# Complementarities in Consumption and the Consumer Demand for Advertising<sup>\*</sup>

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#### Abstract

The standard paradigm in the empirical literature is to treat consumers as passive recipients of advertising, with the level of ad exposure determined by firms' targeting technology and the intensity of advertising supplied in the market. This paradigm ignores the fact that consumers may actively choose their consumption of advertising. Endogenous consumption of advertising is common. Consumers can easily choose to change channels to avoid TV ads, click away from paid online video ads, or discard direct mail without reading advertised details. Becker and Murphy (1993) recognized this aspect of demand for advertising and argued that advertising should be treated as a good in consumers' utility functions, thereby effectively creating a role for consumer choice over advertising consumption. They argued that in many cases demand for advertising and demand for products may be linked by complementarities in joint consumption. We leverage access to an unusually rich dataset that links the TV ad consumption behavior of a panel of consumers with their product choice behavior over a long time horizon to measure the co-determination of demand for products and ads. The data suggests an active role for consumer choice of ads, and for complementarities in joint demand. To interpret the patterns in the data, we fit a structural model for both products and advertising consumption that allows for such complementarities. We explain how complementarities are identified. Interpreting the data through the lens of the model enables a precise characterization of the treatment effect of advertising under such endogenous non-compliance, and assessments of the value of targeting advertising. To illustrate the value of the model, we compare advertising, prices and consumer welfare to a series of counterfactual scenarios motivated by the "addressable" future of TV ad-markets in which targeting advertising and prices on the basis of ad-viewing and product purchase behavior is possible. We find that both profits and net consumer welfare can increase, suggesting that it may be possible that both firms and consumers are better off in the new addressable TV environments. We believe our analysis hold implications for interpreting ad-effects in empirical work generally, and for the assessment of ad-effectiveness in many market settings.

*Keywords*: Advertising, complementarities, treatment effects, non-compliance, discrete-continuous demand, consumer welfare.

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"Advertising has always been a difficult subject to introduce into the conventional theory of consumer choice." – Auld (1974), The Quarterly Journal of Economics.

# 1 Introduction

Markets for advertising now make up a large part of the economy. Advertising revenues in the United States for 2013 totaled \$175 billion, with Internet advertising totaling \$43 billion and TV advertising (Broadcast and Cable TV combined) totaling \$74.5 billion in revenues (IAB 2014). Many modern markets for online search and social networking, broadcast TV, magazine and print media are sustained by advertising revenues. Against this background, the study of advertising and how it affects behavior is now one of the key problems of interest to firms and one of the central questions of Economics and Marketing.

While questions of how to target, measure and determine mechanisms to sell advertising have been studied by academics, the question of how recipients of ads choose to consume advertising has received scarce attention. Although exposure to advertising is at least partially under the control of firms, the *consumption* of advertising is ultimately under the control of consumers. Consumers can discard direct mail they do not value, skip TV ads they do not enjoy, or scroll away from online video ads they find a nuisance. Surprisingly however, most of the empirical literature has ignored the role of consumer choice over ad consumption, treating the level of advertising an agent sees as determined primarily by the sophistication of firms' targeting technology and the *supply* of advertising, ignoring consumer *demand* for the ads. Viewing ad consumption as a choice by consumers changes the way we assess the effects of advertising. This papers presents new data on TV advertising consumption to show evidence for ad choice by consumers, develops a model for the choice of advertising consumption based on Becker and Murphy's (1993) theory of complementarities, and presents estimates from the model to illustrate the role of ad choice in assessing advertising effects and welfare.

A formal treatment of advertising consumption requires a precise model of the decision problem a consumer solves in order to determine whether or not to see an ad. The framework developed by Becker and Murphy has a lot of intuitive appeal for this reason. The Becker-Murphy approach is to treat advertising as an explicit good in consumers' utility functions, thereby effectively creating a role for consumer choice over advertising consumption. In this framework, advertising affects product demand due to complementarities in the joint consumption of advertising and products, rather than by shifting consumers' tastes or providing information that changes consumers' beliefs.<sup>1</sup> Complementarities imply

<sup>&</sup>lt;sup>1</sup>Past frameworks for handling the micro-foundations of advertising include the informative model that posits that advertising affects demand by communicating information about products to consumers (Nelson 1970; 1974), or the socalled persuasive model, in which advertising is incorporated into the utility from *product* consumption and viewed as a means of creating brand loyalty (please see Bagwell 2007 for a comprehensive review of the literature). The informative view is not a good description of ad consumption in our study. The product category we study is a fast moving consumer packaged good that has been on the market for years with no new brand entry during the time period of our data. Like Ackerberg (2001), we find that advertising continues to affect the purchase behavior of experienced consumers in the data even after significant product trial, suggesting its primary role is not to convey information about existence, attributes or

that consuming more of the advertised product increases the marginal utility from ad consumption, so observed purchase quantities are informative in explaining advertising consumption. Thus, utilizing the theory helps us leverage the predictive power of purchase data in an internally consistent way to understand ad choice. The link to a well-defined utility maximization problem also makes a precise characterization of the consumer demand for advertising possible.

Viewing the advertising choice problem this way has three main implications for empirical analysis. As we explain in the next section, firstly, a formal assessment of who would consume ads and the extent to which the ad-consumption shifts purchase behavior is important for the individual-level targeting of advertising. Such targeting is increasingly becoming common in TV ad-markets. As TV becomes more addressable (for example, via set-top boxes and internet IP-address enabled viewing devices), TV ad-markets are increasingly allowing advertisers to target advertisements to specific consumers based on their observed historical product purchase and ad-viewing behavior (see Perlman 2014; O'Connor 2014). Compared to the traditional demographic categories of age and gender, companies like Nielsen Catalina Solutions now merge credit card data from shopper loyalty cards with TV viewership data, and provide advertisers and networks a behavior-based profile of what kind of viewer is buying each type of consumer packaged good. DirectTV Group Inc. and Dish Network Corp., the two biggest satellite-TV providers in the US, now offer direct access to chosen households to whom a 30-s ad-spot can be targeted (see O'Connor 2014). These ad-spots can be bought in real-time by advertisers via "programmatic" adexchanges (essentially, computer-mediated markets where TV network inventory is sold via auction), thus facilitating a high degree of dynamic, behavior-based targeting (Peterson and Kantrowitz 2014). The codetermination of the consumption of targeted advertising and advertised products is key to the targeting problem, and for valuing inventory in such markets. Viewing targeting through the lens of a model with complementarities changes the typical intuition about ad-targeting. The conventional wisdom is that those that do not like advertising  $\left(\frac{\partial U}{\partial A} < 0\right)$  should not be targeted. However, it is possible that some consumers get negative utility from advertising, but that advertising still increases their marginal utility from consumption (i.e.,  $\frac{\partial U}{\partial A} < 0$ ,  $\frac{\partial^2 U}{\partial A \partial Q} > 0$ ). Becker-Murphy give the example of fashion- and fitnessrelated ads that makes some consumers feel worse about themselves due to unflattering peer comparisons, but still cause them to buy more cosmetics or fitness-related equipment. The assessment of targeting thus depends on the measurement of complementariness encapsulated in the cross-partials of utility.

Second, modeling advertising as a choice changes the assessment of the welfare effects of ads. In our framework, advertising is an endogenous avoidable choice. Individuals consume ads only if it increases their net individual welfare. Consumers who do not obtain value from consuming ads would simply avoid them upon exposure. Thus, in our set-up, ad exposures and ad consumption are separate constructs. The typical treatment of advertising assumes ad consumption is the same as ad exposure (because the

match values. In the persuasive stream, advertising is usually treated as a taste shifter in utility, and there is usually no specific theoretical justification for its inclusion in the utility function.

choice to consume advertising is not modeled). Compared to the typical set-up, our model presents a more positive role for advertisements because active avoidance of annoying ads reduces the potential for welfare losses, and consumption of advertisements positively affects welfare by inducing higher product demand and consumption.

Thirdly, as we explain in more detail in the section below, the fact that advertising is actively chosen by consumers complicates the assessment of the causal effects of advertising by inducing the problem of non-compliance. Even if ad exposure is randomized, the treatment - ad consumption - is not, because those that are more likely to like the ads end up seeing them. A formal model of who takes up treatment is then very useful to assess the full distribution of treatment effects, as well as to precisely characterize the sub-populations to which the measured treatment effects apply.

The key to such an analysis is disaggregate micro-data that tracks both ad consumption and purchases. Until now, such data had not been easily available. We leverage access to a new dataset of this sort that tracks both the exposure to and consumption of TV advertising by a large panel of households along with all the purchases made by those households of products in the advertised category. The data are collected by AC Nielsen, a large market-research company. The purchase data records the product bought, day of purchase, its price, package size, number of units purchased, brand, and manufacturer information for each household. The TV advertising data is recorded down to the minute of the exposure for each household. In addition, the brand associated with each advertising exposure is recorded, enabling us to track the sequence of purchases and ad exposures over a long period of time for a given household. There are over 100,000 purchase occasions and about 1.5M advertising exposures captured in total. No channel, show or network characteristics associated with the ads are available. But, importantly, the data record a variable that tracks the fraction of an ad that was played on the TV screen, conditional on an exposure to a TV ad.<sup>2</sup> This variable equals one if the entire commercial was displayed on screen, and is a fraction if the consumer changed the channel or turned off the TV during the commercial. Henceforth, when we refer to "advertising consumption", we refer to this viewing variable. This viewing variable is very powerful because it reflects more clearly consumer demand for ads and suffers less from the endogeneity issues that are often problematic when looking at ad exposures in a non-randomized setting. Such endogeneity arises when firms set prices and advertising expenditures simultaneously or target advertising to consumers who tend to buy a lot. In our data, conditional on being exposed, the consumer chooses how much of the ad to consume. Thus, we are able to focus our analysis on a component of advertising consumption over which consumers have agency. This makes the data unique compared to traditional "single-source" advertising

 $<sup>^{2}</sup>$ These collected Nielsen PeopleMeters. data using The following are excerpt from http://en.wikipedia.org/wiki/People\_meter (accessed Sept 18, 2014) describes the technology: "A people meter is an audience measurement tool used to measure the viewing habits of TV and cable audiences. The People Meter is a 'box' about the size of a paperback book. The box is hooked up to each television set and is accompanied by a remote control unit. Each family member in a sample household is assigned a personal 'viewing button'. It identifies each household member's age and sex. If the TV is turned on and the viewer doesn't identify themselves, the meter flashes to remind them. Additional buttons on the People Meter enable guests to participate in the sample by recording their age, sex and viewing status into the system. For an overview, please see: http://www.nielsen.com/content/corporate/us/en/solutions/measurement/television.html.

panels.

We first use the data to test for evidence that advertising is complementary to consumption. An important implication of the model is that since advertising enters the consumer's utility function along with other goods, advertising must satisfy the symmetry conditions of utility theory. In particular, this implies that complementarities must go both ways. More ad consumption should raise the demand for consumption *and* greater consumption of advertised goods should raise the marginal utility from advertising. This is a testable implication of the model.

The main concern in implementing the test is that unobservable tastes that cause individuals to buy more of a product also cause them to view more ads for the product. We leverage the richness of the panel data to control flexibly for such unobserved heterogeneity. In a battery of specifications, we find that higher ad consumption increases quantities purchased, and that higher quantities purchased increases ad consumption on the margin. To interpret these effects, we then develop a parametric, discrete-continuous model of demand along the lines of Wales and Woodland (1983); Kim, Allenby and Rossi (2002); Bhat (2005); and Lee, Kim and Allenby (2013), which we estimate jointly with a model of ad-choices. The econometric model allows for complementarities, but does not impose them. To identify the model, we leverage the rich variation observed in data on the product prices faced by a given household over time. Under the exclusion restriction that these prices do not directly affect the utility from ad-skipping, the observed covariance in the data between low prices paid in the past and future ad-skipping rates identifies complementarities (we discuss our identification strategy in more detail later in the paper). The estimates from the model suggest complementarities between advertising and consumption and show significant heterogeneity in these effects across households.

To assess the effects of advertising, we simulate the response to a change in ad exposures, tracking changes in advertising and product consumption in response to changes in the number of exposures. We find significantly different implied take-ups of ads across various types of consumers, and find larger effects on purchase incidence and quantity purchased amongst those with higher take-ups, underscoring the need for a precise way of handling endogenous compliance with advertising. Finally, motivated by the "addressable" future of TV ad-markets in which targeting advertising on the basis of ad-viewing and product purchase behavior is possible, we use the model and estimates to simulate a series of counterfactuals. We simulate how demand, welfare and profits would change if an advertiser could target ads to consumers (a) on the basis of anticipated skipping behavior (which in the presence of complementarities indirectly selects high demand-consumers); (b) on the basis of the full model of ad-and-product demand; and (c) on the basis of the full model of ad-and-product demand while also implementing targeted first-degree price discrimination. We find that profits are higher under all ad and price targeting scenarios considered; but that targeting on the basis of ad-viewing alone makes up for 40.4% of total potential increase in profits, suggesting the value of this policy for advertisers. Importantly, we find that net consumer welfare can also rise in the new targeted environments, primarily derived from the increased surplus accruing to high-volume consumers, suggesting that it may be possible that firms and consumers are both better off in the new addressable TV environments.

We believe our results have implications for how researchers view advertising effects and welfare as discussed above, and also for revenue-models in new ad-driven markets. Monetization of advertising based on the active choices by consumers to view the advertising targeted to them is increasingly becoming the norm in online markets. For example, YouTube now utilizes an advertising format called TrueView In-Stream in which an advertisement plays first for a few seconds, after which a viewer can choose to skip to the video or watch the rest of the ad. An advertiser using TrueView In-Stream only pays for an impression if the viewer watches a minimum of 30 seconds of the ad before skipping to their intended video (YouTube 2014). Similarly, advertisers pay for Promoted Videos on Twitter only when a user plays the video (Regan 2014). Thus, increasingly, understanding which users choose to consume ads is of relevance to advertisers in digital media. As it becomes easier to track which ads are skipped and which are watched to completion, it may become possible to better understand consumers' preferences for advertisements, and to relate them to preferences for products like we do here, so as to develop a richer understanding of the demand for advertising and products. Comparison to literature (TBD).

The rest of the paper is organized as follows. Section 2 discusses issues related to the measurement of advertising effects in the presence of ad-choice in more detail. Section 3 introduces the dataset used for the empirical application and presents evidence of complementarities. Section 4 formalizes a model that allows for complementarities. Sections 5 through 7 present the estimation and simulation results. Finally, Section 8 concludes.

# 2 Advertising As a Choice: The Econometric Implications of Endogenous Non-Compliance

Our approach to measuring advertising effects is to develop and estimate a structural simultaneous equations model of the decision to consume products and advertising, and to assess advertising effects through the lens of this model. This section explains in more detail why a model of this sort is useful to assess causal effects of advertising in settings in which advertising is a choice variable for the consumer. We first discuss why randomization alone may not be sufficient to measure advertising effects with policyrelevant economic content in such settings, and then discuss the implications of consumer ad-choice for advertiser and TV networks' policies.

The main econometric challenge in measuring causal effects of advertising in settings with skipping is non-compliance. Using the terminology of program evaluation, if one views advertising as the "treatment," randomization alone cannot measure the *treatment effect* of advertising for all sub-populations of consumers of interest. To set up some notation, denote an individual by i and let  $d_i$  be an indicator of whether i consumes an ad associated with a brand. Let  $y_{i0}$  be i's outcome if no ad is consumed, and  $y_{i1}$  the outcome when the ad is consumed. Define the treatment effect of advertising,  $\theta_i$  as,

$$\theta_i = y_{i1} - y_{i0} \tag{1}$$

If  $d_i$  is randomized to consumers, we can measure the average effect of the ad across all i,

$$ATE = \mathbb{E}\left[y_{i1} - y_{i0}\right] = \underbrace{\mathbb{E}\left[y_{i1}\right] - \mathbb{E}\left[y_{i0}\right]}_{\text{linearity}} = \underbrace{\mathbb{E}\left[y_{i1}|d_i = 1\right] - \mathbb{E}\left[y_{i0}|d_i = 0\right]}_{\text{randomization}} = \mathbb{E}\left[y_i|d_i = 1\right] - \mathbb{E}\left[y_i|d_i = 0\right]$$
(2)

where the first equality obtains because of the linearity of expectations, and the second from the fact that the treatment  $d_i$  is randomized, and therefore  $d_i \perp \{y_{i0}, y_{i1}\}$ . Now suppose that consumers actively choose to see the ad conditional on being randomized into the ad-condition (the situation considered here). Let  $\tilde{d}_i$  denote whether *i* was assigned to the ad condition, and  $d_i$  denote whether *i* actually *chose* to see the ad. Those with higher  $\theta_i$  (for example, those that value the brand more) will tend to choose  $d_i = 1$  (i.e., will be more likely to choose to consume the ad). From (1), these high  $\theta_i$  individuals will tend to have higher potential outcome differences. Because of this differential compliance, even though  $\tilde{d}_i \perp \{y_{i0}, y_{i1}\}$ , now  $d_i$  is no longer independent of  $\{y_{i0}, y_{i1}\}$ . Essentially, the "treatment" is no longer randomized and the decomposition in (2) no longer obtains. Since the set of individuals who see the ad are different from the control, even with randomization we cannot measure the treatment effect of ad consumption.

Non-compliance was first recognized in the medical field as a statistical problem that confounded estimation of treatment effects because some patients assigned the treatment refused to consume the drug, or dropped out of the treatment intervention. The concern is that those who drop out are different from those who stay because they perceive the benefits of treatment to be lower. Medical researchers solved this problem by "double blinding," so the set of treated patients who refuse to comply with the treatment are not aware if they are assigned the treatment drug or the placebo. When non-complying patients do not know they are in the treated or control groups, there is no reason to believe that noncompliers are more averse to treatment than compliers, so this does not confound the measurement of treatment effects. However, in advertising situations, the double blinding strategy is not feasible, because a consumer always sees an ad before deciding to skip it or to see it fully.

Why would an advertiser or a TV network care about non-compliance? Randomization does after all, identify the intent to treat effect of advertising (ITT) (the average effect of being assigned to the ad condition) even with non-compliance, and the ITT is a sufficient metric with economic content for some questions. In particular, the ITT is sufficient for assessing the return from an advertising campaign run on the entire population. If an advertiser randomizes viewers into an ad campaign and wishes to assess overall campaign effectiveness, it is sufficient to know the net profit from those assigned to the campaign relative to those not, without having to know the differential intervention effects for individuals with different compliance types.

In other situations – most notably, those involving the targeting of advertising to individual consummers - the advertiser may care about knowing differential ad-responses for individuals as well as their anticipated compliance. As we explained in the introduction, such targeting at the individual-level is increasingly becoming common in TV ad-markets. For individual-level targeting, a model of whether the targeted viewer will comply, as well as an assessment of the actual treatment effect of the ad is important. For instance, an advertiser may decide to target a given digital video ad-unit to consumers who are more likely to watch it, or to those for who the response from the ad is the highest. For this, the ITT alone is not sufficient. In other situations, the advertising firm may care not just about the total number of units sold in response to advertising, but about the *composition* of buyers per se. Credit-cards, insurance and other financial product markets are leading examples, because the cost curve facing the firm is a function of the composition of customer types, and not just the total number of card or policy holders. Hence, credit-card companies and auto insurance firms - two sets high-spending TV advertisers in the US – care about the *type* of customers who respond to their advertising because they would like to avoid attracting high-cost, high-risk agents to their customer pool. This requires knowing who out of the targeted sub-population will respond to the advertising.<sup>3</sup> In all these contexts, we would like to learn the entire distribution of advertising effects, not just the mean effect of ad exposure as in Equation (2). In the endogenous compliance case with heterogeneous consumer response, it is difficult to characterize which sub-population will consume the ad, the distribution of treatment effects for all sub-populations of interest, or even the mean treatment effect for all consuming sub-populations, from randomization alone.<sup>4</sup> All of these are of policy interest but difficult to address without a well-posed model with heterogeneity that characterize these sub-populations and articulates the effects precisely.

From the TV network's perspective, non-skippable ads by advertisers may be favored all else equal, because it reduces the chance that consumers may switch away from TV during commercials. Hence, a TV ad-network may be willing to entertain price discounts on ad targeted to consumers who are less likely to skip them. More generally, a TV network that would like to assess the price to charge advertisers for specific sub-populations of its viewers would find it useful to know which subsets of viewers and of what type actually see the ads targeted at them, and what the effect on the advertiser's sales and revenue were from each subset's exposure to those ads. This requires measurement of actual treatment effects. In other situations, it may be of separate interest to a firm to measure what proportion of consumers of a given type who are assigned to an ad actually view it, so as to measure consumers' taste for privacy,

 $<sup>^{3}</sup>$ Evidence in the literature suggests a link between those who respond to advertising and risk in such markets. Using randomized trials on direct-mail advertising, Ausubel (1999) documents that customer pools resulting from credit card offers with inferior terms (e.g., a higher introductory interest rate, a shorter duration for the introductory offer) have worse observable credit-risk characteristics and are more likely to default than solicitations offering superior terms.

<sup>&</sup>lt;sup>4</sup>To see this, note that when advertising consumption is an explicit choice, the treatment assignment of individual i,  $\tilde{d}_i$ , should properly be viewed as an instrument for  $d_i$ , and randomization facilitates an instrumental variables (IV) estimator of the effect of advertising. Following Imbens and Angrist (1994), with heterogeneous treatment effects, IV measures a local average treatment effect for a specific sub-population of compliers – i.e., a set of consumers that are induced by assignment to the ad-condition to change their decision to consume ads. Unfortunately, this sub-population cannot be characterized without additional assumptions, nor can the measured effect be extrapolated to any other sub-populations of interest.

or to assess their nuisance value of advertising. Or it may be of interest to a researcher to measure the efficacy of advertising *per se* ("what would happen if an agent saw the ad") as opposed to assessing the effectiveness of an ad-campaign ("will the campaign work when some viewers could plausibly skip ads"?). For situations such as these, the ITT metric and the randomization strategy alone is insufficient. A related question is why do some sub-populations respond and others not? This requires recognizing that the decision to take up treatment is a function of anticipated gains from the treatment, along with a clearly understood mechanism for why heterogeneous consumers decide to consume advertising.

Taken together, in our view, a well-posed and empirically realistic model of ad consumption is important to interpret advertising effects in such situations.

By placing advertising consumption in the same footing as product consumption, Becker and Murphy's framework provides an elegant framework to handle ad and product choice with a clear link to micro-foundations. Becker-Murphy's theoretical framework has to be modified in four ways when confronting household-level panel data on TV ad consumption as we do in our empirical application. First, it needs to be augmented to allow for stock effects of advertising (as opposed to purely flow effects) when considering the panel level variation over time. Stock effects are required to handle the carryover effects of advertising that have been extensively documented in past empirical work (e.g., Naik et al. 1998). Second, we need to have a definition of "advertising consumption" that can be sensibly interpreted as reflecting consumer demand for TV ads. Third, TV ads are at the brand-level, and hence the model has to be modified to allow for choice over brand-level quantity and advertising consumption. Fourth, a common budget constraint over product and advertisement consumption of the type envisaged by Becker-Murphy may not hold in TV ad consumption contexts where the product purchase and ad consumption decisions are separated in time.

# 3 Data Description

As mentioned in the introduction, the dataset used in our empirical analysis comprises a long panel of household-level matched purchase and advertising data from a large sample of households in a Western European country. The data is collected by AC Nielsen. The data covers purchases and advertising exposure and consumption for all brands sold in a product category. The product category is described as a fast moving consumer packaged good that is primarily sold in brick and mortar stores. For privacy reasons the identities of the product category and origin country are not revealed. The sample is not entirely representative in that it slightly over-samples households with elderly people and households that have internet connections. Purchases are recorded at the household-brand-day level and ads are captured at the household-brand-exposure level.<sup>5</sup> Finally, demographic information about the households, including the number of family members, number of children, and average income and education levels

<sup>&</sup>lt;sup>5</sup>Per the recommendation of the company sponsor that provided the data, we define a brand using the variable denoted "umbrella brand" in the dataset. This is also the level at which the advertising data is collected.

Table 1: Summary Statistics

Full Panel Starts	6/14/10
Full Panel Ends	12/31/11
Brands	11
Households	$6,\!437$
Purchase Occasions	117,516
TV Ad Exposures	$1,\!445,\!389$

Table 2: Across Household Variation in Purchases

	Ν	Min	Median	Mean	Max
HH Purchase Occasions	6,272	1	13	19	304
HH Brand Count	$6,\!272$	1	4	4	11

Note: Reported for the 6,272 HHs who made at least one purchase.

of the head of household is also available to track heterogeneity. Much of TV advertising in this category are not informative (providing details of product attributes, sales or prices). Rather, ads are focused on associating the brand with the pleasure of product consumption, documenting scenarios where individuals consume the product in a variety of settings. We believe that advertising works in this category by building associations in consumer memory between the brand and the felt-utility from consumption in the category as documented in the applied psychology literature (e.g., Anderson 1983; Wyer and Srull 1989; Isen 1992).<sup>6</sup>

Table 1 presents aggregate summary statistics for the purchase and advertising data. The data runs from June 14, 2010 through December 31, 2011 covering about 6,500 households (i.e., a balanced panel with T = 557 days). 58 distinct brands are purchased. We focus our analysis on the 11 brands with the largest market share. There are over 100,000 purchase occasions and about 1.4M advertising exposures captured for these brands.

Table 2 provides summary statistics on the distribution of purchases for those households who made at least one purchase in the category. The median household made 13 purchases and bought 4 different brands in the category (mean inter-purchase time of 29 days). Figure 1a shows the distribution across households of the total number of purchases, and Figure 1b shows the distribution across households of

<sup>&</sup>lt;sup>6</sup>Paraphrasing Keller (1993): Cognitive psychologists conceptualize the "associative network" model of semantic memory as a set of nodes and links. Nodes are stored information connected by links that vary in strength. Retrieval of internal information from long-term memory or encoding of external information activates nodes, which spread to other linked nodes, till a threshold level is reached, at which point, information is recalled. Thus, the strength of association between an activated node and other linked nodes determines the extent of "spreading activation" and the extent to which information can be retrieved from memory. The strength of the association depends on both the quantity and quality of the processing an information receives at encoding. For example, in considering a soft drink purchase, a consumer may think of Pepsi because of its strong association with the product category. Consumption of many, memorable ads featuring Pepsi that affects both the quantity and quality of information encoded in memory increases the strength of association between the product category node and the Pepsi brand node, thus making the brand salient in the consumer's mind, leading such advertising to increase that's brand purchases. We believe associations of this sort play an important role in explaining the complementarity that Becker and Murphy (1993) postulated as arising in utility from the consumption of products and non-information related, branding-and-lifestyle oriented ads.

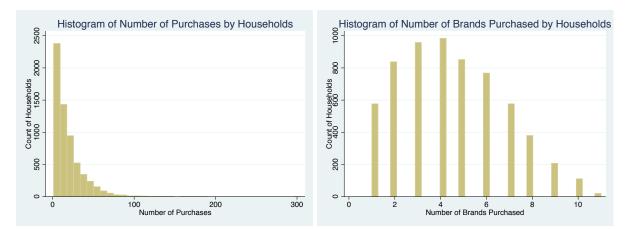
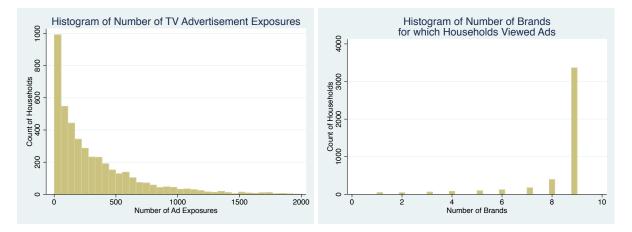


Figure 1: Distribution of Purchases and Number of Brands Purchased by Household

Figure 2: Distribution of Ad Exposures by Household



the total number of brands purchased, over the course of the panel. There is significant heterogeneity in both brand preferences as well as the total frequency of purchase.

Table 3 provides the analogous summary statistics for advertisement exposures. The median household views 199 TV ads and views a TV ad for 9 different brands in the category (mean = 0.6 exposures per day). Figure 2a shows the distribution of advertisement exposures across households. There is a spike in the distribution at the lower end, but there is extensive variation in the number of exposures across households. Figure 2b summarizes the number of brands for which households viewed at least one advertisement. Two of the 11 brands in our analyses do not advertise, so the maximum number of

Table 3: Across Household Variation in TV Advertisement Exposures

	Ν	Min	Median	Mean	Max
HH TV Ad Exposures	4,401	1	199	328	3,808
HH Brand Ad Count	4,401	1	9	8	9

Note: Reported for HHs who viewed at least one TV advertisement.

advertised brands is 9.

Part of the identification of complementarities derives from the extent to which quantity purchased responds to advertising consumption, so it is also interesting to document the variation in quantity purchased in the data. Because of the company sponsor's desire to remain anonymous, the amount of a product purchased is reported in units of equivalent volume, without specifying exactly what scale these map to (we cannot convert it to say grams, pounds or liters). Table 4 reports summary statistics for the quantity purchased on a given purchase occasion defined as the number of units bought in a day of a given brand times the equivalent volume of that unit (i.e., the total package volume of all purchases of brand j made by household i in day t expressed in equivalent units). Figure 3 shows a histogram of the same variable across households. The mean equivalent volume purchased of a brand is about 2,000 units, and there is extensive variation in purchase quantity across purchase occasions.

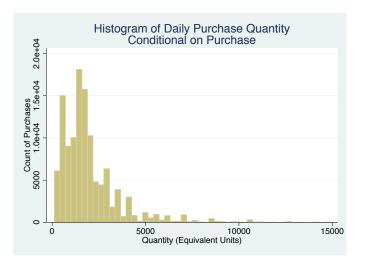


Figure 3: Histogram of Daily Purchase Quantity, Conditional on Purchase

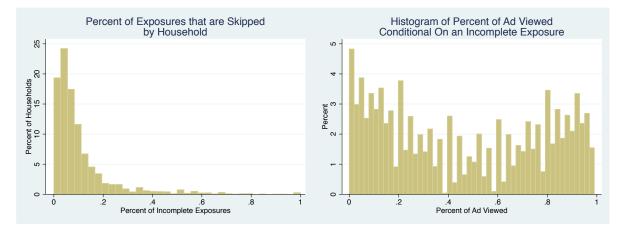
Turning to advertising consumption, Figure 4a shows a histogram of ad-skipping rates across households. We define a household's ad-skip rate as the proportion of that household's total ad exposures over the observed length of the panel that are not watched to completion (i.e., the proportion of exposures for which the corresponding ad consumption variable is less than 1). The histogram shows large heterogeneity in skip-rates across households with some skipping more than 60% of the ads to which they are exposed. The median household skips about 10% of the ads it sees. Ignoring the household-level variation, if we look at all ad exposures across all households, we find that about 5% of the ads are skipped. Figure 4b shows a histogram of the variation in ad consumption for the subset of skipped exposures (there are about 72,000 such observations, i.e.,  $\approx 5\%$  of 1.4M). We see wide variation in how much of an ad is viewed conditional on the decision to skip it.

The 5% ad-skip rate warrants some discussion as it may seem small compared to casual intuition about

	N	Min	Median	Mean	Max
Purchase Quantity	$117,\!516$	119	$1,\!617$	2,015	42,000

Table 4: Variation in Daily Purchase Quantity, Conditional on Purchase

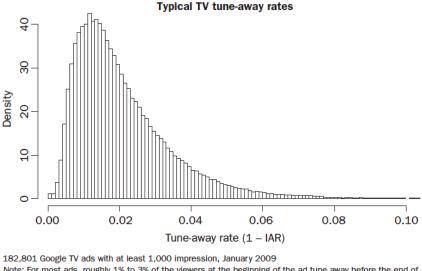
Figure 4: Distribution of Ad Skip Rates and Percentage Watched



ad-skipping, especially in relation to online ads.<sup>7</sup> The skip-rates we observe in our TV data are consistent with those reported in the few academic papers we know that have access to data on household-level TV ad-skipping rates, as well as some of the trade press. For instance, Story (2006) reports that prime time shows on Broadcast TV in the US lose roughly about 5% of viewers during commercials. This article quotes a 2005 study by the American Association of Advertising Agencies and the Association of National Advertisers on non-DVR households that found a similar average -5.6% — of viewers 18 to 49 years old actually skip commercials. Other trade press articles that rely on self-reported survey data often suggest much higher skipping rates (e.g., around 47% in a survey conducted by Jupiter Media reported in Green, 2007), especially for recorded shows played on DVR-s. However, recent academic research that has explored actual TiVo log-data shows that actual ad-skipping rates are much lower than suggested in such self-reported data. Analyzing 46,620 total ad exposures amongst TiVo owning households, Bronnenberg et al. (2010) report that only 3,034 are fast-forwarded (implying a mean skip rate of 6.5%). Similar rates are reported in research from Google using TV set-top data. Figure 5 reproduces ad-skipping rates reported in Interian et al. (2009a), and Zigmod et al. (2009) based on data acquired by Google from the DISH Network in the US, describing the second-by-second tuning behavior of television set-top boxes in millions of US households. Analyzing 182,801 ad-placements, they report mean "tune-away" rates - defined as the proportion of the audience that starts viewing an ad that tune away from it without

<sup>&</sup>lt;sup>7</sup>Typically, reported skip-rates of TV ads are lower than skip-rates of online ads. This difference may arise because the effort required to skip an ad online (ignoring a banner ad or clicking to skip a YouTube TrueView ad) is generally less than the effort required to skip a TV commercial (changing the channel and monitoring when to return to the program). Some advertising executives we spoke to stated that it could be because the passive default option for an online consumer is to ignore the ad, while the action that involves some effort on his part is to click on it. In television advertising, this is reversed: the passive default option for the consumer is to view the ad, while the action that involves some effort on his part is to change the channel.

Figure 5: Distribution of Ad Tune-away rates in Dish Network Set-Top Data (reproduced from Interian et al. (2009a), and Zigmod et al. (2009))



Note: For most ads, roughly 1% to 3% of the viewers at the beginning of the ad tune away before the end of the ad.

watching it completely - of 1%-3%. These data are more credible than the self-reports used in trade press surveys because they reflect actual ad-skipping behaviors collected in an unobtrusive manner. Our data are consistent with these numbers.<sup>8</sup>

Finally, in more recent research using data from TiVo logs, Deng (2014, Table 1) analyzes the extent to which factors such as the brand of the ad, show genre, network in which the ad airs, product category, location of the commercial break within the show and the slot within the break, day of week and hour of show explain the variation in ad-skipping across exposures. She finds that each of these factors explain less than 1% of the observed variation. Rather, the bulk of the variation in ad-skipping is explained by household fixed-effects (20.4%) and past observed propensity to skip ads (11.9%). In addition, if household fixed effects are replaced by a set of demographic variables, the demographics account for only 3.2% of the variation in ad-skipping, suggesting that unobserved household-level heterogeneity is significant in explaining ad-skipping. Analogous results are reported in Zigmond et al. (2009), who report that a "user-behavior model" which predicts ad-skipping rates using the observed skipping behavior of viewers an hour before the airing of a show performs better than models that use only network, weekday, day-part, and ad duration variables, or uses only demographics. Using the same individual-level set-top data, Interian et al. (2009b) also report an interesting correlation that the more often viewers have seen an ad over the last month, the less likely they are to tune away.

<sup>&</sup>lt;sup>8</sup>A related question is the extent to which observed skip rates are understated by measurement error. For instance, it could be that households lose attention or simply look away when an ad is playing, a form of non-consumption that is not captured by Nielsen's Peoplemeters. This kind of measurement error is possible and our results should be seen with this caveat, though it is likely to be of second-order as it takes the form of less econometrically problematic measurement error in the *y*-variable (ad-skip).

In our data, we do not observe TV network or show information, but we do observe data on household characteristics. Like Deng, we explore what percent of the variation in ad exposure and ad skipping can be explained by these characteristics by regressing household exposures and skip rates on the set of observed consumer characteristics. In general, the observed characteristics explain little of the variation in ad exposures and skip rates across households. Later, we will show that historical behavior predicts ad-skipping better than these demographics. Larger households tend to be exposed to more ads, but all else equal, household size does not correlate with skip rates. Homeowners, people over 50 and those with higher levels of income and education tend to see fewer exposures and have higher skip rates. The fact that wealthier, more educated people are more likely to skip an ad is consistent with the interpretation of the cost of an advertisement as the opportunity cost of one's time.

These stylized facts from previous research provide face validity for our analysis, which focuses on household preferences over product and advertising consumption as the main explanation for ad-skipping variability, as opposed to show, network and other TV-environment specific characteristics. In our setup, the heterogeneity in household skip-rates will be explained by preferences over ad consumption, and by the observed quantities consumed by those consumers of the brands featured in those ads. The codependence of ad-skipping rates on the quantity consumed of the advertised products is a novel feature of our empirical model that provides a mechanism for the observed state dependence reported in product and advertising consumption in past studies. For instance, the correlation reported in Interian et al. (2009b) can be explained if ads have a positive effect on quantities purchased, and quantities in turn have a positive effect on ad views.

With this exposition of the dataset, we now report on the relationship between purchased quantities and ad consumption in our data, and discuss our strategy to identify complementarities.

### 3.1 Relating Advertising and Product Demand

First Cut: Cross-sectional Analysis The implication of a joint model of complementarities is that more consumption of an ad induces more consumption of the product, *and*, more consumption of the product induces more consumption of the ad. At a minimum, support for such a model requires seeing a positive covariation between quantities and ads in the data. To see if there is preliminary evidence for complementarities, we start by checking whether households who view more advertisements also purchase more on average. We report the joint distribution of total quantity purchased and total category ad consumption at the household level. First, we split the sample of households into three buckets – the lowest quartile, the middle two quartiles, and the upper quartile of the distribution of total ad consumption.<sup>9</sup> These buckets correspond to households who viewed between 0 and 65 ads, between 65 and 448 ads, and 448+ ads, respectively. Then we non-parametrically estimate the density of purchase

 $<sup>^{9}</sup>$ Going forward, we restrict our analyses to only include the households who made at least one purchase and were exposed to at least one ad.

	TV Ad Exposures	HH Skip Rate
Income	-51.2714***	0.0092***
	(7.3563)	(0.0024)
Unemployed	-0.9489	0.0178***
1 0	(14.0883)	(0.0046)
Part Time Employed	-22.0769	-0.0049
- •	(14.3128)	(0.0047)
Higher Education	-104.980***	0.0289***
	(13.5961)	(0.0044)
Age 29 and Under	-37.7855	0.0013
	(24.1770)	(0.0079)
Age 55 and Over	-49.3056***	0.0290***
~	(15.1363)	(0.0049)
Children	-45.1231**	-0.0007
	(19.0153)	(0.0062)
HH Size	85.5733***	-0.0074
	(8.0448)	(0.0026)
Urban	6.4441	0.0054
	(12.7932)	(0.0042)
Homeowner	-85.2041***	0.0123***
	(12.4223)	(0.0040)
Constant	360.777***	0.0875***
	(23.018)	(0.0054)
R-Squared	0.0902	0.0635
Observations	4,221	4,221
Standar	d errors in parenthese	es
*** p<0	.01, ** p<0.05, * p<0	).1

Table 5: Regression of Household Ad Exposure and Ad Skip Rate on Observed Characteristics

Note: Income is a categorical variable taking on values of 1, 2, 3, and 4 for increasing levels of household income. Unemployed and Part Time Employed are dummy variables indicating employment status. Higher Education is a dummy variable indicating some education beyond the high school level. The Under 29 and Over 55 dummies indicate the age of the head of household. Children is a dummy variable recording whether there are children under the age of 18 living in the home. Household Size records the number of people in the household. Urban is a dummy variable indicating a town population larger than 100,000. Homeowner is a dummy variable indicating the residence is owned.

	]	Purchase Quantity	7
Ad Consumption	25th Percentile	50th Percentile	75th Percentile
$0 < A \le 65$	7.78	19.54	41.30
$65 < A \le 448$	11.42	25.90	53.38
A>448	13.54	31.54	62.49

Table 6: Conditional Distribution of Purchase Quantity by Ad Consumption Quartile

Note: Purchase quantities reported in 1,000's of equivalent units.

quantities for each of these groups separately. Table 6 summarizes the estimated kernel distributions. The quartiles of the purchase quantity distribution are larger for households in higher ad quartiles. Twosample Kolmogorov-Smirnov tests reject the null hypotheses that these samples come from the same distribution.<sup>10</sup>

Based on this cross-sectional analysis, we cannot conclude that advertising *per se* induces this shift out in the purchase density, or that purchase quantities *per se* induce more ad consumption. For example, it could be that larger households are i) more likely to view more advertisements because they are more likely to own multiple TVs and ii) more likely to buy more because they have a higher demand for the product. Household size is just one of many observable household characteristics that could potentially be related to both advertising and purchase behavior. While we can control for these observable characteristics, there may also be unobservable characteristics of households that may generate spurious correlation. In general, any "correlated unobservable" that affects both the propensity to buy a brand and view ads of that brand can confound the effect of complementarities.

Identification of Complementarities Our identification strategy is two-fold. First, we leverage the panel aspect of the data to use only the within-household variation to test for the effect of purchase quantities and ad consumption on each other. The use of the within-variation controls for correlated time-invariant unobserved heterogeneity. Second, we utilize the exclusion restriction that product prices affect quantity purchased, but do not directly affect the percentage of ads watched. We believe this exclusion restriction is reasonable. In essence, we ask whether all things equal, are more of a product's ads consumed if the household paid low prices in the past? In the panel analysis below, we show that this is the case, as we find that prices affect quantity purchased, and that quantity purchased affects subsequent ad consumption. Under the maintained exclusion restriction, these identify complementarities. Our strategy is analogous to Gentzkow's (2007) contribution on measuring substitution and complementarity between online and offline newspapers. Fox and Lazatti (2014, section 2) provide a formal proof of identification along these lines, showing that access to a variable that shifts the utility from consumption of one good which can be excluded from the utility from consumption of another identifies complementarities in a

<sup>&</sup>lt;sup>10</sup>We reject the null hypothesis that the observed purchase quantities for households in the bottom quartile and middle two quartiles of the ad consumption distribution are drawn from the same distribution  $(p = 9 \times 10^{-9})$ ; we also reject the null for the comparison between the middle two quartiles and the upper quartile of the ad consumption distribution  $(p = 1.9 \times 10^{-4})$ .

2-goods model. Product prices in our data serve the role of this variable.<sup>11</sup> We now present the panel analysis in more detail.

**Panel Analysis** We use the panel data to test if within-household variation over time in purchase quantity for a brand is related to cumulative past advertising consumption by that household of that brand. We define cumulative past advertising consumption as the sum of the percentage watched of the advertisements to which the consumer was previously exposed. We construct this variable for the preceding 1, 2, 3, and 4 weeks, and regress household i's day t purchase quantity of brand j on household i's cumulative past advertising consumption of ads for brand j. Each observation in the regression is a household-brand-day. This regression is estimated unconditional on purchase, meaning that we include days with no-purchase in the analysis setting quantity equal to 0. We also control for the price per unit of brand j. Because we only observe prices when a purchase is made, we reconstruct the price series for the 11 most frequently purchased brands in the data and restrict all our analyses to these brands. Appendix A describes in detail how we constructed the price series for these brands.

Given that we have panel data over a long time horizon, we include *household-brand* fixed effects to control for unobserved heterogeneity. Thus, our coefficients are estimated off of within household-brand variation over time rather than across household variation and across brand variation which could have endogeneity concerns as discussed above. In particular, we estimate the following specification.

$$q_{ijt} = \beta_{0ij} + \beta_1 A_{ijt} + \beta_2 p_{ijt} + \beta_3 Time_t + \epsilon_{ijt} \tag{3}$$

where  $q_{ijt}$  is daily purchase quantity in equivalent units,  $A_{ijt}$  is a cumulative ad-duration variable (defined more precisely in Table 7), and  $Time_t$  is a time-trend counting the days since June 14, 2010 (the first day of the data). Table 7 presents the results. Consistent with our cross-sectional findings, the coefficient on cumulative past advertising consumption is positive and statistically significant across all time windows we consider, suggesting that households tend to buy more quantity when they have spent more time watching commercials in the past. To interpret the magnitudes of the ad-effect, we also report in the last row, the effect on daily quantity demanded of a 1 SD increase in the cumulative ad-consumption variables over the past 1, 2, 3, and 4 weeks. Across specifications, we find that a 1 SD increase in ad-consumption over the past 1-4 weeks increases the mean daily quantity demanded by 3.5-6%. For instance, looking at the last 3 rows of Table 7, the mean daily quantity demanded is 7.93 equivalent units. A 1 SD increase in  $A_{ijt,7}$ — the ad-consumption over the last one week — increases the mean daily quantity demanded by 3.53%.

We now present some robustness checks to these regressions. We report these in Table 8, in which we use our preferred specification in which past ad consumption is defined over the preceding two weeks

<sup>&</sup>lt;sup>11</sup>The success of this identification strategy depends on utilizing extensive within household price variation in the data. The *within*-household variation in prices paid over time in the data is large. Figure (18) in Appendix B presents a histogram across households of the standard deviation in price paid per unit over time split by brand to document the extensive within-household price variability.

	(1)	(2)	(3)	(4)
	Quantity	Quantity	Quantity	Quantity
$A_{ijt,7}$	0.2254***			
	(0.0337)			
$A_{ijt,14}$		$0.1549^{***}$		
		(0.0210)		
$A_{ijt,21}$			$0.1194^{***}$	
• /			(0.0160)	
$A_{ijt,28}$				$0.1241^{***}$
• /				(0.0139)
Price Per Unit	-14.931***	$-14.279^{***}$	-13.012***	-13.386***
	(2.6402)	(2.6848)	(2.6924)	(2.7727)
Time Trend	$0.0014^{***}$	$0.0011^{***}$	$0.0008^{***}$	0.0001
	(0.0003)	(0.0003)	(0.0003)	(0.0003)
Observations	29,590,182	29,218,979	28,847,776	28,476,573
HH-Brand FE	Υ	Υ	Υ	Υ
Ave Quantity	7.93	7.95	7.96	8.02
Effect of $+1$ SD Ads	0.28	0.34	0.37	0.48
Percent of Mean D.V.	3.53%	4.28%	4.65%	5.99%
		rs in parenthe		
**	<sup>**</sup> p<0.01, **	p<0.05, * p<	< 0.1	

Table 7: Regression of Daily Purchase Quantity on Cumulative Ad Consumption

(Column 2, Table 7). Our first robustness check address a concern there may be unobserved, time-varying shocks driving both purchases and ad consumption that remain even after including household-brand fixed effects.<sup>12</sup> Column 1 adds a dummy for observations in the month of December to check if the results are robust to a story that there may be increased demand for the product around the holidays and, at the same, the intensity of advertising may be higher and ad content may be more engaging during those times. We find our results are robust to this additional control. Another story along these lines could be that when a consumer goes out of town, we might observe zero purchases and zero ad consumption, which could create spurious correlation between purchase quantity and ad consumption. To check whether our results are driven by such a scenario, we re-estimate the same model, restricting the data to days in which a household purchased at least one brand. Again, we continue to estimate a positive relationship between purchase quantity and cumulative ad consumption.

We also estimate the analogous model as above on the ad side, but now treating current advertising consumption as the dependent variable and cumulative product consumption of the related brand as a

Note:  $A_{ijt,\tau}$  records the cumulative time household *i* spent watching ads for brand *j* in the  $\tau$  days preceding day *t*. Robust standard errors clustered at the household level.

 $<sup>^{12}</sup>$ In general we would have an endogeneity problem if the firm coordinated prices and advertising quality over time. For example, if low prices lead to high purchase quantities and high-quality ads lead to a higher propensity to watch, we might over-state the relationship between purchase quantity and ad consumption. However, our understanding is that due to the fact that there is little local TV in the Western European country where the data comes from, the manufacturer sets almost all TV advertising at the national level while prices are set by individual retailers at the local level. This suggests that such systematic coordination is unlikely.

	(1)	(2)
	Quantity	Quantity
$A_{ijt,14}$	$0.1943^{***}$	$2.5047^{***}$
•	(0.0211)	(0.5053)
Price Per Unit	-13.5245***	-343.43***
	(2.6878)	(21.621)
December	3.4976***	× ,
	(0.1471)	
Time Trend	-0.0001	$0.0270^{***}$
	(0.0003)	(0.0058)
Observations	29,218,979	883,736
HH-Brand FE	Υ	Υ
Ave Quantity	7.95	262.74
Effect of $+1$ SD Ads	0.43	5.82
Percent of Mean D.V.	5.41%	2.21%
Standard erro	ors in parenthe	ses
*** p<0.01, **	* p<0.05, * p<	(0.1

Table 8: Robustness Checks of Regression of Daily Purchase Quantity on Cumulative Ad Consumption

Note:  $A_{ijt,14}$  records the cumulative time household *i* spent watching ads for brand *j* in the 14 days preceding day *t*. Robust standard errors clustered at the household level.

regressor. As noted before, whether or not a consumer is exposed to an ad is determined by firms' supply of advertisements and consumers' show-preferences. However, conditional on being exposed to an ad, consumers have agency over how much of the ad to watch. Hence, we believe the percentage of the ad watched conditional on exposure is a better metric of advertising demand. In the regressions below, we treat the percent watched (range: 0-1) of the  $r^{th}$  ad exposure for brand j watched by household i in day t on household i's cumulative past consumption of brand j,  $a_{ijrt}$ , as the dependent variable. Each row in the regression is a household-brand-day-exposure combination. This specification is summarized in equation 4.

$$a_{ijrt} = \theta_{0i} + \theta_{0j} + \theta_1 Q_{ijt} + \theta_2 Time_t + \epsilon_{ijrt} \tag{4}$$

Here,  $Q_{ijt}$  is the cumulative quantity variable, and  $Time_t$  is a similar time-trend counting the days since June 14, 2010 (the first day of the data). Table 9 reports the results. Column 1 reports on the effect of cumulative quantity purchased over the past 2 weeks on the percentage of an ad viewed by a household, conditional on exposure. Since we are doing everything conditional on exposure, we lose power and can include only brand and household fixed effects as opposed to brand-household fixed effects. Looking at column 1, past quantity is seen to have a positive and significant effect on ad-skipping (we report marginal effects below). For completeness, we also run the analogous regression using the percentage of the ad viewed unconditional on exposure as the dependent variable. Here, days in which there are no exposures to a given brand's ad for a household are also included as rows in the data with the value of

	(1)	(2)
	Percent Ad Watched	Percent Ad Watched
	Conditional on Exposure	Unconditional on Exposure
$Q_{ijt,14}$	$3.83e-07^{*}$	6.01e-07***
	(2.10e-07)	(1.53e-07)
Time Trend	$5.47e-05^{***}$	9.08e-05***
	(1.75e-06)	(2.20e-06)
Observations	1,436,400	29,778,304
HH-Brand FE	Ν	Y
HH FE	Y	Ν
Brand FE	Y	Ν
	Standard errors in par	entheses
	*** p<0.01, ** p<0.05	* p<0.1

Table 9: Regressions of Ad Consumption on Cumulative Quantity

Note: Regression estimated at the household-brand-day-exposure level. The dependent variable Percent Ad Watched records the percentage of the exposure that was watched and ranges between 0 and 1. Column 1 is estimated conditional on an exposure. In column 2, the dependent variable is recorded as a 0 for days in which no ads were viewed.  $Q_{ijt,14}$  records the cumulative package volume household *i* purchased of brand *j* in the 14 days preceding day *t*. Robust standard errors clustered at the household level.

the dependent variable set equal to 0. Hence, this regression explores the effect of past purchase quantity on both the propensity to be exposed to an ad, and the propensity to view it for longer conditional on exposure. Looking at column 2, we see the effect of past quantity is positive, though this regression is hard to interpret as it mixes the supply and the demand for ads.

$\begin{array}{cccc} 07 & 6.06e-07 \\ 07) & (2.36e-06) \\ 2^{***} & 0.000180^{***} \\ 06) & (1.38e-05) \\ 42 & 57,764 \\ 9 & 0.064 \\ 89\% & Y \\ Y & Y \\ 0.064 & 89\% \\ & Y \\ 0.06\% & 0.09\% \\ \% & 0.09\% \\ \% & 0.09\% \\ \% & 0.09\% \\ \% & 0.09\% \end{array}$	COLUMN: DEPENDENT VARIABLE: HH AD-SKIP RATE IS >	% Ad Watched 0		$\% { m Ad Watched} 0.05$	(4) % Ad Watched 0.10	(5) % Ad Watched 0.15	(6) % Ad Watched 0.20	$\% { m Ad Watched} \ 0.25$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$Q_{iit,14}$	3.83e-07*	3.85e-07*	4.34e-07	7.67e-07	6.06e-07	2.73e-06	6.54e-06
Trend $5.47e-05***$ $5.67e-05***$ $8.96e-05***$ $0.000142^{***}$ $0.000180^{***}$ (1.75e-06) $(1.80e-06)$ $(3.57e-06)$ $(7.72e-06)$ $(1.38e-05)vations 1,436,400 1,369,030 558,595 159,242 57,7641.38e-05)$		(2.10e-07)	(2.19e-07)	(4.12e-07)	(9.52e-07)	(2.36e-06)	(3.87e-06)	(6.21e-06)
$ \begin{array}{c cccc} (1.75e-06) & (1.80e-06) & (3.57e-06) & (7.72e-06) & (1.38e-05) \\ \mbox{vations} & 1,436,400 & 1,369,030 & 558,595 & 159,242 & 57,764 \\ \mbox{ared} & 0.039 & 0.038 & 0.036 & 0.049 & 0.064 \\ \mbox{Dep Var} & 97\% & 97\% & 95\% & 92\% & 89\% \\ \mbox{E} & Y & Y & Y & Y & Y & Y \\ \mbox{FE} & Marginal Effect of an Additional Purchase of Various Quantities on Expected Percentage Wat \\ \mbox{onog} & 0.08\% & 0.08\% & 0.04\% & 0.016\% & 0.012\% \\ \mbox{Percentile} & 0.09\% & 0.04\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \mbox{Percentile} & 0.00\% & 0.00\% & 0.01\% & 0.01\% & 0.00\% & 0$	Time Trend	$5.47e-05^{***}$	$5.67e-05^{***}$	$8.96e-05^{***}$	$0.000142^{***}$	$0.000180^{***}$	$0.000227^{***}$	$0.000283^{***}$
		(1.75e-06)	(1.80e-06)	(3.57e-06)	(7.72e-06)	(1.38e-05)	(2.52e-05)	(3.76e-05)
ared $0.039  0.038  0.036  0.049  0.064$ Dep Var $97\%  97\%  95\%  92\%  89\% \\ F \\ E \\ Y \\ Y$	Observations	1,436,400	1,369,030	558, 595	159,242	57,764	23,522	12, 327
$ \begin{array}{c ccccc} {\rm Dep}  {\rm Var} & 97\% & 95\% & 92\% & 89\% \\ {\rm E} & {\rm Y} \\ {\rm FE} & {\rm Marginal}  {\rm Effect}  {\rm of}  {\rm an}  {\rm Additional}  {\rm Purchase}  {\rm of}  {\rm Various}  {\rm Quantities}  {\rm on}  {\rm Expected}  {\rm Percentage}  {\rm Watch} \\ & & 0.08\% & 0.08\% & 0.09\% & 0.16\% & 0.12\% \\ {\rm Percentile} & 0.03\% & 0.04\% & 0.07\% & 0.01\% & 0.09\% \\ {\rm Percentile} & 0.09\% & 0.07\% & 0.11\% & 0.09\% \\ {\rm Percentile} & 0.09\% & 0.07\% & 0.11\% & 0.09\% \\ {\rm Percentile} & 0.09\% & 0.07\% & 0.17\% & 0.14\% \\ \end{array} $	R-squared	0.039	0.038	0.036	0.049	0.064	0.083	0.096
$ \begin{array}{c cccc} F & Y & Y & Y & Y & Y \\ FE & Y & Y & Y & Y & Y \\ \hline FE & Marginal Effect of an Additional Purchase of Various Quantities on Expected Percentage Watch \\ \hline 0.08\% & 0.09\% & 0.016\% & 0.12\% \\ \hline 0.03\% & 0.04\% & 0.04\% & 0.07\% & 0.06\% \\ \hline 0.06\% & 0.04\% & 0.07\% & 0.01\% & 0.09\% \\ \hline 0.09\% & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.09\% \\ \hline 0.09\% & 0.09\% & 0.00\% & 0.01\% & 0.01\% & 0.04\% \\ \hline 0.00\% & 0.00\% & 0.01\% & 0.01\% & 0.04\% \\ \hline 0.00\% & 0.00\% & 0.01\% & 0.01\% & 0.01\% \\ \hline 0.00\% & 0.00\% & 0.01\% & 0.01\% & 0.01\% \\ \hline 0.00\% & 0.00\% & 0.01\% & 0.01\% & 0.01\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.01\% & 0.01\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.01\% & 0.01\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0.00\% \\ \hline 0.00\% & 0.00\% & 0.00\% & 0$	Mean Dep Var	97%	%26	95%	92%	89%	85%	81%
$ \begin{array}{c cccc} FE & Y & Y & Y & Y & Y \\ \hline \mbox{Marginal Effect of an Additional Purchase of Various Quantities on Expected Percentage Watch} \\ \hline \mbox{Marginal Effect of an Additional Purchase of Various Quantities on Expected Percentage Watch} \\ \hline \mbox{Outbox} & 0.08\% & 0.09\% & 0.16\% & 0.12\% \\ \hline \mbox{Percentile} & 0.03\% & 0.04\% & 0.04\% & 0.07\% & 0.06\% \\ \hline \mbox{Percentile} & 0.06\% & 0.07\% & 0.11\% & 0.09\% \\ \hline \mbox{Percentile} & 0.09\% & 0.00\% & 0.11\% & 0.09\% \\ \hline \mbox{Standard errors in parentheses} \\ \hline \mbox{Standard errors in parentheses} \\ \hline \end{tabular} $	HH FE	Υ	Υ	Υ	Υ	Υ	Υ	Υ
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Brand FE	Υ	Υ	Υ	Υ	Υ	Υ	Υ
$\begin{array}{ccccc} 0.08\% & 0.08\% & 0.09\% & 0.16\% & 0.12\% \\ \hline \mbox{Percentile} & 0.03\% & 0.04\% & 0.04\% & 0.07\% & 0.06\% \\ \hline \mbox{Percentile} & 0.06\% & 0.07\% & 0.11\% & 0.09\% \\ \hline \mbox{Percentile} & 0.09\% & 0.09\% & 0.10\% & 0.17\% & 0.14\% \\ \hline \mbox{Standard errors in parentheses} & & & & & & & & & & & & & & & & & & &$	W	larginal Effect of	an Additional Pure	chase of Various (	<b>Quantities on Exp</b>	ected Percentage	Watched	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Mean	0.08%	0.08%	0.09%	0.16%	0.12%	0.55%	1.31%
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	25th Percentile	0.03%	0.04%	0.04%	0.07%	0.06%	0.26%	0.60%
0.09% 0.09% 0.10% 0.17% 0.14% Standard errors in parentheses ***001 **001	50th Percentile	0.06%	0.06%	0.07%	0.11%	0.09%	0.40%	0.96%
Standard errors in parentheses	75th Percentile	0.09%	0.09%	0.10%	0.17%	0.14%	0.62%	1.48%
			Sta ***	ndard errors in pe p<0.01, ** p<0.0	arentheses $5, * p < 0.1$			

higher skip rates. Panel 2 reports the expected increase in ad percentage watched from an additional purchase in the preceding 14 days. The marginal effects are reported for

different quartiles of the purchase quantity distribution. Robust standard errors clustered at the household level.

Table 10: Regressions of Daily Ad Consumption on Cumulative Quantity for Households with Different Skip Rates

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To explore the heterogeneity in advertising consumption effects, we repeat the same regression separately for households of different observed ad-skip rates. Table 10 repeats the regression from Column 1 of Table 9 separately for households with mean observed ad-skip rates over the entire data that are greater than 1%, 5%, 10%, 15%, 20% and 25%, respectively. All regressions report the effect of cumulative quantity purchased over the past 2 weeks on the percentage of an ad viewed by a household, conditional on exposure, while including household and brand fixed effects. For ease of comparison, Column 1 in Table 10 repeats the results from Column 1 of Table 9. Although we lose power when focusing on only households that have high skip-rates, the effects of past product consumption is positive for all subgroups of households, and the marginal effect of quantity on ad consumption is higher for those with higher observed skip-rates.

To interpret these numbers, in the bottom panel of Table 10, we also report the effect on ad consumption of an increase in the quantity consumed of the product over the past two weeks for each of these subgroups. To do this, for each subgroup, we calculate the mean, 25th, 50th and 75th percentiles of the quantity purchased by households in that subgroup on days with a purchase. Then, we report how much ad consumption would change if a household in each subgroup increased its quantity purchased over the last two weeks by these values. For instance, column 7 reports the results for households with an ad-skip rate > 25%. Denote the mean, 25th, 50th and 75th percentile of the quantity purchased by households in that subgroup conditional on purchase as  $(\bar{q}, q_{.25}, q_{.5}, q_{.75})$  respectively. Looking at the bottom panel of column 7, we see that if the quantity purchased over the previous two weeks is increased by  $\bar{q}$ , households in that subgroup are likely to watch 1.31% more of the brand's ad, conditional on exposure. If the quantity purchased over the previous two weeks is increased by  $\bar{q}$ , households in that subgroup are likely to watch 1.31% more of the brand's ad, conditional on exposure. If the quantity purchased over the previous two weeks is increased by  $q_{.5}$ , households in that subgroup are likely to watch 0.96% more of an ad, conditional on exposure; and if the quantity purchased over the previous two weeks is increased by  $q_{.75}$ , households in that subgroup are likely to watch 1.48% more of an ad, conditional on exposure.

The above regressions pooled data across households including household and brand specific intercepts, but restricted the slope coefficients to be the same. Different households may have different sensitivities in how their purchase quantity is related to their ad consumption, and the un-modeled slope heterogeneity may be a source of spurious within-household correlation. To address this, we also run the above regressions separately for each household, in essence allowing the coefficients on cumulative quantity and cumulative ad time to differ for each household. For each household, we separately estimate the following regressions,

$$q_{jt} = \beta_{0j} + \beta_1 A_{jt,14} + \beta_2 p_{jt} + \beta_3 Time_t + \epsilon_{jt}$$

$$\tag{5}$$

$$a_{jrt} = \theta_0 + \theta_1 Q_{jt,14} + \theta_2 Time_t + \epsilon_{jrt} \tag{6}$$

where the index i for household is suppressed for brevity. The implicit assumption here is that parameters

Coeff	Brand FE	-	n.s.	+
$\beta_{1i}$	Y	3%	87%	10%
$\theta_{1i}$	Y	10%	74%	16%

Table 11: Significance of Household Specific Coefficients

are time-invariant for a given household. Table 11 records the percentage of coefficients that are positive and significant, negative and significant, and not statistically significant at the 5% level, and Figure 6 plots the empirical CDFs of the two sets of coefficients across households. Though we lose power especially for households with few purchase and ad exposure observations, we see that the cross effects of quantity and advertising are positive for a large subset.

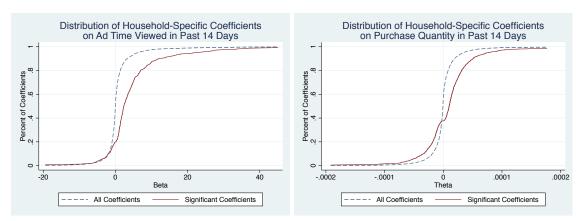


Figure 6: Histogram of Household-Specific  $\hat{\beta}_{1i}$  and  $\hat{\theta}_{1i}$  from Model with Brand Fixed Effects

**Summary** To summarize our results so far, we find that households tend to buy more when they have been exposed to more advertising in the past, and also tend to watch more advertising on the margin when they have purchased more in the past. These patterns do not seem to be driven by correlated tastes that drive both product purchase and ad viewing behavior. The fact that past purchase quantities seem to systematically explain the within-household variation in ad consumption rates also suggests that advertising is not simply playing a reminder role, and that complementarities are the more likely explanation.

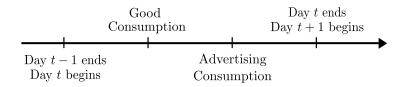
In the following section we present a model of demand for goods and advertising that allows for, but does not impose, complementarities between purchases and advertising. The model will allow us to relate the demand for products and advertising to a well defined utility maximization problem; to implement more efficient joint estimation of the simultaneous equations system defining advertising and product demand; to control for correlated unobserved heterogeneity and allow for stock effects; and to evaluate counterfactual scenarios that explore the response in purchases and welfare to changes in advertising while taking into account the co-dependence between purchases and ads.

# 4 A Model with Complementarities

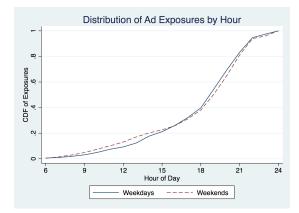
We describe an empirical model of consumption of goods and of advertisements. The consumption of goods is central to most utility maximization problems and is described in the next section. The consumption of advertisements is novel, however, and requires some attention. Unlike the case of regular consumption of goods, exposure to advertisements is not always deliberate. Advertising seeps into consumers' daily activities, often interrupting their favorite television programs and internet browsing. While consumers may not be able to choose which commercials they are exposed to on TV, they do have agency over whether to watch a whole commercial or decide to skip it. This is the decision that we model: We investigate whether consumers skip advertisements conditional on advertising exposure. Before we describe the main parts of the model, we present two key assumptions.

Assumption 1: Sequential Choices We treat the good and advertising consumption decisions as related but separate. While we allow the decisions to be interdependent (through complementarities or common factors that affect both actions), the decisions of which goods to buy and how much of each advertisement to watch are separated in time and location and unlikely to be simultaneously made. It is more likely that consumers make these decisions sequentially during the day. Hence, we use a sequential model of daily decisions as depicted in Figure 7. We assume that within a given day, the purchase decision of goods takes place before the decision of advertising consumption. Our data suggest this assumption is reasonable. Our advertising data includes a timestamp which indicates the exact second of each exposure. Looking at Figure 8 below, we see that 70% of ad exposures occur after 5 PM. Unfortunately, we do not observe a time stamp in the purchase data to test the time of purchase of products, but it is not unreasonable to assume that many in-store purchases occur during the day.

Figure 7: Timing of Consumption Decisions



Assumption 2: Myopic Behavior We also assume that consumers make purchase and ad-skipping decisions myopically, without formally incorporating the effect of their purchase decisions on subsequent ad consumption utility, and without incorporating the effect of their ad-skipping decisions on subsequent product purchase utility. Consumers face a number of decision problems during the course of a day and it is hard to imagine that they would, for example, deliberately decide to buy more of a good in order



#### Figure 8: Ad Exposures by Time of Day

to enjoy its advertisements more later in the same day. Further, it seems implausible that ad-skipping is driven significantly by the fact that a consumer may anticipate that seeing more or less of an ad may change his utility from *future* product consumption. While possible to incorporate at substantially higher computational cost, to us, entertaining such degree of forward-looking decision-making in this context seems unrealistic.

Finally, in the structural model of ad-skipping below, we model only a binary variable of the consumer's action of seeing the ad fully versus skipping it towards another channel or activity. Thus, on the ad-side, we model only the skip or not decision, and not the continuous decision of *how much* of an ad to watch. We think this modeling choice is more consistent with the actual trade-off consumers make.<sup>13</sup> Further, because the key preference over advertisements is revealed in the decision to skip the ad or not, we believe the continuous metric would be a more noisy measure to infer underlying ad- preferences. We do, however, think that the amount of ad time watched by consumers may be correlated with purchase outcomes. For example, if a commercial is informative or contains a catchy jingle, then even if the ad was not watched to completion (was "skipped"), a consumer who saw 95% of the ad may be more likely to make a purchase than a consumer who saw only 5% of the ad, i.e., an ad may have an effect on purchases even if it was skipped. To capture this, we use the continuous variation in ad consumption when modeling purchase decisions.

In the following, we utilize a discrete-continuous model of demand following Wales and Woodland (1983) and Kim, Allenby and Rossi (2002). The discrete-continuous model has the advantage of flexibly handling corner solutions and continuous purchase quantities across brands while retaining a clear link to a direct utility function (please see Chintagunta and Nair 2011 for a review). Later, when we present our counterfactuals, we show that explicitly handling the quantity decisions in the model has a material effect on our welfare and profit assessments. Within this set-up, we use Bhat (2005) and Lee, Kim and Allenby's (2013) parametrization of utility, which has been documented to fit scanner panel data well.

<sup>&</sup>lt;sup>13</sup>In Appendix C, we show that our results are not sensitive to local changes to how we define an ad as "skipped". Our results remain unaffected if we define an ad as "skipped" if the fraction viewed is < 1, 0.95, 0.9, 0.8 or 0.75.

We relax the models above by introducing the possibility of advertising acting as a complement to the advertised product.

### 4.1 Consumption Utility of Goods

On day t, an agent decides whether and how much to consume of each of J + 1 goods by maximizing a direct utility function conditional on past advertising exposures:

$$\max_{\boldsymbol{x}_t \ge 0} \quad U^G \left( \boldsymbol{x}_t | \boldsymbol{A}_{t-1} \right) \quad s.t. \ \boldsymbol{p}_t^G \cdot \boldsymbol{x}_t \le E_t^G \tag{7}$$

where  $\boldsymbol{x}_t$  is a vector of product quantities  $x_{0t} \dots x_{Jt}$ ,  $\boldsymbol{p}_t^G$  is a vector of prices  $p_{0t}^G \dots p_{Jt}^G$ , and  $E_t^G$  is the consumer's total expenditure. Here,  $\boldsymbol{A}_{t-1}$  is a vector of the stock of ad consumption over the preceding 14 days for each product,  $A_{1,t-1} \dots A_{J,t-1}$ . Some amount of the outside good (j = 0) is always consumed, and its price is normalized to one. The total direct utility from consumption is divided into two subutility functions as shown below, where  $U_0^G$  captures the utility from consuming the outside good and  $U_1^G$  captures the direct utility from consumption of the remaining goods as well as complementarity effects with advertising exposure:

$$U^{G}(x_{0t}...x_{Jt}|\mathbf{A}_{t-1}) = U_{0}^{G}(\cdot) + U_{1}^{G}(\cdot)$$
(8)

where

$$U_0^G(x_{0t}) = e^{\gamma_0} x_{0t} \times e^{\mu \varepsilon_{0t}^G}$$

$$\tag{9}$$

$$U_1^G(x_{1t}...x_{Jt} | \mathbf{A}_{t-1}) = \sum_{j=1}^J \exp\left[\gamma_j + \beta \log\left(1 + A_{jt-1}\right)\right] \log\left(1 + x_{jt}\right) \times e^{\mu\varepsilon_{jt}^G}$$
(10)

Above,  $\varepsilon_{0t}^G \dots \varepsilon_{Jt}^G$  are i.i.d. stochastic shocks that are known to the consumer but not to the econometrician, assumed to be T1EV distributed and  $\mu$  is a scale parameter. We introduce  $\mu$  so the expected demands implied by the problem are well defined (discussed in more detail below). The utility is quasilinear which is consistent with the fact that the category in question makes up a small percentage of household income, and income effects are not first-order.<sup>14</sup> The utility function is additively separable across brands, reflecting the assumption that the brands are substitutes. However, non-separability in utility between ads and quantities implies complementarities between consumption and advertising of a given brand are allowed. The implied marginal utility of consumption is,

$$\frac{\partial U^G}{\partial x_{jt}} = \frac{\exp\left[\gamma_j + \beta \log\left(1 + A_{jt-1}\right)\right]}{(1 + x_{jt})} \times e^{\mu \varepsilon_{jt}^G} \tag{11}$$

The marginal utility of consuming  $x_{jt}$  depends on the quantity purchased  $x_{jt}$  and on the quantity of advertising the individual consumed for that good in the past 14 days,  $A_{jt-1}$ . The complementary effect of advertising on the marginal utility from consumption of good j is given by,

<sup>&</sup>lt;sup>14</sup>Equation (8) is quasilinear because by dividing through by  $e^{\gamma_0 + \mu \epsilon_{0t}^G}$ , a monotone transformation of  $U^G$  (.), we can write  $U(x_{0t}...x_{Jt} | \mathbf{A}_{t-1}) = x_{0t} + U(x_{1t}...x_{Jt} | \mathbf{A}_{t-1})$ .

$$\frac{\partial}{\partial A_{jt-1}} \frac{\partial U^G}{\partial x_{jt}} = \frac{\beta \exp\left[\gamma_j + \beta \log\left(1 + A_{jt-1}\right)\right]}{\left(1 + x_{jt}\right)\left(1 + A_{jt-1}\right)} \times e^{\mu \varepsilon_{jt}^G}$$
(12)

An attractive feature of this model is that the sign of  $\beta$  determines whether advertising complements or substitutes the consumption of good j.

The agent maximizes her utility subject to the budget constraint in the domain of positive quantities. The Lagrangian for the utility maximization problem is,

$$\max_{\boldsymbol{x}_t \ge 0, \lambda_t \ge 0} \mathbb{L}^G \left( x_t, \lambda \right) = U^G \left( \boldsymbol{x}_t | \boldsymbol{A}_{t-1} \right) + \lambda_t \left( E_t^G - \sum_j \boldsymbol{p}_{jt} \cdot \boldsymbol{x}_{jt} \right)$$
(13)

which can be solved by the Karush-Kuhn-Tucker (KKT) optimality conditions. One goal of the demand model is to accommodate the mixed distribution of zeros (corner solutions) and positive quantities observed in the data. A generic observation comprises no-purchase of some brands, and positive quantities of the others by a household in a given day. Without loss of generality, let the first K goods be consumed, and the consumption of the rest be zero. Then the KKT conditions for this observation imply that,

$$\frac{\partial \mathbb{L}^G}{\partial x_{jt}} = 0, \quad x_{jt}^* > 0, \ j \in (0, .., K)$$

$$\tag{14}$$

$$\frac{\partial \mathbb{L}^G}{\partial x_{jt}} \leq 0, \quad x_{jt}^* = 0, \ j \in (K+1, .., J)$$

$$\tag{15}$$

$$x_{0t}, x_{jt}, \lambda_t \ge 0, \quad j \in (0, .., J)$$

$$\tag{16}$$

The problem above can be simplified by solving the optimality conditions with respect to  $\lambda_t$  and taking logarithms. Following the standard procedure of differencing with respect to the outside good (see for e.g., Kim, Allenby and Rossi 2002), it follows that at the optimum,

$$\mathcal{V}_{0t} - \mathcal{V}_{jt} = \varepsilon_{jt} - \varepsilon_{0t}, \, x_{jt}^* > 0, \, j \in (1, .., K)$$

$$(17)$$

$$\mathcal{V}_{0t} - \mathcal{V}_{jt} \geq \varepsilon_{jt} - \varepsilon_{0t}, \, x_{jt}^* = 0, \, j \in (K+1, ..., J)$$

$$\tag{18}$$

where,

$$\mathcal{V}_{0t} = \frac{\gamma_0}{\mu} \tag{19}$$

$$\mathcal{V}_{jt} = \frac{1}{\mu} \left[ \gamma_j + \beta \log \left( 1 + A_{jt-1} \right) + \log \left( 1 + x_{jt} \right) - \log \left( p_{jt} \right) \right]$$
(20)

In the remainder of this section, we derive in sequence the expected demand; the expected consumer surplus, and the joint probabilities of purchase and density of quantities implied by this model. In our empirical analysis, we will use the expected demand and consumer surplus to calculate expected profits and welfare from counterfactual targeting scenarios; and the implied joint density of purchase/quantities to form a maximum likelihood estimator of the consumer's utility parameters.

#### 4.1.1 Expected Demand

To simplify notation, we define  $\eta_{jt} \equiv \varepsilon_{jt}^G - \varepsilon_{0t}^G$ .  $\eta_{jt}$  is the difference between two T1EV random variables and has a logistic distribution with location 0 and scale 1. Equation (18) of the KKT conditions implies that if the consumer buys any units of brand j, it has to be his marginal utility for the initial unit of good j is higher than that of the outside good, i.e.,

$$\mathcal{V}_{jt}|_{x_{jt}=0} + \varepsilon_{jt}^G > \mathcal{V}_{0t} - \varepsilon_{0t}^G \tag{21}$$

$$\iff \eta_{jt} > \underline{\eta} \left( A_{jt-1}, p_{jt} \right) \equiv -\frac{1}{\mu} \left[ \gamma_j - \gamma_0 + \beta \log \left( 1 + A_{jt-1} \right) - \log \left( p_{jt} \right) \right]$$
(22)

We can combine this with Equation (17) to characterize the quantity of good j at time t bought by a given consumer conditional on demand shock  $\eta_{jt}$ :

$$x_{jt}^{*}(A_{jt-1}, p_{jt}, \eta_{jt}) = \begin{cases} \frac{\kappa_{jt}(A_{jt-1})}{p_{jt}} \exp\left(\mu\eta_{jt}\right) - 1, & \eta_{jt} \ge \underline{\eta}\left(A_{jt-1}, p_{jt}\right) \\ 0 & \eta_{jt} < \underline{\eta}\left(A_{jt-1}, p_{jt}\right) \end{cases}$$
(23)

where  $\kappa_{jt}(A_{jt-1}) = \exp\left[\gamma_j - \gamma_0 + \beta \log\left(1 + A_{jt-1}\right)\right]$ .

Integrating over the demand shock  $\eta_{jt}$  yields the expected demand for the consumer,

1

$$\mathbb{E}_{\eta}\left[x^{*}\left(A_{jt-1}, p_{jt}, \eta_{jt}\right)\right] = \int_{\underline{\eta}(A_{jt-1}, p_{jt})}^{\infty} \left(\frac{\kappa_{jt}\left(A_{jt-1}\right)}{p_{jt}}\exp\left(\mu\eta_{jt}\right) - 1\right) d\mathcal{F}_{\eta}\left(\eta_{jt}\right)$$
(24)

where  $\mathcal{F}_{\eta}$  is the c.d.f. of a standard logistic distribution. Given the properties of the logistic distribution, a necessary condition for the first moment of quantity to be finite is that  $\mu < 1$ .

#### 4.1.2 Consumer Welfare

Given the quasilinear utility, the consumer surplus is an exact representation of consumer welfare (representing both equivalent or compensating variation; see, for eg., Varian 1992, chap. 10). We calculate the consumer surplus by integrating the demand function in (23) from the observed price up to the reservation price at which the consumers' demand becomes zero. From equation (23), we can see that the reservation price  $\bar{\mathfrak{p}}(A_{jt-1}, \eta_{jt})$  at which demand  $x^*(A_{jt-1}, \bar{\mathfrak{p}}_{jt}, \eta_{jt}) = 0$  is,

$$\bar{\mathfrak{p}}\left(A_{jt-1},\eta_{jt}\right) = \kappa_{jt}\left(A_{jt-1}\right)\exp\left(\mu\eta_{jt}\right) \tag{25}$$

which defines the maximum price the consumer is willing to pay for a unit of good j at time t. Therefore, the surplus to the consumer from buying inside good j conditional on the demand shock  $\eta_{it}$  is,

$$\mathcal{C}S_{jt}\left(A_{jt-1}, p_{jt}, \eta_{jt}\right) = \mathbb{I}\left[\bar{\mathfrak{p}}\left(A_{jt-1}, \eta_{jt}\right) > p_{jt}\right] \int_{p_{jt}}^{\mathfrak{p}\left(A_{jt-1}, \eta_{jt}\right)} \left[\kappa_{jt}\left(A_{jt-1}\right) \frac{\exp\left(\mu\eta_{jt}\right)}{p} - 1\right] dp \tag{26}$$

where  $p_{jt}$  is the per-unit price of good j available to consumer i at time t. The integral above sums the consumer surplus from the price observed in the data  $(p_{jt})$  up to the reservation price  $\bar{\mathfrak{p}}(A_{jt-1}, \eta_{jt})$ . The indicator function  $\mathbb{I}[\bar{\mathfrak{p}}(A_{jt-1}, \eta_{jt}) > p_{jt}]$  ensures that consumer surplus is only calculated when a positive quantity of the good is bought. The expected consumer surplus (unconditional on  $\eta_{jt}$ ) is,

$$\mathcal{ECS}_{jt} = \mathbb{E}_{\eta} \left[ \mathcal{CS}_{jt} \left( A_{jt-1}, p_{jt}, \eta_{jt} \right) \right] = \int_{-\infty}^{\infty} \mathcal{CS}_{jt} \left( A_{jt-1}, p_{jt}, \eta_{jt} \right) d\mathcal{F}_{\eta} \left( \eta_{jt} \right)$$
(27)

Calculating the integral in (26) with respect to prices, and substituting into (27), the total expected consumer surplus is,

$$\mathcal{ECS}_{jt} = \int_{\widetilde{\eta}_{jt}(A_{jt-1}, p_{jt})}^{\infty} \left\{ p_{jt} + \kappa_{jt} \left( A_{jt-1} \right) \exp\left(\mu \eta_{jt} \right) \left[ \mu \eta_{jt} + \log\left(\frac{\kappa_{jt} \left( A_{jt-1} \right)}{p_{jt}} \right) - 1 \right] \right\} d\mathcal{F}_{\eta} \left( \eta_{jt} \right)$$
(28)

where the lower bound for the demand shocks,  $\tilde{\eta}_{jt}(A_{jt-1}, p_{jt}) = \frac{1}{\mu} \log \frac{p_{jt}}{\kappa_{jt}(A_{jt-1})}$  arises from the indicator function in (26).

Neither the expected demand in equation (24) nor the expected consumer surplus in equation (28) can be computed analytically; in our counterfactuals, we compute these integrals by simulation.

#### 4.1.3 Probability of Purchase and Density of Quantities

We close this section by presenting the probabilities of purchase and the associated density of quantities implied by the model, which we use later to construct the likelihood. To do this, we refer back to the KKT conditions for a generic observation in the data for which the first K goods are purchased, and the consumption of the rest are zero. For the goods not purchased, equation (18) implies that their unobserved stochastic utility components relative to the outside good,  $\eta_{jt}$ , cannot be larger than  $\mathcal{V}_{0t} - \mathcal{V}_{jt}$ . The corresponding probability that these goods are not purchased can be obtained by integrating the density of  $\eta_{jt}$  over the truncated support consistent with no-purchase. For the goods purchased, the model implies optimal quantities purchased are determined by trading off the marginal utility from consumption of the brand with that of the outside good. Equation (17) captures this tradeoff. Since the quantity purchased depend on  $\eta_{jt}$ , the distribution of  $\eta_{jt}$  induce a distribution on quantities, which can be derived by change-of-variables calculus using equation (17). Assuming  $\varepsilon_{0t}^G, ..., \varepsilon_{Jt}^G$  follow a Type-1 extreme-value distribution, the mixed discrete-continuous density of a generic observation for an agent at time t conditional on a vector of price and past advertising becomes,

$$l(x_{0t}^{*},...,x_{Kt}^{*},0,...,0|\boldsymbol{p}_{t},\boldsymbol{A}_{t-1}) = K! \times \frac{\prod_{j=0}^{K} \exp\left(\mathcal{V}_{jt}\right)}{\left[\sum_{j=0}^{J} \exp\left(\mathcal{V}_{jt}\right)\right]^{K+1}} \times |\mathcal{J}_{t}|$$
(29)

where  $V_{t}$  are as defined in Equations (19 and 20) and  $|\mathcal{J}_{t}|$  is the determinant of the Jacobian induced by the nonlinear change in variables transformation from the density of the error terms to the density of the purchased quantities. The Jacobian  $\mathcal{J}_{t}$  is diagonal with elements,

$$\left[\mathcal{J}_{t}\right]_{l,k} = \frac{\partial \left(\mathcal{V}_{0t} - \mathcal{V}_{lt}\right)}{\partial x_{kt}^{*}} = \mathbf{1} \left(l = k\right) \frac{1}{\mu \left(1 + x_{kt}^{*}\right)}, \, l, k = 1..K$$
(30)

and its determinant is  $[\Pi_{k=1}^{K} f_{kt}]$ , where  $f_{kt} = \frac{1}{\mu(1+x_{kt}^*)}$ , k = 1, ..., K (see Bhat 2005; Chintagunta and Nair 2011; or Lee, Kim and Allenbys 2013 for derivations in related setups).

### 4.2 Consumption Utility of Advertising

We now introduce the consumer's advertising consumption decision. We assume that advertising exposures are independent and take place sequentially within a day. An observation is a household-dayexposure combination. At advertising exposure s for brand j in day t, the consumer decides  $a_{sjt} \in \{0, 1\}$ where a zero means the advertisement is skipped (opt for the outside option) and a one means that the consumer watches the advertisement in its entirety. We view the ad-skip decision as the outcome of a time-allocation problem. We assume that an ad is  $\tau$  seconds long, so not skipping it consumes  $\tau$  seconds out of T total seconds allocated by the consumer for watching the show airing at a given occasion. Skipping the ad takes the consumer to the outside option (e.g., watching the show) of which some continuous quantity is always consumed. The number of seconds allocated to the outside option is denoted  $w_{st}$ . Conditional on an advertising exposure, the consumer's problem is given by,

$$\max_{w_{st}\in\mathbb{R}^+, a_{sjt}\in(0,1)} \quad U^A\left(w_{st}, a_{sjt} | X_{jt}\right) \quad s.t. \ w_{st} + \tau a_{sjt} \le T \tag{31}$$

where  $X_{jt}$  is a stock of past product consumption in the preceding 14 days (including the purchase earlier in day t, if applicable). Analogous to the purchase-side model, we assume the direct utility  $U^A(.)$  is,

$$U^{A}(w_{st}, a_{sjt} | X_{jt}) = U_{0}^{A}(w_{st}) + U_{1}^{A}(a_{sjt} | X_{jt})$$
(32)

where,

$$U_0^A(w_{st}) = e^{\alpha_0} w_{st} \times e^{\varepsilon_{s0t}^A}$$
  
$$U_1^A(a_{sjt} | X_{jt}) = a_{sjt} \exp[\alpha_j + \theta \log(1 + X_{jt})] \times e^{\varepsilon_{sj}^A}$$

and  $\varepsilon_{s0t}^A$ ,  $\varepsilon_{sjt}^A$  are IID Type-1 extreme value errors that shift the value from skipping versus seeing the ads. To solve for the choices induced by this program, recall,  $a_{st}$  is binary. If the consumer decides to skip exposure s,  $a_{sjt} = 0$ . Substituting for  $a_{sjt}$  into the time-constraint in Equation (31),  $w_{st} + \tau \times 0 = T$ , so  $w_{st} = T$  when  $a_{sjt} = 0$ . We can now obtain the conditional indirect utility from skipping by evaluating the direct utility (31) at  $w_{st} = T$  and  $a_{sjt} = 0$ ,

$$\mathcal{V}^{A}(a_{sjt} = 0|X_{jt}) = U_{0}^{A}(w_{st} = T) + U_{1}^{A}(a_{sjt} = 0|X_{jt})$$
  
=  $e^{\alpha_{0}}T \times e^{\varepsilon_{s0t}^{A}}$ 

If the consumer decides to watch ad-exposure s,  $a_{sjt} = 1$ . Substituting again for  $a_{sjt}$  into the timeconstraint in Equation (31),  $w_{st} + \tau \times 1 = T$ , so  $w_{st} = T - \tau$  when  $a_{sjt} = 1$ . We can obtain the corresponding conditional indirect utility from not skipping by evaluating the direct utility (31) at  $w_{st} = T - \tau$  and  $a_{sjt} = 1$ ,

$$\mathcal{V}^{A} (a_{sjt} = 1 | X_{jt}) = U_{0}^{A} (w_{st} = T - \tau) + U_{1}^{A} (a_{sjt} = 1 | X_{jt}) = e^{\alpha_{0}} (T - \tau) \times e^{\varepsilon_{s0t}^{A}} + \exp [\alpha_{j} + \theta \log (1 + X_{jt})] \times e^{\varepsilon_{sjt}^{A}}$$

The consumer will not skip the ad if  $\mathcal{V}^A(a_{sjt} = 1 | X_{jt}) > \mathcal{V}^A(a_{sjt} = 0 | X_{jt})$ . Taking logarithms on both sides of this inequality, and letting  $\mathcal{W}_{sjt} = \alpha_j + \theta \log(1 + X_{jt})$ , and  $\mathcal{W}_{s0t} = \alpha_0 + \log(\tau)$ , the consumer will watch the advertisement if,

$$\mathcal{W}_{sjt} - \mathcal{W}_{s0t} \ge \varepsilon_{s0t}^A - \varepsilon_{sjt}^A \tag{33}$$

The implied probability of watching an advertisement is a logit,<sup>15</sup>

$$Pr(a_{sjt} = 1) = \frac{\exp(\mathcal{W}_{sjt})}{\exp(\mathcal{W}_{s0t}) + \exp(\mathcal{W}_{sjt})}$$
(34)

Evidence of complementarities for the agent can be found if,

$$\frac{\partial}{\partial X_{jt}} \left[ U_1^A \left( a_{sjt} = 1 | X_{jt} \right) - U_1^A \left( a_{sjt} = 0 | X_{jt} \right) \right] = \frac{\theta}{1 + X_{jt}} \exp\left[ \alpha_j + \theta \log\left( 1 + X_{jt} \right) \right] \cdot \mathrm{e}^{\varepsilon_{sjt}^A} \ge 0 \qquad (35)$$

Again, an attractive feature of the specification is that the interaction parameter  $\theta$  captures the direction of complementarity/substitution between advertising and consumption. A positive value of  $\theta$  would indicate that the consumption of good j increases the utility of its advertisements.

### 4.3 Maximum Likelihood

**Purchase Likelihood** The mixed discrete-continuous density of a purchase observation for an individual was presented in Equation (29). Because purchase incidence is infrequent at the daily level, we estimate the model conditional on purchase of at least one of the inside goods. This keeps the empirical model from losing precision from having to fit a large number of zeros. This also means we only identify parameters from the quantity and brand-choice variation, and not from the buy vs. not-buy decision. Let  $B_t$  indicate the purchase of an inside good s.t.,

$$B_t = \begin{cases} 1 \text{ if } x_{jt} > 0 \text{ for at least one } j \in \{1, .., J\} \\ 0 \text{ otherwise} \end{cases}$$

To handle the selection conditional on purchase, we derive the corresponding conditional purchase likelihood as the unconditional likelihood divided by the probability of purchase,

$$\mathcal{L}_{t}^{G} = l\left(x_{0t}^{*}, ..., x_{Kt}^{*}, 0, ..., 0 | B_{t} = 1\right) = \frac{l\left(x_{0t}^{*}, ..., x_{Kt}^{*}, 0, ..., 0\right)}{1 - Pr(B_{t} = 0)}$$

where,

$$Pr(B_t = 0) = \frac{\exp(\mathcal{V}_{0t})}{\sum_{j=0}^{J} \exp(\mathcal{V}_{jt})}$$

Ad Consumption Likelihood While we infer ad consumption purely from the consumer's decision to skip or watch an ad, for predictive purposes it is useful to have a statistical model for the number of ad exposures and the brand of each exposure. Let  $z_t$  denote the total number of ads a consumer is exposed to in day t,  $b_{1t},...,b_{zt}$  indicate the brand of each ad exposure, and  $a_{1t},...,a_{zt}$  indicate the consumer's binary decision to skip or watch those exposures. The joint likelihood of observing the ad-exposures, the brand content of the exposures, and the ad-skip decisions is,

$$\mathcal{L}_{t}^{A} = Pr(z_{t}, b_{1t}, ..., b_{zt}, a_{1t}^{*}, ..., a_{zt}^{*} | X_{t})$$

$$= \underbrace{Pr(a_{1t}^{*}, ..., a_{zt}^{*} | z_{t}, b_{1t}, ..., b_{zt}, X_{t})}_{\text{Logit}} \underbrace{Pr(b_{1t}, ..., b_{zt} | z_{t}, X_{t})}_{\text{Ind. Draws from pmf}} \underbrace{Pr(z_{t} | X_{t})}_{\text{Poisson}}$$

 $<sup>^{15}\</sup>tau$  is not identified separately from the intercept and is absorbed into  $\alpha_0$ .

We choose statistical distributions to fit the observed exposures and ad-brand content across agents in the data. To accommodate the large number of zero exposures, we assume  $z_t$  follows a zero-inflated Poisson distribution with probability mass function:

$$Pr(z_t = h) = \begin{cases} \pi + (1 - \pi)e^{-\lambda}, & h = 0\\ (1 - \pi)\frac{\lambda^h e^{-\lambda}}{h!}, & h \ge 1 \end{cases}$$
(36)

where  $\lambda$  and  $\pi$  are parameters to be estimated. Next, let  $b_{st} \in (1, ..., J)$  denote the brand corresponding to exposure s on day t. We assume  $b_{st}$  follows a multinomial distribution with parameters  $\phi_j$  s.t.  $\sum_{j=1}^{J} \phi_j =$ 1. Assuming the probabilities are independent across exposures, the likelihood of viewing the observed distribution of ad-brand content, conditional on  $z_t$  exposures is,

$$Pr(b_{1t},...,b_{zt}|z_t,X_t) = \prod_{s=1}^{z_t} \prod_{j=1}^J \phi_j^{1(b_{st}=j)}$$
(37)

Finally, under the assumption that  $\varepsilon^A$  is T1EV, the problem of maximizing utility through the consumption of advertisements is a discrete choice model much in the spirit of McFadden (1974) and subsequent classical applications. The last component of the conditional likelihood of the consumer's choices is a logit likelihood,

$$l(a_{0t}^{*},...,a_{zt}^{*}|z_{t},b_{1t},...,b_{zt},X_{t-1}) = \prod_{s=1}^{z_{t}} \left(\frac{\exp\left(\mathcal{W}_{sjt}\right)}{\exp\left(\mathcal{W}_{s0t}\right) + \exp\left(\mathcal{W}_{sjt}\right)}\right)^{a_{sjt}} \left(\frac{\exp\left(\mathcal{W}_{s0t}\right)}{\exp\left(\mathcal{W}_{s0t}\right) + \exp\left(\mathcal{W}_{sjt}\right)}\right)^{1-a_{sjt}}$$
(38)

To reiterate, our ad-utility parameters  $(\alpha, \theta)$  are identified purely off the conditional skip/no-skip decision (equation 38), and not from the observed distribution of exposures and ad-brand content across agents in the data. We fit a distribution to exposures and brand-content purely for prediction purposes.

**Deriving the Joint Likelihood** Putting the two pieces together, we arrive at the joint likelihood of observing the purchase and advertising data given the model parameters,

$$\mathcal{L}_t = \mathcal{L}_t^G \times \mathcal{L}_t^A$$

We allow for a random effects specification of heterogeneity. To reflect this, we now introduce the index i for agent. Collect the key parameters of interest in a vector  $\Theta_i = (\gamma_i, \beta_i, \alpha_i, \theta_i)$ . We assume that

$$\Theta_i \sim MVN(\bar{\Theta}, \Sigma)$$

We allow the key parameters pinning down the complementarities in the purchase and ad consumption models  $\beta$  and  $\theta$ , to covary with each other. This allows us to capture the fact that households whose consumption utility is sensitive to their level of ad consumption may also derive ad utility that is sensitive to the level of product consumption. Such households may experience a feed-back loop whereby viewing a large number of ads leads them to buy a lot of the product which in turn leads them to view more ads. In addition, we allow for the fact that households who tend to have a high preference for brand j may also derive higher utility from advertisements for brand j by allowing  $\gamma_j$  and  $\alpha_j$  to be correlated. Accordingly,  $\Sigma$  is specified to be a block-diagonal matrix with the exception of non-zero covariance terms that we estimate on the  $\gamma_j \times \alpha_j$  and  $\beta \times \theta$  off-diagonals. We estimate this joint model via maximum simulated likelihood using 1,000 draws of  $\Theta_i$  per household to integrate over the implied random effects distribution. The mixed discrete-continuous density implied by the purchase model and the joint covariance with the advertising model combined with the large panel duration per household (557 days) makes the likelihood function complicated to maximize. We use the efficient SNOPT solver with a likelihood tolerance of 10e - 6 to facilitate the maximization. On an AMD 64-bit Unix server equipped with a 24-core Intel Xeon X5650 chip running at 2.67GHz and parallelized across 12 workers, the model takes roughly 7 days to maximize.

# 5 Estimation Results

The estimates for the parameters of the product and ad consumption equations are presented in Table 12. The estimates of the remaining auxiliary parameters  $(\lambda, \pi, \phi_i)$  are included in a table in Appendix D. Looking first at the estimates for the product consumption model in Table 12, the  $\bar{\gamma}$  parameters reflect brand-level purchase incidence in the data, such that brands with a more negative coefficient are those that are purchased less frequently. The negative values reflect the large share of no-purchase at the daily level. The relatively large magnitude of the  $\sigma^{\gamma}$  parameters indicates significant heterogeneity in purchase frequency across households. The complementary coefficient on advertising  $\bar{\beta}$  is estimated to be positive and the relatively small magnitude of  $\sigma^{\beta}$  implies a low probability of having a negative  $\beta$ . Turning to the estimates for the advertising consumption model, recall that brands 1 and 6 do not advertise in our data, so we do not estimate parameters for these brands. The intercept parameters  $\bar{\alpha}$  pin down ad-skip rates; so add for brands with a higher intercept correspond to those observed to be skipped less frequently. The relatively large magnitude of the  $\sigma^{\alpha}$  parameters shows that there is extensive heterogeneity in adskip rates across households. The mean complementarity parameter  $\bar{\theta}$  in the ad consumption model is estimated to be positive. The standard deviation  $\sigma^{\theta}$  is relatively large so there is a reasonably large chance of having a negative  $\theta$ . Finally, the estimated correlation between  $\beta$  and  $\theta$  is captured by  $\rho$  and is estimated to be positive but small. Finally, though the scale parameter  $\mu$  is theoretically identified, we were unable to pin it down across a range of specifications we tried. We calibrate  $\mu = \frac{1}{2}$  and estimate the remaining parameters of the model fixing  $\mu$  at this value. We re-estimated the model at various values of  $\mu \in (-1, 1)$  and found the fit of the model to the data to be roughly the same.

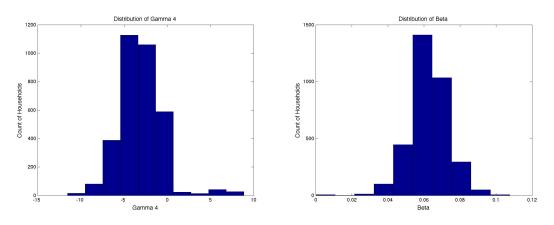
After estimating the demand system, we use the estimated mean  $\hat{\Theta}$  and variance  $\hat{\Sigma}$  parameters to characterize the distribution of household-specific tastes. We apply Bayes Rule to calculate an estimate of each household's expected preference parameters conditional on the household's purchase history and

Brand	$ar{\gamma}_{ij}$	$\sigma_{ij}^{\gamma}$	$\bar{lpha}_{ij}$	$\sigma^{lpha}_{ij}$
1	-2.9555	0.3624	-	-
	(0.0218)	(0.0286)	-	-
2	-2.9506	0.4092	3.1207	0.3823
	(0.0208)	(0.0241)	(0.0175)	(0.0304)
3	-0.8893	3.4289	3.3273	0.2131
	(0.0229)	(0.0151)	(0.0178)	(0.0449)
4	-2.7595	3.2278	3.2790	0.2908
	(0.0384)	(0.0378)	(0.0162)	(0.0307)
5	-2.4964	0.3820	2.7269	0.5520
	(0.0181)	(0.0219)	(0.0177)	(0.0275)
6	-1.8154	2.5924	_	_
Ŭ	(0.0377)	(0.0321)	_	_
7	-2.3140	0.1944	2.7002	0.6312
	(0.0162)	(0.0241)	(0.0140)	(0.0195)
8	-4.9805	4.3027	2.8935	0.6482
	(0.0571)	(0.0416)	(0.0155)	(0.0215)
9	-2.6011	0.1427	2.5248	0.7409
-	(0.0174)	(0.0346)	(0.0160)	(0.0206)
10	-2.6809	0.1969	2.6825	0.5932
	(0.0205)	(0.0430)	(0.0161)	(0.0227)
11	-2.1150	3.8769	2.9105	0.4888
	(0.0296)	(0.0227)	(0.0174)	(0.0256)
	( )	( )	( )	()
$\bar{eta}$	$\sigma^{eta}$	$\bar{ heta}$	$\sigma^{ heta}$	ρ
0.0631	0.0138	0.0012	0.0332	0.0004
(0.0067)	(0.0107)	(0.0040)	(0.0059)	(0.0003)

Table 12: Parameter Estimates for the Joint Model Estimated on the Full Sample

${ m LL}=$ -1,699,456.84				
No. of Households $= 4,221$				
No. of Observations $= 2,325,771$				

Notes: Table reports results from joint maximum likelihood estimation of the purchase and advertising model across households. Random effects allowed on  $\Theta_i = (\gamma_i, \beta_i, \alpha_i, \theta_i)$  as  $\Theta_i \sim MVN(\bar{\Theta}, \Sigma)$ , where,  $\Sigma$  is a block-diagonal matrix with the exception of non-zero covariance terms on the  $\gamma_j \times \alpha_j$  and  $\beta \times \theta$  off-diagonals. The parameter  $\rho$  measures the covariance between  $\beta$  and  $\theta$ . The covariances between  $\gamma_j$  and  $\alpha_j$  are estimated but not reported. Model estimated via maximum simulated likelihood using 1,000 draws of  $\Theta_i$  per household to integrate over the implied random effects distribution. The scale  $\mu$  is fixed at 0.5.



#### Figure 9: Household Specific Estimates of $\gamma_4$ and $\beta$

the estimated distribution of the population's preference parameters as,

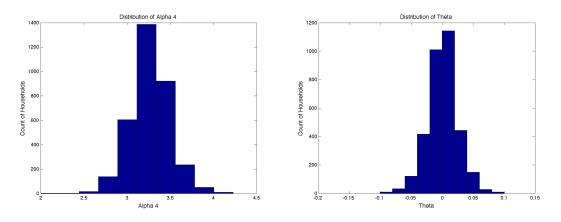
$$\hat{\Theta}_{i} = \int \Theta_{i} \frac{\mathcal{L}_{i}(\Theta_{i}|\bar{\Theta},\hat{\Sigma})}{\mathcal{L}_{i}(\bar{\Theta},\hat{\Sigma})} d\Phi(\Theta_{i}|\bar{\Theta},\hat{\Sigma})$$
(39)

We use this "approximate Bayesian" approach (Allenby and Rossi 1999; Revelet and Train 2001; Chintagunta et al. 2005) to recover household level parameter estimates that can be used to measure heterogenous treatment effects.

### 5.1 Model Simulations

Assessing intuition regarding the implication of estimates in a nonlinear model with random effects like our model above is best done with simulation. We would like to document what the estimates predict about changes in demand in response to changes in advertising and to illustrate endogenous non-compliance with treatment. We first forward-simulate daily advertising and purchase outcomes for the last quarter of the data using our estimated household-specific coefficients at the observed prices and level of ad exposure. We then simulate advertising and purchase outcomes increasing the number of ad exposures for each brand separately by one standard deviation, holding everything else fixed. For illustration, we discuss in detail below the output of this simulation for brand 4. To aid in the interpretation of the simulations, we include plots of the household specific preference parameters for brand 4 in Figures 9 and 10. Table 14 at the end of this section summarizes the analogous results for all the advertised brands in the data.

Consider brand 4. We simulate the increase in exposures for the brand as follows. We first calculate the total number of exposures each household saw for brand 4 in the last quarter of 2011. The mean number of exposures over the three months is 24. We then increase each household's number of ad exposures by the standard deviation of the number of brand 4 Q4-2011 ad exposures across households. This turns out to be 28 exposures. We allocate the additional 28 exposures evenly across the days in



#### Figure 10: Household Specific Estimates of $\alpha_4$ and $\theta$

which each household was observed to view an ad for brand 4 in the data. Our simulation only includes the 2,751 households who viewed at least one advertisement for brand 4 during these 3 months.

**Endogenous Non-Compliance: Ad Skipping** Figure 11 shows the model-predicted household skip rates (fraction of ad exposures that are skipped) at the observed level of exposures. The median household skip rate is 0.0351 which is similar to the overall percent of ads that are observed to be skipped in the data. These skip rates can be thought of as a measure of the uptake of treatment. All households are "offered" the same treatment of an additional 28 advertisements, but households vary in the extent to which they are treated, and, importantly, this variation in treatment is an endogenous outcome of the model.

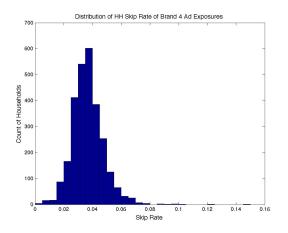
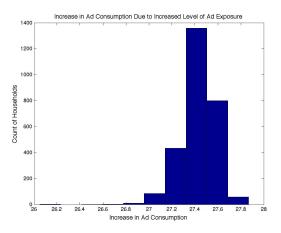


Figure 11: Distribution of Simulated Household Ad Skip Rates of Brand 4 Exposures

Figure 12 plots a histogram across households of the increase in realized total ad consumption resulting from the increased level of exposures. The realized total ad consumption for a household is calculated by computing the model-predicted proportion of each of the 28 additional exposures that is consumed, and Figure 12: Increase in Consumption of Brand 4 Ads Resulting from an Increase in Number of Ad Exposures



then adding up these proportions across the 28 incremental exposures. This plot shows that the extensive heterogeneity in skip rates documented in Figure 11 results in differences in the level of treatment across households. Although all households were shown an additional 28 ad exposures, the average increase in ad consumption ranges from 26.1 to 27.9 ads across households. This reflects the differential uptake of treatment across households.

**Endogenous Non-Compliance: Measuring Treatment Effects** We now discuss the implications of the differential uptake of advertising on purchase quantity and welfare. In order to measure the responsiveness of demand and welfare to changes in advertising, we calculate advertising elasticities. Looking at equation (40), we calculate the advertising elasticity of demand for each household as the household's percent increase in total predicted purchase quantity of brand 4 over the three months in the simulation, divided by the corresponding percent increase in predicted ad consumption. For each household, we calculate welfare in a given day using equation (27). We sum this consumer surplus across the last three months of the data to obtain the total surplus from consumption. We report a "welfare elasticity" calculated as the percent increase in total consumer surplus over the three months in the simulation divided by the percent increase in ad consumption.

$$\eta_{ij}^{q} = \frac{\Delta Q_{ij}/Q_{ij}}{\Delta A_{ij}/A_{ij}}, \eta_{ij}^{u} = \frac{\Delta C S_{i}/C S_{i}}{\Delta A_{ij}/A_{ij}}$$
(40)

Appendix 5.1 describes the simulation steps in greater detail. To the extent these measure tabulate in outcomes over a quarter in response changes in the consumed advertising stock, these elasticities capture medium to long-run effects of advertising. Figure 13 shows the distributions of model-predicted demand and welfare elasticities with respect to advertising across households. The median advertising elasticity

of demand is 0.0561 and the median advertising elasticity of welfare is 0.0639.<sup>16</sup> There is significant heterogeneity across households.

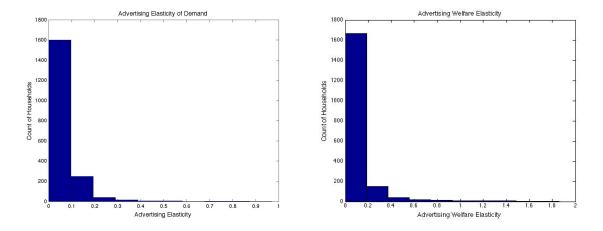


Figure 13: Advertising Elasticities

The model implies that households self-select into receiving more or less treatment by choosing to skip ads. Logically, we should also expect that those households who watch more ads are the ones that show larger purchase quantity and welfare changes. We assess informally whether households who are predicted by the model to consume more incremental ad-time also see larger increases in these outcome variables of interest, thereby assessing whether endogenous non-compliance with advertising matters at the estimated parameters. We regress the increase in predicted quantity and utility on the model-predicted incremental ad time watched. The results are in Table 13. In both, the effect of incremental ad time is positive -i.e., those who consume more ad-time see larger increases in purchase outcomes and welfare. In effect, this illustrates that the change in ad consumption can explain the cross-sectional change in purchase quantity and welfare, but the change in ad exposures cannot (because in this exercise, all households are subject to the same change in ad exposures). This is important for firms who may be considering the profitability of a targeted ad campaign because the efficacy of the campaign will depend on the compliance of the sub-population that is targeted. A positive take-away is that if the advertiser can observe individual-level ad consumption and not just ad exposure, the firm can identify the subset of households whose purchases and welfare will likely change in response to the campaign. We explore this further in the targeting counterfactuals below.

### 6 Counterfactuals

Our counterfactuals are motivated by the future of TV ad-markets referred to in the introduction, in which new digital streaming and set-top box technology has made televisions individually addressable

 $<sup>^{16}</sup>$  Across brands, we estimate average advertising elasticities of demand ranging from 0.02 to 0.11. These estimates are comparable to the advertising elasticity of demand of 0.15 that Ackerberg (2001) estimates in his analysis of TV commercials and demand for yogurt.

	(1)	(2)						
	$\Delta$ Purchase Quantity_i	$\Delta$ Consumer Surplus <sub>i</sub>						
$\Delta A_i$	293.15***	4,004.69***						
	(88.33)	(1,274.10)						
Constant	-7,984.63***	-109,114.60***						
	(2, 422.91)	(34, 949.98)						
Observations	2,751	2,751						
Mean Dep Var.	56.75	737.03						
Standard errors in parentheses								
*** p<0.01, ** p<0.05, * p<0.1								

Table 13: Regression of Increase in Purchase Outcomes on Increase in Ad Consumption

Note: Regression estimated at the household level. The independent variable  $\Delta A_i$  records each household's model-predicted increase in ad consumption between the two simulations. The dependent variables record the model-predicted increase in purchase quantity and consumer surplus between the two simulations.

Brand	$\Delta$ Ad Exposures	$\Delta$ Ad Consumption	Demand Elasticity	Welfare Elasticity
2	8	7.8190	0.0234	0.0252
		(0.0687)	(0.0227)	(0.0281)
3	25	24.5314	0.0328	0.0366
		(0.1080)	(0.0451)	(0.1079)
4	28	27.4511	0.0561	0.0639
		(0.1467)	(0.2572)	(1.9588)
5	9	8.7017	0.0265	0.0274
		(0.1374)	(0.0219)	(0.0260)
7	11	10.6414	0.0297	0.0318
		(0.2058)	(0.0258)	(0.0305)
8	0	-	-	-
		-	-	-
9	12	11.5512	0.0299	0.0322
		(0.3053)	(0.0280)	(0.0325)
10	6	5.8014	0.0157	0.0158
		(0.1106)	(0.0134)	(0.0179)
11	10	9.7280	0.0207	0.0221
		(0.1181)	(0.0649)	(0.2663)

Table 14: Summary of Simulation Results by Brand

*Note:* Table reports the median and standard deviation of outcomes across households. Each simulation includes the set of households who were exposed to at least one ad for the focal brand. One outlier household dropped in the simulation for brand 4.

(via their IP-address or unique set-top box ids). Compared to online advertising markets, TV-ad markets currently provide only limited targeting ability, allowing advertisers to buy ad-spots on TV shows whose attractiveness in reaching desired audiences can be assessed on the basis of only limited, aggregated audience data split by coarse demographics (age and gender). The coming addressability of TV in the future will imply that advertisers will be able to target ads to individual consumers *directly*, rather than to the shows they watch. Further, increasingly, as TV-viewing data gets merged with data on the corresponding product purchases of the viewers, advertisers will be able to target audiences more finely on the basis of their preferences for products (as revealed in their historical purchase data), rather than on the basis of coarse demographics like age and gender. More broadly, in this addressable ecosystem advertisers could contemplate targeting ads to consumers who are less likely to skip them and who are more likely to respond favorably to advertising consumption by increasing product demand. Non-skipped ads may be favored by the TV network all things equal, as they reduce the chance that targeted consumers move away from the show being viewed or from viewing TV to other entertainment options. Further, as product purchases are tracked, it may be feasible to target both advertising and prices (say via targeted discounts or coupons) to pinpointed audiences, and to track track subsequent redemptions and purchases. The structural model we built that endogenizes product purchase and ad-consumption behavior in a fully specified setup with heterogeneous preferences is useful to assess how demand, consumer welfare and profitability may change in such an addressable market.

We use our model to assess a variety of targeting scenarios. First, we consider how demand, welfare and profits would change if an advertiser could target ads to consumers who are less likely to skip them (i.e., essentially those with  $\frac{\partial U}{\partial A} > 0$ ), where we assess expected skip-rates using our model and estimated parameters. To the extent that it is possible for advertisers to assess skip-rates of consumers by leveraging only TV-viewing data, and to the extent that TV-networks may prefer that ads are targeted to the subset of their viewers who are less likely to skip them; this kind of targeting policy is likely to be increasingly used by firms in the future. Complementarities in demand for products and advertisements imply that those that like the ads are also more likely to prefer the advertisers' products; hence, this is ad-targeting strategy benefits the advertiser by indirectly picking consumers who are favorably disposed to its product.

This targeting policy could be improved if the advertiser also incorporated product preference information. Because some who do not skip ads may have bought the product anyway even in the absence of ad-exposure, it is possible that targeting on ad skip-rates alone allocates advertising to infra-marginal consumers and misses marginal consumers. Since we have estimated product and ad preferences, we can use the full model to assess outcomes if the advertiser targets consumers who are less likely to skip the targeted ads and for who the marginal ad induces a change in purchase behavior  $(\frac{\partial^2 U}{\partial A \partial Q} > 0)$ .

Finally, complementarities imply that prices play a stronger role in moderating advertising effects in our model compared to other setups for handling advertising. Because the propensity to consume advertising is mediated by how much of the product is consumed, low prices can induce higher adconsumption on the margin via its effect on product quantity demanded. The increased ad-consumption in turn affects product demand, and so on, resulting in potential feedback and large returns to low prices. To assess this, we simulate a third counterfactual in which we allow the advertiser to target both individual prices and ads to consumers, essentially implementing first-degree price and advertising discrimination. For each, we compute demand, consumer welfare and profitability across the consumers in the data.

Our results are meant to illustrate the importance of considering demand-side complementarities and the value of endogenizing the decision to consume advertising in assessing these targeting scenarios. An important caveat to the counterfactuals is that we do not incorporate competitive price and advertising response in reaction to the improved price and advertising targeting by the focal advertiser. Thus, our results are not meant to speak to equilibrium outcomes in a market with improved addressability and targeting. Doing so would require specifying a supply-side model of price and advertising competition, which is beyond the scope of the current demand-side analysis.

#### 6.1 Simulation Setup and Procedure

We implement our counterfactuals using the 2011 Fall season in our data as a benchmark. We first forward-simulate daily advertising and purchase outcomes for the 106 days starting Sept 17, 2011 through Dec 31, 2011 for all households in our data using our estimated household-specific coefficients at the observed prices and level of ad exposure. Then we ask how outcomes look under various counterfactual ad and price targeting scenarios compared to this benchmark. Because we have no way to predict how many customers will watch TV on a given day (and hence can potentially be targeted with ad exposures), we implement all our counterfactuals holding the number and the timing of total exposures fixed at the allocation observed in Fall 2011 in the data. Thus, we implicitly hold the sequence of "opportunities" to deliver exposures, as well as the total number of possible exposures fixed under all counterfactual comparisons. To fix ideas, suppose we observe  $b_{jt}$  exposures by advertiser j on day t = 1, ..., 106 in Fall 2011 in the data. In our counterfactuals, we hold  $b_{jt}$  fixed for each t and vary how the  $b_{jt}$  exposures are allocated across different sets of consumers. Thus, the ad-side control variable for the firm in all our counterfactuals is a set of indicators  $\left\{\tilde{b}_{ijt}; i=1,..,N\right\}$  such that  $\tilde{b}_{ijt}=1$  if consumer *i* gets allocated ads on day t, and 0 otherwise, and such that  $\sum_{i=1}^{N} \tilde{b}_{ijt} = b_{jt} \forall t$ . In counterfactuals in which price targeting is also considered, the control variable for the firm also includes  $\tilde{p}_{ijt}$ , an individual price to offer consumer i for purchase of product j in period t. We treat firms as myopic, by not incorporating into their price and advertising decisions that current controls can affect future profits through their effects on the consumer's consumption stocks. This implies that our static assessments will understate the potential profits impacts from targeting. Finally, we assume that the manufacturer earns a margin set at 30% on a constant marginal cost implying an expected profit from consumer i on day t of,

$$\pi_{ijt} = 0.3p_{ijt} \times \mathbb{E}\left[x_{ijt}^*\right] - cb_{ijt} \tag{41}$$

where  $\mathbb{E}\left[x_{ijt}^*\right]$  is as defined in equation (24) and c is the cost of an ad-exposure. The total profit at time t is,

$$\pi_{jt} = 0.3 \sum_{i=1}^{N} p_{ijt} \times \mathbb{E}\left[x_{ijt}^*\right] - cb_{jt}$$

$$\tag{42}$$

In counterfactuals invoking only ad-targeting, we allocate ads to consumers on the basis of their expected skip-probability, where the probability is implied by the ad-skip model in equation (34). In counterfactuals involving only targeted pricing, we solve for the optimal prices for each consumer by maximizing the profit function in equation (41) with respect to prices. Because the optimization with respect to  $\mathbb{E}\left[x_{ijt}^*\right]$  is complex, we utilize an approximation to the optimal price by solving a first-order Taylor-series approximation to the first-order conditions implied by the pricing problem. Appendix F provides exact details. In counterfactuals in which we consider ad- and price-targeting, we use the full ad-and-product side demand model, computing the expected profit to the firm for each consumer when targeting an optimal price, taking into account that the consumer can choose to skip or watch the ad with some probability implied by the model. Appendix G presents exact details of the ad-and-price reallocation algorithms along with pseudo-code for the steps implemented.

#### 6.2 Results

Table 15 presents the profits and consumer welfare levels for the different scenarios normalized relative to the benchmark. The top panel of the table keeps the firm's prices constant, whereas the second panel allows the firm to reoptimize its prices to each consumer as discussed above. For illustration, the results below report in detail on simulations in which the advertising and pricing polices of brand 4 are assessed (the results for all other brands are currently being computed and available on request). The profit numbers reported sum the total profit across all the consumers. The welfare numbers reported sum the expected consumer surplus (equation 28) across all consumers. Note, this welfare measure only captures the surplus from the consumption of the product. The model implies consumers also derive utility directly from the consumption of advertisements. While we can measure this latter utility using our model estimates, we do not have a way to convert these into money metric terms.<sup>17</sup> Hence, we report only the more conventional metric representing the surplus from product-consumption, noting that this understates the overall welfare implied by the model because it omits the direct surplus from the consumption of advertising and prices affect this metric though their effect on product quantities.

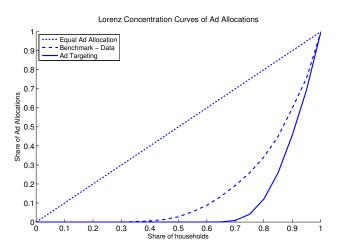
We first discuss the case with no price discrimination. Looking at Table 15, we see that when prices are fixed, being able to target households based on their expected ad-viewing ("Targeting: Ad Viewing" case) increases profits by 4.4% relative to the benchmark. Targeting on the basis of ad-viewing *and* product-purchase behavior ("Targeting: Full Information" case) increases profits by 10.9% and welfare by

 $<sup>^{17}</sup>$ To do this, we would have to measure the marginal cost in dollars of a unit of time spent on watching the ad. Using hourly wages or salaries to value time did not seem to us as reasonable ways to assess this cost.

	Firm Profit	Consumer Welfare	Profit Change (%)	Welfare Change (%)
Prices Fixed at Values in Data				
No Ad Targeting	100.0	100.0	_	_
Targeting: Ad Viewing	104.4	105.5	4.4%	5.5%
Targeting: Full Information	110.9	111.9	10.9%	11.9%
Prices Optimized to Each Consumer				
No Ad Targeting	224.3	91.5	124.3%	-8.5%
Targeting: Ad Viewing	233.9	95.5	133.9%	-4.5%
Targeting: Full Information	247.2	101.8	147.2%	1.8%

Table 15: Effects of Ad and Price Targeting on Profits and Welfare

Figure 14: Lorenz Curves of Benchmark and Ad-viewing Targeting



11.9%. Taken together, targeting on the basis of ad-viewing alone makes up for 40.4% ( $4.4\% \div 10.9\%$ ) of total potential increase in profits, revealing that ad-viewing behavior is likely to be an important targeting indicator for firms.

To understand what's driving these results, Figure 14 presents the Lorenz curves associated with the benchmark and the ad-viewing targeting cases. To interpret this figure, note that higher levels of concentration of ad exposures across households are associated with lower Lorenz curves (more to the right). The straight line denotes the case where ads are equally distributed across consumers. The benchmark case (dashed line) denotes the concentration of ads distributed to consumers with regular ad targeting technologies. Finally, the targeting case (solid line) denotes the concentration of ad allocations to consumers by ad-viewing behavior. It is clear that ad targeting concentrates ad exposures on a relatively small portion of households over time. In particular, before the targeting policy is considered, about 20% of households received roughly 65% of ad exposures. With ad-viewing based targeting, the percentage of ad exposures to the households increases to about 90% of the total number of ad exposures. To obtain intuition for which subset of households the new policies allocate advertising to, we check

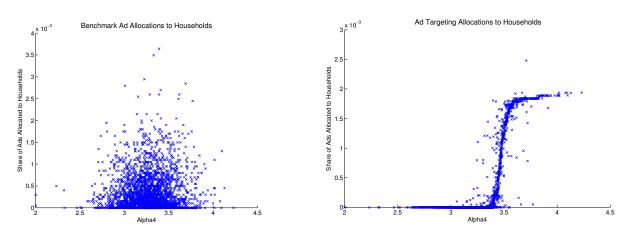


Figure 15: Ad Allocations to Households on Parameter  $\alpha_{i4}$ 

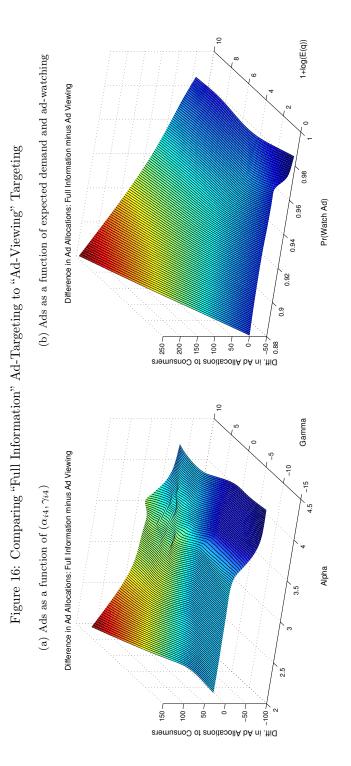
the correlation of the share of advertising allocated to a household under the counterfactual with its estimated parameters. We find that it is mainly driven by parameter  $\alpha_{i4}$ , the intercept in the ad-skip model in equation (34). The correlation between ad watching behavior of a household and its  $\alpha_{i4}$  and  $\gamma_{i4}$  parameter (the intercept in the purchase quantity model) is 72% and 18.6%, respectively. Because in this case the firm is not targeting on  $\gamma_{i4}$  directly, it is clear that the profit effects of targeting ads to consumers through their ad viewing behavior is generated by the correlation between their ad viewing behavior and their inclination to buy the product.

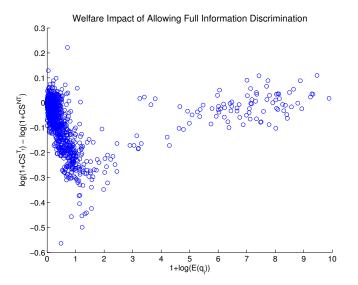
Figure (15) plots the share of ads allocated to a household (y-axis) against  $\alpha_{i4}$  (x-axis) for the benchmark case on the left, and for the ad-viewing allocation case on the right. The probability of watching an ad at baseline for the same households range from 0.9 to 0.99. While there is no relationship between ad exposures and ad-viewing behavior in the benchmark case (left plot), this changes considerably after targeting is implemented (right plot). Households are sorted on  $\alpha_{i4}$ , and in particular are assigned more exposures when they are more likely to watch the advertisement. The 'S-shape' suggests cutoff rules based on estimates of  $\alpha_{i4}$  or the probability of watching the ad may form useful heuristics for an advertiser seeking to allocate a fixed number of advertisements across consumers.

We now discuss how the full information targeting policy ("Targeting: Full Information" case) improves on the ad-viewing policy discussed above. Figures 16a and b plot the difference in the ad allocation to a consumer under the full information targeting policy relative to the ad-viewing policy (z-axis) against their ( $\alpha_{i4}, \gamma_{i4}$ ) parameters (Figure a), and against their expected probability of watching the ad and expected demands (Figure b). Looking at Figures 16a and b, we see that the full information policy trades off the probability of watching an ad with the incremental effect of that ad consumption on quantities. In particular, Figures 16a shows there exists some consumers with high  $\alpha_{i4}$  (high probability of watching the ad), but with low  $\gamma_{i4}$ ; these consumers will not purchase much of the product even though they like the ad. There also exists a set of consumers who have low  $\alpha_{i4}$  (low probability of watching the ad), but high  $\gamma_{i4}$ ; though these consumers may have a high probability of skipping the ad, conditional on consuming the ad, they purchase large quantities. Figures 16a shows the full information policy reallocates ads away from the first type of households towards the second. Looking at Figures 16b, we see that at high expected quantities, ad-allocations are driven primarily by the quantity response, while at low expected quantities, allocations are driven primarily by the ad-response. The net effect of these reallocations is a 6.2% (110.9÷104.4) improvement for the full information policy over the ad-viewing only targeting policy.

Finally, allowing the firm to also optimize on prices (bottom panel of Table 15) produces very large impacts on profitability. Price discrimination alone with no ad targeting improves on the benchmark case profits by 124.3%. As before, the ability to target ads on the basis of the full product and ad-side model is superior (profit of 247.2% relative to benchmark) to targeting ads on the basis of ad-viewing alone (profit of 233.9% relative to benchmark).

Turning to the welfare assessments, we see overall consumer welfare increases with the ability of the firm to target households with ads, holding prices fixed. Note, advertising in the model cannot reduce welfare (because consumers can always skip away from the ads they do not like), so targeting advertising to individuals would are more likely to buy, or less likely to skip increases welfare, because they buy more quantity. However, this increase does not apply uniformly to all consumers. For example, some consumers may just have a preference for advertisements, but that preference may not translate into a preference for the product. In these cases, targeting on the basis of ad-viewing alone may end up allocating advertisements of little consequence to these households. These welfare advantages disappear when prices are allowed to adjust to each consumers' preferences. Allowing for individual price discrimination with no ad-targeting or when allowing for ad-targeting on the basis of skip-rates alone is detrimental for welfare (comparing row-by-row between the first and second panels of Table 15). Interestingly, we find that welfare would increase in the full information case where the firm can set prices and ads to each consumer on the basis of the full model (last row last column in Table 15). To obtain intuition, we denote by  $CS^{NT}$  the consumer surplus in the benchmark case and by  $CS^{T_f}$  the surplus in the full-information case with individually-targeted prices, and plot the incremental surplus,  $CS^{T_f} - CS^{NT}$  for each household against their expected quantity demanded at benchmark. We see that high demand consumers benefit under the full information targeting case. Intuitively, under the full information, high-demand consumers are allocated more ads, and buy many units. However, because the firm is constrained to linear prices, it is unable to appropriate all the consumer surplus generated over all units of quantity purchased with a single price. Overall, our results suggest that in the absence of more complex pricing schemes, it may be possible that *both* firms and consumers are better off in new addressable TV environments.





#### Figure 17: Effect of Allowing Full Information Discrimination on Welfare

## 7 Conclusions

An empirical assessment of the demand for advertising is presented. The model views advertising consumed by an agent as a deliberate choice. Following Becker and Murphy's (1993) theory of complementarities, we assume this choice is co-determined with the choice of consumption of the advertised products. Further, following the theory, joint consumption of advertising and products are allowed to generate complementarities in utility. Using data on detailed product purchase and TV ad-viewing choices by consumers, we document that advertising significantly shifts quantity demanded. We also document that many ads are skipped, and that the skip-rates are explained by the quantity of the advertised product that was purchased recently. These results provide support for a model of complementarities.

We then present and estimate a structural model of demand for products and advertising that allows for such complementarities. Viewing advertising as a deliberate choice by consumers changes the way we assess the "treatment" effect of advertising. The model facilitates assessing these precisely. Using the model, we document that endogenous advertising effects are important to consider for understanding the way advertising works and for assessing its effects on consumer welfare.

Motivated by the "addressable" future of TV ad-markets in which targeting advertising on the basis of ad-viewing and product purchase behavior is possible, we use the model and estimates to simulate a series of counterfactuals. We simulate how demand, welfare and profits would change if an advertiser could target ads to consumers (a) on the basis of anticipated skipping behavior (which in the presence of complementarities indirectly selects high demand-consumers); (b) on the basis of the full model of ad-and-product demand; and (c) on the basis of the full model of ad-and-product demand while also implementing targeted first-degree price discrimination. We find that profits are higher under all ad and price targeting scenarios considered; but that targeting on the basis of ad-viewing alone makes up for 40.4% of total potential increase in profits, suggesting the value of this policy for advertisers. Importantly, we find that net consumer welfare can also rise in the new targeted environments, primarily derived from the increased surplus accruing to high-volume consumers, suggesting that it may be possible that firms and consumers are both better off in the new addressable TV environments.

A limitation of the current analysis is the perfunctory treatment of ad-content (except for the brand of the message). Unfortunately, data on the content of the ads could not be obtained from the corporate sponsor in this study to facilitate this analysis. Our data does contain a copy id unique to each creative. In regressions of the percent of ad watched on a set of copy id fixed effects, we found many of the copy id fixed effects significant suggesting that some creatives are watched significantly longer than others. This persists when including household fixed effects (to control for the fact that the kinds of households viewing different copy ids may be different) and brand fixed effects, suggesting that there is variation in propensity to watch within ads for a given brand and the variation is not being driven by the fact that some brands are more popular than others. We view studying the role of advertising content in ad-skipping and consumption an important area of future research.

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# 9 Appendices

# A Price Series Construction

The purchase data records the price paid and package volume of transactions at the barcode level. We only observe the prices of purchased products, but in order to estimate the model, we need to reconstruct the price series of the alternatives that were not purchased. Additionally, our model is at the brand level, so we need to transform barcode level prices into brand level prices. Our approach is to reconstruct a barcode-store-week level price series using all observed transactions and weight by purchase volume to create a brand-store-week price per unit. As we do not model store choice, the final step in the price series construction is to create a hhid-brand-week level price series by creating a weighted average of the store-brand-week price series using the frequency of a household's store visits as the weights. The steps below describe the process used to construct the price series.

- 1. Although we only observe purchase and TV advertising data for 6,552 households, we observe purchase data for 22,670 households. The entire purchase database is used in the construction of the price series.
- 2. We observe at least one purchase of 58 different brands in the transaction data. In order to make the model more tractable, we restrict the analysis to the set of brands that have the largest purchase market shares. We focus on the brands that collectively cover 90% of the market. The "other" or smaller brands category has the largest purchase market share (61.72%). These brands generally do not advertise, and because we cannot be sure whether the ads we do observe in the data correspond to the same brands that were purchased in this category, we do not include the "other" brands in the analysis. This leaves us with 12 brands. The remaining brand with the largest market share is brand 195 with 8.49% of all purchases observed in the database.
- 3. We keep transactions for barcodes that make up at least 5% of brand sales. Brand 839 does not have any barcodes that have at least 5% of sales, so this brand is dropped from the analysis.
- 4. We keep transactions for stores that have at least 1,000 purchases of the barcodes identified in step 3.
- 5. A barcode-store-week level price series is constructed by taking the median/mode observed price of the transactions in that week. If there are not any observed purchases in a week, we fill in the median/mode observed price from the previous week. If no transactions are observed to date, we fill in the median/mode observed price in the week of the first observed transaction.
- 6. Because barcodes correspond to different package volumes, we create a barcode-store-week level price per unit time series by dividing weekly prices by package volume.
- 7. A brand-store-week level price series is constructed by weighting the barcode level price per unit time series by the total volume of that barcode bought at that store over the sample.
- 8. Finally, we create a brand-hh-week price series by averaging the brand-store-week level price series across stores and weighting by household *i*'s count of purchases at store *s* over the sample period.
- 9. Using this store weighting procedure, some households do not have a price series for all 11 brands because sometimes a household has never been to a store where a given brand is sold. For example, household 135075 only ever purchases at store 1. Brand 195 is not sold at store 1.

The idea behind weighting the store price series by frequency of store visits is intended to reflect a household's best belief about a brand's current price. In the instances where a household never visits a store where a brand is sold, we assume that that household's beliefs about the price of that brand will be equal to the maximum price for that brand across all stores that week.

# **B** Within-Household Variation in Prices Paid

Figure (18) documents a histogram of the within-household variation in prices-paid in the data split by brand.

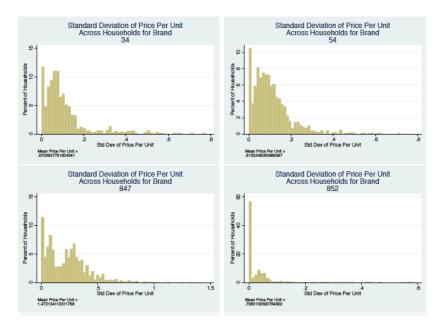


Figure 18: Histogram of  $\mathrm{SD}_{\mathrm{time}}(\mathrm{price}~\mathrm{per}~\mathrm{unit})$  Across HH-s

# C Ad Consumption Model Sensitivity Estimates

Table 16:	Logit	Regression	of Ad	Watched	Dummy	on	Cumulative	Purchase	Quantity

	(1)	(2)	(3)						
	Ad Watched $100\%$	Ad Watched $95\%$	Ad Watched $75\%$						
$Q_{ijt,14}$	1.8e-05***	$1.67e-05^{**}$	$2.27e-05^{***}$						
	(6.56e-06)	(6.56e-06)	(7.73e-06)						
Time Trend	$0.0024^{***}$	$0.0024^{***}$	$0.0022^{***}$						
	(2.8e-05)	(2.9e-05)	(3.23e-05)						
Observations	1,436,400	1,436,400	1,436,400						
HH-Brand RE Y Y Y									
Standard errors in parentheses									
*** p<0.01, ** p<0.05, * p<0.1									

Note: Logit model estimated at the household-brand-day-exposure level, conditional on an ad exposure. The dependent variable is a dummy variable with value 1 if the ad is watched and 0 if the ad is skipped. In column 1, we consider an ad to be watched if 100% of the exposure is displayed. In columns 2, we consider an ad to be watched if at least 95% of the exposure is displayed, and in column 3 if at least 75% of the exposure is displayed.  $Q_{ijt,14}$  records the cumulative package volume household *i* purchased of brand *j* in the 14 days preceding day *t*.

In our model, we consider an ad to be skipped if it is not watched to completion. In this section we explore the sensitivity of our results to different definitions of skipping. Table 16 reports the results of a

logit model in which we regress the binary decision of whether to watch an ad on cumulative purchase quantity in the previous two weeks. We consider alternative definitions of ad consumption where an ad is considered skipped if a) less than 100% of the ad is watched (the ad is not watched to completion), b) less than 95% of the ad is watched, and 3) less than 75% of the ad is watched. The regression is estimated at the household-brand-day level and household-brand random effects are included to control for heterogeneity across households. The magnitudes of the coefficients on product quantity are similar, showing that our results are not sensitive to our specific definition of ad skipping.

## D Additional Ad-Model Parameter Estimates

The tables below report on estimates for the zero inflated Poisson model for the number of ad-exposures observed across households in equation (36), as well as the multinomial model in equation (37) for the brand of each ad-exposure.

Table 17: Estimates for Zero-Inflated Poisson Model for the Number of Ad-Exposures
--

$\pi$	$\lambda$
0.6565	7.9383
(0.0007)	(0.0137)

Table 18: Estimates for Multinomial Model of Ad-Exposure Brand Content

Brand	1	2	3	4	5	6	7	8	9	10	11
$\hat{arphi}_j$	-	0.0961	0.1063	0.1232	0.0741	-	0.1552	0.1496	0.1106	0.0956	0.0892
$\operatorname{SE}(\hat{\varphi}_j)$	-	(-0.0466)	(-0.0503)	(-0.0564)	(-0.0384)	-	(-0.0679)	(-0.0659)	(-0.0519)	(-0.0464)	(-0.0002)

## **E** Simulation Procedure

Here, we discuss in more detail how we implement the simulations in section 5.1 to measure "long-run" elasticities in the model. For each household we simulate purchase and advertising consumption outcomes over the last 3 months in the data at the observed levels of prices and advertising exposures. We compare the model-predicted results to the results of a second simulation in which we allocate additional exposures to each household. The steps below outline the simulation procedure.

- 1. Restrict the sample to the set of households who viewed at least one ad for brand j.
- 2. Allocate the additional ad exposures for brand j to the first household, spreading the additional exposures evenly across days in which the household viewed an ad for brand j.
- 3. Conditional on the initial observed stock of product consumption, take error draws for the ad-skip model and predict ad consumption decisions for each of the s' exposures for all brands on day t.
- 4. If an ad is skipped, draw the percentage of the ad watched independently from the observed distribution of ad durations in the data. If an ad is not skipped, set the percentage equal to 1.
- 5. Update the advertising stock  $\overrightarrow{A}$  using the simulated ad percentages in t.
- 6. Conditional on the observed prices in the data and the advertising stock  $\overrightarrow{A}$ , take error draws for the product purchase model and predict product consumption for all brands on day t.
- 7. Update the consumption stock  $\overrightarrow{Q}$  using the simulated purchase quantities in t. Set t = t+1. Return to step 3.

8. Repeat the forward simulation procedure in steps 3 - 7 for R = 100 paths of error shocks and average the statistics of interest (total purchase quantity, ad consumption, and consumer surplus) over all simulations. Repeat this procedure for all households.

## F Approximating the Solution to the Pricing Problem

This section discusses how we approximate the solution to the optimal pricing problem in our counterfactuals. When the firm is capable of price discriminating across consumers, it chooses  $p_{ijt}^*$  by solving the problem,

$$\max_{p_{ijt\geq 0}} (p_{ijt} - c_{jt}) \times \mathbb{E} \left[ x_{ijt}^* \left( p_{ijt}, A_{jt-1} \right) \right] - c b_{ijt}$$

$$\tag{43}$$

here  $\mathbb{E}\left[x_{ijt}^{*}\left(p_{ijt}, A_{jt-1}\right)\right]$  is the expected demand from consumer *i* as defined in equation (24). Differentiating the expression above using Leibniz's integral rule yields the first-order condition with respect to prices for profit maximization,

$$\frac{c_{jt}}{p_{ijt}} + \frac{\exp(2\gamma_0)p_{ijt}(p_{ijt} - c_{jt})}{\exp(2\gamma_j)(1 + A_{jt-1})^{2\beta} + \exp(2\gamma_0)p_{ijt}^2} + \frac{c_{jt}\left(\pi - 2\tan^{-1}\left(p_{ijt}(1 + A_{jt-1})^{-\beta}\exp(\gamma_0 - \gamma_j)\right)\right)}{2(1 + A_{jt-1})^{-\beta}\exp(\gamma_0 - \gamma_j)p_{ijt}^2} - 1 = 0$$
(44)

We choose  $c_{jt}$  to fix the variable margin of the firm at 0.7, i.e.,  $c_{jt} = 0.7p_{ijt}$ . To reduce execution time, we use a first-order Taylor series approximation of the left-hand side of first-order condition (44) in order to solve for the optimal price  $p_{ijt}^*$ . In particular we solve the problem,

$$f_{\Omega_{jt}}(p_0) + f'_{\Omega_{jt}}(p_0) \left( p^*_{jt} - p_0 \right) = 0$$
(45)

for  $p_{jt}^*$ , where  $f_{\Omega_{jt}}(\cdot)$  is the left-hand side of the first-order condition (44) given  $\Omega_{jt} = \{c_{jt}, A_{jt-1}, \gamma_0, \gamma_j, \beta_j\}$ and  $p_0$  is set to the average price  $p_{jt}$  observed in the data for product j on date t.

# G Steps for Allocation of Ads to Consumers in Counterfactual Analysis

#### G.1 Summary

This procedure reallocates advertisements by households efficiently. Because ad exposures cannot be allocated in a fractional manner, we optimize the problem of the firm by assigning blocks of ads among households. For example, if at time t the firm is observed to allocate 1,000 exposures to 300 households in the data, then the average exposure intensity is equal to  $3.\overline{3}$ . We solve the targeting problem of the firm allocating an integer number of ads across consumers, i.e., in the case above the firm seeks to allocate 3 -  $floor(3.\overline{3})$  - advertisements to 333 households (as many households as possible given the ad intensity), and one last ad exposure is assigned to household number 334. The problem of the firm is then to identify, based on advertising consumption and product purchasing patterns, which households to target with advertisements.

#### G.2 General Steps

Below we outline the basic steps in the procedure. A pseudocode listing follows.

- 1. At t, calculate initial advertisement and quantity stocks, A0 and Q0.
- 2. Define  $n\_exp$  as the total number of advertising exposures at time t, and  $n\_hh$  as the number of households exposed to ads in the data at time t. In order to reallocate (whole) ad exposures, define  $mean\_exp$  as "floor( $n\_exp / n\_hh$ )", i.e., the result of the integer division of total exposures by households exposed.
- 3. Re-define  $n\_hh$  as the number of households that can be assigned  $n\_exp$  ads, given that the first  $n\_hh$ -1 households will be exposed to mean\\_exp ads, and the  $n\_hh^{th}$  household will be assigned last ads. (See pseudocode for definitions.)

- 4. For each household, calculate the expected daily consumption if it were exposed to mean\_exp advertisements.
- 5. Then, for each household, calculate the expected daily consumption if it were not exposed to any advertisements.
- 6. Calculate the marginal effect of average ad exposure on consumption by differencing the results from steps 4 and 5, and rank households in descending order of ad effects.
- 7. Starting from the top of the list, assign  $mean\_exp$  ads to the first  $n\_hh$  households. Then, assign last ads to the n  $hh^{th}$  household.
- 8. Calculate expected sales given the assignment, and the resulting expected profits.
- 9. Update the advertising and quantity stocks A0 and Q0, but use shock realizations rather than expectations. Set t = t + 1 and go back to step 1. Stop at t = 106.

With endogenous prices, steps 3 and 4 incorporate total expenditure by consumers rather than just consumption. The firm optimizes over price individually, taking into account each consumer's expected demand. The remaining steps are not affected. Finally, while the firm maximizes *expected* profits, in reality advertising and good consumption take place at particular draws of the random variables. This is especially important for step 9, which ensures that the program is simulated forward by taking realizations of consumption of the ad and of the good, rather than their expectations. Hence, in order to extract expected profits appropriately several paths with different draws of ad and consumption shocks are simulated, and the statistics of interest (consumer surplus and profits) are averaged over those simulations.

#### G.3 Pseudocode

Variable Definitions:

- 1. A0 is a vector  $(N \times 1)$  with each household's initial stock of ad consumption
- 2. Q0 is a vector  $(N \times 1)$  with each household's stock of good consumption
- 3. a t is a matrix  $(N \times T)$  with each household's ad exposures in the data
- 4. q t is a matrix  $(N \times T)$  with each household's ad exposures in the data
- 5. hh id is a vector  $(N \times 1)$  with a unique id for each household
- 6. profit is a scalar initiated at zero

```
// main function
```

```
// calculate ad stocks and n.people exposed
n_exp = sum(a_t(..,t));
i_hh = a_t(..,t) > 0;
n_hh = sum(i_hh);
// calculate number of people to expose ads to, and n.exposures
mean_exp = floor(n_exp / n_hh);
ads_left = n_exp - mean_exp * n_hh;
n_hh = n_hh + floor(ads_left / mean_exp) + 1;
last_exp = n_exp - (n_hh - 1) * mean_exp;
// calculate exp. marginal effects of showing ads
// (hh_id is a vector of unique id's for individuals)
qt_pred0 = calc_Eq(hh_id, A0);
qt_pred1 = calc_exp_q(hh_id, mean_exp);
```

```
delta_qt = qt_pred1 - qt_pred0;
  // find households with highest expected increment
  mat1 = [delta_qt, hh_id];
  mat1 = sort(mat1, 1); // sort by column 1
  target_hh = mat1(.., 2); target_hh = target_hh(1..n_hh);
  last_target_id = target_hh(n_hh);
  // update advertising and quantity with realizations
  a_t(.., t) = 0; a_t(target_hh, t) = ad_viewing(target_hh, mean_exp);
  a_t(last_target_id,t) = ad_viewing(last_target_id, last_exp);
  A0 = A0 + a_t(..., t) - a_t(..., t-14);
  q_t(..,t) = calc_q(hh_id, A0);
  Q0 = Q0 + q_t(..., t) - q_t(..., t-14);
  // calculate statistics of interest (here profits are exemplified)
  profits = profits + calc_profits();
  // go to next period
  t = t + 1:
// Other Functions:
// function calc_profits()
// updates profit
function calc_profits(){
  // get dollar value
  margin = 0.3 * p_j(t);
  return margin * sum(q_t(...,t));
}
// function calc_exp_q(hh_id, ad_exp)
// calculates expected q for households with id's
// in hh_id, exposed to ad_exp ads
function calc_exp_q(hh_id, ad_exp){
  Eq_pred = 0;
  for i=1..nsimul{
     n_ads = ad_viewing(hh_id, ad_exp);
     Eq_pred(i) = Eq_pred(i) + calc_Eq(hh_id, A0(hh_id) + n_ads);
  }
  return Eq_pred / nsimul;
}
// function ad_viewing(target_hh, ad_exp)
// predicts a draw of number of ads watched given exposure to ad_exp ads
// Note: (unif_rand return a vector of uniform r.v.'s with
// the same number of elements as target_hh)
function ad_viewing(target_hh, ad_exp){
  for j=1..ad_exp{
     n_ads = n_ads + (unif_rand(0,1) < pr_watch_ad(target_hh));</pre>
  }
```

```
return n_ads;
}
// External Functions:
// function pr_watch_ad();
// calculates the expected ad consumption for each household
// function calc_q(hh_id, Ad_Stock);
// calculates a draw of product consumption for each household
// function calc_Eq(hh_id, Ad_Stock);
// returns E [x<sup>*</sup><sub>ijt</sub> (p<sub>ijt</sub>, A<sub>jt-1</sub>)] for individuals in hh_id, according to equation (24)
//in the paper.
```

\_ \_