

How Loan Modifications Influence the Prevalence of Mortgage Defaults

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Abstract

How much can government-driven mortgage modification programs reduce the mortgage default rate? I compare an economy without a modification option to one with easy modifications, and evaluate the impact of these loan modifications on the foreclosure rate. Through loan modification, mortgage servicers can mitigate their losses and households can improve their financial positions without having to walk away from their homes.

When modifying loan contracts is prohibitively costly, the default rate increases 1.5 percentage points in response to a 2007-style unexpected drop in housing prices of 30%. I calibrate the cost of modification after the financial crisis to match the Home Affordable Modification Program (HAMP) modification rate of 0.68%. My quantitative exercises show that current government efforts to promote mortgage modifications reduce the mortgage default rate by 0.63 percentage points.

Keywords: Mortgage default, Mortgage modification, Home Affordable Modification Program (HAMP), Financial crisis

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1 Introduction

Starting in late 2007, the US housing market entered a period of decline. One of the more notable changes observed was the sudden increase in the mortgage default rate. Many papers have examined the potential causes of the housing crisis and the scope for policy to improve the financial situation of affected households. In this paper, I propose a quantitative model of mortgage default that allows for modification of mortgage terms and analyze the impact of government-driven modification programs in reducing mortgage defaults.

As shown in Figure 1, commercial banks' residential mortgage charge-off rate abruptly increased after 2007. In turn, the US government, along with many financial sector entities, attempted to mitigate losses incurred by creditors and to reduce foreclosure rates among debtors ([18]Gerardi and Li (2010)). While the number of modifications has increased since late 2007, as shown in Figure 1, the effectiveness of loan modification programs are still unclear.

[Figure 1 here]

In this paper, I formulate a mortgage default model with loan modification under symmetric information between creditors and debtors. When housing price declines are correlated with negative income shocks, the budget set of mortgage holders shrinks. When drops in housing prices are large enough to produce negative housing equity, a financially constrained mortgage holder has less incentive to repay debt or sell the house to relieve budget tightness. While default is always an option, default has further costs, including a bad credit history. When a household chooses to default, the mortgage servicer forecloses on the house and resells it in the market. However, at the same time, the financial intermediary needs to pay a foreclosure cost, estimated as 30% of the housing price ([36]Posner and Zingales (2009)). Since household default incurs costs for both parties, there may be room for a mutually beneficial renegotiation of the loan contract. By reducing payments in order to prevent mortgage default, mortgage servicers could mitigate foreclosure costs. At the same time, mortgage holders might be better off by staying in their current houses and saving their default-related costs.

In order to evaluate the benefit of modifications, I construct a dynamic model where a household makes saving decisions, housing purchase decisions, housing size decisions, down payment decisions, and selling and default decisions. For simplicity, there is only one type of mortgage contract: a 30-year fixed rate mortgage. Each household faces three types of

exogenous shocks: income, housing price, and moving shocks. For example, a household might move for their children’s education or for health reasons. Once a household receives a moving shock, it has to vacate its home immediately after closing housing-related contracts. Hence, the moving shock captures involuntary home exit and the indirect cost of owning a house.

In addition to this basic model, I introduce a loan modification option to examine its effect on the household default rate. Once a household initially wants to default on its mortgage debt, a mortgage servicer can reduce current mortgage debt up to the point where the value of defaulting and the value of repaying are the same. Lenders find this optimal whenever the expected present value of cash inflows under the modified loan is larger than the value of the defaulted housing net of any foreclosure costs. Under this modification scheme, the contract is efficient and all the surplus is taken by the mortgage servicer. Since there is no information asymmetry between creditors and debtors, *ex ante* it is a state-contingent contract, and *ex post* it is a loan modification contract. The structure of my modification contract is similar to [21]Harris and Holmstrom (1982), where the contracted long-term wage jumps up when the outside market wage goes up.

I calibrate the model to pre-crisis data using the model without loan modification. This choice reflects the fact that modifying mortgage contracts before the housing crisis was likely prohibitively costly, possibly due to contract or securitization frictions ([27]Keys et al. (2013)). Through securitization, the ownership of mortgage bonds is transferred from loan originators to investors. Hence, mortgage servicers, which manage the mortgage schedules and coordinate the multiple investor relationships created through securitization, have less incentive to initiate any modifications. In fact, loan modification was a very rare event before the housing crisis. According to the Office of the Comptroller of the Currency, “[the] number of modifications done prior to 2008 was very small, almost negligible.” Further, “As a loss mitigation tool, modifications were not typically used as assistance prior to the downturn in the economy and housing market focused on assisting homeowners with short-term credit repair.”

However, after the outbreak of the housing crisis, the costs of modifying mortgage terms may have became lower due to government-driven foreclosure prevention policies. A financially constrained household that is struggling to make mortgage payments may be eligible for its loans to be modified under the Home Affordable Modification Program (HAMP). Also, financial institutions that participate in the program receive monetary incentives from

the government. These efforts to promote mortgage modifications reduce the effective costs of modifying loan contracts.

In my model, when a mortgage servicer modifies a loan contract, it incurs costs proportional to the debt principal. If the costs of modifying the loan contract are large enough, the mortgage servicer will let households default and recover the house value net of foreclosure costs, rather than renegotiate the loan contracts. In turn, no modifications occur in the steady-state. Through calibration, I match the steady-state default rate to the pre-crisis mortgage default rate, which is 1.5%. Under the other extreme case where there are no modification frictions, the steady-state default rate is almost zero. Depending on the modification cost, the steady-state default rate varies between these two extreme cases.

Next, I conduct an experiment mirroring recent events in the housing market. Starting in late 2007, the Case-Shiller index declined around 30% through the recession. At the same time, the mortgage foreclosure rate almost tripled (from 1.5% to 4.2%). Motivated by this observed decline in house prices, I calculate the response of the mortgage default rate from an unexpected drop in average housing prices of 30%. Since financially constrained households are more likely to default on their mortgages after an unexpected house price shock, the mortgage default rate suddenly increases. When modifying loan contracts is prohibitively costly, the default rate increases 1.5 percentage points in response to an unexpected drop in housing prices of 30%, and increases 1.6 percentage points in response to unexpected simultaneous drops in house prices of 30% and income of 10%.

The loan modification structure presented here is similar to certain aspects of the HAMP, which began in 2009. I calibrate the cost of modification after the financial crisis to match the 2011 HAMP modification rate of 0.68%. My quantitative results show that this type of mortgage modification program reduces the mortgage default rate by 0.63 percentage points. When the government doubles program spending, the mortgage default rate can be decreased by additional 0.37 percentage points.

1.1 Related Literature

After the housing market crash, several foreclosure prevention policies were introduced ([18]Gerardi and Li (2010) and [39]Robinson (2009)). However, the potential ability of further mortgage loan modifications to complement and improve on these initiatives continued to be emphasized. ([28]Levitin (2009) and [42]White (2008)).

Then, why are financial institutions hesitant to modify loan contracts? Mortgage holders' strategic behavior, especially "redefault" and "self-cure" risk, might be one reason ([1]Adelino et al. (2013) and [16]Foote et al. (2009)). Redefaults are when a borrower who receives a modification still ends up in delinquency or default. Self-cure refers to delinquent borrowers who would become "current" in their repayment schedule without receiving any modification. However, empirical studies draw different conclusions regarding the potential for such strategic household behavior. [24]Haughwout et al. (2010) finds that the redefault rate decreases as monthly payments or the debt principal are reduced. Conversely, [30]Mayer et al. (2014), and [16]Foote et al. (2009) find the opposite.

Contract frictions between borrowers and lenders, especially as generated by securitization, is another obstacle that hinders loan modification. Empirical research shows that securitized loans are less likely to be renegotiated than non-securitized loans ([35]Piskorski et al. (2010), [2]Agarwal et al. (2011), and [27]Keys et al. (2013)). Similarly, the performance of securitized loans is worse than that of non-securitized loans ([15]Elul (2011) and [26]Jiang et al. (2014)). Again, however, the literature is split here as well, with some arguing mortgage loan securitization does not affect loan renegotiation ([1]Adelino et al. (2013)).

One general agreement within this literature is that modifying loans incurs some cost. Depending on how big the cost is, a loan modification program may or may not be effective. This speaks to the need for a quantitative approach. However, to the best of my knowledge, there are no papers that analyze the effect of loan modification on reducing the foreclosure rate in a quantitative manner. In this paper, I compare an economy without a modification option to one with easy modifications and I evaluate the effectiveness of government-driven modification programs in reducing foreclosures.

In a related vein, there is a literature quantitatively examining what drove the observed increase in mortgage defaults. The introduction of unconventional mortgage contracts was the main reason according to [6]Campbell and Cocco (forthcoming) and [11]Corbae and Quintin (2015). Others claim that a positive housing supply shock, along with credit constraints and delays in the foreclosure process, was the driving force ([9][10]Chatterjee and Eyigungor (2009, forthcoming)). In this paper, the main driving force of the huge increase in the foreclosure rate is the optimistic belief in the future housing market and the interest rate subsidies to low-income households followed by an unexpected drop in house prices. Related to this mechanism, [5]Burnside et al. (forthcoming) explained the boom and bust of housing market through agents' expectation about long-run fundamentals. In addition, [33]Paul

(2008), [38]Roberts (2008), and [32]Mian and Sufi (2009) emphasized that low-income households could easily take out mortgages with low prices before the housing market crash. I will examine the model impact of this potential unwarranted optimism and subsidies to low-income households in this paper.

The model structure presented here is similar to [11]Corbae and Quintin (2015), [9][10]Chatterjee and Eyigungor (2009, forthcoming), [22]Hatchondo et al. (2014), [25]Jeske et al. (2013), [20]Guler (forthcoming), and [3]Arslan et al. (forthcoming). ([12]Davis and Van Nieuwerburgh (forthcoming) review extensive empirical and theoretical housing market papers.) However, these papers do not have a loan modification option and thus both my steady-state and transition exercises are somewhat unique in comparison ([12]Davis and Van Nieuwerburgh (forthcoming)).

The remainder of the paper is organized as follows. Section 2 introduces a basic model that does not have a loan modification option. Section 3 introduces a model with a loan modification option. Section 4 calibrates the model. Section 5 reports the steady-state results. Section 6 presents households' responses from a sudden drop in housing prices. Section 7 analyzes the effectiveness of current US housing policies. Section 8 concludes the paper.

2 Model with No Modification

Time is discrete and infinite. There are two market participants: households and mortgage servicers (or financial intermediaries). Households either young or old. Households stochastically move from young to old with a probability of ρ_O , and then die with a probability of ρ_D . The total measure of households is constant over time. The household's expected utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, s_t)$$

where c_t is consumption, $h_t \in \{h_S, h_L\}$ is housing size with $h_S < h_L$, small and large, and $s_t \in \{0, 1\}$ is an indicator function that is 1 (or H) if a household is a homeowner, and 0 (or R) if a household is a renter. When a household is a renter, the only available housing size is h_S . However, prospective homeowners can choose to buy either $\{h_S, h_L\}$. The household's

utility is defined by

$$u(c, h, s) = \begin{cases} \frac{[c^{1-\omega} h^\omega]^{1-\xi}}{1-\xi} & \text{if a renter, } h = h_S \\ \frac{[c^{1-\omega} \{h(1+\kappa)\}^\omega]^{1-\xi}}{1-\xi} & \text{if a homeowner, } h \in \{h_S, h_L\} \end{cases}$$

where κ is an extra utility gain when a household lives in owner-occupied housing. With a larger house, a household receives an extra utility gain. A household can own at most one house.

A household is born with zero assets and starts life as a renter. While young, a household receives stochastic income (e) and accumulates financial assets (a). Also, the household decides whether to buy a house and assume a mortgage contract or remain a renter. If a household decides to remain a renter, it keeps accumulating financial assets. If a household decides to buy a house, it chooses the housing size and the fraction of down payment $\eta \in [0, 1]$. A transaction cost, which is χ_B fraction of the housing value, is also paid. There is only a fixed-rate mortgage (FRM) with N contract periods available in the mortgage market.¹ A mortgage contract is only for buying housing. There is no refinancing option or junior liens of a mortgage.

Once a household becomes a homeowner, it chooses one of three options: making periodic payments (x) while staying in the house, selling the house and becoming a renter, and defaulting on its mortgage debt. If a household defaults, it becomes a renter and is not eligible to buy a house for some period of time. With a probability of γ , the defaulted household recovers a good credit record and becomes a renter eligible to buy a house. When a household decides to sell a house, it has to pay a transaction cost, which is χ_S fraction of the housing value. If a household repays all of the mortgage debt before becoming old, it can stay in or sell the house. Since there is no remaining debt at this point, a default option is not needed.

While owning a house, a household receives a moving shock with a probability of μ . This moving shock captures the household's involuntary move to other places. When a household with mortgage debt receives a moving shock, it is forced to sell the house or default on its mortgage debt. When a household without mortgage debt receives a moving shock, it is forced to sell the house and become a renter.

¹According to the Survey of Consumer Finances (SCF), a 30-year fixed-rate mortgage is the most prevalent type of mortgage product.

When a household becomes old, it becomes a renter, staying in the h_S housing size unit, provided by the government (or a nursing home). Hence, it is assumed that old households do not need to pay rental costs. Also, it is assumed that the old household receives a fixed amount of periodic income and dissaves financial assets before dying. Since there is no bequest motive, old age is a period for spending all of the household's remaining wealth. A fair annuity market exists during the old-age period.

In this model, the house supplier has large enough resources, and the housing supply is infinitely elastic. The house supplier prices each house with a unit housing price of $p \in \wp$. Hence, a household can choose any type of house if it can afford without affecting market house prices. However, households face a so-called housing price shock, where the unit housing price stochastically changes over time following a Markov process. Applying the law of large numbers, the measure of supplied houses with a unit price of $p \in \wp$ is constant. The house supplier also provides rental housing with a unit rental price of $\theta(p)$. For each sub-market of housing with a unit price of $p \in \wp$, there should be no arbitrage opportunity from providing rental housing. Then, for each unit housing price $p \in \wp$, the unit rental price is determined by

$$p = \theta(p) + \frac{\theta(p)}{1 + r_f} + \frac{\theta(p)}{(1 + r_f)^2} + \dots$$

where r_f is the risk-free interest rate.

Overall, the model has three types of exogenous shocks: an income shock, a unit housing price shock, and a moving shock. A moving shock affects the margin whether a homeowner exits his/her house or not. However, a unit housing price shock does not directly affect the home exit margin. Instead, it affects the margin of whether to sell a house or default on mortgage debt conditional on home exit.

2.1 Young Household

Young households can have one of four statuses: a renter who is eligible to buy a house (V_R^Y), a renter who is not eligible to buy a house because of a previous history of mortgage default (V_D^Y), a homeowner with mortgage debt (V_H^Y), or a homeowner who has repaid all of the mortgage debt (V_F^Y).

(Renter who is eligible to buy a house) A renter has two options: remaining a renter (V_{RR}^Y), or becoming a homeowner with a mortgage loan contract (V_{RH}^Y). Then, a

young renter solves the following problem:

$$V_R^Y(a, e, p) = \max \{V_{RR}^Y(a, e, p), V_{RH}^Y(a, e, p)\}$$

If a household chooses to remain a renter, its value is given by

$$\begin{aligned} V_{RR}^Y(a, e, p) &= \max_{a'} u(c, h_S, R) + \beta [(1 - \rho_O) EV_R^Y(a', e', p') + \rho_O V^O(a')] \\ &\quad s.t. \\ &\quad c + a' + \theta(p) h_S = (1 + r_f) a + e \\ &\quad c \geq 0, a' \geq 0 \end{aligned}$$

where V^O is the value for old households, which will be defined later.

If a household chooses to buy a house with a mortgage contract, it has to choose a housing size (h), the down payment fraction (η), and the amount of saving (a'). Then, the homeowner's state in the next period will be (a, e, p, h, n, x, r_m) , where (n, x, r_m) is the mortgage contract term, which indicates mortgage age (n), periodic payment (x), and mortgage interest rate (r_m). A renter can buy a house if the renter's initial assets are greater than the sum of the down payment, transaction cost, and one periodic repayment.² The mortgage contract terms are endogenously determined by the household's choices and states and will be specified in the mortgage servicer's problem.

Next period, with a probability of $(1 - \rho_O)$, the household will stay young. Conditional on being young, a household receives an exogenous moving shock with a probability of μ . Then, the household sells the house or defaults on its debt. If a household does not receive a moving shock, a household will remain a homeowner with mortgage debt. With a probability of ρ_O , the household becomes old. If a household becomes old with remaining mortgage debt, it chooses one of two options: having housing equity net of the mortgage debt or giving up net housing equity.

²This means that a renter can buy a house if $(\eta + \chi_B) ph + x \leq (1 + r_f) a$. The assumption prevents zero-asset/very low asset households from buying a house. According to "How to Buy a Home With a Low Down Payment" from the Federal Citizen Information Center (FCIC), a household that wants to buy a house with a low down payment should have enough cash to cover the down payment, related expenses, and two months of periodic payments. This inequality captures such a constraint.

$$\begin{aligned}
& V_{RH}^Y(a, e, p) \\
= & \max_{\substack{a', h \in \{h_S, h_L\}, \\ \eta \in [0, 1]}} u(c, h, H) + \beta \left[\begin{array}{l} (1 - \rho_O) \mu E \max \left\{ \begin{array}{l} V_{HS}^Y(a', e', p', h, 1, x, r_m), \\ V_D^Y(a', e', p') \end{array} \right\} \\ + (1 - \rho_O)(1 - \mu) EV_H^Y(a', e', p', h, 1, x, r_m) \\ + \rho_O E \max \{ V^O(a' + p'h - d'), V^O(a') \} \end{array} \right] \\
& \quad s.t. \\
& c + a' + (\eta + \chi_B) ph + x = (1 + r_f) a + e \\
& (\eta + \chi_B) ph + x \leq (1 + r_f) a \\
& x = (1 - \eta) ph \frac{r_m}{1 + r_m} \left[1 - \frac{1}{(1 + r_m)^N} \right]^{-1} \\
& d' = x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-1}} \right] \\
& x = x(a, e, p, h, \eta), \quad r_m = r_m(a, e, p, h, \eta) \\
& c \geq 0, a' \geq 0
\end{aligned}$$

(Homeowner without mortgage debt) When a homeowner repays all of the remaining mortgage debt before becoming old, it chooses either to stay in (V_{FK}^Y) or to sell the house (V_{FS}^Y). A household without mortgage debt solves the following problem:

$$V_F^Y(a, e, p, h) = \max \{ V_{FK}^Y(a, e, p, h), V_{FS}^Y(a, e, p, h) \}$$

Once a household chooses to keep the house, the value is given by

$$\begin{aligned}
V_{FK}^Y(a, e, p, h) = & \max_{a'} u(c, h, H) + \beta \left[\begin{array}{l} (1 - \rho_O) \mu E V_{FS}^Y(a', e', p', h) \\ + (1 - \rho_O)(1 - \mu) E V_F^Y(a', e', p', h) \\ + \rho_O E V^O(a' + p'h) \end{array} \right] \\
& \quad s.t. \\
& c + a' = (1 + r_f) a + e \\
& c \geq 0, a' \geq 0
\end{aligned}$$

Since there is no remaining mortgage debt, a default option is not needed. When a homeowner receives a moving shock with a probability of μ , it is forced to sell the house.

If a homeowner sells the house and becomes a renter, her value is given by³

$$\begin{aligned}
V_{FS}^Y(a, e, p, h) &= \max_{a'} u(c, h_S, R) + \beta \left[(1 - \rho_O) EV_R^Y(a', e', p') + \rho_O V^O(a') \right] \\
&\quad s.t. \\
c + a' + \theta(p)h_S &= (1 + r_f)a + e + (1 - \chi_S)ph \\
c \geq 0, a' \geq 0
\end{aligned}$$

(Homeowner with mortgage debt) Once a household becomes a homeowner (V_H^Y) with mortgage debt ($n \leq N - 1$), it has three options: repaying the debt (V_{HP}^Y), selling the house (V_{HS}^Y), or defaulting (V_D^Y). Then, a household with mortgage debt solves the following problem:

$$V_H^Y(a, e, p, h, n, x, r_m) = \max \left\{ \begin{array}{l} V_{HP}^Y(a, e, p, h, n, x, r_m), \\ V_{HS}^Y(a, e, p, h, n, x, r_m), \\ V_D^Y(a, e, p) \end{array} \right\}$$

If a household chooses to repay its debt, with $n < N - 1$, the value is given by

$$\begin{aligned}
&V_{HP}^Y(a, e, p, h, n, x, r_m) \\
&= \max_{a'} u(c, h, H) + \beta \left[\begin{array}{l} (1 - \rho_0) \mu E \max \left\{ \begin{array}{l} V_{HS}^Y(a', e', p', h, n + 1, x, r_m), \\ V_D^Y(a', e', p') \end{array} \right\} \\ + (1 - \rho_O)(1 - \mu) EV_H^Y(a', e', p', h, n + 1, x, r_m) \\ + \rho_O E \max \{ V^O(a' + p'h - d'), V^O(a') \} \end{array} \right] \\
&\quad s.t. \\
c + a' + x &= (1 + r_f)a + e \\
d' &= x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n-1}} \right] \\
c \geq 0, a' \geq 0
\end{aligned}$$

³When a homeowner decides to sell a house, the household receives the housing price (net of transaction cost) and vacates the house. The household then moves to a rental house and pays periodic rent. Hence, a household is a homeowner in the first half of a period, and a renter in the last half of a period. I assume that the current utility is $u(c, h_S, R)$, which is the utility of the last half of a period.

When a household receives a moving shock which occurs with probability μ , it chooses to sell the house or default on its mortgage debt. Conditional on not receiving a moving shock, the household's value in the next period is given by V_H^Y .

When $n = N - 1$, a household repays the last periodic payment. Then, it solves the following problem:

$$\begin{aligned}
& V_{HP}^Y(a, e, p, h, N - 1, x, r_m) \\
&= \max_{a'} u(c, h, H) + \beta \left[\begin{array}{l} (1 - \rho_O) \mu EV_{FS}^Y(a', e', p', h) \\ + (1 - \rho_O) (1 - \mu) EV_F^Y(a', e', p', h) \\ + \rho_O EV^O(a' + p'h) \end{array} \right] \\
& \quad s.t. \\
& \quad c + a' + x = (1 + r_f) a + e \\
& \quad c \geq 0, a' \geq 0
\end{aligned}$$

When $n \leq N - 1$, if a household with mortgage debt chooses to sell the current house and repay all of the remaining debt, the household becomes a renter eligible to buy a house.

$$\begin{aligned}
V_{HS}^Y(a, e, p, h, n, x, r_m) &= \max_{a'} u(c, h_S, R) + \beta \left[(1 - \rho_O) EV_R^Y(a', e', p') + \rho_O V^O(a') \right] \\
& \quad s.t. \\
& \quad c + a' + \theta(p) h_S + d = (1 + r_f) a + e + (1 - \chi_S) ph \\
& \quad d = x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \\
& \quad c \geq 0, a' \geq 0
\end{aligned}$$

If a household defaults on its mortgage debt, it is not eligible to buy a house for some period of time as a default penalty. With a probability of γ , a defaulted household becomes eligible to make a new mortgage contract.

$$V_D^Y(a, e, p) = \max_{a'} u(c, h_S, R) + \beta \begin{bmatrix} (1 - \rho_O) \gamma EV_R^Y(a', e', p') \\ (1 - \rho_O)(1 - \gamma) EV_D^Y(a', e', p') \\ + \rho_O V^O(a') \end{bmatrix}$$

s.t.

$$c + a' + \theta(p) h_S = (1 + r_f) a + e$$

$$c \geq 0, a' \geq 0$$

2.2 Old Household

When a household becomes old, it sells any housing equity or defaults and becomes a renter, staying in a small house h_S . For simplicity, it is assumed that there is no rental cost for old people. If a household has accumulated large amounts of wealth while young, it gets utility in old age from spending on consumption goods, rather than from spending on housing services. Old people have a fixed amount of income (e_O). With a probability of ρ_D , the old household dies and there is no bequest motive. The interest rate for assets is given by $\frac{1+r_f}{1-\rho_D}$, which incorporates the annuity value.

$$V^O(a) = \max_{a'} u(c, h_S, R) + \beta (1 - \rho_D) V^O(a')$$

s.t.

$$c + a' = \frac{1 + r_f}{1 - \rho_D} a + e_O$$

$$c \geq 0, a' \geq 0$$

2.3 Mortgage Servicer's Expected Profit

Assume that the mortgage servicing market is competitive and the expected profit of mortgage servicers is zero. Mortgage servicers can freely borrow money at a risk-free interest rate of r_f . Such borrowing by households is not allowed. It is also assumed that there is no information asymmetry between borrowers and lenders.⁴

⁴If we model that there is information asymmetry between borrowers and lenders, we need to change the model structure significantly. For example, suppose mortgage servicers do not observe households' current

The mortgage servicer's expected profit by contracting with an (a, e, p) -type household that chooses a housing size of h and a down payment of η at the time of the loan contract is

$$\Pi^0(a, e, p, h, \eta) = -(1 - \eta)ph + x(a, e, p, h, \eta) + \frac{E\Pi(a', e', p', h, 1, x, r_m)}{1 + r_f}$$

where the first term shows the total outstanding loans and the second term is the periodic repayment after making the loan contract. The last term is the expected cash inflow from the second period of the contract. For notational simplicity, let $\Delta \equiv (a, e, p, h, n, x, r_m)$ and $\Delta' \equiv (a', e', p', h, n + 1, x, r_m)$. After the first period of the contract, the mortgage servicer's expected cash inflow is:

$$\begin{aligned} \Pi(\Delta) = & (1 - \rho_O) \mu I_{MS}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + (1 - \rho_O) \mu I_{MD}(\Delta) \{ (1 - \chi_D) ph \} \\ & + (1 - \rho_O) (1 - \mu) I_P(\Delta) \left\{ x + \frac{E\Pi(\Delta')}{1 + r_f} \right\} \\ & + (1 - \rho_O) (1 - \mu) I_S(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + (1 - \rho_O) (1 - \mu) I_D(\Delta) \{ (1 - \chi_D) ph \} \\ & + \rho_O I_{OP}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + \rho_O I_{OD}(\Delta) \{ (1 - \chi_D) ph \} \end{aligned}$$

where χ_D is the foreclosure cost incurred by a mortgage servicer. $\Pi(\Delta)$ is the expected cash inflow after realizing the household's income and housing price but before realizing the household's age and moving shock status. With a probability of $1 - \rho_O$, a household stays young. With a probability of μ , a household receives a moving shock. Conditional on being young with a moving shock, a household chooses either to sell or to default. $I_{MS}(\Delta)$ and $I_{MD}(\Delta)$ are indicator functions that are 1 if a household chooses to sell or default,

income. Then, mortgage servicers can infer each household's income by observing a household's optimal decisions (saving, default, home purchase, down payment, and selling decisions). That is, though there is information asymmetry between borrowers and lenders, households reveal their types (or hidden information) to mortgage servicers automatically under this model structure with a simple addition of information asymmetry. Future potentially fruitful research in this thread may include a model with information asymmetry between borrowers and lenders involving Bayesian approaches, such as [8]Chatterjee et al. (2011) or [14]D'Erasco (2011).

respectively, conditional on a moving shock, and 0 otherwise. If a household decides to sell the house, the mortgage servicer recovers the entire loan, $x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$. If a household defaults following the moving shock, the mortgage servicer recovers the collateral value (housing value) net of the foreclosure cost, $(1 - \chi_D) ph$.

Conditional on being young and not receiving a moving shock, a household chooses either repayment, selling, or default. $I_P(\Delta)$, $I_S(\Delta)$, and $I_D(\Delta)$ are indicator functions that are 1 if a household chooses to repay, sell, or default, respectively, and 0 otherwise. If a household repays its debt, the expected cash inflow is $x + \frac{E\Pi(\Delta')}{1+r_f}$. The first term is the household's periodic payment, and the second term is the expected cash inflow when the mortgage contract persists. If a household sells the house, the expected cash inflow is $x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$. If a household defaults on its debt, the cash inflow is $(1 - \chi_D) ph$.

A household becomes old with a probability of ρ_O . $I_{OP}(\Delta)$ and $I_{OD}(\Delta)$ are indicator functions that are 1 if a household repays, or defaults on its debt right after being old, respectively, and 0 otherwise.⁵ Then, the mortgage servicer can recover the entire mortgage loan, $x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$, conditional on the household's repayment. If the household chooses not to repay its debt, due to having negative equity in their house, the mortgage servicer's cash inflow is given by $(1 - \chi_D) ph$.

If a household repays all of the remaining debt, $n \geq N$, the expected cash inflow is zero. That is,

$$\Pi(\Delta) = 0 \text{ if } n \geq N$$

Since the mortgage market is competitive, the mortgage servicer's expected profit is zero for every feasible state.

$$\Pi^0(a, e, p, h, \eta) = 0$$

In addition to the zero profit condition, the periodic payment and interest rate are pinned down by the following fixed-rate mortgage condition:⁶

$$(1 - \eta) ph = x(a, e, p, h, \eta) + \frac{x(a, e, p, h, \eta)}{1 + r_m(a, e, p, h, \eta)} + \frac{x(a, e, p, h, \eta)}{(1 + r_m(a, e, p, h, \eta))^2} + \dots + \frac{x(a, e, p, h, \eta)}{(1 + r_m(a, e, p, h, \eta))^{N-1}}$$

⁵ $I_{OP}(\Delta) = 1$ if $ph \geq x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$, and 0 otherwise. $I_{OD}(\Delta) = 1$ if $ph < x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$, and 0 otherwise.

⁶ I follow the equilibrium pricing concept in [11]Corbae and Quintin (2015).

2.4 Definition of a Steady-State Equilibrium

A steady-state equilibrium consists of value functions, household policy functions, mortgage contract schedules, and an invariant distribution Ψ such that

1. Household policies are optimal given the mortgage contract schedule.
2. The mortgage servicer's zero profit condition and the fixed-rate mortgage condition hold for every state (a, e, p) and every feasible mortgage contract term $\{r_m(a, e, p, h(a, e, p), \eta(a, e, p)), x(a, e, p, h(a, e, p), \eta(a, e, p))\}$.
3. The cross-sectional distribution Ψ is invariant given optimal policies and mortgage contract schedules.

3 Loan Modification Model

In this section, I add a loan modification option to the basic model. The main aim of introducing the possibility of loan modification is to evaluate the effectiveness of modifications in reducing mortgage defaults when the average housing price suddenly drops, as in the recent recession.

When a mortgage holder wants to default on her mortgage debt, a mortgage servicer has the option to reduce the mortgage debt principal to the point where the values of defaulting and not defaulting are the same. However, the amount of written-off debt must be small enough that the expected present value of cash inflows with a modified loan is larger than the housing value (or collateral value) net of foreclosure costs. This type of modification rescues the marginal defaulters from walking away from their homes. At the same time, mortgage servicers mitigate their losses. Since there is no information asymmetry between borrowers and lenders, this is a state-contingent contract at the time of the loan contract, *ex ante*. When a household receives a loan modification after experiencing any type of bad shock, it is a contract with a loan modification, *ex post*.

Conditional on the household choosing default without receiving a moving shock, mortgage servicers reduce the current debt burden x to \tilde{x}_1 , which satisfies the following:

$$V_D^Y(a, e, p) = V_{HP}^Y(a, e, p, h, n, \tilde{x}_1, r_m) \quad (1)$$

Since the values of defaulting and repaying are now the same, households can stay in their home by repaying a modified amount \tilde{x}_1 . However, mortgage servicers have an incentive to

agree to modification only when they expect a larger return after the modification. That is, the modified amount \tilde{x}_1 has a lower bound:

$$\underbrace{(1 - \chi_D) ph}_{\text{Expected cash inflow with default}} \leq \tilde{x}_1 + \underbrace{\frac{E\Pi(a', e', p', h, n + 1, \tilde{x}_1, r_m)}{1 + r_f} - \alpha A}_{\text{Expected cash inflow with modification}} \quad (2)$$

When a mortgage servicer modifies a loan contract, it incurs costs proportional to the debt principal. Let $A \left(= x \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}}\right]\right)$ be the current debt principal. Then, the modification cost is given by αA . Note that a' on the right-hand side is the household's saving after modifying the contract.

Conditional on the household's choice of defaulting after the moving shock, mortgage servicers reduce the current debt burden x to \tilde{x}_2 , which is determined by:

$$V_D^Y(a, e, p) = V_{HS}^Y(a, e, p, h, n, \tilde{x}_2, r_m) \quad (3)$$

The modified loan must again provide a larger cash inflow to the mortgage servicers than the original contract. That is,

$$\underbrace{(1 - \chi_D) ph}_{\text{Expected cash inflow with default}} \leq \tilde{x}_2 \underbrace{\frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}}\right] - \alpha A}_{\text{Expected cash inflow with modification}} \quad (4)$$

Given the household's state, a household chooses either to repay, sell, or default. Suppose a household initially chooses default. If condition (2) or (4) does not hold, such households will not receive a modification and end up defaulting. If condition (2) or (4) holds, the loan terms are modified and households get the value of the modified contract, which is equal to the value of default as shown in (1) and (3). Therefore, regardless of whether a household's loan is modified, *ex post*, the household's optimal policy functions are the same as in the base model, *ex ante*, conditional on the loan rate schedule. For simplicity, I assume that a loan is modified only when a household is young.

Expected profit when the mortgage is originated is:

$$\Pi^0(a, e, p, h, \eta) = -(1 - \eta) ph + x(a, e, p, h, \eta) + \frac{E\Pi(a', e', p', h, 1, x, r_m)}{1 + r_f}$$

For notational simplicity, let $\tilde{\Delta}'(\Delta) \equiv (a', e', p', h, n + 1, \tilde{x}_1(\Delta), r_m)$ be the set of state variables after modifying a loan contract. Then, after the mortgage contract is signed, the

expected cash inflow is given by:

$$\begin{aligned}
\Pi(\Delta) = & (1 - \rho_O) \mu I_{MS}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\
& + (1 - \rho_O) \mu I_{MD}(\Delta) \left[\max \left\{ \tilde{x}_2(\Delta) \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right] - \alpha A(\Delta), \right. \right. \\
& \quad \left. \left. (1 - \chi_D) ph, \tilde{x}_2(\Delta) \frac{1+r_m}{r_m} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right] - \alpha A(\Delta) \right\} \right] \\
& + (1 - \rho_O) (1 - \mu) I_P(\Delta) \left\{ x + \frac{E\Pi(\Delta')}{1 + r_f} \right\} \\
& + (1 - \rho_O) (1 - \mu) I_S(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\
& + (1 - \rho_O) (1 - \mu) I_D(\Delta) \max \left\{ \tilde{x}_1(\Delta) + \frac{(1 - \chi_D) ph}{1 + r_f}, \tilde{x}_1(\Delta) + \frac{E\Pi(\tilde{\Delta}'(\Delta))}{1 + r_f} - \alpha A(\Delta) \right\} \\
& + \rho_O I_{OP}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\
& + \rho_O I_{OD}(\Delta) \{(1 - \chi_D) ph\}
\end{aligned}$$

where $A(\Delta)$ is the debt principal in state Δ . $\tilde{x}_1(\Delta)$ and $\tilde{x}_2(\Delta)$ are the modified periodic repayments, which are endogenously determined by (1) and (3).⁷

When the modification cost (α) is large enough, the above problem converges to the “no modification” case. Let $\bar{\alpha}$ be the minimum modification cost that generates zero modification in a steady state. Then, for every α with $\alpha \geq \bar{\alpha}$, loan modifications will not occur in the steady state.

When $n \geq N$, the expected cash inflow is zero.

$$\Pi(\Delta) = 0 \text{ if } n \geq N$$

⁷Since mortgage servicers write off mortgage debt up to the point where the value of defaulting and not defaulting are the same, we can interpret this modification scheme as a Nash bargain where the entire bargaining power is held by the mortgage servicer. We can also extend the model to allow households to have some bargaining power, as in [43]Yue (2010). This extension increases computation time dramatically. Within the problem this means, given mortgage rate schedules, households solve new optimal problems. Mortgage servicers then solve the mortgage rate schedule again, given households’ decisions. This procedure iterates until both sets of optimal solutions converge. Since [43]Yue (2010) assumed a one-period bond contract, the computational work was doable. Unlike her paper, the model suggested here assumes a multi-period contract and in this situation extending the model to give households some bargaining power makes computation prohibitively time consuming. Hence, I simply assume that the mortgage servicer holds all bargaining power, which is also consistent with the HAMP structure as will be explained in section 7.

Since the mortgage servicing market is competitive, the expected profit of mortgage servicers is zero for every feasible state:

$$\Pi^0(a, e, p, h, \eta) = 0$$

In addition to the zero profit condition, the mortgage interest rate and the periodic payment are pinned down by the following fixed-rate mortgage condition:

$$(1 - \eta) ph = x(a, e, p, h, \eta) + \frac{x(a, e, p, h, \eta)}{1 + r_m(a, e, p, h, \eta)} + \frac{x(a, e, p, h, \eta)}{(1 + r_m(a, e, p, h, \eta))^2} + \dots + \frac{x(a, e, p, h, \eta)}{(1 + r_m(a, e, p, h, \eta))^{N-1}}$$

After the housing market crisis, government-driven mortgage modification programs were introduced, which reduced the effective cost of modification. This is captured by the reduction in the modification cost from $\bar{\alpha}$ (or equivalently ∞) to α_1 , where α_1 is less than $\bar{\alpha}$. The reduction in the modification cost should be financed by outside sources, for example, by tax, which could be modelled in several ways. For computational simplicity, I assumed that the modification cost reduction is financed through tax paid by the old households.⁸ Then, the old households' problem under an economy with modification options is the following:

$$\begin{aligned} V^O(a) = \max_{a'} u(c, h_S, R) + \beta(1 - \rho_D) V^O(a') \\ \text{s.t.} \\ c + a' = \frac{1 + r_f}{1 - \rho_D} a + e_O - \tau \\ c \geq 0, a' \geq 0 \end{aligned}$$

where every old household pays a lump-sum tax of τ . The lump-sum tax is determined by the following market clearing condition:

$$\underbrace{\int_{\Delta \in \text{Modified households}} (\bar{\alpha} - \alpha_1) A(\Delta) \Psi(\Delta)}_{\text{Reduction in the modification cost (or government subsidy)}} = \underbrace{\int_{\Delta \in \text{Old households}} \tau \Psi(\Delta)}_{\text{Total modification program funding}} \quad (5)$$

⁸Young households understand that, though they may receive “modification benefits” as homeowners, they need to pay the tax when they are old. In the model, young households who inhabit owner-occupied houses are suffering from a drop in house prices. Since the goal of the modification program is to rescue such financially vulnerable young households, the tax burden is delayed until these households become old.

where Ψ is the stationary distribution.

Before moving to the calibration section, one more thing should be noted. A household who experiences a moving shock will always get a loan modification if $\alpha = 0$. More specifically, the following proposition holds.

Proposition 1. *Suppose $V_D^Y(a, e, p) > V_{HS}^Y(a, e, p, h, n, x, r_m)$, $\chi_D > \chi_S$, and $\alpha = 0$. Then, the following holds:*

$$\begin{aligned} & \max \left\{ (1 - \chi_D) ph, \tilde{x}_2(\Delta) \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ &= \tilde{x}_2(\Delta) \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \end{aligned}$$

Proof. See appendix □

Hence, when the cost of modification is low enough, households can easily get modifications, especially after facing a moving shock.

3.1 Definition of a Steady-State Equilibrium

A steady-state equilibrium consists of value functions, household policy functions, mortgage contract schedules, mortgage modification schedules, tax, and an invariant distribution Ψ such that

1. Household policies are optimal given the mortgage contract schedule.
2. The mortgage servicer's zero profit condition and the fixed-rate mortgage condition hold for every renter's state (a, e, p) and every feasible mortgage contract $\{r_m(a, e, p, h(a, e, p), \eta(a, e, p)), x(a, e, p, h(a, e, p), \eta(a, e, p))\}$.
3. The loan modification decision follows (2) and (4), and the modified amount is determined by (1) and (3).
4. The cost of modification is financed by lump-sum tax paid by old households, as shown in (5).
5. The cross-sectional distribution Ψ is invariant given optimal policies and mortgage contract schedules.

4 Calibration

I choose parameters to match the pre-crisis economy using the model without loan modification. As mentioned in the introduction, loan modification was very costly and was a very rare event before the housing crisis. Hence, the model without loan modification is chosen as my benchmark.

A period in this model is two years. Demographic parameters are set as $(\rho_O, \rho_D) = (\frac{1}{22}, \frac{1}{10})$. On average, agents are young for 44 years starting from age 20 and old age lasts for 20 years after the young period.

There are three income grid points for young households, $\{e_u, e_l, e_h\}$. A household receives e_u when unemployed and receives one of $\{e_l, e_h\}$ when employed. Income for either type of employed household, $\{e_l, e_h\}$, follows an AR(1) process:

$$\log(e_{t+1}) = (1 - \rho_e) \log(\bar{e}) + \rho_e \log(e_t) + v_t$$

where $v_t \sim i.i.d. N(0, \sigma_v^2)$ and $\bar{e} = 1$ in the benchmark. $(\rho_e, \sigma_v^2) = (0.9801, 0.0318)$ are taken from [41] Storesletten et al. (2004). Income for unemployed households is zero, $e_u = 0$.⁹

The income transition matrix captures both the probability of switching employment states and how income evolves within each state.

$$\begin{array}{ccc} & e'_u & e'_l & e'_h \\ e_u & \left[\begin{array}{ccc} \pi_{u,u} & 1 - \pi_{u,u} & 0 \\ \pi_{l,u} & \pi_{l,l} & \pi_{l,h} \\ \pi_{h,u} & \pi_{h,l} & \pi_{h,h} \end{array} \right] \\ e_l & & \\ e_h & & \end{array}$$

where the following sub-matrix jointly follows an AR(1) process:

$$\left[\begin{array}{cc} \pi_{l,l} & \pi_{l,h} \\ \pi_{h,l} & \pi_{h,h} \end{array} \right]$$

$\pi_{l,u}$ is the probability that a low-income household becomes unemployed and similarly $\pi_{h,u}$ is the probability that a high-income household becomes unemployed. For simplicity, I assume that $\pi_{l,u} = \pi_{h,u}$. I choose $\pi_{l,u}$ and $\pi_{h,u}$ to match the unemployment rate between 2000

⁹In computation, I use a very small positive number for the unemployed households' income, $e_u = 10^{-4}$. If a renter does not have enough money to pay the periodic rent, I assume that (s)he can stay in a rental house for free in that period, consuming e_u and saving zero.

and 2004, which is around 5%. $1 - \pi_{u,u}$ is the probability that an unemployed household becomes employed, which I match with [40]Shimer (2012).¹⁰ ¹¹ Income for old agents is around 60% of the median income for young agents (2004 SCF).

Following [22]Hatchondo et al. (2014), the unit housing price process follows an AR(1) process.

$$\log(p_{t+1}) = (1 - \rho_p) \log(\bar{p}) + \rho_p \log(p_t) + \varepsilon_t$$

where \bar{p} is the mean housing price and $\varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon^2)$.¹² The persistence parameter and variance of the housing price process are given by $(\rho_p, \sigma_\varepsilon^2) = (0.9409, 0.5861)$. Using the Tauchen method, I discretize the housing price process with five grid points.

Following [19]Gruber and Martin (2003), the transaction costs of buying and selling a house are 2.5% and 7% of the housing price, respectively. From [34]Pennington-Cross (2006), the foreclosure cost incurred by a mortgage servicer is 22% of the housing value.

There is no consensus as to how long mortgage defaulters are excluded from the mortgage market. For unsecured, or credit card, debt, a bad credit record lasts for 10 years ([7]Chatterjee et al. (2007)). [9]Chatterjee and Eyigungor (2009) assume that mortgage defaulters are excluded from credit markets for 3.33 years. [10]Chatterjee and Eyigungor (forthcoming) assume 4 years and [20]Guler (forthcoming) assumes 7 years. I choose 4 years as the average exclusion period ($\gamma = 0.5$).

¹⁰This data was constructed by Robert Shimer. For additional details, please see [40]Shimer (2012) and his webpage <http://sites.google.com/site/robertshimer/research/flows>.

¹¹Since one model period is two years, the average duration of unemployment is calculated by $2/(1 - \pi_{u,u}) (= 2(1 - \pi_{u,u}) + 4\pi_{u,u}(1 - \pi_{u,u}) + 6\pi_{u,u}^2(1 - \pi_{u,u}) + \dots)$. Given the calibrated value of $1 - \pi_{u,u}$ as 0.5997, the average duration of unemployment is 3.34 years, which is much longer than actual unemployment spell. According to the Current Population Reports from the Census, the mean (median) unemployment spell duration per unemployed worker is 1.5 (1.8) months between 2004 and 2007. Hence, the unemployment shock in the model seems to be much severer than the real world shock. However, the model does not include households' expenditure shocks, such as medical expenditure, divorce, or child shocks, as explained in [29]Livshits et al. (2007). The unemployment shock implicitly contains those unmodelled expenditure shocks. There is also a technical reason that I need such a harsh unemployment shock. I initially calculated the model without including an unemployment shock. In that case, defaults are driven almost exclusively by moving shocks, not by income shocks, which I think is not a natural result.

¹²[22]Hatchondo et al. (2014) and [6]Campbell and Cocco (forthcoming) calibrate a process for ε , which is correlated with the persistent component of income. In those papers, housing size is uniform, which necessitates a correlation between housing price and income. In this paper, since the household endogenously chooses housing size, I can relax this assumption.

Table 1: Calibration

Parameter	Description	Value	Target / Source
<i>Non-target parameters</i>			
ρ_O	Probability of being old	$\frac{1}{22}$	44 years of young life
ρ_D	Probability of dying	$\frac{1}{10}$	20 years of old life
ρ_e	Persistence in income process	0.9801	Storesletten et al. (2004)
σ_v^2	Variance of income process	0.0318	Storesletten et al. (2004)
ρ_p	Persistence in housing price	0.9409	Hatchondo et al. (2011)
σ_ε^2	Variance of housing price	0.5861	Hatchondo et al. (2011)
ω	Utility parameter	0.24	Chatterjee and Eyigungor (2009)
ξ	Utility parameter	2	Chatterjee and Eyigungor (2009)
χ_B	Transaction cost - Buying	0.025	Gruber and Martin (2003)
χ_S	Transaction cost - Selling	0.07	Gruber and Martin (2003)
χ_D	Foreclosure cost	0.22	Pennington-Cross (2006)
γ	Credit recovery rate	0.5	4 years of exclusion
N	Contract periods	15	30 year contract
r_f	Risk free interest rate	0.08	2-year risk-free rate
e_O	Income for old households	0.6	2004 SCF
h_S	Small housing size	1	Normalized to one
\bar{e}	Average income	1	Normalized to one
$\pi_{l,u} = \pi_{h,u}$	Prob. of being unemployed	0.0315	Unemployment rate 5%
$1 - \pi_{u,u}$	Prob. of being employed	0.5997	Shimer (2007)
<i>Target parameters</i>			
β	Discount factor	0.7	Financial asset-to-income ratio
μ	Moving shock	0.08	Mortgage default rate
κ	Homeowner's extra utility gain	4	Homeownership rate
\bar{p}	Average housing price	1.2	Rent-to-income ratio
h_L	Big housing size	1.2	House value-to-income ratio

The risk-free interest rate is 4% per year ($r_f = 0.08$). The preference parameters $(\omega, \xi) = (0.24, 2)$ are chosen from [9]Chatterjee and Eyigungor (2009). The small housing size is normalized to 1. The mortgage contract lasts for 30 years (or $N = 15$).

Six free parameters remain: discount factor (β) , moving shock (μ) , homeowner's extra utility gain from owning (κ) , average unit housing price (\bar{p}) , big housing size (h_L) , and

modification cost (α). Since there are no modification options in the benchmark model, the modification cost parameter is not used in the benchmark calibration (or we can find the minimum value of α that makes steady state modification rate to zero, $\bar{\alpha}$).¹³ I jointly match the mortgage default rate, which is 1.5% per year before the housing crisis ([11]Corbae and Quintin(2015)), the homeownership rate, which is 68.1% (2001-2004 Census), the ratio of average annual rent to average annual income for renters, which is 0.21 (2004 SCF), the ratio of average housing value to average annual income for homeowners, which is 2.71 (2004 SCF), and the ratio of average financial assets to average annual income, which is 1.65 (2004 SCF). Table 1 summarizes all model parameters and targets.

5 Steady-State Results

In this section, I report quantitative results with and without loan modification. First, I compare steady-state economies with different modification costs. I then analyze the financial characteristics of households that decide to either default or sell their house and subsequently enter a new owner-occupied house, using model-generated data. This allows me to compare the financial characteristics of households that receive loan modifications to similar households that cannot receive loan modifications. Finally, the effects of the moving shock, and foreclosure costs on the steady state are analyzed.

5.1 Steady State

Table 2 contrasts steady-state economies with and without loan modification options. When the modification cost α is greater than or equal to 0.53 (or $\bar{\alpha} = 0.53$), the steady-state modification rate is zero, which represents the pre-crisis environment. When the modification cost is prohibitively high, the calibrated results show that the annual mortgage default rate is 1.47% (1.5% in the data), the homeownership rate is 68.7% (68.1% in the data), the ratio of average housing value to average income is 2.71 (2.71 in the data), the ratio of average annual rent to average annual income is 0.22 (0.21 in the data), and the ratio of average financial assets to average income is 1.66 (1.65 in the data). These are the target moments that I matched through calibration.

I now compare a steady-state economy without a modification option to one with easy

¹³When $\alpha \geq 0.53$, given parameters in table 1, the steady state modification rate is zero.

Table 2: Steady state

	Data	No Mod ($\alpha \geq 0.53$)	Costly Mod ($\alpha = 0.265$)	Costless Mod ($\alpha = 0$)
<i>Targeted statistics with no modification model</i>				
Annual default rate	1.5%	1.47%	1.14%	0.04%
Homeownership rate	68.1%	68.7%	69.3%	72.03%
Average(Housing price) Average(Annual income for homeowners)	2.71	2.71	2.71	2.80
Average(Rent) Average(Annual income for renters)	0.21	0.22	0.23	0.23
Average(Financial asset) Average(Annual income)	1.65	1.66	1.68	1.75
<i>Non-targeted statistics</i>				
Modification acceptance rate	0.45	0	0.41	0.98
Avg interest rate (30 year FRM real rate, 95-04)	4.87%	6.20%	6.49%	6.27%
Average(Originated loan) Average(Annual income)	2.72	2.77	2.97	3.63
Average(Annual periodic payment) Average(Annual income)	0.14	0.18	0.19	0.22
Coeff of variation (Housing value)	0.74	0.70	0.70	0.70
Loan-to-value ratio (Loan originators)	0.86	0.86	0.91	0.99
Loan-to-value ratio (Defaulting households)	1.62	2.1	1.83	1.12
Home exit rate	15%	13.47%	13.40%	13.73%
Fraction of housing exit driven by selling		83.73%	87.11%	99.57%
Fraction of housing exit driven by moving shock		49.03%	49.65%	48.04%
Fraction of default driven by moving shock		81.73%	94.3%	0%
Annual modification rate		0%	0.80%	2.88%
Fraction of negative equity households		29.65%	32.08%	33.62%

modification (or $\alpha = 0$).¹⁴ The annual mortgage default rate is reduced from 1.47% to 0.04% when a costless modification option is introduced. With costless modification, the homeownership rate increases from 68.7% to 72.03% (68.1% in the data), and the average interest rate increases from 6.2% to 6.27% (4.87% in the data). The average loan-to-value (LTV) ratio at the time of loan origination is 0.86 (0.99), and the average LTV ratio of defaulting households is 2.1 (1.12) in a “no modification” (“costless modification”) model, whereas in the data LTV at origination is around 0.86,¹⁵ and the median LTV of defaulting

¹⁴When $\alpha = 0$, the lump-sum tax paid by old households is given by $\tau = 0.075$. Or, it is 12.5% ($= \frac{\tau}{e_O} = \frac{0.075}{0.6}$) of the old households’ income.

¹⁵According to [20] Guler (forthcoming), the average down payment is 20% in 2002-2006 and 25% in 1992-

households is 1.62.¹⁶ The home exit rate among homeowners is around 13% in both models, which is close to the data.¹⁷ The fraction of housing exit driven by selling is 99.57% in a “costless modification” model, and 83.73% in a “no modification” model (the fraction of housing exit driven by default is 0.43% in a “costless modification” model, and 16.27% in a “no modification” model). The fraction of housing exit driven by a moving shock is around 49% in both models. (The fraction of voluntary housing exit is 51%.) The fraction of negative equity households among mortgage holders is 33.62% in a “costless modification” model, and 29.65% in a “no modification” model.

In the second column, the cost of modification is halved (or $\alpha = 0.265$).¹⁸ As the modification cost falls, the default rate decreases. At the same time, the modification rate increases. Unsurprisingly, the results for this scenario are consistent between those of the baseline and costless modification economies.

To better understand these quantitative results, we need to see the mortgage servicers’ profit function. In a model with modification, the mortgage servicers’ expected present value of cash inflows at the time of the loan contract is higher than without modification, *ceteris paribus*, since all the modification surplus accrues to the mortgage servicers. Then, with modification, the competitive mortgage servicers will reduce the interest rate by meeting the zero profit condition. Since the interest rate schedule with loan modifications is lower, renters

1995, whereas [23]Haughwout et al. (2008) find that the median combined initial loan-to-value ratio of subprime loans is 0.8 in 2001 and 2002, 0.84 in 2003, 0.85 in 2004, 0.87 in 2005, 0.9 in 2006, and 0.87 in 2007. [27]Keys et al. (2013) report that both the average loan-to-value ratio and the average combined loan-to-value ratio are 0.85 in 2001. [13]Demyanyk and Van Hemert (2011) find that the combined loan-to-value ratio of the first lien subprime loans is 0.794 in 2001, 0.801 in 2002, 0.82 in 2003, 0.836 in 2004, 0.849 in 2005, 0.859 in 2006, and 0.828 in 2007.

¹⁶[4]Bhutta et al. (2010) report that the median borrower does not strategically default until the loan-to-value ratio is 1.62 after distinguishing between defaults induced by job losses and income shocks from those induced purely by negative equity. Since they used data that only cover Arizona, California, Florida, and Nevada between 2006 and 2009, the data moment is not exactly comparable to the model-generated statistics. Also, I report the average loan-to-value ratio of defaulted households, without distinguishing defaults driven by job losses and income shocks from those purely driven by negative housing equity. Hence, it is hard to compare the data and the model-generated statistics directly. However, it remains a rough benchmark to compare the data and model moment.

¹⁷The PSID shows that home exit rates by homeowners are 12.10% between 1999 and 2001, and 15% between 2001 and 2003.

¹⁸When $\alpha = 0.265$, the lump-sum tax paid by old households is $\tau = 0.0082$. Or, it is 1.4% $\left(= \frac{\tau}{e_O} = \frac{0.0082}{0.6}\right)$ of the old households’ income.

prefer to make a smaller down payment and defer more payments. Panel (a) in Figure 2 compares the mortgage interest rate schedules with and without the loan modification option. As explained above, the mortgage interest rate schedule in a “no modification” model is higher than in a “modification” model, *ceteris paribus*. Therefore, with loan modification, loan originations are higher, the fraction of negative equity households is higher, the LTV ratio is higher, and the homeownership rate is higher than with no modification scheme. Further, the number of mortgage holders under a costless modification economy is 10% higher than that under a no modification economy (not shown in the table). Though the mortgage interest rate schedule is lower in a “modification” model, households take out larger amounts of mortgage debt, which increases the average interest rate.

[Figure 2 here]

Figure 2 also shows mortgage interest rate schedules faced by different household states and decisions in a model with no modification option. As household income increases, the mortgage interest rate schedule shifts down (see (b) of Figure 2). When a household decides to buy a bigger home, the mortgage interest rate rises (see (c) of Figure 2). When a household makes a larger down payment, the mortgage interest rate schedule shifts down. (see (d) of Figure 2).

Note that these interest rate schedules are not uniquely determined. For each possible renter’s state (a, e, p) , a renter chooses a specific housing size and down payment among the possible set of menus (h, η) , or remains as a renter. The options not chosen are off the equilibrium paths. The interest rate schedules include those off-equilibrium paths and they can be defined in diverse ways without affecting on-equilibrium paths. Hence, the interest rate schedule that I present here is one example among the multiple on- and off-equilibrium paths.

In Figure 3, I present the renter’s optimal decision rules in a no modification economy. The figure shows the renter’s housing purchase decision, optimal housing size decision, down payment decision, and the equilibrium mortgage interest rates, conditional on assets, income, and unit housing prices. When assets are close to zero, a renter cannot take on a mortgage contract and therefore cannot buy a house, so the mortgage loan interest rate does not exist (zero in the figure). As assets increase, a renter can buy a small house with no down payment and a mortgage interest rate of 6.5% per year. With more assets, a renter chooses to buy a big house with no down payment. With yet more assets, a household makes a full down payment. Given the down payment and housing size, the equilibrium interest rate goes down

as household assets increase.

[Figure 3 here]

5.2 Default, Selling, and Repayment Decision

In this subsection, I analyze the financial characteristics of households that choose to default, sell, or repay using model-generated data. I used the “no modification” model to generate the data.¹⁹ Conditional on receiving a moving shock, a household has two options: selling or default. Without a moving shock, a household has three options: repayment, selling, or default. First, I analyze the household’s binary choice conditional on a moving shock. The dependent variable is defined by

$$I_1 = \begin{cases} 1 & \text{if a mortgage holder defaults after a moving shock} \\ 0 & \text{if a mortgage holder sells a house after a moving shock} \end{cases}$$

Table 3 shows the results. Households are more likely to default as housing value and financial assets decline, and as the remaining mortgage debt principal increases. Unemployed households are more likely to default. The signs of the coefficients are the same using the logit and probit models.

When a household does not face a moving shock, it can choose one of three alternatives. Hence, I used multinomial logit and probit models to analyze the propensity of selling and default. The dependent variable is defined by

$$I_2 = \begin{cases} 1 & \text{if a mortgage holder repays} \\ 2 & \text{if a mortgage holder sells a house} \\ 3 & \text{if a mortgage holder defaults} \end{cases}$$

I take the base category in the dependent variable as repayment (or $I_2 = 1$). Table 4 shows that households are more likely to sell their house, rather than repay, as housing value and debt principal increase, and as financial assets decrease. Households are more likely to default, rather than repay, as housing value and financial assets decrease, and as the debt principal increases. Unemployed households are more likely to sell the house or default on their mortgage. The signs of all coefficients are the same under the multinomial logit and probit models.

¹⁹I also carried out the same econometric exercise using data generated by costly and costless modification economies. The qualitative results are the same.

Table 3: Default propensity

	No Modification	
	(1) Logit	(2) Probit
Housing value	-16.45* (0.33)	-9.18* (0.17)
Financial asset	-7.51* (0.22)	-4.07* (0.11)
Debt principal	19.33* (0.41)	10.80* (0.21)
$I\{\text{Unemployed}\}$	4.16* (0.20)	2.45* (0.10)
Constant	1.97* (0.12)	0.99* (0.06)
Pseudo R2	0.8419	0.843

Note: Dependent variable is one if a household chooses default conditional on a moving shock, and zero if a household chooses selling conditional on a moving shock. Standard error in parenthesis.

*: p-value < 0.01

In the model, negative housing equity is not a sufficient condition for default. Figure 4 shows the distribution of loan-to-value among home sellers with and without a moving shock. Conditional on a moving shock, some households decide to sell their house, rather than default, even when they have negative housing equity (or $LTV > 1$). If the default penalty is large enough, a household might decide to sell their house and repay the mortgage debt which is larger than the housing value. However, a household that sells their house without receiving a moving shock always has positive housing equity (or $LTV < 1$).

[Figure 4 here]

5.3 Home Entering Decision

I now analyze the financial characteristics of renters that choose either to enter an owner-occupied house or not to enter. As I did in the previous subsection, I used the “no modification” model to generate the data.²⁰ I define an indicator function which is one if a renter

²⁰The qualitative results are the same when we use the costly and costless modification models.

Table 4: Default, selling, and repayment propensity

	No Modification	
	(1) Multinomial Logit	(2) Multinomial Probit
	Selling	
Housing value	2.97* (0.02)	2.39* (0.01)
Financial asset	-6.99* (0.06)	-5.47* (0.04)
Debt principal	3.58* (0.04)	2.80* (0.03)
$I \{ \text{Unemployed} \}$	4.69* (0.05)	3.74* (0.04)
Constant	-9.54* (0.07)	-7.70* (0.05)
	Default	
Housing value	-9.62* (0.23)	-6.19* (0.15)
Financial asset	-10.11* (0.20)	-6.90* (0.13)
Debt principal	11.45* (0.20)	7.73* (0.13)
$I \{ \text{Unemployed} \}$	6.17* (0.11)	4.35* (0.08)
Constant	-7.63* (0.19)	-5.44* (0.13)
Pseudo R2	0.6564	

Note: Without a moving shock, a mortgage holder chooses repayment, selling, or default. The base category in the dependent variable, I_2 , is repayment. Standard error in parenthesis. * if p-value < 0.01

decides to buy an owner-occupied house, and zero if she stays as a renter.

$$I_3 = \begin{cases} 1 & \text{if a renter enters to an owner-occupied house} \\ 0 & \text{if a renter does not enter to an owner-occupied house} \end{cases}$$

Table 5 shows the results. Households facing low house prices are more likely to buy houses. Similarly unsurprisingly, as financial assets and income increase, households are also

Table 5: Home entering propensity

	No Modification	
	(1) Logit	(2) Probit
Unit housing value	-7.69* (0.06)	-3.65* (0.02)
Financial asset	4.23* (0.04)	2.13* (0.02)
Income	5.70* (0.06)	2.70* (0.03)
Constant	7.06* (0.06)	3.09* (0.02)
Pseudo R2	0.88	0.87

Note: Dependent variable is one if a household chooses to enter an owner-occupied house, and zero if a household chooses to stay as a renter. Standard error in parenthesis. *: p-value < 0.01

Table 6: Financial characteristics of households that decide to enter an owner-occupied house

	No Mod	$\alpha = 0.265$	$\alpha = 0$
<u>Average(Home entering households' consumption)</u> Average(Economy wide consumption)	0.97	0.99	1.00
<u>Average(Home entering households' financial asset)</u> Average(Economy wide financial asset)	1.46	1.44	1.40
<u>Average(Home entering households' unit house price)</u> Average(Economy wide unit house price)	0.76	0.77	0.84

more likely to buy a house. The signs of coefficients are the same under the logit and probit models.²¹

Table 6 shows how the cost of modification affects the financial characteristics of households that decide to enter owner-occupied houses. As the cost of modification decreases, households can buy a house with lower mortgage loan rates. Hence, increasingly low-asset households can buy a house, even when they face a relatively high housing price shock. Since the mortgage financing cost declines as the cost of modification decreases, home buyers' average consumption increases.²²

²¹When I compare the average consumption of households that decide to buy an owner-occupied house to those staying in a rental house, the former is higher than the latter. This is because a home buyer has relatively high income and financial assets, despite the need for a down payment.

²²Average income of home entrants is almost same when the cost of modification changes.

5.4 Financial Characteristics of Modified and Non-Modified Households

I now compare the financial characteristics of households that receive a loan modification to households that do not receive a loan modification. Only financially troubled households default on their mortgages in a modification economy. Once a household defaults on its mortgages, a mortgage servicer decides whether to provide a loan modification or not, depending on its expected future cash inflow. Thus, some households are rescued with a modification while others default on their mortgages.

Table 7 shows the financial characteristics of households that are either marginal defaulters or get a loan modification under the no modification ($\alpha = 0.53$), costly ($\alpha = 0.265$), and costless ($\alpha = 0$) modification economies. Regardless of the modification cost, households that default on their mortgages (or do not receive a loan modification) have lower income and financial assets than households that receive a loan modification. Since households with low income and savings are more likely to default even after modifying mortgage terms, due to the persistence of the income process, mortgage servicers let them default rather than provide a modification. In turn, households that default consume and save less than households that receive loan modifications.

Households that have large mortgages are less likely to receive loan modifications, conditional on financial characteristics, such as income, financial assets, and housing value.²³ Since the cost of modification is proportional to the loan principal, households with large loans are less likely to receive loan modifications. However, if we consider the unconditional average of mortgage loans, households that receive loan modifications tend to have larger mortgages than those that fail to get modifications. This result comes from the heterogeneity in households' financial statuses in the steady state.

As the cost of modification decreases (or modification becomes easier), only the more financially troubled households default on their mortgages. Under a costless modification economy, a large number of households are rescued from defaults. Only households that are particularly financially vulnerable default in the end. Hence, regardless of whether default is induced by a moving shock, defaulting households' income, asset position, consumption, and saving under a costless modification economy are lower than those under a costly modification economy. Furthermore, when a household receives a loan modification, its periodic mortgage

²³This comes from the probit analysis by using the model generated data.

Table 7: Financial characteristics of households that receive and do not receive a loan modification

	Default after a moving shock			Modification after a moving shock		
	No Mod	$\alpha = 0.265$	$\alpha = 0$	No Mod	$\alpha = 0.265$	$\alpha = 0$
<u>Average(Modified/Defaulted households' income)</u>	0.95	0.88	0.59	N/A	1.31	1.02
<u>Average(Economy-wide income)</u>						
<u>Average(Modified/Defaulted households' financial assets)</u>	1.01	0.96	0.68	N/A	1.36	1.15
<u>Average(Economy-wide financial assets)</u>						
<u>Average(Modified/Defaulted households' outstanding loans)</u>	1.40	1.48	1.01	N/A	1.51	1.90
<u>Average(Economy-wide financial assets)</u>						
<u>Average(Amount of reduced periodic burden)</u>	N/A	N/A	N/A	N/A	0.05	0.10
<u>Average(Modified/Defaulted households' income)</u>						
<u>Average(Modified/Defaulted Households' consumption)</u>	0.95	0.86	0.57	N/A	1.29	1.02
<u>Average(Economy-wide consumption)</u>						
<u>Average(Modified/Defaulted Households' saving)</u>	1.02	0.96	0.70	N/A	1.02	0.97
<u>Average(Economy-wide saving)</u>						
	Default without a moving shock			Modification without a moving shock		
<u>Average(Modified/Defaulted households' income)</u>	0.38	0.00	0.00	N/A	0.55	0.67
<u>Average(Economy-wide income)</u>						
<u>Average(Modified/Defaulted households' financial assets)</u>	0.89	0.36	0.20	N/A	0.98	1.11
<u>Average(Economy-wide financial assets)</u>						
<u>Average(Modified/Defaulted households' outstanding loans)</u>	1.77	1.18	0.89	N/A	1.96	2.34
<u>Average(Economy-wide financial assets)</u>						
<u>Average(Amount of reduced periodic burden)</u>	N/A	N/A	N/A	N/A	0.07	0.07
<u>Average(Modified/Defaulted households' income)</u>						
<u>Average(Modified/Defaulted Households' consumption)</u>	0.51	0.05	0.03	N/A	0.52	0.63
<u>Average(Economy-wide consumption)</u>						
<u>Average(Modified/Defaulted Households' saving)</u>	0.74	0.26	0.12	N/A	0.78	0.89
<u>Average(Economy-wide saving)</u>						

burden decreases by around 5-10% of their current income (or $\frac{x-\tilde{x}}{e}$).

5.5 Analysis of a Moving Shock

To understand the role of the moving shock, I turn off the moving shock to see the effects on the steady-state statistics. Columns with $\mu = 0$ in table 8 show the steady-state statistics without a moving shock. The annual default rate decreases from 1.47% to 0.18% in the “no modification” model. Since many defaults are triggered by a moving shock, the default rate decreases as the moving shock effect is turned off. Under a costless modification model, the default rate is close to zero and the modification rate decreases from 2.86% to 1.5%, as the moving shock effect is eliminated.²⁴ The homeownership rate increases and the home exit rate decreases in both models. Since the major source of default risk is eliminated, the average mortgage interest rate goes down.²⁵

5.6 Analysis of Foreclosure Costs

Columns with $\chi_D = 0$ in Table 8 present steady-state statistics when the mortgage servicer incurs zero foreclosure costs. In the data, a mortgage servicer loses 22% of the housing value when a household defaults and the mortgage servicer resells it. If the mortgage servicer instead recovers 100% of the defaulted housing value (or $\chi_D = 0$), the mortgage default rate goes up and the loan modification rate goes down in a “costless modification” model ($\alpha = 0$).²⁶ Since a mortgage servicer with low foreclosure costs recovers more when a household defaults, they have less incentive to modify loans. Such a mortgage servicer lets more households default and recovers the housing value without incurring any foreclosure-related costs. Since the mortgage servicer’s expected cash inflow improves, the average interest rate goes down. Since the interest rate schedule shifts down, the average originated loan increases.

A similar interpretation can be made in a “no modification” model. In a “no modification” model, a low-income and low-asset renter can access the mortgage market, taking

²⁴The lump-sum tax τ is 0.075 in a costless modification economy with $\mu = 0$. The only difference between the second and the fourth columns is the moving shock parameter.

²⁵I also considered the case where the cost of modification is 0.265. It turns out that all statistical moments land between those of the no modification and costless modification economies.

²⁶The lump-sum tax τ is 0.075 in a costless modification economy with $\chi_D = 0$. The only difference between the second and the sixth columns is the foreclosure cost parameter.

Table 8: Changes in the moving shock and foreclosure cost

	Benchmark		No moving shock ($\mu = 0$)		No foreclosure cost ($\chi_D = 0$)	
	No Mod	$\alpha = 0$	No Mod	$\alpha = 0$	No Mod	$\alpha = 0$
Annual default rate	1.47%	0.04%	0.18%	0.06%	1.52%	0.77%
Homeownership rate	68.7%	72.03%	77.10%	77.79%	68.9%	71.34%
Average(Housing price)	2.71	2.80	2.79	2.89	2.71	2.81
Average(Annual income for homeowners)	0.22	0.23	0.27	0.27	0.22	0.23
Average(Average(Rent))	1.66	1.75	1.48	1.69	1.66	1.75
Average(Annual income for renters)	N/A	0.98	N/A	0.96	N/A	0.75
Modification acceptance rate	Average(Financial asset)	Average(Annual income)	Avg interest rate (30 year FRM real rate, 95-04)	6.20%	6.27%	5.08%
Avg interest rate (30 year FRM real rate, 95-04)	2.77	3.63	2.74	3.98	2.86	3.69
Average(Originated loan)	Average(Annual income)	Average(Annual periodic payment)	Average(Annual income)	0.18	0.22	0.15
Coef of variation (Housing value)	0.70	0.70	0.72	0.71	0.71	0.70
Loan-to-value ratio (Loan originators)	0.86	0.99	0.87	0.99	0.90	0.99
Loan-to-value ratio (Defaulting households)	2.1	1.12	2.65	1.05	2.1	1.38
Home exit rate	13.47%	13.73%	7.16%	8.34%	13.36%	13.70%
Fraction of housing exit driven by selling	83.73%	99.57%	95.91%	98.82%	82.75%	91.11%
Fraction of housing exit driven by moving shock	49.03%	48.04%	N/A	N/A	49.45%	47.76%
Fraction of default driven by moving shock	81.73%	0%	0%	0%	81.56%	91.74%
Annual modification rate	N/A	2.88%	N/A	1.50%	N/A	2.29%
Fraction of negative equity households	29.65%	33.62%	26.48%	34.74%	29.74%	34.31%

advantage of the low mortgage interest rate schedule under $\chi_D = 0$. This leads to an increase in the default rate and the loan-to-value ratio, and a decrease in the average interest rate.²⁷

6 Transition Analysis with No Modifications

Starting in late 2007, average US housing prices abruptly dropped. From April 2007 to April 2009, the Case-Shiller composite-20 index dropped by 30%. Motivated by this observed decline in housing prices, I calculate the transition path of the model following an unexpected drop in the average unit house price \bar{p} of 30%. I also consider the case where average income \bar{e} unexpectedly drops by 10% and where both income and housing prices drop by these amounts simultaneously.

Figure 5 shows the transition paths in response to a permanent drop in average housing prices or income under a “no modification” model. The black solid line shows the response to the joint shocks, the dotted blue line is for the housing price shock only, and the dashed red line is for the income shock only. When agents unexpectedly face both shocks, the annual mortgage default rate increases from 1.47% to 2.73% over the next two years. With a negative housing price shock (income shock), the default rate increases from 1.47% to 2.36% (1.55%).

[Figure 5 here]

The homeownership rate goes up with the negative housing price shock and goes down with the negative income shock. When both types of shocks hit simultaneously, the response in the homeownership rate is then mixed. When the average housing price decreases, the inflow from renters to homeowners goes up. Households holding few financial assets can suddenly afford to buy a house.²⁸ Conversely, when average income decreases, holding average housing prices constant, renters find it more difficult to buy a house. Only households with high assets can purchase houses when they face an unexpected income shock.²⁹ At the same time, financially constrained homeowners start to sell their houses to relieve financial tightness, which leads to a decrease in the homeownership rate. Model generated panel

²⁷I also considered the steady-state economy with $\alpha = 0.265$. All moments are again in the middle between the no modification and costless modification economies.

²⁸Since the average house price has fallen, the average consumption of period 2 home buyers increases by 0.4%, compared to the home buyer’s consumption under the steady state.

²⁹Because of a drop in average income, the average consumption of home buyers decreases by 7% compared to the home buyer’s average consumption before the income shock.

data commonly shows that households that decide to enter owner-occupied houses in period 2 have higher incomes and financial assets, and face lower housing prices than those that decide not to buy houses, consistent with steady-state results.³⁰

The average interest rate increases over time in both scenarios, especially from the house price shock. Since the mortgage contract is a long-term contract, the average interest rate does not respond quickly. When the housing price drops, low-asset renters start buying houses. This increases the default risk and therefore the average interest rate.

When there is a negative housing price shock, the fraction of negative equity households among mortgage holders suddenly increases. It jumps from 29.6% to 45.7% when both shocks hit, and to 45.1% with solely the house price shock. According to [31]Mayer et al. (2009), the negative equity ratio of subprime loans jumped to more than 50% in California, Florida, Arizona, and Nevada in 2008. In Ohio, Michigan, and Indiana, the ratio jumped to around 30%. The income shock has almost no effect on the fraction of negative equity households.

With a negative income shock, the home exit rate suddenly increases. However, with a negative housing price shock, it suddenly decreases. When average income goes down, financially constrained households start to sell their houses voluntarily, which increases the home exit rate. When the average housing price goes down, homeowners are less likely to sell their homes voluntarily, given the reduced and possibly negative capital gain incurred. Homeowners simply enjoy the housing service utility by staying in their homes.³¹ The reduction in home selling dominates the home exit triggered by defaults. Hence, the home exit rate falls on net.

The PSID shows that households have become more likely to move over time. Using two-year probabilities due to the PSID, 11.8% of homeowners moved between 1997 and 1999, 12.1% between 1999 and 2001, 15.0% between 2001 and 2003, 16.9% between 2003 and 2005, and 17.1% between 2005 and 2007. But, after the housing crisis, the home exit rate decreased to 12.7%. This is consistent with the model-generated numbers following just the negative housing price shock or both shocks. That is, the home exit rate decreases from 13.47% to 11.47% with both shocks and to 9.88% with solely a housing price shock.

³⁰This comes from the probit analysis by using the model generated panel data, as I similarly carried out in section 5.3.

³¹Model generated data shows that only households with serious financial troubles choose to sell their homes. After an unexpected drop in house prices, the average income and financial assets of home sellers in period 2 decreases by 0.2% and 8%, respectively, compared to sellers before the house price shock.

In the model, home exit occurs in four ways: 1) selling after a moving shock, 2) selling without a moving shock, 3) default after a moving shock, and 4) default without a moving shock. First, with a negative income shock, the fraction of home exits triggered by a moving shock decreases. When there is a negative income shock, households are more likely to sell their houses voluntarily. That is, the fraction of home exits driven by a moving shock goes down. In the case of a negative housing price shock, households seldom sell their houses voluntarily. However, the number of defaults triggered by a moving shock increases. Hence, the fraction of home exits driven by a moving shock rises.

Following a negative housing price shock, the fraction of home exits through (voluntary or involuntary) selling goes down, while home exits due to default increase. With a negative income shock, home sales rise along with a small increase in defaults, so the fraction of exits through selling slightly increases.³²

Last, households can default on their debt with or without a moving shock. As I showed in Table 2, many defaults are triggered by a moving shock. In Figure 5, the fraction of defaults driven by a moving shock suddenly decreases with a negative house price shock. That is, households are more likely to default on their mortgages voluntarily without being affected by a moving shock.

7 Analysis of US Housing Policy

In this section, I evaluate the effectiveness of government-driven mortgage modification programs in reducing the mortgage default rate. The US government introduced several foreclosure prevention policies after the outbreak of the housing crisis. The Streamlined Foreclosure and Loss Avoidance Framework, the Federal Deposit Insurance Corporation (FDIC) Loan Modification Program, the Hope for Homeowners (H4H) refinancing program, the Streamlined Modification Program, and the Homeownership Preservation Policy Program were introduced between late 2007 and early 2009 ([18]Gerardi and Li (2010) and [39]Robinson (2009)). However, the success of those programs is still questionable.

In March 2009, the Obama administration launched a new initiative called the Home

³²When house prices fall, home sellers recover less after selling their homes. Hence, their consumption in period 2 decreases by 14%, compared to their consumption before the housing price shock. After the unexpected income shock, consumption of households that sell their home decreases by 1%, compared to before the income shock.

Affordable Modification Program (HAMP). The prior model with the modification option generally represents the structure of the HAMP. In particular, it requires lending institutions to calculate the expected net present value of cash inflows with and without modification before deciding whether to provide a loan modification. Mortgage servicers follow the same logic in my model. Financial institutions are also forced to participate in the modification program both in the model and through the HAMP program, after receiving government subsidies.

Using the modification technology introduced above, I quantitatively analyze how government-driven modification programs reduce mortgage defaults following a 2007-style house price decline. However, according to the OCC Mortgage Metrics Report, the annual initiated foreclosure rate was around 4.2% in 2009 and 2010. As reported in the previous section, even in a no modification environment, the model generates smaller mortgage default rate responses to the unexpected housing price shock (an increase to 2.36%). This motivates me to extend my model in two ways to amplify the simulated mortgage default rate from an unexpected drop in house prices.

7.1 Optimistic House Price Expectations

One possible reason that the model does not generate enough mortgage defaults from an unexpected drop in house price is the steep interest rate schedule in the benchmark model, as shown in Figure 2. Given the interest rate schedule, low-income and low-asset renters face high interest rates or cannot access the mortgage market at all. Hence, these households take out small mortgages or simply never buy houses. Therefore, they are less vulnerable to unexpected shocks.

The slope of the interest rate schedule is endogenously decreased by introducing optimistic expectations about housing prices. In the previous section, the average unit house price unexpectedly dropped by 30%, as shown in Figure 6(a). Unlike in the previous section, I now assume the average unit house price is expected to increase *ex ante*, as shown in Figure 6(b). The model expectation and the realization of average unit house prices coincide through 2007. However, every agent expects the average unit house price to continue increasing until 2009. Contrary to the agents' expectation, the average unit house price drops by 30% after the end of 2007. By introducing this optimistic house price expectation, the interest rate schedules in 2005 and 2007 shift down, reflecting reduced default risks. Hence, households

can finance more mortgage debt, taking advantage of the low interest rates, and thus become more vulnerable to negative shocks. This mechanism is consistent with [5]Burnside et al. (forthcoming) and [17]Foote et al. (2012), who argue that the main driver of the foreclosure crisis was optimistic beliefs about house prices.

[Figure 6 here]

One issue here is to calibrate how people think about future house prices. For simplicity, I assume that the expected percentage changes in the average unit house price from 2005 to 2007 and from 2007 to 2009 are the same. I match the combined loan-to-value ratio of the model in 2007 to the data, which is approximately 0.95 during the housing boom ([27]Keys et al. (2013)). While there are no junior liens on mortgages in the model, the combined loan-to-value ratio is the effective level of mortgage burden (or housing equity). An expectation of a 15% increase in the average unit house price per two years is required to match this moment. That is, \bar{p} is the initial average unit house price, and $1.15\bar{p}$ and $1.15^2\bar{p}$ are the expected average unit house prices in 2007 and 2009, respectively.

In the appendix, I report how households' optimistic expectation about their housing value affects mortgage interest rate schedules. My quantitative exercise shows that the mortgage default rate jumps to 2.55% (2.78%) from an unexpected drop in house price of 30% (along with a drop in income of 10%). This, as discussed, drives the improved quantitative fit of the model.

7.2 Interest Rate Subsidy

Prior to the housing market crash, low-income households could easily access the (subprime) mortgage market with low interest rates, possibly because of the Community Reinvestment Act (CRA). The CRA was designed to encourage financial institutions to extend mortgage, small business, and other types of credit to low- and moderate-income households. Though the CRA was initially introduced in 1977, it may have particularly increased the availability of mortgage financing to low-income households as the housing market boomed. Congressman [33]Paul (2008) said in an interview with CNN that the CRA “[requires] banks to make loans to previously underserved segments of their communities, thus forcing banks to lend to people who normally would be rejected as bad credit risks.” Similarly, [38]Roberts (2008) wrote in the Wall Street Journal that a policy such as the CRA encourages financial institutions to lend money to low- and moderate-income families.

The loose lending standard used by financial intermediaries before the housing crisis, or alternatively particularly low financing costs incurred by financial intermediaries pre-crisis, possibly because of securitization, might be the other force that triggered the foreclosure crisis ([13]Demyanyk and Van Hemert (2011), [32]Mian and Sufi (2009), [37]Purnanandam (2011), [27]Keys et al. (2013)).³³

Collectively, this motivates me to model low-income households as having easy access to the mortgage market prior to the housing market crash. In particular, when mortgage servicers lend money to low-income households, I assume they could finance these loans at the rate of $r_f - \lambda$. λ captures the mortgage servicers' low financing cost only for low-income households. However, when mortgage servicers lend money to high-income households, the financing cost is the risk-free rate, r_f . Since the financing cost is reduced to $r_f - \lambda$ for lower-income households, the risk-neutral mortgage servicers discount future cash inflows with a rate of $r_f - \lambda$. When mortgage servicers lend money to high-income households, they discount future cash inflows with the risk-free rate. I cannot find a good calibration target for λ . As a quantitative exercise, I choose $\lambda = 0.04$, or 2% per year. I assume that there are no more interest rate subsidies to low income households after the housing market crash. This model of subsidies increases the mortgage default rate to 2.9% (3.1%) from an unexpected drop in house prices of 30% (along with a drop in income of 10%).³⁴ Details about the interest rate subsidy structure and the mortgage servicer's profit function for low-income households are presented in the appendix.³⁵

³³In particular, [32]Mian and Sufi (2009) suggest that income and mortgage credit growth were negatively correlated between 2002 and 2005, due to the securitization boom.

³⁴Widespread origination of unconventional mortgages, such as adjusted-rate mortgages, interest-only mortgages, and jumbo loans, might have been an important factor in the foreclosure crisis between 2007 and 2009. Since I model only fixed-rate mortgages, the mortgage default rate responses from an unexpected drop in house prices might be understated.

³⁵Since the model assumes exogenous house price processes, it cannot account for any possible feedback mechanism between house prices and lending standards. For example, loose lending standards leading to an increase in new home buyers, which fuels a run-up in house prices. This, in turn, makes it easier again for low-income households to finance a mortgage. (U.S. Financial Crisis Inquiry Commission 2011) With such a mechanism in the model, the default rate responses from an exogenous shock would likely be amplified again.

7.3 Government Mortgage Modification Program

I now analyze the effectiveness of a HAMP-like mortgage modification program introduced in 2009 in reducing the mortgage default rate. The transition timing is as follows. The steady-state economy without modification in table 2 represents the pre-crisis economy, or the 2005 economy. At the end of 2005, every agent in the market suddenly expects that average house prices will go up, as shown in Figure 6(b). At the same time, unemployed and low-income households receive the interest rate subsidy. At the end of 2007, contrary to general expectation, the average house price declines by 30% and the subsidy to low-income households is terminated. At the end of 2009, the loan modification program is suddenly introduced, which represents the HAMP.

An important issue here is how to set a new cost of modification after introducing the government-driven mortgage modification program. I pick the new cost of modification (α_1) after initiating the government program to match the mortgage modification rate through the HAMP, which is 0.68% in 2011. This yields a post-HAMP cost of modification of $\alpha_1 = 0.18$. That is, the cost of modification before the crisis was 53% ($\bar{\alpha} = 0.53$) of the loan principal. After the crisis, the cost of modification decreases by 35% p ($= 53\% - 18\%$) of the mortgage principal. This scenario is my benchmark.³⁶

I also consider a counter-factual economy where the modification program is not introduced after the housing crisis. That is, the cost of modification remains 0.53 over the transition path. By comparing the benchmark and the counter-factual scenario, I can evaluate the short- and long-run effects of HAMP-like programs in reducing the mortgage default rate.

Figure 7 shows the transition of the default and modification rates from an unexpected drop in average house prices of 30%. The black solid line is the benchmark economy and the red dotted line is the counter-factual economy where the modification program is absent after the housing crisis.³⁷ The default rate gap between the two lines after a drop in house

³⁶In the previous version of this paper, I chose the new cost of modification (α_1) to match actual US government spending on the housing program in 2011. According to the US Department of the Treasury, the US government spent approximately 1.9 billion dollars on HAMP and similar housing programs in 2011. Based on this amount, I chose the new cost of modification to match the ratio of government subsidies to household income in the HAMP program, which is around 0.1. In this scenario, the new cost of modification is 0.48.

³⁷The lump sum tax τ changes over the transition path by covering the cost of modification.

prices represents the policy effect. The default rate decreases by 0.63% p , 0.54% p , and 0.46% p in 2011, 2013, and 2015, respectively. The benchmark modification rate is around 0.65% over the transition, while the counter-factual modification rate is 0.12%, 0.06%, and 0.03% in 2011, 2013, and 2015, respectively. This suggests the HAMP mortgage modification program made a significant dent in the mortgage default rate.

[Figure 7 here]

I also consider a counter-factual economy where the US government's subsidy is doubled (decreases by half).³⁸ Then, the new cost of modification decreases further to $\alpha_1 = 0.06$ (or increases to $\alpha_1 = 0.26$). The blue-dashed line (green dot-and-dash line) shows the default/modification rate responses when the government subsidy is doubled (decreases by half). The default rate in 2011 decreases by 0.37 percentage points (increases by 0.23 percentage points), compared to the benchmark, and the modification rate in 2011 increases up to 1.00 percent (decreases to 0.44 percent).

7.4 Analysis of Households' Decisions

I continue by analyzing how government mortgage modification programs affect households' optimal decisions. Specifically, I compare the benchmark transition to the counter-factual transition where the cost of modification does not change over the transition path. In Table 9, I compare the financial characteristics of households that are marginal for receiving loan modification to those that fail to receive loan modification in 2011. As I reported in section 5.4, households that default on mortgages in 2011 have lower income and fewer financial assets than households that receive loan modification. Under the benchmark transition, households whose loans were modified held larger mortgages than households that defaulted on their mortgages. However, when I control for households' financial characteristics, either through probit or logit analysis, households having more debt are more likely to default as opposed to receiving a modification. The model generated data also show that defaulting households have lower consumption and savings than households that receive loan modification, regardless of the cost of modification. Lastly, once households obtain a modification, the periodic mortgage burden decreases by between 8 to 15 percent of their income, depending on their financial status at the time of default.

Defaulting households' income and financial assets under the counter-factual economy are

³⁸In the model, the government's total subsidy is defined by $\int_{\Delta \in \text{Modified households}} (\bar{\alpha} - \alpha_1) A(\Delta) \Psi(\Delta)$.

Table 9: Financial characteristics of households over transition

	Default after a moving shock in 2011		Modification after a moving shock in 2011	
	Benchmark	Counter-factual	Benchmark	Counter-factual
<u>Average(Modified/Defaulted households' income)</u>	0.91	0.97	1.34	N/A
<u>Average(Economy-wide income)</u>				
<u>Average(Modified/Defaulted households' financial assets)</u>	1.02	1.09	1.54	N/A
<u>Average(Economy-wide financial assets)</u>				
<u>Average(Modified/Defaulted households' outstanding loans)</u>	1.72	1.75	1.95	N/A
<u>Average(Amount of reduced periodic burden)</u>	N/A	N/A	0.08	N/A
<u>Average(Modified households' income)</u>				
<u>Average(Modified/Defaulted households' consumption)</u>	0.90	0.98	1.36	N/A
<u>Average(Economy-wide consumption)</u>				
<u>Average(Modified/Defaulted households' saving)</u>	1.04	1.10	1.23	N/A
<u>Average(Economy-wide saving)</u>				
	Default without a moving shock in 2011		Modification without a moving shock in 2011	
	Benchmark	Counter-factual	Benchmark	Counter-factual
<u>Average(Modified/Defaulted households' income)</u>	0.00	0.26	0.45	0.79
<u>Average(Economy-wide income)</u>				
<u>Average(Modified/Defaulted households' financial assets)</u>	0.58	0.93	1.09	0.94
<u>Average(Economy-wide financial assets)</u>				
<u>Average(Modified/Defaulted households' outstanding loans)</u>	1.71	2.32	2.57	2.05
<u>Average(Economy-wide financial assets)</u>				
<u>Average(Amount of reduced periodic burden)</u>	N/A	N/A	0.15	0.07
<u>Average(Modified households' income)</u>				
<u>Average(Modified/Defaulted households' consumption)</u>	0.08	0.40	0.48	0.65
<u>Average(Economy-wide consumption)</u>				
<u>Average(Modified/Defaulted households' saving)</u>	0.43	0.75	0.84	0.89
<u>Average(Economy-wide saving)</u>				

higher than those in the benchmark economy. Under the benchmark economy, mortgages are more easily modified. Hence, households that default are more financially troubled. This is also why households that default under the benchmark have lower consumption and savings than the counter-factual economy.

Under the benchmark transition, the homeownership rate is higher than the counter-factual transition (not shown in the paper). When mortgages are easily modified, mortgage loan rate schedules shift down. Hence, households can take out loans with low interest rates. Furthermore, financially troubled households do not necessarily move out of their homes once they successfully receive loan modification. These two forces increase the homeownership rate under the benchmark transition. When I compare financial characteristics of households that enter an owner-occupied house under the benchmark scenario to such households in the counterfactual scenario, households in the benchmark case can buy a house even when they have fewer financial assets, or when they face higher high price shocks.³⁹

8 Conclusion

In this paper, I compare an economy without a loan modification option to an economy with fairly easy modification, and evaluate the effect of loan modification on the foreclosure rate. Through loan modification, mortgage servicers can mitigate their losses and households can improve their financial positions without having to walk away from their homes. Since household default imposes costs on both parties, there is room for a mutually beneficial renegotiation of the loan contract. The quantitative results show that the steady-state default rate varies from almost zero with costless modification to a 1.5% default rate when modification is extremely costly.

Motivated by the observed decline in housing prices during the recent recession, I experiment with how the default rate responds to an unexpected drop in house prices of 30%. The default rate increases up to 1.5 percentage points under a “no modification” model from unexpected shocks mimicking the recession. I subsequently evaluate the effectiveness of government-driven mortgage modification programs, like the Home Affordable Modification Program, in reducing mortgage defaults. My quantitative exercise shows that the modelled mortgage modification program reduces mortgage default rates by 0.63 percentage

³⁹The mortgage modification program also affects the home exit margin, usually through the default, rather than the selling, margin.

points. I also consider several counter-factual economies where the government's subsidies to promote mortgage modifications are different. Notably, doubling expenditures on subsidies decreases mortgages defaults by an additional 0.37 percentage points. I conclude that government mortgage modification programs have likely reduced mortgage defaults by a significant amount.

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Appendices

A Optimistic House Price Expectations and Unexpected Shocks

In this section, I analyze how optimistic beliefs in future housing prices followed by an unexpected drop in house prices amplifies mortgage defaults in the model, compared to the scenario without such optimistic beliefs (see section 7.1).

Consider the following transition scenario. The initial distribution in 2005 is the same as the benchmark distribution, as shown in Table 2. At the end of 2005, every agent in the market suddenly expects that the average unit house price will increase by 15% per two years until 2009, and then stay constant forever, as shown in Figure 6(b). Their prior expectation and the realization of average unit house prices coincide in 2007. However, unlike their *ex ante* expectation, the average house price decreases by 30% in 2009.

Figure 8 shows the transition of the mortgage default rate under a model without loan modification from the unexpected house price shock. The initial default rate is 1.47%, as shown in Table 2. In 2007, the default rate is almost same as the rate in 2005. At the end of 2007, the average unit house price unexpectedly decreases by 30% (along with a drop in income of 10%). This pushes the default rate from 1.43% to 2.55% (2.78%).

[Figure 8 here]

When agents expect that house prices will increase, they can take out larger amounts of debt at lower interest rates. Figure 9 shows the *ex ante* interest rate schedules in 2005, 2007, and 2009. The interest rate schedule becomes steeper over time. In 2005, mortgage servicers expect that households will be less likely to default in the future. This follows since as they believe the average housing price will go up in the future, households that need to move will sell their houses, rather than default on their mortgage debt. Hence, the interest rate schedule reflects their perception of a smaller default risk. In 2007, the average unit house price is still expected to increase for two more years, which leads to a steeper interest rate schedule than that of 2005 but flatter than that of 2009. Given the period-by-period interest rate schedule, the average loan-to-value ratio in 2007 is 0.95, with an average originating interest rate of 4.35%. Since households take out larger amounts of debt with low interest rates during the housing boom period, they are more vulnerable to the unexpected house

price shock.

[Figure 9 here]

B Interest Rate Subsidy

In section 7.2, I model that mortgage servicers receive subsidies when they lend money to low-income (and unemployed) households. That is, when mortgage servicers lend money to those low-income households, their financing cost is reduced to $r_f - \lambda$. However, when mortgage servicers lend money to high-income households, they can only finance money with the risk-free rate. Then, mortgage servicers' profit function in financing low-income households changes to the following (under the no modification option):

$$\Pi^0(a, e_L, p, h, \eta) = -(1 - \eta)ph + x(a, e_L, p, h, \eta) + \frac{E\Pi_L(a', e', p', h, 1, x, r_m)}{1 + r_f - \lambda}$$

where $\Pi_L(\Delta)$ is defined by

$$\begin{aligned} \Pi_L(\Delta) = & (1 - \rho_O) \mu I_{MS}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + (1 - \rho_O) \mu I_{MD}(\Delta) \{ (1 - \chi_D) ph \} \\ & + (1 - \rho_O) (1 - \mu) I_P(\Delta) \left\{ x + \frac{E\Pi(\Delta')}{1 + r_f - \lambda} \right\} \\ & + (1 - \rho_O) (1 - \mu) I_S(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + (1 - \rho_O) (1 - \mu) I_D(\Delta) \{ (1 - \chi_D) ph \} \\ & + \rho_O I_{OP}(\Delta) \left\{ x \frac{1 + r_m}{r_m} \left[1 - \frac{1}{(1 + r_m)^{N-n}} \right] \right\} \\ & + \rho_O I_{OD}(\Delta) \{ (1 - \chi_D) ph \} \end{aligned}$$

Since the financing cost is reduced to $r_f - \lambda$, the risk-neutral mortgage servicers discount future cash inflows with a rate of $r_f - \lambda$. When mortgage servicers lend money to high-income households, they discount future cash inflows with the risk-free rate.

This forces me to assume that there is no borrowing/lending interest rate arbitrage opportunity. That is, $r_m \geq r_f$ for every feasible state. (Note that households' saving interest rate is the risk-free rate, r_f .) To satisfy the non-arbitrage opportunity, the zero profit

condition cannot always hold. More specifically, since the mortgage servicers' financing cost for low-income households is less than the risk-free rate, some households with low income and significant assets might face an interest rate of $r_m \in [r_f - \lambda, r_f)$ under the zero profit condition. Those households would borrow as much as possible at a low interest rate and save money at the risk-free rate, which is higher than their borrowing rate. A no arbitrage assumption rules out such cases by relaxing the zero profit condition. Hence, I assume that the lower bound for the mortgage interest rate is the risk-free rate, r_f . This allows mortgage servicers to possibly make a positive expected profit in some states.

$$\Pi^0(a, e_j, p, h, \eta) \left\{ \begin{array}{ll} \geq 0 & \text{if } e_j = e_u \text{ or } e_l \\ = 0 & \text{if } e_j = e_h \end{array} \right\} \quad (6)$$

To see the effect of the interest rate subsidy on defaults from the unexpected house price shock, I study the transition in the "no modification" model. In 2005, the initial distribution is given by the benchmark distribution. At the end of 2005, every agent in the market suddenly expects that the average house price will go up by 15% per two years until 2009, as shown in Figure 6(b). At the same time, low-income households receive the interest rate subsidy, $\lambda = 0.04$. At the end of 2007, the average unit house price unexpectedly decreases by 30%. Also, the interest rate subsidy to low-income households ceases, $\lambda = 0$.

The red dashed line in Figure 10 shows the transition of the default rate under this interest rate subsidy model. I allow both unemployed and low-income households to receive interest rate subsidies. In this case, the default rate increases by 1.5 percentage points from an unexpected drop in house prices of 30%. When agents face an unexpected drop in house prices of 30% along with a drop in income of 10%, the default rate increases by 1.6 percentage points (blue dotted line). This serves as an improvement to the quantitative fit of the model.

[Figure 10 here]

C Analysis of House and Rental Prices

In the model, I assume that the unit rental price is proportional to the unit housing price. The unit rental price is given by $\theta(p) = \frac{pr_f}{1+r_f}$. Hence, when I calculate the transition path from an unexpected drop in house prices of 30%, the unit rental price also decreases proportionally. However, under an incomplete market structure with non-convexities, the unit rental price is not necessarily proportional to the unit house price. In this section, I consider a transition

path where the average housing price unexpectedly decreases by 30%, while the average unit rental price does not change over the transition.⁴⁰

Figure 11 compares responses to the drop in average housing prices of 30% under changing (black solid line) and constant (blue dotted line) average rental prices.⁴¹ The default rate for the experimental transition (blue dotted line) is lower than that under the benchmark transition. This results because holding rental prices constant implies an increase in their relative price when housing prices fall. When the relative rental price goes up, the cost of staying in a rental house increases. This can be interpreted as an increase in default penalty. Since the default cost is now higher, the mortgage interest rate path is lower in this experiment than under the benchmark. Further, the increase in relative rental prices leads to an increase in homeownership rates and a decrease in home exit rates. Therefore, depending on the assumption about the relationship between owner-occupied house prices and rental prices, responses from the unexpected shocks mirroring the 2007 financial crisis are significantly different.⁴²

[Figure 11 here]

D Proposition Proof

Proof. Let $A \left(= x^{\frac{1+r_m}{r_m}} \left[1 - \frac{1}{(1+r_m)^{N-n}}\right]\right)$ be the remaining debt principal in state Δ . Also, let $v (> 0)$ be the household's consumption equivalent default penalty. When a household initially chooses to default after a moving shock, the mortgage servicer decides whether to provide a loan modification. If the mortgage servicer does not modify the loan and lets the

⁴⁰When I calculate this transition path, I used five unit house price grid points, p_1, p_2, p_3, p_4, p_5 . After facing an unexpected drop in house prices, the unit house price grids decline to $0.7p_1, 0.7p_2, 0.7p_3, 0.7p_4, 0.7p_5$. In my original model, the unit rental price also declines by 30%. In this exercise, I model that the unit rental price is always θ_i over the transition path when a household faces the i -th unit house price grid point. That is, the average unit rental price does not decline even after housing prices fall. Only owner-occupied housing prices decline by 30%, not rental rates.

⁴¹The black solid line in Figure 11 is the same as the blue dotted line in Figure 5

⁴²With the relatively higher rental price in this experiment, the default rate, average mortgage interest rate, and home exit rate will all decrease by more, and the homeownership rate will increase by more, than in the scenario with constant average rental prices.

household default, the net cash benefit for both parties is:

	Household	Servicer
Default	$-v$	$-A + (1 - \chi_D) ph$

Since the household does not repay its remaining debt A when defaulting, the net benefit of the household is the default penalty, $-v$. A mortgage servicer loses the household's debt but recovers the house value net of foreclosure costs.

Let ι be the principal reduction from a modification. That is, $A - \iota = \tilde{x}_2(\Delta)^{\frac{1+r_m}{r_m}} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right]$. If the mortgage servicer modifies the loan, the net cash benefit of both parties is:

	Household	Servicer
Modification & Sell	$-A + \iota + (1 - \chi_S) ph$	$-\iota$

After modifying the loan terms, a household repays the modified amount of debt $A - \iota$, receives the sale price of the house net of transaction costs, and then becomes a renter. The mortgage servicer recovers the original debt but loses the value of the debt reduction.

Since the mortgage servicer reduces the debt principal up to the point where the values of defaulting and not defaulting are equal, the amount of debt reduction ι is determined by:

$$-v = -A + \iota + (1 - \chi_S) ph$$

Since v is the consumption equivalent utility value, the left- and right-hand sides are comparable. Then, we have:

$$-\iota = -A + v + (1 - \chi_S) ph > -A + (1 - \chi_D) ph$$

The inequality comes from the assumption of $\chi_D > \chi_S$ and $v > 0$. Using the original notation, $\tilde{x}_2(\Delta)^{\frac{1+r_m}{r_m}} \left[1 - \frac{1}{(1+r_m)^{N-n}} \right] = A - \iota$, we have $\max \{(1 - \chi_D) ph, A - \iota\} = A - \iota$. Hence, a mortgage servicer always provides a loan modification. \square

E Computational Method

E.1 Steady State of No Modification Model

1. There are three income grid points: $e \in \{e_1, e_2, e_3\}$ where $e_3 > e_2 > e_1$. There are five unit house price grid points: $p \in \{p_1, p_2, p_3, p_4, p_5\}$ where $p_5 > p_4 > p_3 > p_2 > p_1$. I use

a 200-point asset grid. There are 60 equally spaced asset grid points between 0 and e_3 , 30 equally spaced asset grid points between e_3 and h_{LP5} , 40 equally spaced asset grid points between h_{LP5} and $3.5h_{LP5}$, and another 70 equally spaced asset grid points from $3.5h_{LP5}$ to the point where asset choice decisions do not bind. (When I refined the grid with additional points, the steady-state statistics did not change.)

2. Solve the old household's problem $V^O(a)$ using value function iteration.
3. Guess a mortgage loan interest rate schedule, $r_m(a, e, p, h, \eta) = r_f$
4. Guess the renter's value function, $V_R^Y(a, e, p) = 0$.
5. Given $V^O(a)$ and $V_R^Y(a, e, p)$, solve the defaulter's value function, $V_D^Y(a, e, p)$
6. Given $V^O(a)$ and $V_R^Y(a, e, p)$, solve the value functions for a homeowner without mortgage debt, $V_F^Y(a, e, p, h)$, $V_{FK}^Y(a, e, p, h)$, $V_{FS}^Y(a, e, p, h)$
7. Given $V^O(a)$, $V_R^Y(a, e, p)$, $V_F^Y(a, e, p, h)$, $V_{FK}^Y(a, e, p, h)$, and $V_{FS}^Y(a, e, p, h)$, solve the value functions for a homeowner with $(N - 1)$ -aged mortgage debt, $V_H^Y(a, e, p, h, N - 1, x, r_m)$, $V_{HP}^Y(a, e, p, h, N - 1, x, r_m)$, and $V_{HS}^Y(a, e, p, h, N - 1, x, r_m)$.
8. Given $V_H^Y(a, e, p, h, N - 1, x, r_m)$, $V_{HP}^Y(a, e, p, h, N - 1, x, r_m)$, and $V_{HS}^Y(a, e, p, h, N - 1, x, r_m)$, solve the life-cycle problem. That is, solve the value functions for a homeowner with $(N - 2)$ -aged mortgage debt. Then, using those value functions, solve the value functions for a homeowner with $(N - 3)$ -aged mortgage debt, and so on.
9. Given $V_H^Y(a, e, p, h, 1, x, r_m)$, $V_{HP}^Y(a, e, p, h, 1, x, r_m)$, $V_{HS}^Y(a, e, p, h, 1, x, r_m)$, $V_D^Y(a, e, p)$, and $V^O(a)$, solve the renter's value function $V_R^Y(a, e, p)$. Then, update the renter's value function and go back to step 4. If every value function converges go to the next step.
10. Calculate the mortgage servicers' profit function, $\Pi^0(a, e, p, h, \eta)$ using the life-cycle method. If the equilibrium profit is $\Pi^0(a, e, p, h, \eta) < 0$, slightly increase the mortgage loan interest rate, $r_m(a, e, p, h, \eta) = r_m(a, e, p, h, \eta) + \varepsilon$. I chose $\varepsilon = 0.2\%$. Then, go back to step 3. If the equilibrium profit is non-negative for every feasible solution, go to the next step.
11. Calculate the stationary distribution.

E.2 Transition Dynamics with Optimistic House Price Expectation

1. Let $t = 0$ be the initial period. Every agent expects that the average unit house price will go up for two consecutive periods, $t = 1$ and 2, and then be stable from $t = 3$ as shown in

Figure 6(b).

2. Solve the optimal policy, interest rate, and value functions at $t = 2$ as I did in the previous subsection. Let $V^{t=2}(\cdot)$ be the value function at $t = 2$.

3. Given value functions at $t = 2$, solve the value functions and interest rate schedules at $t = 1$. That is, for every value function at $t = 1$, $V^{t=1}$, solve the problem in this way:

$$V^{t=1}(\Delta) = \max u(c, h, \cdot) + \beta E V^{t=2}(\Delta')$$

4. Given value functions at $t = 1$, solve the value functions and interest rate schedules at $t = 0$ as I did in the previous step.

5. Given the initial distribution, every household's decision is ruled by the optimal policy at $t = 0$ and the value functions at $t = 1$, along with the interest rate schedules at $t = 0$. That is, the transition is ruled by the following dynamics:

$$V^{t=0}(\Delta) = \max u(c, h, \cdot) + \beta E V^{t=1}(\Delta')$$

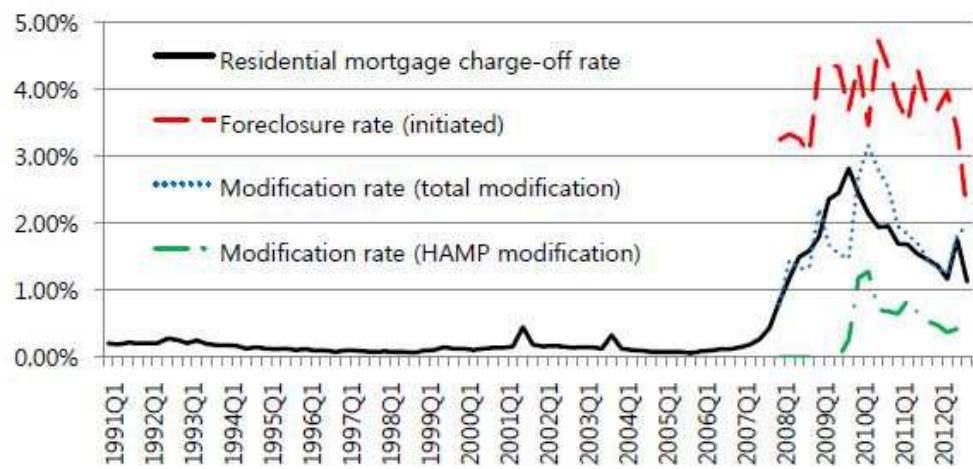
where Δ is the initially given distribution (or state).

6. At the end of period 1 (or at the start of period 2), the average unit house price unexpectedly drops. Calculate the new optimal policy, interest rate, and value functions with the new average house price level, as I did in the previous section.

7. Given the distribution in step 5, every agent's decision follows the optimal policy calculated in step 6, rather than the policy calculated in step 3.

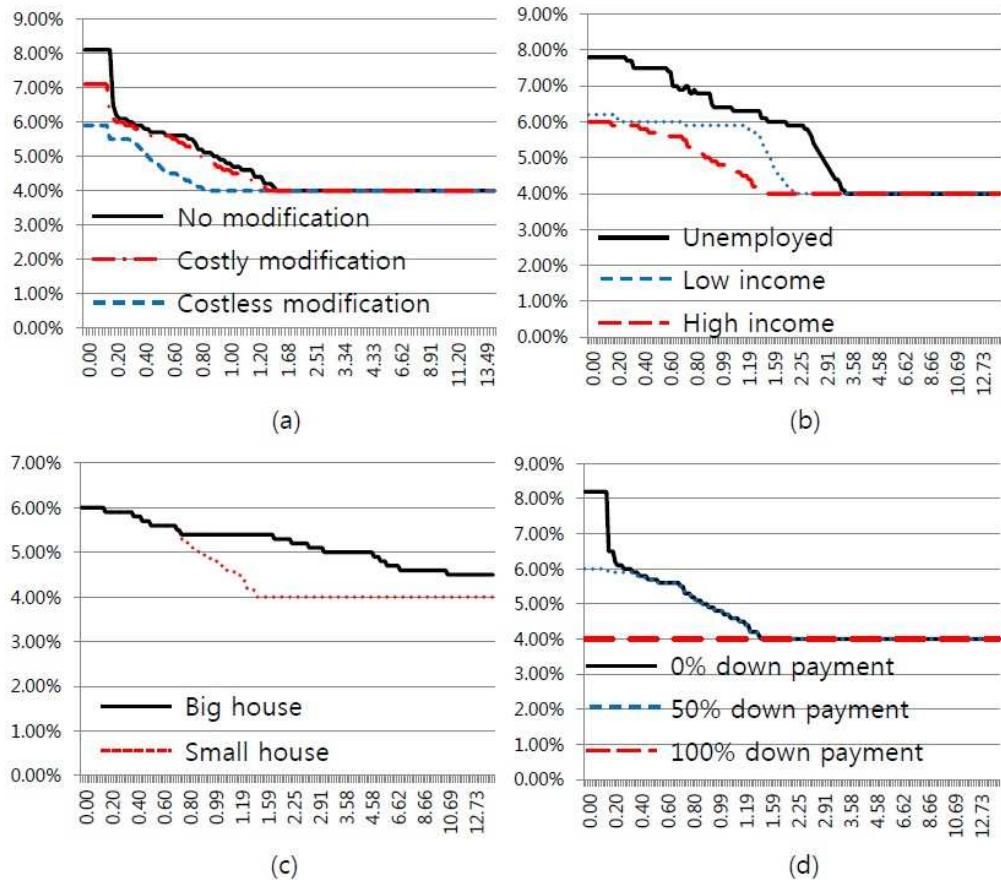
(Note: Since the modification option is absent in this transition, we do not need to consider the tax over the transition path.)

Figure 1: Quarterly residential mortgage charge-off rate, foreclosure rate, loan modification rate, and HAMP modification rate (All rates are annualized)



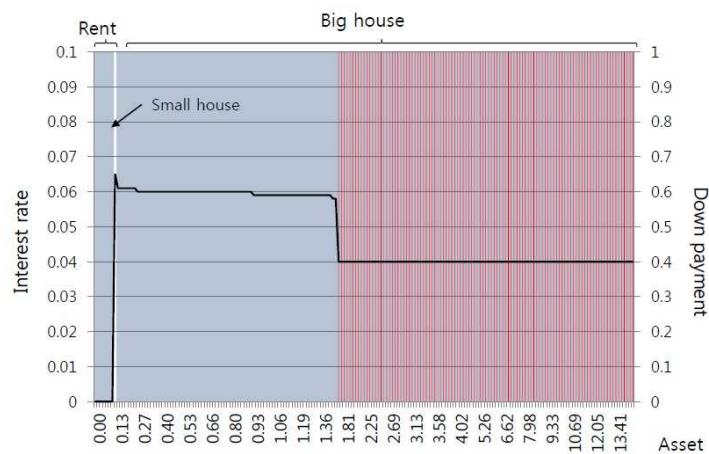
Source: Federal Reserve, OCC Mortgage Metrics Report

Figure 2: Mortgage interest rate schedules



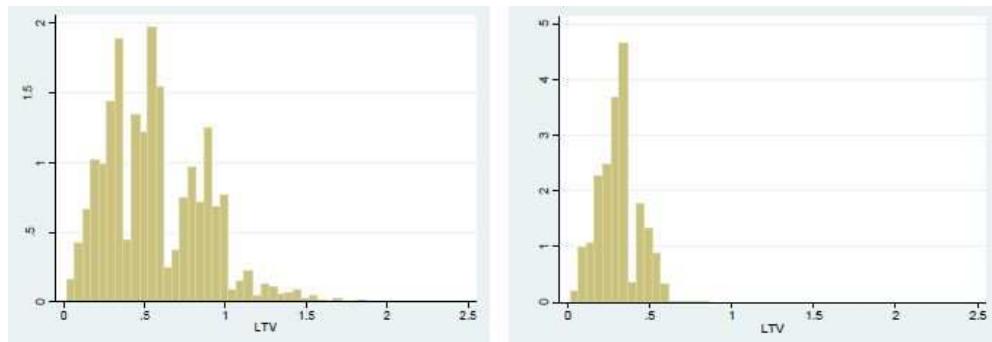
Note: The figure shows the mortgage interest rate schedule, given state variables. The horizontal axis is the asset grids.

Figure 3: Renter's optimal decisions



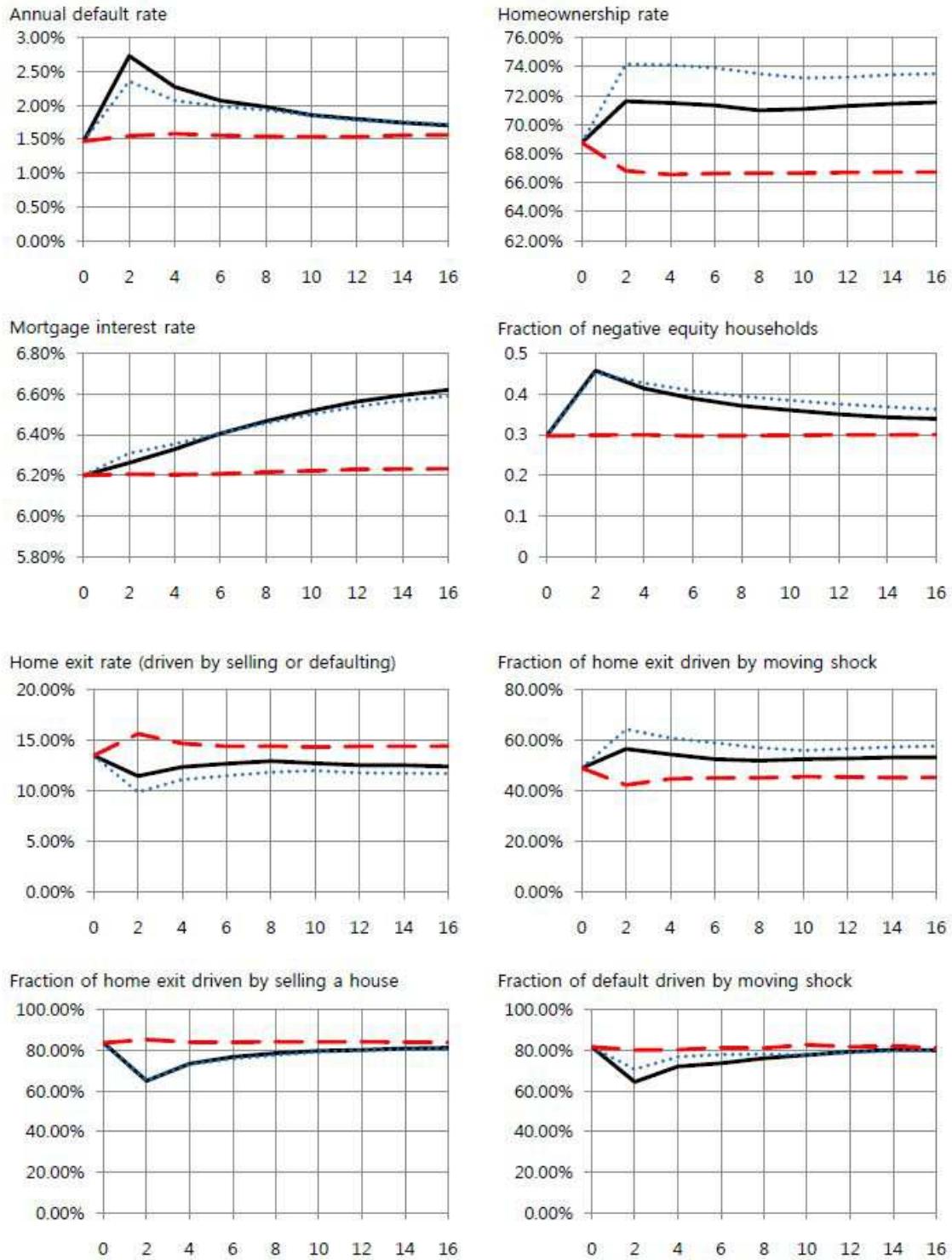
Note: Renter's optimal housing purchase, housing size, and down payment decisions, as well as equilibrium mortgage interest rates. The horizontal axis is the asset grids. The black line is the mortgage interest rate (left axis), the red vertical line is the down payment decision (right axis), and the blue shaded area is the housing purchase/housing size decision.

Figure 4: Loan-to-value ratio distribution



Note: (Left) Seller's loan-to-value ratio distribution, conditional on a moving shock. (Right) Seller's loan-to-value ratio distribution, without a moving shock.

Figure 5: Responses to the drop in average housing price, income, and both



Note: The horizontal axis is the year. The black solid line is both shocks, the blue dotted line is the housing price shock, and the red dashed line is the income shock.

Figure 6: Average unit house price (\bar{p}) expectation and realization



Note: The dashed blue line is the expected average unit house price at time 0 *ex ante*. The red solid line is the realized average unit house price *ex post*. The horizontal axis is the year. The average unit house price unexpectedly drops by 30% between 2007 and 2009.

Figure 7: Analysis of the government-driven mortgage modification program

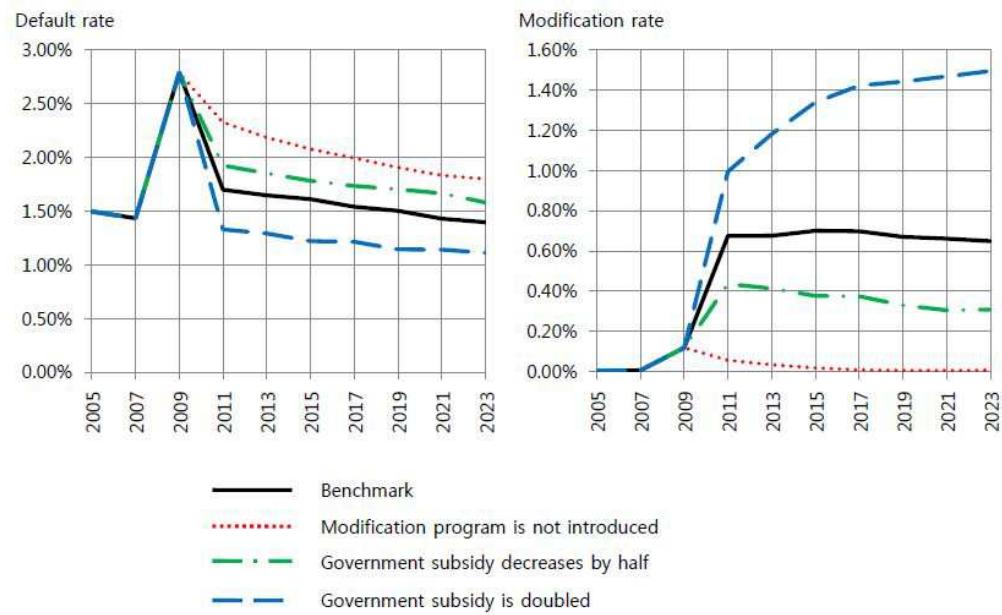


Figure 8: Mortgage default rate with optimistic belief in the housing market

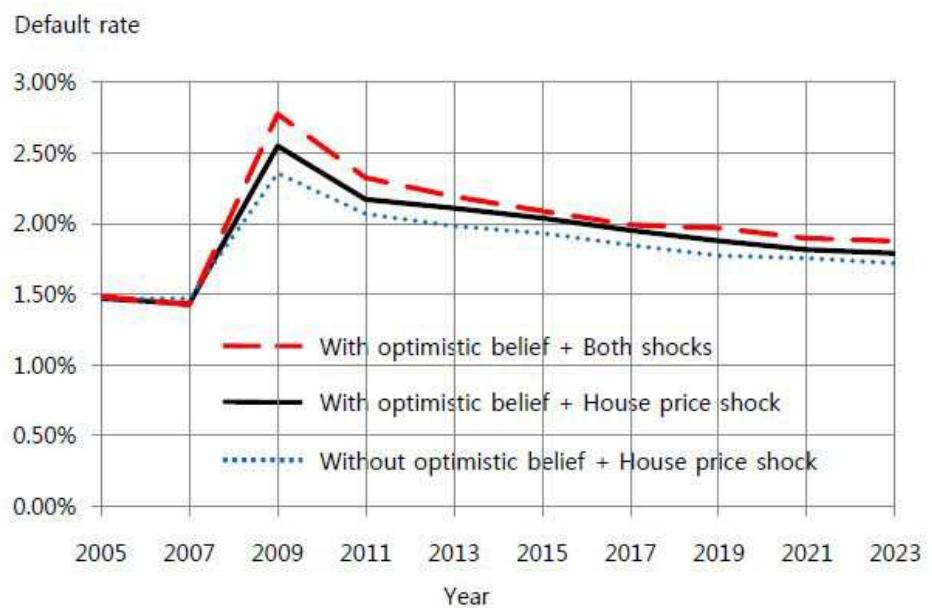
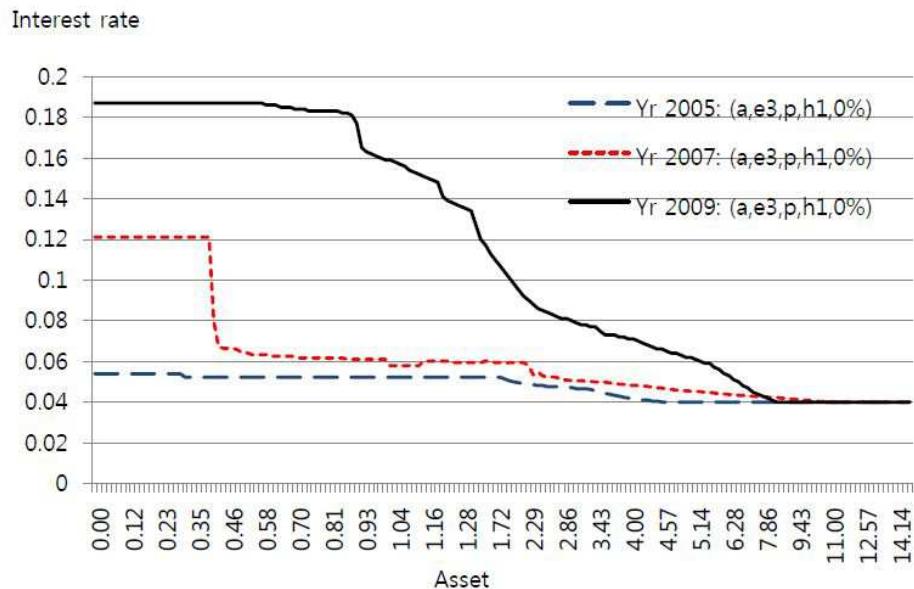


Figure 9: Interest rate schedule with optimistic belief in the housing market



Note: Each interest rate schedule is a function of assets (a) conditional on income (e), unit house price (p), house size (h), and down payment (η). $e3$ indicates the third grid among three income grids; $h1$ indicates the first grid among two house size grids; 0% indicates the down payment as a percentage of house price. The unit house price p is fixed.

Figure 10: Mortgage default rate with interest rate subsidy

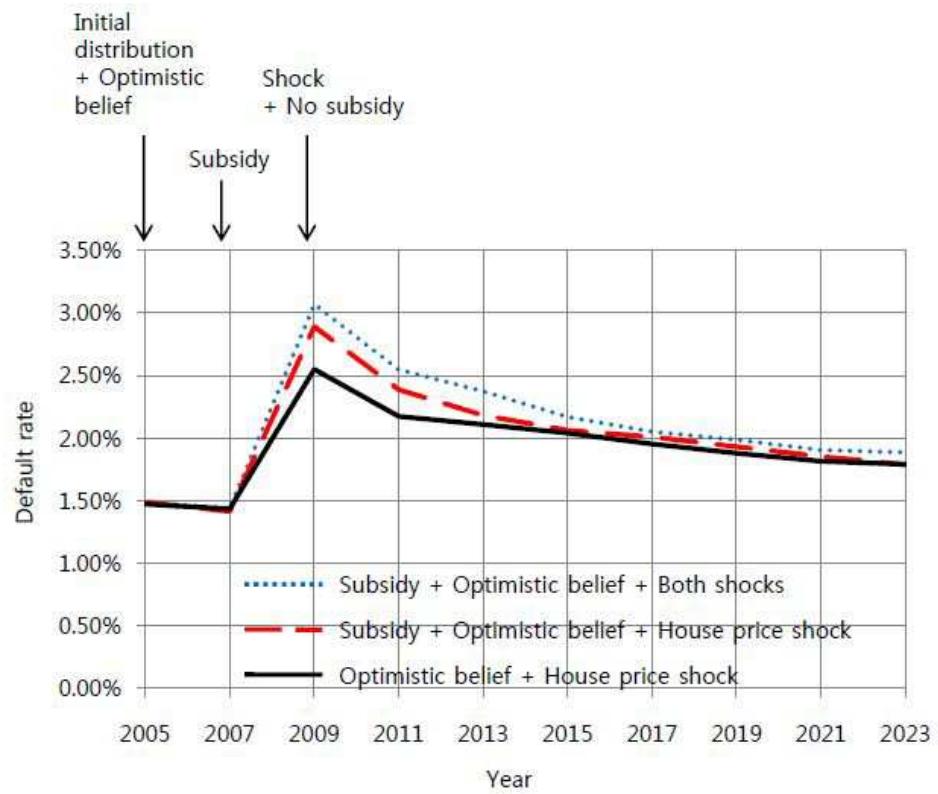
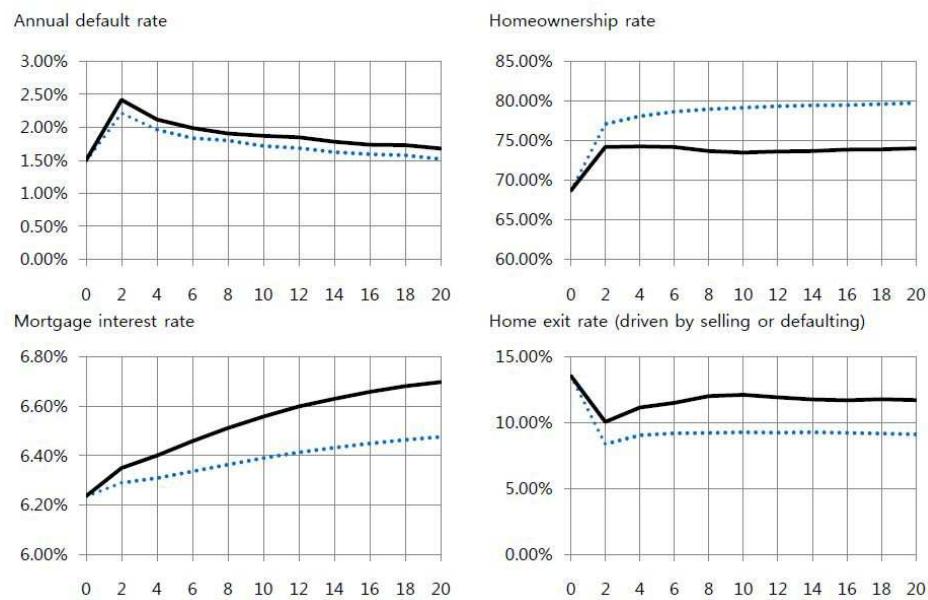


Figure 11: Responses to the drop in housing prices with constant rental prices



Note: The horizontal axis is the year. The black solid line is the transition where rental rates change proportionally with housing prices. The blue dotted line is the transition when rental prices are constant.