Securitization Networks and Endogenous Financial Norms in U.S. Mortgage Markets[*†]

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PRELIMINARY DRAFT

Abstract

We develop a theoretical model of a network of intermediaries, which we apply to the U.S. mortgage market. In our model, heterogeneous financial norms and systemic vulnerabilities arise endogenously. Intuitively, the optimal behavior of each intermediary, in terms of its attitude toward risk, the quality of the projects that it undertakes, and the intermediaries it chooses to interact with, is influenced by the behavior of its prospective counterparties. These network effects, together with intrinsic quality differences between intermediaries, jointly determine financial health and systemic vulnerability at the aggregate level as well as for individual intermediaries. We apply our model to the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than one million private-label mortgages originated and securitized in 2006. We then track the ex-post foreclosure performance of each loan in the network and compare the evolution of credit risk by vintage with the model's predictions. We find that credit risk evolves in a concentrated manner among highly linked nodes, defined by the geography of the network and the interactions between originator and counterparty over time. This confirms that network effects are of vital importance for understanding the U.S. mortgage market.

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1 Introduction

Several recent studies highlight the importance of network linkages between intermediaries and financial institutions in explaining systemic risk in financial markets (see, for example, Allen and Gale, 2000; Allen, Babus, and Carletti, 2012; Cabrales, Gottardi, and Vega-Redondo, 2014; Glasserman and Young, 2013; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015; Elliott, Golub, and Jackson, 2014). These studies show that financial networks may create resilience against shocks in a market via diversification and insurance, but also contagion and systemic vulnerabilities by allowing shocks to propagate and amplify. The network structure is thus a pivotal determinant of the riskiness of a financial market.

These theoretical studies of networks and risk in financial markets typically focus on how the network redistributes risk between participants, and on the consequences for the system's solvency and liquidity after a shock. This is an ex post effect of the financial network. One may also expect ex-ante effects to be important. Specifically, the presence and structure of a financial network should affect—and be affected by—the actions of individual intermediaries and financial institutions, even before shocks are realized. Understanding the equilibrium interaction between network structure, the actions taken by market participants, and the market's riskiness is the main theoretical focus of our study.

We build upon the approach in Stanton, Walden, and Wallace (2014), who study the mortgage market from a network perspective and find that, despite the large total number of firms, the market is highly concentrated, with significant inter-firm linkages among the loan originators, aggregators,[1] special purpose entities (SPEs), securitization shelves,[2] and shelf holding companies. For example, the private-label mortgage originations in 2006 were sourced from 11,103 mortgage originators of record, their loans were assembled by 2,030 aggregators, and these in turn sold the newly originated loans to SPEs that belonged to 146 separate securitization shelves. These shelves were controlled by only 56 holding companies. Of the 1.4 million first-lien private-label mortgages originated in 2006, sales of the loans among affiliated entities (i.e., when the lender of record, the aggregator, and the holding company were subsidiaries of the same firm) accounted for 47.41% of transactions, whereas

[1] Aggregators assemble the loans for sale to special purpose entities (SPEs) (see Inside Mortgage Finance, 2015).

[2] When private-label issuers file a registration statement to register an issuance of a REMIC security, they typically use a "shelf registration." The sponsor first files a disclosure document, known as the "core" or "base" prospectus, that outlines the parameters of the various types of REMIC securities offerings that will be conducted in the future through the sponsor's shelf registration. The rules governing shelf issuance are part of the Secondary Mortgage Market Enhancement Act (SMMEA) (see Simplification of Registration Procedures for Primary Securities Offerings, Release No. 33-6964, Oct. 22, 1992, and SEC Staff Report: Enhancing Disclosure in the Mortgage-Backed Securities Markets, January, 2003, http://www.sec.gov/news/studies/mortgagebacked.htm#secii).

52.59% of the loan sales were between unaffiliated firms.

In 2006, the preponderance of the loan sales into SPEs occurred within sixty days of the origination date of the loan due to the contractual structure of the wholesale lending mechanisms used to fund mortgage origination.[3] These contractual funding structures assign the cash flows of the originated mortgages forward to each purchaser. However, the lender of record, or in some cases the aggregator, retains a contractual put-back option on the loan, which makes the risk structure of the loan flows bidirectional. Stanton et al. (2014) find that the mortgage market can be represented as a network, where links are loan-cash-flow sales, and the performance of an individual node is closely related to the performance of the node's neighbors in the network.

We introduce a model with multiple agents representing financial intermediaries, who are connected in a network. Network structure in our model, in addition to determining the ex post riskiness of the financial system, also affects—and is affected by—what we call the *financial norms* in the network, inspired by the literature on influence and endogenous evolution of opinions and social norms in networks (see, for example, Friedkin and Johnsen, 1999; Jackson and López-Pintado, 2013; López-Pintado, 2012). The financial norms are defined as the quality and riskiness of the actions agents take, which in turn are influenced by the actions of other agents in the network.

Our model is parsimonious, in that the strategic action space of agents, as well as the contract space, is limited. Links in the network represent risk-sharing agreements, as in Allen et al. (2012). Agents may add and sever links, in line with the concept of pairwise stability in games on networks (see Jackson and Wolinsky, 1996), and also have the binary decision of whether to invest in a costly screening technology that improves the quality of the projects they undertake.

The equilibrium concept used is subgame-perfect Nash. In an equilibrium network, each agent optimally chooses to accept the network structure, as well as whether to invest in the screening technology, having correct beliefs about all other agents' actions and risk. Shocks are then realized and distributed among market participants according to a clearing mechanism similar to that introduced in Eisenberg and Noe (2001). As in Elliott et al. (2014), we assume that there are costs associated with the insolvency of an intermediary, potentially creating contagion and propagation of shocks through the clearing mechanism, and thereby making the market systemically vulnerable. The model is simple enough to

[3]The two most important of these funding mechanisms were, and continue to be, 1) the master repurchase agreement, a form of repo, which received safe-harbor protections under the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) (Pub.L. 109-8, 119 Stat. 23) enacted April 20, 2005; and 2) extendable asset-backed commercial paper programs. In 2006, both of these had forty-five day "repurchase" maturities.

allow us to analyze the equilibrium properties of large-scale networks computationally, using numerical approximation methods.

Our model has three general implications. First, network structure is related to financial norms. Given that an agent's actions influence and are influenced by the actions of those that the agent interacts with, this result is natural and intuitive. Importantly, an agent's actions affect not only his direct counterparties but also those who are indirectly connected through a sequence of links. As a consequence, there is a rich relationship between equilibrium financial norms and network structure, in turn suggesting a further relationship between the network and the financial health of the market, beyond the mechanical relationship generated by shock propagation.

Second, heterogeneous financial norms may coexist in the network in equilibrium. Thus, two intermediaries that are ex ante identical may be very different when their network position is taken into account, not just in how they are influenced by the rest of the network but also in their actions. Empirically, this suggests that network structure is an important determinant not only of the aggregate properties of the economy but also of the actions and performance of individual intermediaries.

Third, proximity in the network is related to financial norms: nodes that are close tend to develop similar norms, just like in the literature on social norms in networks. This result suggests the possibility of decomposing the market's financial network into "good" and "bad" parts, and addressing vulnerabilities generated by the latter. We view this as a promising topic for future research.

We analyze the mortgage-origination and securitization network of financial intermediaries, using a large data set of private-label mortgages originated and securitized in 2006. Our approach is to use loan flows to identify the network structure of this market, letting ex-post foreclosure rates of these loans measure performance, and using the model to estimate the evolution of credit risk. Our empirical results suggest that both ex ante and ex post network effects are of vital importance for understanding the U.S. mortgage market and its systemic vulnerabilities.

The rest of the paper is organized as follows. Section 2 describes the structure of the U.S. residential mortgage market and available data. Section 3 introduces the model. Section 4 analyzes the properties of equilibrium, and Section 5 discusses how to estimate equilibrium from observed data in a large-scale network. Section 6 applies our approach to the 2006 U.S. private-label mortgage market, and Section 7 concludes. The appendix contains a detailed description of the mortgage data, as well as of the network game in the model.

2 Structure of the U.S. residential mortgage market

Following Bresnahan and Levin (2012) and Jacobides (2005), we focus on the economic and organizational forces underlying the system of disintegrated exchange that characterizes the U.S. residential mortgage market. This system is coordinated explicitly through contracts, such as funding commitments and put-back options for loans found to be poorly performing ex post; and implicitly through standardized institutional structures and norms that are difficult to monitor contemporaneously, such as the underwriting-quality choices of loan originators.

There is a small literature that considers the economic factors leading to disintegrated exchange systems.[4] In his famous essay on the nature of the firm, Coase (1937) describes why and how economic activity divides between firms and markets. He argues that firms exist to reduce the costs of transacting through markets. A key element of Williamson's transaction cost theory of integration (see Williamson, 1971, 1975, 1979) is that vertical integration is an efficient outcome because contracts are inherently incomplete, and this limitation may be especially severe when complexity or uncertainty make it difficult to specify contractual safeguards, or when parties cannot walk away without incurring substantial costs. The property-rights theories of vertical integration (see Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995) have focused on how integration changes the incentives to make specific investments and find that ownership strengthens a party's bargaining position. Incentive theories (see Holmström and Milgrom, 1994; Holmström, 1999) have shown that under certain conditions, asset ownership by the agent (e.g., non-integration) can be complementary to providing high-powered financial incentives.

Jacobides (2005) builds on these ideas based on an extensive case study of vertical disintegration in the U.S. mortgage banking industry from 1970 through 1998. He argues that transaction-cost theories, decision and incentive theories provide an inadequate explanation for how the mortgage-banking industry evolved. Both Jacobides (2005) and Bresnahan and Levin (2012) argue that the disintegration of the (previously highly vertically integrated) mortgage origination and funding systems that existed prior to 1970 was caused by three institutional changes. First, the federal government introduced standardized securitization systems through the GSEs and the Government National Mortgage Association (GNMA) and allowed non-depository mortgage banks to issue and service loans under GSE criteria. To fund loans temporarily before sales to the GSE securitizers, mortgage banks would use

[4]This literature includes theoretical treatments (see Jacobides, 2005; Chen, 2005; Bresnahan and Levin, 2012); empirical studies (see Langlois and Robertson, 1992; Holmes, 1999; Chen, 2005); and studies of the sociological and institutional norms, or pre-conditions, needed to engender disintegrated market structures (see Cooter, 2000; MacKenzie and Millo, 2003; Fligstein, 2001; Jacobides, 2005).

lines of credit obtained from commercial banks (see Fabozzi and Modigliani, 1992). The second major change occurred as a result of the recession of 1979–81, when banks and S&Ls laid off their loan-origination staff and then re-established long-term relationships with independent loan brokers (often with the same staff). The loan-brokerage vertical disintegration was funded with lines of credit from mortgage banks, commercial banks or S&Ls, or by an alternative model whereby the brokers merely served as agents that matched borrowers with loan products without making the underwriting or funding decisions. The third major change was the vertical disintegration of loan servicing from loan origination through the creation of a market for mortgage servicing rights.

Each of these stages of vertical disintegration in the mortgage market led to the creation of highly specialized entities: mortgage brokers with highly specialized local market knowledge; mortgage bankers with specialized knowledge concerning capital-market funding and managing pre-securitization pipeline risk; depository institutions who did some origination but increasing specialized in funding downstream originators through short-term lines of credit and repurchase agreements, both of which require the management of liquidity and roll-over risks; mortgage servicers specialized in the management of interest rate and prepayment risk; and mortgage securitizers with specialized capital-market knowledge associated with accessing the mortgage investors needed to purchase the mortgage-backed securities. In addition, the mortgage market became characterized by potential gains from trade, resulting from the imbalances associated with the specialization of existing participants along the value chain.

2.1 Network structure of the mortgage market

The pre-crisis residential-mortgage origination market comprised thousands of firms and subsidiaries, including commercial banks, savings banks, investment banks, savings and loan institutions (S&Ls), mortgage banks and companies, real estate investment trusts (REITs), mortgage brokers and credit unions.[5] An important distinction among the firms in the industry is between retail and wholesale originators (see Inside Mortgage Finance, 2015). For retail originators, the underwriting and funding processes are carried out by the labor and capital of either a single originator or the consolidated subsidiary of a single originator. In contrast, the origination and underwriting processes of wholesale originators are handled in whole or in part by the labor and capital of another party. Wholesale originators are also distinguished by the degree of autonomy that the originating party exercises over the underwriting and funding processes. Wholesale-broker lending usually involves a more lim-

[5] Other than the closure of the Office of Thrift Supervision, the types of firm in the U.S. mortgage market are largely the same today.

ited level of autonomy, because brokers generally neither make the final credit decision nor fund the loan. Correspondent wholesale originators — who can be subsidiaries of mortgage companies, REITs, or depositories — originate and deliver loans determined by defined underwriting standards (usually an advance commitment on the loan structure and price), and they exercise full control over the underwriting and funding processes of loan origination (legally they are the creditor of record). In wholesale and correspondent lending, usually the originators of record fund their mortgage origination using credit facilities (lines of credit) provided by the subsidiaries of large commercial banks, large mortgage companies, S&Ls, insurance companies, and investment banks.

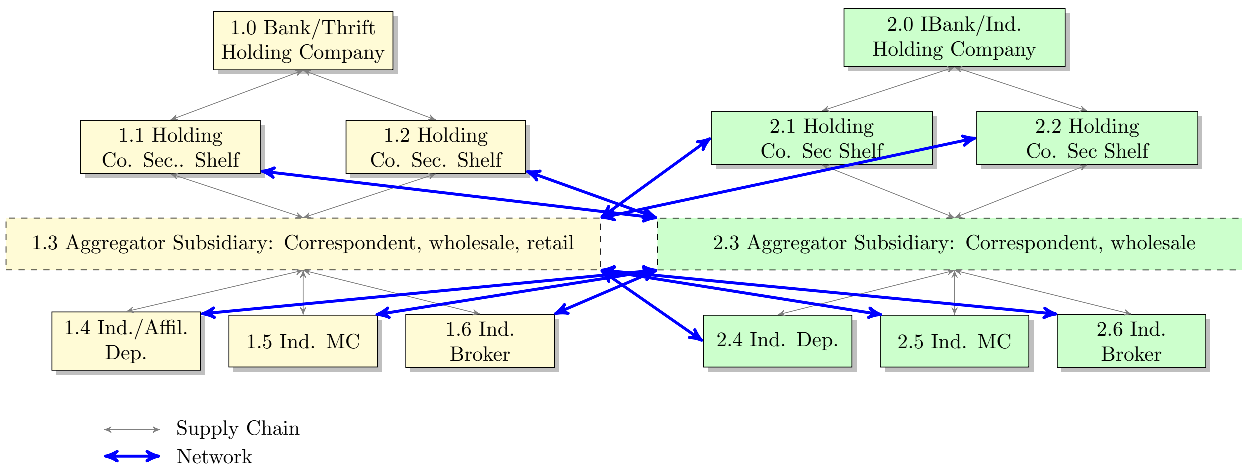

Figure 1: The residential mortgage market structure: Loan sales within affiliated firms (gray arrows) versus loan sales between unaffiliated firms (blue arrows)

Figure 1 presents the private-label mortgage-market structure for loan sales from the mortgage originator of record to the aggregator of the loans (either the correspondent or the warehouse lender), then to the securitization shelf and to the holding company that owns the securitization shelf. Figure 1 portrays two possible holding-company types. The left-hand side of the schematic, shown in yellow, represents bank or thrift holding company operations whereas the right-hand side of the schematic, shown in green, represents investment bank or large independent mortgage company operations in the pre-crisis period. The gray double-headed arrows represent loan sales between entities that are subsidiaries (they may or may not be fully consolidated) of the same holding company (the arrows are double headed because of the put-back liability associated with each loan) and the blue double-headed arrows represent loan sales and the symmetric put-back liability between two unaffiliated firms and holding companies.[6] The graph of the gray double arrows represents a tree (i.e., the graph has no cycles), whereas when the links represented by the double blue arrows are added, the market is a network (a general graph).

Table 1 presents a network of loan sales for a sample of seven loans originated and securitized in 2006. Each of the seven loans was aggregated by the same subsidiary of Bank of America. As shown in the table, Bank of America (BofA) aggregated loans from independent mortgage companies (Accubanc, Ameriquest, GMAC, Taylor, Bean & Whittaker Mortgage (TB&W Mtg)), from a S&L (World Savings), from a Bank of America branch, and from the subsidiary of another bank depository (Wells Fargo Bank). Bank of America then sold the mortgages to REMICs in five different shelf-registration facilities. Three of these shelves were owned by Lehman Brothers and two by Bear Stearns.

Loan Number	County	State	Lender of Record	Aggregator	Shelf Name	Holding Company
1	Virginia Beach	VA	Accubanc	B of A	333-133985-08	Lehman Brothers
2	Queens	NY	Ameriquest	B of A	333-129480-11	Lehman Brothers
3	Baltimore	MD	B of A	B of A	333-131374-59	Bear Stearns
4	Hillsborough	FL	GMAC	B of A	333-129480-11	Lehman Brothers
5	Union	GA	TB&W Mtg	B of A	333-133985-74	Lehman Brothers
6	Ventura	CA	World Savings	B of A	333-129480-11	Lehman Brothers
7	Salt Lake	UT	Wells Fargo	B of A	333-132232-17	Bear Stearns

Table 1: Example of a network loan sales for a sample of seven loans that were originated and securitized in 2006. Bank of America (B of A) was the aggregator for all seven loans. The geographic location of the loans varied as did the lender of record, the shelf registration of the loan, and the holding company that owned the shelf.

Figure 2 presents the tree structure for a subsample of the intermediaries in the market, including 21 lenders, 8 aggregators, and 4 holding companies, which are connected via loans originated and securitized in 2006. We restrict the graph to allow only for a tree structure originating from each of the four headquarters, along the lines of a supply-chain interpretation of the market. Specifically, we only include the link representing the largest number of loans from each node, leading to a *weighted minimal spanning tree* for each of the holding companies.

In Figure 3, by contrast, all the links between these nodes (and the loans these links represent) are included, leading to a network representation of the market. This network

[6]For visual clarity, we do not show the sales between entities within the investment bank or independent mortgage holding company (the green entity on the right-hand side of Figure 1) to the bank and thrift entities, but these sales would also exist. There would also be sales between bank/thrift entities and investment bank/mortgage company entities.

representation has 61 links, compared with the 28 links in the tree representation. Thus, more than half of the links are unaccounted for in the tree representation. For the full market of intermediaries, in 2006 consisting of 11,103 lenders, 2,030 aggregators, and 56 holding companies, the tree representation only accounts for about 20% of the links. The fraction of loans accounted for is higher—about 50%—since the links with the highest number of loans are chosen in the weighted minimal spanning tree, but still a significant fraction of the market is unaccounted for when using the tree representation.

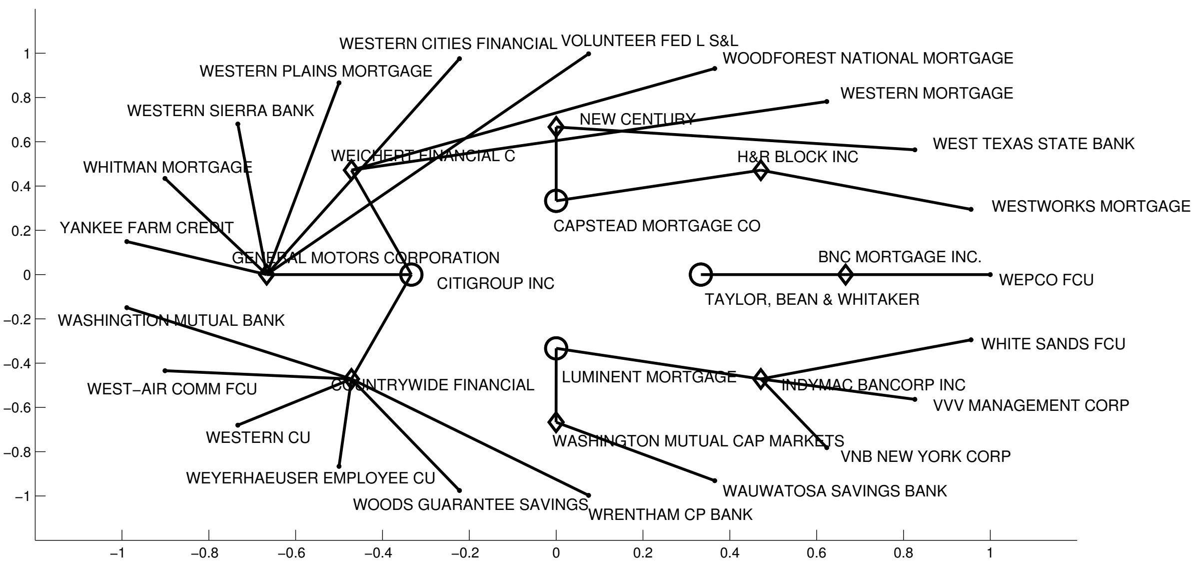

Figure 2: Within-firm trees for a sub-sample of private-label mortgage originations in 2006

Table 2 presents the results of a logistic regression of the probability of loan default, defined as loans that became delinquent at any time between their origination date and December, 2013, on the physical geography of the loan collateral and the quality of the performance of the neighboring loan originators, the performance of the neighboring loan aggregators, and the neighboring holding companies, controlling for loan characteristics and firm fixed effects for the aggregators and holding companies. As discussed in Appendix A, we develop our mortgage network using mortgage origination and performance data from a merger of data from ABSNet, Dataquick, and REMIC prospectuses. Our data set includes 1,152,312 private-label mortgages originated in 2006, all with complete origination-characteristic data. The table shows that although the default rate of a loan depends significantly on the average mortgage default rate within the same zip code (excluding the sample loan), it also depends significantly (both economically and statistically) on the average default rate for neighboring lenders, aggregators and holding companies within the loan's network. To better evaluate

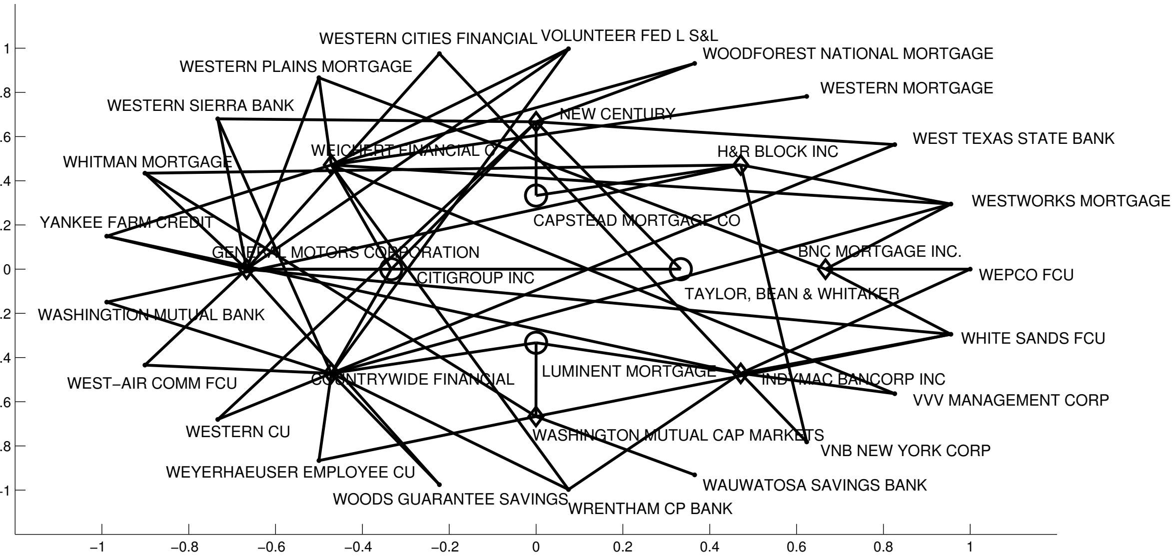

Figure 3: Between-firm networks for a subsample of private-label mortgage originations in 2006

the marginal effect of our network measures, our results indicate that a one standard deviation increase in the average default performance of neighboring lenders would increase the loan default probability by 96% of its pre-increase value, a one standard deviation increase in the average default performance of neighboring aggregators would increase the loan default probability by 1% of its pre-increase value, and a one standard deviation increase in the average default performance of neighboring holding companies would increase the loan default probability by 5% of its pre-increase value. By comparison, a one standard deviation increase in the average default level of zip code of the loan would increase the loan default probability by 11% of its pre-increase value.

To summarize, our analysis of the private-label mortgage market shows that it was well represented by a network, in which risk was shared bilaterally between connected nodes, and in which network position was at least as important in explaining loan performance as geography. We take this as a starting point for the network model that we introduce in the next section.

3 A Network Model of Intermediaries

We introduce the equilibrium network model with the fundamental properties that agents 1) act strategically when entering into contractual agreements among themselves, 2) are

	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
Intercept	-1.0957***	0.0518	-1.4097 ***	0.051
Geographic effects (excluding subject loan)				
Loan Zip Code Average Default	0.5323 ***	0.0024	0.5352 ***	0.0024
Network effects (excluding subject loan)				
Neighboring Lenders Average Default Rate	4.8118 ***	0.1307		
Neighboring Aggregators Average Default Rate	0.0361 **	0.0132		
Neighboring Holding Company Average Default Rate	0.2665 ***	0.0094		
Loan Origination Cost	0.0238 ***	0.0018	0.0209***	0.0018
Original Loan Balance	0.0547 ***	0.0026	0.0533 ***	0.0027
Original Cumulative Loan to Value Ratio	0.4486 ***	0.0025	0.4482 ***	0.0025
Original Mortgage Contract Rate	0.1147 ***	0.0027	0.1130 ***	0.0027
Original Mortgage Amortization Term	0.1574 ***	0.0022	0.1566 ***	0.0022
Conventional Conforming Loan	-0.1127 ***	0.0073	-0.1135 ***	0.0063
Firm fixed effects	Yes		Yes	
N =	1152312		1152312	
Pseudo R^2	0.1948		0.1933	
*** (prob > $\chi^2 \leq 0.0001$)				
** (prob > $\chi^2 \leq 0.01$)				

Table 2: Logistic regression of loan-level default on the average default rate within the loan's zip code (excluding the subject loan) and the average default rate within the loan's network neighborhood (excluding the subject loan).

influenced by the actions of others to whom they are only indirectly connected, 3) make unobservable quality choices that impact outcomes, locally in the network, as well as potentially in aggregate.

We introduce the model in several steps. We first describe the risk environment and possible actions of agents that affect project payoffs, and analyze the outcome when agents act in isolation. We then study in detail the case when two agents interact, before analyzing equilibrium in the general N-agent model. A detailed description of the strategic game between agents is provided in the appendix.

3.1 Intermediaries and projects

There are N intermediaries with limited liability, each owned by a different risk-neutral agent. Each intermediary initially has full ownership of a project that generates risky cash flows at $t = 1$, $\widetilde{CF}^n_P$, and may moreover incur some costs at $t = 0$. The one-period discount rate is normalized to 0. Agent n's objective is to maximize the expectation at $t = 0$ of the value of the intermediary's cash flows at $t = 1$, $\widetilde{CF}_1^n$, net any costs incurred at time 0.[7]

$$V^n = E_0[\widetilde{CF}_1^n] - C_0^n.$$

The risky project has scale $s^n > 0$, with two possible returns, represented by the Bernoulli-distributed random variables ξ^n, so that $R^n = R_H$ if $\xi^n = 1$, and $R^n = R_L$ if $\xi^n = 0$. The probability, p, that $\xi^n = 0$ is exogenous, with $0 < p \ll 1$. We assume complete symmetry of risks in that the probability is the same for each project of the N projects.[8] Although obviously a simplification, this assumption qualitatively captures the bidirectional risk structure between intermediaries, discussed in the previous section.

Each agent has the option to invest a fixed amount, $C_0^n = cs^n$ at $t = 0$, where $c > 0$, to increase the quality of the project. This cost is raised externally at $t = 0$. If the agent invests, then in case of the low realization, $\xi^n = 0$, the return on the project is increased by $\Delta R > c$, to $R_L + \Delta R$. This investment cost could, for example, represent an investment in a screening procedure that allows the agent to filter out the parts of the project that are most vulnerable to shocks. We represent this investment choice by the variable $q^n \in \{0, 1\}$, where $q^n = 1$ denotes that the intermediary invests in quality improvement. Intermediaries that choose $q = 1$ are said to be of high quality, whereas those that choose $q = 0$ are said to be of low quality. For the time being, we assume that c is the same for all intermediaries. We will subsequently allow c to vary across intermediaries, representing exogenous quality variation (as opposed to the quality differences that arise endogenously because of intermediaries' investment decisions).[9]

There is a threshold, $d > 0$, such that if the return on the investment for an intermediary falls below d, additional costs are immediately imposed, and no cash flows can be recovered by the agent.[10] For simplicity, we call these costs "insolvency costs," in line with Nier, Yang, Yorulmazer, and Alentorn (2008), who note that systemic events typically originate from insolvency shocks, although they are often also associated with liquidity constraints. We view these additional costs of insolvency as wasteful, in that they impose a real cost on

[7]The intermediary's cash flow $\widetilde{CF}_1^n$ may differ from $\widetilde{CF}_P^n$ because intermediaries may enter into risk-sharing agreements with each other, as will be explained subsequently.

[8]Note that each project may be viewed as a representative project for a portfolio of a large number of small projects with idiosyncratic risks that cancel out, and an aggregate risk component measured by ξ^n.

[9]Several variations of the model are possible, e.g., assuming that screening costs, c, are part of the $t = 1$ cash flows, and fixing the scale of all intermediaries to $s \equiv 1$, all leading to qualitatively similar results. The version presented here was chosen for its tractability in combination with empirical relevance.

[10]This assumption, similar to assumptions made, for example, in Elliott et al. (2014) and Acemoglu et al. (2015), is a stylized way of modeling the additional costs related to risk for insolvency, e.g., direct costs of bankruptcy, costs of fire sales, loss of human capital, customer and supplier relationship capital, etc. It could also more generally represent other types of convex costs of capital faced by a firm with low capitalization, along the lines described in Froot, Scharsfstein, and Stein (1993).

society rather than representing a transfer between agents.

It will be convenient to define the functions

$$X(z) = \begin{cases} 1, & z > d, \\ 0, & z \leq d, \end{cases} \qquad \text{and} \qquad Y(z) = X(z)z.$$

We also make the following parameter restrictions:

$$0 \leq R_L < d < 1 < (1-p)R_H, \qquad \text{and} \tag{1}$$

$$R_L + \Delta R < R_H. \tag{2}$$

The first restriction, among other things, implies that there will be insolvency costs for a low-quality intermediary after a low realization. The second restriction states that the outcome in the high realization is always higher than in the low realization, even for a high-quality intermediary.

3.2 Isolated intermediaries

We first focus on the setting in which intermediaries do not interact, and study the choice of an intermediary whether to be of high or low quality. The cash flows generated by project $n \in \{1, 2\}$ are then

$$\widetilde{CF}^n_P(\xi, q) \stackrel{\text{def}}{=} \begin{cases} s^n R_H, & \xi = 1, \\ s^n(R_L + q\Delta R), & \xi = 0, \end{cases} \tag{3}$$

the total cash flows generated to the owner after insolvency costs are accounted for are

$$\widetilde{CF}^n_1(\xi, q) = s^n Y\left(\frac{\widetilde{CF}_P(\xi, q)}{s^n}\right) = \begin{cases} s^n Y(R_H), & \xi = 1, \\ s^n Y(R_L + q\Delta R), & \xi = 0, \end{cases} \tag{4}$$

and the $t = 0$ value of the intermediary is

$$V^n = V^n(q) = E_0[\widetilde{CF}^n_1] - C^n_0 = s^n\left((1-p)Y(R_H) + pY(R_L + q\Delta R) - qc\right). \tag{5}$$

Given the parameter restrictions (1), we have $V^n(0) = s^n(1-p)R_H$. We make the technical assumption that if an intermediary is indifferent between being high and low quality, it chooses to become low quality. Therefore, $q = 1$, if and only if $V^n(1) > s^n(1-p)R_H$, immediately leading to the following result:

Proposition 1. *An intermediary chooses to be of high quality, $q = 1$, if and only if*

$$R_L + \Delta R > \max\left(d, \frac{c}{p}\right). \tag{6}$$

Proposition 1 is very intuitive, implying that increases in the probability of a high outcome and costs of being insolvent, as well as decreases in the costs of information acquisition, make it less attractive for an intermediary to be of high quality. The first argument in the maximum function on the RHS ensures that a high-quality firm avoids insolvency in the low state. If the condition is not satisfied, there is no benefit to being high quality even in the low state. The second argument ensures that the expected increase of cash flows in the low state outweighs the cost of investing in quality. The value of the intermediary when acting in isolation and following the rule (6) is then $V_I^n = s^n V_I$, where

$$V_I = \begin{cases} pR_H, & q = 0, \\ pR_H + (1-p)(R_L + \Delta R) - c, & q = 1. \end{cases}$$

Note that the objective functions of the agents coincide with that of society in this special case. Specifically, given that society has the social welfare function, $\mathcal{V} = \sum_n V^n$, under the constraint that intermediaries act in isolation, the socially optimal outcome is realized by the intermediaries' joint actions. We obviously do not expect this to be the case in general, when agents interact.

3.3 Two intermediaries

We explore the case with two interacting intermediaries, allows us to gain intuition in a fairly simple setting, before introducing the general N-agent network model. Intermediaries may enter into contracts that transfer risk. These contracts are settled according to a market-clearing system along the lines of that in Eisenberg and Noe (2001). Because of the high dimensionality of the problem when we allow agents to act strategically, we necessarily have to assume a very limited contract space between intermediaries. Specifically, we assume that the contracts available are such that intermediaries swap claims on the aggregate cash flows generated by their projects in a one-to-one fashion, similar to what is assumed in Allen et al. (2012), and consistent with our discussion about bidirectional risk sharing.

We focus on the case when the two intermediaries have the same scale ($s^1 = s^2 = 1$). The contract is then such that intermediary 1 agrees to deliver $\pi \times \widetilde{CF}_P^1$ to intermediary 2 at $t = 1$, and in turn receive $\pi \times \widetilde{CF}_P^2$ from intermediary 2, for some $0 \leq \pi \leq 1$. Our focus is on the two cases when project risks are shared equally ($\pi = 0.5$) and when intermediaries act in isolation ($\pi = 0$). We use the general π notation, since in the general case with N agents that we will analyze subsequently, π will typically take on other values.

The probabilities for the possible realizations of (ξ^1, ξ^2) are[11]

$$\mathbb{P}(\xi^1 = 0, \xi^2 = 0) = p_2, \qquad \mathbb{P}(\xi^1 = 1, \xi^2 = 0) = p_1,$$
$$\mathbb{P}(\xi^1 = 0, \xi^2 = 1) = p_1, \qquad \mathbb{P}(\xi^1 = 1, \xi^2 = 1) = 1 - 2p_1 - p_2.$$

Consider a situation in which the intermediaries choose qualities q^1 and q^2, respectively, and let $f^n(\xi^1, \xi^2)$ denote the binary variable that takes on value 0 if intermediary $n \in \{1, 2\}$ is insolvent in state (ξ^1, ξ^2), and 1 otherwise. Define

$$z^1(\xi^1, \xi^2 | q^1, q^2, \pi) = (1 - \pi) f^1(\xi^1, \xi^2) \widetilde{CF}^1_P(\xi^1, q^1) + \pi f^2(\xi^1, \xi^2) \widetilde{CF}^2_P(\xi^2, q^2), \tag{7}$$
$$z^2(\xi^1, \xi^2 | q^1, q^2, \pi) = \pi f^1(\xi^1, \xi^2) \widetilde{CF}^1_P(\xi^1, q^1) + (1 - \pi) f^2(\xi^1, \xi^2) \widetilde{CF}^2_P(\xi^2, q^2). \tag{8}$$

Because of insolvency costs, it follows that

$$\widetilde{CF}^n_1(\xi^1, \xi^2 | q^1, q^2, \pi) = f^n(\xi^1, \xi^2) z^n(\xi^1, \xi^2 | q^1, q^2, \pi), \tag{9}$$

and

$$f^n(\xi^1, \xi^2) = X\left(\frac{z^n(\xi^1, \xi^2 | q^1, q^2)}{s^n}\right). \tag{10}$$

A realization of cash flows and insolvency that satisfies Equations (7)–(10) in each state is said to be an outcome of the clearing mechanism. The time-0 value of an intermediary is then

$$V^n(q^1, q^2 | \pi) = \sum_{x_1, x_2 \in \{0,1\}} \widetilde{CF}^n_1(x_1, x_2 | q^1, q^2, \pi) \mathbb{P}(\xi^1 = x_1, \xi^2 = x_2) - qcs^n.$$

Equations (7)–(10) provide the adaptation of the clearing system in Eisenberg and Noe (2001) to our setting. This specification is almost identical to theirs (state by state), except for the important difference that insolvency is costly in our setting, represented by $d > 0$.

As a consequence of insolvency costs, there may be multiple solutions to the clearing mechanism (7, 9, 10) that lead to different net cash flows to intermediaries. This is because the insolvency of one intermediary can trigger the insolvency of another in a self-generating circular fashion. Along similar lines to Elliott et al. (2014), who also introduce solvency costs in their clearing mechanism, we focus on the unique outcome that minimizes the number of insolvencies. As noted in their study, since insolvencies are complements, all intermediaries,

[11] Note here that the subscripts of p refer to the number of projects that yield low realizations, *not* to which investment, n, is considered (which is not needed since we assume symmetric probabilities).

as well as society, agree that their number should be minimized. Briefly, their method initially assumes no insolvencies and then calculates which nodes become insolvent iteratively, taking into account that the insolvency of one node may trigger that of another.

We use an algorithm similar to Elliott et al. (2014) to define the outcome of the clearing mechanism. The algorithm also leads to a natural shock-propagation mechanism, through which insolvencies spread step-by-step. With two intermediaries, the propagation mechanism is of course simple: either the insolvency of one intermediary triggers that of the other, or it does not. In the general case an iterative algorithm is needed, which we define in Appendix B. We stress that although our clearing mechanism is similar to that in earlier work, what distinguishes or work is our focus on the endogenous development and coexistence of heterogeneous financial norms (represented by the q's), and how these norms are influenced by and influence the equilibrium network structure.

Note that the contracts offer a simple form of risk-sharing, potentially making it beneficial for agents to interact. Agents are risk neutral, but the cost of insolvency introduces a motive for avoiding low outcomes that trigger solvency costs, effectively generating risk aversion. By sharing risks, the negative effects of a low realization for the two agents can be limited.

Note also that the incentive for an agent to invest in high quality is affected by the interaction with other agents. One may conjecture several potential effects, depending on the economic environment. An agent's incentive to invest in quality may decrease compared with the case of no interaction, since the benefits of quality are shared whereas the costs are not. The agent's incentive to invest in quality may also increase, because risk sharing allows insolvency to be avoided for high-quality intermediaries that interact, although it cannot be avoided when they act in isolation. Thus our stylized contracting environment potentially allows for rich equilibrium behavior.

3.3.1 Equilibrium

Intermediaries can either act in isolation ($\pi = 0$) or share risks ($\pi = 1/2$). For a risk-sharing outcome to be an equilibrium, both agents must have correct beliefs about the quality decisions made by their counterparties. We make the standard assumptions that each agent may unilaterally decide to sever a link to the other agent, and that bilaterally the two agents can decide to add a link between themselves. For an outcome with risk sharing to be an equilibrium, it follows that neither agent can be made better off by acting in isolation. For an isolated outcome to be an equilibrium, it cannot be that both agents are better off by sharing risk.

We model the mechanism by a strategic game with the sequence of events described in Figure 4, where we have formulated the game for the general N-agent case. At $t = -2$, given

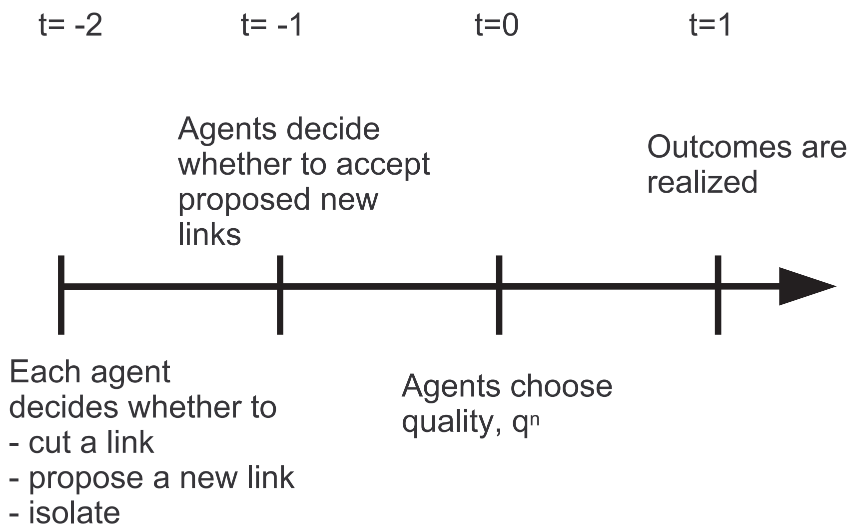

Figure 4: Sequence of events in network formation game with endogenous financial norms.

that $\pi = 1/2$, each agent may unilaterally decide to sever the link to the other agent and switch to $\pi = 0$, leading to the isolated outcome. If, on the other hand, $\pi = 0$, each agent can propose to switch to $\pi = 1/2$, in which case the other agent has the option to accept or decline at $t = -1$. Then, after the resulting network is determined, agents choose quality and outcomes are realized. Note that we implicitly assume that the actual quality decision is not contractible.[12]

An equilibrium is now described by (q^1, q^2) and π, such that neither agent has an incentive to sever the link (in the case $\pi = 1/2$), and it is not the case that both agents have an incentive to form a link (in the case $\pi = 0$). Moreover, each agent's belief about the other agent's actions, both in the case when π remains the same and in the case when it switches because of actions at $t = -2$ and $t = -1$, need to be correct.

For the action $q \in \{0, 1\}$, we let $\neg q$ denote the complementary action ($\neg q = 1 - q$). It follows that the three numbers, q^1, q^2 and $\pi > 0$, describe an equilibrium with risk sharing

[12]In our stylized model, the quality decision can of course be inferred from the realization of project cash flows. This issue would be avoided by assuming a small positive probability for $\Delta R = 0$ in case of a low realization. For simplicity, we assume that contracts are restricted to being linear in realized project cash flows.

if:

$$
\begin{aligned}
V^1(q^1, q^2|\pi) &\geq V^1(\neg q^1, q^2|\pi), \\
V^2(q^1, q^2|\pi) &\geq V^2(q^1, \neg q^2|\pi), \\
V^1(q^1, q^2|\pi) &\geq V_I^1, \\
V^2(q^1, q^2|\pi) &\geq V_I^2.
\end{aligned}
$$

The first two conditions ensure that it is incentive compatible for both agents to choose the suggested investment strategies given that they share risks, whereas the latter two state that risk sharing dominates acting in isolation for both agents.

Interesting dynamics arise already in this network with only two intermediaries, as seen in the following example. We choose parameter values $R_H = 1.2$, $R_L = 0.1$, $\Delta R = 0.5$, $c = 0.05$, $p_1 = 0.1$, $p_2 = 0.05$, $\pi = 0.5$, and vary the insolvency threshold, d. Since the setting is symmetric, it follows that $V_I^1 = V_I^2 = V_I$, and $V^1(q_1, q_2) = V^2(q_2, q_1)$, reducing the number of constraints that need to be considered. The resulting value functions are shown in Figure 5.

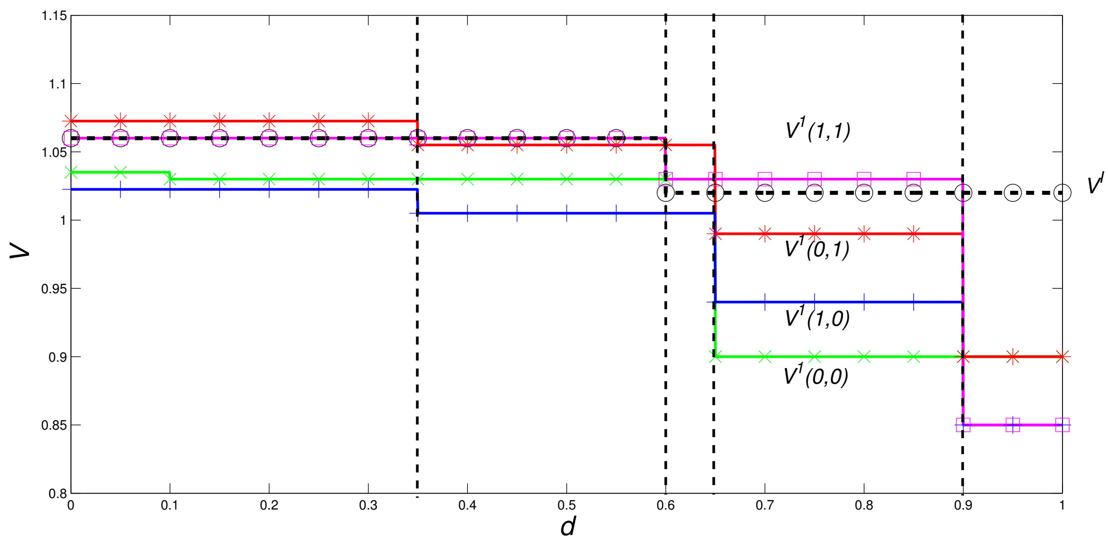

Figure 5: Value functions in economy with two intermediaries, as a function insolvency threshold, d. The following value functions are shown: V_I (circles, dotted black line), $V^1(1,1)$ (squares, magenta), $V^1(0,1)$ (stars, red), $V^1(1,0)$ (pluses, blue), and $V^1(0,0)$ (crosses, green).

There are five different regions with qualitatively different equilibrium behavior. In the first region, $0 < d < 0.35$, the unique equilibrium is the one where both agents invest in quality, $(q^1, q^2) = (1, 1)$, and there is no risk-sharing, leading to values V_I for both intermediaries. No intermediary ever becomes insolvent in this case (since $R_L + \Delta R > d$). The outcome where both agents invest and share risk would lead to the same values, but cannot be an equilibrium because each agent would deviate and choose to avoid investments in this case, given that the other agent invests. For example, if intermediary 1 does not invest, but intermediary 2 does, agent 1 reaches $V^1(0, 1) > V^1(1, 1)$ by avoiding the cost of investment but still capturing the benefits of not becoming insolvent after a low realization. Therefore, $V^1(1, 1)$ cannot be sustained in equilibrium. Now, $V^1(0, 1)$ can of course not be an equilibrium either, since under this arrangement intermediary 2 is on the (blue) $V^1(1, 0)$ line, which is inferior to V_I. So only the isolated outcome survives as an equilibrium.

In the second region, $0.35 \le d < 0.6$, there are two equilibria, both with investments, $(q^1, q^2) = (1, 1)$, and the same value for both intermediaries, V_I. In addition to the isolated outcome, the outcome with risk-sharing and investments in quality by both agents is now an equilibrium. The reason is that the solvency threshold has now become so high that agent 1 has an incentive to invest in the risk-sharing outcome when agent 2 invests, to avoid insolvency which otherwise occurs if both $\xi^1 = 0$, and $\xi^2 = 0$.

The third region is $0.6 \le d < 0.65$, in which the unique equilibrium is for intermediaries to share risk and not invest, $(q^1, q^2) = (0, 0)$, leading to value $V^1(0, 0)$ for both agents. Indeed, this strategy dominates the value under isolation, V_I, which for $d \ge 0.6$ entails the strategy of not investing in quality since in that region insolvency occurs even when such investments are made (this is the reason for the discontinuity in V_I at $d = 0.6$). Note that $V^1(0, 0)$ is dominated by $V^1(1, 1)$ and $V^1(0, 1)$, though neither can constitute an equilibrium. The outcome $V^1(1, 1)$ is not sustainable, since it is better for either agent to switch to low quality, as it is for agent 2 under $V^1(0, 1)$. So the only equilibrium is the one with risk-sharing.

When $0.65 \le d < 0.9$, i.e., in the fourth region, $V^1(0, 0)$ decreases substantially compared with the third region, because for such high levels of the default threshold, both intermediaries become insolvent if there is one low realization, whereas two low realizations were needed in the third region. This makes $V^1(0, 0)$ inferior to the isolated outcome, V_I (in which both agents choose not to invest since d is so high), because of a contagion effect. When risks are shared, a low realization for one intermediary not only causes that intermediary to become insolvent but also triggers the insolvency of the other intermediary. Thus, the only remaining equilibrium is now $V^1(1, 1)$, i.e., for agents to share risk and for both to invest in quality, $(q^1, q^2) = (1, 1)$.

Finally, when $d \ge 0.9$, the isolated equilibrium without quality investments, $(q^1, q^2) = (0, 0)$, is the only remaining equilibrium, since any risk sharing equilibrium will lead to contagion.

We note that equilibrium quality choice is non-monotone in d, in contrast to the isolated case in which q is naturally non-increasing in d (see (1)). With interaction between intermediaries, the unique quality choice in the third region, $0.6 \leq d < 0.65$, is $(q^1, q^2) = (0, 0)$, whereas in the fourth region, $0.65 \leq d < 0.9$, $(q^1, q^2) = (1, 1)$ in equilibrium, as the prospects for high-quality investments increases with d in parts of the domain in this case. Then, for $d > 0.9$, investments in quality again become inferior, leading to $(q^1, q^2) = (0, 0)$.

We also note that all equilibrium outcomes have $q^1 = q^2$. This is natural for the isolated equilibrium, but also occurs for the risk-sharing equilibria. It suggests that the "financial norm"—defined as the quality an intermediary chooses—depends on the financial norms of the intermediary with which it interacts, in line with the intuition that norms are jointly determined among interacting agents. We wish to explore this effect in more complex financial networks.

3.4 General networks of $N \geq 2$ intermediaries

We represent the network by the graph $\mathcal{G} = (\mathcal{N}, E)$, $\mathcal{N} = \{1, \ldots, N\}$. The relation $E \subset \mathcal{N} \times \mathcal{N}$ describes which intermediaries are connected in the network. Specifically, the edge $e = (n, n') \in E$, if and only if there is a connection (edge, link) between intermediary n and n'. No intermediary is connected to itself, $(n, n) \notin E$ for all n, i.e., E is irreflexive. We define the transpose of the link $(n, n')^T = (n, n')$, and assume that connections are bidirectional, i.e., $e \in E \Leftrightarrow e^T \in E$. The operation $E + e = E \cup \{e, e^T\}$, augments the link e (and its transpose) to the network, whereas $E - e = E \backslash \{e, e^T\}$ severs the link if it exists. The number of neighbors of node n is $Z_n(E) = |\{(n, n') \in E\}|$.

Intermediaries will in general have different scale and number of neighbors, and therefore choose to share different amounts of risk among themselves. Similar to the case with two intermediaries, we choose a simple sharing rule, represented by the sharing matrix $\widehat{\Pi} \in \mathbb{R}_+^{N \times N}$, where $0 \leq (\widehat{\Pi})_{nn'} \leq s^n$ is the amount of risk that is swapped between intermediary n and n', with the summing up constraint that $\widehat{\Pi}\mathbf{1} = s$, where $s = (s^1, \ldots, s^N)'$. It will be more convenient to characterize the *fraction* of risk that agent n' shares with agent n, which is represented by the matrix $\Pi = \widehat{\Pi}\Lambda_s^{-1}$, implying that $s = \Pi s$. Here, we have used the notation that for a general vector, $v \in \mathbb{R}^N$, we define the diagonal matrix $\Lambda_v = diag(v) \in \mathbb{R}^{N \times N}$, with diagonal elements $(\Lambda_v)_{ii} = v_i$. We also use the notation that δ_n represents a vector of zeros, except for the nth element which is 1. In our previous example with two intermediaries of unit scale, $s = (1, 1)'$ and the sharing matrix is

$$\Pi = \begin{bmatrix} 1 - \pi & \pi \\ \pi & 1 - \pi \end{bmatrix}.$$

The network represents a restriction on which sharing rules are feasible. As we will discuss, this restriction can be self-imposed by intermediaries in equilibrium, who could choose not to interact even if they may, or it could be exogenous. Specifically, for a sharing rule to be feasible it must be that every off-diagonal element in the sharing matrix that is strictly positive is associated with a pair of agents who are linked, $\widehat{\Pi}_{nn'} > 0 \Rightarrow (n, n') \in E$.

We choose to study a specifically simple class of sharing rules that ensure that all weights are nonnegative and that each intermediary keeps some of its own project risk, namely

$$(\widehat{\Pi})_{nn'} = \min\left\{\frac{s^n}{1+Z_n(E)}, \frac{s^{n'}}{1+Z_{n'}(E)}\right\}, \qquad n \neq n', \tag{11}$$

and

$$\left(\widehat{\Pi}\right)_{nn} = s^n - \sum_{n' \neq n} \widehat{\Pi}_{nn'}. \tag{12}$$

We write $\widehat{\Pi}(E)$ when stressing the underlying network from which the sharing rules is constructed.

The joint quality decision of all agents is represented by the vector $q = (q^1, \ldots, q^N) \in \{0,1\}^N$. In the general case, the cost of investing in quality may vary across intermediaries, represented by the vector $c = (c^1, \ldots, c^N) \in \mathbb{R}^N_{++}$. The state realization is represented by the vector $\xi = (\xi^1, \ldots, \xi^N) \in \{0,1\}^N$. We will work with a limited state space, assuming that $\xi \in \Omega \subset \{0,1\}^N$ where Ω is a strict subset of $\{0,1\}^N$, and mainly focus on two such sets: The first set is $\Omega^1 = \left\{\xi \in \{0,1\}^N : \xi'\mathbf{1} \geq N-1\right\}$, with $\mathbb{P}(\xi = \mathbf{1} - \delta_n) = p_1$, $1 \leq n \leq N$, and associated probability space $\mathbb{P} : \Omega^1 \to [0,1]$. Here, $\mathbf{1} = (1, \ldots, 1)^T$ is an N-vector of ones, and we also define $\mathbf{0} = (0, \ldots, 0)^T$. For this set, either zero or one low realization occurs, and the probability for a low realization is the same for all intermediaries. The second state space is $\Omega^2 = \left\{\xi \in \{0,1\}^N : \xi'\mathbf{1} \geq N-2\right\}$, for which no more than two realizations may be low, with full symmetry across intermediaries, so that

$$\begin{aligned}
\mathbb{P}(\xi = \mathbf{1} - \delta_n) &= p_1, \qquad 1 \leq n \leq N,\\
\mathbb{P}(\xi = \mathbf{1} - \delta_n - \delta_{n'}) &= p_2, \qquad 1 \leq n < n' \leq N,\\
\mathbb{P}(\xi = \mathbf{1}) &= 1 - Np_1 - \frac{N(N-1)}{2}p_2.
\end{aligned}$$

Solvency is represented by the vector $f \in \{0,1\}^N$, and realized cash flows to agents by the random vector $\widetilde{CF} = (\widetilde{CF}^1, \ldots, \widetilde{CF}^N)' \in \mathbb{R}^N_+$. The realized project cash-flows are then represented by the vector $\widetilde{CF}_P = (\widetilde{CF}^1_P, \ldots, \widetilde{CF}^N_P)'$, where

$$\widetilde{CF}^n_P(\xi, q) = s^n\left(R_H\xi^n + (R_L + q^n\Delta R)(1-\xi^n)\right), \qquad n = 1, \ldots, N.$$

The general network version of the clearing system that calculates a mapping $\widetilde{CF}_1(\xi, q) = \mathcal{CM}[\widetilde{CF}_P(\xi, q)]$, is described by the algorithm described in detail in Appendix B. The outcome of the algorithm is the solution that minimizes the number of insolvencies, or equivalently maximizes the total cash-flows to agents. In matrix form, this can be written as

$$\widetilde{CF}_1 = \mathcal{CM}[\widetilde{CF}_P] = \max_f (\Lambda_f \Pi \Lambda_f) \times \widetilde{CF}_P, \qquad \text{s.t.} \tag{13}$$

$$f = X\left(\Lambda_s^{-1} \times \widetilde{CF}_1\right). \tag{14}$$

Here, X operates element-wise in (14), $X(v) = (X(v^1), X(v^2), \ldots, X(v^N))'$. The net cash flows to the intermediaries are then given by the vector

$$w(\xi|q, E) = \widetilde{CF}_1(\xi) - \Lambda_c \Lambda_s q. \tag{15}$$

This is the general network version of equations (7-10). The $t = 0$ value vector of intermediaries, given quality investments, q, and network E is then given by

$$V(q|E) = \sum_{\xi \in \Omega} w(\xi|q, E)\mathbb{P}(\xi). \tag{16}$$

We can view the clearing mechanism as an iterative algorithm that propagates insolvencies in each iteration until it terminates. Specifically, if only insolvencies that occur within the initial $\bar{m}$ steps of the propagation are considered, (see the description in Appendix B), the cash flows are $\widetilde{CF}_{1,m} = \mathcal{CM}[\widetilde{CF}_P|\bar{m}]$. It immediately follows that $\widetilde{CF}_1 = \widetilde{CF}_{1,infty} = \mathcal{CM}[\widetilde{CF}_P] = \mathcal{CM}[\widetilde{CF}_P|\infty]$. We will use the $\bar{m}$-step propagation version of the clearing mechanism in the empirical section, assuming observations in an intermediate step of the mechanism.

3.4.1 Equilibrium

Let E^* denote the complete network in which all agents are connected. We assume that there is a maximum possible network, $\bar{E} \subset E^*$, such that only links that belong to $\bar{E}$ may exist in the sharing network. This restriction on feasible networks could, for example, represent environments in which it is impossible for some agents to credibly commit to deliver upon a contract written with some other agents, due to low relationship capital, limited contract enforcement across jurisdictions, etc. A network, E, is feasible if $E \subset \bar{E}$. If all agents who may be linked actually choose to be linked in equilibrium, i.e., if $E = \bar{E}$, we say that the equilibrium network is *maximal*. It may also be the case that E is a strict subnetwork of $\bar{E}$, just as was the case in the economy with two intermediaries where $\bar{E} = E^*$, but $E = \emptyset$ for some parameter values because agents chose the isolated outcome in equilibrium.

To define equilibrium, we build upon the pairwise stability concept of Jackson and Wolinsky (1996). We also require equilibrium in this multistage game to be subgame perfect. The game and definition of a stable equilibrium are explained in detail in the appendix. Here we provide a summary. The sequence of events is as in Figure 4. Consider a candidate equilibrium, represented by a network E and quality choices q. Each agent, n, has the opportunity to accept the sharing rule, $\hat{\Pi}(E)$, as is, by neither severing nor proposing new links at $t = -2$. But, in line with the pairwise stability concept, any agent n can also unilaterally decide to sever a link with one neighbor, n', leading to the sharing network $E' = E - (n, n')$, and corresponding sharing rule $\hat{\Pi}(E')$. Also, any agent can propose an augmentation of another link $(n, n') \in \bar{E} \backslash E$, which if agent n' accepts leads to the sharing network $E'' = E + (n, n')$ with sharing rule $\hat{\Pi}(E'')$. Finally, we assume that each agent can unilaterally choose the isolated outcome, V_I^n, by severing its links to all other agents.

The possibility to unilaterally sever all links in a sharing network—although not technically part of the standard definition of pairwise stability—is natural, in line with there being a participation constraint that no intermediary can be forced to violate. It provides a minor extension of the strategy space.

The severance, proposal, and acceptance/rejection of links occur at $t = -2$ and $t = -1$. The agents then decide whether to invest in quality or not at $t = 0$, each agent choosing $q^n \in \{0, 1\}$. A pair (q, E), where $E \subset \bar{E}$, is now defined to be an equilibrium, if agents given network structure E choose investment strategy q, if no agent given beliefs about other agent's actions—under the current network structure as well as under all other feasible network structures in $\bar{E}$—has an incentive to either propose new links or sever links, and if every agent's beliefs about other agents actions under network E as well as under all possible alternative network formations are correct.

4 Analysis of equilibrium

We wish to understand how the quality choices—the financial norms—of agents affect and are affected by equilibrium network structure. To gain some intuition, we first study a specific example with $N = 8$ intermediaries, all of which have scale equal to unity, $s = \mathbf{1}$, and with the shock structure Ω^2. The maximal network, $\bar{E}$, which is also an equilibrium network is shown in panel A of Figure 6. As shown in panel A, for low ΔR the equilibrium outcome is for all intermediaries to be of low quality. This is simply because investing in quality does not increase the payoff in low states sufficiently to avoid insolvency for any intermediary for low ΔR.

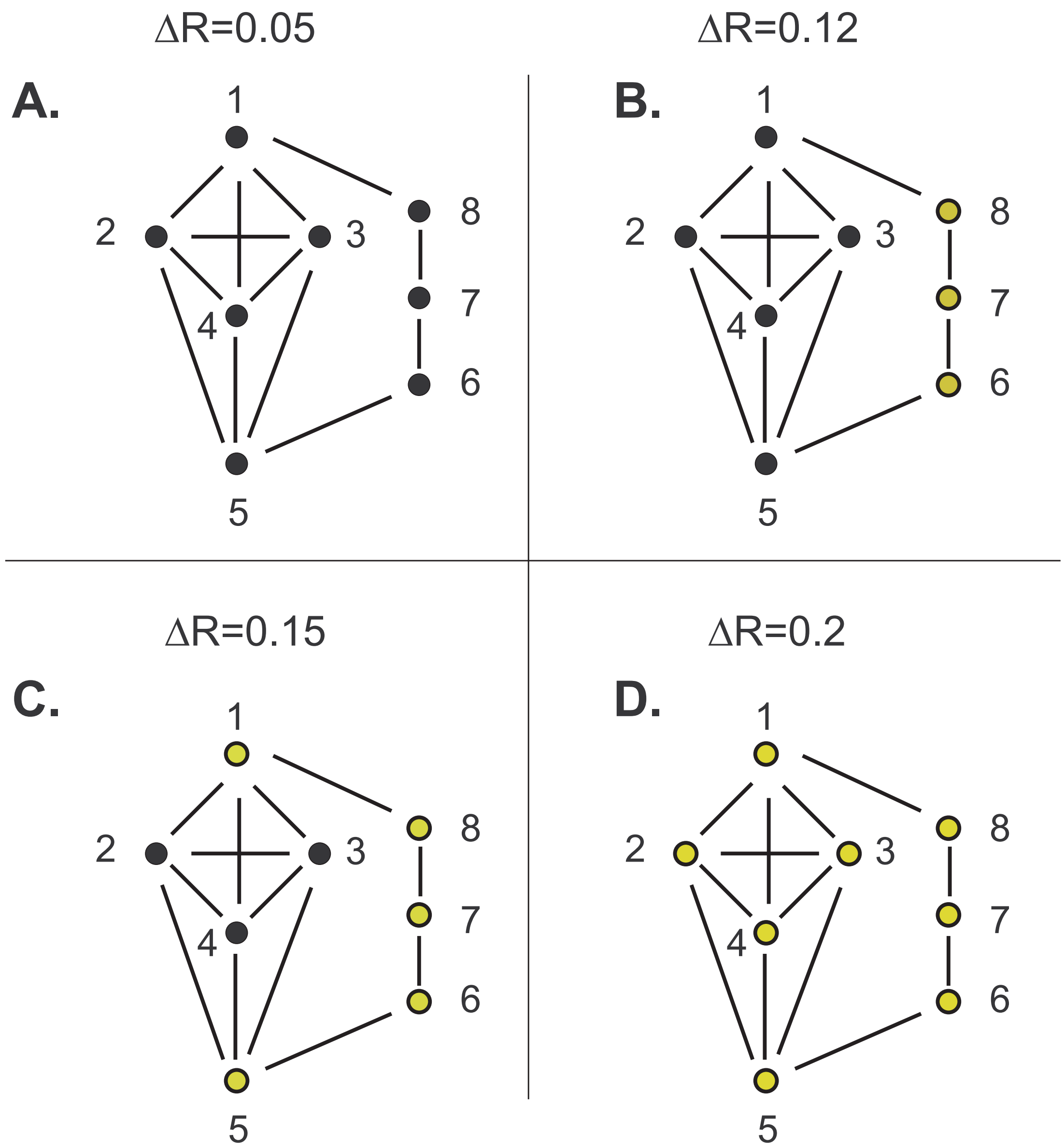

Figure 6: Equilibrium outcomes for different ΔR in network with 8 nodes. Low quality nodes are marked in black, whereas high quality nodes are yellow (gray). In panel A (upper left corner), $\Delta R = 0.05$, and all nodes are of low quality. In panel B (upper right corner), $\Delta R = 0.12$, and nodes 6–8 are of high quality. In panel C (lower left corner), $\Delta R = 0.15$, and nodes 1, 5–8 are of high quality. In panel D, $\Delta R = 0.2$ and all nodes are of high quality. Parameters: $c = 0.0025$, $R_H = 1.2$, $R_L = 0.1$, $d = 0.68$, $p_1 = 0.0625$, $p_2 = x$, $\bar{E} = E$. Shock structure Ω^2, and scale $s = \mathbf{1}$.

Average	Number in network	$q(neighbors)$	Number of neighbors	cost
$q = 1$	6.34	0.79	4.32	0.011
$q = 0$	2.66	0.4	3.12	0.016

Table 3: Summary statistics of high- and low-quality intermediaries. Number of simulations: 1,000. Parameters: $R_H = 1.1$, $R_L = 0.2$, $\Delta R = 0.3$, $d = 0.75$, $p_1 = 4/90$, $p_2 = 1/90$, $c \sim U(0, 0.025)$. Shock structure Ω^2, an scale $s = \mathbf{1}$.

For $\Delta R = 0.12$, shown in Panel B, by investing in quality intermediaries can now avoid insolvency. This is what intermediaries 6–8 do in equilibrium. However, intermediaries 1–5 cannot sustain an equilibrium in which they also invest in quality, since it is too tempting for them to free-ride on the investments by others. This is because they have so many neighbors that their individual investment decisions are not pivotal in avoiding insolvency.

For $\Delta R = 0.15$, shown in Panel C, it becomes an equilibrium strategy also for nodes 1 and 5 to invest in quality. This is because the nodes can now avoid insolvency for several cases in which two shocks hit the network, in which case they are pivotal. Nodes 2–4 still cannot sustain quality investments in equilibrium, though.

Finally, when $\Delta R = 0.2$, nodes 2–4 can also be made to invest in quality in equilibrium, since they are now also pivotal in avoiding insolvency after two shocks in a sufficient number of states. Thus, as shown in Panel B and C, heterogeneous financial norms may coexist in equilibrium, and intermediaries with the same quality tend to be linked (or at least to be close in the network).

We wish to explore this relationship between network structure and quality choice for a larger class of networks. We simulate 1,000 networks for which equilibrium exists, each with $N = 9$ nodes. We use the Erdös-Renyi random graph generation model, in which the probability that there is a link between any two nodes is i.i.d., with the probability 0.25 for a link between any two nodes, and we also randomly vary c across intermediaries, $c^n \sim U(0, 0.025)$. For computational reasons, we focus on networks in which equilibria are maximal, $E = \bar{E}$.

Table 3 shows summary statistics for nodes that are of high quality compared with those of low quality in equilibrium. We see that there are on average more high-quality nodes in equilibrium. Also, the average cost of investing in quality for high-quality nodes is lower than for low-quality nodes. More interestingly, the average number of neighbors of high-quality nodes is higher, and the average quality of neighbors of high-quality nodes is higher than of low-quality nodes. All these differences are statistically significant.

The last result is especially important, since it shows that the financial norms that arise in the network are indeed closely related to network position, i.e., that different clusters exist in which nodes have different norms. Another way of measuring whether such clusters exist is to partition each network into a high-quality and a low-quality component, and study whether the number of links between these two clusters is lower than it would be if quality were randomly generated across nodes. Specifically, consider a network with a total of $K = |\{(n, n') \in E\}|$ links, and a partition of the nodes into two clusters: $\mathcal{N} = \mathcal{N}^A \cup \mathcal{N}^B$, of size $N^A = |\mathcal{N}^A|$ and $N^B = |\mathcal{N}^B| = N - N^A$, respectively, and the number of links between the two components: $M = |\{(n, n') \in E : n \in \mathcal{N}^A, n' \in \mathcal{N}^B\}|$. In the terminology of graphs, M is the size of the cut-set, and is lower the more disjoint the two clusters are. The number of links one would expect between the two clusters, if links were randomly generated, would be $W = \frac{1}{N(N-1)} N^A N^B K$, so if the average M in the simulations is significantly lower than the average W, this provides further evidence that financial norms are clustered. Indeed, the average M in our simulations is 12.2, substantially lower than the average W which is 14.2, corroborating that network position is related to financial norms.

5 Computation of Equilibrium

Although the equilibrium calculations are straightforward in economies with networks of up to about 15 nodes, they become computationally infeasible for real-world large-scale networks—potentially with thousands of intermediaries. In addition, in practice some or all of the model parameters (R_L, R_H, ΔR, d, and p) may be unobservable, and therefore have to be estimated from observed dynamics. We introduce a numerical method that addresses these two issues jointly, by approximating an equilibrium that optimally matches observed insolvency dynamics, and that can be applied to large-scale problems. Our focus is on the quality choice vector, q, and we therefore take the network as given, i.e., we focus on the equilibrium constraints imposed on q at time $t = 0$ in the game.

We assume that w, E, and c are observable, whereas the parameter values R_L, R_H, ΔR, d, and p, and the quality vector $q \in \{0,1\}^N \stackrel{\text{def}}{=} \mathcal{D}$ are not. We also assume that the shock structure is Ω^1. The value of project cash-flows, given a state realization ξ are:

$$\widetilde{CF}_P(\xi, q) = \Lambda_s(R_H \Lambda_\xi \mathbf{1} + \Lambda_{\mathbf{1}-\xi}(R_L \mathbf{1} + \Delta R q)).$$

The cash flows are assumed to be observed after $\bar{m}$ steps in the clearing mechanism, where we set $\bar{m} = 2$, leading to $w(\xi|q, E) = \mathcal{CM}[\widetilde{CF}_P|2] - \Lambda_c \Lambda_s q$.

We conjecture that the equilibrium quality vector is q. We can then calculate the best response for agents (at time 0), given the actions of other agents,

$$\mathcal{F}(q) = \mathcal{F}(q|R_L, R_H, p, \Delta R, d, c) = \{\hat{q} : \hat{q}^n \in \arg\max_{x\in\{0,1\}} V^n(x, \hat{q}^{-n})\},$$

where we as before assume that indifferent agents choose low quality, so that $\mathcal{F}$ is a single valued function. For q to be consistent with equilibrium, it must be that $q = \mathcal{F}(q)$. We define the equilibrium set $\mathbb{Q} \subset \mathcal{D}$, as

$$\mathbb{Q}(R_L, R_H, p, \Delta R, d, c) = \{q : q = \mathcal{F}(q)\}.$$

Assume that $v^n = w^n + \epsilon$, $n = 1, \ldots, N$, are observed after a shock, where the measurement errors, ϵ^n are i.i.d. An estimate of the unobservable quality vector, q, is then given by solving problem:

$$\min_{\Delta R, R_L, R_H, p, d, q\in\mathbb{Q}(R_L, R_H, p, \Delta R, d, c)} \min_{\xi\in\Omega^1} ||v - \mathcal{CM}[\widetilde{CF}_P(\xi, q|2)] + \Lambda_s\Lambda_c q||. \tag{17}$$

Here, the mean-square (L^2) norm or the least absolute deviations (L^1) norm, for example, could be used.

The program (17) is an integer optimization problem over a nontrivial domain $\mathbb{Q}$. A simplification is given by replacing the constraint $q \in \mathbb{Q}$ with the penalty function $A||q - \mathcal{F}(q)||$, where $A > 0$ is a constant, leading to the relaxed problem:

$$\min_{\Delta R, R_L, R_H, p, d, q\in\mathcal{D}} \min_{\xi\in\Omega^1} ||v - \mathcal{CM}[\widetilde{CF}_P(\xi, q|2)] + \Lambda_s\Lambda_c q|| + A||q - \mathcal{F}(q|\Delta R, R_L, R_H, p, d, c)||. \tag{18}$$

The properties of the solutions to the relaxed problem (18) vary with the parameter A. For large A, the equilibrium constraint becomes relatively more important, and for sufficiently large A the solution will be the same as the solution to (17). For $A = 0$, no equilibrium constraint is imposed. The advantage of choosing a small A is that a closer match to v will be found. The disadvantage is that the equilibrium constraints imposed by requiring agents to act consistently are weakened. An immediate effect of choosing a low A is that the calculated q may become volatile across nodes for small A, since the equilibrium restrictions become weak.

In our computations, we use the formulation (18), with the least square norm for the first term and the least absolute deviations norm for the second, and parameter value $A = 1$. An advantage with using the least absolute deviations norm for the second error term in (18) is that the term then has the natural interpretation of being the number of agents whose quality choices are inconsistent with equilibrium.

6 Model Application to the 2006 U.S. Private-label Mortgage Data

We apply the computational algorithm in the previous section to the 2006 mortgage data. Using the visualization approach in Stanton et al. (2014), in which the high and low foreclosure segments of the network are plotted separately, Figure 7 presents the network with a low foreclosure part (left panel) and a high foreclosure part (right panel). As is clear from Figure 7, the high foreclosure segment of the network constitutes a concentrated part of the graph.

We estimate the cost vector, c, for each county/lender-type node using the origination cost for each loan. For each node we compute the average of all costs of loans flowing through that node. As a proxy for v^n, we use $1 - r^n$, where r^n is the observed foreclosure rate for loans flowing through a node. The network E is created by forming a link between any two nodes between which a loan flows. Using the algorithm of Section 4, we calculate the quality vector, q, and parameter estimates, R_L, R_H, ΔR, d, and p.

In total, there are 515 low-quality nodes in the estimated network. In Figure 8, we show the estimated low-quality part of the network. Black nodes are those of high quality, whereas red nodes are of low quality. We also draw the links that exist between any pair of nodes of low quality. The low-quality part of the network constitutes a distinct subnetwork. For example, the number of links between the nodes in 2006, M, and the number of links one would expect between nodes, if the partition of the network was random, W (introduced in Section 4), are 41,062 and 30,216, respectively. Thus, M is substantially lower than it would be under a random partition of the network.

We use the estimated equilibrium network to calculate the distribution of the number of insolvencies, as shown in Figure 9. Recall that if a shock hits, it hits either one or two nodes. For the vast majority of such shocks, the cumulative propagation is fairly limited: With 99.97% chance, at most 20 nodes become insolvent. However, there are a few shocks whose cumulative effects reach a threshold and then, in line with a phase transition mechanism, affect the whole network. In this case, with 0.03% chance, about two thirds of the nodes become insolvent.

Figure 10 shows an example of a limited shock, initially hitting a holding company, and ultimately spreading to 17 other nodes. Figure 11, in contrast, shows a shock that propagates throughout the network. It initially hits one holding company and one aggregator. It then "spreads" step by step. Each step here represents a new round of insolvencies in the clearing mechanism, when the amount of capital goes below the insolvency threshold, d, for a new set of nodes, bringing them down and in turn further affecting their counterparties in the

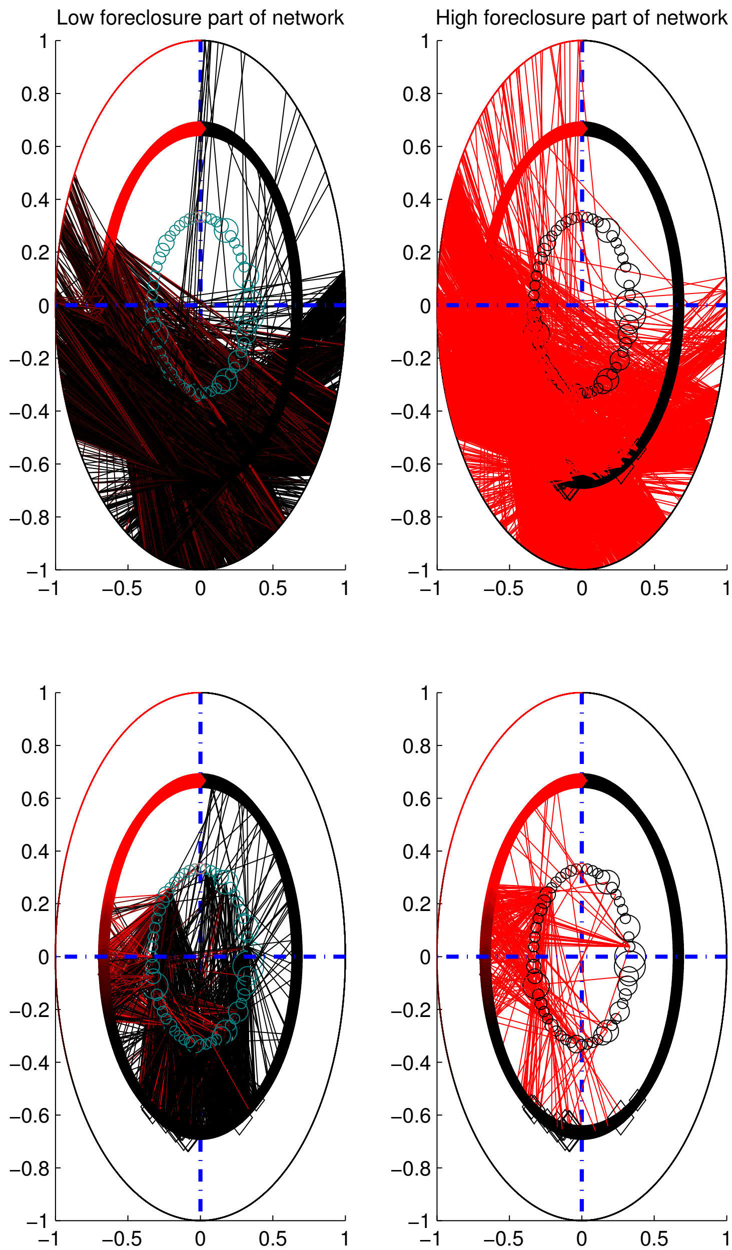

Figure 7: High (left) and low (right) foreclosure part of network. The outer circle of nodes represents originators of record (11,103 in total in 2006), sorted clockwise in increasing order of quality of loans. The middle circle represents aggregators (2,030 in total in 2006) with the same ordering. The inner circle represents the holding companies (56 in total in 2006). The cutoff is made so that links with a higher than 80% foreclosure rate are shown in the right panel, whereas links with a lower foreclosure rate are shown in the left panel. Year: 2006.

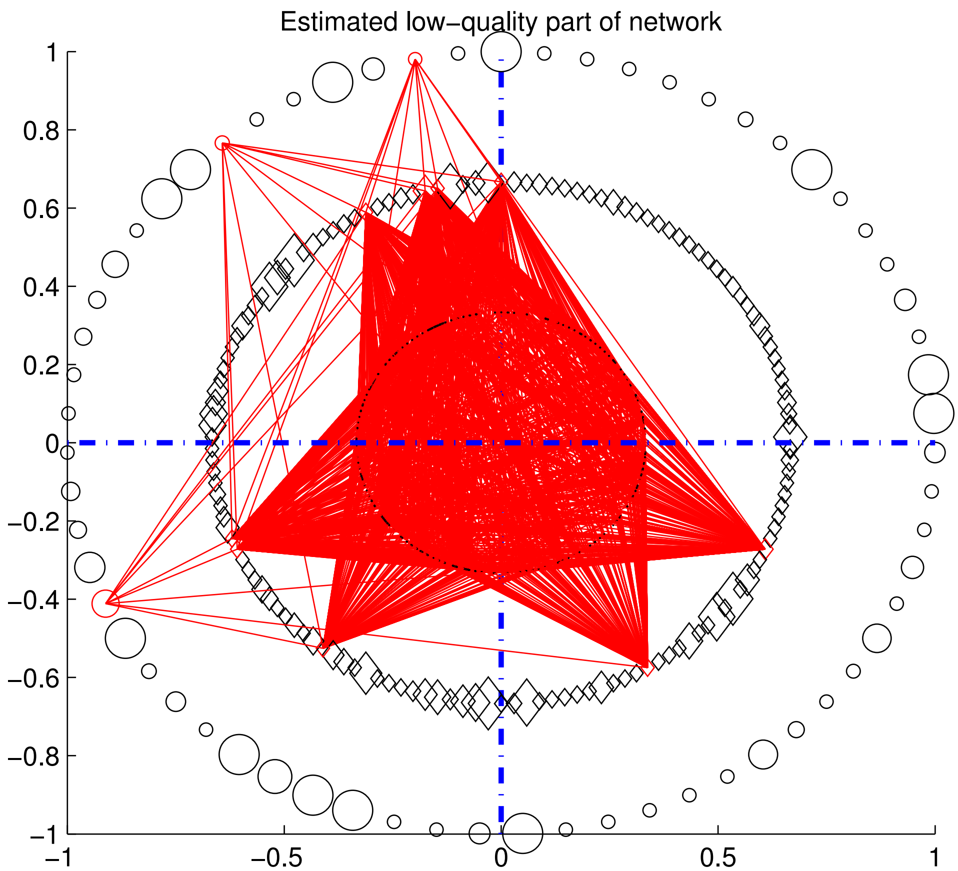

Figure 8: Red nodes in the figure represent those that are estimated as being of low quality, using the algorithm described in Section 4. Links between pairs of nodes of low quality are also shown. Equilibrium parameter estimates: $R_L = 0.54$, $d = 0.663$, $\Delta R = 0.1220$, $p = 0.42$. There are 3,371 high-quality and 515 low-quality nodes in the network. Year: 2006.

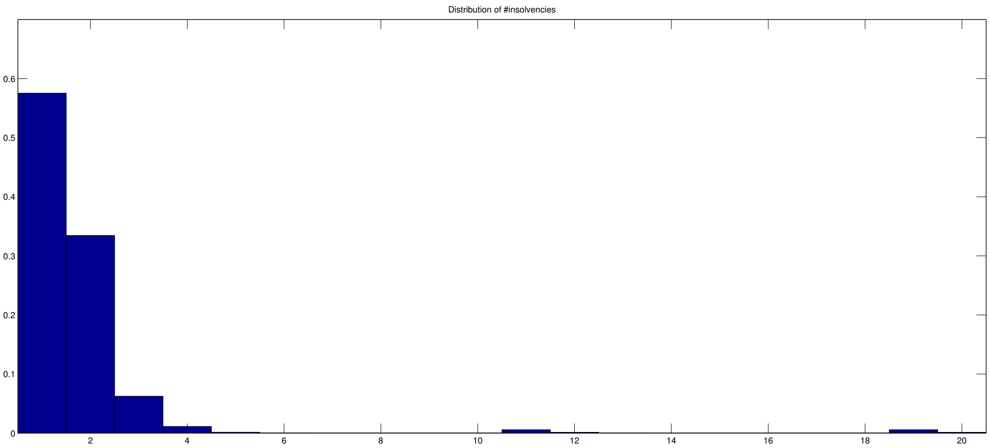

Figure 9: Distribution of number of insolvent nodes. The probability is 99.97% that at most 20 nodes become insolvent. There is a 0.03% chance that 2,731-2,733 nodes become insolvent. Year: 2006.

next step. The shock is still fairly limited after 4 steps, but once step 6 is reached, its effects are global. It ultimately causes almost all the aggregators, as well as most lenders, to become insolvent, whereas the holding companies show higher resilience (according to the assumptions of the model), as shown in step 8 (beyond which no further insolvencies occur).

Overall, our application of the computational algorithm, using the realized network structure and foreclosure outcomes of the 2006 mortgage data, generates predicted equilibrium quality metrics, q, that appear to be concentrated among an inner core of nodes, the holding companies of the securitization shelves. Using this implied equilibrium quality structure, our analysis suggests that in most states of the world the network is quite resilient to shocks, since the full effect of a shock is limited to a relatively limited number of nodes. However, the model computations also suggest that the network is prone, with positive probability, to a cascading outcome in which almost all the shelves, as well as most of the aggregators and lenders become insolvent. This outcome with cascading insolvencies may be used as a definition of systemic vulnerability of the financial market, being caused by an ex ante small shock that propagates and is amplified within the system, eventually affecting the whole market.

Our estimates of the inherent vulnerability of the 2006 network may thus provide a potentially powerful ex ante indicator of the systemic risks of such networks. Specifically, the computational algorithm may potentially be applied to an observed network structure together with ex ante predictions of mortgage foreclosure, solving for the implied unobservable quality measures of intermediaries, and subsequently stress testing the system with various

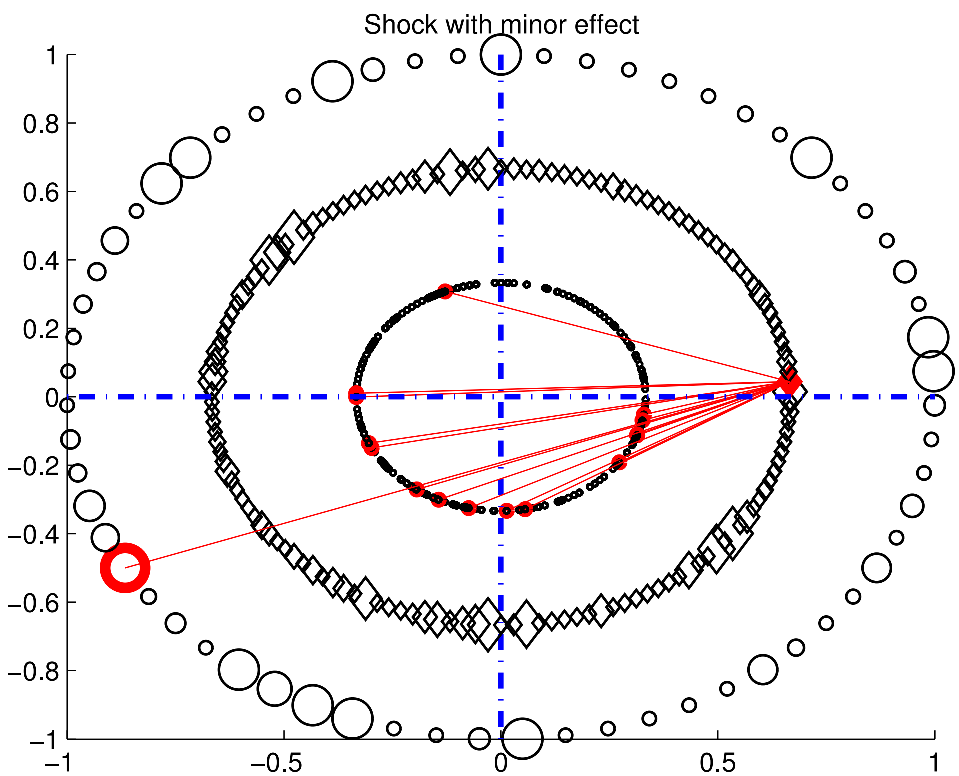

Figure 10: Example of minor shock. In total 18 nodes become insolvent after a shock initially hits the holding company in the lower left part of the figure. Year: 2006.

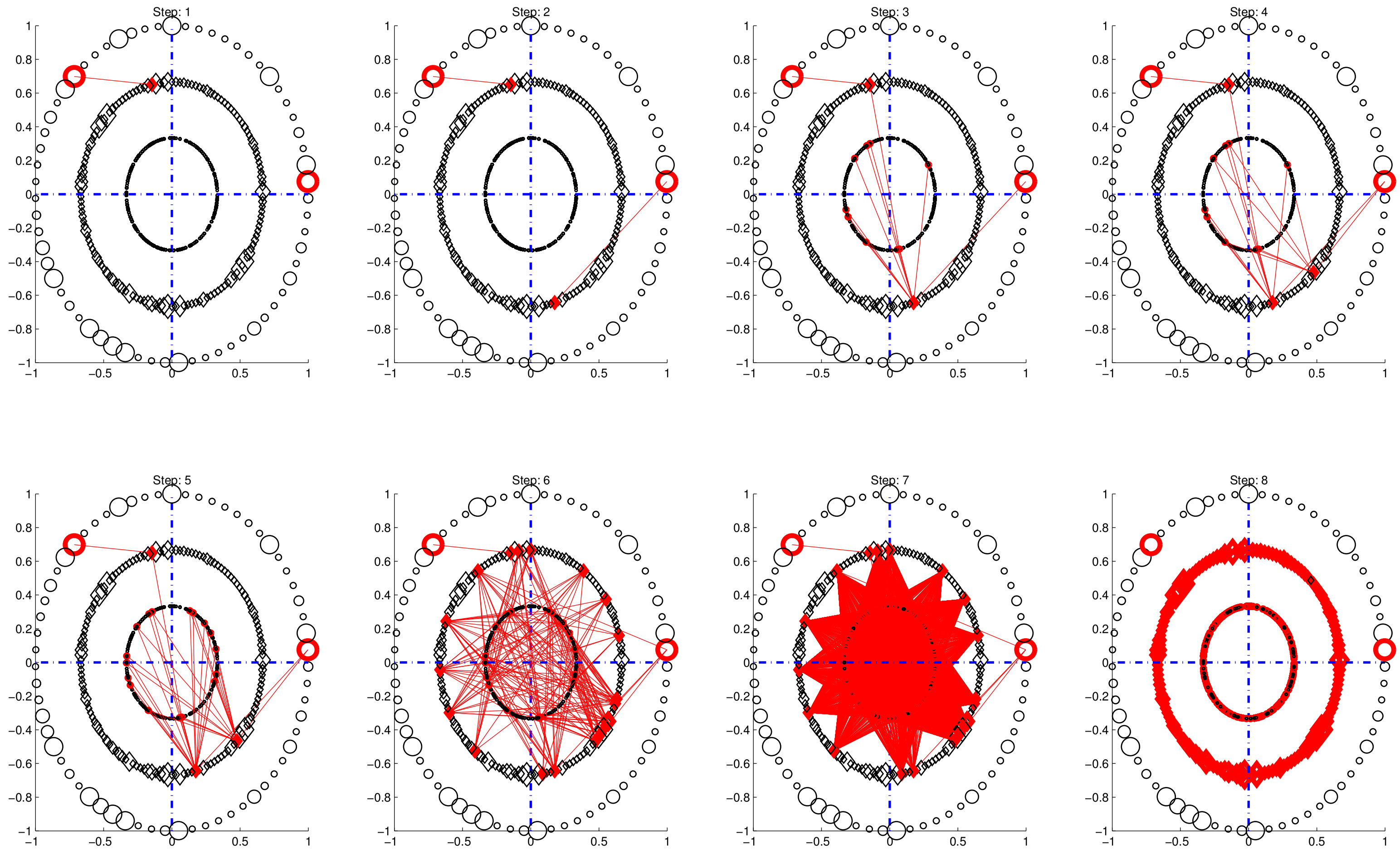

Figure 11: Example of major shock. In total 2,732 nodes become insolvent after an initial shock that jointly hits one holding and one shelf company. The shock propagates step-by-step, causing new nodes to become insolvent, until ultimately (after 8 steps) most shelf and county-type nodes are insolvent. The holding companies are more resilient. Note that links are not shown in step 8, for expositional reasons. Year: 2006.

shocks.

7 Conclusions

In our theoretical network model of intermediaries, heterogeneous financial norms and systemic vulnerabilities arise endogenously. The optimal behavior of each intermediary, in terms of its attitude toward risk, the quality of the projects it undertakes, and the intermediaries it chooses to interact with, are influenced by the behavior of its prospective counterparties. These network effects, together with the intrinsic differences between intermediaries, jointly determine financial health, quality, and systemic vulnerability, at the aggregate level of the market, as well as for individual intermediaries.

We apply the model to the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than a million mortgages originated and securitized through the private-label market in 2006. The market operates in a highly interlinked network, and network position of an intermediary is strongly related to the foreclosure rates of its loans above and beyond geographical effects. Using a computational algorithm to estimate intermediaries' unobserved quality choices, we find that risk propagated in a concentrated manner among highly linked nodes in the network. Altogether, our results suggest that network effects are indeed of vital importance for understanding the U.S. mortgage market. A potential application of the model is in detecting vulnerabilities to systemic shocks before they become severe.

A Mortgage Data Appendix

A significant challenge in network analysis of the U.S. mortgage market is the fact that there is no unique mortgage identifier that can be used to track individual mortgages when they are sold. Thus, identifying the network path of a given loan from the address of the houses that collateralize the loans, the identity of the legally recorded loan originator, the aggregator of the loan, the pools or special purpose entities (SPE) in which the loans are securitized, and the holding companies that exercise the control rights to structure the SPEs requires complicated merging protocols between disparate data sets. Another challenge is measuring the life-of-loan performance of individual loans in the mortgage network. To address the limitations of existing single data sets, we create a new data set that merges together three different data sets. These data sets are: 1) the Dataquick Historical Transaction data that records the legal originator of record, the recording date and the loan principal but has no other information about the mortgages such as the performance of the loan or who it was sold to; 2) the newly updated ABSNet loan- and pool-level origination and transaction data that includes detailed information about the contract structure of the individual loans as well as the aggregator for each loan and the REMIC pool into which the loan was securitized; 3) the prospectuses for all the pools in the ABSNet data that provide the legal names of all the agents involved in the securitization and the legal name of the SPE which we obtained from the Securities and Exchange Commission (SEC) website.

Since our interest is in securitized loan networks, we merge the Dataquick lien data with loan-level data obtained from ABSNet. The ABSNet data set includes detailed information about private-label securitized mortgages, including the initial loan balance, loan contract features, the loan zip code, the origination date, and the identifiers for the special purpose entity (SPE) in which the loan is securitized. ABSNet records a total of 13,453,796 first-lien loan originations between 2002 and 2007. We successfully merge 9,099,280 of these loan records with the Dataquick Historical Transaction data, giving us an originator of record and a lender type for each merged loan. We then downloaded all the prospectuses for all the residential mortgage-backed security deals from the SEC website to obtain the full deal name for each SPE in the ABSNet data from each prospectus. We then hand-searched the SEC deal names to obtain the name of the shelf registration for the SPE and the identity of the holding company that had made the shelf registration. For each loan, we track the month-by-month payment performance from the origination date until December 2013 (the end of our performance data). We thus identify all the ex post foreclosure outcomes for each loan over our performance period.

The first five rows of Table 4 provides summary statistics for the loan level origination for first lien mortgages that were originated in 2006 and securitized in private-label mortgage-backed securities. As shown Table 4, the average loan balance was $277,425 and the average cumulative loan-to-value ratio was 79.32%. The average coupon on the mortgages was 7.09% and the average maturity was 32.75 years. 32.75% of the loans were conventional conforming loans.

A.1 Mortgage Origination Costs

In addition to information on the flow of mortgages through the private-label mortgage market, we also need information on the costs of mortgage origination for the lenders. To obtain these costs we collect data from two sources. For the banks, thrifts, and credit unions that primarily engaged in retail lending, we obtain data from Bankrate.com[13] that carries out an annual survey of about 306 institutions that are primarily banks and thrifts in the U.S. The costs reported by Bankrate.com include the lender fees, third-party fees, and government fees that are obtained from their lender surveys. Bankrate.com reports the survey results as averages of total origination costs by state.[14]

For the wholesale lenders, we use the loan-level origination costs obtained from the New Century origination data set, and including all the cost categories reported by Bankrate.com.[15] Since New Century was one of the largest mortgage companies operating in the U.S. over this period, we have loan-level origination cost data for all states in the U.S., and these costs have a comparable composition to those of Bankrate.com. We assume that the New Century costs are a good proxy for the other mortgage companies with which they competed, including the mortgage companies that were part of bank holding companies. We assign the origination costs to the lenders by their type and by the state in which the loan was originated.

The sixth row of Table 4 presents summary statistics for the loan costs. As shown, the average loan cost was 14.7% of the balance, with a standard deviation of 4.02% of the balance. The lower section of Table 4 presents summary statistics for the average geographic and network performance for each loan. The average zip code default level, excluding the subject loan, was 37.44% for all of the zip codes in our data. The next three rows of the lower section of Table 4 presents the default performance within the local network position

[13] `http://www.bankrate.com/`

[14] The Lender fees include the: points in dollars, administration fee; commitment fee; document preparation; funding fee; lender fee; processing fees; tax service fee; underwriting fee; and wire transfer fee. The third-party fees include the: appraisal fee; the attorney or settlement fees; credit report; flood certification; pest & other inspection fees; postage/courier fees; survey; title insurance; and title work (title search, plat drawing, name search). The government fees include the: recording fee; city/county/state tax stamps and intangible taxes.

[15] These data were made available from the bankruptcy trustee of New Century Financial Corporation.

Variable Name	Mean	Standard Deviation	Minimum	Maximum
Original Loan Balance ($)	277,425	224,182	50,000	989,560
Original Cumulative Loan to Value Ratio	0.7932	0.1618	0.1360	1.6500
Original Mortgage Contract Rate	0.0709	0.0259	0.0350	0.1294
Original Mortgage Amortization Term (years)	31.38	4.4092	10.0000	40.1667
Conventional Conforming Loan	0.3275	0.4693	0.0000	1.0000
Loan Origination Cost	0.1467	0.0402	0.0700	0.280
Geographic effects (excluding subject loan)				
Loan Zip Code Average Default	0.3744	0.1256	0.000	1.0000
Network effects (excluding subject loan)				
Neighboring Lenders Average Default Rate	0.4348	0.0074	0.0000	1.0000
Neighboring Aggregators Average Default Rate	0.3779	0.0069	0.2354	0.5293
Neighboring Holding Company Average Default Rate	0.3847	0.0197	0.03437	0.4890
Number of Loans	1,152,312			

Table 4: Summary statistics for the loan origination characteristics, the default performance of the loans (90 days delinquent), the performance of loans in the loan's zip code (excluding subject loan), and the performance over the network path of the loan (excluding subject loan).

of the loan, again excluding the subject loan. As shown, the average lender default level for neighboring network lenders was 43.48%. The average default level for neighboring aggregators to the subject loan's aggregator was 37.79%, and the average default level for the neighboring holding company was 38.47%.

B Clearing Algorithm for general network model

Algorithm 1 (Clearing mechanism)**.**

1. *Set iteration* $m = 0$ *and the initial insolvency vector* $f_1 = \mathbf{1}$.
2. *Repeat:*
   - *Set* $m = m + 1$.
   - *Calculate* $z_m = \Lambda_{f_m} \Pi \Lambda_{f_m} \widetilde{CF}_P$.
   - *Calculate* $f^n_{m+1} = X\left(\frac{z^n_M}{s^n}\right)$, $n = 1, \ldots, N$.
3. *Until* $f_{m+1} = f_m$.
4. *Calculate the* $t = 1$ *cash flow as* $\widetilde{CF}_1 = z_m$.

The iteration over M can be viewed as showing the gradual propagation of insolvencies, where $f_M - f_{M-1}$ show the insolvencies that are triggered in step M, by the insolvencies that occurred in step $M - 1$.

We can write the algorithm using returns, defining $R = \Lambda_s^{-1} \widetilde{CF}_P$, and $\Gamma = \Lambda_s^{-1} \hat{\Pi}$. The algorithm then becomes even simpler:

Algorithm 2 (Clearing mechanism on return form)**.**

1. *Set iteration* $m = 0$ *and the initial insolvency vector* $f_1 = \mathbf{1}$.
2. *Repeat:*
    - *Set* $m = m + 1$.
    - *Calculate* $z_m = \Lambda_{f_m} \Gamma \Lambda_{f_m} R$.
    - *Calculate* $f_{m+1} = X(z_m)$.
3. *Until* $f_{m+1} = f_m$.
4. *Calculate the* $t = 1$ *cash flow as* $\widetilde{CF}_1^n = \Lambda_s z_m$.

It is straightforward to show that the two algorithms are equivalent, given that $\hat{\Pi} = \Lambda_s^{-1}\Gamma = \Pi\Lambda_s^{-1}$.

C Network formation game

The sequence of events are as in Figure 4. We use subgame perfect, pairwise stable Nash as the equilibrium concept. We make one extension of the pairwise stability concept in the definition of agents' action space. Specifically, agents are allowed to unilaterally decide to become completely isolated by severing links to all agents they are connected to. In contrast, with the standard definition of pairwise stability agents are only allowed to sever exactly one link, or propose the addition of one link. The assumption that agents can choose to become isolated can thus be viewed as a network participation constraint.

The sequence of events is as follows: At $t = -2$—the proposal/severance stage of the game—there is a given initial network, $E \subset \bar{E}$. Recall that $\bar{E}$ here is the maximal network in the economy, which arises if all possible links are present. Each agent, $n = 1, \ldots, N$, simultaneously chooses from the following mutually exclusive set of actions:

1. Sever links to all other agents and become completely isolated.
2. Sever exactly one existing link to another agent, $(n, n') \in E$.
3. Propose the formation of a new link to (exactly) one other agent, $(n, n') \in \bar{E} \backslash E$.
4. Do nothing.

In contrast to actions 1. and 2., which are unilateral, agent n' needs to agree for the links to actually be added to the network under action 3.

The set of networks that can potentially arise from this process is denoted by $\mathcal{E}$. We note that $E' \subset \bar{E}$ for all $E' \in \mathcal{E}$. The set of actual proposals for addition of links, generated by actions 3., is denoted by L^A. The set of links that are actually severed, generated by actions 1. and 2., is L^S. The total set of potential link modifications is $\mathcal{L} = \{(L^A, L^S)\}$.

At $t = -1$—the acceptance/decline stage—L^A and L^S are revealed to all agents, who then simultaneously choose whether to accept or decline proposed links. Formally, for each proposed link, $\ell = (n, n') \in L^A$, agent n' chooses an action $a_\ell \in \{D, A\}$ (representing the actions of Declining or Accepting the proposed link). The total set of actions is then $A = \{a_\ell : \ell \in L_A\} \in \{D, A\}^{L_A}$, which for each n' can be decomposed into $A^{n'} \cup A^{-n'}$, where $A^{n'} = \{(n, n') \in L_A\}$ represents the actions taken by agent n', and $A^{-n'} = A \backslash A^{n'}$ the actions taken by all other agents. Altogether, E, L_A, L_D, and A then determines the resulting network, E', after the first two stages of the game, at $t = 0$.

At $t = 0$—the quality choice stage—each agent, $n = 1, \ldots, N$, simultaneously chooses the quality $q^n \in \{0, 1\}$. The joint quality actions of all agents are summarized in the action vector $q \in \{0, 1\}^N$.

At $t = 1$, shocks, ξ, are realized, leading to realized net cash flows $w^n(\xi|q, E')$ as defined by equations (13-14). The value of intermediary n at $t = 0$ is thus

$$V^n(q|E') = \sum_{\xi \in \Omega} w^n(\xi|E', q)\mathbb{P}(\xi), \qquad n = 1, \ldots, N. \tag{19}$$

Equilibrium

A stable equilibrium to the network formation game is an initial network and quality strategies, together with a set of beliefs about agent actions for other feasible network structures, such that no agent has an incentive to add or sever links, given that no other agents do so, and agents' have consistent beliefs about each others' behavior on and off the equilibrium path.

Specifically, the action-network pair (q,E), together with $t = -1$ acceptance strategies: $A : \mathcal{L} \to \{D, A\}^{L_A}$, and $t = 0$ quality strategies $Q : \mathcal{E} \to \{0, 1\}^N$ constitute an equilibrium, if

1. At $t = 0$, strategies are consistent in that for each $E' \in \mathcal{E}$, and $q = Q(E')$,

$$q^n \in \arg\max_{x \in \{0,1\}} V^n((x, q^{-n})|E),$$

   for all $n = 1, \ldots, N$, i.e., it is optimal for each agent, n, to choose strategy q^n, given that the other agents choose q^{-n}.
2. At $t = -1$, strategies are consistent in that for each $(L_A, L_D) \in \mathcal{L}$, $A^n(L_A, L_D)$ is the optimal action for each agent, n, given that the other agents choose $A^{-n}(L_A, L_D)$. Optimal here, means that the action maximizes V^n at $t = 0$.
3. At $t = -2$, the strategy $L = \emptyset$ is consistent, i.e., for each agent n, given that no other

agent severs links or proposes additional links, it is optimal for agent n not to do so either, given the value such actions would lead to at $t = -1$.

A stable equilibrium is said to be maximal if $E = \bar{E}$. We note that since we focus on pure strategies, the existence of stable equilibrium is not guaranteed, which has not been an issue in the examples we have studied. Neither is uniqueness of stable equilibrium guaranteed.

It follows that the following conditions are necessary and sufficient for there to exists acceptance and quality strategies such that (q, E) is an equilibrium:

1. For all n, $V^n(q|E) \geq V^n((\neg q^n, q^{-n})|E)$.
2. For all n, $V^n(q|E) \geq V_I^n$.
3. For all $(n, n') \in E$, $\exists q' \in \{L, H\}^N$ such that
   - $V^n(q|E) \geq V^n(q'|E - (n, n'))$,
   - For all n'', $V^{n''}(q'|E - (n, n')) \geq V^{n''}((\neg(q')^{n''}, (q')^{-n''})|E - (n, n'))$,
4. For all $(n, n') \in \bar{E} \backslash E$, $\exists q' \in \{L, H\}^N$ such that
   - $V^n(q|E) \geq V^n(q'|E + (n, n'))$ or $V^{n'}(q|E) \geq V^{n'}(q'|E + (n, n'))$,
   - For all n'', $V^{n''}(q'|E + (n, n')) \geq V^{n''}((\neg(q')^{n''}, (q')^{-n''})|E + (n, n'))$,

The first condition ensures that each agent makes the optimal quality choice at $t = 0$, given that no change to the network is made. The second condition ensures that it is not optimal for any agent to sever all links and become isolated. The third condition ensures that there are consistent beliefs about future actions, such that no agent has an incentive to sever a link at $t = -2$. The fourth condition ensures that there are consistent beliefs about future actions, so that no two agents can be made jointly better off by adding a link.

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