The International CAPM Redux

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Abstract

We provide evidence that international equity investors are compensated for bearing currency risk. Three factors — a global equity factor denominated in local currencies, and two currency factors, dollar and carry — account for a wide cross-section of equity returns from 46 developed and emerging countries from 1976 to the present. They are also useful at explaining the risks of international mutual funds and hedge funds. A simple complete-markets model replicates our empirical findings. The model implies a novel perspective on optimal currency hedging.

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1   Introduction

Are international equity investors compensated for bearing exchange rate risk? This question is increasingly relevant in light of the rapid acceleration of international financial integration over the past three decades. Figure 1 shows that aggregate foreign equity holdings as a percentage of global gross domestic product have increased steadily from roughly 3% in the 1980s to approximately 30% in 2011, alongside a steady dismantling of de-jure cross-border capital flow restrictions. The magnitudes are large, highlighting the importance of this issue – in the United States, for example, foreign equity holdings are currently worth roughly US$ 6 trillion.

In theory, exchange rate risk should matter to the pricing of equities and other risky assets in a world of real rigidities and deviations from purchasing power parity. These deviations affect the consumption of international investors, who invest abroad, but consume at home. In most plausible theoretical international asset pricing models, this effect on consumption leads to equilibrium compensation for the risk of low returns on foreign investments once these returns are expressed in real domestic terms. Reasonable though this rationale is, empirical work in international asset pricing, with a few notable exceptions, has not been able to provide convincing evidence that currency risk is priced in international equity markets.

In this paper, we present a new three-factor model to capture the risks in international equity portfolios. These three factors are the return on a world market portfolio denominated in local currency terms, and two currency factors which effectively summarize variation in a broad cross-section of bilateral exchange rates, namely, the dollar

Figure 1
Portfolios of Foreign Equity Assets and Financial Openness

This figure shows the portfolio equity assets (bar plots) for 39 developed and emerging countries along with a de jure financial openness index. In each year, portfolio assets of all countries are summed up and divided by the corresponding world GDP. The financial openness index is based on the restrictions in capital flows listed by the IMF over time; a world index is obtained by GDP-weighting country indices. Estimates of foreign assets come from Lane and Milesi-Ferretti (2007). The financial openness index comes from Chinn and Ito (2008). The set of developed countries comprises Australia, Austria, Belgium, Canada, Hong Kong, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Arab Emirates, United Kingdom, and United States. The data are annual and the sample period is 1971 to 2011.
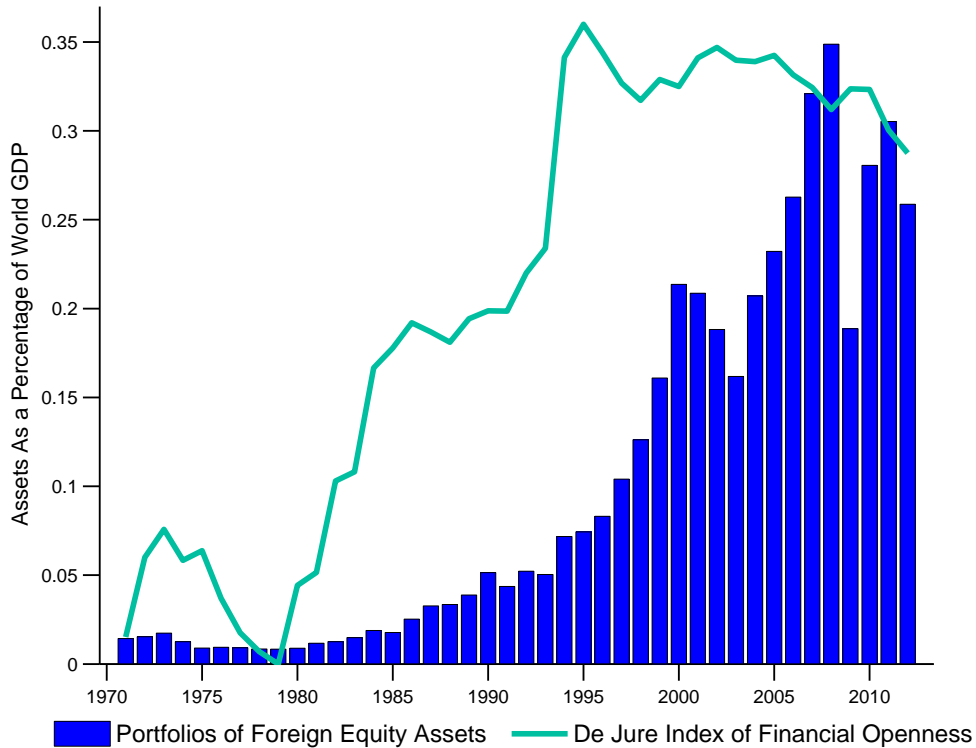
factor and the carry factor.[1]

To test the model, we compile a comprehensive dataset of equity returns spanning value, size, and country aggregate portfolio returns from 25 developed markets and 21 emerging markets between February 1976 and April 2013. When estimated on these data, the model delivers an important role for currency risk in the cross-section of equity returns.

Importantly, in conditional asset pricing tests, our model outperforms the single-global factor World CAPM as well as the International CAPM estimated by Dumas and Solnik (1995), and delivers comparable performance to the global versions of the popular Fama and French (2012) factors. We show that our model is also relevant to those interested in delegated portfolios of international assets. It is often difficult to infer the amount of currency risk taken on by international investment managers, and our factors offer a simple metric of the extent of this issue, helping to explain a significant fraction of the variation in international mutual and hedge fund returns.

To better understand the dynamics captured by the empirical risk factors, we build a simple theoretical model in which exchange rates, currency risk factors, and equity market returns are all precisely defined. In leading international equity asset pricing models, exchange rate variation arises exogenously. In contrast, we assume that exchange rates are driven by the same shocks that drive the pricing kernels. To better explore the impacts of this important difference, we simplify all other aspects of the model, assuming that financial markets are complete, and writing down the law of motion of the lognormal stochastic discount factors (SDFs) in all countries.

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[1] Verdelhan (2014) shows that a substantial fraction of the variation in bilateral exchange rates can be captured by these two factors. The dollar factor is the average excess return earned by an investor that borrows in the U.S. and invests in a broad portfolio of foreign currencies. The carry factor is the average excess return earned by an investor that goes short (long) in a portfolio of low (high) interest rate currencies.

In our setup, country SDFs depend on country-specific shocks, as well as three global shocks, and each country's aggregate dividend growth rate also depends on the same set of shocks. To create a role for a pure equity risk factor (which affects all countries in the same way), we allow one of the global shocks to affect all SDFs similarly. Since changes in exchange rates are differences in log SDFs, this global shock does not show up in currency markets, resulting in a degree of segmentation between equity and currency markets. This simple innovation to the basic complete markets model, namely, the addition of the "equity" global shock, allows us to work in a very tractable framework in which (realized and expected) equity and currency returns can be written down in closed form.

The model reveals that the world aggregate equity return expressed in a common currency actually does contain all relevant information necessary for pricing international assets, meaning that there should be no role for bilateral exchange rates. The twist is that time-variation in the prices of global shocks – necessary to account for time-varying currency risk premia – confounds empirical estimation using a single-factor model, especially when the relevant state variables are unknown to the econometrician. In other words, a conditional version of the world CAPM holds in the model, but in practice the conditioning variables are unknown. We show that including the currency risk factors in our three-factor empirical model arises as a natural solution to the challenges of empirical estimation in this framework.

We use the model to shed light on our main empirical results, calibrating it to match a large set of equity and currency moments, and generating simulated data. We apply exactly the same estimation procedure as we do on real data to these simulated data, and replicate our empirical finding that the new "International CAPM Redux" outperforms both the World CAPM of Sharpe (1964) and Lintner (1965), and the "Classic" International CAPM of Adler and Dumas (1983). We note that time-variation in the prices of risk appears key to this result.

Finally, we use the model to revisit a longstanding and important issue in international finance, namely, optimal currency hedging in international portfolios.[2] The prevailing wisdom arises from the mean-variance efficient framework of Black (1989), in which the optimal amount of currency hedging depends on the mean and standard deviation of world market returns, as well as on exchange rate volatility. In other words, Black (1989) shows that the optimal amount of currency hedging is constant through time and across investor location. In contrast, in our model, in which exchange rates are driven by many of the same risks affecting equities, the optimal amount of investment in currency portfolios such as carry and dollar is time-varying and investor-location-specific, and depends on the prices of risk of both country-specific and global shocks.

Our paper relates to two very large strands of literature on international equity markets and on currency risk. A short paragraph in the introduction of this paper would not do justice to this literature. Instead, we propose a four-page review of the most relevant work. In the interest of space, the material is placed at the start of the Appendix.

The paper is organized as follows. Section 2 shows the empirical specifications of the various international asset pricing models that we test. Section 3 describes the data on which we conduct these asset pricing tests, and Section 4 discusses the results. Section 5 presents our simple theoretical model which endogenizes exchange rates in complete markets and gives rise to the empirical specification of the International CAPM Redux. Section 6 shows how we can use the model to shed light on optimal hedging in a world with endogenous exchange rates, and Section 7 concludes. All robustness checks and

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[2]This continues to be a hot topic for both institutional and retail investors. For example, four Exchange-Traded Funds (ETFs) that seek to completely hedge the currency risk component of international equity returns are among the ten fastest growing ETFs over the last six months, collecting more than 30 billion dollars over this short period.

additional results that are mentioned but not reported in the paper are provided in the Online Appendix.

2 Specifying and Testing International Asset Pricing Models

We compare the performance of our three-factor model against a range of alternative asset pricing models: the World CAPM, the International CAPM, and a global version of the Fama-French-Carhart model. In this section, we briefly present the empirical specifications of these leading international asset pricing models and contrast them with our three-factor specification.

2.1 The World CAPM and the International CAPM

The World CAPM is a simple extension of the CAPM to global markets. The additional assumption necessary in the global context is that PPP always holds, meaning that currency risk is rendered irrelevant. In the World CAPM, global market risk is the single source of systematic risk driving asset prices, and international investors should only earn a premium for exposure to this source of risk. Empirically, the measure of global market risk has generally been the excess return on the world equity market portfolio, $WMKT$, denominated in a common currency, generally U.S. dollars. Writing $r^{\$}_{i,t+1} - r_{f,t}$ for the excess return of asset i at time $t+1$ expressed in dollars, the empirical specification of this model is:

$$r^{\$}_{i,t+1} - r_{f,t} = \alpha^i_{WCAPM} + \beta^i_{WMKT} WMKT_{t+1} + \epsilon_{t+1}. \tag{1}$$

In the international CAPM of Adler and Dumas (1983), PPP does not hold instantaneously, and exchange rates are an additional source of exogenous risk. This model

is operationalized in the empirical work of Dumas and Solnik (1995). With the same notation as before, and r_{t+1}^{GBP}, r_{t+1}^{JPY}, r_{t+1}^{DEM} representing excess currency returns denominated in Pound Sterling, Japanese Yen, and Euro/Deutsche Mark, respectively, the (unconditional) model specification is:

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\begin{aligned}
r_{i,t+1}^{\$} - r_{f,t} &= \alpha_{ICAPM}^{i} + \beta_{WMKT}^{i} WMKT_{t+1} \\
&+ \beta_{GBP}^{i} r_{t+1}^{GBP} + \beta_{JPY}^{i} r_{t+1}^{JPY} + \beta_{DEM}^{i} r_{t+1}^{DEM} + \epsilon_{t+1}.
\end{aligned} \tag{2}
$$

2.2 Fama-French Global Factor Model

Since their discovery, the Fama and French (1993) three factors and the Carhart (1997) four factors, although not based on a particular theoretical model, have become standard benchmarks in empirical asset pricing. These models ignore exchange rate risk, and offer an explanation for patterns in international average stock returns based on loadings on size, value, and momentum premia:

$$
\begin{aligned}
r_{i,t+1}^{\$} - r_{f,t} &= \alpha_{FF}^{i} + \beta_{WMKT}^{i} WMKT_{t+1} \\
&+ \beta_{SMB}^{i} SMB_{t+1} + \beta_{HML}^{i} HML_{t+1} + \beta_{WML}^{i} WML_{t+1} + \epsilon_{t+1}
\end{aligned} \tag{3}
$$

where $WMKT$ is defined as before, SMB is small minus big, capturing the size premium, HML is high minus low (book-to-market), capturing the value premium, and WML is (short-term) winners minus losers, capturing the effect of momentum.

2.3 The International CAPM Redux

We present now intuitively our three-factor specification. A proper derivation is presented in Section 5.

The International CAPM read literally recommends the use of all bilateral exchange

rates as additional risk factors, which is somewhat cumbersome empirically. However, we know from recent research in currency markets that a large set of bilateral exchange rates can be summarized using carry and dollar factors, which are well able to capture systematic variation in bilateral exchange rates. In addition to these currency factors, one may need additional factors to summarize equity risk unrelated to currency risk.

This heuristic description is simply here to introduce our empirical model, and we spend a great deal of time rationalizing the model, following the discussion of our empirical results. For now, we describe our international CAPM Redux model simply as:

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\begin{aligned}
r_{i,t+1}^{\$} - r_{f,t} &= \alpha_{CAPMredux}^{i} + \beta_{LWMKT}^{i} LWMKT_{t+1} \\
&+ \beta_{Dollar}^{i} Dollar_{t+1} + \beta_{Carry}^{i} Carry_{t+1} + \epsilon_{t+1},
\end{aligned}
\tag{4}
$$

where $LWMKT_{t+1}$ denotes the excess return on the world market portfolio denominated in local currencies, and the construction of $Carry_{t+1}$ and $Dollar_{t+1}$ is described below.

2.4 Time-varying Quantities and Risk Prices

While we write all these models in their unconditional form, we estimate all of them conditionally, using rolling windows, to account for the possibility that betas and market prices of risk vary over time. Time-variation in the models' parameters is not a luxury, but a key feature of any international asset pricing exercise. To see this point clearly, let us assume that financial markets are complete.

When the law of one price holds on financial markets and investors can form portfolios freely, there exists a SDF M_{t+1} that prices any return R_{t+1}^{i} such that $E_t\left(M_{t+1} R_{t+1}^{i}\right) =$

1.[3] The same condition holds for the risk-free rate R_f. Assuming that the returns and SDF are lognormal, the Euler equation implies:

$$E_t \left(r_{t+1}^i - r_{f,t} + \frac{1}{2} var_t(r_{t+1}^i) \right) = \underbrace{- \frac{cov_t(m_{t+1}, r_{t+1}^i)}{var_t(m_{t+1})}}_{\beta_t^i} \underbrace{var_t(m_{t+1})}_{\Lambda_t} \tag{5}$$

where lower letters denote logs. Expected excess returns are the product of the quantity of risk, β_t^i, which is asset-specific, and the market price of risk, Λ_t.

It is well-known since Bekaert (1996) and Bansal (1997) that in a lognormal model in complete markets, the log currency risk premium equals half the difference between the conditional volatilities of the log domestic and foreign SDFs. Since currency risk premia are time-varying (as shown by the large literature on uncovered interest rate parity and the forward premium puzzle), log SDFs must be heteroskedastic.[4] That is, empirical estimation must (at least) account for time-varying market prices of risk (Λ_t).

With this set of empirical specifications in hand, we turn now to the data.

3 Data

This section describes our test assets and risk factors.

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[3] The law of one price on financial markets implies that for any payoffs X and Y, their prices satisfy $P(aX + bY) = aP(X) + bP(Y)$ for any real numbers a and b.

[4] When SDFs and returns are not lognormal, a similar result implies that the higher moments of the SDF must be time-varying. Lustig and Verdelhan (2015) extend the Bekaert (1996) and Bansal (1997) result to incomplete markets: in the case of lognormal shocks, the expected currency risk premium depends additionally on the conditional volatility of the incomplete market wedge.

3.1 International Equity Returns

Our equity return data span 46 countries, comprising 225 different indices, over the period from January 1976 to April 2013. The coverage of countries follows the constituents of the 2011 Morgan Stanley Capital International (MSCI) Global Investable Market Indices. Following MSCI's approach, countries are classified into two categories, namely, 25 developed markets and 21 emerging markets. For each of these countries, Datastream reports daily total return series denominated in U.S. dollars for five different MSCI indices, namely, (i) the aggregate market, (ii) an index of growth stocks, (iii) an index of value stocks, (iv) an index of large market-capitalization stocks, and (v), an index of small market-capitalization stocks. Monthly returns are obtained from end-month to end-month. The risk-free rate is the U.S. 30-day Treasury bill rate, obtained from Kenneth French's website. Countries and asset types enter the equity data set at different points in time, depending on data availability. There are 54 test assets at the beginning of the sample in 1976, covering 18 developed markets and three asset types, namely, aggregate market, value, and growth. The size of the cross-section progressively increases from 1986 onwards.

The MSCI portfolios offer a challenging cross-section of returns to explain. Figure 2 provides a pictorial description of this fact – in the figure, for each asset type, countries are sorted according to their average market excess returns. The figure shows that these test assets exhibit large cross-sectional variation in average aggregate equity excess returns, across both developed and emerging countries, and across the different types of indices.

Our two global equity factors, $WMKT_{t+1}$ in U.S. dollars and $LWMKT_{t+1}$ in local currencies are monthly series, and we construct them using the daily MSCI World Index series denominated either in U.S. dollars or in local currrencies, obtained from Datastream. Monthly returns are computed from end-month to end-month; excess

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Figure 2

Annualized Average Excess Returns by Country Group and Asset Type

This figure shows annualized average excess returns (in percentage) for Developed Markets (left plot) and Emerging Markets (right plot). Monthly returns are computed on five types of MSCI country equity indices (aggregate market, growth stock, value stock, big cap and small cap indices) and reported in excess of the U.S. one month Treasury bill rate. In both graphs, sorting is based on aggregate market excess returns: countries are ranked from the highest to the lowest value of those returns. Average excess returns are computed over different samples due to data availability. For data coverage refer to Table A.1. Daily indices are from Datastream; monthly returns are computed from end-of-month series; annualized returns are obtained multiplying monthly sample average excess returns by 12. The sample is February 1976 to April 2013.

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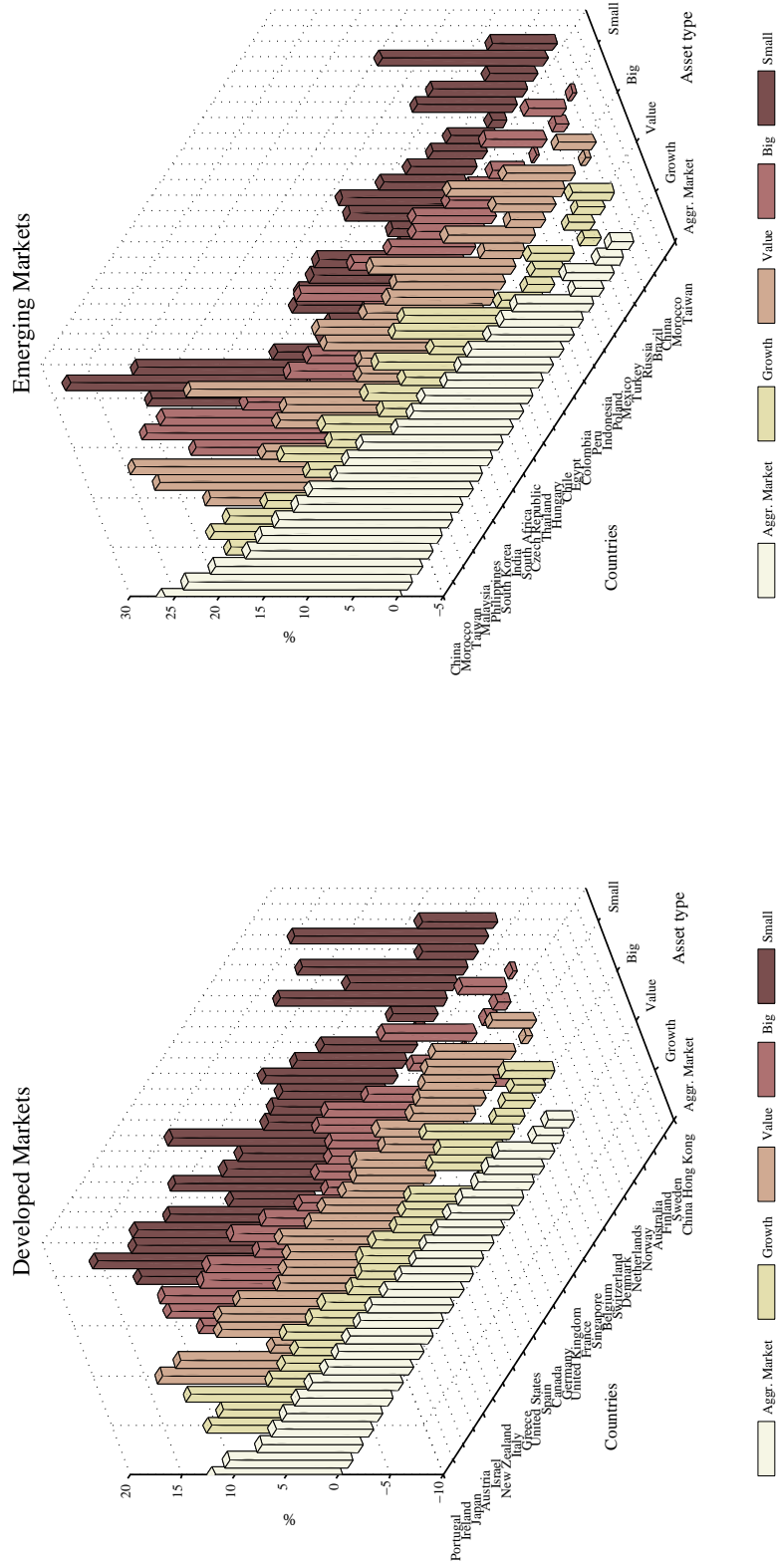
returns simply subtract off the U.S. 30-day Treasury bill rate. The size, value, and momentum international Fama and French (2012) factors and the U.S. size, value and momentum Fama and French factors are obtained from Kenneth French's website. All these series are denominated in U.S. dollars. International series are available from July 1990, except for the momentum series that start in November 1990.

Our use of a world equity index built without any exchange rate data is to avoid any misattribution of currency risk to equities and vice versa. This "synthetic" local factor may be raise concerns about real-world implementability of our model. To address such concerns, we show that this factor can easily be replicated using existing mutual fund returns. The Appendix presents details about the construction of a factor mimicking portfolio (FMP) for $LWMKT$, and shows that this FMP is highly correlated with $LWMKT$, and delivers virtually identical returns. The Appendix also shows that the pricing errors from estimating the model using the FMP rather than $LWMKT$ are virtually identical to the ones we obtain in our benchmark results.

We turn next to describing our exchange rate data.

3.2   Exchange Rates, Carry, and Dollar

We obtain daily spot and one-month forward exchange rate series (midpoint quotes) quoted in British pounds for the same set of countries as above by merging data from Datastream, Reuters, Barclays, and additional sources. The Online Appendix describes these series in detail.

Assuming that the covered interest parity condition (CIP) holds, the difference between the (log) forward and spot exchange rates (i.e., the forward discount) is equal to the interest rate differential (in log-form) between the foreign and domestic nominal one-month risk-free rates. Countries enter the currency data set at different points in time according to the availability of their forward rate series. There are 15 currencies

at the beginning of the sample in 1976 and 28 at the end. The maximum monthly coverage is 34, as the Euro replaces national Euro area currencies from January 1999 onwards.

Following Lustig and Verdelhan (2005, 2007), at each time t, we create six currency portfolios by sorting all available currencies in our data set by their forward discounts. These portfolios are rebalanced at the end of each month. The excess returns on the carry factor in each month t, denoted $Carry_t$, are constructed as the difference between the returns on the top portfolio minus the return on the bottom portfolio constructed in this fashion. The average excess return earned by a U.S. investor on the carry trade strategy is 7.65%.

To construct the dollar excess return, we assume that in each period an investor borrows in the U.S. and invests in all other currencies in our data set. The excess returns on this strategy in each month are denoted by $Dollar_t$. The correlation between the dollar factor and the carry factor has historically been low – in our data set, it is 0.18 over the full sample period, but rises to 0.40 in the post-1990 period, driven primarily by the incidence of currency and financial crises in this latter period.

We also expand our set of test assets beyond equity markets by adding two sets of six currency portfolios, either sorting countries by their short-term interest rates (i.e., Carry portfolios) or by their dollar betas (i.e., Dollar portfolios). The construction of these portfolios is described in greater detail in Verdelhan (2014).

3.3   Mutual Fund and Hedge Fund Returns

In our empirical work, we also use our model to evaluate the exposure of international mutual funds and hedge funds to currency risk.

Monthly mutual fund data are from CRSP. The sample includes all funds classified as "Foreign Equity Funds" according to the CRSP fund style code. The sample period

is 11/1990–4/2013, that is the horizon over which the Fama-French global factors are constructed. This choice ensures a large cross-section of mutual fund returns. At each time t we compute returns only if CRSP provides the corresponding total net asset value (NAV) under management. There are 74 funds with 5 years of past returns as of October 1995, but up to 1148 funds by the end of the sample, which collectively manage roughly US\$ 800 billion.

Monthly hedge fund data are from the updated version of the consolidated hedge fund database built by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013).[5] The sample period for these data is 1/1994–4/2013. From this universe, we select funds with self-reported strategies falling into "Macro" or "Emerging" categories, and we do not select any funds-of-funds. The database includes fund returns net of management and incentive fees, and fund assets under management (AUM). All series are denominated in U.S. dollars. The number of hedge funds varies over time – with 85 funds at the beginning of the sample and 362 towards the end of the sample, which collectively manage roughly US\$ 149 billion.[6]

We now turn to the results of our asset pricing tests.

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[5]Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013) consolidate data from the TASS, HFR, CISDM, Morningstar, and BarclayHedge databases.

[6]For mutual fund series, when NAV are annual or quarterly, we linearly interpolate monthly values. We adopt the same procedure for single missing observations, which we interpolate using the two adjacent values. Likewise, for hedge fund series, single missing observations of the AUM are linearly interpolated. We never interpolate returns.

4 Time-series and Cross-sectional Asset Pricing Tests

4.1 Time-series Tests

As described above, there are numerous reasons to expect that capturing the role of currency risk will require allowing for time-variation in the prices and quantities of risk. This importance of using a conditional model is also consistent with the findings of Dumas and Solnik (1995) when testing international asset pricing models.

Harvey (1991) and Dumas and Solnik (1995) capture time-variation in factor risk premia by conditioning on a set of instruments. As Cochrane (2001) notes, since the econometrician does not know the true set of state variables, conditional asset pricing tests are joint tests of the set of variables employed as conditioning information and whether the asset pricing model minimizes pricing errors.

Our principal approach in our asset pricing tests, therefore, is to estimate time-variation in factor loadings using simple rolling window regressions in the spirit of Lewellen and Nagel (2006). Following their implementation, we use 60-month rolling windows for our regressions. This choice means that the maximum number of rolling regressions we run for a single country is 388, and the minimum is 161 – this variation is a result of country-specific data availability. In the Online Appendix, we verify that our results are robust to the use of other window sizes (namely, 36- and 72-month windows).[7]

Figure 3 shows that there is indeed substantial time-variation in the factor betas estimated using our model across rolling windows. For each country in the data set and each risk factor in the model, the figures report the average rolling factor loading (the central dot in each figure), as well as the range between the minimum and the

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[7]Results from the unconditional estimations are omitted to save space, but intercepts $\hat{\alpha}$, adjusted R-squared statistics and beta estimates are available upon request.

maximum estimated rolling factor loadings (the two ends of the line in each figure).[8] Across countries, especially for the currency factors, there are significant differences both in the magnitude of the risk exposures, and the degree to which they vary over time. While these features are evident in both sets of markets, they are more pronounced in emerging markets.

With few exceptions (primarily in developed countries), estimated carry loadings switch sign over time, and dollar betas are more volatile than carry betas. Extreme examples of time variation include the dollar loadings of Indonesia, and the carry loadings of Turkey. The Netherlands and the United States show the most stable exposures to currency risk. The heterogeneity across countries is so pronounced that it is difficult to identify common patterns. For example, while Japan and Switzerland are typical carry-trade funding countries, the carry factor loadings of their equity returns often move in opposite directions.

Figure 4 presents a high-level overview of the time-series of the estimated coefficients from our international CAPM Redux model. The figure shows results from 60-month rolling-window regressions of test asset returns on the factors in our new model. Developed markets are on the left- and emerging markets on the right-hand side of the figure.

The top panel in both columns shows the number of test assets in our asset pricing tests. Until late 1991, the set of test assets (including aggregate country indexes and value and size sorted equity portfolios) is restricted to those from the developed countries. Emerging countries begin entering the data in 1991, but really only constitute

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[8]Country-by-country time-series of the betas, along with Newey and West (1987) standard error bands computed with the optimal number of lags according to Andrews (1991), are available upon request. They confirm that time-variation in these risk-exposures is not driven by outliers, and is usually statistically significant.

Figure 3

Time-Varying Factor Betas

This figure shows, for each country in the dataset, the interval between the minimum and the maximum value (black arrow) of a time-series of 60-month rolling factor betas along with their average values (white dot). Monthly equity excess returns are regressed on a constant, the global equity factor (LWMKT) and the dollar and carry factor over 60-month rolling windows. Test assets are excess returns on MSCI aggregate market indices denominated in U.S. dollars. The sample period is February 1976 to April 2013.
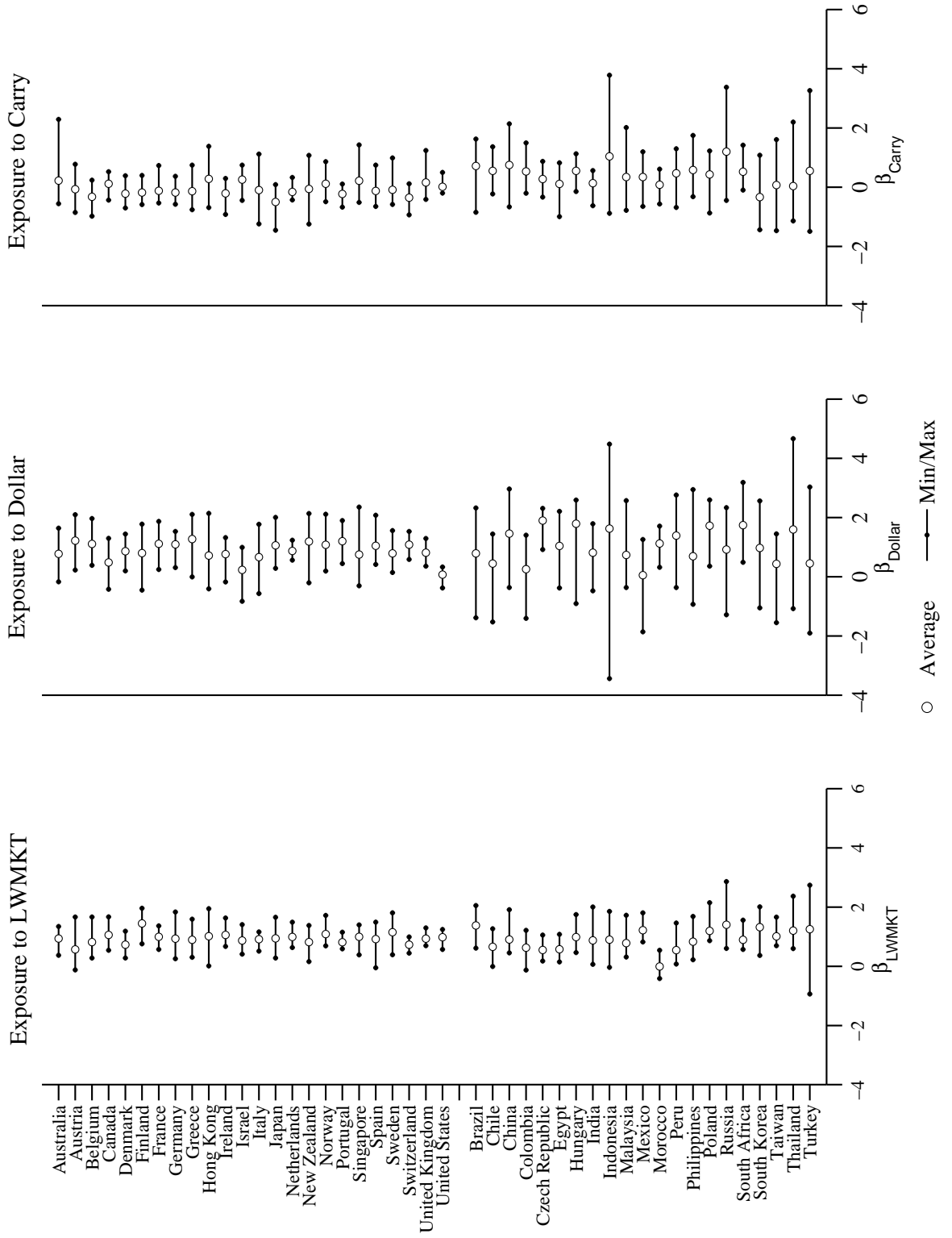

Exposure to LWMKT

Exposure to Dollar

Exposure to Carry

Figure 4
CAPM Redux: Significant Time-Varying Exposure to Global Factors

This figure reports the time-series of the share of test assets with significant exposure to the global equity and currency factors (β_t^{**}). Monthly equity excess returns are regressed on a constant and the global equity, dollar and carry factor over 60-month rolling windows. The size of the cross-section varies across time according to data coverage (# Test Assets). Test assets are equity excess returns on MSCI aggregate market, value, growth, small cap and big cap indices for developed (left graphs) and emerging (right graphs) markets. Standard errors are Newey-West. Statistical significance is tested at the 5% level. The sample period is February 1976 to April 2013.

a significant fraction of the data from the late 1990s onwards. The set of assets continues expanding well into the 2000s, posing some challenges for time-series tests, since we suffer from large N, small T problems in our unbalanced panel. We explain how we conduct GRS tests in this setting below.

The panels below show the percentage of betas from the model that are statistically significant in each rolling window. Despite the overlap between these windows and the mechanical persistence of these estimates over shorter periods, it is clear that there is considerable time-variation in the statistical significance of these beta estimates. The equity factor, $LWMKT$ is virtually always significant for the developed markets, and by the end of the sample period, for the emerging markets as well. The Dollar factor also shows considerable statistical significance for the developed markets, with over 50% of the set of test assets having statistically significant loadings on this factor, and has increasing significance as an explanatory variable for emerging market assets, reaching statistical significance in 75% of test asset regressions by the end of the sample. Finally, the Carry factor shows substantial time variation in its statistical significance, peaking after crises, with statistical significance seen for between 10% and 50% of test assets depending on the rolling window. The fact that these factors are statistically significant is important, especially in light of recent literature in asset pricing which casts doubt on second-stage results from standard two-pass cross-sectional asset pricing tests when first-stage betas are statistically insignificant (see, for example, Bryzgalova, 2015).

Table 1 shows the first comparison between our model and the competitor international asset pricing models, using the usual "GRS" F-test of Gibbons, Ross, and Shanken (1989) on each model to test whether all test-asset intercepts are jointly zero. The table compares the models by reporting the share of rolling windows in which the GRS test rejects the null hypothesis that rolling alphas implied by a given asset pricing model are jointly zero, at the 5% level. To combat the "large N, small T" issue, we re-

Table 1

Rolling GRS Tests

This table compares the World CAPM, the International CAPM, the Fama-French four-factor model (4FF) and the CAPM Redux by reporting the share of rolling windows in which the GRS test rejects the null hypothesis that rolling alphas implied by a given asset pricing model are jointly zero at the 5% level. Intercepts are estimated by regressing country-i equity excess returns on a constant and the appropriate set of factors over rolling-windows of 60 months. Under the assumption of normality, the GRS test statistic is F-distributed and defined as $\frac{T-N-K}{N}\left(1 + E_T(f)'\hat{\Omega}^{-1}E_T(f)\right)^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \sim F_{(N,T-N-K)}$, where T, N and K denote the sample size, the number of test assets, and the number of factors, respectively, $\hat{\Omega}$ is the variance-covariance matrix of the factors f and $\hat{\Sigma}$ is the variance-covariance matrix of the estimated residuals. Five sets of test assets are considered (i.e., excess returns on MSCI aggregate market/value/growth indices and small/large capitalization indices). Developed and emerging markets are treated separately in Panel I and II and jointly in Panel III. Fama-French factors are obtained by combining U.S. factors with their global counterparts. The sample period is February 1976 to April 2013.

Model	Test assets				
	Aggr. Market	Value	Growth	Small	Big
I: Developed Markets					
World CAPM	28.68	3.62	1.29	4.91	1.81
Int. CAPM	29.72	4.91	0.26	4.91	1.03
4FF	6.20	2.07	4.65	0.00	4.13
CAPM Redux	8.01	2.58	1.81	2.84	2.33
II: Emerging Markets					
World CAPM	10.59	3.88	5.17	3.36	3.10
Int. CAPM	10.34	0.78	2.84	5.43	3.10
4FF	3.88	2.58	0.00	2.07	3.10
CAPM Redux	6.72	0.00	0.26	1.03	5.68
III: All Markets					
World CAPM	11.89	1.03	2.07	6.46	3.62
Int. CAPM	17.31	2.84	1.81	6.72	2.33
4FF	2.07	2.07	2.84	3.10	4.39
CAPM Redux	5.43	0.00	2.58	5.17	1.29

port this fraction separately for each category of test assets, i.e., the country indexes, as well as all value, growth, small, and big stock portfolios across developed and emerging markets, as well as for all markets together.

The table shows that our model exhibits far lower fractions of rejections of the GRS null than the World CAPM and the International CAPM, especially when applied to the country indexes. Interestingly, this outperformance of our model in the time series domain is particularly pronounced for the developed markets rather than for the emerging markets. The Fama-French-Carhart four factor model has performance comparable to that of our model across both emerging and developed markets, but for the set of Value test assets, our model generally beats even this model, despite the fact that the Fama-French model was first constructed to explain the value and size premiums.

Using the time-series estimates of factor betas, we turn to the factor risk premiums in the cross-section of test asset returns.

4.2 Cross-sectional Tests

As a preliminary exercise, Figure 5 provides a pictorial representation of the relative performance of the models in the cross-section. The vertical axis in these plots is common to all models, and reports the realized average excess returns (in percentage) of our entire cross-section of test assets over the full sample period. The horizontal axis reports the average excess returns of the same test assets as predicted by each model, and varies across the four models we inspect. For the purposes of these plots, we compute predicted returns by using the factor loadings for each asset estimated using the 60-month rolling windows described above. We then multiply these estimated factor loadings by factor means computed over the same time window, and average these conditional predictions across all periods. As usual, better performing models will

generate points which lie close to the 45° line, and deviations from this line indicate pricing errors.

The figure shows that the World CAPM and the International CAPM underestimate realized average excess returns: the large cross-sectional variation observed in the data is not matched because there is little cross-sectional variation in the predictions of these models, leading to a more vertical line. From the figure it is apparent that this poor performance is not driven by a particular type of equity asset; the differences between predictions and realizations are similar for the various different types of assets. The Fama-French-Carhart four-factor model does do substantially better than the more theoretically grounded models, but there is still significant deviation from the 45° line.

Our model significantly improves the visual relationship between predicted and realized average returns. These improvements come from two sources. First, we explicitly model the currency component embedded in foreign equity market returns. Second, we reduce the noise in measured currency risks by relying on two global currency risk factors rather than a selected few currency excess returns.

We confirm this result in a number of ways in the Online Appendix. First, we find that the cross-sectional average $\bar{R}^2$ from these time-series regressions is generally higher for our model than for the competition. At each date in the sample, the global equity, dollar, and carry factors explain a larger share of the time-series variation in international equity returns than the world CAPM. We also find that our model outperforms the International CAPM as we move from the distant past towards the recent past. This latter finding suggests that the increasing integration of global markets might account for the increasing explanatory power of global currency factors.

Next, we relax the no-arbitrage condition that pins down the market prices of risk and estimate them using the cross-section of excess returns. We conduct our cross-sectional tests using the standard two-pass cross-sectional approach of Fama and Mac-Beth (1973, henceforth FMB). For each of the models in our comparisons, we run FMB

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Figure 5
Realized versus Predicted Average Excess Returns

This figure plots realized average excess returns against those predicted by the World CAPM, the International CAPM, the Fama-French four-factor model and the CAPM Redux. All models are estimated conditionally. For each country the predicted excess returns are computed as follows: a) Time-varying factor betas are estimated using 60-month rolling windows; b) At each time t estimated conditional betas are multiplied by the corresponding factor means computed over the same time-window (from time t-59 to time t); c) Predicted values are averaged. For each country the realized average excess returns are computed as follows: a) In each 60-month rolling window actual excess returns are averaged and b) 60-month rolling return means are averaged. Test assets are equity excess returns for all asset types and all countries, six currency portfolios sorted on forward discounts (Carry portfolios) and six currency portfolios sorted on dollar exposure (Dollar portfolios). Average returns are in percentage. The straight line is the 45-degree line through the origin. Fama -French factors are U.S. (global) factors prior to (from) November 1990. The sample is February 1981 to April 2013.
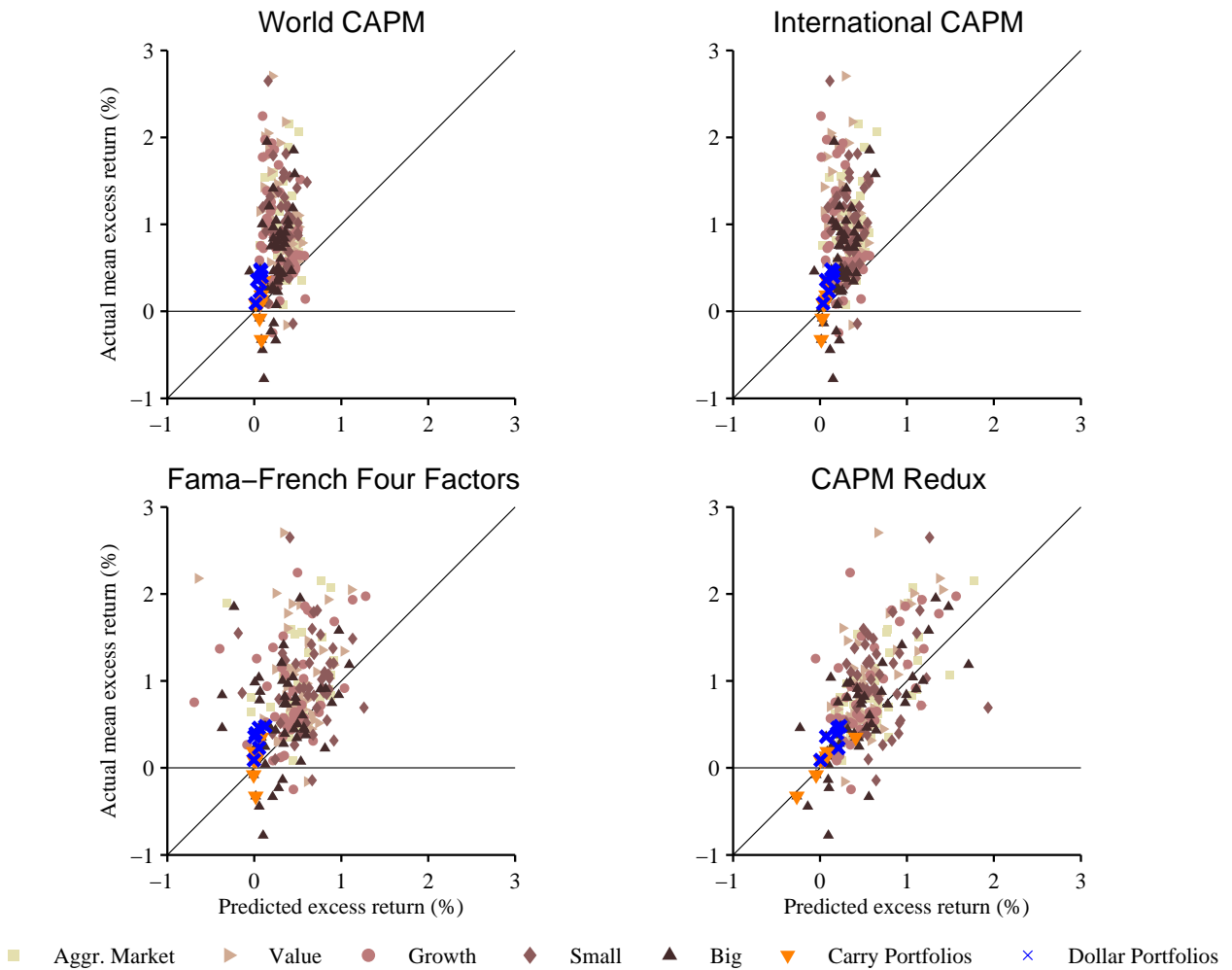
tests in three different ways. In the first variant of these FMB tests (which we denote as FMB^1), we estimate the factor loadings using unconditional time-series regressions over the full sample. We then compute market prices of risk (λ) via a cross-sectional regression of average excess returns of the test assets on these unconditional factor loadings. In the second variant (which we denote FMB^2), first-stage betas continue to be obtained using unconditional time-series regressions over the full sample as in FMB^1. However, in the second stage, we run T cross-sectional regressions, one for each time period, of country excess returns on these estimated factor loadings. The average market prices of risk are then computed as simple averages of the slope coefficients obtained from these T cross-sectional regressions. In the third variant (denoted FMB^{TV}), we obtain time-varying factor loadings using our rolling regressions over 60-month windows. In each period $t+1$, we estimate market prices of risk λ_{t+1} using cross-sectional regressions of test asset returns on these time-varying factor loadings estimated using windows ending in period t. Average market prices of risk are once again simple averages of λ_{t+1} over all periods T. In all three variants of these tests, we omit constants in the second stage of the FMB procedure. To ensure that the three tests are comparable, the FMB^1 and FMB^2 tests are carried out on the second-stage estimation sample of the FMB^{TV} procedure.

Table 2 reports the estimates of the average market prices of risk along with Shanken (1992)-corrected standard errors (in parentheses) from these tests across models which are in blocks of rows. The columns identify the set of test assets on which we run these tests. The first set of test assets includes aggregate market excess returns, and value and size-sorted portfolios for the developed markets in the sample (120 assets), as well as 12 currency portfolios. The second set of test assets expands the equity cross-section to include assets from emerging markets, leading to a total of 225 equity assets plus 12 currency portfolios.

The table shows that across all model specifications, cross-sections of test assets,

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Table 2

Asset Pricing Fama-MacBeth Tests

This table reports results from the Fama-MacBeth asset pricing procedure for the World CAPM, the International CAPM, the Fama-French four-factor model (4FF, U.S. and global factors are combined) and the CAPM Redux. In FMB^1, the average market prices of risks (λ) are obtained via a cross-sectional regression of average excess returns on the (unconditional) first-step betas. In FMB^2, the λs correspond to the average across T cross-sectional regressions of excess returns on the same unconditional betas. In FMB^{TV}, in the first step of the procedure time-varying (TV) betas are estimated over 60-month rolling windows ending at time t. In the second-step, the market prices of risk are estimated at each time t+1 via a cross-sectional regression of country excess returns on the first-step conditional betas. The λs are obtained as average of these second-stage estimates. In all cases, the second stage of the procedure does not include a constant and factors are added to the set of test assets. $MAPE$ ($RMSE$) denotes the Mean Absolute Pricing Error (Root of Mean Square pricing Errors). Shanken (1992)-corrected standard errors are in brackets. *, **, *** denote statistical significance of the average prices of risk at the 10%, 5% and 1% level, respectively. Test assets are country equity excess returns for developed markets (DM, left panel) or all markets (right panel), six currency portfolios sorted on forward discounts (Carry portfolios) and six currency portfolios sorted on dollar exposure (Dollar portfolios). The sample period is February 1981 to April 2013.

Model		DM Equity + FX Portfolios			All Equity + FX Portfolios			Factor
		FMB¹	FMB²	FMB^TV	FMB¹	FMB²	FMB^TV	Means
World	λ_{WMKT}	0.623**	0.679***	0.683***	0.735***	0.803***	0.811***	0.518
		(0.258)	(0.262)	(0.263)	(0.266)	(0.269)	(0.261)	(0.252)
CAPM	$MAPE$	0.251	0.260	0.308	0.320	0.328	0.441	
	$RMSE$	0.333	0.352	0.408	0.425	0.438	0.603	
	λ_{WMKT}	0.661**	0.698***	0.597**	0.755***	0.825***	0.736***	0.518
		(0.257)	(0.261)	(0.254)	(0.269)	(0.276)	(0.253)	(0.252)
	λ_{FX}^{GBP}	0.203	0.188	0.387**	0.309	0.12	0.468***	0.190
		(0.198)	(0.215)	(0.166)	(0.214)	(0.255)	(0.174)	(0.160)
Int.	λ_{FX}^{JPY}	0.074	0.032	−0.043	−0.143	−0.123	0.095	-0.012
CAPM		(0.246)	(0.281)	(0.210)	(0.274)	(0.348)	(0.205)	(0.183)
	λ_{FX}^{EUR}	0.018	0.039	0.108	−0.105	−0.160	0.146	0.118
		(0.219)	(0.250)	(0.188)	(0.221)	(0.254)	(0.177)	(0.177)
	$MAPE$	0.239	0.238	0.286	0.303	0.305	0.393	
	$RMSE$	0.326	0.331	0.410	0.402	0.406	0.550	
	λ_{WMKT}	0.620**	0.682**	0.709**	0.699***	0.771***	0.780***	0.459
		(0.264)	(0.270)	(0.288)	(0.267)	(0.271)	(0.283)	(0.257)
	λ_{SMB}	0.158	0.013	0.317**	0.189	0.261	0.321**	-0.036
4FM		(0.154)	(0.222)	(0.146)	(0.173)	(0.249)	(0.141)	(0.121)
(Combined)	λ_{HML}	−0.147	−0.118	−0.164	−0.221	−0.244	−0.148	0.324
		(0.178)	(0.198)	(0.152)	(0.187)	(0.196)	(0.142)	(0.130)
	λ_{MOM}	0.197	0.239	0.399	−0.049	0.016	0.204	0.672
		(0.259)	(0.260)	(0.245)	(0.332)	(0.375)	(0.238)	(0.216)
	$MAPE$	0.251	0.234	0.284	0.315	0.330	0.357	
	$RMSE$	0.331	0.315	0.377	0.414	0.434	0.496	
	λ_{LWMKT}	0.542**	0.588**	0.526**	0.680***	0.720***	0.572**	0.418
		(0.248)	(0.256)	(0.242)	(0.253)	(0.264)	(0.241)	(0.235)
CAPM	λ_{Dollar}	0.187	0.189	0.241*	0.101	0.132	0.293**	0.234
Redux		(0.143)	(0.157)	(0.130)	(0.147)	(0.159)	(0.126)	(0.119)
	λ_{Carry}	0.436**	0.493*	0.450***	0.428**	0.699**	0.418***	0.615
		(0.207)	(0.255)	(0.169)	(0.185)	(0.282)	(0.149)	(0.130)
	$MAPE$	0.237	0.235	0.288	0.302	0.289	0.362	
	$RMSE$	0.328	0.333	0.412	0.403	0.394	0.500	

and testing approaches, the world equity factor is priced, whether it is measured in local currency or U.S. dollar terms. The statistical significance of this result holds at the five percent level or better. This finding supports the early evidence in the international finance literature that international investors are compensated for taking on risk that is correlated with returns on the world equity market portfolio.

In contrast, there is little evidence to support the pricing of currency risk in models other than our own. Looking across the three currencies included in the International CAPM, only the British pound appears to carry a significant currency premium in our sample. Moreover, this result holds only when time-variation is taken into account (FMB^{TV}). The importance of using a conditional model is consistent with the findings of Dumas and Solnik (1995), however even allowing for time-variation in factor loadings and risk premia, the German mark/Euro and Japanese Yen are not priced over the sample period for the wider cross-section of assets. This result is potentially attributable to the longer sample period, the larger cross-section of test assets, and our use of a methodology based on rolling windows instead of instrumental variables to model time-variation.

The table also shows that there is no statistical evidence for a value or momentum premium in the cross-section of test assets, either conditionally or unconditionally. However there is some evidence to support the existence of a size premium, especially when we evaluate the Fama-French model allowing for time-variation in factor loadings.

In contrast with these results, we find evidence to support the pricing of currency risk when we measure this risk using the dollar and carry factors. This is even true to some extent unconditionally, in the sense that the carry factor is priced using FMB^1 across developed and developed plus emerging cross-sections, and using FMB^2 in the broader developed plus emerging cross-section.

The results supporting our model become substantially stronger when we account for time-variation in factor loadings as well as in risk premia, using FMB^{TV}. The prices

of dollar and carry risk are all significantly different from zero at the 5% level or better, in both cross-sections.

Additional evidence on the models is provided when we inspect the prices of risk of the equity, dollar, and carry factors and compare them with the average excess returns of those factors which are provided in the final column of Table 2. The no-arbitrage condition implies, since the beta of each factor on itself is obviously one, that the market price of risk of each factor should be equal to the average of the factor. While the sample is short, leading to difficulties in estimating this relationship precisely, we do see that these factor means are relatively close to the estimated factor risk premia. For the other models, the price of equity risk is much higher, further removed from its sample mean. As noted, however, the sample is indeed short, leading to substantial imprecision, and susceptibility to Daniel and Titman's (1997) critique. We cannot of course rule out a characteristics-based behavioral explanation of the cross-sectional variation in test asset returns.

For each model, Table 2 also reports the cross-sectional Mean Absolute Pricing Error (MAPE) and the cross-sectional Root Mean Square Error (RMSE). Consistent with the evidence discussed above, our factor model delivers relatively smaller RMSEs than the competition, although these differences are not substantial.

The Online Appendix reports a number of additional robustness checks including re-estimating the model on samples of expanding sizes, either moving forwards in time, beginning in 2/1976 or moving backwards in time, beginning in 12/2013. The dollar and carry factors appear priced even in samples that exclude the recent financial crisis (i.e., before 2007). Small perturbations in the estimation sample do not imply abrupt changes in the estimates. The magnitude of these currency premia varies smoothly over time suggesting that the price of risk (not merely risk exposure) is time-varying. We also find that the significance of the pricing results for the model increases over time. This pattern is certainly related to the increase in power arising from a broader

cross-section of asset returns available to test the model, as well as the increasing global capital market integration over time.

One obvious question is whether our strong results are simply a consequence of the currency return component embedded in dollar-denominated test asset returns, and the pricing of these currency components by the carry and dollar factors. Figure 6 shows that a simple story of this nature would be insufficient to explain the somewhat involved dynamics of equity and currency risk.

Figure 6

Cross-Sectional Average Time-Varying Correlation Between Equity Returns in Local Currency and Each Currency Factor

This figure shows, for each 60-month rolling window, the average cross-sectional correlation between country aggregate market returns expressed in local currency and each currency factor (dollar/carry factor in the left/right graph, respectively). Specifically, at each time t we compute pairwise correlations between the equity return of each country-i and a given currency factor and report the average correlation across countries. In each rolling window, a 95% confidence interval (dash-dotted line) is obtained via pairwise bootstrapping (1000 bootstrap samples). The lower/upper bound corresponds to the 2.5^{th} and 97.5^{th} percentile of the bootstrap distribution of the cross-sectional average correlation in that time-window. The sample is February 1976 to April 2013.
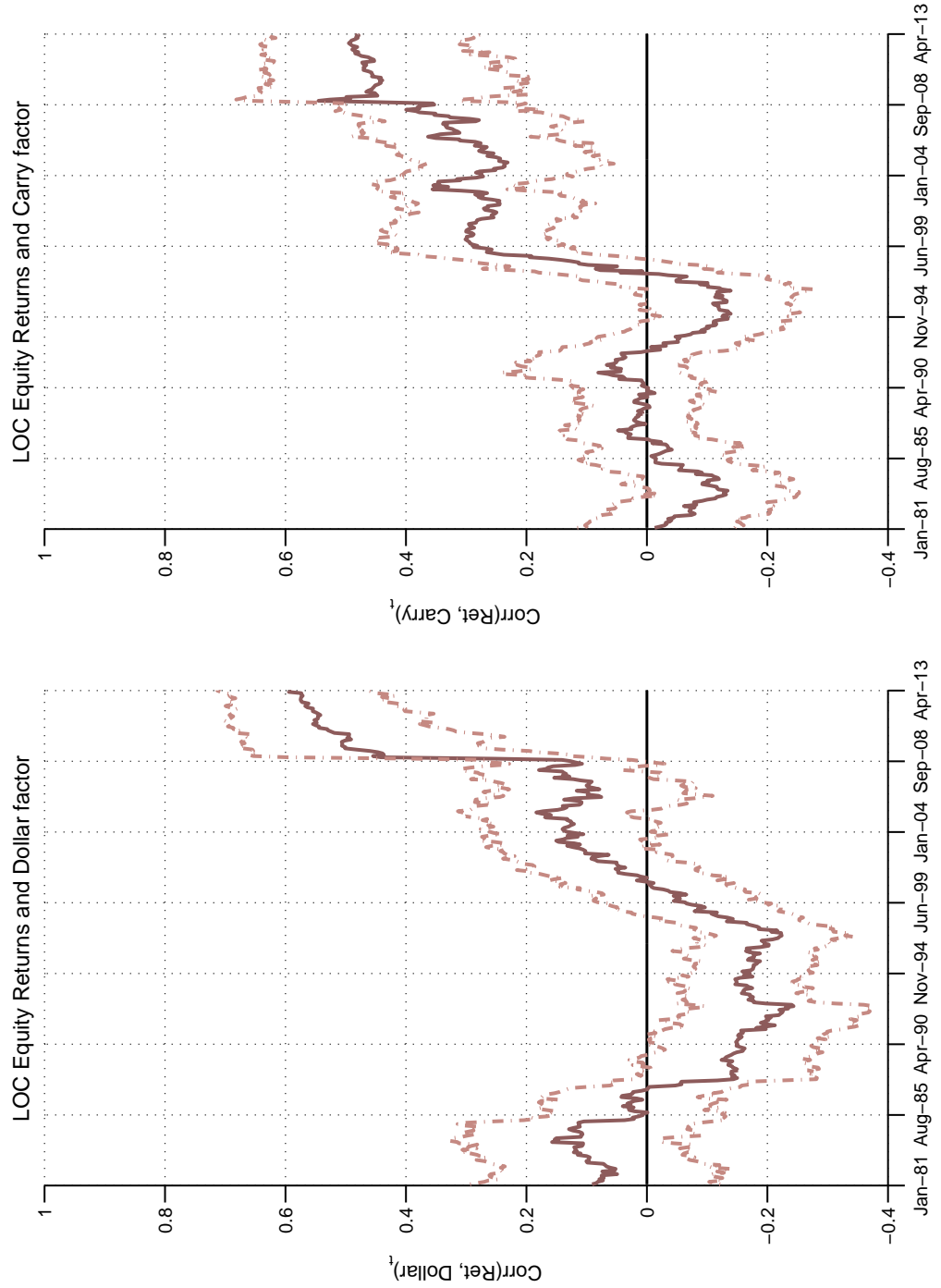
The figure reports the cross-sectional average correlation coefficient between country aggregate market equity returns expressed in local currency and the dollar factor (left panel) and the carry factor (right panel). Clearly, equity and currency risk are not orthogonal to one another, and demonstrate significant time-variation in their relationship. Particularly over the second half of the sample period that we consider, these correlations are significant and increasing, sometimes non-linearly. These correlations reach 60% (50%) at the end of our sample period for the dollar (carry) factor. We discuss this issue in more detail when we present the model in the next section.

Next, we show that a large proportion of mutual and hedge funds investing internationally are also exposed to currency risk, which we are well able to detect using our model.

4.3 Global Risk in Mutual and Hedge Fund Returns

Mutual fund managers can choose whether or not to hedge the currency exposure in their foreign equity investments.[9] While many mutual fund brochures remain vague about currency risk, we find that their realized returns reveal a clear and economically significant exposure to exchange rate fluctuations.

As an initial exercise, we simply document the explanatory power of our model for these returns over the entire sample period. A clear pattern emerges in the time-series, as shown in the top panel of Figure 7: over time, despite the number of international funds rising throughout the sample period, there is a steady increase in the percentage of funds exposed to the global equity, dollar, and carry factors in the data. The percentage of mutual funds significantly exposed to dollar or carry risk increases from roughly 60%

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[9]According to the Wall Street Journal (August 4, 2013), for example, Fidelity Overseas manages $2.3 billion without hedging its foreign currency risk, while Oakmark International, which has $19.5 billion in assets, "hedges up to 80% of its exposure to a given currency when the currency's exchange rate against the dollar is more than 20% above what the management team considers fair value." It is not clear whether this fair value is known to outside investors or not.

of funds levels in the mid 1990s to close to 100% of funds by the end of the sample. The average cross-sectional R^2 at each date increases from around 60% in the early 1990s to more than 90% at the end of our sample. Clearly, international mutual fund returns are exposed to currency risk, and as a consequence, their investors bear significant currency risk as well.[10]

The bottom panel of the same figure assesses the growth in international hedge funds' exposures to the factors in our model. While exposure to the global equity factor appears stable throughout the period, the combined exposure to the dollar and carry factors has increased notably: very few hedge funds load significantly on the currency factors in the 1990s but close to half the funds do so by the end of the sample.

Overall, while hedge fund returns appear to be less exposed to the factors in our model than mutual funds, it is nonetheless true that a large number of hedge fund returns load significantly on our risk factors. This means that their exposure to currency risk can potentially be reproduced at a much lower cost than by incurring the substantial fees involved in investing in a hedge fund. On average across the sample, one can reject the null joint hypothesis that their returns do not load on the equity and currency factors for close to 60% of international hedge funds, representing a similar share of the money invested in the "Macro" and "Emerging" hedge fund sectors.

Figure 8 presents a simple comparison across the models that we consider, regressing monthly mutual fund and hedge fund returns on the competing models, and plotting CDFs of the t-statistics of (positive) alphas (using bootstrap standard errors) obtained from these different models. For mutual funds, in the left panel, the figure reveals

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[10]We check the robustness of our findings on longer rolling windows of 120 months, instead of 60 months. The number of funds with available data decreases. There are in this case only 568 funds at the end of the sample, but those funds as a group still manage more than $620 billions. The results are broadly similar as above. On average over the whole period, 80% of the mutual funds are significantly exposed to currency risk, representing 91% of the total funds under management in our dataset.

Figure 7
Mutual Funds' and Hedge Funds' Significant Exposure to Global Factors

This figure plots the time-series of the share of hedge funds (top three graphs) and mutual funds (bottom three graphs) with significant exposure to the CAPM Redux factors (LWMKT, Dollar and Carry). At each point in time monthly hedge fund/mutual fund returns are regressed on a constant and the three global factors over rolling-windows of 60 months. The first and fourth panels report the number of funds with available data. The second and fifth panels report the percentage of funds with significance exposure to LWMKT at the 5% significance level (i.e., t-statistic on the equity factor above 1.96 in absolute value). Standard errors are obtained via bootstrapping (1000 bootstrap samples). The third and last panel report the percentage of funds with significance exposure to both the dollar and the carry factor at the 5% significance level (i.e. p-value of the F-test below 5% under the null hypothesis that both FX loadings are zero). Funds are equally-weighted. Hedge fund data are from the updated version of the consolidated hedge fund database by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013). The sample includes all funds classified as "Macro" or "Emerging" according to the strategy code. The sample period is January 1994 to April 2013. The mutual fund sample includes all funds classified as "Foreign Equity Funds" according to the CRSP fund style code. The sample period is November 1990 to April 2013. All data are monthly.

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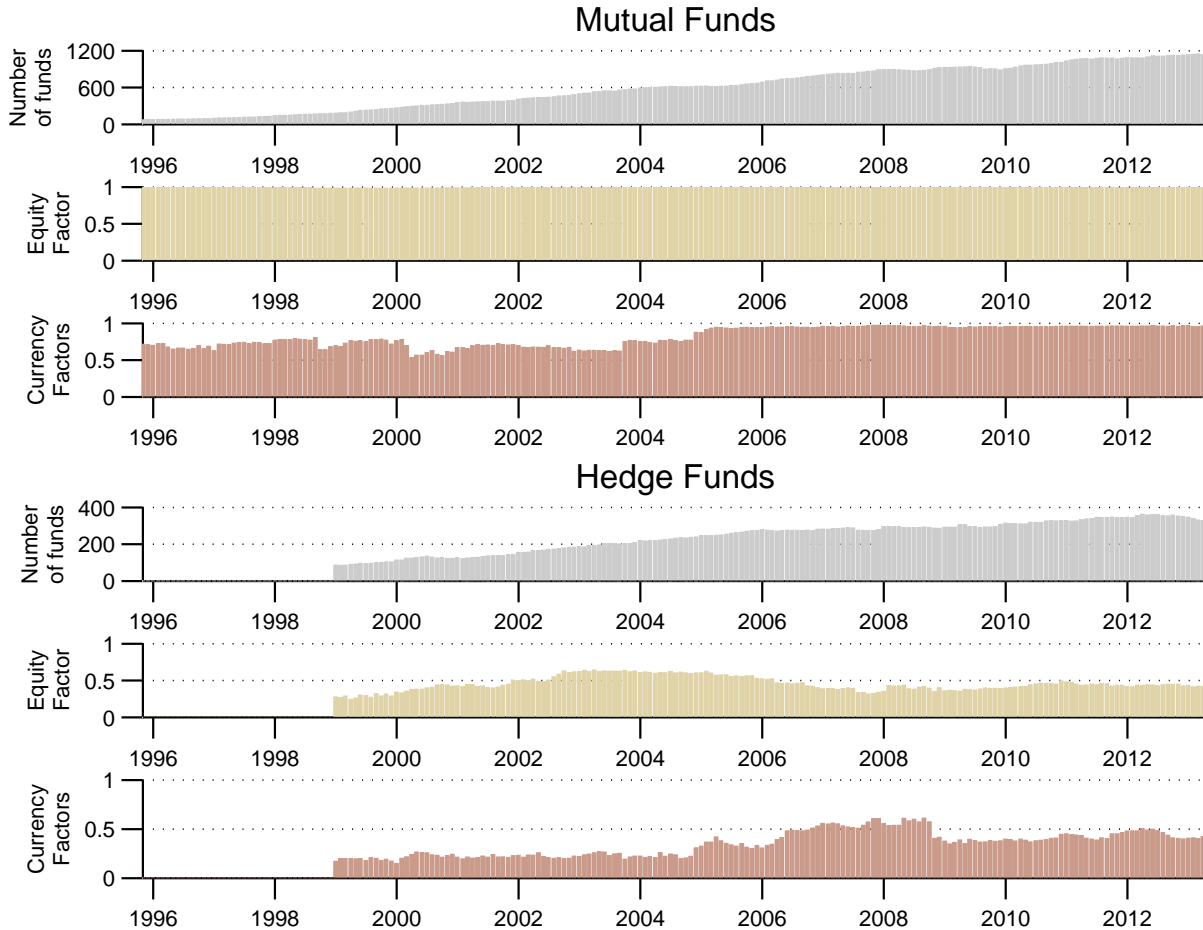
that the performance of our model is better than that of the World CAPM and the International CAPM, but not as good as that of the Carhart four-factor model.[11].

The right-hand panel of the figure shows the same CDF of t-statistics of positive alphas for hedge funds. The figure reveals that the performance of our model is better even than that of the Carhart four-factor model, delivering a slightly lower number of funds with statistically significant alphas.

Having described our empirical results, we now turn to describing a simple theoretical model that helps to explain our results.

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[11]As we document in the Online Appendix, the MSCI World indices include only developed countries. We can account for all emerging markets in our dataset by constructing our own market index using lagged market capitalization as weighting scheme. Using this extended global equity factor we have verified that our model and the Carhart four-factor model share similar performances.

Figure 8

Statistical Significance of Mutual Funds' and Hedge Funds' Alphas

This figure shows the empirical cumulative distribution function (ECDF) of t-statistics of positive hedge funds' (left graph) and mutual funds' (right graph) rolling alphas. Alphas are obtained by estimating the World CAPM, the International CAPM, the Fama-French four-global factor model (4FF) and the CAPM Redux over rolling-windows of 60 months. Standard errors are obtained via bootstrapping (1000 bootstrap samples). Hedge fund data are from the updated version of the consolidated hedge fund database by Ramadorai (2013) and Patton, Ramadorai and Streatfield (2013). The sample includes all funds classified as "Macro" or "Emerging" according to the strategy code. The sample period is January 1994 to April 2013. The mutual fund sample includes all funds classified as "Foreign Equity Funds" according to the CRSP fund style code. The sample period is November 1990 to April 2013. All data are monthly.
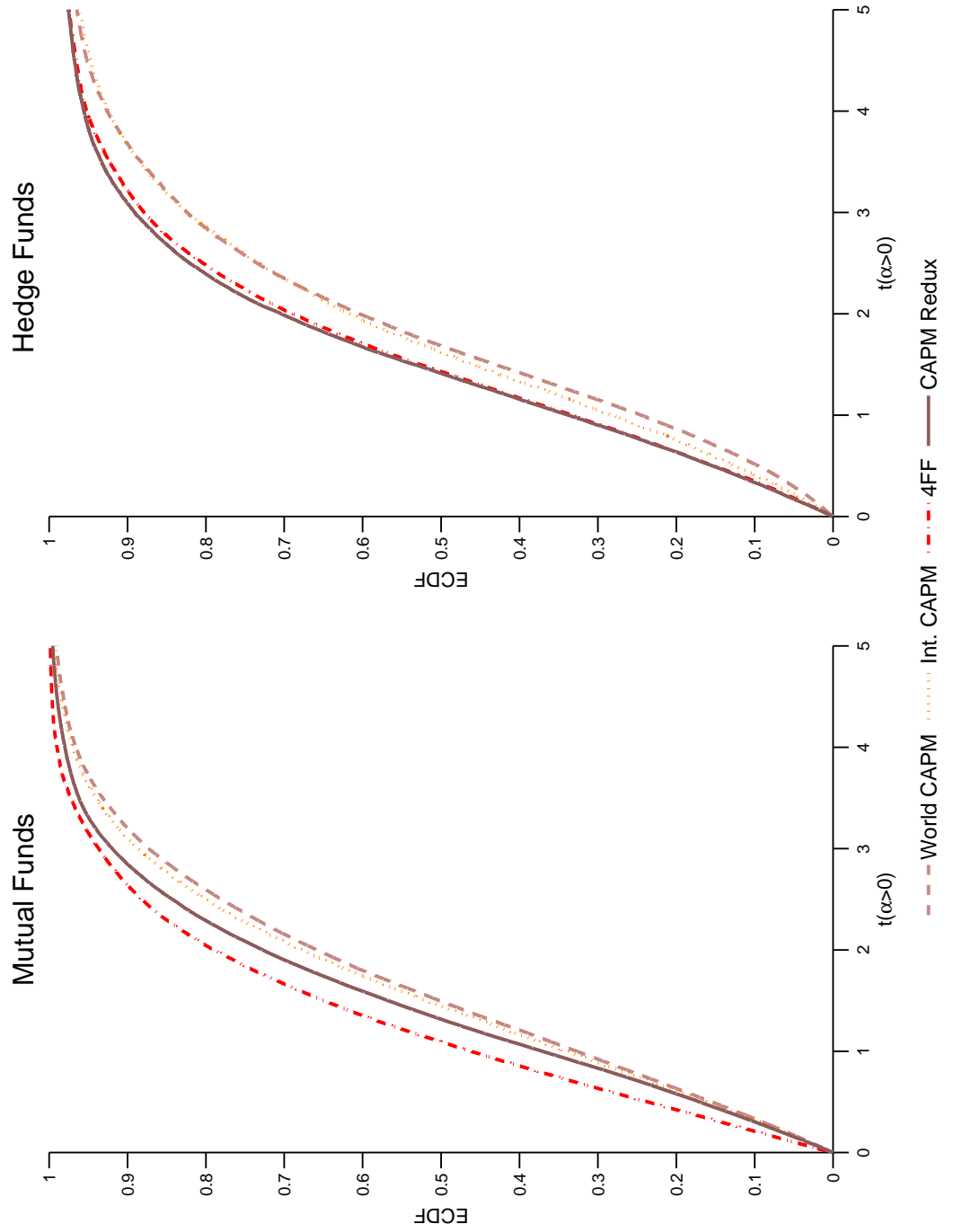
5 The International CAPM Redux Model

We begin by discussing the theoretical literature on international asset pricing, to provide context about how our simple model fits in to the broader asset pricing literature. We then move to a formal presentation of the model.

The International CAPM is very general, but with this strength comes some costs. The most important one is that exchange rate shocks, while priced, are exogenous to the model, despite potentially being linked to world market returns. This can be seen clearly in Adler and Dumas (1983), which forms the basis of the empirical work of Dumas and Solnik (1995).

Our simple International CAPM Redux model is a modest attempt to plug this gap in the literature. In our model, exchange rates, currency risk factors, and equity market returns all depend on the same shocks. Our focus is to explore what this change buys us in a relatively stripped-down setting. In our model, financial markets are complete, and we specify the law of motion of the lognormal stochastic discount factors (SDFs) in all countries. The SDFs are posited to depend on country-specific shocks, as well as three global shocks. We assume that each country's aggregate dividend growth rate depends on the same shocks as the country-specific SDF. In order to specify a role for a pure equity risk factor affecting all countries in the same way, we posit that one global shock affects all SDFs in the same way.

To preview the main insight obtained from the model, we find that the world stock market return in U.S. dollars depends on all global shocks, meaning that there should be no role for bilateral exchange rates, i.e., the World CAPM should work. Yet the model shows that the world equity return expressed in U.S. dollars bundles state variables with different time-dynamics together. This makes it difficult to use this single factor to uncover risk exposures in international asset pricing since the state variables are unknown to the econometrician. We find in this setting that our three factor empirical

model is the best way to capture the various sources of global risk posited in the theory. We now turn to describing our model more formally.

5.1 SDFs

In the tradition of Backus, Foresi, and Telmer (2001), we assume that pricing kernels m_{t+1}^i are exponentially affine:

$$
\begin{aligned}
-m_{t+1}^i &= \alpha + \chi z_t^i + \sqrt{\gamma z_t^i} u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w} u_{t+1}^w + \sqrt{\kappa z_t^i} u_{t+1}^g + \sqrt{\omega z_t^w} u_{t+1}^c, \\
z_{t+1}^i &= (1-\phi)\theta + \phi z_t^i - \sigma \sqrt{z_t^i} u_{t+1}^i, \\
z_{t+1}^w &= (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w \sqrt{z_t^w} u_{t+1}^w
\end{aligned}
$$

where $u_{t+1}^i, u_{t+1}^w, u_{t+1}^g, u_{t+1}^c$ are i.i.d, mean-zero, variance-one Gaussian shocks, m_{t+1}^i is the log SDF of country i, and z_t^i and z_t^w are the state variables that govern the conditional volatility of the SDF. Each SDF is heteroskedastic because currency risk premia are driven by the conditional variances of the SDFs. Inflation is not a priced risk in this model – the SDFs can be interpreted as nominal SDFs. We use the U.S. dollar as the base currency and drop the superscript i to describe any U.S. variable.

A similar model is studied in Lustig, Roussanov, and Verdelhan (2011, 2014) and in Verdelhan (2014).[12] The key distinguishing feature of our model is the introduction of equity-specific shocks u_{t+1}^c. As we shall see, these shocks affect both dividends and SDFs, but not exchange rates, and drive the world equity return factor.

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[12]In those papers, the prices of risk (i.e., the square roots in the law of motion of the SDFs), depend on both the country-specific and global state variables in order to differentiate between unconditional and conditional currency risk premia. For the sake of clarity, we leave this difference aside here, but the model can be easily extended in this direction. Earlier examples of affine models in international finance include Frachot (1996) and Brennan and Xia (2006).

The SDFs depend on country-specific shocks, u_{t+1}^i, and three global shocks, u_{t+1}^w, u_{t+1}^g, and u_{t+1}^c. We refer to the volatilities of the SDF related to these three shocks (namely, $\sqrt{\delta^i z_t^w}$, $\sqrt{\kappa z_t^i}$, and $\sqrt{\omega z_t^w}$) as the market prices of risk of these shocks.

The first shock u_{t+1}^w is priced similarly in each country up to a scaling factor, denoted δ^i. Examples of such a shock might be a global financial crisis which affects prices in all countries in a perfectly correlated fashion, but with differential intensity. To be parsimonious, we model differences in exposure δ^i as the only source of heterogeneity in countries' SDFs, and fix all the other parameters of the SDFs to be the same across countries. Countries also differ in their aggregate dividend growth rates.

The model thus features two sources of heterogeneity: one in the SDF and one in the dividend growth rates. The first source of heterogeneity is necessary to account for the cross-section of interest rates and currency excess returns. As Lustig, Roussanov, and Verdelhan (2011) show, high interest rate countries must be characterized by low exposure (δ^i) to world shocks for the high interest rate currency to depreciate in bad times — the key mechanism of any risk-based explanation of carry trade profits. This first source of heterogeneity entails differences in global equity betas, but if it were not for the differences in dividend growth rates, the equity betas would line up with the carry betas. In the data, they do not, consistent with our modeling choice of two sources of cross-country differences.

The second shock u_{t+1}^g is priced differently across countries, even if countries share the same exposure κ. An example of this might be a productivity shock that affects some economies more than others. Finally, as described earlier, the third shock u_{t+1}^c is priced in exactly the same way in all countries.

5.2 Exchange Rates

When markets are complete, log changes in exchange rates correspond to the differences between domestic and foreign log pricing kernels (Bekaert, 1996, Bansal, 1997):[13]

$$
\begin{aligned}
\Delta s_{t+1}^i &= m_{t+1} - m_{t+1}^i, \\
&= \chi(z_t^i - z_t) + \sqrt{\gamma z_t^i} u_{t+1}^i - \sqrt{\gamma z_t} u_{t+1} \\
&+ (\sqrt{\delta^i} - \sqrt{\delta})\sqrt{z_t^w} u_{t+1}^w + \sqrt{\kappa}(\sqrt{z_t^i} - \sqrt{z_t}) u_{t+1}^g,
\end{aligned}
$$

where the exchange rate is defined in foreign currency per U.S. dollar. Therefore, an increase in the exchange rate corresponds to an appreciation of the U.S. dollar. Although financial markets are complete, real exchange rates are not necessarily constant as soon as some frictions exist in the goods markets (e.g., non-traded goods, or trading costs). The exchange rate between country i and the domestic economy depends on the country-specific shocks u^i and u, as well as on the global shocks u^w and u^g, but not on the global shocks u^c since their prices of risk are the same across countries.

---

[13]This result derives from the Euler equations of the domestic and foreign investors buying any asset R^i that pays off in foreign currency: $E_t[M_{t+1}R^i S_t^i / S_{t+1}^i] = 1$ and $E_t[M_{t+1}^i R^i] = 1$. When markets are complete, the pricing kernel is unique and thus exchange rates are defined as $S_{t+1}^i / S_t^i = M_{t+1}/M_{t+1}^i$, or in logs $\Delta s_{t+1}^i = m_{t+1} - m_{t+1}^i$.

5.3 Equity Returns

In order to define equity returns, the model posits a dividend growth process in each country i:

$$\Delta d_{t+1}^i = \mu_D + \psi z_t^i + \psi_w z_t^w + \sigma_D \sqrt{z_t^i} u_{t+1}^i + \sigma_D^w \sqrt{z_t^w} u_{t+1}^w + \sigma_D^g \sqrt{z_t^i} u_{t+1}^g + \sigma_D^{c,i} \sqrt{z_t^w} u_{t+1}^c,$$

where the innovations are the same as those described above. Dividend growth rates respond to both country-specific and global shocks. The only systematic difference across countries comes from the impact of global shocks u_{t+1}^c on dividend growth, governed by the parameters $\sigma_D^{c,i}$. This source of heterogeneity drives the differences in global equity market betas.

Innovations to the log gross equity excess return are then:[14]

$$
\begin{aligned}
r_{t+1}^{e,i} - E_t(r_{t+1}^{e,i}) &= (\sigma_D - k_1 B_{pd}^i \sigma)\sqrt{z_t^i} u_{t+1}^i \\
&+ (\sigma_D^w - k_1 C_{pd}^i \sigma^w)\sqrt{z_t^w} u_{t+1}^w + \sigma_D^g \sqrt{z_t^i} u_{t+1}^g + \sigma_D^{c,i} \sqrt{z_t^w} u_{t+1}^c,
\end{aligned}
$$

---

[14]In this model, the log price-dividend ratio is affine in the state variables, z_t^i and z_t^w:

$$pd_t^i = A_{pd}^i + B_{pd}^i z_t^i + C_{pd}^i z_t^w,$$

where the constants A_{pd}^i, B_{pd}^i, and C_{pd}^i are defined as function of the SDF and dividend growth parameters. The model is solved in closed form using the standard log-linear approximation for the log gross return on the aggregate dividend claim:

$$r_{t+1}^{e,i} \approx k_0 + k_1 pd_{t+1}^i - pd_t^i + \Delta d_{t+1}^i,$$

where k_0 and k_1 are defined by the Taylor approximation of the log price-dividend ratio pd_t^i around its mean. More precisely, the Euler equation applied to the stock market return implies that the coefficients A_{pd}^i, B_{pd}^i and C_{pd}^i are defined by:

$$
\begin{aligned}
A_{pd}^i &= -\alpha + k_0 + k_1 A_{pd}^i + k_1 B_{pd}^i (1-\phi)\theta + k_1 C_{pd}^i (1-\phi^w)\theta^w + \mu_D, \\
B_{pd}^i &= k_1 B_{pd}^i \phi + \psi - \chi + \frac{1}{2}(\sqrt{\gamma} + k_1 B_{pd}^i \sigma - \sigma_D)^2 + \frac{1}{2}(\sigma_D^g - \sqrt{\kappa})^2, \\
C_{pd}^i &= k_1 C_{pd}^i \phi^w + \psi_w - \tau + \frac{1}{2}(\sqrt{\delta^i} + k_1 C_{pd}^i \sigma^w - \sigma_D^w)^2 + \frac{1}{2}(\sqrt{\omega} - \sigma_D^{c,i})^2.
\end{aligned}
$$

where $r_{t+1}^{e,i}$ is the logarithmic gross rate of return on each country's stock market index denominated in that country's currency.

5.4   International CAPM Redux

The equity return of country i expressed in U.S. dollars, denoted $r_{t+1}^{e,i,\$}$, is simply derived from the equity return of country i in local currency and the change in the exchange rate:

$$
\begin{aligned}
r_{t+1}^{e,i,\$} - E_t\left(r_{t+1}^{e,i,\$}\right) &= \sqrt{\gamma z_t}u_{t+1} + \left(\sigma_D - k_1 B_{pd}^i \sigma - \sqrt{\gamma}\right)\sqrt{z_t^i}u_{t+1}^i \\
&+ \left(\sigma_D^w - k_1 C_{pd}^i \sigma^w - \sqrt{\delta^i} + \sqrt{\delta}\right)\sqrt{z_t^w}u_{t+1}^w + \sigma_D^{c,i}\sqrt{z_t^w}u_{t+1}^c \\
&+ \left((\sigma_D^g - \sqrt{\kappa})\sqrt{z_t^i} + \sqrt{\kappa z_t}\right)u_{t+1}^g.
\end{aligned}
$$

The foreign equity return expressed in U.S. dollars therefore depends on the foreign and U.S. specific shocks (u^i and u), as well as the global shock captured by the carry and dollar factors (u^w) and (u^g) and the world equity shock (u^c).

The expected equity excess return from the perspective of a U.S. investor is:

$$
\begin{aligned}
E_t r_{t+1}^{e,i,\$} - r_{f,t} + \frac{1}{2}Var_t\left[r_{t+1}^{e,i,\$}\right] &= -Cov_t\left[m_{t+1}, r_{t+1}^{e,i,\$}\right] \\
&= \gamma z_t + \left(\sigma_D^w - k_1 C_{pd}^i \sigma^w - \sqrt{\delta^i} + \sqrt{\delta}\right)\sqrt{\delta}z_t^w + \sigma_D^{c,i}\sqrt{\omega}z_t^w \\
&- \left((\sigma_D^g - \sqrt{\kappa})\sqrt{z_t^i} + \sqrt{\kappa z_t}\right)\sqrt{\kappa z_t}
\end{aligned}
\tag{6}
$$

Note that our assumption of complete markets implies that we only need to verify the Euler condition for one country's investor. As soon as the Euler equation is satisfied for the U.S. investor, for example, it implies that the Euler condition for any foreign investor is also satisfied. To see this point, recall that the Euler condition for the U.S. investor is $E_t\left[M_{t+1}R_{t+1}^i S_t^i / S_{t+1}^i\right] = 1$, which implies $E_t\left[M_{t+1}^i R_{t+1}^i\right] = 1$ — the Euler condition of the representative investor in country i — as well as $E_t\left[M_{t+1}^j R_{t+1}^i (S_t^i/S_{t+1}^i)(S_{t+1}^j/S_t^j)\right] =$

1, the Euler condition of the representative investor in country j.

In order to better understand our empirical approach, next, we express various factors in the language of the model, namely, the world equity return in U.S. dollars $WMKT_{t+1}$, the world equity return in local currencies $LWMKT_{t+1}$, and the carry and dollar factors.

5.5 Equity Factors

The innovations to the average world equity market return in local currency terms, which we define for ease of exposition as the simple average of local equity returns, are:

$$LWMKT_{t+1} = \overline{r_{t+1}^{e,i}} - E_t(\overline{r_{t+1}^{e,i}}) = (\sigma_D^w - k_1\overline{C_{pd}^i}\sigma^w)\sqrt{z_t^w}u_{t+1}^w + \sigma_D^g\overline{\sqrt{z_t^i}}u_{t+1}^g + \overline{\sigma_D^{c,i}}\sqrt{z_t^w}u_{t+1}^c,$$

where $\bar{x}$ denotes the cross-country average of a variable x.

Country-specific shocks average out, and the world equity market return is only driven by the global shocks u_{t+1}^w, u_{t+1}^g, and u_{t+1}^c. If the law of large numbers applies, the cross-sectional mean of z_t^i is constant (and equal to θ). The world equity return in U.S. dollars is:

$$
\begin{aligned}
WMKT_{t+1} = \overline{r_{t+1}^{e,i,\$}} - E_t\left(\overline{r_{t+1}^{e,i,\$}}\right) =\ & \sqrt{\gamma z_t}u_{t+1} + \left(\sigma_D^w - k_1\overline{C_{pd}^i}\sigma^w - \overline{\sqrt{\delta^i}} + \sqrt{\delta}\right)\sqrt{z_t^w}u_{t+1}^w \\
& + \overline{\sigma_D^{c,i}}\sqrt{z_t^w}u_{t+1}^c + \left((\sigma_D^g - \sqrt{\kappa})\overline{\sqrt{z_t^i}} + \sqrt{\kappa z_t}\right)u_{t+1}^g.
\end{aligned}
$$

5.6 Currency Factors

We first express currency excess returns, which are returns on the following strategy: the investor borrows at the domestic risk-free rate, converts the amount into foreign currency and lends at the foreign risk-free rate, converting back the proceeds at the end of the investment period and paying back the debt. The log currency excess return is

thus:[15]

$$rx_{t+1}^i = r_{f,t}^i - r_{f,t} - \Delta s_{t+1}^i.$$

The systematic components of currency excess returns are driven by at least two risk factors (Lustig, Roussanov, and Verdelhan, 2011, 2014, and Verdelhan, 2014), i.e., carry and dollar factors.

The carry factor is the excess returns of a strategy that invests in high- and borrows in low-interest rate currencies:

$$Carry_{t+1} = \frac{1}{N_H} \sum_{i \in H} rx_{t+1}^i - \frac{1}{N_L} \sum_{i \in L} rx_{t+1}^i,$$

where N_H (N_L) denotes the number of high (low) interest rate currencies in the sample.

For the sake of exposition, assume that most of the cross-country difference in interest rates is due to their exposure (δ^i) to the world state variable. In this case, baskets of high and low interest rate currencies will exhibit the same level of country-specific

---

[15]The risk-free rate in country i, denoted $r_{f,t}^i$, is given by:

$$r_{f,t}^i = -E_t[m_{t+1}^i] - \frac{1}{2}Var_t[m_{t+1}^i] = \alpha + \left(\chi - \frac{1}{2}(\gamma + \kappa)\right)z_t^i + \left(\tau - \frac{1}{2}(\delta^i + \omega)\right)z_t^w.$$

The interest rate difference, or forward discount, between country i and the U.S is therefore equal to:

$$r_{f,t}^i - r_{f,t} = \left(\chi - \frac{1}{2}(\gamma + \kappa)\right)(z_t^i - z_t) - \frac{1}{2}\left(\delta^i - \delta\right)z_t^w.$$

The expected currency excess return is therefore:

$$
\begin{aligned}
E_t\left(rx_{t+1}^i\right) &= r_{f,t}^i - r_{f,t} - E_t\left(\Delta s_{t+1}^i\right) = \frac{1}{2}Var_t(m_{t+1}) - \frac{1}{2}Var_t(m_{t+1}^i) \\
&= \frac{1}{2}(\gamma + \kappa)(z_t - z_t^i) + \frac{1}{2}(\delta - \delta^i)z_t^w.
\end{aligned}
$$

If the SDFs were not hetereoscedastic, the expected currency excess returns would be constant and the uncovered interest rate parity, which is strongly rejected in the data, would be satisfied in the model.

volatility, assuming that the law of large numbers holds.[16] Under this assumption, innovations to the carry factor only depend on shocks u^w, not on shocks u^g.

$$Carry_{t+1} - E_t\left(Carry_{t+1}\right) = \left(\overline{\sqrt{\delta^i}}^L - \overline{\sqrt{\delta^i}}^H\right)\sqrt{z_t^w}u_{t+1}^w,$$

where $\overline{x^i}^H$ and $\overline{x^i}^L$ denote the cross-sectional average of the variable x across countries in the high- and low-interest rate portfolios: $\overline{x^i}^H = \frac{1}{N_H}\sum_{i\in H}x^i$, $\overline{x^i}^L = \frac{1}{N_L}\sum_{i\in L}x^i$.

The dollar risk factor is the average of all currency excess returns defined in U.S. dollars:

$$Dollar_{t+1} = \frac{1}{N}\sum_i rx_{t+1}^i,$$

where N denotes the number of currencies in the sample. In large baskets of currencies, foreign country-specific shocks average out (again assuming that there are enough currencies in the baskets for the law of large numbers to apply). As a result, innovations to the dollar risk factor depend on both U.S.-specific and world shocks, but not on country-specific shocks:

$$
\begin{aligned}
Dollar_{t+1} - E_t\left(Dollar_{t+1}\right) = {} & \sqrt{\gamma z_t}u_{t+1}\\
& + \left(\sqrt{\delta} - \overline{\sqrt{\delta^i}}\right)\sqrt{z_t^w}u_{t+1}^w + \sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right)u_{t+1}^g.
\end{aligned}
$$

If the U.S. SDF exhibits the average exposure to shocks u^w (i.e., $\overline{\delta^i} = \delta$), the dollar factor is orthogonal to the carry factor. Under that assumption, the dollar factor captures shocks u^g, while the carry factor captures shocks u^w.

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[16]In practice of course, the number of currencies is small (the dataset used in this paper contains at most 39 currencies). As a result, the law of large numbers is only an approximation used here to provide intuition.

5.7 Connecting the Model to the Empirics

The world equity return, $LWMKT_{t+1}$, built from returns in local currencies, does not span all systematic shocks: it depends on shocks u^w, u^g, and u^c, but not on the U.S. shocks u. These shocks u are systematic from the perspective of the U.S. investor, although not from the perspective of other investors. They appear in the cross-section of equity excess returns because the returns need to be expressed in one common currency (e.g., the U.S. dollar).

In contrast, the world equity return in U.S. dollars $WMKT_{t+1}$ contains all the systematic shocks that drive foreign equity returns. At first sight, it appears as a sufficient tool to measure aggregate risk, without the need to add any bilateral exchange rates. This result contrasts with the findings of Adler and Dumas (1983) because the exchange rate is endogenous in our setup. In practice, however, even in this simple model, the world equity return in U.S. dollars is an imperfect measure of risk in the (obviously general) case when the econometrician does not know the country-specific and global state variables.

To see this point, consider the time-variation in the quantity and market price of aggregate risk, starting with the betas. The betas on the global shocks u^w and u^c are constant, while the betas on the global shocks u^g are not. The total beta on the world equity return in U.S. dollars $WMKT_{t+1}$ is time-varying, following the dynamics of the country-specific state variables z and z^i:

$$\beta^i_{WMKT,t} = 1 + \frac{\sigma^w_D - k_1 C^i_{pd}\sigma^w - \sqrt{\delta^i} + \sqrt{\delta}}{\sigma^w_D - k_1 \overline{C^i_{pd}}\sigma^w - \overline{\sqrt{\delta^i}} + \sqrt{\delta}} + \frac{\sigma^{c,i}_D}{\sigma^{c,i}_D} + \frac{(\sigma^g_D - \sqrt{\kappa})\sqrt{z^i_t} + \sqrt{\kappa z_t}}{(\sigma^g_D - \sqrt{\kappa})\overline{\sqrt{z^i_t}} + \sqrt{\kappa z_t}} \qquad (7)$$

In our simple setup, all country-specific state variables z^i are characterized by the same persistence and volatility. However, in actual data, country-specific state variables seem very likely to evolve at different frequencies, confounding estimation.

Market prices of risk are also time-varying in our simple model. Expected excess

44

returns are the product of betas and market prices of risk, i.e., the market price of world equity risk is the ratio of the expected excess return, defined in Equation (6), to the beta in Equation (7). The price of risk thus also varies with the country-specific and global state variables (z, z^i, and z^w). If those state variables were known to the econometrician, a conditional asset pricing experiment could recover the time-variation in the quantities and prices of risk. In practice however, the state variables are not known, forcing reliance on rolling windows or other such empirical approaches.

The issue becomes even more acute if the heteroskedasticity of the u^w or u^c shocks were to depend on both global and country-specific state variables.[17] In that case, the aggregate market beta $\beta^i_{WMKT,t}$ also depends on the global state variable z^w. The critical issue here is that if the local and global state variables evolve at different frequencies, it becomes impossible to use a single beta to perfectly summarize the time-variation in two state variables.

Taken together, the world equity return expressed in U.S. dollars bundles variables with different time-dynamics together. This makes it difficult to use this single factor to uncover risk exposures in international asset pricing. While our simple model with endogenous exchange rates suggests that we should go back to the World CAPM, the heteroskedasticity of the SDF implies that this single-factor model would struggle to accurately capture the risk-return tradeoff in practice.

This motivates our choice of a multiple-factor model. With $LWMKT$, $Carry$, and $Dollar$, we can summarize all the shocks in the system, and we can allow all quantities

---

[17]Lustig, Roussanov, and Verdelhan (2011, 2014) and Verdelhan (2014) consider this variation in order to reproduce some features of the currency markets. In the context of our model, the law of motion of the log SDF would be:

$$-m^i_{t+1} = \alpha + \chi z^i_t + \sqrt{\gamma z^i_t} u^i_{t+1} + \tau z^w_t + \sqrt{\delta^i z^w_t + \lambda z^i_t} u^w_{t+1} + \sqrt{\kappa z^i_t} u^g_{t+1} + \sqrt{\omega z^w_t} u^c_{t+1}$$

We do not consider that variation here as it does not admit a closed-form solution for equity returns and betas.

and prices of risk to have their own time-dynamics. When we empirically implement the model in the data, the only difference lies in the approach to aggregation, i.e., we weight countries and currencies by stock market capitalizations, instead of the equal weights that we employ in the model for ease of exposition.

The model has three additional implications that we check in the data. First, the carry and dollar factors should matter even for equity excess returns expressed in local currency. Likewise, the equity factor $LWMKT_{t+1}$, built without introducing exchange rates, should be correlated with the carry and dollar factors. Second, despite exchange rates being endogenous, exchange rate shocks in this model do not span equity returns. Shocks u^c affect equity returns but do not affect exchange rates, because their impact is exactly the same across countries. Equity and currency markets therefore appear segmented to an extent. Third, while using bilateral exchange rates as risk factors directly is consistent with the model, they are also driven by country-specific shocks that are irrelevant to asset pricing and thus weaken the identification of aggregate risk.

5.8   Shortcomings of the Model

The model is parsimonious and can be solved in closed form. But its simplicity entails some shortcomings when compared to the data. The main weakness of the model lies in its implied betas.

In the data, global equity and currency betas are clearly time-varying. Moreover, the global equity, carry, and dollar betas are not highly correlated. The time-series correlations (obtained country-by-country) range from -0.7 to 0.7 for the $LWMKT - Dollar$, $LWMKT - Carry$, and $Dollar - Carry$ pairs. Across countries, these correlations are close to zero. Our asset pricing estimates allow for these factor-specific dynamics in the quantities of risk.

But in the model, betas are either constant or too smooth. A univariate regression

of equity excess returns in U.S. dollars on the carry factor, for example, delivers a constant beta; the data suggest otherwise. Time-variation in all betas can be obtained in the model by assuming that the market prices of risk (i.e., the square roots in the SDFs) depend not only on one but on two state variables. In this case, however, the model does not admit a closed-form. For the sake of clarity, we choose to focus on a model that delivers all objects of interest in closed-form.

5.9 Simulating the Model

We simulate the model in order to better understand the drivers of our empirical results. We begin by calibrating key parameters of the model, and present these calibration parameters in Table 3.

All parameters governing the state variable dynamics and the parameters τ, χ and γ governing the stochastic discount factor are from Lustig, Roussanov, and Verdelhan (2014). The boundaries of the parameter δ (i.e., δ_l and δ_h) are determined to match the mean interest-rate differential of the Japanese yen and the Australian dollar against the U.S. dollar over the period January 1976 to April 2013 (-3.18 % and 3.27 %, respectively). The parameter α is calibrated to match an average annualized nominal risk-free rate of 4.5% given the parameter values for χ, γ, κ, τ, ω and the average δ. The remaining parameters are set to match the empirical pairwise correlations between the equity and currency factors (reported in the Online Appendix). The model delivers reasonable interest rates, exchange rates, equity, and currency excess returns as can be seen in Table 4 below.

Figure 9 shows what happens when we estimate our empirical specifications on these simulated data from the model – it is the counterpart to Figure 5, but estimated using simulated data from the model. The vertical axis corresponds to average excess returns from simulated data. The horizontal axis corresponds to predicted excess returns from

Table 3
Parameter Values

This table reports the parameter values used to simulate the model. The boundaries of the parameter δ (i.e., δ_l and δ_h) are determined to match the mean interest-rate differential of the Japanese yen and the Australian dollar against the U.S. dollar over the period January 1976 to April 2013 (-3.18 % and 3.27 %, respectively). All parameters governing the state variable dynamics and the parameters τ, χ and γ governing the stochastic discount factor are from Lustig, Roussanov and Verdelhan (2014). The parameter α is calibrated to match an average annualized nominal risk-free rate of 4.5% given the parameter values for χ, γ, κ, τ, ω and the average δ. The remaining parameters are set to match the empirical pairwise correlations between the equity and currency factors. The law of motion of the stochastic discount factor (Panel A), the state variable dynamics (Panel B) and the dividend growth process (Panel C) are reported at the top of each panel.

Panel A: Stochastic discount factor

$$-m_{t+1}^i = \alpha + \chi z_t^i + \sqrt{\gamma z_t^i} u_{t+1}^i + \tau z_t^w + \sqrt{\delta^i z_t^w} u_{t+1}^w + \sqrt{\kappa z_t^i} u_{t+1}^g + \sqrt{\omega z_t^w} u_{t+1}^c$$

	SDF						Heterogeneity		
α (%)	χ	γ	κ	τ	ω		δ_{HC}	δ_L	δ_H
4.5	0.89	0.04	2.2	0.06	2.2		0.36	0.10	0.61

Panel B: State variable dynamics

$$z_{t+1}^i = (1-\phi)\theta + \phi z_t^i - \sigma\sqrt{z_t^i} u_{t+1}^i \qquad z_{t+1}^w = (1-\phi^w)\theta^w + \phi^w z_t^w - \sigma^w\sqrt{z_t^w} u_{t+1}^w$$

ϕ	θ (%)	σ (%)		ϕ^w	θ^w (%)	σ^w (%)
0.91	0.77	0.68		0.99	2.09	0.28

Panel C: Dividend growth rate

$$\Delta d_{t+1}^i = \mu_D + \psi z_t^i + \psi_w z_t^w + \sigma_D\sqrt{z_t^i} u_{t+1}^i + \sigma_D^g\sqrt{z_t^i} u_{t+1}^g + \sigma_D^w\sqrt{z_t^w} u_{t+1}^w + \sigma_D^{c,i}\sqrt{z_t^w} u_{t+1}^c$$

	Dividends						Heterogeneity		
μ_D (%)	ψ	ψ_w	σ_D	σ_D^g	σ_D^w		$\bar{\sigma}_D^c$	$\sigma_D^{c,L}$	$\sigma_D^{c,H}$
2.71	0	-1.1	0.12	0.05	0.01		0.19	0.16	0.23

Table 4
Descriptive Statistics of Simulated Equity and Foreign Exchange Series

This table compares cross-sectional moments of simulated equity and foreign exchange series (Panel A) with their empirical counterparts (Panel B). Moments are defined as the cross-sectional mean and standard deviation of the time-series mean, standard deviation, and first-order autocorrelation (AC1) of the following variables: country equity returns denominated in either local currency (r^{ei}) or U.S. Dollars ($r^{e\$}$), the price-dividend ratio in log-form ($p-d$), the dividend growth rate (Δd), the country risk-free rates (r_f^i), the log changes in bilateral exchange rates (Δs) and the interest rate differences between a foreign and the domestic currency ($i^* - i$). The last column reports time-series moments for the United States. The model is calibrated using parameter values reported in Table 3. It is simulated for a population of 45 countries and one million periods. Empirical moments refer to developed countries. Except for the price-dividend ratio, means and standard deviations are annualized and reported in percentage terms.

Moments	A: Simulation		B: Data		
	Mean	Std	Mean	Std	US
$E(r^{ei})$	10.39	1.83	10.05	2.43	10.29
$\sigma(r^{ei})$	9.93	0.84	20.38	3.73	15.23
$AC1(r^{ei})$	0.02	0.00	0.11	0.06	0.06
$E(r^{e\$})$	10.36	1.83	10.34	1.75	10.29
$\sigma(r^{e\$})$	14.88	2.37	22.84	3.83	15.23
$AC1(r^{e\$})$	0.00	0.00	0.07	0.05	0.06
$E(p-d)$	5.46	0.37	3.53	0.36	3.60
$\sigma(p-d)$	0.03	0.01	0.40	0.11	0.47
$AC1(p-d)$	0.99	0.01	0.98	0.01	0.99
$E(\Delta d)$	4.95	0.01	7.65	2.60	6.02
$\sigma(\Delta d)$	9.96	0.81	14.79	5.18	7.18
$AC1(\Delta d)$	0.02	0.00	0.20	0.17	0.46
$E(r_f^i)$	4.50	1.93	5.10	2.17	5.03
$\sigma(r_f^i)$	1.64	0.10	1.33	0.50	0.97
$AC1(r_f^i)$	0.99	0.00	0.84	0.15	0.97
$\sigma(\Delta s)$	12.48	1.82	10.15	2.91	-
$AC1(\Delta s)$	-0.01	0.00	0.00	0.09	-
$E(i^* - i)$	0.00	1.93	0.93	2.82	-
$\sigma(i^* - i)$	0.19	0.04	1.03	0.37	-
$AC1(i^* - i)$	0.93	0.01	0.73	0.24	-

the World CAPM, the International CAPM, and our three-factor model all estimated using the simulated data, using rolling-windows in order to capture time-variation in quantities and prices of risk. The figure presents a striking demonstration of the difference: the World CAPM and the International CAPM both fit the simulated data rather poorly, while our model accurately describes the cross-section of returns.

Our next step is to attempt to use our model framework to explore the portfolio choice and hedging strategy of global investors.

6   Optimal Currency Hedging

Black (1989) derives a striking result: the optimal amount of currency hedging is the same for every investor in the world. It depends on only three variables: the average world market portfolio expected excess return (μ_m), the average world market portfolio excess return variance (σ_m^2), and the average exchange rate variance (σ_e^2).

Black (1989) shows that the fraction of exchange risk hedged should be equal to:

$$\frac{\mu_m - \sigma_m^2}{\mu_m - \frac{1}{2}\sigma_e^2}.$$

This striking result continues to serve as something of a benchmark to this day. But it is derived in a mean-variance efficient world that does not encompass our model. We thus revisit the issue in our framework, by solving for the optimal portfolio and then inspecting the optimal currency portfolio investment strategy of an atomistic international investor.

6.1   Optimal Portfolio Problem

In order to define an optimal portfolio, we naturally need to start with an objective. We interpret each stochastic discount factor of country i as describing the inter-temporal

Figure 9

Realized vs Predicted Average Excess Returns: Simulated Data

This figure plots annualized realized average excess returns against those predicted by the World CAPM, the International CAPM and the CAPM Redux from simulated series. For each country the predicted excess returns are computed as follows: a) Time-varying factor betas are estimated using 60-month rolling windows; b) At each time t estimated conditional betas are multiplied by the corresponding factor means computed over the same time-window (from time *t-59* to time *t*); c) Predicted values are averaged. For each country the realized average excess returns are computed as follows: a) In each 60-month rolling window actual excess returns are averaged and b) 60-month rolling return means are averaged. Average returns are in percentage. The straight line is the 45-degree line through the origin. The sample has length T=447 and a cross-section of 45 countries
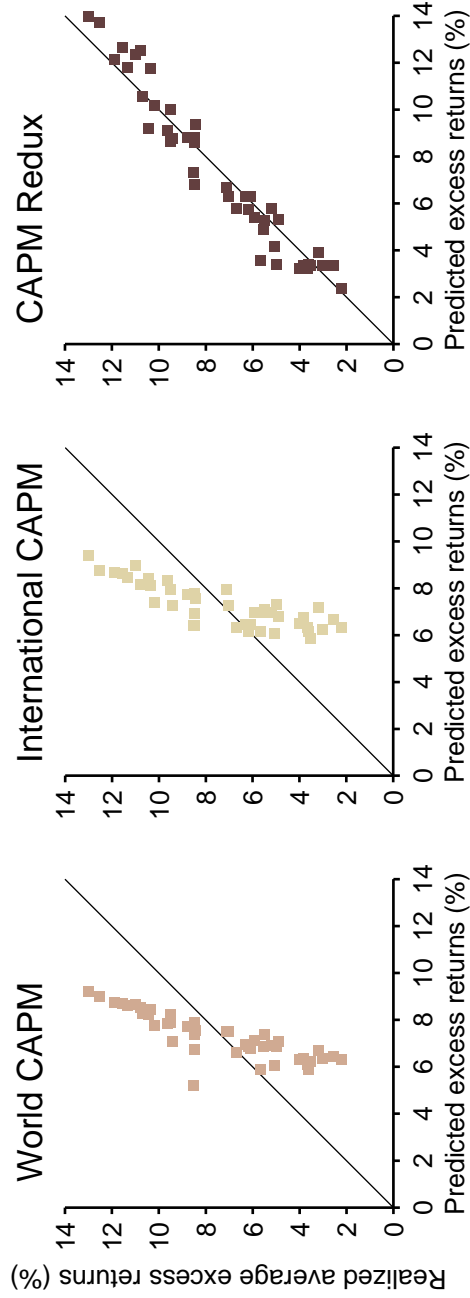
marginal rate of substitution of a representative investor in country i characterized by constant relative risk-aversion Γ.

Consider the optimal one-period portfolio problem of each investor with initial wealth W^i, in the absence of labor income:

$$max_{X^i} \ E[u(X^i)] \text{ subject to } E(M^i X^i) = W^i,$$

where X^i denotes the payoffs next period. The optimal portfolio is therefore (Cochrane, 2005):

$$\widehat{X^i} = W^i \frac{M^{-\frac{1}{\Gamma}}}{E(M^{1-\frac{1}{\Gamma}})}.$$

Solving for the optimal return $\widehat{R^i} = \frac{\widehat{X^i}}{W^i}$ and expressing it in log terms leads to:

$$\widehat{r^i_{t+1}} = -\frac{1}{\Gamma} m^i_{t+1} - \left(1 - \frac{1}{\Gamma}\right) E_t(m^i_{t+1}) - \frac{1}{2}\left(1 - \frac{1}{\Gamma}\right)^2 Var_t(m^i_{t+1}). \quad (8)$$

Substituting in the expressions for our pricing kernels m, the optimal return is:

$$
\begin{aligned}
\widehat{r^i_{t+1}} &= \frac{1}{\Gamma}\left(\sqrt{\gamma z^i_t}u^i_{t+1} + \sqrt{\delta^i z^w_t}u^w_{t+1} + \sqrt{\kappa z^i_t}u^g_{t+1} + \sqrt{\omega z^w_t}u^c_{t+1}\right) \\
&+ \left(\alpha + \chi z^i_t + \tau z^w_t\right) - \frac{1}{2}\left(1 - \frac{1}{\Gamma}\right)^2 \left(\gamma z^i_t + \delta^i z^w_t + \kappa z^i_t + \omega z^w_t\right).
\end{aligned} \quad (9)
$$

The optimal return depends on the three global shocks in the model as well as the country-specific shock u^i.

6.2 Optimal Portfolio Solution and Hedging

Let us assume that each investor has access to a risk-free asset and four ETFs which replicate risky returns. In the case of the U.S. investor, the four ETFs are $LWMKT$, $Dollar$, $Carry$, and MKT_{US} (the U.S. stock market return in U.S. Dollars). A U.K. investor would similarly have access to $LWMKT$, $Pound$, $Carry$, and MKT_{UK}. We

find the optimal wealth allocation $(\omega^{LWMKT}, \omega^{Dollar}, \omega^{Carry}, \omega^{MKT_i}, \omega^{rf_i})$ by assuming that each investor wants to replicate the unexpected component of the optimal return by investing in all these five assets. The optimal wealth allocation therefore satisfies:

$$
\begin{aligned}
\widehat{r_{t+1}^i} - E_t\left(\widehat{r_{t+1}^i}\right) &= \omega^{LWMKT} LWMKT_{t+1} + \omega^{Dollar} Dollar_{t+1} + \omega^{Carry} Carry_{t+1} \\
&\quad + \omega^{MKT_i} r_{t+1}^{MKT,i} + \omega_{rf_i} r_{f,t}^i
\end{aligned}
$$

where $\omega^{LWMKT} + \omega^{Dollar} + \omega^{Carry} + \omega^{MKT_i} + \omega^{rf_i} = 1$.

We ignore the small expected component of the optimal return, but it could be replicated by adding for example a bond to the portfolio.[18] Since all returns are known in closed-form, we find that the optimal wealth allocation for the U.S. investor at time t is:

$$
\omega_t^{LWMKT} = \frac{\frac{\sqrt{\gamma}\sigma_D^c}{\Gamma}\left(\sqrt{\kappa z_t} - \left(1 - \frac{\sqrt{\omega}}{\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\right)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) - \frac{\sqrt{\omega}}{\sigma_D^c}\sigma_D^g\sqrt{z_t}\right)}{\overline{\sigma_D^{c,i}}(\sigma_D - k_1 B_{pd}\sigma)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) - \sqrt{\gamma}\sigma_D^g\left(\overline{\sigma_D^{c,i}}\sqrt{z_t} - \sigma_D^c\overline{\sqrt{z_t^i}}\right)},
$$

$$
\omega_t^{MKT} = \frac{\frac{1}{\Gamma}\sqrt{\omega} - \omega_t^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c},
$$

$$
\omega_t^{Dollar} = \frac{1}{\Gamma} - \frac{1}{\sqrt{\gamma}}\left(\frac{\frac{1}{\Gamma}\sqrt{\omega} - \omega_t^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\right)(\sigma_D - k_1 B_{pd}\sigma),
$$

$$
\omega_t^{Carry} = \frac{\frac{1}{\Gamma}\sqrt{\delta} - \omega_t^{LWMKT}(\sigma_D^w - k_1\overline{C_{pd}^i}\sigma^w) - \omega_t^{MKT}(\sigma_D^w - k_1 C_{pd}\sigma^w)}{\left(\overline{\sqrt{\delta^i}}^L - \overline{\sqrt{\delta^i}}^H\right)},
$$

$$
\omega_t^{rf} = 1 - \omega_t^{LWMKT} - \omega_t^{Dollar} - \omega_t^{Carry} - \omega_t^{MKT}.
$$

Note that these shares of wealth in the different assets are time-varying. Date by date, the investor can replicate the unexpected component of the optimal portfolio. Doing

---

[18]Our basis of assets is incomplete as it does not replicate the expected component of the optimal return. Moreover, we do not impose the market clearing conditions (equity should be in strictly positive supply and currency and bonds in net zero supply). The basis of assets is here purely illustrative.

so, however, requires the estimation of the model and a measure of the country-specific market price of risk (the variable z in the example of the U.S. investor).

In this framework, optimal portfolio returns are clearly defined, but in the absence of additional frictions, optimal currency hedging is not. The same global shocks affect both equity and currency returns. Investing without any exchange rate risk, for example in a world index of local-currency equity returns, would still expose the investor to many of the same shocks that affect exchange rates. It is thus not clear why a given shock should be hedged when it affects exchange rates but not when it affects equity returns.

One can, however, distinguish between the ETFs that are explicitly investing in currency portfolios versus those that are investing in global or local markets without any exchange rate risk. We define the following hedge ratio to summarize this allocation:

$$\text{Hedge ratio}_t = \frac{\omega_t^{LWMKT} + \omega_t^{rf} + \omega_t^{MKT}}{\omega_t^{LWMKT} + \omega_t^{Dollar} + \omega_t^{Carry} + \omega_t^{rf} + \omega_t^{MKT}} = \omega_t^{LWMKT} + \omega_t^{rf} + \omega_t^{MKT}.$$

It is the ratio of the optimal exchange rate insensitive component of the portfolio to the total size of the portfolio. In our setup, this fraction is investor- and time-specific, which contrasts with the universal hedging result of Black (1989). While the optimal portfolio return does not depend on the set of traded assets but only on the risk-aversion and SDF of the representative investor in each country, the hedge ratio depends on the set of traded assets used to implement the optimal portfolio.

Our framework abstracts from real-word frictions that could be added at the cost of loosing the tractability of closed-form results. Through simulations, the model could be extended notably to add short-sale and base-currency constraints.

7 Conclusion

In this paper, we show that currency risk is priced in international equity portfolios. We do so using a new set of three factors, namely, a global equity factor denominated in local currencies, and two currency factors, dollar and carry, which are constructed from the space of global currency excess returns. Our results on the pricing of currency risk using this model are obtained from a comprehensive set of equities from 46 developed and emerging countries spanning value, growth, and country index returns from 1976 to the present. We also find that our model outperforms the World and International CAPM, as well as the Fama-French three and four factor models in our sample, and find evidence of substantial exposure to currency risk in the universe of global mutual funds and hedge funds, suggesting that whether exposure to international markets is direct or indirect, currency risk is a significant factor affecting the returns of international investors.

We set up a simple complete-markets model of international equities with endogenous exchange rates, and show that this model is well able to replicate our key empirical findings. The model also enables us to derive the optimal asset portfolio weights for an atomistic international investor. In our framework, the optimal investment in currency assets for such an investor varies over time and across investor locations, contrary to the famous result of Black (1989) who operates in a simpler mean-variance preference setting.

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Appendix

A   A Brief Review of the Literature on International Asset Pricing

Early theoretical work in international asset pricing generally assumed that consumption and investment opportunity sets do not differ across countries. This restrictive assumption was relaxed in later work, which eliminated the assumption of investor indifference between domestic and foreign markets.[19] With country-specific investor heterogeneity in preferences, or homogenous preferences but differences in relative prices faced by investors in different countries, currency risk matters for asset pricing. Such relative price differences naturally arise in a world in which there are deviations from purchasing power parity (PPP); moreover in such a world, investing in foreign currencies is risky, since adverse exchange rate movements imply low income from foreign investment expressed in domestic currency terms.[20] In this context, Solnik (1974), Sercu (1980), Stulz (1981), and Adler and Dumas (1983) incorporated currency risk into theoretical asset pricing models, allowing for differences across countries in consumption opportunity sets.

Specifically, Solnik (1974) modeled exchange rates as cross-country relative prices of consumption baskets, and Sercu (1980) generalized the approach, allowing stock returns expressed in local currencies to be correlated with exchange rates (we refer to

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[19]See Glen and Jorion (1993) and Stulz (1995) for a comprehensive survey of the theoretical and empirical literature on international portfolio choice and asset pricing. More recently, Campbell, de Medeiros, and Viceira (2010) consider optimal currency hedging in international equity investment, while Kroencke, Schindler and Schrimpf (2013) study the optimal portfolio allocation across equity and exchange rate investment styles.

[20]Empirical work suggests that PPP holds only in the long run. See Froot and Rogoff (1995) for a comprehensive survey of the early literature on PPP.

this henceforth as the Solnik-Sercu model). Stulz (1981) and Adler and Dumas (1983) assumed stochastic country-specific inflation in addition to deviations from PPP.[21]

Early empirical work in international asset pricing generally extended the standard Capital Asset Pricing Model (CAPM) (Sharpe, 1964 and Lintner, 1965) to global markets, or simply augmented the domestic CAPM with the addition of a few international factors. In this early literature, unconditional tests of these models generally proved inconclusive (see, for example, Stehle, 1977, Solnik, 1974 and Korajczyk and Viallet, 1989).

Later conditional studies yielded more promising results: for example, Bekaert and Harvey (1995) provided evidence that countries' capital markets become increasingly globally integrated over time, Chan, Karolyi and Stulz (1992) identified a time-varying global market premium in U.S. equity markets, and Karolyi and Stulz (1996) studied co-movement in Japanese and U.S. stock markets.[22] Ferson and Harvey (1993) studied the predictability of foreign equity returns and showed that most of it is related to time-variation in global risk premia, and Bekaert and Hodrick (1992), and Bekaert (1995, 1996) studied the predictability of equity and currency returns.

Harvey (1991) introduced a novel approach to the literature, modelling time-variation in both the exposure and the price of risk by conditioning on common and country-specific set of instruments, and finding that time-variation reveals differences across countries, but fails to fully predict conditional expected returns. Implementing a variation on this methodology, and incorporating currency risk explicitly, Dumas and Solnik (1995) found that an exchange rate risk model outperforms a simple "World CAPM" with global market equity returns but no explicit role for currency risk. In their empirical work, Dumas and Solnik (1995) assumed that currency risk for a U.S. investor is well captured by three major currencies: the Japanese Yen, the British Pound, and the German Mark. Providing evidence that currency risk is priced in global capital markets, they attributed the widespread failure of the World CAPM to a misspecification problem. These findings were subsequently corroborated by De Santis and Gerard

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[21]In the model of Adler and Dumas (1983), there are $L+1$ countries, and a set of $m = n + L + 1$ assets – other than the base-currency deposit – comprised of n portfolios of equities, L foreign currency deposits, and the world market portfolio. In that model, the expected return on asset j is:

$$E_t \left(r_{j,t+1} - r_{f,t} \right) = \sum_{i=1}^{L} \lambda_{i,t} cov_t(r_{j,t+1}, r_{n+i,t+1}) + \lambda_{m,t} cov_t(r_{j,t+1}, r_{m,t+1}) \qquad (10)$$

where $r_{j,t+1} - r_{f,t}$ is the nominal return on the equity portfolio j in excess of the risk-free rate (denoted $r_{f,t}$) of the currency in which returns are measured, and $r_{m,t+1}$ is the excess return on the world market portfolio. The covariance terms $cov_t(r_{j,t+1}, r_{n+i,t+1})$ measure the quantity of exchange rate risk. The time-varying coefficients $\lambda_{i,t}$ are the world prices of exchange rate risk. The time-varying coefficient $\lambda_{m,t}$ is the world price of market risk. The unconditional version of this model is presented later in this section.

[22]A large related literature studies the integration of emerging and developed equity markets [see, e.g., Bekaert and Harvey (2000), Bekaert, Harvey and Lundblad (2003), Bekaert, Harvey, Lundblad, and Siegel (2007), Bekaert, Hodrick and Zhang (2009), Bekaert, Harvey, Lundblad and Siegel (2011), and Bekaert and Harvey (2014)].

(1998), who extended the analysis to account for volatility dynamics. Harvey, Solnik and Zhou (2002) also provided support for currency risk in international equity returns. Using latent factors, they find that their first factor premium resembles the expected return on the world market portfolio, while their second factor premium is related to foreign exchange risk. These studies provided strong empirical support for the predictions of international asset pricing theory, but were generally implemented on a small sample of assets from developed countries, over relatively short sample periods.[23]

More recently, a variety of alternate explanations have been proposed for differences in international average equity returns (see Lewis (2011) for the most complete and recent survey of international asset pricing). These explanations include global economic risks (Ferson and Harvey, 1994); inflation risk (Chaieb and Errunza, 2007); liquidity risk (Karolyi, Lee, and van Dijk, 2012, Bekaert, Harvey, and Lundblad, 2007, Malkhozov, Mueller, Vedolin, and Venter, 2014) factors including momentum and a global cash-flow-to-price factor (Hou, Karolyi, and Kho, 2011); and investability restrictions (Karolyi and Wu, 2014). These specifications do not account for currency risk; nevertheless they significantly outperform the World CAPM. This is also a feature of the work of Fama and French (2012), who present international versions of both the Fama and French (1993) three-factor model, and the Carhart (1997) four-factor model. Fama and French (2012) find that their global models perform reasonably well in unconditional time-series tests on global size and book-to-market sorted portfolios as well as size and momentum sorted portfolios, but perform worse when the same portfolios are constructed at a regional level (i.e., North America, Japan, Asia Pacific, and Europe).

It is worth mentioning here that our work has close links with recent advances in research on currencies. It is well-known (see Meese and Rogoff, 1983) that economic models of exchange rate determination generally lack empirical support in the short-run, with few identifiable links between nominal exchange rates and economic fundamentals.[24] However, a related literature has attempted to explain returns on portfolios of currencies as compensation for risk. Lustig, Roussanov, and Verdelhan (2011) show that the cross-section of currency portfolios sorted by interest rates can be well-explained by a "slope" factor – which corresponds to the "carry" factor that we employ in this paper. Lustig, Roussanov, and Verdelhan (2011), Menkhoff, Sarno, Schmeling and Schrimpf (2012), Maggiori (2012), and Lettau, Maggiori and Weber (2013) link the currency factors to measures of volatility and downside risk in equity and currency markets. Verdelhan (2014) shows that the carry factor and the "dollar" factor together account for a large share of the variation in bilateral exchange rates, and provides evidence that dollar risk is priced in the cross-section of currencies. These

---

[23]Dumas and Solnik (1995) considered a set of four countries (United Kingdom, Germany, Japan and the United States) from March 1970 to December 1991. De Santis and Gerard (1998) take into account the same countries from June 1973 to December 1994.

[24]At high frequencies, a number of papers, including Evans and Lyons (2002), and Froot and Ramadorai (2005) show that order flow in exchange rate markets is helpful at predicting and explaining exchange rate movements, but there is considerable debate about the source of this explanatory power, with both rational and behavioral explanations rationalizing these findings.

two currency risk factors are nearly orthogonal to one other, and capture two distinct sources of currency risk relevant for explaining variation in currency returns. Our use of carry and dollar as risk-factors in our international asset pricing model is a product of these applications of risk-based arbitrage pricing (Ross, 1976) to currency markets.

B   Factor Mimicking Portfolio

Figure 10 reports the characteristics of the a factor mimicking portfolio for the global equity factor denominated in local currencies.

Table 1 compares the pricing errors obtained with the LWMKT factor or its factor mimicking portfolio.

<div align="center">

Table 1

FMP vs. LWMKT: Pricing Error Tests

</div>

This table evaluates the CAPM Redux pricing errors when the global equity factor is defined as either the LWMKT factor or its factor mimicking portfolio. Pricing errors are compared along four statistical metrics: the Mean Pricing Error (MPE), the Mean Absolute Pricing Error (MAPE), the Time-Series Mean Absolute Pricing Error (TS-MAPE) and the Root Mean Squared Error (RMSE). For TS-MAPE: At each point in time, we compute the cross-sectional average absolute pricing error and report the time-series average of this statistic. For all other metrics: we compute the metrics at country level and report the cross-sectional average of each statistic. Details about the construction of the FMP are reported in Table 10. The sample period is April 1978 to April 2013.

	FMP	LWMKT
MPE	0.271	0.280
MAPE	0.782	0.777
TS MAPE	0.538	0.540
RMSE	0.845	0.846

Figure 10
Factor Mimicking Portfolio For LWMKT (Equity Factor)

This figure presents details about a factor mimicking portfolio (FMP) for the global equity factor denominated in local currencies (LWMKT). The FMP is built over 60-month non-overlapping windows in two steps. In the first step LWMKT is regressed on a constant, the excess returns on the eight extreme Fama-French U.S. portfolios and the excess returns on a set of maximum 30 mutual funds with no exposure to the currency factors and high statistical exposure to LWMKT over the same time window. The coefficients are estimated under two constraints: they sum up to one and are all no greater than one in absolute value. In the second step, the FMP is obtained as a weighted average of the excess returns on the same set of traded assets, where weights are given by the first-step coefficients. Panel A shows the number of mutual funds embedded in the FMP in each non-overlapping window. Panel B shows the correlation between the FMP series and LWMKT series over the same horizon. Panel C plots the difference in excess returns between FMP and LWMKT at each point in time. The correlation between these two series is reported at the bottom of the graph. The sample period is April 1978 to April 2013.

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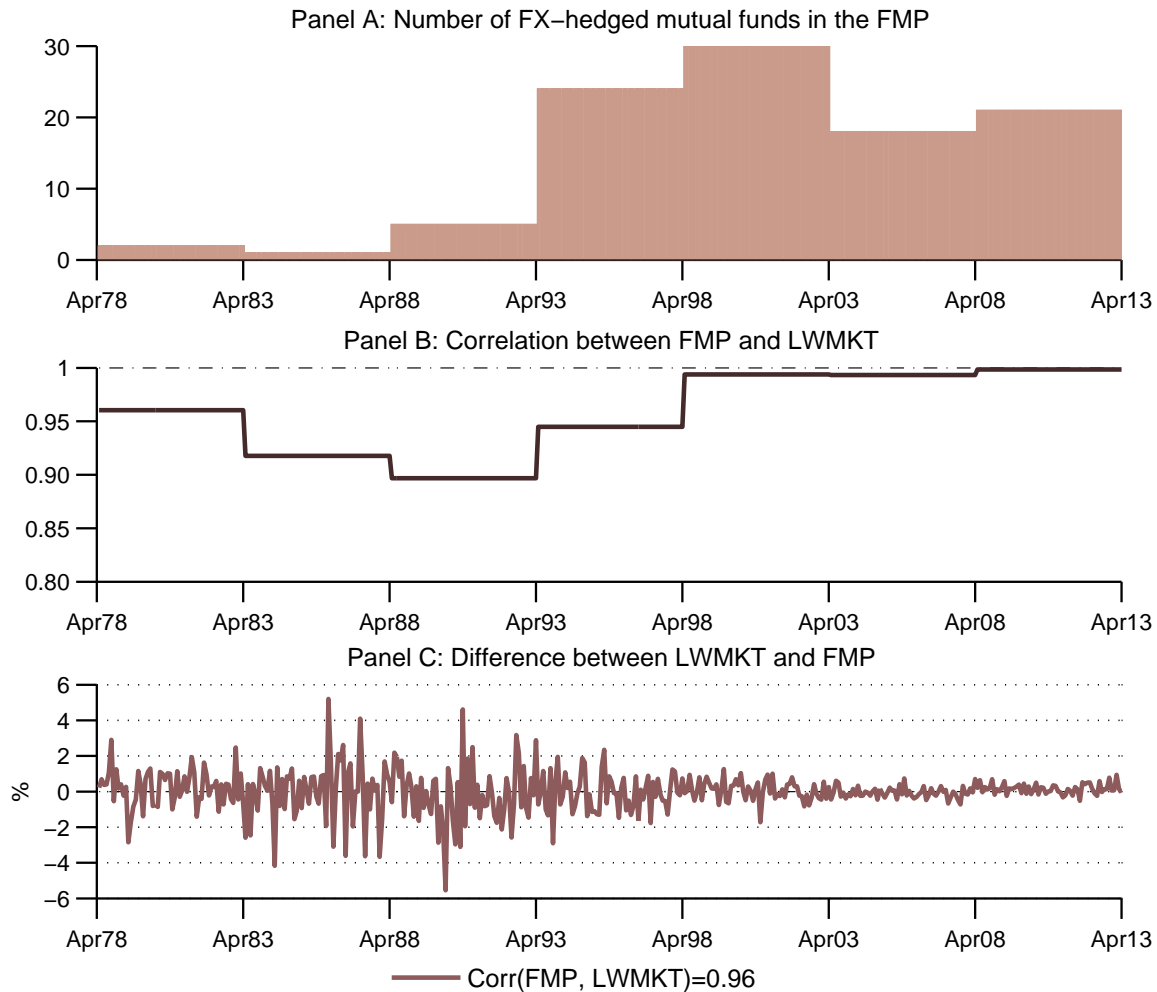
C   Solving The Model

Under the standard assumption that asset returns and the SDF are jointly log-normal, the Euler equation implies that the expected equity return of any country i, r_{t+1}^{ei}, satisfies:

$$0 = \ln E_t \, \exp \{m_{t+1}^i + r_{t+1}^{ei}\} = E_t[m_{t+1}^i + r_{t+1}^{ei}] + \frac{1}{2} Var_t[m_{t+1}^i + r_{t+1}^{ei}]. \tag{11}$$

To solve for the price-dividend ratio of the aggregate dividend claim, we proceed in three steps.

First, we rely on the standard log-linear approximation for the log gross return on that claim:

$$r_{t+1}^{ei} \approx k_0 + k_1 pd_{t+1}^i - pd_t^i + \Delta d_{t+1}^i, \tag{12}$$

where r_{t+1}^{ei} denotes the logarithmic gross real rate of return on each country's stock market index denominated in that country's currency and k_0 and k_1 are the coefficients for the expansion of the log price-dividend ratio pd_t^i around its mean, $\bar{pd}^{25}$.

Second, we conjecture that the log price-dividend ratio is affine in the country-specific and the global state variables, z_t^i and z_t^w:

$$pd_t^i = A_{pd}^i + B_{pd}^i z_t^i + C_{pd}^i z_t^w. \tag{13}$$

Third, we posit that the dividend growth process of country i follows:

$$\Delta d_{t+1}^i = \mu_D + \psi z_t^i + \psi_w z_t^w + \sigma_D \sqrt{z_t^i} u_{t+1}^i + \sigma_D^g \sqrt{z_t^i} u_{t+1}^g + \sigma_D^w \sqrt{z_t^w} u_{t+1}^w + \sigma_D^{c,i} \sqrt{z_t^w} u_{t+1}^c, \tag{14}$$

Substituting Equation (13) and Equation (14) into Equation (12) yields the following expression for the country-i return in local currency:

$$r_{t+1}^{ei} \approx k_0 + k_1(A_{pd}^i + B_{pd}^i z_{t+1}^i + C_{pd}^i z_{t+1}^w) - pd_t^i + \mu_D + \psi z_t^i + \psi_w z_t^w + \sigma_D \sqrt{z_t^i} u_{t+1}^i +$$
$$+ \sigma_D^g \sqrt{z_t^i} u_{t+1}^g + \sigma_D^{c,i} \sqrt{z_t^w} u_{t+1}^c + \sigma_D^w \sqrt{z_t^w} u_{t+1}^w. \tag{15}$$

Combining the law of motion of each state variable with this expression (Equation 15), substituting back into Equation (11) and solving for pd_t^i yield:

$$pd_t^i = -\alpha + k_0 + k_1 A_{pd}^i + k_1 B_{pd}^i(1 - \phi)\theta + k_1 C_{pd}^i(1 - \phi^w)\theta^w + \mu_D + [k_1 B_{pd}^i \phi + \psi - \chi$$
$$+ \frac{1}{2}(\sqrt{\gamma} + k_1 B_{pd}^i \sigma - \sigma_D)^2 + \frac{1}{2}(\sigma_D^g - \sqrt{\kappa})^2]z_t^i +$$
$$[k_1 C_{pd}^i \phi^w + \psi_w - \tau + \frac{1}{2}(\sqrt{\delta^i} + k_1 C_{pd}^i \sigma^w - \sigma_D^w)^2 + \frac{1}{2}(\sqrt{\omega} - \sigma_D^{c,i})^2]z_t^w.$$

---

[25]The coefficients k_0 and k_1 are given by $k_0 = \log(1 + exp(\bar{pd})) - \frac{\bar{pd} \, exp(\bar{pd})}{1 + exp(\bar{pd})}$ and $k_1 = \frac{exp(\bar{pd})}{1 + exp(\bar{pd})}$, where $\bar{pd}$ denotes the mean of the log price-dividend ratio.

The linear coefficients A_{pd}^i, B_{pd}^i and C_{pd}^i can be obtained by solving 3 equations in 3 unknowns:

$$
\begin{aligned}
A_{pd}^i &= -\alpha + k_0 + k_1 A_{pd}^i + k_1 B_{pd}^i(1-\phi)\theta + k_1 C_{pd}^i(1-\phi^w)\theta^w + \mu_D, \\
B_{pd}^i &= k_1 B_{pd}^i \phi + \psi - \chi + \frac{1}{2}(\sqrt{\gamma} + k_1 B_{pd}^i \sigma - \sigma_D)^2 + \frac{1}{2}(\sigma_D^g - \sqrt{\kappa})^2, \\
C_{pd}^i &= k_1 C_{pd}^i \phi^w + \psi_w - \tau + \frac{1}{2}(\sqrt{\delta^i} + k_1 C_{pd}^i \sigma^w - \sigma_D^w)^2 + \frac{1}{2}(\sqrt{\omega} - \sigma_D^{c,i})^2.
\end{aligned}
\tag{16}
$$

Solving yields:

$$
A_{pd}^i = \frac{-\alpha + k_0 + \mu_D + k_1[B_{pd}^i(1-\phi)\theta + C_{pd}^i(1-\phi^w)\theta^w]}{(1-k_1)},
\tag{17}
$$

and two solutions each for B_{pd}^i and C_{pd}^i:

$$
\begin{aligned}
B_{pd}^i &= \frac{1 - k_1(\phi + \sqrt{\gamma}\sigma - \sigma\sigma_D) \pm \sqrt{[1 - k_1(\phi + \sqrt{\gamma}\sigma - \sigma\sigma_D)]^2 + 2k_1^2\sigma^2(\chi - \psi - \frac{1}{2}(\gamma + \kappa) + \dots}}{\kappa_1^2\sigma^2} \\
&\quad \frac{\sqrt{\dots - \frac{1}{2}\sigma_D^2 + \sqrt{\gamma}\sigma_D - \frac{1}{2}\sigma_D^{g2} + \sigma_D^g\sqrt{\kappa})}}{\kappa_1^2\sigma^2}, \\
C_{pd}^i &= \frac{1 - k_1(\phi^w + \sqrt{\delta^i}\sigma^w - \sigma^w\sigma_D^w) - \sqrt{[1 - k_1(\phi^w + \sqrt{\delta^i}\sigma^w - \sigma^w\sigma_D^w)]^2 + 2k_1^2\sigma_w^2(\tau - \psi_w + \dots}}{k_1^2\sigma_w^2} \\
&\quad \frac{\sqrt{\dots - \frac{1}{2}\delta^i - \frac{1}{2}\sigma_D^{2,w} + \sqrt{\delta^i}\sigma_D^w - \frac{1}{2}(\sqrt{\omega} - \sigma_D^{c,i})^2)}}{k_1^2\sigma_w^2}
\end{aligned}
\tag{18}
$$

In both cases, we consider only the smallest root, as the other one is not consistent with the assumption that $E[pd^i] = \overline{pd^i}$.

Optimal Hedging: Full Solution for the U.S. investor   The set of optimal conditions for the U.S. (domestic) investor is:

[1] $\omega^{LWMKT}(\sigma_D^w - k_1\overline{C_{pd}^i}\sigma^w) + \omega^{Carry}\left(\overline{\sqrt{\delta^i}}^L - \overline{\sqrt{\delta^i}}^H\right) + \omega^{MKT}(\sigma_D^w - k_1 C_{pd}\sigma^w) = \frac{1}{\Gamma}\sqrt{\delta}$ $[u_{t+1}^w]$

[2] $\omega^{LWMKT}\overline{\sigma_D^{c,i}} + \omega^{MKT}\sigma_D^c = \frac{1}{\Gamma}\sqrt{\omega}$ $[u_{t+1}^c]$

[3] $\omega^{LWMKT}\sigma_D^g\overline{\sqrt{z_t^i}} + \omega^{Dollar}\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) + \omega^{MKT}\sigma_D^g\sqrt{z_t} = \frac{1}{\Gamma}\sqrt{\kappa z_t}$ $[u_{t+1}^g]$

[4] $\omega^{MKT}(\sigma_D - k_1 B_{pd}\sigma) + \omega^{Dollar}\sqrt{\gamma} = \frac{1}{\Gamma}\sqrt{\gamma}$ $[u_{t+1}]$

[5] $\omega^{LWMKT} + \omega^{Dollar} + \omega^{Carry} + \omega^{MKT} + \omega^{rf} = 1.$

That is:

[2] $\omega^{MKT} = \dfrac{\frac{1}{\Gamma}\sqrt{\omega} - \omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c},$

[4] $\omega^{Dollar} = \dfrac{1}{\Gamma} - \dfrac{1}{\sqrt{\gamma}}\left(\dfrac{\frac{1}{\Gamma}\sqrt{\omega} - \omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\right)(\sigma_D - k_1 B_{pd}\sigma)$

[3] $\omega^{LWMKT}\sigma_D^g\overline{\sqrt{z_t^i}} + \left(\dfrac{1}{\Gamma} - \dfrac{1}{\sqrt{\gamma}}\left(\dfrac{\frac{1}{\Gamma}\sqrt{\omega} - \omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\right)(\sigma_D - k_1 B_{pd}\sigma)\right)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right)$

$+\dfrac{\frac{1}{\Gamma}\sqrt{\omega} - \omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\sigma_D^g\sqrt{z_t} = \frac{1}{\Gamma}\sqrt{\kappa z_t},$

[1] $\omega^{Carry} = \dfrac{\frac{1}{\Gamma}\sqrt{\delta} - \omega^{LWMKT}(\sigma_D^w - k_1\overline{C_{pd}^i}\sigma^w) - \omega^{MKT}(\sigma_D^w - k_1 C_{pd}\sigma^w)}{\left(\overline{\sqrt{\delta^i}}^L - \overline{\sqrt{\delta^i}}^H\right)},$

[5] $\omega^{LWMKT} + \omega^{Dollar} + \omega^{Carry} + \omega^{MKT} + \omega^{rf} = 1.$

Solving equation [3] for ω^{LWMKT} yields:

$\omega^{LWMKT}\sigma_D^g\overline{\sqrt{z_t^i}} + \left(\dfrac{1}{\Gamma} - \dfrac{\sqrt{\omega}}{\Gamma\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\right)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) +$

$+\dfrac{\omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) + \dfrac{\sqrt{\omega}}{\Gamma\sigma_D^c}\sigma_D^g\sqrt{z_t} - \dfrac{\omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\sigma_D^g\sqrt{z_t} = \frac{1}{\Gamma}\sqrt{\kappa z_t}.$

That is,

$$\omega^{LWMKT}\sigma_D^g\overline{\sqrt{z_t^i}} + \frac{\omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) - \frac{\omega^{LWMKT}\overline{\sigma_D^{c,i}}}{\sigma_D^c}\sigma_D^g\sqrt{z_t}$$

$$= \frac{1}{\Gamma}\left(\sqrt{\kappa z_t} - \left(1 - \frac{\sqrt{\omega}}{\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\right)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) - \frac{\sqrt{\omega}}{\sigma_D^c}\sigma_D^g\sqrt{z_t}\right).$$

That is,

$$\omega^{LWMKT}\left(\sqrt{\gamma}\sigma_D^g\left(\sigma_D^c\overline{\sqrt{z_t^i}} - \overline{\sigma_D^{c,i}}\sqrt{z_t}\right) + \overline{\sigma_D^{c,i}}(\sigma_D - k_1 B_{pd}\sigma)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right)\right)$$

$$= \frac{\sqrt{\gamma}\sigma_D^c}{\Gamma}\left(\sqrt{\kappa z_t} - \left(1 - \frac{\sqrt{\omega}}{\sqrt{\gamma}\sigma_D^c}(\sigma_D - k_1 B_{pd}\sigma)\right)\sqrt{\kappa}\left(\sqrt{z_t} - \overline{\sqrt{z_t^i}}\right) - \frac{\sqrt{\omega}}{\sigma_D^c}\sigma_D^g\sqrt{z_t}\right).$$

This system of linear equations leads to the optimal wealth allocation for the U.S. investor described in the main text.