

Unspanned Macroeconomic Risks in Oil Futures

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Abstract

This paper constructs a macro-finance model for commodity futures. Model estimates suggest a feedback relationship between crude oil prices and U.S. real activity. Moreover, the channel from real activity to oil prices is unspanned – meaning not identified in current futures prices – consistent with oil futures as a hedge asset against supply shocks. Relative to a benchmark spanned-risk model, incorporating unspanned real activity raises the volatility of the estimated oil risk premium tenfold and raises the value of real options by 35 to 400%.

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1 Introduction

Commodity futures are claims on direct inputs into production and consumption and are among the most active markets in the world.¹ Affine pricing models accurately fit the dynamics of many commodity futures markets. Although they do not explicitly consider the interaction of commodity prices with the macroeconomy, these models impose strong assumptions on those interactions. This paper empirically tests those assumptions, finds that they are rejected by the data, and proposes an alternative.

I construct an affine model with both pricing and macroeconomic factors. The approach is tractable and includes benchmark futures pricing models as special cases. It can be applied to any commodity and any set of economic data. I concentrate on oil because it is the single most important commodity to the U.S. and world economy as reflected in its trading volume, media coverage and academic and industry attention.

I find that real economic activity is an important unspanned risk factor in oil futures. Benchmark models assume that all relevant risks are spanned by futures on that commodity, which imposes strong restrictions on the joint behavior of macroeconomic variables and futures prices. In particular, it implies that conditional on current futures prices no other information can forecast futures prices or returns. I find that this restriction is strongly rejected by the data. Real activity forecasts oil prices and futures returns, over and above the information in the current futures curve. Equivalently, the spot risk premium in oil futures has an unspanned procyclical component. This pattern aligns with recent evidence (e.g. [Ludvigson and Ng \(2009\)](#); [Duffee \(2011\)](#); [Joslin, Priebsch and Singleton \(2014\)](#)) of unspanned countercyclical risk premia in Treasury bonds, but in the opposite direction. I

¹In October 2014 the average trading volume across the two benchmark crude oil futures, WTI and Brent, was \$120 billion per day compared to \$129 billion per day across all NYSE and NASDAQ stocks.

argue that the pattern is consistent with oil futures as a hedge asset against oil supply shocks.

Estimates of spanned-risk models like those employed by [Casassus and Collin-Dufresne \(2005\)](#) and [Hamilton and Wu \(2014\)](#) imply that the spot risk premium in oil is slow moving and does not covary with the business cycle. This is a consequence of the spanning assumption plus the fact that the correlation of oil prices with real activity is low. Adding unspanned real activity produces an estimated oil risk premium that is ten times as volatile and covaries strongly with the business cycle, which suggests that spanned-risk models may miss the majority of variation in the oil risk premium.

By construction unspanned macro risks cannot affect the prices of financial options or other derivatives, but they can affect the valuation and exercise of real options. In a calibrated example I find that adding real activity as an unspanned macro factor increases the value of a hypothetical oil well over the nested spanned-risk model by 35% to 400% depending on the well's current cost of extraction. There are two channels by which unspanned macro risks raise real option values: their dynamics with futures prices and their risk premiums. In the example the dynamics effect dominates, while the effect of the risk premium of real activity on real option value is much smaller.

The approach lets us use the full panel of futures price data in a simple and consistent way and delivers other insights that are not available from a VAR using macroeconomic factors and the spot price of oil. In particular the estimates suggest a dynamic feedback relationship between oil prices and real activity. A high oil price forecasts lower real activity consistent with the evidence from VARs that oil shocks forecast recessions. I find that this relationship is conditional on the market's forecast of how long the oil shock will last: the more persistent the market forecasts a given spot price hocks to be, the stronger and longer-lasting is its effect on real activity. Conversely, a high level of real activity forecasts higher oil prices. I

find that although shocks to real activity dissipate in less than a year, the market forecasts that their effect on oil prices will persist for decades, perhaps because oil is a nonrenewable resource.

1.1 Related Literature

There are two strands of the literature in commodity futures that this paper builds upon. In the first, commodity futures prices are modeled as affine functions of latent state variables. Classic examples are [Gibson and Schwartz \(1990\)](#), [Schwartz \(1997\)](#), and [Casassus and Collin-Dufresne \(2005\)](#). More recent examples include [Casassus, Liu and Tang \(2013\)](#) and [Hamilton and Wu \(2014\)](#). Studies of this type do not incorporate explicit macroeconomic data. More subtly, they implicitly assume that all relevant information in the economy is spanned by futures prices and no other information can contribute incremental forecasting power. I find that real economic activity has material effects on risk premiums and forecasts of oil prices that are unspanned in the oil futures curve.

The second strand uses VARs to explore the time series relations of oil prices with the real economy; examples include [Hamilton \(1983\)](#); [Hamilton \(2003\)](#); [Kilian \(2009\)](#); [Alquist and Kilian \(2010\)](#); [Kilian and Vega \(2011\)](#). These studies include a single state variable based on the spot price of oil or a short-dated futures price. A limitation of this approach is that it does not incorporate the full panel of futures prices of different maturities. The model in this paper imposes the additional assumption that risk premiums are “essentially affine” in the state variables which lets us bring the full futures curve to bear on returns, price forecasts, and the spanning of macroeconomic risks.

[Fama and French \(1987\)](#), [Bessembinder and Chan \(1992\)](#), [Singleton \(2013\)](#) and [Hamilton and Wu \(Forthcoming\)](#) run return forecasting regressions for individual futures returns;

Szymanowska et al. (2013) decompose individual futures returns into a spot premium and a term premium. The model in this paper contributes to this literature as well, as it offers a simple and internally consistent way to make use of the full panel of futures price data. The approach also allows us to distinguish between forecastability that is spanned versus unspanned by commodity derivatives markets.

Chiang et al. (forthcoming) extract spanned factors from oil futures and a volatility factor from oil options, and find that exposure to these factors is priced in equity markets. They establish that “...oil is an important determinant of cross-sectional asset prices.” By contrast, I examine the interaction of macroeconomic data with risk premia and price forecasts in futures markets. I document that 1) the business cycle is an important yet unspanned determinant of oil risk premia and price forecasts and 2) the market’s forecast of the oil price is an important determinant of real activity.

2 Data and Forecasting Regressions

In this section I describe the data, which consist of futures prices and time series data, and investigate to what extent the macroeconomic time series are spanned by futures prices. The distinction between spanned and unspanned risks drives the modelling strategy. I conclude that, first, two linear factors suffice to summarize the oil futures curve, and second, the macro factors (in particular real activity) contain relevant information that is unspanned by the oil futures curve.

2.1 Futures Price Data

I use closing prices for West Texas Intermediate (WTI) oil futures with maturities of one to twelve months, on the last business day of each month from January 1986 to July 2013. The futures price data is denoted

$$f_t^j = \log(F_t^j), \quad j = 1 \dots J, \quad t = 1 \dots T$$

$$f_t = \left[f_t^1 \quad f_t^2 \quad \dots \quad f_t^J \right]'$$

where F_t^j is the closing price at end of month t of the future that expires in month $t + j$, $t = 1$ corresponds to 1/1986, $T = 331$ corresponds to 7/2013, and $J = 12$. The maximum futures maturity of twelve months is because longer dated futures were seldom traded in the early years of the sample. The results do not change significantly if I extend J to, e.g., 24 months maturity.

2.2 Macro Factors

I use the Chicago Fed National Activity Index, hereafter labelled *GRO*, as the first macro factor. The index is released toward the end of each month and is a weighted combination of 74 U.S. economic indicators, similar in spirit to the real economic activity indexes of [Stock and Watson \(1999\)](#) and [Ludvigson and Ng \(2009\)](#). The index is intended as a forward-looking indicator of U.S. economic activity and is used in macro-finance models of bond yields (cf. [Joslin, Priebsch and Singleton \(2014\)](#)). From January 2001 onward I use the real-time values of the index; results using the revised values are very similar. I use the Conference Board's Leading Economic Index (LEI) as an alternative index of real activity and obtain similar

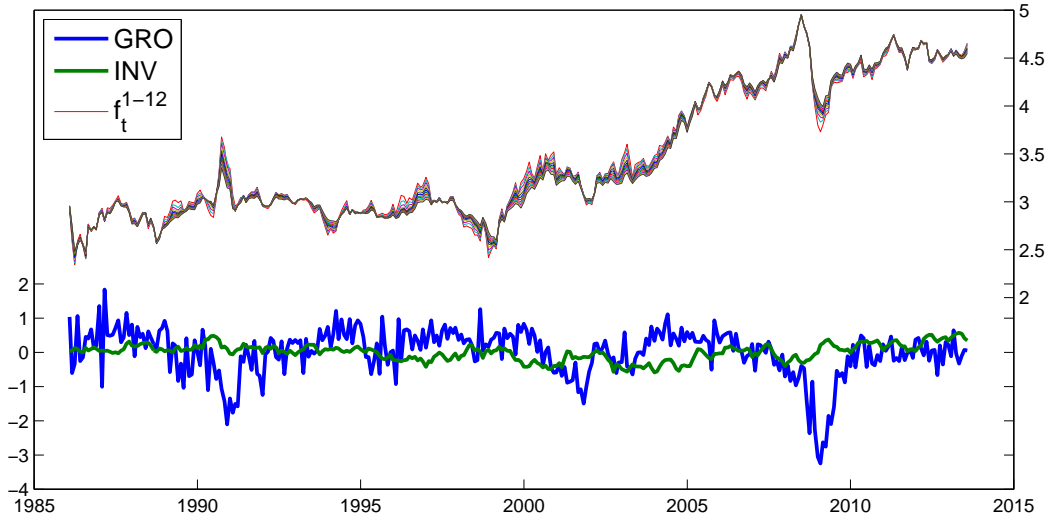


Figure 1: The figure plots the time series of log futures prices for Nymex crude oil f_t^{1-12} , the Chicago Fed National Activity Index GRO , and the log of the EIA's monthly U.S. oil inventory INV .

results.

The second macro factor is the inventory, or quantity in readily available storage. I use the log of the Energy Information Administration (EIA)'s "Total Stocks of Commercial Crude Oil excluding the Strategic Petroleum Reserve" as a measure of the readily available U.S. inventory of crude oil, hereafter labelled INV . The EIA's storage report is released weekly, and I use the most recent data as of the last business day of each month. The macro factors are thus $M_t = [GRO_t, INV_t]'$. Figure 1 plots the time series of log oil futures prices and the macro factors GRO and INV .

2.3 Evidence for Unspanned Macro Risks

Previous futures pricing models assume complete spanning of the state variables and are estimated using financial data only. As [Duffee \(2011\)](#) and [Joslin, Le and Singleton \(2013\)](#) observe in the context of bond yields, this assumption has strong implications for the joint behavior of futures prices and the economy. First, it implies that the state vector can be rotated so that the state variables equal the prices of arbitrary linearly independent portfolios of futures contracts. Second, it implies that those portfolios explain log futures prices up to idiosyncratic pricing errors. Third, it implies that all relevant information is fully summarized by those portfolios' prices and no other information can contribute incremental forecasting power.

I first document that the first two principal components, level and slope, account for more than 99% of the variation in levels and changes of log oil futures prices. There are more than two sources of aggregate uncertainty in the world, so the natural hypothesis is that some relevant economic state variables may be unspanned by oil futures.

A) Oil futures prices display a low dimensional factor structure

Figure 2 plots the loadings of the first three principal components (PCs) of log oil futures prices. The figure also displays the fraction of the variance that is accounted for by the PCs. The first two PCs – level and slope – account for 99.9% of the variation in log price levels and 99.2% of variation in log price changes.

Second, I find that the macro factors M_t , in particular GRO , are not well summarized by futures prices.

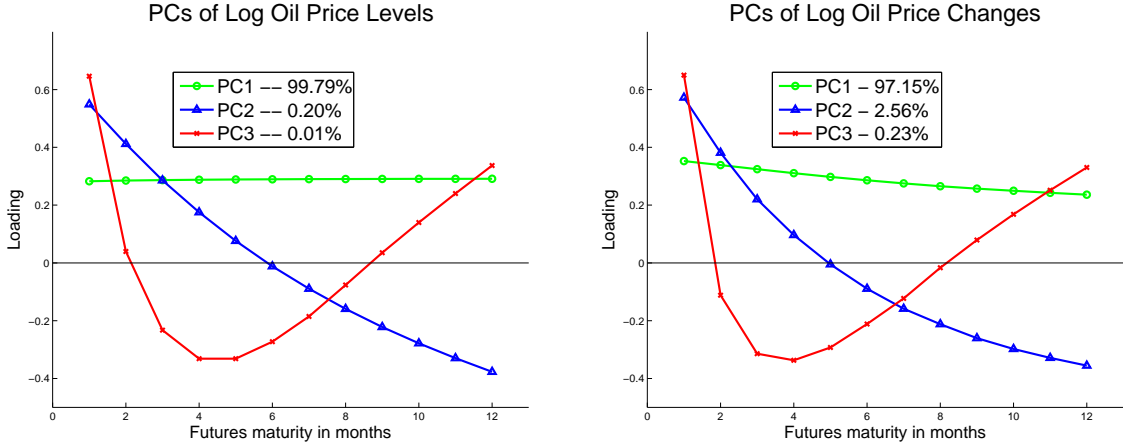


Figure 2: Loadings of the first three principal components of the levels (panel A) and changes (panel B) of log oil futures prices, monthly from 1/1986 - 7/2013. The legend displays the fraction of total variance explained by each of the principal components.

B) M_t is mostly unspanned by oil futures

I project M_t on the time series of the first two principal components of log oil futures prices, and label the residual UM_t :

$$M_t = \alpha + \gamma_{1,2}PC_t^{1,2} + UM_t$$

The R^2 of the projections for $[GRO, INV]$ are $[6.4\%, 27.5\%]$. Projected on the first five PCs the R^2 are $[14.5\%, 30.0\%]$; projected on the log prices of all 12 futures maturities the R^2 are $[18.9\%, 30.9\%]$. Thus, much of the variation in M_t is unspanned by variation in oil futures prices. The AR(1) coefficients of the projection residuals are 62% for GRO and 89% for INV , so much of the unspanned variation in M_t is persistent.

However, M_t might be measured with error or some subcomponent of M_t may be irrelevant to the oil market. The main test of whether M_t is unspanned is not the projection R^2 but whether M_t is economically meaningful over and above the information in futures

prices. I find that M_t contributes incremental forecasting power for oil prices and returns.

C) M_t forecasts returns over and above information in the futures curve

Table 1 Panel A shows the results of forecasting log returns to oil futures contracts, using information from the current futures curve and then adding the macro variables M_t . The results show that M_t contributes additional forecasting power, over and above the information in the futures curve, for returns to the first nearby contract and average returns to all traded futures that month (Panel A) and for changes in the level factor in oil futures prices (Panel B). The adjusted R^2 s increase significantly in every case. The coefficients on f^{1-12} have different signs on adjacent maturities, clear evidence of overfitting, yet M_t still contributes substantial forecasting power.

The magnitudes are economically large. A one percent increase in GRO – about two standard deviations – forecasts a return to the second nearby oil future that is 2.6% higher over the next month. The results are not driven by the huge swing in 2008-2009: Appendix F Table 8 shows they are slightly stronger when estimated on a subsample that ends in 2007. Thus, real activity appears to be a first-order determinant of the oil price forecast.

Kilian and Vega (2011) generally find that macroeconomic news does not forecast changes in the spot price of crude oil at a monthly horizon and conclude that relevant information is more or less immediately reflected in the price. Kilian and Vega examine a variety of macroeconomic data that are mostly backward looking; they do not examine the Chicago Fed National Activity Index. For the most closely related forward-looking time series that they examine – the Conference Board’s Leading Economic Index – they find that it does in fact forecast oil price changes at a monthly horizon ($p < 0.01$) and I replicate that finding in unreported results.

Table 1: Panel A shows the results of forecasting returns to oil futures: r_{t+1}^2 is the log excess return to the second nearby oil futures contract. \bar{r}_{t+1} is the average log excess return to all active futures contracts with maturities up to 12 months. Panel B shows the results of forecasting changes in the level factor PC^1 in oil futures. The forecasting variables are 1) three sets of 'reduced-form' state variables \mathcal{P}_t based on oil futures prices and 2) the Chicago Fed National Activity Index GRO_t and log U.S. oil inventory INV_t . The data are monthly from from 1/1986 to 7/2013. Newey-West standard errors with six lags are in parentheses.

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	r_{t+1}^2			\bar{r}_{t+1}		
GRO_t	0.0263** (0.0110)	0.0247** (0.0114)	0.0220* (0.0117)	0.0186** (0.0091)	0.0187** (0.0091)	0.0167* (0.0093)
INV_t	0.033 (0.095)	0.028 (0.096)	0.025 (0.095)	0.018 (0.078)	0.008 (0.077)	0.004 (0.076)
Spanned Factors \mathcal{P}_t :	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
Adjusted $R^2(\mathcal{P}_t)$	0.4%	0.7%	4.5%	0.1%	0.0%	3.7%
Adj. $R^2(\mathcal{P}_t + UM_t)$	3.3%	3.0%	6.1%	2.2%	1.9%	5.1%
F -ratio	5.9***	4.8**	3.7**	4.5**	4.1**	3.2**

Panel B: Forecasting the Level Factor

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV}M_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	ΔPC^1 (Change in Level)		
GRO_t	0.067** (0.032)	0.068** (0.033)	0.061* (0.034)
INV_t	0.073 (0.264)	0.040 (0.263)	0.026 (0.265)
Spanned Factors \mathcal{P}_t :	$PC^{1,2}$	PC^{1-5}	f^{1-12}
Adjusted $R^2(\mathcal{P}_t)$	-0.5%	-0.6%	2.9%
Adj. $R^2(\mathcal{P}_t + UM_t)$	2.0%	1.7%	4.5%
F -ratio	5.1***	4.7**	3.8**

3 Model

Motivated by the facts in Section 2, in this section I develop a macro-finance model for commodity futures that admits unspanned macroeconomic risks. Let X_t denote a vector of N state variables that summarize the economy. X_t includes macroeconomic risk factors such as expected economic growth, and factors specific to the commodity such as hedging pressure, inventories, and expectations of supply and demand. The stochastic discount factor is given by

$$e^{\Lambda_t' \epsilon_{t+1}} = e^{(\Lambda_0 + \Lambda_1 X_t)' \epsilon_{t+1}} \quad (1)$$

The state vector follows a Gaussian VAR,

$$X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \quad (2)$$

where $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, 1_N)$.

Previous models such as [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Casassus and Collin-Dufresne \(2005\)](#) assume that X_t is spanned i.e. fully reflected in contemporaneous futures prices. As is well known for bond yields ([Duffie and Kan \(1996\)](#)), Appendix B shows that the spanning assumption implies that X_t can be replaced by an arbitrary set of linear combinations of log futures prices:

$$\mathcal{P}_t^N = W f_t$$

where W is any full rank real valued $N \times J$ matrix. This assumption has the following implications:

1. Futures prices are described up to idiosyncratic errors by the N factors \mathcal{P}_t^N .
2. The projection of X_t on \mathcal{P}_t^N has R^2 of one.

3. Conditional on \mathcal{P}_t^N , no other information forecasts X_t or futures prices or returns.

I instead assume that a subspace of X_t is spanned, while its complement is unspanned but observed by the econometrician. Suppose that contemporaneous futures prices are determined by a set of linear combinations $L_t = Vf_t$ where V is a real valued $N_L \times J$ matrix and $N_L < N$. That is, the spot price and its evolution under the risk neutral measure are:

$$s_t = \delta_0 + \delta_1' L_t \quad (3)$$

$$L_{t+1} = K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \quad (4)$$

where $\epsilon_{t+1}^{\mathbb{Q}} \sim N(0, 1_{N_L})$ and $\Sigma_L = V\Sigma_X$. The spanned components L_t may be observed or latent.

Finally, I assume that the unspanned components L_t^\perp of X_t are observed by the econometrician. There are $N_M = N - N_L$ of these factors. Label them $UM_t = L_t^\perp$ – the unspanned components of observed macroeconomic information – and rewrite

$$\begin{bmatrix} L_t \\ UM_t \end{bmatrix} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} \begin{bmatrix} L_t \\ UM_t \end{bmatrix} + \Sigma \epsilon_{t+1}^{\mathbb{P}}$$

By construction, the factors UM_t are not spanned by contemporaneous futures prices: this specification is in the class of macro-finance models explored by [Diebold, Rudebusch and Aruoba \(2006\)](#); [Duffee \(2011\)](#); [Joslin, Priebsch and Singleton \(2014\)](#) for bonds. By the same rationale as before, we can replace L_t with N_L linear combinations of log prices,

$$\mathcal{P}_t^L = W_L f_t$$

where W_L is any full rank $N_L \times J$ real valued matrix. In contrast to the spanned-risk formulation, this model has the implications:

1. Futures prices are described up to idiosyncratic errors by $N_L < N$ factors.
2. The projection of X_t on \mathcal{P}_t^L has R^2 less than one.
3. Conditional on \mathcal{P}_t^L , other information may forecast X_t or futures prices or returns.

Motivated by the variance decomposition in the previous section, I assume the number of spanned state variables $N_L = 2$. Appendix B describes the parametrization and estimation of the model. After estimating, I rotate and translate so that the state variables are the model implied spot price and cost of carry (s_t, c_t) and the macroeconomic series $M_t = SM_t + UM_t$.²

The model can then be described in just two equations:

- 1) the law of motion for the state variables:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \\ M_{t+1} \end{bmatrix} = \begin{bmatrix} K_{0sc}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{sc,sc}^{\mathbb{P}} & K_{sc,M}^{\mathbb{P}} \\ K_{M,sc}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} + \Sigma \epsilon_{t+1}^{\mathbb{P}} \quad (5)$$

- 2) the dynamics of (s_t, c_t) under the risk neutral measure:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \end{bmatrix} = K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \Sigma_{sc} \epsilon_{t+1}^{\mathbb{Q}} \quad (6)$$

where

- s_t is the spot price and c_t is the one-period cost of carry

²Appendix A.2 presents the definitions of s_t and c_t .

- M_t are the macro state variables
- $K_{0sc}^{\mathbb{P}}, K_{0M}^{\mathbb{P}}$ are 2×1 and $N_M \times 1$ real valued matrices
- $K_{sc,sc}^{\mathbb{P}}, K_{sc,M}^{\mathbb{P}}, K_{M,sc}^{\mathbb{P}}, K_{MM}^{\mathbb{P}}$ are 2×2 , $2 \times N_M$, $N_M \times 2$ and $N_M \times N_M$ real valued matrices
- $K_0^{\mathbb{Q}}, K_1^{\mathbb{Q}}$ are 2×1 and 2×2 real valued matrices
- Σ is $N_M + 2 \times N_M + 2$, lower triangular, and Σ_{sc} is the upper left 2×2 submatrix of Σ .

The model is a canonical form, that is, any affine model with two spanned state variables and $N_M \geq 0$ macroeconomic variables can be written in the form above. Extending the model to more than two spanned state variables is straightforward.

3.1 Constant Volatility

The Gaussian assumption in the model is a strong one, because the volatility of commodity futures markets varies over time ([Trolle and Schwartz \(2009\)](#)). Time varying volatility affects futures prices directly via the convexity term and could affect price forecasts or expected returns. If these effects are present, in general they will be reflected in the reduced-form (pricing) factors because the model identifies the spanned state variables directly from futures prices in an agnostic way. Thus, spanned effects of stochastic volatility are compatible with the model estimates and do not confound the findings.

Unspanned stochastic volatility does not appear to drive the association between real activity and oil futures. Appendix F Table 10 shows the results of forecasting regressions that include the real activity index *GRO* and indexes of crude oil futures volatility. All three volatility indexes are insignificant and more importantly they do not subsume the forecasting

power of GRO . This is also consistent with the finding of [Chiang et al. \(forthcoming\)](#) that exposure to crude oil volatility has a risk premium attached to it in equity returns but not in returns to oil futures or options.

4 Model Estimates

This section presents the estimates of the macro-finance model with two spanned factors and two macroeconomic factors: the monthly Chicago Fed National Activity Index (GRO) and log U.S. oil inventories (INV).

Figure 3 Panel A plots the spanned and unspanned components of GRO as well as the log spot price. We see that essentially all of the monthly and yearly variation in GRO appears in the unspanned component. Figure 3 Panel B plots the spanned and unspanned components of log oil inventories INV . Compared to GRO , much more of the monthly and yearly variation in INV is spanned by futures prices. The spanned component of inventory loads exclusively and strongly on the cost of carry c_t .

Table 2 presents the maximum likelihood estimate of the model using the monthly data from January 1986 to July 2013.

4.1 Model Fit

The risk neutral parameters $K_0^{\mathbb{Q}}$, $K_1^{\mathbb{Q}}$ are estimated relatively precisely for two reasons. First, they use the entire futures curve data; on each date, the set of futures prices represents a “snapshot” of risk-neutral expectations. Second, a linear factor model holds closely in the futures curve: our two affine factors fit the futures curve closely, so the pricing errors relative to the fitted model are small and idiosyncratic. By contrast, the historical measure

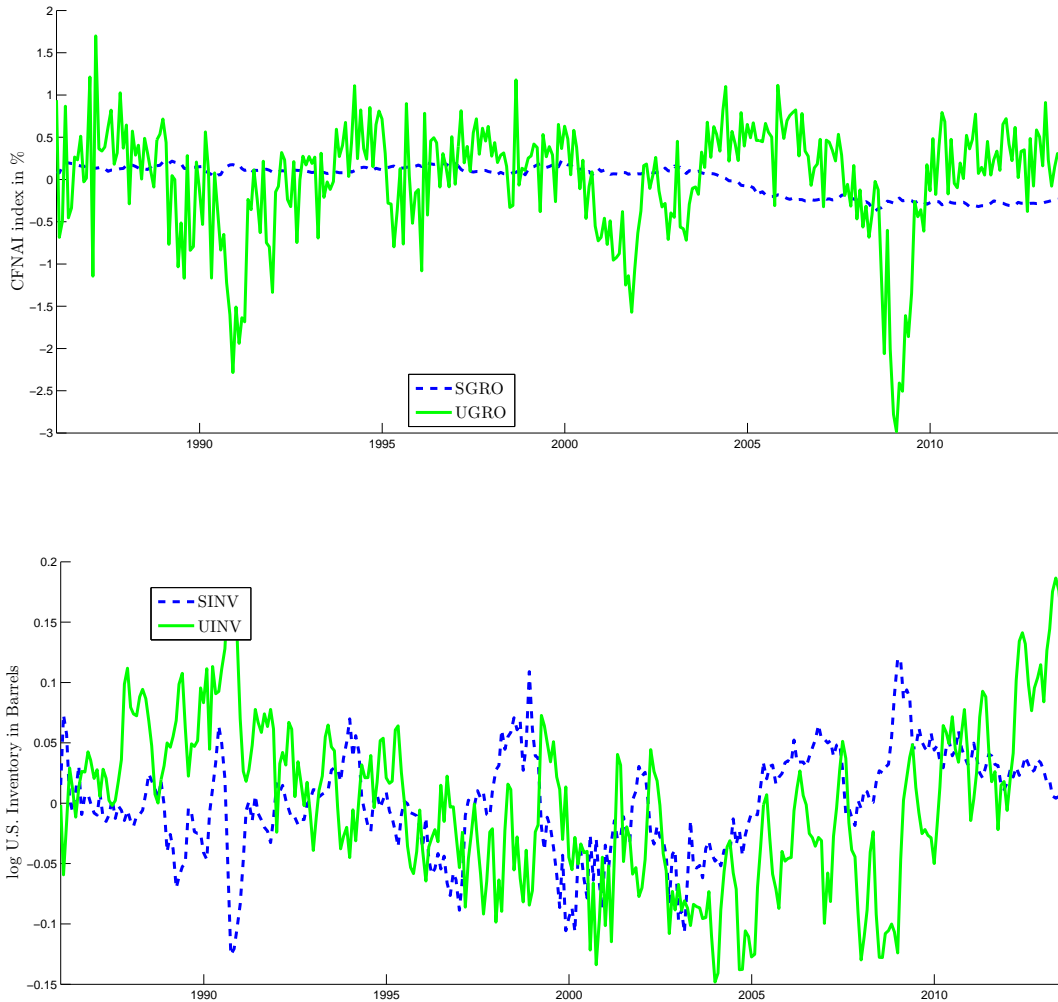


Figure 3: Panel A plots the components of the monthly Chicago Fed National Activity Index that are spanned ($SGRO$) and unspanned ($UGRO$) by oil futures prices. Panel B plots the components of monthly log U.S. oil inventories that are spanned ($SINV$) and unspanned ($UINV$) by oil futures prices.

Table 2: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures. s , c are the spot price and annualized cost of carry respectively. GRO is the monthly Chicago Fed National Activity Index. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meanned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

	Historical (\mathbb{P}) Measure				
	$K_0^{\mathbb{P}}$	s_t	c_t	GRO_t	INV_t
s_{t+1}	0.008 (0.006)	0.994 (0.009)	0.061 (0.033)	0.025 (0.008)	0.038 (0.082)
c_{t+1}	-0.007 (0.005)	0.016 (0.008)	0.874 (0.031)	-0.015 (0.008)	-0.045 (0.078)
GRO_{t+1}	0.001 (0.030)	-0.115 (0.046)	0.051 (0.171)	0.618 (0.043)	-0.194 (0.433)
INV_{t+1}	0.002 (0.002)	0.000 (0.002)	0.032 (0.009)	-0.002 (0.002)	0.888 (0.023)

	Risk Neutral (\mathbb{Q}) Measure		
	$K_0^{\mathbb{Q}}$	s_t	c_t
s_{t+1}	-0.004 (0.008)	1.000 (0.004)	0.083 (0.011)
c_{t+1}	0.001 (0.012)	-0.004 (0.010)	0.892 (0.029)

	Shock Volatilities			
	[off-diagonal = % correlations]			
	s	c	GRO	INV
s	0.103			
c	-81%	0.057		
GRO	5%	-2%	0.530	
INV	-22%	27%	4%	0.028

parameters $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}, \Sigma$ are estimated less precisely, from the time series of the state variables. Risk prices $\Lambda = K^{\mathbb{P}} - K^{\mathbb{Q}}$ inherit the lower precision of $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$.

The two spanned (pricing) factors do a good job of summarizing oil futures prices³. The model's fitted values explain 99.97% of variation in futures prices and the residuals (pricing errors) explain 0.03%. The root mean squared pricing error (RMSE) is 54 basis points corresponding to a typical pricing error of 54 cents on a futures contract worth \$100.00.

Another test of the model's validity is to examine the forecasting power of the pricing errors. The pricing errors – regardless of their true time-series or cross sectional structure – should be purely observation errors. If meaningful spanned information is omitted from the model then the pricing errors should forecast \mathcal{P}_t or M_t .⁴ When I regress $\Delta\mathcal{P}_t$ and ΔM_t on the lagged pricing errors, neither the individual coefficients nor the joint F -test reject the null at the 10% level. Thus, the pricing errors do not appear to be economically important.

4.2 Historical Dynamics

The models of [Schwartz \(1997\)](#) and [Schwartz and Smith \(2000\)](#) impose that s_t is unit-root. Without that restriction and using data from 1990 to 2003, [Casassus and Collin-Dufresne \(2005\)](#) estimate that s_t has a long run mean to which it reverts with a halflife of around two years, so the expected spot price of oil in ten years' time is effectively constant. Our unrestricted estimate which adds ten years of subsequent data is more consistent with [Schwartz \(1997\)](#). The AR(1) coefficient for s_t of 0.994, which is close to the largest eigenvalue

³This does not contradict the conclusions of [Schwartz \(1997\)](#) and [Casassus and Collin-Dufresne \(2005\)](#) that a three-factor model is necessary to summarize commodity futures prices. The three-factor models in those papers have two latent factors – spot price and convenience yield – and a spanned interest rate that is estimated separately. Interest rates are very slow moving compared to futures prices, so they contribute almost no explanatory power.

⁴I am grateful to Peter Bossaerts for pointing this out.

of $K_1^{\mathbb{P}}$, is very close to unity.⁵

The cost of carry reverts to a slightly negative mean with a half-life of five months. Shocks to the spot price and the cost of carry are strongly negatively correlated ($\rho = -81\%$), so a higher spot price is accompanied by a more downward sloping curve, but spot price shocks are essentially permanent while the cost of carry shock decays within a few years. As a result, about half of a typical move in the oil spot price disappears after two to three years, while the other half is expected to persist effectively forever.

4.2.1 Oil Futures and Real Activity

Shocks to real activity are almost uncorrelated with shocks to the spot price and the cost of carry. A one percent shock to real activity predicts a 2.5% higher spot oil price the next month but only a 1.5% lower cost of carry. Thus, the effects of real activity on oil prices are forecast by the market to be persistent – higher activity raises both the short run and the expected long run price of oil.

A higher spot price of oil predicts lower real activity. A higher cost of carry – higher expected prices in future – forecasts slightly higher real activity, but the effect is not significant, and c_t also forecasts a higher spot price. The impulse response functions in section 4.2.3 make clear that the net effect of c_t on *GRO* is negative. As a result, a shock to the spot price of oil that the market expects to persist has a more negative effect on growth than a shock that is expected to be transitory. These results are not driven by the big swings in 2008-2009: Appendix F shows the results are similar when I estimate on a subsample that ends in 2007.

⁵The estimates all use nominal futures prices. Inflation was relatively constant over the period from 1986 to 2013, relative to the movement in oil prices, so it does not drive the high AR(1) coefficient of s_t . Using futures prices deflated by the CPI or PPI does not materially change any of the results.

Taken together, there is a negative feedback relationship between the spot price of oil and growth. A positive growth shock forecasts persistent higher oil prices, while a positive oil price shock forecasts lower real activity, and the effect is stronger for oil price shocks that the market expects to persist.

4.2.2 Oil Futures and Inventories

Shocks to log inventories are negatively correlated with the spot price and positively correlated with the cost of carry. Both of these observations are consistent with the Theory of Storage – higher inventories signal that the market is moving up the supply-of-storage curve. The correlation between shocks to inventory and the cost of carry (27%) is relatively modest; in the frictionless storage model of [Working \(1949\)](#) and others, INV_t and c_t are collinear. A higher cost of carry strongly predicts higher inventories the next month. This relationship further suggests adjustment costs in physical storage: the futures curve adjusts to relevant information first and inventories respond with a lag.

Looking down the last column of the transition matrix, unspanned crude oil inventory does not forecast any of the other variables. In particular, periods of higher inventory do not have much effect on the forecast of either the spot price or the cost of carry. This finding is consistent with the fundamental drivers of oil inventory such as precautionary storage and expected physical supply and demand being fully spanned by oil futures prices.

4.2.3 Impulse Response Functions

Figure 4 plots the impulse response functions (IRFs) to shocks to oil prices and economic activity. The ordering of the variables for the impulse response functions is (GRO, s_t, c_t, INV) . GRO is first because innovations in the unspanned component, which dominates the varia-

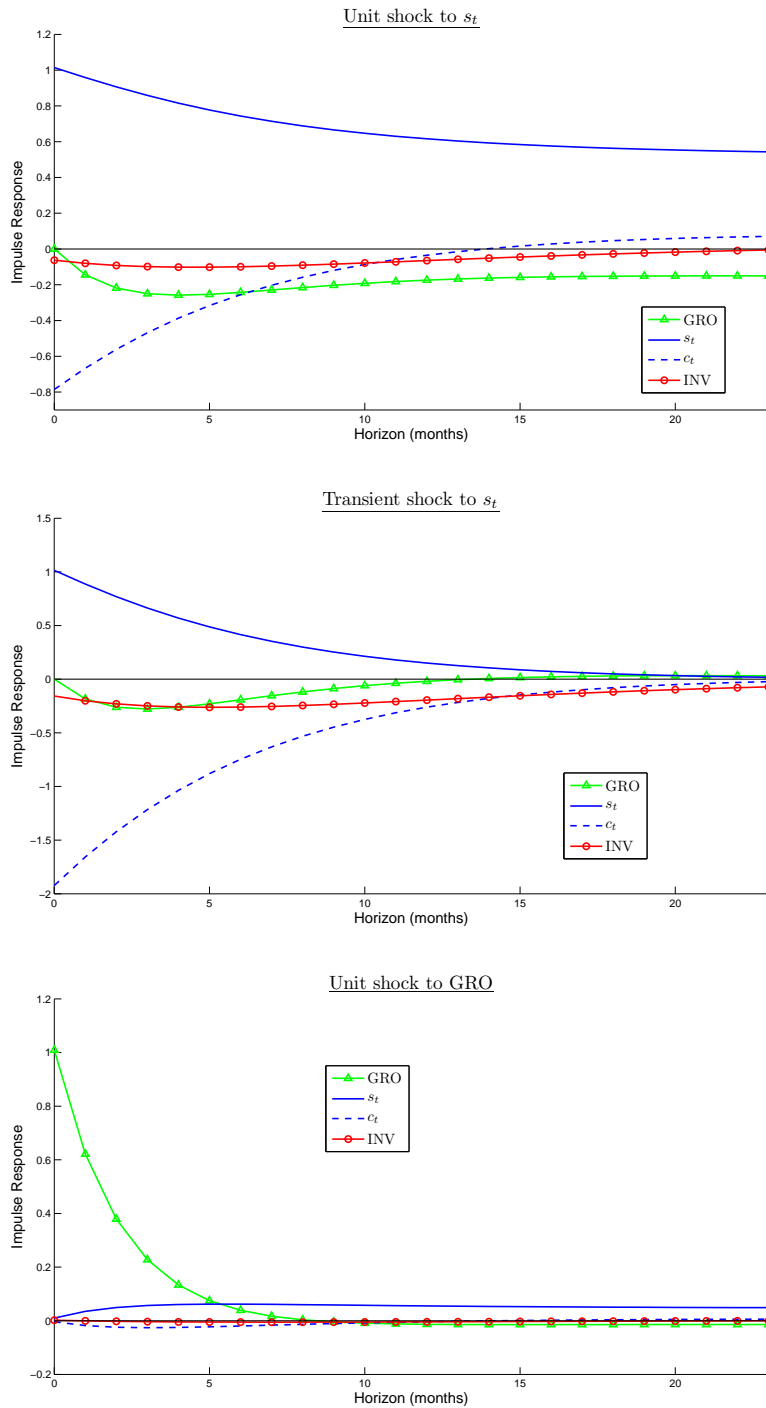


Figure 4: Panel A shows the impulse response functions (IRFs) of the four state variables to a unit shock to the log spot price of oil s_t . Panel B shows the IRFs for a transient shock for which the spot price of oil fully reverts to the baseline. Panel C shows the IRFs for a unit shock to economic growth, GRO . The order of the variables is (GRO, s_t, c_t, INV) .

tion in GRO , can be thought of as exogenous to contemporaneous oil prices and inventories. We analyze s_t and c_t simultaneously so their relative ordering is not important. Finally, it is intuitive and also supported by the estimates and regressions that the oil futures curve adjusts to new information faster than physical inventory does.

Panel A plots the response to a unit shock to the log spot price, which is correlated with a negative shock to the cost of carry and a more downward-sloping curve. A unit shock to s_t means a doubling of the spot price of oil. About half of the increase decays within two years, while the other half is effectively permanent, and forecasts an economic activity index that is 0.2% lower effectively forever. This effect is material: the index averaged -1.66% in 2009 during the depths of the financial crisis, while it averaged 0.02% in 2006. The higher spot price and lower cost of carry also produce a fall in inventories.

Panel B plots the response to a joint shock to s_t and c_t such that the spot price is expected to fully revert to the pre-shock baseline. The response of economic activity is transient as well, and in fact GRO recovers to the baseline faster than s_t does. Comparing to Panel A, which only differs in the size of the shock to c_t , makes clear that the net effect of c_t on expected growth is negative. Note that the fact that the forecast of the long-run spot price is unchanged in Panel B does not mean that long maturity futures prices will be unchanged – the two are equivalent only in the case that oil risk premiums are non time varying. Thus, a VAR that includes a long-maturity futures price or spread will not in general recover the correct dynamics of the state variables.

Panel C plots the response to a shock to economic activity. The index mean reverts rapidly and the shock decays back to the baseline within a year. However, a transient shock to GRO produces a near permanently higher spot price of oil – perhaps because oil is a nonrenewable resource. The magnitude of the effect is large: a one-period shock to economic

Table 3: Maximum Likelihood (ML) estimates of risk premiums in the macro-finance model for U.S. crude oil futures. s , c are the spot price and annualized cost of carry respectively. GRO and INV are the Chicago Fed National Activity Index and log U.S. crude oil inventory respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the state variables are de-meanned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

$$\begin{bmatrix} \Lambda^s \\ \Lambda^c \end{bmatrix}_t = \Lambda_0 + \Lambda_1 \begin{bmatrix} s_t & c_t & M_t \end{bmatrix}'$$

	Λ_0	Λ_1			
		s	c	GRO	INV
Λ^s	0.012 (0.014)	-0.001 (0.002)	-0.001 (0.003)	0.013 (0.004)	0.001 (0.002)
Λ^c	-0.010 (0.017)	0.003 (0.002)	-0.001 (0.004)	-0.008 (0.004)	-0.001 (0.002)

activity of one percent produces a spot price of oil that is 5.1% higher than the baseline, ten years later.

4.3 Risk Premiums

Table 3 displays the estimates of the parameters governing risk premiums. The unconditional spot risk premium is positive, while the unconditional cost-of-carry risk premium is negative⁶. Only one entry in the time-varying loadings of risk premiums Λ_1 is statistically significant: higher economic activity is associated with a higher spot risk premium in oil. Similar results obtain using the LEI as an alternative proxy for growth, including longer maturity futures prices, or estimating on a subsample that stops in 2007 and omits the wide swings of 2008-9.

The effect of economic activity on the estimated spot risk premium in oil futures is

⁶Szymanowska et al. (2013) decompose futures returns into a spot premium and a term premium. Appendix A describes the correspondence of their decomposition to the risk premiums in the model. Their spot premium equals the risk premium attached to the spot price plus a small convexity term, while their term premium equals the risk premium attached to the cost of carry minus the conditional expected cost of carry.

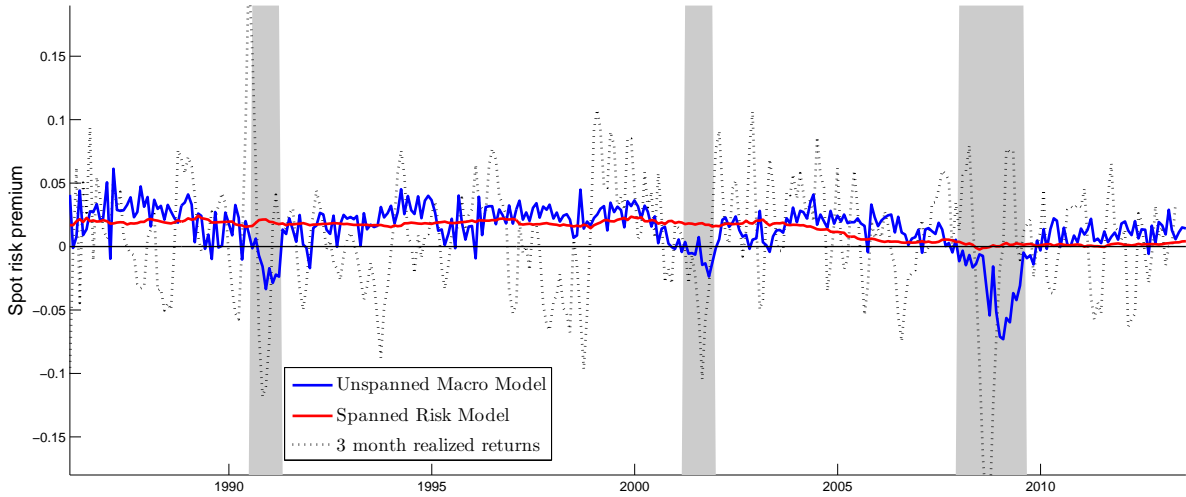


Figure 5: The figure compares the model implied spot premium in oil futures for the model with unspanned macro factors versus the nested model that enforces spanning. Also plotted is the realized average return to oil futures with maturities from 1 to 24 months over the following three months. NBER recessions are shaded in grey.

material. Figure 5 plots the implied spot premiums for the macro-finance model and the two-factor nested model that enforces spanning⁷, as well as the average realized returns for oil futures in the sample over the following three months. The model predictions differ most noticeably during 1990-1991, 2001-2002 and 2008-2009: slumps in real activity forecast slumps in the oil price. This unspanned procyclical component dominates the variation in the estimated spot risk premium; the standard deviation of changes in Λ_t^s in the unspanned macro model is 1.5% per month compared to 0.16% per month in the spanned-risk model, an increase of nearly tenfold.

The forecast is attached to the unspanned component of real activity because it is not reflected in the futures curve at the time. Per the estimates in Table 2, a fall in GRO is weakly correlated with a rise, not a fall, in the cost of carry. In other words, in economic

⁷That is, the unrestricted two factor model with $UM_t = \emptyset$.

downturns the oil futures curve “fails” to forecast the subsequent fall in the spot price.

This observation aligns with the findings of unspanned countercyclical risk premia in bonds by [Ludvigson and Ng \(2009\)](#), [Duffee \(2011\)](#) and [Joslin, Priebsch and Singleton \(2014\)](#), but in the opposite direction. However, just as [Duffee \(2011\)](#) emphasizes in bond yields, it is easy to tell a story consistent with the facts in oil futures. Conditional on news about economic growth, higher oil prices reflect negative supply shocks ([Kilian \(2009\)](#)) and forecast lower growth ([Hamilton \(1983\)](#); [Hamilton \(2003\)](#)). In an ICAPM where the state variables are total wealth and the quantity of oil in the world, asset returns obey

$$E[r_{t+1}^i] = \beta_{mkt}^i \lambda_t^{mkt} + \beta_{oil}^i \lambda_t^{oil}$$

where λ_t^{mkt} , λ_t^{oil} are the risk premiums for exposure to shocks to the wealth portfolio and the supply of oil. We can replace shocks to the oil supply by inverse shocks to the spot price of oil S :

$$E[r_{t+1}^i] = \beta_{mkt}^i \lambda_t^{mkt} - \beta_S^i \lambda_t^S$$

Oil futures of all maturities have a beta to the spot price that is near unity: a long position in oil futures is a hedge against supply shocks. The magnitude of λ_t^S is plausibly countercyclical: in economic slumps when uncertainty is greater or the consumption-investment tradeoff is steeper, investors have more demand for the oil hedge. As a result the expected return to a long position in oil futures is procyclical. However, as oil futures have a positive beta with the market portfolio the unconditional expected return may be positive or negative.

Both the regressions and model estimates suggest that innovations to growth are unspanned by oil futures prices. The existence of state variables that are material for yield forecasts yet unspanned by current yields is an active question in the term structure liter-

ature (Duffee (2011); Joslin, Priebsch and Singleton (2014)). In our setting, it corresponds to the state variable having offsetting effects on oil risk premiums and the oil price forecast. This is again consistent with futures as a hedge against oil supply shocks. A negative growth shock raises risk premiums including λ_t^S , which raises oil futures prices. At the same time, it forecasts reduced demand and lower spot prices. If these effects are of comparable magnitude and duration then the net effect on the futures curve could be small enough that we do not detect it.

4.3.1 Positive vs Negative Growth Regimes

The effects of economic activity on the spot risk premium in oil appear to be concentrated in economic downturns. To investigate this possibility I split GRO into two components: GRO^+ equals GRO in months when its value is positive and zero otherwise, while GRO^- equals GRO in months when its value is negative, and zero otherwise. This split lets the coefficients of risk premiums GRO differ when the world is in a positive-growth regime versus a negative-growth regime⁸.

Table (4) presents the estimated risk prices when GRO is split in this way. The size of the coefficients on GRO^+ and GRO^- are not directly comparable to the previous table because all of the coefficients are standardized to reflect a one standard deviation change, and GRO^+ and GRO^- naturally have lower standard deviations than GRO . The message of the table is that the response of the spot risk premium to growth shocks is symmetric on the upside and the downside: the coefficient on GRO^- is 0.008 per month compared to 0.007 per month for GRO^+ . Contrary to our impression from Figure 5, the effect of growth on the oil price forecast is relatively symmetric in good times versus bad.

⁸As the unspanned macro factors do not enter the pricing relation they need not be Gaussian.

Table 4: Estimates of risk premiums in the macro-finance model in which s , c are the spot price and annualized cost of carry in oil futures and GRO^+ and GRO^- are the monthly Chicago Fed National Activity Index in months when the index is positive and negative respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the state variables are de-meant. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

$$\begin{bmatrix} \Lambda_s \\ \Lambda_c \end{bmatrix}_t = \Lambda_0 + \Lambda_1 \begin{bmatrix} s_t & c_t & M_t \end{bmatrix}'$$

	Λ_0	Λ_1			
		s	c	GRO^+	GRO^-
Λ_s	0.013 (0.017)	-0.001 (0.002)	-0.001 (0.002)	0.007 (0.006)	0.008 (0.004)
Λ_c	-0.011 (0.020)	0.003 (0.002)	-0.002 (0.004)	-0.003 (0.006)	-0.005 (0.004)

5 Real Options Valuation

Firms' capacity to adjust their investment or production ex post make up a substantial part of firm value, and evaluating and managing these adjustments is a primary role of firm management (Pindyck (1988); Berk, Green and Naik (2004)). Previous studies (Brennan and Schwartz (1985); Schwartz (1997); Casassus and Collin-Dufresne (2005)) have explored what commodity derivatives can tell us about the value of real options. These studies using spanned-risk models make the strong assumption that the economy is spanned by commodity futures. In Trolle and Schwartz (2009) and Chiang et al. (forthcoming), a latent volatility factor is unspanned by futures but identified in the prices of oil options.

By contrast, if a factor M_t is unspanned in the sense of this paper then it does not affect *any* derivatives prices on the underlying commodity. Unspanned factors of this type are still relevant to real options, however, if the option's payoff depends on M_t . For example, an oil well is often modeled as the right to pump oil out of the ground at a fixed cost per barrel,

equivalent to a purely financial option. But for a real oil well the costs of extraction are uncertain. [Moel and Tufano \(2002\)](#) find that for gold mines, changing extraction costs over time are a significant predictor of mine openings and closings after controlling for factors estimated from both futures and options on gold. More generally, commodity prices are only one element of the firm’s decision process. For example, in an airline’s decision to purchase more fuel efficient planes, the cost savings will vary with oil prices while revenues will vary with aggregate economic activity. [Pindyck \(1993\)](#) makes this argument and points out that all relevant risk factors will also affect real option valuation⁹.

To illustrate the effects of unspanned macroeconomic risks on real options valuation, I model an oil well as a strip of European options on an oil field that produces 1000 barrels of oil per month when open. The oil is extracted at lifting cost L_t and sold at the spot price s_t each month that it is open. Thus, it is open whenever $s_t > l_t$. In the model that is used to generate the data, the log lifting cost l_t has both spanned and unspanned components plus idiosyncratic noise. The dynamics of the state variables (s, c, GRO) are a simplified version of the estimates presented earlier. [Appendix E](#) describes the setting in detail.

[Figure 6](#) plots the value of wells with different current lifting costs L_0 using different models. The lower two lines represent option values for spanned-risk models in which all relevant risks are assumed to be spanned by oil futures. This means l_t must be a linear combination of s_t and c_t plus an error term ([Joslin, Le and Singleton \(2013\)](#)). Whether the error term is modelled as an i.i.d. or AR(1) process is essentially irrelevant to option value.

The upper two lines represent option values from models with unspanned macro risk. We see that the spanned-risk models miss a large component of option value. To emphasize,

⁹“... *this effect [of uncertainty on option value and exercise] is magnified when fluctuations in construction costs are correlated with the economy, or, in the context of the Capital Asset Pricing Model, when the ‘beta’ of cost is high... [A] higher beta raises the discount rate applied to expected future costs, which raises the value of the investment opportunity as well as the benefit from waiting rather than investing now.*”

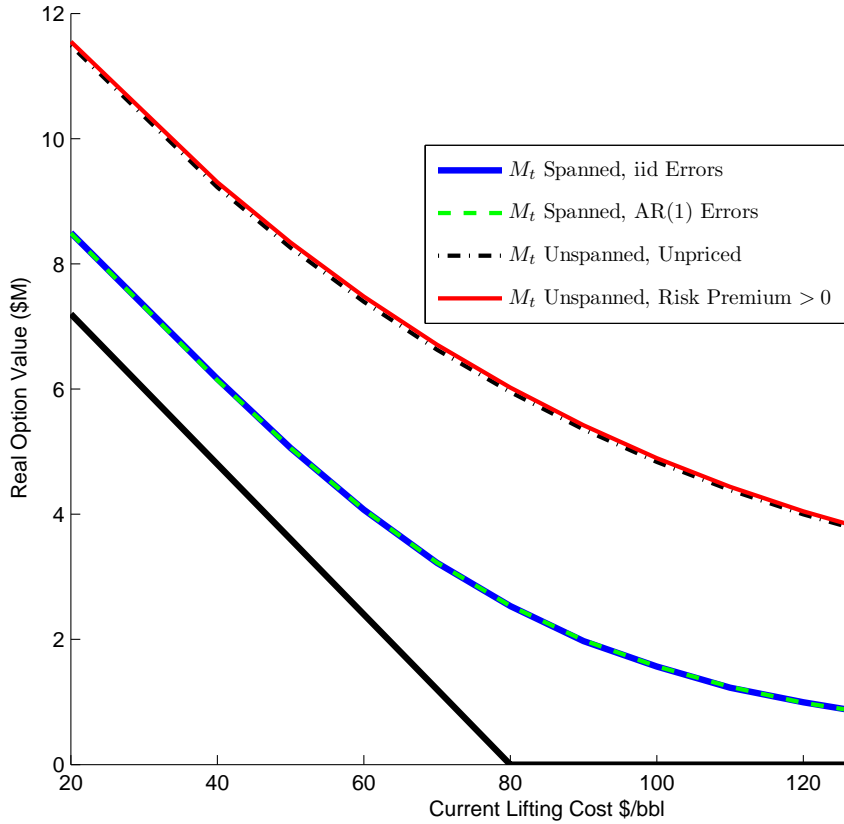


Figure 6: Examples of real options valuation with unspanned risks. An oil well is modelled as a strip of European options that are exercised when the stochastic log extraction cost l_t is less than the log spot price s_t . l_t covaries with s_t and the unspanned macro risk GRO_t . The current spot price of oil is \$80, and the x-axis indexes the current lifting cost L_0 of different oil wells.

the monthly volatility of shocks to l_t in the spanned-risk models is the same as it is in the unspanned-risk models. The difference is that l_t 's dependence on GRO adds persistent time variation in lifting costs that covaries with the spot price and cost of carry. This addition has a large effect on option valuation: Adding the unpriced ($\lambda = 0$) unspanned macro risk raises the real option value by 35% for an 'in the money' well with current lifting cost = \$20 per barrel and 405% for an 'out of the money' well with current lifting cost of \$150 per barrel.

The risk premium (Pindyck) effect is that the option value is higher when GRO , and hence L_t , carries a positive risk premium ($\lambda > 0$). This effect on valuation is present but minor in the example, increasing the well's value by only 0.99% for the 'in the money' well with $L_0 = 20$ and by 1.27% for the 'out of the money' well with $L_0 = 150$.

6 The Cost of Carry

6.1 Comparison with the Basis

The model estimates say that s_t and c_t are precisely pinned down by the data. Empirical studies usually proxy for these quantities with futures prices and spreads. [Fama and French \(1987\)](#); [Gorton, Hayashi and Rouwenhorst \(2013\)](#); [Singleton \(2013\)](#) and others use some version of the log spread or "basis", $f_t^2 - f_t^1$, as a proxy for the cost of carry.¹⁰

By definition, c_t equals the difference between the model implied values of f_t^1 and s_t . The model fits an AR(1) process to (s_t, c_t) using the full futures curve at each date. The basis does not model the behavior or number of state variables but instead assumes that the

¹⁰[Fama and French \(1987\)](#) and others subtract a short term Treasury bill rate from the spread. The short rate is slow moving relative to the spread, and defining the basis as $(f_t^2 - f_t^1) - r_t^f$ gives the same results.

errors on f_t^1 and f_t^2 are zero and that $f_t^2 - f_t^1$ is closely correlated with c_t .

The normalization and estimation in this paper, based on the recent advances of [Joslin, Singleton and Zhu \(2011\)](#) in term structure modelling, are exceedingly stable and tractable: estimates converge in a few seconds. Thus it is practical to use the model estimated c_t instead of the basis. There are several reasons why fitting the model could yield more accurate estimates of the cost of carry. First, producers and consumers plan and hedge their activities more than two months in advance, in which case longer dated futures prices will contain relevant information. Second, market microstructure issues like congestion at delivery points or financial order flows could add noise to individual prices. Third, because the cost of carry is mean reverting, when the futures curve slopes in either direction $f_t^2 - f_t^1$ is biased toward the mean as a proxy for $f_t^1 - s_t$.

Figure 7 Panel A plots the model implied log spot price s_t against the one month futures log price f_t^1 in the sample. The two are almost collinear with a correlation in levels (monthly changes) of 0.999 (0.995). Thus, the one month futures prices is a close proxy for the spot price although they occasionally differ by as much as 5%.

Figure 7 Panel B plots the model implied cost of carry c_t against the annualized basis. The two series have similar unconditional averages but their correlation in levels (monthly changes) is 0.79 (0.60) and their values differ significantly throughout the sample. In particular, c_t is much more slow moving. Monthly innovations in c_t have a standard deviation of 10.0% compared to 16.9% for the basis. The AR(1) coefficient of c_{t+1} on c_t is 0.88 (half-life of 5.5 months) compared to 0.74 (half-life of 2.3 months) for the basis, and the differences in variances and AR(1) coefficients are significant at the 1% and 5% level respectively. Thus, c_t implies that the net convenience yield for oil varies less and returns to its mean more slowly than the basis implies.

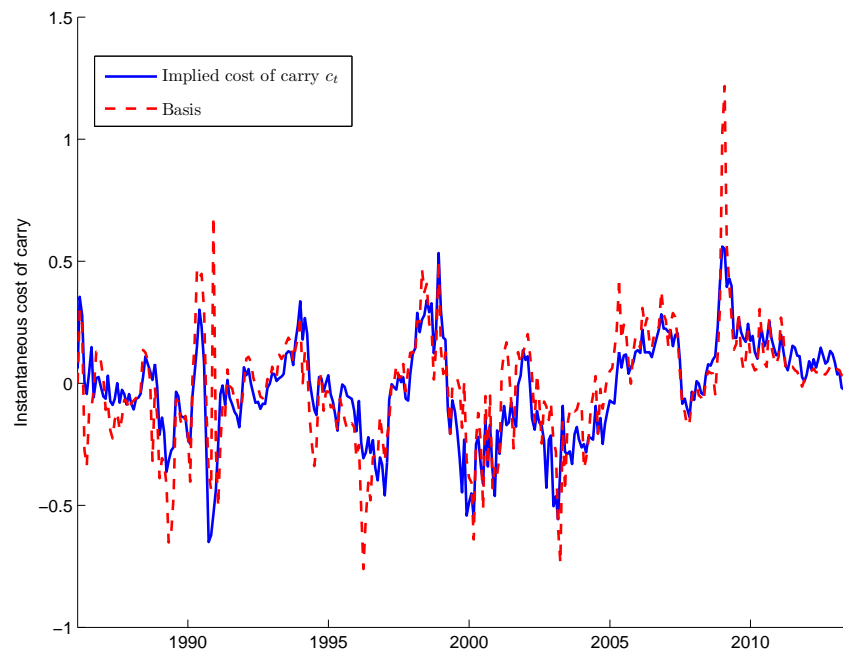
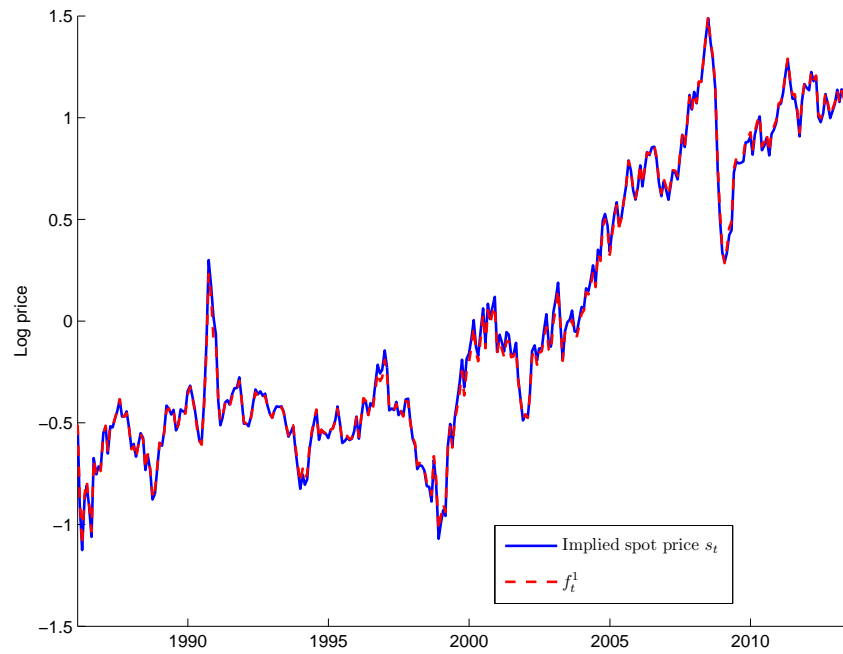


Figure 7: Panel A plots the model implied log spot price of oil s_t and the one-month log futures price f_t^1 . Panel B plots the model implied cost of carry c_t and the annualized basis $12 \times (f_t^2 - f_t^1)$.

Table 5: Comparison between the model implied cost of carry c_t and the basis $12 \times (f_t^2 - f_t^1)$ as predictors of log U.S. inventories of crude oil INV . The standard errors are Newey-West with six lags. * : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$.

Panel A: Cost of Carry from WTI Futures, 1/1986-7/2013

	ΔINV_{t+1}		
c_t	0.031*** (0.008)		0.038*** (0.010)
$basis_t$		0.016* (0.009)	-0.009 (0.012)
INV_t	-0.112*** (0.022)	-0.088*** (0.023)	-0.112*** (0.022)
$adj. R^2$	6.4%	3.9%	6.2%
T	330	330	330

Panel B: Cost of Carry from Brent Futures, 1/1990-7/2013

	ΔINV_{t+1}		
c_t^{BRENT}	0.039*** (0.011)	-0.027 (0.037)	0.035** (0.015)
c_t		0.062** (0.030)	
$basis_t$			0.004 (0.013)
INV_t	-0.105*** (0.026)	-0.127*** (0.023)	-0.107*** (0.028)
$adj. R^2$	6.3%	7.4%	6.0%
T	281	281	281

Whether c_t or the basis is a better measure of the cost of carry is answered by linking them with the inventory data. In models of storage without adjustment costs, the cost of carry and the quantity stored are perfectly correlated. c_t is modestly more correlated with contemporaneous inventories INV_t than the basis is: $corr(c_t, INV_t)$ is 0.52 while $corr(basis_t, INV_t)$ is 0.42, and the difference in correlations is significant at the 10% level. Table 5 Panel A presents a horse race regressing the change in inventories on c_t , the basis, and the current inventory level. We see that c_t is a stronger predictor of future inventories, and in the joint regression c_t drives out the basis entirely. Thus, c_t is more tightly linked to both present and future storage decisions than the basis is. Similar results obtain if I winsorize the basis at the 1% or 5% level in both tails, indicating that the stronger performance of c_t is not driven by a few extreme observations in the basis.

To further investigate the validity of the model implied cost of carry, I estimate the model using the prices of Brent crude oil futures from one to twelve months maturity, from January 1990 to July 2013. The WTI contract delivers oil in Cushing, Oklahoma, while the Brent contract delivers oil on shipboard in the North Sea approximately 4,500 miles away. The two markets are naturally linked, but can diverge materially. The correlation of the WTI basis with the Brent basis is 77.8%, while the correlation of c_t with c_t^{BRENT} is 95.2%. Thus, there is considerable market-specific variation in the basis, while the slopes of the futures curves are more closely linked. The question is whether the market-specific variation in the basis is economically meaningful in relation to storage. Table (5) Panel B, Column 1 shows that c_t^{BRENT} is a strong predictor of future U.S. inventories. Column 2 shows that c_t^{BRENT} is driven out entirely by c_t computed from U.S. oil futures, so some variation between the markets is relevant to storage. But Column 3 shows that c_t^{BRENT} completely drives out the U.S. futures basis as a predictor of U.S. inventories. These results suggest, first, that the

model's estimation of c_t picks up as much or more of the market-specific variation in the cost of carry than the basis does, and second, that the additional market-specific variation in the basis is not related to storage.

7 Conclusion

This paper develops an affine macro-finance model for futures that admits unspanned macroeconomic variables. The model includes benchmark futures pricing models as special cases, and is a bridge between vector autoregressions (VARs) on the one hand and affine latent-factor models on the other. The framework can be applied to any commodity futures market and any set of macro factors.

I apply the framework to crude oil futures prices to investigate their interaction with real economic activity and oil inventories. There is a negative feedback relationship between oil prices and real activity. Higher real activity forecasts higher oil prices, and this effect is unspanned in contemporaneous futures prices. At the same time, higher oil prices forecast lower real activity, especially when the price increase is forecast by the market to be persistent. Thus, there is a negative feedback relationship between oil prices and the economy. The implied spot risk premium in the estimate differs particularly in recessions from the spot risk premium in a model that assumes perfect risk spanning.

The estimates highlight the importance of using information beyond that contained in the futures curve when studying futures returns and price forecasts. The main conclusion is that standard futures pricing models extract information exclusively from the cross-section of futures prices, and this limitation seems to miss much of the variation in futures risk premia.

The estimates also have implications for the valuation of real options. By construction, unspanned macro factors do not affect the prices of commodity derivatives. However, when the payoff of a real option such as an oil well depends on macroeconomic factors beyond the oil price then unspanned macro factors can have a large effect on option value and exercise. In a calibrated example I show that both the dynamics and the risk premiums of unspanned macro risks raise the values of a hypothetical real option drastically relative to a benchmark spanned-risk model.

The model estimates further imply that the spot price and cost of carry in the oil market are precisely pinned down by futures prices. The model cost of carry differs significantly from the basis, which is commonly used as a proxy for the cost of carry. In particular, the model cost of carry is 40% less volatile month-to-month, and reverts to its mean more than twice as slowly as the basis does. The model cost of carry is more strongly related to both current and future oil inventories than the basis, and the cost of carry based on North Sea futures is more connected to U.S. inventories than the U.S. basis. Thus, the model estimates imply that the convenience yield is much less volatile than the basis is, and we obtain a better measure by fitting a pricing model to the full futures curve than we do from a single calendar spread.

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A Model Specification and Risk Premiums

Consider a Gaussian model where the log spot price s_t of a commodity is a function of N_L spanned state variables L_t , which may be latent or observed, and N_M unspanned state variables M_t that are observed:

$$\begin{aligned} \begin{bmatrix} L_{t+1} \\ M_{t+1} \end{bmatrix} &= K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \delta_0 + \delta_1' L_t \end{aligned} \tag{7}$$

where

- \mathbb{P} denotes dynamics under the historical or data generating measure
- \mathbb{Q} denotes dynamics under the risk neutral measure
- $\epsilon_{L,t+1}^{\mathbb{Q}} \sim N(0, I_{N_L})$, $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, I_N)$
- Σ_L is the top left $N_L \times N_L$ block of Σ_X ; Σ_L, Σ_X are lower triangular

(7) is equivalent to specifying the equation for s_t and the \mathbb{P} -dynamics plus a lognormal affine discount factor with 'essentially affine' prices of risk as in [Duffee \(2002\)](#). For $N_M = 0$ the framework includes models such as [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Schwartz and Smith \(2000\)](#) as special cases (see [Appendix D](#)). Standard recursions show that (7) implies affine log prices for futures,

$$f_t = A + BX_t \tag{8}$$

$$f_t = \begin{bmatrix} f_t^1 & f_t^2 & \dots & f_t^J \end{bmatrix}'$$

where f_t^j is the price of a j period future and J is the number of futures with different maturities.

Estimating the model as written presents difficulties; with two latent factors and two macro factors there are 40 free parameters. Different sets of parameter values are observationally equivalent due to rotational indeterminacy of the latent factors. Discussing models of the form (7) for bond yields, [Hamilton and Wu \(2012\)](#) refer to “*tremendous numerical challenges in estimating the necessary parameters from the data due to highly nonlinear and badly behaved likelihood surfaces.*” In general, affine models for futures identify the model by specifying dynamics that are less general than (7).

[Joslin, Priebsch and Singleton \(2014\)](#) note that if N_L linear combinations of bond yields are measured without error, then any model of yields of the form (7) implies a model with observable factors in place of the latent factors. They construct a minimal parametrization where no sets of parameters are redundant - models in the “JPS form” are unique. Thus the likelihood surface is well behaved and contains a single global maximum. Their results hold to a very close approximation if the linear combinations of yields are observed with relatively small and idiosyncratic errors.

Section B demonstrates the same result for futures pricing: if N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t = W f_t \tag{9}$$

for any full rank $N_L \times J$ matrix W , then any model of the form (7) is observationally equivalent to a unique model of the form

$$\begin{aligned}
\begin{bmatrix} \Delta \mathcal{P}_{t+1} \\ \Delta UM_{t+1} \end{bmatrix} &= \Delta Z_{t+1} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}} \\
\Delta \mathcal{P}_{t+1} &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\
s_t &= \rho_0 + \rho_1 \mathcal{P}_t
\end{aligned} \tag{10}$$

parametrized by $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$, where

- $\lambda^{\mathbb{Q}}$ are the N_L ordered eigenvalues of $K_1^{\mathbb{Q}}$
- p_{∞} is a scalar intercept
- Σ_Z is the lower triangular Cholesky decomposition of the covariance matrix of innovations in the state variables
- $\Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} = [\Sigma_Z \Sigma'_Z]_{N_L}$, the top left $N_L \times N_L$ block of $\Sigma_Z \Sigma'_Z$

A.1 \mathcal{P}_t Measured Without Error

In this paper I assume that while each of the log futures maturities is observed with iid measurement error, the pricing factors \mathcal{P}_t^1 and \mathcal{P}_t^2 are measured without error.

$$f_t^j = A_j + B_j \mathcal{P}_t + \nu_t^j, \quad \nu_t^j \sim N(0, \zeta_j^2)$$

The use of the first two PCs of log price levels is not important: in unreported results I find that all estimates and results are effectively identical using other alternatives such as the

first two PCs of log price changes or of returns, or a priori weights such as

$$W = \begin{bmatrix} 1 & \dots & 1 \\ 0 & \dots & 12 \end{bmatrix}$$

The identifying assumption that N_L linear combinations of yields are measured without error is common in the bond yields literature beginning with [Chen and Scott \(1993\)](#). Given the model parameters, values of the latent factors at each date are then extracted by inverting the relation (8). The same assumption is used to identify previous latent factor models for commodity futures (see [Gibson and Schwartz \(1990\)](#); [Casassus and Collin-Dufresne \(2005\)](#); [Hamilton and Wu \(2014\)](#)). In unreported results I find that all estimates and results are effectively identical if the pricing factors are estimated via the Kalman filter.

A.2 Rotating to s_t and c_t

Once the model is estimated in the JPS form, I rotate $(\mathcal{P}_t^1, \mathcal{P}_t^2)$ to be the model implied log spot price and instantaneous cost of carry, (s_t, c_t) . For s_t this is immediate:

$$s_t = \rho_0 + \rho_1 \mathcal{P}_t$$

For c_t the definition is as follows. Any agent with access to a storage technology can buy the spot commodity, sell a one month future, store for one month and make delivery. Add up all the costs and benefits of doing so (including interest, costs of storage, and convenience yield) and express them as quantity c_t where the total cost in dollar terms = $S_t(e^{c_t} - 1)$.

Then in the absence of arbitrage it must be the case that

$$F_t^1 = S_t e^{c_t}$$

$$f_t^1 = s_t + c_t = E_t^{\mathbb{Q}}[s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$c_t = E_t^{\mathbb{Q}}[\Delta s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$= \rho_1[K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}}\mathcal{P}_t] + \frac{1}{2}\sigma_s^2$$

A.3 Risk premiums and s_t , c_t :

Szymanowska et al. (2013) define the per-period log basis $y_t^n \equiv f_t^n - s_t$, and define two risk premiums based on different futures trading strategies; the spot premium $\pi_{s,t}$ and the term premium $\pi_{y,t}^n$.

The spot premium is defined as

$$\begin{aligned} \pi_{s,t} &\equiv E_t[s_{t+1} - s_t] - y_t^1 \\ &= E_t[s_{t+1}] - f_t^1 = E_t^{\mathbb{P}}[s_{t+1}] - E_t^{\mathbb{Q}}[s_{t+1}] - \frac{1}{2}\sigma_s^2 \\ &\Rightarrow \pi_{s,t} = \Lambda_{s_t} - \frac{1}{2}\sigma_s^2 \end{aligned}$$

The term premium is defined as

$$\begin{aligned} \pi_{y,t}^n &\equiv y_t^1 + (n-1)E_t[y_{t+1}^{n-1}] - n y_t^n \\ &= f_t^1 + (n-1)E_t[f_{t+1}^{n-1}] - n f_t^n \end{aligned}$$

The one month term premium is always zero, because storing for one month is riskless.

$$\pi_{y,t}^{(1)} = f_t^1 + 0 - f_t^1 = 0$$

$$\pi_{y,t}^{(2)} = c_t + E_t^{\mathbb{P}} [c_{t+1}] - 2E_t^{\mathbb{Q}} [s_{t+2} - s_{t+1} + s_{t+1} - s_t]$$

$$= E_t^{\mathbb{P}} [c_{t+1}] - E_t^{\mathbb{Q}} [c_{t+1}] - E_t^{\mathbb{Q}} [s_{t+2} - s_{t+1}] - c_t$$

$$\Rightarrow \pi_{y,t}^{(2)} = \Lambda_{c_t} - (c_t + E_t^{\mathbb{Q}} [c_{t+1}])$$

Thus the spot premium and term premium of [Szymanowska et al. \(2013\)](#) each have a natural expression in our affine framework. The spot premium is exactly the risk premium attached to shocks to the log spot price s_t plus a small constant. The term premium is the risk premium attached to shocks to the cost of carry minus the (risk-neutral) total expected cost of carry.

B JPS Parametrization

I assume that N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t^L = W f_t$$

for any full-rank real valued $N_L \times J$ matrix W , and show that any model of the form

$$\begin{aligned} \begin{bmatrix} \Delta L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} &= \Delta X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ \Delta L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} X_t + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \\ s_t &= \delta_0 + \delta_1' X_t \end{aligned} \tag{11}$$

is observationally equivalent to a unique model of the form

$$\begin{aligned} \begin{bmatrix} \Delta \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} &= \Delta Z_{t+1} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{Z,t+1}^{\mathbb{P}} \\ \Delta \mathcal{P}_{t+1}^L &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} Z_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \rho_0 + \rho_1' Z_t \end{aligned} \tag{12}$$

which is parametrized by $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$.

The proof that follows is essentially the same as that of [Joslin, Priebsch and Singleton \(2014\)](#). [Joslin, Singleton and Zhu \(2011\)](#) demonstrates the result for all cases including zero, repeated and complex eigenvalues.

Assume the model (11) under consideration is nonredundant, that is, there is no observationally equivalent model with fewer than N state variables. If there is such a model, switch to it and proceed.

B.1 Observational Equivalence

Given any model of the form (11), the $J \times 1$ vector of log futures prices f_t is affine in L_t ,

$$f_t = A_L + B_L L_t$$

Hence the set of N_L linear combinations of futures prices, \mathcal{P}_t^L , is as well:

$$\mathcal{P}_t^L = W_L f_t = W_L A_L + W_L B_L L_t$$

Assume that the N_L ordered elements of $\lambda^{\mathbb{Q}}$, the eigenvalues of $K_{1L}^{\mathbb{Q}}$, are real, distinct and nonzero. There exists a matrix C such that $K_{1L}^{\mathbb{Q}} = C \text{diag}(\lambda^{\mathbb{Q}}) C^{-1}$. Define $D = C \text{diag}(\delta_1) C^{-1}$, $D^{-1} = C \text{diag}(\delta_1)^{-1} C^{-1}$ and

$$\begin{aligned} Y_t &= D[L_t + (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}}] \\ \Rightarrow L_t &= D^{-1} Y_t - (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} \end{aligned}$$

Then

$$\begin{aligned} \Delta Y_{t+1} &= D \Delta L_{t+1} \\ &= D[K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}}(D^{-1} Y_t - (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}}) + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}] \\ &= \text{diag}(\lambda^{\mathbb{Q}}) Y_t + D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \end{aligned}$$

and

$$\begin{aligned} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} &= \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} [K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} \left(\begin{bmatrix} D^{-1} & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} - \begin{bmatrix} (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} \\ 0 \end{bmatrix} \right) + \Sigma_X \epsilon_{t+1}^{\mathbb{P}}] \\ &= K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \end{aligned}$$

and

$$p_t = \delta_0 + \delta_1' L_t = \delta_0 + \delta_1' D^{-1} Y_t - \delta_1' (K_{1L}^{\mathbb{Q}})^{-1} K_{0L}^{\mathbb{Q}} = p_{\infty} + \iota \cdot Y_t$$

where ι is a row of N_L ones.

$$f_t = A_Y + B_Y Y_t$$

$$\mathcal{P}_t^L = W f_t = W A_Y + W B_Y Y_t$$

The model is nonredundant $\Rightarrow W B_Y$ is invertible:

$$Y_t = (W B_Y)^{-1} \mathcal{P}_t^L - (W B_Y)^{-1} W A_Y$$

$$\cdot \mathcal{P}_{t+1}^L = W B_Y \Delta Y_{t+1} = W B_Y \text{diag}(\lambda^{\mathbb{Q}}) [(W B_Y)^{-1} \mathcal{P}_t^L - (W B_Y)^{-1} W A_Y] + W B_Y D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}$$

$$= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t^L + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}}$$

Further,

$$\begin{aligned}
\Delta Z_{t+1} &= \begin{bmatrix} \cdot \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} \\
&= \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \left(K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \right) \\
&= K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}}
\end{aligned}$$

$$p_t = p_{\infty} + \iota \cdot Y_t = p_{\infty} + \iota \cdot (WB_Y)^{-1} \mathcal{P}_t^L - \iota \cdot (WB_Y)^{-1} W A_Y = \rho_0 + \rho_1' \mathcal{P}_t^L$$

QED. Collecting the formulas: given any model of the form (7), there is an observationally equivalent model of the form (10), parametrized by $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$, where

- $D = C \text{diag}(\delta_1)^{-1} C^{-1}$
- $\Sigma_Z = \begin{bmatrix} WB_Y D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X, \Sigma_{\mathcal{P}} = [\Sigma_Z]_{\mathcal{L}\mathcal{L}}$
- $B_Y = \begin{bmatrix} \iota' [I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})] \\ \vdots \\ \iota' [I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})]^J \end{bmatrix}$
- $A_Y = \begin{bmatrix} p_{\infty} + \frac{1}{2} \iota' \Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}' \iota \\ \vdots \\ A_{Y,J-1} + \frac{1}{2} B_{Y,J-1} \Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}' B_{Y,J-1}' \end{bmatrix}$
- $K_1^{\mathbb{Q}} = WB_Y \text{diag}(\lambda^{\mathbb{Q}}) (WB_Y)^{-1}, K_0^{\mathbb{Q}} = -K_1^{\mathbb{Q}} W A_Y$
- $\rho_0 = p_{\infty} - \iota \cdot (WB_Y)^{-1} W A_Y, \rho_1' = \iota \cdot (WB_Y)^{-1}$

In estimation I adopt the alternate form

- $\Delta Y_{t+1} = \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \text{diag}(\lambda^{\mathbb{Q}})Y_t + D\Sigma_X\epsilon_{t+1}^{\mathbb{Q}}$
- $p_t = \iota \cdot Y_t$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota'\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}}\iota \\ \vdots \\ A_{Y,J-1} + B_{Y,J-1} \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \frac{1}{2}B_{Y,J-1}\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}}B'_{Y,J-1} \end{bmatrix}$
- $K_1^{\mathbb{Q}} = WB_Y\text{diag}(\lambda^{\mathbb{Q}})(WB_Y)^{-1}$, $K_0^{\mathbb{Q}} = WB_Y \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} - K_1^{\mathbb{Q}}WA_Y$
- $\rho_0 = -\iota \cdot (WB_Y)^{-1}WA_Y$, $\rho'_1 = \iota \cdot (WB_Y)^{-1}$

which is numerically stable when $\lambda^{\mathbb{Q}}(1) \rightarrow 0$. See the online supplement to [JSZ 2011](#).

B.2 Uniqueness

We consider two models of the form (10) with parameters θ and $\hat{\theta} = (\hat{\lambda}^{\mathbb{Q}}, \hat{p}_\infty, \hat{\Sigma}_Z, \hat{K}_0^{\mathbb{P}}, \hat{K}_1^{\mathbb{P}})$ that are observationally equivalent and show that this implies $\theta = \hat{\theta}$.

Since $Z_t = \begin{bmatrix} \mathcal{P}_t^L \\ M_t \end{bmatrix}$ are all observed, $\{\Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}}\} = \{\hat{\Sigma}_Z, \hat{K}_0^{\mathbb{P}}, \hat{K}_1^{\mathbb{P}}\}$.

Since $f_t = A + BZ_t$ are observed, $A(\theta) = A(\hat{\theta})$, $B(\theta) = B(\hat{\theta})$.

Suppose $\lambda^{\mathbb{Q}} \neq \hat{\lambda}^{\mathbb{Q}}$. Then by the uniqueness of the ordered eigenvalue decomposition,

$$B_Y^j(\lambda) \neq B_Y^j(\hat{\lambda}) \forall j$$

$$\Rightarrow WB_Y(\lambda) \neq WB_Y(\hat{\lambda}) \Rightarrow (WB_Y(\lambda))^{-1} \neq (WB_Y(\hat{\lambda}))^{-1}$$

$$\Rightarrow \rho_1(\lambda) \neq \rho_1(\hat{\lambda}) \Rightarrow B(\lambda) \neq B(\hat{\lambda})$$

, a contradiction. Hence $\lambda^{\mathbb{Q}} = \hat{\lambda}^{\mathbb{Q}}$. Then $A(\lambda^{\mathbb{Q}}, p^{\infty}) = A(\hat{\lambda}^{\mathbb{Q}}, \hat{p}^{\infty}) \Rightarrow p^{\infty} = \hat{p}^{\infty}$.

C Estimation

Given the futures prices and macroeconomic time series $\{f_t, M_t\}_{t=1, \dots, T}$ and the set of portfolio weights W that define the pricing factors:

$$\mathcal{P}_t = W f_t$$

we need to estimate the minimal parameters $\theta = (\lambda^{\mathbb{Q}}, p_{\infty}, \Sigma_Z, K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ in the JPS form. The estimation is carried out by maximum likelihood (ML). If no restrictions are imposed (i.e. we are estimating the canonical model (11)), then $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$ do not affect futures pricing and are estimated consistently via OLS. Otherwise $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$ are obtained by GLS taking the restrictions into account. The OLS estimate of Σ_Z is used as a starting value, and the starting value for p_{∞} is the unconditional average of the nearest-maturity log futures price. Both were always close to their ML value. Finally I search over a range of values for the eigenvalues $\lambda^{\mathbb{Q}}$.

After the ML estimate of the model in the JPS form is found, I rotate and translate the spanned factors from $\mathcal{P}_t^1, \mathcal{P}_t^2$ to s_t, c_t as described in A.2. I rotate and translate UM_t to M_t , so that the estimate reflects the behavior of the time series M_t :

$$\begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \frac{1}{2}\sigma_s^2 + \rho_1 K_0^{\mathbb{Q}} \\ \alpha_{M\mathcal{P}} \end{bmatrix} + \begin{bmatrix} \rho_1 & 0_{1 \times N_M} \\ \rho_1 K_1^{\mathbb{Q}} & 0_{1 \times N_M} \\ 0_{N_M \times 1} & \beta_{M\mathcal{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_t \\ UM_t \end{bmatrix}$$

where

$$M_t = \alpha_{MP} + \beta_{MP}\mathcal{P}_t + UM_t$$

D Comparison with other Futures Models

The model (7) is a canonical affine Gaussian model, so any affine Gaussian model is a special case. For example, discretized, the [Gibson and Schwartz \(1990\)](#); [Schwartz \(1997\)](#); [Schwartz and Smith \(2000\)](#) two factor model can be written

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} \mu \\ \kappa\alpha \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{P}} \quad (13)$$

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} r \\ \kappa\alpha - \lambda \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{Q}} \quad (14)$$

which is clearly a special case of (7).

The [Casassus and Collin-Dufresne \(2005\)](#) model, discretized, is:

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa_X^P \theta_X^P + \kappa_{Xr}^P \theta_r^P + \kappa_{X\delta}^P \theta_\delta^P \\ \kappa_\delta^P \theta_\delta^P \\ \kappa_r^P \theta_r^P \end{bmatrix} + \begin{bmatrix} -\kappa_X^P & -\kappa_{X\delta}^P & -\kappa_{Xr}^P \\ 0 & -\kappa_\delta^P & 0 \\ 0 & 0 & -\kappa_r^P \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 \\ \rho_{X\delta} & 1 \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{P}} \quad (15)$$

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_X \theta_X^Q + (\alpha_r - 1) \theta_r^Q + \theta_\delta^Q \\ \kappa_\delta^Q \theta_\delta^Q \\ \kappa_r^Q \theta_r^Q \end{bmatrix} + \begin{bmatrix} -\alpha_X & -1 & 1 - \alpha_r \\ 0 & -\kappa_\delta^Q & 0 \\ 0 & 0 & -\kappa_r^Q \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 \\ \rho_{X\delta} & 1 \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon_{t+1}^{\mathbb{Q}} \quad (16)$$

(see their formulas 7, 12, 13 and 27, 28, 30) which is the Schwartz three factor model with more flexible risk premiums.

Table 6: Parameters of the calibration for computing real option values

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			Σ		
		s_t	c_t	GRO_t	s	c	GRO
s_{t+1}	0.00	1.00	0.083	0.03	s	0.10	
c_{t+1}	0.00	0.00	0.90	0.00	c	-0.08	0.06
GRO_{t+1}	0.00	-0.10	0.00	0.60	GRO	0	0.50

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$		
		s_t	c_t	GRO_t
s_{t+1}	0.00	1.00	0.08	0.00
c_{t+1}	0.00	0.00	0.90	0.00
GRO_{t+1}	$-\lambda$	-0.10	0.00	0.60

E Real Option Valuation

The lifting cost is

$$l_t = \kappa_l + 0.1s_t + 0.01GRO_t + \epsilon_t^l, \quad \epsilon_t^l \sim N(0, \sigma_l)$$

, that is, l_t varies with both s_t and GRO_t as well as having an i.i.d idiosyncratic component. The other parameters in the simulated data are in Table 6. Notice the third row of $K_1^{\mathbb{Q}}$, which was not present in the actual estimates. When we consider assets with payoffs that depend on M_t the risk neutral dynamics of M_t are material although M_t is still constrained not to affect the drifts of s_t and c_t . In principle one could estimate the dynamics with a tracking portfolio for GRO (e.g. Lamont 2001), but here I simply assume that exposure to GRO carries a fixed risk premium λ .

I compute option values for different starting values of $L_0 = \exp(l_0)$, with $S_0 = \exp(s_0)$ equal to \$80 per barrel and $c_0 = 0$. This is meant to mimic an oil firm evaluating wells that differ in their current cost of extraction, conditional on a spot price of \$80 and a flat futures curve.

F Robustness Checks

Upon inspecting Figure 1 a natural question is whether the findings in this paper are driven by the huge swings in oil prices and real activity during 2008-2009. Table 7 presents the model estimate on a subsample from January 1986 to December 2007. Overall the subsample estimate is very similar to the full-sample estimate, and the key coefficients of GRO_{t+1} on s_t , s_{t+1} on GRO , and INV_{t+1} on c_t are virtually unchanged and remain statistically significant at the 5% level. The most notable differences relative to the full sample results are that in the subsample, c_t and INV_t appear to have significant forecasting power for future values of GRO .

Table 8 shows that the forecasting regressions are also very similar, and indeed the incremental forecasting power of GRO is slightly stronger, when we omit the financial crisis.

F.1 Time Varying Volatility

This section examines the results of the forecasting regressions in Table 1 when I add three indexes of time-varying volatility in crude oil prices. $garchvol_t$ is the conditional volatility of Δf_{t+1}^1 estimated as a GARCH(1,1) process. $optvol_t$ is the implied volatility based on the prices of at-the-money options on one month futures. $sqchg_t$ is the squared change $(\Delta f_t^1)^2$ of the front-month futures contract last month. Table 9 shows that the crude oil volatility indexes are all negatively correlated with GRO – oil price volatility is higher when real activity is lower. If higher volatility means a greater hedge premium and a lower expected return to oil futures, then oil price volatility might be an omitted factor that explains the positive association between real activity and the oil price forecast.

This possibility is not borne out in the data, however. Table 10 shows that one of the

Table 7: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures, using data from 1/1986 to 12/2007. s , c are the spot price and annualized cost of carry respectively. GRO is the monthly Chicago Fed National Activity Index. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

	Historical (\mathbb{P}) Measure				
	$K_0^{\mathbb{P}}$	s_t	c_t	GRO_t	INV_t
s_{t+1}	0.011 (0.007)	0.996 (0.014)	0.058 (0.036)	0.029 (0.011)	-0.006 (0.107)
c_{t+1}	-0.013 (0.007)	0.026 (0.015)	0.864 (0.037)	-0.012 (0.011)	0.031 (0.111)
GRO_{t+1}	0.056 (0.035)	-0.171 (0.070)	0.633 (0.178)	0.427 (0.054)	-1.571 (0.533)
INV_{t+1}	0.002 (0.002)	-0.008 (0.004)	0.031 (0.010)	-0.003 (0.003)	0.855 (0.031)

	Risk Neutral (\mathbb{Q}) Measure		
	$K_0^{\mathbb{Q}}$	s_t	c_t
s_{t+1}	-0.003 (0.007)	1.000 (0.005)	0.083 (0.011)
c_{t+1}	0.000 (0.012)	-0.008 (0.013)	0.885 (0.031)

	Shock Volatilities			
	[off-diagonal = % correlations]			
	s	c	GRO	INV
s	0.102			
c	-85%	0.057		
GRO	7%	0%	0.500	
INV	-23%	31%	4%	0.028

Table 8: The table shows the results of forecasting returns to U.S. oil futures. The data are monthly from from 1/1986 to 12/2007. The forecasting variables are 1) three sets of 'reduced-form' state variables \mathcal{P}_t that are based on oil futures prices and 2) the Chicago Fed National Activity Index GRO_t and log U.S. oil inventory INV_t .

Panel A: Forecasting Futures Returns

$$r_{t+1} = \alpha + \beta_{GRO,INV}UM_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	r_{t+1}^2			\bar{r}_{t+1}		
β_{GRO}	0.030*** (0.009)	0.029*** (0.009)	0.023** (0.009)	0.022*** (0.006)	0.021*** (0.006)	0.017*** (0.006)
β_{INV}	-0.043 (0.118)	0.007 (0.125)	-0.007 (0.131)	0.009 (0.076)	0.026 (0.078)	0.036 (0.084)
Spanned Factors \mathcal{P}_t :	$PC^{1,2}$	PC^{1-5}	f^{1-12}	$PC^{1,2}$	PC^{1-5}	f^{1-12}
Adjusted $R^2(\mathcal{P}_t)$	-0.5%	0.2%	5.8%	-0.1%	-0.3%	6.8%
Adj. $R^2(\mathcal{P}_t + M_t)$	2.2%	2.4%	7.0%	2.8%	2.5%	8.3%
F -ratio	4.6**	4.0**	2.7*	4.8***	4.6**	3.0*

Robust standard errors are in parentheses. * : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$

Panel B: Forecasting the Level Factor

$$\Delta PC_{t+1} = \alpha + \beta_{GRO,INV}UM_t + \beta_{\mathcal{P}}\mathcal{P}_t + \epsilon_{t+1}$$

	ΔPC^1 (Change in Level)		
β_{GRO}	0.107*** (0.028)	0.106*** (0.028)	0.087*** (0.028)
β_{INV}	0.047 (0.335)	0.107 (0.353)	0.123 (0.391)
Spanned Factors \mathcal{P}_t :	$PC^{1,2}$	PC^{1-5}	f^{1-12}
Adjusted $R^2(\mathcal{P}_t)$	-0.5%	-1.1%	4.1%
Adj. $R^2(\mathcal{P}_t + M_t)$	3.0%	2.2%	6.1%
F -ratio	5.7***	5.4***	3.7**

Robust standard errors are in parentheses.

* : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$

Table 9: The table shows the correlations of the monthly real activity index GRO and three indexes of time varying volatility in crude oil prices. The data are monthly from 1/1989 to 7/2013. $garchvol_t$ is the conditional volatility of Δf_{t+1}^1 estimated as a GARCH(1,1) process. $optvol_t$ is the implied volatility based on the prices of at-the-money options on one month futures. $sqchg_t$ is the squared change $(\Delta f_t^1)^2$ of the front-month futures contract last month.

	GRO_t	$sqchg_t$	$optvol_t$	$garchvol_t$
GRO_t	1			
$sqchg_t$	-24.8%	1		
$optvol_t$	-54.9%	50.7%	1	
$garchvol_t$	-51.8%	27.6%	68.7%	1

volatility factors is significant in the forecasting regressions, none of them significantly raises the adjusted R^2 , and their inclusion does not alter the forecasting power of GRO .

Table 10: The table shows the results of forecasting returns to oil futures. The data are monthly from 1/1986 to 7/2013, except *optvol* which is monthly from 1/1989 to 7/2013. r_{t+1}^2 is the log excess return to the second nearby oil futures contract. \bar{r}_{t+1} is the average log excess return to all active futures contracts with maturities up to 12 months. The forecasting variables are the Chicago Fed National Activity Index GRO_t , the first two PCs of log oil futures prices, and three indexes of crude oil volatility. $garchvol_t$ is the conditional volatility of Δf_{t+1}^1 estimated as a GARCH(1,1) process. $optvol_t$ is the implied volatility based on the prices of at-the-money options on one month futures. $sqchg_t$ is the lagged squared change $(\Delta f_t^1)^2$ of the first nearby futures contract. The standard errors are Newey-West with six lags.

$$r_{t+1} = \alpha + \beta_{GRO}M_t + \beta_P PC_t^{1,2} + \beta_{VOL}VOL_t + \epsilon_{t+1}$$

	r_{t+1}^2			\bar{r}_{t+1}		
<i>GRO</i> _{<i>t</i>}	0.023** (0.011)	0.028*** (0.011)	0.029*** (0.012)	0.018** (0.009)	0.019** (0.009)	0.022** (0.009)
<i>optvol</i> _{<i>t</i>}	-0.009 (0.018)			0.000 (0.014)		
<i>sqchg</i> _{<i>t</i>}		-0.166 (0.422)			-0.159 (0.309)	
<i>garchvol</i> _{<i>t</i>}			0.001 (0.008)			0.004 (0.006)
Adjusted $R^2(\mathcal{P}_t + GRO_t)$	4.1%	4.5%	4.5%	2.9%	3.1%	3.1%
Adj. $R^2(\mathcal{P}_t + GRO_t + VOL_t)$	3.9%	4.4%	4.3%	2.6%	3.0%	3.0%
<i>F</i> -ratio	0.4	0.4	0.0	0.0	0.5	0.5

Robust standard errors are in parentheses.* : $p < 0.10$, ** : $p < 0.05$, *** : $p < 0.01$