Global Collateral:

How Financial Innovation Drives Capital Flows and Increases Financial Instability

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Abstract

We show that cross-border financial flows arise when levels of financial innovation differ across countries. Financial integration is a way of sharing scarce collateral. The ability of one country to leverage and tranche assets provides attractive financial contracts to investors in the other country, and general equilibrium effects on prices create opportunities for investors in the sophisticated country to invest abroad. Foreign demand for collateral and for collateral-backed financial promises increases the collateral value of domestic assets, and cheap foreign assets provide attractive returns to investors who do not demand collateral to issue promises. Gross global flows respond dynamically to fundamentals, exporting and amplifying financial volatility.

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1 Introduction

Recent decades have exhibited a proliferation of financial innovation and dramatic increases in gross international financial flows. Economists have begun to recognize that the rapid growth of gross global financial flows poses serious risks for macroeconomic and financial stability,[4] but the causes and consequences of these flows are not entirely understood. What drives these flows, and what are the consequences?

The main contribution of this paper is to show that cross-border differences in the ability to collateralize financial promises are enough to generate international capital flows. This is because international financial trade is a way of sharing scarce collateral. The central element of our analysis is repayment enforceability problems: we suppose that agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance. Agents need to post collateral in order to issue promises. We define financial innovation as the use of new kinds of collateral, or new kinds of promises that can be backed by collateral. Our model provides precise predictions on the direction of capital flows and their effects on asset prices and financial instability.

To conduct our analysis we focus on a special class of models, C-models, introduced by Geanakoplos (2003). C-models are characterized by two states of nature and a continuum of risk neutral agents with heterogenous beliefs. There are two assets, a risky and a riskless asset, that pay dividends in units of the only consumption good. Final consumption is entirely derived from asset dividends. We consider two countries, Home and Foreign, which are identical except in their ability to collateralize financial promises. The Home country has an advanced financial system that can collateralize and tranche the risky domestic asset (which can be sold domestically or abroad); the Foreign country cannot collateralize its assets at all.

We first consider a static version of the model with only two periods and we study two environments: i) Home can issue debt against the risky asset (Home can leverage the asset)

[4]See especially Obstfeld (2012). Shin (2012) has stressed the role of gross cross-border banking flows to understand the recent financial and economic crises in the US and Europe.

while Foreign cannot, ii) Home can issue contingent promises against the risky asset (Home can tranche the asset) while Foreign cannot. The key results are that (1) financial integration allows Foreign investors to buy Home collateral and to make financial promises unavailable before, and (2) with integration assets with and without collateral value are priced relative to each other, so that even assets which cannot be used as collateral behave similarly to collateralizable assets. Flows arise even though there are no interest rate differentials, nor hedging or risk-sharing motives to trade assets (agents are risk-neutral and assets have identical payoffs).

More precisely, when financial integration takes place we observe the following trade patterns. First, foreign investors may demand Home collateral in order to issue financial contracts, increasing the demand for domestic assets. Second, foreign investors may demand collateral-backed financial promises, increasing the values of those contracts supplied by Home and the value of Home assets used as collateral. Third, domestic investors without strong demands to issue financial promises may demand foreign assets with payoff properties similar to domestic assets. Foreign assets with similar payoffs, but lower prices owing to a lack of collateral value, can have attractive returns, increasing the required return on the domestic asset. Hence, flows arise in both directions. These financial flows will tend to increase asset prices. The Home asset's price tends to increase given its attractiveness to international agents in its role as collateral. Foreign asset prices increase by virtue of being priced relative to a higher valued alternative (the over-valuation coming from the Home asset's role as collateral).

In a second step we conduct the same analysis in a dynamic setting in order to study the effects of financial integration on volatility. In a dynamic setting, global flows respond to changes in fundamentals and as a consequence prices change dynamically. When Home can leverage its risky asset whereas Foreign cannot, we find that Home can almost totally export its domestic Leverage Cycle to the Foreign country, while Home remains volatile itself. Asset markets in Foreign become much more volatile as the attractiveness of alternative assets (leveraged Home assets) fluctuates; it is as if Home is exporting the effects of financial innovation. When Home can tranche its risky asset while Foreign cannot, we find that Foreign demand for collateral-backed financial promises increases the collateral value of Home assets, amplifying pricing fluctuations driven by financial innovations. In this case asset prices in both countries become more volatile as a result of financial integration; financial flows amplify volatility.[5]

Our mechanism, that financial flows arise as a way to share collateral and contingent financial contracts, has several attractive features and important implications. First, our model explicitly models the expansion of securitization, leverage, and asset tranching that occurred in the previous decades. Second, collateral sharing leads to gross international flows in both directions—both the financially sophisticated country and the less financially developed country choose to hold assets from the other country—which is a salient feature of the data. Third, our story has important implications for global financial stability and crisis transmission. Collateral sharing and tranching exports volatility to the less developed financial sector and at the same time increases volatility for the developed financial sector.

The rest of the paper is organized as follows. Section 2 presents a basic general equilibrium model with collateralized borrowing. Section 3 studies the effects of financial integration on asset prices in a static model in which countries have different levels of financial innovation. Section 4 uses a 3-period model to consider the dynamic consequences of financial innovation and capital flows on asset prices and volatility. Section 5 concludes.

Related Literature

In this paper we follow the model of collateral equilibrium developed in Geanakoplos (2003), Fostel and Geanakoplos (2008, 2011, 2012, 2015a,b), Phelan (2015), Gong and Phelan (2015) and Geanakoplos and Zame (2014). This literature focuses on the effect of collateral on asset prices and investment. Our paper adds to this work in studying the effect of collateral and financial innovation on global flows and international asset prices.

[5]This is a complementary result to Fostel and Geanakoplos (2008), which shows that when the *same* marginal buyer uses leverage to buy two *independent* assets, the condition of one market spills over to the other.

Our paper is also related to a large literature that studies global imbalances. Research on how differential financial innovation drives capital flows has tended to focus on how *net* capital flows arise from interest rate and risk-sharing differentials. Caballero et al. (2008) show how the ability of countries to supply financial assets based on real assets can lead to persistent current account imbalances, cross border asset flows, and low global interest rates. Differential asset supplies create differential savings demand across countries, producing different autarkic interest rates, and interest rates converge when capital markets are open. Similarly, Quadrini et al. (2009) and Angeletos and Panousi (2011) show how net capital flows arise when the developed country can better insure idiosyncratic investment risk. Risk sharing has consequences for buffer-stock savings and equilibrium interest rates. In all these papers, financial flows are driven primarily by interest rate differentials, and financial integration leads to a convergence in savings levels and interest rates. As well, current account deficits can be financed indefinitely because the financially "deep" country earns intermediation rents. Finally, this literature is not exclusively about interest rates, but risk-premium as well. Maggiori (2013) provides a model in which Home financiers can take on greater financial risk as a result of funding advantages. This leads Home to run persistent current account deficits financed by the risk-premium earned by its financial sector.[6]

While these interest rate and risk-sharing mechanisms are clearly important for understanding global flows and imbalances, we instead emphasize the role of collateral to facilitate global flows. In our model, agents are risk-neutral, assets are perfectly correlated, and interest rates do not change with financial integration (they are always zero). Instead, because leverage and tranching create contingent payoffs from underlying collateral, international flows arise as a way for agents to share collateral, not risk, and to buy securities that provide more specialized cash flows than are available domestically.

Our story also relates to the "banking glut hypothesis" put forth in Shin (2012). Shin

[6]Within this literature, Phelan and Toda (2015) study how the risk-sharing qualities of securitized markets affect international capital flows, growth, and welfare in a two country general equilibrium model. They find that capital flows from the high- to low-margin country, leading to high investment levels and economic growth in the latter. Despite low growth, the high-margin country substantially gains in terms of welfare through better risk sharing opportunities.

documents how European banks greatly expanded their balance sheets, depressing spreads in the U.S. (and periphery Europe), by increasing both U.S. assets and liabilities: gross flows were massive even though net flows were negligible, generally canceling out and thus absent from the current account. Shin argues that the regulatory environment in Europe and the advent of the Euro enabled banks to easily expand their balance sheets. The question remains: why did European banks expand by intermediating U.S. assets and liabilities as much as they did? We propose that differences in financial innovation between the U.S. and Europe contributed to the expansion of European banks' balance sheets. In particular, the ability of the U.S. financial system to leverage and tranche U.S. assets (especially mortgages) created securities in demand by European banks. While Shin (2012) emphasizes how expanded intermediation by European banks depresses credit spreads in the U.S., we document how global banking flows and financial linkages of the type seen between the U.S. and Europe can create serious spillovers, exporting U.S. volatility to European markets and greatly increasing the fragility of the global banking system.

2 General Equilibrium Model with Collateral

Our analysis focuses on a special class of models with collateral, C-models, introduced by Geanakoplos (2003). These economies are complex enough to allow for the possibility that financial innovation can have a big effect on prices and global international flows. But they are simple enough to be tractable and to generate unambiguous (as well as intuitive) results.[7]

Time and Assets

The model is a two-period general equilibrium model, with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, D\}$ with a root $s = 0$ at time 0 and two states of nature

[7]None of the results depend on risk neutrality or heterogeneous priors. By assuming common probabilities and strictly concave utilities, and adding large endowments in state D vs state U for agents with high i and low endowments in state D vs state U for agents with low i, we could reproduce the distribution of marginal utilities we get from differences in prior probabilities. We have chosen to replace the usual marginal analysis of consumers who have interior consumption with a continuum of agents and a marginal buyer. Our view is that the slightly unconventional modeling is a small price to pay for the simple tractability of the analysis.

$s = U, D$ at time 1.

There are two assets in the economy which produce dividends of the consumption good c at time 1. The riskless asset X produces $d_U^X = d_D^X = d^X$ unit of the consumption good in each state, and the risky asset Y produces d_U^Y unit in state U and $0 < d_D^Y < d_U^Y$ unit of the consumption good in state D. Figure 1 shows asset payoffs.

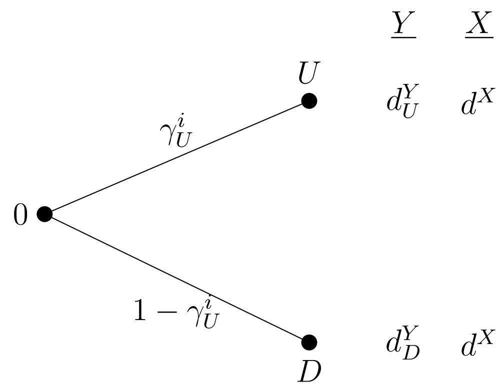

Figure 1: Asset Payoffs.

We can always normalize one price in each state $s \in S = \{0, U, D\}$, so we take the price of X in state 0 and the price of consumption in each state U,D to be one. Thus X is both riskless and the numeraire; hence, it is in some ways analogous to a durable consumption good like gold, or to money, in our one commodity model. The price of the risky asset Y at time 0 is denoted by p.

Agents

We suppose that agents are uniformly distributed in $(0, 1)$ described by Lebesgue measure. Each investor in the continuum $i \in I = (0, 1)$ is risk-neutral and characterized by a linear utility for consumption of the single consumption good c at time 1, and subjective probabilities, $(\gamma_U^i, \gamma_D^i) = (\gamma(i), 1 - \gamma(i))$. The von-Neumann-Morgenstern expected utility to agent i is

$$U^i(c_U, c_D) = \gamma_U^i c_U + \gamma_D^i c_D. \tag{1}$$

We shall suppose that $\gamma(i)$ is strictly monotonically increasing and continuous in i. Since only the output of Y depends on the state and $0 < d_D^Y < d_U^Y$, higher i denotes more optimism. Heterogeneity among the agents stems entirely from the dependence of $\gamma(i)$ on i.

Each investor $i \in (0,1)$ has endowments e^X and e^Y of assets X and Y respectively at time 0 and nothing else. Hence, consumption at time 1 is entirely derived from asset dividends.

Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. We explicitly incorporate repayment enforceability problems, but exclude cash flow problems.[8] Agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance. Agents need to post collateral in the form of durable assets in order to issue promises. But there is no doubt what the collateral will pay, conditional on the future state of nature.

At time 0 agents can trade financial contracts. A financial contract $j = ((j_U, j_D), C_j)$ consists of both a promise, (j_U, j_D) of repayment in terms of the consumption good at each state, and an asset acting as collateral backing it, $C_j \in \{X, Y\}$.

The lender has the right to seize as much of the collateral as will make him whole once the promise comes due, but no more: the contract therefore delivers

$$(\min(j_U, d_U^{C_j}), \min(j_D, d_D^{C_j})),$$

in the two states.

Notice that there are no cash flow problems: the value of the collateral in each future state does not depend on the size of the promise, or on what other choices the seller makes, or on who owns the asset at the very end. This eliminates any issues associated with hidden effort or unobservability.

We shall suppose every contract is collateralized either by one unit of X or by one unit of Y. The set of promises j backed by one unit of X is denoted by J^X and the set of contracts backed by one unit of Y is denoted by J^Y. We denote the total set of contracts by $J = J^X \cup J^Y$.

[8] For an extensive analysis on the of the implications on asset prices, leverage and production arising from the distinction see Fostel and Geanakoplos (2015a,b).

We shall denote the sale of promise j by $\varphi_j > 0$ and the purchase of the same contract by $\varphi_j < 0$. The sale of a contract corresponds to borrowing the sale price, and the purchase of a promise is tantamount to lending the price in return for the promise. The sale of $\varphi_j > 0$ units of contract type $j \in J^{C_j}$ requires the ownership of φ_j units of C_j, whereas the purchase of the same number of contracts does not require any ownership of C_j.

Each contract $j \in J$ will trade for a price π^j. An investor can borrow π^j today by selling contract j in exchange for a promise of (j_U, j_D) tomorrow, provided she owns C_j.

Budget Set

Given asset and contract prices at time 0, $(p, (\pi^j)_{j \in J})$, each agent $i \in I$ choses her asset holdings x of X and y of Y and contract trades φ_j in state 0, and consumption in final states c_U, c_D, in order to maximize utility (1) subject to the budget set defined by

$$B^i(p, \pi) = \{(x, y, \varphi, c_U, c_D) \in R_+ \times R_+ \times R^J \times R_+ \times R_+ :$$

$$(x - e^X) + p(y - e^Y) \leq \textstyle\sum_{j \in J} \varphi_j \pi^j$$

$$\textstyle\sum_{j \in J^X} \max(0, \varphi_j) \leq x, \sum_{j \in J^Y} \max(0, \varphi_j) \leq y$$

$$c_s = x d^X + y d_s^Y - \textstyle\sum_{j \in J} \varphi_j \min(j_s^j, d_s^{C_j}), s = U, D\}$$

At time 0 expenditures on the assets purchased (or sold) can be at most equal to the money borrowed selling contracts using the assets as collateral. The assets put up as collateral must indeed be owned. In the final states, consumption must equal dividends of the assets held minus debt repayment.

Collateral Equilibrium

A Collateral Equilibrium in this economy is a price of asset Y, contract prices, asset purchases, contract trades and consumption decisions by all the agents $((p, \pi), (x^i, y^i, \varphi^i, c_U^i, c_D^i)_{i \in I}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R^J \times R_+ \times R_+)^H$ such that

1. $\int_0^1 x^i di = e^X$
2. $\int_0^1 y^i di = e^Y$
3. $\int_0^1 \varphi_j^i di = 0 \; \forall j \in J$
4. $(x^i, y^i, \varphi^i, c_U^i, c_D^i) \in B^i(p, \pi), \forall i$
5. $(x, y, \varphi, c_U, c_D) \in B^i(p, \pi) \Rightarrow U^i(x) \leq U^i(x^i), \forall i$

All markets clear in equilibrium, and agents optimize their utility in their budget sets. As shown by Geanakoplos and Zame (2014), equilibrium in this model always exists under the assumptions we made so far.

Collateral and Financial Innovation

A vitally important source of financial innovation involves the possibility of using assets as collateral to back promises. We define financial innovation as the use of new kinds of collateral, or new kinds of promises that can be backed by the same collateral. Financial innovation in our model then, is described by a different set J. In Sections 3 and 4 we will analyze different economies obtained by varying the set $J = J^X \cup J^Y$.

These simplifications are to reflect in the simplest setting key differences in available financial instruments. Financial systems differ in a myriad of both subtle and complex ways—for example, the level of insurance and risk sharing—but the salient features that we are focusing on are the ability to leverage, securitize and tranche assets, which is reflected in the financial structures we assume. We model investors as borrowing directly against assets and issuing tranches against the assets, but this could capture the role of financial intermediary in producing the financial assets that correspond to these cash flows.

3 A Static Model of Global Flows

We now consider a model with two countries, Home and Foreign, each defined as the economy in Section 2. Both countries are identical in every way except for the feasible contracts available in each country J^H and J^F. In both countries there is a risky asset Y^H and Y^F and a riskless asset X^H and X^F with identical state-payoffs as defined in Figure 1. In each country there is a unit continuum of investors, denoted by I^H and I^F. Risky assets prices in each country are denoted by p^H, p^F.

We suppose that Home has a more advanced financial system than Foreign, and to capture this we will assume that agents in the foreign country are not able to use assets as collateral to back promises using contracts, that is will let $J^F = \emptyset$. In contrast, at Home agents will be able to use the risky asset as collateral and hence $J^H = J^Y \neq \emptyset$. We will consider two different forms of financial innovations at Home: leverage and tranching. In the first agents can use the risky asset Y^H to issue non-contingent promises (debt), whereas in the second agents can issue contingent promises against the risky asset. Hence J^H will vary to reflect financial innovation at Home.

More precisely, we consider two variations when analyzing financial integration: (1) Home can leverage assets and countries can trade bonds and assets, and (2) Home can tranche assets and countries can trade tranches and assets.[9]

The main insight coming from the static model of this section is that the difference in financial systems as expressed in J^H and J^F, as well as the financial innovations at home expressed in changes in the set J^H, are enough to generate international capital flows. In our model, the are no interest rate differentials, nor hedging or risk-sharing motives to trade assets (agents are risk-neutral and assets are perfectly correlated). International financial trade is a way of internationally sharing scarce collateral. Moreover our model provides precise predictions on the direction of capital flows and their effect on asset prices.

Finally, in the remainder of the paper we will solve for equilibrium for the following set of parameters: agents utilities and endowments in both countries are given by $\gamma(i) = 1-(1-i)^2$, $e^Y = e^X = 1$, and asset payoffs in both countries are given by and $d^X = d^Y_U = 1$ and $d^Y_D = .2$.[10]

[9]In the Appendix we consider when the Home can tranche assets and countries can trade tranches but not the risky assets, and we consider when the Foreign financial sector can leverage assets, but only against debt, while the Home financial sector can tranche assets.

[10]The results are robust to the choice of parameters, see for Example Fostel and Geanakoplos (2012).

3.1 Leverage and Global Flows

The financial sector in Home is able to issue non-contingent promises using the asset as collateral. In this case $J^H = J^Y = \{j : j = ((j, j), 1)\}$. As mentioned before $J^F = \emptyset$. We first describe autarky equilibrium in each country and then describe the equilibrium with financial integration.

3.1.1 Autarky: Foreign

In autarky, Foreign agents can only trade assets Y^F and X^F; they cannot borrow using the assets as collateral. Because of the strict monotonicity and continuity of $\gamma(i)$ in i, and the linear utilities and the connectedness of the set of agents $I = (0, 1)$, at state $s = 0$, in equilibrium there is a unique marginal buyer, i^{Y^F}, who is indifferent between holding Y^F and X^F. All agents $i > i^{Y^F}$ buy all they can afford of Y^F while selling all their endowment of X. Agents $i < i^{Y^F}$ sell all their endowment of Y^F and hold X^F.

Equilibrium is described by a system of two equations in two unknowns: the price of the asset, p^F, and the marginal buyer, i^{Y^F}. The equations are

$$1 = (1 - i^{Y^F})\frac{(1 + p^F)}{p^F}, \tag{2}$$

and

$$p^F = \gamma(i^{Y^F})1 + (1 - \gamma(i^{Y^F}))d_D^Y. \tag{3}$$

Equation (2) is market clearing for the risky asset; equation (3) is the optimality condition, which says that the marginal buyer is willing to buy the asset (as a result, all other agents strictly prefer their portfolio choices). For the parameters discussed before equilibrium is given by $i^{Y^F} = .54$ and $p^F = .83$.

3.1.2 Autarky: Home

Home agents are allowed to borrow by issuing non-contingent promises using the risky asset as collateral. The following result regarding leverage holds in the C-model considered in

Section 2.

Proposition 1: *Suppose that in equilibrium the max min contract* $j^* = min_{s=U,D}\{d_s^Y\} = d_D^Y$ *is available to be traded, that is* $j^* \in J = J^Y$. *Then* j^* *is the only contract traded, and the risk-less interest rate is equal to zero, this is,* $\pi^{j^*} = j^* = d_D^Y$.

Proof: See Geanakoplos (2003) and Fostel and Geanakoplos (2011, 2012).

Leverage is endogenously determined in equilibrium. In particular, the proposition derives the conclusion that although all contracts will be priced in equilibrium, the only contract actively traded is the max min contract, ruling out default in equilibrium.

Knowing which contract is traded in equilibrium greatly simplifies equilibrium calculations. As before, there is a marginal buyer, i^{Y^H}, who is indifferent between buying Y^H on margin and holding X^H. All agents $i > i^{Y^H}$ buy all they can afford of Y^H, i.e., they sell all their endowment of the X and borrowd_D^Y using Y^H as collateral. Notice that when agents buy the asset Y^H on margin, they are effectively buying the Arrow U security, thus they end up consuming only at state U. Agents $i < i^{Y^H}$ sell all their endowment of Y^H and lend to the more optimistic investors, holding the riskless bond and X^H.

Equilibrium is described by a system of two equations in two unknowns: the price of the asset, p^H, and the marginal buyer, i^{Y^H}. Equations are

$$1 = (1 - i^{Y^H})\frac{(1 + p^H)}{p^H - d_D^Y}, \tag{4}$$

$$p^H = \gamma(i^{Y^H})1 + (1 - \gamma(i^{Y^H}))d_D^Y. \tag{5}$$

Notice how equation (4) differs from equation (2). Optimists now can borrow d_D^Y, implying that in equilibrium a fewer number of optimists can afford to buy all the asset in the economy. Hence, the marginal buyer in the Home economy will be someone more optimistic than the marginal buyer in the Foreign economy.

For the parameters discussed before equilibrium is given by $i^{Y^H} = .63$ and $p^H = .89$. Notice that the both the marginal buyer and the asset price are higher at Home than at

Foreign in autarky. This is due to the fact that Y^H has a collateral value.[11]

3.1.3 Financial Integration with Leverage

We now suppose that Home and Foreign agents can trade assets and bonds, and any agent can use the Home asset Y^H as collateral in order to borrow. As we will see, financial integration will affect equilibrium even though assets in both countries have identical payoffs and agents are identical. This is because, crucially, Y^H can serve as collateral. Investors that buy Y^H are able to borrow against their asset purchases and leverage their positions. Optimistic investors in Foreign believe that U is very likely—they would prefer to buy Y^H with leverage than Y^F. At the same time, moderate investors in Home that are unwilling to buy Y^H with leverage (remember the price is high) would be willing to buy Y^F at a lower price without leverage. Thus, cheap Foreign assets are attractive to Home investors who are otherwise priced out of the Home market by optimistic, leveraged investors.

We denote equilibrium variables after financial integration by a 'hat' ($\hat{}$) to distinguish them from their autarky counterparts. In equilibrium, in each country there is a marginal investor $\hat{i}^{Y^H}$, who is indifferent between buying Y^H with leverage and buying Y^F without leverage, and there is a marginal investor $\hat{i}^{Y^F}$, who is indifferent between buying Y^F and holding risk-free assets (X or debt). The marginal investors in each country are the same because the assets are have identical payoffs and agents have the same beliefs $\gamma(i)$. Investors $i \geq \hat{i}^{Y^H}$ in both countries buy Y^H on margin (hence holding the Arrow U security); investors $i \in (\hat{i}^{Y^F}, \hat{i}^{Y^H})$ in both countries buy Y^F with cash; and the remaining investors hold X^H and X^F, and diskless debt. The equilibrium is shown in Figure 2.

Equilibrium is described by a system of four equations in four unknowns: the prices of the assets, $\hat{p}^F$ and $\hat{p}^H$, and the marginal buyers, $\hat{i}^{Y^F}$ and $\hat{i}^{Y^H}$. The marginal buyer $\hat{i}^{Y^H}$ is indifferent between the leveraged return on Y^H and the un-leveraged return Y^F; hence

$$\frac{\gamma(\hat{i}^{Y^H})(1-d_D^Y)}{\hat{p}^H - d_D^Y} = \frac{\gamma(\hat{i}^{Y^H}) + (1-\gamma(\hat{i}^{Y^H}))d_D^Y}{\hat{p}^F}. \tag{6}$$

[11]See Geanakoplos (2003) and Fostel and Geanakoplos (2008) for an early treatment of Collateral Value.

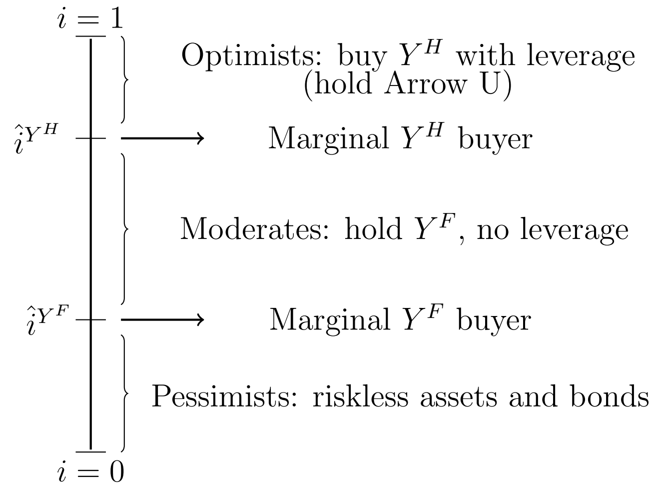

Figure 2: Equilibrium Regime with Leverage and Financial Integration.

The marginal buyer $\hat{i}^{Y^F}$ is indifferent between the un-leveraged return on Y^F and holding the risk-free asset (and bond); hence,

$$\hat{p}^F = \gamma(\hat{i}^{Y^F})1 + (1 - \gamma(\hat{i}^{Y^F}))d_D^Y. \tag{7}$$

Market clearing conditions for Y^H and Y^F are given by[12]

$$1 = (1 - \hat{i}^{Y^H})\frac{\left(2 + \hat{p}^H + \hat{p}^F\right)}{\hat{p}^H - d_D^Y}, \tag{8}$$

[12] To calculate the market clearing condition for Y^H, remember that investors will buy $\frac{1}{\hat{p}^H - d_D^Y}$ units of the asset for every unit of wealth. Investors in H are endowed with X^H and Y^H and thus have wealth $1 + \hat{p}^H$; similarly, investors in F have wealth $1 + \hat{p}^F$. Since all investors $i \geq \hat{i}^{Y^H}$ in each country buy the Home asset, the total global demand is given by

$$\int_{\hat{i}^{Y^H}}^{1} \left(\frac{1 + \hat{p}^H}{\hat{p}^H - d_D^Y}\right) di + \int_{\hat{i}^{Y^H}}^{1} \left(\frac{1 + \hat{p}^F}{\hat{p}^H - d_D^Y}\right) di = (1 - \hat{i}^{Y^H})\left(\frac{2 + \hat{p}^H + \hat{p}^F}{\hat{p}^H - d_D^Y}\right)$$

Equating with the unit supply of Y^H yields the market clearing condition for the Home asset. The same exercise yields the result for Y^F.

and

$$1 = (\hat{i}^{Y^H} - \hat{i}^{Y^F}) \frac{(2 + \hat{p}^H + \hat{p}^F)}{\hat{p}^F}. \tag{9}$$

Notice that compared to (4), the market clearing condition (8) includes wealth from *both* countries. Therefore, the marginal buyer $\hat{i}^{Y^H}$ increases as a result of trade. Notice that compared to equation (5), in equation (6) the alternative return to leveraging Y^H is not a risk-free return but rather the un-leveraged return on Y^F. For investor $\hat{i}^{Y^H}$ the un-leveraged return on Y^F is greater than 1—it is a more attractive alternative than cash. This effect will tend to increase the return to leveraging Y^H, which will decrease the price $\hat{p}^H$ even though the marginal investor will increase.

For the same parameter values in equilibrium we get $\hat{p}^H = .886$, $\hat{p}^F = .864$, $\hat{i}^{Y^H} = .82$, and $\hat{i}^{Y^F} = .59$. Table 1 summarizes the results of Section 3.1.

Table 1: Equilibrium Prices and Marginal Investors with Leverage.

	Autarky	Financial Integration ($\hat{}$)
Home Asset Price: p^H	.893	.886
Foreign Asset Price: p^F	.834	.864
Marginal Y^H Investor: i^{Y^H}	.63	.82
Marginal Y^F Investor i^{Y^F}	.545	.59

The effects of financial integration on marginal investors and financial flows are pictured in Figure 3

Leverage and Global Flows

We can summarize the main results on this section as follows.

First, differences in the ability to collateralize financial promises across borders are enough to generate international capital flows. In our model, the are no interest rate differentials, nor hedging or risk-sharing motives to trade assets (agents are risk-neutral and assets have identical payoffs). International financial trade is a way of internationally sharing scarce collateral.

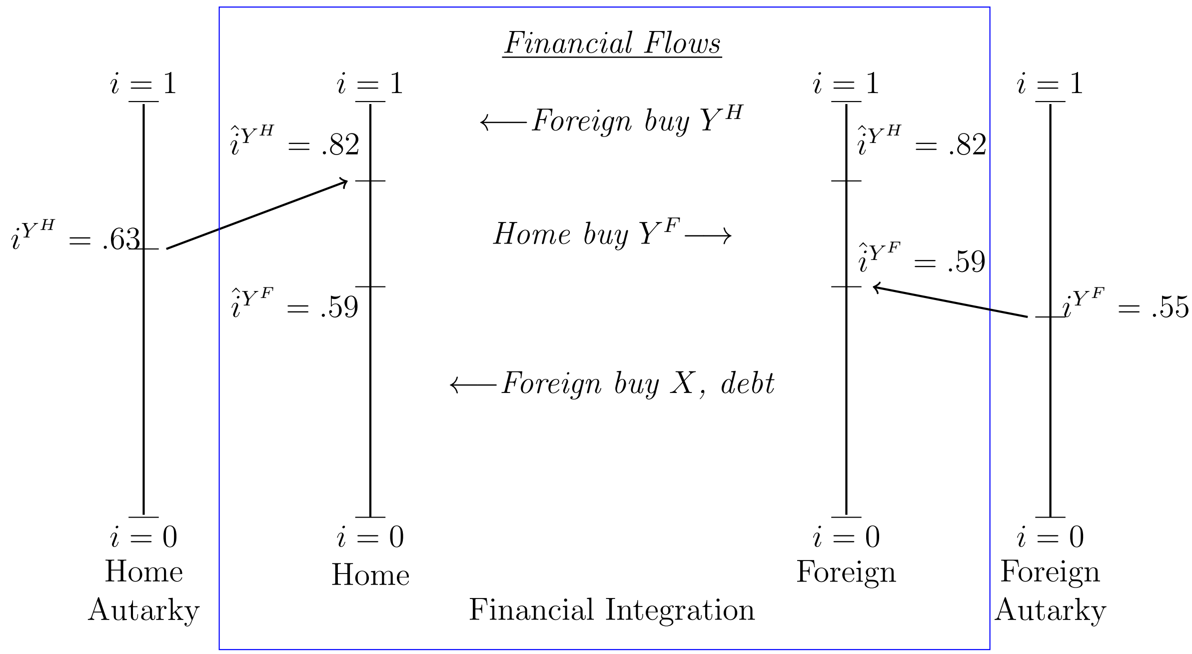

Figure 3: Equilibrium with Financial Integration and Leverage.

Second, our model provides precise predictions on the direction of capital flows and their effect on asset prices. Global capital flows in equilibrium are: (1) optimistic Foreign investors buy Home assets (capital flows to Home and assets flow to Foreign); (2) moderate Home investors buy Foreign assets (capital flows to Foreign); (3) pessimistic Foreign investors buy risk-free assets from Home (capital flows to Home).[13] The effect on asset valuation is subtle. The asset price at Home stays pretty stable while the asset price at Foreign increases after trade.

To understand better these effects, let us look at the effect of financial trade on marginal buyers. As a result of financial integration, the marginal buyers of *both* assets increase. First, optimistic Foreign investors desire to buy the Home asset with leverage, and this increases the marginal buyer. Investors in both countries achieve the same amount of leverage, but

[13]Notice that Foreign wealth has increased ($1 + p^F < 1 + \hat{p}^F$) and more investors hold risk-free assets ($i^{Y^F} < \hat{i}^{Y^F}$), implying a greater demand for risk-free assets. Since the supply of X^F has not changed, the demand is being met by debt backed by the Home asset.

investors in Home, who are endowed with the more expensive asset, buy more units of the asset because they start with more wealth ($\hat{p}^H > \hat{p}^F$). However, while the marginal buyer of the Home asset increases, the effect on the price of the Home asset is complicated by the fact that the alternative investment (the Foreign asset) is more attractive. Leveraged investors are now comparing the leveraged return to a (very attractive to them) un-leveraged return in the Foreign asset. Thus, even though the marginal investor is more optimistic, the required return is higher and so the asset price decreases. There are two opposing forces: greater optimism and a more attractive investment alternative. The overall effect on the Home price is ambiguous.[14] Finally, moderate Home investors (though more optimistic than the autarkic marginal buyer of Y^F) seek to buy the Foreign asset, which increases both the marginal buyer and the asset price.

Notice that still after financial integration there is a gap in prices between Home and Foreign assets due to the Home asset role as collateral. However, this gap is smaller than the one observed in autarky.

3.2 Tranching and Global Flows

We now suppose that the Home financial sector can create tranches using the risky asset as collateral. We suppose that $J^H = J^Y$ consists of the single promise $j^T = ((0, d_D^Y), 1)$. A moment's reflection should convince the reader that in our two state economy, completely tranching Y is tantamount to allowing the asset to back a promise of d_D^Y in the down state. The asset holder on net then retains the U Arrow security. By buying y units of Y and selling off y units of the tranche $j^T = (0, d_D^Y)$, and also buying z/d_D^Y units of the down tranche (perhaps created by somebody else), any agent who has enough wealth can effectively purchase the arbitrary consumption $c_U = y, c_D = z$. If it were possible to create different tranches beyond the two Arrow securities, no agent would have anything to gain by doing so. In the end his new tranches would not offer a potential buyer anything the buyer could not obtain for himself via the Arrow tranches, as we just saw. With unlimited and

[14] The "alternative return" force tends to dominate when the function $\gamma(i)$ is concave. The Home price can increase if $\gamma(i)$ is sufficiently convex.

costless tranching, tranching into Arrow securities always drives out all alternative tranching schemes, a point made in Geanakoplos and Zame (2014). Since $j^T = (0, d_D^Y)$ already embodies Arrow tranching, there is no reason to consider any more complicated tranching schemes.

As we did with the case of leverage, we first consider the autarky equilibrium and then move on to consider the financial integration equilibrium. The autarky Foreign equilibrium is as described in Section 3.1.1, since changes in J^H do not affect the Foreign economy.

3.2.1 Autarky: Home

Consider the autarkic equilibrium at Home. Equilibrium in this case is more subtle. There are two marginal buyers i^{Y^H} and i^T. All agents $i > i^{Y^H}$ buy all of Y^H, and sell the down tranche $j^T = (0, d_D^Y)$, hence effectively holding the Arrow U security. Agents $i^T < i < i^{Y^H}$ sell all their endowment of Y^H and purchase all of the durable consumption good X. Finally, agents $i < i^T$ sell their assets Y^H and X and buy the down tranche $j^T = (0, d_D^Y)$, from the most optimistic investors (hence effectively buying the Arrow D security). The equilibrium regime is shown in Figure 4.

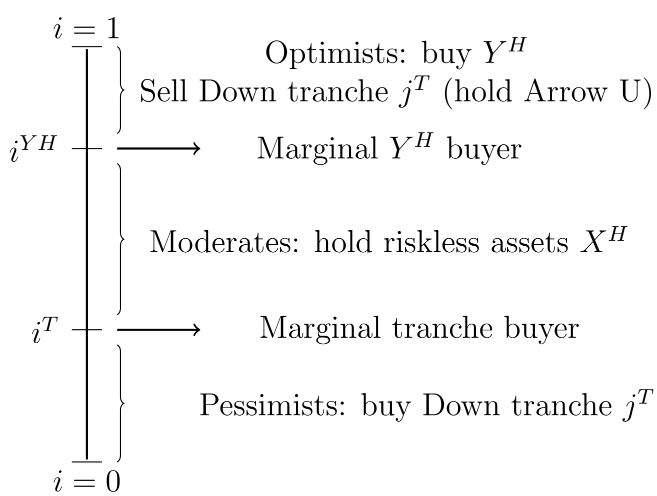

Figure 4: Equilibrium Regime with Tranching in Autarky.

Equilibrium is described by a system of four equations and four unknowns: the price of the asset, p^H, the price of the down tranche π^T, and the two marginal buyers, i^{Y^H} and i^T.

Market clearing for the asset Y^H and the tranche j^T are

$$1 = (1 - i^{Y^H}) \frac{(1 + p^H)}{p^H - \pi^T}, \tag{10}$$

and

$$i^T (1 + p^H) = \pi^T. \tag{11}$$

The top $1 - i^{Y^H}$ agents are buying the asset and selling off the down tranche: they each have wealth $1 + p^H$ plus the revenue from the tranche sale π^T. The bottom i^T agents spend their endowments to buy the tranche. There is 1 unit of total supply of the asset, implying there is one unit supply of the tranche.

Optimality conditions are given by

$$\gamma(i^{Y^H}) = p - \pi^T, \tag{12}$$

and

$$(1 - \gamma(i^T)) d_D^Y = \pi^T, \tag{13}$$

in each state the value of the Arrow U and D securities created through tranching are correctly priced by the marginal agent. Finally, notice that despite the fact that both Arrow securities can be created through tranching the asset, markets are not complete. Arrow securities are created through the risky asset only. Hence, agents cannot sell all the Arrow securities they desire and the Arrow-Debreu allocation cannot be implemented. Tranching the risky asset is not enough to complete markets.[15]

For the same parameter values in equilibrium we get $i^{Y^H} = .58$, $i^T = .08$, $p^H = 1$, and $\pi^T = .17$.

The asset price is much higher even than it was with leverage. The tranching of the asset Y that was begun with leverage is perfected by tranching into Arrow securities. Leverage is

[15]In Arrow Debreu Equilibrium there is a unique marginal buyer, above which all agents buy Arrow U, and below which all agents buy Arrow D. The reason why collateral equilibrium cannot implement Arrow Debreu is that the riskless asset cash flows cannot be tranched into Arrow securities.

a precursor or primitive form of tranching. With leverage the asset can be used as collateral to issue non-contingent promises, and hence its cash flows are tranched into an Arrow U security and a diskless bond. In the tranching economy, the asset can be used as collateral to issue contingent promises, and hence its cash flows are trenched into Arrow securities. This more sophisticated tranching scheme increases even further its value as collateral.

3.2.2 Financial Integration with Tranching

We now suppose that Home and Foreign agents can trade assets and tranches, and any agent can issue tranches by using the Home asset Y^H collateral.

In equilibrium in each country there are three marginal buyers: $\hat{i}^{Y^H}$, who is indifferent between buying Y^H against a tranche and buying Y^F with cash, $\hat{i}^{Y^F}$, who is indifferent between buying Y^F with cash and holding risk-free assets, and $\hat{i}^T$ who is indifferent between buying the Down tranche and holding risk-free assets. Investors in both countries with $i \geq \hat{i}^{Y^H}$ buy Y^H and issue the down tranche using the asset as collateral, and hence hold the Arrow U security. Investors $i \in [\hat{i}^{Y^F}, \hat{i}^{Y^H})$ buy Y^F with cash. Investors with $i \in (\hat{i}^T, \hat{i}^{Y^F})$ hold the riskless assets X^H and X^F. Finally, investors $i \leq \hat{i}^T$ buy the down tranche (hence holding the Arrow D).

Equilibrium is described by a system of six equations in six unknowns: the prices of the assets and the tranche, $\hat{p}^H$, $\hat{p}^F$, and $\hat{\pi}^T$, and the marginal buyers, $\hat{i}^{Y^H}$, $\hat{i}^{Y^F}$, and $\hat{i}^T$. The equations for the marginal valuation and market clearing for the Foreign asset and for the tranche are given by equations (7), (9), (13), and (14).

$$\hat{i}^T \left(2 + \hat{p}^H + \hat{p}^F\right) = \hat{\pi}^T. \tag{14}$$

The marginal buyer $\hat{i}^{Y^H}$ is indifferent between the leveraged return on Y^H, issuing a down tranche, and the un-leveraged return Y^F. We have

$$\frac{\gamma(\hat{i}^{Y^H}) \times 1}{\hat{p}^H - \hat{\pi}^T} = \frac{\gamma(\hat{i}^{Y^H}) + (1 - \gamma(\hat{i}^{Y^H}))d_D^Y}{\hat{p}^F}. \tag{15}$$

The market clearing condition for the Home asset is

$$1 = (1 - \hat{i}^{Y^H}) \frac{\left(2 + \hat{p}^H + \hat{p}^F\right)}{\hat{p}^H - \hat{\pi}^T}. \tag{16}$$

The equilibrium for the same parameter values is summarized in Table 2.

Table 2: Equilibrium Prices and Marginal Investors with Tranching.

	Autarky	Financial Integration ($\hat{}$)
Home Asset Price: p^H	1	1.021
Foreign Asset Price: p^F	.834	.848
Down Tranche Price: π^T	.168	.182
Marginal Y^H Investor: i^{Y^H}	.585	.783
Marginal Y^F Investor: i^{Y^F}	.545	.564
Marginal Tranche Investor: i^T	.084	.0467

The effects of financial integration on marginal investors and financial flows are pictured in Figure 5

Tranching and Global Flows

As with the case of leverage differences in the ability to collateralize financial promises across borders are enough to generate international capital flows, and our model provides precise prediction on the direction of capital flows and effect on asset prices.

First, financial flows that result from integration are as follows: (1) optimistic Foreign investors buy Home assets and sell the down tranche (capital flows to Home and assets flow to Foreign); (2) moderate Home investors buy Foreign assets (capital flows to Foreign); (3) pessimistic Foreign investors buy tranches backed by Home assets and some of the risksless Home asset. Second, both risky asset prices increase after financial integration.

There are two main differences with respect to financial integration with leverage in Section 3.1: Foreign investors buy Down tranches from home and the price of the Home risky asset increases compared to autarky. To understand better these effects, let us look

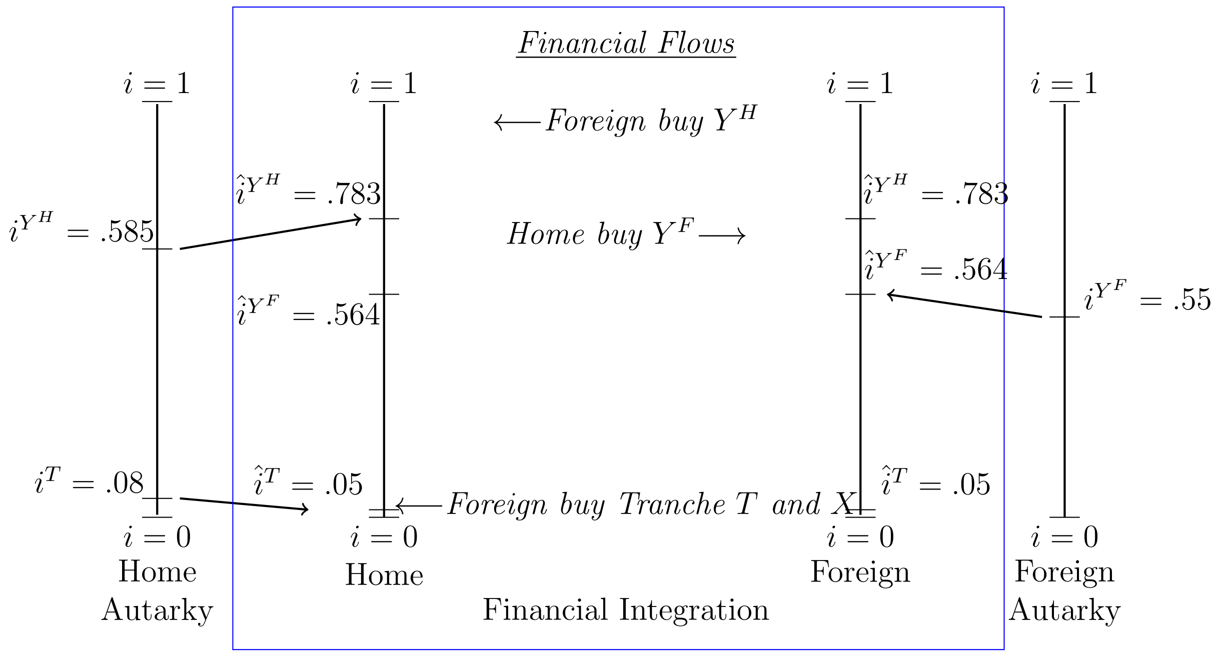

Figure 5: Equilibrium with Financial Integration and Tranching.

at the effect of financial trade on marginal buyers. As before, the marginal buyers of both risky assets increase, and the marginal buyer of the tranche decreases, which increases the price of the tranche and increases the collateral value of the Home asset.

To see the change in the asset price, notice that the asset price is given by

$$\hat{p}^H = \gamma(\hat{i}^{Y^H})1 + (1 - \gamma(\hat{i}^T))d_D^Y, \tag{17}$$

which we get from combining equations (12) and (13). Foreign demand for the asset increases $\hat{i}^{Y^H}$, and foreign demand for the tranche decreases $\hat{i}^T$. The increase in the collateral value of the Home asset due to financial integration outweighs the "alternative investment return" effect we discussed before. As a result of financial integration, pessimistic Foreign investors demand down tranches, which pushes up their price; with leverage, the demand for risk-free assets did not change the risk-free price. Now, more expensive tranches increases the value of the Home asset. In other words, financial integration increases the value of the promise backed by collateral when the promise is contingent, but with only leverage financial integration does not increase the collateral value of the asset because financial integration does not increase the value of risk-free debt.

Notice that in equilibrium the price of the Home asset is above what every agent in the economy think is worth. This result parallels the result in Harrison and Kreps (1978), who defined a bubble as a situation in which an asset is priced higher than any agent thinks its cash flows are worth. But we obtain the result in a static context without resale of the asset. They displayed bubbles in a dynamic context with heterogeneous agents: the most optimistic agent would buy the asset, but next period instead of suffering bad cash flows he could resell the asset to a different agent. In our context, tranching alone, without resale, creates collateral value and potentially bubbles (see Fostel and Geanakoplos (2012)).

Finally, while the Down tranche is a state-contingent (and therefore very risky) security, we prefer to interpret the creation of Down tranches as related to the the creation of "safe assets," which as it turns out tend to increase in value in bad states of the world. First, long-maturity bonds increase in price in bad states because long-term interest rates decline (even as the face value of the promised payoffs remain the same). Second, because the US Dollar tends to appreciate during crises, dollar-denominated bonds provide a natural hedge for foreign buyers. This is a point made in Maggiori (2013), which, in a different financial context, describes the rest of the world buying "down state" Arrow securities from Home in order to achieve safer portfolios.

4 A Dynamic Model of Global Flows

The static models illustrate how financial integration affects financial flows and how prices and marginal investors are affected. In this section we examine how the behavior of international flows affect volatility. We consider a dynamic model following Geanakoplos (2003), which demonstrates that asset prices become more volatile when assets can be used as collateral, and the collateral value of assets fluctuates in response to fundamentals. The main results of this section are that financial integration exports volatility from Home to Foreign, and when financial integration involves trade in tranches, it amplifies overall volatility.

We consider a dynamic variation of the C-model in Section 2 with three periods, $t = 0, 1, 2$. Uncertainty in the payoffs of Y^H and Y^F is represented by a tree $S = \{0, U, D, UU, DU, DD\}$ illustrated in Figure 6. The assets in each country pay identical dividends in each state, which come only in $t = 2$.

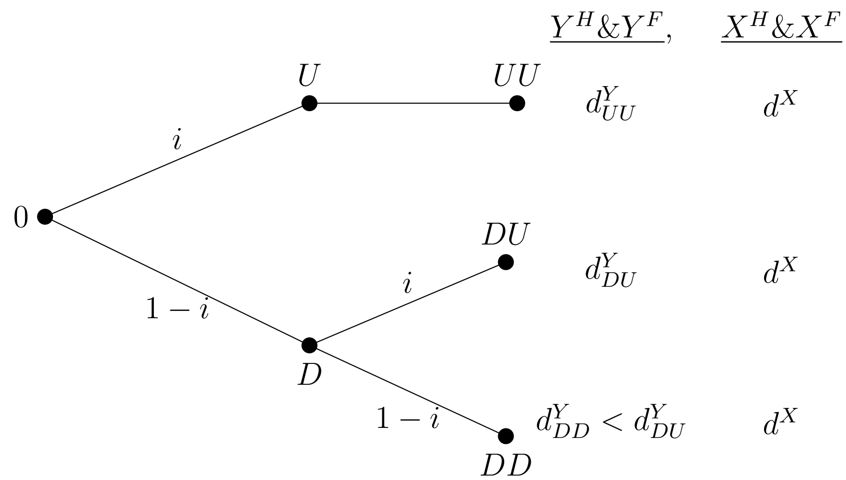

Figure 6: Asset Payoffs with Three Periods.

Agents consume only at the terminal nodes. The von-Neumann-Morgenstern expected utility to agent i is

$$U^i(c_{UU}, c_{DU}, c_{DD}) = ic_{UU} + i(1-i)c_{DU} + (1-i)^2 c_{DD}. \tag{18}$$

As in Section 2, agents have endowments of the assets only at time 0.

In the remainder of the paper we will solve and present equilibrium for the following set of parameters: agents utilities and endowments in both countries are given by $\gamma(i) = i$.[16]. Agents endowments at 0 are as before, $e^Y = e^X = 1$, and asset payoffs in both countries are given by and $d^X = d^Y_{UU} = d^Y_{DU} = 1$ and $d^Y_{DD} = .2$. As in Section 3 we will first study the case of leverage and then the case in which Home can tranche the risky asset.

[16]Notice that the beliefs at $s = 0$ about payoffs at $t = 2$ correspond to the static models with beliefs $\gamma(i) = 1 - (1-i)^2$.

4.1 Leverage Cycle and Dynamic Global Flows

We first consider when the Home economy can leverage the risky asset Y^H to issue non-contingent promises. We first describe the autarky equilibrium in each country and then describe the equilibrium with financial integration. For expositional ease, all equations for this section are provided in Appendix B.

4.1.1 Autarky: Foreign

Even though the economy is dynamic, the autarky equilibrium for Foreign is essentially the same as the static equilibrium described in Section 3. Denote the asset price in each state by p_0^F and p_D^F. Agents choose portfolios of Y^F and X at $s = 0$ and $s = D$, taking into account that in D they can rebalance their portfolio with an asset price p_D^F. However, given the monotonicity of beliefs, in $s = 0$ investors will hold all their wealth either in X or the risky asset. In $s = D$, the wealth of investors who bought the risky asset will have decreased, but these investors are the optimistic agents who continue to want to hold the risky asset, and so they do not choose to sell the asset. Because all of their wealth is in the asset, they do not have any other resources they can use to buy more of the asset at its low price. In other words, all agents will hold the same portfolios in D as they did at $s = 0$.

As a result, in both states there is a single marginal agent, $i_0^{Y^F} = i_D^{Y^F}$, and all agents $i > i_0^{Y^F}$ buy-and-hold the risky asset to maturity. The asset is thus priced by the marginal agent according to the payoff distribution at each state. For the parameter values considered before in equilibrium we get $i_0^{Y^F} = .545$, $p_0^F = .834$, and $p_D^F = .636$. The price falls in D because the bad payoff is more likely; however, there are no amplifying mechanisms. The price crashes by 23.77%.

4.1.2 Autarky: Home

With leverage the dynamic equilibrium is essentially different from the static equilibrium, though the equilibrium regimes in each state resemble the equilibrium regime in the static economy of Section 3. Denote the asset price in each state by p_0^H and p_D^H. The implication of Proposition 1 in the dynamic model is that the only contract traded at time $s = 0$ is the the max min $j^* = p^H_D$, and at $s = D$ is $j^* = d^Y_D$ (the worst case scenario avoiding default in both cases).

In $s = 0$ there is a marginal buyer $i^{Y^H}_0$ such that all investors with $i > i^{Y^H}_0$ buy Y^H on margin promising p^H_D. Crucially, in state D all investors that bought the risky asset on margin go bankrupt (they owe all the asset value and have no other wealth). This means that the asset must be bought by more pessimistic investors $i \in [0, i^{Y^H}_0]$. In state D there is a marginal buyer $i^{Y^H}_D$, such that all remaining investors with $i > i^{Y^H}_D$ buy Y^H with leverage by borrowing d^Y_D against each unit of the asset. For the same parameters the equilibrium is given by:

$$p^H_0 = .958, \quad p^H_D = .69, \quad i^{Y^H}_0 = .86, \quad i^{Y^H}_D = .61.$$

The price crashes by 27.91%, greater than the crash in Foreign precisely because of the amplifying mechanisms due to leverage. This is what Geanakoplos (2003) called the Leverage Cycle. The asset price falls in the Down state for three reasons. First, the price of the asset falls because fundamentals are worse (a bad payoff is more likely). Second, the equilibrium margin increases. In period 0 agents can borrow more against the asset than in state D, and so investors use less leverage and the marginal buyer is therefore less optimistic. Third, investors who used leverage in the first period go bankrupt in the Down state as a result of the margin call. These optimistic buyers—the most optimistic buyers—have no wealth available after repaying their debt, which leaves less optimistic investors to buy the asset. The changes in the marginal buyer and in equilibrium leverage create excess volatility; the price drops by more than can be attributed to fundamentals alone.[17]

[17] In an equivalent model with two agents with heterogeneous risk-aversion or endowments, there is an equivalent mechanism which corresponds to a change in optimism of the marginal buyer. In a two-agent model, the marginal buyer is the same but the wealth of the marginal buyer decreases in the bad state, increasing the marginal utility of consumption and causing the investor to discount future payments by more. See for example Fostel-Geanakoplos (2008).

4.1.3 Leverage Cycle and Financial Integration

We now suppose that Home and Foreign agents can trade assets and bonds, and any agent can use the Home asset Y^H as collateral in order to borrow. As in the static model, some investors in both countries will choose to hold Y^H with leverage, while some will choose to hold Y^F without leverage. In $s = D$ the amplifying mechanisms from leverage will cause the marginal buyer of Y^H to fall as leveraged agents go bankrupt and margins increase. Importantly, a decrease in the marginal investor in Y^H will decrease the marginal investor for Y^F, even though the Foreign asset is not leveraged. The amplification in the volatility of the Foreign asset is entirely driven by dynamic global flows.

In period 0 there are two marginal buyers $\hat{i}_0^{Y^H}$ and $\hat{i}_0^{Y^F}$ in both countries. The most optimistic investors, with $i \geq \hat{i}_0^{Y^H}$, buy Y^H with leverage. Intermediate buyers with $i \in [\hat{i}_0^{Y^F}, \hat{i}_0^{Y^H})$ hold Y^F un-leveraged; some of these investors hold Y^F to maturity, while some will sell Y^F to buy Y^H in state D. The most pessimistic investors, with $i < \hat{i}_0^{Y^F}$, hold risk-free claims, either X or riskless debt.

In D there are two marginal buyers $\hat{i}_D^{Y^H}$ and $\hat{i}_D^{Y^F}$ in both countries. The optimists holding Y^H go bankrupt because of the margin call. The new buyers of Y^H, with $i \in [\hat{i}_D^{Y^H}, \hat{i}_0^{Y^H})$, are investors who held Y^F in 0. However, their purchasing power is impaired because the price of the Foreign asset has decreased. Some of the Y^F investors continue to hold their investment, but the most optimistic of the Y^F investors dump the asset in D to buy Y^H. The remaining Y^F is bought by investors, with $i \in [\hat{i}_D^{Y^F}, \hat{i}_0^{Y^F})$ who held riskless claims at $s = 0$.

Denote the asset prices in each state by $\hat{p}_0^H$, $\hat{p}_0^F$, $\hat{p}_D^H$, $\hat{p}_D^F$. Table 3 shows the equilibrium asset prices and marginal investors in autarky and with financial integration. Figure 7 plots the asset prices, and Figure 8 displays marginal investors.

Table 3: Leverage Cycle and Financial Integration.

$s = 0$	Autarky	Financial Integration ($\hat{}$)
p_0^H	.958	.948
p_0^F	.834	.888
$i_0^{Y^H}$	.86	.934
$i_0^{Y^F}$	.545	.702

$s = D$	Autarky	Financial Integration ($\hat{}$)
p_D^H	.69	.696
p_D^F	.636	.659
$i_D^{Y^H}$	.61	.76
$i_D^{Y^F}$	.545	.574

Price Crash	Autarky	Financial Integration
crash in Y^H	27.91%	26.63%
crash in Y^F	23.77%	25.8%
total	51.69%	52.43%

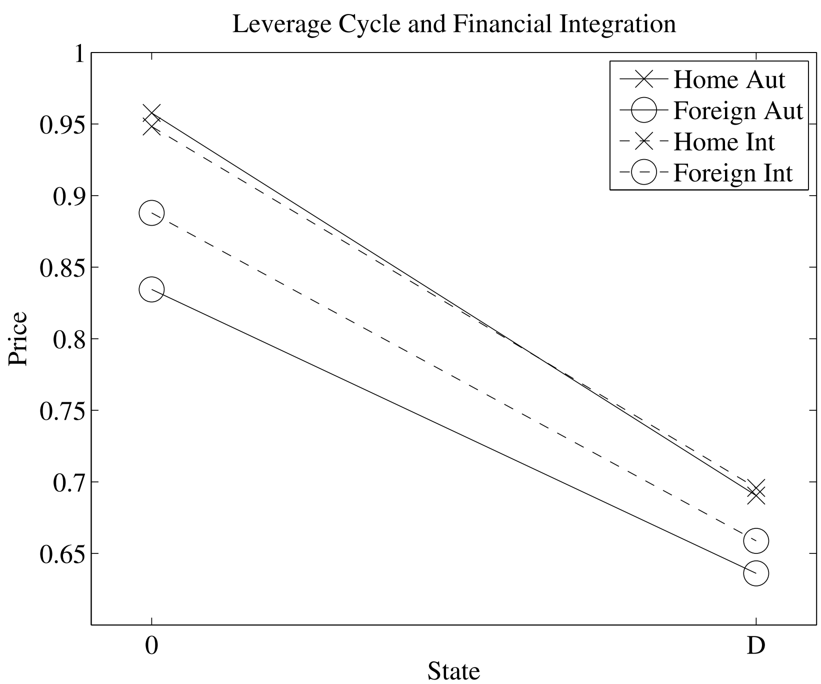

Figure 7: Dynamic Asset Prices with Financial Integration and Leverage.

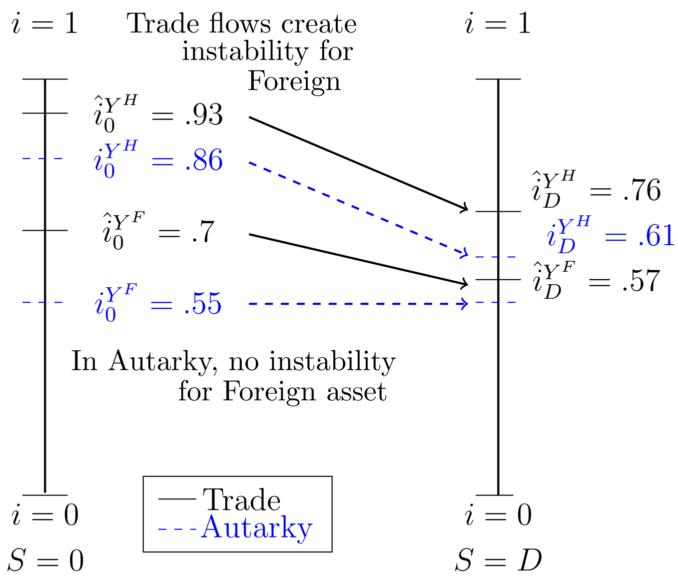

Figure 8: Marginal Investors with Financial Integration and Leverage.

The crash is slightly moderated in the domestic country, but it is exacerbated in the Foreign country. The total volatility in autarky is $27.91 + 23.77 = 51.69$ and the total volatility with trade is slightly higher, $26.63 + 25.8 = 52.43$. Even though the Foreign asset is not leveraged, the price crash is almost as large as it is in the domestic asset. The leverage cycle becomes global. As the price of Y^H fluctuates, the relative attractiveness of investing in Y^F fluctuates as well. Since Y^H has excess volatility because it is bought with leverage, Y^F has excess volatility. Notice that the state-0 Home asset price is lower with financial integration than in autarky, as was also true in the static model. However, the state-D Home asset price is higher with financial integration. The relative attractiveness of the Foreign asset is lower in D than it was at 0, or in other words, collateral becomes more valuable when margins become tougher.

4.2 Tranching and Dynamic Global Flows

We now consider the ability of Home to tranche the risky asset. We first characterize the autarky equilibrium in Home to demonstrate how tranching affects dynamics, and then we consider the equilibrium with financial integration. The autarky equilibrium in Foreign is the same as in Section 4.1.1.

4.2.1 Autarky: Home

As in the leverage economy, the marginal investors will change from $s = 0$ to $s = D$ as a result of bankruptcy and wealth distribution among agents. As we see now the Tranching cycle is even more volatile than the Leverage cycle of Section 4.1.2.

In equilibrium there are two marginal buyers in each state. In $s = 0$, investors $i > i_0^{Y^H}$ buy the risky asset and issue a down tranche promising p_D^H; investors with $i < i_0^T$ buy the down tranche, which has a price π_0^T; the remaining moderate investors hold X. In $s = D$, investors $i > i_0^{Y^H}$ go bankrupt (they owe the whole value of the asset). There is a marginal buyer $i_D^{Y^H}$ such that all remaining investors $i > i_D^{Y^H}$ buy the asset and issue a down tranche promising d_D^Y. There is a marginal buyer i_D^T such that all investors $i < i_D^T$ buy the down tranche, which has a price π_D^T. The remaining investors hold X as before. For the same parameter values we get the following equilibrium:

$$p_0^H = 1.132, \quad p_D^H = .669, \quad i_0^{Y^H} = .708, \quad i_D^{Y^H} = .482,$$

$$\pi_0^T = .509, \quad \pi_D^T = .186, \quad i_0^T = .239, \quad i_D^T = .067.$$

The price crash is now 41%, much larger than in the leverage economy. Fostel and Geanakoplos (2012) show that tranching adds an additional kick to the leverage cycle due to fluctuations in the the collateral value. The asset price with tranching price starts out higher than with leverage, 1.132 compared to .95, due to its higher collateral value. As time goes and bad news occur, the asset price all fall. Notice that at $s = D$ the asset price in both economies (tranching and leverage) is nearly the same, hence the fall is inversely related to the starting point.

4.2.2 Financial Integration with Tranching

We now suppose that Home and Foreign agents can trade assets and tranches, and any agent can issue tranches by using the Home asset Y^H as collateral.

In equilibrium in period 0 there are three marginal buyers $\hat{i}_0^{Y^H}$, $\hat{i}_0^{Y^F}$, and $\hat{i}_0^T$ in both countries. The most optimistic investors, with $i \geq \hat{i}_0^{Y^H}$, buy Y^H selling the down tranche. Intermediate buyers with $i \in [\hat{i}_0^{Y^F}, \hat{i}_0^{Y^H})$ hold Y^F outright.The most pessimistic investors, with $i < \hat{i}_0^{Y^T}$, buy down tranches, and the rest hold risk-free X^H and X^F.

After bad news, important wealth distributions takes place. In D there are three marginal buyers $\hat{i}_D^{Y^H}$, $\hat{i}_D^{Y^F}$, and $\hat{i}_D^T$ in both countries. The optimists holding Y^H go bankrupt. The new buyers of Y^H, with $i \in [\hat{i}_D^{Y^H}, \hat{i}_0^{Y^H})$, are investors who held Y^F in 0. However, their purchasing power is impaired because the price of the Foreign asset has decreased. Some of the Y^F investors continue to hold their investment, but the most optimistic of the Y^F investors dump the asset in D to buy Y^H selling the down tranche. The remaining Y^F is bought by investors, with $i \in [\hat{i}_D^{Y^F}, \hat{i}_0^{Y^F})$ who held riskless assets at $s = 0$. The most pessimistic investors, with $i < \hat{i}_D^{Y^T}$, buy down tranches.

The results are shown in Table 6. Figure 9 plots the asset prices and Figure 10 illustrates the marginal investors.

Table 4: Tranching Cycle and Financial Integration.

$s = 0$	Autarky	Financial Integration ($\hat{}$)
p_0^H	1.132	1.33
p_0^F	.834	.837
π_0^T	.509	.62
$i_0^{Y^H}$	.708	.831
$i_0^{Y^F}$	.545	.63
i_0^T	.239	.15

$s = D$	Autarky	Financial Integration ($\hat{}$)
p_D^H	.669	.734
p_D^F	.636	.6
π_D^T	.186	.192
$i_D^{Y^H}$	.482	.65
$i_D^{Y^F}$	.545	.5
i_D^T	.067	.039

Price Crash	Autarky	Financial Integration
crash in Y^H	41%	44.8%
crash in Y^F	23.8%	28.3%
total	64.7%	73.1%

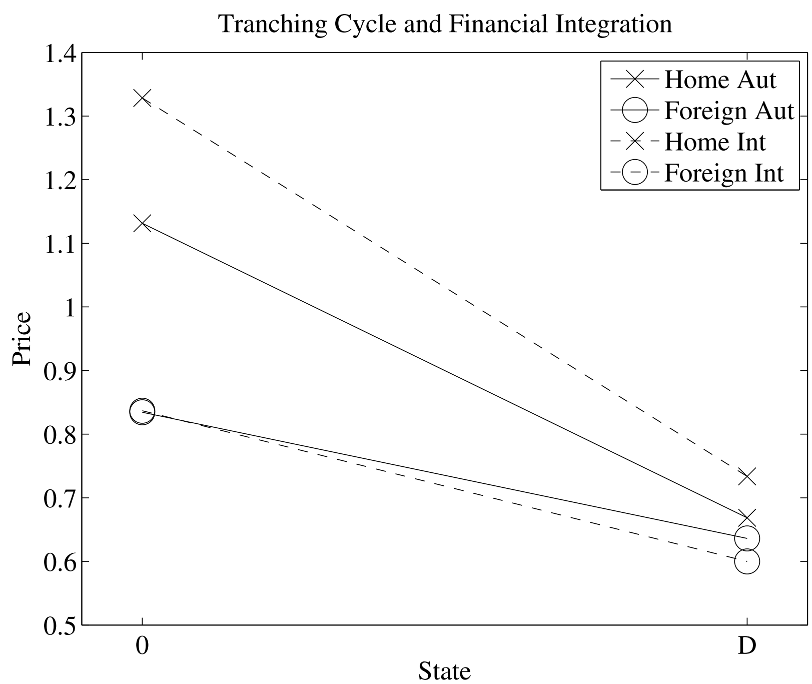

Figure 9: Dynamic Asset Prices with Financial Integration and Tranching.

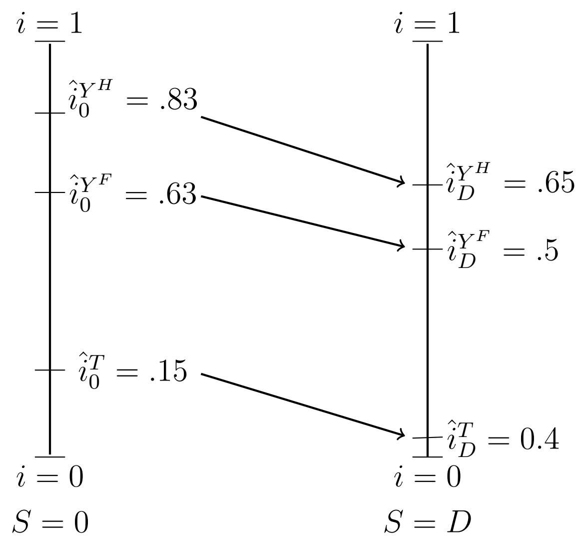

Figure 10: Marginal Investors with Financial Integration and Tranching.

There is an even larger crash in the Home asset, and the crash in the Foreign asset price is larger than it was without tranching—larger even than the crash in the leverage economy in autarky. Tranching increases volatility everywhere. Financial integration has increased the value of tranching, which increased the collateral value of the Home asset and therefore increased the price volatility of the Home asset. Because the Foreign asset is priced relative to the Home asset, the Foreign asset is much more volatile as well.[18]

5 Conclusion

We have presented a two country general equilibrium model with collateralized lending and tranching in which global capital flows are driven by different levels of financial innovation across countries. Financial integration provides Foreign access to attractive Home financial assets, and cross-border flows arise in both directions in response to changes in the available

[18]To isolate the effect of the increase in the collateral value, in Appendix A.1 we consider when countries can trade only in tranches but not in the risky assets. Because there is no trade in Y^F, the volatility of the Foreign asset does not increase relative to autarky. However, the volatility of the Home asset increases because of dynamic trade in tranches.

financial contracts and as a result of general equilibrium changes in the prices of currently available assets. These flows arise as a way for countries to share scarce collateral and to trade contingent claims, not to share risk or to gain access to risk-free assets. The resulting flows have important consequences for financial stability because global flows are dynamic and adjust in response to fundamentals.

Our results have important policy implications. First, we highlight the importance of monitoring cross-border gross banking flows, since it is precisely the changes in these flows, even when they net out, that exacerbate and transmit volatility. Second, this is a word of caution to countries whose financial systems are less developed. While there are benefits to accessing financial markets in financially developed countries (as other authors have pointed out), these benefits come at the cost of importing financial instability that naturally arises as a result of leverage. Macro-prudential policy, even global macro-prudential policy, becomes critical.

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Appendices

A Other Financial Environments

A.1 Financial Integration with Trade in Tranches but Not Assets

Suppose that countries can trade financial assets—promises backed by collateral—but not the asset itself. In this case, only Home investors can buy the Home asset, but there is Foreign demand for the tranches backed by the asset, and so there will be trade in tranches. There will be trade in the down tranche, so we refer to this case as "T-Trade." This will have no effect on the price of the Foreign asset, which is purchased without leverage, but it will increase the price of the Home asset. The Home asset is bought by issuing a down tranche against the collateral. Trade in the tranche will increase the price of the tranche and thus increase the amount of assets that *Home* investors can purchase using tranching.

Equilibrium looks as follows: there is a marginal Home investor for the Home asset, i^{Y^H}, and all Home investors with $i \geq i^{Y^H}$ buy Y^H by issuing a down tranche in order to borrow. There is a marginal investor for the Foreign asset i^{Y^F} and all Foreign investors with $i \geq i^{Y^F}$ buy Y^F without leverage. There is a marginal investor i^T such that all Home *and* Foreign investors with $i \leq i^T$ buy the down tranche. Intermediate (moderate) Home investors with $i \in (i^T, i^{Y^H})$ and Foreign investors $i \in (i^T, i^{Y^F})$ hold the riskless assets X^H, X^F. Optimists buy assets, moderates hold debt, pessimists hold tranches.

Let us describe the system of equations that summarizes equilibrium. Because there is no trade in assets, the marginal valuations and market clearing conditions for Y^H, Y^F are as they were in the respective situations in autarky, i.e., given by equations (2), (3), (10), and (12). The conditions for the tranche are given by equations (13) and (14).

The equilibrium with $\gamma(i) = 1 - (1-i)^2$ and $d^Y_D = .2$ is summarized in Table 5.

Table 5: Equilibrium Prices and Marginal Investors when Home Country Tranches Assets.

	Autarky	T-Trade
p^H	1	1.011
p^F	.834	.834
p^{AD}	.84	.91
i^{Y^H}	.585	.587
i^{Y^F}	.545	.545
i^T	.084	.0472

The dynamic model yields the following results.

Table 6: Tranching Cycle with T-Trade and TY-Trade.

Time-0	Autarky	T-Trade	TY-Trade
p_0^H	1.132	1.172	1.33
p_0^F	.834	.834	.837
p_0^{AD}	.761	.861	.85
$i_0^{Y^H}$	.708	.717	.831
$i_0^{Y^F}$	.545	.545	.63
i_0^T	.239	.139	.15

State-D	Autarky	T-Trade	TY-Trade
p_D^H	.669	.646	.734
p_D^F	.636	.636	.6
p_D^{AD}	.933	.959	.96
$i_D^{Y^H}$	.482	.454	.65
$i_D^{Y^F}$	.545	.545	.5
i_D^T	.067	.041	.039

	Autarky	T-Trade	TY-Trade
Crash in Y^H	41%	44.9%	44.8%
Crash in Y^F	23.8%	23.8%	28.3%
total	64.7%	68.7%	73.1%

With trade in tranches but not assets (T-trade), excess volatility is being driven by the change in the marginal buyer of the down security. In state D, the tranche gets bid up because (1) pessimists believe the ultimate bad state is more likely and (2) pessimists are wealthy because they bought a down security last period, which just paid off big. The equilibrium allocations are as in the 2-period static economy, but the marginal buyers for Y^H for the tranche change in state D. Because there is no trade in assets, there is no effect on the marginal buyer or the price of the Foreign asset. The marginal buyer of the Home asset changes because of the leverage cycle. How much the marginal leveraged investor changes depends on trade because the price of the tranche changes, reflecting foreign demand for the tranche.

A.2 Leverage, Tranching and Global Flows

In this section the Home financial sector can tranche assets and the Foreign financial sector can leverage, but not tranche, assets, and countries can trade both assets and tranches. We refer to this case as "TL-trade." Unlike in the other environments, we do not have segmented markets for Home and Foreign assets. To see this, suppose an investor is considering buying Y^H with leverage against a tranche and buying Y^F with leverage against debt. The return to these options are

$$\gamma(i)\frac{1}{p^H - p^{AD}d_D^Y}, \quad \gamma(i)\frac{1-d_D^Y}{p^F - d_D^Y}$$

In particular, the return in both cases in multiplicative in the investor's belief. So if any investor i^Y prefers Y^H over Y^F, every investor with $i > i^Y$ will, too, and no investor would prefer the alternative. Thus, in equilibrium both leverage and tranched-leverage must coexist. There is a marginal buyer i^Y that is willing to hold *both* assets. This requires that

$$\frac{1}{p^H - p^{AD}d_D^Y} = \frac{1-d_D^Y}{p^F - d_D^Y} \tag{19}$$

In addition, investors with $i \geq i^Y$ are willing to hold both assets. To define the fractions ϕ^H, ϕ^F of wealth devoted to each asset, market clearing requires

$$(1-i^Y)\phi^H\frac{2+p^H+p^F}{p^H - p^{AD}d_D^Y} = 1, \quad (1-i^Y)\phi^F\frac{2+p^H+p^F}{p^F - d_D^Y} = 1$$

With the previous equation, this implies that the shares satisfy

$$\phi^H = \frac{1-d_D^Y}{2-d_D^Y}, \quad \phi^F = \frac{1}{2-d_D^Y}$$

With these modifications to equilibrium conditions, we have

Table 7: Equilibrium when Home Tranches Assets and Foreign Leverages Assets.

	Autarky	TL-Trade
p^H	1	.965
p^F	.893	.827
p^{AD}	.84	.907
i^{Y^H}	.63	.535
i^{Y^F}	.545	.535
i^T	.084	.0478

Surprisingly, trade decreases the prices of all assets. This is because in equilibrium leveraged and tranched trades must coexist and all investors must be willing to hold either asset. The attractiveness of alternative investments increases for both Home and Foreign investors since both investors compare investing in the asset to a leveraged return rather than cash (either "leveraging against a tranche" or leveraging against debt).

Since high prices are often symptomatic of financial fragility (propped up by leverage), one might be inclined to think that this economy is more stable. After all, prices are not propped up as much and perhaps have less far to fall. However, the static effects mask the dynamic consequences.

Dynamic Leverage and Tranching Cycle

When Home can tranche assets and Foreign can leverage but only against debt, this situation creates important dynamics because now every investor, whether buying the Home or Foreign assets, goes bankrupt in D. Since initial prices are low and the marginal investor is low, *more* optimistic investors go bankrupt than before. We saw in the two-period model that the marginal investor is much lower with trade in this situation, and this means that there are even more defaults in D than ever before. Equilibrium now is as follows

Table 8: Leverage and Tranching Cycle.

Time-0	Autarky	TL-Trade
p_0^H	1.132	1.10
p_0^F	.958	.801
p_0^{AD}	.761	.907
$i_0^{Y^H}$	.708	.673
$i_0^{Y^F}$	.86	.673
i_0^T	.239	.137

State-D	Autarky	TL-Trade
p_D^H	.669	.619
p_D^F	.69	.542
p_D^{AD}	.933	.958
$i_D^{Y^H}$	.482	.43
$i_D^{Y^F}$	.61	.43
i_D^T	.041	.042

	Autarky	TL-Trade
Crash in Y^H	41%	43.8%
Crash in Y^F	27.91%	32.4%
total	68.91%	76.2%

Prices are more volatile in both countries. This is a warning that macro-prudential policy must consider the nature of financial entanglements and cannot simply look at asset prices as a proxy for brewing instability. If anything, it is low initial prices, allowing more investors to buy the assets with leverage, that plant the seeds of instability.

B Equilibrium Conditions for Dynamic Economies

Autarky Leverage

Let $J = J^Y$. Denote by $i_0^{Y^H}, i_D^{Y^H}$ the marginal buyers in periods 0 and 1. The equilibrium conditions with leverage and no trade are

$$(1 - i_0^{Y^H})\frac{1 + p_0}{p_0 - p_D} = 1 \tag{20}$$

$$(i_0^{Y^H} - i_D^{Y^H})\frac{1 + p_0}{p_D - d_D^Y} = 1 \tag{21}$$

$$\frac{i_0^{Y^H}(1 - p_D)}{p_0 - p_D} = i_0^{Y^H} + (1 - i_0^{Y^H})\frac{i_0^{Y^H}(1 - d_D^Y)}{p_D - d_D^Y} \tag{22}$$

$$i_D^{Y^H} + (1 - i_D^{Y^H})d_D^Y = p_D \tag{23}$$

The four equations are: market clearing for Y^H at $s = 0$, market clearing for at $s = D$, marginal valuation of Y^H at $s = 0$, and marginal valuation at $s = D$. Notice that the marginal valuation at $s = 0$ compares the value to leveraging the asset—in which case the investor gets a payment only if the next state is up—compared to holding cash and then leveraging the asset in state D.

Y-Trade

With trade, the marginal buyers in period 0 are $(\hat{i}_0^{Y^H}, \hat{i}_0^{Y^F})$ and in D are $(\hat{i}_D^{Y^H}, \hat{i}_D^{Y^F})$. The period-0 conditions are straightforward from before:

$$(1 - \hat{i}_0^{Y^H})\frac{2 + \hat{p}_0^H + \hat{p}_0^F}{\hat{p}_0^H - \hat{p}_D^H} = 1 \tag{24}$$

$$(\hat{i}_0^{Y^H} - \hat{i}_0^{Y^F})\frac{2 + \hat{p}_0^H + \hat{p}_0^F}{\hat{p}_0^F} = 1 \tag{25}$$

$$\frac{\hat{i}_0^{Y^H}(1 - \hat{p}_D^H)}{\hat{p}_0^H - \hat{p}_D^H} = \frac{\hat{i}_0^{Y^H}}{\hat{p}_0^F} + \frac{(1 - \hat{i}_0^{Y^H})\hat{p}_D^F}{\hat{p}_0^F}\frac{\hat{i}_0^{Y^H}(1 - d_D^Y)}{\hat{p}_D^H - d_D^Y} \tag{26}$$

$$\frac{1 - (1 - \hat{i}_0^{Y^F})^2(1 - d_D^Y)}{\hat{p}_0^F} = \hat{i}_0^{Y^F} + (1 - \hat{i}_0^{Y^F})\frac{\hat{i}_0^{Y^F} + (1 - \hat{i}_0^{Y^F})d_D^Y}{\hat{p}_D^F} \tag{27}$$

The first two equations are market clearing for Y^H and Y^F in period 0. The second two equations are the marginal valuation equations for Y^H and Y^F respectively. In the first case, the investor is indifferent between leveraging Y^H and investing in Y^F without leverage and then in D selling Y^F to leverage Y^H. The marginal valuation for Y^F says the investor is indifferent between holding Y^F until the end or holding cash for a period and investing in Y^F in the down state (at a better price).

In period 0, the interim buyers hold a risky asset (without leverage). Thus, the amount they can spend on asset purchases in state D depends on the asset price. Importantly, in equilibrium there are investors who hold Y^F until maturity, which means that $\hat{i}_D^{Y^H} > \hat{i}_0^{Y^F}$. All the new buyers of Y^H are agents who held Y^F last period. Thus equilibrium conditions are

$$\frac{\hat{i}_0^{Y^H} - \hat{i}_D^{Y^H}}{\hat{i}_0^{Y^H} - \hat{i}_0^{Y^F}} \frac{\hat{p}_D^F}{\hat{p}_D^H - d_D^Y} = 1 \tag{28}$$

$$\frac{\hat{i}_D^{Y^H} - \hat{i}_0^{Y^F}}{\hat{i}_0^{Y^H} - \hat{i}_0^{Y^F}} + \frac{\hat{i}_0^{Y^F} - i_D^{Y^F}}{p_D^F}(2 + \hat{p}_0^H + \hat{p}_0^F) = 1 \tag{29}$$

$$\frac{\hat{i}_D^{Y^H}(1 - d_D^Y)}{\hat{p}_D^H - d_D^Y} = \frac{\hat{i}_D^{Y^H} + (1 - \hat{i}_D^{Y^H})d_D^Y}{\hat{p}_D^F} \tag{30}$$

$$\hat{i}_D^{Y^F} + (1 - \hat{i}_D^{Y^F})d_D^Y = \hat{p}_D^F \tag{31}$$

The first equation is market clearing for Y^H. The second equation is market clearing for Y^F: the first term are agents who continue to hold Y^F and the second term are agents who sell cash for Y^F. The last two equations are marginal valuation equations. Investors in Y^H prefer the leveraged investment over un-leveraged investment in Y^F, and Y^F investors prefer that over cash. In the equilibria we find, there are some investors who bought Y^F in state 0 who continue to hold Y^F in state D, and thus no cash investors buy Y^H in D.

Autarky Tranching

Denote by $i_0^{Y^H}, i_0^T, i_D^{Y^H}, i_D^T$ the marginal buyers in state 0 and D. The equilibrium conditions with tranching and no trade are

$$(1 - i_0^{Y^H})\frac{(1+p_0)}{p_0 - p_D p_0^{AD}} = 1 \tag{32}$$

$$i_0^T(1+p_0) = p_D p_0^{AD} \tag{33}$$

$$\frac{i_0^{Y^H}}{p_0 - p_D p_0^{AD}} = i_0^{Y^H} + (1 - i_0^{Y^H})\frac{i_0^{Y^H}}{p_D - d_D^Y p_D^{AD}} \tag{34}$$

$$1 - i_0^T = p_0^{AD} \tag{35}$$

$$p_D - d_D^Y p_D^{AD} = (i_0^{Y^H} - i_D^{Y^H})(1+p_0) \tag{36}$$

$$\frac{i_D^T}{i_0^T} p_D = d_D^Y p_D^{AD} \tag{37}$$

$$\frac{i_D^{Y^H}}{p_D - d_D^Y p_D^{AD}} = 1 \tag{38}$$

$$1 - i_D^T = p_D^{AD} \tag{39}$$

In $s = 0$ leveraged buyers buy the asset against a down tranche of value p_D—they can promise up to the value of the asset in the down state. The price of this tranche is $\pi_0^T = p_D p_0^{AD}$. Market clearing for the tranche in D is modified to reflect that investors who bought the tranche at zero have gotten a great payoff in D and so their wealth is multiplied by $\frac{1}{p_D}$.

T-Trade

Eleven variables: $\hat{i}_0^{Y^H}, \hat{i}_D^{Y^H}, \hat{i}_0^{Y^F}, \hat{i}_0^T, \hat{i}_D^T, \hat{p}_0^H, \hat{p}_0^F, \hat{p}_0^{AD}, \hat{p}_D^H, \hat{p}_D^F, \hat{p}_D^{AD}$. Because the Foreign asset cannot be leveraged, the marginal buyer in 0 and D are the same. 3 Market clearing in 0, 3 valuation in 0, 2 market clearing in D, 3 valuation in D.

$$(1-\hat{i}_0^{Y^H})\frac{(1+\hat{p}_0^H)}{\hat{p}_0^H-\hat{p}_D^H\hat{p}_0^{AD}}=1 \tag{40}$$

$$p_0^F=(1-\hat{i}_0^{Y^F})(1+\hat{p}_0^F) \tag{41}$$

$$\hat{i}_0^T(2+\hat{p}_0^H+\hat{p}_0^F)=\hat{p}_D^H\hat{p}_0^{AD} \tag{42}$$

$$\frac{\hat{i}_0^{Y^H}}{\hat{p}_0^H-\hat{p}_D^H\hat{p}_0^{AD}}=\hat{i}_0^{Y^H}+(1-\hat{i}_0^{Y^H})\frac{\hat{i}_0^{Y^H}}{\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}} \tag{43}$$

$$\hat{p}_0^F=1-(1-\hat{i}^{Y^F})^2d_D^Y \tag{44}$$

$$1-\hat{i}_0^T=\hat{p}_0^{AD} \tag{45}$$

$$\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}=(\hat{i}_0^{Y^H}-\hat{i}_D^{Y^H})(1+\hat{p}_0^H) \tag{46}$$

$$\frac{\hat{i}_D^T}{\hat{i}_0^T}\hat{p}_D^H=d_D^Y\hat{p}_D^{AD} \tag{47}$$

$$\frac{\hat{i}_D^{Y^H}}{\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}}=1 \tag{48}$$

$$\hat{p}_D^F=\hat{i}^{Y^F}+(1-\hat{i}^{Y^F})d_D^Y \tag{49}$$

$$1-\hat{i}_D^T=\hat{p}_D^{AD}; \tag{50}$$

TY-Trade

Twelve variables: $\hat{i}_0^{Y^H}, \hat{i}_D^{Y^H}, \hat{i}_0^{Y^F}, \hat{i}_D^{Y^F}, \hat{i}_0^T, \hat{i}_D^T, \hat{p}_0^H, \hat{p}_0^F, \hat{p}_0^{AD}, \hat{p}_D^H, \hat{p}_D^F, \hat{p}_D^{AD}$. 3 Market clearing in 0, 3 valuation in 0, 3 market clearing in D, 3 valuation in D.

$$(1-\hat{i}_0^{Y^H})\frac{2+\hat{p}_0^H+\hat{p}_0^F}{\hat{p}_0^H-\hat{p}_D^H\hat{p}_0^{AD}}=1 \tag{51}$$

$$\hat{p}_0^F=(\hat{i}_0^{Y^H}-\hat{i}_0^{Y^F})(2+\hat{p}_0^H+\hat{p}_0^F) \tag{52}$$

$$\hat{i}_0^T(2+\hat{p}_0^H+\hat{p}_0^F)=\hat{p}_D^H\hat{p}_0^{AD} \tag{53}$$

$$\frac{\hat{i}_0^{Y^H}}{\hat{p}_0^H-\hat{p}_D^H\hat{p}_0^{AD}}=\frac{\hat{i}_0^{Y^H}}{\hat{p}_0^F}+\frac{1-\hat{i}_0^{Y^H}}{\hat{p}_0^F}\frac{\hat{i}_0^{Y^H}}{\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}} \tag{54}$$

$$\frac{1-(1-\hat{i}^{Y^F})^2d_D^Y}{\hat{p}_0^F}=\hat{i}_0^{Y^F}+(1-\hat{i}_0^{Y^F})\frac{\hat{i}_0^{Y^F}+(1-\hat{i}_0^{Y^F})d_D^Y}{\hat{p}_D^F} \tag{55}$$

$$1-\hat{i}_0^T=\hat{p}_0^{AD} \tag{56}$$

$$\frac{\hat{i}_0^{Y^H}-\hat{i}_D^{Y^H}}{\hat{i}_0^{Y^H}-\hat{i}_0^{Y^F}}\frac{\hat{p}_D^F}{\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}}=1 \tag{57}$$

$$\frac{\hat{i}_D^{Y^H}-\hat{i}_0^{Y^F}}{\hat{i}_0^{Y^H}-\hat{i}_0^{Y^F}}+\frac{\hat{i}_0^{Y^F}-\hat{i}_D^{Y^F}}{\hat{p}_D^F}(2+\hat{p}_0^H+\hat{p}_0^F)=1 \tag{58}$$

$$\frac{\hat{i}_D^T}{\hat{i}_0^T}\hat{p}_D^H=d_D^Y\hat{p}_D^{AD} \tag{59}$$

$$\frac{\hat{i}_D^{Y^H}}{\hat{p}_D^H-d_D^Y\hat{p}_D^{AD}}=\frac{\hat{i}_D^{Y^H}+(1-\hat{i}_D^{Y^H})d_D^Y}{\hat{p}_D^F} \tag{60}$$

$$\hat{p}_D^F=\hat{i}^{Y^F}+(1-\hat{i}^{Y^F})d_D^Y \tag{61}$$

$$1-\hat{i}_D^T=\hat{p}_D^{AD}; \tag{62}$$

Foreign Leverage and Home Tranching

10 variables: $\hat{i}^Y_0, \hat{i}^Y_D, \hat{i}^T_0, \hat{i}^T_D, \hat{p}^H_0, \hat{p}^F_0, \hat{p}^{AD}_0, \hat{p}^H_D, \hat{p}^F_D, \hat{p}^{AD}_D$. 2 Market clearing in 0, 3 valuation in 0, 2 market clearing in D, 3 valuation in D. With $\phi^H = \frac{1-d^Y_D}{2-d^Y_D}$.

$$(1-\hat{i}^Y_0)\phi^H \frac{2+\hat{p}^H_0+\hat{p}^F_0}{\hat{p}^H_0-\hat{p}^H_D\hat{p}^{AD}_0} = 1 \tag{63}$$

$$\hat{i}^T_0(2+\hat{p}^H_0+\hat{p}^F_0) = \hat{p}^H_D\hat{p}^{AD}_0 \tag{64}$$

$$\frac{\hat{i}^Y_0}{\hat{p}^H_0-\hat{p}^H_D\hat{p}^{AD}_0} = \hat{i}^Y_0 + (1-\hat{i}^Y_0)\frac{\hat{i}^Y_0}{\hat{p}^H_D-d^Y_D\hat{p}^{AD}_D} \tag{65}$$

$$\frac{\hat{i}^Y_0}{\hat{p}^H_0-\hat{p}^H_D\hat{p}^{AD}_0} = \frac{\hat{i}^Y_0(1-\hat{p}^F_D)}{\hat{p}^F_0-\hat{p}^F_D} \tag{66}$$

$$1-\hat{i}^T_0 = \hat{p}^{AD}_0 \tag{67}$$

$$(\hat{i}^Y_0-\hat{i}^Y_D)\phi^H \frac{2+\hat{p}^H_0+\hat{p}^F_0}{\hat{p}^H_D-d^Y_D\hat{p}^{AD}_D} = 1 \tag{68}$$

$$\frac{\hat{i}^T_D}{\hat{i}^T_0}\hat{p}^H_D = d^Y_D\hat{p}^{AD}_D \tag{69}$$

$$\frac{\hat{i}^Y_D}{\hat{p}^H_D-d^Y_D\hat{p}^{AD}_D} = 1 \tag{70}$$

$$\frac{\hat{i}^Y_D}{\hat{p}^H_D-d^Y_D\hat{p}^{AD}_D} = \frac{\hat{i}^Y_D(1-d^Y_D)}{\hat{p}^F_D-d^Y_D} \tag{71}$$

$$1-\hat{i}^T_D = \hat{p}^{AD}_D; \tag{72}$$