Indeterminacy in Sovereign Debt Markets:
A Quantitative Analysis[*]

Luigi Bocola

Northwestern University

Alessandro Dovis

Pennsylvania State University

and

NBER

July 2015

Abstract

This paper shows how a benchmark model of sovereign debt can be used to measure the importance of non-fundamental risk in driving interest rate spreads. The model has three main ingredients: choices regarding the maturity of debt, risk averse lenders and rollover crises á la Cole and Kehoe (2000). In this environment, lenders' expectations of a default can be self-fulfilling, and market sentiments contribute to variation in interest rate spreads along with economic fundamentals. We show that maturity choices of the government are informative about the prospect of a future self-fulfilling crisis. The government can, in fact, partially insure from these inefficient runs by lengthening the maturity of its debt. Hence, when rollover problems are pressing, we should observe an increase in debt duration. We calibrate the model to fit key features of the euro-area sovereign debt crisis. The calibrated model is used, along with the observed pattern of debt duration, to measure the importance of rollover risk over the episode. Our results have implications for the effects of the liquidity provisions announced by the European Central Bank during the summer of 2012.

1 Introduction

The summer of 2012 marked one of the major developments of the Eurozone sovereign debt crisis. After a period of sharp increases, in August 2012, interest rate spreads of peripheral countries declined to almost their pre-crisis level. These declines have been attributed to the establishment of the Outright Monetary Transaction (OMT) program, a framework that gives the European Central Bank (ECB) powers to purchase sovereign bonds in order to prop up their prices. One reading of these events is that the establishment of the OMT program was successful in dealing with coordination failures among bondholders. By promising to act as a lender of last resort, the argument goes, the ECB reduced the scope for self-fulfilling debt crises, bringing back bond prices to the value justified by economic fundamentals.

This is not, however, the only interpretation. Indeed, the high interest rate spreads observed in Europe could have purely been the results of poor economic conditions. A credible announcement by the ECB to sustain prices in secondary bond markets would still produce a decline in interest rate spreads, because it would eliminate downside risk for bondholders. However, it would also generate inefficient moral hazard. Unlike the coordination failure view, this moral hazard view implies excessive borrowing by governments and the potential of balance sheet risk for the ECB. Therefore, any assessment of these interventions needs first to address a basic question: were interest rate spreads in the euro-area periphery the result of confidence driven fluctuations, or were they due to bad economic fundamentals? This paper takes a first step toward answering this question by bringing a benchmark model of sovereign borrowing with self-fulfilling debt crisis to the data and applying it to the debt crisis in the euro area.

We consider the canonical model of sovereign borrowing in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). In our environment, a government issues debt of different maturities in order to smooth out endowment risk. We follow Cole and Kehoe (2000) and assume that the government cannot commit to repaying its debt within the period. This opens the door to self-fulfilling debt crises: if lenders expect the government to default and do not buy new bonds, the government may find it too costly to service the stock of debt coming due, thus validating the expectations of lenders. This can happen despite the fact that a default would not be triggered if lenders held more optimistic expectations about the government's willingness to repay. These rollover crises can arise in the model when the stock of debt coming due is sufficiently large and economic fundamentals are sufficiently weak.

As commonly done in the literature, we assume that this indeterminacy in the *crisis*

zone is resolved by the realization of a coordination device.[1] Our selection rule consists of a Markov process for the probability that lenders coordinate on the bad equilibrium when the economy falls in the crisis zone. Conditional on this selection rule, the equilibrium is unique. In our set up, default risk varies over time because of "fundamental" and "non-fundamental" uncertainty. More specifically, default risk may be high because lenders expect the government to be insolvent in the near future- they expect that the government will default irrespective of their behavior. Or, it may be high because of the expectation of a future inefficient rollover crisis. The goal of our exercise is to distinguish these different sources of default risk.

The first contribution of this paper is to point out that the maturity choices of governments provide key information to accomplish this goal. Our argument builds on two properties of canonical sovereign default models. First, if default risk mostly reflects the prospect of a future rollover crisis, then the government has incentives to *lengthen* the maturity of its debt: by doing so, it reduces the amount of debt that needs to be serviced in the near future, thus decreasing the possibility of a self-fulfilling debt crisis. Second, if default risk is purely the result of bad economic fundamentals, the government has incentives to *shorten* the maturity of its debt. This is due to the combination of two effects. On the one hand, as emphasized by Arellano and Ramanarayanan (2012) and Aguiar and Amador (2014), short term debt is more effective in providing the government with incentives to repay. By shortening the maturity of its debt, the government can obtain better terms from the lenders, and this is desirable as the government is credit constrained during a debt crisis. On the other hand, Dovis (2014) shows that the need to hold long term debt for insurance reasons falls when default risk increases. These two properties imply that fundamental and non-fundamental sources of default risk predict a different comovement pattern between interest rate spreads and debt duration during a crises: the two variables are positively associated when rollover risk is an important driver of interest rate spreads, they are negatively correlated otherwise. Their joint behavior is therefore informative about the underlying sources of default risk.

The second contribution of our paper is to make this insight operational. Indeed, a key problem in using these identifying restrictions is that the relationship between interest rate spreads and debt duration is not only a product of government's incentives, but also depends on the lenders' attitude toward risk. Broner et al. (2013) document that sovereign debt crisis are typically accompanied by an increase in term premia. Neglecting these shifts could undermine our identification strategy: rollover risk could be an important driving force of interest rate spreads and yet we could observe a shortening of debt du-

---

[1]The term *crisis zone* indicates the region of the state space where self-fulfilling crises are possible.

ration simply because lenders demand high compensation to hold long term risky bonds. To address this issue, we deviate from most of the quantitative literature on sovereign debt and allow for time-varying term premia by introducing shocks to the lenders' stochastic discount factor. In doing so we follow a large literature on affine models of the term structure of interest rates (Piazzesi, 2010), specifically the approach in Duffie et al. (1996), Backus et al. (2001), and Bekaert and Grenadier (1999).[2]

We apply our framework to the recent sovereign debt crisis in Italy. We estimate the stochastic process governing the country's output and the lenders' stochastic discount factor using the method of simulated moments. Specifically, we ask the implied pricing model to reproduce key statistical features of the joint behavior of Italian detrended real GDP, the term structure of German's zero coupon bonds and the stock price-consumption ratio for the euro area. Implicit in our approach is the assumption that financial markets in the euro area are sufficiently integrated and that the lenders in the model are the marginal investors for other assets beside Italian government securities. The pricing model considered is parsimonious, but it is flexible enough to capture the episode of low yields over short term riskless assets and high yields over long term assets (both stocks and bonds) recently observed in our sample. The parameters of the government's decision problem are borrowed from previous research in the area.

We next measure the importance of non-fundamental risk in driving Italian spreads during the recent sovereign debt crises. Using a preliminary calibration, we apply a filter to our model and we estimate the path of the model's state variable over our sample. Given this path, we are able to decompose observed interest rate spreads into a component reflecting the expectation of a future rollover crisis and a component due purely to fundamental shocks. We document that the combination of high risk premia and bad domestic fundamentals can account for the most of the run-up in interest rate spreads observed during the 2011-2012 period. However, we show that neglecting the informational content of maturity choices results in substantial uncertainty over the split between fundamental and non-fundamental sources of default risk, as the model lacks identifying restrictions to discipline the risk of a rollover crisis.

Finally, we show how our results can be used to understand the implications of the OMT announcements. We model OMT as a price floor schedule implemented by the ECB. These interventions can eliminate the possibility of rollover crises and they do not

---

[2]In a related paper, Borri and Verdhelan (2013) study a sovereign debt model where lenders have time-varying risk aversion *á la* Campbell and Cochrane (1999). In a previous version of the paper we followed this route considering a more flexible specification of the external habit model that allows for time-variation in term premia (Wachter, 2006; Bakaert et al., 2009). Such formulation delivers similar results to the one currently used, but it is computationally more challenging.

require the ECB to ever intervene in bond markets along the equilibrium path, resulting in a Pareto improvement. Our question is whether the ECB followed this benchmark. To test for this hypothesis, we use the model to construct a fundamental value for the Italian spread- the value that would prevail in the absence of coordination failures- and we compare it with the actual spread observed after the ECB announcement. We document that this counterfactual fundamental spread is roughly 150 basis points higher than the actual interest rate spread observed in the second half of 2012. This result indicates that the sharp decline in interest rate spreads observed after these announcements to a large extent reflects a prospective subsidy offered by the ECB to the peripheral countries.

This paper contributes to the literature on multiplicity of equilibria in sovereign debt models. Previous works in this area like Alesina et al. (1989), Cole and Kehoe (2000), Calvo (1988), and Lorenzoni and Werning (2013) have been qualitative in nature. More recently, Conesa and Kehoe (2012), Roch and Uhlig (2014), Stangebye (2014) and Navarro et al. (2015) considered more quantitative models featuring multiple equilibria. To best of our knowledge, this is the first paper in the literature that conducts a quantitative assessment of the importance of rollover risk in driving interest rate spreads.[3] The main innovation relative to the existing literature is our identification strategy based on the behavior of debt duration around default crises.

More generally, the paper is related to quantitative analysis of sovereign debt models. Papers that are related to our work include Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2013), Hatchondo and Martinez (2009), Bianchi et al. (2014) and Borri and Verdhelan (2013). Relative to the existing literature, this is the first paper to consider a model with rollover risk, endogenous maturity choice and risk aversion on the side of the lenders. Our analysis shows that the behavior of debt duration is necessary for the identification of rollover risk, while shocks to the stochastic discount factor of the lenders are necessary to control for confounding demand factors that may undermine our identification strategy. Our modeling of the maturity choices differ from previous research and builds on recent work by Sanchez et al. (2015) and Bai et al. (2014). Specifically, the government in our model issues portfolios of zero coupon bonds with an exponentially decaying duration. The maturity choice is discrete, and it consists on the choice of the decaying factor. This modeling feature simplifies the numerical analysis of the model relative to the canonical formulation of Arellano and Ramanarayanan (2012).

Our analysis on the effects of liquidity provisions is related to Roch and Uhlig (2014) and Corsetti and Dedola (2014). These papers show that these policies can eliminate self-fulfilling debt crisis when appropriately designed. We contribute to this literature by using

---

[3]There is also a reduced form literature that addresses this issue, see De Grauwe and Ji (2013).

our calibrated model to test whether the drop in interest rates spreads observed after the announcement of OMT is consistent with the implementation of such policy or whether it signals a prospective subsidy paid by the ECB.

Finally, our paper is related to the literature on the quantitative analysis of indeterminacy in macroeconomic models, see the contributions of Jovanovic (1989), Farmer and Guo (1995) and Lubik and Schorfheide (2004). The closest in methodology is Aruoba et al. (2014) who use a calibrated New Keynesian model solved with global methods to measure the importance of confidence driven fluctuations for the U.S. and Japanese economy.

Layout. The paper is organized as follows. Section 2 presents the model. Section 3 discusses our key identifying restriction, and it presents an historical example supporting our approach. Section 4 describes the calibration of the model and presents an analysis of its fit. Section 5 uses the calibrated model to measure the importance of rollover risk. Section 6 analyzes the OMT program. Section 7 concludes.

2 Model

2.1 Environment

Preferences and Endowments: Time is discrete, $t \in \{0, 1, 2, ...\}$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition matrix $\mu\left(\cdot | s_{t-1}\right)$. The exogenous state has two types of variables: *fundamental*, $s_{1,t}$, and *non-fundamental*, $s_{2,t}$. The fundamental states are stochastic shifters of endowments and preferences while the non-fundamental states are random variables on which agents can coordinate. These coordination devices are orthogonal to fundamentals.

The economy is populated by lenders and a domestic government. The lenders value flows according to the stochastic discount factor $M(s_t, s_{t+1})$. Hence the value of a stochastic stream of payments $\{d\}_{t=0}^{\infty}$ from time zero perspective is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \tag{1}$$

where $M_{0,t} = \prod_{j=0}^{t} M_{j-1,j}$.

The government receives an endowment (tax revenues) $Y_t = Y(s_t)$ every period and decides the path of spending G_t. The government values a stochastic stream of spending

$\{G_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(G_t\right), \tag{2}$$

where the period utility function U is strictly increasing, concave, and it satisfies the usual assumptions.

Market Structure: The government can issue a portfolio of non-contingent defaultable bonds to lenders in order to smooth fluctuations in G_t. For tractability, we restrict the portfolios that the government can issue to be portfolios of zero-coupon bonds indexed by (B_t, λ_t). A portfolio (B_t, λ_t) at the end of period t corresponds to issuance of $(1 - \lambda_t)^{j-1} B_t$ zero-coupon bond of maturity j. The parameter $\lambda \in [0,1]$ captures the duration of the government stock of debt, and it can be interpreted as its decay factor. Higher λ implies that debt payments are concentrated at shorter maturities. For instance, if $\lambda = 1$, then all the debt is due next period. B_t controls for the face value of debt. Specifically, the total face value of debt is B_t / λ_t.

If we let $q_{t,j}$ be the price of a zero-coupon bond of maturity j at time t, the value of a portfolio (B_t, λ_t) is

$$q_t\left(\lambda_t\right) B_t = \sum_{j=1}^{\infty} q_{t,j}(1 - \lambda_t)^{j-1} B_t$$

where $q_t\left(\lambda_t\right)$ is the per-unit value of the portfolio.

The timing of events within the period follows Cole and Kehoe (2000): the government issues a new amount of debt, lenders choose the price of newly issued debt, and finally the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. Differently from the timing in Eaton and Gersovitz (1981), the government does not have the ability to commit not to default within the current period. As we will see, this opens the door to self-fulfilling debt crisis.

The budget constraint for the government when he does not default is

$$G_t + B_t \leq Y_t + \Delta_t, \tag{3}$$

where Δ_t is the net issuance of new debt given by

$$\Delta_t = \sum_{j=1}^{\infty} q_{t,j} \left[(1 - \lambda_{t+1})^{j-1} B_{t+1} - (1 - \lambda_t)^j B_t \right] \tag{4}$$

$$= q_t\left(\lambda_{t+1}\right) B_{t+1} - q_t\left(\lambda_t\right)\left(1 - \lambda_t\right) B_t \tag{5}$$

Note that if a government enters the period with a portfolio (B_t, λ_t) and want to exit the

period with a portfolio (B_{t+1}, λ_{t+1}) the government must issue additional $(1 - \lambda_{t+1})^{j-1}B_{t+1} - (1 - \lambda_t)^j B_t$ zero coupon bonds of maturity j.[4]

We assume that if the government defaults, he is permanently excluded from financial markets and he suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value for the government conditional on a default. Lenders that hold inherited debt and the new debt just issued do not receive any repayment.[5]

2.2 Recursive Equilibrium

2.2.1 Definition

We now consider a recursive formulation of the equilibrium. Let $\mathbf{S} = (B, \lambda, s)$ be the state today and $\mathbf{S}'$ the state tomorrow. The problem for a government that has not defaulted yet is

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', \lambda', G} \delta \left\{ U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}] \right\} + (1 - \delta)\underline{V}(s_1) \tag{6}$$

subject to

$$
\begin{aligned}
G + B &\leq Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\
\Delta(\mathbf{S}, B', \lambda') &= q(s, B', \lambda' | \lambda') B' - q(s, B', \lambda' | \lambda)(1 - \lambda)B,
\end{aligned}
$$

where $q(s, B', \lambda' | \lambda)$ is the per unit value of a portfolio with decay parameter λ after the exogenous state s is realized and the government new portfolio is (B', λ').

The lender's no-arbitrage condition requires that

$$q(s, B', \lambda' | \lambda) = \delta(\mathbf{S}) \mathbb{E} \left\{ M(s_1, s_1') \delta(\mathbf{S}') \left[1 + (1 - \lambda)q(s, B'', \lambda'' | \lambda) \right] | \mathbf{S} \right\} \tag{7}$$

where $B'' = B'(s', B', \lambda')$ and $\lambda'' = \lambda'(s', B', \lambda')$. The presence of $\delta(\mathbf{S})$ in equation (7) means that new lenders receive a payout of zero in the event of a default today.

A *recursive equilibrium* is value function for the borrower V, associated decision rules δ, B', λ', G and a pricing function q such that $V, \delta, B', \lambda', G$ are a solution of the government

---

[4]If $(1 - \lambda_{t+1})^{j-1}B_{t+1} - (1 - \lambda_t)^j B_t$ is negative the government is buying back the ZCB of maturity j. Notice that with our formulation, if the government wants to shorten the duration it must buy back some debt at sufficiently long horizon. One may argue that such feature is unrealistic as governments rarely buy back bonds. We follow this formulation for the sake of numerical tractability.

[5]This is a small departure from Cole and Kehoe (2000), since they assume that the government can use the funds raised in the issuance stage. Our formulation simplifies the problem and it should not change its qualitative features. The same formulation has been adopted in other works, for instance Aguiar and Amador (2014).

problem (6) and the pricing function satisfies the no-arbitrage condition (7).

2.2.2 Multiplicity of equilibria and Markov selection

We now show that there are multiple recursive equilibria in this model. Following Cole and Kehoe (2000), when inherited debt is sufficiently high, a coordination problem can generate a "run" on debt, whereby it is optimal for an atomistic investor not to lend to the government whenever the other investors decide not to buy the bonds. This can happen despite the fact that the atomistic investor would lend to the government if the other lenders would.

This form of strategic complementarity gives rise to a multiplicity of equilibria. In fact, suppose that lenders expect the government to default today so that the price of government debt is $q = 0$. The expectation of the lenders is validated in equilibrium if default is an optimal choice of the government. This can happen in this economy if[6]

$$U(Y - B) + \beta \mathbb{E}\left[V\left((1-\lambda)B, \lambda, s'\right) | \mathbf{S}\right] < \underline{V}(s_1). \tag{8}$$

From the above expression it is clear that for each (λ, s) there exists a level of inherited debt B sufficiently high such that the inequality in (8) is satisfied. Condition (8) depends on V, an equilibrium object. In the Appendix, we show that we can however find sufficient conditions on the primitives of the model for (8) to be satisfied, thus establishing the presence of multiple equilibria.

Debt crisis may thus be self-fulfilling for sufficiently high level of debt: lenders may lend to the sovereign and there will be no default, or the lenders may not roll-over government debt, in which case the sovereign would find it optimal to default. Therefore, the outcomes are *indeterminate* in this region of the state space. We follow most of the literature and use a parametric mechanism that selects among these possible outcomes. In order to explain our selection mechanism, it is useful to partition the state space in the three region. Following the terminology in Cole and Kehoe (2000), we say that the borrower is in the *safe zone*, $\mathcal{S}^{\text{safe}}$, if the government does not default even if lenders do not rollover his debt. That is,

$$\mathcal{S}^{\text{safe}} = \left\{ \mathbf{S} : U(Y - B) + \beta \mathbb{E}V\left((1-\lambda)B, \lambda, s'\right) \geq \underline{V}(s_1) \right\}.$$

---

[6]If condition (8) is not satisfied, instead, no coordination problem among lenders can arise. This is because if lenders decide to run, and so $q = 0$, it is still optimal for the government to repay his debt. Thus, lenders have no incentive to run: it is optimal for an individual lender to lend at a positive price even if other lenders do not and so $q = 0$ cannot be an equilibrium price.

We say that the borrower is in the *crisis zone*, $\mathcal{S}^{\text{crisis}}$, if the initial state is such that it is not optimal to repay debt during a rollover crisis but the government finds it optimal to repay whenever lenders roll-over its debt. That is,

$$\mathcal{S}^{\text{crisis}} = \left\{ \mathbf{S} : U\left(Y - B\right) + \beta\mathbb{E}\left[V\left((1-\lambda)B,\lambda,s'\right)|\mathbf{S}\right] < \underline{V}\left(s_1\right) \text{ and} \right.$$
$$\left. \max_{B',\lambda'} U\left(Y - B + \Delta\left(\mathbf{S},B',\lambda'\right)\right) + \beta\mathbb{E}\left[V\left(B',\lambda',s'\right)|\mathbf{S}\right] \geq \underline{V}\left(s_1\right) \right\}.$$

Finally, the residual region of the state space, the *default zone*, $\mathcal{S}^{\text{default}}$ is the region of the state space in which the government defaults on his debt regardless of lenders' behavior. That is,

$$\mathcal{S}^{\text{default}} = \left\{ \mathbf{S} : \max_{B',\lambda'} U\left(Y - B + \Delta\left(\mathbf{S},B',\lambda'\right)\right) + \beta\mathbb{E}\left[V\left(B',\lambda',s'\right)|\mathbf{S}\right] < \underline{V}\left(s_1\right) \right\}.$$

Indeterminacy in outcomes arises only in the crisis zone.

The selection mechanism works as follows. Without loss of generality, let the non-fundamental state, s_2, be $s_2 = (p, \xi)$. Whenever the economy is in the crisis zone, lenders roll-over the debt if $\xi \geq p$. In this case, there are no run on debt and $\delta(\mathbf{S}) = 1$ by our definition of crisis zone. If $\xi < p$, instead, the lenders do not roll-over the government debt. We will assume that ξ is an i.i.d. uniform on the unit interval while p follows a first order Markov process, $p' \sim \mu_p(.|p)$. Given these restrictions, we can interpret p as the probability of having a rollover crises this period conditional on the economy being in the crisis zone. While p is relevant for selecting between outcomes in the crisis zone today, the relevant state variable that determines how perspective rollover risk affects interest spreads today is the expectations of p in the next period. We will denote this quantity by $\pi = \mathbb{E}\left(p'|p\right)$.

Note that the outcome of the debt auctions are unique in the crisis zone once we adopt this selection rule. However, even with this selection, we cannot assure that the equilibrium value function, decision rules and pricing functions are unique as the operator that implicitly defines a recursive equilibrium may have multiple fixed points. In order to overcome these issues, we restrict our attention to the limit of the finite horizon version of the model. Under our selection rule, the finite horizon model features a unique equilibrium and so does its limit.

The equilibrium outcome is a stochastic process

$$\mathbf{y} = \{\delta(s^t, B_0, \lambda_0), B(s^t, B_0), \delta(s^t, B_0, \lambda_0), G(s^t, B_0, \lambda_0), q(s^t, B_0, \lambda_0)\}_{t=0}^{\infty}$$

naturally induced by the recursive equilibrium objects. The outcome path depends on properties of the selection, i.e. the process for $\{p_t\}$, and on the realization of the non-fundamental state s_2. The goal of our exercise is to understand how properties of the process for $\{p_t\}$ affect the equilibrium outcome path, then estimate such process and filter out its realizations to assess whether rollover risk was an important driver of Italian spreads in the recent crisis.

3 Maturity Choices and Sources of Default Risk

In this section, we explain why the joint distribution of interest rate spreads and debt duration provides information that is useful to distinguish between fundamental and non-fundamental sources of default risk. The key insight is that if rollover risk is large then the government has an incentive to "exit" from the crisis zone. As first showed in Cole and Kehoe (2000), to achieve this objective the government can lengthen the maturity of his debt since long term debt is less susceptible to runs. Hence, if rollover risk is a major driver of default risk in the model then we should expect the duration of the debt to lengthen when interest rate spreads increase. On the contrary, previous research - for instance Arellano and Ramanarayanan (2012), Aguiar and Amador (2014) and Dovis (2014) - has shown that a shortening of maturity is typically an optimal response of the sovereign when facing a default crises driven by fundamental shocks: a negative comovement between spreads and duration would then indicate a more limited role for rollover risk.

In what follows we illustrate these insights using numerical illustrations from a calibrated version of our model with risk neutral investors, $M_{t,t+1} = 1/(1+r)$.

3.1 Maturity choices in absence of rollover risk

We start from the case in which rollover risk is absent, $\{p_t\}$ is identically equal to zero. Previous works on incomplete market models without commitment have emphasized two channels as the main determinants of the maturity composition of debt in the face of default risk: insurance and incentives not to dilute outstanding debt.

The *insurance channel* refers to the fact that long term debt is a better asset than short term debt to provide the government with insurance against shocks. More specifically, capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities. This gives an incentive for the government to issue bonds of longer duration.

While insurance generates a motive for the government to issue long term debt, *incentives not to dilute outstanding debt* pushes the government to issue relatively more short term debt. When agents follow Markovian strategies,[7] short term debt is relatively more attractive because it is not prone to be diluted. That is, the government cannot reduce ex-post its value absent a default, while he can dilute long term debt by increasing issuance relative to the ex-ante expectations of lenders. This capital loss realized by lenders is tantamount to a partial default on existing debt. Since the government cannot commit to future issuance of debt, lenders will demand a compensation for this dilution risk. This makes the price of long term debt more sensitive to new issuance relative to the price of short term debt, which makes this latter a better instrument for the government to raise resources from creditors.

The optimal maturity structure of the debt portfolio issued by the government balances these two motives. Figure 1 below shows the maturity choice as a function of inherited debt and the realization of Y for a given inherited λ. Darker colors stand in for shorter maturities (high $1/\lambda'$). The white area in the figure is the default zone. We can see that for high level of inherited debt and low level of income the maturity of government debt is shorter relative to states with lower default risk.

Figure 1: **Maturity choices in absence of rollover risk**
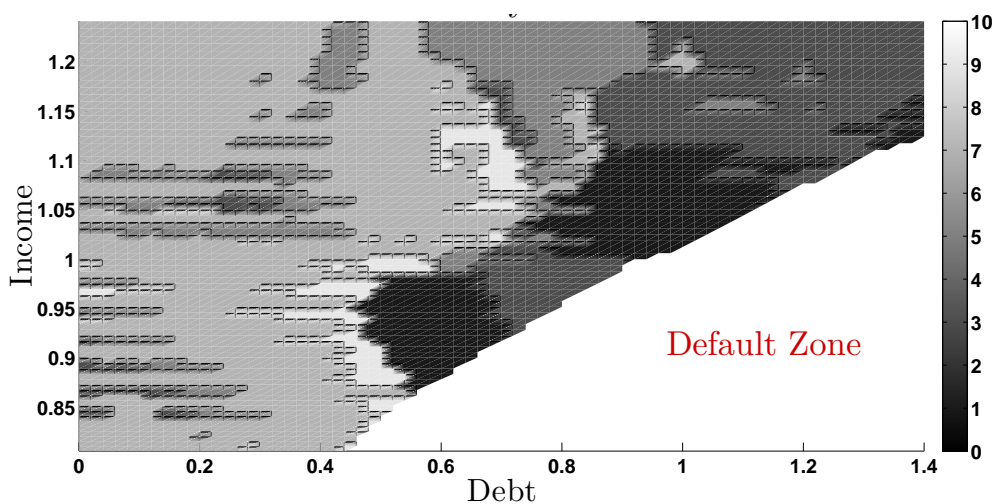
This relative preference for shorter maturities in the face of "fundamental" default risk is the result of two forces. First, incentives not to dilute outstanding debt are stronger the higher is the risk of default. Indeed, in states when output is low and/or inherited debt is high, the government would like to issue more debt in order to smooth out consumption.

---

[7]In infinite horizon, the debt-dilution problem is not present if we consider the best SPE (which is history dependent).

As argued earlier, with dilution and no rollover risk, the value of issuance is maximized for all new debt being short term, since short term debt allows the government to commit not to issue too much debt in the future. This helps to keep the price of debt high today. See Aguiar and Amador (2014) for a similar argument.

Second, the need to hold long term debt for insurance reasons falls when default risk increases. As discussed in Dovis (2014), this happens because pricing functions become more sensitive to shocks when the economy is approaching the default region. Hence the same amount of long term debt provides more insurance.

The decision rules in Figure 1 imply that when interest rates spreads are high because of fundamental default risk we should observe the government issuing more short term debt. This pattern can be verified by considering the response of interest rate spreads and debt duration to a negative income shock in the model. As the solid line in Figure 2 shows, a sufficiently negative income shock that raises the prospect of a default generates negative comovement between interest rate spreads and debt duration.

3.2 Maturity choices with rollover risk

We now turn to the analysis of the maturity choice in presence of rollover risk. Alongside insurance and incentives motives, the government has now an additional reason to actively manage the maturity of its debt. When $\pi = \mathbb{E}[p'|p] > 0$, a rollover crisis can occur with positive probability if the economy happens to be in the crisis zone next period. Since these outcomes are inefficient from the government's perspective, there is an incentive for the government to take actions in order to reduce the likelihood of falling into the crisis zone next period. As emphasized in Cole and Kehoe (2000), this can be achieved by reducing debt issuance and/or by lengthening the maturity of issued debt.

The logic of why lengthening the maturity of debt issued today helps avoiding the crisis zone in the next period can be best understood by looking at the condition defining the crisis zone,

$$U(Y - B) + \beta\mathbb{E}[V((1 - \lambda)B, \lambda, s')|\mathbf{S}] < \underline{V}(s_1). \tag{9}$$

An increase in the duration of debt holding the face value constant can be achieved by lowering B and reducing λ. By doing so, the government can reduce the payments coming due in the next period. This increases the first term on the left hand side of (9) at the expenses of a lower continuation value as more debt is due in the following periods. It is easy to show that this variation increases the left hand side of (9). The borrower is "credit constrained" and the marginal utility of consumption next period when there is

no rollover crises is higher than the marginal reduction in expected utility from two period onward. Therefore, lengthening the maturity of the debt reduces the likelihood of falling into the crisis zone next period.

The circled line in Figure 2 describes this effect by reporting impulse response functions to a p_t shock. In contrast to what described in the previous section, the p_t shock generates positive comovement between interest rate spreads and debt duration.

Figure 2: **The dynamics of interest rate spreads and debt duration**
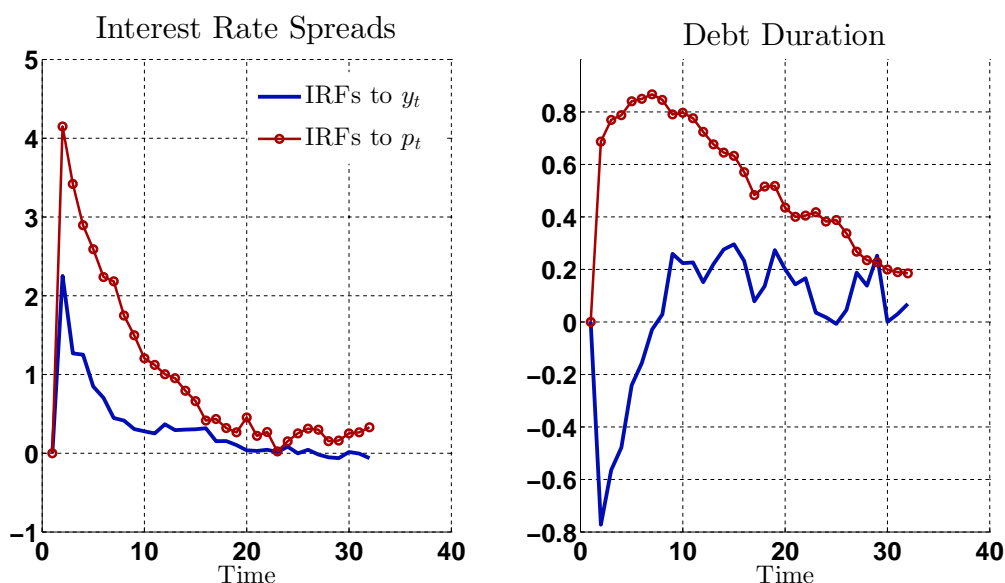
Notes: The blue solid line reports impulse response functions (IRFs) of interest rate spreads and debt duration to a 3 standard deviations income shock in the model without rollover risk. The red circled line reports IRFs to a 3 standard deviation increase in p_t. IRFs are calculated by simulation, and they are expressed as deviation from the ergodic mean. Interest rate spreads are expressed in annualized percentages while debt duration in years.

This discussion suggests that when the sources of default risk are fundamental, interest rate spreads increase and the duration of debt declines: short term debt provides more incentives for the government to repay in the future, and these incentives are very valuable when a country is facing a solvency crises. When default risk arises because of the prospect of a rollover crisis, instead, the government lenghtens the maturity of its debt in order to be less prone to future runs. In our quantitative analysis we are going to use these different implications regarding maturity choices in order to disentangle these two sources of default risk.

3.3 A case study: Italy in the early 1980s

Before turning to the quantitative analysis it is useful to discuss in more details our main identifying restriction. Our approach is based on the presumption that governments act strategically and manage the maturity of debt in order to avoid future roll-over problems. However, previous cross-country studies have documented that the maturity of new issuances typically shortens around default crises (Broner et al., 2013; Arellano and Ramanarayanan, 2012). One may wonder whether there are historical episodes that provide support for our identification strategy. One of such examples is given by the Italian experience of the early 1980s. Indeed, there are different signs that the Italian government was facing roll-over problems at the time. First, the Italian economy entered the 1980s with a redemption profile concentrated at short horizons: at the end of 1981, the average term to maturity of Italian government debt was 1.13 years.[8] Second, and starting from 1981, the Italian government intensified efforts to increase the independence of the Bank of Italy by freeing it from the obligation of buying unsold public debt in auctions (Tabellini et al., 1987). This effectively meant that the government had to rely primarily on auctions in order to refinance its maturing debt and its spending needs. Finally, there is evidence that the Italian government had issues in raising funds from primary dealers at the time. Table x reports the average difference between the bonds demanded by primary dealers and the maximal quantity issued by the treasury as a percentage of the quantity issued. A negative number means that the demand by primary dealers was below the target set by the treasury. In all years but in 1984 the Italian treasury was not able able to fully issue all the debt it wanted.

[INSERT TABLE 1 HERE]

Consistent with these three facts, the early 1980s saw a rapid increase in interest rate differentials between Italian and German government securities: as we can see from the left panel of Figure 3, between January 1980 and March 1983, interest rate spreads rose from 500 basis points to 1300 basis points.[9] In light of the extremely short maturity of the stock of Italian government debt, the institutional changes happening at the time, and the low demand for debt in treasury auctions, it is plausible to believe that these tensions in the Italian bond markets were mostly reflecting rollover risk. In this respect,

---

[8]These extremely low values were the results of the chronic inflation of the 1970s, which discouraged investors from holding long duration bonds that were unprotected from inflation risk. During the 1970s, the average term to maturity of the Italian public debt went from roughly 6 years to 1 years. See the volume edited by Giavazzi and Spaventa (1988) for a discussion of this historical period.

[9]Mention inflation, sme, etc.

the response of the Italian treasury to this situation is consistent with the prediction of our model. Indeed, the treasury followed throughout the early 1980s a policy directed to increase the maturity of government debt and to smooth out its redemption profile. As documented in Alesina et al. (1989), for example, the Italian treasury introduced a new type of bonds indexed to the prevailing nominal interest rate. These new instruments protected bondholders from inflation risk and they were instrumental for the treasury's efforts to lengthen the maturity of the debt. The left panel of Figure 3 shows that the average term to maturity of Italian government debt more than tripled within the span of five years, going from 1.13 years in 1981 to 3.88 years in 1986.

Figure 3: **Debt duration and Interest rate spreads: 1980s vs 2010s**
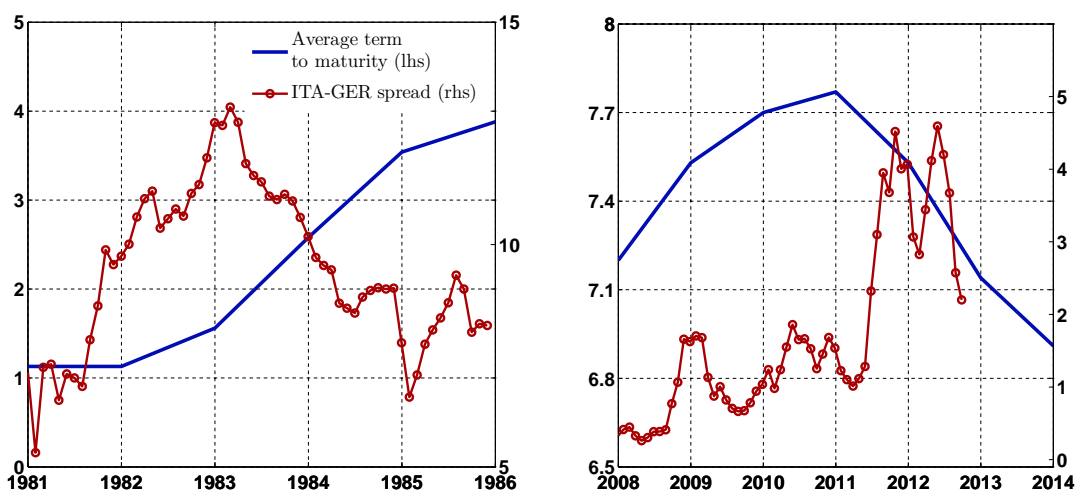
Notes: The solid line stands for the average term to maturity of the outstanding central government debt. Magnitudes are reported in years (right hand side). The circled line reports the interest rate differential between an Italian and a German zero coupon government bond with a duration of twelve months. Magnitudes are reported in annualized percentages (left hand side).

Through the lens of the model, the actions of the Italian treasury reduced its exposure to rollover risk without increasing the incentive to inflate away the debt. Consistent with this view, we can observe from Figure 3 that as the maturity of the stock of debt increased the interest rate spreads paid by the Italian government started to decline in 1983. Overall, this episode provides support to our identification strategy: when rollover problems are pressing, governments have incentives to actively manage the duration of their debt in order to minimize the risk of facing a run.

For comparison, Table x and Figure 3 report these variables during the latest years. We can verify that the dynamics of auctions of government debt, interest rate spreads and debt duration appear different from the experience of the 1980s. To start, auctions of government debt do not show signs of lack of demand, as the demand of both short

term debt (BOT, with maturity between three months and one year), and that of longer term bonds (BTP, with maturity between three years and thirty years) above the minimal price was always well above the amount that the treasury planned to issue. Moreover, the right panel of Figure 3 shows that the average term to maturity of government debt decreased by roughly one years during the 2011-2014 period. Given the discussion of this section, this cursory look at the data suggests that the recent experience does not square well with an interpretation that emphasizes roll-over problems as the major source of the current crisis. In what follows, we will make the analysis more formal and we will use the structural model to measure the contribution of rollover risk in the run-up of Italian spreads.

4   Quantitative Analysis

We now apply our framework to Italian data. This section proceeds in three steps. Section 4.1 describes the parametrization of the model and our empirical strategy. Section 4.2 describes the data. Section 4.3 reports the results of our calibration and some indicators of model fit.

4.1   Parametrization and Empirical Strategy

4.1.1   Lenders' stochastic discount factor and endowment process

It is common practice in the sovereign debt literature to assume risk neutrality on the lenders' side. This specification, however, is not desirable given our objectives. First, several authors have argued that risk premia are quantitatively important to account for the volatility of sovereign spreads (Borri and Verdhelan, 2013; Longstaff et al., 2011). Assuming risk neutrality implies that other unobserved factors in the model, for instance p_t, would have to absorb the variations in this component of the spread. Second, sovereign debt crisis are typically accompanied by a significant increase in term premia (Broner et al., 2013). Neglecting these shifts could undermine our identification strategy: rollover risk could be an important driving force for interest rate spreads of peripheral countries in the euro area and yet we could observe a shortening in the duration of debt simply because high term premia made short term borrowing cheaper during the crises.

Therefore, we deviate from most of the existing literature and we introduce a stochastic discount factor that allows us to fit the behavior of asset prices observed in Europe over the period of analysis. Specifically, we closely follow Duffie et al. (1996), Backus et al.

(2001), and Bekaert and Grenadier (1999) and assume that $m_{t,t+1} = \log M_{t,t+1}$ is given by a "square-root" process

$$m_{t,t+1} = \mu_m + \chi_t + \sigma_m \max\{\chi_t, 0\}\, \varepsilon_{\chi,t+1}, \tag{10}$$

where the latent factor χ_t follows

$$\chi_{t+1} = \mu_\chi(1 - \rho_\chi) + \rho_\chi \chi_t + \sigma_\chi \max\{\chi_t, 0\}\, \varepsilon_{\chi,t},$$

and $\varepsilon_{\chi,t}$ is an i.i.d. standard normal variable. When enriched with a stochastic process for payouts, one can use the implied law of motion for $m_{t,t+1}$ along with the pricing formula in equation (1) to express asset prices as a function of parameters and state variables. Appendix x discusses the properties of this pricing model. Specifically, we show that if $\sigma_m < 0$, then the model produces on average a risk premium on long term non-defaultable zero coupon bonds. These risk premia are time-varying: an increase in χ_t implies an increase in the conditional volatility of the stochastic discount factor, which translate into higher premia demanded by the lenders for holding risky assets. Moreover, because a high χ_t is associated to low interest rates today, we can see that our set up can in principle fit an episode of low yields over short term riskless assets and high yields over long term risky assets as the one observed in Europe during the period of analysis.

In the Appendix we show that The country's endowment $Y_t = \exp\{y_t\}$ follows the stochastic process

$$y_{t+1} = \rho_y y_t + \rho_{y\chi}(\chi_t - \mu_\chi) + \sigma_y \varepsilon_{y,t+1} + \sigma_{y\chi}\varepsilon_{\chi,t+1}.$$

Note that we allow output to depend on the latent factor χ_t and on its innovations. We do so to allow for a correlation between periods of high price of risk and the real economy.

We let $\theta_1 = [\mu_m, \sigma_m, \mu_\chi, \rho_\chi, \sigma_\chi, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}]$ denote the parameters governing the joint behavior of the lenders' stochastic discount factor and the country's endowment.

4.1.2 Government's decision problem and $\{p_t\}$

The government period utility function is CRRA

$$U^{\text{gov}}(G_t) = \frac{G_t^{1-\sigma} - 1}{1 - \sigma},$$

with σ being the coefficient of relative risk aversion. The government discounts future flow utility at the rate β. If the government enters a default state, he is excluded from international capital markets and he suffers an output loss τ_t. These costs of default are a function of the country's income, and they are parametrized following Chatterjee and Eyigungor (2013),

$$\tau_t = \max\{0, d_0 e^{y_t} + d_1 e^{2y_t}\}.$$

If $d_1 > 0$, then the output losses are larger when income realizations are above average.[10] We also assume that, while in autarky, the sovereign has a probability of reentering capital markets ψ. If the government reenters capital markets, he pays the default costs and he starts his decision problem with zero debt.

The probability of lenders not rolling over the debt in the crises zone follows the stochastic process $p_t = \frac{\exp\{\tilde{p}_t\}}{1 + \exp\{\tilde{p}_t\}}$, with $\tilde{p}_t$ given by

$$\tilde{p}_{t+1} = (1 - \rho_p)p^* + \rho_p \tilde{p}_t + \sigma_p \varepsilon_{p,t}.$$

We let $\theta_2 = [\sigma, \beta, d_0, d_1, \psi, p^*, \rho_p, \sigma_p]$ denote the parameters associated to the government decision problem. The innovations $\{\varepsilon_{p,t}\}$ are i.i.d. standard normal random variables.

4.1.3 Empirical strategy

Broadly speaking, our empirical strategy consists in choosing $\theta = [\theta_1, \theta_2]$ in two steps. In the first step, we jointly estimate the parameters of the lenders' stochastic discount factor and of the endowment process using information from Italian detrended real GDP, the term structure of German zero coupon bonds and the stock price-consumption ratio for the euro-area. This step makes sure that our model is sensible in pricing risky and riskless assets at different maturities. Implicit in our approach is the assumption that the lenders are "marginal" for pricing other financial assets in the euro area beside Italian government securities and that the stochastic discount factor is exogenous to the issuance of government debt and to default decisions. In the second step, and conditional on the parameters governing the lenders' behavior, we calibrate θ_2 by matching some basic facts about Italian public finances. In view of our previous discussion, we place empirical discipline on the $\{p_t\}$ process by making sure that the calibrated model replicates the joint behavior of interest rate spreads and the duration of debt for the Italian economy.

---

[10]This feature makes it easier for the model to match the empirical observation that sovereign spreads are countercyclical. With convex output costs, in fact, the sovereign has more incentives to default in presence of bad income realization, see Arellano (2008).

4.1.4 Numerical Solution

The model is solved numerically using global methods. Let $\mathbf{S} = [B, \lambda, y_t, \chi_t, p_t]$ be the vector collecting the model's state variables. The objects we approximate are the value of repaying, $V^R(\mathbf{S})$, the value of defaulting, $V^D(y_t, \chi_t)$, the pricing schedule and the area defining whether the economy is currently in the safe zone. This section describes some key steps of the numerical solution, while the Appendix describes it at length. We consider the choice of λ to be discrete, from the set $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_N\}$. The value functions are approximated using Chebyshev's polynomials. For example, consider $V^R(\mathbf{S})$, and let $\tilde{\mathbf{S}} = [B, y_t, \chi_t, p_t]$ be the set of states that excludes λ. We approximate $V^R(\lambda_j, \tilde{\mathbf{S}})$ as follows:

$$V^R(\lambda_j, \tilde{\mathbf{S}}) = \psi_{\lambda_j}^{R}{}' T(\tilde{\mathbf{S}}),$$

where $T(.)$ are Chebyshev's polynomials and $\psi_{\lambda_j}^R$ is a vector collecting the associated coefficients. We evaluate this value at a set of collocation points, $\Lambda \times \#\tilde{\mathbf{S}}$. The collocation points of $\tilde{\mathbf{S}}$ and the associated polynomials are constructed using a Smolyak grid with dimension $\mu = 6$, while the grid for λ contains 6 values: $+/- 2$ years around the average term to maturity of seven years (the Italian pre-crisis level), and one year. The approximation for the value of defaulting follows a similar construction. The pricing schedule and the safe zone are calculated on the same grid.

Given an initial guess for $\{\psi_{\lambda_j}^R, \psi^D, q, safe\}$, we update it using the decision problem of the government, the lenders' no arbitrage condition and the definition of safe zone. The numerical solution of the model consists in iterating over $\{\psi_{\lambda_j}^R, \psi^D, q, safe\}$ until we achieve convergence.

4.2 Data

Our sample starts in 1999:Q1 and ends in 2012:Q4. We collect quarterly data on private consumption expenditures for members of the euro area from the ECB-SDW database.[11] We construct a quarterly series for the stock-price consumption ratio in the euro area by scaling the Dow Jones Euro Stoxx 50 with our consumption series. Nominal bond yields for Germany (1 year and 5 year maturity) at a monthly frequency are from Bundesbank, while monthly data on CPI inflation in the euro area are from Eurostat. We convert monthly series at a quarterly frequency using simple averages. The endowment process y_t

---

[11]In what follows, the euro-area is defined as the 18 members of the monetary union as of December 2013. CPI inflation is calculated by Eurostat as a weighted average of CPI inflation in these countries (changing composition).

is mapped to linearly detrended log real Italian GDP. The quarterly GDP series is obtained from OECD. These data series are used in the first step of our procedure to estimate θ_1.

The interest rate spread series is the annualized difference between yields on Italian government debt of a one year maturity and the yields on German bonds of the same duration. We will map this data series to the interest rate spread on a portfolio with an average duration of one year. Our indicator for debt duration is the average term to maturity of outstanding bonds issued by the Italian central government. This indicator, obtained at quarterly frequencies from the Italian treasury, is mapped in the model to $\frac{1}{\lambda'}$. These data series will be used in the second step of our procedure to calibrate θ_2.

4.3 Results

The results are organized in two sections. First, we describe the estimation of the pricing model. Then, we discuss the calibration of the parameters governing the government decision problem.

4.3.1 Estimation of the pricing model

The estimation of θ_1 is accomplished by fitting our pricing model to the detrended Italian real GDP series, the yield curve for German nominal zero coupon bonds and our time series on the price-consumption ratio in the euro area. In order to accomplish this purpose, we enrich the process of $[y_t, \chi_t]$ with inflation and consumption growth. Let $\mathbf{Y}_t = [\text{infl}_t, \Delta c_t, y_t, \chi_t]'$. We assume that $\mathbf{Y}_{t+1}$ follows the process

$$\mathbf{Y}_{t+1} = \mu + A\mathbf{Y}_t + (\Sigma_F \mathbf{F}_t + \Sigma_H)\,\varepsilon_{t+1}, \tag{11}$$

where $\mathbf{F}_t$ is a diagonal matrix with an element (i, i) given by $\max\{Y_{i,t}, 0\}^2$. For tractability, we impose restrictions on $\mathbf{Y}_t$. First, $[y_t, \chi_t]$ do not depend on $[\text{infl}_t, \Delta c_t]$, and it satisfies the restrictions described in section 4.1.1. This assumption makes sure that $[\text{infl}_t, \Delta c_t]$ are not state variables when solving the decision problem on the government. Second, we assume that $[\text{infl}_t, \Delta c_t]$ are homoskedastic.

Given the law of motion for $\mathbf{Y}_t$, the process for $m_{t,t+1}$ in equation (10) and the lenders' Euler equation, we can express yields on nominal non-defaultable zero coupon bonds and the stock price-consumption ratio as a function of model's parameters and states $\mathbf{Y}_t$. Specifically, our formulation implies that these variables are linear in $\mathbf{Y}_t$, see Bekaert and Grenadier (1999). The parameters of this pricing model, $[\mu, A, \Sigma_F, \Sigma_H]$ are estimated using

the method of simulated moments. Specifically, the vector of moments to match includes the sample mean, standard deviation, autocorrelation and cross-correlation matrix for euro-area inflation and consumption growth, the yields on German government securities, the stock price-consumption ratio and the Italian detrended real GDP. The corresponding model implied statistics are computed on a long simulation (N = 10000).[12]

Table 1: **Model fit: Yields, price-consumption ratio and Italian output**

	$\mu(r^{\$}_{4,t})$	$\mu(r^{\$}_{20,t} - r^{\$}_{4,t})$	$\sigma(r^{\$}_{4,t})$	$\sigma(r^{\$}_{20,t} - r^{\$}_{4,t})$	$\sigma(\mathrm{pc}_t)$
Data	2.50	0.77	1.47	0.51	0.33
Model	2.49	0.72	1.36	0.39	0.29
	$\mathrm{Acorr}(r^{\$}_{4,t})$	$\mathrm{Acorr}(r^{\$}_{20,t} - r^{\$}_{4,t})$	$\mathrm{Acorr}(\mathrm{pc}_t)$	$\rho(r^{\$}_{4,t}, r^{\$}_{20,t} - r^{\$}_{4,t})$	$\rho(r^{\$}_{4,t}, \mathrm{pc}_t)$
Data	0.95	0.88	0.96	-0.60	0.87
Model	0.96	0.87	0.99	-0.82	0.51
	$\sigma(y_t)$	$\mathrm{Acorr}(y_t)$	$\rho(r^{\$}_{4,t}, y_t)$	$\rho(r^{\$}_{20,t} - r^{\$}_{4,t}, y_t)$	$\rho(\mathrm{pc}_t, y_t)$
Data	0.03	0.96	0.53	-0.58	0.22
Model	0.03	0.85	0.49	-0.50	0.31

Notes: $\mu(.)$ represents the mean, $\sigma(.)$ the standard deviation, $\mathrm{Acorr}(.)$ the first order autocorrelation and $\rho(.)$ the correlation. $r^{\$}_{j,t}$ stands for nominal yields on a zero coupon bonds maturing in j quarters.

Table 1 reports the fit of the model regarding the key variables of interest while Table 2 reports the point estimates for the parameters governing the joint behavior of $[y_t, \chi_t]$. There are two things to emphasize. First, we estimate σ_m to be negative. As explained earlier, this implies that the model generates risk premia on long term assets, a feature that allows us to match a positive average slope for the German yield curve (see first row, second column of Table 1). Second, we estimate $\rho_{y\chi}$ to be negative. That is, an high price of risk today (high χ_t) forecast Italian output to be below average in the future. This feature allows the model to match the fact that periods of depressed asset prices in our sample are periods during which Italian output is below average (see the last row of Table 1). Moreover, it implies that Italian government bonds are risky from the lenders' perspective, because high χ_t today forecast bad times for the lenders and higher incentives to default for the sovereign.

We can gain further insights into the interpretation of the χ_t shock by looking at its estimated path over the sample period. Figure 4 reports the behavior of the nominal yields on zero coupon bonds of 1 year, an indicator of the slope of the yield curve (the

---

[12]The weighting matrix is diagonal, with non-zero entries given by the absolute value of the moment to match. This implies that the objective function equals sum of percentage deviations of model implied moments relative to the data.

difference between 5 and 1 years), and the stock price-consumption ratio over the 2005-2012 period. The red solid line reports the data series while the blue dotted line the model implied series obtained by applying the particle filter to our pricing model.[13] The bottom right panel of the figure reports the implied path for χ_t along with a 90% confidence interval. The figure shows how the increase in χ_t leads to a "flight to quality" whereby yields on short term safe assets decline while yields on long term risky assets (both stocks and bonds) increase.

Figure 4: **Asset prices and the χ_t process**
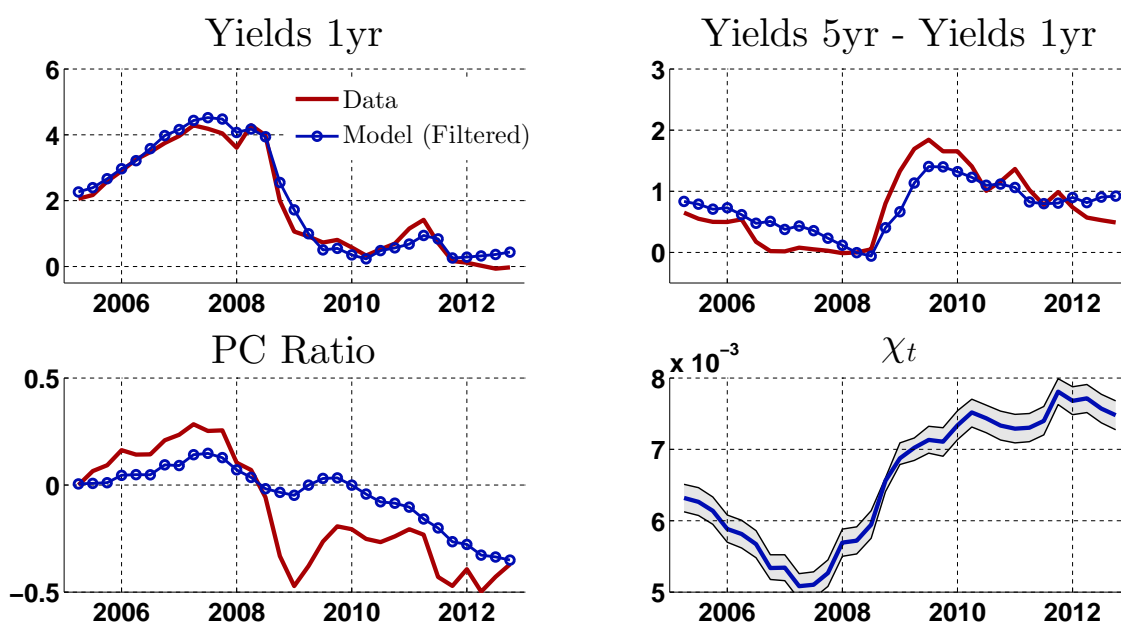
Notes: *The red line reports the data. The blue circled line reports the model implied corresponding variable computed using the particle filter. The solid line in the bottom-right panel reports the filtered series for χ_t (mean) along with its 90% confidence interval.*

4.3.2 The government's decision problem

We next turn to the calibration of $\theta_2 = [\sigma, \beta, d_0, d_1, \psi, p^*, \rho_p, \sigma_p]$. We fix σ to 2, a conventional value in the literature, and we set $\psi = 0.0492$, a value that implies an average exclusion from capital markets of 5.1 years following a sovereign default, in line with the evidence in Cruces and Trebesch (2013).

While in a future draft we plan of calibrating the remaining parameters to match basic

---

[13]Specifically, we run the particle filter on the six time series used in estimation. As the model features less shocks (four) than observables (six), we introduce Gaussian errors in the measurement equation in order to avoid stochastic singularity. The variance of these measurement errors equal 5% of the sample variance of the observables.

facts about the price, quantity and duration of Italian public debt, in this draft we borrow their value from previous research.

Table 2: **Model Calibration**

	Numerical Value	**Source**
μ_m	-0.031	Estimated by SMM
σ_m	-2.778	Estimated by SMM
μ_χ	0.006	Estimated by SMM
ρ_χ	0.960	Estimated by SMM
σ_χ	0.003	Estimated by SMM
ρ_y	0.716	Estimated by SMM
$\rho_{y\chi}$	-10.731	Estimated by SMM
σ_y	0.010	Estimated by SMM
$\sigma_{y\chi}$	0.011	Estimated by SMM
σ	2.000	Conventional value
ψ	0.049	Cruces and Trebesch (2013)
β	0.950	Chatterjee and Eyigungor (2013)
d_0	-0.180	Chatterjee and Eyigungor (2013)
d_1	0.240	Chatterjee and Eyigungor (2013)
$\frac{\exp\{p^*\}}{1+\exp\{p^*\}}$	0.005	Numerical Exploration
ρ_p	0.950	Numerical Exploration
σ_p	0.500	Numerical Exploration

5   Decomposing Italian Spreads

We now use the calibrated model to measure the importance of non-fundamental risk in driving Italian spreads during the period of analysis. We proceed in two steps. In the first step, we use our calibrated model along with the data presented in Section 4 to estimate a time series for the exogenous shocks, $\{y_t, \chi_t, p_t\}$. In the second step, we use the estimated path for the state variables and the model equilibrium conditions to calculate the component of interest rate spreads due to rollover risk. In order to highlight the informational content of maturity choices, Section 5.1 first performs this experiment disregarding debt duration data when estimating the path of exogenous shocks. We then incorporate, in Section 5.2, debt duration data in the filtering stage. Finally, Section 5.3 discusses the role of risk averse lenders for generating our results.

5.1 Measuring rollover risk disregarding maturity choices

Our model defines the nonlinear state space system

$$
\begin{aligned}
\mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t; \theta) + \eta_t \\
\mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t; \theta),
\end{aligned}
\tag{12}
$$

with $\mathbf{Y}_t$ being a vector of measurements, η_t classical measurement errors, the state vector is $\mathbf{S}_t = [b_t, \lambda_t, y_t, \chi_t, p_t]$ and ε_t are innovations to structural shocks. The first part of the system collects measurement equations, describing the behavior of observable variables while the second part of the system collects transition equations, regulating the law of motion for the potentially unobserved states. We estimate the realization of the model state variables by applying the particle filter to the above system.[14] The set of measurements $\mathbf{Y}_t$ includes the time series for interest rate spreads, linearly detrended real GDP and the previously estimated series for χ_t. As we have shown earlier, this latter series summarizes information on the behavior of financial market indicators in Europe over our sample period. It is important to notice that the estimation of $[y_t, \chi_t]$ is disciplined by "actual" observations because the measurement equation incorporates empirical counterparts of these shocks. The truly unobservable process in our exercise is the realization of p_t. Absent data on debt duration, the model does not have clear identifying restrictions that can be used to discipline p_t, and it will tend to attribute to this term variation in interest rate spreads that can not be accounted by $[y_t, \chi_t]$.

Equipped with the estimated path for the model state variables, we next use the structural model to measure the contribution of rollover risk to interest rate spreads. For illustrative purposes, we can express interest rate spreads on a zero coupon bond maturing next period as follows

$$
\begin{aligned}
\frac{r_{\lambda=1,t} - r_t^*}{r_{\lambda=1,t}} &= \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}] \\
&\quad - \operatorname{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right).
\end{aligned}
\tag{13}
$$

The equation tells us that interest rate spreads can be decomposed into three parts. The first two components represent the different sources of default risk in the model, and they add up to the conditional probability that the government defaults at $t+1$. As discussed in Section 2, this can happen because of two events. First, if $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}$, the sovereign

---

[14]The measurement errors are Gaussian, and their variance is set to 10% of the sample variance of the observables. The number of particles adopted is 500000.

finds it optimal to default irrespective of the behavior of lenders. Second, the sovereign may be in the crisis zone next period, in which case the conditional probability of observing a default in this region of the state space is $\mathbb{E}_t[p_{t+1}]$. Finally, $\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right)$ reflects a premium required by the lenders to hold risky government securities. Our objective is to construct a time series for the contribution of rollover risk to interest rate spreads, $\text{Pr}_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}]$.

Because we do not have an empirical counterpart to the $\lambda = 1$ portfolio, we construct our object of interest with a counterfactual exercise. That is, we feed the model with a realization for the structural shocks that is equivalent to the point estimates obtained earlier, with the exception that p_t is set to 0 throughout the sample. The difference between the actual and this counterfactual time series isolates the component of interest rate spreads that is due to rollover risk. The left panel of Figure 5 reports the result of this decomposition. The red shaded area represents the component of the spreads that is explained purely by the fundamental shocks $[y_t, \chi_t]$ while the blue shaded area represents the component due to rollover risk. From the figure, we can see that the bulk of the variation in interest rate spreads over our episode can be explained by the fundamental shocks, without the need to rely on rollover risk. This latter component appears to be relevant toward the end of the sample, but, on average, it does not account for more than 25% of interest rate spreads in a given quarter.

Figure 5: **Contribution of rollover risk to interest rate spreads**
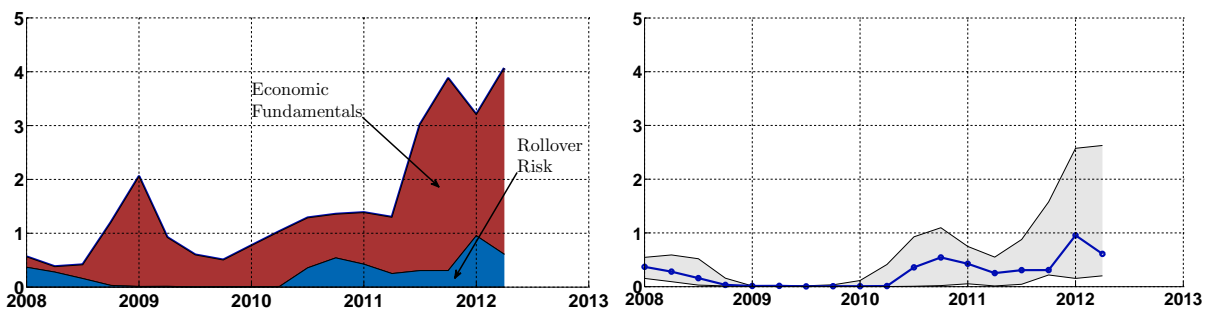
Notes: *The left panel reports the decomposition of the filtered interest rate spreads into the component due to fundamental shocks (red area) and the component due to rollover risk (blue area). The right panel plots the point estimate for the component due to rollover risk, along with a 90% confidence interval.*

The right panel of Figure 5 shows, however, that there is a large degree of uncertainty for this reading of the events. The panel plots the point estimate for the contribution of rollover risk along with its 90% confidence interval.[15] The figure shows that there are

---

[15]Even though we are fixing parameters, there is uncertainty over the different components of the spreads because of the uncertainty in the estimation of the state vector.

realizations of the state vector for which rollover risk could account for more than two-thirds of the run-up of interest rate spreads. This latter result is due to the combination of two factors. First, pricing schedules in models of sovereign debt are highly nonlinear: hence, small variations in $[y_t, \chi_t]$ have fairly large effects on interest rate spreads. Second, the lack of discipline on p_t implies that the model uses this shock to fit variation in interest rate spreads that is not explained by the fundamental shocks. These two factors imply that small degrees of uncertainty over $[y_t, \chi_t]$, generated in our experiment by the presence of measurement errors, translates into high uncertainty over the rollover risk component. We now turn to the full blown experiment and discuss how the behavior of debt duration disciplines this component.

5.2 The informational content of maturity choices

To be completed.

5.3 The role of risk averse lenders

To be completed.

6 Evaluating OMT Announcements

As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright transactions in secondary, sovereign bond markets. The technical framework of these operations was formulated on September 6 of the same year. The Outright Monetary Transaction (OMT) program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.

OMTs consist in direct purchases of sovereign bonds of members of the euro-area in secondary markets.[16] These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions.[17] The Governing Council decides on the start, continuation and suspension of OMTs in full discretion

---

[16]Transactions are focused on the shorter part of the yield curve, and in particular on sovereign bonds with a maturity of between one and three years. The liquidity created through OMTs is fully sterilized.

[17]A necessary condition for OMTs is a *conditionality* attached to a European Financial Stability Facility/European Stability Mechanism (EFSF/ESM) macroeconomic adjustment or precautionary programs. For a country to be eligible for OMTs, these programs should include the possibility of EFSF/ESM primary market purchases.

and acting in accordance with its monetary policy mandate. There are two important characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, the ECB accepts the same (*pari passu*) treatment as private or other creditors with respect to bonds issued by euro area countries and purchased through OMTs.

Even though the ECB has not yet implemented OMTs, the mere announcement of the program had significant effects on interest rate spreads of peripheral countries. Altavilla et al. (2014) estimate that OMT announcements decreased the Italian and Spanish 2 years government bonds by 200 basis points. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing non-fundamental inefficient fluctuations in sovereign bond markets of euro-area peripheral countries. Accordingly, OMT has been regarded thus far as a very successful program. The aim of this section is to put this interpretation under scrutiny.

We introduce OMTs in our model as a price floor schedule implemented by a Central Bank. Section 6.1 shows that an appropriate design of this schedule i) can eliminate the bad equilibria in our model, and ii) it does not require the Central Bank to ever intervene in bond markets. Therefore, along the equilibrium path the Central Bank can achieve a Pareto improvement without taking risk for their balance sheet. However, we also show that alternative formulations of the price floor may induce the sovereign to ask for assistance in the face of bad fundamental shocks. *Ex-ante*, this option leads the sovereign to overborrow. Under both of these scenarios, interest rate spreads decline once the Central Bank announces the price floor schedule: in the first scenario, the reduction in interest rate spreads is due the elimination of rollover risk. In the second scenario, this reduction reflects the option for bondholders to resell the security to the Central Bank whenever the sovereign is approaching a solvency crises. Section 6.2 proposes a simple procedure to test which of these two hypothesis better characterizes the observed behavior of Italian spreads after the announcements of the OMT program.

6.1   Modeling OMT

We model OMT as follows. At the beginning of each period, after all uncertainty is realized, the government can ask for assistance. In such case the Central Bank (CB) commits to buy government bonds in secondary markets at a price $q_{CB}(\mathbf{S}, B', \lambda'|\lambda)$ that may depend on the state of the economy, $\mathbf{S}$, and on the quantity of debt issued, B', and the maturity of the portfolio, λ'. We assume that assistance is conditional on the fact that total debt issued is below a cap $\bar{B}_{CB}(\mathbf{S}, \lambda') < \infty$ also set by the CB. The limit can depend on the state of the economy and on the duration of the stock of debt issued by the government. This

limit captures the conditionality of the assistance in the secondary markets. Moreover, it rules out Ponzi-scheme on the central bank. Hence OMT is fully characterized by a policy rule $(q_{CB}(\mathbf{S}, B', \lambda'|\lambda), \bar{B}_{CB}(\mathbf{S}, \lambda'))$. We assume that the CB finances such transactions with a lump sum tax levied on the lenders. We further assume that such transfers are small enough that they do not affect the stochastic discount factor $M_{t,t+1}$.

The problem for the government described in (6) changes as follows. We let $a \in \{0, 1\}$ be the decision to request CB assistance, with $a = 1$ for the case in which assistance is requested. Then we have:

$$V(\mathbf{S}) = \max_{\delta, B', \lambda', G, a} \delta \left\{ U(G) + \beta \mathbb{E}[V(\mathbf{S}')|\mathbf{S}] \right\} + (1 - \delta)\underline{V}(s_1) \tag{14}$$

subject to

$$
\begin{aligned}
G + B &\leq Y(s_1) + \Delta(\mathbf{S}, a, B', \lambda'), \\
\Delta(\mathbf{S}, B', \lambda') &= q(s, a, B', \lambda'|\lambda')B' - q(s, a, B', \lambda'|\lambda)(1 - \lambda)B,
\end{aligned}
$$

$$B' \leq \bar{B}_{CB}(\mathbf{S}, \lambda') \quad \text{if } a = 1.$$

The lenders have the option to resell government bonds to the CB at the price q_{CB} in case the government asks for assistance. The no-arbitrage condition for the lenders (7) is modified as follows:

$$
\begin{aligned}
q(\mathbf{S}, a, B', \lambda'|\lambda) = \; &\max\{aq_{CB}(\mathbf{S}, B', \lambda'|\lambda); \tag{15} \\
&\delta(\mathbf{S}) \mathbb{E}\{\delta(\mathbf{S}) \mathbb{E}\{M(s_1, s_1')\delta(\mathbf{S}')[1 + (1 - \lambda)q(s, a', B'', \lambda''|\lambda)]|\mathbf{S}\}\}.
\end{aligned}
$$

where $B'' = B'(s', B', \lambda')$, $\lambda'' = \lambda'(s', B', \lambda')$, and $a' = a(s', B', \lambda')$. Note that the bonds prices now depend also on the decision of the government to activate assistance because only in that situation the CB stands ready to buy the bonds.

Given a policy rule $(q_{CB}, \bar{B}_{CB})$, a *recursive competitive equilibrium with OMT* is value function for the borrower V, associated decision rules δ, B', λ', G and a pricing function q such that V, δ, B', G are a solution of the government problem (14) and the pricing function satisfies the no-arbitrage condition (15).

As a theoretical benchmark, it is convenient first to define the *fundamental equilibrium outcome*, $\mathbf{y}^* = \{\delta_t, B_{t+1}, \lambda_{t+1}, G_t, q_t(\lambda)\}$ as the optimal choice associated with the solution

to the following functional equation that attains the higher value:[18]

$$V^* (B_0, s) = \max_{\{\delta_t, B_{t+1}, \lambda_{t+1}, G_t, q_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \{\delta_t \beta^t U(G_t) + (1 - \delta_t) \underline{V}_t\} \tag{16}$$

subject to

$$G_t + B_t \leq Y_t + [q_t(\lambda_{t+1}) B_{t+1} - q_t(\lambda_t)(1 - \lambda_t) B_t]$$

$$q_t(\lambda) = \mathbb{E}_t \{M_{t,t+1} \delta_{t+1} [1 + (1 - \lambda) q_{t+1}(\lambda)]\}$$

and if $\delta_t = 1$

$$\mathbb{E}_t \sum_{j=0}^{\infty} \{\delta_{t+j} \beta^{t+j} U(G_{t+j}) + (1 - \delta_{t+j}) \underline{V}_{t+j}\} \geq V^* (B_t, s_t)$$

It is clear that such outcome can be implemented as a recursive competitive equilibrium outcome. We denote the objects of a recursive competitive equilibrium associated with the fundamental outcome with a superscript "$*$"

We now turn to show that an appropriately designed policy rule can uniquely implement the fundamental equilibrium outcome defined in (16), our normative benchmark.[19]

Proposition 1. *The OMT rule can be chosen such that the fundamental equilibrium is the unique equilibrium and assistance is never activated along the equilibrium path. In such case, OMT is a weak Pareto improvement relative to the equilibrium without OMT (strict if the private equilibrium does not coincide with the fundamental equilibrium).*

Proof. An obvious way to uniquely implement the fundamental equilibrium outcome is to set $q_{CB}(\mathbf{S}, B', \lambda' | \lambda) = q^*(s, B', \lambda' | \lambda)$ and $\bar{B}_{CB}(\mathbf{S}, \lambda) \leq B^{*\prime}(\mathbf{S})$ if $\lambda = \lambda^{*\prime}(\mathbf{S})$ and zero otherwise. Such construction is not necessary. A less extreme alternative is to design policies such that for all $\mathbf{S}$ for which there is no default in the fundamental equilibrium, $\delta^*(\mathbf{S}) = 1$, there exists at least one (B', λ') with $B' \leq \bar{B}_{CB}(\mathbf{S}, \lambda')$ such that

$$U(Y - B + \Delta(\mathbf{S}, 1, B', \lambda')) + \beta \mathbb{E} V^*(B', \lambda', s') \geq \underline{V}(s_1), \tag{17}$$

---

and for all (B', λ') such that $B' \leq \bar{B}_{CB}(\mathbf{S}, \lambda')$ the fundamental equilibrium is always preferable, in that

$$U\left(Y - B + \Delta\left(\mathbf{S}, 1, B', \lambda'\right)\right) + \beta \mathbb{E} V^*\left(B', \lambda', s'\right) \leq V^*\left(\mathbf{S}\right). \tag{18}$$

Under (17) and (18), it is clear that no self-fulfilling run is possible and there is no over-borrowing. Hence (17) and (18) are set of sufficient conditions to eliminate runs and to uniquely implement the fundamental equilibrium outcome. □

Note that quantity limits (conditionality) are necessary to uniquely implement the fundamental equilibrium. In absence of $\bar{B}_{CB}$, because the CB guarantees a price q_{CB} then borrower act as a price taker and so it will issue new debt such that $\partial \mathbb{E} V(s', B') / \partial B' = q_{CB} U'(G)$. That is, under assistance it is optimal for the government to choose a B' that is larger than the one in the fundamental equilibrium,

$$\begin{aligned}
\frac{\partial \mathbb{E} V(s', B')}{\partial B'} &= \left\{\sum_{j=0}^{\infty} q_j^*(1-\lambda')^j + \sum_{j=0}^{\infty} \frac{\partial q_j^*}{\partial B'}\left[(1-\lambda')^j B' - (1-\lambda)^{j+1} B\right]\right\} U'(G) \\
&< q^*\left(\lambda'\right) U'(G) = q_{CB}\left(\lambda'\right) U'(G).
\end{aligned}$$

So a limit to B' when the government asks for assistance is needed to prevent overborrowing while uniquely implement the desired outcome.

Proposition 1 gives us the most benevolent interpretation of the drop in Italian spreads after OMT was announced. If OMT follows the rule described in the proof of Proposition 1 and it uniquely implements the fundamental equilibrium outcome. In this case the observed drop in spreads is due to the fact that lenders anticipate that no run can happen along the equilibrium path and resulting in lower default probability and hence lower spreads.

However, the central bank does not want to support bond prices if they are low because of fundamental reasons. This entails a subsidy from the lenders to the borrower, reducing welfare for the lenders relative to the equilibrium without OMT (assuming lenders are the ones that have to pay for the losses of the bailout authority). Even in this scenario, bond prices may decline. To see this, suppose that in a given state the fundamental price for the portfolio of debt is $q^{*'}$. Suppose now that the ECB sets an assistance price $q'_{CB} > q'^*$. It is clear from (15) that the price today increases (the spread drops) relative to a counterfactual world without OMT.

Thus, decline in price not informative on whether ECB is following the benchmark rule, or whether it is providing some subsidy to peripheral countries. Next we use the model to test between these two alternatives.

6.2 A Simple Test

We now test for the hypothesis that the ECB did follow the policy described in Proposition 1. The logic of our approach goes as follows. Suppose that the Central Bank credibly commits to our normative benchmark. The announcement of this intervention would eliminate all extrinsic uncertainty, and the spreads today would jump to their "fundamental" value, i.e. the value that would arise if rollover crisis were not conceivable from that point onward. This fundamental level of the interest rate spread represents a lower bound on the post-OMT spread under the null hypothesis that the program was directed exclusively to prevent runs on Italian debt. Our test consists in comparing the spreads observed after the OMT announcements to their fundamental value: if the latter is higher than the observed ones, it would be evidence against the null hypothesis that the ECB followed the policy described in Proposition 1.

We perform this test using our calibrated model. Our procedure consists in three steps:

1. Obtain decision rules from the "fundamental" equilibrium defined in (16).

2. Feed these decision rules with our estimates for the fundamental shocks $\{\chi_t, v_t, y_t\}$. Obtain counterfactual post-OMT spreads justified purely by economic fundamentals.[20]

3. Compare post-OMT spreads with the counterfactual ones.

Table 3 shows the results. The first column reports the Italian spreads observed after the OMT announcements, while the second column presents the counterfactual spreads constructed with the help of our model. We can verify that the observed spreads lie below the one justified by economic fundamentals under the most "optimistic" interpretation of OMT. In 2012:Q4, the observed interest rate spread on Italian debt was 222.25 basis points, while our model suggests that the spread should have been 354.60 basis points if the program was exclusively eliminating rollover risk. Therefore, our model suggests that the decline in the spreads observed after the OMT announcements partly reflects the anticipation of a future intervention of the ECB in secondary sovereign debt markets. This is not surprising given our result in Section 5: since rollover risk was almost negligible in 2012:Q2, the observed drastic reduction in the spreads should partly reflect the value of an implicit put option for holders of Italian debt guaranteed by the ECB.

Clearly, it would be interesting to use our model to dig deeper into the implications of the OMT program. For example, we could try to measure the put option implicit in this

---

[20]The estimates of the state vector ends in 2012:Q2. For the 2012:Q3-2012:Q4 period, we set y_t equal to linearly detrended Italian output and we filter out $\{\chi_t, v_t\}$ using our pricing model along with the data on the German yield curve and the euro-area price-consumption ratio.

Table 3: **Actual and Fundamental Sovereign Interest Rate Spreads in Italy**

	Actual spreads	Spreads justified by fundamentals
2012:Q3	348.24	379.80
2012:Q4	222.25	354.60

intervention, to calculate the amount of resources that the ECB is implicitly committing under this policy or we could assess the moral hazard implications associated to this policy. This would not be an uncontroversial task, as it would require us to i) specify the policy rule followed by the ECB and to ii) specify how the selection mechanism responds to the policy intervention. The test we have described in this section is robust to these caveats, and we regard it as a first step for the evaluation of this type of interventions in sovereign debt models with multiple equilibria.

7   Conclusion

In this paper, we studied how important was non-fundamental risk in driving interest rate spreads during the euro-area sovereign debt crisis. We show that the joint behavior of interest rate spreads and debt duration provides key information for this purpose. Our preliminary results indicate that non-fundamental risk accounted for a modest fraction of the increase in interest rate spreads during the recent sovereign debt crisis.

Our analysis is limited to non-fundamental risk that arises from rollover risk as introduced in Cole and Kehoe (2000). We did not consider the type of multiplicity emphasized in Calvo (1988) and recently revived by Lorenzoni and Werning (2013) and Navarro et al. (2015). Future research should investigate which feature of the data can be used to discipline empirically these other sources of multiplicity.

References

Aguiar, Mark and Manuel Amador, "Take the Short Route: How to Repay and Restructure Sovereign Debt with Multiple Maturities," 2014. Manuscript, Federal Reserve Bank of Minneapolis.

Alesina, Alberto, Alessandro Prati, and Guido Tabellini, "Public Confidence and Debt Management: A Modela and a Case Study of Italy," 1989. NBER Working Paper, No. 3135.

Altavilla, Carlo, Domenico Giannone, and Michele Lenza, "The Financial and Macroeconomic Effects of OMT Announcements," 2014. ECB Working Paper Series, No 1707.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 2008, *98* (3), 690–712.

— **and A. Ramanarayanan**, "Default and the Maturity Structure in Sovereign Debt," *Journal of Political Economy*, 2012, *120*, 187–232.

Aruoba, Boragan, Pablo Cuba-Borda, and Frank Schorfheide, "Macroeconomic Dynamics near the ZLB: A Tale of Two Countries," 2014. Manuscript, University of Pennsylvania.

Auclert, Adrien and Matthew Rognlie, "Unique Equilibrium in the Eaton-Gersovitz Model of Sovereign Debt," 2014. Manuscript, M.I.T.

Backus, David K, Silverio Foresi, and Chris I Telmer, "Affine term structure models and the forward premium anomaly," *Journal of finance*, 2001, pp. 279–304.

Bai, Yan, Seon Tae Kim, and Gabriel Mihalache, "The Maturity and Payment Schedule of Sovereign Debt," 2014. Manuscript, University of Rochester.

Bakaert, G., E. Engstrom, and Y. Xing, "Risk, Uncertainty and Asset Prices," *Journal of Financial Economics*, 2009, *91*, 59–82.

Bekaert, Geert and Steven R Grenadier, "Stock and bond pricing in an affine economy," Technical Report, National Bureau of Economic Research 1999.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez, "International Reserves and Rollover Risk," 2014. Manuscript, Wisconsin University.

Borri, Nicola and Adrien Verdhelan, "Sovereign Risk Premia," 2013. Manuscript, MIT Sloan.

Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler, "Why Do Emerging Economies Borrow Short Term?," *Journal of the European Economic Association*, 2013, *11*, 67–100.

Calvo, Guillermo, "Servicing the Public Debt: The Role of Expectations," *The American Economic Review*, 1988, *78*, 647–661.

Campbell, John and John Cochrane, "By force of habit: a consumption-based explanation of aggreagte shock market behavior," *Journal of Political Economy*, 1999, *107*, 205–251.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness and Default Risk," *The American Economic Review*, 2013, *6*, 2674–2699.

Cole, Harald L. and Timothy J. Kehoe, "Self-Fulfilling Debt Crisis," *Review of Economic Studies*, 2000, *67*, 91–116.

Conesa, Juan Carlos and Timothy J. Kehoe, "Gambling for Redemption and Self-Fulfilling Crises," 2012. Research Department Staff Report 465, Federal Reserve Bank of Minneapolis.

Corsetti, Giancarlo and Luca Dedola, "The "Mystery of the Printing Press". Monetary Policy and Self-fulfilling Debt Crises," 2014. Manuscript, Cambridge University.

Cruces, Juan J. and Christoph Trebesch, "Sovereign defauls: the price of haircuts," *American Economic Journal: Macroeconomics*, 2013, *5*, 85–117.

De Grauwe, Paul and Yuemei Ji, "Self-filfilling Crises in the Eurozone: An Empirical Test," *Journal of International Money and Finance*, 2013, *34*, 15–36.

Dovis, Alessandro, "Efficient Sovereign Default," 2014. Manuscript, Penn State University.

Duffie, Darrell, Rui Kan et al., "A yield-factor model of interest rates," *Mathematical finance*, 1996, *6*, 379–406.

Eaton, Jonathan and Mark Gersovitz, "Debt with potential repudiation: Theoretical and empirical analysis," *Review of Economic Studies*, 1981, *48*, 289–309.

Farmer, Roger E. A. and Jang-Ting Guo, "The econometrics of indeterminacy: an applied study," *Carnegie-Rochester Conference Series on Public Policy*, 1995, *43*, 225–271.

Giavazzi, Francesco and Luigi Spaventa, *High public debt: the Italian experience*, CUP Archive, 1988.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long Duration Bonds and Sovereign Defaults," *Journal of International Economics*, 2009, *79*, 117–125.

Jovanovic, Boyan, "Observable Implications of Models with Multiple Equilibria," *Econometrica*, 1989, *57*, 1431–1437.

Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton, "How Sovereign is Sovereign Credit Risk?," *American Economic Journal: Macroeconomics*, 2011, *3*, 75–103.

Lorenzoni, Guido and Ivan Werning, "Slow Moving Debt Crisis," 2013. Manuscript, Northwestern University.

Lubik, Thomas A. and Frank Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 2004, *94*, 190–217.

Navarro, Gaston, Junapa Nicolini, and Pedro Teles, "Sovereign Defaults: The Role of Expectations," 2015. Manuscript, Federal Reserve Bank of Minneapolis.

Piazzesi, Monika, "Affine Term Structure Models," in Y. Ait-Sahalia and L. P. Hansen, eds., *Handbook of Financial Econometrics: Tools and Techniques*, Elsevier, 2010.

Roch, Francisco and Harald Uhlig, "The Dynamics of Sovereign Debt Crises and Bailouts," 2014. Manuscript, University of Chicago.

Sanchez, Juan M., Horazio Sapriza, and Emircan Yurdagul, "Sovereign Default and Maturity Choice," 2015. Manuscript, University of Rochester.

Stangebye, Zachary, "Dynamic Panics: Theory and Application to the Eurozone," 2014. Manuscript, University of Pennsylvania.

Tabellini, Guido et al., "Monetary and Fiscal Policy Coordination With a High Public Debt," Technical Report, UCLA Department of Economics 1987.

Wachter, Jessica A., "A Consumption-based model of the term structure of interest rates," *Journal of Financial Economics*, 2006, *79*, 365–399.