International Financial Integration and Crisis Contagion *

Michael B. Devereux † Changhua Yu‡

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Abstract

International financial integration helps to diversify risk but also may increase the transmission of crises across countries. We provide a quantitative analysis of this trade-off in a two-country general equilibrium model with endogenous portfolio choice and collateral-constrained borrowing. Borrowing constraints bind occasionally, depending upon the state of the economy and levels of inherited debt. We examine different degrees of international financial integration, moving from financial autarky, to bond market integration, and equity market integration. Financial integration leads to a significant increase in global leverage, doubles the probability of balance sheet crises for any one country, and dramatically increases the degree of 'contagion' across countries. Outside of crises, the impact of financial integration on macro aggregates is relatively small. But the impact of a crisis with integrated international financial markets is much less severe than that under financial market autarky. Thus, a trade-off emerges between the probability of crises and the severity of crises. Financial integration can raise or lower welfare, depending on the scale of macroeconomic risk. In particular, in a low risk environment, the increased leverage resulting from financial integration can reduce welfare of investors.

Keywords: International Financial Integration, Occasionally Binding Constraints, Financial Contagion, Leverage

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†Vancouver School of Economics, University of British Columbia, NBER and CEPR. Address: 997 – 1873 East Mall, Vancouver, BC, Canada, $V6T1Z1$. E-mail: devm@mail.ubc.ca

‡School of Banking and Finance, University of International Business and Economics. Address: No. 10 Huixin East Rd., Chaoyang District, Beijing, China, 100029. E-mail: changhuay@gmail.com.

1 Introduction

In recent years there has been a re-evaluation of the benefits of international financial market integration. While financial integration offers welfare gains, it may also carry substantial risks. Financial linkages between countries have been critical to the rapid transmission of banking and financial crises across national borders. A large empirical and theoretical literature has explored the nature of this transmission (see for instance, Reinhart and Rogoff, 2009; Mishkin, 2011; Campello, Graham and Harvey, 2010).[1]

Many of these papers present detailed accounts of the recent global financial crisis. While most observers (e.g., Lane, 2013; Eichengreen, 2010) argue that the roots of the crisis were tied to regulatory failures and misperceptions about the concentration of risk, it is clear that cross-border capital flows facilitated by the globalization of financial markets were a factor in the generation of these circumstances. In addition, linkages between financial markets were critical to the propagation of the crisis (see e.g. Imbs, 2010; Lane, 2013).

More generally, open capital markets have historically been associated with a higher incidences of financial crises. For instance, Reinhart and Rogoff (2009) argue that:

"*Periods of high international capital mobility have repeatedly produced international banking crises, not only famously, as they did in the 1990s, but historically..*"(p. 155).

Reinhart and Rogoff also find that the probability of a banking crisis conditional on a previous capital flow bonanza is substantially higher than the unconditional probability. In a similar vein Demirguç-Kunt and Detragiache (1998) find a significant association between financial liberalizations over the 1980-1995 period and subsequent banking crises among a sample of 53 countries. Eichengreen (2004) notes the heightened risks of financial liberalization in the presence of fragile domestic financial systems based on evidence of financial crises from the 1980s and 1990s.

This paper investigates the effects of international financial integration on the incidence of financial crises, their correlation across countries, and the severity of crises. We explore these issues within a stochastic general equilibrium model where financial integration facilitates international risk sharing, but also alters the incentives and willingness of agents to make risky investments financed by borrowing.

It is widely acknowledged that high leverage, both inside and outside the financial system, was

[1]Others include Almunia, Benetrix, Eichengreen, ORourke and Rua (2010), Puri, Rocholl and Steffen (2011), Chudik and Fratzscher (2011), Perri and Quadrini (2011), Eaton, Kortum, Neiman and Romalis (2011), Kalemli-Ozcan, Papaioannou and Perri (2013) and Fratzscher (2012).

critical to the origin and propagation of the 2008-2009 crisis. The scaling-up of leverage took place within a global financial system of interconnected financial institutions. Eichengreen (2010) notes that competitive pressures on large banks in the years before the crisis, alongside the asset price inflation facilitated by global capital markets, allowed for unprecedented growth in leverage. Lane (2013) points out that much of the increased leverage among US institutions was directly financed by European banks. Borio and Disyatat (2011) argue that financial globalization increased the 'elasticity' the financial system, facilitating large, globally coordinated credit booms.

International capital flows helped to finance the global credit build-up in the early part of the century. Lane and McQuade (2014) find a strong positive relationship between net inflows and domestic credit growth in a wide sample of countries for the period prior to the financial crisis, much of this growth intermediated through banks relying on non-deposit funding through debt instruments. But it is also important to note that large credit booms can take place without large net flows of capital across countries. For instance, Calderon and Kubota (2012) show that growth in leverage is more closely tied to gross capital inflows than net inflows.

While crises may be more likely in a globalized financial system, financial linkages may reduce their severity. Bordo, Eichengreen, Klingebiel and Martinez-Peria (2001) provide evidence on the frequency and severity of crises after the opening up of financial markets following the collapse of the Bretton Woods agreement. They find that the frequency of crises among a large group of OECD economies doubled during this period. Interestingly however, the depth and severity of crises did not increase at all. In a discussion of the global financial crisis, Lane (2013) points out a number of mechanisms through which international financial connections operated as a buffer for the impact of the major shocks hitting the financial system. Rose (2012) argues that countries with capital market linkages to the US suffered less severe effects following the 2008-2009 crisis. Along related lines, Gourinchas, Rey and Govillot (2010) put forward the notion of 'Exorbitant Duty', capturing the idea that the US acts as an 'insurer of last resort' in the international financial system during a global crisis. They point out US net foreign assets to GDP fell by 19% during the crisis, helping to cushion the impact of the crisis on the rest of the world.

In our model, international financial integration occurs in the presence of market failures within the domestic financial system. Our investigation is built around a two-country general equilibrium model with endogenous portfolio choice and borrowing constrained by the value of collateral. Collateral constraints bind occasionally in one country or both, depending on inherited debt burdens,

shocks to productivity, and linkages between national financial markets. We allow for three stages of financial integration: financial autarky, bond market integration and equity market integration. In each type of financial regime, an investor must raise external funds from domestic or foreign lenders to invest in a project, but faces a collateral constraint because of default risk. In financial autarky, an investor borrows only from domestic lenders. In bond market integration, an investor obtains funding from a global bank that accepts deposits from savers in all countries. In equity market integration, investors borrow from a global bank but can also make investments in domestic or foreign projects.

The aim of the paper is to explore how different levels of international financial market integration effect the level of risk-taking that investor-borrowers are willing to engage in, to explore how financial integration affects the probability of financial crises and the international transmission of crises, and to ask what financial market linkages imply for the nature and severity of crises. Given answers to these questions, we can investigate the welfare effects of financial market integration within a framework of endogenous financial crises. A novel feature of the study is that we explore these questions within a full multi-country general equilibrium model, where world interest rates, asset prices, and capital flows are all endogenously determined. In this model, crises can be specific to one country, or can occur in all countries simultaneously.

The model embodies the central trade-off inherent in the above discussion of the nature of financial markets and international financial crises. Integrated financial markets facilitate intertemporal capital flows and portfolio diversification, and by doing so, help to defray country-specific risk. But at the same time, more open capital markets may increase the probability of financial crises and the contagion of crises across countries.

Our results closely reflect this two-fold nature of financial market integration. First, we find that financial integration tends to increase investor leverage and risk-taking in all countries. Thus financial integration is associated with global increases in credit availability. Two channels are critical for this linkage between financial opening and increased risk-taking. First, by increasing the value of existing asset holdings, financial integration increases the collateral value of investors' portfolios and facilitates an increase in borrowing capacity. Second, by reducing overall consumption risk, financial integration reduces precautionary savings and leads to an increase in investors willingness to acquire debt.[2]

[2] Eichengreen (2010) highlights these two factors - the increase in risk-taking among financial institutions, and the increase in the value of assets in global integration, as important elements linking the Global Financial Crisis to the

As a result of the increase in global leverage, we find that financial market integration increases the unconditional probability of financial crises. In addition, due to the linkage of borrowing costs and asset prices through international financial markets, the contagion of crises across countries is markedly higher after financial market liberalization. Because investors do not take account of how their borrowing and investment decisions impact the probability of financial crises, this represents a negative externality which reduces the welfare benefits of financial liberalization.

It is important to note that while this increased global leverage is associated with large gross asset flows across borders, in our model net flows are on average quite small, given equal preferences and technologies across countries. Leverage growth and credit booms take place primarily due to a greater willingness to invest in risky assets and a reduced precautionary saving among investors.

While we find that financial crises are more likely in an integrated world financial market, crises are much less severe in terms of lost output and consumption than those in financial autarky. During 'normal times' (or in the absence of crises), the impact of financial integration is rather small - financial market openness improves allocative efficiency modestly and has a benefit in terms of slightly lower output and consumption volatility. But in a financial crisis, the output and consumption losses are much greater in an environment of financial autarky. Hence, while financial integration increases the probability and co-movement of crises, crises are distinctly milder events, and the costs are more evenly spread amongst countries.

In welfare terms, we can ask whether, given the existence of a trade-off between the probability of crises and the severity of crises, there is always a net gain from financial market integration. Our results indicate that this depends on the overall level of technology risk. In an environment of high risk, the benefits of diversification exceed the costs of increased crisis occurrence, and both investors and savers are always better off with open capital markets. But with a lower risk environment, the induced effects of financial integration on leverage and crisis probability can be more important, and we find that investors can be worse off with open financial markets than in financial market autarky.

Our paper contributes to several branches of the academic literature. The question of the international transmission of crises has attracted much recent attention.[3] Recently, several authors have developed models of crisis transmission in a two-country framework with financial frictions.[4]

pre-crisis trend towards financial globalization.

[3]See recent work on the global financial crisis by Cetorelli and Goldberg (2010), de Haas and van Lelyveld (2010), Aizenman (2011), Aizenman and Hutchison (2012) and others; on 1998 Russian default by Schnabl (2012).

[4]See for instance, Mendoza, Quadrini and Rios-Rull (2009), Devereux and Sutherland (2010) and Dedola and

A paper closely related to our work is Perri and Quadrini (2011). They assume that investors can perfectly share their income risk across borders and consequently investors in both countries simultaneously face either slack collateral constraints or tight collateral constraints. In another words, the conditional probability of one country being in a crisis given that the other is in a crisis is one. There are two main differences between their work and ours. First, we investigate endogenous portfolio decisions made by investors, and risk sharing is imperfect between investors across borders, while they focus on perfect risk sharing for investors. Second, the mechanism is quite different. We study a channel of fire sales, in which both asset prices and quantities of assets adjust endogenously to exogenous shocks, while they focus only on the quantity adjustment of assets. Another related paper is Kalemli-Ozcan, Papaioannou and Perri (2013). They study a global banker who lends to firms in two countries and focus on a bank lending channel. Firms in both countries need to finance their working capital via borrowing from a banker in advance. Variation in the interest payment for working capital charged by the global banker in both countries delivers a transmission of crises across borders. Our model is quite different from theirs and emphasizes the balance sheet channel of firms (investors). Moreover, we provide a model of endogenous portfolio choice by investors (firms) and bankers, and study explicitly the transmission of crises through the fire sale of assets. Finally, two more recent papers by Mendoza and Quadrini (2010) and Mendoza and Smith (2014) examine the role of capital mobility in crisis propagation, a theme common to our paper. In Mendoza and Quadrini (2010), using an extension of the model of Mendoza, Quadrini and Rios-Rull (2009), they show that capital market integration leads to an increase in both the domestic credit and net foreign debt of the most financially developed country, and magnifies the cross-border spillovers of a shock to bank capital. In contrast to our model, their paper focuses on a model with idiosyncratic but not aggregate uncertainty. Hence they do not explore how financial integration affects the probability of crises. By contrast, Mendoza and Smith (2014) examines the impact of financial liberalization within a small economy that can trade with the outside world in both equity and debt markets, in the presence of financial frictions. They find that financial liberalization leads to an 'overshooting' of the probability of crises. Their model differs from ours principally in that we focus on a more symmetric and fully general equilibrium world economy.

Our paper also has some relevance for the recent discussion of the macro-prudential aspects of capital controls in the presence of borrowing constraints. A growing literature has developed

Lombardo (2012).

normative models for evaluating the desirability of capital controls as macro-prudential tools in open economies with incomplete markets. In Bianchi (2011), private sector borrowing is constrained partly by the size of the non-tradable sector. Private agents don't internalize the effects of their borrowing on asset prices in recession, which leads to overborrowing ex-ante, and offers a rationale for a capital infow tax. Bianchi and Mendoza (2010) develop state-contingent capital inflow taxes to prevent overborrowing (see also Bianchi and Mendoza (2013) for a discussion of time-consistency issues).[5] Korinek (2011b) and Lorenzoni (2015) provide comprehensive reviews on borrowing and macroprudential policies during financial crises in recent research. Our paper provides some welfare results suggesting that macro-prudential tools may play a role in a full world general equilibrium context, but for reasons of space we do not provide an analysis of optimal policy design.

Finally, the paper is complementary to a literature on international portfolio choice (see Devereux and Sutherland, 2011a; Devereux and Sutherland, 2010; Tille and van Wincoop, 2010 and others). Compared to these work with approximation around a deterministic steady state, we develop a model with occasionally binding collateral constraints and solve this model using a global solution method based on Dumas and Lyasoff (2012) and Judd, Kubler and Schmedders (2002). In the model, we obtain a stochastic steady state of portfolios. In terms of model setups, this paper is a variation of Devereux and Yetman (2010) and Devereux and Sutherland (2011b). They focus on a case where collateral constraints are always binding, while we consider a model with occasionally binding collateral constraints and address a different set of issues.

The paper is organized as follows. Section 2 analyzes a two-country financial integration model with equity market integration, bond market integration and financial autarky. The algorithm for solving the model, some computational issues, and calibration assumptions are discussed in section 3. A perfect foresight special case of the model is presented in section 4. Section 5 provides the main results. The last section concludes. All detailed issues related to the solution of the model are contained in a Technical Appendix at the end of the paper.

[5] This state-contingent taxation can be interpreted as a Pigouvian corrective tax, as discussed in Jeanne and Korinek (2010). Schmitt-Grohe and Uribe (2013) construct a model with downward wage rigidity, when policy-makers face an exogenous constraint on exchange rate adjustment, and establish the benefit of an ex ante tax on capital inflows. Overborrowing is not always an outcome however, and depends on the details of the economy and the borrowing constraints(see Benigno, Chen, Otrok, Rebucci and Young, 2013).

2 A Two-country General Equilibrium Model of Investment and Leverage

Here we set out a basic model where there are two countries, each of which contains borrowers and lenders, and lenders make risky levered investments subject to constraints on their total borrowing.[6] Country 1 (home) and 2 (foreign) each have a set of firm-investors with a measure of population n who consume and borrow from banks to invest in equity markets.[7] Investors also supply labor and earn wage income. A remaining population of $1-n$ workers (savers) operate capital in the informal backyard production sector, supply labor, and save in the form of risk-free debt. There is a competitive banking sector that operates in both countries. Bankers raise funds from workers and lend to investors. We look at varying degrees of financial market integration between the two countries. In financial autarky, savers lend to domestic banks, who in turn lend to home investors, and investors can only make investments in the domestic technology (or domestic equity). In bond market integration, there is a global bank that raises funds from informal savers in both countries, and extends loans to investors. But investors are still restricted to investing in the domestic equity. Finally, in equity market integration, investors borrow from the global bankers but may make investments in the equity of either country. In all environments, there is a fixed stock of capital which may be allocated to the informal backyard sector or the domestic investment technology. Capital cannot be physically transferred across countries.

2.1 Equity Market Integration

It is convenient to first set out the model in the case of full equity market integration, and then later describe the restrictions for case of bond market integration, or financial autarky.

The budget constraint for a representative firm-investor in country $l = 1, 2$, reads as

$$-\frac{b_{l,t+1}}{R_{t+1}} + c_{l,t} + q_{1,t}k^l_{1,t+1} + q_{2,t}k^l_{2,t+1} = d_{l,t} + W_{l,t}h_{l,t} + k^l_{1,t}(q_{1,t} + R_{1k,t}) + k^l_{2,t}(q_{2,t} + R_{2k,t}) - b_{l,t}. \quad (1)$$

The right hand side of the budget constraint states income sources including labor income $W_{l,t}h_{l,t}$, profit from the ownership of domestic firms, $d_{l,t}$, gross return on equities issued by country 1 and

[6] The baseline is similar to Devereux and Yetman (2010) and Devereux and Sutherland (2011b), which is essentially an two-country version of Kiyotaki and Moore (1997), except extended to allow for uncertainty in investment returns.

[7] We could also think of them as investing in real projects, but since there is no idiosyncratic distribution of returns by investors, this would be equivalent to investing in an economy-wide equity market.

held by investor l, $k^l_{1,t}(q_{1,t} + R_{1k,t})$, gross return on equity 2, $k^l_{2,t}(q_{2,t} + R_{2k,t})$, less debt owed to the bank $b_{l,t}$. The left hand side denotes the investor's consumption $c_{l,t}$, and portfolio decisions, $(k^l_{1,t+1}, k^l_{2,t+1}, b_{l,t+1})$. Asset prices for country 1 and 2 equities and the international bond are $q_{1,t}$, $q_{2,t}$, $\frac{1}{R_{t+1}}$, respectively. Dividends for equities come from the marginal product of capital, $R_{1k,t}$ and $R_{2k,t}$.

Profit $d_{l,t}$ is then defined as

$$d_{l,t} \equiv \frac{1}{n}\left[F(A_{l,t}, H_{l,t}, K_{l,t}) - W_{l,t}H_{l,t} - K_{l,t}R_{lk,t}\right],$$

where the total cost of labor services is $W_{l,t}H_{l,t}$.

Investors borrow to finance consumption and investment. They face a collateral (or leverage) constraint as in Kiyotaki and Moore (1997) when borrowing from a bank

$$b_{l,t+1} \leq \kappa E_t \left\{ q_{1,t+1}k^l_{1,t+1} + q_{2,t+1}k^l_{2,t+1} \right\}, \tag{2}$$

where κ characterizes the upper bound for loan-to-value.

Domestic and foreign equity assets are perfect substitutes when they are used to obtaining external funds for investors in either of the countries.[8] Preferences of investors are given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_l^t U(c_{l,t}, h_{l,t}) \right\},$$

where $0 < \beta_l < 1$ is investors' subjective discount factor and $U(c_{l,t}, h_{l,t})$ is their utility function. Investors' labor supply is determined by

$$-\frac{U_h(c_{l,t}, h_{l,t})}{U_c(c_{l,t}, h_{l,t})} = W_{l,t}\ , \ l = 1, 2. \tag{3}$$

Let the Lagrange multiplier for the collateral constraint (2) be $\mu_{l,t}$. The optimal holdings of equity for investors must satisfy the following conditions,

$$q_{i,t} = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1})(q_{i,t+1} + R_{ik,t+1}) + \mu_{l,t}\kappa E_t q_{i,t+1}}{U_c(c_{l,t}, h_{l,t})}\ , \ i = 1, 2. \tag{4}$$

The left hand side is the cost of one unit of equity at time t. The right hand side indicates that

[8] Several recent studies explore asymmetric efficiency of channeling funds through financial markets (Mendoza, Quadrini and Rios-Rull, 2009) or through financial intermediations (Maggiori, 2012) across countries.

the benefit of an additional unit of equity is twofold; there is a direct increase in wealth in the next period from the return on capital plus the value of equity, and in addition, holding one more unit of equity relaxes the collateral constraint (2). If $\mu_t > 0$, then this increases the borrowing limit at time t.

The optimal choice of bond holdings must satisfy

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1}) + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})}. \tag{5}$$

When the collateral constraint (2) is binding, reducing one unit of borrowing has an extra benefit $\mu_{l,t}$, by relaxing the constraint. Rearranging the equation above yields,

$$1 = \frac{\beta_l E_t U_c(c_{l,t+1}, h_{l,t+1}) R_{t+1}}{U_c(c_{l,t}, h_{l,t})} + EFP_{l,t+1},$$

where $EFP_{l,t+1} \equiv \frac{\mu_{l,t} R_{t+1}}{U_c(c_{l,t}, h_{l,t})}$ measures the external finance premium in terms of consumption in period t faced by investors in country l.

The complementary slackness condition for the collateral constraint (2) can be written as

$$\left(\kappa E_t \left\{q_{1,t+1} k^l_{1,t+1} + q_{2,t+1} k^l_{2,t+1}\right\} - b_{l,t+1}\right) \mu_{l,t} = 0, \tag{6}$$

with $\mu_{l,t} \geq 0$.

We investigate the extent to which constraint (2) binds in an equilibrium that is represented by a stationary distribution of decision rules made by savers and investors. The fact that productivity is stochastic, inducing riskiness in the return on equities to investors, is critical. In a deterministic environment, the constraint would always bind in a steady state equilibrium (with any degree of financial market integration). This is because following Kiyotaki and Moore (1997), we assume that investors are more impatient than savers. Thus, in an infinite horizon budgeting plan, investors will wish to front-load their consumption so much that (2) will always bind. But this is not true generally, in a stochastic economy, since then investors will have a precautionary savings motive that leads them to defer consumption as a way to self-insure against low productivity (and binding collateral constraints) states in the future.

2.1.1 Global Bankers

In both countries, there are worker/savers who supply labor, earn income, employ capital to use in home production, and save. They save by making deposits in a 'bank', which in turn makes loans to investors. Workers are subject to country specific risk coming from fluctuations in wage rates and in the price of domestic capital. Because idiosyncratic variation in workers' consumption and savings decisions plays no essential role in the transmission of productivity shocks across countries, we make a simplification that aids the model solution by assuming that with financial market integration (either bond market integration or equity market integration), workers' preferences are subsumed by a 'global banker' who receives their deposits and chooses investment and lending to maximize utility of the global representative worker. Hence, there is full risk-sharing across countries among worker/savers. This assumption substantially simplifies the computation of equilibrium without changing the nature of the results in any essential way.[9]

Hence, there is a representative banker in the world financial market. The banker runs two branches, one branch in each country with which the representative worker/saver conducts business. As we noted, the worker receives a competitive wage rate in the local labor market and uses capital in informal production. The objective of the banker is to maximize a representative worker's lifetime utility. Let subscript or superscript 3 indicate variables for the banker. The budget constraint for the global banker can be written as

$$-\frac{b_{3,t+1}}{R_{t+1}}+\frac{c^3_{1,t}+c^3_{2,t}}{2}+\frac{q_{1,t}k^3_{1,t+1}+q_{2,t}k^3_{2,t+1}}{2}=\frac{W_{1,t}h^3_{1,t}+W_{2,t}h^3_{2,t}}{2}+\frac{q_{1,t}k^3_{1,t}+q_{2,t}k^3_{2,t}}{2}$$

$$-b_{3,t}+\frac{G(k^3_{1,t})+G(k^3_{2,t})}{2}. \tag{7}$$

The left hand side of the equation above gives expenditures for a representative worker in the bank, including borrowing $\frac{b_{3,t+1}}{R_{t+1}}$, consumption $c^3_{l,t}$ and physical capital $\frac{q_{1,t}k^3_{1,t+1}+q_{2,t}k^3_{2,t+1}}{2}$ employed in informal production sectors. The right hand side describes labor income per worker $\frac{W_{1,t}h^3_{1,t}+W_{2,t}h^3_{2,t}}{2}$, the value of existing capital holdings $\frac{q_{1,t}k^3_{1,t}+q_{2,t}k^3_{2,t}}{2}$, debt repayment $b_{3,t}$ and home

[9] To see why this assumption is innocuous, note that it is the interaction between financial integration and borrowing constraints that represents the key trade-off in the paper. Savers are not limited by borrowing constraints, so altering their ability to engage in risk-sharing would have no qualitative implications for our results. Moreover, since it is well-known that trade in non-contingent debt (without financial constraints) closely approximates a complete markets allocation, it is very likely that this simplifying assumption has negligible quantitative implications.

production $\frac{G(k^3_{1,t})+G(k^3_{2,t})}{2}$. $G(k^3_{1,t})$ and $G(k^3_{2,t})$ denote the home production technology of savers in country 1 and 2 with physical capital $k^3_{1,t}$ and $k^3_{2,t}$ as inputs, respectively. We assume that $G(.)$ is increasing and concave.

The global banker internalizes the preferences of worker savers, maximizing an objective function given by

$$E_0\left\{\left(\frac{1}{2}\right)\sum_{t=0}^{\infty}\beta_3^t\left\{U(c^3_{1,t},h^3_{1,t})+U(c^3_{2,t},h^3_{2,t})\right\}\right\},$$

where β_3 stands for the subjective discount factor for a worker. As noted above, we assume that $\beta_l < \beta_3 < 1$.

The optimality condition for labor supply in each country is

$$-\frac{U_h(c^3_{l,t},h^3_{l,t})}{U_c(c^3_{l,t},h^3_{l,t})} = W_{l,t}\ ,\ l=1,2. \tag{8}$$

In addition, through the global banker, consumption risk-sharing among worker/savers across borders is attained, so that

$$U_c(c^3_{1,t},h^3_{1,t}) = U_c(c^3_{2,t},h^3_{2,t}). \tag{9}$$

The optimal choices of capital and bond holdings for the banker are represented as:

$$q_{i,t} = \frac{\beta_3 E_t U_c(c^3_{l,t+1},h^3_{l,t+1})(q_{i,t+1}+G'(k^3_{i,t+1}))}{U_c(c^3_{l,t},h^3_{l,t})}\ ,\ i=1,2, \tag{10}$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t U_c(c^3_{l,t+1},h^3_{l,t+1})}{U_c(c^3_{l,t},h^3_{l,t})}. \tag{11}$$

We note that the global banker faces no separate borrowing constraints such as (2).

2.1.2 Production and Market Clearing Conditions

In the formal sector of country i with $i = 1,2$, goods are produced by competitive goods producers, who hire domestic labor services and physical capital in competitive factor markets. Taking the formal sector production function as $Y_{i,t} = F(A_{i,t},H_{i,t},K_{i,t})$, where $A_{i,t}$ represents an exogenous productivity coefficient, we have in equilibrium, the wage rate equaling the marginal

product of labor and the return on capital being equal to the marginal product of capital

$$W_{i,t} = F_h(A_{i,t}, H_{i,t}, K_{i,t}) \ , \ i = 1, 2, \tag{12}$$

$$R_{ik,t} = F_k(A_{i,t}, H_{i,t}, K_{i,t}) \ , \ i = 1, 2. \tag{13}$$

Labor market and rental market clearing conditions are written as

$$H_{i,t} = nh_{i,t} + (1-n)h^3_{i,t} \ , \ i = 1, 2. \tag{14}$$

Total labor employed is the sum of employment of investors and savers. Capital employed in the formal sector is the sum of equity holdings of the domestic capital stock by both domestic and foreign investors:

$$K_{i,t} = n(k^1_{i,t} + k^2_{i,t}) \ , \ i = 1, 2. \tag{15}$$

Asset market clearing conditions are given by

$$nk^1_{1,t+1} + nk^2_{1,t+1} + (1-n)k^3_{1,t+1} = 1, \qquad nk^1_{2,t+1} + nk^2_{2,t+1} + (1-n)k^3_{2,t+1} = 1, \tag{16}$$

$$nb_{1,t+1} + nb_{2,t+1} + 2(1-n)b_{3,t+1} = 0. \tag{17}$$

The top line says that equity markets in each country must clear, which is equivalent to equilibrium in the market for capital, while the bottom line says that bond market clearing ensures that the positive bond position of the global bank equals the sum of the bond liabilities of investors in both countries.

Finally, there is only one world good, so the global resource constraint can be written as

$$n(c_{1,t} + c_{2,t}) + (1-n)(c^3_{1,t} + c^3_{2,t}) = F(A_{1,t}, H_{1,t}, K_{1,t}) + F(A_{2,t}, H_{2,t}, K_{2,t}) +$$

$$(1-n)\left(G(k^3_{1,t}) + G(k^3_{2,t})\right). \tag{18}$$

2.1.3 A Competitive Recursive Stationary Equilibrium

We define a competitive equilibrium which consists of a sequence of allocations $\{c_{l,t}\}_{t=0,1,2,\cdots}$, $\{c^3_{l,t}\}_{t=0,1,2,\cdots}$, $\{k^i_{l,t}\}_{t=0,1,2,\cdots}$, $\{b_{i,t}\}_{t=0,1,2,\cdots}$,$\{h_{l,t}\}_{t=0,1,2\cdots}$,$\{h^3_{l,t}\}_{t=0,1,2\cdots}$, $\{H_{l,t}\}_{t=0,1,2,\cdots}$, $\{K_{l,t}\}_{t=0,1,2,\cdots}$,

a sequence of prices $\{q_{i,t}\}_{t=0,1,2,\cdots}$, $\{W_{l,t}\}_{t=0,1,2,\cdots}$ and $\{R_{lk,t}\}_{t=0,1,2,\cdots}$, and a sequence of Lagrange multipliers $\{\mu_{l,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, $i = 1, 2, 3$, such that (a) consumption $\{c_{l,t}\}_{t=0,1,2,\cdots}$, $\{c^3_{l,t}\}_{t=0,1,2,\cdots}$, labor supply, $\{h_{l,t}\}_{t=0,1,2\cdots}$, $\{h^3_{l,t}\}_{t=0,1,2\cdots}$, with $l = 1, 2$, and portfolios $\{k^i_{l,t}\}_{t=0,1,2,\cdots}$, $\{b_{i,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, $i = 1, 2, 3$, solve the investors' and bankers' problem; (b) labor demand $\{H_{l,t}\}_{t=0,1,2,\cdots}$ and physical capital demand $\{K_{l,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, solve for firms' problem; (c) wages $\{W_{l,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, clear labor markets and $\{R_{lk,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, clear physical capital markets; (d) asset prices $\{q_{i,t}\}_{t=0,1,2,\cdots}$, with $i = 1, 2, 3$, clear the corresponding equity markets and bond markets; (e) the associated Lagrange multipliers $\{\mu_{l,t}\}_{t=0,1,2,\cdots}$, with $l = 1, 2$, satisfy the complementary slackness conditions.

Our interest is in developing a global solution to the model, where the collateral constraint may alternate between binding and non-binding states. A description of the solution approach is contained in Section 3 below, and fully exposited in the Technical Appendix.

2.2 Bond Market Integration

We wish to compare the equilibrium with fully integrated global equity markets with one where there is restricted financial market integration. Take the case where there is a global bond market, but equity holdings are restricted to domestic agents. All returns on capital in the formal sector must accrue to domestic firm-investors, although they can finance investment by borrowing from the global bank. To save space, we outline only the equations that differ from the case of equity market integration.

A representative firm-investor's budget constraint in the bond market integration case is given by

$$-\frac{b_{l,t+1}}{R_{t+1}} + c_{l,t} + q_{l,t}k^l_{l,t+1} = d_{l,t} + W_{l,t}h_{l,t} + k^l_{l,t}(q_{l,t} + R_{lk,t}) - b_{l,t}. \tag{19}$$

The collateral constraint now depends only on domestic equity values

$$b_{l,t+1} \leq \kappa E_t \left\{ q_{l,t+1}k^l_{l,t+1} \right\}. \tag{20}$$

A firm-investor maximizes his life-time utility

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t_l U(c_{l,t}, h_{l,t}) \right\}.$$

Consumption Euler equations for portfolio holdings imply

$$q_{l,t} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \left(q_{l,t+1} + R_{lk,t+1} \right) \right\} + \mu_{l,t} \kappa E_t \left\{ q_{l,t+1} \right\}}{U_c(c_{l,t}, h_{l,t})} \ , \ l = 1, 2, \tag{21}$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \right\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})}. \tag{22}$$

The complementary slackness condition implied by the collateral constraint (20) can be written as

$$\left(\kappa E_t \left\{ q_{l,t+1} k^l_{l,t+1} \right\} - b_{l,t+1} \right) \mu_{l,t} = 0, \tag{23}$$

with $\mu_{l,t} \geq 0$.

The global banker's problem is identical to the condition under equity market integration, and so is omitted here.

Market clearing conditions for rental and equity assets are as follows

$$K_{l,t} = n k^l_{l,t} \ , \ l = 1, 2, \tag{24}$$

$$n k^1_{1,t+1} + (1-n) k^3_{1,t+1} = 1, \qquad n k^2_{2,t+1} + (1-n) k^3_{2,t+1} = 1. \tag{25}$$

A competitive recursive stationary equilibrium in bond market integration is similar to equity market integration in section 2.1.3.

2.3 Financial Autarky

Finally, financial autarky represents a market structure without any financial linkages between countries at all. Investors obtain external funds only from local bankers and hold only local equity assets. Therefore, their budget constraints and collateral constraints are the same as in bond market integration (equation (19)-(20)). Now, local bankers receive deposits only from local savers.

A representative local banker's problem in country $l = 1, 2$ is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_3^t U(c^3_{l,t}, h^3_{l,t}) \right\},$$

subject to

$$-\frac{b^l_{3,t+1}}{R_{t+1}} + c^3_{l,t} + q_{l,t}k^3_{l,t+1} = W_{l,t}h^3_{l,t} + q_{l,t}k^3_{l,t} - b^l_{3,t} + G(k^3_{l,t}). \tag{26}$$

The optimality conditions yield

$$-\frac{U_h(c^3_{l,t}, h^3_{l,t})}{U_c(c^3_{l,t}, h^3_{l,t})} = W_{l,t}\ , \ l = 1, 2, \tag{27}$$

$$q_{l,t} = \frac{\beta_3 E_t U_c(c^3_{l,t+1}, h^3_{l,t+1})(q_{l,t+1} + G'(k^3_{l,t+1}))}{U_c(c^3_{l,t}, h^3_{l,t})}, \tag{28}$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t U_c(c^3_{l,t+1}, h^3_{l,t+1})}{U_c(c^3_{l,t}, h^3_{l,t})}. \tag{29}$$

The market clearing condition for the domestic bond market now becomes

$$nb_{l,t+1} + (1-n)b^l_{3,t+1} = 0. \tag{30}$$

The resource constraint in financial autarky is written as

$$nc_{l,t} + (1-n)c^3_{l,t} = F(A_{l,t}, H_{l,t}, K_{l,t}) + (1-n)G(k^3_{l,t}). \tag{31}$$

A competitive recursive stationary equilibrium in financial autarky is similar to equity market integration in section 2.1.3.

3 Calibration and Model Solution

3.1 Specific Functional Forms

We make the following set of assumptions regarding functional forms for preferences and technology. First, all agents are assumed to have GHH preferences (Greenwood, Hercowitz and Huffman, 1988) so that

$$U(c, h) = \frac{[c - v(h)]^{1-\sigma} - 1}{1-\sigma},$$

with

$$v(h) = \chi\frac{h^{1+\nu}}{1+\nu}.$$

In addition, the formal good production function is Cobb-Douglas with the form

$$F(A_{i,t}, H_{i,t}, K_{i,t}) = A_{i,t} H_{i,t}^{\alpha} K_{i,t}^{1-\alpha} \ , \ i = 1, 2. \tag{32}$$

Home production has a technology of $G(k_{i,t}^3) = Z(k_{i,t}^3)^{\gamma}$. Parameter Z denotes a constant productivity in the informal sector.

3.2 Solution Method

The solution of the model with stochastic productivity shocks, occasionally binding collateral constraints, multiple state variables for capital holdings and debt, and endogenous asset prices and world interest rates, represents a difficult computational exercise. The solution approach is described at length in the Technical Appendix. The key facilitating feature of the solution is that the model structure allows us to follow the approach of Dumas and Lyasoff (2012). Their method involves a process of backward induction on an event tree. Current period consumption shares in total world GDP are treated as endogenous state variables. The construction of equilibrium is done by a change of variable, so that the equilibrium conditions determine future consumption and end of period portfolios as functions of current exogenous and endogenous state variables. From these functions, using consumption-Euler equations we can recursively compute asset prices and the end of period financial wealth. We then iterate on this process using backward induction until we obtain time-invariant policy functions. Once we have these policy functions, we can make use of the initial conditions, including initial exogenous shocks and initial portfolios, and of equilibrium conditions in the first period to pin down consumption, end of period portfolios, output and asset prices in the initial period.

3.3 Calibration

The model has relatively few parameters. The period of measurement is one year. Parameter values in the baseline model are completely standard, following existing literature, and are listed in table 1.[10] The population of investors in each country is $n = 0.5$. The coefficient of relative risk aversion σ is set equal to 2.

The key features of the calibration involve the productivity shock processes. This is done in a

[10] In the deterministic steady state, both financial integration regimes have the same values for aggregate variables.

Table 1: Parameter values

Parameter		Value
Preferences		
n	Population size	0.5
σ	Relative risk aversion	2
$\beta_l, l = 1, 2$	Subjective discount factor for investors	0.954
β_3	Subjective discount factor for workers	0.96
ν	Inverse of Frisch labor supply elasticity	0.5
χ	Parameter in labor supply (H=1 at steady state)	0.58
κ	Loan-to-value ratio for inter-period borrowing	0.5
Production		
α	Labor share in formal production	0.64
γ	Informal production	0.3
Z_l, l=1,2	TFP level in informal production (capital share in the formal production=0.8)	0.7
ρ_z	Persistence of TFP shocks	0.65
σ_ϵ	Standard deviation of TFP shocks	0.02
D	Disaster risk	-0.1054
π	Probability of disaster risk	0.025
K	Normalized physical capital stock	1

two-fold manner. First, we specify a conventional AR process for the shock. But since we wish to allow for unlikely but large economic downturns, we append to this process a small probability of a large negative shock (a 'rare disaster'). The $AR(1)$ component of the shock can be represented as:

$$\ln(A_{l,t+1}) = (1 - \rho_z)\ln(A_l) + \rho_z \ln(A_{l,t}) + D\phi_{l,t+1} + \epsilon_{l,t+1},\ l = 1, 2, \tag{33}$$

where A_l is the unconditional mean of $A_{l,t}$, ρ_z characterizes the persistence of the shock, and $\epsilon_{l,t+1}$ denotes an innovation in period $t + 1$, which is assumed to follow a normal distribution with zero mean and standard deviation σ_ϵ.

The disaster component of the shock is captured by the $\phi_{l,t+1}$ term. This follows a Bernoulli distribution and takes either value of $\{-\pi, (1 - \pi)\}$, with probability of $\{1 - \pi, \pi\}$ respectively, $0 < \pi < 1$. The scale parameter $D < 0$ measures a disaster risk in productivity.[11] We take $\rho_z = 0.65$ and $\sigma_\epsilon = 0.02$ as in Bianchi (2011). We assume that the cross-country correlation of productivity shocks is zero. The mean of each country's productivity shock is normalized to be one. The distribution of $\phi_{l,t+1}$ is taken from the rare disaster literature (see for instance, Barro and

[11] The mean of the disaster risk is zero $E(\phi_{l,t}) = 0$ and its standard deviation equals $-D\sqrt{\pi(1-\pi)}$. Compared to the standard $AR(1)$ process without disaster risk (say $D = 0$), the standard deviation of innovations increases by $-D\sqrt{\pi(1-\pi)}$.

Ursua, 2012) so that $\pi = 0.025$, which implies that the probability of an economy entering a disaster state is 2.5% per year. Once an economy enters a disaster state, productivity will experience a drop by 10%, that is, $D = \ln(0.9)$. The number is chosen such that investor's consumption will drop by around 30% in disasters relative to that in normal times.

With this calibration, the unconditional standard deviation of TFP shocks is 4 percent per annum. In some of the analysis below, we look at a low-risk case, where the unconditional standard deviation of the TFP shocks is 2 percent per annum. While this has some important implications for welfare comparisons, all the qualitative results discussed in the paper are robust to a change from the high-risk to a low-risk economy.

In solving the model, we discretize the continuous state $AR(1)$ process above into a finite number of exogenous states. The Technical Appendix describes in detail how we accomplish this task. In the baseline model, we choose three grid points for technological levels $A_{l,t}$. The grid points and the associated transition matrix in each country are as follows

$$A_l \equiv \begin{bmatrix} A_L \\ A_M \\ A_H \end{bmatrix} = \begin{bmatrix} 0.9271 \\ 0.9925 \\ 1.0269 \end{bmatrix}, \qquad \Pi_l = \begin{bmatrix} 0.6311 & 0.2723 & 0.0967 \\ 0.1312 & 0.5739 & 0.2949 \\ 0.0078 & 0.2321 & 0.7601 \end{bmatrix}.$$

Given this specification, productivity in each country will visit its lowest state 0.9271 with a probability of 15%. The lowest state is associated with the disaster state but because the continuous distribution is projected onto a three state Markov Chain, this is not identical to the disaster itself. The exogenous state of the world economy in financial integration is characterized by a pair $(A_{1,t}, A_{2,t})$, which takes nine possible values. Its associated transition matrix is simply the Kronecker product of transition matrices in both countries, $\Pi \equiv \Pi_1 \otimes \Pi_2$, since the shocks are independent across countries.

The loan-to-value ratio parameter is set to be $\kappa = 0.5$, which states that the maximal leverage is 2 in the stationary distribution of the economy. This leverage ratio is consistent with evidence from non-financial corporations in the United States (Graham, Leary and Roberts, 2014). The subjective discount factor for bankers is $\beta_3 = 0.96$, which implies an annualized risk free rate of 4%. Investors are less patient, and their subjective discount factor is chosen to be $\beta_l = 0.954$. Together with the productivity shock process and the loan-to-value ratio of $\kappa = 0.5$, this implies that in financial autarky, the economy visits the state where the collateral constraint is binding and productivity is

at its lowest level with a probability of around 3.5%, or approximately every 30 years.

In the preference specification, the inverse of the Frisch labor supply elasticity is set to be 0.5, which is consistent with business cycle observations (Cooley, ed, 1995). We normalize the steady state labor supply to be $H = 1$, which implies the parameter $\chi = 0.58$.

The share of labor in formal production is set to be $\alpha = 0.64$. In the informal backyard production, the marginal product of capital is characterized by parameter $\gamma = 0.3$, which is lower than that in the formal production sector. The level of productivity in the informal backyard production is set to be $Z_l = 0.7$. This implies that around 80% of physical capital is employed in the formal production in the stationary distribution of the economy.[12]

4 A Perfect Foresight Special Case

Before we present the main results of the paper, it is worthwhile to explore the workings of the model in a simpler environment. Here we look at the impact of productivity shocks in a deterministic version of the model, under financial autarky. This can give some insight into the conditions under which the collateral constraint will bind. Although the constraint will always bind in a deterministic steady state, the dynamic response to productivity shocks may involve periods under which the constraint is not binding.

Figure 1 illustrates the impact of an unanticipated 5% shock to productivity that lasts for 3 periods. The continuous line denotes a negative shock, or a fall in productivity, while the dotted line shows the response to a positive shock. An unanticipated fall in productivity leads to fall in the price of capital. This causes a tightening of the collateral constraint, illustrated by a jump in the Lagrange multiplier μ_t. Investors are forced to de-lever, reducing borrowing and investment, and there is a large reallocation of capital out of the final goods sector and into the backyard production sector. This is followed by a large and persistent fall in aggregate output.

On the other hand, the response to a positive shock is to increase the price of capital. As Figure 1 shows, this leads to a relaxation of the collateral constraint, which ceases to bind for a considerable period. Borrowing increases, capital moves into the formal sector, and aggregate output increases.

[12] The solution of the model also requires some constraints on trading strategies in order to rule out paths in which agents acquire unbounded debts. This is common in the literature on general equilibrium model with incomplete markets (GEI). Without imposing some additional constraints, an equilibrium may not exist (e.g. Krebs, 2004). The appendix describes how this is done, following Judd, Kubler and Schmedders (2002) in imposing a utility penalty on holdings of assets.

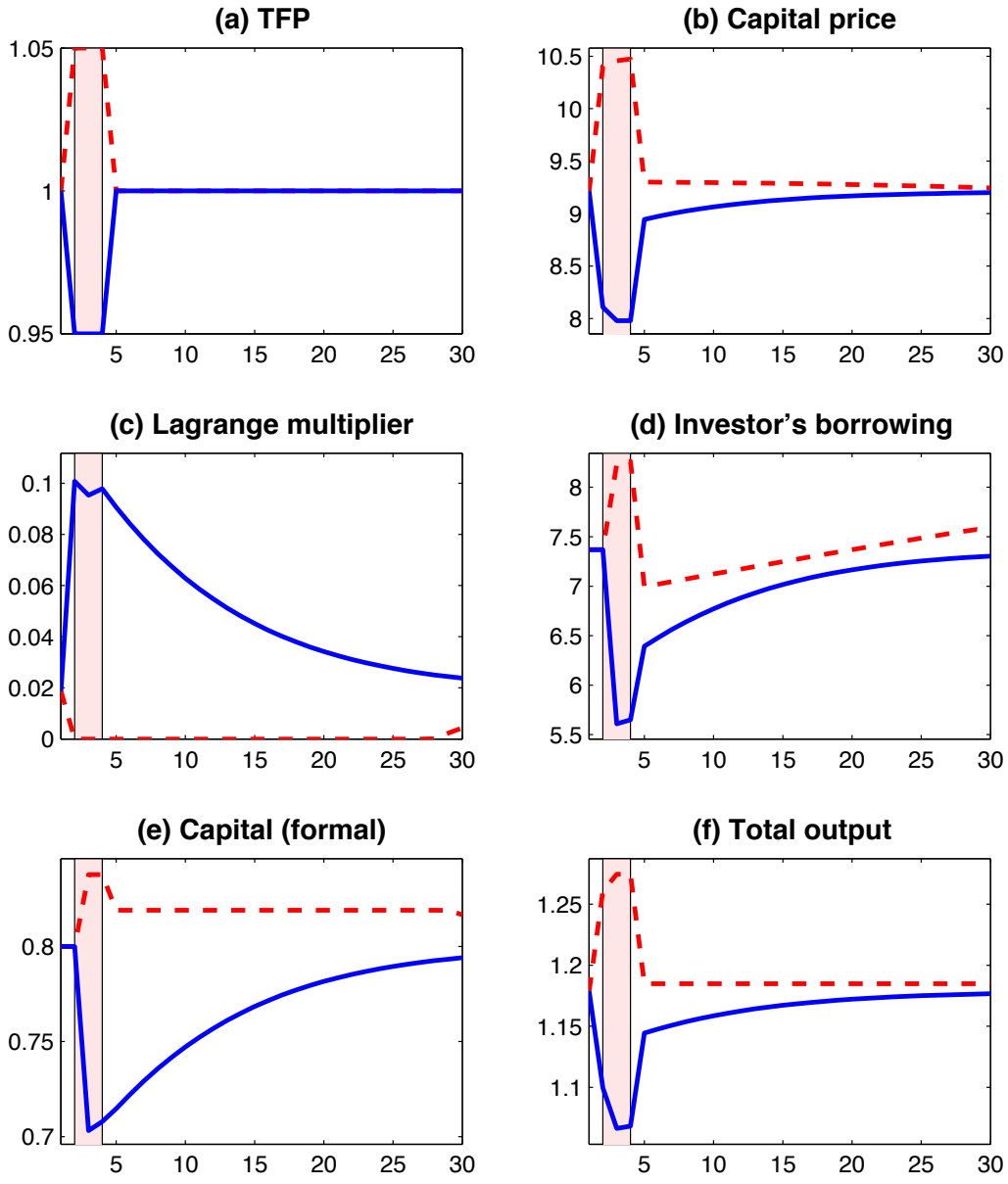

Figure 1: This figure reports responses of capital price, Lagrange multiplier, investor's borrowing, capital in the formal production and total output to an *unanticipated* negative (denoted by the solid blue line) or positive (denoted by the dotted red line) productivity (TFP) shock. The economy stays at its steady state in period 1. TFP shock occurs unexpectedly in period 2 and lasts for 3 periods (the shaded region), and it returns to its steady state from period 5 onwards.

Although the absolute size of the shock is equal in the negative and positive shock case, the responses are asymmetric. The forced de-leveraging in response to a temporary negative productivity shock is over 50% greater than the peak increase in debt accumulation in the case of a temporary positive productivity shock. Likewise, the fall in aggregate output for a negative shock significantly exceeds the increase in output in response to a positive shock. Intuitively, the dynamics of the economy when bound by the collateral constraint involves a financial accelerator as in Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). In response to the positive shock, investors are unconstrained, and the movement of capital into the formal sector is less than the movement outwards after a negative shock.

We note also that there is much less persistence in response to the positive shock. Aggregate output and investor borrowing return to steady state much more quickly in the absence of the binding collateral constraint.

Although this example pertains only to a one time shock in a deterministic environment, the asymmetric pattern of responses is mirrored in the stochastic simulation results shown below. This example also shows that, even though an undisturbed economy will always settle down to a steady state with binding constraints, the dynamic adjustment to shocks may involve long episodes in which the constraints do not bind, so capital is allocated efficiently between the formal and home production sector.

5 Results: Simulations over Alternative Financial Regimes

5.1 The Effect of Financial Integration on Balance Sheet Constraints

We simulate the stationary policy rules obtained from the model, using the stochastic processes for productivity defined in the previous section, for the three different financial regimes described above. The simulations are done for T=210,000 periods, with the first $10,000$ periods dropped from the sample. The first issue of interest is the degree to which the leverage constraint binds, and how this differs across the different financial regimes. Figure 2 provides an illustration and contrast between the different regimes. The figure presents illustrations of the fraction of time spent at the leverage constraints, and the degree to which the leverage constraints bind simultaneously in the two countries. Starting with the financial autarky case, we find that under the calibration and the distribution of productivity shocks in the baseline model, the leverage constraint binds only 10

percent of the time.

We noted above that investor's impatience will lead them to borrow right up the limit of the leverage constraint in a perfectly certain environment. But when wage income and the return on capital is uncertain, their desire to front-load consumption is tempered by the uncertainty of future income, generating an offsetting precautionary savings motive. Of course this is not a small open economy, so a rise in savings of investors must be matched with a fall in savings of the savers. But the desire for precautionary saving is higher, the greater the uncertainty of future consumption. When the leverage constraint binds, investor's consumption variability is amplified by the financial accelerator dynamics. As a result, in the stochastic environment, investor's precautionary saving motive exceeds that of savers. Investors then reduce their borrowing to the extent that the leverage constraint is generally non-binding.

In financial autarky, given that productivity shocks are independent across countries, there is a very small probability that the constraint binds simultaneously in both countries - there is no 'contagion' in leverage crises in the absence of financial linkages.

How does opening up to international financial markets affect the likelihood of leverage crises? Allowing for trade in a global bond market, the unconditional probability that the leverage constraint binds now increases to 19 percent. Moreover, this increase in the probability of leverage crises is associated with a large jump in the correlation of crises across countries. The probability that the constraint binds in both countries simultaneously is now 13 percent (relative to 1 percent in financial autarky). Conditional on a crisis in one country, the probability that its neighbour experiences a crisis rises from near zero under financial autarky to 71 percent under bond market integration.

What drives the high correlation in leverage crises across countries with bond market integration? Under global bond integration, equities are not tradable across countries, so it is only the linkages in debt markets that lead to connections between investment financing constraints across countries. As we will see below in more detail, a negative shock to productivity in the foreign country will tighten the collateral constraint in that country, increasing the world interest rate on debt. The rise in the world interest rate leads to a fall in the home country price of capital, tightening the home country collateral constraint. This indirect effect is strong enough to generate high co-movement in leverage crises events across countries.

Besides being increasingly correlated, the unconditional probability of crises increases under

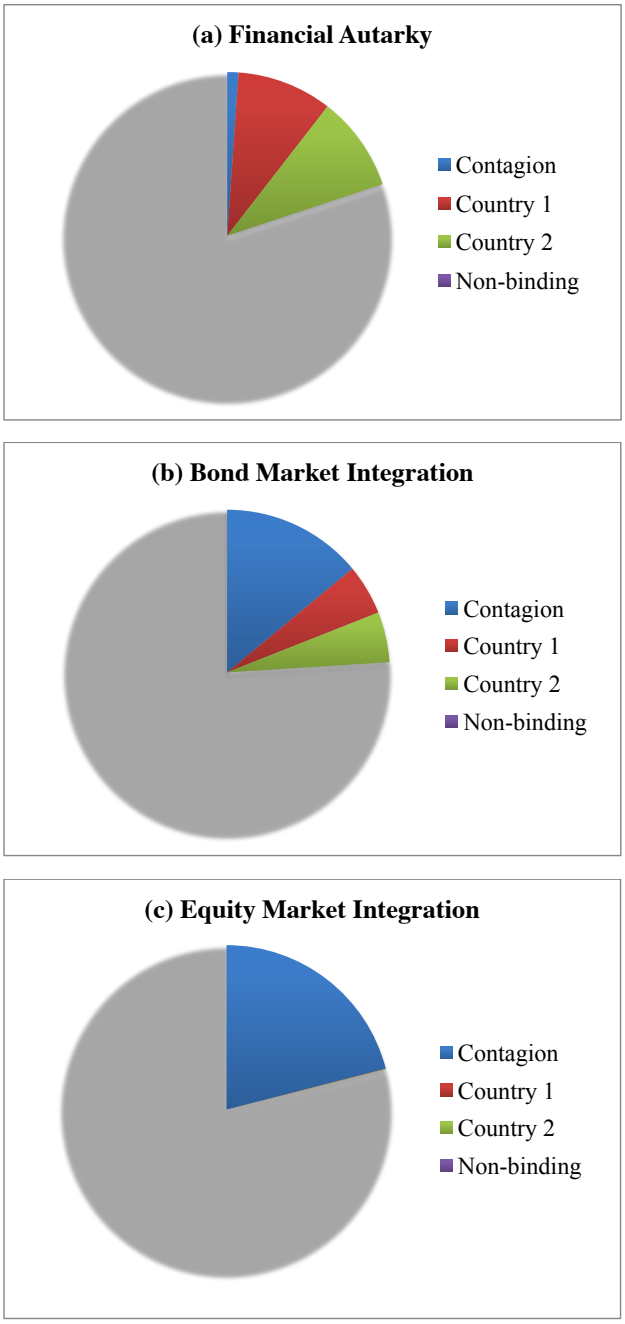

Figure 2: This chart shows the joint distribution of collateral constraints being binding or not in financial autarky (panel a), bond market integration (panel b) and equity market integration (panel c). Contagion is defined as prob$(\mu_1 > 0, \mu_2 > 0)$, Country i in the chart denotes prob$(\mu_i > 0, \mu_j = 0)$ with $i \neq j$, and Non-binding is prob$(\mu_1 = 0, \mu_2 = 0)$.

global bond integration. What accounts for this? The key reason is due to the scale of overall borrowing. In financial autarky, investors limit their debt accumulation due to precautionary motives. Because of this, the leverage constraint binds on average only 10 percent of the time. When bond markets become integrated, the magnitude of consumption risk falls significantly, as we will see below. More importantly, consumption volatility during crisis events (i.e. when the leverage constraint is binding) is significantly less than that under financial autarky. As a result, there is a significant fall in the motive for precautionary saving, increasing the willingness to borrow, raising leverage closer to the limit implied by equation (20).[13] This increases the unconditional probability for the leverage constraint to bind.

Hence the two key features of financial integration are a substantial increase in the correlation of crises across countries, and an increased willingness to assume a higher level of leverage. Figure 2 illustrates now the impact of moving from integration in bond markets alone to a full integration of equity markets and bond markets.[14] With integrated equity markets, the correlation of leverage crises across countries becomes effectively complete. Conditional on one country being constrained, the probability of the second being constrained is above 99 percent. The unconditional probability of crises is not much affected however, relative to the bond market integration case (21 percent relative to 19 percent). But the key difference is that there is effectively zero probability that a country will be subject to a balance sheet constraint on its own.

The key feature that effects the linkage of financial contagion with equity integration relative to bond market integration is the direct interdependence of balance sheets. As illustrated by equation (2), with equity market integration, the collateral value of investors portfolios are linked via the prices of domestic and foreign equity. Investors in one country hold a diversified portfolio, and shocks which affect foreign equity prices directly impact on the value of domestic collateral, independent of the world interest rate. This leads to a dramatic increase in the degree of financial contagion in the equity market equilibrium relative to the bond market equilibrium.

Since the leverage constraint depends on the collateral value of capital, we expect that the constraint is more likely to be binding in low productivity states. Figure 3 shows how the probability

[13] It is important to note that since the two countries are ex-ante identical, increased leverage in financial integration is on average financed by domestic savers. The fall in risk associated with integration does also impact on domestic savers' motive for precautionary saving. But as noted above, the impact of financial constraints on the volatility of investors' consumption means that the precautionary savings motive for that group exceeds that of savers. As a result, increasing financial integration leads to an increase in investors' borrowing and savers' lending, within each country.

[14] We note that although equity markets enhance the possibilities for cross-country risk-sharing relative to bond market integration alone, this still falls short of a full set of Arrow-Debreu markets for risk-sharing.

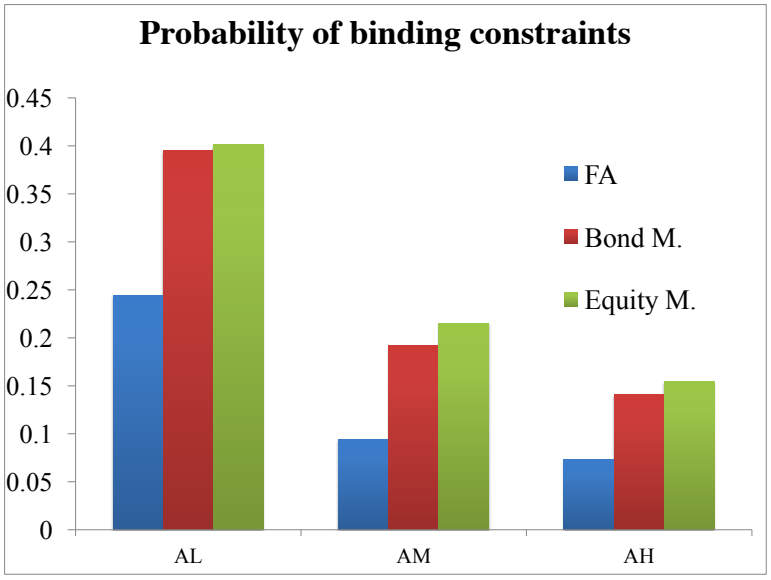

Figure 3: This chart shows the conditional distribution of collateral constraints being binding or not in financial autarky (FA), bond market integration (Bond M.) and equity market integration (Equity M.) under different shock states. *AL*, *AM* and *AH* denote the low, medium and high state of productivity *A*, respectively.

of binding constraints depends on the state of productivity in any one country, contrasting this across all three financial regimes. In financial autarky, the probability of being constrained is much higher in the low productivity state. Conditional on a low state, the probability of the constraint binding is about 25 percent. The corresponding probabilities under the medium and high states are 9 percent and 7 percent respectively. But when we open up international bond markets and international equity markets, it is much more likely for a country to be leverage constrained in medium or high productivity states, as well as states of low productivity. In the low productivity state (for one country alone), the leverage constraint is binding 40 percent of the time under financial integration. But the corresponding probability for the medium and high productivity states are 20 and 15 percent, approximately. Hence, financial integration doubles the probability of leverage crises, independent of the underlying productivity states.

5.2 Moment Analysis

We now look more closely at the implications of alternative degrees of financial integration on economic outcomes in the two countries. The first thing to note is that financial integration affects means or first moments as well as standard deviations of each variable. The first moment effects

are critical for welfare analysis, but as we see below, they also provide an important element in understanding the overall effects of financial integration.

The impact of alternative degrees of financial integration can be seen in Tables 2 and 3. Table 2 shows the simulated mean levels of a range of macro and credit aggregates, as well as asset prices.[15] Table 3 shows standard deviations, in percentage terms, as well as the cross-country correlations, under each financial regime. In each case, we report first the moments over the whole sample, then the moments restricted to states where the leverage constraint binds in both countries simultaneously (or in the case of financial autarky, just cases where the leverage constraint is binding). The realization of the productivity draw is unconstrained in this comparison. In the discussion below, we will also look at a more restrictive case where the leverage constraint binds, and productivity in one country is at its lowest level.

In the discussion above, we noted that financial integration leads to an increase in the overall probability of leverage crises, as well as an increase in the cross-country correlation of crises. The first factor comes about because financial integration leads to a substantial increase in the leverage of investors in both countries. Table 2 shows that the average level of investor borrowing increases by about 30 percent when we move from financial autarky to bond or equity integration. This increased borrowing occurs in both countries. As we noted above, since the countries are exactly symmetric there is no increase in net foreign indebtedness on average. In net terms the increased borrowing is financed by domestic savers. The rise in indebtedness translates into a shift from an average leverage rate of 1.46 under autarky to a leverage of 1.67 and 1.69 with financial integration under bonds and equities, respectively.

The rise in leverage after financial integration is driven by the fall in consumption risk. Table 3 shows that financial integration reduces the standard deviation of investor's consumption from 4.6 under financial autarky to 3.8 and 3.6 in bond and equity integration, respectively. This reduces precautionary savings and increases the willingness on behalf of investors to accumulate debt, (given that the precautionary savings motive for savers is weaker). But there is an amplifying secondary effect, through a rise in the average level of equity prices. The reduction in risk leads to a fall in the equity risk premium, as shown in Table 2. The fall in the required return on equity relative to debt leads to an equilibrium rise in the price of equity, thus increasing the ability to service debt without violating the leverage constraint.

[15] We note that our measure of output also includes output produced in the informal sector. The results when comparisons for market output are used instead are very similar to those in the Tables.

Tables 2 and 3 show that when simulations are averaged over the full sample, including both episodes when the leverage constraint is binding and non-binding, the effects of financial integration on means and volatilities of macro aggregates are relatively small, apart from the sharp increase in average credit levels. Table 2 shows that average investor consumption levels fall when moving to bond or equity integration. This is because, in a stationary equilibrium, impatient investors must service a higher debt burden, since they wish to front-load their consumption stream. Mean output rises slightly, as the increased allocation of capital to the formal sector increases production. Mean employment is nearly unchanged. The access to world capital markets reduces the volatility of consumption, as we have noted, since investors can now borrow at more stable interest rates. There is a small drop in output and employment volatility in bond and equity integration. As we noted above the equity risk premium falls with integration, but the external finance premium rises, since investors are holding more debt in equilibrium.

These results are more or less in accord with with standard results in open economy macro literature (e.g., Baxter and Crucini, 1995; Heathcote and Perri, 2002; Gourinchas and Jeanne, 2006)- international financial integration has limited implications for macroeconomic aggregates. But if we now restrict the sample to a comparison during leverage crises, we find a very big divergence. During episodes when leverage constraints bind, there is a dramatic effect of financial market integration. While integration increases the frequency of crises, as shown before in Figures 2-3, it significantly reduces their severity. Focusing on first moments, we see that mean levels of output, consumption and employment during crises are all significantly higher when financial markets are integrated. Compared to autarky, output is on average 2 percent higher under bond integration, and 3 percent higher with equity integration during a crisis. This reflects a higher level of mean employment and a much larger fraction of capital use in the formal sector. As a result, investor consumption in crises is 4 percent higher under bond integration and 7 percent higher with equity integration, despite the fact that in the full sample, average investor consumption is higher in financial autarky. Because investors are leverage constrained in a crisis, the movement in equity prices is an important constraint on their borrowing and consumption. While equity prices fall during a crisis, the drop under financial autarky is higher than that under bond or equity integration: 8 percent, as opposed to 7 percent and 6 percent, respectively. In addition, the average rise in interest rates in crises, adding to the costs of borrowing, is less with financial market integration than in financial autarky. We also note that the external finance premium, capturing the excess cost of investor borrowing, is

higher during a crisis in financial autarky than with financial integration, despite the fact that over the whole sample, the premium is lower under financial autarky.

Thus, international capital markets have relatively minor impacts during 'normal' times, but act so as to substantially reduce the severity of leverage crises. We see the same features in comparing second moments. The volatility of output, consumption and employment, conditional on a crisis, are substantially lower under financial integration than in financial autarky. Under financial autarky, average output volatility jumps by 40 percent during a crisis. With financial integration under bond and equity trade, the increase in volatility is only about half as much (26 percent).

What accounts for the major difference between 'normal times' and 'crisis times' as regards the effects of financial market openness? We know from previous literature that a binding collateral constraint introduces a 'financial accelerator', so that a negative shock to productivity leads to an amplified fall in equity prices, borrowing, and output through the process of forced de-leveraging. The same process is taking place in this model. In the comparison of the performance during leverage crises, the financial accelerator is in operation under all degrees of financial market integration. But because financial markets allow for diversification, when the underlying fundamental shocks are not perfectly correlated across countries, they also allow the multiplier effects of these shocks to be cushioned through a smaller volatility in world interest rates and asset prices. Our results indicate that equity prices are less pro-cyclical under financial integration than in financial autarky, and interest rates are distinctly less counter-cyclical.[16] Thus, while volatility is magnified during leverage crises under all regimes, the impact is much greater in the absence of this international diversification.

With these observations, we can look back at Table 2 and more clearly understand why international financial integration leads to such a rise in the mean level of borrowing and leverage within countries. Investors are willing to increase their leverage not simply because average consumption risk is lower in a financially integrated environment, but because the consequences of crises in terms of the level and volatility of consumption are less severe.

Why is it that first moments are also lower during a leverage crisis, under financial autarky? This is due to the asymmetry between positive and negative shocks, as we pointed out in Figure 1 above. Since a negative productivity shock is more likely to lead to binding collateral constraints, and the response to a negative shock will be greater under financial autarky than with international

[16] Results are available upon request.

financial integration, it follows that international financial markets facilitate higher average levels of consumption, output and employment, even during episodes of leverage crises. Table 2 indicates that the rise in interest rates, affecting the borrowing costs facing investors, is significantly larger in leverage crises in the financial autarky environment than when capital markets are open.

Hence, while on average, international capital markets lead to a rise in the probability of binding leverage constraints, and an increase in financial contagion, they have the benefit that crises are much less severe with financial market integration than under financial autarky. This points to a clear trade-off between the benefits of integration and the increased preponderance of balance sheet crises under integration. In section 5.6 below, we explore the welfare implications of this trade-off.

5.3 Comparison of Financial Autarky and Financial Integration under Low States of Productivity.

In the previous section, we defined a leverage crisis as a state where the leverage constraint binds in one or both countries. But this is not necessarily associated with the lowest outcome for productivity. Moreover, as Figure 3 indicates, the greater frequency of crises with financial integration comes partly because there are more crises under medium and high states for productivity. Thus, one may ask whether the comparison of crisis events between financial autarky and financial integration is robust to an alternative definition of crises. Are crises in financial autarky still more severe than those under financial integration when we define a crisis event as one where leverage constraints bind in both countries, and in addition, productivity is at its lowest level?

The bottom panels in Table 2-3 show that this is indeed the case. These tables illustrate the comparison of first and second moments for the three different financial regimes when leverage constraints bind, and productivity in the home country is at its lowest level (i.e. $A = A_L$). We see that average levels of consumption, output and employment in the home country are still significantly higher under either bond market integration or equity integration than with financial autarky. Likewise, consumption, output and employment volatility are lower in the presence of international financial market integration. Hence, even when the comparison is restricted to the low productivity state, we find nonetheless that there remains a major cushioning effect of financial markets in times of crises.

Table 2: Simulated means

	Autarky	Bond		Equity	
		Home	Foreign	Home	Foreign
Panel A: The whole sample					
Total output	1.193	1.196		1.196	
Investor consumption	1.064	1.020		1.017	
Capital stock K_1	0.776	0.783		0.783	
Investor borrowing	4.520	5.834		5.918	
Leverage	1.464	1.671		1.686	
Labor	1.008	1.012		1.013	
Equity price	9.186	9.284		9.286	
Interest rate	1.0417	1.0417		1.0417	
External finance premium (%)	0.22	0.33		0.34	
Equity risk premium (%)	0.69	0.52		0.52	
Panel B: A subsample with $\mu_1 > 0$ and $\mu_2 > 0$					
Total output	1.116	1.137		1.150	
Investor consumption	0.826	0.862		0.884	
Capital stock K_1	0.667	0.699		0.719	
Investor borrowing	5.838	6.222		6.461	
Leverage	2.000	2.000		2.000	
Labor	0.924	0.948		0.962	
Equity price	8.551	8.739		8.862	
Interest rate	1.0497	1.0486		1.0474	
External finance premium (%)	2.07	1.96		1.61	
Equity risk premium (%)	1.73	1.42		1.25	
Panel C: A subsample with $\mu_1 > 0$, $\mu_2 > 0$ and $A_1 = A_L$					
Total output	1.025	1.040	1.151	1.049	1.159
Investor consumption	0.721	0.769	0.859	0.804	0.852
Capital stock K_1	0.600	0.642	0.694	0.665	0.700
Investor borrowing	4.715	5.338	6.026	5.875	5.801
Leverage	2.000	2.000	2.000	2.000	1.989
Labor	0.848	0.867	0.961	0.879	0.968
Equity price	7.492	8.057	8.430	8.273	8.333
Interest rate	1.0760	1.0610	1.0610	1.0617	1.0617
External finance premium (%)	2.62	2.47	2.15	2.06	2.06
Equity risk premium (%)	2.30	1.70	1.52	1.50	1.51

Notes: This table reports simulated means for variables of interest in the model economies. Column Autarky, Bond and Equity denote financial autarky, bond market integration and equity market integration respectively. In financial autarky, Home and Foreign countries are symmetric. Model parameters are the same as the baseline model. The results are obtained through simulating the model economy $210,000$ periods and the first $10,000$ periods are discarded to get rid of the impact of initial conditions. A_L denotes the low state of productivity. All model economies use the same realized shock sequences.

Table 3: Simulated standard deviations

	Autarky	Bond	Equity	Corr.(bond)	Corr.(equity)
Panel A: The whole sample					
Total output	3.91	3.86	3.89	0.03	0.02
Investor consumption	4.61	3.76	3.63	0.64	0.78
Capital stock K_1	1.99	1.65	1.67	0.57	0.67
Investor borrowing	35.01	28.11	26.18	0.57	1.00
Labor	3.16	3.04	3.07	0.07	0.07
Equity price	47.18	34.73	34.62	0.96	1.00
Interest rate	1.77	1.23	1.23		
External finance premium	0.18	0.21	0.21	0.42	1.00
Equity risk premium	5.60	4.01	3.98	0.94	0.99
Panel B: A subsample with $\mu_1 > 0$*, and* $\mu_2 > 0$					
Total output	5.42	4.80	4.77	0.03	0.05
Investor consumption	6.36	4.79	4.43	0.49	0.78
Capital stock K_1	4.16	3.12	2.99	0.60	0.75
Investor borrowing	68.92	50.27	47.39	0.72	1.00
Labor	4.61	3.96	3.93	0.09	0.11
Equity price	63.41	43.03	42.53	0.89	1.00
Interest rate	1.81	1.15	1.20		
External finance premium	0.55	0.50	0.43	0.38	1.00
Equity risk premium	6.15	4.48	4.37	0.91	0.99
Panel C: A subsample with $\mu_1 > 0$*,* $\mu_2 > 0$ *and* $A_1 = A_L$					
Total output	2.38	1.86	1.71	0.27	0.33
Investor consumption	3.85	3.53	3.46	0.56	0.92
Capital stock K_1	4.19	3.67	3.50	0.57	0.91
Investor borrowing	42.82	43.59	46.91	0.61	1.00
Labor	3.08	2.54	2.39	0.36	0.48
Equity price	15.36	27.31	29.62	0.82	1.00
Interest rate	1.02	0.95	0.97		
External finance premium	0.96	0.77	0.73	0.52	1.00
Equity risk premium	7.72	5.23	5.09	0.92	0.99

Notes: This table reports simulated standard deviations in percentage terms for variables of interest in the model economies. Column Autarky, Bond and Equity denote financial autarky, bond market integration and equity market integration respectively. Corr.(bond) and Corr.(Equity) are for cross correlations in bond and equity market integration. Model parameters are the same as the baseline model. The results are obtained through simulating the model economy $210,000$ periods and the first $10,000$ periods are discarded to get rid of the impact of initial conditions. A_L denotes the low state of productivity. All model economies use the same realized shock sequences. The model period is one year and variables are HP-filtered with parameter $\lambda = 100$.

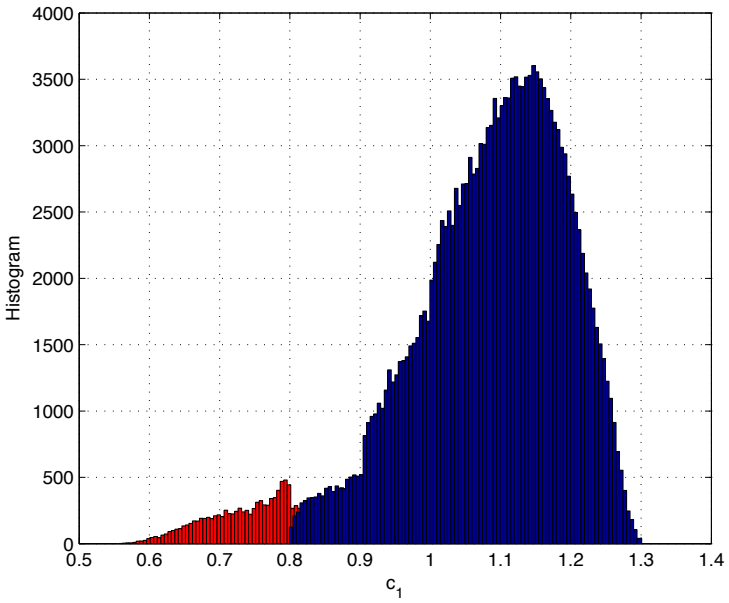

Figure 4: Investors' consumption distribution along a simulated path of 200, 000 periods in financial autarky. The blue region denotes the distribution of consumption when collateral constraints don't bind and the red region is for binding collateral constraints. There are also overlapping areas in the middle. Model parameters are the same as the baseline model.

5.4 The Distribution of Investors' Consumption under Alternative Financial Market Outcomes.

Figures 4-6 illustrate the empirical distributions of investors' consumption from the model simulations. In financial autarky, Figure 4 shows that investors' consumption has a bi-modal characteristic with a fat left tail. The blue shades indicate states where the leverage constraint is non-binding, while the red shaded areas indicate states with the binding constraint. Clearly consumption is lower in the latter case. But the presence of the fat left tail of the distribution indicates that constrained states tend to be more persistent than unconstrained states. This is consistent with previous analysis (e.g. Brunnermeier and Sannikov, 2014). Once the economy is in a debt constrained equilibrium, the convergence to a steady state slows down significantly.

Figures 5 and 6 illustrate the joint consumption distribution for country 1 and country 2 in four separate panels, depending upon whether they are both unconstrained, both constrained, or just one country constrained. Figure 5 illustrates the distribution under bond market integration. Again, when both countries are unconstrained, consumption is substantially higher on average, and the distribution of consumption is narrower. When both countries are constrained, consumption

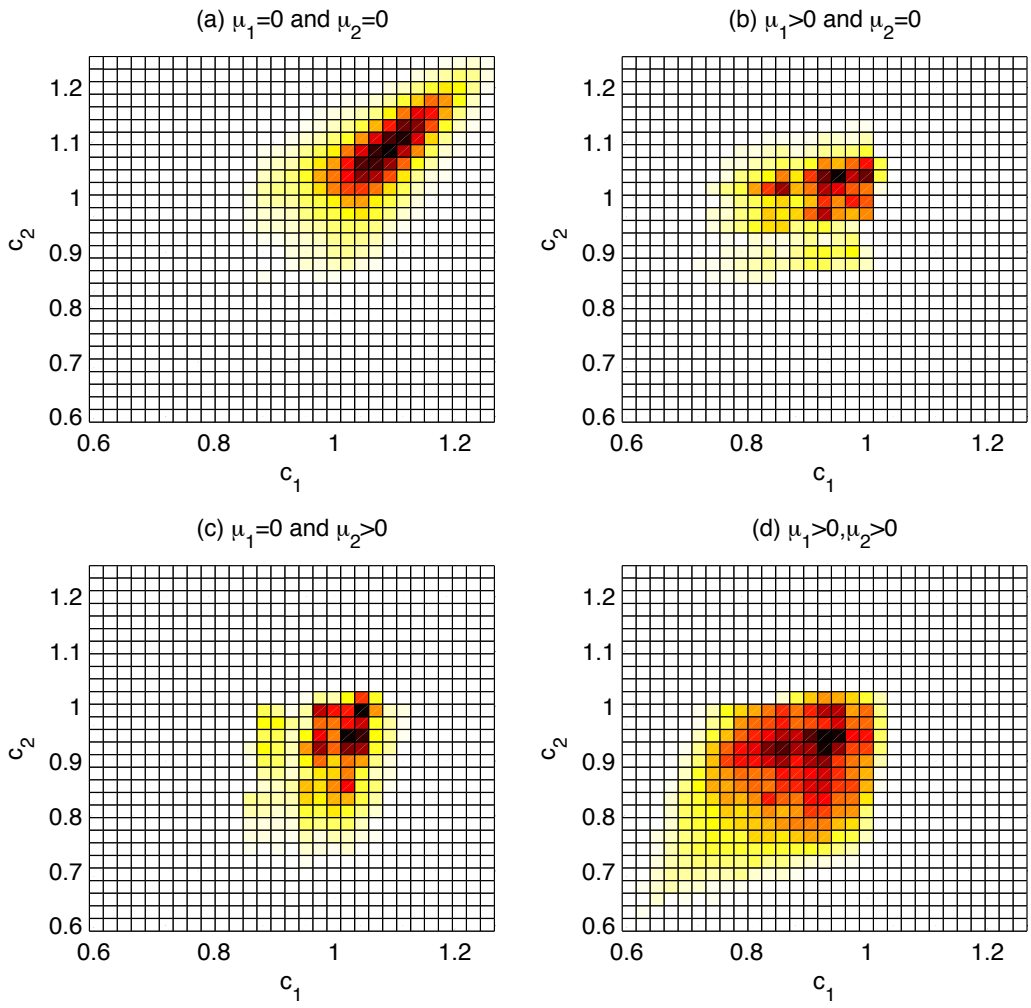

Figure 5: Investors' consumption distribution along a simulated path of 200,000 periods in bond market integration. The darker an area is, the higher the frequency becomes. Panel (a) displays the distribution of consumption when investors in both countries are financially unconstrained, panel (b) for only investors in country 1 constrained, panel (c) for only investors in country 2 constrained, and , panel (d) for investors in both countries constrained. Model parameters are the same as the baseline model.

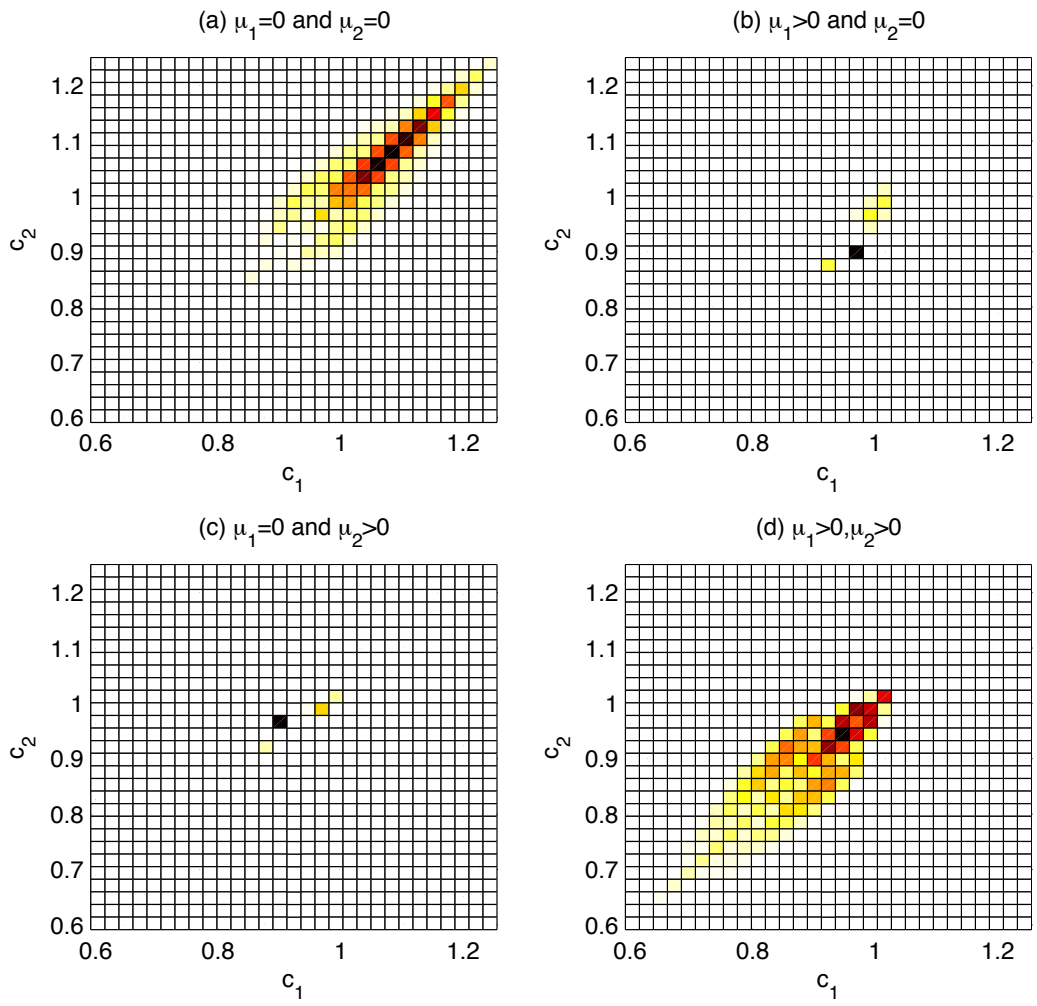

Figure 6: Investors' consumption distribution along a simulated path of 200,000 periods in equity market integration. The darker an area is, the higher the frequency becomes. Panel (a) displays the distribution of consumption when investors in both countries are financially unconstrained, panel (b) for only investors in country 1 constrained, panel (c) for only investors in country 2 constrained, and panel (d) for investors in both countries constrained. Model parameters are the same as the baseline model.

rates are lower, and the distribution is more spread out in both directions. More generally, we see a mutli-modal characteristic of the consumption distribution in this case, where depending on constraints that bind, there are multiple local maxima in the distribution of consumption. Figure 5 also indicates two features of the nature of leverage crises under bond market equilibrium. First, as in the financial autarky case, there is substantially more persistence in the economy constrained by leverage limits (i.e. fat left tails). But also, given that the consumption distribution is more laterally spread, the impact of leverage constraints substantially reduces the degree to which bond market integration facilitates international consumption risk-sharing. This point can be seen equivalently in going back to Table 3, where we see that the cross country consumption correlation drops from 0.64 in the full sample to 0.49 in the case where leverage constraints bind.

Finally, Figure 6 shows the joint consumption distribution but now under full equity integration. The sparseness of the distributions with just one constraint binding confirms our previous results that financial contagion is almost complete in the equity integration case. Again we find that the distribution is shifted down substantially when the leverage constraints are binding, and tends to be more spread out. But unlike the bond market case, the distribution is not noticeably more laterally spread out when leverage constraints are binding. The implication of this is that unlike the bond market equilibrium, the presence of binding leverage constraints does not clearly interfere with cross country consumption risk sharing when equity markets are integrated. While leverage crises reduce average consumption rates, they do not limit the degree to which equity markets can share risk across countries. In fact, Table 3 establishes that the cross country correlation of consumption is not reduced in simulations that are restricted to the binding leverage states relative to those from the overall sample.

5.5 Event Analysis

We now organize the simulation results in terms of an event analysis. Because the responses to shocks to productivity in the model depend upon the existing states, it is not possible to conduct conventional impulse response calculations as in models analyzed with linear approximations. Instead, we define a particular event characterized by a particular set of criteria, group together all simulation runs in the model which satisfy this criteria, and then take an average of these runs over the whole sample. In this instance, we define the event as a situation where the home country (country 1) leverage constraint is initially non-binding (for at least two periods), then binds for

three successive periods, and in addition, the home country experiences the lowest productivity outcome in the middle of these periods. All other variables are left unconstrained, including all variables pertaining to the foreign country.[17] Figure 7-9 organizes the simulation results in terms of an 'Event Analysis'.

Using this definition for each of the three financial market regimes, we then compare the outcomes in Figure 7-9. For the case of financial autarky, we show only the home country responses, because under financial autarky, the two country's responses are independent of one another.

A crisis event as defined here is associated with a shift from an unconstrained to a constrained borrowing position. We note that although the event only requires that the constraint is binding in period -1, 0 and 1 in the Figures, in fact the constraint remains binding for a persistent period afterwards, in all degrees of financial integration. Thus, as documented above, leverage crises become endogenously persistent in this model. We can measure the severity of the crisis by the size of the Lagrange multiplier. Under autarky, this is highest, confirming again that the crisis is more severe in the absence of financial integration. With full equity market integration, the multiplier is equated across countries.

The third and fourth panel of Figure 7 indicates that as soon as the leverage constraint becomes binding, investors undertake a process of de-leveraging, although this begins from different levels of initial debt, depending on the degree of financial integration. Crisis events under all financial environments follow episodes of credit booms. Under financial autarky, debt is lower on average, but despite this, the proportionate debt reduction during the crisis episode (from period -2 to 0) is significantly greater in autarky than with bond or equity integration. Likewise, in the bottom panels of the Figure, we see that the crisis is associated with a successive fall in home country output, beginning at the time the leverage constraint binds. Again, while output starts from a lower level under autarky, the proportionate fall during the event is higher than with financial integration.

Figure 8 shows the response of consumption, asset prices and interest rates to the event. Asset prices fall by substantially more in autarky, and interest rates rise by more. Again, the rise in interest rates and fall in asset prices is triggered as soon as the leverage constraints become binding in period -1. Consumption of both investors and workers falls by more in autarky than with financial integration. With bond and equity market integration, there is a higher correlation of macro aggregates across the two countries, but the amplitude of response to the crisis event is

[17] We experimented with other definitions of an event, for instance assuming only that the leverage constraint binds without specifying the productivity state. The results were similar to those in Figure 7-9.

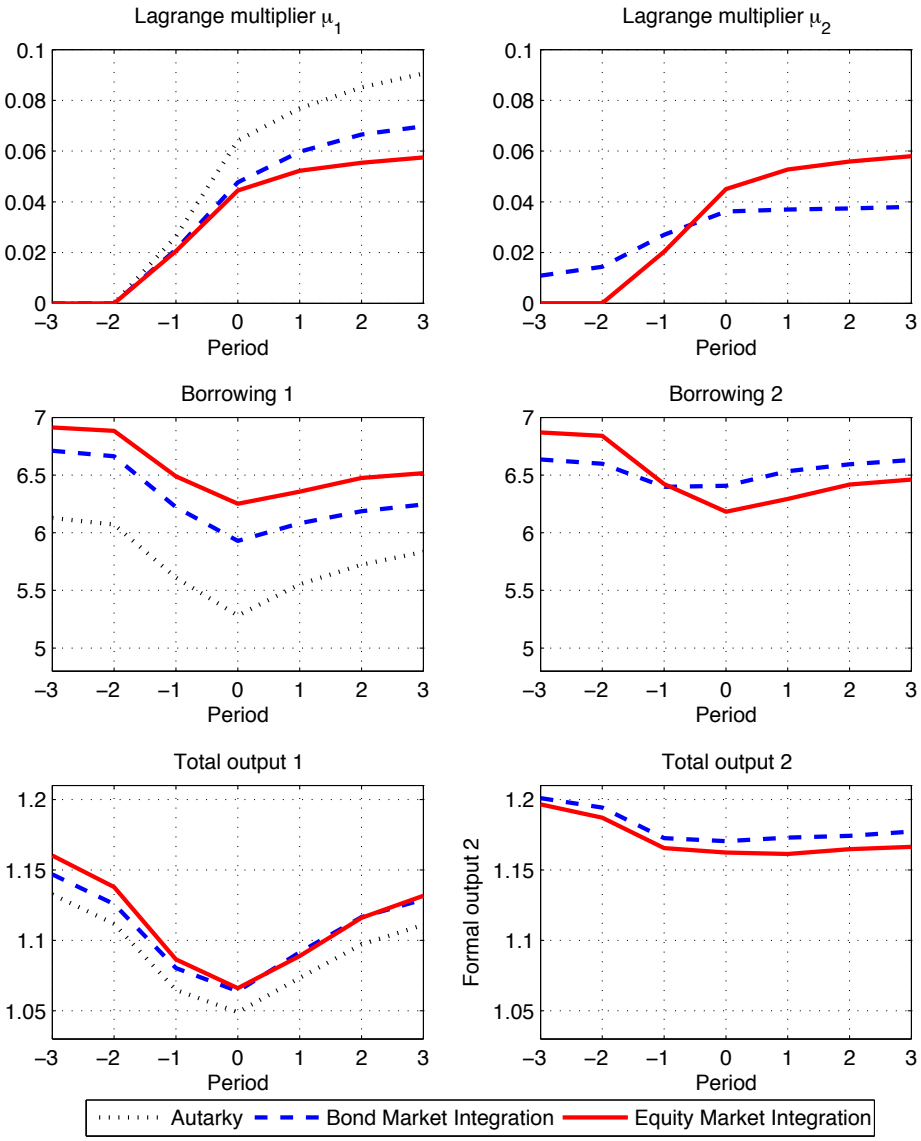

Figure 7: Event analysis in financial autarky (black dots), bond market integration (blue dashed lines) and equity market integration (solid red lines). The figure shows an average of events with a seven-period window along a simulated path with 200,000 periods. The selection of a seven-period window satisfies that (a) collateral constraints don't bind in period -3 and -2 and bind in the following three periods (period -1, 0, 1) in country 1, (b) country 1 experiences the lowest productivity in the period 0, and (c) no restrictions are imposed in the last periods (period 2 and 3). '1' denotes country 1 and '2' for country 2 in panel titles.

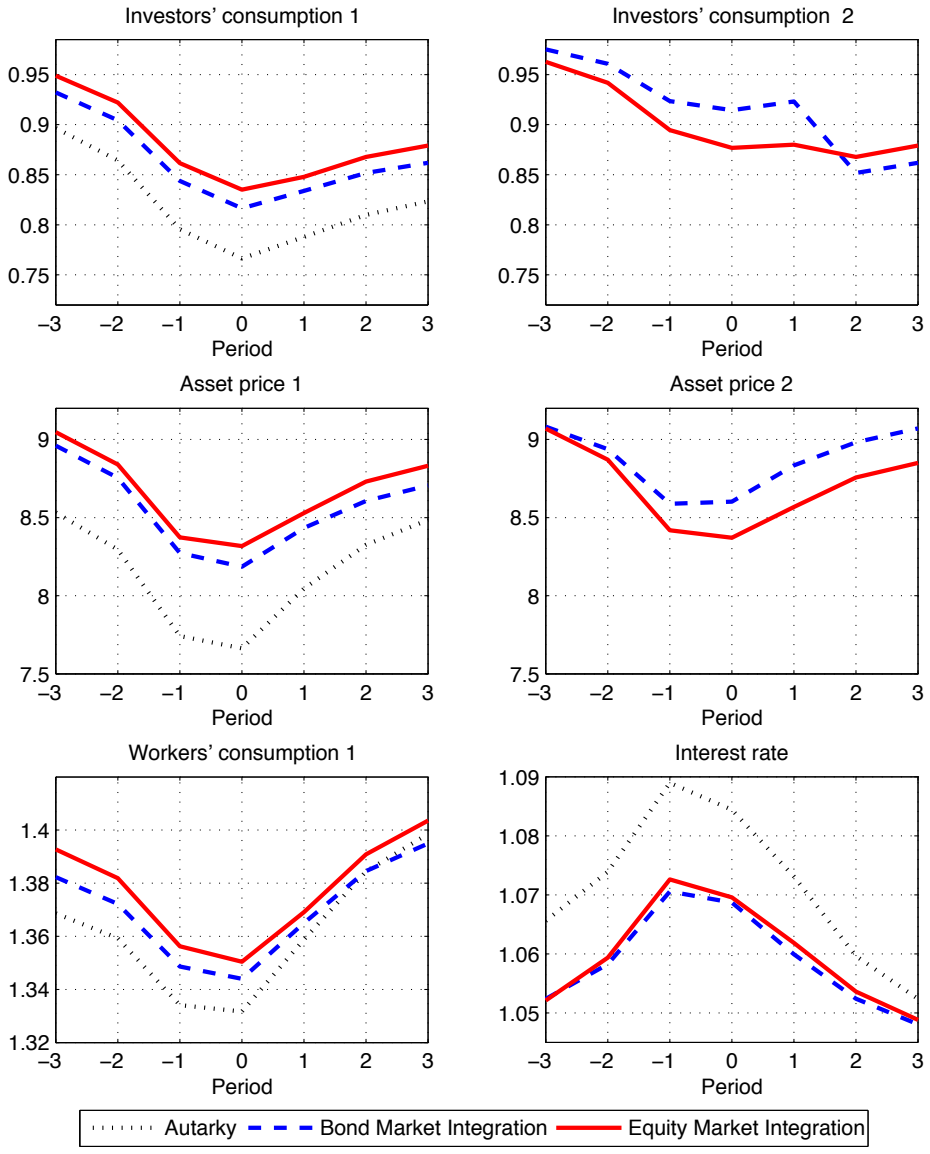

Figure 8: Event analysis in financial autarky (black dots), bond market integration (blue dashed lines) and equity market integration (solid red lines). The figure shows an average of events with a seven-period window along a simulated path with 200,000 periods. The selection of a seven-period window satisfies that (a) collateral constraints don't bind in period -3 and -2, and bind in the following three periods (period -1, 0, 1) in country 1, (b) country 1 experiences the lowest productivity in the period 0, and (c) no restrictions are imposed in the last periods (period 2 and 3). '1' denotes country 1 and '2' for country 2 in panel titles.

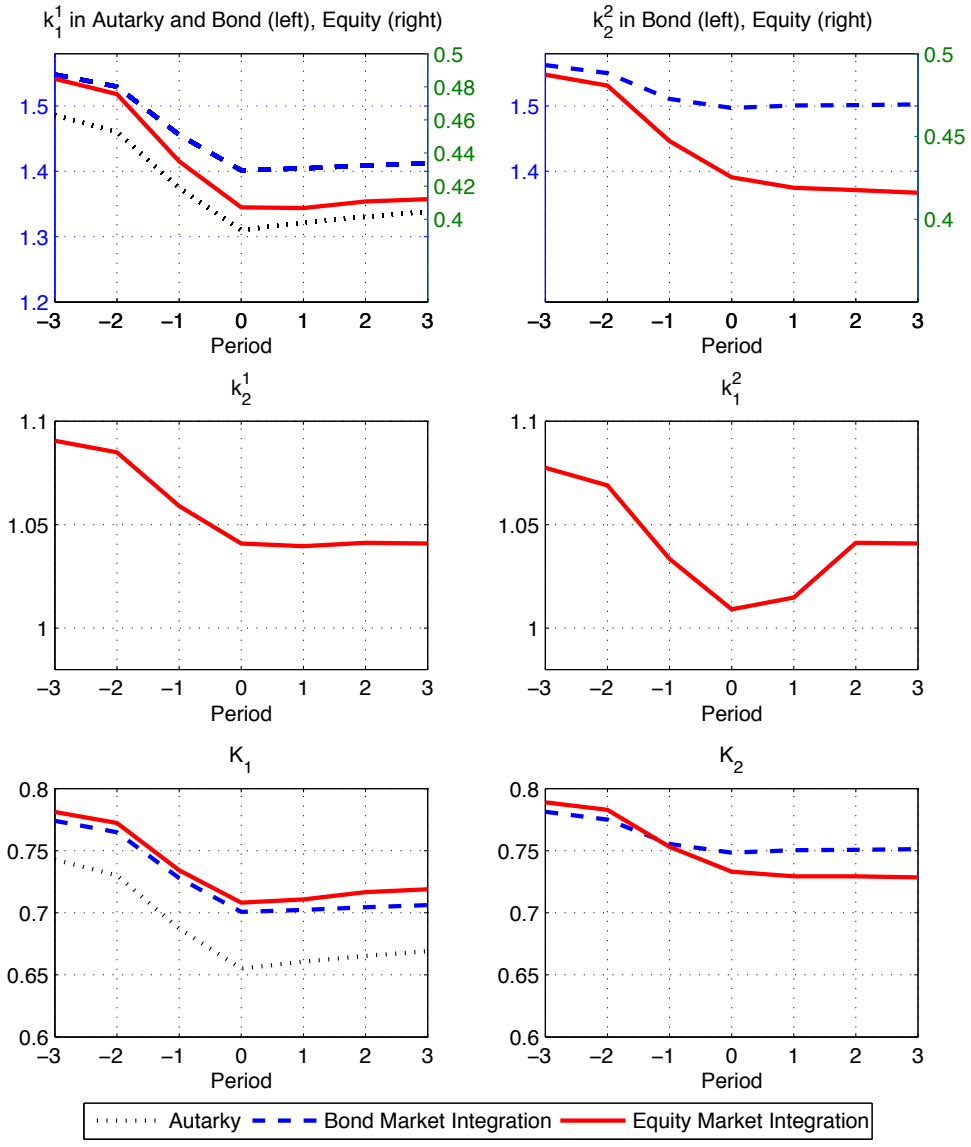

Figure 9: Event analysis in financial autarky (black dots), bond market integration (blue dashed lines) and equity market integration (solid red lines). The figure shows an average of events with a seven-period window along a simulated path with 200,000 periods. The selection of a seven-period window satisfies that (a) collateral constraints don't bind in period -3 and -2 and bind in the following three periods (period -1, 0, 1) in country 1, (b) country 1 experiences the lowest productivity in the period 0, and (c) no restrictions are imposed in the last periods (period 2 and 3). '1' denotes country 1 and '2' for country 2 in panel titles.

smaller. In general, with bond market integration, the correlation of responses is less than with equity market integration. With bond market integration, output falls by more in the home country, which is the source of the 'event', and also borrowing, investor's consumption, and asset prices fall by more. Bond integration helps insulate the country from the shock, but by less than can be achieved by full equity market integration.

Finally, Figure 9 shows the impact of the crisis event on portfolio holdings. We see that both home and foreign investors undertake a period of retrenchment, reducing both their holdings of both domestic equity as well as their offshore equity holdings. Thus, there is a scale back of gross equity positions in both countries, despite the fact that the crisis event is centred around the home country alone.

5.6 Welfare Analysis

What are the welfare implications of financial market integration in this model? The answer is not immediately obvious. Leverage constraints in each country prior to financial opening represent a pre-existing financial distortion, so increasing the number of financial markets is not necessarily welfare enhancing. More precisely, a number of authors in recent literature have pointed out (e.g. Bianchi, 2011; Bianchi and Mendoza, 2010; Korinek, 2011a; Korinek and Simsek, 2014; Jeanne and Korinek, 2010) that the existence of balance sheet constraints in financial markets introduces a pecuniary externality associated with the failure of individual investors to take account of their trading activity on the constraints faced by other traders. Hence, in general, investment and portfolio choices will fail to be socially optimal, and as a result it is not guaranteed that opening up financial markets will raise welfare. More generally, the presence of balance sheet constraints opens up the possibility for macro-prudential policy instruments applied to bond or equity trading that may improve upon the unrestricted free market outcomes.

Given that this is a heterogeneous agent model within each country, the evaluation of welfare must involve weighting preferences in some way. This introduces some complications with respect to the nature of the social welfare function, given that investors have a higher rate of time preference than savers. In particular, it would not be a valid comparison to focus only on a stationary or unconditional measure of welfare, since, as shown in Table 2, in a stationary equilibrium with open international financial markets, investors will have lower mean consumption levels, given that they reduce precautionary savings and tilt consumption more toward the present and away from the

future. An unconditional measure of welfare solely based on the stationary distribution will neglect the additional welfare benefits that investors obtain from being able to consume earlier, as a result of the lower-volatility environment induced by international financial integration.

To take account of this we compute welfare from a conditional distribution. Specifically, we compute welfare under all three different financial environments using the equilibrium policy functions for each environment, conditional on the same initial values for investor's portfolios and debt, where these initial portfolio and debt liabilities are determined at their mean levels under financial autarky. By using this conditional measure of welfare, we incorporate the full transitional benefits that agents receive, following the opening up of bond or equity markets. In addition, we assume that the regime change is unanticipated, and once announced, is taken as permanent.

The conditional utility for a representative agent l ($= 1, 2$ for an investor in either country or $= 3$ for a banker) is defined as

$$Wel_l \equiv E_0 \left\{ \sum_{t=1}^{\infty} \beta_l^{t-1} U(c_{l,t}, h_{l,t}) \right\}. \tag{34}$$

We focus on the certainty equivalence of effective consumption $\tilde{c}_l$, which is given by[18]

$$Wel_l = E_0 \left\{ \sum_{t=1}^{\infty} \beta_l^{t-1} \frac{[c_{l,t} - v(h_{l,t})]^{1-\sigma} - 1}{1-\sigma} \right\} = \frac{\tilde{c}_l^{1-\sigma} - 1}{1-\sigma} \frac{1}{1-\beta_l}.$$

Rearranging the equation above, yields

$$\tilde{c}_l = [Wel_l(1-\sigma)(1-\beta_l) + 1]^{\frac{1}{1-\sigma}}. \tag{35}$$

Suppose that economy-wide social welfare is defined as the equally weighted sum of utilities for all agents in an economy, and then we have a measure of economy-wide welfare

$$Wel = nWel_l + (1-n)Wel_3, l = 1, 2. \tag{36}$$

We compute both the conditional welfare and the effective consumption certainty equivalence for each set of agents (investors and savers) in all financial integration regimes. Computation of

[18] Certainty equivalences of effective consumption across different financial integration regimes are comparable. Let the $\tilde{c}_l^r$ denote effective consumption certainty equivalence in regime r, $r = 1, 2, 3$ for financial autarky, bond market integration and equity market integration, and then $\left(\tilde{c}_l^j / \tilde{c}_l^i - 1\right) \times 100$ measures the percentage increase of effective consumption in regime i such that welfare for agent l in regime i is the same as that in regime j.

Table 4: Conditional welfare in the baseline models

	Utility Wel_l	Certainty equivalence $\tilde{c}_l$	Change (%)
Panel A: Financial autarky			
Investor	-10.2859	0.6788	
Worker	-2.3124	0.9153	
Economy-wide	-6.2992	0.7971	
Panel B: Bond market integration			
Investor	-10.2725	0.6791	0.042
Worker	-2.1492	0.9208	0.601
Economy-wide	-6.2109	0.8000	
Panel C: Equity market integration			
Investor	-10.2667	0.6792	0.018
Worker	-2.1406	0.9211	0.032
Economy-wide	-6.2036	0.8002	

Notes: This table reports the conditional welfare and certainty equivalence of effective consumption for all agents in various financial integration regimes. The volatility of shocks are the same as in the baseline model. Wel_l denotes the life time discounted utility. Economy wide welfare is a weighted average of the life time discounted utility for investors and savers (workers). The initial portfolio and shock in period 0 read $k^1_{1,0} = k^2_{2,0} = 1.6$, $b_{1,0} = 4.5$ and $A_{1,0} = A_{2,0} = A_M$ where A_M is the middle state of productivity. The last column shows the percentage change of effective consumption in regime j relative to the previous regime i, $(\tilde{c}^j_l/\tilde{c}^i_l - 1) \times 100$.

Table 5: Conditional welfare in the low risk economies

	Utility Wel_l	Certainty equivalence $\tilde{c}_l$	Change (%)
Panel A: Financial autarky			
Investor	-10.1504	0.6817	
Worker	-2.2617	0.9170	
Economy-wide	-6.2060	0.7994	
Panel B: Bond market integration			
Investor	-10.1780	0.6811	-0.086
Worker	-2.1260	0.9216	0.500
Economy-wide	-6.1520	0.8014	
Panel C: Equity market integration			
Investor	-10.1775	0.6811	0.001
Worker	-2.1242	0.9217	0.007
Economy-wide	-6.1508	0.8014	

Notes: This table reports the conditional welfare and certainty equivalence of effective consumption for all agents in various financial integration regimes. The standard deviation of shocks is one-half of the baseline shocks (2% vs. 4%). Wel_l denotes the life time discounted utility. Economy wide welfare is a weighted average of the life time discounted utility for investors and savers (workers). The initial portfolio and shock in period 0 read $k^1_{1,0} = k^2_{2,0} = 1.6$, $b_{1,0} = 4.5$ and $A_{1,0} = A_{2,0} = A_M$ where A_M is the middle state of productivity. The last column shows the percentage change of effective consumption in regime j relative to the previous regime i, $(\tilde{c}^j_l/\tilde{c}^i_l - 1) \times 100$.

conditional welfare is done through two steps. First, we compute the stationary value function for each agent in each regime. Second, starting at the same initial conditions mentioned above in period 0, we apply the transition matrix and the value function to calculate the conditional welfare for each agent Wel_l in each financial integration regime.

Table 4 indicates the results of the comparison of welfare across the different regimes for the baseline calibration of the model. We find that overall social welfare as measured by (36) is higher when we move from financial autarky to bond integration or equity integration. This is also true separately for investors and savers/workers, although as to be expected from previous literature, the gains are very small. Investors gain in terms of consumption certainly equivalence by 0.04% moving from autarky to bond market integration, and then by about another 0.02% moving from bond market integration to equity market integration. The gain for savers/workers is significantly larger, equal to 0.6% and 0.03% respectively. Thus, for the baseline (or high-risk) economy, although there is no theoretical guarantee that financial market integration will raise welfare, we find that this is indeed the case. Welfare rises by a small amount when financial markets are opened, and most of the gains can be accrued by opening international bond markets alone.

But these results are likely to depend on the overall scale of risk in the economy. In the previous discussion, we highlighted the trade-off between the benefits of diversification and the costs of an increase in the probability of binding balance sheet constraints. A fall in overall risk may be expected to tilt the calculation towards an increase in the importance of the costs of financial distortions and away from the benefits of risk sharing. To explore this, Table 5 reports the same welfare calculations in a low-risk economy. This is defined in the same way as before, but now assuming that the unconditional standard deviation of productivity shocks is 2% instead of 4%.[19] Now we find that again, overall social welfare increases with financial integration, as before. But this overall measure masks a conflict among groups. Savers/workers are better off in either bond market integration or equity integration than under financial autarky. But now, investors are slightly worse off with integrated financial markets, both in bond integration and equity integration. As expected, the magnitude of welfare changes in terms of effective consumption equivalencies is again very small.[20] Nevertheless, the negative effects of financial market integration for the welfare of

[19] Aside from the welfare differences, the qualitative results of the paper do not change under this alternative low-risk analysis.

[20] It is important to note that these welfare calculations are not approximation errors. In calculating welfare effects, we choose a tolerance size and the number of grid points for endogenous state variables so that the approximation error is several orders of magnitude smaller than calculated differences in welfare across financial market regimes.

investors indicates that the presence of balance sheet externalities are important enough to reverse the normal presumption of welfare gains from financial market integration for this version of the model.

6 Conclusions

This paper constructs and solves a two-country general equilibrium model with endogenous portfolio choice, occasionally binding collateral constraints, and within and across country trade in equity and bond assets. Leverage is time varying and will depend on the nature of international financial markets. The paper finds that international financial integration introduces a trade-off between the frequency and severity of financial crises. Opening up financial markets leads to a higher degree of global leverage, increasing the frequency of financial crises for any one country and dramatically increasing the correlation (or contagion) of crises across countries. But crises in an open world capital market are less severe than in closed economies. In terms of welfare, financial market integration may be positive or negative.

The paper naturally suggests a number of extensions. One major question we have not addressed is the role for economic policy, whether it terms of macro-prudential tools that affect leverage and investment choices of agents, or other more general tools of fiscal or monetary policy. In addition, we have focused on shocks coming from real economic fundamentals - productivity. An obvious further question would be how shocks arising from the financial system itself would affect the nature and workings of international financial markets. Finally, we have not allowed a role for aggregate demand deficiencies following crises. These would naturally arise in an extended model that incorporated slow price or wage adjustment.

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Not For Publication Technical Appendix

A Asset Constraints and Model Solution

The text discussed the need for imposing asset trading constraints as part of the solution of the general equilibrium model with incomplete markets. Here we report how these asset market constraints affect the first order conditions of investors and bankers in the model. The portfolio penalties lead investors to have utility functions over consumption and portfolio holdings of the form

$$E_0\left\{\sum_{t=0}^{\infty}\beta_l^t\left[U(c_{l,t},h_{l,t})-\rho^l(k^l_{1,t+1},k^l_{2,t+1})\right]\right\}.$$

The penalty function for investor l has a form of

$$\rho^l(k^l_{1,t},k^l_{2,t})=\kappa_1\min(0,k^l_{1,t}-\underline{k}^l_1)^4+\kappa_2\min(0,k^l_{2,t}-\underline{k}^l_2)^4,$$

where $\underline{k}^l_i$, with $i=1,2$, denotes the lower bound for holding equity i by investor l, and $\kappa_i>0$ is a penalty parameter. Whenever equity holdings are lower than their lower bounds, investors will receive a penalty. We use a power of 4 here to make sure that the first-order partial derivative of ρ^l with respect to its any argument is twice continuously differentiable.

In the case of full equity market integration, investors first order condition for equity holding are given by:

$$q_{i,t}=\frac{\beta_l E_t\left\{U_c(c_{l,t+1},h_{l,t+1})(q_{i,t+1}+R_{ik,t+1})\right\}+\mu_{l,t}\kappa E_t\left\{q_{i,t+1}\right\}-\rho^l_{i,t+1}}{U_c(c_{l,t},h_{l,t})},\ i=1,2, \tag{A.1}$$

where $\rho^l_{i,t+1}$ is defined as

$$\rho^l_{i,t+1}=\frac{\partial\rho^l(k^l_{1,t+1},k^l_{2,t+1})}{\partial k^l_{i,t+1}}=4\kappa_i\min(0,k^l_{i,t+1}-\underline{k}^l_i)^3\leq 0.$$

The numerator of equation (A.1) states three types of gains for an investor from increasing an additional unit of equity holdings: (a) increasing consumption tomorrow, (b) relaxing a borrowing constraint for inter-period loans, (c) reducing penalties of hitting lower bounds of equity holdings.

That analogous optimality condition for bond holdings reads

$$q_{3,t}\equiv\frac{1}{R_{t+1}}=\frac{\beta_l E_t\left\{U_c(c_{l,t+1},h_{l,t+1})\right\}+\mu_{l,t}}{U_c(c_{l,t},h_{l,t})}. \tag{A.2}$$

The preference for a representative global banker takes a form of

$$E_0\left\{\sum_{t=0}^{\infty}\beta_3^t\left(\frac{1}{2}\right)\left[U(c^3_{1,t},h^3_{1,t})+U(c^3_{2,t},h^3_{2,t})-\rho^3(k^3_{1,t+1},k^3_{2,t+1},b_{3,t+1})\right]\right\},$$

with a penalty function

$$\rho^3(k^3_{1,t}, k^3_{2,t}, b_{3,t}) = \kappa_1 \min(0, k^3_{1,t} - \underline{k}^3_1)^4 + \kappa_2 \min(0, k^3_{2,t} - \underline{k}^3_2)^4 + 2\kappa_b \min(0, \bar{b}_3 - b_{3,t})^4.$$

where $\kappa_b > 0$, $\underline{k}^3_i$ with $i = 1, 2$ is the lower bound for physical capital held by the banker, and $\bar{b}_3$ is the upper bound for borrowing by the banker.

For the global banker under equity market integration, the optimality condition for equity holdings are

$$q_{i,t} = \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1})(q_{i,t+1} + G'(k^3_{i,t+1})) \right\} - \rho^3_{i,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})}, \; i = 1, 2, \tag{A.3}$$

where the marginal penalty for holding physical capital $\rho^3_{i,t+1}$ is similar to $\rho^1_{i,t+1}$.

Again, for the banker under equity market integration, the optimality condition for bond holdings is as follows

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1}) \right\} + \frac{1}{2}\rho^3_{3,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})}, \tag{A.4}$$

where $\rho^3_{3,t+1}$ is defined as

$$\rho^3_{3,t+1} = \frac{\partial \rho^3(k^3_{1,t+1}, k^3_{2,t+1}, b_{3,t+1})}{\partial b_{3,t+1}} = -8\kappa_b \min(0, \bar{b}_3 - b_{3,t+1})^3 \geq 0.$$

Under bond market integration, the optimality conditions for equity holdings are

$$q_{l,t} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \left(q_{l,t+1} + R_{lk,t+1} \right) \right\} + \mu_{l,t} \kappa E_t \left\{ q_{l,t+1} \right\}}{U_c(c_{l,t}, h_{l,t})}, \; l = 1, 2, \tag{A.5}$$

and for bond holding, the optimality condition is

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \right\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})}. \tag{A.6}$$

The global banker's problem under bond market integration is the same as under equity market integration.

Finally, in financial autarky, the investor's optimality conditions are the same as that under bond market integration. The optimality conditions for the banker's problem (now a domestic banker) yield

$$q_{l,t} = \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1})(q_{l,t+1} + G'(k^3_{l,t+1})) \right\} - \rho^3_{l,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})}, \tag{A.7}$$

$$q_{3,t} \equiv \frac{1}{R_{t+1}} = \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1}) \right\} + \rho^3_{3,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})}, \tag{A.8}$$

with penalty on excessive holdings of a portfolio

$$\rho^3_{l,t+1}(k^3_{l,t+1}, b^l_{3,t+1}) = 4\kappa_l \min(0, k^3_{l,t} - \underline{k}^3_l)^3,$$

$$\rho^3_{3,t+1} = \frac{\partial \rho^3(k^3_{l,t+1}, b^l_{3,t+1})}{\partial b^l_{3,t+1}} = -4\kappa_b \min(0, \overline{b}^l_3 - b^l_{3,t+1})^3 \geq 0.$$

B An Event Tree Approach to Solving the Model

This section shows in detail how we solve a model with equity market integration. The other two financial integration regimes can be solved similarly and are omitted here. Basically, we need to rewrite the forward-looking recursive competitive equilibrium in terms of a backward-looking system. The event tree approach developed by Dumas and Lyasoff (2012) is used. We use B-splines smooth functions with degree three to interpolate and approximate policy functions on discrete state grid points. Accordingly, the transformed system of equations in equilibrium are twice continuously differentiable by construction. Therefore, a Newton-type method can be used. Next, we calculate the system based on a discrete consumption share distribution, which lies between zero and one, and exogenous shocks $A_{l,t}$. Let S be the number of exogenous states and J be the number of consumption distribution grid points in the economy. After simplifying the dynamic system, there are $2S+6$ ($2S+4$ or $S+2$) number of equations in equity market integration (bond market integration or financial autarky). Let's take equity market integration for example. In equity market integration, we need to solve $2S+6$ nonlinear equations at $S \times J$ different nodes in each iteration. As is well known, it is not a trivial task to solve a system of nonlinear equations using Newton method, particularly at so many different nodes, unless we provide the solver with an extremely good initial guess at each node. In practice, we adopt a method similar to that in Judd, Kubler and Schmedders (2002), which uses a homotopy path-following algorithm to solve a system of nonlinear equations. Open sources for FORTRAN supply necessary subroutines to use a homotopy path-following algorithm, such as HOMPACK77/90, as well as subroutines for nonlinear solvers. Of course the key challenge for using this algorithm is in constructing a homotopy function.

The homotopy path-following algorithm requires certain conditions to be satisfied (see, Schmedders, 1998; Eaves and Schmedders, 1999; Watson, 1990). In practice, it is not easy to verify these conditions, particularly when the system is complicated, like the system here. So we take a practical perspective. That is, we will use the algorithm as long as it leads us to find optimal policy functions. First of all, we need to construct a proper homotopy function such that the homotopy path-following algorithm works effectively. In the GEI literature, several ways are proposed to use the homotopy path-following algorithm. Basically, in these exercises, researchers attempt to find current optimal consumption and exiting portfolio of assets given *current portfolio of assets* and shocks. In order to solve these models, it is normal to introduce a penalty function for asset trading, which guarantees continuity in excess demand functions. Conditional on continuous excess demand functions, the approach rephrases agents' objective functions such that financial autarky for each agent in each period is a good start for the homotopy path-following algorithm. However, the way of constructing homotopy functions in these papers seems improper in our model, because we don't

have any information on the *current portfolio of assets*. The main fact about the system here is that it should have a solution under a proper set of parameter values and that the Jacobian of the system should have full rank. After trying several ways of homotopy functions, we found that the Newton homotopy works quite well in the current model.

To illustrate, let $F(x)$ be a system of nonlinear equations with an endogenous variable vector x. $F(x)$ contains all of the equilibrium conditions in the model and it is a square nonlinear system. A homotopy function $H(x, \lambda)$ is defined as

$$H(x, \lambda) \equiv \lambda F(x) + (1 - \lambda)(F(x) - F(x_0)), \tag{B.1}$$

where x_0 is a starting point for the homotopy path-following and $0 \leq \lambda \leq 1$. When $\lambda = 0$, the homotopy function is degenerated to a simple system $H(x_0, 0) = F(x_0) - F(x_0) = 0$. This simple system $F(x) - F(x_0) = 0$ should have a unique and robust solution at $x = x_0$. When $\lambda = 1$, the homotopy function becomes the original function $H(x, 1) = F(x)$. Observe that if the Jacobian of $F(x)$ has a full rank, the Jacobian of the homotopy function $H(x, \lambda)$ also has a full row rank. Based on this constructed homotopy function, we can solve the transformed system for the optimal policy functions. We next need to simulate a path for exogenous shocks to pin down other endogenous variables. Combining the optimal policy functions with the initial conditions for shocks and portfolios, we can solve for all endogenous variables along the simulation path.

B.1 Equilibrium Conditions

Following the algorithm in Dumas and Lyasoff (2012), we rewrite the whole equilibrium system in the period $t + 1$ and get rid of all endogenous variables in period t.

The so-called 'marketability conditions' (budget constraints) of Dumas and Lyasoff (2012) are as follows

$$\tilde{c}_{l,t+1} + F_{l,t+1} = NW_{l,t+1} - v(H_{l,t+1}) + k^l_{1,t+1}(q_{1,t+1} + R_{1k,t+1}) + k^l_{2,t+1}(q_{2,t+1} + R_{2k,t+1}) - b_{l,t+1}, \tag{B.2}$$

where exiting wealth is $F_{l,t+1} \equiv q_{1,t+1}k^l_{1,t+2} + q_{2,t+1}k^l_{2,t+2} - b_{l,t+2}/R_{t+2}$ with $l = 1, 2$, and effective consumption is $\tilde{c}_{l,t+1} \equiv c_{l,t+1} - v(H_{l,t+1})$. The effective non-financial income becomes $NW_{l,t+1} - v(H_{l,t+1})$, where $NW_{l,t+1}$ is defined as

$$\begin{aligned} NW_{l,t+1} &\equiv \frac{1}{n}\left[F(A_{l,t+1}, H_{l,t+1}, K_{l,t+1}) - W_{l,t+1}H_{l,t+1} - K_{l,t+1}R_{lk,t+1}\right] + W_{l,t+1}H_{l,t+1} \\ &= W_{l,t+1}H_{l,t+1}, \end{aligned}$$

where we have used the fact that the formal production is CRS.

Kernel conditions read

$$q_{i,t} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \left(q_{i,t+1} + R_{ik,t+1} \right) \right\} + \mu_{l,t} \kappa E_t \left\{ q_{i,t+1} \right\} - \rho^l_{i,t+1}}{U_c(c_{l,t}, h_{l,t})}$$
$$= \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1})(q_{i,t+1} + G'(k^3_{i,t+1})) \right\} - \rho^3_{i,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})}. \tag{B.3}$$

with $l = 1, 2$ and $i = 1, 2$. And

$$q_{3,t} = \frac{\beta_l E_t \left\{ U_c(c_{l,t+1}, h_{l,t+1}) \right\} + \mu_{l,t}}{U_c(c_{l,t}, h_{l,t})} = \frac{\beta_3 E_t \left\{ U_c(c^3_{l,t+1}, h^3_{l,t+1}) \right\} + \frac{1}{2} \rho^3_{3,t+1}}{U_c(c^3_{l,t}, h^3_{l,t})} \ , \ l = 1, 2. \tag{B.4}$$

The collateral constraint reads

$$\left(\kappa E_t \left\{ q_{1,t+1} k^l_{1,t+1} + q_{2,t+1} k^l_{2,t+1} \right\} - b_{l,t+1} \right) \mu_{l,t} = 0. \tag{B.5}$$

Equilibrium labor yields

$$H_{l,t+1} = \left[\frac{\alpha A_{l,t+1} K^{1-\alpha}_{l,t+1}}{\chi} \right]^{\frac{1}{1+\nu-\alpha}}. \tag{B.6}$$

Let the global output be

$$Y_t \equiv A_{1,t} H^{\alpha}_{1,t} K^{1-\alpha}_{1,t} + A_{2,t} H^{\alpha}_{2,t} K^{1-\alpha}_{2,t} + (1-n) \left(Z(k^3_{1,t})^{\gamma} + Z(k^3_{2,t})^{\gamma} \right),$$

and the effective global output is given by

$$\tilde{Y}_t \equiv Y_t - v(H_{1,t}) - v(H_{2,t}).$$

B.2 Scaled Equilibrium Conditions

The share of investors' effective consumption in country l is defined as

$$\omega_{l,t+1} \equiv \frac{n \tilde{c}_{l,t+1}}{\tilde{Y}_{t+1}} \ , \ l = 1, 2. \tag{B.7}$$

Then bankers' effective consumption share becomes $\omega_{3,t+1} = \frac{2(1-n)\tilde{c}^3_{1,t+1}}{\tilde{Y}_{t+1}} = 1 - \omega_{1,t+1} - \omega_{2,t+1}$. Effective consumption for each agent thus reads

$$\tilde{c}_{1,t+1} = \frac{\omega_{1,t+1} \tilde{Y}_{t+1}}{n}, \qquad \tilde{c}_{2,t+1} = \frac{\omega_{2,t+1} \tilde{Y}_{t+1}}{n}, \qquad \tilde{c}^3_{1,t+1} = \frac{(1 - \omega_{1,t+1} - \omega_{2,t+1}) \tilde{Y}_{t+1}}{2(1-n)}.$$

Therefore, consumption levels can be obtained as

$$c_{1,t+1} = \tilde{c}_{1,t+1} + v(H_{1,t+1}), \qquad c_{2,t+1} = \tilde{c}_{2,t+1} + v(H_{2,t+1}),$$
$$c^3_{1,t+1} = \tilde{c}^3_{1,t+1} + v(H_{1,t+1}), \qquad c^3_{2,t+1} = \tilde{c}^3_{1,t+1} + v(H_{2,t+1}).$$

Notice that current output $\tilde{Y}_t$ is an endogenous variable, we then scale asset prices and exiting wealth by $1/\tilde{Y}_t^{\sigma}$

$$\tilde{q}_{i,t} = \frac{q_{i,t}}{\tilde{Y}_t^{\sigma}} \ , \ i = 1,2,3, \qquad\qquad \tilde{F}_{l,t} = \frac{F_{l,t}}{\tilde{Y}_t^{\sigma}}, \ l = 1,2. \tag{B.8}$$

Substituting the effective consumption share $\{\omega_1, \omega_2\}$, scaled asset prices $\tilde{q}_i$ and exiting financial wealth $\tilde{F}_l$ in period t and $t+1$ into the system of equations (B.2-B.6) to replace the effective consumption, asset prices and exiting financial wealth, we then obtain a transformed system of equations, which are listed as follows

$$\begin{aligned}\omega_{l,t+1} + \frac{n}{\tilde{Y}_{t+1}}\tilde{Y}_{t+1}^{\sigma}\tilde{F}_{l,t+1} = \frac{n}{\tilde{Y}_{t+1}}\left(NW_{l,t+1} - v(H_{l,t+1})\right) + \frac{n}{\tilde{Y}_{t+1}}k_{1,t+1}^{l}(\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{1,t+1} + R_{1k,t+1}) + \\ \frac{n}{\tilde{Y}_{t+1}}k_{2,t+1}^{l}(\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{2,t+1} + R_{2k,t+1}) - \frac{n}{\tilde{Y}_{t+1}}b_{l,t+1} \ , \ l = 1,2,\end{aligned} \tag{B.9}$$

$$\begin{aligned}\tilde{q}_{i,t} &= \frac{\beta_l E_t\left\{\left(\frac{\omega_{l,t+1}\tilde{Y}_{t+1}}{n}\right)^{-\sigma}\left(\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{i,t+1} + R_{ik,t+1}\right)\right\} + \mu_{l,t}\kappa E_t\left\{\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{i,t+1}\right\} - \rho_{i,t+1}^{l}}{\left(\frac{\omega_{l,t}}{n}\right)^{-\sigma}} \\ &= \frac{\beta_3 E_t\left\{\left(\frac{\omega_{3,t+1}\tilde{Y}_{t+1}}{2(1-n)}\right)^{-\sigma}\left(\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{i,t+1} + G'(k_{i,t+1}^{3})\right)\right\} - \rho_{i,t+1}^{3}}{\left(\frac{\omega_{3,t}}{2(1-n)}\right)^{-\sigma}},\end{aligned} \tag{B.10}$$

with $l = 1,2$ and $i = 1,2$.

$$\tilde{q}_{3,t} = \frac{\beta_l E_t\left\{\left(\frac{\omega_{l,t+1}\tilde{Y}_{t+1}}{n}\right)^{-\sigma}\right\} + \mu_{l,t}}{\left(\frac{\omega_{l,t}}{n}\right)^{-\sigma}} = \frac{\beta_3 E_t\left\{\left(\frac{\omega_{3,t+1}Y_{t+1}}{2(1-n)}\right)^{-\sigma}\right\} + \frac{1}{2}\rho_{3,t+1}^{3}}{\left(\frac{\omega_{3,t}}{2(1-n)}\right)^{-\sigma}} \ , \ l = 1,2, \tag{B.11}$$

$$\mu_{l,t}\left(\kappa E_t\left\{\tilde{Y}_{t+1}^{\sigma}\tilde{q}_{1,t+1}k_{1,t+1}^{l} + \tilde{Y}_{t+1}^{\sigma}\tilde{q}_{2,t+1}k_{2,t+1}^{l}\right\} - b_{l,t+1}\right) = 0 \ , \ l = 1,2, \tag{B.12}$$

$$H_{l,t+1} = \left[\frac{\alpha A_{l,t+1}K_{l,t+1}^{1-\alpha}}{\chi}\right]^{\frac{1}{1+\nu-\alpha}}. \tag{B.13}$$

Consumption share simplex reads

$$\Omega \equiv \left\{(\omega_{1,t}, \omega_{2,t}, \omega_{3,t}) : \omega_{l,t} > 0, \ l = 1,2,3, \text{ and } \sum_{l=1}^{3}\omega_{l,t} = 1 \text{ for all } t\right\}. \tag{B.14}$$

Let the exogenous shock vector be $A_t = (A_{1,t}, A_{2,t})$. Policy functions for variables of interest can be expressed as functions of investors' effective consumption distribution $(\omega_{1,t}, \omega_{2,t})$. Given future policy functions $\{\tilde{q}_{i,t+1}(\omega_{1,t+1}, \omega_{2,t+1}; A_{t+1}), \tilde{F}_{l,t+1}(\omega_{1,t+1}, \omega_{2,t+1}; A_{t+1})\}$, with $i = 1,2,3$, and

$l = 1, 2$, and current effective consumption shares $\{\omega_{1,t}, \omega_{2,t}\}$, we solve for future state-contingent effective consumption shares $\{\omega_{l,t+1}(\omega_{1,t}, \omega_{2,t}; A_t; A_{t+1})\}_{l=1,2}$, the Lagrange multiplier for the inter-period collateral constraint $\{\mu_{l,t}(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$, state-contingent equilibrium labor $\{H_{l,t+1}(\omega_{1,t}, \omega_{2,t}; A_t; A_{t+1})\}_{l=1,2}$, end-of-period t equity portfolio $\{k^i_{l,t+1}(\omega_{1,t}, \omega_{2,t}; A_t)\}_{i=1,2,3}$ and bond portfolio $\{b_{i,t+1}(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$. Asset price and exiting financial wealth are updated via corresponding conditions $\{\tilde{q}_i(\omega_{1,t}, \omega_{2,t}; A_t)\}_{i=1,2,3}$ and $\{\tilde{F}_l(\omega_{1,t}, \omega_{2,t}; A_t)\}_{l=1,2}$, which are derived in the following subsection in detail.

Let S be the number of exogenous states in the economy. there are then $2S + 7 + 2S = 4S + 7$ equations and variables to be solved at each grid point and at each iteration.

B.3 Dealing with Inequality Constraints

Following Judd, Kubler and Schmedders (2002), we make the following transformation

$$\mu_{l,t} = (\max\{0, \eta_{l,t}\})^L \ , \ l = 1, 2, \tag{B.15}$$

$$\kappa E_t \left\{ q_{1,t+1} k^l_{1,t+1} + q_{2,t+1} k^l_{2,t+1} \right\} - b_{l,t+1} = (\max\{0, -\eta_{l,t}\})^L \ , \ l = 1, 2,$$

where $\eta_{l,t}$ is a real number and L is an integer, usually taking $L = 3$. These two equations are equivalent to the slackness conditions in the system. Notice that function $(\max\{0, \eta_{l,t}\})^L$ is a $(L-1)$th continuously differentiable function. Therefore, the transformed equilibrium system is twice continuously differentiable, and a Newton method can be applied here.

Rearranging the collateral constraint above, yields

$$b_{l,t+1} = \kappa E_t \left\{ \tilde{Y}^\sigma_{t+1} \tilde{q}_{1,t+1} k^l_{1,t+1} + \tilde{Y}^\sigma_{t+1} \tilde{q}_{2,t+1} k^l_{2,t+1} \right\} - (\max\{0, -\eta_{l,t}\})^L \ , \ l = 1, 2. \tag{B.16}$$

Accordingly, we use $\eta_{l,t}$ to replace $\mu_{l,t}$ in the computation.

B.4 Simplifying the System

Notice that the sum of consumption shares equals unity. We can use only consumption shares for investors as independent state variables here. In addition, we can also get rid of bond holdings $b_{l,t+1}$ by using equation (B.16). Asset market clearing conditions imply that one agent's portfolio of assets is pinned down by the portfolios of the rest of agents. Consequently, we have a sequence of independent variables $\{\omega_{l,t+1,s}\}_{l=1,2;s=1,\cdots,S}$, $\left\{k^l_{i,t}\right\}_{l=1,2;i=1,2}$, $\{\eta_{l,t}\}_{l=1,2}$, totally $2S + 6$ variables. The system of equations consists of $2S + 6$ equations (B.9)-(B.12), with $l = 1, 2$, $i = 1, 2$.

B.5 Updating Asset Prices and Exiting Financial Wealth

Once solving the system of equations above, we update asset prices according to equation (B.10)-(B.11). Multiplying $\beta_l U_c(c_{l,t+1}, h_{l,t+1})$ on both sides of equation (B.2), taking expectations conditional on information up to period t, and replacing relevant terms with the ones in corresponding consumption Euler equations and complementary slackness conditions for collateral constraints,

yields

$$F_{l,t} = \frac{1}{U_c(c_{l,t}, h_{l,t})} \begin{Bmatrix} \beta_l E_t \left[U_c(c_{l,t+1}, h_{l,t+1})(F_{l,t+1} + \tilde{c}_{l,t+1} - NW_{l,t+1} + v(H_{l,t+1})) \right] \\ -k^l_{1,t+1}\rho^l_{1,t+1} - k^l_{2,t+1}\rho^l_{2,t+1} \end{Bmatrix},$$

where net non-financial income $NW_{l,t+1}$ can be written as

$$NW_{l,t+1} = W_{l,t+1}H_{l,t+1}.$$

Normalizing this equation by Y_t^σ, yields

$$\tilde{F}_{l,t} = \frac{1}{\left(\frac{\omega_{l,t}}{n}\right)^{-\sigma}} \begin{Bmatrix} \beta_l E_t \left[\left(\frac{\omega_{l,t+1}\tilde{Y}_{t+1}}{n}\right)^{-\sigma} \left(\tilde{Y}^\sigma_{t+1}\tilde{F}_{l,t+1} + \frac{\omega_{l,t+1}\tilde{Y}_{t+1}}{n} - NW_{l,t+1} + v(H_{l,t+1}) \right) \right] \\ -k^l_{1,t+1}\rho^l_{1,t+1} - k^l_{2,t+1}\rho^l_{2,t+1} \end{Bmatrix}, \; l = 1, 2. \tag{B.17}$$

B.6 The Initial Period t=0 and Simulated Paths

At period $t \geq 1$, we can solve for endogenous variables $\{H_{l,t+1},\, K_{l,t+1},\, k^i_{l,t+1}, b_{l,t+1},\, \mu_{l,t},\, \omega_{l,t+1}\}$, with $l = 1, 2$, $i = 1, 2, 3$, as functions of consumption share distribution in period t (also in period $t+1$ for state-contingent variables). Asset prices $\tilde{q}_{i,t}$ and exiting financial wealth $\tilde{F}_{i,t}$ can be updated based on the corresponding consumption Euler equations. Although we can't prove the existence of an equilibrium in such a complicated model as ours, we take a more practical approach as in most of the literature. As long as policy functions of interest converge after a long enough period of time, we assume that an equilibrium exists over an appropriate domain. Nevertheless, the economy starts with some initial conditions such as initial portfolio, $k^i_{l,0}, b_{l,0}$, initial interest rate R_0 and shocks A_0. The path for variables of interest should be calculated given these initial conditions. We first solve for $\omega_{l,0}, H_{l,0}$ with $l = 1, 2$, based on the following four equations.

$$\omega_{l,0} + \frac{n}{\tilde{Y}_0}\tilde{Y}_0^\sigma \tilde{F}_{l,0} = \frac{n}{\tilde{Y}_0}(NW_{l,0} - v(H_{l,0})) + \frac{n}{\tilde{Y}_0}k^l_{1,0}(\tilde{Y}_0^\sigma \tilde{q}_{1,0} + R_{1k,0}) +$$

$$\frac{n}{\tilde{Y}_0}k^l_{2,0}(\tilde{Y}_0^\sigma \tilde{q}_{2,0} + R_{2k,0}) - \frac{n}{\tilde{Y}_0}b_{l,0} \;, \; l = 1, 2, 3, \tag{B.18}$$

$$H_{l,0} = \left[\frac{\alpha A_{l,0} K_{l,0}^{1-\alpha}}{\chi} \right]^{\frac{1}{1+\nu-\alpha}}. \tag{B.19}$$

Notice that equilibrium labor is a function of state variables and becomes known at the beginning of a period. We need essentially to solve two budget constraints for consumption share distribution $\{\omega_{1,0}, \omega_{2,0}\}$. Once obtaining the current consumption distribution $\{\omega_{1,0}, \omega_{2,0}\}$, the end-of-period portfolio $k^i_{l,1}, b_{l,1}$ are obtained via interpolating relevant policy functions. We then move the process forwardly by redoing the calculation for $\{\omega_{1,t}, \omega_{2,t}\}$ based on four equations in period t and given portfolio $k^i_{l,t}, b_{l,t}$ with $t \geq 1$ along the simulation path. Other endogenous variables can be found accordingly along the simulation path.

B.7 The Algorithm for Solving the Model

Assume that exogenous shocks $(A_{1,t}, A_{2,t})$ follow a Markovian process. We can use time-iteration (backward induction) to solve the system. At the last period T, $\tilde{q}_{i,T} = \tilde{F}_{l,T} = 0$ with $i = 1, 2$ and $l = 1, 2$. The algorithm is summarized as follows

Step 1. Choose an appropriate function tolerance ϵ. In the baseline model we use $\epsilon = 10^{-5}$. Discretize the exogenous state space $(A_{1,t}, A_{2,t})$ into S grid points $\{(a_{1,s}, a_{2,s})\}_{s=1,\cdots,S}$ and endogenous state space Ω into $J = n_x n_y$ grid points $\{(\omega_{1,i}, \omega_{2,j})\}_{j=1,\cdots,n_y}^{i=1,\cdots,n_x}$. Set period T long enough.

Step 2. Given asset price functions $\tilde{q}_{i,t+1}$ and exiting wealth functions $\tilde{F}_{l,t+1}$ with $i = 1, 2, 3$ and $l = 1, 2$, $t = T-1, T-2, \cdots$, for each grid point $\{(a_{1,s}, a_{2,s}; \omega_{1,i}, \omega_{2,j})\}$, we solve equation (B.9)-(B.11) with $l = 1, 2$, $i = 1, 2$ for state consumption share $\{\omega_{l,t+1}\}_{l=1,2}$, current portfolio $\left\{k_{i,t+1}^{l}\right\}_{i=1,2}^{l=1,2}$ and current Lagrange multipliers $\{\eta_{l,t}\}_{l=1,2}$. Asset price $\tilde{q}_{i,t+1}$ and exiting wealth $\tilde{F}_{l,t+1}$ are obtained through interpolation at a specific point of $\{(a_{1,t+1}, a_{2,t+1}; \omega_{1,t+1}, \omega_{2,t+1})\}$. Current asset prices $\tilde{q}_{i,t}$, $i = 1, 2, 3$, are updated through equation (B.10)-(B.11), and exiting wealth $\tilde{F}_{l,t}$, $l = 1, 2$, through equation (B.17).

Step 3. Compare the distance between two consecutive asset prices and exiting wealths

$$dist = \max\{\mid k_{i,t+1}^{l} - k_{i,t}^{l} \mid, \mid \tilde{q}_{i,t+1} - \tilde{q}_{i,t} \mid, \mid \tilde{F}_{l,t+1} - \tilde{F}_{l,t} \mid\}_{l=1,2;i=1,2,3}.$$

If $dist \geq \epsilon$, go to step 2; otherwise terminate the calculation and go to step 4.

Step 4. Once obtaining a convergent solution, we simulate the model forwardly for given initial conditions $\left\{k_{i,0}^{l}\right\}_{i=1,2}^{l=1,2,3}$, $\{b_{l,0}\}_{l=1,2,3}$ and shock A_0 to obtain state consumption levels $\{c_{l,t}\}_{l=1,2,3}$, labor $\{H_{l,t}\}_{l=1,2}$, portfolios $\left\{k_{i,t+1}^{l}\right\}_{i=1,2}^{l=1,2,3}$, $\{b_{l,t+1}\}_{l=1,2,3}$, Lagrange multipliers $\{\mu_{l,t}\}_{l=1,2}$, asset prices $\{q_{i,t}\}_{i=1,2,3}$ and exiting wealth $\{F_{l,t}\}_{l=1,2}$, for $t = 0, 1, 2, \cdots$.

C Interpolation and Approximation

C.1 Consumption Simplex

In the model, the domain of consumption shares is a triangle, which is not easy to cope with directly in computation. There are several methods to investigate this issue in numerical analysis, for instance, Barycentric coordinates on surfaces. However, we will avoid this computational issue by making a one-to-one mapping between a triangle and a rectangle.[21] Consumption share simplex in the economy is rewritten here for convenience

$$\Omega \equiv \left\{(\omega_1, \omega_2, \omega_3) : \omega_l > 0,\ l = 1, 2, 3, \text{ and } \sum_{l=1}^{3} \omega_l = 1\right\}.$$

[21] We thank Hiroyuki Kasahara for his helpful suggestion.

Here, we treat (ω_1, ω_2) as a pair of free states. The consumption share simplex is equivalent to $0 < \omega_i < 1$, $i = 1, 2$, $\omega_l + \omega_2 < 1$ and $\omega_3 = 1 - \omega_1 - \omega_2$. We employ a trick in the following way. First, write (ω_1, ω_2) as functions of two other variables, say, z, w

$$\omega_1 = \frac{z}{1+z+w}, \qquad \omega_2 = \frac{w}{1+z+w}.$$

Here, ω_1 is increasing in z and decreasing in w. ω_2 is increasing in w and decreasing in z. These two functions map $(0, +\infty) \bigcup (0, +\infty)$ onto the consumption share simplex Ω. Second, finding a mapping between a rectangle and $(0, +\infty) \bigcup (0, +\infty)$, we use two new variables (θ_1, θ_2) which are defined over $(0, 1) \bigcup (0, 1)$, and two new functions here

$$z = -\log(\theta_1), \qquad w = -\log(\theta_2).$$

Consequently, we build a one-to-one mapping from a rectangle $(0, 1) \bigcup (0, 1)$ onto the simplex Ω.[22]

C.2 B-spline Interpolation and Approximation

In the algorithm, we need to know the policy functions $\tilde{q}_{h,t+1}(\theta_{1,t+1}, \theta_{2,t+1}, s_{t+1})$ and $\tilde{F}_{l,t+1}$ $(\theta_{1,t+1}, \theta_{2,t+1}, s_{t+1})$ with $h = 1, 2, 3$, $l = 1, 2$ and s_{t+1} is the state of Nature. Since we cannot obtain closed-form expressions for these functions, we take use of B-spline smooth functions with degree 3 to approximate policy functions. Therefore the approximated functions are twice continuously differentiable. We approximate asset prices $\tilde{q}_h(\theta_1, \theta_2, s)$ with $h = 1, 2, 3$ and exiting wealth $\tilde{F}_l(\theta_1, \theta_2, s)$ with $l = 1, 2$ parametrically by functions

$$\hat{\tilde{q}}_h(\theta_1, \theta_2, s) = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} a_{ij}^{qhs} B_i(\theta_1) B_j(\theta_2),$$

$$\hat{\tilde{F}}_l(\theta_1, \theta_2, s) = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} a_{ij}^{fls} B_i(\theta_1) B_j(\theta_2),$$

[22] Of course, there are many other transformations. Let's discuss some of them. Example 1: $(\theta_1, \theta_2) \in (0, \frac{\pi}{2}) \bigcup (0, \frac{\pi}{2})$

$$z = \tan(\theta_1), \qquad w = \tan(\theta_2)$$

Example 2: $(\theta_1, \theta_2) \in (0, 1) \bigcup (0, 1)$

$$z = \frac{1}{\theta_1} - 1, \qquad w = \frac{1}{\theta_2} - 1$$

Example 3: $(\theta_1, \theta_2) \in (0.5, 1) \bigcup (0.5, 1)$

$$z = \log(\theta_1) - \log(1 - \theta_1), \qquad w = \log(\theta_2) - \log(1 - \theta_2)$$

Example 4: $(\theta_1, \theta_2) \in (-1, 1) \bigcup (-1, 1)$

$$\omega_1 = \frac{1 + \theta_1}{2}, \qquad \omega_2 = \frac{1}{4}(1 + \theta_2)(1 - \theta_1)$$

The key differences among these transformations are the density of consumption shares on the simplex Ω even when grid points for (θ_1, θ_2) are equidistant.

where a_{ij}^{qhs} and a_{ij}^{fls} are coefficients in B-spline approximations.

B-splines are piecewise polynomial. The B-splines of degree 0 are defined as

$$B_i^0(x) = \begin{cases} 0 & \text{if } x < x_i \\ 1 & \text{if } x_i \leq x < x_{i+1} \\ 0 & \text{if } x_{i+1} \leq x \end{cases}$$

where $\{x_i\}_{i=1,\cdots,n}$ are grid points on x. The B-splines of degree 1 follow as

$$B_i^1(x) = \begin{cases} 0 & \text{if } x < x_i \\ \frac{x-x_i}{x_{i+1}-x_i} & \text{if } x_i \leq x < x_{i+1} \\ \frac{x_{i+2}-x}{x_{i+2}-x_{i+1}} & \text{if } x_{i+1} \leq x < x_{i+2} \\ 0 & \text{if } x_{i+2} \leq x \end{cases}$$

The B-splines of degree k have a recursive form of

$$B_i^k(x) = \frac{x - x_i}{x_{i+k} - x_i} B_i^{k-1}(x) + \frac{x_{i+k+1} - x}{x_{i+k+1} - x_{i+1}} B_{i+1}^{k-1}(x) \ , \ k \geq 1.$$

The B-splines of degree k require $(n + k)$ grid points. In the algorithm, we need to implement interpolation and approximation of policy functions. In the interpolation part, asset prices and exiting wealth are obtained through interpolating approximated asset price functions $\hat{\tilde{q}}_h(\theta_1, \theta_2, s)$ and end-of-period wealth $\hat{\tilde{F}}_h(\theta_1, \theta_2, s)$. Once we have updated current asset prices and exiting wealth through equation (B.10) and (B.17), coefficients in the approximated functions $\{a_{ij}^{qhs}, a_{ij}^{fls}\}$ can be obtained through interpolation.

C.3 Interpolation on a Bounded Set

In interpolation, we want to find an asset pricing (exiting wealth) function $f(state, \cdot) : \Omega \to \Omega$, where Ω is a bounded set. In practice, we might only use a subset of Ω to enhance the accuracy given the number of nodes (the nodes are more dense on a subset of Ω for a given number of grid points). The grid points we are mainly concerned with are boundary points. For instance, when they are patient and have high consumption shares today, say at the boundary points in the domain, investors will hold more assets to smooth consumption whenever they have the chance, say, exogenous constraints on asset holdings are not tight. It would be the case that their optimal consumption shares are higher than the upper bound of the bounded set. The zero-finding problem then becomes truncated and it's possible that the nonlinear system doesn't have a solution. There are at least three ways to deal with this situation.

One is increasing the range of the bounded set. The problem is how large the highest consumption share is since the domain in the original problem is an open set (say, Ω). For given preference parameters, there should exist, we think, an upper bound for consumption shares. But as a matter of fact, this approach will produce a policy function surface with many curvatures, which in turn requires very dense grid points to obtain an accurate solution.

A second way is to make investors more patient, in which investors prefer consuming today to tomorrow. Therefore, consumption share domain is smaller than that in the original problem. When they are impatient enough, investors would borrow a lot and they will face binding borrowing constraints in equilibrium. Thereby, given other parameter values unchanged, an economy with more impatient investors would make collateral constraints bind more frequently.

Another way to deal with unbounded asset holdings is to allow for large penalties when asset holdings exceed some bounds. For instance, if we don't allow for short positions (or small positive positions) in assets, agents can not accumulate a large position in any of assets. In this case, their consumption shares will lie in a narrow range of its natural domain.

Our approach takes a practical perspective. First, we set the penalties for over holdings of portfolios are large enough, choose a relatively large tolerance size, and try a relatively large domain for consumption shares to obtain a stationary solution to the model. Note that the achieved solution might be inaccurate. We then narrow the domain for consumption shares based on simulations, use the obtained stationary solution as an new initial guess, and then rerun the solution procedure above to obtain an accurate solution.

C.4 Discretizing an AR(1) Process with a Disaster Risk

We discretize the continuous state technological shock Markov process into a discrete and finite state Markov process. Researchers usually make use of Hussey and Tauchen (1991)'s method to find a finite state Markov process based on a standard AR(1) Markov process. However their method doesn't apply directly here because of the additional disaster risk ϕ_t. We use the following approach instead.[23]

Assume that the finite state space for the exogenous state variable is A_l in country l. For the sake of simplicity but without loss of generality, we choose three states to characterize the exogenous process (C.1) in each country,

$$\ln(A_{l,t+1}) = (1-\rho_z)\ln(A_l) + \rho_z \ln(A_{l,t}) + D\phi_{l,t+1} + \epsilon_{l,t+1},\ l = 1, 2, \tag{C.1}$$

$A_l = \{lo, mid, hi\}$. These three states could be any numbers around the unconditional mean of $A_{l,t}$. We set the middle state mid equals the mean of $A_{l,t}$, the lowest state $lo = mid - 2 \times std(A_{l,t})$, and the highest state $hi = mid + std(A_{l,t})$, where std stands for the unconditional standard deviation of $A_{l,t}$. Now we need an associated transition matrix Π such that the discrete state process generates the same moments as the original $AR(1)$ process with a disaster shock. Since Π is a 3 by 3 matrix and the sum of each row is one, there are 6 free unknowns in Π. Consequently, we need 6 moments (constraints) to pin down these unknowns.

In the calculation, we choose the first three unconditional moments including mean, variance and skewness, and three auto-correlations with lagged 1, 2 and 3 periods. Then we simulate the original exogenous process (C.1) with 2500 periods (discard the first 500 periods) and 5000 times to calculate the 6 unconditional moments for equation (C.1). Next we then use nonlinear solvers to find 6 unknowns in matrix Π such that the unconditional moments generated by matrix Π are very

[23] We thank Victor Rios-Rull for his valuable suggestion and encouragement.

closed to the original process.

Some remarks are in order. First, the discrete state Markov process characterizes the original continuous state Markov process in terms of unconditional moments. There might be other moments one could choose, but we prefer these six moments in the calculation. Second, we want to associate the lowest technological state with an event of disaster, but it isn't necessarily the disaster state itself. One could use a much lower value for the lowest state, say, $lo = mid - 3 \times std(A_{l,t})$ and obtain a different transition matrix. The choice of state values doesn't affect the business cycle property of the model. It only affects how we define a recession scenario.

C.5 Accuracy

Once obtaining policy functions for asset prices and exiting wealth, we can implement a simulation starting from the initial period (given the portfolio and state of Nature in the first period) to obtain all other variables. Along a simulated path, we have a sequence of consumption levels and a sequence of portfolio of assets for each agent in the world economy. The accuracy of the solution is based on relative consumption between actual consumption $\tilde{c}_{l,t}$ along the simulated path and the consumption $\hat{\tilde{c}}_{l,t}$ that is derived from Euler equations, given current portfolio choices and future state contingent consumption and asset prices,

$$\hat{\tilde{c}}_{l,t} = \left\{ \frac{\beta_l E_t \left\{ U'(\tilde{c}_{l,t+1})(q_{i,t+1} + R_{ik,t+1}) \right\} + \mu_{l,t} \kappa E_t \left\{ q_{i,t+1} \right\} - \rho^l_{i,t+1}}{q_{i,t}} \right\}^{-\frac{1}{\sigma}} \text{ with } l = 1, 2, i = 1, 2, 3.$$

The banker's Euler equations deliver similar current consumption $\hat{\tilde{c}}_{3,t}$. The absolute relative error is defined as

$$\epsilon_{l,i} = \left| \frac{\hat{\tilde{c}}_{l,t}}{\tilde{c}_{l,t}} - 1 \right| \text{ with } l, i = 1, 2, 3.$$

When the mean and maximum of $\epsilon_{l,i}$ along the simulated is small enough, the solution to the model is accurate. In the baseline model, we obtain an average error of $\log(\epsilon_{l,i}) < -6.98$ and maximal error of $\log(\epsilon_{l,i}) < -2.8$.

D Value Functions and Welfare

Once obtaining policy functions for variables of interest, we arrive at the policy function for consumption at period t as $\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$. Then the value function for agent i (investors and workers in either country) is defined as,

$$\begin{aligned} V_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1}) &= \max_{\{\tilde{c}_{i,t}\}} \left\{ U(\tilde{c}_{i,t}) + \beta_i E \left[V_i(\omega_{1,t+1}, \omega_{2,t+1}, A_{t+1}, A_t) \right] \right\} \\ &= U(\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})) + \beta_i E \left[V_i(\omega_{1,t+1}, \omega_{2,t+1}, A_{t+1}, A_t) \right], \end{aligned} \tag{D.1}$$

where the second equality uses the optimal effective consumption $\tilde{c}_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$. The time-invariant function $V_i(\omega_{1,t}, \omega_{2,t}, A_t, A_{t-1})$ satisfying the equation above is the value function we look for. Note that the value function can also be expressed as a function of predetermined portfolios

and exogenous shocks $V_i(\{k^i_{j,t}, b_{i,t}\}^{i=1,2,3}_{j=1,2}, A_t)$.

D.1 Unconditional Welfare Evaluation

Investors are less patient than workers, so they will front-load their consumption by borrowing more from workers. In the long run ergodic distribution, investors accumulate higher debts and consume less, while workers' consumption will have the opposite feature. Financial integration which enhances risk-sharing for investors is associated with higher indebtedness and lower consumption in the long run. How does financial integration alter risk sharing in the long run? We can characterize the unconditional stationary value of welfare by calculating the unconditional mean of value function V_i. To economize on notation, we use V_i^r as such an unconditional mean in financial integration regime $r = u, b, e$, with u standing for financial autarky, b for bond market integration and e for equity market integration. The unconditional mean of value function V_i^r is calculated based on 100 simulation runs, each of which contains 210000 periods (the first 10000 periods are discarded). Along these simulation paths, portfolio choices never exceed their preset lower bounds.

Specifically, in order to make welfare gains from financial integration comparable to consumption changes, we apply the certainty equivalent of effective consumption $\tilde{c}_i$, which is defined as,

$$V_i^r = \sum_{t=0}^{\infty} \beta_i^t U(\tilde{c}_i^r). \tag{D.2}$$

Given the preference specification in the main text, the certainty equivalent of consumption $\tilde{c}_i^r$ reads,

$$\tilde{c}_i^r = [V_i^r(1-\sigma)(1-\beta_i)+1]^{\frac{1}{1-\sigma}}. \tag{D.3}$$

Unconditional welfare gains now can be written as effective consumption changes across different financial integration regimes. Let $\lambda_{r1,r2,i}$ be the change of effective consumption from regime $r1$ to regime $r2$ for agent i,

$$\lambda_{r1,r2,i} = \frac{\tilde{c}_i^{r2} - \tilde{c}_i^{r1}}{\tilde{c}_i^{r1}}. \tag{D.4}$$

When $\lambda_{r1,r2,i} > 0$, agent i is better off from regime $r1$ to regime $r2$ in the long run.

D.2 Conditional Welfare Evaluation

In order to properly evaluate the gains from financial integration, it is necessary to calculate welfare conditional on initial conditions.[24] Starting from an common initial state, including initial portfolios for all agents and exogenous shocks, assume that there is an unanticipated change in regime from $r1$ to $r2$. Accordingly, agents in the economy optimize their consumption and portfolio paths in each integration regime after they observe the realization of shocks. Specifically, assume that the economy starts at period 0 with end-of-period portfolio $\{k^j_{i,0}, b_{j,0}\}^{j=1,2,3}_{i=1,2}$ and exogenous state A_0. The switch of regimes happens unexpectedly at period 1 and the economy stays in that regime from period 1 onwards. Nevertheless, the exogenous shocks in period $t = 1, 2$ are unknown

[24] This is because unconditional measures of welfare ignore the transitory gains in utility that investors get from early consumption.

for agents in period 0. Let the welfare in period 0 be $V^r_{0,j}$ for integration regime r and agent j,

$$V^r_{j,0}(\{k^j_{i,0}, b_{j,0}\}^{j=1,2,3}_{i=1,2}, A_0) \equiv E_0 \left\{ U(\tilde{c}^*_j(\{k^j_{i,0}, b_{j,0}\}^{j=1,2,3}_{i=1,2}, A_1)) + \beta_j E_1[V^r_j(\{k^j_{i,1}, b_{j,1}\}^{j=1,2,3}_{i=1,2}, A_2)] \right\}, \tag{D.5}$$

where $\tilde{c}^*_j(\{k^j_{i,0}, b_{j,0}\}^{j=1,2,3}_{i=1,2}, A_1)$ is agent j's optimal effective consumption given endogenous state $\{k^j_{i,0}, b_{j,0}\}^{j=1,2,3}_{i=1,2}$ and exogenous state A_1 at period 1, V^r_j denotes the value function and E_t represents conditional expectations over exogenous state A_{t+1} with $t = 0, 1$.

Once obtaining the effective consumption $\tilde{c}^*_j$ and value function V^r_j, we can calculate the conditional welfare for agent j, $V^r_{j,0}$, based on the transition matrix Π. Similarly, the certainty equivalence of effective consumption in the short run $\tilde{c}^{r,s}_j$ is defined as,

$$\tilde{c}^{r,s}_j = \left[V^r_{j,0}(1-\sigma)(1-\beta_j) + 1\right]^{\frac{1}{1-\sigma}}. \tag{D.6}$$