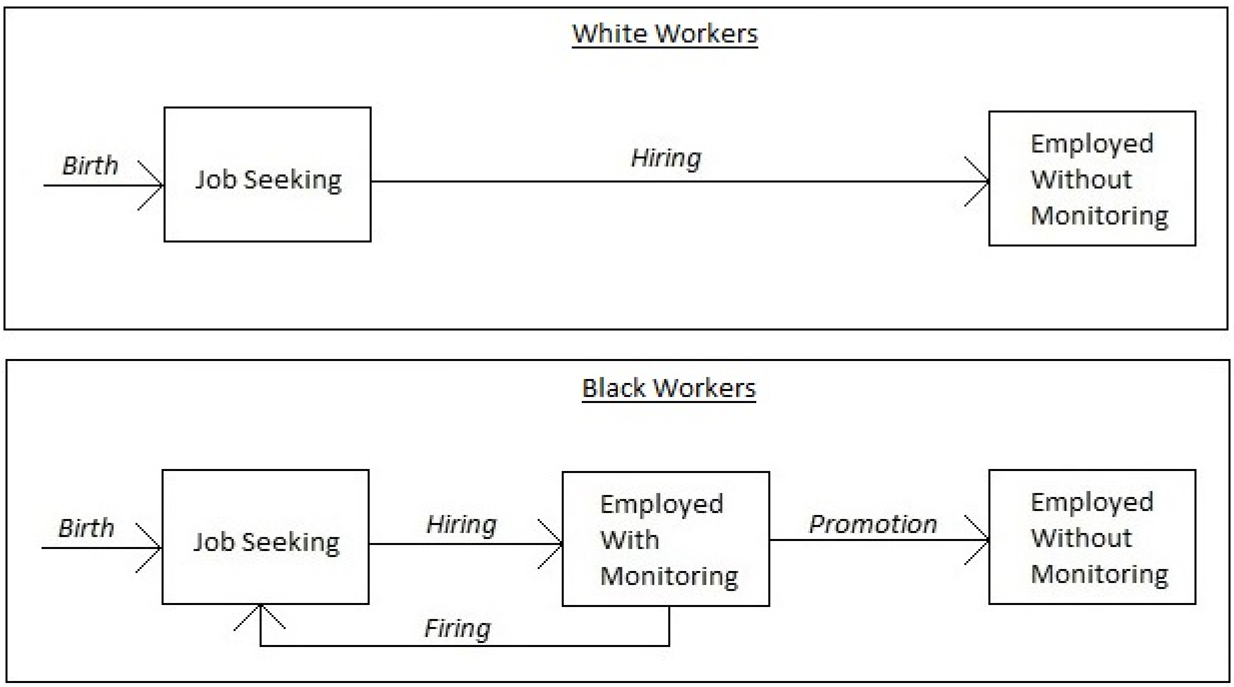

Figure 1: White workers' perpetual employment, black workers' churning cycle

Although we do not wish to overstate the predictive power of the model, we note that until around 1940, blacks and whites had similar unemployment rates (Fairlie and Sundstrom, 1999), while blacks faced lower wages. This is consistent with a setting in which, due to low human capital investments, blacks were assumed to have low productivity at most jobs and therefore not monitored for quality. 'Churning' of the black labor market would not begin until human capital investments were sufficiently high.

Such churning equilibria are hard to escape. This is disheartening since *policy succeeding at convergence of group characteristics may fail to equate labor market outcomes*. Only if the skill level of blacks is raised sufficiently above that of whites (technically the proportion of good workers is sufficiently high), does the bad equilibrium cease to exist and white and black workers receive similar treatment. At the same time, there is a potential role for policy. Policies aimed at reducing the rate at which poorly matched blacks are returned to the unemployment pool, temporarily reduce output by increasing the rate of mismatch. However, they increase output in the long run by moving the labor market from the bad to the good equilibrium. In essence, the current practice of firing mismatched workers leads to excess monitoring of new workers.

Formally, we assume that the market cannot distinguish between new labor market entrants and job seekers who have experienced one or more previous employment spells. Obviously, it is implausible that the market would get no signal of a worker's age and therefore of her likely prior work history. We do not view this as problematic provided that some workers separate into unemployment randomly. Then our implicit assumption is that the market has difficulty distinguishing between a recent high school graduate who has not yet found a job and one who has had an unsuccessful employment experience. And similarly, the market has difficulty distinguishing between a thirty-year old who lost a job in a mass layoff but held a new job briefly before returning to unemployment and one who has been continuously unemployed since the layoff.

In an informal extension to the model, we argue that black job seekers who have experienced enough turnover will be permanently relegated to low-skill, low-wage jobs, although for this group the model does not imply that such workers will have shorter employment duration than white workers do.

2 A simple example of churning

To provide some intuition, we first consider a simple discrete-time market in which we abstract from wage bargaining and vacancy creation decisions. These will play a central role in the full model. Suppose each worker is either "α," producing $q_\alpha = 1$ unit per period or "β," producing $q_\beta = 0$ per period and that output is not visible to the firm even ex post. Let newly created workers have a $2/3$ probability of being α and $1/3$ of being β and normalize the mass of workers entering the labor market in each period to 1. Suppose furthermore that the wage, endogenized in the full model, of a worker is set exogenously to $w = 1/3$ per period,[2] that matches do not dissolve naturally and that the discount factor is $\delta = .95$.

Workers who are unemployed at the end of a period and new workers are randomly matched at the beginning of the next period to a firm with which they have not previously been matched. Lastly, employers are given the option to test or monitor a newly hired employee at a cost of $4/3$, producing a signal G with probability 1 if the employee is α, and with probability .5 if the employee is β, and the signal $\neg G$ otherwise. This imperfect test is performed prior to production, so that fired workers do not produce and are not paid in the current period. This simplifies the arithmetic but is otherwise unimportant.

Consider first a job market comprised solely of first-time job seekers. When matched, the firm can employ the worker without monitoring, a strategy that will pay off

$$(2/3 * q_\alpha + 1/3 * q_\beta - w)/r = (2/3 - 1/3) * 20 = 20/3$$

[2] This is half the expected surplus of a retained worker.

or it can monitor to fire revealed βs, so that half the β workers end up producing and the other half get fired, while all matches incur the test cost $b = 4/3$, resulting in an expected firm payoff of

$$(2/3 * q_\alpha + 1/6 * q_\beta - 5/6 * w)/r - b = (2/3 - 5/18) * 20 - 4/3 = 58/9.$$

So, a firm will prefer not to monitor newly hired workers. Consequently, no workers are ever fired in this market, and thus, as assumed, all unemployed workers are first-time job seekers.

Now consider a market that has been churned by the monitoring technology. In each period, half the β workers who got a job the previous period are fired and return to the job-seeking pool where they join a new batch of β workers of size 1/3 and α workers of size 2/3. In steady state, the number of β job-seekers obeys the equation

$$B = \frac{1}{2}B + \frac{1}{3},$$

so that the steady-state mass of βs in job-seeking status is 2/3. The mass of newly-minted α workers is also 2/3, hence βs comprise one half of all job seekers. An employer who does not monitor will get a payoff of

$$(1/2 * q_\alpha + 1/2 * q_\beta - w)/r = (1/2 - 1/3) * 20 = 10/3$$

whereas one who does monitor expects a payoff of

$$(1/2 * q_\alpha + 1/4 * q_\beta - 3/4 * w)/r - b = (1/2 - 1/4) * 20 - 4/3 = 11/3$$

thus workers in the second, or *'churned'*, market are monitored and can be fired.

This simplistic model demonstrates how two groups with the same underlying abilities can face very different treatment, and that this process can be self-enforcing. It captures discrimination via churning. Since whites are employed immediately, blacks have longer unemployment durations. Alternatively, if we view the testing period as employment, the example generates shorter employment durations for blacks. But, in either case, it cannot address wage differentials. The rest of this paper remedies this by nesting the intuitions of this example in a search model in the spirit of Mortensen and Pissarides (1994) but with a variant of Rubinstein (1982) bargaining rather than Nash bargaining. The main model relies on a fully revealing test and match-specific productivity, but we briefly discuss imperfect tests without match-specific productivity in Section 5.5.

We will only address dynamics informally in our treatment of the full model, but this simple example also helps demonstrate an important point. It is readily confirmed that the market switches from monitoring to not monitoring when the proportion of αs in the labor market surpasses .6 and that firms will make a loss if this proportion is less than .28. Consider a group for which historically the proportion α was less than .28 and was therefore employed in some other type of job. Now let improvements in human capital lead new entrants in period t to have a proportion of α equal to .3; also, let this proportion grow to .7 in period $t + 1$ and remain at this level thereafter. We assume, for strictness, that workers born before t never compete in this labor market. Nevertheless, we show that the group never exits the churning equilibrium.

When the first very high human capital generation enters the labor market, the proportion of α workers is $.7/(.35+1) = .52$, and thus firms continue to monitor workers. The process approaches its asymptotic value of $.7/(.3+1) = .538$ very rapidly. Despite a legacy of only one generation in which the quality of the inflow favored churning, the group would remain stuck in the churning equilibrium until some time after the proportion α in the new generation exceeded .75.

3 The Model

3.1 Setup

There are two population groups, 'blacks' and 'whites'. Race is directly observable and therefore known to the worker and prospective employers. The critical assumption is that employers cannot perfectly observe a worker's employment history because workers can hide previous jobs from which they were dismissed, and therefore employers use current unemployment status to infer productivity. Formally, we assume that employment history cannot be credibly conveyed to prospective employers.

In essence, we require that employers cannot acquire sufficiently precise information to distinguish recent labor market entrants or those displaced exogenously from those who were employed previously and proved to be part of a bad match. At a more informal level, we believe that workers have some ability to hide their employment history and that they will not report information speaking to their own low ability. A worker who claims to have been continuously unemployed for, say, four months following what appears to be a random layoff, may, in fact, have begun an unemployment spell four months ago, been employed for two months, fired for cause, and again become unemployed. If workers can only imperfectly hide their employment history, certain histories will reveal that the worker is highly likely to be a low quality worker. Later in the paper, we consider an informal extension of the model in which the employment history of unemployed workers is revealed stochastically and such revelation relegates the worker to 'dead-end jobs.'

We will show that when prospective employers statistically infer past employment based on race, this can lead to monitoring and potential firing for blacks but not whites. As fired workers are on average less productive than retained workers, the average quality of black job-seekers will be lower than that of white job-seekers, inducing firms to monitor newly hired blacks but not whites.

At all times a steady flow of new workers is born into each population group.[3] A proportion $g \in (0,1)$ of new workers are type α, who are good at every job.[4] The rest, referred to as type β, have probability $\beta \in (0,1)$ of being good at any particular job. The probability of a worker being good at a job, conditional on his type, is independent across jobs. A worker, but not firms, knows

[3] We do not allow for death but could do so at the cost of a little added complexity.

[4] Having type α workers perform well at every job does not appear to be essential to the argument but does appear to be essential to having comprehensible mathematics.

if he is of type α or β. Newly created workers are immediately unemployed. We define the average probability of being good at a particular job among new job seekers as

$$\theta_0 = g + (1-g)\,\beta. \tag{1}$$

3.2 Match Quality

Production, the payment of wages and the use of the monitoring technology occur in continuous time using a common discount rate r.

Workers can be either well-suited to a task ('good'), producing q per unit time; or ill-suited ('bad'), producing expected output $q - \lambda c$ per unit time. We interpret the lower productivity of bad workers as errors or missed opportunities, each costing the firm c, that arrive at Poisson rate λ. Under this interpretation, opportunities for error are also opportunities for the firm to learn whether the worker is good or bad at this job since good workers are observed to avoid errors. Then, if the employee is being monitored to observe such errors, the expected time until the parties learn whether the worker is good at this job is $1/\lambda$.[5]

For monitoring to ever be sensible, it must be that a match revealed to be bad must end. To this end, we make the sufficient and simple assumption that such a match is unproductive:

$$\text{(C1)} \qquad q - \lambda c \leq 0.$$

It is *much* stronger than necessary. In general, if the worker and firm know that the match is bad, it will be efficient for the worker to experience some unemployment in order to try a new match; this is a consequence of productivity conditional on worker type being match-specific. Assumption (C1) ensures that such separation in search of a better match is efficient regardless of the expected duration of unemployment. If the inequality were reversed, there might be parameter values for which a fraction of workers revealed to be bad at a particular job would nevertheless remain there.[6]

Neither the employer nor a type β worker can know the match quality without monitoring. The parties can agree to a costly regime of monitoring that may produce a fully informative, bilaterally observable signal about match quality. In keeping with the opportunities-for-errors interpretation, we assume the signal arrives at a Poisson rate λ.[7] The monitoring technology costs b per unit time to utilize, so that the expected cost of information is $\int_0^\infty be^{-\lambda t}dt = b/\lambda$ and its expected discounted cost is $\int_0^\infty (e^{-rt}b)e^{-\lambda t}dt = b/(\lambda + r)$. The principal benefit of a Poisson signal, rather

[5] Note that under the opportunities interpretation, workers have a flow rate of productivity of $q-\lambda c$. Opportunities of value c arrive at rate λ and only type α workers succeed in taking advantage of the opportunity. Alternatively, we could assume that the flows are $q-d$ and q with $d \equiv \lambda c$ and that λ is the arrival rate of opportunities to measure the flows.

[6] The cases ruled out here are not of interest; if workers are sometimes retained after revelation of a bad match, no equilibrium with monitoring can exist as the information from costly monitoring adds no value.

[7] This allows for a certain stationarity in the model. So long as no signal has arrived, the underlying incentives do not change.

than one that arrives deterministically, is that it makes the employment survival function more realistic. In addition, we model wages as determined by a variant of Rubinstein bargaining with no commitment. If monitoring required a fixed period, we would need the firm and worker to commit to monitoring and compensation during the monitoring period. Note that the benefit of monitoring is the opportunity to discover that the worker is mismatched and is likely to be better matched at a different job. If the worker is almost always well-matched, it will not be worth paying b/λ to check for a bad match. Similarly, if the worker is almost never a good match, then the opportunity to try a different match has little value, and monitoring is inefficient. Of course, by (C1), if the probability of a good match is sufficiently low, the worker will never be employed in jobs of this type and will be permanently relegated either to nonemployment or to a different type of job, if one exists.

3.3 Job Search

When a worker is born or her match is terminated, she becomes unemployed. Unemployed workers are stochastically matched to firms, which occurs at a Poisson rate μ. For the moment, we treat this rate as exogenous; it will be endogenized in Section 5.4.

In the unemployed state, workers merely search for new jobs; we normalize the flow utility from this state to 0. The value from unemployment is thus simply the appropriately discounted expected utility from job-finding and is invariant to history. The discount on job-finding is $\int_0^\infty e^{-rt}\mu e^{-\mu t}dt = \mu/\left(\mu + r\right)$; the value of a new job will depend on the equilibrium. We denote value of the job-finding state as U_α for type α workers and U_β for type β workers.

When a match dissolves, transfers cease and the worker becomes unemployed. A firm does not recoup a vacancy and therefore receives a payoff of 0 on termination.[8]

3.4 Bargaining

3.4.1 Informal Description

Modeling wage determination in this setting is essential to addressing whether discrimination persists in equilibrium. However, because of the dynamic nature of the game, our discussion ends up being more technical than we expect to be of interest to readers who are primarily interested in discrimination. We therefore begin this section with a brief intuitive discussion which we hope will be adequate to permit such readers to skip the technical discussion.

We cannot use Nash bargaining because there is no accepted model of Nash bargaining with asymmetric information. Instead, we use a bargaining model in which workers and firms make alternating offers. We assume that the parties may unilaterally reopen bargaining at any time but with a one-period delay. Offers take the form of a wage and monitoring regime. If the regime involves no monitoring, no new information arises. There should be no reason to reopen bargaining,

[8] This occurs naturally due to free entry when vacancy creation is endogenized; see Section 5.4.

and the wage should be constant over time. If the regime involves monitoring, workers who are shown to be bad will leave, and those shown to be good will reopen negotiations and get a higher wage with no monitoring.

A critical question is whether the bargaining can reveal workers' private information about their type. Intuitively, firms might offer a monitoring contract that would attract one type and a no monitoring contract that would attract the other. The problem is that if separation occurs, the α workers should immediately renegotiate to a no-monitoring contract with a high wage reflecting the fact that they are known to be a good match. But this is also the best possible outcome for a β worker. So, knowing that renegotiation will occur immediately, β workers will pretend to be αs, and there will be no separating contract.

In general, separation requires different behavior by α and β workers over a period of time. Metaphorically, an α can demonstrate his type by burning a huge pile of money. But doing this requires time; so the instant he lights the match, everyone knows he is an α, and therefore he puts the match out. Only if he can commit to burning the entire pile of money even though everyone already knows his type, can he signal his type. But allowing renegotiation rules out this type of commitment.

Since, they do not wish to reveal themselves $\beta - types$ negotiate as if they were $\alpha - types$. The firm therefore does not know with which type it is renegotiating and acts as if it is negotiating with an average of the two types. As is typical in Rubinstein bargaining, the outside option is the utility flow while bargaining, which is zero.

When there is no monitoring and the quality of the worker is unknown, we have the equivalent of the zero-outside-option equal-weights Nash bargaining solution where the worker receives w/r and the firm receives $\left(q - (1-\theta)\lambda c - w\right)/r$. This is smoothed over time with a wage of

$$w_{N\theta} = .5\left(q - (1-\theta)\lambda c\right).$$

If the worker is known to be a good match, this becomes

$$w = .5q. \tag{2}$$

Lemma 1 below allows us to determine that whenever monitoring shows a match to be good, the parties will renegotiate and agree on this wage. Therefore, when monitoring to determine worker quality takes place, an α worker expects to get

$$\frac{w_{M\theta}}{\lambda + r} + \frac{\lambda}{\lambda + r}\frac{.5q}{r} \tag{3}$$

where the first term is the present value of the wage until revelation and the second term is the present value of the wage after revelation. The firm expects to get

$$\frac{(q-(1-\theta)\lambda c - w_{M\theta})}{\lambda + r} + \theta\frac{\lambda}{\lambda + r}\frac{.5q}{r}. \tag{4}$$

The first term is the discounted profit until revelation and the second term the expected profit post-revelation. Bargaining results in a wage of

$$w_{M\theta} = \frac{1}{2}(q - b - \lambda c(1-\theta)) - \frac{(1-\theta)}{2}\lambda\frac{q}{2r}.$$

which is once again also the Nash bargaining outcome of an α worker bargaining with a firm with belief θ that the match is good, each with an outside option of 0.

Note that workers receive less than half the flow value of output. In effect, because all workers bargain as α who know they will be revealed to be good matches, but firms assume that the worker with whom they are bargaining is an average worker, workers are more impatient to get to revelation and therefore bargain as if delays were more costly. This means that monitored workers bear not only their share of the monitoring cost but an additional "Pooling Penalty." As under Nash bargaining, the full model will choose between the no-monitoring and monitoring regimes efficiently from the standpoint of firms and α workers.

In the next subsubsections we impose conditions to ensure that βs do not wish to reveal themselves and derive these equilibria as the limit of the full alternating-offers bargaining model when the time between offers goes to 0. To derive this as the unique equilibrium we require strong stationarity assumptions. We restrict strategies and beliefs so that if in a purported equilibrium an α or firm proposer can make an offer that makes the proposer better off and is either better for the receiver than the continuation payoff he would get in that 'equilibrium' or is better than the one-period discounted payoff that the receiver would get from rejecting the offer and resuming the equilibrium path, then the purported equilibrium is not an equilibrium.[9] Readers who are less interested in the technical details may wish to skim the material until Section 4.

3.4.2 The Formal Bargaining Model

Wage and monitoring contracts are determined by Rubinstein bargaining with a delay of Δ.[10] An offer is a pair $(w, m) \in \mathbb{R} \times \{0, 1\}$ comprised of a wage w paid per unit time continuously and a

[9] This is similar to Nash's requirement that bargaining outcomes be Pareto efficient.

[10] Although the standard Mortensen and Pissarides (1994) model uses Nash bagaining, it requires symmetric information and therefore is unusable in our setting. Rubinstein bargaining on the other hand produces tractable pooling equilibria under reasonable assumptions. Evidence from Milgrom and Hall (2008) suggests that their own Rubinstein variant (their "credible bargaining" model introduces pecuniary costs of delay alongside the time costs) is able to produce far more realistic unemployment predictions than Nash barganing. Our model shares the feature that enables this prediction (workers' outside option not dampening firm payoff fluctuations), enhancing our ability to address unemployment duration.

policy m of either using or neglecting the monitoring technology.[11]

When a match is first formed, a first proposer is chosen with equal probability on the firm and worker. Production, monitoring and wages cease during bargaining. We are interested in solutions when Δ is low.

Most importantly, either partner may *unilaterally* choose to re-open negotiations at any time by causing a single delay of length Δ during which production and wages are suspended. Once this delay expires, the party instigating renegotiation is placed in the role of proposer.[12] The choice to reopen negotiation is logically simultaneous at each time, and if both partners wish to reopen negotiations at the same instant they each assume the role of proposer with probability 1/2.

Thus, there is no commitment to any agreement. This is important. If the wage is independent of worker type, it will generally fall between the wages that would be negotiated by a known β and by a known α. Therefore, starting from a common wage, if a worker is revealed to be an α, she will renegotiate to raise her wage, while the firm will renegotiate a lower wage if the workers is revealed to be a β. This creates an environment hostile to separating equilibria as the firm cannot commit to compensate those β workers who reveal their type.

With the impermanence of deals, however, we now open ourselves to repeated-games type equilibria where the acceptance of bad offers, and intransigence in insisting on them, is enforced by off-path punishment. To recover the uniqueness of Rubinstein bargaining from this, we make an assumption:

(S0) Stationarity: Consider histories where beliefs put probability 1 on a certain worker type or match quality. There are no deviating offers at such histories that if not renegotiated (in the case of uncertain match quality, until revelation) improve[13] the payoff of the proposer while giving the receiver more than the once-discounted expected value at their previous offer (or, if this is the first offer, the receiver's once-discounted value of offering first).

Stationarity allows for precisely the kind of argument present in standard Rubinstein bargaining. A party who makes offers it values at x should be willing to accept offers it values at $e^{-r\Delta}x$. This further allows us to dispense with repeated-games type wasteful behavior, such as strategies that waste most of the surplus under the threat that they'll waste even more of the surplus.

At this point we want to assume that the bargaining delay itself does not cause extreme inefficiency. It must be small enough that it is worthwhile to utilize renegotiation to cease monitoring

[11] We assume that offers entail constant wages and monitoring, a limitation. Allowing time-varying wage profiles to be offered does not affect our findings but results in the loss of some elegance. We can show that our results hold for the average wage over a small interval that is nevertheless large relative to the bargaining delay but cannot rule out wages that, for example, alternate between a high and low wage with each wage maintained for a period equal to the bargaining delay. If we further assume that wages and monitoring can be contingent on the signal arriving, we require additional assumptions on the delay, Δ, to preserve our results; at the cost of considerable complexity, the equilibrium derived here is essentially unique as $\Delta \downarrow 0$.

[12] This delay on renegotiation ensures that disagreeable offers are rejected rather than accepted with the intent to renegotiate instantly.

[13] A delay caused by rejecting an equilibrium offer or reopening negotiations is of course factored in to deciding whether a deviating proposal is payoff-improving to the proposer.

in a match revealed to be good (where monitoring is of no more use). A match with monitoring produces a flow of $q-b$ per unit time, so that the greatest the less-well-off partner can get is $\frac{1}{2}(q-b)$; a party that instigates renegotiation will get the first-proposer's symmetric-information payoff with a one-period delay, which is $e^{-r\Delta}q/\left(1+e^{-r\Delta}\right)$.[14] For renegotiation to be worthwhile for at least one party, we therefore need

$$\text{(C2)} \qquad (1+e^{-r\Delta})b > (1-e^{-r\Delta})q.$$

In fact, we want to think of the bargaining delay as being vanishingly small and do our analysis in Sections 5 and 6 treating it as such, in which case (C2) holds trivially whenever $b > 0$.

Bearing this in mind, we additionally postulate that

$$\text{(C3)} \qquad e^{-r\Delta} > \frac{\mu}{\mu + r}$$

to ensure that for a worker, rejecting an offer and making a counter offer is, in expectation, faster than separating in order to find a new match where the worker might be the first proposer (for simplicity, we formalize this as though he will be the first proposer). Counter-offering is quicker than finding a new employer to make an offer to. Again, this condition must always be satisfied for sufficiently small Δ.

3.4.3 Bargaining solution with symmetric information

First, let us find the subgame perfect Nash equilibrium (SPNE) solution where bargaining occurs after all relevant information is revealed.

1. **Known Bad Match.** The worker is known to be bad at this job. Then by (C1), the match produces a negative flow value. As any bargaining outcome will deliver a negative payoff to at least one party, instead of doing so, they separate, as is efficient.
2. **Known Good Match.** The worker is commonly known to be good at this job (either revealed as type α or observed to be good at this job via monitoring). The total match surplus is the discounted value of producing q for all time, which would be $\int_0^\infty qe^{-rt}\,dt = q/r$. Using S0, the parties make stationary offers; therefore, they split the output according to the Rubinstein bargaining weights. As the delay is Δ, discounting following a rejected offer is $e^{-r\Delta}$. The proposer will therefore earn
$$\frac{1}{1+e^{-r\Delta}} \cdot \frac{q}{r}.$$
In the limit as $\Delta \downarrow 0$ each party will achieve $.5\frac{q}{r}$, via a wage of $.5q$.

Lemma 1 *If monitoring reveals a match to be good, both parties request renegotiation; they*

[14] This is established more rigorously by Lemma 1.

each expect a payoff of $e^{-\Delta r}q/(2r)$ *upon such revelation.*

Proof. See A.1 ■

3. **Known β, no monitoring.** Worker type is commonly known to be β, and the match is of unknown quality. Note that when worker type is known, the monitoring decision will always be efficient from the perspective of the worker/firm pair, and therefore can be decoupled from the wage bargaining. We analyze first the case of no monitoring. The average cost of errors per unit time in a bad match is λc; so the present match, which is bad with probability $(1-\beta)$, will lose an average of $(1-\beta)\lambda c$ to errors per unit time. Total match surplus is

$$S_{N\beta} = 2\frac{q-(1-\beta)\lambda c}{r}. \tag{5}$$

The first proposer therefore receives

$$\frac{1}{1+e^{-r\Delta}} \cdot \frac{q-(1-\beta)\lambda c}{r}. \tag{6}$$

The average[15] wage and firm flow profit is therefore $\bar{w}_{N\beta} = .5\left(q-(1-\beta)\lambda c\right).$

4. **Known β, monitoring.** We continue to analyze the case where worker type is commonly known to be β, but now assume that there is monitoring. If the match is revealed by the signal to be bad, separation ensues; the firm receives 0 while the worker receives the value of unemployment to a β worker, U_β. Conversely, if the match is revealed to be good, each player expects a payoff of $qe^{-r\Delta}/(2r)$ by Lemma 1. The expected discount on revelation of match quality is the expectation of the discount, e^{-rt}, with respect to $t \sim Poisson(\lambda)$, so it is $\int_0^\infty e^{-rt}\lambda e^{-\lambda t}\,dt = \lambda/(\lambda+r)$. For expected discounted pre-revelation total wages W, the worker's total expected payoff is

$$W + \frac{\lambda}{\lambda+r}\Big(\beta\frac{qe^{-r\Delta}}{2r} + \Big(1-\beta\Big)U_\beta\Big). \tag{7}$$

The firm's payoff, remembering a termination gives it 0, is

$$\frac{q-(1-\beta)\lambda c - b}{\lambda+r} - W + \frac{\lambda}{r+\lambda}\Big(\beta\frac{qe^{-r\Delta}}{2r} + \Big(1-\beta\Big)\cdot 0\Big). \tag{8}$$

The total surplus from the match is therefore

$$S_{M\beta} = \frac{q-b}{\lambda+r} - \frac{(1-\beta)\lambda c}{\lambda+r} + \frac{\lambda\beta}{\lambda+r}\frac{qe^{-r\Delta}}{r} + \frac{\lambda(1-\beta)}{\lambda+r}U_\beta. \tag{9}$$

We can solve for the expected discounted wages by equating (7) and the $\frac{e^{-r\Delta}}{1+e^{-r\Delta}}$ and $\frac{1}{1+e^{-r\Delta}}$ Rubinstein shares of (9) to find the pre-revelation wages W_1 and W_2 the worker expects to get if proposing first or second. Finding the instantaneous wages $\int_0^\infty e^{-\lambda t}e^{-rt}w_i = W_i$ and recalling that the worker and firm each have a probability 1/2 of being first proposer, we can

[15] Averaged over which party proposes first. Also happens to be the limit as $\Delta \downarrow 0$.

find the average expected instantaneous wage of a known β who is monitored:

$$w_{M\beta} = .5\left(q - (1-\beta)\lambda c - b\right) - .5\left(1-\beta\right)\lambda U_\beta. \tag{10}$$

Note that the solution has the somewhat disturbing property that the worker's payment following separation *lowers* the wage. In essence, Stationarity has the property that the worker is impatient for the opportunity to 'try again' if she turns out to be bad at the job. In contrast with Nash bargaining, where better outside opportunities lead to a more favorable outcome, in the standard application of the Rubinstein model, outside options have no effect unless they are binding; the term appears here not in its capacity as the outside option but as a potential outcome of monitoring for type β workers. In our equilibrium, known βs are never retained. Hence, we do not observe wages with this property.

In each of these equilibria, continuation play if an off-path offer has been made or is in place is for both players to request immediate renegotiation and propose the equilibrium shares, unless both players are receiving greater than the receiver's share by the current offer (in which case the current offer is executed until revelation).

3.5 Steady State

A steady state of a labor market is a state of the world in which the mass of job-seekers and the proportion of α and β types among them as well as mass of monitored β workers do not change with time and that can be supported by equilibrium strategies for firms and workers that form an equilibrium. We are interested in two kinds of stable steady states: those in which all employees are monitored until match quality is revealed, and those in which no monitoring occurs.[16]

Consider the case where no employees are monitored: the white labor market. As no information is ever gained, matches never deteriorate and therefore the only source of job seekers is newly born workers. In this scenario, a firm just matched with an employee infers his probability of being of type α is the population prevalence g; denoting this as the "white equilibrium," the chance of a white job-seeker being good at a job to which he is matched is therefore

$$\theta_W = \theta_0 = g + (1-g)\beta.$$

Now suppose instead that all newly hired black employees are monitored and all those found to be "bad" at the current job are fired (this is necessarily the case as (C1) implies such a match has a negative total surplus).

Lemma 2 *The probability a newly hired black worker is in a good match is*

[16] A steady state in which only some workers are monitored until revelation is not stable as it necessarily implies indifference and a mixed strategy for the firm. A perturbation in θ will lead to either complete or no monitoring, causing movement away from the steady state.

$$\theta_B = \frac{\beta}{\beta g + (1-g)} < \theta_W. \tag{11}$$

Proof. See A.2 ■

Therefore, although monitoring may be individually prudent for each matched pair, it creates a negative externality by feeding a stream of workers who are worse than the population average (i.e. containing more β types) back into the job-seeker pool. Surprisingly, the steady state θ_B of this process does not depend on the rate of information λ, the worker-vacancy matching rate μ or the rate at which new workers enter the market.[17] This deleterious cycle is referred to as the 'churning' of the labor market.

3.6 Solution Concept

We are interested in solutions that fulfill the following criteria in addition to S0:

S1 Steady State: The labor market is in steady state; that is, the composition of the job-seeker market and the matching rate are constant.

S2 PBE: Firms and worker strategies form a perfect Bayesian equilibrium.

S3 Stationarity/No Dictatorial Beliefs: At no history with firm beliefs $\theta_h \in (\beta, 1)$ on the match being good can a deviating offer be made that, should it stay in place until revelation and beliefs be fixed at θ_h until revelation:

- a) strictly improves the payoff of a proposing firm or α worker, and either
- b1) if the first offer in a match, improves on the lesser of the receiving firm or α worker's once-discounted first-proposer payoff or equilibrium payoff at the current node, or
- b2) If a subsequent offer, it gives higher payoff to a receiving firm or α worker than their once-discounted expected payoff at their previous offer.

S4 Cho-Kreps Intuitive Criterion: At histories where types do not separate at t, if one worker type would gain by truthfully declaring its type, the other type would gain by declaring that type as well.

Restriction S3 requires some explanation. Its primary purpose is to provide uniqueness. While stated in terms of offers, it is also a plausible restriction on beliefs. It allows α workers to make off-equilibrium offers that are beneficial to them without having to worry about the offers' effect on beliefs. It furthermore allows firms to make offers that the best workers should accept without those workers worrying about a deleterious effect acceptance has on beliefs.

This restriction ensures that the bargaining protocol will produce real bargaining and Rubinstein-like solutions rather than dogmatic offers backed by the threat of belief change or punishment off-path. Lacking S3, low wages could be maintained by the firm believing any deviation is due to the worker being type β. By providing for deviations from such situations without belief ramifications,

[17] This is an artifact of the assumption that workers are infinitely lived.

we eliminate these distasteful equilibria.[18]

Requirement S4 is a restatement of the Cho-Kreps Intuitive Criterion for this setting.

3.7 Parametric Assumptions

Now we impose certain restrictions on the joint values of parameters sufficient to ensure that pooling equilibria with monitoring and without monitoring both exist.

For an equilibrium with no monitoring to exist for white workers, we want to assume that monitoring costs are not too low.

Initially, we want to abstract from bargaining frictions; suppose $\Delta \downarrow 0$ to gain some additional intuition. An α worker can estimate a wage increase of $((1-\theta_W)\lambda c)/2r$ from the no-monitoring baseline if revelation occurs as he is no longer penalized for expected errors. The employer sees a revelation as increasing the wage by that same amount with probability θ_W and with probability $1-\theta_W$ expects a bad employee to be terminated, which would yield $\frac{\lambda c}{r}-\frac{q}{r}$ in foregone losses and an additional $q/2r-(1-\theta_W)\lambda c/2r$ in foregone wages. We therefore want the limiting condition to show that monitoring costs $\frac{b}{\lambda}$, on the left, exceed the sum of these benefits to the firm and α worker.

$$\underbrace{\frac{b}{\lambda}}_{\text{Mon. cost}} > \underbrace{\overbrace{(1-\theta_W)\frac{\lambda c}{2r}}^{\text{Wage Increase}}}_{\text{Revelation Gain to }\alpha} \quad \underbrace{+\overbrace{(1-\theta_W)(\frac{\lambda c}{r}-\frac{q}{2r}-(1-\theta_W)\frac{\lambda c}{2r})}^{\text{Bad Match End Benefit}}-\overbrace{\theta_W(1-\theta_W)\frac{\lambda c}{2r}}^{\text{Wage Increase}}}_{\text{Revelation Gain to Firm}} \tag{12}$$

Restating this limiting condition to constrain β and g rather than the monitoring costs and recalling that $\lambda c - q/2$ is guaranteed to be positive due to (C1), we get

$$\theta_W = g+(1-g)\beta \geq 1-\frac{r}{\lambda}\cdot\frac{b}{\lambda c-\frac{q}{2}} \Rightarrow \underbrace{(1-\beta)(1-g)}_{\text{Probability Match is Bad}} < \frac{\frac{b}{\lambda}}{\frac{\lambda c}{r}-\frac{q}{2r}}. \tag{13}$$

However, as we wish to derive equilibria when bargaining delays are nonzero, things are a bit more complicated. Expression (12), when accounting for bargaining frictions, transforms into the rather less interpretable

[18] To put this in a simpler setting. Suppose that there is no monitoring so that worker type will never be revealed. Type αs have a flow rate of output of 20 and βs a flow rate of output of 10 and there are equal numbers of the two types. Absent a mechanism by which αs can reveal themselves, the, to us, natural equilibrium is for firms to negotiate as if with someone who produces 15 and to reach an agreement on a flow wage rate of 7.5 (as $\Delta \to 0$). However, in the absence of S3, firms could believe that anyone who suggets a wage of, say, more than 5.5 is a β. Knowing that the equilibrium wage for a known β would be 5, no worker would offer a wage above 5.5. We wish to avoid such dictatorial beliefs. Therefore we assume that if a worker offers $5.5+\varepsilon$ instead, the firm would accept it, since it prefers to pay the slightly higher wage immediately than to have 5.5 accepted with a delay, and it does not infer from the offer than the worker is a β since an α would also prefer the higher wage.

$$\text{(C4)} \qquad \frac{b}{\lambda} > (1-\theta_W)\frac{\lambda c}{r} - \frac{q(2-e^{-r\Delta}(1+\theta_W))}{2r}.$$

Unsurprisingly, this condition is still of the form *"monitoring costs must not be too low."* On the left hand side, we have the instantaneous cost of information, $\frac{b}{\lambda}$. On the right hand side, we have the losses to the employer that result from bad matches, which would end if monitoring is successful, minus a measure of production lost to both firing bad workers and delays due to renegotiation when match quality revelation occurs. As an α worker and firm hold different beliefs about the probability of a bad match, the lost production is estimated using their average beliefs. Strictness of the inequality ensures persistence and stability.

This condition ensures that a type α worker (who is more eager for monitoring) does not want to propose monitoring if, having done so, the employer will not update his beliefs. In other words, when $\theta = \theta_W$, αs must prefer not to be monitored rather than using revelation via monitoring to separate themselves from bad workers.

Our second condition ensures that the monitoring equilibrium also exists.

$$\text{(C5)} \qquad \frac{b}{\lambda} < (1-\theta_B)\frac{\lambda c}{r} - \frac{q(2-e^{-r\Delta}(1+\theta_B))}{2r}$$

Antisymmetric to (C4), this condition ensures that, in a 'churned' labor market with a pooling monitoring equilibrium, it is not profitable for α types to deviate to a no-monitoring offer if this would not change the employer's belief. In other words θ_B, the belief about the average ability in the black unemployed pool, must be sufficiently low that α workers prefer to be monitored. Strictness of the inequality ensures that switching to an unchurned market is not simply a matter of switching equilibria (as the non-monitoring one will not exist here). Equivalently to (C4), this condition can be phrased as *"monitoring costs must not be too high."*

For additional intuition, in the limit as $\Delta \downarrow 0$ one can restate this condition as a restriction on β and g:

$$\theta_B = \frac{\beta}{\beta g + (1-g)} < 1 - \frac{r}{\lambda} \cdot \frac{b}{\lambda c - \frac{q}{2}} \Leftrightarrow \underbrace{\frac{(1-\beta)(1-g)}{\beta g + (1-g)}}_{\text{Bad match probability}} > \frac{\frac{b}{\lambda}}{\frac{\lambda c}{r} - \frac{q}{2r}}. \tag{14}$$

Combining the limiting forms of conditions (C4) and (C5), we get

$$(1-\beta)(1-g) < \frac{\frac{b}{\lambda}}{\frac{\lambda c}{r} - \frac{q}{2r}} < \frac{(1-\beta)(1-g)}{\beta g + (1-g)}. \tag{15}$$

The above expression requires that the ratio of the average costs of monitoring to a measure of its benefits lies strictly between the rate of bad matches in the two markets.

The two remaining conditions rule out the complexities associated with partial pooling.

$$\max\{S_{N\beta}, 0\} < e^{-r\Delta} S_{M\beta} \tag{C6}$$

where $S_{N\beta}$ is the surplus from a known β match without monitoring and $S_{M\beta}$ is the surplus from a known β match with monitoring, both derived in Section 3.4.1.

This condition ensures that if the worker is revealed to be of type β, the firm and worker would agree to monitoring, even if it would take a single bargaining period to put this regime in place. It further allows us to say that there is positive surplus from these matches; it is not detrimental to social welfare that β workers are employed at all. The two sides of this equation can be replaced with the primitives of the model at the cost of some clarity. We do this in section 6 where we show that there are parameter values for which all conditions hold and thus that both equilibria can exist.

Assuming (C6) facilitates the proofs in two ways. Since αs benefit more than βs from monitoring, if βs want to reveal themselves in order to be monitored, αs will want to pretend to be βs. This will allow us to dismiss some deviations more easily. Second, in the absence of (C6), it is difficult to rule out mixed strategy equilibria where some β workers reveal themselves to avoid monitoring costs. Furthermore, (C6) will ensure that firms will strive to monitor β workers at least as much as α workers.

Finally, we require that the black equilibrium pooling wage is no lower than the wage β workers could get by revealing their type:

$$\frac{e^{-r\Delta}(q - b - \lambda(1-\theta_B)) - (1 - e^{-r\Delta}\theta_B)qe^{-r\Delta}/\,(2r)}{(1 + e^{-r\Delta})} > w_{M\beta} \tag{C7}$$

This condition says that when the worker types are pooled and monitored, the βs must prefer this to revealing their type and being monitored. As we discuss in more detail later, βs gain from pooling with αs because the firm assigns them a higher expected productivity but lose because it is costly for them to bargain as if they were αs. This Pooling Penalty (C7) can be rewritten as a restriction on g. If g were near 0, θ_B will also be near 0, and the value of the increased expected productivity will be small.

Note that if (C6) and/or (C7) is violated, it will still not be an equilibrium for all βs to reveal themselves. If it were, those not revealing themselves to be βs would be known to be αs and therefore would not be monitored and would earn $.5q$. But this is strictly preferred to any possible outcome for βs. Therefore βs would not reveal themselves. Thus the benefit of these conditions is that they allow us to rule out equilibria in which some, but not all, βs reveal their type. As far as we have been able to determine, allowing for such equilibria would contribute only complexity.

It is not self-evident that (C4)-(C7) can hold simultaneously. In Section 6 we provide an example with plausible parameters in which they do.

4 Solution

We present the main results of the paper: existence and essential uniqueness of equilibria in the two markets that perpetuate their associated steady states.

4.1 The Non-Monitored Market

Proposition 1 *Assuming (C1)-(C7), the white (non-churned) labor market has an employment equilibrium where the monitoring technology is never used and the worker, regardless of type, receives as his total payoff his Rubinstein share of the surplus, with a wage in the limit as* $\Delta \downarrow 0$ *of*

$$w_{N\theta_w} = .5[q - (1 - \theta_W)\lambda c]. \tag{16}$$

Proposition 2 *The above equilibrium is unique.*

All proofs are in the appendix.

The main intuition here flows from (C4), the lack of commitment and S3. On the one hand, (C4) tells us that α workers would only really want to deviate to a monitoring offer if they could affect beliefs by doing so - beliefs are high enough that the wage is already good in equilibrium. On the other hand, if it were possible for α workers to reveal themselves by making a monitoring offer, then as soon as they made it, beliefs would change, and S3 would allow them to make a new, improved no-monitoring offer that β workers would prefer over the equilibrium as well.

Interestingly, since the firm cannot learn the worker's type in this non-churned equilibrium, type has no effect on wages. Instead, all prospective workers are employed indefinitely with identical wages. Beliefs about average worker quality (θ_W) are sufficiently high that sorting the good from the bad is a waste of resources, and even the good workers cannot gain from being monitored to reveal their productivity.

4.2 The Monitored Market

Whether or not the equilibrium involves monitoring, αs are always willing to pay more than βs to be monitored since they know that monitoring will (eventually) reveal them to be good at the job. In the monitored (black) labor market, average job-seeker quality, θ_B, is sufficiently low that the penalty to wages from the firm's expectation of error costs $\lambda c(1 - \theta_B)$ is high and, as part of it is passed on to the workers, this makes αs push for monitoring in order to prove the goodness of the match. But then βs must follow suit lest they be revealed. Therefore, in the monitored equilibrium, all workers must bargain as if they were α types. Since the firm knows that all workers bargain as if they were αs, it treats them as the average "θ_B type " bargaining as αs.

Proposition 3 *Assuming (C1)-(C7), the black (churned) labor market has a monitoring employment equilibrium with a wage limiting to*

$$w_{M\theta_B} = \frac{1}{2}\left[q - b - \lambda c(1-\theta_B)\right] - \frac{(1-\theta_B)}{2}\lambda\frac{q}{2r}. \tag{17}$$

Proposition 4 *This equilibrium is unique.*

Note that since the worker has to bargain as type α regardless of type, he acts as though the probability of promotion is 1 even though the firm treats him as being of average type θ_B. If the worker were truly a "type θ_B," with probability of matching well of θ_B, known to be one and bargained as one, the Rubinstein bargaining solution would substitute the value of unemployment as a θ_B for q/r in the final term. By bargaining as αs, βs allow the firm to extract additional surplus. The additional surplus extracted in the model over the baseline of a "θ_B" type limits to $.5(1-\theta_B)\lambda(.5q - U_{\theta_B})$, the pooling penalty.[19] In contrast, type αs are hurt by pooling with βs not only because the firm expects the flow of output to be lower, as captured by the $\lambda c\,(1-\theta_B)$ term, but also because of the last term, which reflects the difference in beliefs between them and the firm about the probability of the match being good.

As the equilibrium strategies induce full monitoring, employees who are revealed to be in bad matches separate from the firm. This sends only β workers back into the job-seeking pool, giving us the churning process that gives rise to a market quality of θ_B.

The equilibria are summarized in Figure 1.

5 Implications for Labor Markets

The previous sections establish conditions under which there are two distinct steady-states of the labor market. In this section, we compare labor market outcomes for workers in these steady states. Our comparative statics are performed in the limit as the bargaining delay goes to 0.

5.1 Persistence of Discrimination

A key result of the churning mechanism developed in this paper is that deleterious steady states are persistent. In this section we show just how hard it is to transition to a good steady state. We regard this as illustrating the difficulty of addressing labor market discrimination in the context of policy, particularly policy aimed at improving the skills of black workers.

Recall the conjunction of the limits of (C4) and (C5):

$$(1-\beta)(1-g) \leq \frac{r}{\lambda}\frac{b}{\lambda c - \frac{q}{2}} \leq \frac{(1-\beta)(1-g)}{\beta g + (1-g)}. \tag{18}$$

[19] The Pooling Penalty is always positive as the unemployment value of the worker is never higher than the payoff to matching well.

This double inequality also allows us to talk about *persistence* of the deleterious equilibrium. Up to now, we have assumed that average skill levels for the two population groups are identical. Suppose instead that skill levels are $g_B \neq g_W$ and the initial equilibrium for blacks has monitoring but that for whites does not. Monitoring will persist as the equilibrium in the black labor market until the second inequality is reversed, that is to say until

$$g_B > \frac{\lambda (1-\beta)(2\lambda c - q) - 2rb}{\lambda (1-\beta)(2\lambda c - q) - 2(1-\beta) rb}. \tag{19}$$

Note that if β is close to zero, then essentially all new workers must be αs in order to break the bad equilibrium. In contrast, the no-monitoring equilibrium can persist if

$$g_W \geq \frac{\lambda (1-\beta)(2\lambda c - q) - 2rb}{\lambda (1-\beta)(2\lambda c - q)}. \tag{20}$$

Thus, in principle, we could have a setting with discrimination in which the ratio of good workers in the black population to good workers in the white population was as high as

$$\frac{g_B}{g_W} = \frac{\lambda (1-\beta)(2\lambda c - q)}{\lambda (1-\beta)(2\lambda c - q) - 2(1-\beta) rb}. \tag{21}$$

Put differently, we can have the black workers in the bad equilibrium and the white workers in the good equilibrium provided that

$$g_B \leq \frac{g_W}{\beta + (1-\beta) g_W}. \tag{22}$$

To set ideas, suppose that g_W and β both equal .5, then we could observe the black workers in the bad equilibrium if g_B is as large as 2/3. In short, not only may discriminatory markets persist when skill levels for whites and blacks are identical, but they may persist *even when black skill levels are significantly higher*. Policy aimed at accomplishing convergence of labor market outcomes via changes in population skill may fail to clear the hurdle of inertia.

5.2 Wages

Wages are lower for black workers at the point of hiring. Not only do they pay a share of the monitoring cost, they also pay what we dubbed the Pooling Penalty. In addition, each type expects lower lifetime earnings than its white counterparts. To see this, consider the following:

(i) Rearranging (C4), we have that the payoff to αs is higher in the unchurned market for a no-monitoring strategy:

$$\frac{q - \lambda c(1-\theta_W)}{2r} > \frac{q - \lambda c(1-\theta_W) - b + \lambda(1+\theta_W)\frac{q}{2r}}{2(r+\lambda)}. \tag{23}$$

But as the right-hand side of that inequality is increasing in θ, we further have

$$\begin{aligned} & \frac{q - \lambda c(1-\theta_W) - b + \lambda(1+\theta_W)\frac{q}{2r}}{2(r+\lambda)} \\ > \; & \frac{q - \lambda c(1-\theta_B) - b + \lambda(1+\theta_B)\frac{q}{2r}}{2(r+\lambda)} \end{aligned} \tag{24}$$

and therefore

$$\frac{q - \lambda c(1-\theta_W)}{2r} > \frac{q - \lambda c(1-\theta_B) - b + \lambda(1+\theta_B)\frac{q}{2r}}{2(r+\lambda)}, \tag{25}$$

which implies that white αs have a higher ex-ante payoff compared to their black counterparts. As all worker payoff derives from wages, this means that *lifetime wages are lower for black α workers.*

(ii) On the other hand, white α and β workers expect the same lifetime wages. Since β workers value monitoring strictly less than α workers and black α workers are worse off than white ones, black β workers must expect lower lifetime wages than their white counterparts.

Significantly, the model predicts that the realized strategies produce payoffs that maximize the joint surplus of αs and θ-belief firms. This implies that as a function of market θ, payoff to newly matched firms and type α workers is continuous, being an upper envelope of linear functions. However, the strategy shift produces a discontinuity in the payoff to β types forced to follow suit. Figure 2 illustrates this jump.

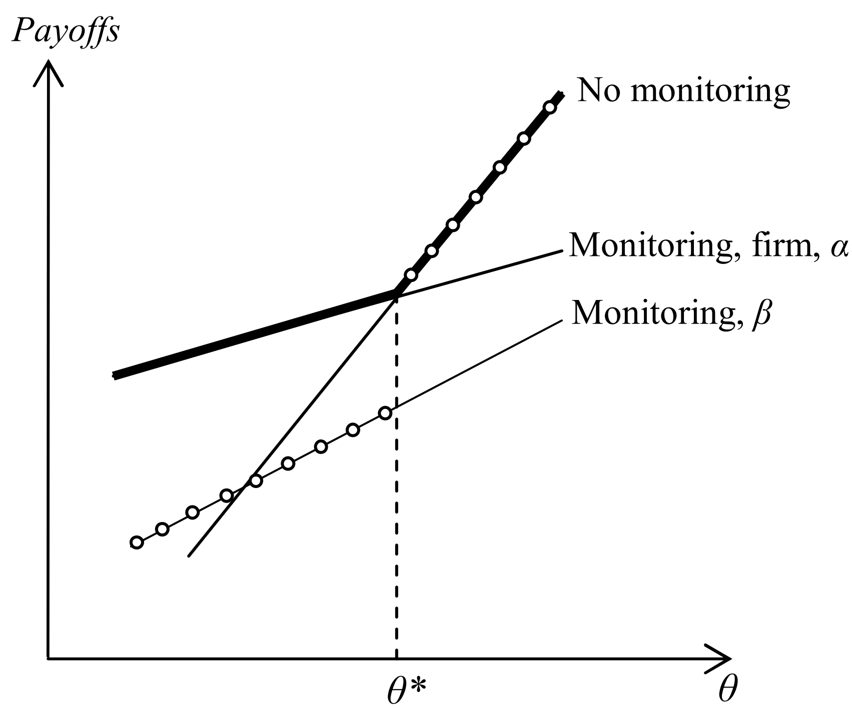

Figure 2: Equilibrium firm and α payoffs: thick line; β payoff: line with circles.

In a sense, because the cost of the sharp discontinuity in the earnings of βs, the model predicts that the return to skill is higher for blacks than for whites, consistent with the empirical findings in

Neal and Johnson (1996) and Lang and Manove (2011). We are reluctant to push this point strongly because the evidence is about either observable ability in the form of education or potentially observable ability in the form of performance on the Armed Forces Qualifying Test. In section 5.5 we consider the case of observable investments.

At the same time, the model predicts that as time goes to infinity, the probability that any single black worker has been revealed to be at a good match and therefore receives a higher wage than white workers goes to 1. We do not regard this property as a proper prediction of the model since it is an artefact of the information structure and infinite lifetimes. In the simplified model of Section 2, worker types are 'perfect' - that is, good workers always enter good matches and bad workers always enter bad matches - but the test is imperfect, producing false positives. In that setting, white workers always receive the high wage that black workers achieve only following successful monitoring. Models with imperfect testing are discussed in Section 5.5.4. This prediction also depends on the bargaining model, in that outside options do not affect the outcome. With a monitoring regime to look forward to at any future job, a black worker revealed to be in a good match could receive less than an unrevealed white worker if the wage varied enough with the outside option. Alternatively, introducing a Poisson death process along with a process by which match quality is revealed even without monitoring albeit at a much slower rate will also promote white workers to the 'revealed good' state at the cost of adding considerable complexity. Moreover, in Section 5.5.5, we discuss a variant of the model in which unlucky black workers can get stuck in low-wage jobs.

5.3 Labor Turnover

When workers are not monitored, there is no new information to dissolve the match. Therefore, taken literally, the model implies no turnover in the white equilibrium. In contrast, when workers are monitored, some workers prove to be ill-suited for the job and return to the unemployment pool. We interpret this as a prediction that black workers will experience lower employment duration on average. Recall that the workers who return to the unemployment pool are all type β workers. Therefore, turnover is even higher than it would be if only new entrants were monitored.

5.4 Unemployment Duration

We have so far treated the workers' matching rate, μ, as exogenous. Making the standard assumption of free entry, we now allow firms to post and maintain vacancies at a cost of k per unit time. When a firm creates a vacancy, it can choose to direct its search. This can take several forms, most notably the choice to locate production operations in an area with specific population characteristics or advertising the vacancy in different areas and through different media. In general, a firm can target markets indexed by i where a proportion ρ_i of unemployed workers are white. The open vacancy cost k is invariant to this target choice. We assume that in each market i the bargaining equilibria and population group steady states break down along the discriminatory lines described so far.

Let the worker job-finding rate function follow the commonly assumed form

$$\mu(\phi) = m\phi^{\gamma} \tag{26}$$

for constants $m > 0$ and $0 < \gamma < 1$. Note that if firms expect a match to be worth V, the free-entry level of ϕ in such a market sets

$$\frac{\mu(\phi)}{\phi}V - k = 0. \tag{27}$$

So

$$\phi = (\frac{Vm}{k})^{\frac{1}{1-\gamma}}. \tag{28}$$

Therefore, ϕ is an increasing function of V.

Assuming that (C6) and (C7) hold for the entire breadth of derived matching rates, we can now derive the free-entry equilibrium level of μ_{ρ_i} for each market i. The payoff to a firm for matching is the same as for an α worker, that is, when hiring from pool i, the firm expects a successful match to pay

$$V_i = \rho_i \frac{q - \lambda c(1-\theta_W)}{2r} + (1-\rho_i)\frac{q - \lambda c(1-\theta_B) - b + \lambda(1+\theta_B)\frac{q}{2r}}{2(r+\lambda)}. \tag{29}$$

Since the payoff to white α workers is higher than for blacks, the expression above is increasing in ρ. Therefore, markets with more black workers will have a lower expected payoff for a filled vacancy. Therefore, the free-entry $\phi(\rho_i)$ and $\mu(\phi(\rho_i))$ are increasing in ρ_i. As average unemployment duration is $\frac{1}{\mu}$, this implies that *markets with higher black concentration will experience higher average unemployment duration.* In the extreme case where markets are fully segregated, that is $\rho_i \in \{0, 1\}$, we can derive the ratio of the matching rates in the two markets:

$$\frac{\mu(\phi(0))}{\mu(\phi(1)} = \left(\frac{q - \lambda c(1-\theta_B) - b + \lambda(1+\theta_B)\frac{q}{2r}}{q - \lambda c(1-\theta_W)} \frac{r}{r+\lambda}\right)^{\frac{\gamma}{1-\gamma}} < 1. \tag{30}$$

5.5 Extensions

We have assumed unrealistically that the match quality of workers who are not monitored is never revealed. More plausibly, heightened scrutiny speeds the rate at which match quality is revealed. In the model in which workers live forever, this change considered in isolation would eliminate our result because the composition of the jobless pool is independent of the rate at which bad matches are revealed. However, if workers do not live forever, then reducing the rate at which match quality is revealed does affect the quality of the unemployment pool, and our basic results go through.

5.5.1 Dead-end jobs

Further, we can allow for observable heterogeneity among workers. If there are groups of workers for whom g is high regardless of race, only the no-monitoring equilibrium will exist for these groups, regardless of race. This will also be true at very low g and very low β (although we have assumed away this case to simplify the proofs). The first result is consistent with similar outcomes for blacks and whites with high levels of skill as measured by education or the Armed Forces Qualifying Test (Neal and Johnson, 1996; Lang and Manove, 2011). The latter is consistent with some evidence that the bottom of the labor market is similarly bad for blacks and whites. On the other hand, Lang and Manove find that the market learns the productivity of white but not black high school dropouts. This is consistent with an equilibrium in which white dropouts are, on average, more skilled than black dropouts and therefore in which white but not black dropouts are monitored. Nevertheless, without additional, largely *ad hoc* assumptions, this story cannot account for the very high unemployment rate among black dropouts.

5.5.2 Investment in unobservable skills

We have heretofore postulated that the proportion of α types is exogenous. Assume instead that some fraction of workers are innately of type α. Others can transform themselves from βs into αs at some cost ω with cdf $F(\omega)$. Provided that the fraction of natural αs satisfies (C4) and (C5), both equilibria will continue to exist. However, since in the no-monitoring equilibrium αs and βs receive the same wage, there is no incentive to invest in becoming an α. In contrast, in the monitoring equilibrium, lifetime earnings are strictly higher for αs than for βs. Thus, some individuals will have an incentive to make the investment.[20] This prediction is in contrast with the model in Coate and Loury (1993), where black workers are less willing to invest in skills.

5.5.3 Education

Suppose now that there exists a signal,[21] which we identify with education, that α workers can purchase at some personal cost $\kappa \sim F(\kappa)$. Assume doing so assures that any employer will be immediately aware that the worker is indeed type α. A worker of either population will then anticipate a lifetime utility of $V_{Educ}(\kappa) = \mu q / (2r(\mu + r)) - \kappa$. In Section 5.2 we showed that unrevealed white α workers receive a higher lifetime payoff than their black counterparts; therefore, the incentive for the latter to invest in education is greater. As this implies that $\kappa_W \equiv max\{\kappa : V_{Educ}(\kappa) \geq V_W^\alpha\} < max\{\kappa : V_{Educ}(\kappa) \geq V_B^\alpha\} \equiv \kappa_B$, we must have that $F(\kappa_W) < F(\kappa_B)$ and

[20] It might appear that the incentive to undertake such investments would unravel the monitored equilibrium. However, if this were the case, no worker would have an incentive to invest. This raises messy dynamic issues which we sidestep by assuming that the fraction of additional workers who would choose to invest is insufficient to overturn (C5).

[21] We analyze the case of a pure signal. if education can also turn an α into a β, the analysis is a combination of the anlysis in this and the prior subsection since productive investment increases the fraction of workers who are α but investment that reveals workers to be α reduces the fraction of unrevealed workers who are α.

therefore more black workers will purchase education. In particular, there exists some range of idiosyncratic costs for which black workers will purchase education but white workers will not. This is consistent with the finding in Lang and Manove (2011) that, conditional on past test scores, blacks get more education than whites do. The intuition here is simple; if a worker of high skill is treated as if she has the average hire's skill for her group, she has a greater incentive to reveal her high skill if that average is lower.[22]

Perhaps equally importantly, this extension suggests that blacks and whites with high observable skills will have similar outcomes as discussed in the previous subsubsection.

5.5.4 Imperfect monitoring

The astute reader will have noted that the intuitive example presented first is distinct from the main model where monitoring resolves all uncertainty about worker type. As the example demonstrates, it is possible to write a very similar model in which beta workers always match badly but monitoring can result in false positive good matches. Given a wage-determination mechanism with outcomes similar to our bargaining protocol, much of the analysis would remain unchanged.[23] Parameters would exist that would force monitoring on blacks but not whites, the black labor market would churn, and it would produce higher unemployment duration and lower lifetime wages for blacks. In this formulation, black workers succeeding at monitoring would only be as good as whites who had never been monitored; therefore a churned market does not necessarily produce better long-run matches or higher wages for the successfully monitored.

However, this alternate model would imply that some workers are purely parasitic and cannot be matched well, but rather aspire simply to find a job where their lack of productivity is undiscovered. An equivalent of (C6) cannot hold here and as a result we cannot rule out equilibria where negotiations sometimes break down and separation occurs without monitoring producing information.

5.5.5 Stigma and degeneration into lower-skilled jobs

Our model unrealistically assumes that employers have no information regarding the time that workers have been in the labor market or the number of jobs they have held. If the other aspects of our model were a rough representation of reality, it is implausible that firms would not recognize that some workers were unlikely to be new entrants and therefore very likely to β types. So far we have assumed that βs remain with the firm and are monitored. However, suppose instead that if a worker is sufficiently likely to be a β, it is not efficient to employ or monitor him. Then workers

[22] Strictly speaking, this creates a feedback loop from lower wages for the uneducated to a greater measure of education. The right assumptions on F rule out associated complexities.

[23] Unfortunately, this alternate model would add a lot of complexity and require additional assumptions for uniqueness, due to the lack of a single posterior following succesful monitoring. Barganing strategies would have a much more tangled relation to beliefs and wages, and S3 would not be an apt tool to facilitate the task.

who do not find a good match sufficiently quickly will be permanently barred from the monitoring sector.

Somewhat more formally, as an extension to the model, we can relax the assumption that past history is entirely unobservable. Assume instead that each separation has a probability ζ of becoming public common knowledge. Any worker who has a revealed separation is known to be of type β in any new match. Thus, a newly hired worker who does not have such a stigma will be of average quality $\theta'_B = [\beta + g\zeta(1-\beta)] \,/\, [g\beta + (1-g) + g\zeta(1-\beta)]$. If we assume θ'_B satisfies (C5), churning can persist but will be primarily a phenomenon for relatively young workers.

But what will happen to workers revealed to be βs? It is straightforward to extend the model to allow for a second occupation type (q', c') lacking monitoring technology[24] that is less skill intensive than the task described so far, i.e. $q > q'$ and $q - \lambda c < q' - \lambda c'$. As unrevealed β types are strictly better off than revealed ones in a new match of the first task, there must be q', c' such that the revealed β types prefer to enter the job market for the second occupation but the unrevealed ones do not.

In this scenario, a fraction of black workers are relegated to low-wage jobs while white workers with similar skills can always get better jobs. Furthermore, since the low-wage jobs are not monitored, they are a terminal state, with no possibility of promotion or escape.

6 Example

Here we provide a simple numerical example satisfying our conditions.

Take $r = .05$, suggesting a unit of time of about a year. $\lambda = 2$ so that the average unsuccessful job lasts six months. $\beta = .2$ and $g = .95$, implying that most workers are good matches. $c = .5$ and $q = 1$ so that bad matches produce expected flow output of 0, making separation efficient regardless of unemployment duration. Finally, $b = 1$ so that $b/\lambda = 1/2$, making the expected cost of monitoring roughly equal to the value of six months of output from a well-matched worker.

It is readily verified numerically that all four conditions hold.

These parameters result in a θ_W of .96 and churning produces a θ_B of .833. The value of a filled vacancy in the white market is 9.6 and in the black labor market 8.9. Postulating a Cobb-Douglas matching function with elasticity $\eta = .75$, the model predicts a black-white unemployment duration ratio of 1.25. We can now compute the ratio of white to black income PDV at birth to be 1.11.

This example illustrates that a churning equilibrium is possible even if the proportion of type β workers is quite low in the population, and can generate reasonable income and unemployment disparities while doing so.

[24] Or, more palatably, the same technology but without the incentives to use it, as in the case of a small enough c'.

7 Conclusions

Our model in some ways resembles models of adverse selection in the labor market. Displaced workers are worse, on average, than a randomly selected worker. However, in contrast with standard adverse selection models, firms cannot distinguish between displaced workers and other unemployed workers. Therefore displaced workers depress the wage of all unemployed workers. At the same time, our approach does not generate the asymmetry of information among firms that drives adverse selection models. If the worker is known to be good at a particular job, he will not leave for another job even if he knows that, on average, he will be good at other jobs. If the worker is bad at this particular job, he separates immediately.

To keep the analysis simple, our model assumes that workers who turn out to be well-matched remain employed forever. At first blush, this suggests that it applies only to new entrant unemployment because the market will surely recognize that a fifty-year old worker is not a new entrant. We believe it is more realistic to assume that the market cannot tell whether a fifty-year old worker who was laid-off six months ago has just been unlucky and not had any matches or has had a match that turned out to be bad. The market often cannot tell how long the worker has been unemployed. Thus we think the model is more general. In addition, it provides some insight into the scarring effect of unemployment although the simple framework we have used does not allow for such effects.

Unlike most, perhaps all, existing models, ours can explain a number of empirical regularities regarding discrimination simultaneously:

1. Black workers have longer unemployment durations.
2. Black workers have shorter employment durations.
3. Black workers have lower lifetime earnings.

As written, the model has infinitely lived matches and agents so there is no unemployment "rate." Allowing for deaths, we would have well-defined unemployment rates and would predict the one for black workers is higher. If one is also in the world of extension 5.5.1, black workers of type α and lucky βs will be paid much more than their unfortunate counterparts, consistent with some evidence that the return to skill is higher for black workers than for white workers especially at the upper end of the skill distribution.

Our model also strongly suggests that history matters and that equality of opportunity is not enough to eliminate racial disparities in the labor market even if this concept is used very expansively. The fact that blacks historically had low skills leads to an equilibrium in which the pool of black job seekers has lower skills than the pool of white job seekers even when the distribution of skills among all workers is identical for blacks and whites. While, over time, a human capital-based policy could eliminate labor market discrimination, achieving equality in human capital is not sufficient to racial disparities in the labor market.

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A Appendix

A.1 Lemma 1: Payoff after revelation of a good match

Consider an equilibrium match that has just been revealed to be good at t. For revelation to have just occurred, the currently active offer involves monitoring.

If renegotiation occurs as per case 2 in Section 3.4.1 the proposer will receive $q/\left(r\left(1+e^{-r\Delta}\right)\right)$. The payoff to triggering renegotiations is obtained by discounting this by $e^{-r\Delta}$.

Assume that renegotiation never occurs in equilibrium; then the current monitoring offer persists forever, yielding a total surplus of $\frac{q-b}{r}$. Assuming that neither player wants to reopen negotiations, if the current wage in place is w, we must have that

$$\min\{\frac{w}{r}\ ,\ \frac{q-b-w}{r}\} \geq \frac{1}{1+e^{-r\Delta}}\frac{q}{r}$$

For any current wage w, the greatest $\min\{w/r\ ,\ (q-b-w)/r\}$ can be is $(q-b)/(2r)$; thus for renegotiation to never occur we require that $(q-b)/(2r) \geq q/\left(r\left(1+e^{-r\Delta}\right)\right) \Leftrightarrow (1+e^{-r\Delta})b < (1-e^{-r\Delta})q$, which is the negation of (C2) and therefore a contradiction.

But as one's opponent reopening negotiations gives the receiver's share of the new bargain, $e^{-r\Delta}qe^{-r\Delta}/\left(1+e^{-r\Delta}\right)$, it becomes even harder to satisfy the requirement to not renegotiate instantly with probability 1 if one's opponent may trigger renegotiation; therefore both instantly triggering renegotiation is the only equilibrium. As this means that each player has a probability 1/2 of being first proposer following revelation, each player at the instant of revelation has an expected payoff of

$$\frac{1}{2}\cdot\frac{qe^{-r\Delta}}{1+e^{-r\Delta}}+\frac{1}{2}\cdot e^{-r\Delta}\frac{qe^{-r\Delta}}{1+e^{-r\Delta}}=\frac{qe^{-r\Delta}}{2}.\ \square$$

A.2 Lemma 2: Makeup of the monitored market's job-seeking pool

Define the quantities

ξ	Flow mass of workers born per unit time
A	Mass of unemployed black type α workers
B	Mass of unemployed black type β workers
Λ	Mass of currently monitored black type β workers

As g is the fraction of new workers that is type α and unemployed α workers are becoming employed each at a Poisson rate μ and are never fired, A obeys

$$\frac{dA}{dt} = \xi g - \mu A$$

Similarly, a proportion $(1-g)$ of new workers is type β and such unemployed workers are also being hired at a Poisson rate μ each. However, as Λ workers who are of type β are being monitored, a flow mass $\Lambda\lambda(1-\beta)$ of black β workers are being fired after monitoring reveals a bad match are also coming in to the black unemployed pool. Hence, B obeys

$$\frac{dB}{dt} = \xi(1-g) - \mu B + \Lambda\lambda(1-\beta)$$

.

Finally, unemployed β workers are becoming employed with monitoring at a Poisson rate μ and once they are employed they cease being monitored when match quality is revealed, which occurs at a rate λ. Thus the mass of monitored black β workers Λ must satisfy

$$\frac{d\Lambda}{dt} = \mu B - \Lambda\lambda$$

Steady state implies that

$$\frac{dA}{dt} = \frac{dB}{dt} = \frac{d\Lambda}{dt} = 0$$

Solving, we obtain

$$A = \frac{\xi g}{\mu}$$

$$B = \frac{\xi(1-g)}{\mu\beta}$$

and therefore the proportion of α workers in the unemployed pool is

$$\frac{A}{A+B} = \frac{\frac{\xi g}{\mu}}{\frac{\xi g}{\mu} + \frac{\xi(1-g)}{\mu\beta}} = \frac{g}{g + \frac{1}{\beta}(1-g)}.$$

Thus, a new match from the black job-seeker pool is of average quality

$$\frac{g}{g+\frac{1}{\beta}(1-g)} \cdot 1 + \left(1 - \frac{g}{g+\frac{1}{\beta}(1-g)}\right) \cdot \beta = \frac{\beta}{\beta g + (1-g)} \equiv \theta_B$$

As $\beta < 1$ this is less than θ_W.$\square$

A.3 Proof of Proposition 1

The equilibrium wage proposed is

$$w_{N\theta_W}^{work} = \frac{1}{1+e^{-r\Delta}}(q - \lambda(1-\theta_W)c)$$

if the worker proposes first and

$$w_{N\theta_W}^{firm} = \frac{e^{-r\Delta}}{1+e^{-r\Delta}}(q - \lambda(1-\theta_W)c)$$

if the firm proposes first. As there will be no revelation, these shares split the expected output (using firm beliefs) equally. This equilibrium is supported by firm beliefs that are invariant to all contingencies before revelation.

At a pre-revelation history where an off-path offer is on the table, or where one is already in place, it is accepted/not renegotiated if it does not involve monitoring and the wage w satisfies

$$w_{N\theta_W}^{work} \geq w \geq w_{N\theta_W}^{firm}$$

If this condition does not hold at the off-path history in question, then, if in negotiations, the current proposer plays the equilibrium offer; otherwise, both players' strategy is to instantly reopen negotiations; and when they do, the equilibrium offer will be proposed.

At off-path histories where the match is revealed to be good, if the wage w satisfies $\frac{1}{1+e^{-r\Delta}}q \geq w \geq \frac{e^{-r\Delta}}{1+e^{-r\Delta}}q$ it stays in place; otherwise, play proceeds as in Lemma 1, granting an expected $qe^{-r\Delta}/(2r)$ to each party.

As discussed in Section 3.4.1.1, off-path histories that led to the revelation of a bad match lead to termination of the match.

A party who deviates before revelation can at most, therefore, transition from the receiver's share to the proposer's share of the match surplus, as one's opponent's strategy will not accept worse offers. Doing so, however, occasions a single delay, which discounts the payoff from such a deviation to exactly the receiver's payoff, which is the least the deviator could have started with. Therefore, there is no deviation that will strictly increase the agents' payoff and the strategies described are mutual best responses.

To show that there is no S3-type deviation that proposes monitoring, it remains to show that an α worker or a firm cannot propose a mutually beneficial monitoring regime.

Lemma 1 pins down continuation payoffs from being in a match revealed to be good. Thus, an α worker making an offer of w_u with monitoring in place yields to this worker, in the absence of renegotiation until match quality revelation,

$$V_{M\alpha}(w_u) = \frac{w_u + \lambda\frac{qe^{-r\Delta}}{2r}}{\lambda + r}. \tag{31}$$

The requirement for S3 is stated in terms of beliefs remaining constant, in this case at θ_W. Thus, an employer accepting this offer expects a payoff of

$$F_{M\theta_W}(w_u) = \frac{q - b - (1-\theta_W)\lambda c - w_u + \lambda\theta_W \frac{qe^{-r\Delta}}{2r}}{\lambda + r}. \tag{32}$$

Summing (31) and (32), we get

$$\frac{q - b - (1-\theta_W)\lambda c + \lambda(1+\theta_W)\frac{qe^{-r\Delta}}{2r}}{\lambda + r} \tag{33}$$

For such a deviation to violate S3 necessarily (33) has to be greater than equilibrium payoffs; this can only be the case if

$$\frac{b}{\lambda} < (1-\theta_W)\frac{\lambda c}{r} - \frac{q(2 - e^{-r\Delta}(1+\theta_W))}{2r} \tag{34}$$

which is precluded by assumption (C4).

That this equilibrium does not violate S4 is immediate from the fact that if β workers preferred to reveal themselves, they would be monitored. As monitoring is strictly better for α workers, they too would wish to follow suit. Similarly, if α workers could reveal themselves, following negotiations they would earn a wage as per Lemma 1, which, as it is higher than the equilibrium wage, would be preferable to β workers as well.

In the limit as $\Delta \downarrow 0$, the equilibrium shares of the first proposer and receiver equalize; the limiting wage is $w_{N\theta_W} = .5q - .5(1-\theta_W)\lambda c$.

A.4 Proof of Proposition 2

Consider strategies for the firm and each type of worker that may in principle involve renegotiation and different monitoring for each type of worker. Call $\tilde{m}_\alpha$ and $\tilde{m}_\beta$ the expected discount at the point of match quality revelation for α and β workers given these strategies and let $\tilde{W}_\alpha$ and $\tilde{W}_\beta$ be the total expected discounted wages before such revelation occurs.

Equilibrium requires incentive compatibility: each worker weakly prefers the strategy they actually adopt to the other type's; hence, α's IC requires

$$\tilde{V}_\alpha = \tilde{W}_\alpha + \tilde{m}_\alpha \frac{qe^{-r\Delta}}{2r} \geq \tilde{W}_\beta + \tilde{m}_\beta \frac{qe^{-r\Delta}}{2r} \tag{35}$$

and the β's IC imposes

$$\tilde{V}_\beta = \tilde{W}_\beta + \tilde{m}_\beta\Big(\beta\frac{qe^{-r\Delta}}{2r} + (1-\beta)U_\beta\Big) \geq \tilde{W}_\alpha + \tilde{m}_\alpha\Big(\beta\frac{qe^{-r\Delta}}{2r} + (1-\beta)U_\beta\Big) \tag{36}$$

Combining the two and rearranging terms

$$\begin{aligned}(\tilde{m}_\beta - \tilde{m}_\alpha)(1-\beta)\left(U_\beta - \frac{qe^{-r\Delta}}{2r}\right) &\geq \\ \left(\tilde{W}_\alpha + \tilde{m}_\alpha \frac{qe^{-r\Delta}}{2r}\right) - \left(\tilde{W}_\beta + \tilde{m}_\beta \frac{qe^{-r\Delta}}{2r}\right) &\geq 0 \end{aligned} \tag{37}$$

Since from (C3) $U_\beta < qe^{-r\Delta}/2r$, we have that $\tilde{m}_\alpha \geq \tilde{m}_\beta$.

The firm's payoff in any such equilibrium given $(\tilde{W}_\alpha, \tilde{m}_\alpha, \tilde{W}_\beta, \tilde{m}_\beta)$ is [25]

$$\begin{aligned}\tilde{F} &= g\Big(\frac{q}{r}(1-\tilde{m}_\alpha) - \tilde{W}_\alpha + \tilde{m}_\alpha \frac{qe^{-r\Delta}}{2r} - \tilde{m}_\alpha \frac{b}{\lambda}\Big) + \\ &\quad (1-g)\Big(\frac{q-(1-\beta)\lambda c}{r}(1-\tilde{m}_\beta) - \tilde{W}_\beta + \tilde{m}_\beta \beta \frac{qe^{-r\Delta}}{2r} - \tilde{m}_\beta \frac{b}{\lambda}\Big)\end{aligned} \tag{38}$$

S3 requires that deviating first offers cannot be made that improve the payoff of an α worker and the firm. As the wage can transfer surplus freely, this implies that the sum of candidate equilibrium payoffs are weakly greater than the sum of those feasible by a no-monitoring offer[26]:

$$\tilde{V}_\alpha + \tilde{F} \geq \frac{q-(1-\theta_W)\lambda c}{r} \tag{39}$$

Expanding,

$$\begin{aligned}\tilde{V}_\alpha + \tilde{F} &= (1-g)(\tilde{W}_\alpha - \tilde{W}_\beta) + \tilde{m}_\alpha((1+g)qe^{-r\Delta} - \frac{gb}{\lambda} - \frac{gq}{r}) \\ &\quad + \tilde{m}_\beta(1-g)(-\frac{q-(1-\beta)\lambda c}{r} + \frac{\beta qe^{-r\Delta}}{2r} - \frac{b}{\lambda})\end{aligned} \tag{40}$$

Retrieving

$$\tilde{W}_\alpha - \tilde{W}_\beta \leq (\tilde{m}_\beta - \tilde{m}_\alpha)\Big(\beta \frac{qe^{-r\Delta}}{2r} + (1-\beta)U_\beta\Big)$$

by rearranging (36), we substitute it into (40) we arrive at an expression weakly greater than the LHS of (39) where the coefficient of $\tilde{m}_\beta$ is

$$(1-g)\frac{1}{r}[\lambda c(1-\beta) - q(1-e^{-r\Delta}\beta) + r(1-\beta)U_\beta - \frac{rb}{\lambda}] \tag{41}$$

[25] Here it is revelant to point out that if the expected discount on revelation is m, then the expected cost of monitoring until revelation is mb/λ. This is for the following reason: to say the expected discount on revelation is m is to say that for probability $M(t)$ of monitoring at time t conditional on no revelation by t, $\int_0^\infty e^{-rt}e^{-\int_0^t \lambda M(t')dt'}\lambda M(t)dt = m$; but the amount spent on monitoring at each time is the probability no revelation occurs until that time multiplied by the probability monitoring occurs at that time, the discount and the flow cost of monitoring; hence the total cost is $\int_0^\infty e^{-rt}e^{-\int_0^t \lambda M(t')dt'}bM(t)dt = mb/\lambda$.

[26] Notice that while the candidate equilibrium is allowed to generate value from screening worker types by strategies and therefore apply monitoring more efficiently, the deviations S3 checks against are not. That it turns out such deviations are enough to destroy all equilibria but one is a product of the β workers' incentives to not reveal themselves.

which we know from (C6) is positive. Therefore, up to the constraint imposed by (35) we have an upper bound of the LHS of (39) increasing in $\tilde{m_\beta}$. So if (39) holds for some $(\tilde{m_\alpha}, \tilde{m_\beta})$, it must hold for $(\tilde{m_\alpha}, \tilde{m_\alpha})$. Making this substitution, we arrive at

$$\frac{q-(1-\theta_W)\lambda c}{r}(1-\tilde{m_\alpha}) + \tilde{m_\alpha}\frac{(1+\theta_W)qe^{-r\Delta}}{2r} - \tilde{m_\alpha}\frac{b}{\lambda} \geq \frac{q-(1-\theta_W)\lambda c}{r} \tag{42}$$

which due to (C4) can only occur if $\tilde{m_\alpha} = 0$.

Therefore, regardless of the first proposer, all equilibria in the white labor market lack monitoring for both types of workers. We can further exclude equilibria with delay, as an S3-type deviation giving the receiver his equilibrium utility and the proposer taking the excess would be payoff-increasing in those cases.

Finally, by S3, no deviation by a first receiver that gives the first proposer his payoff when proposing, discounted, is gainful. Therefore, the first proposer's share cannot be greater than $\frac{1}{1+e^{-r\Delta}}$. Similarly, the initial proposer i cannot be getting $x < \frac{1}{1+e^{-r\Delta}}$, lest j have a deviating offer in his own role as first proposer giving i his discounted value, $e^{-r\Delta}x$ and j a share of $1 - xe^{-r\Delta} > \frac{1}{1+e^{-r\Delta}}$.

Thus, all equilibria of the white labor market reach immediate agreement with a no-monitoring offer; the wage splits the surplus along the Rubinstein shares and therefore the equilibrium of Proposition 1 is essentially (up to off-path behavior and beliefs) unique.

A.5 Proof of Proposition 3

The initial equilibrium wage proposed is

$$w^{work}_{M\theta_B} = [q - b - \lambda(1-\theta_B) - (e^{-r\Delta} - \theta_B)qe^{-r\Delta}/\,(2r)]/\left(1 + e^{-r\Delta}\right)$$

if the worker proposes first and

$$w^{firm}_{M\theta_B} = [e^{-r\Delta}(q - b - \lambda(1-\theta_B)) - (1 - e^{-r\Delta}\theta_B)qe^{-r\Delta}/\,(2r)]/\left(1 + e^{-r\Delta}\right)$$

if the firm proposes first. Monitoring is in use until revelation, and no renegotiation takes place until then.

If the worker rejects a firm monitoring with a wage in $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$, opens renegotiation when a monitoring regime with a wage in that interval is in place, or makes or accepts a non-monitoring offer before revelation, the firm immediately believes the worker to be type β. This change is irreversible. Otherwise, the firm has beliefs constant at θ_B.

When the firm starts believing the worker to be type β, both parties immediately renegotiate to the equilibrium in 3.4.1.4 with monitoring and a lower wage.

As long as beliefs are θ_B, agents in the role of proposer offer their $w^i_{M\theta_B}$. Monitoring offers with wages in $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$ wages are accepted by either party without renegotiation until revelation. Other offers are rejected or renegotiated, and the next offer is the proposer's $w^i_{M\theta_B}$.

If revelation occurs, bad matches separate; good matches renegotiate as per Lemma 1.

Clearly, workers don't want to deviate to propose in $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B})$ as they will lead to acceptance but a lower payoff; also, they don't want to wages outside $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$ as the firm will reject and propose $w^{firm}_{M\theta_B}$ in addition to suffering the delay. Thus, always proposing $w^{work}_{M\theta_B}$ is optimal. Workers won't reject offers in $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$ or accept or propose non-monitoring offers as they don't want to be treated as βs as per (C6) and (C7).

Firms know that by the workers' strategy, the highest offer they can get accepted is $w^{firm}_{M\theta_B}$ and that higher ones, or ones below $w^{work}_{M\theta_B}$, will be rejected and that the worker will counter-offer $w^{work}_{M\theta_B}$, in addition to the firm suffering a delay. Firm offers in $(w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$ will be accepted but yield a lower payoff than $w^{firm}_{M\theta_B}$; thus always proposing $w^{firm}_{M\theta_B}$ is optimal for the firm. Given this, the firm accepts offers in $[w^{firm}_{M\theta_B}, w^{work}_{M\theta_B}]$; but will reject higher ones because it can do better as proposer, and lower ones because it knows renegotiation will be imminent once they are in place. If production is occurring, the firm can gain by renegotiating if either (a) the worker will instantly renegotiate, and the firm's first offer here is a lower wage than the worker's (so as above, if $w < w^{firm}_{M\theta_B}$), or if the firm's payoff from making its offer, $w^{firm}_{M\theta_B}$, with delay, is preferable to the current payoff; but that is precisely when the current wage $w > w^{firm}_{M\theta_B}$.

That there is no S3-type deviation that proposes no monitoring follows from (C4).

As the total surplus to an α worker and a firm with belief θ_B in this match is greater than the total surplus to a revealed β match, the wage difference by proposer is larger for the former. Therefore, for β workers not to wish to reveal themselves, satisfying S4, it suffices that they do not wish to do so in the role of receiver. The receiver's share in a revealed β match is $e^{-r\Delta}[q - b - \lambda c(1-\beta) + (1-\beta)U_\beta + \lambda\beta q e^{-r\Delta}/r]/\left(1+e^{-r\Delta}\right)$. That this is less than the value β workers get in the receiver's role in the pooled equilibrium is the content of (C7); therefore S4 is not violated.

In the limit as $\Delta \downarrow 0$, the equilibrium shares of the first proposer and receiver equalize; the limiting wage is $w_{M\theta_B} = \frac{1}{2}\left[q - b - \lambda c(1-\theta_B)\right] - \frac{(1-\theta_B)}{2}\lambda\frac{q}{2r}$.

A.6 Proof of Proposition 4

The proof here proceeds in the same fashion as that in A.4. Instead of comparing to a no-monitoring deviating offer we compare to a monitoring offer; therefore instead of 42 we have

$$\frac{q-(1-\theta_B)\lambda c}{r}(1-\tilde{m}) + \tilde{m}\frac{(1+\theta_B)qe^{-r\Delta}}{2r} - \tilde{m}\frac{b}{\lambda} \geq \frac{q-(1-\theta_B)\lambda c - b + \frac{(1+\theta_B)qe^{-r\Delta}}{2r}}{\lambda + r} \tag{43}$$

which due to (C5) can only be true if $\tilde{m} \geq \frac{\lambda}{\lambda+r}$; but as $\tilde{m}$ is an expected discount of a variable that at most arrives as a Poisson with rate λ, this constitutes an upper bound to $\tilde{m}$ and corresponds to full monitoring and no delay.

Therefore, only fully monitoring equilibria exist in the black labor market. Within such candidate equilibria, S3 would allow for deviation from any initial offer not corresponding to that in Proposition 3, therefore that equilibrium is unique.