Asset Pricing with Countercyclical Household Consumption Risk

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Abstract

We present evidence that shocks to household consumption growth are negatively skewed, persistent, countercyclical, and drive asset prices. We construct a parsimonious model where heterogeneous households have recursive preferences. A single state variable drives the conditional cross-sectional moments of household consumption growth. The estimated model fits well the unconditional cross-sectional moments of household consumption growth and the moments of the risk free rate, equity premium, price-dividend ratio, and aggregate dividend and consumption growth. The model-implied risk free rate and price-dividend ratio are pro-cyclical while the market return has countercyclical mean and variance. Finally, household consumption risk explains the cross-section of excess returns.

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Introduction

There is considerable empirical evidence that households face a substantial amount of uninsurable idiosyncratic labor income risk. The time variation in idiosyncratic labor income risk plays a central role in understanding several observed phenomena in macroeconomics and finance. Earlier studies focused on the cross-sectional variance of the idiosyncratic shocks, arguing that they are countercyclical (e.g., Storesletten, Telmer, and Yaron (2004)) and can account for the high historically observed level of the equity premium (e.g., Constantinides and Duffie (1996)). More recently, Guvenen, Ozkan, and Song (2014), exploiting a very large dataset from the U.S. Social Security Administration, found that the left skewness of the shocks is strongly countercyclical. Contrary to prior research, they find that the cross-sectional variance is not countercyclical and, in fact, remains mostly flat over the business cycle even after controlling for observable characteristics like age and average past earnings. Moreover, Krebs (2007) argued that higher job displacement risk in recessions gives rise to the countercyclical left skewness of earnings shocks and that this can generate a substantial cost of business cycles.

This paper studies the implications of countercyclical left skewness in the cross-sectional distribution of household consumption growth on aggregate asset prices.[1] We construct a parsimonious dynamic equilibrium model with two key ingredients. First, the economy is inhabited by a continuum of heterogeneous households with identical Epstein-Zin (1989) recursive preferences. Second, the heterogeneity among the households arises from their labor income processes that are each modeled as an exponential function of a Poisson mixture of normals distribution. This specification generates higher-order moments in the cross-sectional distribution of household consumption growth in a tractable fashion. In fact, the parameter driving the Poisson process is the single state variable—hereafter referred to as *household consumption risk*—that drives the conditional cross-sectional third central moment of household consumption growth. The aggregate dividend and consumption growth are modeled as *i.i.d.* processes to emphasize that the explanatory power of the model does not derive from such predictability.

[1] In a different context, Ghosh, Julliard, and Taylor (2014), relying on a non-parametric relative entropy minimizing approach to filter the most likely SDF, highlighted the importance of higher moments of the SDF, particularly the skewness, in pricing assets. In particular, they showed that about a quarter of the overall entropy of the most likely SDF is generated by its third and higher order moments, with the third central moment alone accounting for about 18% of the entropy.

We demonstrate that under certain conditions there exists a no-trade equilibrium. To our knowledge, this is the first paper to establish the existence of equilibrium in a heterogeneous agent economy where the investors have recursive preferences. For the log-linearized version of the model, we obtain, in closed form, the equilibrium risk free rate, expected market return, and price-dividend ratio as functions of the single state variable, the household consumption risk.

We estimate the model using the Generalized Method of Moments approach, targeting both the model-implied moments of the market return, risk free rate, and market-wide price-dividend ratio, as well as the first three central moments of the cross-sectional distribution of household consumption growth. The estimated model provides a good fit for the time-series averages of the moments of household consumption growth. The model matches well the unconditional mean, volatility, and autocorrelation of the risk free rate, thereby addressing the risk free rate puzzle. It provides a good fit for the unconditional mean, volatility, and autocorrelation of the market return, thereby addressing the equity premium and excess volatility puzzles. The model matches well the mean, volatility, and autocorrelation of the market price-dividend ratio and the aggregate dividend growth, targets that challenge a number of other models. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are procyclical while the expected market return and its variance and the equity premium are countercyclical. The model is also consistent with the salient features of aggregate consumption growth observed in the data: realistic mean and variance, and lack of predictability. Furthermore, the third central moment of the conditional cross-sectional distribution of household consumption growth explains the cross-section of excess returns of size, book-to-market-equity, and industry-sorted portfolios as well as the three Fama-French factors do.

Figure 1 displays the time series of the skewness of the cross-sectional distribution of quarterly household consumption growth over the period 1982:Q1-2009:Q4. The third central moment is mostly negative and counter-cyclical, with correlation between -24.9% and -21.7% with NBER recessions. Note that our estimates are noisy because of the measurement error in the survey-based CEX database that we use for our analysis. However, the results confirm the findings in Guvenen, Ozkan, and Song (2014). The counter-cyclical nature of the third central moment drives the observed low risk free rate and price-dividend ratio and the high equity premium in recessions.

Shocks to household consumption growth are persistent and so are the estimated moments of the cross-sectional distribution of household consumption growth: the autocorrelation of the third central moment is between 12.4% and 18.0%. The autocorrelation of the 5th percentile of the cross-sectional distribution is between 53.4% and 79.2%. These persistent risks play a pivotal role in matching the data, given that the estimated model implies that households exhibit preference for early resolution of uncertainty, in the context of recursive preferences.

The paper draws on several strands of the literature. It builds upon the empirical evidence by Attanasio and Davis (1996), Blundell, Pistaferri, and Preston (2008), Cochrane (1991), and Townsend (1994) that consumption insurance is incomplete. Constantinides (1982) highlighted the pivotal role of complete consumption insurance, showing that the equilibrium of such an economy with households with heterogeneous endowments and vonNeumann-Morgenstern preferences is isomorphic to the equilibrium of a homogeneous-household economy. Mankiw (1986) showed that, in a two-period economy with incomplete consumption insurance, the concentration of aggregate shocks among the population is an important determinant of the level of the equity premium. Constantinides and Duffie (1996) further showed that, in the absence of complete consumption insurance, given the aggregate income and dividend processes, any given (arbitrage-free) price process can be supported in the equilibrium of a heterogeneous household economy with judiciously chosen persistent idiosyncratic income shocks. Our paper provides empirical evidence that these shocks are negatively skewed, persistent and, more importantly, drive asset prices and excess returns.

The paper draws also on Brav, Constantinides, and Geczy (2002) and Cogley (2002) who addressed the role of incomplete consumption insurance in determining excess returns in the context of economies in which households have power utility. Brav *et al.* presented empirical evidence that the equity and value premia are consistent in the 1982–1996 period with a stochastic discount factor (SDF) obtained as the average of individual households' marginal rates of substitution with low and economically plausible values of the relative risk aversion (RRA) coefficient. Since these premia are not explained with a stochastic discount factor obtained as the per capita marginal rate of substitution with low values of the RRA coefficient, the evidence supports the hypothesis of incomplete consumption insurance. Cogley (2002) calibrated a model with incomplete consumption insurance that recognizes the variance and skewness of the shocks to the households' consumption growth and obtained an annual equity premium of 4.5-5.75% with RRA coefficient of 15. Being couched in terms of economies with households endowed with power utility, neither of these papers allowed for the RRA coefficient and the elasticity of intertemporal substitution (EIS) to be disentangled, a step which appears important in order to address the level and time-series properties of the risk free rate, price-dividend ratio, and market return.

In contrast to the above two papers, the present investigation disentangles the RRA coefficient and the EIS with recursive preferences. The estimated EIS is low and the model is not subject to the criticism in Epstein, Farhi, and Strzalecki (2014) on the extreme implications of models with high EIS regarding the preference for early resolution of uncertainty. In addition, the model addresses the level and time-series properties of the risk free rate, the price-dividend ratio and the market return. Finally, it also addresses the cross-section of size-sorted, book-to-market-equity-sorted, and industry-sorted portfolio returns.

By introducing recursive preferences, the Euler equations of consumption may no longer be written in terms of household consumption growth alone. It becomes necessary to explicitly model the time-series processes of household consumption and express the stochastic discount factor in terms of the consumption-wealth ratio. This complicates the model construction and estimation but has the major side benefit that we sidestep the need to work with the noisy time series of the cross-sectional moments of household consumption growth, working instead with time-series averages of these moments.

The paper also relates to the literature on macroeconomic crises initiated by Rietz (1988) and revisited by Barro (2006) and others as an explanation of the equity premium and related puzzles.[2] This literature builds on domestic and international evidence that macroeconomic crises are associated with a large and sustained drop in aggregate consumption that increases the marginal rate of substitution of the representative consumer. Thus, the basic mechanism of macroeconomic crises is similar in spirit to our paper in that the incidence of a large decline in the consumption of some or all households increases the marginal rates of substitution of these households. The two classes of models part ways in their quantitative implications. As Constantinides (2008) pointed out, Barro (2006) found it necessary to calibrate the model by

[2] Related references include Backus, Chernov, and Martin (2011), Barro and Ursùa (2008), Constantinides (2008), Gabaix (2012), Gourio (2008), Harvey and Siddique (2000), Julliard and Ghosh (2012), Nakamura, Steinsson, Barro, and Ursùa (2013), Veronesi (2004), and Wachter (2013).

treating the peak-to-trough drop in aggregate consumption during macroeconomic crises (which on average last four years) as if this drop occurred in one year, thereby magnifying by a factor of four the size of the observed annual disaster risks. Similar *ad hoc* magnification of the annual aggregate consumption drop during macroeconomic crises is relied upon in a number of papers that follow Barro (2006). Julliard and Ghosh (2012) empirically rejected the rare events explanation of the equity premium puzzle, showing that in order to explain the puzzle with power utility preferences of the representative agent and plausible RRA once the multi-year nature of disasters is correctly taken into account, one should be willing to believe that economic disasters should be happening every 6.6 years. Moreover, Backus, Chernov, and Martin (2011) demonstrated that options imply smaller probabilities of extreme outcomes than the probabilities estimated from international macroeconomic data.

In contrast to these models, our model relies on shocks to *household* consumption growth, with frequency and annual size consistent with empirical observation. These shocks support the observed time-series properties of the risk free rate, market return, and market price-dividend ratio. Furthermore, the shocks to household consumption "average out" across households and do not imply unrealistically large annual shocks to the aggregate consumption growth.

Finally, the paper relates to the literature on the cross section of excess returns. We show that the third central moment of the conditional cross-sectional household consumption growth distribution explains the cross section of excess returns on the size-sorted, book-to-market-equity-sorted, and industry-sorted portfolios. The results from our one-factor model are comparable to those of the three-factor Fama-French model.

The paper is organized as follows. The model and its implications for consumption growth and prices are presented in Section 1. We discuss the data in Section 2. In Section 3, we present summary statistics of the cross-sectional distribution of household consumption growth. The empirical methodology and main results are presented in Section 4. We further interpret the results in Section 5. In Section 6, we present the implications of household consumption risk for the cross-section of excess returns. We conclude in Section 7. Derivations are relegated to the appendices.

1. The Model

We consider an exchange economy with a single nondurable consumption good serving as the numeraire. There are an arbitrary number of traded securities (for example, equities, corporate bonds, default free bonds, and derivatives) in positive or zero net supply. Conspicuously absent are markets for trading the households' wealth portfolios. A household's wealth portfolio is defined as a portfolio with dividend flow equal to the household's consumption flow. It is in this sense that the market is incomplete thereby preventing households from insuring their idiosyncratic income shocks. The sum total of traded securities in positive net supply is referred to as the "market". The market pays net dividend D_t at time *t*, has ex-dividend price P_t, and normalized supply of one unit. We assume that households are endowed with an equal number of market shares at time zero but can trade in these shares and all other securities (except the wealth portfolios) thereafter.

Aggregate consumption is denoted by C_t, log consumption by $c_t \equiv \log(C_t)$, and consumption growth by $\Delta c_{t+1} \equiv c_{t+1} - c_t$. We assume that aggregate consumption growth is *i.i.d* normal: $\Delta c_{t+1} = \mu + \sigma_a \varepsilon_{t+1}$, $\varepsilon_t \sim \mathcal{N}(0,1)$. By construction, aggregate consumption growth has zero auto-correlation, is unpredictable, and is uncorrelated with business cycles. We have also considered the case where the expected growth in aggregate consumption is a function of the state variable that is correlated with the business cycle and obtained similar results. We choose to present the case where the expected growth in aggregate consumption is uncorrelated with the business cycle in order to explore and highlight the role of the variability of household consumption risk along the business cycle. The aggregate labor income is defined as $I_t = C_t - D_t$.

There are an infinite number of distinct households and their number is normalized to be one. Household *i* is endowed with labor income $I_{i,t} = \delta_{i,t} C_t - D_t$ at date *t*, where

$$\delta_{i,t} = \exp\left[\sum_{s=1}^{t}\left\{\left(j_{i,s}^{1/2}\sigma\eta_{i,s} - j_{i,s}\sigma^2/2\right) + \left(\hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\eta}_{i,s} - \hat{j}_{i,s}\hat{\sigma}^2/2\right)\right\}\right] \tag{1}$$

The exponent consists of two terms inside the summation. The first term captures shocks to household income that are related to the business cycle, for example, the event of job loss by the prime wage-earner in the household. The business cycle is tracked by the single state variable in the economy, $\omega_t > 0$, that follows a Markov process to be specified below. The state variable drives the household income shocks through the random variable $j_{i,s}$ that has a Poisson distribution with $prob\left(j_{i,s} = n\right) = e^{-\omega_s}\omega_s^n / n!, \quad n = 0,1,...\infty, \quad E\left(j_{i,s}\right) = \omega_s$, and independent of all primitive random variables in the economy. The random variable $\eta_{i,s} \sim N\left(0,1\right)$ is *i.i.d.* and independent of all primitive random variables in the economy. Thus, the first term is the sum of variables, $j_{i,s}^{1/2}\sigma\eta_{i,s} - j_{i,s}\sigma^2 / 2$, which are normal, conditional on the realization of $j_{i,s}$. The volatility of the conditional normal variable is $j_{i,s}^{1/2}\sigma$ and is driven by the variable $j_{i,s}$ with distribution driven by the state variable.[3] The second term of the exponent captures shocks to household income that are unrelated to the business cycle, for example, the death of the prime wage-earner in the household. It is defined in a similar manner as the first term with the major difference that $\widehat{\omega}$ is a parameter instead of being a state variable.

This particular specification of household income captures several key features of household income and consumption. First, since the income of the i^{th} household at date t is determined by the sum of all past idiosyncratic shocks, household income shocks are permanent, broadly consistent with the empirical evidence that household income shocks are persistent (e.g., Storesletten, Telmer, and Yaron (2004)). Second, the joint assumptions that the number of households is infinite and the income shocks are symmetric across households allow us to apply the law of large numbers and show that the identity $I_t = C_t - D_t$ is respected.[4] Third, this particular specification of household income, combined with the symmetric and homogeneous household preferences to be defined below, is shown to imply that households choose not to trade and household consumption is simply given by $C_{it} = I_{it} + D_t = \delta_{it}C_t$. Finally, the cross-

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[3] The probability distribution of the random variable $j_{i,s}^{1/2}\sigma\eta_{i,s}$ is known as a Poisson mixture of normals. This distribution is tractable because it is normal, conditional on $j_{i,s}$.

[4] The argument is due to Green (1989) and is elaborated in Appendix A.

sectional distribution of the relative household consumption growth, $\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)$, has negative third central moment. Its moments depend on the parameters of the distribution of $j_{i,s}$ which, in turn, are driven by the state variable ω_s. Hereafter, we refer to the state variable as "household risk".

We assume that households have identical recursive preferences:

$$U_{i,t} = \left\{ (1-\delta)\left(C_{i,t}\right)^{1-1/\psi} + \delta \left(E_t\left[\left(U_{i,t+1}\right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{1/(1-1/\psi)} \tag{2}$$

where δ is the subjective discount factor, γ is the RRA coefficient, ψ is the EIS, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$.[5] As shown in Epstein and Zin (1989), the SDF of household *i* is

$$SDF_{i,t+1} = \exp\left(\theta \log\delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + (\theta-1) r_{i,c,t+1} \right) \tag{3}$$

where $\Delta c_{i,t+1} \equiv \log\left(C_{i,t+1}\right) - \log\left(C_{i,t}\right)$ and $r_{i,c,t+1}$ is the log return on the i^{th} household's private valuation of its wealth portfolio. As we discuss in Section 5.3, recursive preferences appear to be necessary to explain the equity premium and risk free rate puzzles.

We conjecture and verify that autarchy is an equilibrium. Autarchy implies that the consumption of household *i* at date *t* is $C_{i,t} = I_{i,t} + D_t = \delta_{i,t} C_t$ and household consumption growth $C_{i,t+1}/C_{i,t} = \delta_{i,t+1} C_{t+1} / \delta_{i,t} C_t$ is independent of the household's consumption level.[6] This, combined

[5] Recursive preferences were introduced by Kreps and Porteus (1978) and adapted in the form used here by Epstein and Zin (1989) and Weil (1990).

[6] Essentially, we build into the model the assumption that the consumption growth of all households in a given period is independent of each household's consumption level. A richer model would allow for the consumption growth of each household in a given period to depend on the household's consumption level, consistent with the empirical findings of Guvenen, Ozkan, and Song (2014). Guvenen *et al.* analyzed the confidential earnings histories of millions of individuals over the period 1978-2010 and found that the earning power of the lowest income workers and the top 1% income workers erodes the most in recessions, compared to other workers.

with the property that the household's utility is homogeneous in the household's consumption level, implies that the return on the household's private valuation of its wealth portfolio is independent of the household's consumption level. The SDF of household i is, therefore, independent of the household's consumption level; it is specific to household i only through the term $\delta_{i,t+1} / \delta_{i,t}$. In pricing any security, other than the households' wealth portfolios, the term $\delta_{i,t+1} / \delta_{i,t}$ is integrated out of the pricing equation and the private valuation of any security is common across households. This verifies the conjecture that autarchy is an equilibrium. We formalize this argument in Appendix B.

In deriving the result that autarchy is an equilibrium and the equilibrium consumption of household i at date t is $C_{i,t} = \delta_{i,t} C_t$, we relied on the assumption that the market is incomplete thereby preventing households from insuring any component of their idiosyncratic income shocks. A reader who finds implausible the assumption that households may not insure any component of their idiosyncratic income shocks and the ensuing implication that autarchy is an equilibrium may simply interpret $C_{i,t} = \delta_{i,t} C_t$ as the *post-trade* consumption of the i^{th} household. Our empirical methodology is consistent with either one of the two interpretations of the relation $C_{i,t} = \delta_{i,t} C_t$ because we use household consumption data and not household income data. The degree of market incompleteness and the relation between household income and household consumption are outside the scope of the present investigation.

The logarithm of the cross-sectional relative household consumption growth is

$$\log\left(\frac{C_{i,t+1} / C_{t+1}}{C_{i,t} / C_t}\right) = \delta_{i,t+1} - \delta_{i,t} = j_{i,t+1}^{1/2} \sigma \eta_{i,t+1} - j_{i,t+1} \sigma^2 / 2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\eta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2 / 2 .$$

Its conditional central moments, derived in Appendix C, are as follows:

$$\mu_1\left(\log\left(\frac{C_{i,t+1} / C_{t+1}}{C_{i,t} / C_t}\right) | \, \omega_{t+1}\right) = -\sigma^2 \omega_{t+1} / 2 - \hat{\sigma}^2 \hat{\omega} / 2 \tag{4}$$

$$\mu_2\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\,\omega_{t+1}\right)=\left(\sigma^2+\sigma^4/4\right)\omega_{t+1}+\left(\hat{\sigma}^2+\hat{\sigma}^4/4\right)\hat{\omega} \tag{5}$$

and

$$\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\,\omega_{t+1}\right)=-\left(3\sigma^4/2+\sigma^6/8\right)\omega_{t+1}-\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega} \tag{6}$$

Equation (5) shows that the variance of the cross-sectional relative household consumption growth distribution increases as household risk increases. Empirically, we find that the variance of the cross-sectional relative household consumption growth is mildly countercyclical.

Equation (6) shows that the third central moment is always negative and becomes more negative as household risk increases.[7] Moreover, the model can generate any desired value of the third central moment, i.e. its maximum achievable absolute value is unbounded from above. Empirically, we find that the third central moment is mostly negative and mildly countercyclical. Guvenen, Ozkan, and Song (2014) found that the skewness of household income shocks is strongly countercyclical. This evidence allows us to associate an increase in household risk with recessions.[8]

For computational convenience, we define the variable x_t in terms of the state variable ω_t as $x_t \equiv \left(e^{\gamma(\gamma-1)\sigma^2/2}-1\right)\omega_t$. Since x_t is proportional to ω_t, we sometimes refer to x_t as the household risk, in place of ω_t. In our estimation, we limit the range of the RRA coefficient to $\gamma > 1$ which implies that $x_t > 0$. We assume that x_t follows an autoregressive gamma process, ARG (1), often referred to as a positive AR (1) process:

[7] The reader may wonder why the model-implied third central moment is always negative. Whereas the third central moment of $j_{i,s}^{1/2}\sigma\eta_{i,s}$ is zero, $\mu_3\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1}\right)|\,\omega_{t+1}\right]=E\left[E\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1}\right)^3|\,j_{i,t+1}\right]|\,\omega_{t+1}\right]=0$, the third central moment of $-j_{i,s}\sigma^2/2$ is negative and this imparts a negative third central moment to the random variable $j_{i,s}^{1/2}\sigma\eta_{i,s}-j_{i,s}\sigma^2/2$.

[8] Equations (4)-(6) show that the mean, variance, and third central moment of the cross-sectional distribution of relative household consumption growth are affine in the single state variable, the household consumption risk, ω_t. Therefore, an increase in the household consumption risk simultaneously results in a decrease in the cross-sectional mean and an increase in the cross-sectional variance and the absolute value of the (negative) third central moment.

$$x_{t+1} = \nu\xi + \rho x_t + \varepsilon_{x,t+1}, \tag{7}$$

where $\nu > 0$, $\xi > 0$, and ρ >0; $\varepsilon_{x,t+1}$ is a martingale difference sequence; $E\left[x_{t+1} \mid x_t\right] = \nu\xi + \rho x_t$; and $var\left(x_{t+1} \mid x_t\right) = \nu\xi^2 + 2\rho\xi x_t$.[9] Thus, the autocorrelation of household risk is ρ. As we show later on, the risk free rate, price-dividend ratio, and expected market return are affine functions of household risk and, therefore, their autocorrelation is also ρ.

The heteroskedasticity of the innovation of household risk implies that the volatility of household risk is increasing in household risk, $var\left(x_{t+1} \mid x_t\right) = \nu\xi^2 + 2\rho\xi x_t$. This property drives key features of the economy. As we shall see shortly, the model implies that the variance of the risk free rate, price-dividend ratio of the stock market, and expected market return are increasing in the household risk and, therefore, are increasing in recessions.

In Appendix D, equation (D.4), we calculate the households' common SDF as

$$\left(SDF\right)_{t+1} = e^{\theta \log\delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma\Delta c_{t+1} + (\theta-1)\left\{h_0 + h_1 A_0 - \left(A_0 + A_1 x_t\right)\right\} + \lambda x_{t+1}} \tag{8}$$

where the parameters A_0, A_1, and λ are defined in Appendix D by equations (D.2), (D.3), and (D.5).

The log risk free rate is calculated in Appendix D, equation (D.6), as

$$\begin{aligned} r_t = {} & -\theta\log\delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \left(\theta - 1\right)\left(h_0 + h_1 A_0 - A_0\right) + \gamma\mu - \gamma^2\sigma_a^2/2 - \lambda\nu\xi - \lambda^2\nu\xi^2/2 \\ & -\left\{\lambda\rho + \lambda^2\rho\xi - \left(\theta - 1\right)A_1\right\}x_t \end{aligned} \tag{9}$$

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[9] The ARG (1) process is the exact discrete-time equivalent of the square-root (CIR) process, and is defined as follows (see e.g., Gourieroux and Jasiak (2006)): $\left(x_{t+1}/\xi\right) \mid y_{t+1} \sim \Gamma\left(\nu + y_{t+1}\right), \nu > 0;\quad y_{t+1} \mid x_t \sim \mathrm{P}\left(\rho x_t/\xi\right), \rho, \xi > 0$, where Γ denotes a gamma distribution, P denotes a Poisson distribution, ξ is a scale parameter, ν is the degree of freedom, ρ is the correlation parameter, and y_{t+1} is the mixing variable. The conditional probability density function of an ARG (1) process, $f\left(x_{t+1} \mid x_t; \nu, \xi, \rho\right)$, is a mixture of gamma densities with Poisson weights. Therefore, the ARG (1) process is strictly positive. Moreover, it also admits the autoregressive representation in equation (7).

Therefore, when household risk is high, the conditional variance of the risk free rate is high. The model also implies that the risk free rate is low when household risk is high since in the estimated model the coefficient of x_t in equation (9) is negative. Thus the model implies that, in recessions, the risk free rate is low and the variance of the risk free rate is high. Both of these implications are consistent with observation.

Finally, the unconditional mean of the risk free rate is

$$\begin{aligned}\bar{r} = {} & -\theta \log \delta - \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - (\theta - 1)\left(h_0 + h_1 A_0 - A_0\right) + \gamma\mu - \gamma^2\sigma_a^2/2 - \lambda\nu\xi - \lambda^2\nu\xi^2/2 \\ & - \left\{\lambda\rho + \lambda^2\rho\xi - (\theta - 1)A_1\right\}\bar{x}\end{aligned} \tag{10}$$

and its unconditional variance is

$$var(r_t) = \left\{\lambda\rho + \lambda^2\rho\xi - (\theta - 1)A_1\right\}^2 var(x_t) \tag{11}$$

In Appendix D, we also show that the real yield curve is upward sloping, downward sloping, or humped, depending on the state. Thus the cross-sectional variation of the idiosyncratic income shocks gives rise to familiar shapes of the yield curve.

We assume that the log dividend growth of the *stock* market follows the process[10]

$$\Delta d_{t+1} = \mu_d + \sigma_d \varepsilon_{d.t+1} \tag{12}$$

where $\varepsilon_{d,t+1} \sim N(0,1)$ is *i.i.d.* and independent of all primitive random variables. By construction, dividend growth has zero auto-correlation, is unpredictable, and is uncorrelated with the business cycle. We have also considered the case where the expected growth in aggregate dividends is a function of the state variable and obtained similar results. We choose to

[10] We draw a distinction between the stock market and the "market" which we defined earlier as the sum total of all assets in the economy. Δd_{t+1} is the log dividend growth of the stock market.

present the case where the expected growth in aggregate dividends is uncorrelated with the business cycle in order to explore and highlight the role of the variability of household consumption risk along the business cycle.

In Appendix D, equation (D.8), we calculate the price-dividend ratio as

$$z_{m,t} = B_0 + B_1 x_t, \tag{13}$$

the expected stock market return (equation (D.11)) as

$$E\left[r_{m,t+1} \mid x_t\right] = k_0 + k_1 B_0 + k_1 B_1 \nu\xi - B_0 + \mu_d + \left\{k_1 B_1 \rho - B_1\right\} x_t \tag{14}$$

and the conditional variance of the stock market return (equation (D.12)) as

$$\operatorname{var}\left(r_{m,t+1} \mid x_t\right) = k_1^2 B_1^2 var\left(x_{t+1} \mid x_t\right) + \sigma_d^2 \tag{15}$$

where the parameters B_0 and B_1 are determined in Appendix D.

In the estimated model, the coefficient of x_t in equation (13) is negative, implying that the price-dividend ratio of the stock market is low in recessions. The coefficient of x_t in equation (14) is positive, implying that the expected return of the stock market is high in recessions. Finally, equation (15) implies that the variance of the stock market is high in recessions. All these implications are consistent with the data.

2. Data Description

2.1. Prices and Dividends

We use monthly data on prices and dividends from January 1929 through December 2009. The proxy for the stock market is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The monthly portfolio return is the sum of the portfolio price and dividends at the end of the month, divided by the portfolio price at the beginning of the month. The annual portfolio return is the sum of the portfolio price at the end of the year and uncompounded dividends over the year, divided by the portfolio price at the beginning of the year. The real annual portfolio return is the above annual portfolio return deflated by the realized growth in the Consumer Price Index.

The proxy for the real annual risk free rate is obtained as in Beeler and Campbell (2012). Specifically, the quarterly nominal yield on 3-month Treasury Bills is deflated using the realized growth in the Consumer Price Index to obtain the *ex post* real 3-month T-Bill rate. The *ex-ante* quarterly risk free rate is then obtained as the fitted value from the regression of the ex post real 3-month T-Bill rate on the 3-month nominal yield and the realized growth in the Consumer Price Index over the previous year. This procedure is equivalent to forecasting inflation and subtracting the inflation forecast from the 3-month nominal yield. Finally, the ex-ante quarterly risk free rate at the beginning of the year is annualized to obtain the ex-ante annual risk free rate.

The above procedure in building an empirical counterpart of the real risk free rate ensures that the real risk free rate for a given period (say, a quarter) is known at the beginning of the period, consistent with its definition in the context of our theoretical model. However, since the real risk free rate is unobservable in the data over the duration of our sample period, we also consider two alternative approaches to constructing the real risk free rate. In the first approach, we deflate the 3-month nominal Treasury bill rate with realized inflation. The mean and volatility of the resulting time series are very similar to those obtained using our baseline procedure. In particular, for the entire sample period 1947:Q1-2009:Q4 over which aggregate quarterly consumption data is available, the mean of the risk free rate is .003 and its volatility is 0.010. These are very close (.003 and .006, respectively) to those obtained using our baseline procedure. Similar conclusions are obtained in the 1982:Q1-2009:Q4 subsample over which household consumption data is available: deflating the nominal 3-month rate using realized inflation gives a mean risk free rate of .006 with volatility .009 that are very close to the corresponding baseline values of .005 and .005, respectively.

In the second approach, we compound the monthly returns on 1-month Treasury bills within a quarter to obtain the quarterly nominal risk free rate and then convert it to real using the realized inflation. This procedure also gives very similar values for the mean and volatility of the real risk free rate. The mean and volatility are .002 and .009, respectively, over the 1947:Q1-2009:Q4 period and .005 and .009, respectively, over the 1982:Q1-2009:Q4 subsample.

These results suggest that our conclusions regarding the fit of the theoretical model for the risk free rate is not sensitive to the precise procedure used to generate an empirical counterpart of the real risk free rate.

The annual price-dividend ratio of the market is the market price at the end of the year, divided by the sum of dividends over the previous twelve months. The dividend growth rate is the sum of dividends over the year, divided by the sum of dividends over the previous year and is deflated using the realized growth in the Consumer Price Index.

2.2. Household Consumption Data[11]

The household-level quarterly consumption data is obtained from the Consumer Expenditure Survey (CEX) produced by the Bureau of Labor Statistics (BLS). This series of cross-sections covers the period since 1980:Q1. Each quarter, roughly 5,000 U.S. households are surveyed, chosen randomly according to stratification criteria determined by the U.S. Census. Each household participates in the survey for five consecutive quarters, one training quarter and four regular ones, during which their recent consumption and other information are recorded. At the end of its fifth quarter, another household, chosen randomly according to stratification criteria determined by the U.S. Census, replaces the household. The cycle of the households is staggered uniformly across the quarters, such that new households replace approximately one-fifth of the participating households each quarter.[12,13] If a household moves away from the sample address, it is dropped from the survey. The new household that moves into this address is screened for eligibility and is included in the survey.

The number of households in the database varies from quarter to quarter. The survey attempts to account for an estimated 95% of all quarterly household expenditures in each

[11] Our description and filters of the household consumption data closely follow Brav, Constantinides, and Geczy (2002).

[12] If we were to exclude the training quarter in classifying a household as being in the panel, then each household would stay in the panel for *four* quarters and new households would replace *one-fourth* of the participating households each quarter.

[13] The constant rotation of the panel makes it impossible to test hypotheses regarding a specific household's behavior through time for more than four quarters. A longer time series of individual households' consumption is available from the PSID database, albeit only for *food* consumption.

consumption category from a highly disaggregated list of consumption goods and services. At the end of the fourth regular quarter, data is also collected on the demographics and financial profiles of the households, including the value of asset holdings as of the month preceding the interview. We use consumption data only from the regular quarters, as we consider the data from the training quarter unreliable. In a significant number of years, the BLS failed to survey households not located near an urban area. Therefore, we consider only urban households.

The CEX survey reports are categorized in three tranches, the *January*, *February*, and *March* tranches. For a given year, the first-quarter consumption of the January tranche corresponds to consumption over January through March; for the February tranche, first-quarter consumption corresponds to consumption over February through April; for the March tranche, first-quarter consumption corresponds to consumption over March through May; and so on for the second, third, and fourth quarter consumption. Whereas the CEX consumption data is presented on a monthly frequency for some consumption categories, the numbers reported as monthly are often simply the quarterly estimates divided by three.[14] Thus, utilizing monthly consumption is not an option.

Following Attanasio and Weber (1995), we discard from our sample the consumption data for the years 1980 and 1981 because they are of questionable quality. Starting in interview period 1986:Q1, the BLS changed its household identification numbering system without providing the correspondence between the 1985:Q4 and 1986:Q1 identification numbers of households interviewed in both quarters. This change in the identification system makes it impossible to match households across the 1985:Q4 - 1986:Q1 gap and results in the loss of some observations. This problem recurs between 1996:Q1 and 1997:Q1 and also in 2005:Q1.

2.3. Definition of the Household Consumption Variables

For each tranche, we calculate each household's quarterly *nondurables and services* (NDS) consumption by aggregating the household's quarterly consumption across the consumption categories that comprise the definition of nondurables and services. We use consumption categories that adhere to the National Income and Product Accounts (NIPA) classification of

[14] See Attanasio and Weber (1995) and Souleles (1999) for further details regarding the database.

NDS consumption. Since the quantity of interest to us is the *relative* household consumption growth, $\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)$, it is unnecessary to either deflate or seasonally adjust consumption.

The *per capita* consumption, C_t of a set of households is calculated as follows. First, the *total consumption* in a given quarter is obtained by summing the nondurables and services consumption of all the households in that quarter. Second, the *per capita consumption* in a given quarter is obtained by dividing the total consumption in that quarter by the sum of the number of family members across all the households in that quarter. The *per capita* consumption *growth* between quarters *t* - 1 and *t* is defined as the *ratio* of the *per capita* consumption in quarters *t* and *t* - 1.

2.4. Household Selection Criteria

In any given quarter, we delete from the sample households that report in that quarter as zero either their total consumption, or their consumption of nondurables and services, or their food consumption. In any given quarter, we also delete from the sample households with missing information on the above items.

We mitigate observation error by subjecting the households to a *consumption growth filter*. The filter consists of the following selection criteria. First, we delete from the sample households with consumption growth reported in fewer than three consecutive quarters. Second, we delete the consumption growth rates $C_{i,t}/C_{i,t-1}$ and $C_{i,t+1}/C_{i,t}$, if $C_{i,t}/C_{i,t-1} < 1/2$ *and* $C_{i,t+1}/C_{i,t} > 2$, and vice versa. Third, we delete the consumption growth $C_{i,t}/C_{i,t-1}$ if it is greater than five.

3. Household Consumption Statistics

We use the panel data on household consumption growth constructed from the CEX database and compute several summary statistics of the cross-sectional distribution of household consumption growth relative to per capita aggregate consumption growth, $\log\left(\frac{C_{i,t+1} / C_{t+1}}{C_{i,t} / C_t}\right)$. These statistics are reported in Table 1 over the period 1982:Q1-2009:Q4. In the first row we present results for the January tranche and in the second row we present results when the households in the January, February, and March tranches are combined to provide a larger cross-section. The surviving sub-sample of households, after the application of the filters described in Section 2, is substantially smaller than the original one. For the January tranche, the maximum number of households in a quarter is 1,310 and the mean is 674.[15] For the combined tranches, the maximum number of households in a quarter is 3,906 and the mean is 2,056.

In the first row we show that, for the January tranche, the sample mean of the cross-sectional distribution of relative household consumption growth, μ_1, is not statistically different from zero, as expected. The sample volatility, $\mu_2^{1/2}$, has a high mean value of .379 and is highly autocorrelated with a first-order autocorrelation coefficient of .772. The sample third central moment, μ_3, is mostly negative, with average value -.025 which is strongly statistically significant; it is also positively auto-correlated with a first-order auto-correlation coefficient of .124. Our finding of negative skewness in the cross-sectional distribution of household consumption growth is consistent with those in Guvenen *et al.* (2014) who reported negative left skewness in the cross-sectional distribution of income growth.

While the third central moment is a commonly used measure of the skewness of a distribution, we also present results for a quantile-based measure that are more robust to outliers. In particular, we report the time-series average of the 5$^{\text{th}}$ percentile of the relative household consumption growth distribution, along with its first-order auto-correlation coefficient. The average 5$^{\text{th}}$ percentile is -.595 and it is strongly positively auto-correlated with a first-order auto-correlation coefficient of .534. The results are largely similar for the February and March tranches and are omitted in the interest of brevity.

Stronger results are obtained in the second row where the households in the January, February, and March tranches are combined to provide a larger cross-section. The third central

[15] The relatively mild minimum asset criterion of $2,000 for a household to be included in the sample eliminates about 80% of the households and eliminates all the households in some quarters. Stricter filters further eliminate households to the point that statistics with a small number of households become unreliable. In the interest of having a large sample, we present our results without imposing a minimum asset filter.

moment is negative on average, strongly statistically significant, and positively auto-correlated. The 5th percentile of the cross-sectional distribution is negative and strongly positively auto-correlated with first-order auto-correlation coefficient of .792. In the second panel of Table 1, we present the statistics implied by the estimated model. We defer discussion of this panel until we present the empirical results.

At each quarter, an indicator variable, I_{rec}, takes the value of one if there is a NBER-designated recession in at least two of the three months of the quarter. In Table 2, we present the correlations of the cross-sectional mean, volatility, third central moment, and the 5th percentile with NBER-designated recessions. In recessions, the third central moment becomes more negative—the correlation between the recession dummy and the third central moment is -.249 for the January tranche (row 1) and -.217 for the combined January, February and March tranches (row 2). The 5th percentile of the cross-sectional distribution also becomes more negative in recessions—the correlations between the recession dummy and the 5th percentile is -.142 and -.087, respectively, for the January tranche and the combined tranches. These findings are also consistent with those reported in Guvenen *et al.* (2014) who find evidence of counter-cyclical left skewness in the cross-sectional distribution of household income growth. In the last panel of Table 2, we present correlations implied by the estimated model. We defer discussion of this panel until we present the empirical results.

Note that the CEX database may contain substantial measurement error because it is based on a survey. Measurement error may contaminate both the levels of the variance and the third central moment of the cross-sectional distribution of household consumption growth as well as their auto-correlations. In order to obtain more reliable estimates of the consumption expenditures of households, Blundell, Pistaferri, and Preston (2008) imputed a measure of nondurables and services consumption for the families contained in the Panel Study of Income Dynamics (PSID) database, by combining information from the PSID and CEX databases. They then used the imputed consumption to construct the time series of the cross-sectional variance of household consumption growth for families in the PSID over the period 1980-1992. Furthermore, they specified a reduced-form model for the joint dynamics of household income and consumption that allows for measurement error in observed consumption. We apply their framework to compute the contribution of the different shocks, including measurement error, to the cross-sectional variance and third central moment of household consumption growth.

Blundell *et al.* (2008) assumed that the observed consumption of household i at time t may be written as the sum of the true unobserved consumption and a measurement error that is independent over time:

$$c^*_{i,t} = c_{i,t} + u_{i,t} \tag{16}$$

Therefore, taking first differences, we have

$$\Delta c^*_{i,t} = \Delta c_{i,t} + u_{i,t} - u_{i,t-1} = \underbrace{\phi \varsigma_{i,t} + \psi \varepsilon_{i,t} + \xi_{i,t}}_{\Delta c_{i,t}} + u_{i,t} - u_{i,t-1} \tag{17}$$

As in Blundell *et al.* (2008), the true (unobserved) consumption growth, $\Delta c_{i,t}$, depends on the permanent shocks to income (ς), the insurance with respect to permanent shocks (ϕ), the transitory shocks to income (ε), the insurance with respect to transitory shocks (ψ), and the shocks to consumption uncorrelated to those in income (ξ), for example, preference shocks or shocks to the higher-order moments of the income process. All the shocks are assumed to be mutually independent.

Given the above structure, we have

$$var\left(\Delta c^*_{i,t}\right) = \phi^2 var\left(\varsigma_{i,t}\right) + \psi^2 var\left(\varepsilon_{i,t}\right) + var\left(\xi_{i,t}\right) + var\left(u_{i,t}\right) + var\left(u_{i,t-1}\right) \tag{18}$$

Note that, using the CEX database, we estimate the time series of the cross-sectional volatility of relative household consumption growth (the square root of the left-hand side of the above equation) over the 1982-2009 period. We find that, at the quarterly frequency, the cross-sectional volatility is 38% on average and varies from 28.4% to 80.5%.

Using the CEX and PSID databases to impute a measure of nondurables and services consumption for the households in the PSID, Blundell *et al.* (2008) showed that, over the period 1980-1992, the cross-sectional volatility is 39.6% on average and varies from 34.6% to 44.9%. Note that these estimates are similar to those obtained using the CEX data alone.

Based on the parameter estimates from Blundell *et al.* (2008), the volatility of the cross-sectional household consumption growth accounted for by the permanent component (the square root of the first term on the right-hand side of the above equation) is 8.7% on average, varying from 6.5% to 11.1%.[16] The volatility of the cross-sectional household consumption growth accounted for by the permanent and transitory components (the square root of the sum of the first and second terms on the right-hand side of the above equation) is 8.8% on average, varying from 6.6% to 11.2%. Thus, the transitory component contributes very little to the overall volatility. Preference shocks and/or shocks to the higher-order moments of the income process (the third term on the right-hand side of the above equation), in conjunction with the permanent and transitory shocks, generate an average cross-sectional volatility of 13.5%, varying from 12.2% to 15.2%. Therefore, measurement error (the fourth and fifth terms on the right-hand side of the above equation) accounts for a large proportion of the overall cross-sectional volatility of household consumption growth.

Turning next to the third central moment, we have

$$E\left[\left(\Delta c^{*}_{i,t}\right)^{3}\right]=\phi^{3}E\left(\varsigma^{3}_{i,t}\right)+\psi^{3}E\left(\varepsilon^{3}_{i,t}\right)+E\left(\xi^{3}_{i,t}\right)+E\left(u^{3}_{i,t}\right)-E\left(u^{3}_{i,t-1}\right) \tag{19}$$

Assuming that the measurement error has a stationary distribution, i.e., $E\left(u^{3}_{i,t}\right)=E\left(u^{3}_{i,t-1}\right)$, the above equation implies that measurement error does not affect the level of the third central moment.

4. Empirical Methodology and Results

The model has thirteen parameters: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the three parameters of the household income shocks, $\sigma, \widehat{\sigma}$, and $\widehat{\omega}$; the three parameters of the dynamics of the state variable, ν, ξ, and ρ; the mean, μ_d, and volatility, σ_d, of aggregate dividend growth; and the three preference parameters, the subjective discount factor, δ, the

[16] See Table 4 of Blundell et al. (2008).

RRA coefficient, γ, and the elasticity of intertemporal substitution, ψ. We estimate the model parameters using GMM to simultaneously match the following moments of aggregate stock market data as well as household level consumption data: the mean and variance of aggregate consumption and dividend growth; the mean, variance, and autocorrelation of the risk free rate, market return, and market-wide price-dividend ratio; and the mean, variance, and third central moment of the cross sectional distribution of household consumption growth relative to per capita aggregate consumption growth. Therefore, we have sixteen moment restrictions in thirteen parameters, providing over-identifying restrictions to test the model specification. We use a diagonal weighting matrix with a weight of one on all the moments except for the unconditional means of the market return and risk free rate that have weights of 100.[17]

Since data on relative household consumption growth is available only at the quarterly frequency since 1982:Q1, we estimate the model at the quarterly frequency over the period 1982:Q1-2009:Q4 in order to test the fit of the model-generated moments of aggregate stock market data and the cross-sectional distribution of relative quarterly household consumption growth to their sample counterparts. The model fit and parameter estimates are presented in Table 3 for the January tranche.[18] The table shows that the parsimonious model with just one state variable fits the sample moments of the risk free rate, market return, price-dividend ratio, aggregate consumption and dividend growth, as well as the first three central moments of the cross-sectional distribution of household consumption growth very well. The *J*-stat is 4.84 and the model is not rejected at the 10% level of significance. The asymptotic 90% critical value is 7.59.

The model generates mean risk free rate 0.6% and stock market return 2.3%, both very close to their sample counterparts of 0.5% and 1.9%, respectively. Therefore, the model provides an explanation of the equity premium and risk free rate puzzles. The model generates volatility 1.2% and first-order autocorrelation 0.877 of the risk free rate, compared to the sample

[17] The pre-specified weighting matrix has two advantages over the efficient weighting matrix. First, it has superior small-sample properties (see e.g., Ahn and Gadarowski (1999), Ferson and Foerster (1994), and Hansen, Heaton, and Yaron (1996)). Second, the moment restrictions included in the GMM have different orders of magnitude, with the mean of the price-dividend ratio being a couple of orders of magnitude larger than the means of the market return and risk free rate. Therefore, placing larger weights on the latter two moments enables the GMM procedure to put equal emphasis in matching all these moments. We repeated our estimation using the efficient weighting matrix and obtained similar results that are available upon request.

[18] Similar results are obtained for the February and March tranches and are available upon request.

counterparts of 0.5% and 0.899, respectively. The model also generates volatility 23.1% and first-order auto-correlation -0.058 of the market return, compared to the sample counterparts of 8.4% and 0.037, respectively. The model-implied mean of the market-wide price-dividend ratio is 3.787, very close to its sample counterpart of 3.759. More importantly, the model generates the high volatility of the price-dividend ratio observed in the data (46.6% versus 41.4%), thereby explaining the excess volatility puzzle. Note that most asset pricing models, including those with long run risks and rare disasters, have difficulty in matching the latter moment and, therefore, at explaining the high volatility of stock prices (see Constantinides and Ghosh (2011)). The model-implied first-order auto-correlation of the market-wide price-dividend ratio is 0.877, close to its sample counterpart of 0.986.

The model matches well the unconditional mean and volatility of the annual aggregate consumption growth rate. Note that models that rely on the incidence of shocks to aggregate, as opposed to household, consumption growth in order to address the equity premium and excess volatility puzzles require unrealistically high variance of the aggregate consumption growth: the Barro (2006) rare disasters model implies aggregate consumption growth volatility of 4.6%. By contrast, the incidence of shocks to household consumption growth, as modeled in our paper, does not affect the volatility of the aggregate consumption growth.

The model generates 0.1% mean and 3.3% volatility of the aggregate dividend growth rate, compared to their sample counterparts of 0.5% and 10.4%, respectively. By construction, the autocorrelation of dividend growth rate in our model is zero, consistent with the broader evidence that dividend growth is unpredictable. This contrasts with long run risks models that rely on implausibly high levels of persistence in the dividend growth process.

The model matches well the unconditional third central moment of the cross-sectional distribution of relative household consumption growth. The model-implied third central moment of -0.024 is almost identical to its sample counterpart of -0.025. The model-implied unconditional volatility of the cross-sectional distribution of household consumption growth is 20.1%. Even though the volatility in the historical data is 37.9%, we argued in Section 3 that the volatility, adjusted for measurement error, is about 15%. Therefore, the model-implied unconditional volatility of the cross-sectional distribution of household consumption growth matches well its error-adjusted empirical counterpart.

The estimated preference parameters are reasonable: the risk aversion coefficient is 1.34 and the EIS is one. The EIS is higher than the inverse of the risk aversion coefficient, thereby highlighting the importance of recursive preferences and pointing towards a preference for early resolution of uncertainty.

The parameters $\nu, \xi,$ and ρ govern household risk. The auto-correlation of household risk is $\rho = 0.877$ and this renders the auto-correlation of the risk free rate and price-dividend ratio to be 0.877 also, close to their sample values. The parameters ξ and ν govern the variance of household risk and render the variance of the risk free rate, market return, and price-dividend ratio close to their sample counterparts.

We use the point estimates of the model parameters in Table 3 to calculate the signs of the coefficients of the household risk in the equations that determine the risk free rate, price-dividend ratio and the conditional expected market return: $r_{f,t} = .009 - 9.63x_t$, $z_{m,t} = 3.89 - 376.0x_t$, and $E\left[r_{m,t+1} \mid x_t\right] = .009 + 53.52\,x_t$. Consistent with empirical evidence, the model implies that the risk free rate and price-dividend ratio are procyclical while the expected market return is countercyclical.

We extract the time series of the model-implied cross-sectional moments of household consumption growth from the observed time series of the risk free rate and market-wide price-dividend ratio.[19] The middle panel of Table 1 displays the model-implied cross-sectional moments of household consumption growth. The volatility is of the same order of magnitude as its sample counterpart. The third central moment is almost identical to its sample counterpart. The first-order auto-correlation of the model-implied volatility is high and of the same order of magnitude as the auto-correlation in the data but the first-order auto-correlation of the third central moment is higher than the auto-correlation in the data, probably due to the small sample size and the quality of the consumption data. The model-implied autocorrelation of the 5$^{\text{th}}$ percentile of the cross-sectional distribution is 86%, more in line with its sample counterpart of 53.4%. The bottom panel of Table 1 displays correlations between the historical and model-implied moments. The model-generated cross-sectional volatility has correlation 49.0% with its

[19] The model implies that the risk free rate and price-dividend ratio are affine functions of the state variable. We use the point estimates of the parameters and extract the current value of the state variable from the observed risk free rate and price-dividend ratio by minimizing the least-squares criterion function. Given the current value of the state variable, we calculate the model-implied cross-sectional moments using equations (4)-(6).

sample counterpart and the cross-sectional third central moment has correlation 37.7% with its sample counterpart.

The bottom panel of Table 2 displays the correlation of the model-implied household consumption growth moments with NBER-designated recessions. The correlation of the model-implied volatility of the cross-sectional distribution with recessions is 26.4%, the correlation of the third central moment with recessions is -26.8%, and the correlation of the 5^{th} percentile with recessions is -23.4%. These are close to the sample values of 13.9%, -24.9%, and -14.2%, respectively, obtained for the January tranche and 11.8%, -21.7%, and -8.7%, respectively, obtained for the combined tranches.

5. Further Interpretation of the Results

We consider several variants of the baseline model and obtain insights into the role played by each of its key ingredient. In Section 5.1, we estimate the model over the entire quarterly sample period, 1947:Q1-2009:Q4, over which aggregate consumption and asset price data is available. In Section 5.2, we examine the performance of the model at the annual frequency over the entire available sample period 1929-2009. In Section 5.3, we investigate the role of recursive preferences by considering a variant of the baseline model where the households have CRRA preferences. In Section 5.4, we consider a version of the model where the shocks to household consumption growth are conditionally lognormal, instead of the Poisson mixture of normals distribution assumed in Section 2. Finally, in Section 5.5, we highlight the significance of shocks unrelated to the business cycle by estimating a version of the model where we suppress household income shocks unrelated to the business cycle.

5.1. Estimation with Quarterly Data, 1947:Q1-2009:Q4

We re-estimate the model using quarterly data over the period 1947:Q1-2009:Q4, the entire period over which quarterly aggregate consumption data is available. Since household consumption data is not available over much of this period, the GMM system consists of only the thirteen moment restrictions on aggregate asset prices and consumption and dividend growth rates.

The model fit and parameter estimates are presented in Table 4. The model matches well the moments of the risk free rate, stock market return, and price-dividend ratio, except that it generates a higher value of the mean market return (.030) than its sample counterpart (.017).

As with the quarterly sub-period in Section 4, the model generates the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return. The model implies that the volatility of the risk free rate, price-dividend ratio, and the conditional expected market return are countercyclical, consistent with observation. We use the point estimates of the model parameters in Table 4 to calculate the coefficients of the household risk in the equations that determine the risk free rate, price-dividend ratio, and the conditional expected market return: $r_{f,t} = .011 - .823 x_t$, $z_{m,t} = 3.88 - 42.36\, x_t$, and $E\left[r_{m,t+1} \mid x_t \right] = .008 + 2.62 x_t$. Consistent with observation, the model implies that the risk free rate and price-dividend ratio are procyclical while the expected market return is countercyclical. The estimated preference parameters are reasonable: the risk aversion coefficient is 5.34 and the EIS is 1.47.

5.2. Estimation with Annual Data, 1929-2009

So far we have assumed that the decision frequency of households is quarterly. In this sub-section, we present results at the annual frequency for the entire available sample period, 1929-2009. As in Section 5.1, the GMM system consists of only the thirteen moment restrictions on aggregate asset prices and aggregate consumption and dividend growth rates.

The results are reported in Table 5. The model matches well the moments of the risk free rate, stock market return, and price-dividend ratio. The model generates the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return. The model implies that the volatility of the risk free rate, price-dividend ratio, and the conditional expected market return are countercyclical; the risk free rate and price-dividend ratio are procyclical; and the expected market return is countercyclical. The estimated preference parameters are reasonable: the risk aversion coefficient is 1.70 and the EIS is 1.00. These results suggest that the model is robust to the assumed decision frequency of the households.

5.3. Estimation with CRRA Preferences

In order to demonstrate the role served by recursive preferences, we discuss the special case of our baseline model with CRRA preferences. In Table 6 we present the counterpart of Tables 3 at the quarterly frequency but with CRRA preferences. We predictably find that the equity premium and risk free rate puzzles arise: the model-implied annual risk free rate is 7.6% and the equity premium is -.06%. In Table 7, we present the counterpart of Table 5 at the annual frequency but with CRRA preferences. Again we find that the equity premium and risk free rate puzzles arise: the model-implied annual risk free rate is 3.9% and the equity premium is -.1%.

5.4. Estimation with Lognormal Shocks

We present the results for the same specification of preferences as in our baseline model but with idiosyncratic shocks to household consumption conditionally lognormal rather than negatively skewed:

$$\delta_{i,t} = \exp\left[\sum_{s=1}^{t}\left\{\left(\sigma_s\eta_{i,s} - \sigma_s^2/2\right) + \left(\hat{\sigma}\hat{\eta}_{i,s} - \hat{\sigma}^2/2\right)\right\}\right], \quad \eta_{i,s}, \hat{\eta}_{i,s} \sim i.i.d.\, N\left(0,1\right) \tag{20}$$

The state variable σ_t^2 is assumed to follow an auto-regressive gamma (positive AR (1)) process:

$$\sigma_{t+1}^2 = \nu_\sigma \xi_\sigma + \rho_\sigma \sigma_t^2 + \varepsilon_{\sigma,t+1} \tag{21}$$

The results are reported in Table 8 over the quarterly subsample 1982:Q1-2009:Q4 for the January tranche. We find that the equity premium and risk free rate puzzles arise: the model-implied annual risk free rate is 6.0% and the equity premium is 3.2%, compared to their sample counterparts of 2.0% and 6.4%, respectively.

More importantly, the model implies that the equilibrium risk free rate, price-dividend ratio, and the conditional expected market return are affine functions of the state variable σ_t^2. We use the point estimates of the model parameters in Table 8 to calculate the coefficients of σ_t^2 in the equations that determine the risk free rate, price-dividend ratio, and the conditional expected market return: $r_{f,t} = .016 - 1.75\sigma_t^2$, $z_{m,t} = 3.85 - 112.0\,\sigma_t^2$, and $E\left[r_{m,t+1} \mid \sigma_t^2\right] = .016 + 12.80\sigma_t^2$. Now, the state variable σ_t^2 also drives the variance of the cross-sectional distribution of household consumption growth. Therefore, in order to generate the empirically observed dynamics of the risk free rate, price-dividend ratio, and stock market return, the model requires that the variance of the cross-sectional household consumption growth be countercyclical. This is improbable, given that Guvenen, Ozkan, and Song (2014) showed that the variance of the cross-sectional household income growth is not countercyclical.

5.5. Estimation without Household Income Shocks Unrelated to the Business Cycle

We explore the significance of shocks to household income that are unrelated to the business cycle, by suppressing the term $\hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\eta}_{i,s} - \hat{j}_{i,s}\hat{\sigma}^2/2$ in equation (1) and writing the household income shock as

$$\delta_{i,t} = \exp\left[\sum_{s=1}^{t}\left\{\left(j_{i,s}^{1/2}\sigma\eta_{i,s} - j_{i,s}\sigma^2/2\right)\right\}\right] \tag{22}$$

The results are reported in Table 9 over the quarterly subsample 1982:Q1-2009:Q4 for the January tranche. The model-implied risk free rate is negative and its volatility is too high. Also the expected market return is too high. These results are inferior to the results in Table 3, thereby highlighting the significance of shocks to household income that are unrelated to the business cycle.

6. Household Consumption Risk and the Cross-Section of Excess Returns

Our empirical results show that household consumption risk, measured by the third central moment of the cross-sectional distribution of household consumption growth, is an important risk factor that drives the time series properties of aggregate quantities: the risk free rate, market return, and market price-dividend ratio. We proceed to show that household consumption risk also explains the cross-section of excess returns.

We follow the standard Fama-Macbeth (1973) methodology. In the first step, we run time series regressions of quarterly excess returns of each asset on the household consumption risk and obtain the factor loading for each asset. In the second step, for each quarter in the second half of the sample, we estimate a cross-sectional regression of the excess asset returns on their estimated factor loadings from the first step and obtain a time series of cross-sectional intercepts and slope coefficients. We present the average of the cross-sectional intercepts, $\hat{a}$, and slope coefficients, $\hat{\lambda}$. We calculate the standard errors of $\hat{a}$ and $\hat{\lambda}$ from the time series of the cross-sectional intercepts and slope coefficients. Given the short length of the time series, we expect and find that the standard errors are large.

We present results for two variations of the first-stage time series regressions. In the first variation ("rolling"), presented in Table 10, each period *t*, starting with the midpoint of the sample, we use all of the returns up to period *t* to estimate the factor loadings as inputs to the cross-sectional regressions. In the second variation ("fixed"), presented in Table 11, we use the first half of the sample to estimate the factor loadings as inputs to the cross-sectional regressions performed on the second half of the sample.

The results with rolling time-series regressions are reported in Table 10. Panels A, B, and C present results when the set of test assets consists of the 25 size and book-to-market sorted equity portfolios of Fama and French (FF), the 30 industry-sorted portfolios, and the combined set of 25 FF and 30 industry-sorted portfolios, respectively. We include the industry portfolios as test assets, in addition to the 25 FF portfolios, because the size and book-to-market sorted equity portfolios have a strong factor structure making it easy for almost any proposed factor to produce a high cross-sectional adjusted R^2 (that we denote throughout by $\overline{R^2}$).[20]

In the first row of each panel, we present the results when the only factor is the household consumption risk, the third central moment of the cross-sectional distribution of household consumption growth. In all three panels, the intercept is both statistically and economically insignificant, as expected. The slope coefficient is positive, as expected, but is not statistically significant given the small size of the sample. The cross-sectional $\overline{R^2}$ is stable, varying from 13.6% to 14.9%.

In the second row of each panel, we present the results when the only factor is the volatility of the cross-sectional distribution of household consumption growth. In all three panels, the intercept is both statistically and economically insignificant, as expected. In Panels A and C, the slope coefficient is negative, as expected, but small; in Panel B the slope coefficient is zero. The cross-sectional $\overline{R^2}$ varies from -6.9% to 40%, suggesting that the results are unstable and possibly spurious. Further evidence against the volatility as a factor is provided in the third row of Panels A, B, and C where we simultaneously consider the household consumption risk and volatility as factors. Whereas in all three panels the slope coefficient of household consumption risk is positive as expected, in Panels B and C the slope of the volatility factor is positive, against expectation.

In the last row of each panel, we present the results for the three FF risk factors. In all three panels the estimated intercept is economically large; it is also statistically significant in Panel A. All slope coefficients are economically insignificant. The cross-sectional $\overline{R^2}$ varies from -22.8% to 59.5%, suggesting that the results are unstable.

The results with fixed time-series regressions are reported in Table 11 and reinforce the above results. When the only factor is the household consumption risk, the intercept is both statistically and economically insignificant, as expected. The slope coefficient is positive, as expected, but is not statistically significant given the small size of the sample. The cross-sectional $\overline{R^2}$ is stable, varying from 7.5% to 21.5%.

When the only factor is the volatility of the cross-sectional distribution of household consumption growth, the intercept is both statistically and economically insignificant, as

[20] See Lewellen, Nagel, and Shanken (2010).

expected. The slope coefficient is positive in Panel B, against expectation; and zero in Panel C, against expectation. The cross-sectional $\overline{R^2}$ varies from -2.0% to 42.8%, suggesting that the results are unstable and possibly spurious.

With the three FF risk factors, the estimated intercept is economically large; it is also statistically significant in Panels B and C. All slope coefficients are economically insignificant. The cross-sectional $\overline{R^2}$ varies from 28.3% to 53.6%.

Overall we conclude that household consumption risk does well in explaining the cross-section of excess returns: the intercept is economically and statistically insignificant, the slope coefficient is consistently positive, as expected, and the cross-sectional adjusted $\overline{R^2}$ is consistently positive.

7. Concluding Remarks

We explore the cross-sectional variation of household income shocks as a channel that drives the time series properties of the risk free rate, market return, and market price-dividend ratio and the cross-section of excess returns. We focus on this channel by suppressing potential predictability of the aggregate consumption and dividend growth rates and modeling them as *i.i.d.* processes. The model is parsimonious with only one state variable that is counter-cyclical and drives the moments of the cross-sectional distribution of household consumption growth. Despite this enforced parsimony, the model fits reasonably well both the unconditional and conditional price moments, particularly the moments of the market price-dividend ratio, a target that has eluded a number of other models. More to the point, the model-generated moments of the cross-sectional distribution of household consumption match well their sample counterparts.

We explore the robustness of model by considering two variants of the baseline model. We re-estimate the model over the entire quarterly sample period, 1947:Q1-2009:Q4, over which aggregate consumption and asset price data is available and obtain similar results. We also re-examine the performance of the model at the annual frequency over the entire available sample period 1929-2009 and conclude that the results are insensitive to the assumed decision frequency of the households.

We also explore the key role played by three ingredients of the model. First, we investigate the role of recursive preferences by considering a variant of the baseline model where the households have CRRA preferences and conclude that recursive preferences are pivotal in addressing the equity premium and risk free rate puzzles. Second, we investigate the role of negative skewness in the shocks to household consumption growth by considering a variant of the baseline model where the shocks to household consumption growth are conditionally lognormal and conclude that negative skewness is pivotal in explaining the data. Third, we explore the significance of shocks to household income that are unrelated to the business cycle, in addition to the shocks that are related to the business cycle, thereby highlighting the significance of shocks to household income that are unrelated to the business cycle.

Finally, we address the cross-sectional variation in the excess returns of size- and book-to-market-sorted equity portfolios. We show that the single factor, the household consumption risk, explains the cross-section of excess returns at least as well as the combined three Fama-French factors do.

Appendix A: Proof that the identity $I_t = C_t - D_t$ is respected

Since the households are symmetric and their number is normalized to equal one, we apply the law of large numbers as in Green (1989) and claim that $I_t = E\left[I_{i,t} \mid C_t, D_t\right]$. Furthermore, since the household shocks are assumed to be conditionally normally distributed and independent of anything else in the economy, we obtain the following:

$$
\begin{aligned}
I_t &= E\left[I_{i,t} \mid C_t, D_t\right] \\
&= E\left[\exp\left(\sum_{s=1}^{t}\left\{j_{i,s}^{1/2}\sigma\eta_{i,s} - j_{i,s}\sigma^2/2 + \hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\eta}_{i,s} - \hat{j}_{i,s}\hat{\sigma}^2/2\right\}\right)\middle| C_t - D_t\right] \\
&= E\left[E\left[\exp\left(\left\{\sum_{s=1}^{t} j_{i,s}^{1/2}\sigma\eta_{i,s} - j_{i,s}\sigma^2/2 + \hat{j}_{i,s}^{1/2}\hat{\sigma}\hat{\eta}_{i,s} - \hat{j}_{i,s}\hat{\sigma}^2/2\right\}\right) \middle| \left\{j_{i,\tau}, \hat{j}_{i,\tau}\right\}_{\tau=1,\ldots,t}\right]\middle| C_t - D_t\right] \\
&= C_t - D_t, \qquad \text{(A.1)}
\end{aligned}
$$

proving the claim.

Appendix B: Proof that autarchy is an equilibrium

We conjecture and verify that autarchy is an equilibrium. The proof follows several steps. First, we calculate the i^{th} household's private valuation of its wealth portfolio. Next we calculate the log return, $r_{i,c,t+1}$, on the i^{th} household's wealth portfolio and substitute this return in the household's SDF, as stated in equation (3). We integrate out of this SDF the household's idiosyncratic income shocks and show that households have common SDF. This implies that the private valuation of any security with given payoffs independent of the idiosyncratic income shocks is the same across households, thereby verifying the conjecture that autarchy is an equilibrium.

Let $P_{i,c,t}$ be the price of the i^{th} household's private valuation of its wealth portfolio, $Z_{i,c,t} \equiv P_{i,c,t} / C_{i,t}$, and $z_{i,c,t} \equiv \log\left(Z_{i,c,t}\right)$. We prove by induction that the price-to-consumption ratio is a function of only the state variable ω_t. We conjecture that $z_{i,c,t+1} = z_{c,t+1}\left(\omega_{t+1}\right)$. The Euler equation for $r_{i,c,t+1}$ is

$$E\left[e^{\theta\log\delta-\frac{\theta}{\psi}\Delta c_{i,t+1}+(\theta-1)r_{i,c,t+1}+r_{i,c,t+1}} \mid \Delta c_t,\omega_t,j_{i,t},\eta_{i,t},\widehat{j}_{i,t},\widehat{\eta}_{i,t}\right]=1 \tag{B.1}$$

We write

$$\begin{aligned} r_{i,c,t+1} &= \log\left(P_{i,c,t+1}+C_{i,t+1}\right)-\log P_{i,c,t} \\ &= \log\left(Z_{i,c,t+1}+1\right)-\log\left(Z_{i,c,t}\right)+\log C_{i,t+1}-\log C_{i,t} \\ &= \log\left(e^{z_{c,t+1}}+1\right)-z_{i,c,t}+\Delta c_{i,t+1} \end{aligned} \tag{B.2}$$

and substitute (B.2) in the Euler equation (B.1):

$$E\left[e^{\theta\log\delta-\frac{\theta}{\psi}\Delta c_{i,t+1}+\theta\left(\log\left(e^{z_{i,c,t+1}}+1\right)-z_{i,c,t}+\Delta c_{i,t+1}\right)} \mid \Delta c_t,\omega_t,j_{i,t},\eta_{i,t},\widehat{j}_{i,t},\widehat{\eta}_{i,t}\right]=1$$

or

$$e^{\theta z_{i,c,t}} = E\left[e^{\theta \log\delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) + \theta\log\left(e^{z_{c,t+1}}+1\right)} \mid \Delta c_t, \omega_t, j_{i,t}, \eta_{i,t}, \hat{j}_{i,t}, \hat{\eta}_{i,t} \right] \tag{B.3}$$

We integrate out of equation (B.3) the random variables $\varepsilon_{t+1}, \eta_{i,t+1}, j_{i,t+1}, \hat{\eta}_{i,t+1}$, and $\hat{j}_{i,t+1}$, leaving $z_{i,c,t}$ as a function of only ω_t, thereby proving the claim that $z_{i,c,t} = z_{c,t}\left(\omega_t\right)$.

Now, the $\left(SDF\right)_{i,t+1}$ of the i^{th} household is

$$\begin{aligned}\left(SDF\right)_{i,t+1} &= \exp\left(\theta\log\delta - \frac{\theta}{\psi}\Delta c_{i,t+1} + \left(\theta-1\right) r_{i,c,t+1}\right)\\ &= \exp\left(\theta\log\delta - \gamma\left(\Delta c_{t+1} + j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right) + \left(\theta-1\right)\left(\log\left(e^{z_{c,t+1}}+1\right) - z_{c,t}\right)\right)\end{aligned} \tag{B.4}$$

In pricing any security, other than the households' wealth portfolios, we integrate out of $\left(SDF\right)_{i,t+1}$ the household-specific random variables $\eta_{i,t+1}, j_{i,t+1}, \hat{\eta}_{i,t+1}$, and $\hat{j}_{i,t+1}$ and obtain a SDF common across households. Therefore, each household's private valuation of any security, *other than the households' wealth portfolios*, is common. This completes the proof that no-trade is an equilibrium.

Appendix C: Derivation of the cross-sectional moments of consumption growth

We use the following result:

$$e^{-\omega}\sum_{n=0}^{\infty} e^{kn}\omega^n / n! = e^{-\omega}\sum_{n=0}^{\infty}\left(e^k\omega\right)^n / n! = e^{-\omega}e^{e^k\omega} \tag{C.1}$$

Differentiating once, twice, and thrice with respect to *k* and setting $k = 0$ we obtain

$$\begin{aligned} e^{-\omega}\sum_{n=0}^{\infty} n\omega^n / n! &= \omega \\ e^{-\omega}\sum_{n=0}^{\infty} n^2\omega^n / n! &= \omega^2 + \omega \\ e^{-\omega}\sum_{n=0}^{\infty} n^3\omega^n / n! &= \omega^3 + 3\omega^2 + \omega \end{aligned} \tag{C.2}$$

We calculate the mean of the cross-sectional distribution of relative household consumption growth as follows:

$$\begin{aligned} \mu_1 &= E\left[\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) \mid \omega_{t+1}\right] \\ &= E\left[E\left[\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right) \mid j_{i,t+1}, \hat{j}_{i,t+1}\right] \mid \omega_{t+1}\right] \\ &= E\left[E\left[j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2 \mid j_{i,t+1}, \hat{j}_{i,t+1}\right] \mid \omega_{t+1}\right] \\ &= E\left[-j_{i,t+1}\sigma^2/2 - \hat{j}_{i,t+1}\hat{\sigma}^2/2 \mid \omega_{t+1}\right] \\ &= -\left(\sigma^2/2\right)\omega_{t+1} - \left(\hat{\sigma}^2/2\right)\hat{\omega} \end{aligned} \tag{C.3}$$

We calculate the variance as follows:

$$
\begin{aligned}
\mu_2 &= \operatorname{var}\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\,\omega_{t+1}\right) \\
&= \operatorname{var}\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)|\,\omega_{t+1}\right] \\
&= \operatorname{var}\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2\right)|\,\omega_{t+1}\right] + \operatorname{var}\left[\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)|\,\omega_{t+1}\right] \\
&= E\left[E\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2\right)^2 |\, j_{i,t+1}\right]|\,\omega_{t+1}\right] - \left(\sigma^2\omega_{t+1}/2\right)^2 \\
&+ E\left[E\left[\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)^2 |\, \hat{j}_{i,t+1}\right]\right] - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \qquad \text{(C.4)} \\
&= E\left[\left(j_{i,t+1}\sigma^2 + j_{i,t+1}^2\sigma^4/4\right)|\,\omega_{t+1}\right] - \left(\sigma^2\omega_{t+1}/2\right)^2 + E\left[\hat{j}_{i,t+1}\hat{\sigma}^2 + \hat{j}_{i,t+1}^2\hat{\sigma}^4/4\right] - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \\
&= \sigma^2\omega_{t+1} + \left(\sigma^4/4\right)\omega_{t+1}\left(1+\omega_{t+1}\right) - \left(\sigma^2\omega_{t+1}/2\right)^2 + \hat{\sigma}^2\hat{\omega} + \left(\hat{\sigma}^4/4\right)\hat{\omega}\left(1+\hat{\omega}\right) - \left(\hat{\sigma}^2\hat{\omega}/2\right)^2 \\
&= \left(\sigma^2 + \sigma^4/4\right)\omega_{t+1} + \left(\hat{\sigma}^2 + \hat{\sigma}^4/4\right)\hat{\omega}
\end{aligned}
$$

We calculate the third central moment as follows:

$$
\begin{aligned}
&\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)|\,\omega_{t+1}\right) \\
&= \mu_3\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)|\,\omega_{t+1}\right] \\
&= \mu_3\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1} - j_{i,t+1}\sigma^2/2\right)|\,\omega_{t+1}\right] + \mu_3\left[\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1} - \hat{j}_{i,t+1}\hat{\sigma}^2/2\right)|\,\omega_{t+1}\right]
\end{aligned}
$$

Now,

$$
\begin{aligned}
&\mu_3\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1}-j_{i,t+1}\sigma^2/2\right)\mid\omega_{t+1}\right]\\
&=E\left[E\left[\left(j_{i,t+1}^{1/2}\sigma\eta_{i,t+1}+\left(\omega_{t+1}-j_{i,t+1}\right)\sigma^2/2\right)^3\mid j_{i,t+1}\right]\mid\omega_{t+1}\right]\\
&=\sigma^3E\left[E\left[\left(j_{i,t+1}^{1/2}\eta_{i,t+1}+\left(\omega_{t+1}-j_{i,t+1}\right)\sigma/2\right)^3\mid j_{i,t+1}\right]\mid\omega_{t+1}\right]\\
&=\sigma^3E\left[E\left[3j_{i,t+1}\left(\omega_{t+1}-j_{i,t+1}\right)\sigma/2+\left(\omega_{t+1}-j_{i,t+1}\right)^3\sigma^3/8\mid j_{i,t+1}\right]\mid\omega_{t+1}\right]\\
&=\left(\sigma^4/2\right)E\left[E\left[3j_{i,t+1}\omega_{t+1}-3j_{i,t+1}^2+\left(\omega_{t+1}^3-3\omega_{t+1}^2j_{i,t+1}+3\omega_{t+1}j_{i,t+1}^2-j_{i,t+1}^3\right)\sigma^2/4\mid j_{i,t+1}\right]\mid\omega_{t+1}\right]\\
&=\left(\sigma^4/2\right)\left\{3\omega_{t+1}^2-3\left(\omega_{t+1}^2+\omega_{t+1}\right)+\left(\omega_{t+1}^3-3\omega_{t+1}^3+3\omega_{t+1}\left(\omega_{t+1}^2+\omega_{t+1}\right)-\left(\omega_{t+1}^3+3\omega_{t+1}^2+\omega_{t+1}\right)\right)\sigma^2/4\right\}\\
&=-\left(3\sigma^4/2+\sigma^6/8\right)\omega_{t+1}
\end{aligned}
$$

Likewise, we show that $\mu_3\left(\hat{j}_{i,t+1}^{1/2}\hat{\sigma}\hat{\eta}_{i,t+1}-\hat{j}_{i,t+1}\hat{\sigma}^2/2\right)=-\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega}$. Therefore

$$
\mu_3\left(\log\left(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}\right)\mid\omega_{t+1}\right)=-\left(3\sigma^4/2+\sigma^6/8\right)\omega_{t+1}-\left(3\hat{\sigma}^4/2+\hat{\sigma}^6/8\right)\hat{\omega} \qquad \text{(C.5)}
$$

Appendix D: Derivation of the common SDF, risk free rate, market price-dividend ratio, and expected market return

Solution for a Household's Consumption-Wealth Ratio

In Appendix B we proved that any household's consumption-wealth ratio is a function of only the state variable, that is, $z_{i,c,t} = z_{c,t}(\omega_t)$. We conjecture and verify that $z_{c,t} = A_0 + A_1 x_t$. We plug $z_{c,t} = A_0 + A_1 x_t$ in the Euler equation (B.3). We also log-linearize the term $\log\left(e^{z_{c,t+1}} + 1\right)$ as in Campbell and Shiller (1988) and obtain $\log\left(e^{z_{c,t+1}} + 1\right) \approx h_0 + h_1 z_{c,t+1}$, where $h_0 \equiv \log\left(e^{\bar{z}_c} + 1\right) - \dfrac{\bar{z}_c e^{\bar{z}_c}}{e^{\bar{z}_c} + 1}$, and $h_1 \equiv \dfrac{e^{\bar{z}_c}}{e^{\bar{z}_c} + 1}$:

$$e^{\theta z_{i,c,t}} = E\left[e^{\theta \log\delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2} \sigma \eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\eta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2/2\right) + \theta \log\left(e^{z_{c,t+1}} + 1\right)} \mid \Delta c_t, \omega_t, j_{i,t}, \eta_{i,t}, \hat{j}_{i,t}, \hat{\eta}_{i,t} \right]$$

or

$$e^{\theta\left(A_0 + A_1 x_t\right)} = E\left[e^{\theta \log\delta + (1-\gamma)\left(\mu + \sigma_a \varepsilon_{t+1} + j_{i,t+1}^{1/2} \sigma \eta_{i,t+1} - j_{i,t+1}\sigma^2/2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\eta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2/2\right) + \theta\left\{h_0 + h_1\left(A_0 + A_1 x_{t+1}\right)\right\}} \mid \Delta c_t, \omega_t, j_{i,t}, \eta_{i,t}, \hat{j}_{i,t}, \hat{\eta}_{i,t} \right]$$

or

$$E\left[e^{\theta \log\delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + \gamma(\gamma-1)\left(j_{i,t+1} + \hat{j}_{i,t+1}\right) + \theta\left\{h_0 + h_1\left(A_0 + A_1 x_{t+1}\right) - A_0 - A_1 x_t\right\}} \mid \omega_t, j_{i,t}, \hat{j}_{i,t} \right] = 1$$

or

$$E\left[e^{\theta \log\delta + (1-\gamma)\mu + (1-\gamma)^2 \sigma_a^2/2 + x_{t+1} + \left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2} - 1\right)\hat{\omega} + \theta\left\{h_0 + h_1\left(A_0 + A_1 x_{t+1}\right) - A_0 - A_1 x_t\right\}} \mid \omega_t \right] = 1$$

since $e^{-\omega} \sum_{n=0}^{\infty} e^{kn} \omega^n / n! = e^{-\omega} \sum_{n=0}^{\infty} \left(e^k \omega\right)^n / n! = e^{-\omega} e^{e^k \omega}$ and $\left(e^{(\gamma-1)\gamma\sigma^2/2} - 1\right)\omega_t \equiv x_t$. Therefore,

$$e^{\theta\log\delta+(1-\gamma)\mu+(1-\gamma)^2\sigma_a^2/2+\left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2}-1\right)\hat{\omega}+\theta\left(h_0+h_1A_0-A_0-A_1x_t\right)}E\left[e^{(1+\theta h_1A_1)x_{t+1}}\mid\omega_t\right]=1$$

or, up to a second-order approximation,

$$e^{\theta\log\delta+(1-\gamma)\mu+(1-\gamma)^2\sigma_a^2/2+\left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2}-1\right)\hat{\omega}+\theta\left(h_0+h_1A_0-A_0-A_1x_t\right)+\left(1+\theta h_1A_1\right)\left(\nu\xi+\rho x_t\right)+\left(1+\theta h_1A_1\right)^2\left(\nu\xi^2+2\rho\xi x_t\right)/2}\approx 1 \tag{D.1}$$

Matching the constant, we obtain:

$$\theta\log\delta+(1-\gamma)\mu+(1-\gamma)^2\sigma_a^2/2+\left(e^{\gamma(\gamma-1)\hat{\sigma}^2/2}-1\right)\hat{\omega}+\theta\left(h_0+h_1A_0-A_0\right)+\left(1+\theta h_1A_1\right)\nu\xi+\frac{1}{2}\left(1+\theta h_1A_1\right)^2\nu\xi^2=0 \tag{D.2}$$

and matching the coefficient of x_t, we obtain:

$$-A_1\theta+\left(1+\theta h_1A_1\right)\rho+\left(1+\theta h_1A_1\right)^2\rho\xi=0 \tag{D.3}$$

The solution of equations (D.2) and (D.3) produces values for the parameters A_0 and A_1 that verify the conjecture that $z_{c,t}=A_0+A_1x_t$.[21] Since $\overline{z_c}=A_0+A_1\overline{x}$, $h_0\equiv\log\left(e^{\overline{z_c}}+1\right)-\frac{\overline{z_c}e^{\overline{z_c}}}{e^{\overline{z_c}}+1}$, and $h_1\equiv\frac{e^{\overline{z_c}}}{e^{\overline{z_c}}+1}$, the parameters h_0 and h_1 are determined in terms of the parameters A_0, A_1, and $\overline{x}$.

Common SDF across Households

[21] Note that equation (D.3) implies that A_1 is the solution of a quadratic. We verified, via simulations, that the economically meaningful root is the smaller of the two.

In pricing any security, other than the households' wealth portfolios, we integrate out of the *SDF* in equation (B.4) the household-specific random variables $\theta_{i,t+1}, j_{i,t+1}, \hat{\theta}_{i,t+1}$, and $\hat{j}_{i,t+1}$ and obtain a SDF common across households:

$$\begin{aligned}(SDF)_{t+1} &= E\left[e^{\theta \log \delta - \gamma\left(\Delta c_{t+1} + j_{i,t+1}^{1/2} \sigma \eta_{i,t+1} - j_{i,t+1} \sigma^2/2 + \hat{j}_{i,t+1}^{1/2} \hat{\sigma} \hat{\eta}_{i,t+1} - \hat{j}_{i,t+1} \hat{\sigma}^2/2\right) + (\theta-1)\left(h_0 + h_1 z_{c,t+1} - z_{c,t}\right)} \mid c_t, c_{t+1}, \omega_t, \omega_{t+1} \right] \\ &= e^{\theta \log \delta - \gamma \Delta c_{t+1} + \omega_{t+1}\left(e^{\gamma(\gamma+1)\sigma^2/2} - 1\right) + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + (\theta-1)\left(h_0 + h_1 z_{c,t+1} - z_{c,t}\right)} \\ &= e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma \Delta c_{t+1} + (\theta-1)\left\{h_0 + h_1 A_0 - \left(A_0 + A_1 x_t\right)\right\} + \lambda x_{t+1}}\end{aligned} \tag{D.4}$$

where

$$\lambda \equiv \frac{e^{\gamma(\gamma+1)\sigma^2/2} - 1}{e^{\gamma(\gamma-1)\sigma^2/2} - 1} + (\theta - 1) h_1 A_1 \tag{D.5}$$

Solution for the Risk Free Rate

The Euler equation for the log risk free rate is

$$E\left[e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) - \gamma \Delta c_{t+1} + (\theta-1)\left\{h_0 + h_1 A_0 - \left(A_0 + A_1 x_t\right)\right\} + \lambda x_{t+1} + r_t} \mid \omega_t \right] = 1$$

or, up to a second-order approximation,

$$e^{\theta \log \delta + \hat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2} - 1\right) + (\theta-1)\left\{h_0 + h_1 A_0 - \left(A_0 + A_1 x_t\right)\right\} - \gamma \mu + \gamma^2 \sigma_a^2/2 + \lambda\left(\nu \xi + \rho x_t\right) + \lambda^2\left(\nu \xi^2 + 2\rho \xi x_t\right)/2 + r_t} \approx 1$$

or

$$r_t = -\theta \log \delta - \widehat{\omega}\left(e^{\gamma(\gamma+1)\widehat{\sigma}^2/2} - 1\right) - (\theta - 1)\left(h_0 + h_1 A_0 - A_0\right) + \gamma\mu - \gamma^2\sigma_a^2/2 - \lambda\nu\xi - \lambda^2\nu\xi^2/2$$
$$-\left\{\lambda\rho + \lambda^2\rho\xi - (\theta - 1)A_1\right\}x_t \tag{D.6}$$

The implications of the model regarding the term structure of interest rates are the same as those of a discretized version of the Cox, Ingersoll, and Ross (1985) model. Recall that x_t follows a heteroscedastic AR (1) process with conditional variance $\nu\xi^2 + 2\rho\xi x_t$. We prove that, under the risk-neutral probability measure Q, x_t follows a heteroscedastic AR (1) process, where the mean of x_{t+1}, conditional on x_t, is shifted by approximately $\lambda\left[\nu\xi^2 + 2\rho\xi x_t\right]$ and the variance is affine in x_t. To see this, note that $e^{r_t}\left(SDF\right)_{t+1}$ is the discrete-time Radon-Nikodym derivative. Under the risk-neutral probability measure Q, the mean of x_{t+1}, conditional on x_t, is $E^Q\left[x_{t+1} \mid x_t\right] = E\left[x_{t+1}e^{r_t}\left(SDF\right)_{t+1} \mid x_t\right] \approx E\left[x_{t+1} \mid x_t\right] + \lambda\left[\nu\xi^2 + 2\rho\xi x_t\right]$ and its variance is affine in x_t. Since the risk free rate is affine in the household risk x_t, the risk free rate also follows a heteroscedastic AR (1) process with variance of the innovation affine in the risk free rate under the risk-neutral probability measure. Then the model is isomorphic to a discretized version of the Cox *et al.* (1985) model and implies that the yield curve is upward sloping, downward sloping, or humped, depending on the state, where the state may be represented by the risk free rate.

Solution for the Price-Dividend Ratio of the Stock Market

We denote the log stock market return as $r_{m,t}$ and the stock market price-dividend ratio as $z_{m,t}$. As in Campbell-Shiller (1988), we write

$$r_{m,t+1} = k_0 + k_1 z_{m,t+1} - z_{m,t} + \Delta d_{t+1} \tag{D.7}$$

where $k_0 \equiv \log\left(e^{\overline{z}_m} + 1\right) - \dfrac{\overline{z}_m e^{\overline{z}_m}}{e^{\overline{z}_m} + 1}$ and $k_1 = \dfrac{e^{\overline{z}_m}}{e^{\overline{z}_m} + 1}$. We conjecture and verify that the price-dividend ratio of the stock market is

$$z_{m,t} = B_0 + B_1 x_t \tag{D.8}$$

and write

$$r_{m,t+1} = k_0 + k_1\left(B_0 + B_1 x_{t+1}\right) - \left(B_0 + B_1 x_t\right) + \mu_d + \sigma_d \varepsilon_{d.t+1}$$

The Euler equation is

$$E\left[e^{\theta\log\delta + \widehat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2}-1\right)+(\theta-1)\left\{h_0+h_1A_0-\left(A_0+A_1x_t\right)\right\}-\gamma\Delta c_{t+1}+\lambda x_{t+1}+r_{m,t+1}} \mid \omega_t\right] = 1$$

or

$$E\left[e^{\theta\log\delta + \widehat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2}-1\right)+(\theta-1)\left\{h_0+h_1A_0-\left(A_0+A_1x_t\right)\right\}-\gamma\Delta c_{t+1}+\lambda x_{t+1}+k_0+k_1\left(B_0+B_1x_{t+1}\right)-\left(B_0+B_1x_t\right)+\mu_d+\sigma_d\varepsilon_{d.t+1}} \mid \omega_t\right] = 1$$

or

$$e^{\theta\log\delta + \widehat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2}-1\right)+(\theta-1)\left\{h_0+h_1A_0-\left(A_0+A_1x_t\right)\right\}-\gamma\mu+\gamma^2\sigma_a^2/2+k_0+k_1B_0-\left(B_0+B_1x_t\right)+\mu_d+\sigma_d^2/2} E\left[e^{\left(\lambda+k_1B_1\right)x_{t+1}} \mid \omega_t\right] = 1$$

or, up to a second-order approximation,

$$\begin{aligned}
&\theta\log\delta + \widehat{\omega}\left(e^{\gamma(\gamma+1)\hat{\sigma}^2/2}-1\right)+(\theta-1)\left\{h_0+h_1A_0-\left(A_0+A_1x_t\right)\right\}-\gamma\mu+\gamma^2\sigma_a^2/2+k_0+k_1B_0 \\
&-\left(B_0+B_1x_t\right)+\mu_d+\sigma_d^2/2+\left(\lambda+k_1B_1\right)\left\{\nu\xi+\rho x_t\right\}+\left(\lambda+k_1B_1\right)^2\left\{\nu\xi^2+2\rho\xi x_t\right\}/2 \\
&\approx 0
\end{aligned}$$

We set the constant and coefficient of x_t equal to zero and obtain two equations that determine the parameters B_0 and B_1:

$$
\begin{aligned}
&\theta \log \delta + \widehat{\omega}\left(e^{\gamma(\gamma+1)\widehat{\sigma}^2/2} - 1\right) + (\theta - 1)\left(h_0 + h_1 A_0 - A_0\right) - \gamma\mu \\
&+\gamma^2 \sigma_a^2 / 2 + k_0 + k_1 B_0 - B_0 + \mu_d + \sigma_d^2 / 2 + \left(\lambda + k_1 B_1\right)\nu\xi + \frac{1}{2}\left(\lambda + \kappa_1 B_1\right)^2 \nu\xi^2 \\
&= 0
\end{aligned}
\tag{D.9}
$$

and

$$
-(\theta - 1) A_1 - B_1 + \left(\lambda + k_1 B_1\right)\rho + \left(\lambda + k_1 B_1\right)^2 \rho\xi = 0 \text{ [22]} \tag{D.10}
$$

Note that the parameters k_0 and k_1 are determined in terms of the parameters B_0, B_1, and $\bar{x}$. Therefore, the expected stock market return is

$$
\begin{aligned}
&E\left[r_{m,t+1} \mid \omega_t\right] = k_0 + k_1 B_0 + k_1 B_1 \left\{\nu\xi + \rho x_t\right\} - \left(B_0 + B_1 x_t\right) + \mu_d \\
&= k_0 + k_1 B_0 + k_1 B_1 \nu\xi - B_0 + \mu_d + \left\{k_1 B_1 \rho - B_1\right\} x_t .
\end{aligned}
\tag{D.11}
$$

[22] Note that equation (D.10) implies that B_1 is the solution of a quadratic equation. We verified, via simulations, that the economically meaningful root is the smaller of the two.

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Table 1: Summary Statistics of Household Consumption Growth, Quarterly Data 1982:Q1-2009:Q4

	μ_1	$\mu_2^{1/2}$	μ_3	$Q_{.05}$	$AC1\left(\mu_2^{1/2}\right)$	$AC1(\mu_3)$	$AC1\left(Q_{.05}\right)$
			Data-Implied Moments				
Jan. tranche	.009 (.006)	.379 (.015)	-.025 (.008)	-.595	.772	.124	.534
All tranches	.005 (.003)	.383 (.015)	-.024 (.006)	-.616	.897	.180	.792
			Model-Implied Moments				
	-.018	.201	-.024	-.086	.877	.877	.860
			Correlation of Data-Implied (All Tranches) and Model-Implied Moments				
	-.269	.490	.377				

The table reports the point estimates of the mean (μ_1), standard deviation ($\mu_2^{1/2}$), third central moment (μ_3), and the 5th percentile of the cross-sectional distribution of quarterly household consumption growth, along with their first-order autocorrelations. Standard errors are in parentheses. The January tranche is the sample of households with first-quarter consumption in January through March; ‘All tranches’ refers to the combined January, February, and March tranches, where the February tranche is the sample of households with first-quarter consumption in February through April and the March tranche is the sample of households with first-quarter consumption in March through May. $AC1$ denotes first-order autocorrelation. For the January tranche, the minimum, maximum, and mean number of households in a quarter is 19, 1310, and 674, respectively. For all tranches, the minimum, maximum, and mean number of households is 64, 3906, and 2056, respectively. The time series of the model-implied first, second, and third central moments are obtained by inverting the expressions for the equilibrium price-dividend ratio and risk free rate to obtain the time series of the state variable and, thereby, computing the time series of the cross-sectional moments as affine functions of the state variable (equations 4-6). The time series of the model-implied percentiles are obtained via simulations. Specifically, the time series of the state variable is obtained as above by inverting the expressions for the equilibrium price-dividend ratio and risk free rate. Using this aggregate time series, we simulate the consumption growth of 10,000 households of the same length as that of the state variable and obtain the time series of the 5th percentile from the cross-sectional distribution of household consumption growth.

Table 2: Correlation of Household Consumption Growth Statistics with Recessions, Quarterly Data 1982:Q1-2009:Q4

	$corr(\mu_1, I_{rec})$	$corr(\mu_2^{1/2}, I_{rec})$	$corr(\mu_3, I_{rec})$	$corr(Q_{.05}, I_{rec})$
Correlation of Data-Implied Statistics with Recessions				
January tranche	-.029	.139	-.249	-.142
All tranches	-.075	.118	-.217	-.087
Correlation of Model-Implied Statistics with Recessions				
	-.268	.264	-.268	-.234

The January tranche is the sample of households with first-quarter consumption in January through March; 'All tranches' refers to the combined January, February, and March tranches, where the February tranche is the sample of households with first-quarter consumption in February through April and the March tranche is the sample of households with first-quarter consumption in March through May. I_{rec} is an indicator variable that takes the value of one if there is a NBER-designated recession in at least two of the three months of the quarter. The time series of the model-implied first, second, and third central moments are obtained by inverting the expressions for the equilibrium price-dividend ratio and risk free rate to obtain the time series of the state variable and, thereby, computing the time series of the cross-sectional moments as affine functions of the state variable (equations 4-6). The time series of the model-implied percentiles are obtained via simulations. Specifically, the time series of the state variable is obtained as above by inverting the expressions for the equilibrium price-dividend ratio and risk free rate. Using this aggregate time series, we simulate the consumption growth of 10,000 households of the same length as that of the state variable and obtain the time series of the 5th percentile from the cross-sectional distribution of household consumption growth.

Table 3: Model Fit and Parameter Estimates, Quarterly Data 1982:Q1-2009:Q4, January Tranche:

	Prices								
	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$AC1(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.005	.005	.899	.019	.084	.037	3.759	.414	.986
	(.001)	(.001)	(.227)	(.009)	(.010)	(.104)	(.066)	(.046)	(.220)
Model	.006	.012	.877	.023	.231	-.058	3.787	.466	.877

	Consumption and Dividends						
	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$\mu_1(\Delta c_{CEX})$	$\mu_2^{1/2}(\Delta c_{CEX})$	$\mu_3(\Delta c_{CEX})$
Data	.005	.004	.005	.104	.009	.379	-.025
	(.0006)	(.0006)	(.006)	(.020)	(.006)	(.016)	(.008)
Model	.007	.004	.001	.033	-.018	.201	-.024

Estimates of Preference Parameters		
γ	ψ	δ
1.34	1.00	.929
(.041)	(.261)	(.027)

Other Parameter Estimates									
μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d	$\hat{\omega}$	$\hat{\sigma}$
.007	.004	.045	.001	.877	.556	.001	.033	.081	.660
(.001)	(.001)	(.121)	(.002)	(.116)	(.005)	(.009)	(.564)	(.073)	(.144)

The table reports parameter estimates and model fit for the baseline model over the quarterly sample period 1982:Q1-2009:Q4. The GMM system consists of 16 moment restrictions (13 aggregate moments and the first 3 central moments of the cross-sectional distribution of household consumption growth) in 13 parameters. $E[r_f], \sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E[r_m], \sigma(r_m)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order auto-correlation of the market return; and $E[p/d], \sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio; $\mu_1(\Delta c_{CEX})$, $\mu_2^{1/2}(\Delta c_{CEX})$, and $\mu_3(\Delta c_{CEX})$ denote the mean, volatility, and third central moment, respectively, of the cross-sectional distribution of household consumption growth. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters governing the dynamics of the state variable, ν, ξ, and ρ; the parameters of the household income shocks, $\sigma, \hat{\sigma}$, and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. The *J*-stat is 4.84 and the model is not rejected at the 10% level of significance. The simulated 90%, 95%, and 99% critical values of the *J*-stat are 7.59, 10.17, and 16.49, respectively. Note that, because we use a pre-specified weighting matrix, the J-stat has a non-standard asymptotic distribution and its critical values are computed via simulation.

Table 4: Model Fit and Parameter Estimates, Quarterly Data 1947:Q1-2009:Q4

Prices

	$E\left[r_f\right]$	$\sigma\left(r_f\right)$	$AC1\left(r_f\right)$	$E\left[r_m\right]$	$\sigma\left(r_m\right)$	$AC1\left(r_m\right)$	$E\left[p/d\right]$	$\sigma\left(p/d\right)$	$AC1\left(p/d\right)$
Data	.003	.006	.916	.017	.084	.090	3.470	.423	.977
	(.001)	(.001)	(.232)	(.006)	(.005)	(.066)	(.046)	(.029)	(.138)
Model	.004	.008	.966	.030	.141	-.002	3.509	.416	.966

Consumption and Dividends

	$E\left[\Delta c\right]$	$\sigma\left(\Delta c\right)$	$E\left[\Delta d\right]$	$\sigma\left(\Delta d\right)$
Data	.005	.004	.005	.138
	(.0004)	(.0003)	(.005)	(.013)
Model	.005	.006	.001	.091

Estimates of Preference Parameter

γ	ψ	δ
5.34	1.47	.986
(1.68)	(4.14)	(.053)

Other Parameter Estimates

μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d	$\hat{\omega}$	$\hat{\sigma}$
.005	.006	.789	.0003	.966	.006	.001	.091	.210	.078
(.0004)	(.0003)	(6.91)	(.0006)	(.139)	(.419)	(.006)	(.128)	(.075)	(.438)

The table reports parameter estimates and model fit for the baseline model over the quarterly sample period 1947:Q1-2009:Q4. The GMM system consists of 13 moment restrictions (only aggregate moments excluding the first 3 central moments of the cross-sectional distribution of household consumption growth) in 13 parameters. $E\left[r_f\right], \sigma\left(r_f\right)$, and $AC1\left(r_f\right)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E\left[r_m\right], \sigma\left(r_m\right)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order autocorrelation of the market return; and $E\left[p/d\right], \sigma\left(p/d\right)$, and $AC1\left(p/d\right)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters of the dynamics of the state variable, ν, ξ, and ρ; the parameters of the household income shocks, $\sigma, \hat{\sigma}$, and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth.

Table 5: Model Fit and Parameter Estimates, Annual Data 1929-2009

	Prices								
	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$AC1(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.006	.030	.672	.062	.198	-.070	3.377	.450	.877
	(.005)	(.005)	(.216)	(.019)	(.017)	(.087)	(.080)	(.051)	(.231)
Model	.005	.030	.806	.055	.260	-.082	3.338	.395	.806

	Consumption and Dividends			
	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$
Data	.020	.021	.010	.117
	(.003)	(.004)	(.013)	(.020)
Model	.019	.018	.020	.093

Estimates of Preference Parameters		
γ	ψ	δ
1.70	1.00	.966
(1201.9)	(1687.8)	(620.9)

Other Parameter Estimates									
μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d	$\hat{\omega}$	$\hat{\sigma}$
.019	.018	.169	.003	.806	.331	.020	.093	.382	.219
(.003)	(.005)	(.335)	(39.5)	(.164)	(640.9)	(.042)	(.095)	(443.5)	(1750.9)

The table reports parameter estimates and model fit for the baseline model over the annual sample period 1929-2009. The GMM system consists of 13 moment restrictions (only aggregate moments excluding the first 3 central moments of the cross-sectional distribution of household consumption growth) in 13 parameters. $E[r_f], \sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E[r_m], \sigma(r_m)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order autocorrelation of the market return; and $E[p/d], \sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters governing the dynamics of the state variable, ν, ξ, and ρ; the parameters of the household income shocks, $\sigma, \hat{\sigma}$, and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth.

Table 6: CRRA Model Fit and Parameter Estimates, Quarterly Data 1982:Q1-2009:Q4, January Tranche

	Prices								
	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$AC1(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.005	.005	.899	.019	.084	.037	3.759	.414	.986
	(.001)	(.001)	(.227)	(.009)	(.010)	(.104)	(.066)	(.046)	(.220)
Model	.019	.012	.911	.019	.021	-.001	4.021	.008	.911

	Consumption and Dividends						
	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$\mu_1(\Delta c_{CEX})$	$\mu_2^{1/2}(\Delta c_{CEX})$	$\mu_3(\Delta c_{CEX})$
Data	.005	.004	.005	.104	.010	.382	-.026
	(.0006)	(.0006)	(.006)	(.020)	(.006)	(.016)	(.008)
Model	.001	.007	.001	.020	-.005	.103	−.0006

Estimates of Preference Parameters	
γ	δ
2.61	.933
(1.03)	(.160)

Other Parameter Estimates									
μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d	$\hat{\omega}$	$\hat{\sigma}$
.001	.007	.00001	.119	.911	.063	.001	.020	.257	.201
(.001)	(.0005)	(.006)	(25.2)	(.132)	(.492)	(.009)	(.542)	(3.40)	(1.47)

The table reports parameter estimates and model fit for the variant of the baseline model with CRRA preferences over the quarterly sample period 1982:Q1-2009:Q4. The GMM system consists of 16 moment restrictions (13 aggregate moments and the first 3 central moments of the cross-sectional distribution of household consumption growth) in 12 parameters. $E[r_f], \sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E[r_m], \sigma(r_m)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order auto-correlation of the market return; and $E[p/d], \sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio; $\mu_1(\Delta c_{CEX})$, $\mu_2^{1/2}(\Delta c_{CEX})$, and $\mu_3(\Delta c_{CEX})$ denote the mean, volatility, and third central moment, respectively, of the cross-sectional distribution of household consumption growth. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters governing the dynamics of the state variable, ν, ξ, and ρ; the parameters of the household income shocks, $\sigma, \hat{\sigma}$, and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. The *J*-stat is 16.41 and the model is rejected at the 1% level of significance. The simulated 90%, 95%, and 99% critical values of the *J*-stat are 6.46, 9.05, and 15.44, respectively. Note that, because we use a pre-specified weighting matrix, the J-stat has a non-standard asymptotic distribution and its critical values are computed via simulation.

Table 7: CRRA Model Fit and Parameter Estimates, Annual Data 1929-2009

Prices

	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$AC1(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.006	.030	.672	.062	.198	-.070	3.377	.450	.877
	(.005)	(.005)	(.216)	(.019)	(.017)	(.087)	(.080)	(.051)	(.231)
Model	.039	.033	.792	.038	.057	-.005	3.395	.020	.792

Consumption and Dividends

	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$
Data	.020	.021	.010	.117
	(.003)	(.004)	(.013)	(.020)
Model	.015	.012	.005	.055

Estimates of Preference Parameters

γ	δ
2.99	.924
(.518)	(.003)

Other Parameter Estimates

μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d	$\hat{\omega}$	$\hat{\sigma}$
.015	.012	.0001	.245	.792	.217	.005	.055	.019	.532
(.003)	(.007)	(.012)	(9.38)	(.171)	(1.60)	(.020)	(.272)	(.013)	(.002)

The table reports parameter estimates and model fit for the variant of the baseline model with CRRA preferences over the annual sample period 1929-2009. The GMM system consists of 13 moment restrictions (only aggregate moments excluding the first 3 central moments of the cross-sectional distribution of household consumption growth) in 12 parameters. $E[r_f], \sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E[r_m], \sigma(r_m)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order autocorrelation of the market return; and $E[p/d], \sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters governing the dynamics of the state variable, ν, ξ, and ρ; the parameters of the household income shocks, $\sigma, \hat{\sigma}$, and $\hat{\omega}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. The *J*-stat is 21.58 and the model is rejected at the 5% level of significance. The simulated 90%, 95%, and 99% critical values of the *J*-stat are 10.07, 14.30, and 24.73, respectively. Note that, because we use a pre-specified weighting matrix, the J-stat has a non-standard asymptotic distribution and its critical values are computed via simulation.

Table 8: Lognormal Model Fit and Parameter Estimates, Quarterly Data 1982:Q1-2009:Q4, January Tranche

	Prices								
	$E\left[r_f\right]$	$\sigma\left(r_f\right)$	$AC1\left(r_f\right)$	$E\left[r_m\right]$	$\sigma\left(r_m\right)$	$AC1\left(r_m\right)$	$E\left[p/d\right]$	$\sigma\left(p/d\right)$	$AC1\left(p/d\right)$
Data	.005	.005	.899	.019	.084	.037	3.759	.414	.986
	(.001)	(.001)	(.227)	(.009)	(.010)	(.104)	(.066)	(.046)	(.220)
Model	.015	.007	.906	.023	.192	-.044	3.792	.443	.906

	Consumption and Dividends						
	$E\left[\Delta c\right]$	$\sigma\left(\Delta c\right)$	$E\left[\Delta d\right]$	$\sigma\left(\Delta d\right)$	$\mu_1\left(\Delta c_{CEX}\right)$	$\mu_2^{1/2}\left(\Delta c_{CEX}\right)$	$\mu_3\left(\Delta c_{CEX}\right)$
Data	.005	.004	.005	.104	.009	.379	-.025
	(.0006)	(.0006)	(.006)	(.020)	(.006)	(.016)	(.008)
Model	.005	.003	.001	.027	-.033	.257	0.0

Estimates of Preference Parameters		
γ	ψ	δ
1.30	1.00	.908
(.049)	(.519)	(.018)

Other Parameter Estimates							
μ	σ_a	ν_σ	ξ_σ	ρ_σ	μ_d	σ_d	$\hat{\sigma}$
.005	.003	.020	.003	.906	.001	.027	.256
(.001)	(.001)	(.097)	(.035)	(.116)	(.009)	(.674)	(.038)

The table reports parameter estimates and model fit for the variant of the baseline model with lognormal shocks over the quarterly sample period 1982:Q1-2009:Q4. The GMM system consists of 15 moment restrictions (13 aggregate moments and the first 2 central moments of the cross-sectional distribution of household consumption growth) in 11 parameters. $E\left[r_f\right], \sigma\left(r_f\right)$, and $AC1\left(r_f\right)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E\left[r_m\right], \sigma\left(r_m\right)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order autocorrelation of the market return; and $E\left[p/d\right], \sigma\left(p/d\right)$, and $AC1\left(p/d\right)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio; $\mu_1\left(\Delta c_{CEX}\right)$, $\mu_2^{1/2}\left(\Delta c_{CEX}\right)$, and $\mu_3\left(\Delta c_{CEX}\right)$ denote the mean, volatility, and third central moment, respectively, of the cross-sectional distribution of household consumption growth. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters of the dynamics of the state variable, ν_σ, ξ_σ, and ρ_σ; the parameter of the household income shocks, $\hat{\sigma}$; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. The *J*-stat is 4.46 and the model is not rejected at the 10% level of significance. The simulated 90%, 95%, and 99% critical values of the *J*-stat are 7.54, 10.11, and 16.37, respectively. Note that, because we use a pre-specified weighting matrix, the J-stat has a non-standard asymptotic distribution and its critical values are computed via simulation.

Table 9: Model Fit and Parameter Estimates, Quarterly Data 1982:Q1-2009:Q4, January Tranche: Only Business Cycle Shock

Prices

	$E[r_f]$	$\sigma(r_f)$	$AC1(r_f)$	$E[r_m]$	$\sigma(r_m)$	$AC1(r_m)$	$E[p/d]$	$\sigma(p/d)$	$AC1(p/d)$
Data	.005	.005	.899	.019	.084	.037	3.759	.414	.986
	(.001)	(.001)	(.227)	(.009)	(.010)	(.104)	(.066)	(.046)	(.220)
Model	-.012	.030	.983	.028	.123	.003	3.727	.396	.983

Consumption and Dividends

	$E[\Delta c]$	$\sigma(\Delta c)$	$E[\Delta d]$	$\sigma(\Delta d)$	$\mu_1(\Delta c_{CEX})$	$\mu_2^{1/2}(\Delta c_{CEX})$	$\mu_3(\Delta c_{CEX})$
Data	.005	.004	.005	.104	.009	.379	-.025
	(.001)	(.001)	(.006)	(.020)	(.006)	(.016)	(.008)
Model	.008	.003	.005	.100	-.007	.133	-.023

Estimates of Preference Parameters

γ	ψ	δ
1.08	1.00	.990
(.543)	(.019)	(.025)

Other Parameter Estimates

μ	σ_a	ν	ξ	ρ	σ	μ_d	σ_d
.008	.003	.979	.00001	.983	.990	.005	.100
(.003)	(1.99)	(1.35)	(.00006)	(.119)	(.611)	(.009)	(.099)

The table reports parameter estimates and model fit for the baseline model over the quarterly sample period 1982:Q1-2009:Q4. The GMM system consists of 16 moment restrictions (13 aggregate moments and the first 3 central moments of the cross-sectional distribution of household consumption growth) in 11 parameters. $E[r_f]$, $\sigma(r_f)$, and $AC1(r_f)$ are the mean, standard deviation, and first-order autocorrelation of the risk free rate; $E[r_m]$, $\sigma(r_m)$, and $AC1(r_m)$ are the mean, standard deviation, and first-order auto-correlation of the market return; and $E[p/d]$, $\sigma(p/d)$, and $AC1(p/d)$ are the mean, standard deviation, and first-order autocorrelation of the price-dividend ratio; $\mu_1(\Delta c_{CEX})$, $\mu_2^{1/2}(\Delta c_{CEX})$, and $\mu_3(\Delta c_{CEX})$ denote the mean, volatility, and third central moment, respectively, of the cross-sectional distribution of household consumption growth. Δc is aggregate consumption growth and Δd is dividend growth. The preference parameters are the RRA coefficient, γ, the elasticity of intertemporal substitution, ψ, and the subjective discount factor, δ. The other parameters are: the mean, μ, and volatility, σ_a, of aggregate consumption growth; the parameters governing the dynamics of the state variable, ν, ξ, and ρ; the parameter of the household income shocks, σ; and the mean, μ_d, and volatility, σ_d, of aggregate dividend growth. The *J*-stat is 8.29 and the model is not rejected at the 5% level of significance. The simulated 90%, 95%, and 99% critical values of the *J*-stat are 8.24, 10.92, and 17.46, respectively. Note that, because we use a pre-specified weighting matrix, the J-stat has a non-standard asymptotic distribution and its critical values are computed via simulation.

Table 10: “Rolling” Fama-MacBeth Regressions, Quarterly Data 1982:Q1-2009:Q4, January Tranche

$\overline{R}^2$	$\hat{\alpha}$	$\hat{\lambda}_{skew}$	$\hat{\lambda}_{std}$	$\hat{\lambda}_{MKT}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$
Panel A: 25 FF portfolios						
13.6%	.01	.73				
	(.01)	(.61)				
40.0%	.01		-.09			
	(.01)		(.07)			
37.4%	.01	.01	-.10			
	(.01)	(.70)	(.07)			
59.5%	.04			-.03	.01	.01
	(.02)			(.03)	(.01)	(.01)
Panel B: 30 Industry portfolios						
14.0%	.01	.34				
	(.01)	(.38)				
-6.9%	.01		.00			
	(.01)		(.07)			
10.4%	.02	.74	.09			
	(.01)	(.38)	(.07)			
-22.8%	.02			-.01	-.01	.005
	(.02)			(.02)	(.01)	(.01)
Panel C: 25 FF and 30 Industry portfolios						
14.9%	.01	.37				
	(.01)	(.37)				
5.0%	.02		-.03			
	(.01)		(.06)			
9.4%	.02	.54	.01			
	(.01)	(.39)	(.06)			
-7.5%	.03			-.01	.004	.01
	(.02)			(.02)	(.01)	(.01)

The table reports Fama-Macbeth cross-sectional regression results using as test assets the 25 Fama-French portfolios (Panel A), 30 industry-sorted portfolios (Panel B), and the combined set of the 25 Fama-French and 30 industry-sorted portfolios (Panel C). The data are quarterly over 1982:Q1-2009:Q4. The adjusted R^2, $\overline{R^2}$, are reported. The standard errors of $\hat{a}$ and $\hat{\lambda}$ are calculated from the time series of the cross-sectional intercepts and slope coefficients. The factor loadings are estimated at each period *t*, starting with the midpoint of the sample, using all the returns up to period *t*.

Table 11: "Fixed" Fama-MacBeth Regressions, Quarterly Data 1982:Q1-2009:Q4, January Tranche

$\overline{R}^2$	$\hat{\alpha}$	$\hat{\lambda}_{skew}$	$\hat{\lambda}_{std}$	$\hat{\lambda}_{MKT}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$
Panel A: 25 FF portfolios						
21.5%	.01	.92				
	(.01)	(.65)				
42.8%	.01		-.11			
	(.01)		(.06)			
41.6%	.01	.16	-.11			
	(.01)	(.90)	(.06)			
53.6%	.04			-.03	.01	.01
	(.03)			(.03)	(.01)	(.01)
Panel B: 30 Industry portfolios						
7.5%	.01	.33				
	(.01)	(.39)				
7.9%	.02		.06			
	(.01)		(.06)			
39.0%	.02	.57	.10			
	(.01)	(.42)	(.06)			
28.3%	.07			-.05	.002	.001
	(.03)			(.03)	(.01)	(.01)
Panel C: 25 FF and 30 Industry portfolios						
9.8%	.01	.40				
	(.01)	(.37)				
-2.0%	.02		.00			
	(.01)		(.05)			
13.2%	.02	.53	.02			
	(.01)	(.41)	(.05)			
30.2%	.07			-.06	.004	.01
	(.03)			(.02)	(.01)	(.01)

The table reports Fama-Macbeth cross-sectional regression results using as test assets the 25 Fama-French portfolios (Panel A), 30 industry-sorted portfolios (Panel B), and the combined set of the 25 Fama-French and 30 industry-sorted portfolios (Panel C). The data are quarterly over 1982:Q1-2009:Q4. The adjusted R^2, $\overline{R^2}$, are reported. The standard errors of $\hat{a}$ and $\hat{\lambda}$ are calculated from the time series of the cross-sectional intercepts and slope coefficients. The factor loadings are estimated on the first half of the sample.

Figure 1: Time Series of the Cross-Sectional Skewness, Quarterly Data 1982:Q1-2009:Q4.